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DYNAMIC BEHAVIOR OF METALS UNDER TENSILE IMPACT

PART II. ANNEALED AND COLD WORKED MATERIALS

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ERRATA

Dynamic Behavior of Metals Under Tensile Impact

Part II: Annealed and Cold Worked Metals
(AFML-TR-69-76)

A numerical error in the computations made to analyze the behavior of OFHC copper caused incorrect results for that material to be reported. The following changes should be made:

Page iii, 3rd sentence: Change "In five of the ten series..." to "In four of the ten series..."

Page 1, 3rd paragraph, 2nd sentence: Change "In five of the ten series..." to "In four of the ten series..."

Page 10, 4th paragraph, 2nd sentence: Change "... negative rate sensitivity" to "... positive rate sensitivity"

Page 34, Figure 18: The curves labelled "Static Prediction," "From V-ε," and "From V-ψ" are not correct.

Page 35, Figure 19: The dynamic stress-strain curve should be above the static, reaching an ultimate stress of 45,900 psi at a strain of 0.27.

Page 39, Table I, OFHC Copper entries: Change the last two entries from 34.3, .95 to 45.9, 1.26.

Both the static stress-strain curve and the experimental observations are correctly reported.

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FOREWORD

This report was prepared by the Department of Materials Engineering, University of Illinois, Chicago, Illinois, under USAF Contract No. F33615-67-C1283. The contract was initiated under Project No. 7351, "Metallic Materials", Task No. 735106, "Behavior of Metals". The work was administered by the Metals and Ceramics Division, Air Force Materials Laboratory, Directorate of Laboratories, Air Force System Command, with Dr. T. Nicholas, MAMD, project scientist.

This report covers work conducted from 1 March 1967 to 15 March 1969. The manuscript of this report was released by the author 15 April 1969 for publication as an AFML Technical Report.

This technical report has been reviewed and is approved.

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ABSTRACT

The mechanical behavior of metals subjected to uniaxial tensile impact at room temperature is reported. Tests were conducted on 1100 aluminum subjected to light and to heavy cold working; 2024 aluminum after annealing and after light and heavy cold working; C1010 steel in these same three conditions; and OFHC copper and 70-30 brass in an annealed state. In five of the ten series the materials were found to posses dynamic stress-strain curves which fell below the static curves. The yield point of the steel was lowered by impact loading. Ratios of dynamic to static ultimate stress were found to range from 0.59 to 1.3.

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SECTION I

INTRODUCTION

This paper presents the results obtained in an experimental study of the room temperature mechanical behavior of five metals subjected to impact loading, and constitutes the second part of a two-part report. Part I [1] described elevated temperature behavior, and both parts represent a continuation of work reported earlier [2,3] in which dynamic behavior is determined by observation of a succession of constant velocity transverse impacts on the center of long thin wire specimens of the materials to be investigated.

The goal of the studies has been to determine the behavior of materials subjected to large uniaxial tensile strains under impact conditions, taking wave propagation phenomena into account. Dynamic behavior may be inferred from observation of how maximum strain in the wire varies with impact velocity or how the deformation angle behind the transverse wave front varies with impact velocity.

Ten sets of results are reported: 1100 aluminum subjected to light and to heavy cold working; 2024 aluminum after annealing, and after light and heavy cold working; C1010 steel in these same three conditions; and OFHC copper and 70-30 brass in an annealed state. In five of these ten series the materials were found to possess dynamic stress-strain curves which fell below the static curves. This negative strain rate sensitivity was also exhibited in several of the series reported on in Part I. In addition, it was found that the yield point of the mild steel is lowered, rather than raised, by impact loading.

The experimental technique used, the equations governing the behavior of the wires upon impact, and the solutions to those equations which are used to relate observations to material behavior were presented in References [2] and [3] and reviewed in Part I. Static tests were performed using an Instron testing machine. The following nomenclature is used:

- σ engineering stress
- ε engineering strain
- ρ mass density
- V impact velocity
- u maximum longitudinal particle velocity
- ψ angle of deformation behind transverse wave front
- c longitudinal wavelet speed
- $\bar{c}$ transverse wave speed

Subsequent sections of the paper present the experimental observations for each of the materials and the behavior which is inferred. The depression of the yield point of mild steel by impact loading is discussed following the presentation of those experimental results. The final section summarizes the behaviors found, and an appendix outlines the modifications required in the calculation procedures to account for the existence of longitudinal shock waves in some of the tests.

SECTION II

RESULTS: 1100 ALUMINUM

The material used in the tests was from the same lot as that used earlier [1,3]. Wire diameter was 0.02 in., obtained by having heavier stock wire commercially drawn to this size. Behavior in the annealed condition was reported in [3], and is shown in Figure 2 for comparison. For the prestressed series, wires were heated to 800°F for 3.5 minutes to anneal, and then subjected to a dead weight stress of 11,100 psi until stretching of the wire ceased, which occurred within 3 minutes of load application. The load was removed prior to impact testing.

<u>Prestressed (Figures 1 and 2)</u> - The experimental observations are shown in Figure 1, along with predictions of behavior based on static stress-strain data. Figure 2 shows static and dynamic stress-strain curves. Static behavior was reproducible. There is some scatter in the observations, but when dynamic behavior was inferred from the velocity-strain observations, the results were consistent with velocity-angle observations.

<u>As Received (Figures 3 and 4)</u> - Static properties exhibited the variability shown, but this had a minor effect on the predicted behavior. Observed strains were too small to measure by the optical technique used, but the dynamic behavior inferred from velocity-angle observations does not indicate observed strains should have been any greater.

Table I contains a summary of the observed behavior of 1100 aluminum. The small amount of cold working of the wires used in the prestressed tests affects the dynamic behavior only slightly. Impact loading of the prestressed wires produces the same changes from static behavior that it does for the annealed wires. The heavy cold working of the wires used in the as-received series, on the other hand, leads to reduced ductility in impact tests, and to a dynamic stress-strain curve which lies below the static. The dynamic ultimate stress is three-fourths that obtained in slow tests, but some of this reduction is the result of the reduced ductility.

Reference [3] contains a summary of the results obtained by other investigators concerning the dynamic behavior of annealed commercially pure aluminum, using several different techniques. The results obtained for this material using the present technique were in agreement. There are only a few reports concerning the effect of cold working on dynamic behavior. Karnes and Ripperger [4] reported a decrease in strain rate sensitivity of high purity aluminum when cold worked up to fifty percent in compression. They found that increasing the strain rate from 10^{-4} to 10^{3} per second caused the stress at two percent strain to increase by a factor of 2.3 in the annealed state and 1.3 after the fifty percent reduction. Wires used in the as-received series were of commercial purity, and were more heavily cold worked than the specimens used by Karnes and Ripperger.

SECTION III

2024 ALUMINUM

The wires used were commercially drawn to 0.02 in. diameter from heavier stock wire. Prior to any tests in the annealed or prestressed series, wires were heated to 600°F and held for 3.5 minutes to anneal. Wires used in the prestressed series were then subjected to a dead weight stress of 25,000 psi until stretching ceased, after which the load was removed.

<u>Annealed (Figures 5 and 6)</u> - Static properties were reproducible. Dynamic behavior was inferred from velocity-strain observations, giving results consistent with velocity-angle observations except in a range of impact velocities near 5000 in. per sec., where angle observations exhibited scatter.

<u>Prestressed (Figures 7 and 8)</u> - Static properties were reproducible, and both velocity-strain and velocity-angle observations were consistent with predictions made from the static stress-strain curve. Three specimens failed on impact at velocities near 4500 in. per sec., but this aspect of behavior was not reproducible.

<u>As Received (Figures 9 and 10)</u> - Dynamic behavior was inferred from velocity-angle observations. Neither the static nor the dynamic stress-strain curve indicated that strains would be large enough to observe, and no measurable strains were observed.

The static and dynamic behavior of 2024 aluminum as obtained from the three series of tests reported is also summarized in Table I. In the prestressed series the only difference between static and dynamic behavior is that the ductility is reduced by impact loading. Otherwise, the material appears rate-insensitive in this series. In the annealed series, for strains larger than one percent, the dynamic stress-strain curve falls under the static, but the dynamic ultimate stress is only slightly reduced. In the as-received series, the dynamic stress-strain curve again falls below the static, with the dynamic ultimate stress being 0.59 times the static. However, some of this reduction is due to the reduced ductility. The stress at a strain of 0.065 is reduced by a factor of 0.7 by impact loading.

Clark and Wood [5] found in impact tests in which wave propagation was not fully accounted for, the ultimate stress of annealed 24S aluminum was raised 33 percent and the ultimate elongation increased by 48 percent by impact loading. In the 24ST state, they found these increases to be 5 and 19 percent, respectively. More recent work on the dynamic behavior of annealed aluminum alloys has indicated that these alloys tend to be rate-insensitive at room temperature. For example, both Lindholm, et al [6] and Green and Babcock [7] report very little rate sensitivity in either tension or compression in 7075-T6 and 6061-T6 aluminum. Holt, et al [8] report the stress of annealed 2024 aluminum at a strain of 0.06 is raised 10 percent by an increase in strain rate from 10^{-3} to 10^{3} per second. These three investigations involved testing under nominally constant strain rate conditions. Negative rate sensitivity for this material at 200°F is reported in Part I [1], and in that case evidence from other investigations confirming this behavior was available.

SECTION IV

C1010 STEEL

The wires used in the annealed and as received series were commercially drawn to 0.02 in. diameter from heavier stock. Prior to testing, wires for the annealed series were heated to 1400°F and held for 15 sec. Longer annealing times did not change the properties of the wire. Tests were made to see that scale formation during annealing had negligible effect on static behavior or the weight per unit length. Scale was removed from the central section of the wires to permit strain measurements to be made by the optical procedure described, and tests insured that scale removal did not affect static behavior. Wires used in the prestressed series were 0.051 in. in diameter, commercially annealed after drawing, and tested as received. Prior to this series of tests, deadweight load was applied until the wires strained uniformly to 0.05, after which the load was removed.

Annealed (Figures 11 and 12) - The unusual shape of the curves of behavior predicted from the static properties, shown in Figure 11, results from longitudinal shock wave propagation. Also shown in that figure is a prediction of velocity-angle data for linear elastic behavior up to a stress of 110,000 psi. Velocity-strain observations exhibit scatter at larger velocities, but angle observations fall on a smooth curve. Three inferences of dynamic behavior are shown; two from velocity-strain observations, marked (A) and (B), and one from velocity-angle observations. The properties inferred from curve (B) are reported in the Table 1 summary, and they are moderately consistent with those inferred from velocity-angle observations. Velocity-angle observations indicate yielding begins dynamically at a stress of 12,000 psi, well below the static yield stress of 50,000 psi. However, strains beyond the dynamic yield point are small, and the differences between static and dynamic behavior in this region can be seen only by magnifying the strain scale. Beyond yield, the tangent modulus is of the order of 10^6 psi.

Prestressed (Figures 13 and 14) - Dynamic behavior was inferred from velocity-angle observations, as strains were too small to measure accurately. The angle observations suggest that yielding occurs at a stress below the static yield stress, but the indications of this are less pronounced here than in the annealed series.

As Received (Figures 15 and 16) - Strains were again too small to measure accurately, and are not shown in Figure 15. The velocity-angle observations infer a dynamic yield stress of approximately 15,000 psi compared to the static yield stress of 80,000 psi. Strains beyond yield are small, and the entire dynamic stress-strain curve lies below the static.

A summary of the behavior of C1010 steel is contained in Table I. Dynamic ultimate strains in all three series are much smaller than those obtained in static tests. The ratio of dynamic to static ultimate stress varies from 0.77 to 1.3. The tests indicate that in two of the series the dynamic yield stress may be less than one-fourth that obtained statically.

Many investigations of the dynamic ultimate stress of annealed mild steel have been conducted. For example, Nadai and Manjoine [9] report an increase in the dynamic ultimate stress over that obtained statically of 44 percent, Clark and Wood [5] 25 percent, Clark and Duwez [10] 45 percent. Suzuki, et al [11] report an increase of 15 percent in the compressive stress at a strain of 0.1. Low values of the dynamic ultimate stress found in the present tests probably result from the brittle behavior exhibited. The conclusion that the yield stress of mild steel is significantly lowered by impact loading, however, deserves careful examination.

The dynamic yield stress of mild steel has been examined by many investigators, and almost without exception, it has been reported that the yield stress is raised by dynamic loading. For example, increases in the yield stress of 85 percent were found by Hopkinson [12] and by Campbell [13], 250 percent by Campbell and Duby [14], and 100 percent by Marsh and Campbell [15].

In the present tests, velocity-angle observations lead to the conclusion that the yield stress is depressed by impact loading. This is clearly indicated in the annealed and as-received test series, and the small amount of data available in the prestressed series show the same trend, although to a lesser degree. The observations infer that strains beyond yield are small until the stress rises to or above the static yield stress. Nevertheless, nonlinear behavior is exhibited at stresses well below the static yield stress. In the annealed series, the departure from linear elastic behavior cannot be seen unless the strain scale is magnified.

The depression of the yield point is indicated by the velocity-angle observations falling above the curve predicted from linear elastic behavior. This does not occur in ten of the twenty-one series at all, lessening the probability that it is caused by a systematic error in the experimental procedures.

Consider the velocity-angle relationship predicted from static linear elastic behavior. The relationship satisfies the equation

$$\tan \psi = - \frac{V}{c_o} \frac{1}{[\varepsilon(1+\varepsilon)]^{\frac{1}{2}} - (\varepsilon-\varepsilon_o)} \qquad (1)$$

where $\rho c_o^2 = E$, Young's modulus. ε_o is the initial strain in the wire caused by the small load necessary to hold the wires in place. The relationship, with parameter values appropriate to the annealed test series, is shown by the dashed curve in Figure 17. Observations could fall above this curve if the dynamic modulus were lower than the static, but this does not constitute a satisfactory explanation for the departure. In the annealed series, the dynamic modulus would have to be 24.5 x 10^6 psi to account on such a basis for observations at $V = 2000$ inches per second. This would imply, for example, $\psi = -9.4°$ at $V = 500$ inches per second, which is not consistent with observations.

Observations will depart from the curve given by Equation (1) when the strain reaches its yield value, ε_y. Suppose that for a range of strains larger than yield the tangent modulus is low enough for the transverse wave speed to exceed any longitudinal wave speeds. Then the yield point is also the transition point for wave speed ordering (see p. 344 of [3]), and

$$c^* = \bar{c}(\varepsilon_y)$$

$$u^* = u(\varepsilon_y)$$

As long as the tangent modulus does not exceed $E^* = \rho c^{*2}$, the velocity-angle relationship beyond yield is given by

$$\tan \psi = - \frac{V}{u^* + (1+\varepsilon_y)c^*} \tag{2}$$

and this relationship depends only on the value of the yield strain. Not until the tangent modulus exceeds E^* is it possible for velocity-angle observations to depart from this constraint. The solid lines of Figure 17 are curves of Equation (2) for various values of ε_y, with corresponding values of E^* indicated. It can be seen that under these circumstances the observations would be a sensitive indicator of ε_y. A doubling of the yield stress would certainly be detected in the experiments. A comparison of present experimental observations with this Figure rules out the possibility that the yield stress is raised by impact loading to the extent reported by others.

The range of V and ψ over which Equation (2) is obeyed depends on the stress-strain relation beyond yield, and may be very small. Observations in these tests indicate that the tangent modulus beyond yield does not decrease enough to change the wave speed ordering at all. This makes the interpretation of the observations more difficult, as their departure from linear elastic predictions is less clear-cut. The figures for the yield stress quoted are reasonable interpretations of the results.

In summary, the velocity-angle observations definitely show that the proportional limit of the mild steel wires used in these tests is not raised by dynamic loading. The observations indicate instead that the limit is depressed, but that the dynamic stress-strain curve beyond yield still has a large slope.

Several possible reasons for the conflicts between these test results and those from other investigations will be discussed. In the analysis of these test results, wave propagation phenomena are accounted for. In many of the other investigations of the dynamic yield stress, results are reported based on the assumption that test specimens are in a state of homogeneous deformation. Wave propagation within the specimen itself is ignored. For that assumption to be justified requires that wave propagation speeds be high enough for waves to reflect from specimen boundaries and traverse the specimen many times during the test. For many materials this is reasonable. However, if the stress-strain curve of mild steel possesses the low values of the tangent modulus at strains in excess of yield often reported, the assumption may become unreasonable. For example, in a test in which strain rate is assumed constant, to strain a specimen to 0.1 at a rate of 10^3 per second requires a test time of 0.1 ms. If the tangent modulus beyond yield were of the order of 1000 psi, about 0.9 ms. would be required for a wave to traverse a one-inch length. Wave propagation phenomena need to be accounted for under these circumstances.

Another possible explanation is that strains in mild steel tend to propagate in the form of longitudinal shock waves if the stress-strain curve approaches that of an ideal bilinear material for small strains. A section of the specimen will experience first a jump in strain from its initial value to the yield point, and later a second jump from the yield strain to a larger plastic strain. The strain rate at a given section will be very high at these two instants and very low at other times. Lateral inertia may therefore play an important role in tests to determine the yield point of mild steel specimens, and failure to achieve a one dimensional state of stress may lead to erroneous conclusions. Neither the present tests nor those of the other investigations cited incorporate this role in the analysis of test results. However, the present tests attempt to overcome this difficulty with the use of thin wire specimens.

In these tests, the velocity-angle measurements provided a sensitive measure of the tangent modulus. Any test which does not provide this would fail to detect the details of yield behavior reported here.

Finally, strain rate history may significantly influence stress-strain relations. The effect of strain rate history on yield stress has been demonstrated in non-ferrous metals by, for example, Lindholm [16], and Klepaczko [17]. Representative studies of strain rate history alteration of the yield stress in mild steel are those of Campbell and Duby [14], Warnock and Pope [18], Campbell and Maiden [19], and Smith [20]. In all of these investigations, it was found that static stress-strain curves obtained after imposing small deformations dynamically fell below the curves for specimens strained only statically. The investigations of the dynamic yield stress of mild steel cited (References [12-15]) were conducted at nominally constant strain rates, with wave propagation in the specimens ignored. In the present tests, wave propagation is accounted for. However, wave propagation causes strain rate variation with both position along the wire and time.

SECTION V

COPPER AND BRASS

Wires of OFHC (CABRA 102) copper were commercially annealed after drawing to 0.04 in. diameter and tested as received. The 70-30 (CABRA 260) brass wires were commercially annealed after drawing to 0.025 in. diameter and tested as received also.

<u>OFHC Copper (Figures 18 and 19)</u> - Considerable scatter appears in the strain observations at larger velocities, but not in the angle observations. However, behavior inferred from angle observations is not consistent at larger velocities with strain observations, despite the scatter. Therefore, both inferences are shown in Figure 18, and both indicate the same qualitative dynamic behavior. Velocity-strain inferences are used in the Table I summary of behavior.

<u>70-30 Brass (Figures 20 and 21)</u> - The observations of behavior agree very well with the predictions based on static stress-strain behavior, except at velocities near critical. At these velocities, dynamic behavior was inferred from the velocity-strain observations, leading to results consistent with velocity-angle observations.

The Table I summary and Figures 19 and 21 show that the brass wires exhibit only a small amount of rate sensitivity at large strains. The copper is found to exhibit negative rate sensitivity. Nadai and Manjoine [9] report a 25 percent increase in the ultimate stress of pure copper under dynamic loading, and Clark and Wood [5] 23 and 38 percent for two different coppers. Suzuki, et al [11] report a 10 percent increase in the stress at a strain of 0.35 for copper and a 15 percent increase for 70-30 brass. Alder and Phillips [16] report negative rate sensitivity in copper at small strains, changing to positive rate sensitivity at larger strains, in agreement with present results.

SECTION VI

DISCUSSION

In the results reported in both Part I and Part II information on the dynamic behavior of the materials tested is derived from measurements by an analysis which accounts for wave propagation in the wires. There are three assumptions made in the analysis: bending effects are negligible; the state of stress is one-dimensional; and the behavior of the materials may be described by a single dynamic stress-strain relation applicable over the range of strain rates encountered in the tests. The strain rate experienced by a section of the wire varies with both position and time, and depends itself upon the dynamic behavior of the material tested.

In Part I, metals subjected to tensile impact loading at elevated temperatures were shown to exhibit a wide range of behavior. Both large positive rate sensitivity and negative rate sensitivity were found. Part II results show both types of behavior exist at room temperature also, except that positive rate sensitivity is less pronounced. Cases of rate insensitivity were also found.

Summarizing, the Part II results indicate that the critical velocity for transverse tensile impact differs from the value predicted from static behavior by factors ranging from 0.58 to 1.4. For critical longitudinal velocity, the factors range from 0.34 to 1.8. Ultimate strains under dynamic loading differ from those found statically by factors ranging from 0.03 to 1.1. The range for ultimate stresses is 0.59 to 1.3. Light cold working, in the form of a modest tensile prestrain, caused a small increase in the rate sensitivity of all three materials so tested (1100 aluminum, 2024 aluminum, and C1010 steel). The heavy cold work produced in wire drawing caused substantial negative rate sensitivity in all three materials.

Two aspects of the Part I and Part II results may be considered unusual. The first is the finding that the yield stress of mild steel is lowered by tensile impact loading. This contradicts the conclusions drawn from many other investigations of the yield behavior of mild steel. Possible explanations for the contradiction were presented in Section IV. The second is the finding of negative rate sensitivity under several circumstances. In most of those cases results of other investigations were available to confirm the finding or to indicate a trend in that direction.

Had there been a single trend in the present results indicating unusual behavior, there might be reason to dismiss the validity of the conclusions drawn. However, the experiments reveal dynamic behavior of many different types, and in the great majority of cases the conclusions drawn from the experiments are in agreement with conclusions drawn from other investigations.

APPENDIX

MODIFICATION TO DATA ANALYSIS SCHEME TO ACCOUNT FOR SHOCK WAVE PROPAGATION

References [1-3] presented the equations which govern the impact response of the wires and the solution to those equations which enables the dynamic stress-strain behavior to be determined from experimental observations. The analyses presented introduced the restriction that the stress-strain curve be concave towards the strain axis;

$$\frac{d^2\sigma}{d\varepsilon^2} \geq 0 \tag{A1}$$

An inspection of the stress-strain curves of the materials dealt with here and in Part I reveals a number of cases in which this restriction is violated. White and Griffis [21] explained how concavity towards the stress axis leads to the formation of plastic shock waves in the material. The passage of a shock wave in the wire is marked by discontinuous jumps in the values of stress, strain, and longitudinal particle velocity. This appendix describes the modifications of the analysis necessary to account for the situation.

<u>Case I: Given $\sigma(\varepsilon)$ Find V, ψ, u</u>

Figure 22 illustrates a stress-strain curve typical of materials in which one-dimensional plastic shock waves propagate. σ_B and ε_B are known, and the corresponding V_B, ψ_B, and the maximum longitudinal particle particle velocity, u_B are to be found. All strains from ε_A to ε_B propagate in a shock wave at speed

$$c_s = \left[\frac{1}{\rho}\left(\frac{\sigma_B - \sigma_A}{\varepsilon_B - \varepsilon_A}\right)\right]^{\frac{1}{2}} \tag{A2}$$

Point A is the point for which the line AB is tangent to the stress strain curve,

$$\frac{\sigma_B - \sigma_A}{\varepsilon_B - \varepsilon_A} = \left(\frac{d\sigma}{d\varepsilon}\right)_A \tag{A3}$$

and therefore may be determined for any given $\sigma(\varepsilon)$ and ε_B.

Once ε_A , σ_A and c_s have been determined, the only modification required in the analysis is to recognize the finite jump in σ, ε, and u during the passage of the shock. For example, in the case all $c > \bar{c}$, the desired quantities are calculated for the equations

$$u_B = u_A - c_s(\varepsilon_B - \varepsilon_A) \tag{A4}$$

$$V_B^2 = -2(1 + \varepsilon_B)\,\bar{c}_B\,u_B - u_B^2 \tag{A5}$$

$$\tan\psi_B = -\frac{V_B}{u_B + (1+\varepsilon_B)\bar{c}_B} \tag{A6}$$

where

$$\bar{c}_B^2 = \frac{\sigma_B}{\rho(1+\varepsilon_B)} \tag{A7}$$

The equations governing the response for other orderings of longitudinal and transverse wave speeds may be modified to account for shock wave propagation in a similar manner.

In the computational work performed in connection with these investigations, stress-strain curves are represented by a succession of short straight line segments rather than by a function possessing a continuous derivative everywhere, as described in the appendix of [2]. ε_A cannot then be determined from Equation (A3), but is selected as that line segment end point which gives minimum slope to the line joining it with ε_B . As the computations proceed, it is necessary to check wave speed orderings, and select the correct set of governing equations. For example, in C1010 steel at 430°F (Part I), for small strains static behavior predicts all $c > \bar{c}$. For larger strains the slope of the stress-strain curve decreases enough to have $\bar{c} <$ some c . For still larger strains, shock waves form and raise longitudinal wave speeds enough to revert to the original ordering.

<u>Case II: Given $V(\varepsilon)$. Find σ, u, ψ</u>

For the ordering all $c > \bar{c}$, for example, the desired quantities may be calculated using Equation (18) of [2]. The modification required to account for the possibility of shock waves is to select as candidates for ε_A many values of $\varepsilon < \varepsilon_B$. The equation is solved for each candidate, ε_A again being that value of ε which yields minimum value for c_s .

Case III: Given $V(\psi)$. Find σ, ε, u

For the ordering all $c > \bar{c}$, the desired quantities may be calculated using Equation (16) of [2]. The modification required to account for the possibility of shock waves is the same as that of Case II.

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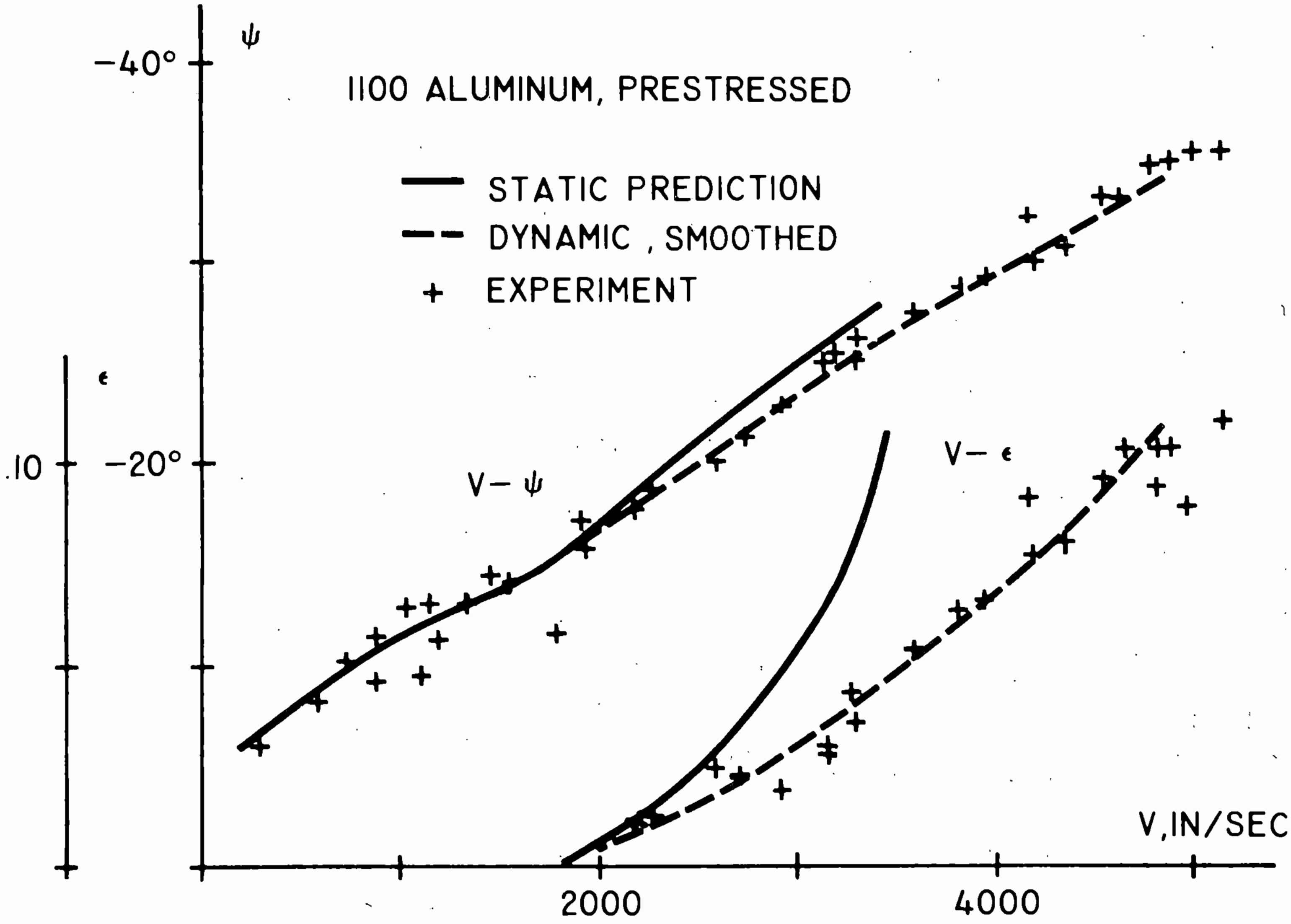

FIG. 1 EXPERIMENTAL OBSERVATIONS, 1100 ALUMINUM, PRESTRESSED

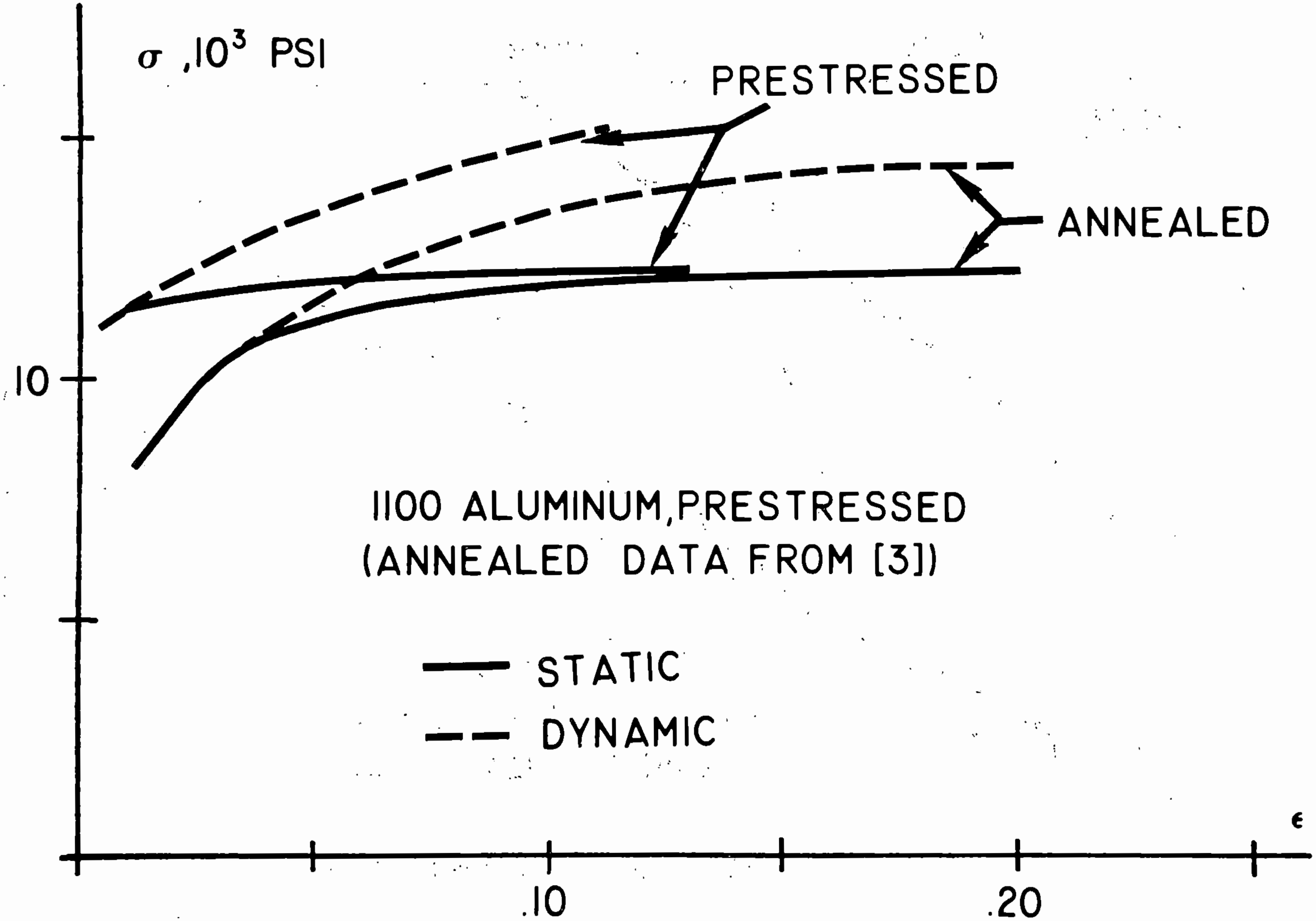

FIG. 2 STRESS-STRAIN DATA, 1100 ALUMINUM, PRESTRESSED

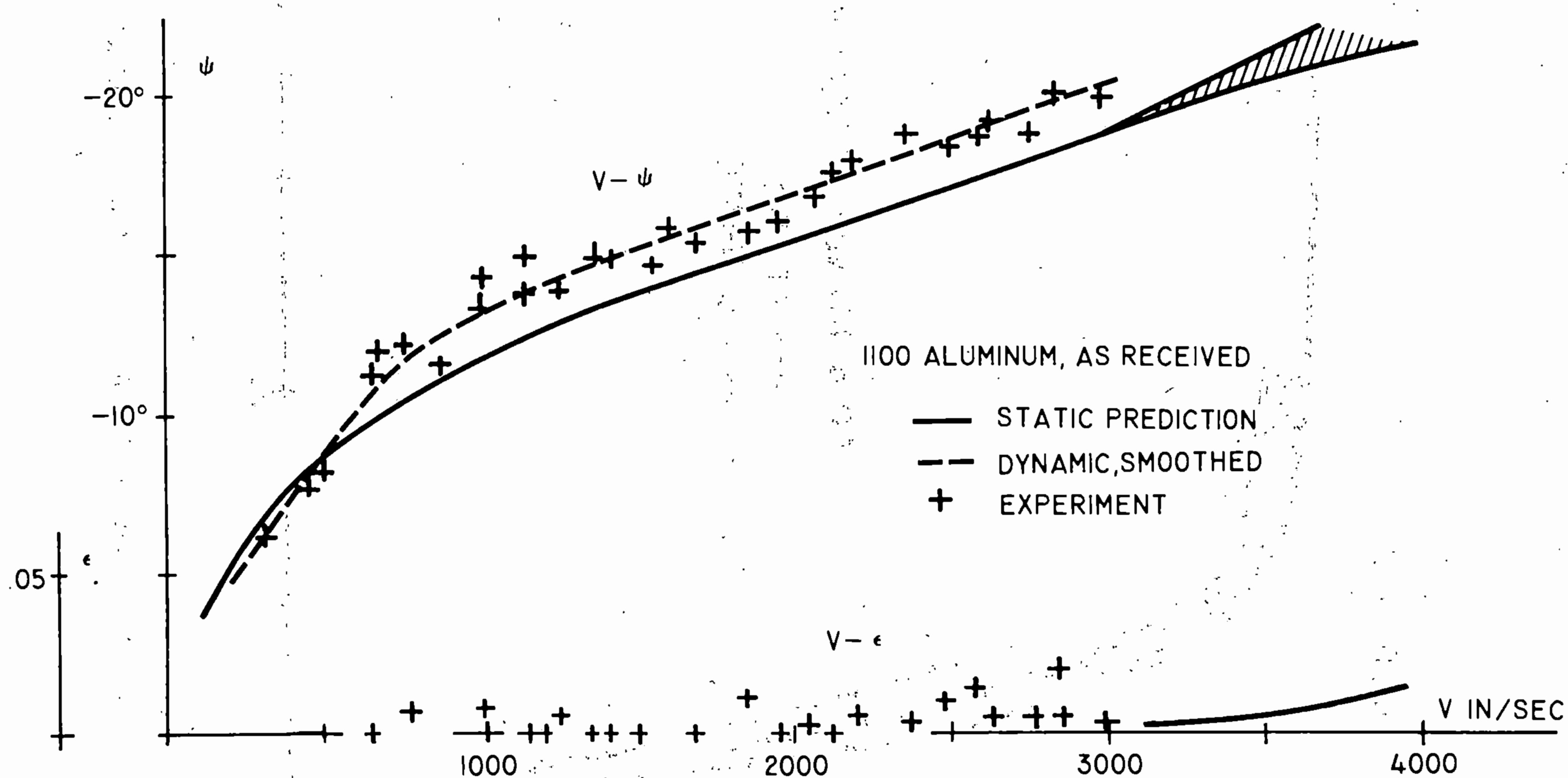

FIG. 3 EXPERIMENTAL OBSERVATIONS, 1100 ALUMINUM, AS RECEIVED

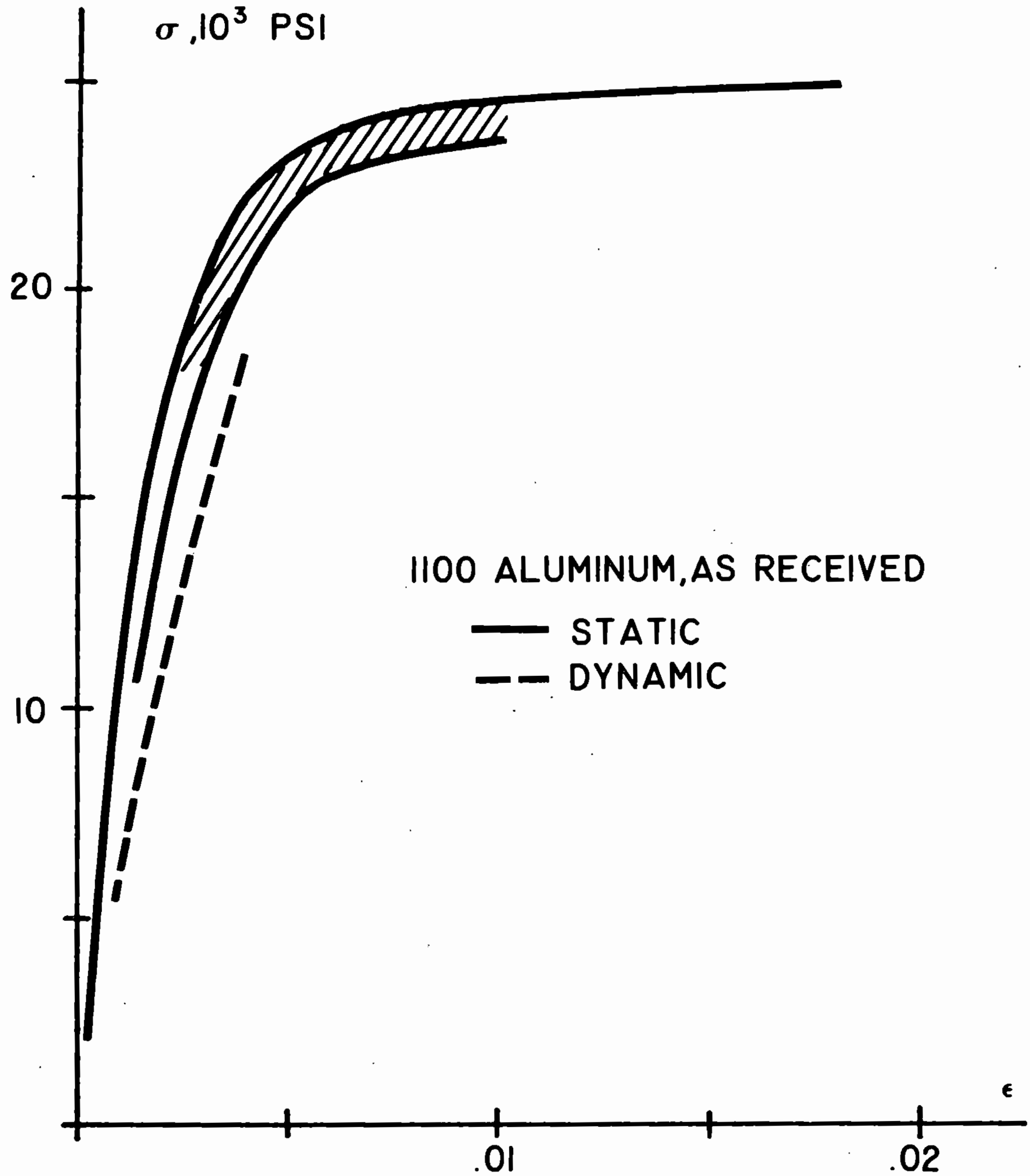

FIG. 4 STRESS-STRAIN DATA, 1100 ALUMINUM, AS RECEIVED

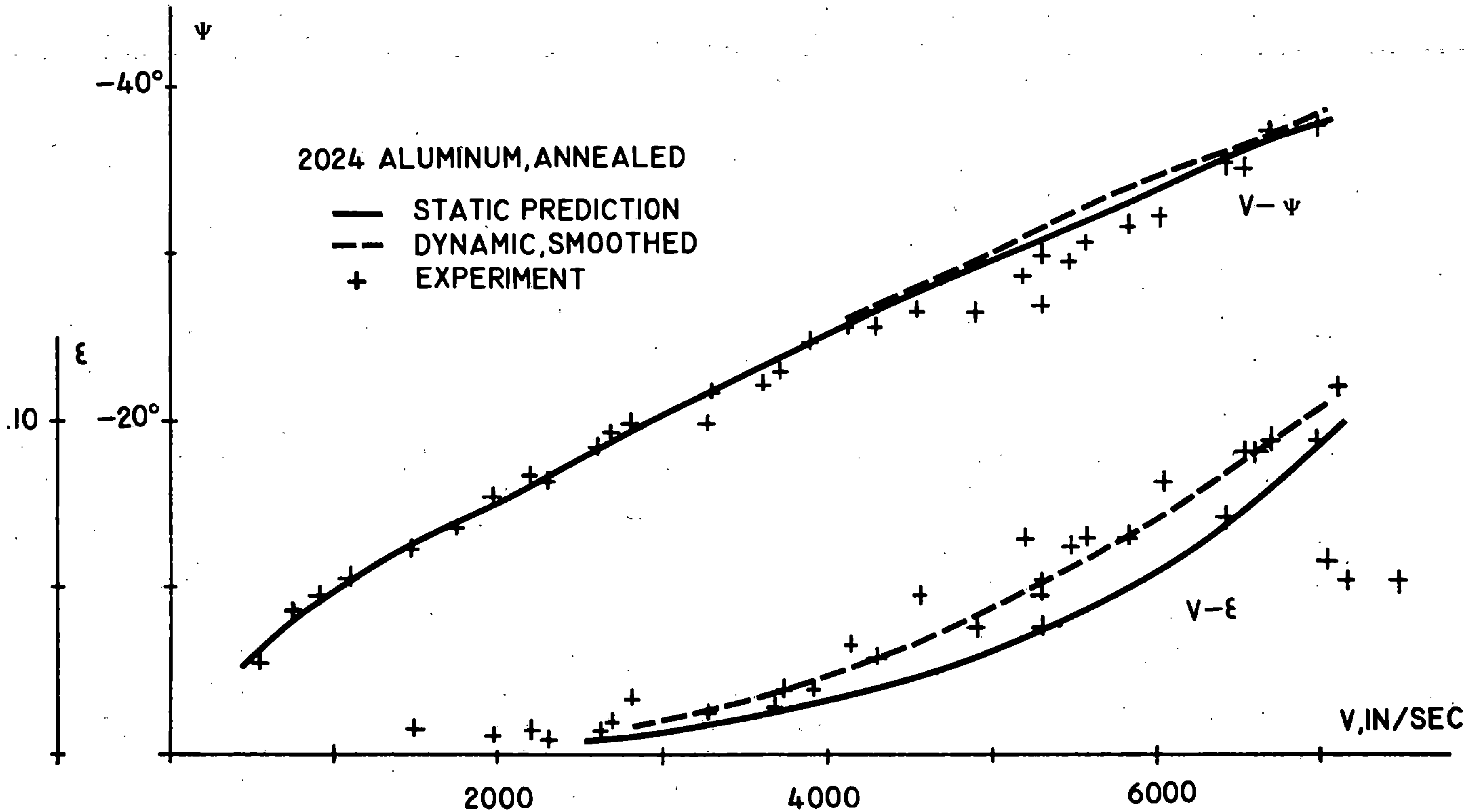

FIG. 5 EXPERIMENTAL OBSERVATIONS, 2024 ALUMINUM, ANNEALED

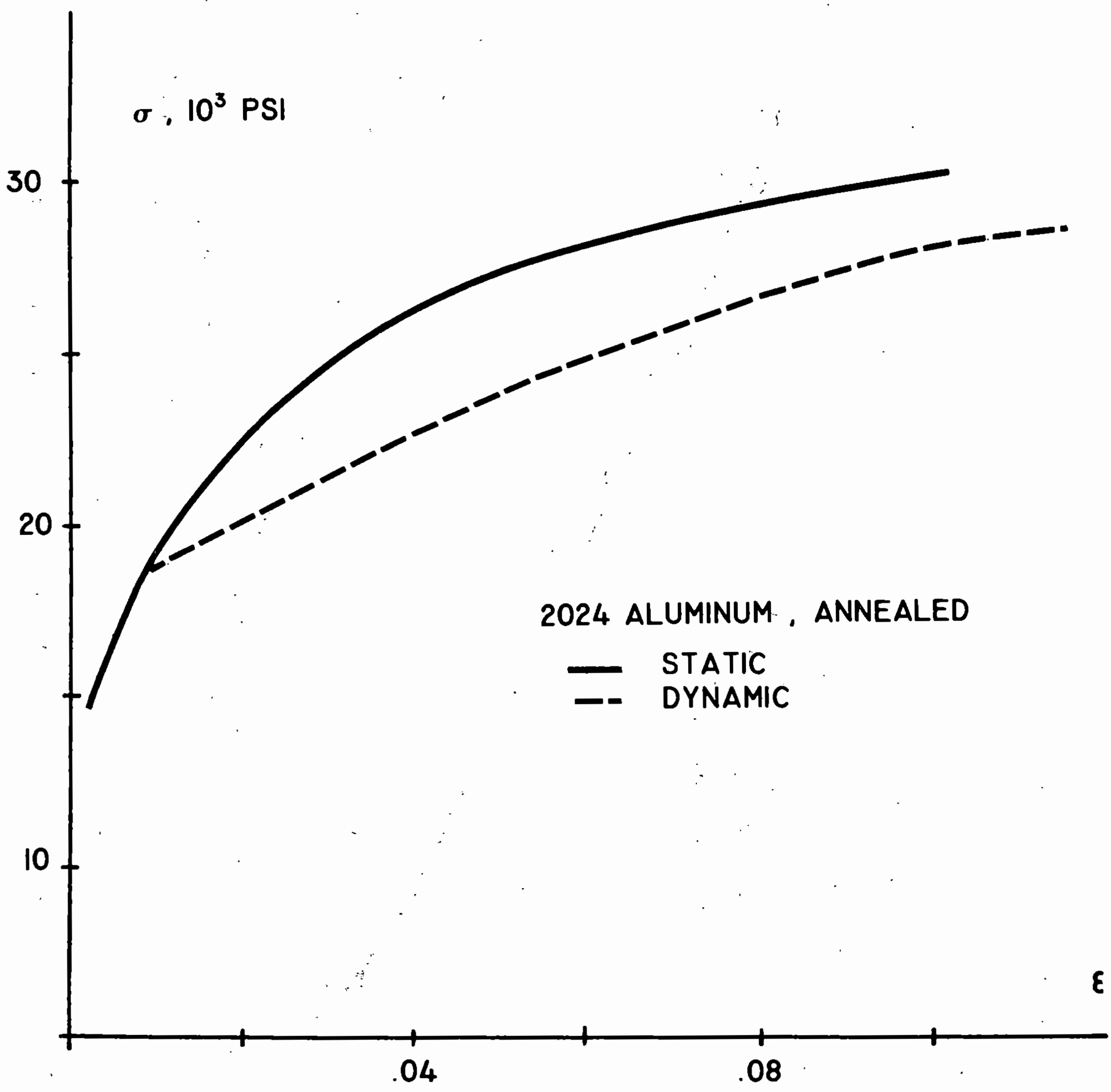

FIG. 6 STRESS-STRAIN DATA, 2024 ALUMINUM, ANNEALED

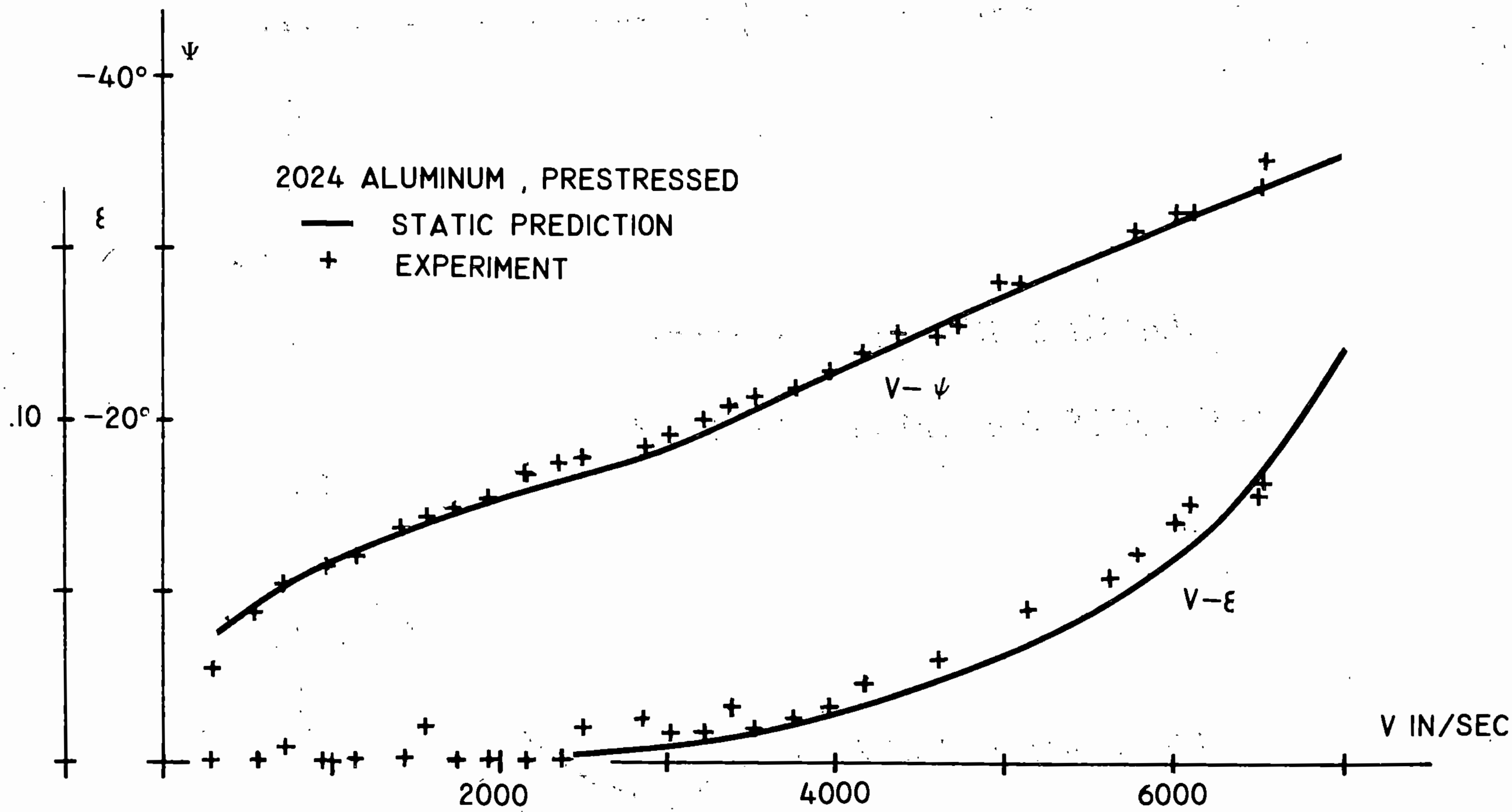

FIG. 7 EXPERIMENTAL OBSERVATIONS, 2024 ALUMINUM, PRESTRESSED

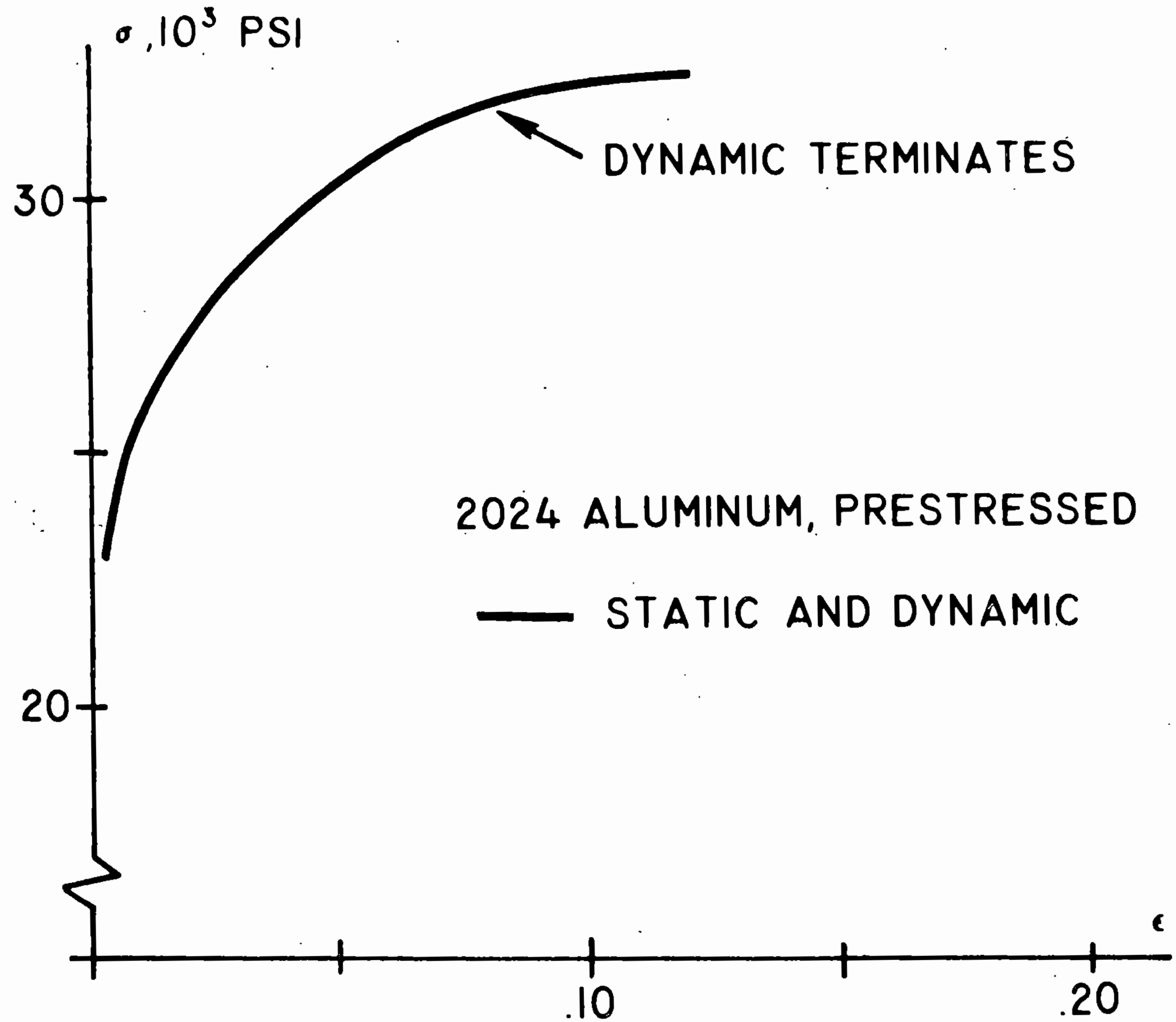

FIG. 8 STRESS-STRAIN DATA, 2024 ALUMINUM, PRESTRESSED

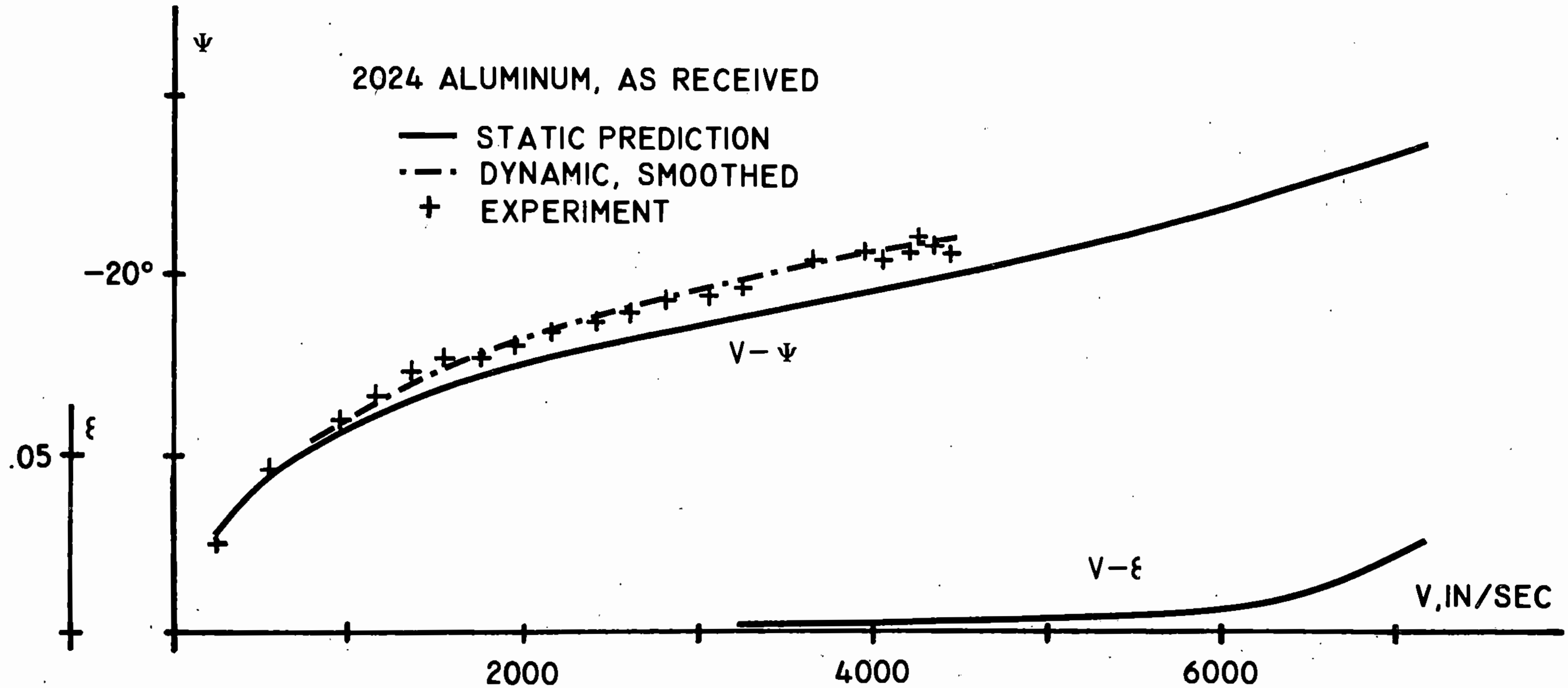

FIG. 9 EXPERIMENTAL OBSERVATIONS, 2024 ALUMINUM, AS RECEIVED

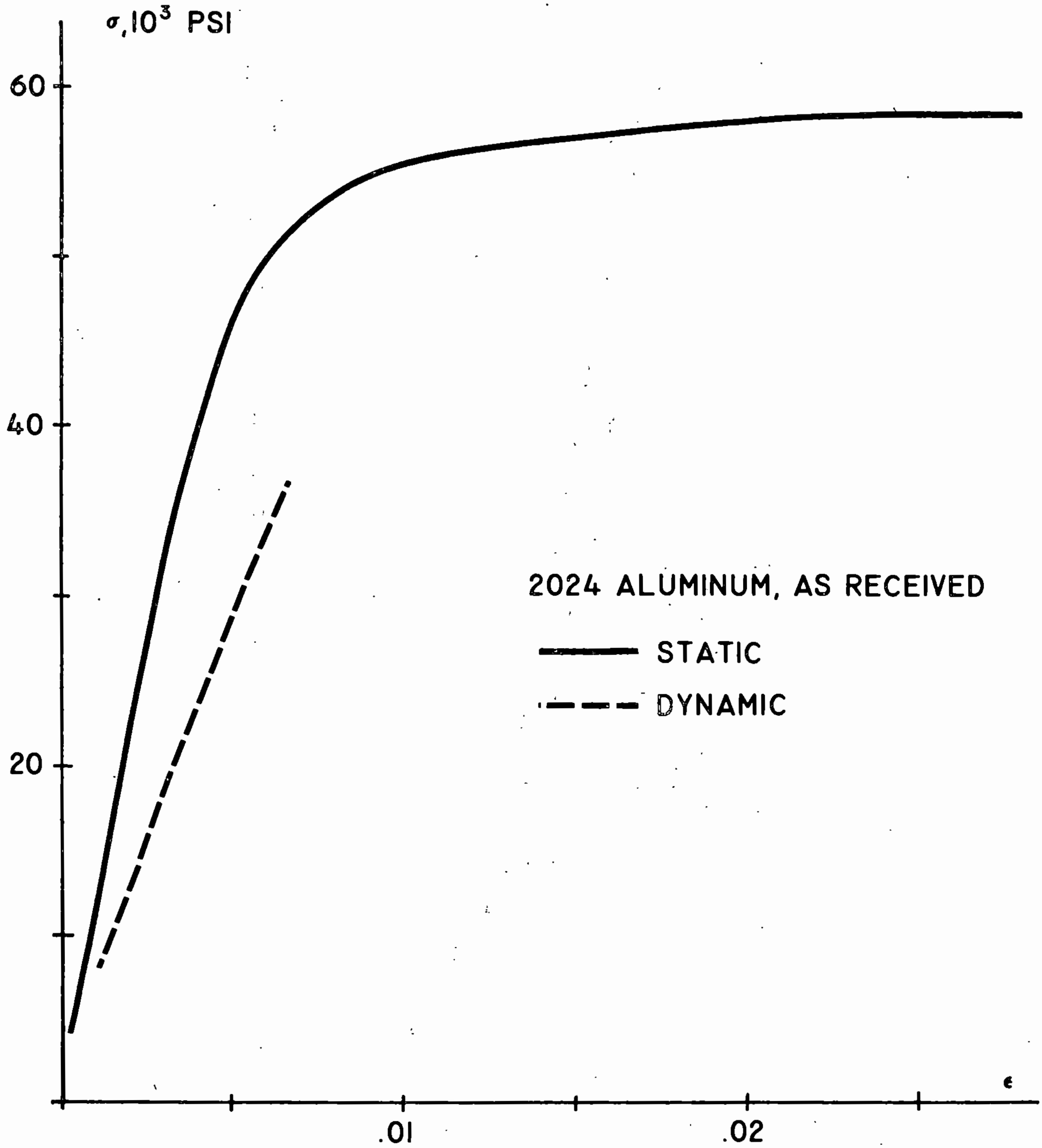

FIG. 10 STRESS-STRAIN DATA, 2024 ALUMINUM, AS RECEIVED

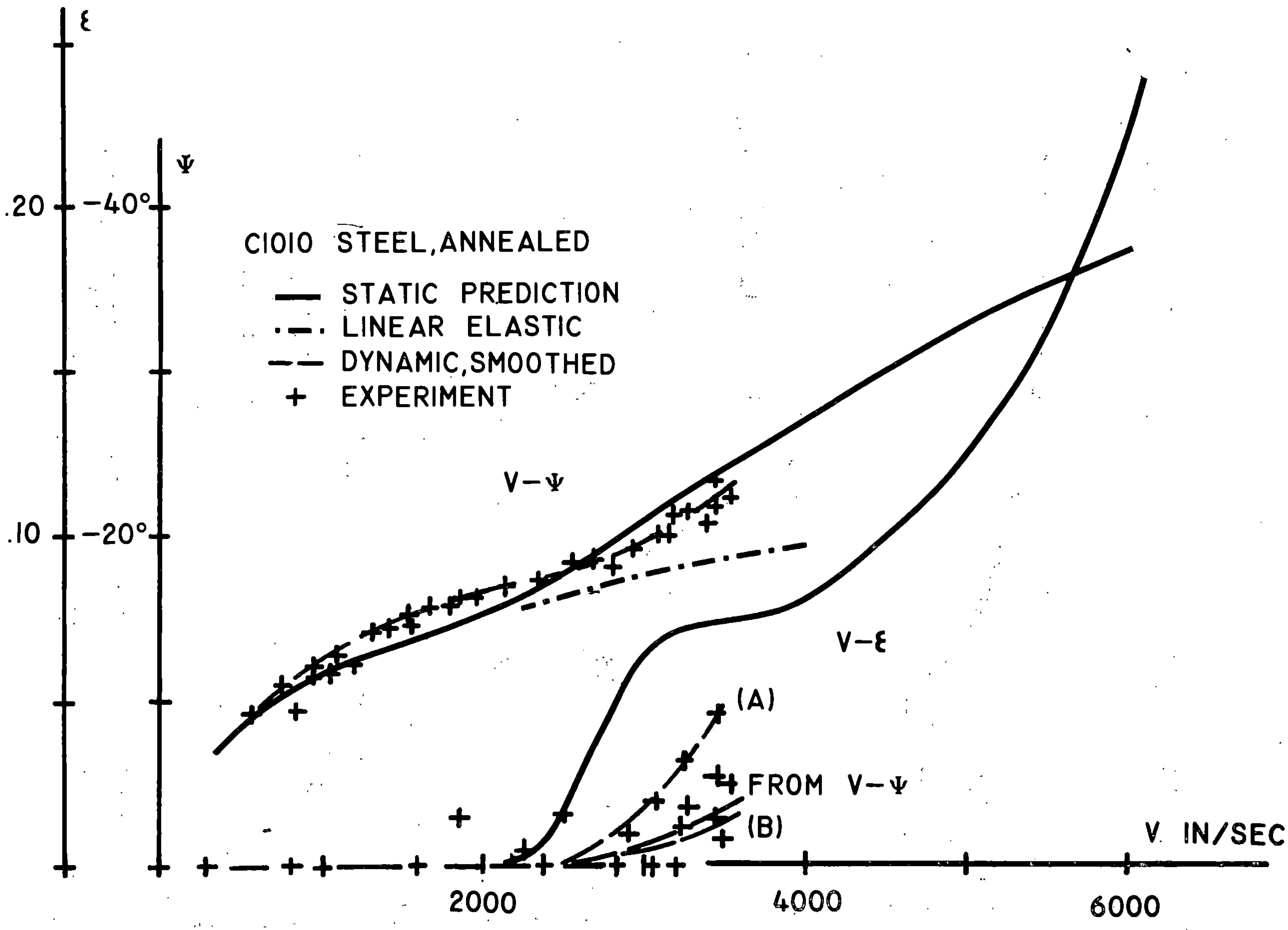

FIG. 11 EXPERIMENTAL OBSERVATIONS, C1010 STEEL, ANNEALED

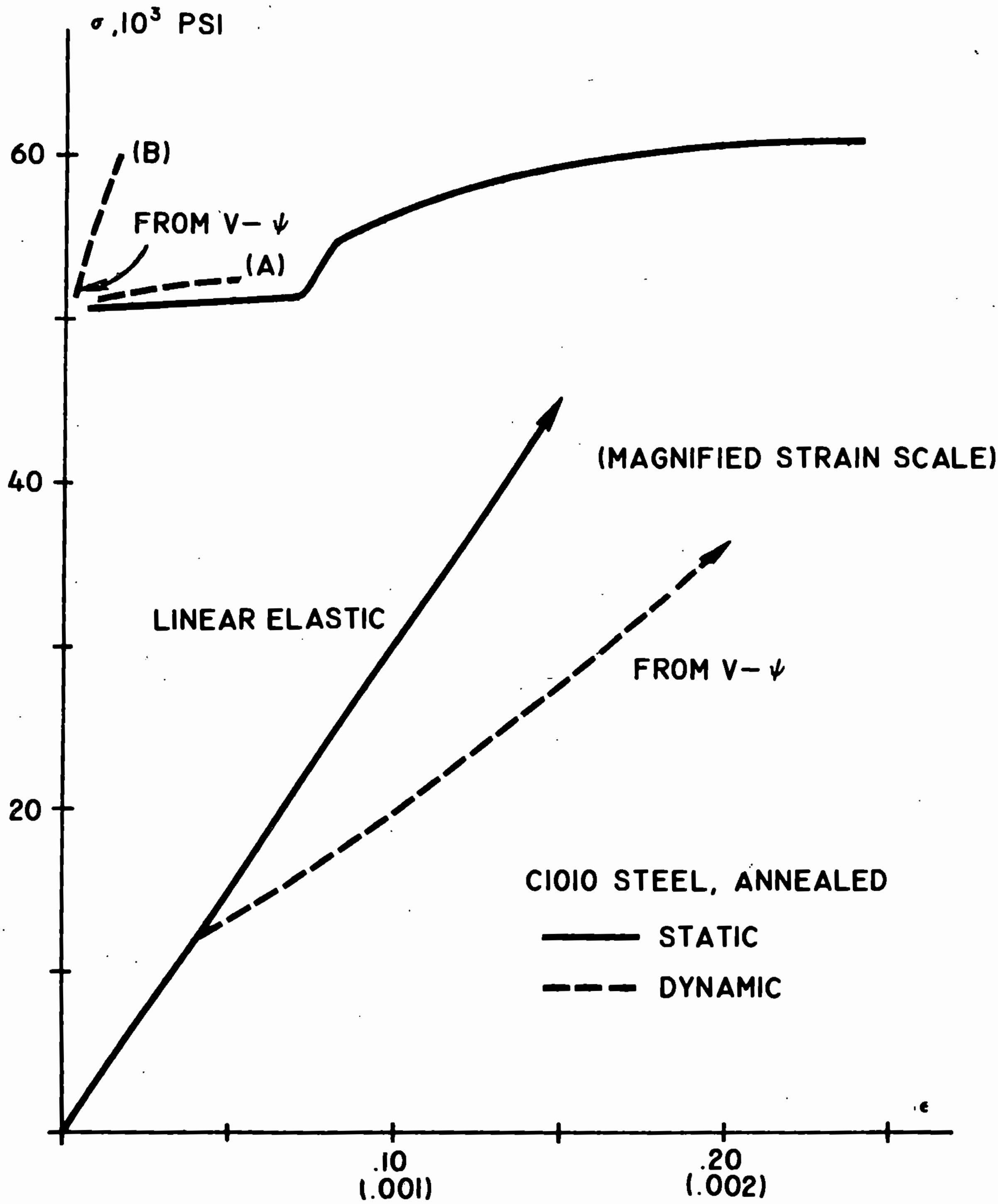

FIG. 12 STRESS-STRAIN DATA, C1010 STEEL, ANNEALED

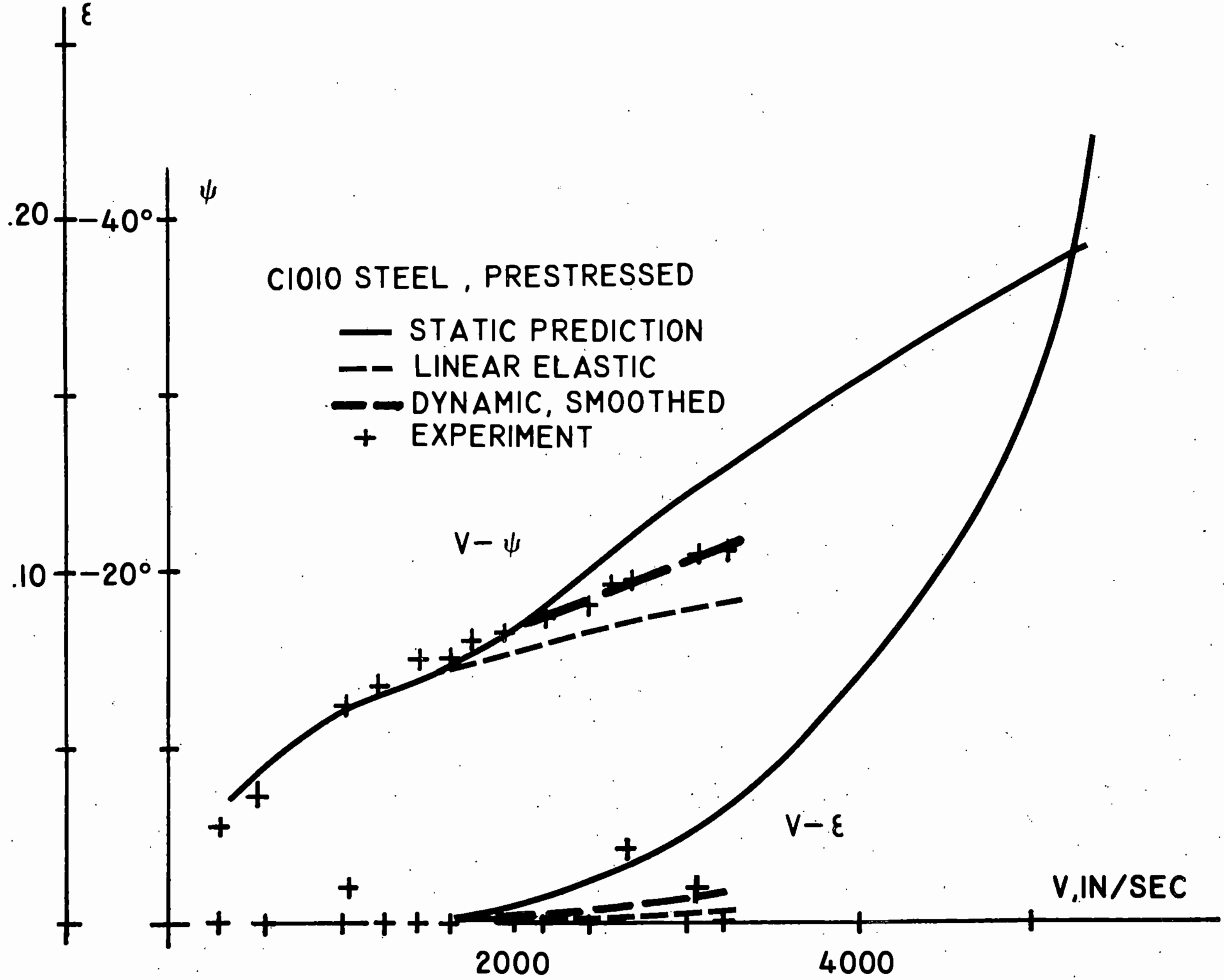

FIG. 13 EXPERIMENTAL OBSERVATIONS, C1010 STEEL, PRESTRESSED

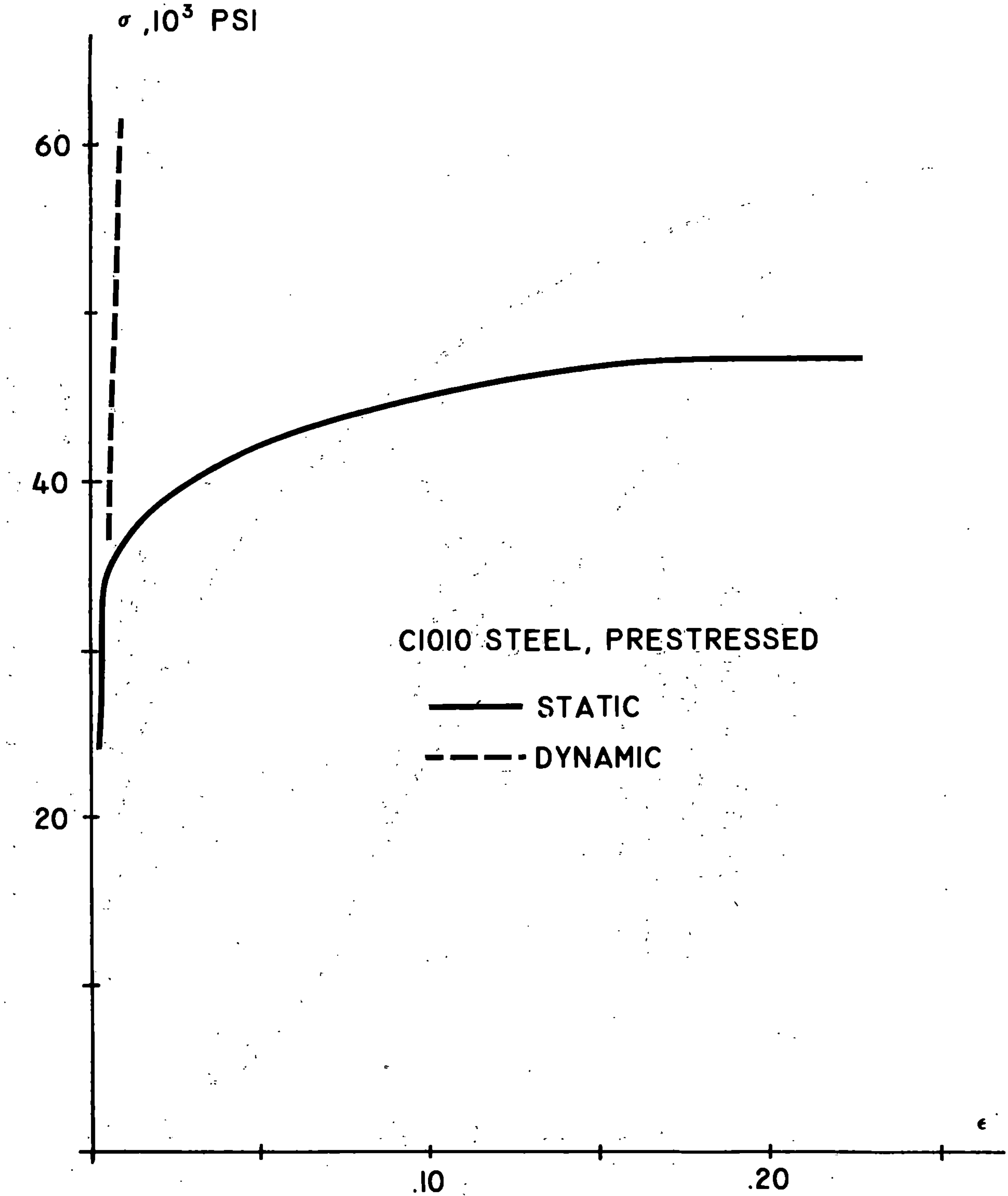

FIG. 14 STRESS-STRAIN DATA, C1010 STEEL, PRESTRESSED

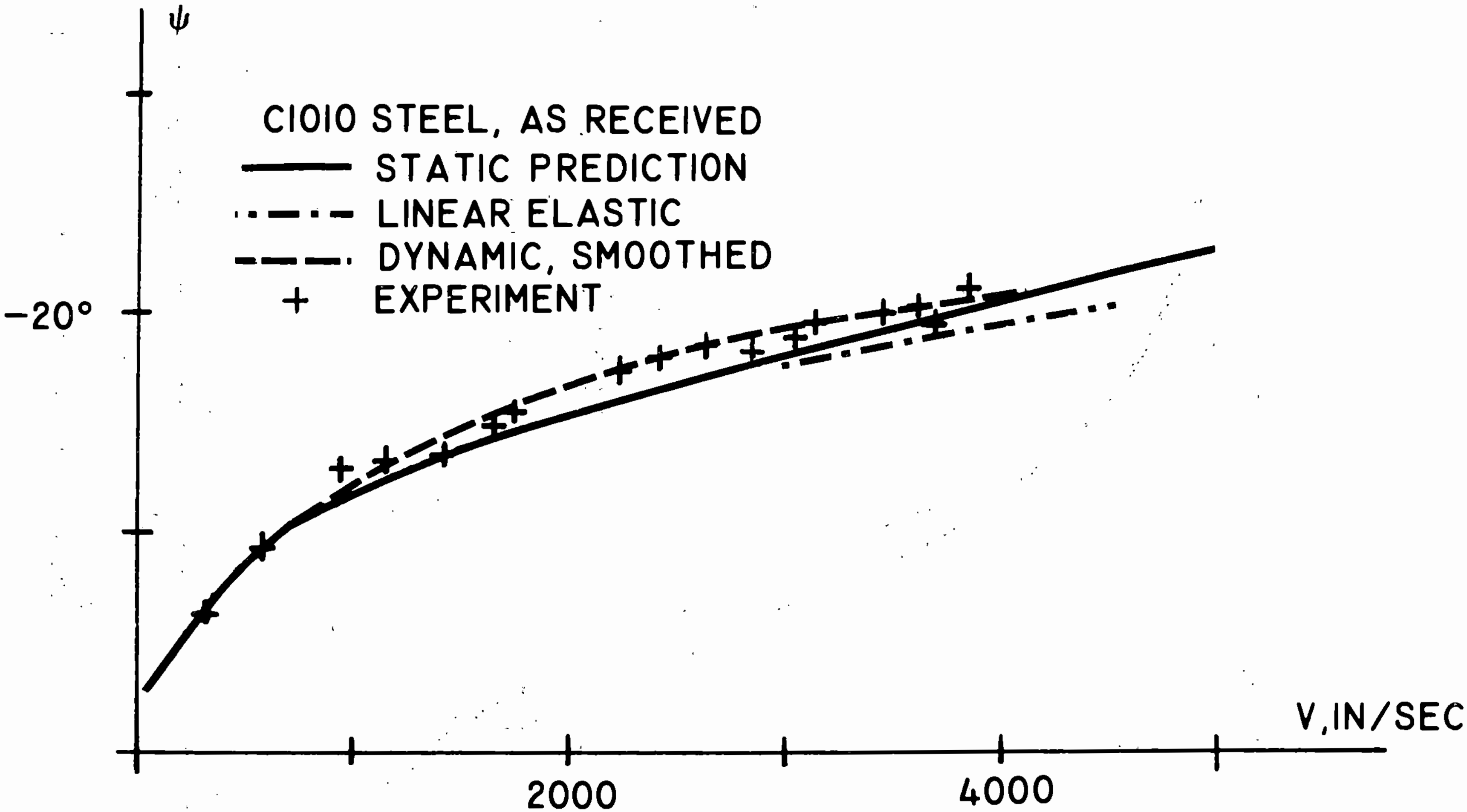

FIG. 15 EXPERIMENTAL OBSERVATIONS, C1010 STEEL, AS RECEIVED

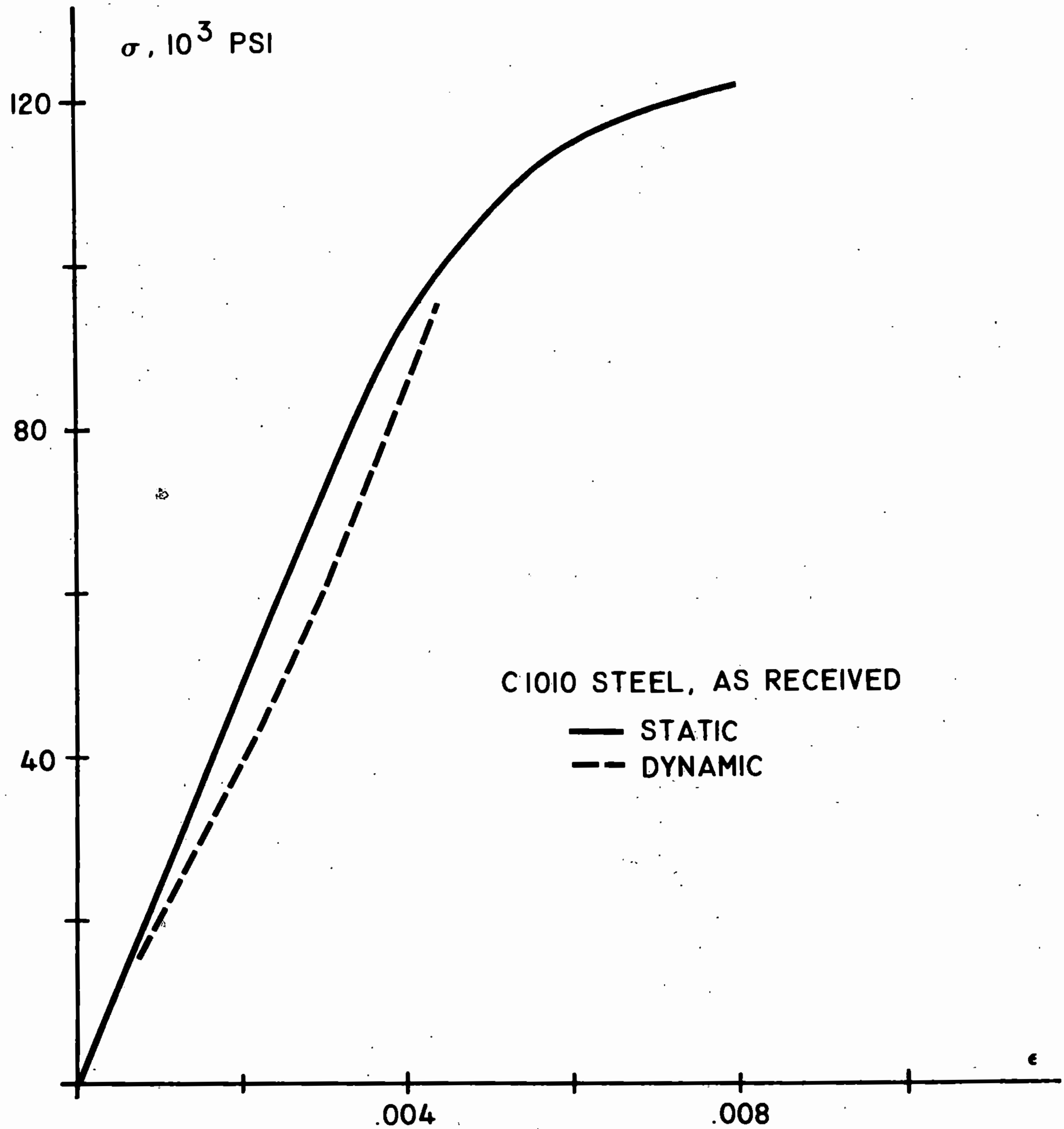

FIG. 16 STRESS-STRAIN DATA, C1010 STEEL, AS RECEIVED

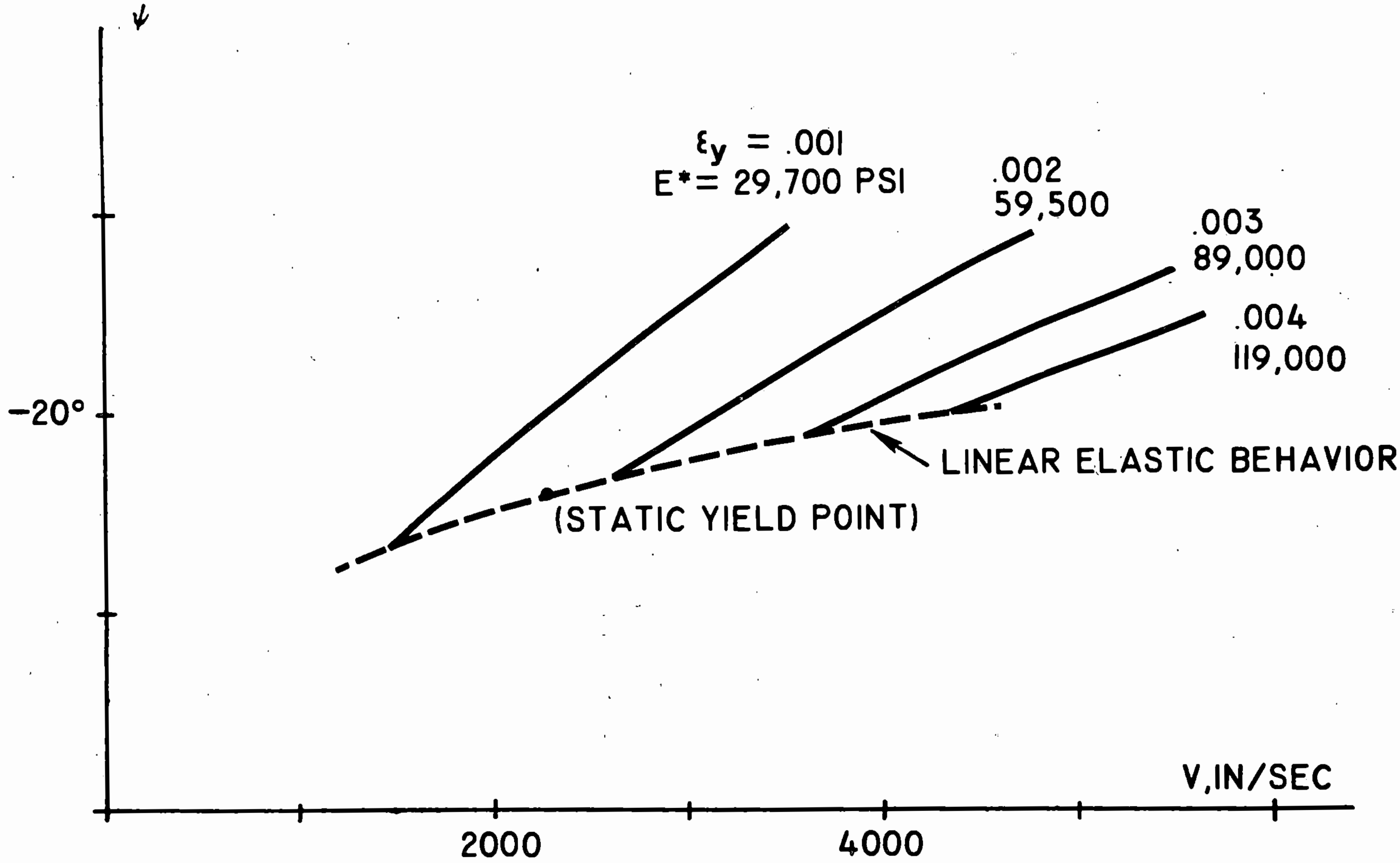

FIG. 17 HYPOTHETICAL STUDY OF DYNAMIC YIELD POINT IN C1010 STEEL

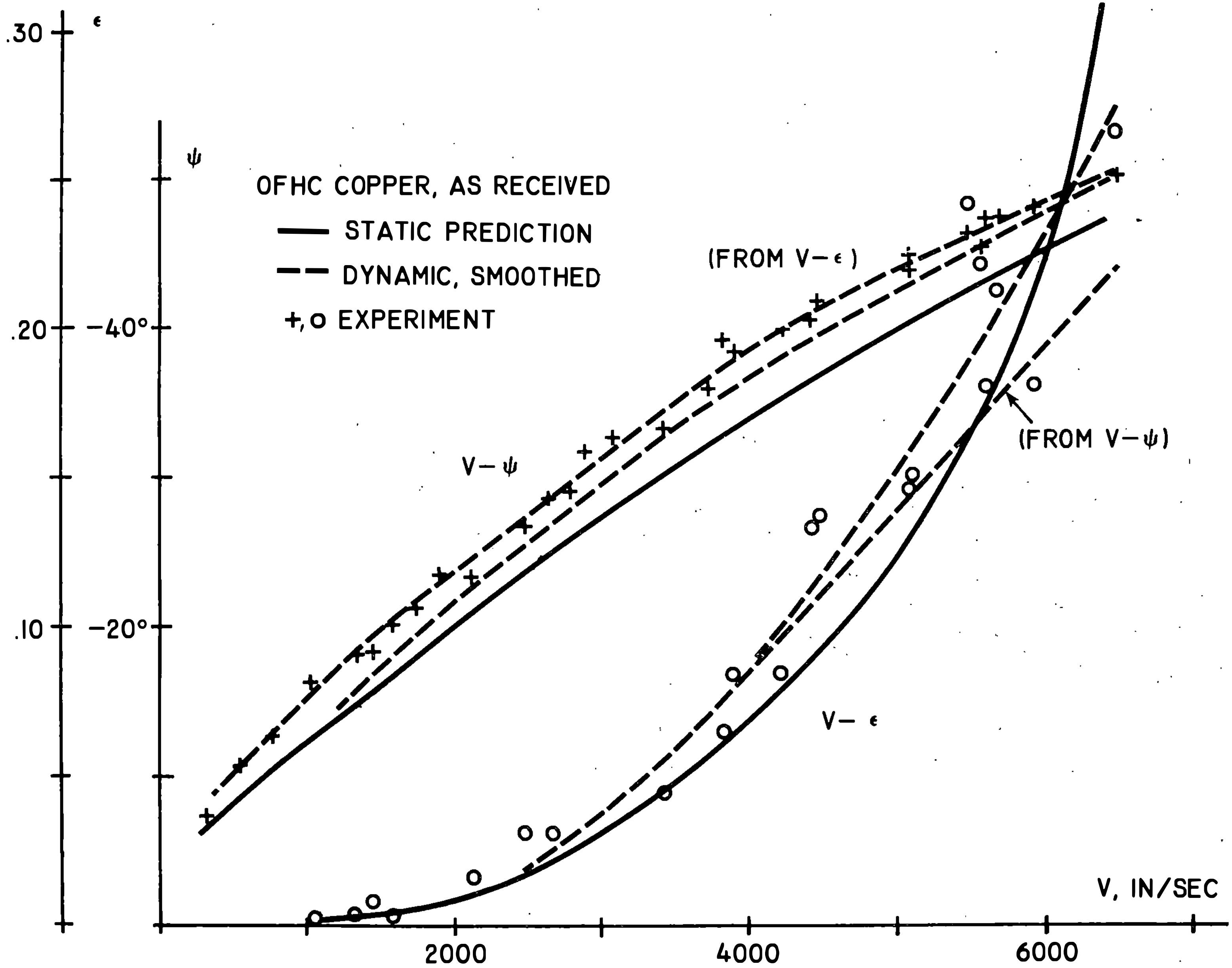

FIG. 18 EXPERIMENTAL OBSERVATIONS, OFHC COPPER, ANNEALED

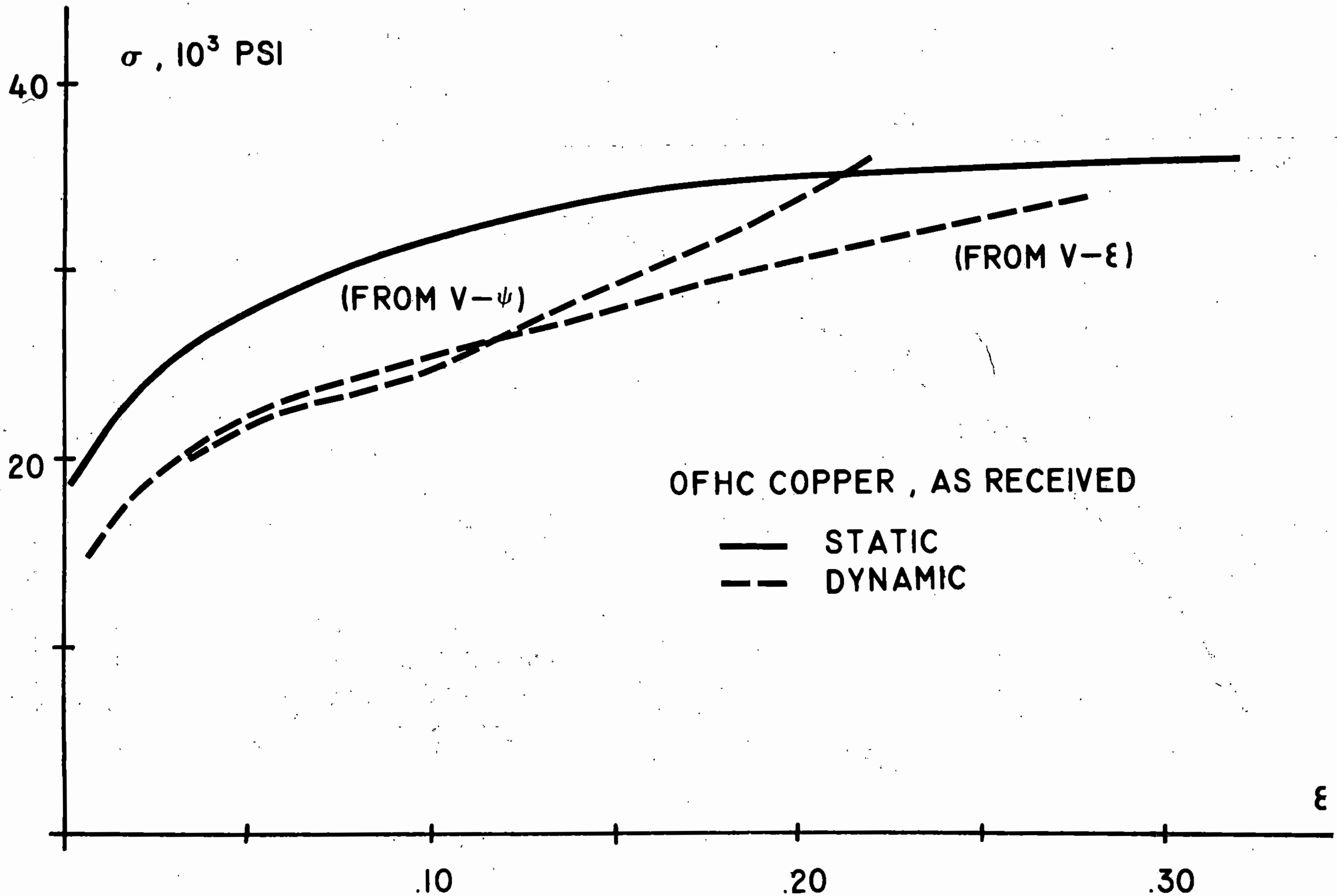

FIG. 19 STRESS-STRAIN DATA, OFHC COPPER, ANNEALED

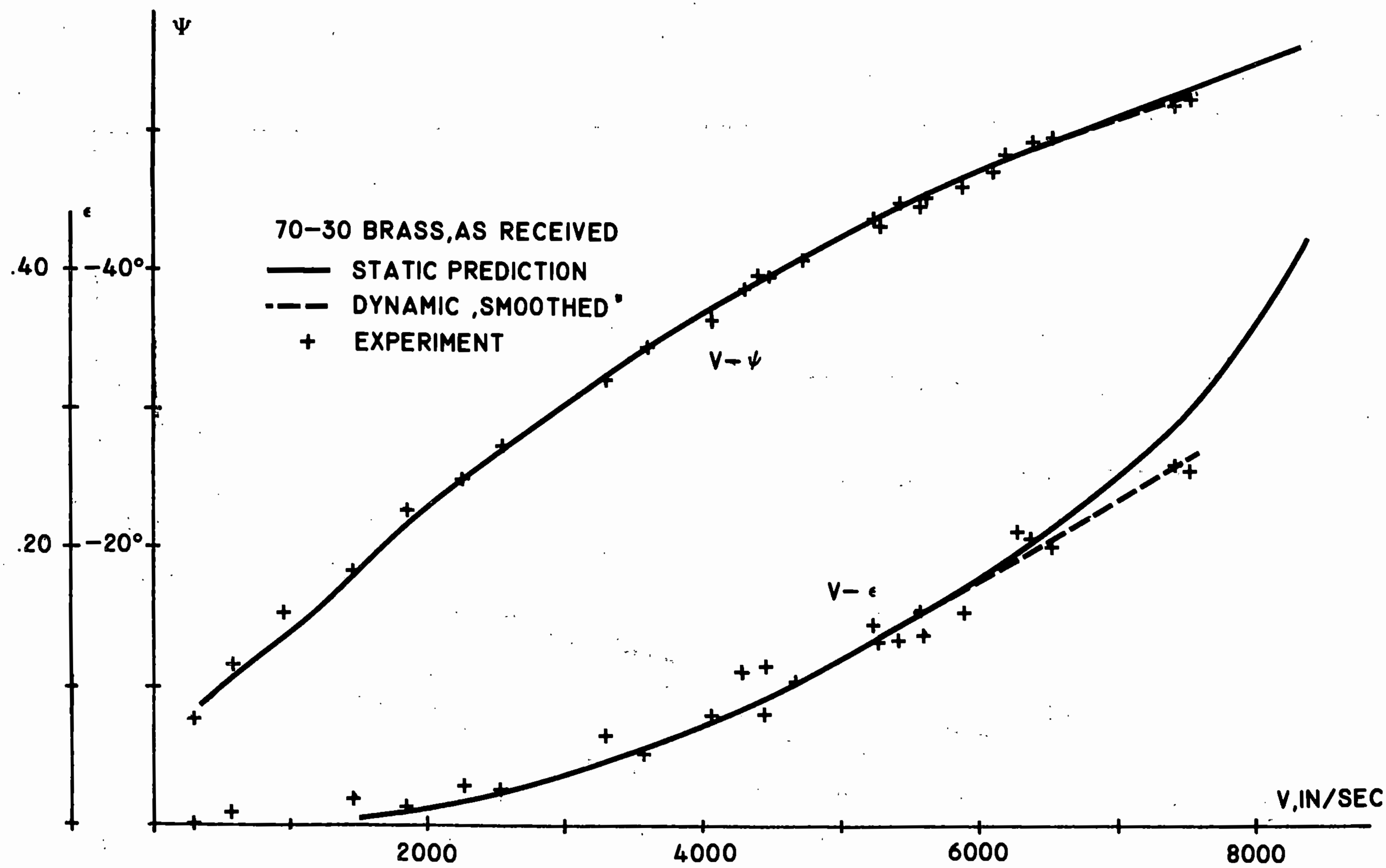

FIG. 20 EXPERIMENTAL OBSERVATIONS, 70-30 BRASS, ANNEALED

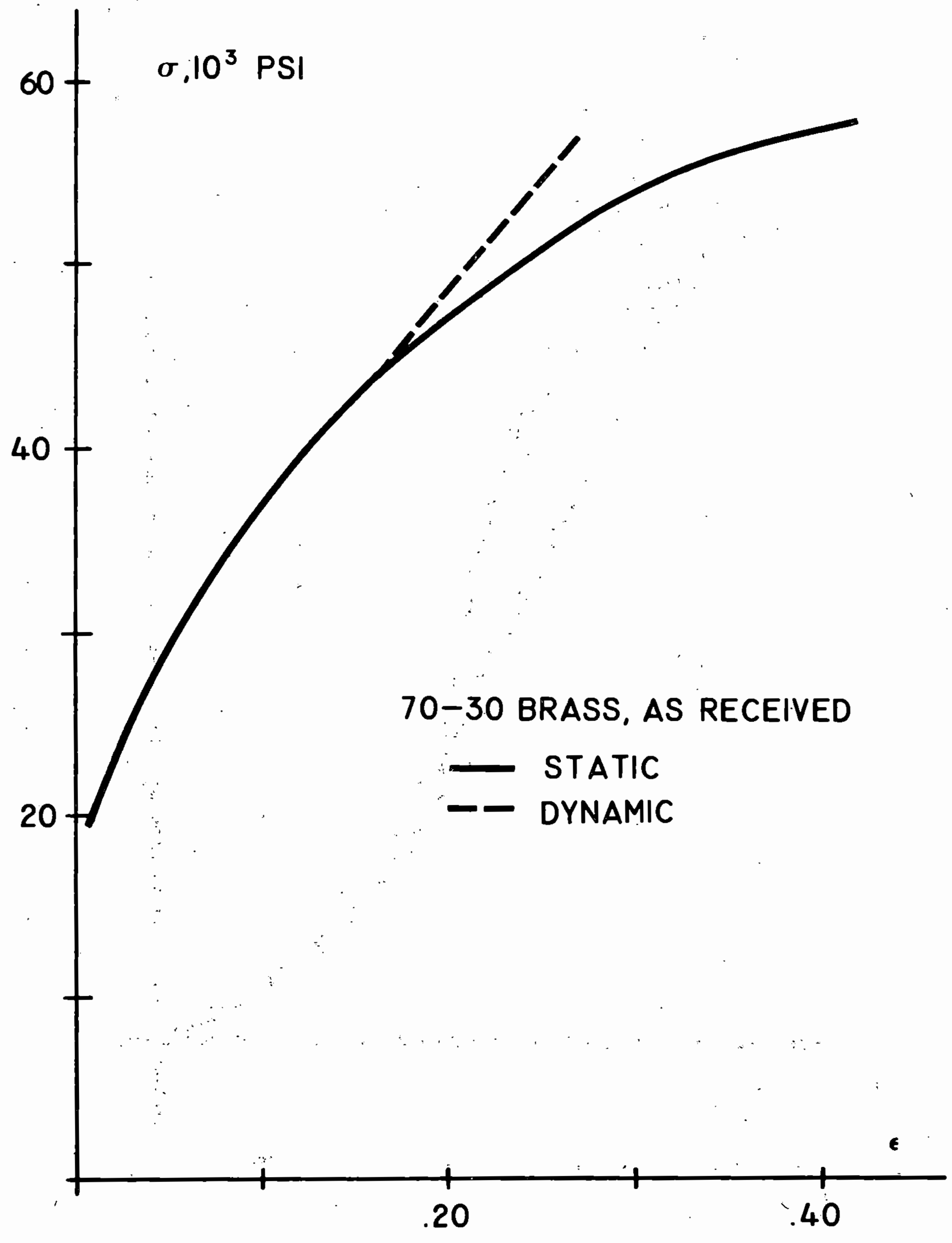

FIG. 21 STRESS-STRAIN DATA, 70-30 BRASS, ANNEALED

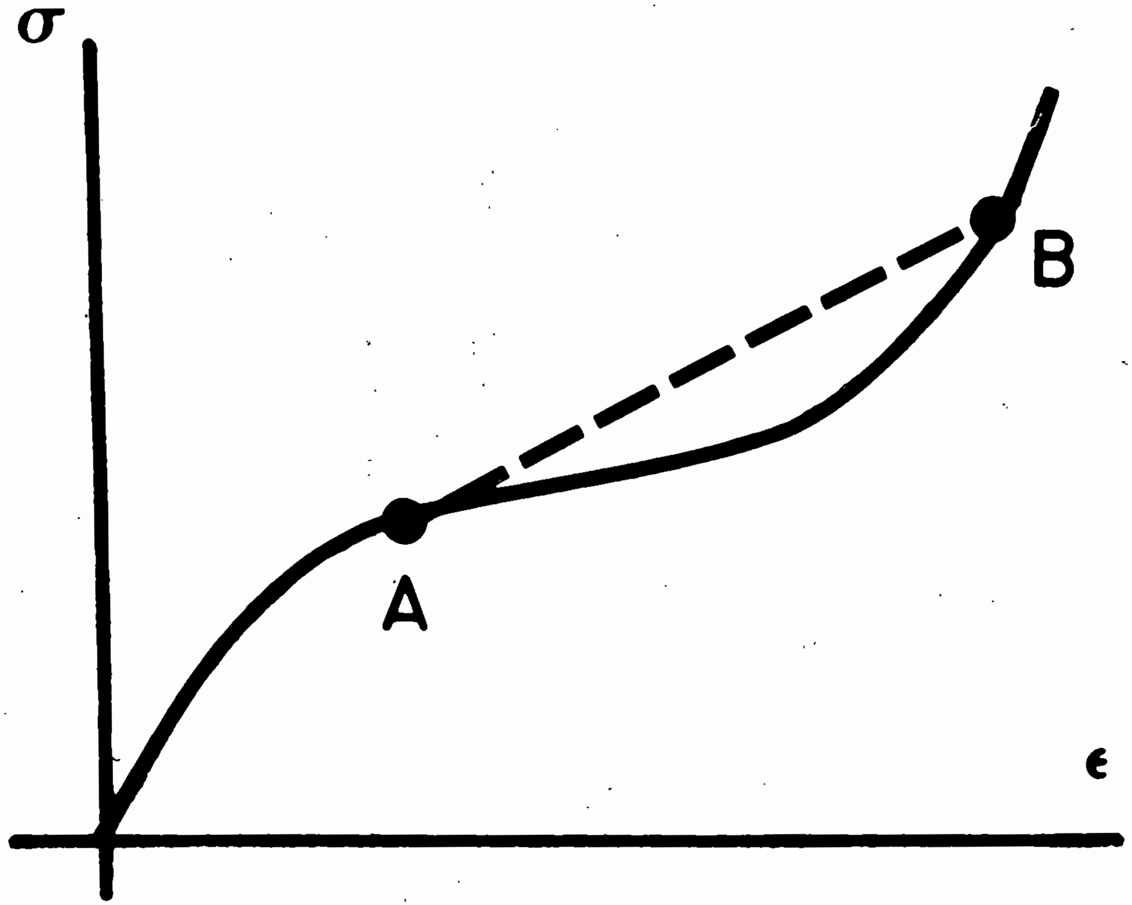

FIG. 22 STATE OF STRESS AND STRAIN WHEN SHOCK WAVES OCCUR

TABLE I SUMMARY OF RESULTS

MATERIAL	CRITICAL TRANSVERSE VELOCITY, IN/SEC			CRITICAL LONG. VELOCITY, IN/SEC			ULTIMATE STRAIN			ULTIMATE STRESS, 10^3 PSI		
	STATIC	DYN.	DYN./STATIC	STATIC	DYN.	DYN./STATIC	STATIC	DYN.	DYN./STATIC	STATIC	DYN.	DYN./STATIC
1100 ALUMINUM												
ANNEALED [3]	5060	6000	1.2	−1940	−2530	1.3	.20	.20	1.0	12.2	14.6	1.2
PRESTRESSED	3470	4840	1.4	−880	−1580	1.8	.13	.11	.85	12.2	15.2	1.25
AS RECEIVED	4000	3000	.75	−860	−540	.63	.018	.004	.22	24.8	18.7	.75
2024 ALUMINUM												
ANNEALED	7110	7140	1.0	−2470	−2560	1.0	.10	.11	1.1	30.8	28.4	.92
PRESTRESSED	7000	6500	.93	−2230	−1970	.88	.12	.08	.67	32.4	31.9	.98
AS RECEIVED	7170	4490	.63	−1760	−890	.50	.028	.006	.21	58.2	34.2	.59
C1010 STEEL												
ANNEALED	6070	3500	.58	−2070	−730	.35	.24	.016	.07	61.0	56.7	.93
PRESTRESSED	5340	3290	.62	−1850	−630	.34	.23	.008	.03	47.0	61.3	1.3
AS RECEIVED	5000	3920	.78	−1020	−710	.70	.008	.004	.50	121	92.7	.77
OFHC COPPER												
AS RECEIVED	6400	6530	1.0	−2780	−3090	1.1	.32	.27	.84	36.3	34.3	.95
70–30 BRASS												
AS RECEIVED	8370	7590	.90	−4450	−3760	.85	.42	.26	.62	57.5	57.1	.99