

INCREASING THE RATIO OF MODULUS OF ELASTICITY TO THE DENSITY OF TITANIUM ALLOYS

WILLIAM H. GRAFT
DAVID W. LEVINSON
WILLIAM ROSTOKER

ARMOUR RESEARCH FOUNDATION

NOVEMBER 1955

MATERIALS LABORATORY
CONTRACT No. AF 33(616)-2355
PROJECT 7351

WRIGHT AIR DEVELOPMENT CENTER
AIR RESEARCH AND DEVELOPMENT COMMAND
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

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FOREWORD

This report was prepared by the Armour Research Foundation under USAF Contract No. AF 33(616)-2355. This contract was initiated under Project No. 7351, "Metallic Materials", Task No. 73510, "Titanium Metal and Alloys", formerly RDO No. 615-11, "Titanium Metal and Alloys", and was administered under the direction of the Materials Laboratory, Directorate of Research, Wright Air Development Center, with Lt J. W. Seeger acting as project engineer.

This report covers work conducted from March 1954 to March 1955.

WADC-TR-55-147

Titanium and its alloys, although possessing many attractive properties, exhibit a somewhat lower elastic moduli to density ratio than other competitive engineering metals. It was the purpose of this investigation to examine alloy additions and structural conditions which might introduce improvement in the elastic modulus to density ratio.

Elastic modulus values were measured by means of a dynamic method employing electrostatic excitation and detection. The effects of variables on the elastic properties were examined at room temperature, 500° and 750°F, and the results evaluated using the elastic modulus (E) and the elastic modulus to density ratio (E/ρ) as the definitive criteria.

Unalloyed titanium was found to exhibit an E of 16.2 x 10⁶ psi for the magnesium reduced variety, and a slightly higher value for the iodide type.

The alpha stabilizing elements oxygen and nitrogen were found to affect the E and E/ρ ratio but slightly at low concentrations. The addition of aluminum, up to 8%, was found to result in marked improvement in E and E/ρ ratio.

The presence of intermediate phases on the E and E/ρ ratio was quite beneficial in some cases. TiC and TiB in their respective binary systems resulted in greatly improved elastic properties. Intermediate phases Ti₃Si₃ and TiBe in their respective binary alloys exhibited only a minor effect. Ti₂Cu produced by eutectoidal decomposition in Ti-Cu alloys up to 11.5% Cu produced no significant increase in E or E/ρ.

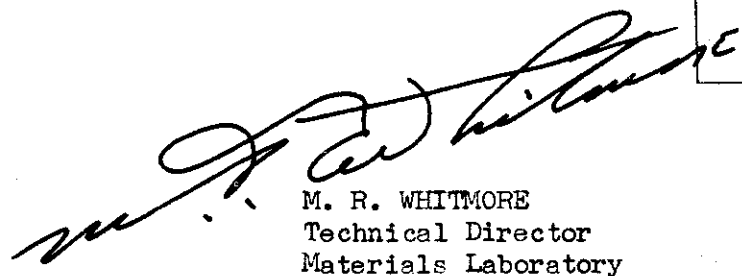
A survey of the E and E/ρ ratio for various crystallographic directions in the base plane of α-Ti in a state of preferred orientation was made. The results showed that E and E/ρ were essentially constant for various directions, and the value of E, 15.2 x 10⁶ psi, was somewhat lower than that for randomly oriented material.

The effect of heat treatment was found to influence the elastic modulus simply in terms of the ratios of proportions of alpha and beta.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER:



M. R. WHITMORE
Technical Director
Materials Laboratory
Directorate of Research

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Contract
INCREASING THE RATIO OF MODULUS OF ELASTICITY
TO THE DENSITY OF TITANIUM ALLOYS

I. INTRODUCTION

This is the Final Report on Contract No. AF 33(616)-2355 summarizing the experimental progress made during the period March 15, 1954 to March 15, 1955. The objective of this study was the investigation of the various factors which affect the ratio of elastic modulus (Young's Modulus) to density of titanium alloys with the ultimate aim of developing an alloy of improved properties. An upper limit of 30% by weight of alloying constituents was imposed by the sponsor, and it was further stipulated that a dynamic method be employed for the determination of elastic modulus values. Other phases of the study included the effect of elevated temperature, preferred orientation, and the relative amounts of α and β on Young's modulus (E) and the ratio of Young's modulus to density (E/ρ).

II. EXPERIMENTAL METHODS

A. Preparation of Alloys

The alloys used in this study were prepared from high purity raw materials. Table I shows the compositions and source of these materials. Alloys containing vanadium, copper, aluminum, boron, beryllium, and silicon were prepared by melting together calculated amounts of the elemental constituents, whereas alloys containing carbon, oxygen, and nitrogen were made using master alloys. The alloys were prepared by the arc melting techniques described in the following paragraphs.

1. First Melting

In the first melting process, the alloy is melted in a water cooled copper crucible using a direct current arc struck between the charge and a tungsten tipped electrode. A negligible amount of tungsten is introduced into the melt in this operation, and the electrode is not consumed. The process is carried out under an atmosphere of argon at slightly above atmospheric pressure. The heat penetration into the charge from the arc is quite shallow, so that, for all but very small ingots, only a portion of the charge is molten at any given time. Reasonably small ingots, e.g. 200 gms, can be made sufficiently homogeneous with repeated flashings of the arc to require no further treatment. However, when a substantial size of ingot is required

TABLE I

COMPOSITION AND SOURCE OF MATERIALS USED
IN THE PREPARATION OF EXPERIMENTAL TITANIUM-BASE ALLOYS

Material	Composition	Source
Titanium Sponge (BHN-121, 3000 Kg load)	C - 0.058% Si - 0.018% Fe - 0.068% N - 0.020% max.	E. I. Dupont
Vanadium Chips 1/4 in. x 12 mesh	C - 0.092% O - 0.040% N - 0.084% H - 0.003%	Electro Metallurgical Company
Carbon Rods 1/8 in. dia. x 12 in.	High purity spectrographic grade	National Carbon Company Inc.
Copper Shot	Reagent grade	General Chemical Division, Allied Chemical & Dye Corporation
Aluminum Sheet	Commercially pure, 2S grade	Aluminum Co. of America
Boron Powder Grade "A", 40 mesh Lot 1710	C - 0.19% Fe - 0.33% B - 99.15%	Cooper Metallurgical Associates, Cleveland, Ohio
Beryllium Shot	Technical Grade Be - 96/99% Fe - .03/.15% Al - .06/.20% Mg - .10/.50% Si - .05%	Brush Beryllium Corporation
Silicon (granules of crushed sintered 30 x 80 mesh powder)	Si - 99.7/99.9% Fe - .005/.015%	Electro Metallurgical Company

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e.g. 2000 gms, a stepwise melting technique employing a number of nominally identical charges is used. The resultant ingot frequently shows inhomogeneities, and it is profitable to subject the alloy to a second melting step.

2. Second Melting

The purpose of the second melting step, as mentioned above, was to improve the homogeneity of the alloy prepared by the first melting technique. The original ingot was scalped of surface irregularities and forged into a long rod about 1 in. in diameter. This rod was then centerless ground to remove any surface contamination occasioned by the forging operation. The rod was then ready for second melting. Second melting resembled the first melting somewhat, except that the electrode was made from the forged rod described above, and therefore the electrode was consumed as melting continued. As before, a protective atmosphere of argon was employed to prevent atmospheric contamination during melting. Alternating current was used in second melting. A small button of the same composition as the alloy being melted was required for striking the arc. This button was usually prepared by first melting at the same time that the large ingot was made. After the second-melted ingot had been prepared, a thin cut was taken from the bottom of the ingot to remove the starting button, since it was not double-melted.

B. Dynamic Elastic Modulus Device

Dynamic elastic modulus determinations involve the application of a periodic stress of variable frequency to a suitably shaped specimen and a means of ascertaining the stress frequency at which maximum strain occurs. Two main subdivisions of dynamic elastic modulus methods can be drawn, vibration in flexure and vibration in extension. In the former, exemplified by the bending of a transversely loaded beam, the stress application is in a direction perpendicular to the specimen length, whereas in the latter the line of action of the stress is along the long axis of the specimen. Flexural periodic stressing, at resonance, results in a condition of maximum strain manifested by the alternate upward and downward bowing of the specimen. Resonant stress application in extension also results in maximum strain, but in this case, the strain appears as displacements along the axis of the ends of the specimen from their equilibrium positions. Each method has certain features to recommend it. Higher stresses can more easily be attained in the flexural method, but the location of the nodes is somewhat tedious, and the mathematics of the data reduction considerably less straightforward than for the extension method. The method of stressing in extension offers the advantages of rapid and easy nodal location and simplified mathematical treatment of data.

Several methods for exciting vibrations in extension are available; electromagnetic, electrostatic and piezoelectric. The electromagnetic and the electrostatic methods are quite similar in action, the chief difference being that in the former, the stressing arises from magnetic attraction, whereas in the second, electrostatic forces produce the stress. It will be appreciated that the electrostatic method is more versatile, since the only requirement for developing drive stresses is that the specimen be an electrical conductor. The electromagnetic method of drive, while capable of

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somewhat greater drive forces, requires the specimen to be either ferromagnetic, or to be provided with end plates that are ferromagnetic. Since in this study we are dealing with titanium alloys which are nonmagnetic, and since the reliable fastening of ferromagnetic end plates to a titanium alloy specimen would pose many problems, the electrostatic method of drive was selected.

The piezoelectric method of driving a specimen in extension involves the fabrication of a composite vibrating element consisting of a specimen rigidly fastened to a short piece of a piezoelectric material. Upon the application of a varying frequency voltage to the piezoelectric unit, the composite will exhibit a strain maximum at some frequency which is a function of the natural frequency of the specimen-driver combination. This method was rejected because it was felt that reliable, strong bonding of drive crystals would be difficult to achieve, particularly at elevated temperatures.

In free-free vibration in extension (specimen held in the middle), the driven end of the specimen is subjected to periodic tensile stresses which generate an accompanying strain, manifested by the displacement of the end of the specimen. This displacement is cyclic, following the impressed stress, and is propagated in a front along the specimen length by interaction with adjacent atoms. The displacement (or strain) wave travels to the far end of the specimen at which point reflection occurs, and the wave retraces its path. The speed of travel of the wave is a function of the elastic modulus (Young's modulus) and the density of the specimen, provided the proper specimen geometry is observed. At drive frequencies off resonance, the reflected wave destructively interferes with the succeeding drive wave, and full or partial cancellation occurs. However, at resonance, constructive interferences of reflected and impressed waves occurs, and large displacement maxima of the ends are developed, limited only by the damping characteristic of the material.

Two conditions of resonance are possible in the case of longitudinal vibrations; i.e. the specimen length can be either an even or odd integer multiple of half of the resonant frequency wave length. This is expressed mathematically as

$$L = \frac{n\lambda}{2}$$

where n is an integer. If n is odd, the bar exhibits displacement maxima at its ends with a node in the center. The end displacements are 180° out of phase, and thus the bar undergoes alternate shortening and lengthening. If n is even, e.g. 2, the center of the bar as well as the ends exhibits displacement maxima, and modal points appear at $\frac{\lambda}{4}$ and $\frac{3\lambda}{4}$, which correspond to positions located one quarter of the specimen length from each end. With n = 2, the end displacements vibrate in phase with each other, and the center displacement is 180° out of phase with the ends. Each half of the bar then alternately shortens and lengthens, one half being in tension while the other is in compression. In this study, the specimen was held in the center, and the length of the specimen for fundamental vibration (n = 1) was equal to one half of the resonant frequency wave length.

C. Dynamic Elastic Modulus Specimen

Two requirements must be met by specimens used in dynamic elastic modulus determinations employing longitudinal vibrations. First, the specimen must have a uniform cross section throughout its entire length, so that all points on the vibration wave travel in a uniform front. Second, the maximum cross sectional dimension of the specimen should be less than one-tenth of the resonant frequency wave length in order to keep the Poisson contraction correction to a negligibly small value. Both of these conditions were satisfied in this study by the use of a cylindrical specimen 1/4 in. in diameter by 3 1/4 in. long. These dimensions were arbitrarily selected, and other values could be used as long as the previously mentioned conditions were fulfilled.

D. Calculation of Dynamic Elastic Modulus from Observed Data

The physical variables involved in determining the modulus of elasticity by dynamic longitudinal vibrations are related by the expression,

$$E = \frac{4\rho L^2 f_n^2}{n^2}$$

where

- E = modulus of elasticity, dynes/cm²
- ρ = density, g/cc
- L = length, cm
- f_n = resonant frequency, cps
- n = order of vibration (n = 1 for fundamental vibration)

Inspection of the above equation discloses that the determination of the ratio of the elastic modulus to density (E/ρ) involves only the length and resonant frequency, since the density disappears from the right side of the expression upon rearrangement. However, density measurements are necessary to arrive at numerical values for Young's modulus. Since the determination of density is relatively easy, and since the actual numerical values of the elastic moduli of experimental alloys would be of value, density measurements were made, using the "loss of weight in water" technique.

The electrostatic dynamic elastic modulus device used in the present study is shown in diagrammatic fashion in Figure 1. The actual apparatus is shown in the photographs of Figures 2 and 3, which show a general view of the device and the associated electronic components, and a detail view of the specimen and holder assembly, respectively. The alternating voltage, of variable frequency, was obtained from a sine wave signal generator acting through a power amplifier and a step-up transformer. Drive voltages of the order of 400/500 rms volts were found to provide satisfactory drive amplitude. Detection of resonance employed the condenser microphone principle in conjunction with a cathode ray oscillograph (CRO) and a preamplifier. Similar electrodes were positioned at each end of the specimen, and thin mica sheet (.001 in.) insulators were cemented to the electrode faces to prevent electrical shorting. The receiver electrode was biased to 225 volts DC to allow the condenser microphone principle to operate. Resonance in the

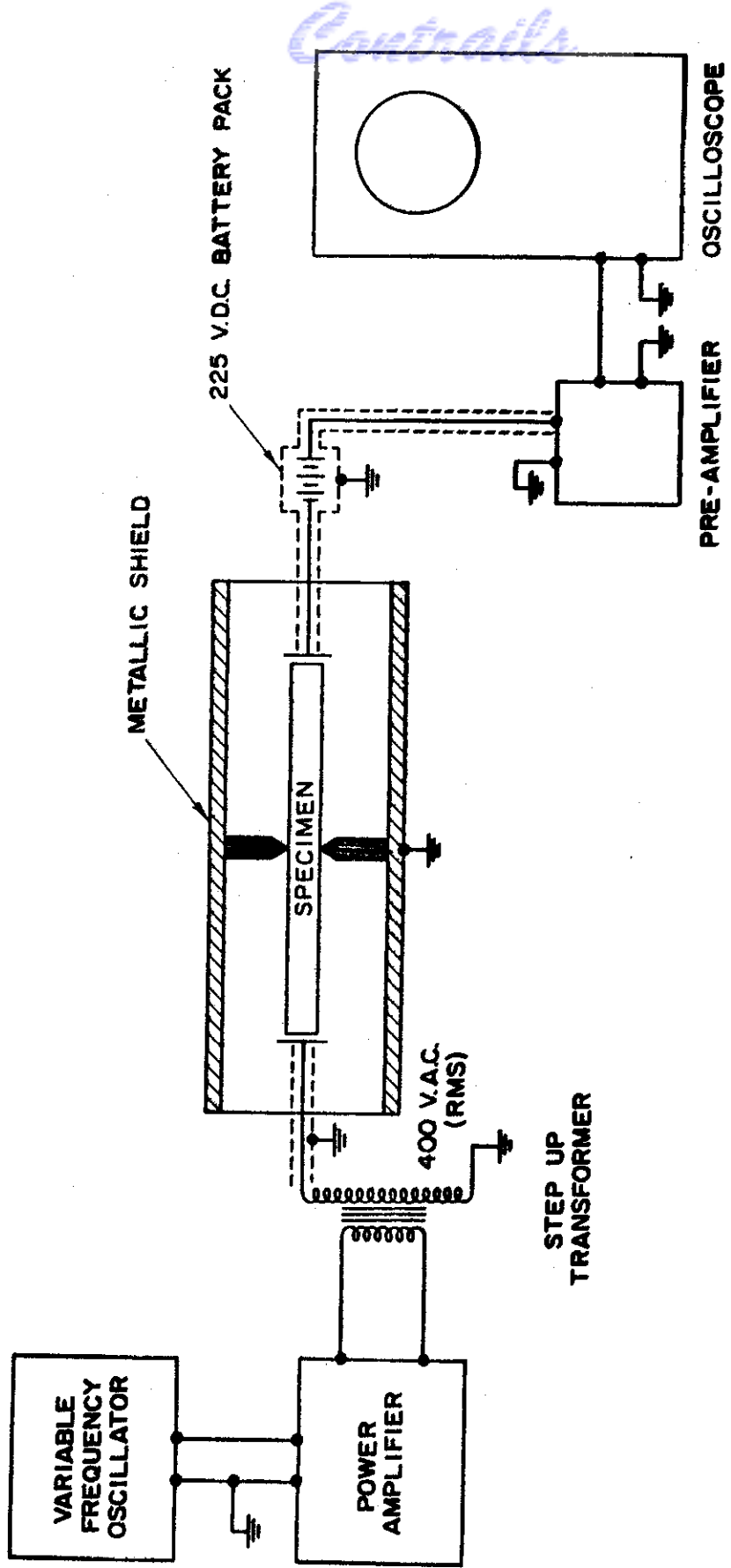


FIG. 1 - BLOCK SCHEMATIC DIAGRAM OF ELECTROSTATIC APPARATUS FOR DYNAMIC MEASUREMENTS OF ELASTIC MODULUS.

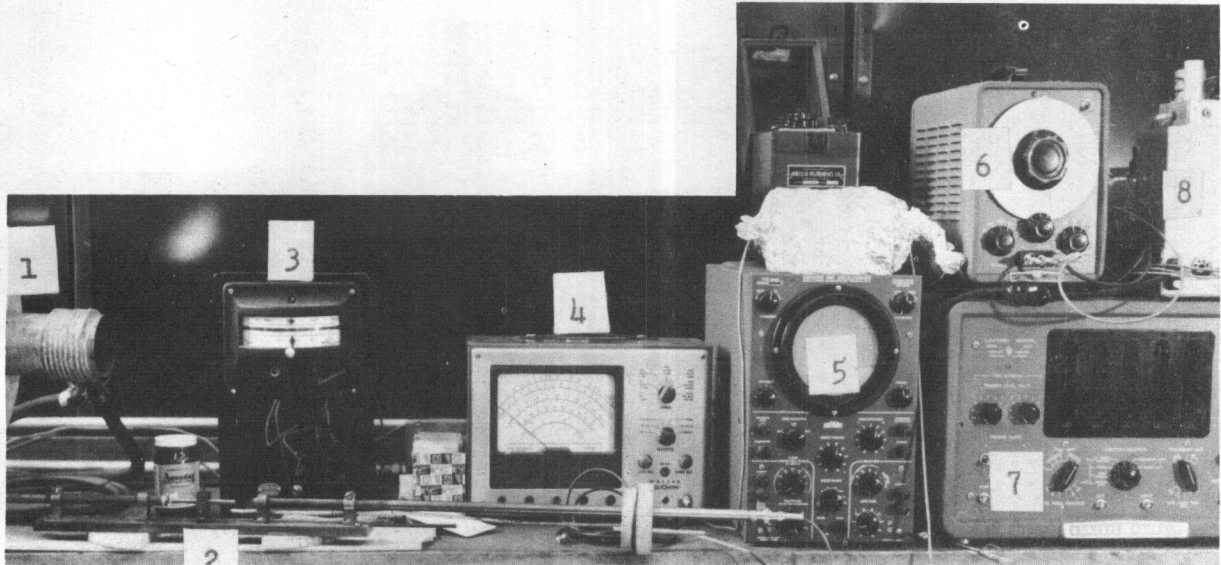


Fig. 2 - Experimental Apparatus for Electrostatic Dynamic Elastic Modulus Determinations.

1. Furnace for elevated temperature measurements
2. Specimen holder and electrode assembly
3. Furnace controller
4. Vacuum tube voltmeter
5. Cathode ray oscillograph
6. Variable frequency signal generator
7. Electronic frequency counter
8. Power amplifier

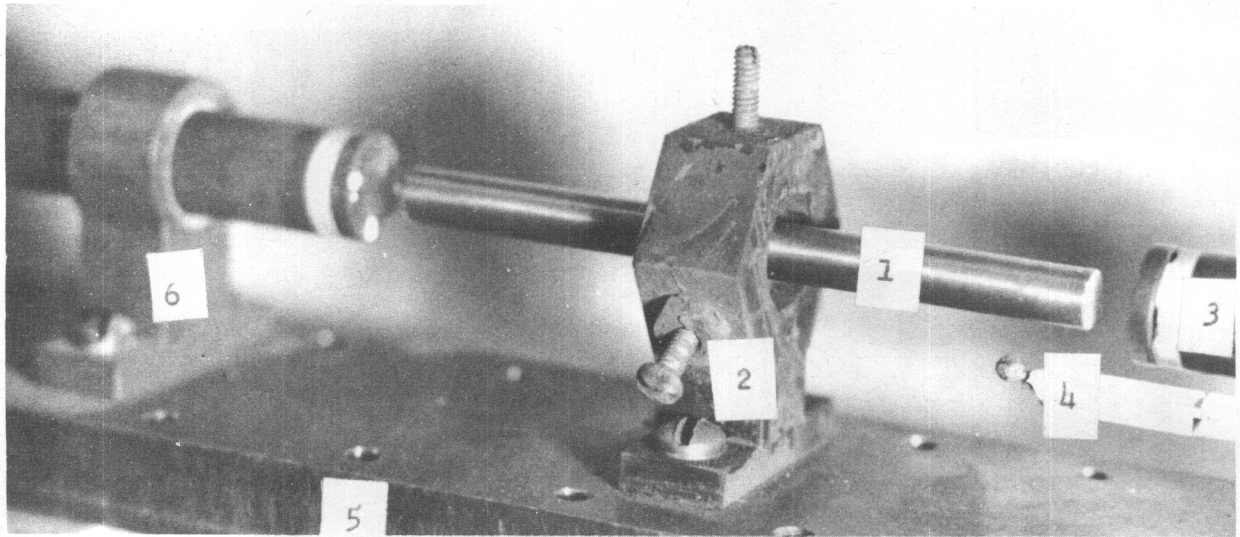


Fig. 3 - Detailed view of Specimen, Specimen Holder and Electrode Assembly for Electrostatic Dynamic Elastic Modulus Apparatus.

1. Specimen
2. Specimen holder
3. Electrode assembly
4. Thermocouple
5. Base
6. Electrode support

specimen was indicated on the CRO by the appearance of a sine wave of appreciable amplitude at resonant frequency as the frequency of the driving voltage was slowly varied. The indication, in most cases, was very sharp, indicating that low damping material was being studied.

Accurate determination of elastic moduli by dynamic methods requires that the resonant frequency be ascertained to the same degree of precision as the other factors involved. In view of this fact, and in view of the fact that no simple method could be found for frequency determination, recourse was made to the use of an electronic frequency counter. This device consists of a number of inter-related electronic circuits which acting together can count the rate (frequency) of signal inputs for variable preset times. The device is capable of direct counting to 100 Kc per second with an inherent accuracy of ± 1 cycle. The result of a counting operation is displayed in a bank of five vertical columns of neon lights, each column running from 0 to 9. The result is thus directly read.

E. Elevated Temperature Elastic Modulus Determinations

The results of elevated temperature elastic modulus determinations given in this report were calculated from observations of the resonant frequency at the given elevated temperature and of the density and length at room temperature. This method of calculating the dynamic modulus, although not theoretically accurate, introduces a negligible error in the results (see Appendix I), and simplifies the determination considerably.

Originally, it was intended to investigate elastic modulus variation with temperature through a range of temperatures up to 1000°F, but the practical limit found for the present type of specimen holder-electrode assembly was 750°F. Basically, two experimental difficulties prevented successful operation at the upper temperature. First, the electrodes showed a tendency toward binding in the supports as higher temperatures were used; this difficulty was reduced somewhat by frequent smoothing of the surfaces followed by lubrication with colloidal graphite. Second, the electrical insulating properties of the porcelain sleeves used to insulate the ungrounded lead in the receiver unit deteriorated with temperature, allowing random currents to flow in the highly biased receiver circuit. This action produced heavy electrical noise on the CRO, rendering accurate determinations of resonant frequency extremely difficult, if not impossible. Several varieties of porcelain insulators were tried, but no success was achieved at 1000°F. It appears at this point that a differently designed electrode assembly will be required for determinations above 750°F.

III. DYNAMIC ELASTIC MODULUS DETERMINATIONS

ON EXPERIMENTAL TITANIUM-BASE ALLOYS

Three main types of experimental titanium-base alloys were investigated in the present study: all alpha, all beta phase, and alpha phase alloys

Continued

containing a dispersed second phase. Some attention was given to the effect of preferred orientation in alpha titanium and to the study of the moduli of Ti-V alloys in which the ratio of the amount of beta to alpha was varied.

A. Alpha Type Alloys

The effect of varying amounts of three different alpha stabilizing elements on the E and E/ ρ ratio of titanium was studied. The interstitial elements oxygen and nitrogen were examined in concentrations up to 0.59 wt % for the former and 0.73 wt % for the latter. The substitutional element aluminum was investigated in concentrations up to 7.9 wt %. Table II lists the composition, heat treatment, and density for these alloys, while Table III summarizes the elastic modulus and ratio of elastic modulus to density determinations made at room temperature, 500° and 750°F. Also shown are the results found for magnesium-reduced and iodide titanium. The information in Table III is presented in graphical form in Figures 4-9, inclusive. The microstructures of these alloys are given in Figures 10-19, inclusive. It may be seen that, in the range studied, neither oxygen nor nitrogen exerts an appreciable effect on either E or the E/ ρ ratio. However, in the case of aluminum, definite improvement in both E and the E/ ρ ratio was found. Also, the addition of aluminum to titanium lowers its density, resulting in an enhanced E/ ρ ratio. This improvement in elastic modulus reflected the solid solution strengthening effect of the aluminum, which was completely dissolved in the alpha solid solution. Investigation of the Ti-Al binary system at concentrations above 7.9% Al was not undertaken due to known ductility limitations.

Closely comparable values of E were found for Mg-reduced and iodide titanium. Only fair agreement in the E/ ρ ratios was seen; this was a result of the slight difference in densities found for the two materials.

The effect of temperatures up to 750°F on the E and E/ ρ ratio of the all alpha type alloys was found to be marked. Both the E and the E/ ρ ratio decreased continuously in the range studied. The rate of decrease with temperature increase was linear and independent of composition (the curves were parallel); see Table IV.

B. Beta Type Alloys

It was desired to investigate the elastic modulus and ratio of elastic modulus to density of beta titanium at room temperature, and the binary Ti-V system was selected for study. Alloys containing nominally 10, 15, 20, 25 and 30% by weight of vanadium were prepared and tested at room temperature in the water quenched condition. Table V summarizes the results of the study, and Figure 20 presents the findings graphically. It will be seen that the data points are shown on two different curves. This arises from the fact that below about 15% V the crystal structure of the water quenched alloy is hexagonal close packed, whereas above this composition the structure is body centered cubic. The microstructures for these alloys are given in Figures 21-23, inclusive. The structures for the 21.5 and 31.3% V are not given, since they are similar to the 25.2% V alloy. It will be noted in Figure 20 that the 10.6% V alloy exhibits a very low modulus of elasticity.

TABLE II
COMPOSITIONS, HEAT TREATMENTS, AND DENSITIES OF ALL-ALPHA TITANIUM-BASE ALLOYS STUDIED
FOR DYNAMIC ELASTIC MODULUS PROPERTIES

Alloy	H (ppm)	O (wt %)	Heat Treatment	Density (g/cu cm)	Density (lbs/cu in)
Unalloyed Ti (Mg reduced)	76	0.109	750°C-1 hr. → AC	4.59	0.166
Unalloyed Ti (Iodide)	165	0.030	900°C-1 hr. → AC	4.54	0.164
Titanium-0.40% oxygen	30	0.403	As-forged	4.54	0.164
Titanium-0.49% oxygen	38	0.486	As-forged	4.54	0.164
Titanium-0.59% oxygen	71	0.589	As-forged	4.54	0.164
Titanium-0.33% nitrogen	35	-	900°C-4 hrs. → AC	4.53	0.164
Titanium-0.48% nitrogen	36	-	900°C-4 hrs. → AC	4.54	0.164
Titanium-0.73% nitrogen	37	-	900°C-4 hrs. → AC	4.54	0.164
Titanium-1.9% aluminum	-	-	850°C-1 hr. → WQ	4.61	0.166
Titanium-4.1% aluminum	111	-	850°C-1 hr. → WQ	4.49	0.162
Titanium-5.9% aluminum	146	-	850°C-1 hr. → WQ	4.46	0.161
Titanium-7.9% aluminum	-	-	850°C-1 hr. → WQ	4.37	0.158

TABLE III

SUMMARY OF DYNAMIC ELASTIC MODULUS MEASUREMENTS AT SEVERAL TEMPERATURES
FOR A NUMBER OF ALL ALPHA TITANIUM-BASE ALLOYS

Alloy	Dynamic Elastic Modulus (psi x 10 ⁶)		Ratio of Dynamic Elastic Modulus to Density (psi/lbs/cu in x 10 ⁴)	
	RT	750°F	RT	750°F
Unalloyed Ti (Mg reduced)	16.2	13.8	9.76	8.31
Unalloyed Ti (Iodide)	16.4	13.8	10.0	8.41
Titanium-0.40% oxygen	16.8	14.5	10.2	8.84
Titanium-0.49% oxygen	17.0	14.8	10.4	9.02
Titanium-0.59% oxygen	17.1	14.7	10.4	8.96
Titanium-0.33% nitrogen	16.9	14.6	10.3	8.90
Titanium-0.48% nitrogen	16.6	14.0	10.1	8.54
Titanium-0.73% nitrogen	16.7	14.2	10.2	8.66
Titanium-1.9% aluminum	16.3	14.3	9.82	8.61
Titanium-4.1% aluminum	17.4	15.3	10.7	9.44
Titanium-5.9% aluminum	17.6	15.7	10.9	9.75
Titanium-7.9% aluminum	18.7	16.8	11.8	10.6
				10.0

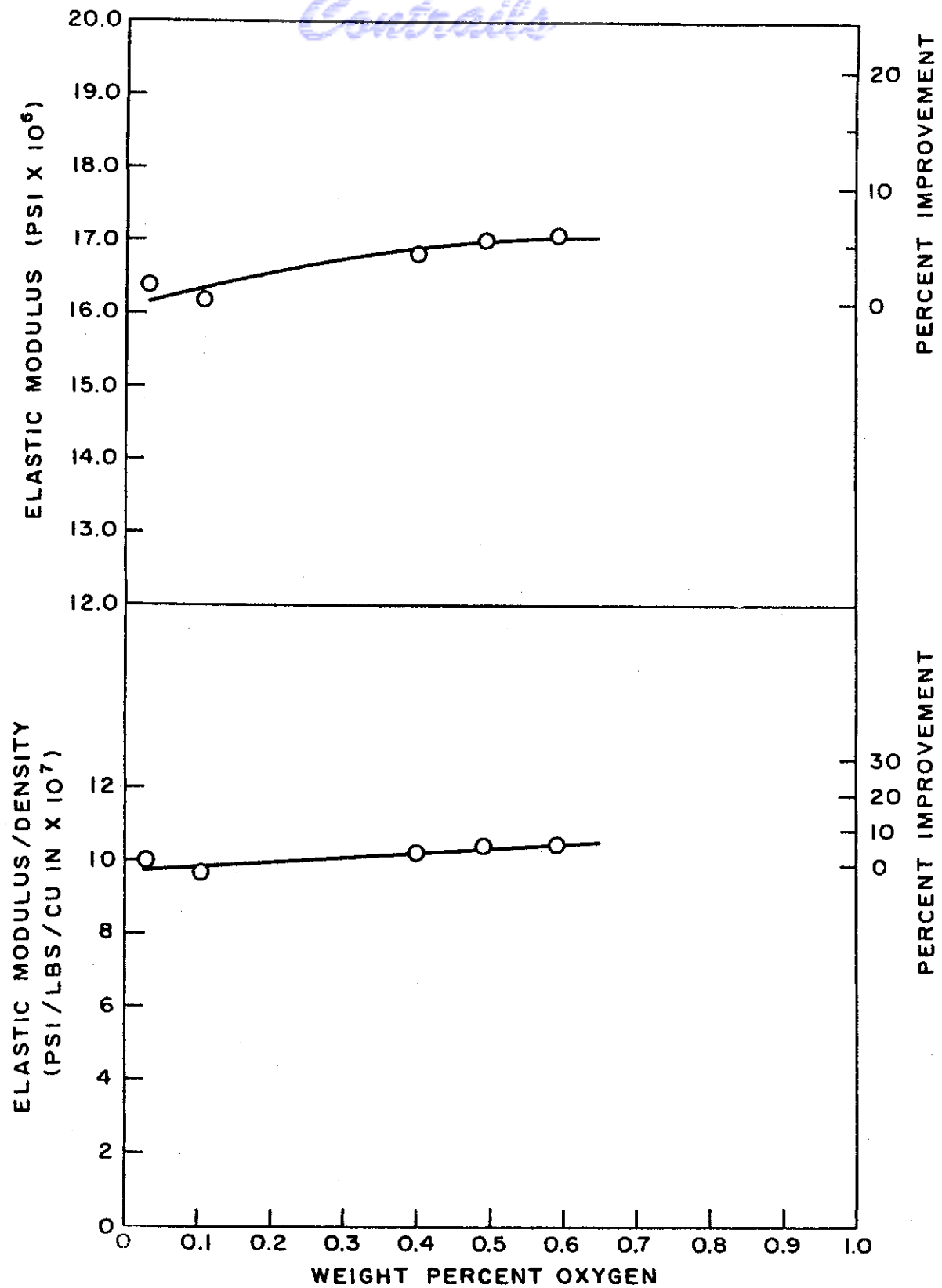


FIG. 4 - VARIATION IN ELASTIC MODULUS AND RATIO OF ELASTIC MODULUS TO DENSITY WITH OXYGEN CONTENT FOR TITANIUM-BASE ALLOYS - ROOM TEMPERATURE

Contracts

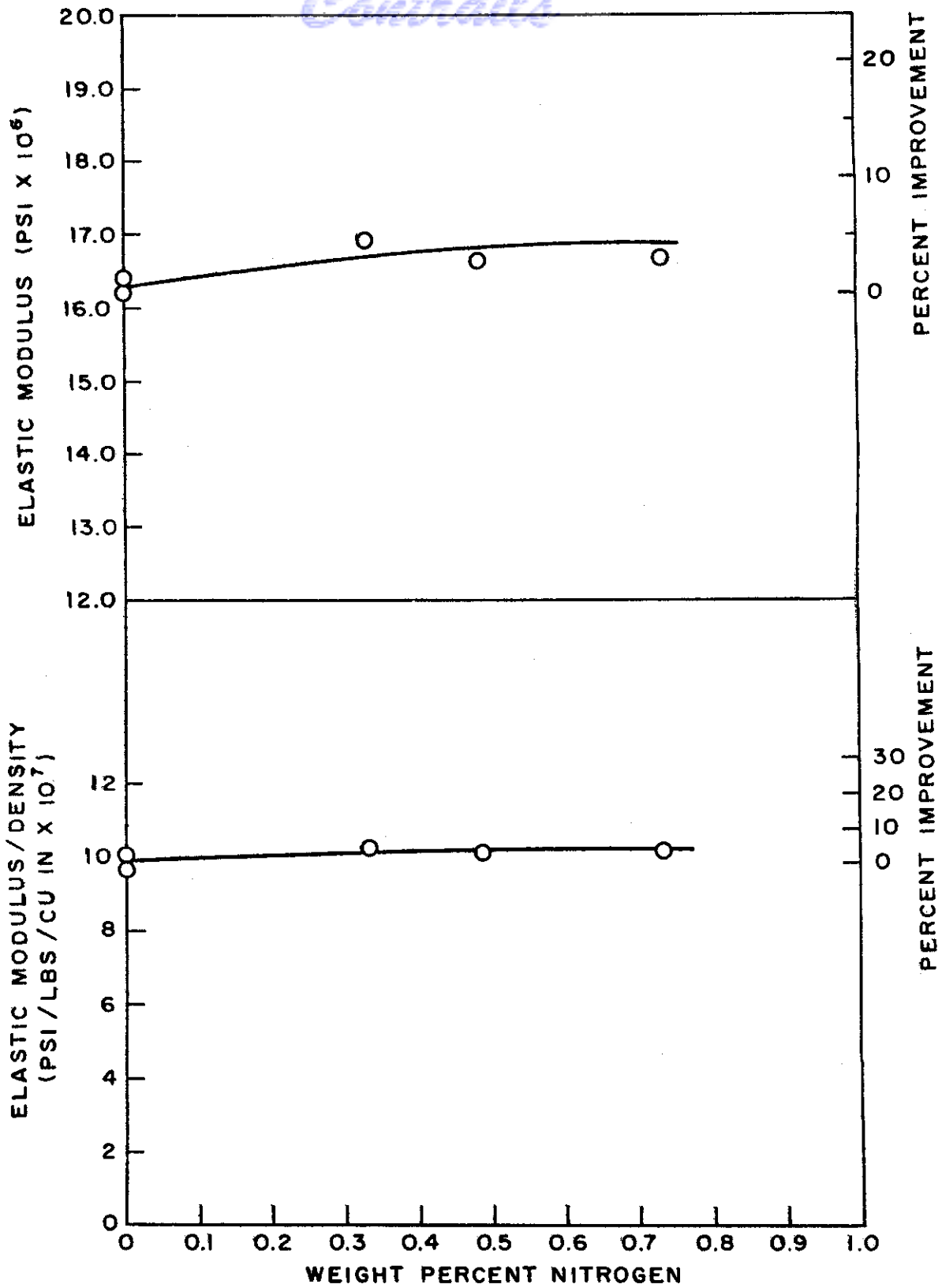


FIG. 5 - VARIATION IN ELASTIC MODULUS AND RATIO OF ELASTIC MODULUS TO DENSITY WITH NITROGEN CONTENT FOR TITANIUM-BASE ALLOYS - ROOM TEMPERATURE

Controls

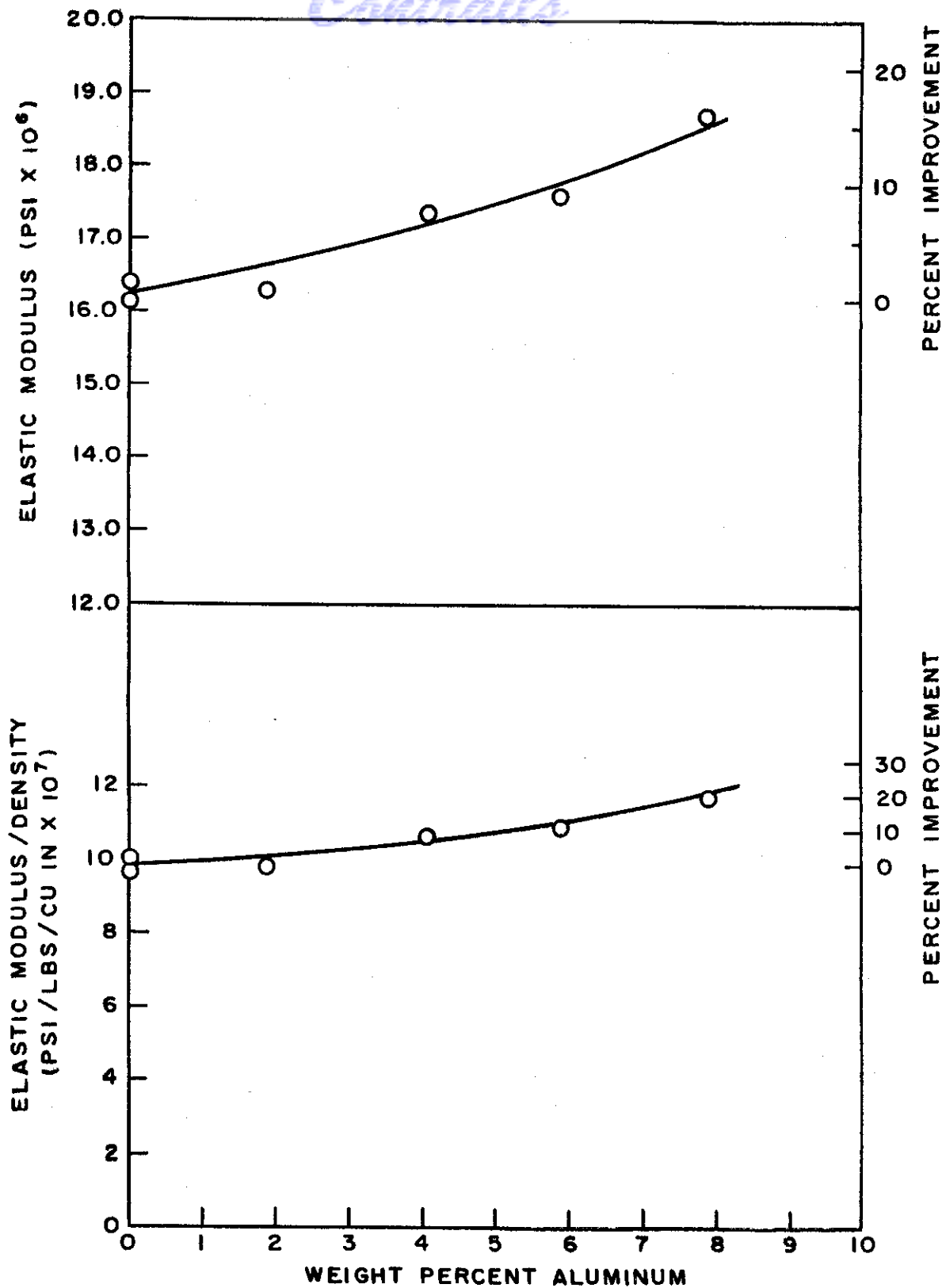


FIG. 6 - VARIATION IN ELASTIC MODULUS AND RATIO OF ELASTIC MODULUS TO DENSITY WITH ALUMINUM CONTENT FOR TITANIUM-BASE ALLOYS - ROOM TEMPERATURE

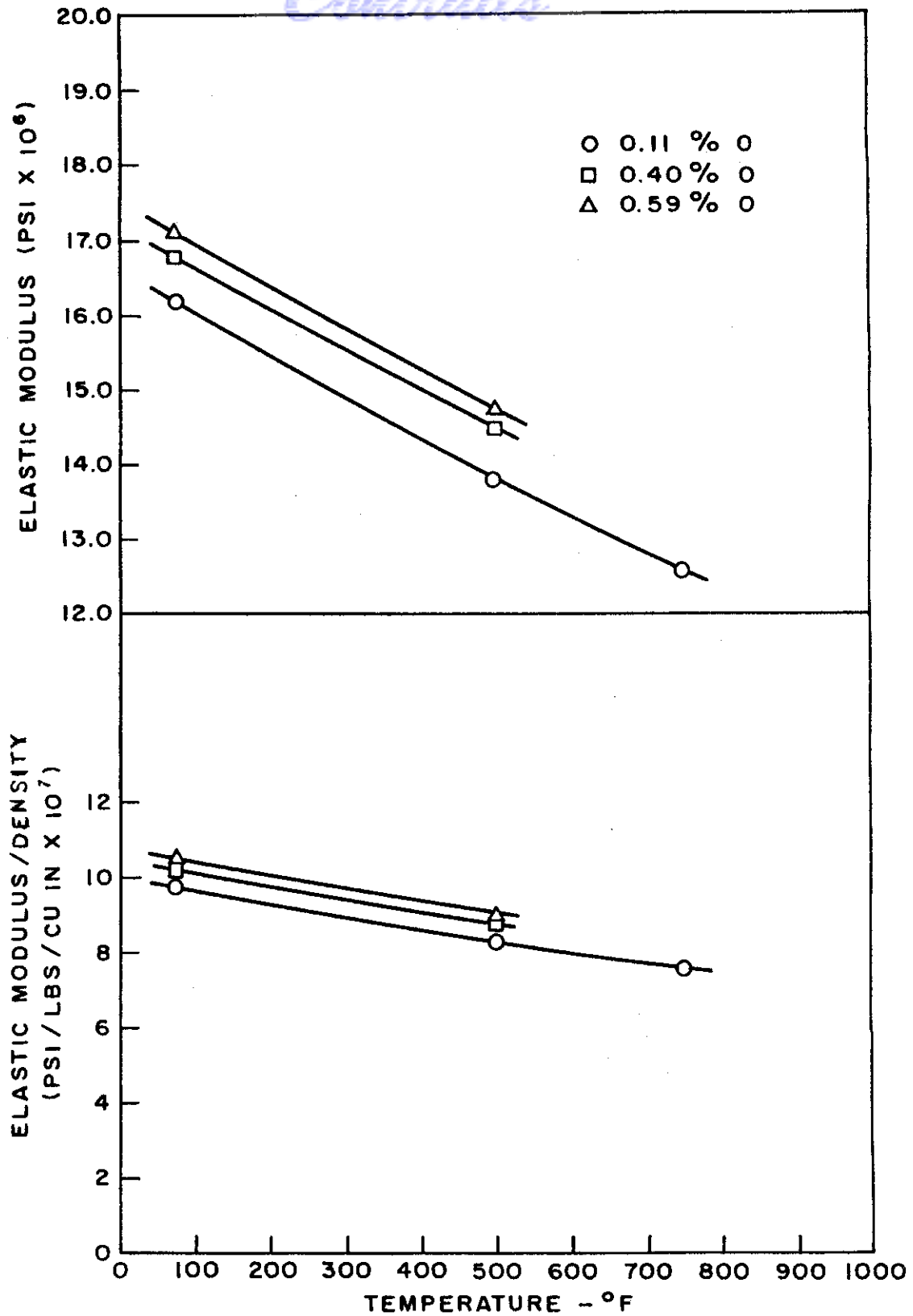


FIG. 7 - VARIATION IN ELASTIC MODULUS AND RATIO OF ELASTIC MODULUS TO DENSITY WITH TEMPERATURE FOR A SERIES OF TITANIUM-BASE ALLOYS CONTAINING OXYGEN AT VARIOUS CONCENTRATIONS

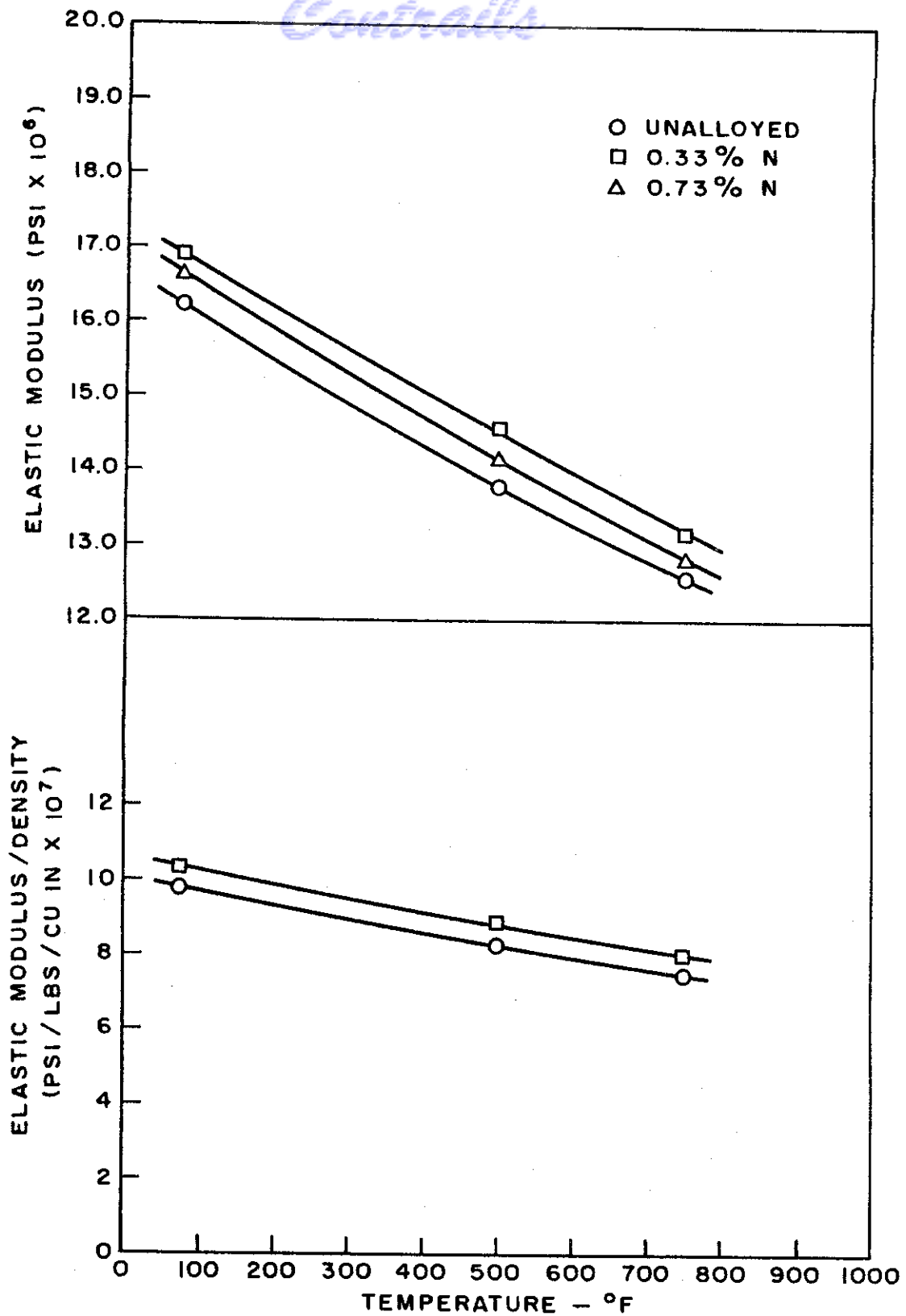


FIG. 8 - VARIATION IN ELASTIC MODULUS AND RATIO OF ELASTIC MODULUS TO DENSITY WITH TEMPERATURE FOR A SERIES OF TITANIUM-BASE ALLOYS CONTAINING NITROGEN AT VARIOUS CONCENTRATIONS

Controls

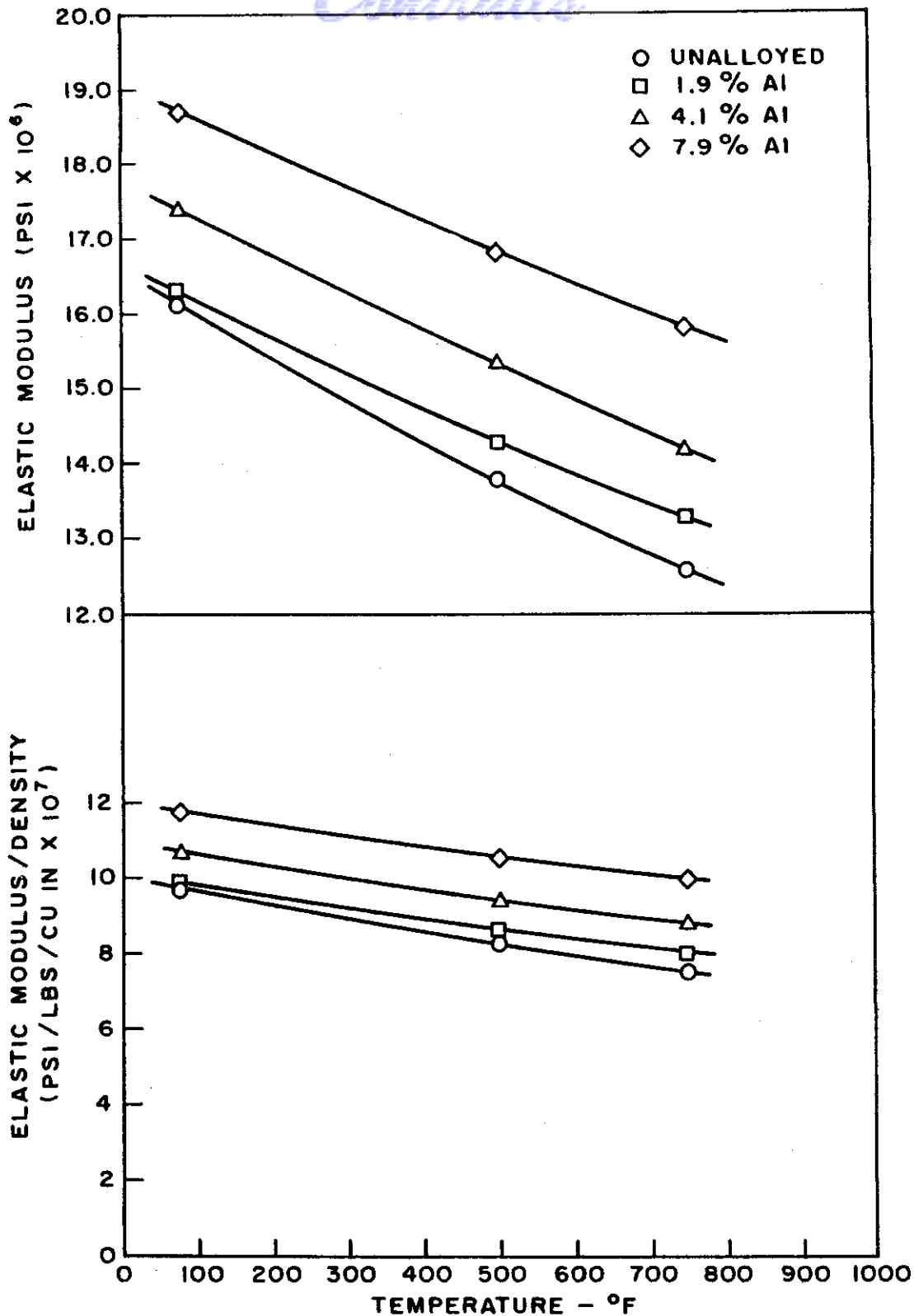


FIG. 9 - VARIATION IN ELASTIC MODULUS AND RATIO OF ELASTIC MODULUS TO DENSITY WITH TEMPERATURE FOR A SERIES OF TITANIUM-BASE ALLOYS CONTAINING ALUMINUM AT VARIOUS CONCENTRATIONS



Neg. No. 10114

X 250

Fig. 10

Mg-reduced Ti, 750°C - 1 hr. → AC.
Transformed α structure.



Neg. No. 10124

X 250

Fig. 11

Iodide Ti, 900°C - 1 hr. → AC.
Transformed α structure.

Etchant: 60 cc glycerine, 20 cc HF, 20 cc HNO₃

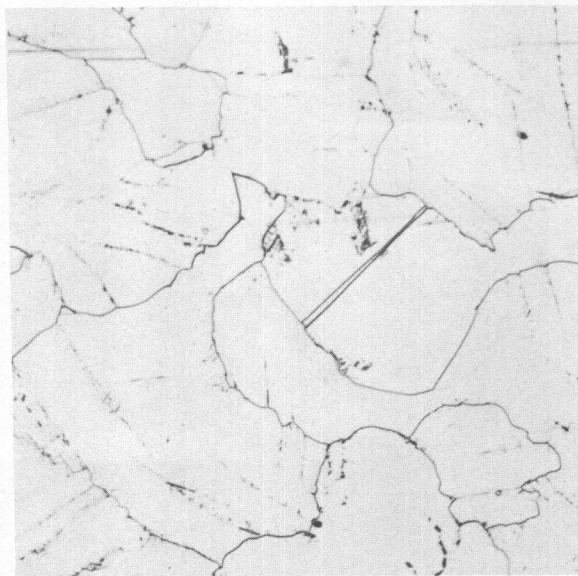


Neg. No. 10388

X 250

Fig. 12

Ti-0.40% O, as-forged. Serrated α structure.



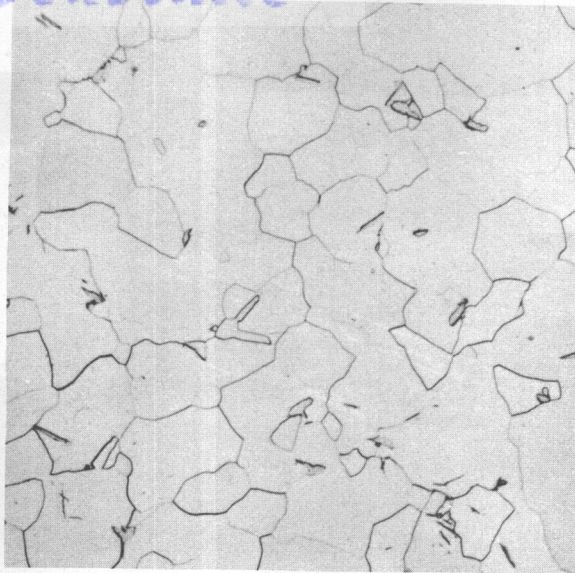
Neg. No. 10389

X 250

Fig. 13

Ti-0.49% O, as-forged. Semi-serrated α structure.

Etchant: 60 cc glycerine, 20 HF, 20 HNO₃

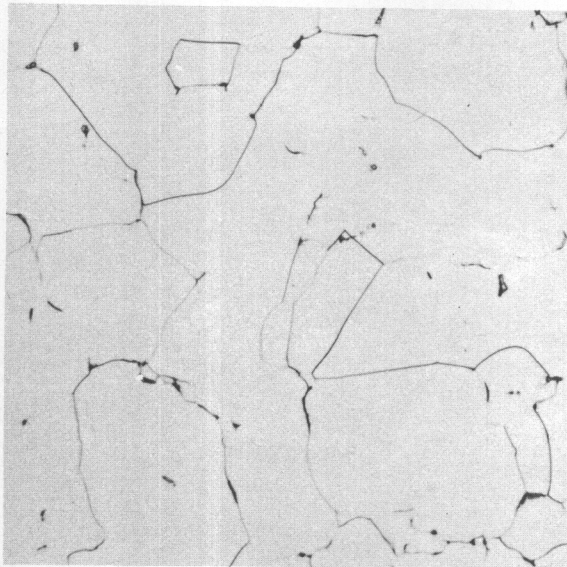


Neg. No. 10390

X 250

Fig. 14

Ti-0.59% O, as-forged. Structure is equiaxed α grains.



Neg. No. 10121

X 250

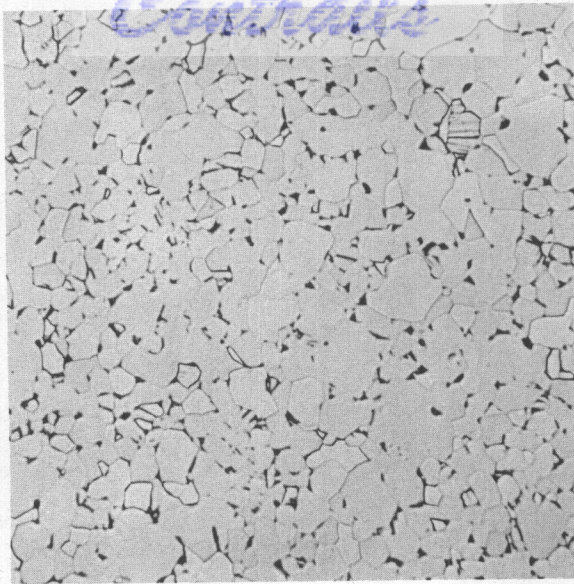
Fig. 15

Ti-0.33% N, 900°C - 4 hrs. \rightarrow AC.
Equiaxed α titanium grains.

Etchant: 60 cc glycerine, 20 HF, 20 HNO₃

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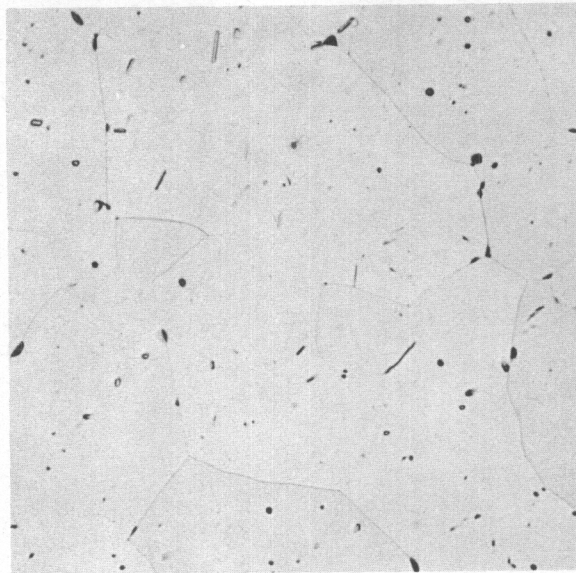


Neg. No. 10123

X 250

Fig. 16

Ti-0.48% N, 900°C - 4 hrs. → AC.
Fine equiaxed α grains.



Neg. No. 10120

X 250

Fig. 17

Ti-0.73% N, 900°C - 4 hrs. → AC,
Identity of particles in equiaxed
 α grains is unknown.

Etchant: 60 cc glycerine, 20 HF, 20 HNO₃

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Neg. No. 10151

X 250

Fig. 18

Ti-1.9% Al, 850°C - 1 hr. → WQ.
Structure is equiaxed grains of α .



Neg. No. 10153

X 250

Fig. 19

Ti-7.9% Al, 850°C - 1 hr. → WQ.
Structure is alpha titanium.

Etchant: 60 cc glycerine, 20 HF, 20 HNO₃

AVERAGE RATE OF CHANGE OF ELASTIC MODULUS AND ELASTIC MODULUS TO DENSITY RATIO
WITH TEMPERATURE IN THE RANGE 75°-750°F FOR A NUMBER OF TITANIUM-BASE ALLOYS

	E/°F (psi/°F)	E/ρ ratio/°F (psi/lbs/cu in/°F)
Unalloyed Ti (Mg reduced)	5340	32,200
Unalloyed Ti (Iodide)	5780	35,200
Ti-0.40% O	5410	32,000
Ti-0.49% O	5180	32,500
Ti-0.59% O	5650	33,800
Ti-0.33% N	5490	33,400
Ti-0.48% N	5930	35,800
Ti-0.73% N	5780	35,600
Ti-1.9% Al	4450	26,800
Ti-4.1% Al	4750	28,800
Ti-5.9% Al	4300	26,200
Ti-7.9% Al	4290	26,700
Ti-0.40% C	5340	32,000
Ti-0.66% C	5480	32,800
Ti-0.85% C	5480	32,900
Ti-1.33% C	5480	33,000
Ti-2.69% C	5190	31,200
Ti-0.48% B	5340	32,400
Ti-0.96% B	5490	33,000
Ti-1.36% B	5490	32,600
Ti-0.56% Si	5340	32,400
Ti-0.96% Si	5340	31,800
Ti-0.55% Si	5340	32,900
Ti-0.44% Be	5640	35,200
Ti-1.41% Be	5340	33,800
Ti-3.02% Cu	5410	32,400
Ti-6.36% Cu	5410	31,800
Ti-11.5% Cu	5180	29,900

TABLE V

SUMMARY OF DYNAMIC ELASTIC MODULUS MEASUREMENTS AT ROOM TEMPERATURE
FOR A SERIES OF TITANIUM-VANADIUM ALLOYS IN THE WATER QUENCHED CONDITION

Alloy*	H (ppm)	O %	Density g/cu cm	Density lbs/cu in	Elastic Modulus (psi x 10 ⁶)	E/ ρ (psi x 10 ⁷ /lbs/cu in)
Titanium-10.6% Vanadium	233	0.296	4.70	.170	9.00	5.29
Titanium-15.3% Vanadium	374	0.200	4.80	.173	17.6	10.2
Titanium-21.5% Vanadium	408	-	4.80	.173	13.1	7.57
Titanium-25.2% Vanadium	178	-	4.86	.175	12.0	6.86
Titanium-31.3% Vanadium	438	-	4.96	.179	13.2	7.38

* Heat treatment for all specimens: 850°C-1 hr. → WQ.

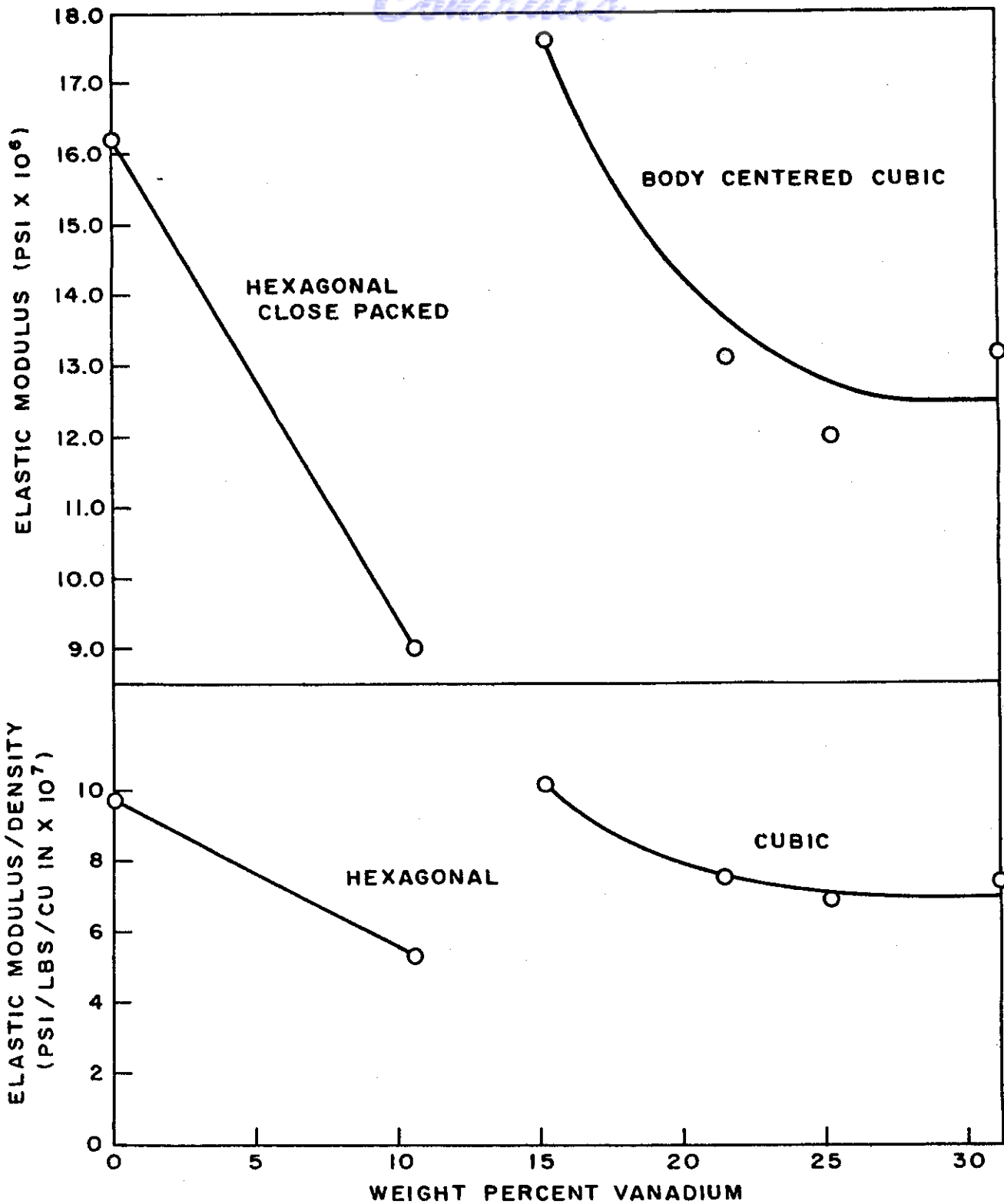
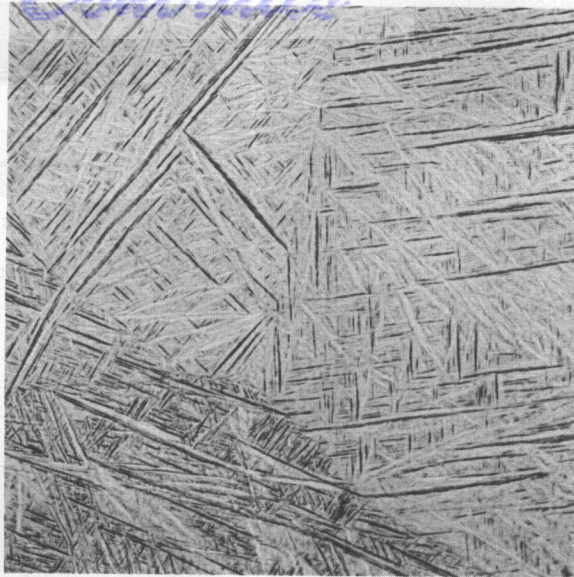


FIG. 20-VARIATION IN ELASTIC MODULUS AND RATIO OF ELASTIC MODULUS TO DENSITY AT ROOM TEMPERATURE FOR A SERIES OF TITANIUM-VANADIUM ALLOYS IN THE WATER QUENCHED CONDITION.

Continued

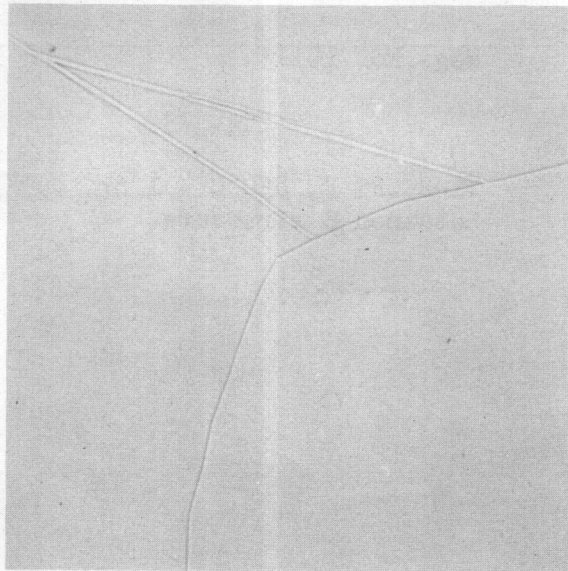


Neg. No. 10115

X 250

Fig. 21

Ti-10.6% V, 850°C - 1 hr. → WQ.
Structure is α' .



Neg. No. 10125

X 250

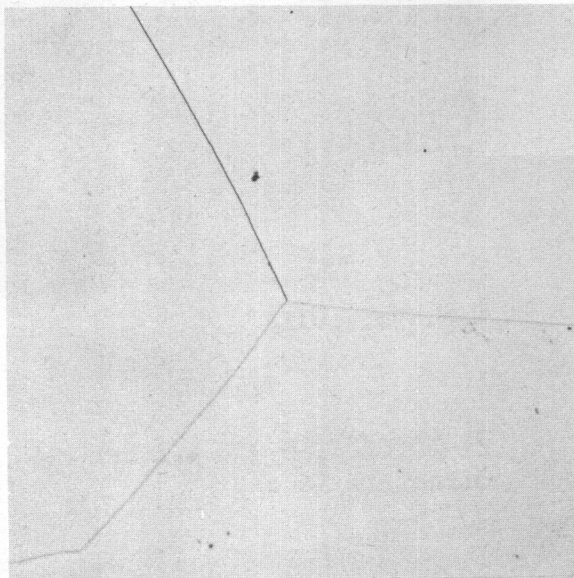
Fig. 22

Ti-15.3% V, 850°C - 1 hr. → WQ.
Structure is retained β showing a
coarse, split α' needle.

Etchant: 60 cc glycerine, 20 HF, 20 HNO₃

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Neg. No. 10113

X 250

Fig. 23

Ti-25.2% V, 850°C - 1 hr. → WQ.
Retained β structure.

Etchant: 60 cc glycerine, 20 HF, 20 HNO₃

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This alloy quenched to α' , which showed an acicular martensitic type of microstructure, and it can be concluded that α' is intrinsically a low modulus material, at least α' that is supersaturated with vanadium. Just above the beta retention composition (approximately 14.9% V*) a fairly high value of the elastic modulus was found, but with increasing amounts of vanadium the modulus dropped rapidly to a steady value of about 12.5×10^6 psi.

It would appear that of the equilibrium phases, beta has the lower modulus. Two other alloys containing 8% Cr and 13% Mo, respectively, in their retained beta state were tested to check the generality of this statement. Beta is retained in the Ti-Cr binary at approximately 5.5% Cr and in the Ti-Mo binary at approximately 10% Mo. The measured elastic moduli of 15.1×10^6 psi (8% Cr) and 14.1×10^6 psi (13% Mo) support the proposition.

The anomalously high modulus of the 15% V alloy in the retained beta state correlates at first thought with the observation by Brotzen et al** that complete suppression of the low temperature transition phase by quenching is difficult. The 8% Cr alloy is an even more striking example of this. In the as-quenched state this alloy, although apparently all-beta, is very hard and brittle. The work of Frost, et al*** has shown that substantial amounts of the transition phase can be developed by low temperature aging. To test the hypothesis that the anomalously high values of elastic modulus in certain all-beta alloys are due to the presence of an insuppressible transition phase, the 8% Cr alloy was aged at 750°F for two times 24 hours and 96 hours, respectively. According to the work of Frost and co-workers, these anneals should develop in the first instance a maximum amount of the transition phase and in the second case an overaged state of almost equilibrium alpha and beta. After the 24 hour aging treatment, the 8% Cr alloy developed the outstanding elastic modulus of 20.0×10^6 psi. On averaging, the value of the modulus dropped to 17.2×10^6 psi. It is, therefore, clear that the transition phase has a very high intrinsic modulus.

C. Alpha-Beta Alloys

A series of Ti-V binary alloys was heat treated in such a fashion as to produce microstructures of various ratios of alpha and beta phases. Elastic modulus determinations were then made to evaluate the effect of the dispersed alpha phase in the beta matrix. Table VI lists the compositions studied, the heat treatments employed and the dynamic elastic modulus values. The microstructure for the water quenched specimen is shown in Figure 21. Those transformed at 500°, 600° and 700°C are given in Figures 24-29, inclusive. It will be seen from a study of Table VI that only a minor effect is found for varying α/β ratios for the two higher vanadium content alloys. In the 10:6% V composition, the previously noted low modulus value for the water quenched specimens was confirmed, and a rather significant increase in elastic

* WADC-TR-53-41, "Constitution of Titanium Alloy Systems", p. 113, Reference (3).

** F. R. Brotzen, E. L. Harmon, A. R. Troiano, J. Metals, Feb. 1955, 413.

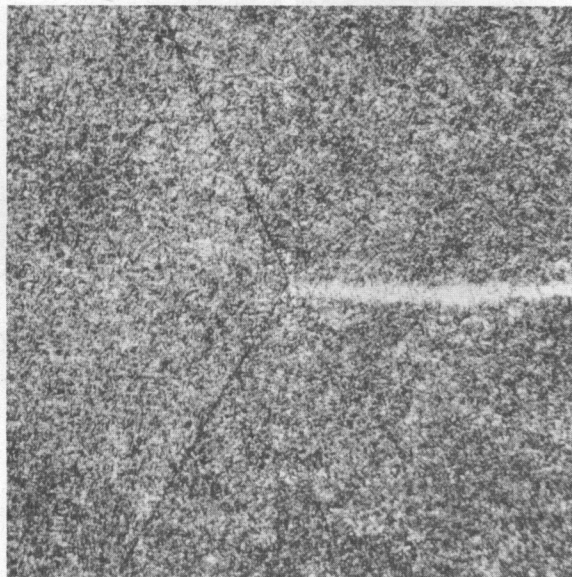
*** Battelle Memorial Institute, WADC-TR-54-355, AF 33(616)-445.

TABLE VI

SUMMARY OF DYNAMIC ELASTIC MODULUS DETERMINATIONS
FOR A SERIES OF Ti-V BINARY ALLOYS HEAT TREATED
TO GIVE A VARYING ALPHA/BETA RATIO

Alloy	Dynamic Elastic Modulus (psi x 10 ⁶)			
	WQ	700°C	600°C	500°C
Titanium-10.6% Vanadium	9.39	12.7	-	15.0
Titanium-21.5% Vanadium	12.0	-	12.4	13.4
Titanium-31.3% Vanadium	12.9	-	13.0	13.2

* Specimens were solution treated at 850°C for 1 hour and transformed isothermally at the temperature shown. Transformation time at 700°C was 5 hours, at 600°C was 16 hours, and at 500°C was 24 hours.



Neg. No. 10823

X 250

Fig. 24

Ti-10.6% V, 850°C - 1 hr. → 500°C -
24 hrs. → WQ. Fine α in β matrix.

Etchant: 60 cc glycerine, 20 HF, 20 HNO₃

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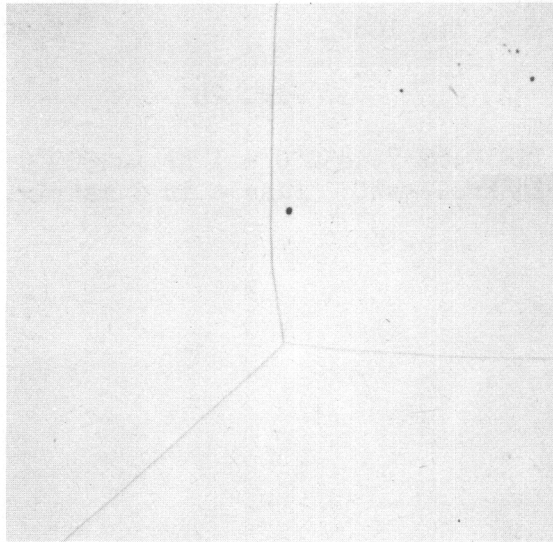


Neg. No. 10460

X 250

Fig. 25

Ti-10.6% V, 850°C - 1 hr. → 700°C -
5 hrs. → WQ. Coarse α in matrix of β .



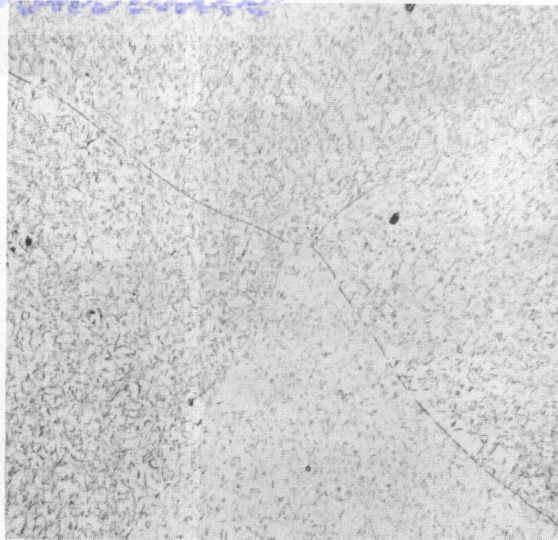
Neg. No. 10461

X 250

Fig. 26

Ti-21.5% V, 850°C - 1 hr. → WQ.
Retained β structure.

Etchant: 60 cc glycerine, 20 HF, 20 HNO₃

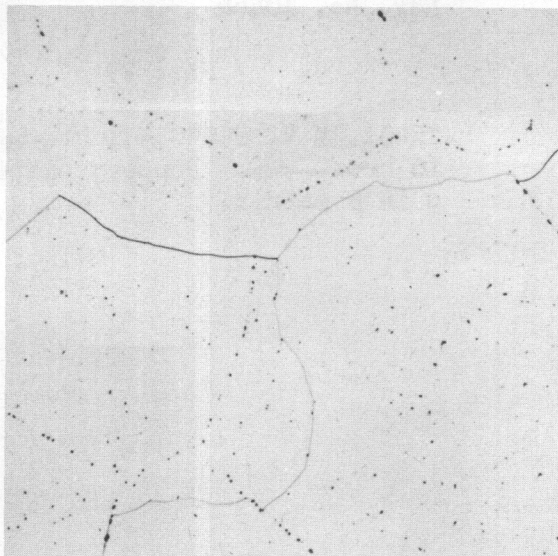


Neg. No. 10462

X 250

Fig. 27

Ti-21.5% V, 850°C - 1 hr. → 600°C -
16 hrs. → WQ. Fine dispersion of α
in β matrix.



Neg. No. 10463

X 250

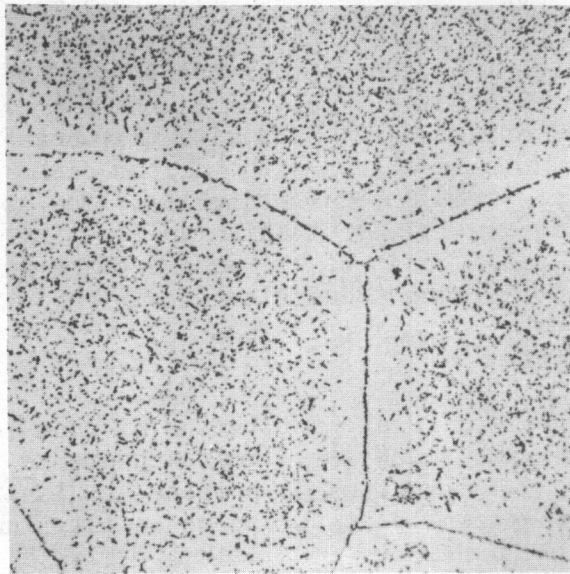
Fig. 28

Ti-31.3% V, 850°C - 1 hr. → WQ.
Retained β structure. Identity of
small dark particles unknown.

Etchant: 60 cc glycerine, 20 HF, 20 HNO₃

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Neg. No. 10464

X 250

Fig. 29

Ti-31.3% V, 850°C - 1 hr. → 600°C -
16 hrs. → WQ. Peppery dispersion of
 α in β matrix.

Etchant: 60 cc glycerine, 20 HF, 20 HNO₃

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modulus was noted as the transformation temperature was decreased. This reflects the very marked difference in elastic modulus between low alloyed α' and highly alloyed α' .

D. Dispersed Phase Titanium Base Alloys

The modulus of elasticity of a metal or alloy can be increased by dispersing in it a second phase of higher elastic modulus. To a first approximation, the contributions of the two phases would depend upon the intrinsic modulus of each constituent and the volume fraction of each present.* Also, the nature of the dispersed phase, i.e., size, shape and distribution would be expected to exert an effect.** It was with the above principle in mind that a series of titanium base alloys with various dispersed second phases was prepared for dynamic elastic modulus studies. The second phase in each case was developed by adding to the titanium an alloying element that formed an intermediate phase. The amount of the second phase was controlled by varying the concentration of the added element.

1. Titanium-Carbon System

The solubility of carbon in alpha titanium at room temperature is quite low, and any excess over the solubility limit appears as the intermediate phase TiC. This compound is reported to possess an elastic modulus of about 46×10^6 psi,*** and consequently its presence in titanium should be very beneficial in improving the elastic modulus. A series of alloys containing increasing amounts of carbon was made, and the expected improvement in modulus was noted; see Table VII and Figure 30. Tests were run at elevated temperatures up to 750°F, and, as in other titanium-base alloys, the modulus was found to decrease linearly with rising temperature; see Table IV and Figure 31. The microstructures for these alloys are presented in Figures 32 to 36, inclusive. Inspection of Figure 30 shows that both E and the E/ ρ ratio increase appreciably with carbon content, the 2.69% C composition exhibiting an E in excess of 20×10^6 psi at room temperature. This composition was forged into 3/8 diameter rods from pancake ingots of about 3 inches diameter by 1/2 inch thick without undue difficulty.

2. Titanium-Boron System

Boron, like carbon, is only slightly soluble in alpha titanium at room temperature,**** any excess over the solid solubility limit appearing in the intermediate phase TiB. A series of alloys was prepared which contained increasing amounts of boron, and E and E/ ρ determinations were made at room

* N. Dudzinski, J. R. Murray, B. W. Mott, B. Chalmers, J. Inst. Metals 74, 1947-8, 291.

N. Dudzinski, J. Inst. Metals 81, 1953-3, 49.

** Interim Project Reports 1, 2 and 3 on Contract No. AF 33(616)-2296, Kaiser Aluminum and Chemical Corporation.

*** Smithells, C. J. Metals Reference Book, Butterworth, 1949.

**** WADC-TR-53-41, "Constitution of Titanium Alloy Systems".

TABLE VII
SUMMARY OF ELASTIC MODULUS AND RATIO OF ELASTIC MODULUS TO DENSITY
FOR A NUMBER OF DISPERSED SECOND PHASE TITANIUM-BASE ALLOYS
MEASURED AT ROOM TEMPERATURE, 500°F, and 750°F

Alloy	H (ppm)	g/cu cm	Density lbs/cu in	Elastic Modulus (psi x 10 ⁶)			Elastic Modulus/Density (psi x 10 ⁶ /lbs/cu in)		
				RT	500°F	750°F	RT	500°F	750°F
Ti-0.40% C	53	4.58	0.166	17.6	15.2	14.0	10.6	9.16	8.44
Ti-0.66% C	85	4.54	0.164	18.1	15.6	14.4	11.0	9.52	8.79
Ti-0.85% C	90	4.53	0.164	18.9	16.5	15.2	11.5	10.1	9.28
Ti-1.33% C	74	4.52	0.163	18.8	16.5	15.1	11.5	10.1	9.27
Ti-2.69% C	59	4.52	0.163	20.4	18.3	16.9	12.5	11.2	10.4
Ti-0.48% B	61	4.53	0.164	17.9	15.6	14.3	10.9	9.51	8.72
Ti-0.96% B	82	4.53	0.164	18.9	16.5	15.2	11.5	10.1	9.27
Ti-1.36% B	82	4.53	0.164	20.2	17.8	16.5	12.3	10.9	10.1
Ti-0.56% Si	34	4.52	0.163	16.5	14.2	12.9	10.1	8.72	7.92
Ti-0.96% Si	34	4.52	0.163	16.7	14.4	13.1	10.2	8.84	8.05
Ti-1.55% Si	33	4.52	0.163	17.4	15.1	13.8	10.7	9.26	8.48
Ti-0.44% Be	23	4.49	0.162	16.8	14.4	13.0	10.4	8.90	8.03
Ti-1.41% Be	28	4.43	0.160	17.4	15.1	13.8	10.9	9.44	8.62
Ti-3.02% Cu	65	4.63	0.167	16.5	14.2	-	9.88	8.50	-
Ti-6.36% Cu	54	4.71	0.170	16.9	14.6	-	9.94	8.59	-
Ti-11.5% Cu	46	4.80	0.173	17.3	15.1	-	10.0	8.73	-

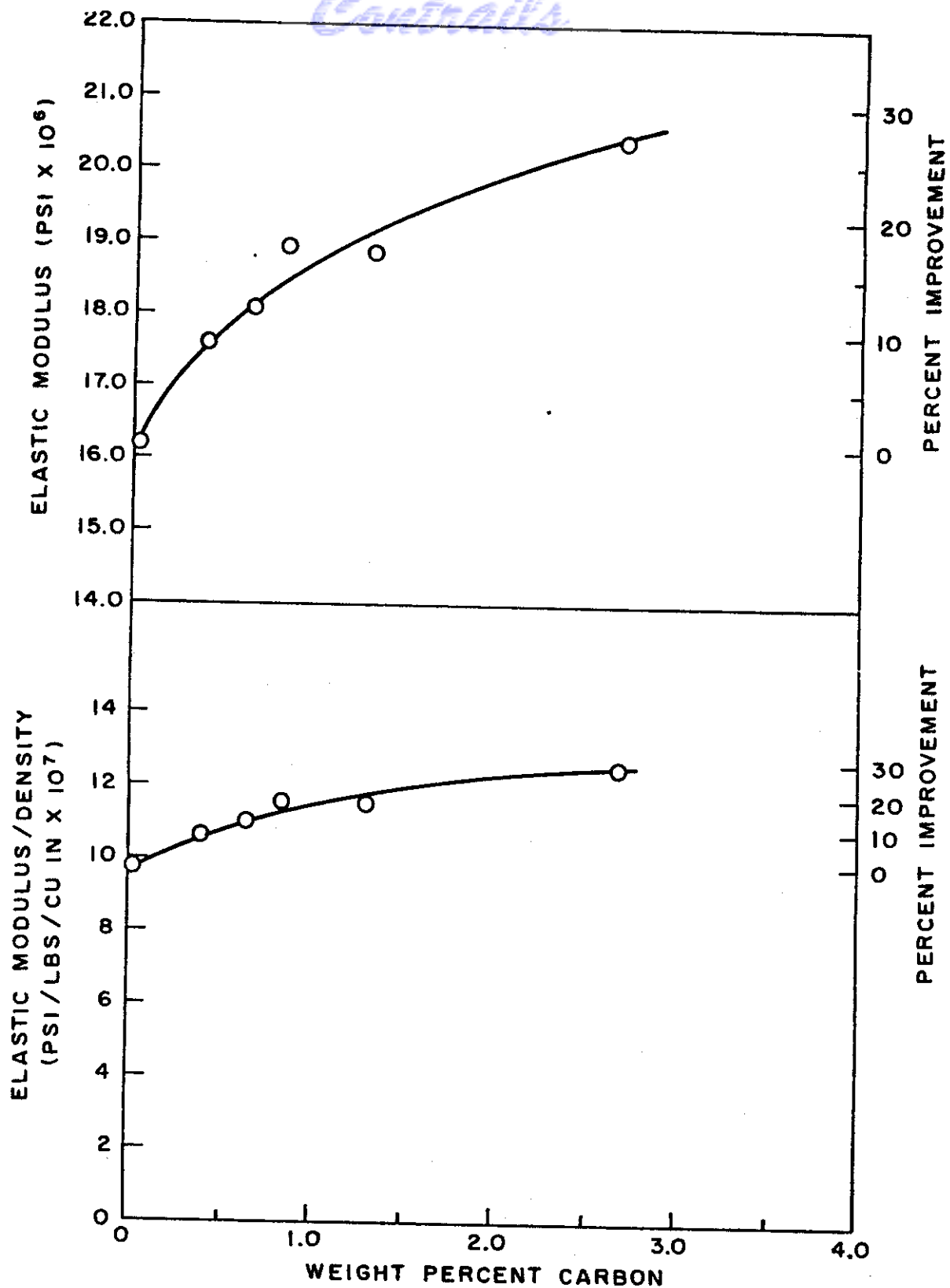


FIG. 30 - VARIATION IN ELASTIC MODULUS AND RATIO OF ELASTIC MODULUS TO DENSITY WITH CARBON CONTENT FOR TITANIUM-BASE ALLOYS - ROOM TEMPERATURE

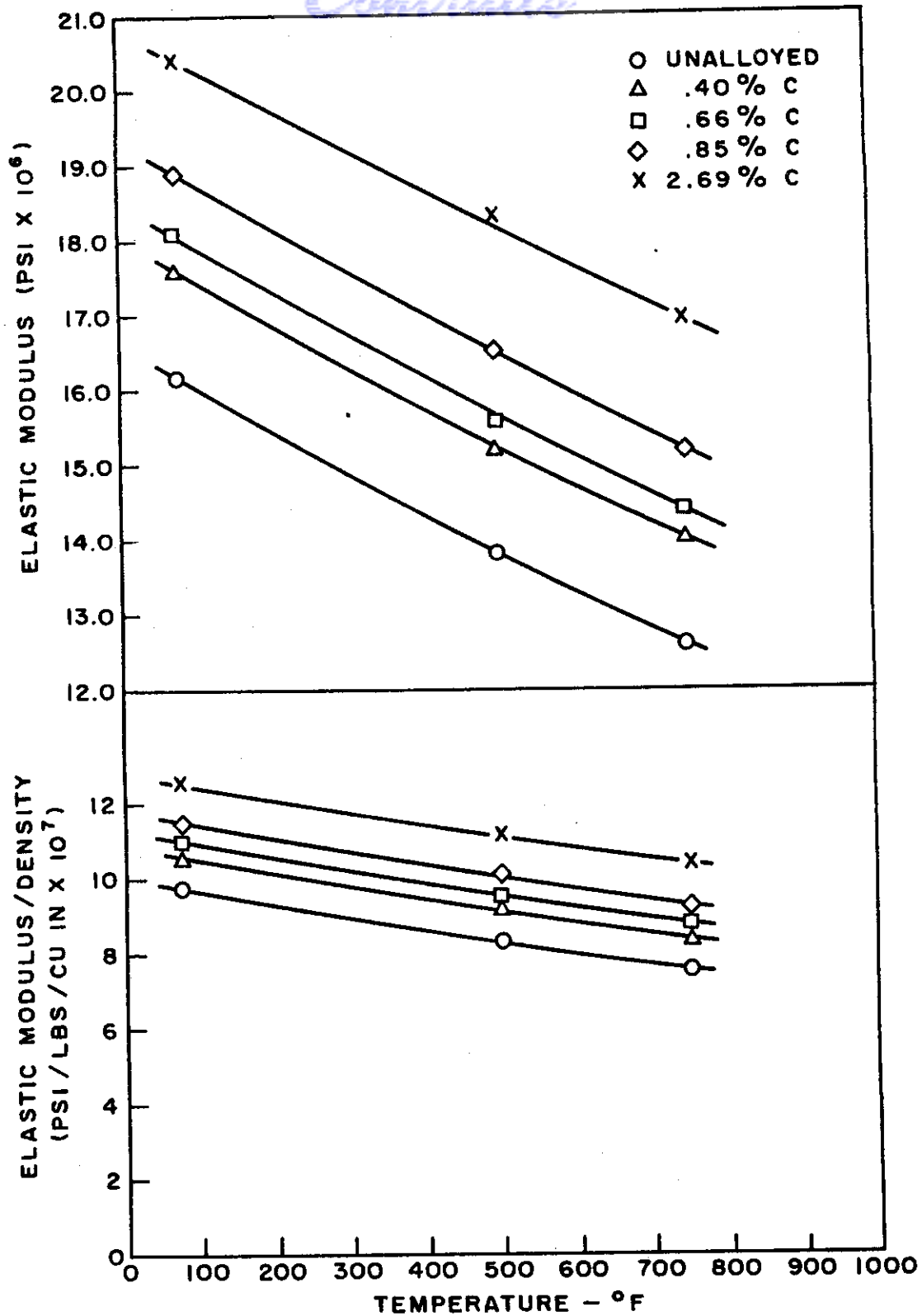
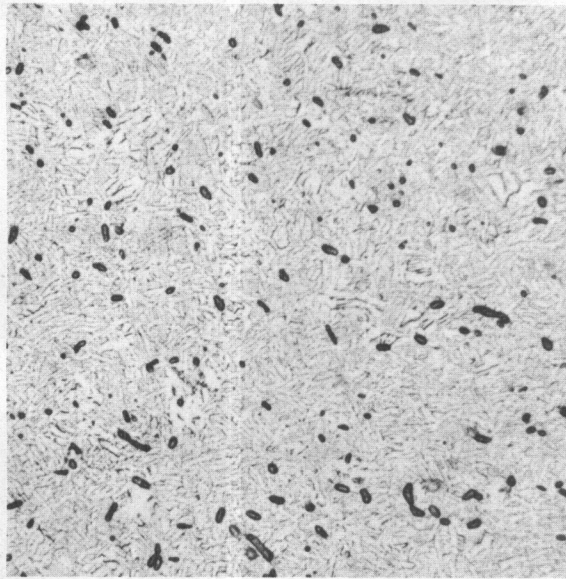


FIG. 31 - VARIATION IN ELASTIC MODULUS AND RATIO OF ELASTIC MODULUS TO DENSITY WITH TEMPERATURE FOR A SERIES OF TITANIUM-BASE ALLOYS CONTAINING CARBON AT VARIOUS CONCENTRATIONS



Neg. No. 10129

X 250

Fig. 32

Ti-0.40% C, 750°C - 1 hr. → AC.
TiC particles in α titanium.

Etchant: 60 cc glycerine, 20 HF, 20 HNO₃

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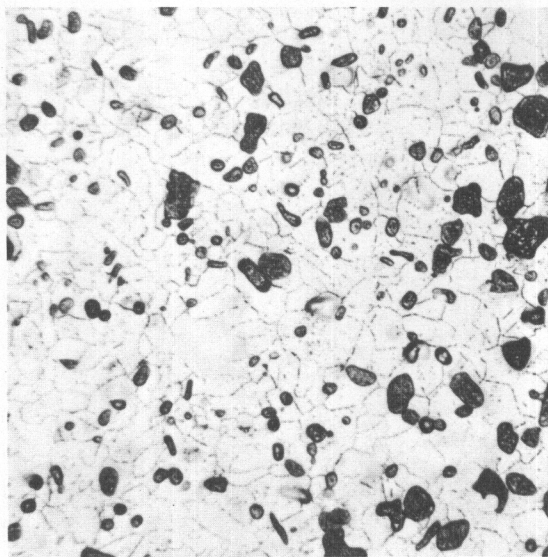


Neg. No. 10130

X 250

Fig. 33

Ti-0.66% C, 750°C - 1 hr. → AC.
Coarse TiC particles in α titanium.



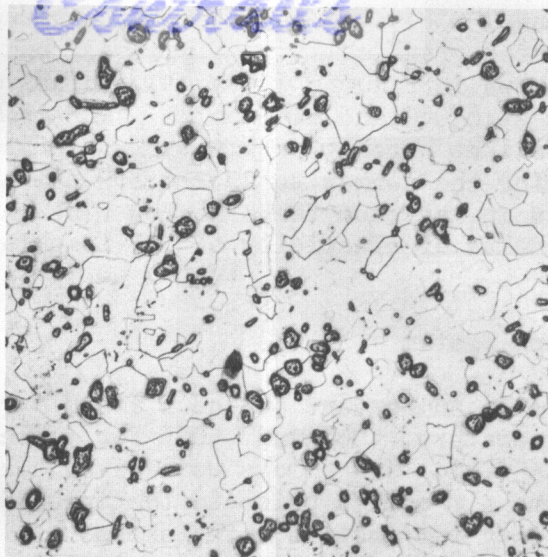
Neg. No. 10131

X 250

Fig. 34

Ti-0.85% C, 750°C - 1 hr. → AC.
Coarse TiC particles in α titanium.

Etchant: 60 cc glycerine, 20 HF, 20 HNO₃

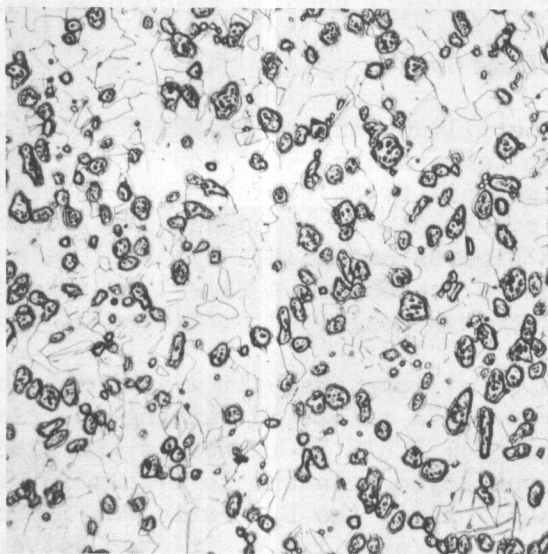


Neg. No. 10385

X 250

Fig. 35

Ti-1.33% C, 850°C - 1 hr. → AC.
Coarse TiC particles in α titanium.



Neg. No. 10386

X 250

Fig. 36

Ti-2.69% C, 850°C - 1 hr. → AC.
Heavy dispersion of coarse TiC particles
in α titanium.

Etchant: 60 cc glycerine, 20 HF, 20 HNO₃

temperature, 500° and 750°F. The results of this study are summarized in Table VII and presented graphically in Figures 37 and 38. The microstructures of the alloys are shown in Figures 39 to 41, inclusive. Specimens were annealed for 1 hour at 850°C and air cooled prior to testing. Inspection of Figure 37 reveals that boron exerts a marked influence on both E and E/ρ, 1.36% boron increasing the former to 20.2 x 10⁶ psi and the latter to 12.3 x 10⁷ psi/lb/cu in. at room temperature. The inference to be drawn from this is that the modulus of elasticity of TiB is quite high, of the order of that of TiC.

3. Titanium-Silicon System

The effect of silicon content on the E and E/ρ ratio of titanium was studied at room temperature, 500° and 750°F for three levels of silicon - 0.56%, 0.96% and 1.55% Si. The alloys were given a homogenizing anneal at 900°C for 4 hours and air cooled prior to testing. Silicon is only slightly soluble in α-Ti at room temperature,* the excess of silicon in these alloys being present as Ti₅Si₃. Figures 42, 43 and 44 illustrate the type of microstructures obtained in these alloys.

Only a small increase in the elastic modulus was found for silicon additions up to 1.55% Si; see Table VII and Figure 45. The E/ρ ratio also was found to rise slowly with increasing silicon content. Since little more silicon could be added without impairing the forgeability, it does not appear that silicon can be profitably used in developing high modulus alloys.

4. Titanium-Beryllium System

The effect of beryllium content on the E and E/ρ ratio of α-Ti was investigated at room temperature, 500° and 750°F. Beryllium, like silicon, has a very limited solubility in titanium,** the intermediate phase TiBe appearing when the solubility is exceeded. The microstructures of these alloys are shown in Figures 46 and 47. It will be noted that in the 0.44% Be alloy the phase TiBe is present in a fine line-like dispersion, whereas in the 1.41% Be alloy, coarse coalesced particles of TiBe are seen.

As in the case of silicon, beryllium additions up to about 1.5% by weight exerted only a small effect in raising the elastic modulus (see Table VII and Figure 48); 1.41% Be increased the elastic modulus from 16.2 x 10⁶ psi for unalloyed titanium to 17.4 x 10⁶ psi. This represents an increase of only 7.4%. Also, the E/ρ ratio was improved slightly by beryllium additions. As in the case of silicon, it does not appear that beryllium can be usefully applied to high modulus alloys.

5. Titanium-Copper System

This system is reported under the dispersed second phase group of experimental alloys since it is one in which a eutectoid reaction occurs in a reasonably short time upon isothermal heat treatment. The second phase developed by heat treatment is Ti₂Cu, and it was thought that this phase might prove beneficial in raising² the elastic modulus. Accordingly, a series

* WADC-TR-53-41, "Constitution of Titanium Alloy Systems"

** WADC-TR-53-41, "Constitution of Titanium Alloy Systems"

Controls

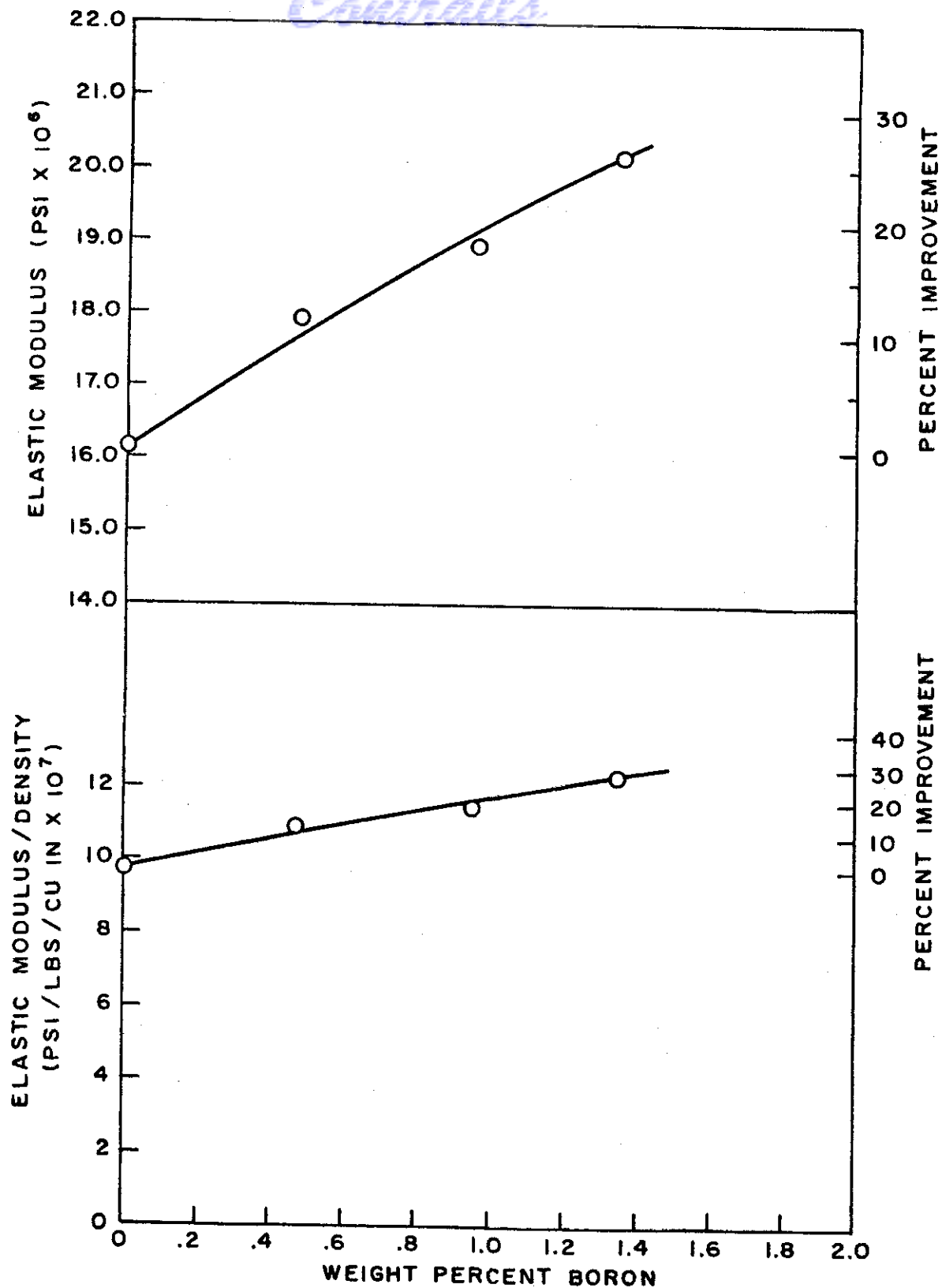


FIG. 37- VARIATION IN ELASTIC MODULUS AND RATIO OF ELASTIC MODULUS TO DENSITY WITH BORON CONTENT FOR TITANIUM-BASE ALLOYS — ROOM TEMPERATURE

Contract

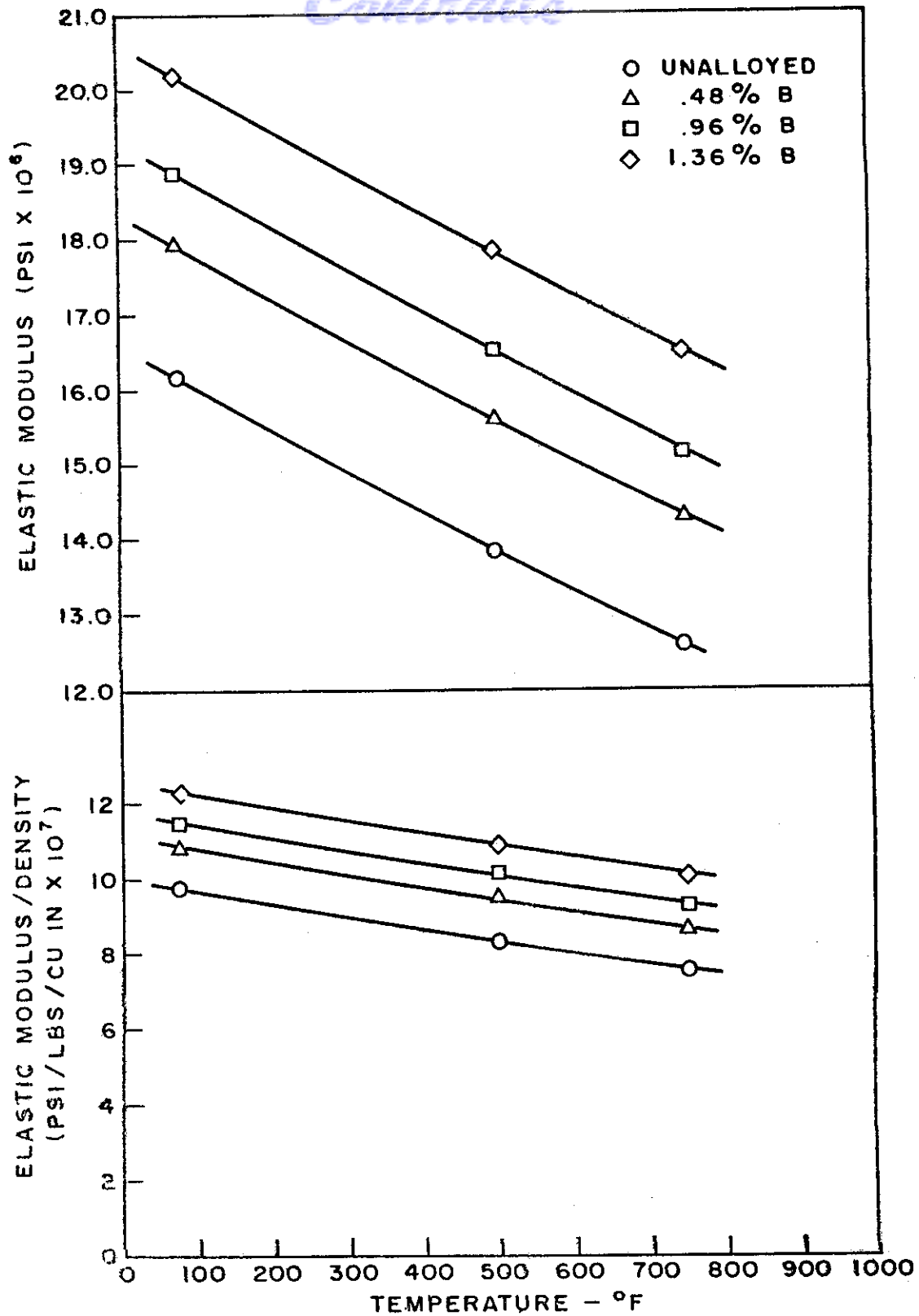


FIG. 38—VARIATION IN ELASTIC MODULUS AND RATIO OF ELASTIC MODULUS TO DENSITY WITH TEMPERATURE FOR A SERIES OF TITANIUM-BASE ALLOYS CONTAINING BORON AT VARIOUS CONCENTRATIONS

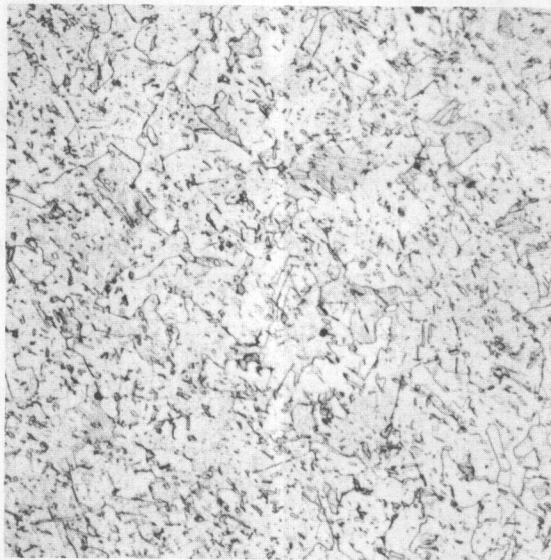


Neg. No. 10391

X 250

Fig. 39

Ti-0.48% B, 850°C - 1 hr. → AC. Small amount of TiB in a titanium matrix.



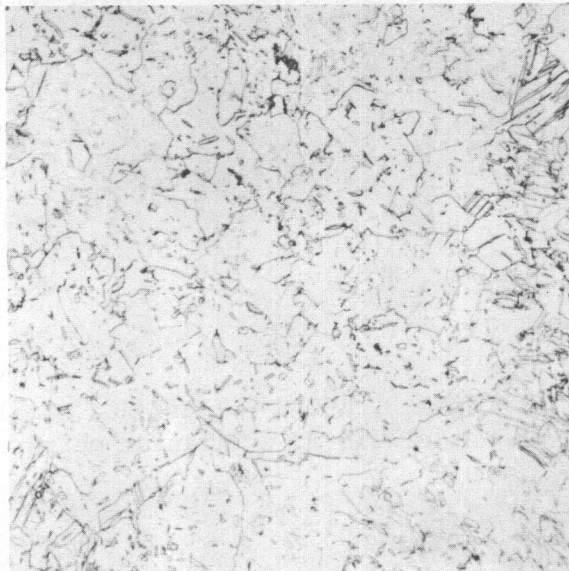
Neg. No. 10392

X 250

Fig. 40

Ti-0.96% B, 850°C - 1 hr. → AC. TiB particles in a titanium.

Etchant: 60 cc glycerine, 20 HF, 20 HNO₃



Neg. No. 10393

X 250

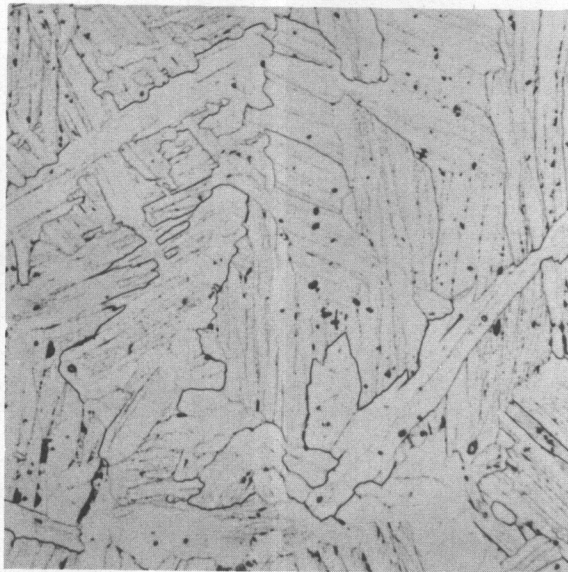
Fig. 41

Ti-1.36% B, 850°C - 1 hr. → AC. TiB
particles in α titanium.

Etchant: 60 cc glycerine, 20 HF, 20 HNO₃

WADC-TR-55-147

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Neg. No. 10126

X 250

Fig. 42

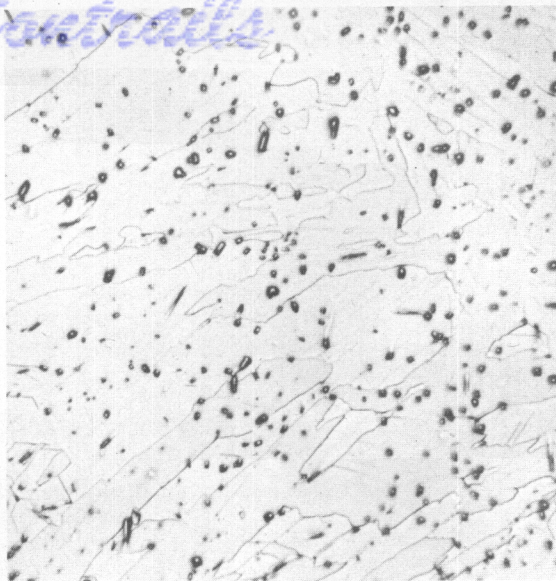
Ti-0.56% Si, 900°C - 4 hrs. → AC.
Alpha Ti grains showing fine dispersion
of Ti_5Si_3 .

Etchant: 60 cc glycerine, 20 HF, 20 HNO_3

WADC-TR-55-147

47

Contrails

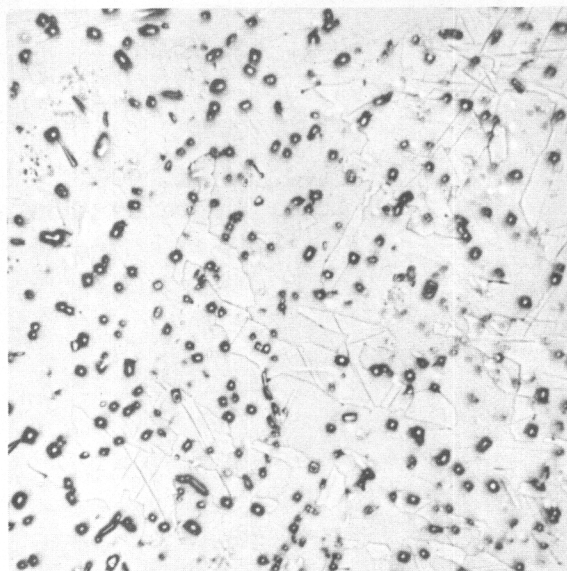


Neg. No. 10127

X 250

Fig. 43

Ti-0.96% Si, 900°C - 4 hrs. → AC.
Coalesced Ti₅Si₃ dispersion in alpha grains.



Neg. No. 10128

X 250

Fig. 44

Ti-1.55% Si, 900°C - 4 hrs. → AC.
Coalesced Ti₅Si₃ dispersion in alpha grains.

Etchant: 60 cc glycerine, 20 HF, 20 HNO₃

WADC-TR-55-147

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Contracts

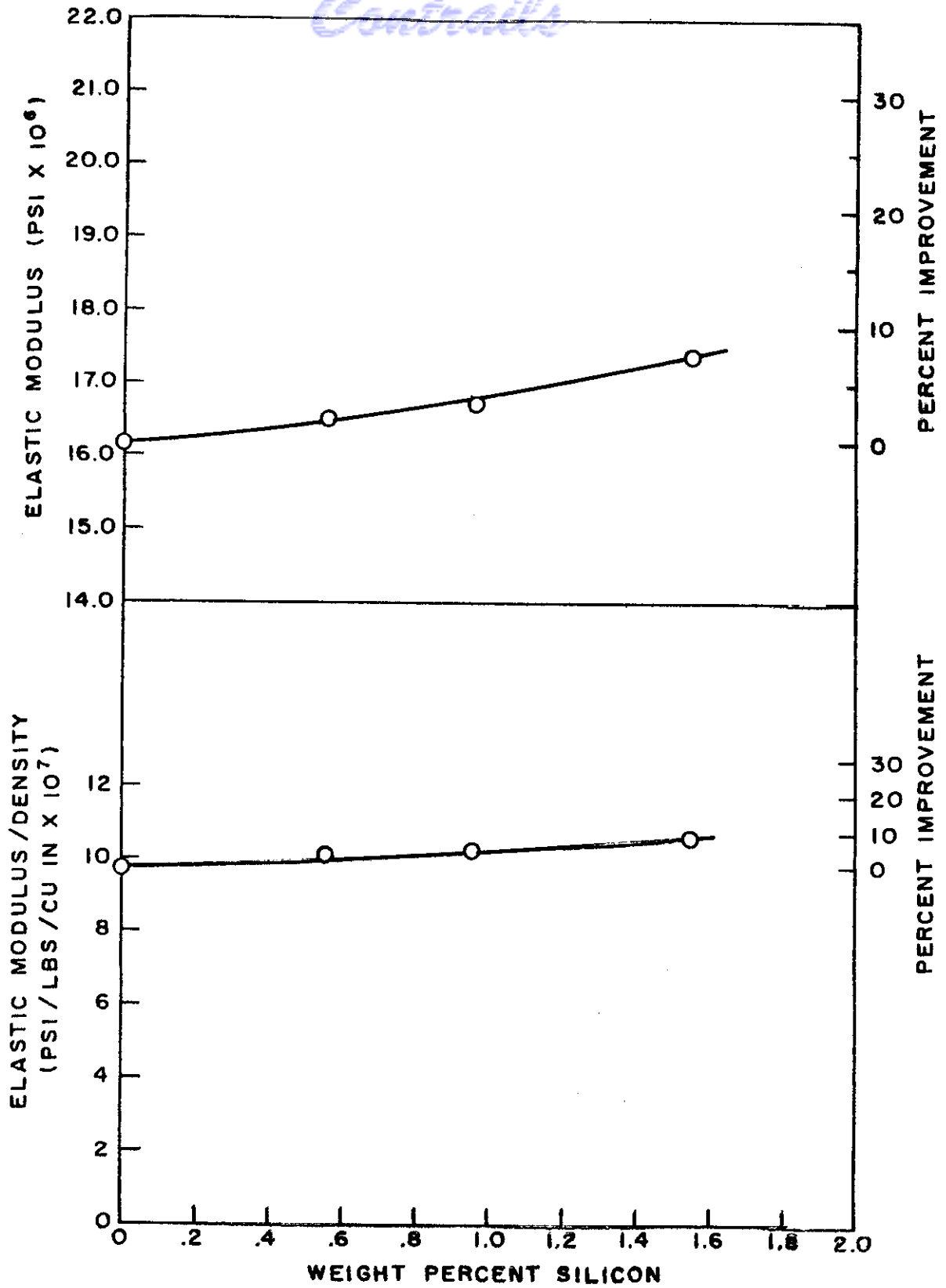
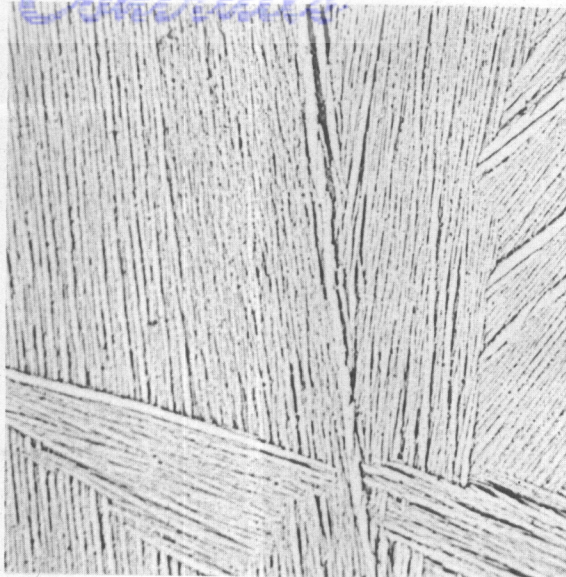


FIG. 45- VARIATION IN ELASTIC MODULUS AND RATIO OF ELASTIC MODULUS TO DENSITY WITH SILICON CONTENT FOR TITANIUM-BASE ALLOYS — ROOM TEMPERATURE

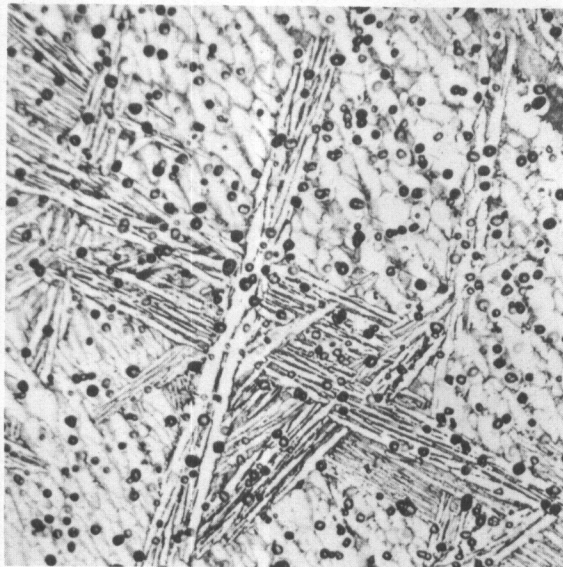


Neg. No. 10122

X 250

Fig. 46

Ti-0.44% Be, 900°C - 4 hrs. → AC.
Fine uncoalesced dispersion of TiBe
in α -Ti.



Neg. No. 10111

X 250

Fig. 47

Ti-1.41% Be, 900°C - 4 hrs. → AC.
Coalesced TiBe particles in background
of fine dispersion of TiBe in α Ti.

Etchant: 60 cc glycerine, 20 HF, 20 HNO₃

WADC-TR-55-147

50

Contrails

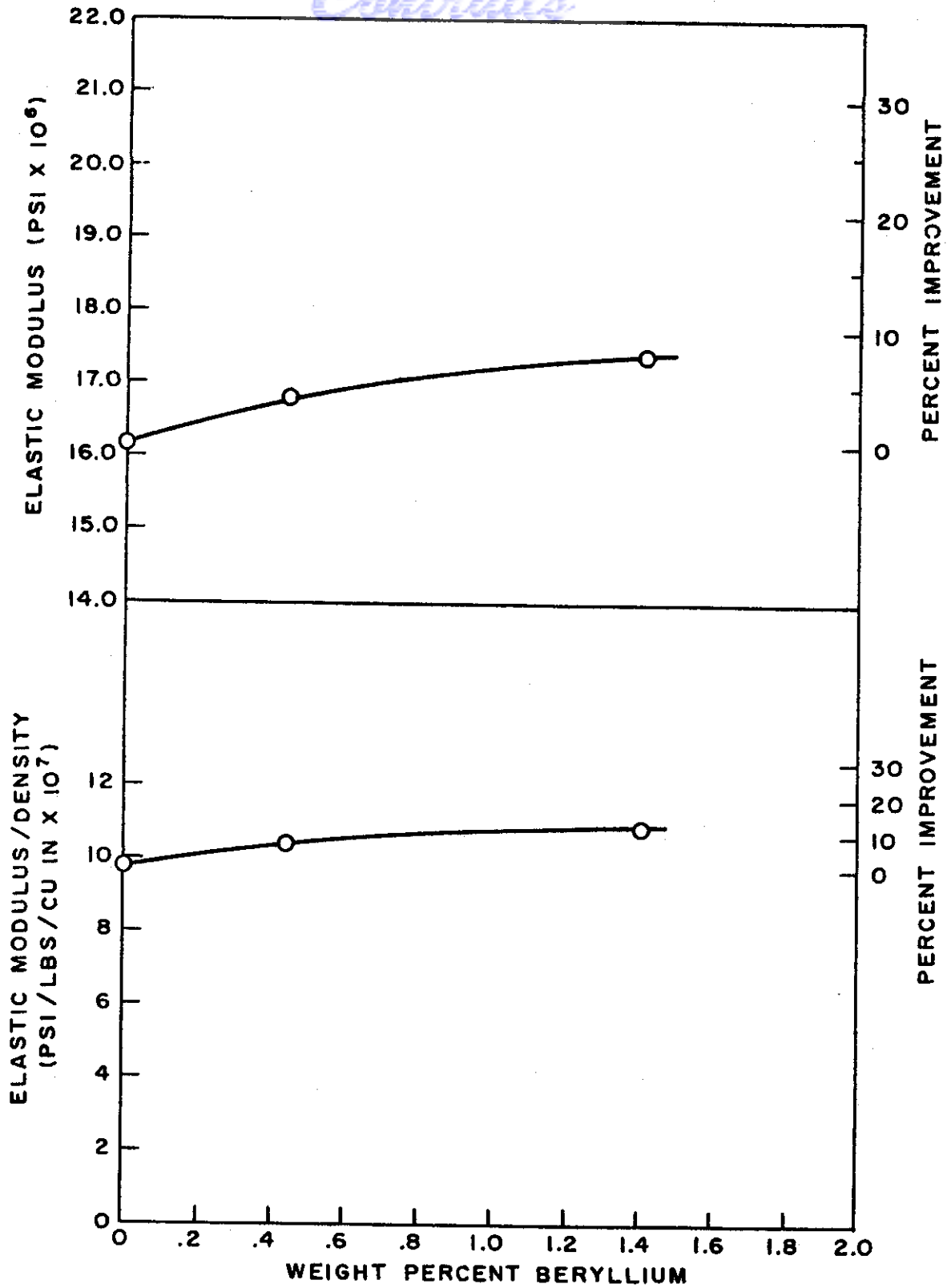


FIG. 48 - VARIATION IN ELASTIC MODULUS AND RATIO OF ELASTIC MODULUS TO DENSITY WITH BERYLLIUM CONTENT FOR TITANIUM-BASE ALLOYS - ROOM TEMPERATURE

of Ti-Cu binary alloys containing 3.02%, 6.36% and 11.5% Cu was prepared for study. Heat treatment consisted of a solution anneal at 950°C for 1 hour followed by an isothermal anneal at 700°C for 2 hours after which the specimens were water quenched. The microstructures produced by this heat treatment of the three alloys are shown in Figures 49, 50 and 51.

The variation in elastic properties with copper content is summarized in Table VII, and the room temperature results are shown graphically in Figure 52. Copper additions up to 11.5% by weight increased the value of E only slightly. The E/ ρ ratio was found to be nearly constant in the range studied, since the increase in E occasioned by the copper addition was offset by the increased density of the alloy.

Elastic modulus measurements were also made for these alloys at 500°F and are reported in Table VI. The elevated temperature behavior was similar to the other alloys studied in this program, i.e. the modulus decreased linearly with increasing temperature. (See Table IV.)

Apparently the elastic modulus of the Ti₂Cu phase is not extremely high, since only a small increase in elastic modulus was observed for alloys containing as much as 11.5% copper.

E. Ti-Al-C Ternary Alloys

Two ternary Ti-Al-C alloys were prepared and tested for elastic modulus at room temperature. This was done in order to determine if the moduli of the ternary alloys could be predicted on the basis of additivity from the previously determined modulus values for the Ti-Al and Ti-C binary alloys. The results of this work are given in Table VIII. It will be seen that the agreement between the calculated and observed modulus values is fairly good, being somewhat better for the higher aluminum alloy. One may conclude from this test that, to a first approximation, additivity effects of aluminum and carbon on the elastic modulus of titanium occur in ternary alloys of the three elements.

F. Preferred Orientation Modulus Studies in Alpha Titanium

The elastic modulus is probably anisotropic to some extent like most other mechanical properties. In normal polycrystalline, randomly oriented structures, the measured modulus is some average value and, as such, is isotropic. This value, of course, gives no indication of maximum and minimum limits with varying crystallographic direction. The measurement of the elastic moduli on materials with strongly developed preferred orientations offers a means of estimating the order of magnitude of the anisotropy.

Preferred orientations can be developed in rods or wire by drawing, swaging or rolling, and in sheet by rolling. For a high degree of perfection of orientation, large total cold reductions are always necessary. A new texture is usually developed by recrystallization of metal having a strong cold worked orientation. Since cold work itself has little influence on the elastic modulus, the cold work and recrystallization textures offer two comparable sources of highly resolved orientations.

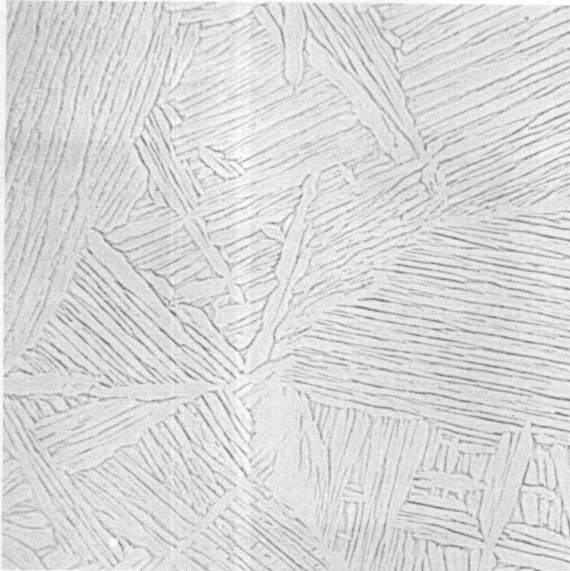


Neg. No. 10152

X 250

Fig. 49

Ti-3.0% Cu, 950°C - 1 hr. → 700°C -
2 hrs. → WQ. Ti₂Cu dispersion in
α-Ti.



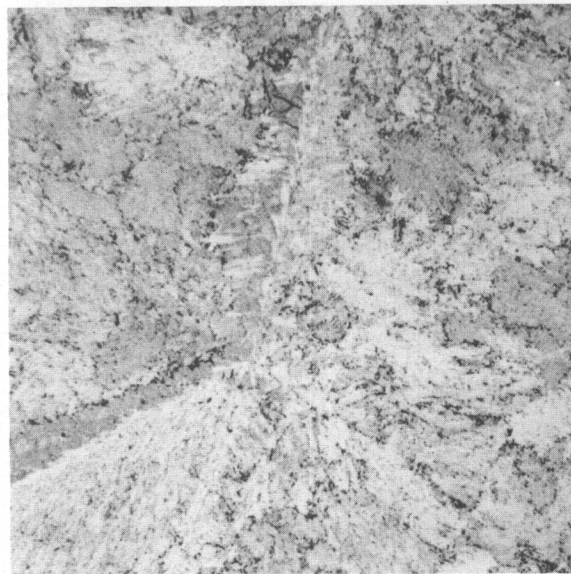
Neg. No. 10154

X 250

Fig. 50

Ti-6.4% Cu, 950°C - 1 hr. → 700°C -
2 hrs. → WQ. Coarse Ti₂Cu plates in
α-Ti.

Etchant: 60 cc glycerine, 20 HF, 20 HNO₃



Neg. No. 10155

X 250

Fig. 51

Ti-11.5% Cu, 950°C - 1 hr. → 700°C -
2 hrs. → WQ. Fine dispersion of Ti₂Cu
in α-Ti.

Etchant: 60 cc glycerine, 20 HF, 20 HNO₃

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Continuity

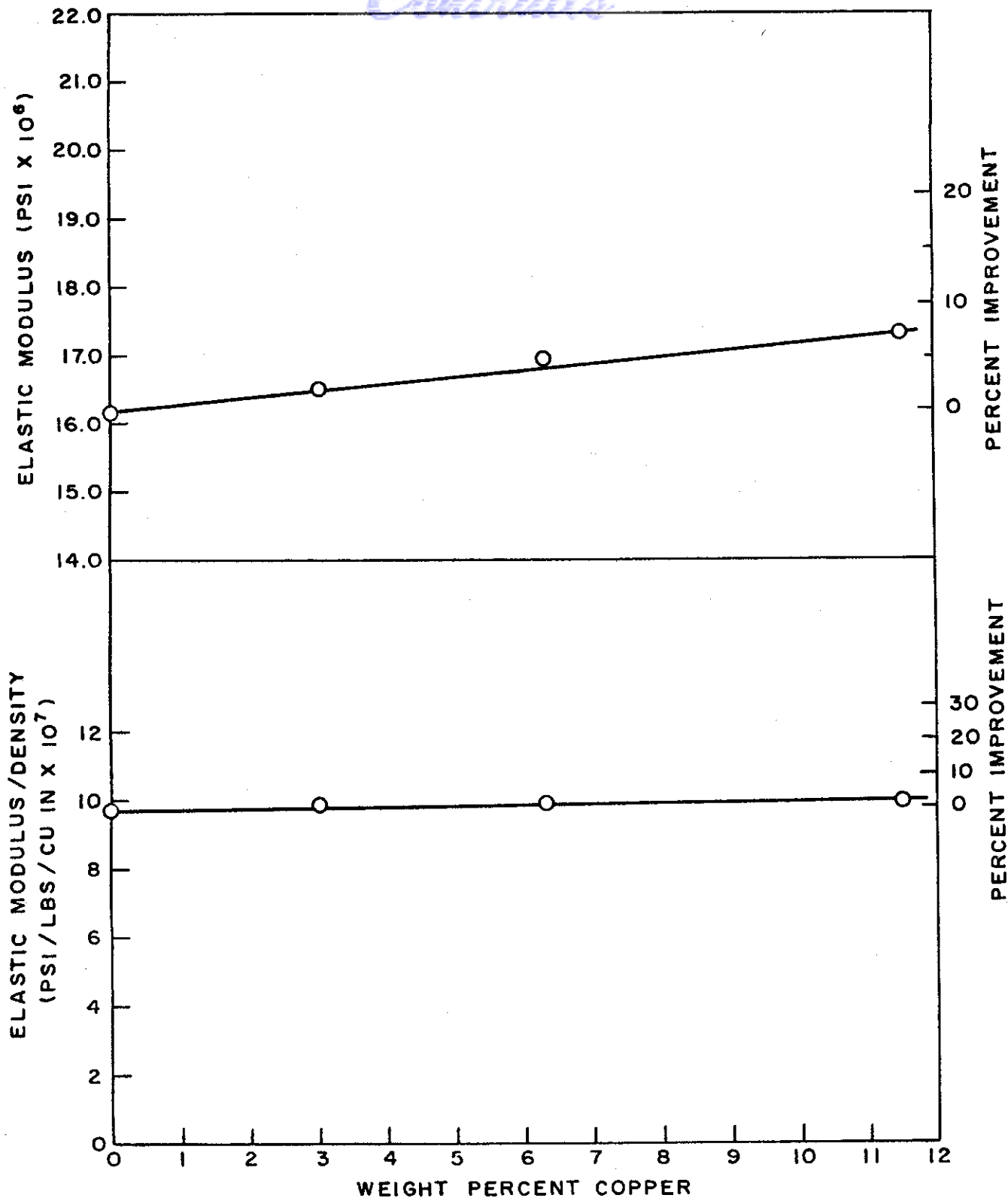


FIG. 52- VARIATION IN ELASTIC MODULUS AND RATIO OF ELASTIC MODULUS TO DENSITY WITH COPPER CONTENT FOR TITANIUM-BASE ALLOYS - ROOM TEMPERATURE. Ti₂Cu PHASE DEVELOPED BY HEAT TREATMENT

TABLE VIII

RESULTS OF ROOM TEMPERATURE ELASTIC MODULUS DETERMINATIONS
FOR TWO Ti-AL-C TERNARY ALLOYS COMPARING OBSERVED VALUES OF ELASTIC MODULUS
WITH CALCULATED VALUES BASED ON ADDITIVITY EFFECTS OF THE ALLOYING CONSTITUENTS

Alloy	Density		ΔE (psi x 10 ⁶)		Calculated E (psi x 10 ⁶)	Observed E (psi x 10 ⁶)
	g/cu cm	lbs/cu in	(from Al)	(from C)		
Ti-4.1% Al-0.78% C	4.45	0.161	1.2	2.2	19.6	19.0
Ti-5.9% Al-0.58% C	4.39	0.159	1.4	1.8	19.4	19.2

Note: Modulus of elasticity for unalloyed Ti = 16.2 x 10⁶ psi.

Contrails

The actual textures for α -Ti reported in Contract No. AF 33(038)-19574 from the University of Kentucky are as follows:

α -Ti (cold worked) = $[10\bar{1}0]$ for wire
(0002) rotated 25° from the plane of the sheet/ $[10\bar{1}0]$ in direction of rolling

α -Ti (recrystallized) = $[11\bar{2}0]$ for wire
(0002)/ $[11\bar{2}0]$ for sheet

Specimens for dynamic elastic modulus determinations on α -Ti in a state of preferred orientation were prepared from a plate of sponge-base, unalloyed titanium which had been subjected to a cold reduction of 82% and subsequently recrystallized at 750°C for 1 hour and air cooled. This treatment, according to the findings of the University of Kentucky, produced a texture in which the basal plane (0001) lay in the plane of the material, and the crystallographic direction $[11\bar{2}0]$ paralleled the rolling direction (see Figure 53). Specimens were cut from the prepared plate according to Figure 54, and modulus data were taken at room temperature. The results are summarized in Table IX. The elastic modulus along various directions in the basal plane is essentially unaffected by crystallographic direction, and is somewhat lower than the value found for randomly oriented α -Ti. This indicates that the elastic modulus in directions of the type $[h\ k\ i\ 1]$ where $l > 0$ are somewhat greater than that for the randomly oriented material, since the latter represents an average value, and directions in the base plane were below the average.

IV. SUMMARY

A. An improved device for conducting dynamic elastic modulus determinations employing electrostatic excitation has been developed. The device is capable of a high degree of precision of measurement and has been used successfully at elevated temperatures up to 750°F .

B. The effect of the interstitial elements oxygen and nitrogen on the elastic modulus of α -Ti is small, with oxygen concentrations up to 0.59 wt % and nitrogen to 0.73 wt %.

C. The substitutional element aluminum exerts a marked effect on the elastic modulus and the ratio of elastic modulus to density of α -Ti. In the range studied, up to 7.9% Al, progressive improvement in the E and E/ ρ ratio was noted. The addition of 7.9% Al increased the room temperature E from 16.2×10^6 psi to 18.7×10^6 psi, representing an increase of 15.4%. The increase in the E/ ρ ratio for the same alloy was 20.4%. Both the E and E/ ρ ratio for these alloys decreased uniformly with temperature increases up to 750°F . The rate of decrease with temperature was found to be essentially composition independent.

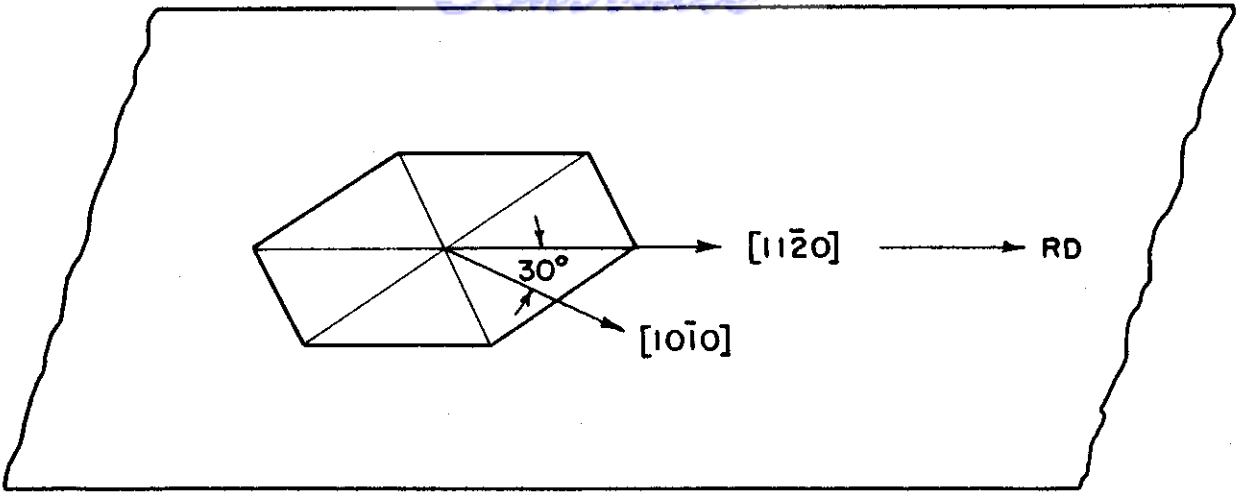


FIG. 53- PREFERRED ORIENTATION DEVELOPED IN α -Ti BY COLD REDUCTION AND RECRYSTALLIZATION.

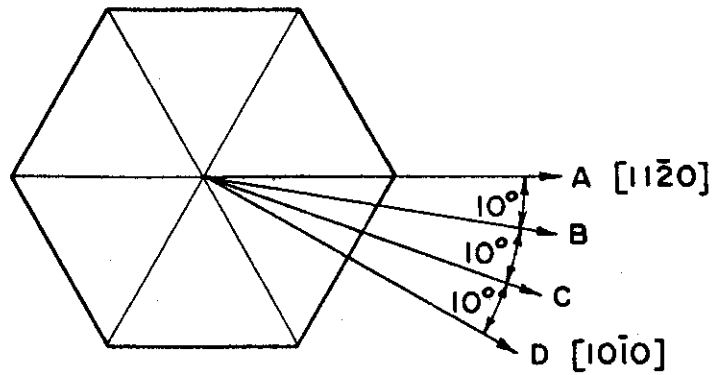


FIG. 54 - METHOD OF SPECIMEN SELECTION FOR ELASTIC MODULUS DETERMINATIONS IN α -Ti HAVING PREFERRED ORIENTATION.

ELASTIC MODULUS DATA FOR SPECIMENS CUT
AT VARIOUS CRYSTALLOGRAPHIC DIRECTIONS
IN THE BASAL PLANE IN TEXTURED α -Ti. (ROOM TEMPERATURE)

Specimen*	Density		Crystallographic Direction	Elastic Modulus (psi x 10 ⁶)
	g/cu cm	lbs/cu in		
A	4.56	0.165	[11 $\bar{2}$ 0]	15.2
B	4.55	0.164		15.2
C	4.55	0.164		15.3
D	4.56	0.165	[10 $\bar{1}$ 0]	15.3
Untextured	4.59	0.166	-	16.2

* Refer to Figure 19 for meaning of specimen designation.

D. The elastic modulus and E/ρ ratio of a water quenched Ti-V binary alloy containing 10.6% V were found to be extremely low in comparison to unalloyed titanium. The microstructure of this alloy showed an all α' structure, and it may be inferred that the low value of modulus seen for this alloy is characteristic of the martensitic modification of titanium.

E. The E and E/ρ ratio of all-beta binary Ti-V alloys were maximum near the beta retention composition (15.3%) and fell from this maximum value of 17.6×10^6 psi for E and 10.2×10^7 psi/lbs/cu in. for the E/ρ ratio to 13.2×10^6 psi for the former and 7.38×10^7 psi/lbs/cu in. for the latter as the vanadium content was raised to 31.3%.

The elastic moduli of all-beta 13% Mo and 8% Cr alloys support the statement that the beta modification is intrinsically of low elastic modulus. Low temperature aging of the 8% Cr alloy showed that the transition phase has an unusually high elastic modulus.

F. Varying the ratio of the amount of alpha to beta phase in a series of Ti-V binary alloys of nominally 10%, 20% and 30% V was found to have a negligible effect on the E and E/ρ ratio of the 30% alloy and a somewhat more pronounced effect on the two lower vanadium alloys. The indications are that such changes as occurred reflect the adjustment of the relative proportions of alpha and beta.

G. The presence of the intermediate phase TiC was found to exert a marked beneficial effect on the E and E/ρ ratio of titanium. At room temperature, addition of carbon in the amount of 2.69% increased the E to 20.4×10^6 psi. This amounts to an increase of nearly 26% of the value for unalloyed titanium. The E/ρ ratio for the same alloy increased 27.6%. The effect of elevated temperature up to 750°F was to cause a uniform decrease in the E and E/ρ ratio as the temperature was increased. As in the case for Ti-Al alloys, the rate of decrease with temperature was essentially composition independent.

H. Boron added to titanium in excess of the solid solubility limit enhances elastic properties by development of the intermediate phase TiB. Alloys containing respectively 0.48%, 0.96% and 1.36% B exhibited progressively higher elastic moduli and E/ρ ratios. The highest alloy, under room temperature testing, exhibited an E that was nearly 25% greater than that for unalloyed titanium, while the E/ρ ratio for this alloy was 26.8% greater. As with other alloys studied, Ti-B binary alloys showed a linear decrease in E and E/ρ as the temperature was increased. Temperatures up to 750°F were studied, and the rate of decrease was again found to be composition independent.

I. The improvement in E and E/ρ ratio in titanium alloys containing either of the dispersed phases Ti_5Si_3 or TiBe was very small. Binary alloys of Ti-Si and Ti-Be containing, respectively, Si up to 1.55% and Be up to 1.41% showed improvements in elastic properties of less than 10%.

J. The presence of Ti_2Cu as a eutectoid dispersion did not result in any marked increase in the value of E for titanium. Binary Ti-Cu alloys

Conclusions

containing up to 11.5% Cu were isothermally treated at 700°C for 2 hours to develop the eutectoid structure. The E/ ρ ratio was essentially constant over the range 0 to 11.5% Cu.

K. The rate of change of elastic modulus and ratio of elastic modulus to density with temperature was found to be nearly uniform in all the experimental alloys studied except for the Ti-Al binary alloys. These alloys exhibited rates that were significantly lower than the others.

L. The elastic modulus of α -Ti in various crystallographic directions in the basal plane (0001) is nearly constant and averages about 15.2×10^6 psi.

V. CONCLUDING DISCUSSION

The work described in this report was designed to reveal the basic principles upon which an alloy development program for the development of alloys of improved elastic modulus to density may be based. The results have clearly established that such an improved alloy must be predominantly alpha phase; that the alpha phase alloyed with aluminum shows markedly increased elastic modulus. As an alternative to the use of aluminum as a solid solution addition, the modulus to density of unalloyed titanium may be increased by addition of carbon or boron occurring as dispersions of carbides or borides. The concurrent addition of aluminum and a compound former exerts approximately additive benefits. Since aluminum in large amounts increases the stiffness of titanium, its use must be limited where formable sheet is required. In this instance, combination of aluminum and carbon or aluminum and boron could be used to achieve the desired degree of improvement with concurrently low flow stresses. Dispersions of intermetallic compounds have something of a bad reputation as regards ductility. It must be remarked that no difficulty was encountered in forging any of the Ti-C, Ti-B and Ti-Al-C alloys. Furthermore, in other work, carbon in amounts up to 1% was shown not to decrease tensile ductilities below tolerable limits.

Now all of the alloys discussed above are not heat treatable. In order to develop heat treatability, beta stabilizing elements must be added. The problem of design of high modulus, heat treatable titanium alloys is one of alloying to produce a minimum amount of beta consistent with the necessary maximum-minimum strength requirements. Such alloys will, in all probability, contain 4-8% Al and 1-4% of V, Mo, Cr, Mn, Fe or combinations of these. The negative effect of the beta phase constituent can be counterbalanced by the addition of carbon or boron in appropriate amounts. The additions of Sn and/or Zr as substitutes for aluminum warrant investigation.

In general, it appears to be entirely feasible to design alloys with satisfactory forming characteristics, heat treatability and a 20% increase in the elastic modulus to density ratio.

* WADC-TR-54-244, Structural Changes of Commercial Titanium and Titanium-Base Alloys on Heat Treatment.

MATHEMATICAL DERIVATION OF CORRECTION FACTORS TO YIELD ELEVATED TEMPERATURE DYNAMIC ELASTIC MODULUS VALUES FROM OBSERVATIONS OF DENSITY AND LENGTH AT ROOM TEMPERATURE AND RESONANT FREQUENCY AT ELEVATED TEMPERATURES

The physical variables involved in the determination of dynamic elastic modulus values employing longitudinal vibrations are related at resonance by the expression

$$E = \frac{4}{n^2} \rho L^2 f_n^2$$

where: E = elastic modulus, dynes per sq. cm
 n = a small integer indicating the order of resonance
 ρ = density, g per cu cm
 L = length, cm
 f_n = resonant frequency in cps at order of resonance n

For a specimen held in the center, $n = 1$ and the bar length is equal to half of the resonant frequency wave length. The above expression then becomes

$$E = \rho L^2 f_r^2$$

where f_r = resonant frequency at $n = 1$

If E/ρ values are desired, the rearranged form of the expression can be used

$$\frac{E}{\rho} = 4L^2 f_r^2$$

The above two equations require that all variables be measured at the same temperature for a given determination. Since elevated temperature studies of elastic modulus data are required, it would be desirable to eliminate the necessity of elevated temperature density and length measurements, if possible, to simplify the procedure. This can be done by developing correction factors which allow room temperature density and length measurements to be used in the calculation of elevated temperature data. The mathematical derivation of these correction factors is shown below:

Calculated Correction Factor for Elastic Modulus Measurements

- i) Let subscript o denote the temperature at which density and length are measured.
- ii) Let subscript t denote the elevated temperature at which resonant frequency is determined.

Corrections

iii) Let $T = t - 0$

Then $E_0 = 4\rho_0 L_0^2 f_0^2$ and $\rho_0 = \frac{M}{V_0}$ by definition.

and $E_t = 4\rho_t L_t^2 f_t^2$ and $\rho_t = \frac{M}{V_t}$ by definition.

and $V_t = V_0 (1 + 3\alpha T)$ where α = coefficient of linear expansion.

$$\therefore \rho_t = \frac{M}{V_0 (1 + 3\alpha T)} = \frac{M}{V_0} \cdot \frac{1}{1 + 3\alpha T} = \frac{\rho_0}{1 + 3\alpha T}$$

also $L_t = L_0 (1 + \alpha T)$, so $L_t^2 = L_0^2 (1 + 2\alpha T)$ (neglecting the third term $\alpha^2 T^2$ which approaches zero).

$$\therefore E_t = \frac{4\rho_t}{1 + 3\alpha T} \cdot L_0^2 (1 + 2\alpha T) f_t^2$$

or
$$E_t = 4\rho_0 L_0^2 f_t^2 \cdot \frac{1 + 2\alpha T}{1 + 3\alpha T}$$

The correction factor to be used, then, at any elevated temperature, t , is

$$\frac{1 + 2\alpha T}{1 + 3\alpha T}$$

Calculated Correction Factor for E/ρ Measurements at Elevated Temperature

i) Let subscript 0 denote the temperature at which density and length are measured.

ii) Let subscript t denote the elevated temperature at which resonant and frequency is determined.

iii) Let $T = t - 0$

Then $E_t = 4\rho_t L_t^2 f_t^2$

and $L_t = L_0 (1 + \alpha T)$ where α = coefficient of linear expansion.

$$L_t^2 = L_0^2 (1 + 2\alpha T) \text{ again neglecting third term } \alpha^2 T^2.$$

$$\therefore E_t = 4\rho_t L_0^2 (1 + 2\alpha T) f_t^2$$

or
$$\frac{E_t}{\rho_t} = 4L_0^2 f_t^2 (1 + 2\alpha T)$$

Contrails

The correction factor, thus, at any elevated temperature, t , is $(1 + 2\alpha T)$.

A plot of the two correction factors versus temperature is shown in Figure 55, using the value 9×10^{-6} cm/cm as an average value for α in the range 0 - 500°C, and a reference room temperature of 25°C.

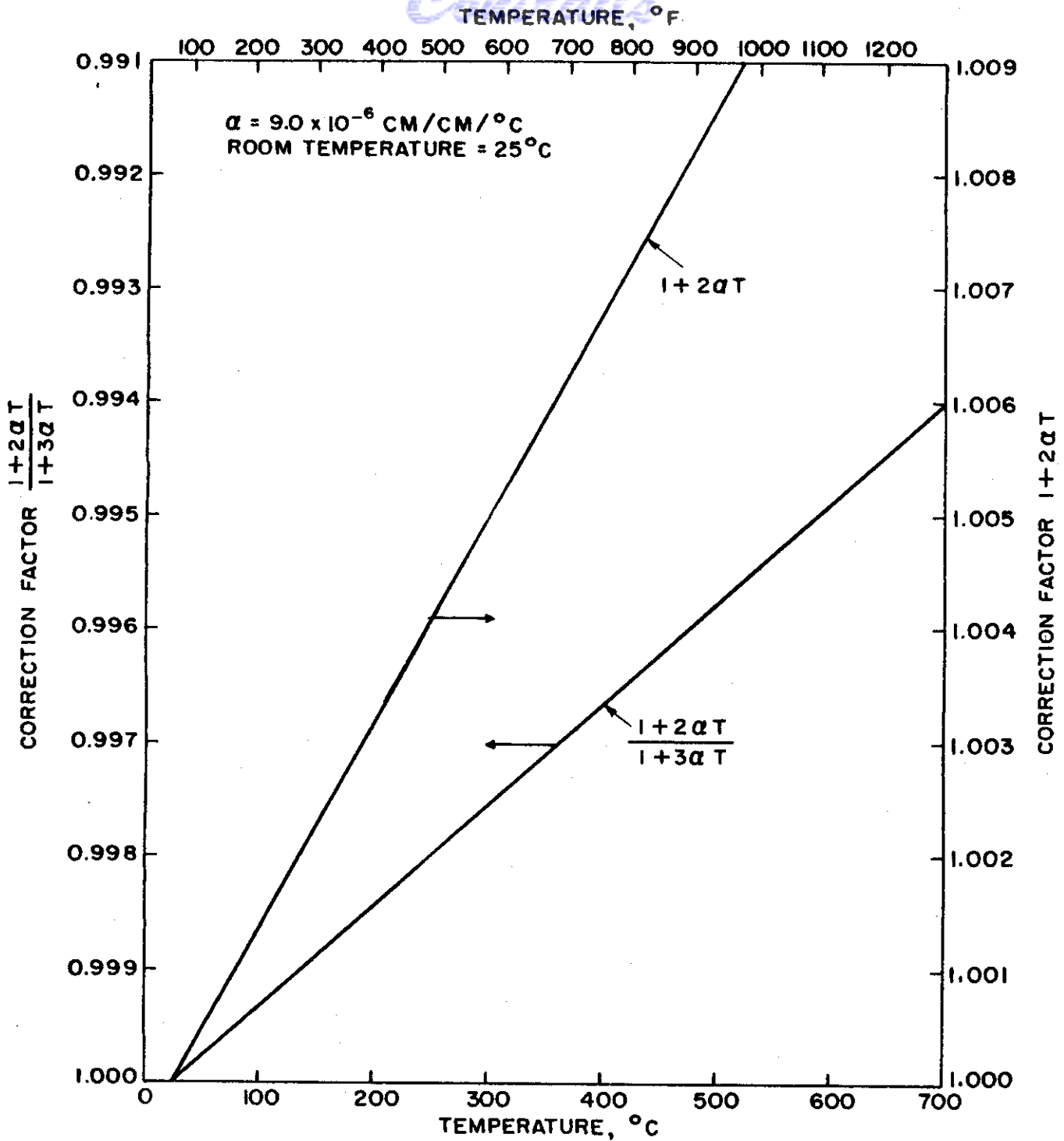


FIG. 55 - CALCULATED CORRECTION FACTORS FOR DERIVING ELEVATED TEMPERATURE ELASTIC MODULUS DATA FROM OBSERVATIONS OF DENSITY AND LENGTH AT ROOM TEMPERATURE AND RESONANT FREQUENCY AT ELEVATED TEMPERATURE.

Contrails