ADAPTIVE CHARACTERISTICS OF THE HUMAN CONTROLLER OF TIME-VARYING SYSTEMS

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FOREWORD

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ABSTRACT

Models are proposed for the decision and control processes involved in the adaptation by the human controller to sudden changes in the dynamics that he is controlling. The decision processes postulated are detection of a change of dynamics, identification of the new dynamics and selection of the appropriate new control strategy. The control processes postulated are steady-state tracking, modification, transient tracking, vernier adjustment and then steady-state tracking with the new dynamics.

A Bayesian model is proposed for the detection and identification processes and is tested in experiments with controlled dynamics of the form K/s and K/s^2. Good agreement was achieved between the behavior of the model and the observed behavior of the human controller. The models for steady-state and transient tracking are based upon a simple describing function representation for the human controller. Modification is postulated to be simply a switching of control strategies. Data from experiments with a variety of controlled dynamics are used to substantiate these models.

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LIST OF SYMBOLS

$c(t)$	control movement
$<ce>$	ensemble average of $c(t)$ and $e(t-\tau)$
$C(s)$	transfer function of controlled dynamics
$C(s,t)$	time-varying transfer function of controlled dynamics
$C_0(s)$	transfer function of pre-transition controlled dynamics
$\bar{C}_0(s)$	the dynamics are <u>not</u> $C_0(s)$
$C_1(s)$	transfer function of correct post-transition dynamics
$C_i(s)$	transfer function of $i^{\underline{th}}$ controlled dynamics
$C_{io}(s)$	$C_i(s)/C_0(s)$
CI	control interval
$d(t)$	input disturbance
D	$(\delta\dot{e}_0)^2-(\mu_0\delta\dot{e}_1/\mu_1)^2$ in Eq. (6.19)
D_n	data obtained in the $n^{\underline{th}}$ CI
$D(s)$	Laplace transform of $d(t)$
$e(t)$	system error
$\dot{e}(t)$	derivative of $e(t)$
$<e^2>$	ensemble average of $e^2(t-\tau)$
E	mean-squared error
$E(s)$	Laplace transform of $e(t)$
$G(s)$	$E(s)/D(s)$
$H(s)$	describing function of the human controller
$H_0(s)$	human controller's describing function for steady-state tracking with $C_0(s)$
$HC(s)$	describing function of human controller and controlled dynamics
j	$\sqrt{-1}$
K	number of possible post-transition dynamics
K	gain of controlled element
K_h	human controller's gain

K_i	gain of $C_i(s)$
K_0	gain of $C_0(s)$
K_{i0}	ratio of post- to pre-transition controlled dynamics gain
n	index referring to the CI
n(t)	human controller remnant
$\langle n^2 \rangle$	ensemble average of n(t)
N	number of transitions
o(t)	system output
$\dot{o}(t)$	derivative of o(t)
$o_i(t)$	output expected from $C_i(s)$
$p(D_n)$	probability density of D_n
$p(D_n \mid C_i)$	likelihood of D_n under $C_i(s)$
$p(\Delta\dot{e} \mid c;n)$	probability density of $\Delta\dot{e}$ conditional upon c for n^{th} CI
$p(\Delta\dot{e} \mid C_j,c;n)$	likelihood of $\Delta\dot{e}$ under C_j and c for the n^{th} CI
$P(C_i)$	prior probability of $C_i(s)$
$P(C_i \mid D_n)$	posterior probability of $C_i(s)$ given D_n
$P(C_j;n)$	prior probability of C_j for n^{th} CI
$P(C_j \mid \Delta\dot{e},c;n)$	posterior probability of C_j given $\Delta\dot{e}$ and c for the n^{th} CI
q	probability of transition in a CI
r^2	squared-correlation between $e(t-\tau)$ and c(t)
s	complex frequency, $\sigma+j\omega$
S_{dd}	power density spectrum of d(t)
t	time
t_σ	transition time
t_s	detection signal time
$\bar{t}$	mean identification time
T	duration of a CI
V_i	expected value of choosing $C_i(s)$
V_{ii}	value of correctly choosing $C_i(s)$
V_{ij}	value of choosing $C_i(s)$ when $C_j(s)$ is the dynamics

V_{00}	value of correctly choosing $C_0(s)$
$V_{\bar{0}0}$	value of choosing $\bar{C}_0(s)$ when dynamics is $C_0(s)$ (false alarm)
$V_{0\bar{0}}$	value of choosing $C_0(s)$ when dynamics is $\bar{C}_0(s)$ (a miss)
$V_{\bar{0}\bar{0}}$	value of correctly choosing $\bar{C}_0(s)$
$\delta\dot{e}_i$	deviation of $\Delta\dot{e}$ from μ_i
$\delta\dot{e}_{it}$	$\Delta\dot{e} - \mu_{it}$
$\Delta c(t)$	change in c(t) during a CI
$\Delta\dot{d}$	change in derivative of d(t) during CI
$\Delta e(t)$	change in e(t) during a CI
$\Delta\dot{e}(t)$	change in error rate during a CI
$\Delta\dot{e}_i(t)$	$\Delta\dot{e}$ predicted by human controller with assumption that $C_i(s)$ is the dynamics
ΔK_i	$K_i - k_0$
$\Delta\dot{\delta}_i$	change in $\dot{\delta}_i(t)$ during a CI
μ_i	expected value of $\Delta\dot{e}_i$
μ_{it}	expected value of $\Delta\dot{e}_i$ if there is a transition to $C_i(s)$ during the CI
η	random coefficient representing the fractional error in the prediction of $\Delta\dot{\delta}_i$
σ	real part of complex frequency s.
σ_{it}	standard deviation of $\delta\dot{e}_{it}$
σ_K	standard deviation of K_h
σ_t	standard deviation of identification time
σ_η	standard deviation of η
τ	effective time delay of the human controller
τ_0	τ when $C_0(s)$ are the dynamics
ω	imaginary part of complex frequency s

ω_c	gain crossover frequency
ω_{c0}	ω_c when $C_0(s)$ are the dynamics
$\omega_{c_{opt}}$	optimum value of ω_c

CHAPTER I

INTRODUCTION

The ability of the human controller to adapt his characteristics to changes in the characteristics of the vehicle or process he is controlling is an important factor determining the safety and performance of piloted vehicles and other manually-controlled systems. In aircraft and space vehicle systems, the human pilot often has to change his control characteristics to compensate for the changes that occur in the vehicle's characteristics as the speed and altitude change. The pilot must also maintain control, or take over control from an automatic system, if a malfunction in the flight control system or if a change in the aerodynamic configuration occurs. Clearly, a quantitative description and a mathematical model of the process by which human controllers adapt to changes in vehicle dynamics would be useful for the design of piloted vehicles and for the analysis of their performance, reliability and safety.

The extensive literature on time-invariant manual control systems provides considerable information and a number of useful models for the human controller for a wide variety of time-invariant systems.* These results indicate the

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*Summaries of research on time-invariant manual control systems have been published by McRuer et al,[1] Elkind,[2] McRuer and Krendel,[4] and Licklider.[3]

scope of the human controller's ability to control different dynamic systems, but shed little light on the dynamic processes involved in his adaptation to time-variations of the system dynamics.

The literature on human controller adaptation in time-varying control situations is not nearly so extensive. Research on this problem has been hampered by the lack of a good theoretical framework and good measurement techniques. The first study of human controller adaptation was performed by Sheridan.[5] He measured slowly-varying human controller describing functions in situations where the plant dynamics were also slowly varying. Another early study was performed by Sadoff[6] who investigated the ability of skilled pilots to control both fixed and moving-base simulators in the presence of sudden changes in the controlled-element dynamics. More recently, Young et al,[7] in a study that served as the prelude to the research reported here, sought to determine the speed with which human controllers could adapt to sudden changes in the gain of plants whose dynamics were a pure gain. Knoop and Fu[8] proposed an adaptive model for very simple plants in which the human controller is assumed to have a model of the plant being controlled and in which modification is accomplished by a gradient search type of parameter adjustment. Gould and Fu[9] extended this model by incorporating pattern recognition techniques for process identification. Hess[10,11] investigated adaptation to periodically-varying plants. Weir and Phatak[12] suggested the use of time optimal control models to account for the human's behavior after he has detected and identified a change in dynamics.

This report is a complete account of the principal theoretical and experimental results obtained during a three-year research program that began late in 1962. The objective of this program was to develop models for one aspect of human controller adaptation -- adaptation by trained controllers to sudden changes in the controlled-element dynamics of single-axis compensatory control systems. A number of papers describing intermediate results have already been published. The first of these by Elkind et al[13] proposed and discussed a multi-phase model for the adaptive process. Miller,[14] in a thesis investigation begun as part of this research program, examined in detail the detection phase of this model. Elkind and Miller[15] revised the multi-phase model and proposed detailed models for the identification and modification phases.

In Chapter II of this report the type of time-varying compensatory control situation used in the experiments is described briefly. The principal features of the human controller's adaptive response are illustrated through a detailed examination of a typical response by the human operator to a sudden change in controlled dynamics. The adaptive process is partitioned into a set of decision processes and a set of control processes. Analytic models for each of these sets are developed in Chapter III, and are discussed in detail there.

In Chapter IV is a description of the three principal experiments that were performed during this study. The apparatus, the experimental conditions, the subjects, the forcing function inputs, and the performance measures are described. The results of the three experiments are presented in Chapters V, VI and VII, and are discussed from the point of view of the model that was presented in Chapter III. Chapters V and VI discuss the experiments relevent to the decision processes of adaptation. Chapter VII discusses the experiments relevent to the control processes. The extent to which the results support the basic assumptions underlying the model and the structure of the model are examined in these chapters. In Chapter VIII we summarize the results of this program. The principal findings are delineated. The applicability of the model to realistic control situations is discussed briefly.

CHAPTER II

THE ADAPTIVE CONTROL SITUATION

A. SYSTEM CONFIGURATION

In this study of the process by which human controllers adapt to changes in controlled dynamics we have used highly simplified idealizations of the control systems found in actual piloted-vehicle systems. Our experiments and analyses were performed with conventional single-axis, compensatory control systems whose essential characteristics can be represented by the block diagram of Fig. 1. In such systems the human controller's task is to keep the output, o(t), equal to the input disturbance, d(t), or alternatively, to keep the system error, e(t), zero. He controls the system output by making appropriate movements of a control device. These control movements, c(t), are functions of e(t), the only source of information about the state of the system available to the human controller.

The controlled dynamics are represented in Fig. 1 by a time-varying transfer function, C(s,t). A finite number of different controlled elements were permissible in our experiments with transitions from one of these to another occurring suddenly in a step-like manner. In most of the experiments the controlled dynamics were of the form K, K/s, or K/s^2. Changes of the polarity and magnitude of K, the gain of the controlled dynamics, and changes of the order of the controlled dynamics were the principal transitions studied. In all of the experiments the input disturbance, d(t), was a low-frequency gaussian process. The human controller was well-trained at controlling all of the different dynamics that were permissible in an experiment, and moreover, he was also well-trained at adapting to transitions among these dynamics.

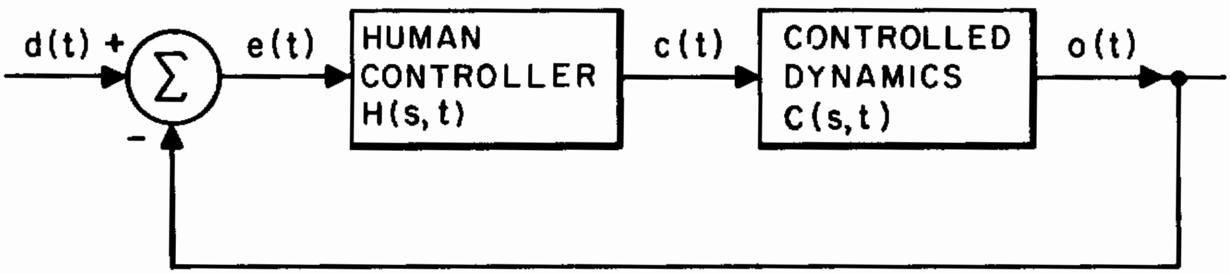

Fig. 1. Simplified block diagram of experimental control system.

B. CHARACTERISTICS OF ADAPTIVE RESPONSES

1. Time History of a Typical Adaptive Response

The nature of the human controller's adaptive process is easily illustrated by examining in detail a typical response by a highly-trained human controller to a change in controlled dynamics. Figure 2 shows the time histories of the input disturbance, the response of the controlled dynamics, the control movement, and the error before and after a transition in controlled dynamics from K/s^2 to $-2K/s^2$. The dynamics changed abruptly at the time t_o indicated on the record. This time, t_o, is the transition time.

In spite of the fact that after t_o the closed-loop system was unstable, the state of the dynamics at t_o and the input after t_o were such that the error and error rate remained small until t_o + 1.2 sec. During the period from t_o to t_o + 1.2, the controller made two large corrective movements, but it is clear from the direction of these and subsequent movements that he had not detected the fact that the dynamics had changed. The error at transition was slightly negative and the controller's movements were directed so as to reduce this error if a change in dynamics had not occurred. Because of the change in the polarity of the controlled dynamics gain, these corrective movements had the effect of making the error more negative. But the input velocity during this period also decreased and compensated for the negative tendency of the error. In fact, the change in input velocity caused the error to drift toward zero, thereby probably giving the controller the impression that his movements were still appropriate.

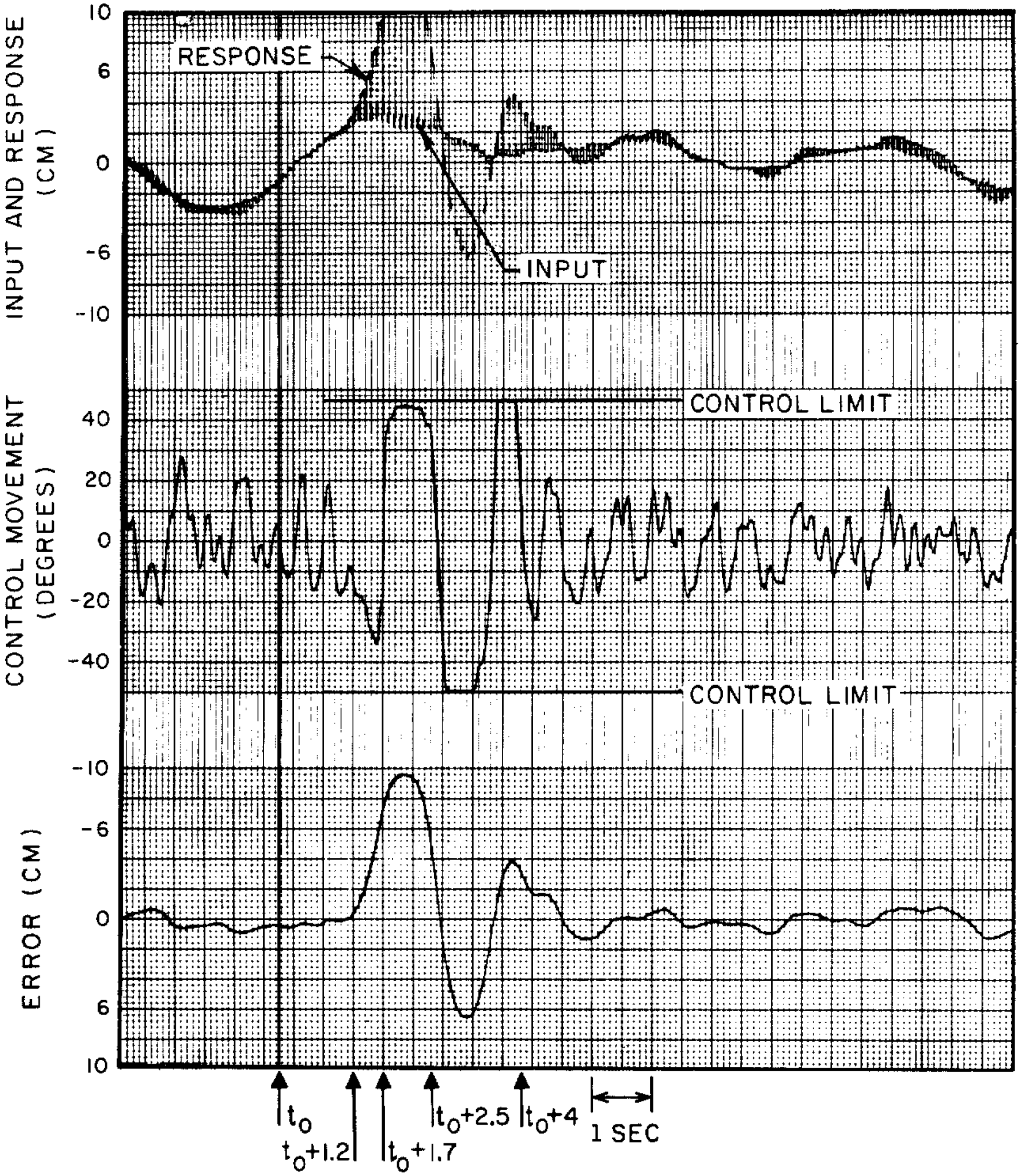

Fig. 2. Time history of input, response, control movement and error immediately before and after a transition from K/s^2 to $-2K/s^2$.

After t_o + 1.2 sec, the error rate became large and the fact that the plant dynamics had changed must have been apparent to the controller. He made one or two more movements, still of the wrong polarity, perhaps to identify plant dynamics, and then at t_o + 1.7 sec he modified his control characteristics by reversing the polarity of his control movements. Since the second overshoot in the error was smaller than the first, he must also have at least partially compensated for the increase in plant gain before the beginning of the large negative stick movement at t_o + 2.5 sec. Thus, in less than 1.3 sec from the time of the probable detection of a transition, the controller had changed the polarity of his movements and had sufficiently lowered his gain so as to render the system stable.

During the interval from t_o + 1.7 sec to about t_o + 4 sec, the error was still large. Prior to the controller's change of the polarity of his movements at t_o + 1.7, a large error and error rate built up. During the ensuing 2.3 sec, the controller was in a transient tracking mode of behavior as he attempted to reduce these errors.

By t_o + 6, the transient errors resulting from the transition had been eliminated and the error signal appears to have stabilized. There is little change in the error characteristics after t_o + 6, and the controller was effectively in a steady-state mode of tracking after this time. Thus, any vernier adjustment of the controller's characteristics must also have been completed by t_o + 6 sec.

2. Describing Functions for a Typical Response

In Fig. 3 are Bode plots of describing functions relating error and control displacement that were computed from five

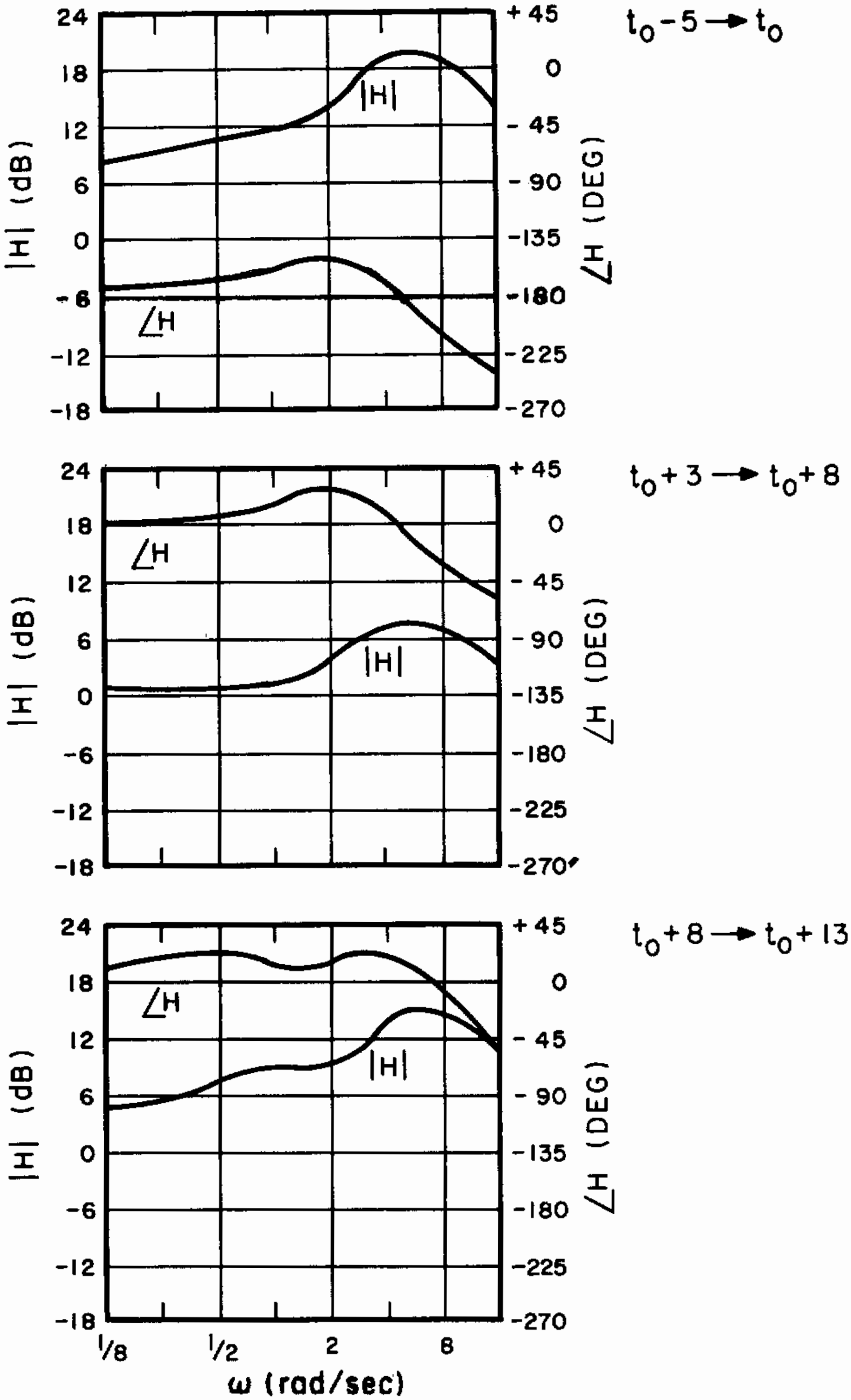

Fig. 3. Bode plots of human controller describing functions obtained from successive five second segments error and stick signals before and after a change in C(s) from K/s^2 to $-2K/s^2$.

sec segments of the error and control signals preceding and following a transition of the same type as that shown in Fig. 2, but from a different run. C(s) changed from K/s^2 to $-2K/s^2$ at time t_o. These describing functions were obtained using a multiple regression analysis method described in previous papers.[16,17] For the 5 sec period preceding the transition (t_o-5 to t_o) the describing function exhibits low-frequency lead that is evident in both the amplitude ratio and the phase. For the 5 sec period starting at $t_o + 3$, the phase was reduced by 180 degrees reflecting the fact that the controller had detected the change of the polarity of the system gain and had reversed the direction of his movements. There is some lead compensation evident in both amplitude ratio and phase, but not as much as before. The amplitude ratio was reduced by 9 or 12 db, indicating the controller had overcompensated for the 6 db increase in the gain of C(s).

For the next 5 sec period, the one beginning at $t_o + 8$, the controller increased his gain so that the total open-loop amplitude ratio, controller plus controlled dynamics, was about the same as it was before the transition. He also increased his lead to make the total open-loop phase characteristics nearly the same as they were before the transition.

3. Speed of Adaptation

The speed with which the human controller adapts to changes in controlled dynamics is a remarkable feature of the adaptive process. The speed of response observed in Figs. 2 and 3 is typical of the speeds we observed in our experiments and those observed by Sheridan,[5] Young et al,[7] and Knoop and Fu[8] in similar studies with controlled dynamics whose form was fairly simple. These studies demonstrate that human controllers who are trained to control all the dynamics that they will encounter and to adapt to transitions among these dynamics can adapt in very short times to a wide range of transitions. The controller can complete the modification of his control strategy within about 1 to 2 sec after the detection of a transition for dynamics up to the second order. The time required for reduction of accumulated errors and optimization of steady-state tracking characteristics depends upon the dynamics. It is about 4 sec or more for some K/s^2 transitions and less time for lower order dynamics.

There is some evidence, however, that in more complex control situations the adaptive process may take longer. Sadoff[6] in his study of pilots' adaptation to failures in flight control systems observed that the times required to stabilize the system and to reduce the accumulated errors were often as long as 30 sec, a considerably longer time than that found in simpler situations. In these experiments the post-transition plant dynamics were unstable and very difficult to control, which may account for the longer adaptation times. Also the frequent occurrence of failures during most experiments that have been performed with simple plant dynamics may have induced an alertness that might have been lacking in Sadoff's experiments (as well as in operational situations) where failures occurred less frequently.

C. ELEMENTS OF THE ADAPTIVE PROCESS

It is evident from Figs. 2 and 3 that the adaptive process consists of a set of decision processes and a related set of control processes. We will consider the decision processes to consist of the following tasks: (1) monitoring to detect changes in the system; (2) identifying the changes when they occur; and (3) selecting control strategies appropriate to the new control situation. We will consider the control processes to consist of the following tasks: (1) steady-state tracking before and after changes in dynamics; (2) modification of control strategy when a transition is identified; (3) transient tracking to null the large errors that result from a change in dynamics; and (4) vernier adjustment of the parameters of the steady-state response to optimize the system's performance in the post-transition period.

This particular partitioning of the adaptive process is not the only one that could be employed. Gibson[18] considers automatic adaptive processes to consist of the functions: identification, decision, and modification. Young et al[7] first pointed out the convenience of considering the human adaptive process to consist of the separable phases: detection, identification, and adjustment. In previous papers[14,15] we have also made use of this concept, but used a somewhat different set of phases.

Although many different partitionings of the adaptive process are possible, separation into a set of decision and a set of control processes is especially convenient for the purposes of model building. It permits us straightforwardly to use results from decision theory and from empirical studies of human decision processes to derive models for the decision processes involved in human controller adaptation. In a similar way, we can use the results from manual control experiments and control theory to derive models for the control process. The particular set of decision and control tasks delineated above is to some extent an arbitrary choice, but it appears to account for most of the identifiable functions performed by the human controller in a time-varying control situation. This partitioning is the one that we will use in the rest of this report.

CHAPTER III

A MODEL FOR HUMAN CONTROLLER ADAPTATION

In this chapter we develop a model for the adaptive characteristics of highly-trained human controllers in an idealized time-varying control situation. The model is based upon concepts taken from decision theory and control theory. The control situation considered is a single-axis, compensatory tracking task like that illustrated in Fig. 1 in which the controlled dynamics may change suddenly. This is the type of situation that we have studied experimentally.

To simplify the development of the model, we will make a number of assumptions about the characteristics of the human controller and about the time-varying system that he is operating. These are discussed first. Next, we discuss the model for the decision processes involved in the human controller's adaptation. Then we consider the model for the control processes.

A. ASSUMPTIONS

The system has a compensatory display which shows only the error signal, e(t). The displayed error and the control movements, c(t), will be assumed to be the only information available to the controller about the state of the system. We will assume that he makes no attempt to predict the input disturbance, d(t). All signals in the system will be assumed to be gaussian. For the control model, the human controller will be assumed to be continuous in his observations of the error and in his movements of the control. For the decision model, however, the human controller will be assumed to be a discrete observer of the error and a continuous observer of his own control movements. Samples of the error will be assumed to be taken periodically every T seconds.* The interval between samples is called the control interval (CI). During the CI, the controller makes control movements designed to reduce the error observed at the beginning of the CI. Although the human controller is not a perfect observer, we will assume that the errors he makes in observing the error signal and control movements are small compared to other sources of variation, and these observation errors will be taken to be zero.

Time-variations in the controlled dynamics will be abrupt, step-like changes, which will be assumed to occur only at the beginning of a CI. Initially, the controlled dynamics will be assumed to be $C_0(s)$. Transitions from $C_0(s)$ to any one of the K other dynamics, $C_1(s)....C_K(s)$, may occur at the beginning of any CI with equal probability q/K.

The human controller will be assumed to be well-trained so that he is able to predict the response of all possible controlled dynamics to his control movements. Moreover, we assume that he is trained to recognize the changes in system output that occur as a result of changes of the dynamics.

*Later in this chapter, in the discussion of the control model, we will associate T with the effective time-delay in the simple crossover model approximation to the human controller's describing function.[1]

B. DECISION MODEL

We postulated that the decision processes are monitoring, identification and selection. Of these three, monitoring to detect a transition and identification of the transition are best described in terms of statistical decision theory. The selection of control strategies can be considered to be a deterministic process in which a pre-stored strategy is retrieved from memory. In this section we first present the basis for the statistical decision theory models for monitoring and identification.* We then use the theory to develop detailed models for the monitoring and identification functions performed by the human controller in a time-varying control situation. Finally, we present a very simple deterministic model for the process used to select control strategies.

1. Basis for Decision Theory Models

Both the monitoring and identification tasks can be represented by the same statistical decision theory model. In both of these tasks we postulate that the human controller takes note of his control movements, c(t), observes the behavior of the error signal, e(t), and from this information decides what the controlled dynamics are. The actual choice is based upon the controller's estimates of the probabilities of the several possible dynamics and upon the values and costs associated with making correct and incorrect choices.

*Green and Swets[19] and Edwards[20,21] give good discussions of the application of statistical decision theory, particularly Bayesian theory, to the modelling of human decision making.

Let us describe this decision process more precisely. The controlled dynamics can be any of $C_0(s)$, $C_1(s) \ldots . C_K(s)$. During the $n^{\underline{th}}$ control interval (CI) the human controller observes the system and gathers some data, D_n, relevant to its behavior. Given D_n, it should be possible for the human controller to estimate the probabilities of each of the $C_i(s)$. These probabilities are written

$$\{P(C_0|D_n), P(C_1|D_n) \ldots . P(C_K|D_n)\}$$

and are called the posterior probabilities of the $C_i(s)$. They are subjective probabilities in the sense that they represent the human controller's estimates of the probabilities of each of the $C_i(s)$ after he has obtained D_n.

The posterior probabilities play a key role in the decision process. For example, if we wanted to adopt as the objective of the decision process the maximization of the probability of a correct decision, we would simply use the decision rule: Choose the $C_i(s)$ whose posterior probability, $P(C_i|D_n)$, is greatest. Most decision objectives commonly used in decision models lead to decision rules that can be expressed in terms of the posterior probabilities.

In our model of the decision processes of adaptation we will choose maximization of expected value as the decision objective. This is a rather versatile decision objective in the sense that many different decision situations can be represented by models designed to achieve this objective. Maximization of probability of a correct decision is a special case of maximization of expected value.

The expected value of choosing $C_i(s)$ is given by the relation

$$V_i(D_n) = V_{i0}P(C_0|D_n)+\ldots.V_{ii}P(C_i|D_n)+\ldots.V_{iK}P(C_K|D_n)$$

$$= \sum_{j=0}^{K} V_{ij}P(C_j|D_n) \qquad (3.1)$$

where $V_i(D_n)$ is the expected value of choosing $C_i(s)$ given the data D_n, and the V_{ij} are the values associated with choosing $C_i(s)$ when, in fact, the controlled dynamics are really $C_j(s)$. Thus, V_{ii} is the value of a correct decision, and the V_{ij}, for $j \neq i$, are the values associated with incorrect decisions.

The expected value of the decision is maximized by selecting the $C_i(s)$ that yields to the highest value of V_i. The decision rule that will achieve the objective is: Compute $V_i(D_n)$ for each of the $C_i(s)$. Select the $C_i(s)$ that gives the largest $V_i(D_n)$.

A simpler expression for V_i is obtained when, as is often the case, the values of all incorrect decisions, V_{ij}, are, or can be, assumed to be equal. In this case, the value of the incorrect decisions can be set arbitrarily to zero without altering the relative values of the correct and incorrect decisions, and therefore without altering the decision outcome. With $V_{ij} = 0$, the expected value of choosing $C_i(s)$ becomes simply

$$V_i(D_n) = V_{ii}P(C_i|D_n) \qquad (3.2)$$

So far, we have said nothing about how to compute the posterior probabilities, $P(C_i|D_n)$, which are essential for computation of $V_i(D_n)$. These probabilities are most easily determined through the use of Bayes Rule.[19] Using this rule, we may express the posterior probabilities as

$$P(C_i|D_n) = \frac{p(D_n|C_i)P(C_i)}{p(D_n)} \tag{3.3}$$

where $p(D_n|C_i)$ is the probability density of D_n given that the dynamics are $C_i(s)$. This probability density is called the likelihood of D_n under $C_i(s)$. $P(C_i)$ is the probability that the dynamics were $C_i(s)$ at the beginning of the CI, before the data D_n were obtained. It is called the prior probability of $C_i(s)$. $p(D_n)$ is the probability density of D_n and it is the same for all $C_i(s)$. All of these probabilities and probability densities are subjective and are the human controller's estimates of the true probabilities.

We can use Eq. (3.3) to express $V_i(D_n)$, the expected value of choosing $C_i(s)$, in terms of the prior probabilities and the likelihoods.

$$V_i(D_n) = \sum_{j=0}^{K} \frac{V_{ij}P(C_j)p(D_n|C_j)}{p(D_n)} \tag{3.4}$$

Since $p(D_n)$ in Eq. (3.4) is the same for all $C_i(s)$, it plays the role of a normalizing constant and its value does not change the relative values of the $V_i(D_n)$. Without loss of generality, we may write

$$V_i'(D_n) = \sum_{j=0}^{K} V_{ij}P(C_j)p(D_n|C_j) \tag{3.5}$$

where $V_i'(D_n)$ and $V_i(D_n)$ differ by the factor $p(D_n)$. Any decision rule based on $V_i'(D_n)$ will give results identical to one based on $V_i(D_n)$, and for simplicity we will often use $V_i'(D_n)$ to denote the computed expected value of a decision.

If the probability densities of D_n given each of the $C_i(s)$ are known, and if the prior probabilities, $P(C_i)$ and the values, V_{ij}, are known for each of the $C_i(s)$, the value of choosing each of the $C_i(s)$ can be computed from Eq. (3.4) or Eq. (3.5). The appropriate decision rule to maximize the expected value is: Select $C_i(s)$ for which

$$V_i(D_n) \geq V_k(D_n)$$

$$\sum_{j=0}^{K} V_{ij} P(C_j|D_n) \geq \sum_{j=0}^{K} V_{kj} P(C_j|D_n) \tag{3.6}$$

$$\sum_{j=0}^{K} V_{ij} P(C_j) p(D_n|C_j) \geq \sum_{j=0}^{K} V_{kj} P(C_j) p(D_n|C_j)$$

for all $k \neq i$

This rule assures the selection of the $C_i(s)$ whose value is greatest.

If the values of all incorrect decisions are equal so that they can be set to zero, $V_{ij}=0$ for $i \neq j$, we obtain a simpler formulation of the decision rule: Select the $C_i(s)$ for which

$$\frac{p(D_n|C_i)}{p(D_n|C_k)} \geq \frac{V_{kk} P(C_k)}{V_{ii} P(C_i)} \quad \text{for all } k \neq i \tag{3.7}$$

where V_{kk} is the value of correctly choosing $C_k(s)$. The ratio on the left side of this equation is called the likelihood ratio of D_n under $C_i(s)$ relative to $C_k(s)$.

All of these alternate expressions for the decision rule are equivalent. Which one is most convenient to use depends upon the particular problem that is to be solved.

This discussion provides the basis for the models that we will develop for the monitoring and identification functions of the adaptive process. Each of these will require some additional assumptions and the separate determination of the appropriate probabilities and values. These issues are discussed in the following sections.

2. The Monitoring Model

The monitoring task requires the human controller to observe the behavior of the system and to decide whether or not the controlled dynamics have changed. He knows that initially the dynamics were $C_0(s)$. His decision task is to choose one of two alternatives: $C_0(s)$ or $\overline{C}_0(s)$. $\overline{C}_0(s)$ denotes that the dynamics are <u>not</u> $C_0(s)$. By virtue of his training, he can predict the response of all of the possible dynamics to his control movements. Since a report that a transition has occurred merely initiates an identification process, the controller loses little by reporting a transition when none has occurred. However, if he misses a transition, his adaptation will be delayed and large tracking errors may result. Thus, there is low cost (high value) associated with a false alarm and a high cost (low value) associated with a miss.

We use V_{00} to denote the value of correctly deciding in favor of $C_0(s)$, $V_{\bar{0}\bar{0}}$ to denote the value of correctly choosing $\overline{C}_0(s)$ $V_{0\bar{0}}$ to denote the value of choosing $C_0(s)$ when $\overline{C_0}(s)$ is correct (a miss), and $V_{\bar{0}0}$ to denote the value of choosing $\bar{C}_0(s)$ when $C_0(s)$ is correct (a false alarm). From Eq. (3.6) we see that a transition should be reported when

$$V_{\bar{0}\bar{0}}P(\bar{C}_0|D_n) + V_{\bar{0}0}P(C_0|D_n) \geq V_{00}P(C_0|D_n) + V_{0\bar{0}}P(\bar{C}_0|D_n) \quad (3.8)$$

$P(\bar{C}_0|D_n)$ and $P(C_0|D_n)$ are the posterior probabilities of $\bar{C}_0(s)$ and $C_0(s)$, respectively. By rearranging terms, we obtain

$$(V_{\bar{0}\bar{0}}-V_{0\bar{0}})P(\bar{C}_0|D_n) \geq (V_{00}-V_{\bar{0}0})P(C_0|D_n) \quad (3.9)$$

But since

$$P(\bar{C}_0|D_n) = \sum_{i=1}^{K} P(C_i|D_n) \quad (3.10)$$

we can write for Eq. (3.9)

$$(V_{\bar{0}\bar{0}}-V_{0\bar{0}}) \sum_{i=1}^{K} P(C_i|D_n) \geq (V_{00}-V_{\bar{0}0})P(C_0|D_n) \quad (3.11)$$

Bayes rule, Eq. (3.3), can be used to expand the posterior probabilities, $P(C_i|D_n)$, and we obtain

$$(V_{\bar{0}\bar{0}}-V_{0\bar{0}}) \sum_{i=1}^{K} p(D_n|C_i)P(C_i) \geq (V_{00}-V_{\bar{0}0})p(D_n|C_0)P(C_0) \quad (3.12)$$

All the terms in summation on the left side of Eq. (3.12) are positive. Therefore, if for any $C_i(s)$

$$(V_{\bar{0}\bar{0}}-V_{0\bar{0}})p(D_n|C_i)P(C_i) \geq (V_{00}-V_{\bar{0}0})p(D_n|C_0)P(C_0) \quad (3.13)$$

Eq. (3.12) will also be true and a transition should be reported. Equation (3.13) is relatively simple to use, and if the densities $p(D_n|C_i)$ do not overlap much, it will lead to decisions that are essentially the same as those made using the complete formulation of Eq. (3.12).

To summarize: A transition should be reported if any one $C_i(s)$ can be found for which Eq. (3.13) is satisfied, or if Eq. (3.11) or Eq. (3.12) are satisfied for the entire set of $C_i(s)$.

In order to specialize the decision theory just presented so as to derive a model for the monitoring function, we must postulate characteristics of the data D_n, of the likelihood functions $p(D_n|C_i)$, of the prior probabilities $P(C_i)$, and of the values, $V_{\overline{0}\overline{0}}$, $V_{0\overline{0}}$, V_{00}, and $V_{\overline{0}0}$. We must then reformulate the decision rule in terms of these postulated quantities. Each of these issues are considered in order in the development which follows.

a. Information Used for Monitoring

The human controller presumably processes the error and stick signals to decide whether or not a transition has occurred. A key problem in the development of a model for the monitoring process is to determine what information must be extracted from these two signals by the model in order to predict the detection performance of the controller with reasonably good accuracy.

For those transitions that result in rapidly increasing errors, such as a large increase in the gain or a change of polarity of the gain of the controlled dynamics, a variety of different functions of the error will result in good predictions of detection performance. In our early studies[13] we found that the detection process could be represented well by a model that reported a transition whenever an excessively large error occurred. The model would have been equally good had we postulated that error rate was used as the basis for detection. Transitions of this kind are usually accompanied by unusually large values of the error and its derivatives and by control movements of large amplitude. The choice of the information to be used for detection is not critical.

Many transitions, such as a decrease in the gain of the controlled dynamics, result in slowly increasing errors. For these the choice of the information to be used for detection becomes more critical. In particular, error magnitude alone appears to be insufficient.[14,15] Miller[14] in a study performed under this contract, suggested the use of error rate as the basis for detection. He proposed a model in which the observed change in the error rate during a control interval, $\Delta\dot{e}$, is compared to the change in error rate expected from the initial dynamics, $C_0(s)$. If the difference between the observed and the expected changes in error rate is greater than a criterion value, the model reports that a transition has occurred.

The use of error rate as the basis for detection is reasonable from several points of view. The error rate is a quantity that is easily perceived by the human controller. Proper control over error rate is essential for stability and good performance.

To achieve stability in a steady-state tracking situation, the human controller must adjust his characteristics so that in the region of the gain crossover frequency

$$HC(s) \doteq \frac{\omega_c}{s} e^{-\tau s} \qquad (3.14)$$

where H(s) is the describing function of the human controller. ω_c is the gain crossover frequency and τ is the effective time delay of the human controller.[1] In many control situations Eq. (3.14) is a good first approximation to the open-loop characteristics of the system over most of the frequency band in which there is a significant input power. For these situations Eq. (3.14) implies that the human controller establishes a well specified relation between the output rate at time 5, and the error τ sec earlier. It will be convenient to set the duration of the control interval CI equal to τ.

In a compensatory control situation the output rate is not available to the controller and he must use the error rate to estimate the output rate. Thus, we may say that to a first approximation the human controller makes control movements so that the error rate at time t will be proportional to the error τ sec earlier. That is,

$$\dot{e}(t) \doteq -\omega_c e(t-\tau) \qquad (3.15)$$

We assumed at the beginning of this development that discrete observations of the error were taken at the end of each CI. If $\dot{e}(t)$ is the primary information quantity for monitoring, the monitoring decision will be based upon a sequence of sampled values of $\dot{e}(t)$. But the error rate at any time t can be expressed as the sum of the increments in the error rate in all previous control intervals, plus the initial value of the error rate. For large values of t, the initial value should not affect the detection decision very much, and, therefore, it may be neglected. Thus we may use $\Delta\dot{e}(t)$, the change in error rate during a CI, instead of $\dot{e}(t)$ for the monitoring decision. Except for the initial value, the use of $\Delta\dot{e}(t)$ will lead to the same results as will the use of the $\dot{e}(t)$. Since the human controller has more direct control over $\Delta\dot{e}(t)$ than over $\dot{e}(t)$, $\Delta\dot{e}(t)$ will depend more upon the control movements made during the current CI than upon those made during previous CI and successive samples of $\Delta\dot{e}(t)$ will tend to be less highly-correlated than successive samples of $\dot{e}(t)$. Therefore, $\Delta\dot{e}(t)$ is a somewhat better quantity to use in the model than $\dot{e}(t)$.

In order to determine the $\Delta\dot{e}(t)$ that is expected from $C_0(s)$ or any of the other dynamics, the human controller must also make use of his knowledge of his control movements, $c(t)$. Thus, we postulate that the data, D_n, consists of the couple $(\Delta\dot{e},c)$. This is what we will use for both the monitoring and the identification models.

b. Posterior Probabilities

We postulated that the monitoring decision is based upon observations of $\Delta\dot{e}$ and $c(t)$. The posterior probabilities previously written as $P(C_i|D_n)$ must be rewritten in terms of these new quantities. They become $P(C_i|\Delta\dot{e}_n,c_n)$, the probability of $C_i(s)$ conditioned on the $\Delta\dot{e}(t)$ and $c(t)$ observed during the n^{th} CI. To simplify notation we will write these probabilities as $P(C_i|\Delta\dot{e},c;n)$, and when the CI index n is not an issue in the development we will use simply $P(C_i|\Delta\dot{e},c)$.

Bayes Rule, Eq. (3.3) must also be rewritten in terms of these new quantities

$$P(C_i|\Delta\dot{e},c;n) = \frac{p(\Delta\dot{e},c|C_i;n)P(C_i;n)}{p(\Delta\dot{e},c;n)} \tag{3.16}$$

where all the probabilities are for the n^{th} CI. We note that $c(t)$ is a quantity that is under the control of the human controller and that the decision problem is to judge whether or not $\Delta\dot{e}(t)$ is appropriate for both the dynamics,

$C_i(s)$, and the control movement, $c(t)$. Thus, it is more useful to write Bayes Rule in the following form, which can be easily shown to be equivalent to Eq. (3.16).

$$P(C_i|\Delta\dot{e},c;n) = \frac{p(\Delta\dot{e}|C_i,c;n)P(C_i;n)}{p(\Delta\dot{e}|c;n)} \qquad (3.17)$$

As before, when the CI interval index n is not of concern, we will use the simpler expression

$$P(C_i|\Delta\dot{e},c) = \frac{p(\Delta\dot{e}|C_i,c)P(C_i)}{p(\Delta\dot{e}|c)} \qquad (3.18)$$

c. Likelihood Functions

To use Eqs. (3.17) or (3.18) we must evaluate the likelihoods $p(\Delta\dot{e}|C_i,c)$. We emphasize that the $p(\Delta\dot{e}|C_i,c)$ are the subjective likelihoods, the human controller's estimates of the true likelihoods.

The $p(\Delta\dot{e}|C_i,c)$ are the probability density functions of the observed values of $\Delta\dot{e}(t)$ when the dynamics are $C_i(s)$ and the control movement is $c(t)$. The properties of these functions are determined by the characteristics of $C_i(s)$ and the input disturbance, $d(t)$. From the relations inherent in the system block diagram of Fig. 1,

$$\Delta\dot{e}(t) = \Delta\dot{d}(t) - \Delta\dot{o}_i(t) \qquad (3.19)$$

where $\Delta\dot{e}$, $\Delta\dot{d}$ and $\Delta\dot{o}_i$ are the changes that occur in the derivatives of the error, the disturbance input, and the output of dynamics $C_i(s)$ during the CI. These increments are defined by relations of the form

$$\Delta\dot{d}(t) = \dot{d}(t) - \dot{d}(t-T). \tag{3.20}$$

Since all the signals in the system were assumed to be normal, the $p(\Delta\dot{e}|C_i,c)$ will be normal probability densities. The mean μ_i and variance σ_i^2, which completely specify these densities, must be determined for each of the $C_i(s)$.

Recall that we assumed that the controller made no attempt to predict the input disturbance, d(t), and that his training was sufficient for him to predict the $\Delta\dot{o}_i$, the change in output rate resulting from his control movements c(t) for each of the possible $C_i(s)$. Under these assumptions the expected value of $\Delta\dot{e}_i$, the $\Delta\dot{e}$ predicted by the human controller under the assumption that $C_i(s)$ are the dynamics and that c(t) is the control movement is

$$E[\Delta\dot{e}_i] = E[\Delta\dot{d} - \Delta\dot{o}_i] = E[-\Delta\dot{o}_i] = -\Delta\dot{o}_i \tag{3.21}$$

But $E[\Delta\dot{e}_i]$ is the mean of $p(\Delta\dot{e}|C_i,c)$, and therefore,

$$\mu_i = -\Delta\dot{o}_i \tag{3.22}$$

Observed values of $\Delta\dot{e}$ will be distributed about μ_i. To indicate this fact, we write

$$\Delta\dot{e} = \mu_i + \delta\dot{e}_i \tag{3.23}$$

where $\delta\dot{e}_i$ is the deviation of $\Delta\dot{e}$ from its mean value when $C_i(s)$ is the dynamics and c(t) is the control movement. The variance of $\Delta\dot{e}$ is just the variance of $\delta\dot{e}_i$.

From Eq. (3.19) it is apparent that $\delta\dot{e}_i$ has two components that we assume are statistically independent. One of these is equal to $\Delta\dot{d}$ and the other to the errors made by the human controller in predicting $\Delta\dot{\delta}_i$. We postulate that the errors of prediction are normal, statistically independent in successive CI, and proportional to the magnitude of $\Delta\dot{\delta}_i$. Thus, we may write

$$\delta\dot{e}_i = \Delta\dot{d} + \eta\Delta\dot{\delta}_i \tag{3.24}$$

where η is the random coefficient representing the fractional error in the prediction of $\Delta\dot{\delta}_i$.

The variance of $\delta\dot{e}_i$ is $\sigma_i^{\;2}$, the desired variance of $p(\Delta\dot{e}|C_i,c)$.

$$\begin{aligned} \sigma_i^{\;2} &= \text{Var}\ [\delta\dot{e}_i] \\ &= \text{Var}[\Delta\dot{d} + \eta\Delta\dot{\delta}_i] \qquad (3.25) \\ &= \sigma_{\Delta\dot{d}}^2 \quad (\Delta\dot{\delta}_i)^2\sigma_\eta^{\;2} \\ &= \sigma_{\Delta\dot{d}}^2 + \mu_i^{\;2}\sigma_\eta^{\;2} \end{aligned}$$

where $\sigma_{\Delta\dot{d}}^2$ is the variance of $\Delta\dot{d}$ and $\sigma_\eta^{\;2}$ is the variance of η. When $\sigma_{\Delta\dot{d}}$ is small compared to $\mu_i\sigma_\eta$, the standard deviation of $\delta\dot{e}_i$ is approximately proportional to the magnitude of its mean.

Given μ_i and σ_i the expression for $p(\Delta\dot{e}|C_i,c)$ can be written as

$$p(\Delta\dot{e}|C_i,c) = \frac{1}{\sqrt{2\pi}\,\sigma_i} e^{-\frac{(\Delta\dot{e} - \mu_i)^2}{2\sigma_i^2}}$$

$$\doteq \frac{1}{\sqrt{2\pi}\,\sigma_\eta |\mu_i|} e^{-\frac{\delta\dot{e}_i^{\,2}}{2\sigma_\eta^{\,2}\mu_i^{\,2}}} \qquad (3.26)$$

where in this context $\delta\dot{e}_i$ is taken to be the deviation of the observed $\Delta\dot{e}$ from μ_i.

Equations (3.22) and (3.25) specify the mean and variance of the distributions $p(\Delta\dot{e}|C_i,c)$. There is only one unknown parameter in these equations, $\sigma_\eta^{\,2}$. All the other quantities can be computed from knowledge of $C_i(s)$, the input statistics and the control movement, $c(t)$. We postulate that the human controller develops estimates of these quantities for each of the possible controlled dynamics and, where necessary, such as the case of first order dynamics, for each possible change of dynamics.

d. Prior Probabilities

The prior probabilities, $P(C_i)$, are not constant for all CI. These probabilities change as the human controller obtains more information about the state of the system.

Consider the control situation that we have investigated in our experiments. Assume that the controlled dynamics has been $C_0(s)$ for a long time. Let $n = 0$ be the index of the present CI. During the period prior to $n = 0$, the human controller's estimates of $P(C_0)$ will converge to unity and his estimates of $P(C_i)$ for $i \neq 0$ will converge to zero. We assumed that a transition could occur at the beginning of any CI with probability of q. Typically, q will be small, of the order of .01. All K possible dynamics are equally likely for the transition.

The prior probabilities for the present CI $P(C_i;0)$, are the posterior probabilities at the end of the previous interval modified to take into account the probability that a transition occurred at the beginning of the present CI. Since the posterior probabilities are 1 and 0, respectively,

$$P(C_0;0) = 1 - q \doteq 1$$

and (3.27)

$$P(C_i;0) = q/K \qquad \text{for } i \neq 0$$

In the first of these equations we took advantage of the assumption that q was very small.

$P(C_i;0)$ will be very small. For the human controller to be able to detect a transition in the CI in which it occurred, $\delta\dot{e}_0$ will have to be large so that the likelihood $p(\Delta\dot{e}|C_0,c)$ will be small.

For the n^{th} CI, the prior probabilities must be expressed in terms of the posterior probabilities for the $(n-1)^{st}$ CI. Thus,

$$P(C_0;n) \doteq P(C_0|\Delta\dot{e},c;n-1)$$

and (3.28)

$$P(C_i;n) \doteq \frac{q}{K}P(C_0|\Delta\dot{e},c;n-1) + P(C_i|\Delta\dot{e},c;n-1)$$

for $i \neq 0$.

The two terms in the expression for $P(C_i;n)$ are, respectively: (1) the probability that the dynamics were $C_0(s)$ in CI n-1 and that a transition occurred at the beginning of CI n; and (2) the probability that the dynamics were $C_i(s)$ in CI (n-1). We assume that the probability of two transitions, i. e., from C_0 to $C_i(s)$ and then from $C_i(s)$ to $C_j(s)$ is negligible.

By making use of Eq. (3.17) a recurrence relation for these probabilities can be obtained.

$$P(C_0;n) \doteq \frac{p(\Delta\dot{e}|C_0,c;n-1)P(C_0;n-1)}{p(\Delta\dot{e}|c;n-1)} \tag{3.29}$$

and

$$P(C_i;n) \doteq \frac{q}{K}\,\frac{p(\Delta\dot{e}|C_0;c;n-1)P(C_0;n-1)}{p(\Delta\dot{e}|c;n-1)} + \frac{p(\Delta\dot{e}|C_i,c;n-1)P(C_i;n-1)}{p(\Delta\dot{e}|c;n-1)} \tag{3.30}$$

for $i \neq 0$

It is apparent from Eq. (3.29) and Eq. (3.30) that the prior probabilities gradually change as more information is obtained from successive CI. Although it will be difficult to detect the transition in the CI in which it occurred, $P(C_1;n)$ will increase and $P(C_0;n)$ will decrease with successive CI following the transition, because the likelihoods $p(\Delta\dot{e}|C_0,c;n-1)$ and $p(\Delta\dot{e}|C_1,c;n-1)$ will reflect the fact that the dynamics have changed from $C_0(s)$ to some $C_1(s)$. Detection in a post-transition CI will, therefore, be easier than detection in the CI in which the transition occurred.

e. Values of Decision Alternatives

In the monitoring situation little penalty is incurred for reporting a transition when, in fact, none occurred. A false alarm will simply lead to a superfluous attempt to identify the controlled-dynamics, a process that should result in $C_0(s)$ being selected as the dynamics. If a transition is missed, however, the penalty in terms of the resulting error may be large, and this is a situation to be avoided.

Recall that by virtue of Eq. (3.12) the monitoring decision depends upon the quantities $(V_{\bar{0}\bar{0}} - V_{0\bar{0}})$ and $(V_{00} - V_{\bar{0}0})$. $V_{\bar{0}\bar{0}}$ is the value of correctly deciding that a transition occurred, and V_{00} the value of deciding correctly no transition occurred, respectively. $V_{0\bar{0}}$ is the value of a miss, and $V_{\bar{0}0}$ is the value of a false alarm. The relative importance of the two kinds of errors suggests that $V_{0\bar{0}}$ should be low and $V_{\bar{0}0}$ should be high. On the other hand, correctly deciding that no transition has occurred means that the controller will not have to be concerned with identification and modification and that there will not be a large increase of the system error which usually occurs following a transition. Thus, it is reasonable to assign a higher value to

V_{00} than to $V_{\bar{0}\bar{0}}$. Thus, the quantities $(V_{\bar{0}\bar{0}}-V_{0\bar{0}})$ and $(V_{00}-V_{\bar{0}0})$ will tend to be more nearly equal than would be expected from consideration of the miss and false alarm values above.

There are a few other considerations to take into account. First, we do not want too many false alarms. Second, when $\delta\dot{e}_0$ (the deviation of $\Delta\dot{e}$ from that expected if $C_0(s)$ were the dynamics) is very small, say of the order of $\sigma_{\Delta\dot{d}}$, (the standard deviation of $\Delta\dot{d}$) we would expect the controller to be content to keep $C_0(s)$ as his choice for the dynamics. Third, as is apparent from Eq. (3.6) the values act in concert with the prior probabilities to determine the decision and it is difficult to separate out the effects of each upon the decision.

Thus, depending upon the prior probabilities, we don't want $(V_{00}-V_{\bar{0}0})$ to be too low with respect to $(V_{\bar{0}\bar{0}}-V_{0\bar{0}})$. In fact, to minimize detections when $\delta\dot{e}_0$ is small, we might like $(V_{00}-V_{\bar{0}0})$ to be somewhat larger than $(V_{\bar{0}\bar{0}}-V_{0\bar{0}})$. Given the uncertainty introduced by the effects of the prior probabilities, let us, for the time being, set $(V_{00}-V_{\bar{0}0})$ equal to $(V_{\bar{0}\bar{0}}-V_{0\bar{0}})$. We will re-evaluate this choice after we have examined the experimental results.

f. Decision Rule

The decision rule given by Eq. (3.12) can now be rewritten in terms of the probabilities and values appropriate to the monitoring situation. First, let us restate the rule. For each CI we postulate that the human controller determines whether or not

$$(V_{\bar{0}\bar{0}}-V_{0\bar{0}}) \sum_{i=1}^{K} p(D_n|C_i)P(C_i) \geq (V_{00}-V_{\bar{0}0})p(D_n|C_0)P(C_0) \tag{3.31}$$

is satisfied. If it is, he decides that a transition has occurred.

We have postulated that

$$(V_{\bar{0}\bar{0}}-V_{0\bar{0}}) = (V_{00}-V_{\bar{0}0}) \qquad (3.32)$$

We have also postulated that the likelihoods are in the form used in Eq. (3.17). Making use of these results we can write for Eq. (3.31)

$$\sum_{i=1}^{K} p(\Delta\dot{e}|C_i,c;n)P(C_i;n) \geq p(\Delta\dot{e}|C_0,c;n)P(C_0;n) \qquad (3.33)$$

where the prior probabilities $P(C_i;n)$ and $P(C_0;n)$ are given by Eqs. (3.27),(3.29) and (3.30).

Two simplifications of Eq. (3.33) can be made. First, we can take advantage of the fact that the inequality condition will be satisfied if for any term of the summation

$$p(\Delta\dot{e}|C_i,c;n)P(C_i;n) \geq p(\Delta\dot{e}|C_0,c;n)P(C_0;n) \qquad (3.34)$$

Second, since the probabilities of all alternative dynamics must sum to unity, we can relate the summation of the left side of Eq. (3.33) to the term on the right.

$$\sum_{i=1}^{K} \frac{p(\Delta\dot{e}|C_i,c;n)P(C_i;n)}{p(\Delta\dot{e}|c;n)} = 1 - \frac{p(\Delta\dot{e}|C_0,c;n)P(C_0;n)}{p(\Delta\dot{e}|c;n)} \qquad (3.35)$$

Using this relation, Eq. (3.33) may be written

$$\frac{p(\Delta\dot{e}|C_0,c;n)P(C_0;n)}{p(\Delta\dot{e}|c;n)} \leq 0.5 \qquad (3.36)$$

The decision rule is then: Report a transition if one of Eq. (3.33), (3.34) or (3.36) is satisfied.

The decision rule in terms of Eq. (3.34) is illustrated in Fig. 4 for a typical detection situation. The distribution of $p(\Delta\hat{e}|C_i,c;n)$ is shown as a function of $\Delta\hat{e}$ for the three choices of $C_i(s)$ in a hypothetical situation in which two dynamics are possible in addition to $C_0(s)$.

In making Fig. 4 we assumed that $P(C_i;n)$ was the same for all three choices. The expected values of $\Delta\hat{e}$ for the three alternatives, μ_0, μ_1, and μ_2, are also given in the figure. $\Delta\hat{e}'$ is a hypothetical observed value of $\Delta\hat{e}$ and the $\delta\hat{e}_0$, $\delta\hat{e}_1$, and $\delta\hat{e}_2$, also shown in the figure, are deviations of $\Delta\hat{e}'$ from the three mean values. Note that the standard deviations of the distributions increase as μ increases. It is clear from the figure that $C_0(s)$ is the decision that should be made and that a transition should not be reported.

Figure 4 also illustrates that the decision rule can also be expressed directly in terms of $\Delta\hat{e}$. A set of regions called acceptance regions can be defined such that a value of $\Delta\hat{e}$ in Region A_i will lead to the choice of $C_i(s)$. In terms of Fig. 4, the definitions are:

$$\begin{aligned} A_0&: \; 0 \le \Delta\hat{e} < c_{02} \\ A_1&: \; \Delta\hat{e} < 0 \\ A_2&: \; c_{02} \le \Delta\hat{e} \end{aligned} \tag{3.37}$$

where c_{02} is the value of $\Delta\hat{e}$ at which $p(\Delta\hat{e}|C_0,c;n)$ and

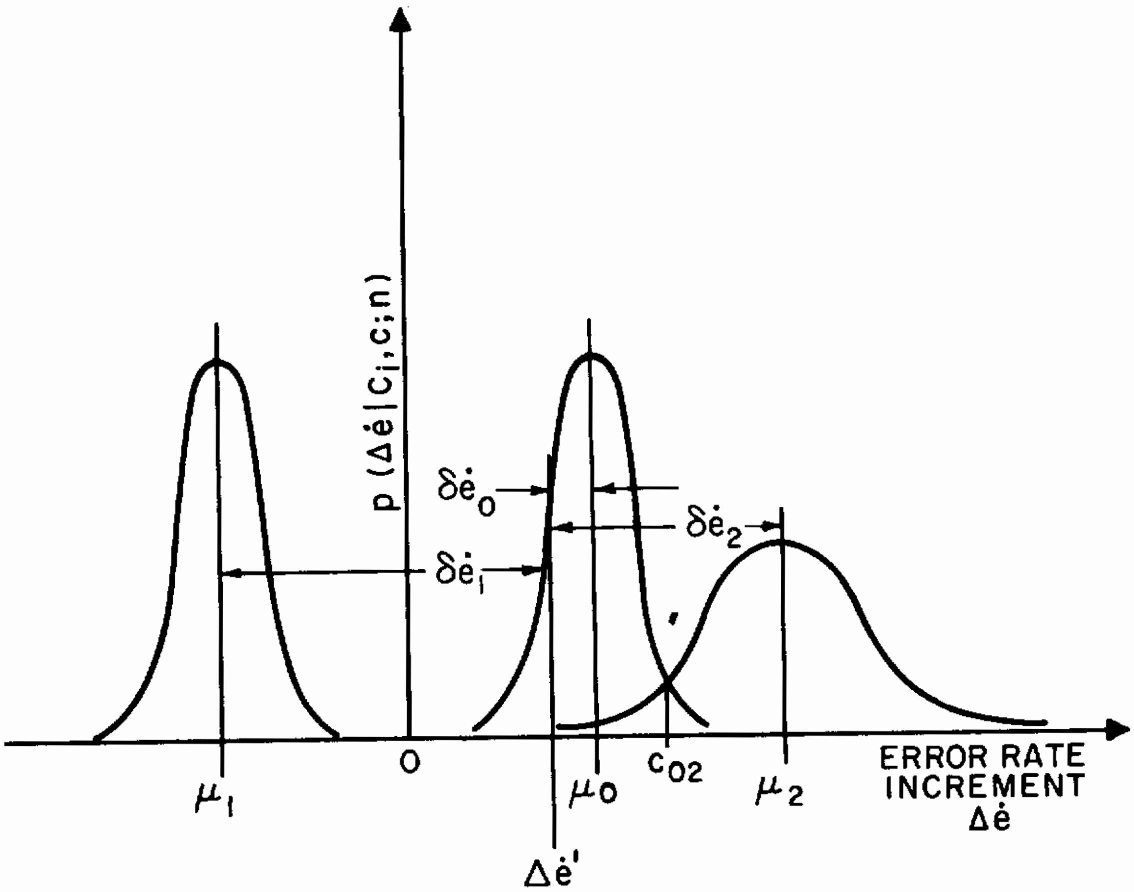

Fig. 4. An illustration of the detection problem.

$p(\Delta\dot{e}|C_2,c;n)$ cross. The acceptance regions can also be defined in terms of the $\delta\dot{e}_1$ by a similar set of equations.

It is perhaps more reasonable to expect the human controller to use a decision rule based on acceptance regions defined in terms of $\Delta\dot{e}$ or $\delta\dot{e}_1$. If $\delta\dot{e}_1$ is small, relative to its standard deviation, and the other $\delta\dot{e}_j$ are large, then a decision in favor of $C_1(s)$ is likely to be correct. It is clear from Fig. 4 that decisions based upon a correctly defined set of acceptance regions will be the same as those based upon Eqs. (3.33), (3.34) and (3.36).

3. Identification Model

After a transition has been detected, the human controller will probably want to confirm the correctness of his detection decision and then determine which of the K dynamics $C_1(s) \ldots. C_K(s)$ are in the system. Thus, the identification task consists of choosing one of K+1 possible control dynamics.

a. Outline of the Identification Model

The decision processes involved in identification are essentially the same as those involved in monitoring, and the model we postulate for identification is very similar to the monitoring model. Most of the results obtained in the development of the monitoring model can be applied directly to the identification model.

For identification we postulate a model whose decision objective is maximization of expected value. The decisions are based on observations of $\Delta\dot{e}$ and $c(t)$ during a CI. The observed value of $\Delta\dot{e}$ is compared with the $\Delta\dot{e}_i$, the values predicted from knowledge of $C_i(s)$ and of $c(t)$. The expected value of choosing $C_i(s)$ is

$$V_i(\Delta\dot{e},c;n)= \sum_{j=0}^{K} V_{ij}P(C_j|\Delta\dot{e},c;n)$$

$$V_i'(\Delta\dot{e},c;n)= \sum_{j=0}^{K} V_{ij}P(C_j;n)p(\Delta\dot{e}|C_j,\rho;n) \tag{3.38}$$

where once again we have chosen to ignore the factor $p(\Delta\dot{e}|c;n)$. The likelihood functions in Eq. (3.38) have the same normal distribution as before. The mean is μ_i and the variance σ_i^2 has two components $\sigma_{\Delta\dot{d}}^2$ and $\mu_i^2\sigma_n^2$.

We will permit the identification model to use the same data as was used for detection. Thus, identification can be made in the same CI as is detection. However, it need not be, and in certain circumstances additional data may be required. This situation may arise if we use different values, V_{ii}, for identification and for monitoring.

In the identification situation there can be a large penalty for incorrectly choosing one of the alternative dynamics. This penalty arises from the fact that the controller will modify his control strategy immediately after identification is made. The modification process takes some time, and if it is based on an incorrect identification, large system errors may result. Still more time will be required until the mistake is corrected and the proper control strategy is established. The controller would be well-advised, therefore, to be conservative and to not modify his characteristics until he is fairly certain that the system is correctly identified.

The identification decision involves finding the $C_i(s)$ for which inequalities of the following form are satisfied:

$$V_i \geq V_k$$

$$\sum_{j=0}^{K} V_{ij} P(C_j) p(\Delta\dot{e}|C_j,c) \geq \sum_{j=0}^{K} V_{kj} P(C_j) p(\Delta\dot{e}|C_j,c) \qquad (3.39)$$

for all $k \neq i$

where we have dropped the index n to simplify the notation. The terms for j=i and j=k can be extracted from the summation and after some rearranging we obtain an expression similar to that derived for the monitoring decision

$$(V_{ii}-V_{ki})P(C_i)p(\Delta\dot{e}|C_i,c) \geq$$

$$(V_{kk}-V_{ik})P(C_k)p(\Delta\dot{e}|C_k,c) + \qquad (3.40)$$

$C_0(s)$ is a possible choice so that if a false alarm is made during monitoring it can be corrected by the identification procedure. In many situations the controller will want to act conservatively and continue controlling as if $C_0(s)$ were the dynamics until he is very sure that they are $C_j(s)$. For these situations V_{0j},the value of selecting $C_0(s)$ when the dynamics are C_j, probably should be high and V_{i0}, the value of selecting $C_i(s)$ when $C_0(s)$ are the dynamics should be low. Furthermore, often V_{kj} and V_{ij} for $j \neq 0,i$, or k can be assumed equal. These are the values of choosing $C_k(s)$ and $C_i(s)$, respectively, when the dynamics are actually neither of these nor $C_0(s)$.

If these assumptions are made, Eq. (3.40) can be written first for K=0

$$(V_{ii}-V_{0i})P(C_i)p(\Delta\dot{e}|C_i,c)\geq(V_{00}-V_{i0})P(C_0)p(\Delta\dot{e}|C_0,c) + \sum_{j\neq i,0}(V_{0j}-V_{ij})P(C_j)p(\Delta\dot{e}|C_j,c) \qquad (3.41)$$

for all $i\neq 0$

and, eliminating $C_0(s)$ as a possible choice,

$$(V_{ii}-V_{ki})P(C_i)p(\Delta\dot{e}|C_i,c)\geq(V_{kk}-V_{ik})P(C_k)p(\Delta\dot{e}|C_k,c) \qquad (3.42)$$

for all $k\neq 0,i$

In order for any dynamics other than $C_0(s)$ to be chosen, Eq. (3.41) must be satisfied. If it is, the $C_i(s)$ for which Eq. (3.42) is satisfied for <u>all</u> other $C_k(s)$ should be selected.

In other control situations special treatment of $C_0(s)$ is not appropriate and equal values can be assigned to V_{0j} and to V_{ij}. In these cases, Eq. (3.40) can be written simply as

$$(V_{ii}-V_{ki})P(C_i)p(\Delta\dot{e}|C_i,c)\geq(V_{kk}-V_{ik})P(C_k)p(\Delta\dot{e}|C_k,c) \qquad (3.43)$$

for all $k\neq i$

where we retain the assumption that $V_{kj} = V_{ij}$ for $j\neq 0,i$, or k. The decision rulle now becomes: Select the $C_i(s)$ for which Eq. (3.43) is satisfied for all $C_k(s)$, $k\neq i$.

In Chapter VI we use a still simpler version of the identification model. In that chapter we assume, in much the same way as we did for the monitoring model, that $(V_{ii}-V_{ki})$ and $(V_{kk}-V_{ik})$ are equal for all values of i and k. By making this assumption, we are treating all dynamics equally and are not associating different values to the different choices of $C_i(s)$. With this simplification, the inequality upon which the decision is based becomes:

$$P(C_i)p(\Delta\dot{e}|C_i,c) \geq P(C_k)p(\Delta\dot{e}|C_k,c) \quad \text{for all } k\neq i \tag{3.44}$$

The decision rule is: Select the $C_i(s)$ which satisfies Eq. (3.44) for all k≠i.

c. Recycling

The identification process may result in the selection of $C_0(s)$ as the dynamics which has highest expected value. If this occurs, no change in control strategy will be made. The model will revert to monitoring mode for the next CI. If a detection of a transition is made during this CI, another identification is attempted. The process will be repeated until one of the $C_i(s)$ is identified, or until steady monitoring is achieved.

d. Partial Identification.

In certain control situations a subset of the alternative dynamics may respond similarly to the control movements, c(t). A set of dynamics differing only slightly in gain is an example. Initially, the identification procedure may select one of this subset from the other dynamics, but not correctly select specific dynamics from the subset. Data obtained from subsequent control intervals may be necessary to make this finer discrimination. As soon as the initial identification has been made, the controller can be expected to modify his characteristics in accordance with his current estimate of the identity of the controlled dynamics. As the identification is refined, the human controller can refine his characteristics accordingly.

e. Incorrect Identification

Under certain circumstances the input disturbance may combine with the output of the control dynamics to produce an error signal that is incorrectly identified. This sometimes happens when a reduction of the gain of the controlled element and a change of the polarity of the gain are two of the possible transitions that might occur. If the input forcing function has a high velocity and a high acceleration of the same sign at the time of transition, the error that results after a gain reduction will increase with each successive CI, in much the same way as it would following a polarity reversal. The model should account for these kinds of errors, and some of the experimental results indicate that it does.

4. Selection Model

Once the identification has been made, the controller must select and employ a new control strategy that is appropriate to the new controlled dynamics. If large errors have built up prior to the time that identification is completed, the control strategy selected will be one appropriate to the tracking of transient disturbances. After the transient errors have been reduced, a further modification of control strategy is required in order to track the steady-state disturbances with the new control dynamics.

Since we have assumed that the controller knows the characteristics of all the possible controlled dynamics to the extent that he is able to predict for each the output that results from his control movements, it is also reasonable to assume that he knows the control strategies to use for transient tracking with each of these dynamics. Once an identification has been made, we postulate that the controller merely selects the appropriate control strategy from his memory. Since it will almost always be the case that the errors at identification will be large, usually it will be appropriate for him to use a transient tracking strategy. After the controller has modified his characteristics and managed to reduce most of the accumulated errors, the control strategy will be changed to one appropriate to steady-state tracking. It is not clear whether the human controller has stored a steady-state control strategy for each of the controlled dynamics which he merely retrieves from his memory at the appropriate time, or gradually adjusts the parameters of his transient control strategy to achieve good steady-state performance.

We postulate that the process of selecting the transient tracking strategy from memory is simple, takes a fixed amount of time, is error free, and can be represented simply by a time delay. This retrieval time will be taken to be equal to the duration of a CI, which, in turn, is equivalent to the human controller's time delay τ shown in Eq. (3.14).

We have not studied in detail the process by which the controller decides to change from a transient to a steady-state tracking strategy. We would expect that the change would occur when the error and error rate became small, but we have no information on which to base the selection of the values of these quantities at which the change would be initiated.

C. CONTROL MODEL

We postulate that the control processes of human controller adaptation are steady-state tracking, modification, transient tracking and vernier adjustment. In this section we first present the empirical and theoretical bases for the steady-state tracking and transient tracking, and develop in detail the models for these processes. A simple mode switching model for the modification process is postulated and some remarks on the vernier adjustment process are given.

The assumptions of Section A of this Chapter will be invoked to simplify the development of the control models. The models we develop will be based heavily upon empirical results from studies of manual control systems and upon concepts taken from optimal control theory.

1. Steady-State Tracking

The characteristics of the human controller in linear, single-axis, time-invariant, compensatory manual control systems are well documented.[1] These results can be used directly to model the human controller's characteristics in steady-state tracking.

It has been demonstrated that in a time-invariant compensatory system, the human controller's characteristics can be represented with good accuracy by a quasi-linear describing function, plus a remnant that accounts for those parts of the controller's response that are not linearly related to the input.[1-4] The characteristics of the human controller's describing function can, in turn, be approximated to the first order by the so-called crossover model suggested by McRuer, et al.[1] The crossover model is a simple model that is based on the observation that the human controller adjusts his characteristics so that in the region of the gain crossover frequency the open-loop transfer function of the system is of the form

$$HC(s) \doteq \frac{\omega_c}{s} e^{-\tau s} \tag{3.45}$$

where H(s) is the describing function of the human controller, C(s) is the transfer function of the controlled dynamics, ω_c is the gain crossover frequency, and τ is the effective time delay of the controller. This relation, together with some rules for adjusting ω_c and τ constitute the crossover model.

Approximate values for ω_c and τ for the dynamics and input signals used in our experiments are given in Table 1. These values are those given by McRuer, et al[1]. For other dynamics or inputs, the reader is referred to McRuer's report.

Table 1

APPROXIMATE VALUES OF ω_c and τ FOR THE CROSSOVER MODEL

Dynamics C(s)	ω_c rad/sec	τ sec
K	5.1	.2
K/s	4.6	.24
K/s^2	3.2	.39

Strictly speaking, the crossover model describes the human controller's characteristics only in the region of the gain crossover frequency. We shall, however, assume that this model is valid over the entire frequency band, and neglect the additional lead and lag terms which are frequently found in the human controller's describing functions at high and low frequencies. By using the crossover model, we introduce errors which would not be present if we used a more detailed model. However, we also achieve considerable simplicity, and for our purposes, this tradeoff seems worthwhile.

The crossover model is useful for describing the human controller's characteristics in two distinctly different phases of the adaptive process. Naturally, it is directly applicable in the interval between transitions, when the system is in steady-state and all transient errors resulting from previous transitions are eliminated. The crossover model can also be used to predict the characteristics of the human controller and of the system in the period between the occurrence of a transition and the time at which the human controller begins to modify his characteristics. In this interval, called the retention phase by Weir and Phatak,[12] the controller presumably retains the control strategy that was appropriate to the pre-transition dynamics, $C_0(s)$, even though the dynamics are $C_1(s)$.

If the human controller's describing function in the pre-transition period is $H_0(s)$, in the post-transition retention phase prior to modification, the describing function of the complete open-loop system is $H_0C_1(s)$. The closed-loop behavior of the system is, of course, determined largely by the roots of $1+H_0C_1(s)$. In particular, the relation between system error and the input disturbance is

$$G(s) = \frac{E(s)}{D(s)} = \frac{1}{1+H_0C_1(s)} \tag{3.46}$$

$$= \frac{1}{1+\frac{C_1}{C_0} H_0C_0(s)}$$

where E(s) and D(s) are the Laplace transforms of the error, e(t), and the input disturbance, d(t), respectively. G(s) is the input to error transfer function. We substitute Eq. (3.45) for $H_0C_0(s)$ and let $C_{10}(s)$ denote the ratio C_1/C_0.

$$G(s) = \frac{1}{1+ \frac{\omega_{c0}C_{10}(s)}{s} e^{-\tau_0 s}} \tag{3.47}$$

where ω_{c0} and τ_0 are the values of ω_c and τ appropriate to $C_0(s)$. A Padé approximation may be used for the time delay without great loss of accuracy.

$$G(s) = \frac{1}{1- \frac{\omega_{c0}(s-2/\tau_0)C_{10}(s)}{s(s+2/\tau_0)}} \tag{3.48}$$

It is clear from Eqs. (3.47) and (3.48) that the behavior of the closed-loop system in the immediate post-transition period depends upon the nature of the change in the controlled dynamics, more than upon either the pre- or post-transition dynamics. More precisely, the closed-loop roots are determined by $C_{10}(s)$, the ratio of the two controlled dynamics and by the characteristics of the crossover model. These roots can be determined from a root locus analysis of Eq. (3.48), or by other methods.

A common kind of transition is a change in the gain of the controlled element for which $C_{10}(s) = K_{10}$. K_{10} is the ratio of the post- and pre-transition gains. In Fig. 5 we show the locus of the closed-loop roots as a function of K_{10} for $\omega_{c0} = 4.6$ rad/sec and $\tau_0 = .24$, values from Table 1 for K/s dynamics. Negative values of K_{10} give rise to positive real roots and to non-oscillatory divergence. Gain increases, if large enough, give rise to oscillatory divergence.

Given $\{e(t_0), \dot{e}(t_0) \ldots e^{(i)}(t_0)\}$, the error and its derivatives at the time of transition, the input, d(t), after the transition, and the closed-loop poles, s_j, we can determine e(t) in the immediate post-transition period. If the poles are distinct the expression for the error is

$$e(t) = \sum_{j=1}^{N} \sum_{i=0}^{N-1} a_{ij}\, e^{(i)}(t_0) \exp\left[-s_j(t-t_0)\right] + \sum_{j=1}^{N} h_j \int_{t_0}^{t} d(\tau) \exp\left[-s_j(t-\tau)\right] d\tau \qquad (3.49)$$

where N is the number of poles, $\exp[\cdot]$ is the exponential function, $e^{(i)}(t_0)$ is the ($i^{\underline{th}}$) derivative of e(t) at t_0. The coefficients a_{ij} and h_j are determined from G(s) using the standard methods.[22] The second term on the right is the convolution of the impulse response relating error to the input disturbance d(t).

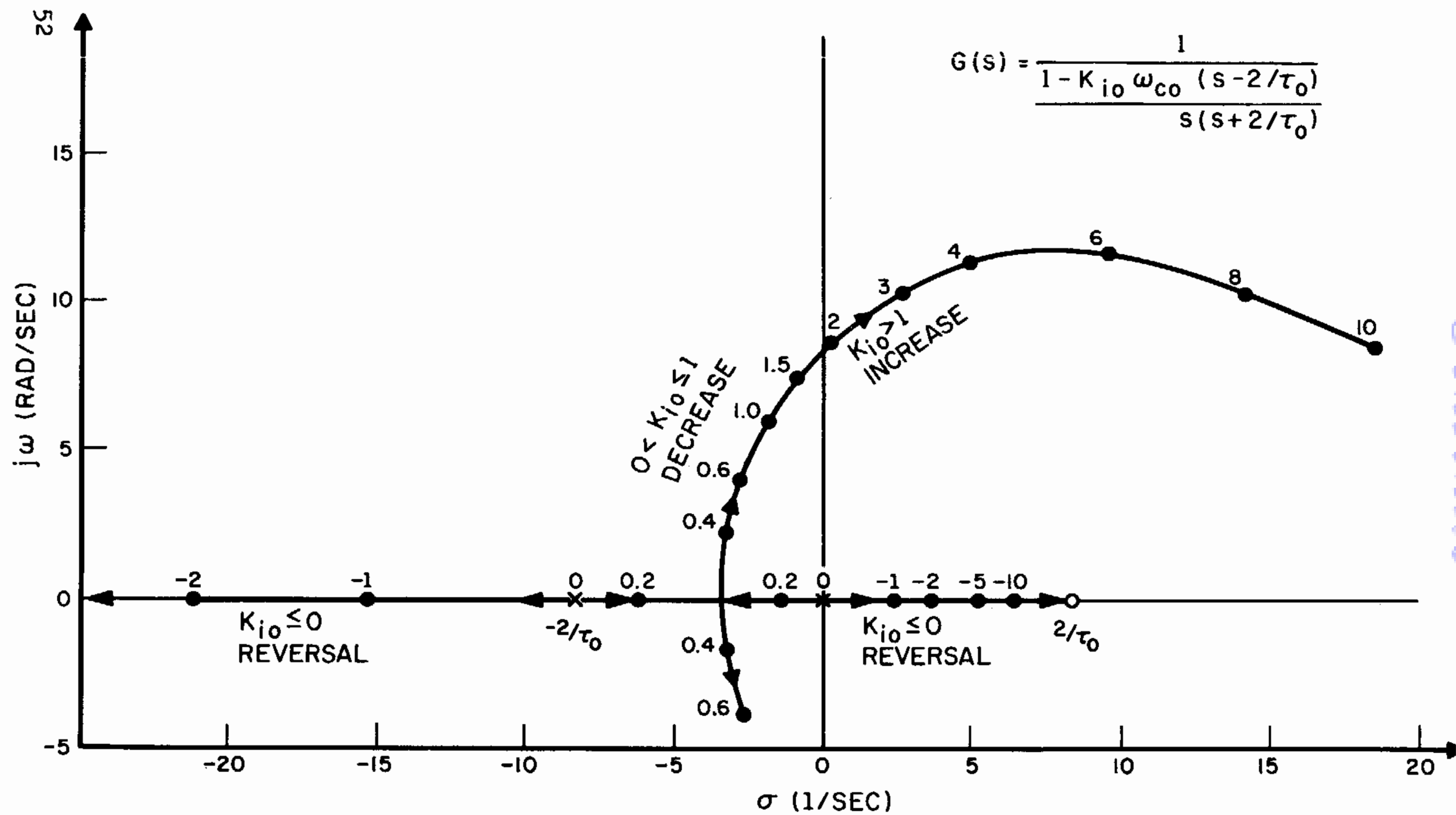

Fig. 5. Locus of roots of G(s) following a change of gain with K/s dynamics as a function of K_{io}, the ratio of post- to pre-transition controlled-element gains.

2. Transient Tracking

We postulate that in the transient tracking phase that begins after modification the human controller adopts the characteristics specified by the crossover model, except that his gain ω_c, is adjusted to give minimum mean-squared error.

The crossover model requires that

$$HC(s) \doteq \frac{\omega_c e^{-\tau s}}{s} \qquad (3.50)$$

where the subscript i is used to indicate that the quantities are appropriate to the post-transition dynamics, $C_1(s)$. We treat the error that exists at the beginning of the transient tracking phase as a step input. The mean-squared error is

$$E^2 = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \left(\frac{D(s)}{1+HC(s)}\right) \left(\frac{D(-s)}{1+HC(-s)}\right) ds \qquad (3.51)$$

where D(s) is the Laplace transform of the disturbance step that was assumed to exist at the beginning of transient tracking. We may choose the step to be of unit amplitude without changing the results, which makes

$$D(s) = \frac{1}{s} \qquad (3.52)$$

A Padé approximation is used for the time delay giving

$$\frac{1}{1+HC(s)} = \frac{s(s+\frac{2}{\tau})}{s(s+\frac{2}{\tau})-\omega_c(s-\frac{2}{\tau})} \qquad (3.53)$$

$$= \frac{s(s+\frac{2}{\tau})}{s^2+(\frac{2}{\tau} - \omega_c)s + \frac{2\omega_c}{\tau}} \qquad (3.54)$$

The substitution of these quantities into Eq. (3.51) yields

$$E^2 = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \frac{(s+\frac{2}{\tau})}{s^2+(\frac{2}{\tau}-\omega_c)s+\frac{2\omega_c}{\tau}} \; \frac{(-s+\frac{2}{\tau})}{s^2-(\frac{2}{\tau}-\omega_c)s+\frac{2\omega_c}{\tau}}\, ds \tag{3.55}$$

The method of Booton, Mathews and Seifert[20] may be used to integrate Eq. (3.55). We find that

$$E^2 = \frac{(\omega_c+\frac{2}{\tau})}{2\omega_c(\frac{2}{\tau}-\omega_c)} \tag{3.56}$$

To find the optimum value of the gain ω_c we differentiate with respect to ω_c and set the result equal to zero. We obtain

$$\omega_{c_{opt}} = .828/\tau \tag{3.57}$$

The closed-loop roots of the optimum system in transient tracking mode are found to be approximately

$$s = -\frac{2}{\tau}\,(.29 \pm j\,\sqrt{.33}) \tag{3.58}$$

This corresponds to a natural frequency $\omega_n = 1.29/\tau$ and a damping factor of 0.45

In Table 2 we give the values of $\omega_{c_{opt}}$ for the dynamics and inputs we have used in our experiments. The values of τ were taken from Table 1. The values of ω_c for steady-state tracking were also from Table 1, and are shown to allow comparison of the two modes of tracking. It is evident from Table 2 that the steady-state ω_c is consistently higher than the corresponding transient $\omega_{c_{opt}}$. Therefore, we would expect the phase margin and damping during steady-state tracking to be considerably less than that during transient tracking.

Table 2

COMPARISON OF ω_c FOR STEADY-STATE AND TRANSIENT TRACKING

Dynamics C(s)	τ sec	Steady State ω_c rad/sec	Transient $\omega_{c_{opt}}$ rad/sec
K	.2	5.1	4.1
K/s	.24	4.6	3.4
K/s^2	.39	3.2	2.1

3. Modification

We postulate a mode switching model for modification. Once the human controller has selected the desired control strategy and retrieved it from memory, he begins to employ it. We will assume that there is a time delay between the retrieval and the activation of the new dynamics. For simplicity this delay will be taken equal to T, the duration of a CI, which, in turn, is taken to be equal to τ, the human controller's effective time delay with post-transition dynamics.

4. Vernier Adjustment

In the vernier adjustment phase, we postulate that the human controller adjusts his gain ω_c (and other parameters that we shall ignore for simplicity) to be the values appropriate to steady-state tracking. We expect this adjustment process to be slow relative to the rate at which the control strategy was changed during modification.

We have not investigated the vernier adjustment process in detail and do not have an analytic model to propose. It seems reasonable to think in terms of a model that employs an iterative procedure for finding the value of ω_c that gives optimal performance in say the minimum mean-squared error sense. A gradient method might be employed in this iterative search process. Standard methods of finding the gradient by introducing perturbation of the gain could be employed.[23]

Presumably the vernier adjustment phase would begin after transient tracking had reduced the error and error rate to a low value. A decision process is involved in determining when to begin vernier adjustment. This was treated briefly in the discussion of the selection part of the decision model.

CHAPTER IV

DESCRIPTION OF EXPERIMENTS

At the beginning of this research program only a limited quantity of data relevant to the human controller's adaptive process was available. Little progress had been made toward achieving a quantitative model for this process. We initiated a series of experiments to enlarge the data base and to increase our understanding of the adaptive process so that we might be better able to develop a model.

The experiments that we performed fall naturally into three groups. The first group of experiments was a detailed study of the ability of the human controller to adapt to known changes in dynamics that occurred at random times. In these experiments the controlled dynamics were switched between two different dynamics, both of which were well known to the controller. A variety of pairs of dynamics were used. These experiments provided basic information about the control processes involved in adaptation.

In the second experiments the effects on the adaptive process of uncertainty about the post-transition dynamics and the effects of an alerting signal were investigated. The results of these experiments provided important information about the identification process.

The third experiment was primarily a detailed study of the decision processes involved in adaptation, although some

Table 3

SUMMARY OF EXPERIMENTAL CONDITIONS

Experiment	Purpose	Subj.	DYNAMICS $C_0(s)$	DYNAMICS $C_1(s)$	Input Trans.	Number of Possible Transitions	Alerted (A) or Non-Alerted (N)	Input Dis-turbance
EXPERIMENT I: STUDIES OF THE CONTROL PROCESS OF ADAPTATION								
IA	Effect of system order on adaptation to gain changes	GK RB	K_0 K_0/s K_0/s^2	K_1 K_1/s K_1/s^2	No	1	N	R1.5
IB	Effect of changes of system order and gain on adaptation	GK RB	K_0 K_0/s K_0/s^2	K_1/s, $K_1, K_1/s^2$ K_1/s	No	1	N	R1.5
EXPERIMENT II: STUDIES OF THE EFFECTS OF TRANSITION UNCERTAINTY AND ALERTING ON THE DECISION PROCESS OF ADAPTATION								
IIA	Effects of C_1 uncertainty and alerting signal	GK RB	K_0/s^2 K_0/s^2 K_0/s^2 K_0/s^2	$K_1, K_1/s, K_1/s^2$ $K_1, K_1/s, K_1/s^2$ $K_1, K_1/s, K_1/s^2$ $K_1, K_1/s, K_1/s^2$	No No No No	1 1 18 18	N A N A	R1.5A R1.5A R1.5A R1.5A
IIB	Effects of input transient	GK RBT	K_0/s K_0/s	K_1/s K_1/s	Yes Yes	1 3	N N	R1.5 R1.5
EXPERIMENT III: DETAILED STUDIES OF DECISION PROCESSES OF ADAPTATION								
IIIA	Studies with K/s dynamics	RBT RGT JCV	K_0/s	K_1/s	No	3	N	R1.5
IIIB	Studies with K/s^2 dynamics	GK	K_0/s^2	K_1/s^2	No	7	N	R1.5A

information about modification and vernier adjustment was obtained from it. The post-transition dynamics were uncertain. In most of the experiments a button was given to the controller for him to report when a transition had occurred. The experimental conditions were carefully controlled to reduce the variation of the adaptive response.

The conditions of each of the experiments are summarized in Table 3. This table gives the purpose of each experiment, the identity and number of subjects used in the experiments, the pre- and post-transition controlled dynamics ($C_0(s)$ and $C_1(s)$, respectively), the number of different controlled dynamics that were possible in a transition, whether or not an input transient could occur instead of a transition, whether or not the subject was alerted that a change of dynamics had occurred, and the input forcing function. The notation used for the forcing functions has the following meaning: R1.5 was a pseudo-gaussian input with rectangular spectrum whose cutoff frequency was 1.5 rad/sec. R1.5A denotes a signal with the same spectrum augmented by a high-frequency shelf composed of low-amplitude components. These inputs are described more fully in Section B.

The procedures followed in the several experiments varied in detail, but were basically similar. Data were taken from a set of experimental sessions. The duration of each session was typically four to eight minutes. During each session a series of transitions were made between the controlled dynamics relevant to the experiment without interrupting the session. Usually between ten and twenty transitions comprised a session. The interval between transitions was not constant, but varied from between 10 and 30 sec. The procedures followed in each experiment are discussed more fully along with the presentation of the experimental results.

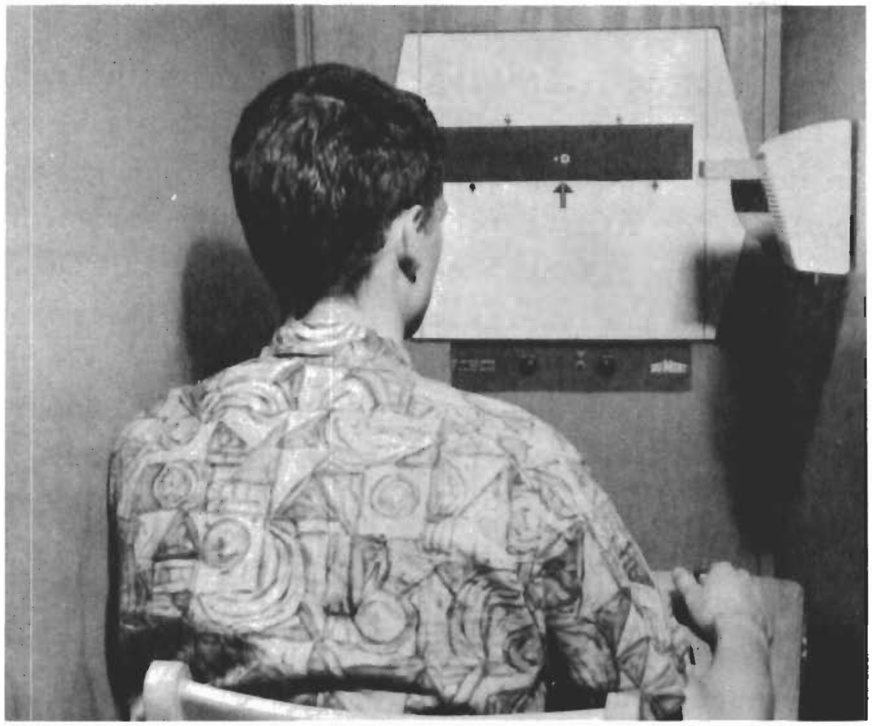

Fig. 6. Photograph of tracking apparatus used in Experiments IA and IB.

Fig. 7. Photograph of tracking apparatus used in Experiments IIB and III.

A. APPARATUS

The apparatus used in the experiments went through several stages of evolution. Initially, we used the apparatus that Young et al[7] employed in their studies with pure gain controlled-element dynamics. This setup shown in Fig. 6 was employed in Experiments IA and IB. Later, we changed the display oscilloscope from an 11x14 inch rectangular display to a 12 cm diameter circular display. In other respects, the apparatus remained the same. This slightly modified setup was used in Experiment IIA. After this experiment we completely redesigned the subject's station. The principal changes were a new enclosure and a new control device. This last setup, shown in Fig. 7, was used for Experiments IIB and III.

1. Initial Apparatus

The apparatus used in Experiments IA and IB is shown in the photograph of Fig. 6. The subject was seated in a small cubicle 6 feet high, 2-1/2 feet wide, and 6 feet long. Placed on the wall directly in front of him was an 11x14 inch oscilloscope whose center was positioned at eye level and approximately 36 inches from the subject. The visual indicators on the display were a 1/2 inch diameter circle and a small dot. The circle remained stationary at the center of the oscilloscope and the horizontal displacement of the small dot was proportional to the error. Mechanical choppers were used in the X and Y axes for presentation of the two visual indicators on the display oscilloscope. The input signals were stored on magnetic tape.

A special display card was used to mask all but the display part of the oscilloscope face, showing only a small rectangular segment of the screen which was 3 inches high and 14 inches wide. At the edge of this display was an arrow marking the horizontal center of the rectangle. Other indications occurred at 3 and 5 inches to the right and left of center. These markings were used to reduce auto-kinetic effects in the tracking cubicle.

The subject made his response by moving a light control stick which protruded through a circular hole in the right arm rest of a student's chair on which the subject was seated. The control stick was spring restrained and easily manipulated by a wrist movement. One pound was required for maximum deflection. The stick could be moved approximately ± 45 degrees from its upright position. The right and left movements of the stick provided the voltages for the input to the controlled dynamics. The stick was free to move in the forward and back directions as well as left and right, but only the latter motion affected the response signal.

The cubicle was lighted indirectly from a fluorescent lamp positioned about one foot above the oscilloscope. The walls of the cubicle were painted dull gray to reduce reflections from the face of the display. The response signal from the subject's control stick was fed to an Electronic Associates Inc. TR-48 analog computer which was used to simulate the controlled dynamics, to control the transition and to construct the displays. The TR-48 computer was programmed to give two parallel channels of controlled dynamics. Each channel could simulate a variety of different controlled dynamics by proper selection of gains and insertion of integrators. While the subject was tracking with one set of dynamics using one channel, the variables of the other channel, which was not connected to the display, were set to give the desired dynamics.

Transitions could occur only when several conditions were satisfied simultaneously: (1) the experimenter had to set an "activate" switch; (2) the input rate had to be of proper sign (usually negative); (3) the input rate had to be greater than some minimum value; and (4) the output of the active channel had to be equal to the output of the inactive channel. The inactive channel was usually held in ground state until the transition occurred, and when this was the case, transitions could occur only when the output was zero. Condition (4) insured continuity of the output through the transition. Conditions (3) and (4) insured that transitions would occur during similar portions of the input so that the responses could be compared more easily.

Pen recordings were made of the input, output, stick and error signals, and of the time of transition. For computation of the ensemble average error following a transition, the tracking error was recorded on magnetic tape and later was played into a Digital Equipment Corporation PDP-1 digital computer.

2. Second Stage Apparatus

For Experiments IIA, the initial apparatus was modified slightly. The large rectangular oscilloscope was replaced by a circular one 12 cm in diameter. The scope was located at eye level approximately 18 inches from the subject. The indicators on the display were a .4 cm diameter circle and a small dot. The circle remained stationary at the center of the oscilloscope and the horizontal displacement of the dot was proportional to the system error.

In Experiment IIA one of the experimental conditions required the use of an audio alerting signal. This signal was a 1000 cps tone of moderate intensity which was presented to the subject through earphones. The signal was turned on when a transition was made from the base condition dynamics and remained on until the transition back to the base condition occurred.

3. Third Stage Apparatus

The apparatus was completely redesigned for Experiments IIB and III. The new setup is shown in Fig. 7. The subject's station was redesigned and placed in a soundproof room. The subject was seated facing an oscilloscope screen whose diameter was 12 cm. A compensatory tracking display was used, but performance feedback was provided the subject by making the diameter of the target circle vary in proportion to the running mean-squared error. The averaging time was 10 sec. The larger the mean-squared error during this period, the larger the circle diameter.

A new control device, a modified Measurement Systems, Inc. Model 435, force-sensitive hand control was installed. The modification to the control consisted of the addition of an 11-inch nylon stick which was attached to the transducer of this control to give an omni-directional, spring-restrained control device. The stick had a spring constant of 2.2×10^6 dynes/cm displacement of the top of the stick.

The control was oriented so that the stick was horizontal and could be moved left-right and up-down in a plane parallel to the scope face. Only the left-right movements controlled the error dot. The controlled dynamics were of the form

$C(s) = K/s$. The stick gain was adjusted so that an error dot velocity of 15 cm/sec resulted from a 1 cm stick displacement.

A signaling device was given the subject with which he could indicate when he thought a transition had occurred. The signal device was held in the subject's left hand. It contained a button which the subject kept depressed until he detected a transition. The release of the button was taken to be an indication that a transition had been detected.

B. FORCING FUNCTION DISTURBANCES

Pseudo-gaussian, low-frequency forcing function inputs were used in the experiments. These inputs were constructed by adding together a large number (at least forty) of sinusoids which were equally spaced in frequency. The spacing was about .04 rad/sec. The amplitudes of the sinusoids were adjusted to give a rectangular power spectrum. The cutoff frequency of this spectrum was 1.5 rad/sec. The RMS displacement of the forcing function was 1.5 inches on the large rectangular oscilloscope and 1.3 cm on the 12 cm scope. Signals having these characteristics are designated R1.5 in Table 3.

In Experiments IIA and IIIB the rectangular spectrum was augmented by the addition of three sinusoids at frequencies of 3, 6, and 12 rad/sec. The RMS displacement of this high-frequency signal was 26 db below the primary rectangular spectrum. The input disturbance augmented in this way is designated R1.5A in Table 3.

In Experiments IIB and IIIA an unaugmented rectangular spectrum with cutoff frequency of 1.5 rad/sec was used. This was the same spectrum as was used in Experiment I. The RMS displacement of this input was 2.1 cm on the 12 cm scope.

C. SUBJECTS

Two subjects (GK and RB) were used in Experiments I and IIB. Two subjects (GK and RBT) were used in Experiment IIB. Three subjects (RBT, RGT, and JCV) were used in Experiment IIIA. One subject (GK) was used in Experiment IIIB.

All of these subjects were male, high school or college students. None had pilot training, all were automobile drivers and all well-trained to be good trackers before data were taken. Subjects GK and RB had between 20 and 30 hours of tracking experience before Experiment I was begun.

Subjects RBT, RGT and JCV were trained until they were equally proficient in tracking each of the alternative dynamics and until their performance appeared to stabilize. Total tracking time with K/s controlled-element dynamics ranged from 3 to 6 hours. They were then allowed to practice transitions between dynamics until their adaptive performance stabilized.

D. PERFORMANCE MEASURES

A number of different time-domain performance measures were used to compare and describe the human controller's adaptive processes in these experiments. Time-domain descriptions are well-suited to describing time-varying systems. The measures were: ensemble average error curves following a transition; descriptive properties of the adaptive response such as estimated times for detection, identification and modification; ensemble average estimates of the human controller's time-varying gain; and, of course, the time histories of the input, error, stick and output signal before and after a transition.

1. Ensemble Average Error Waveforms

The results of Experiment I will be presented principally in the form of ensemble average error waveforms. The ensemble average error waveforms following each type of transition were computed to reveal consistencies in a subject's responses to that type of transition. Individual adaptive responses to transitions of a single kind often showed large variabilities which were attributed in part to variations in the input signal in the region about transition and in part to the variations in the subject's adaptation characteristics.

In Experiment I, however, transitions could occur only when the system output was zero and when the output velocity was greater than some threshold and in a specified direction. Therefore, transitions tended to occur during portions of the input signal that were similar, and the error responses obtained showed a systematic dependence upon control situation and type of transition. The ensemble averaging technique caused those aspects of error not closely associated with the adaptive process to cancel out on the average, and enhanced the consistent features of the adaptive process.

Because of the consistency of the input signal during a transition, a major component of the average error curves was due to the input alone and not to the transition. This component was eliminated by subtracting the average error occurring when the pre- and post-transition dynamics are identical so that the transitions involved no change whatsoever in the controlled-element characteristics. The average error curves presented in the discussion of this experiment were compensated by this process.

A more detailed discussion of the average error computation procedure is given by Young et al.[7] The computations were done on a Digital Equipment Corporation PDP-1 computer.

2. Descriptive Measures

The results of Experiment II are presented principally in terms of descriptive measures which approximately delimit three phases of the adaptive process: identification, modification and transient tracking. The identification phase is concluded when the subject has discovered what the new dynamics are. The modification phase is concluded when he has changed his characteristics so that they are appropriate to the new condition, and the transient tracking is concluded when his tracking performance becomes commensurate with the steady-state level appropriate to the new dynamics. We use the terms identification, modification and adjustment times to refer to the termination of each of these phases. All times are taken relative to the transition time.

It is not easy to determine the termination times of all three phases from time histories, and in Experiments I and II we had to improvise to obtain rough estimates of these quantities. The following are the descriptive measures used for Experiments I and II.

a. Identification Time

We have used the time at which the subject's post-transition tracking behavior starts to differ from his pre-transition tracking behavior as an estimate of the identification time. Since we require that the subject change his behavior in order to register an identification, the identification times we obtained include at least some portion of the modification phase.

b. Peak Error

The peak error is the maximum error observed following a transition. It provides information about the speed and appropriateness of the subject's adaptation.

c. Time of Peak Error

For transitions that lead to unstable closed-loop behavior and for many other transitions, the error peak will occur after modification is completed. Thus, the time of peak error provides an estimate of the modification time.

d. Adjustment Time

When the error following a transition fell below a criterion, we considered the transient tracking phase to have ended, steady-state tracking to have begun, and the subject to have essentially completed his adaptation. For subject GK, the error had to fall below 0.4 cm and remain below .8 cm for 1 sec. For subject RB, .8 cm and 1.2 cm were used. These criteria agreed in most cases with the subjective estimates of when the subject had adjusted to new conditions.

In Experiment III the subjects used a signaling device to indicate when they thought a detection had occurred. The signaling time was taken as an estimate of detection time. The time at which the subject changed the character of his response could then be used as an estimate of identification time. Modification and adjustment times were obtained through the use of an ensemble averaging technique for measuring controller gain which is described below.

3. Human Controller Gain Estimates

When C(s) is equal to K/s, we obtain a very simple representation for the human controller from the crossover model of Eq. (3.45).

$$HC(s) \doteq \frac{\omega_c e^{-\tau s}}{s} \qquad (4.1)$$

If $C(s) = K/s$

$$H(s) \doteq \frac{\omega_c e^{-\tau s}}{K} \doteq K_h e^{-\tau s} \tag{4.2}$$

Direct measurements of K_h can be made from values of the error, $e(t)$, and the stick, $c(t)$, time functions. If we assume Eq. (4.2), then it follows that $c(t)$ can be written as

$$c(t) = K_h(t)e(t-\tau) + n(t) \tag{4.3}$$

where $e(t-\tau)$ is the error delayed by τ sec, and $n(t)$ is the remnant. To obtain stable estimates of K_h we average over an ensemble of correctly synchronized adaptive responses to the same type of transition. Doing this we obtain

$$K_h(t) = \frac{\langle c(t)e(t-\tau)\rangle}{\langle e^2(t-\tau)\rangle} \tag{4.4}$$

where $\langle ce\rangle$ and $\langle e^2\rangle$ are averages taken over the ensemble. The standard deviation of $K_h(t)$, $\sigma_K(t)$, can be estimated at the same time K_h is being computed.[16]

$$\sigma_K^2(t) \doteq \frac{\langle n^2(t)\rangle}{N\langle e^2(t-\tau)\rangle} \tag{4.5}$$

where N is the size of the ensemble and $\langle n^2(t)\rangle$ is the average squared remnant. By definition

$$\langle n^2 \rangle = [1-r^2(t)] \langle c^2(t) \rangle \tag{4.6}$$

where r(t) is the correlation between e(t-τ) and c(t).

$$r^2(t) = \frac{\langle c(t)e(t-\tau) \rangle}{\langle c^2(t) \rangle \langle e^2(t-\tau) \rangle} \tag{4.7}$$

One of the problems encountered in computing ensemble averages is how to synchronize the members of the ensemble in a consistent way. In the analysis of most of our data, we found that synchronizing all the responses with respect to the detection time yielded the most consistent estimates of K_h. For Experiment III, the signaling time was taken to be an indication of the detection time, and it was used for synchronization.

CHAPTER V

PRELIMINARY EVALUATION OF DECISION MODEL

Experiment II was performed primarily to explore the nature of the decision processes of adaptation and to test in a preliminary way models for these processes. In this Chapter we present the results of Experiments IIA and IIB, and then we use these results to evaluate the decision model developed in Chapter III.

Experiment IIA was a study of the effects of transition uncertainty and alerting on the adaptive process. Experiment IIB was a study of the identification in which transition uncertainty was also a factor. Both of these experiments have been discussed at least partially in previous papers.[13,14] However, since writing these papers we have revised the model for the decision processes considerably and have analyzed additional data from these experiments.

A. EXPERIMENT IIA: STUDIES OF THE EFFECTS OF TRANSITION UNCERTAINTY AND ALERTING

1. Summary of Conditions

In most time-varying control situations the human controller does not have complete knowledge of the post-transition dynamics. In other situations he receives warning, or alerting signals that tell him that a transition is about to occur or has just occurred. To determine the effects of these two factors upon the adaptive process, two sets of experiments were performed in which the post-transition dynamics were not completely predictable and in which an audio signal alerted the subject that a transition had occurred.

In Experiment IIA four conditions were tested and compared:

a. Alerted Certain, (AC) -- the audio signal indicated that a transition had occurred and the subject had complete knowledge of the post-transition dynamics.

b. Alerted Uncertain, (AU) -- the audio signal indicated the change, but the subject did not have complete knowledge of the post-transition dynamics.

c. Not Alerted Certain, (NC) -- no alerting signal, but the subject knew what the post-transition dynamics would be.

d. Not Alerted Uncertain, (NU) -- no signal and incomplete knowledge of post-transition dynamics.

The transitions were from a base condition of K_0/s^2 to any one of 18 other dynamics for one subject, and to any one of 12 other dynamics for the other subject. Each transition from the base condition was followed by a transition back to the base condition. The possible dynamics included K_1, K_1/s, and K_1/s^2, where K_1 could be of either polarity and have three magnitudes with each system order.

There were two subjects, GK and RB. Each received basically the same set of experimental conditions. This experiment was carried out after each subject's total tracking experience was approximately 40 to 50 hours.

2. Results of Experiment IIA

Descriptive measures were obtained for the five types of transitions: $+8/s^2$ to $16/s^2$; $+8/s^2$ to $-16/s^2$; $+8/s^2$ to $+16/s$; $+8/s^2$ to $-16/s$; and $+8/s^2$ to $+4$ for each of the four conditions of alerting and certainty. These measures, shown in Table 4 are the average of the measurements made on ten transitions for the certain cases and three transitions for the uncertain cases for each of the two subjects.

a. Identification Times

The identification for all transitions, except $8/s^2$ to $+16/s^2$ are in Table 4a. The control strategies required for $8/s^2$ and $+16/s^2$ were very similar, and it was not possible to determine accurately the times at which the human controller changed his control strategy and had identified the new dynamics. Therefore, the identification times were omitted.

The identification times for the AC condition were generally the shortest. Not only was the overall mean AC time shortest, but the majority of the AC times for each transition were shorter than the times for corresponding transitions under the other conditions. The times for the other three conditions were not ranked in any consistent way. The AC times were significantly shorter than the NC times. A sign test[24] on the mean scores for each type of transition for each of the two subjects indicated a significant difference at the .01 level. Comparisons made on the other pairs of conditions did not give significant results.

Table 4a
AVERAGE IDENTIFICATION TIMES (sec)

TRANSITION	AC	NC	AU	NU
$+8/s^2 \rightarrow -16/s^2$	.83	.90	.92	.82
$+8/s^2 \rightarrow +16/s^2$				
$+8/s^2 \rightarrow -16/s$	.50	.61	.49	.70
$+8/s^2 \rightarrow +16/s$	.64	.79	.60	.59
$+8/s^2 \rightarrow +4$	.30	.51	.45	.39
Mean	.57	.70	.62	.63

AC<NC significant at .01 level.

Table 4b
AVERAGE PEAK ERRORS (cm)

TRANSITION	AC	NC	AU	NU
$+8/s^2 \rightarrow -16/s^2$	3.93	6.83	7.93	10.05
$+8/s^2 \rightarrow +16/s^2$	2.22	2.41	5.10	2.88
$+8/s^2 \rightarrow -16/s$	5.22	7.87	7.22	6.94
$+8/s^2 \rightarrow +16/s$	3.51	4.08	4.55	7.61
$+8/s^2 \rightarrow +4$	4.13	4.21	5.77	5.27
Mean	3.80	5.08	6.11	6.55

AC<NC significant at .05 level
AC<AU significant at .01 level
NC<NU significant at .05 level
C <U significant at .005 level

Table 4c
AVERAGE TIMES OF PEAK ERROR (sec)

TRANSITION	AC	NC	AU	NU
$+8/s^2 \rightarrow -16/s^2$	1.12	1.40	1.82	1.55
$+8/s^2 \rightarrow +16/s^2$	1.08	1.34	1.53	1.53
$+8/s^2 \rightarrow -16/s$	0.59	0.73	0.92	0.95
$+8/s^2 \rightarrow +16/s$	0.53	0.59	0.55	0.93
$+8/s^2 \rightarrow +4$	0.19	0.30	0.25	0.26
Mean	.70	.87	1.01	1.05

AC<NC significant at .01 level.
AC<AU significant at .01 level.
AC<NU significant at .01 level.
C <U significant at .01 level.

Table 4d
AVERAGE ADJUSTMENT TIMES (sec)

TRANSITION	AC	NC	AU	NU
$+8/s^2 \rightarrow -16/s^2$	3.35	4.15	4.75	6.1
$+8/s^2 \rightarrow +16/s^2$	2.35	3.94	4.5	3.1
$+8/s^2 \rightarrow -16/s$	1.79	2.82	2.9	2.8
$+8/s^2 \rightarrow +16/s$	1.77	1.83	1.8	2.4
$+8/s^2 \rightarrow +4$	1.04	0.99	1.3	0.85
Mean	2.06	2.75	3.05	3.05

AC<NC significant at .05 level.
AC<AU significant at .01 level.

b. Peak Error

Table 4b shows that the average peak error scores for each transition under the AC condition were smaller than the scores for the corresponding transitions made under any of the other three conditions. A sign test made on the mean scores for each subject for each transition indicated a significant difference between the AC and NC scores at the .05 level. Although the AU mean was less than the NU mean, the difference was not significant and the two subjects gave inconsistent results. (The mean AU score for RB was greater than his NU score). Thus, we cannot conclude that alerting *per se* leads to smaller peak errors.

The peak error scores for the two certain conditions were smaller than for the two uncertain conditions. The mean AC peaks were about 60 percent of the AU error. The difference was significant at the .01 level. The mean NC peak error was about 75 percent of the NU error, the difference being significant at the .05 level. Combining all the AC and NC transitions of the same type and comparing them with the combined NC and NU transitions, we, of course, also found that the certain scores were significantly lower than the uncertain scores, the significance level being .005. Thus, complete knowledge of the post-transition dynamics appears to result in considerably and significantly smaller peak errors.

c. Time of Peak Error

The times to reach peak error are in Table 4c. Controlled-element dynamics had a greater effect on time of peak error than did either alerting or certainty. The higher the order, the longer was the time to peak error.

The two certain conditions gave shorter times than did the two uncertain conditions. The AC conditions gave shorter times than any of the other transitions. The AC times were significantly shorter than the times for each of the other three conditions. The significance level being .01 for the three comparisons. The times for the two certain conditions combined were significantly shorter (at the .01 level) than the times for the two uncertain conditions.

d. Adjustment times

Adjustment times in Table 4d tended to increase with the system order and with uncertainty. The mean AC was shorter than the mean times for the other three conditions. The AC adjustment times were significantly shorter than the NC and AU times, the significance levels being .05 and .01, respectively

3. Discussion of Experiment IIA

The effects of alerting and knowledge of the post-transition dynamics observed in this experiment should be predicted by the model for the identification process developed in Chapter III. According to this model, the identification decision is based upon Eqs. (3.41) and (3.42) which are repeated here. The decision rule based on these equations is: Select a $C_i(s)$ other than $C_0(s)$ as the dynamics if a $C_i(s)$ can be found for which

$$(V_{ii}-V_{0i})P(C_i;n)p(\Delta\dot{e}|C_i,c;n) \geq (V_{00}-V_{i0})P(C_0;n)p(\Delta\dot{e}|C_0,c;n) + \sum_{j\neq i,0} (V_{0j}-V_{ij})P(C_j;n)p(\Delta\dot{e}|C_j,c;n) \quad (5.1)$$

for $i\neq 0$.

and for which

$$(V_{ii}-V_{ki})P(C_i;n)p(\Delta\dot{e}|C_i,c;n) \geq (V_{kk}-V_{ik})P(C_k;n)p(\Delta\dot{e}|C_k,c;n) \quad \text{for } k\neq 0,i \qquad (5.2)$$

The effect of the alerting signal should be to reduce the prior probability of $C_0(s)$ to zero. Thus, for the trials with the alerting signal

$$P(C_0;0) \doteq 0$$

and

$$P(C_i;0) \doteq \frac{1}{K} \qquad (5.3)$$

As before, we have assumed the transition occurs at the beginning of CI(n=0), and that there are K possible dynamics. We showed in Chapter III (Eq. (3.27)) that without the alerting signal

$$P(C_0;0) \doteq 1$$

and (5.4)

$$P(C_i;0) \doteq \frac{q}{K}$$

The effect of complete knowledge of the post-transition dynamics (the certain condition) is to make all but one of the prior probabilities approach zero. If $C_i(s)$ is certain to be the post-transition dynamics

$$P(C_j;0) \doteq 0 \quad \text{for } j\neq 0,i \qquad (5.5)$$

In this experiment the likelihood functions should not have changed with experimental conditions. The main variables of alerting and certainty should affect the results primarily through the prior probabilities. We use Eq. (3.27) to find these probabilities. For the AC condition

$$\begin{aligned} &P(C_0;0) = P(C_j;0) \doteq 0 \text{ for } j \neq 0,i \\ &P(C_i;0) \doteq 1 \end{aligned} \tag{5.6}$$

For the NC condition

$$\begin{aligned} &P(C_0;0) \doteq 1 \\ &P(C_i;0) \doteq q \\ &P(C_j;0) \doteq 0 \quad \text{for } j \neq 0,i \end{aligned} \tag{5.7}$$

For the AU condition

$$\begin{aligned} &P(C_0;0) \doteq 0 \\ &P(C_i;0) \doteq \frac{1}{K} \quad \text{for } i \neq 0 \end{aligned} \tag{5.8}$$

and for the NU condition

$$\begin{aligned} &P(C_0;0) \doteq 1 \\ &P(C_i;0) = \frac{q}{K} \quad \text{for } i \neq 0 \end{aligned} \tag{5.9}$$

It is clear from these probabilities that the AC condition imposes the least uncertainty upon the human controller, and that, therefore, little, if any, information need be derived from observations of $\Delta\dot{e}(t)$ and $c(t)$ in order to identify the new dynamics and select an appropriate control strategy. As a result we would expect the AC condition to lead to the shortest identification times. This, in fact, was the observed result for most cases, Table 4a).

There was essentially no difference in the AU and the NU mean identification times. In the AU runs, $P(C_0;0)=0$, and $P(C_i;0)$

should have been considerably larger than in the NU runs. We, therefore, would expect that identification to be easier in the AU and to require less time. The fact that the times were about the same suggests that at the time of the alerting signal the second term on the right side, Eq. (5.1), the summation term, was sufficiently large so as to preclude an immediate selection of one of the $C_i(s)$. The summation represents the difference in the expected values of choosing $C_0(s)$ and $C_i(s)$ when, in fact, the dynamics are neither. If the penalty for incorrectly choosing $C_i(s)$ is high, the identification decision will be postponed until sufficient data has been obtained to make the probability of such an error small enough so that Eq. (5.1) is satisfied. This delay would tend to obscure the tendency of the alerting signal to decrease the identification times.

We would expect NC identification times to be shorter than the NU times and the result obtained in this experiment was surprising. The NC times should be shorter because $P(C_i;0)$ for the NC condition is K times larger than for NU condition. However, Eq. (5.1) still must be satisfied in the NC condition and sufficient data must be obtained to reduce the error probability to a sufficiently small value. This requirement would tend to equalize the identification times for the two conditions. The longer times actually observed for the NC were not significantly different statistically from the times for the NU condition, but the fact that they were longer runs counter to the model. Statistical sampling errors may have been the cause. Another possible explanation is that the AC and the NC trials were run before the AU and the NU trials and the subjects may have been more proficient at adapting in the later experiments.

The fact that the AC trials resulted in smaller peak errors and shorter times to peak error is to be expected from the model. Once the identification had been made, the subject knew immediately the correct strategy to adopt and he should not have made errors of identification or modification. Since the identification times for the AC trials were the shortest, the peak error times for this condition should also be shortest. Since the peak error generally increases with the time until a correct identification is made, shorter peak error times should be accompanied by smaller peak errors. Note that the difference between the AC and the NC mean peak error times is about the same as the difference between the AC and NC mean identification times.*

The two certain conditions gave smaller peak errors and shorter peak error times (with one exception)than the two uncertain conditions. The differences were statistically significant. For the certain conditions, once a transition had been detected and confirmed using Eq. (5.1), the choice of post-transition dynamics could be made with little chance of error. For the uncertain conditions more information is required for identification error. This will take additional time and probably lead to larger peak errors and longer peak error times.

The adjustment times are a measure of the time to complete the entire adaptive process. Many factors, including the identification times, peak errors, peak error times, and controlled dynamics affect the adjustment times. It is not surprising, therefore, to find that only the AC condition gave rise to significantly shorter times. The NC times are also shorter than NU time,, but the differences are not significant.

*The peak error times for transitions to +16/s and to +4 are smaller than the corresponding identification times. This apparent anomaly results from the fact that for these transitions the closed-loop roots of the system immediately after these transitions were highly oscillatory. The peak error resulted from the natural oscillation of the system. Modification and identification typically occurred between the first two peaks of this oscillation.

In summary, these results confirm qualitatively several features of the adaptive model proposed in Chapter III. In terms of the performance measures used for this experiment, it appears that complete knowledge of the post-transition dynamics will lead to considerably improved adaptive performance. Alerting without such knowledge does not appear to help much. The combination of alerting and certainty results in performance markedly better than any of the other conditions.

B. EXPERIMENT IIB: FURTHER STUDIES OF IDENTIFICATION

1. Summary of Conditions

In Experiment IIB we continued the study of the identification process. In this experiment the effects of transition uncertainty with K/s dynamics were investigated. Starting from a base condition K_0/s, the gain could decrease by a factor of 5, or change polarity. In addition, a transient signal that simulated the error waveform observed with a polarity change could occur instead of a gain change.

The changes of control situation were presented in a randomized order so that the subject could not predict the change. However, a change from the base condition of K_0/s was always followed by a change back to the base condition. Thus, although the first member of a pair of transitions was unpredictable, the second was always predictable. This procedure allowed us to investigate within the context of a single run the differences between the performance with complete and incomplete knowledge of the post-transition control situation. The use of the input transient allowed us to study how the human controller distinguishes between two events, both of which lead to similar error signals.

Two subjects were used in this experiment. They were both well-trained for steady-state tracking and for adapting to transitions.

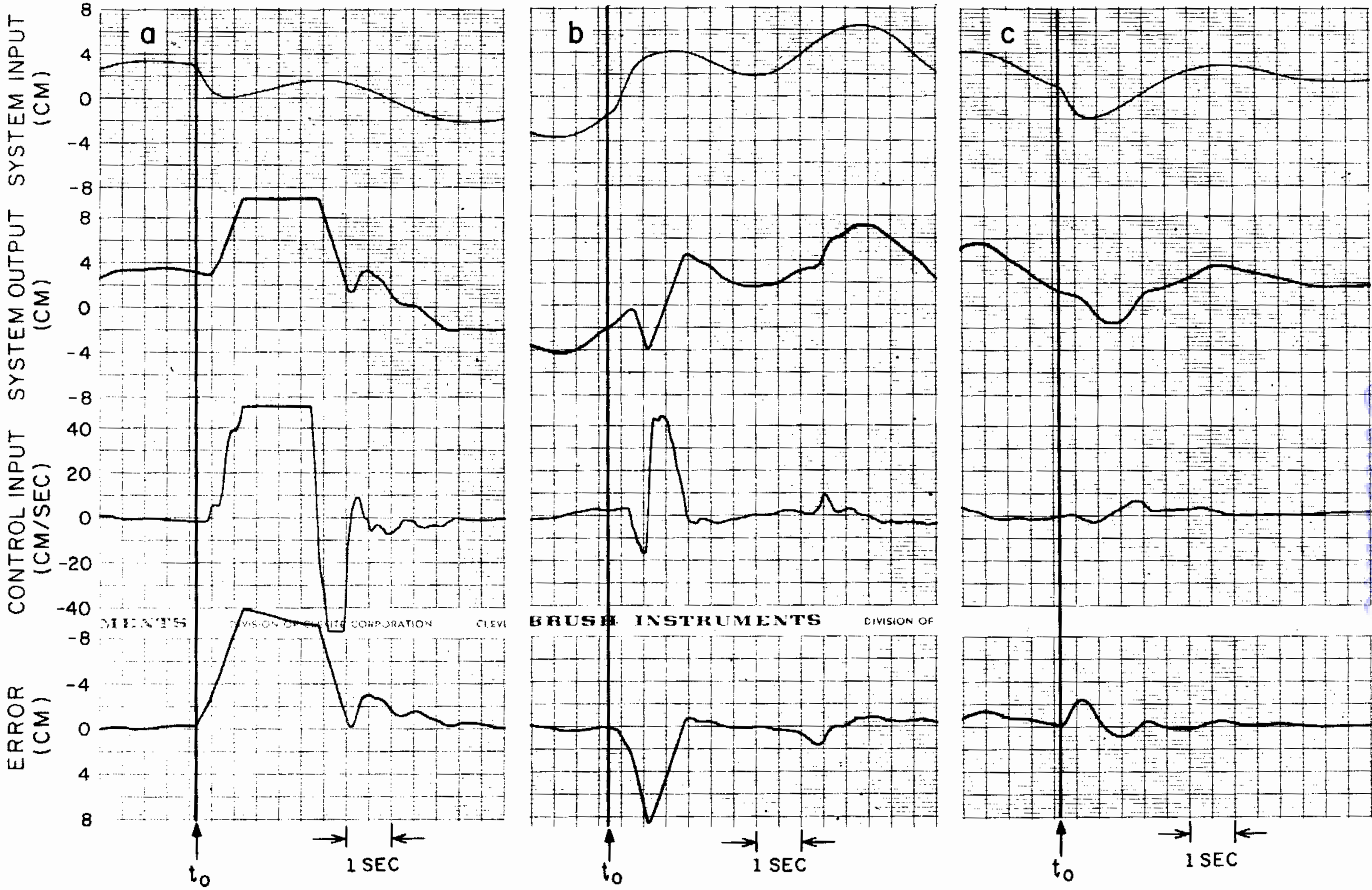

Fig. 8. Development of an identification strategy. Responses to transient input which simulated a gain reversal: (a) before any training, (b) after ten such transitions, (c) after fifty such transitions.

2. Development of an Identification Strategy

In Fig. 8 is an interesting series of adaptive responses obtained during the training period for this experiment. The subject had been trained with polarity reversals and gain decreases prior to the set of runs from which the data in Fig. 8 were taken. Without the subject's knowledge, a pseudo-transition consisting of a transient disturbance input occurred during the portion of the run shown in Fig. 8a, instead of the normal change of controlled dynamics. The transient waveform was designed to produce an error waveform similar to that observed when a polarity reversal occurred. Figure 8a shows the subject's response to this pseudo-transition when he had received no prior training or exposure to such a transition. In Fig. 8b is his response after 10 such transitions, and in Fig. 8c is his response after tracking about 50 such transitions. These pseudo-transitions, which simulated a polarity reversal, were embedded in a set of transitions consisting of polarity reversals and gain decreases.

Early in this training series, the subject assumed that the pseudo-transition was a polarity reversal. He reversed the polarity of his control movements immediately after detection and had to reverse again when he found that his identification was incorrect. This is what we would expect from our identification model. Initially, the prior probability of the pseudo-transition would be very small. The likelihood that the observed $\Delta\dot{e}$ came from a polarity reversal would be high and the prior probability of the reversal would also be relatively high. Thus, the polarity reversal is the best choice given the subject's experience. After considerable experience with this set of transitions, the subject adopted a different strategy. He tracked both the pseudo-transitions and polarity reversals as if they were input disturbances. He would reverse the polarity of his control movements only if the error continued to increase in subsequent control intervals.

This strategy would also be predicted by our identification model. After considerable experience with this set of transitions and pseudo-transitions, the subjective prior probability of the polarity reversal and the pseudo-transition should become approximately equal, since in this experiment both types of transitions could occur with equal probability. However, the error waveforms produced by both transitions were sufficiently similar so that the likelihood of both transitions also should be about equal. However, deciding that the input transient occurred is equivalent to deciding that $C_0(s)$ was still the dynamics. In the identification model, if a higher value were associated with choosing $C_0(s)$ than with the choosing of other dynamics, we would predict that early in the post-transition period, the subject would decide that no change in dynamics had occurred. He would continue to track as if dynamics had not changed until he obtained sufficient information in subsequent control intervals that would lead him to decide that a polarity reversal had indeed occurred.

3. Effects of Transition Uncertainty on Identification

As mentioned earlier, transitions occurred in pairs, the first of the pairs being uncertain and the second being certain. We would expect that the identification times for the uncertain member of each pair would be longer than for the certain member. Although we did not obtain this result in Experiment IIA, the results of Experiment IIB confirm our expectations. The effects of knowledge of the post-transition dynamics are clearly evident in the results given in Table 5. The Table contains the mean approximate identification times, the standard deviation of these times, and the number of correct

Table 5

EFFECTS OF UNCERTAINTY ON MEAN APPROXIMATE IDENTIFICATION TIME
and
NUMBER OF CORRECT AND INCORRECT RESPONSES

		TRANSITION ACTUALLY MADE			
		Certain Reversal	Uncertain Reversal	Certain Input	Uncertain Input
Subject GK's Response	Reversal	N = 33 $\bar{t}$ = .39 sec σ_t = .09 sec	N = 28 $\bar{t}$ = .63 sec σ_t = .1 sec	N = 4 $\bar{t}$ = .64 σ_t = .21	N = 5 $\bar{t}$ = .75 σ_t = .04
	Input	N = 0	N = 5 $\bar{t}$ = 1.08 sec σ_t = .37 sec	N = 20 $\bar{t}$ = .34 σ_t = .05	N = 19 $\bar{t}$ = .38 σ_t = .06
Subject RBT's Response	Reversal	N = 32 $\bar{t}$ = .45 σ_t = .14	N = 32 $\bar{t}$ = .57 σ_t = .13	N = 0	N = 1 t = .7
	Input	N = 1 t = 1.1 sec	N = 1 t = .8 sec	N = 23 $\bar{t}$ = .31 σ_t = .05	N = 22 $\bar{t}$ = .39 σ_t = .11

and incorrect identifications made in the certain and uncertain polarity reversal and transient input cases.

The approximate identification time is an estimate of the time required for the subject to identify the transition once the transition has been detected. It is the interval delimited by the time at which the error first exceeded three times the standard deviation of the pre-transition error and the time at which the error reached its peak value. The first of these times is assumed to approximate the time of detection and the second, the time of identification.

We see from Table 5 that for one subject (GK) about 15 percent of the reversals were identified as transient inputs in the uncertain case, whereas none of the reversals were incorrectly identified in the certain case. For transient inputs, about 20 percent of the transitions were incorrectly identified as reversals in the uncertain case and about 16 percent were incorrect in the certain case. The mean identification time, $\bar{t}$, for uncertain reversals was .24 sec longer than for the certain reversals. The mean identification time for certain transient inputs was slightly less than the time for uncertain transient inputs. Subject RBT showed smaller differences in identification times and made fewer mistakes. For both types of transitions, the certain times are shorter than for the uncertain times.

The small differences in identification times for the certain and uncertain input transient transitions reflect the fact that the subjects adopted the strategy of deciding in favor of $C_0(s)$ until the input transient and the polarity reversal could be clearly distinguished. The larger values of $\bar{t}$ for the uncertain reversal transitions as compared to the certain transitions also reflects this strategy. The difference between the times for the certain and uncertain reversal cases, which amounts to .24 sec for GK and .12 sec for RBT, is comparable to the duration of a control interval, indicating that about one additional movement was required for identification of the polarity reversal when it was uncertain. The number of identification errors made by GK is high in the uncertain cases. To reduce this number of errors, the subject would have to acquire more information and, therefore, presumably take more time to identify than he did.

These results clarify one of the anomalies of Experiment IIA, namely, the lack of a significant difference between the non-alerted uncertain, NU, and the non-alerted certain, NC, condition. In the present experiment, in which better measurements of the identification time were possible because of the choice of dynamics and in which a larger sample size was obtained, we were able to show significant differences between the certain reversal and the uncertain reversal transitions. This result is predicted by the identification model.

CHAPTER VI

DETAILED STUDY OF DECISION MODEL

In this chapter we present and discuss results obtained from two experiments that constituted a detailed study of the monitoring and identification processes of adaptation. The first of these experiments, Experiment IIIA, was performed by Miller[14] to test a model for the monitoring phase that he developed as part of his thesis. We have used his data and many of his results to validate the monitoring and identification model of Chapter III for the case in which the controlled dynamics are of the form K/s*. The second experiment, Experiment IIIB, performed by Elkind and Kelley[13] provides a test of the model with K/s^2 dynamics.

Because Miller's experiment provided the most complete set of data available for testing the model, we present a fairly complete account of his experiment. Several figures are reproduced directly from his thesis.

*Results from this experiment are also used for testing the modification portion of the control model. These are discussed in the next chapter.

A. EXPERIMENT IIIA: MILLER'S STUDY WITH K/s DYNAMICS

1. Conditions

a. Apparatus

Miller's experiment differed from Experiment II in a number of important respects that led to better experimental control and more sensitive analysis of the results. The controlled dynamics were K/s. As a result, the human controller's characteristics could be represented simply by a gain and a delay as in Eq. (3.45). The transitions, which involved only changes in the polarity and magnitude of the gain, were controlled by a signal that was recorded on a channel of the magnetic tape on which the input forcing function was also recorded. This insured that transitions would occur at the same points of the input during each run, thereby eliminating input signal differences as a source of run-to-run variance. Finally, the subject was given a signaling device to indicate when he thought a transition had occurred. This device was a normally-closed pushbutton switch. When the button was released, the marking pen on the event channel of a strip-chart recorder began to oscillate, thus indicating explicitly the time of detection.

Two gain magnitudes were used for the controlled-element: one produced a dot velocity of 15 cm/sec per cm of stick deflection in the presence of no input signal, and the other produced a velocity of 3 cm/sec per cm of stick deflection. The polarity of the gain could be of either sign.

Strip-chart recordings were made of the input, the output, the error, the control displacement, the control signal which triggered the transition, and the output of the signaling device which indicated the subject's detection of a transition.

b. Procedure

The three subjects, all 21-year old, male, MIT students, were first trained in steady-state tracking with each of the controlled dynamics. Next, they practiced tracking with transitions. After about two hours of practice with transitions, they were given the signaling device and allowed to practice transitions using it. They were instructed to use the same tracking strategy with the signaling device as they did without it. Then the data runs were begun.

In each data run there were eighteen points at which a transition could occur. At six of these, the same transitions occurred during each run. These six transitions were the only ones scored. The other twelve transitions were varied randomly from run to run. At some points, where a transition could occur, no transition was made, so that the subject could not learn when to expect a change.

A total of twenty-six data runs each for two subjects, and eighteen runs for the third one, were made with the signaling button. Data from only the last ten runs with each subject were analyzed. At the conclusion of the runs with the signaling device, five to seven runs per subject were made in which the subject did not have to signal at all, but could concentrate on tracking the changes. These data were taken to allow assessment of the extent to which the signaling task interfered with the tracking task.

2. Effects of Button Release on Adaptation

Comparison of the results with and without the signaling device indicated that the signaling task did not interfere materially with monitoring and identification, but may have interfered with modification and transient tracking.

In Table 6 is a comparison of the average times of the peak error and the average mean-squared tracking errors with and without the signaling device. The results in the Table were obtained from seven signaling runs and five no signaling runs with one subject. The standard deviation of the peak error times and the mean-squared error are also given.

Table 6

COMPARISON OF RUNS WITH AND WITHOUT A SIGNALING TASK

Transition No.	Transition Type	Peak Error Times: With Signal Mean (sec)	With Signal Standard Deviation (sec)	Without Signal Mean (sec)	Without Signal Standard Deviation (sec)
7	+3/s → +15/s	.40	.06	.38	.06
8	+15/s → -15/s	.56	.08	.58	.10
15	+3/s → -3/s	.82	.23	.61	.05
9	-15/s → -3/s	.78	.23	.91	.10
14	+15/s → +3/s	2.10	.15	2.28	.22
16	-3/s → -15/s	.93	.11	.68	.22
		Mean-Squared Error: With Signal Mean mm^2	**With Signal Standard Deviation mm^2**	**Without Signal Mean mm^2**	**Without Signal Standard Deviation mm^2**
Average Over Entire Run		25.8	7.4	17.3	1.9

There are no consistent differences in the peak error times between the signaling and no signaling results. Since for K/s dynamics, detection of a transition and identification must be essentially completed prior to the time of peak error, the lack of a difference in peak error times indicates that signaling did not interefere with these two tasks.

A large and significant difference in the mean-squared error scores was found. Most of the mean-squared tracking error is produced by the large errors that result when a transition occurs. Since the time of peak errors was not affected by the signaling task, the larger mean-squared errors observed when signaling was required probably resulted from interference between the signaling task on the one hand and the modification and transient tracking phases of the adaptive process on the other. The modification phase may extend beyond the time of peak error and the greater part of the transient tracking phase will be after the peak of the error. Moreover, the signal response usually occurred during the modification and transient tracking phases. In spite of its apparent effect on modification and transient tracking, the information provided by the signaling device is very important for understanding detection and identification and its use seems justifiable.

3. Temporal Characteristics of the Adaptation

In Figs. 9a through 9f are typical time histories of the input, response, and error signals corresponding to the transitions scored in this experiment. These include two gain increases, two gain decreases and two polarity reversals. Transitions 7, 8, and 15 (a gain increase and two polarity reversals), shown in Figs. 9a, b and e, occurred during regions of the

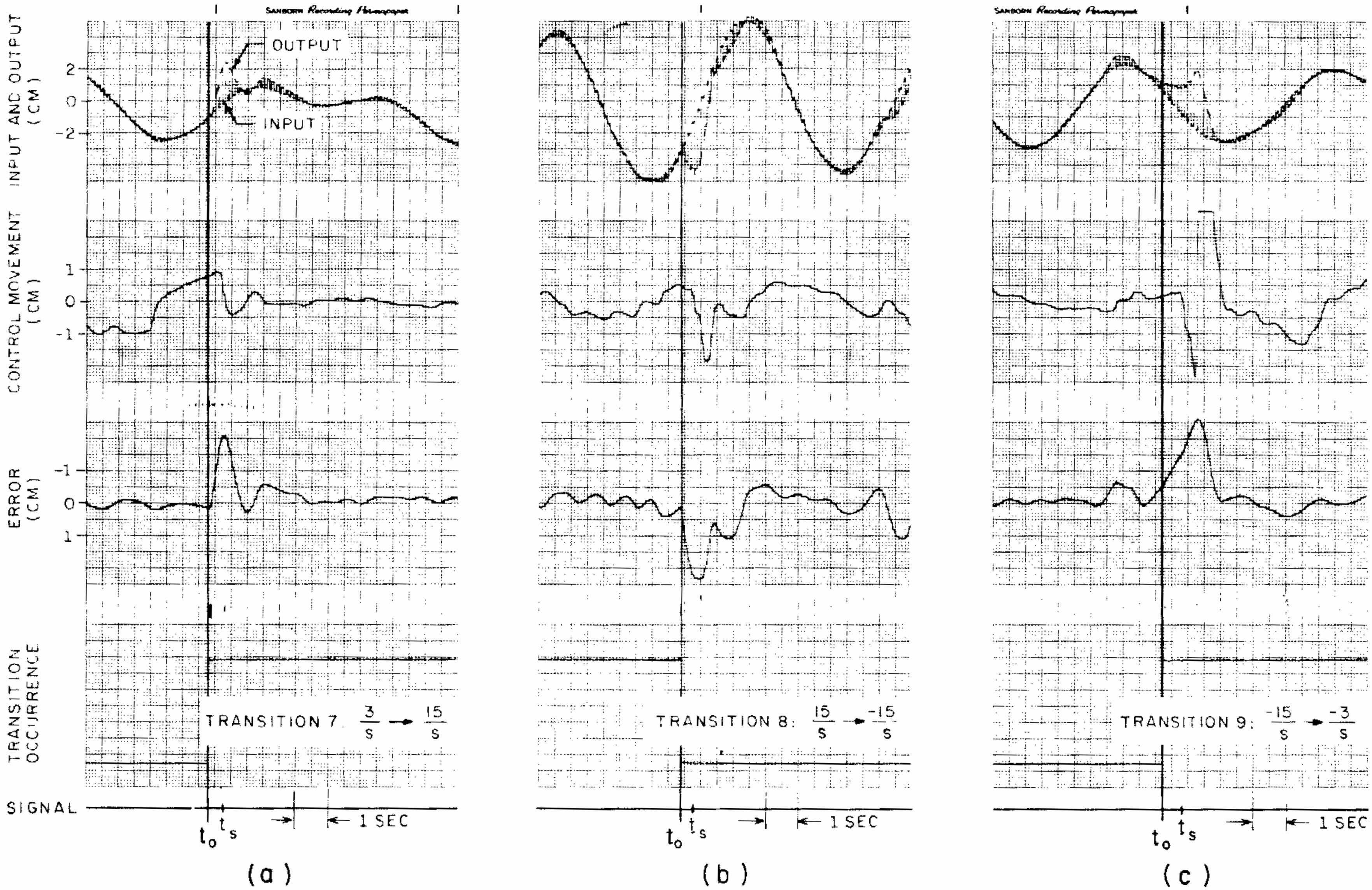

Fig. 9. Typical time histories from Miller's experiment. Samples from each of the six types of transitions are shown. The transition times and the signaling times are denoted by t_o and t_s, respectively.

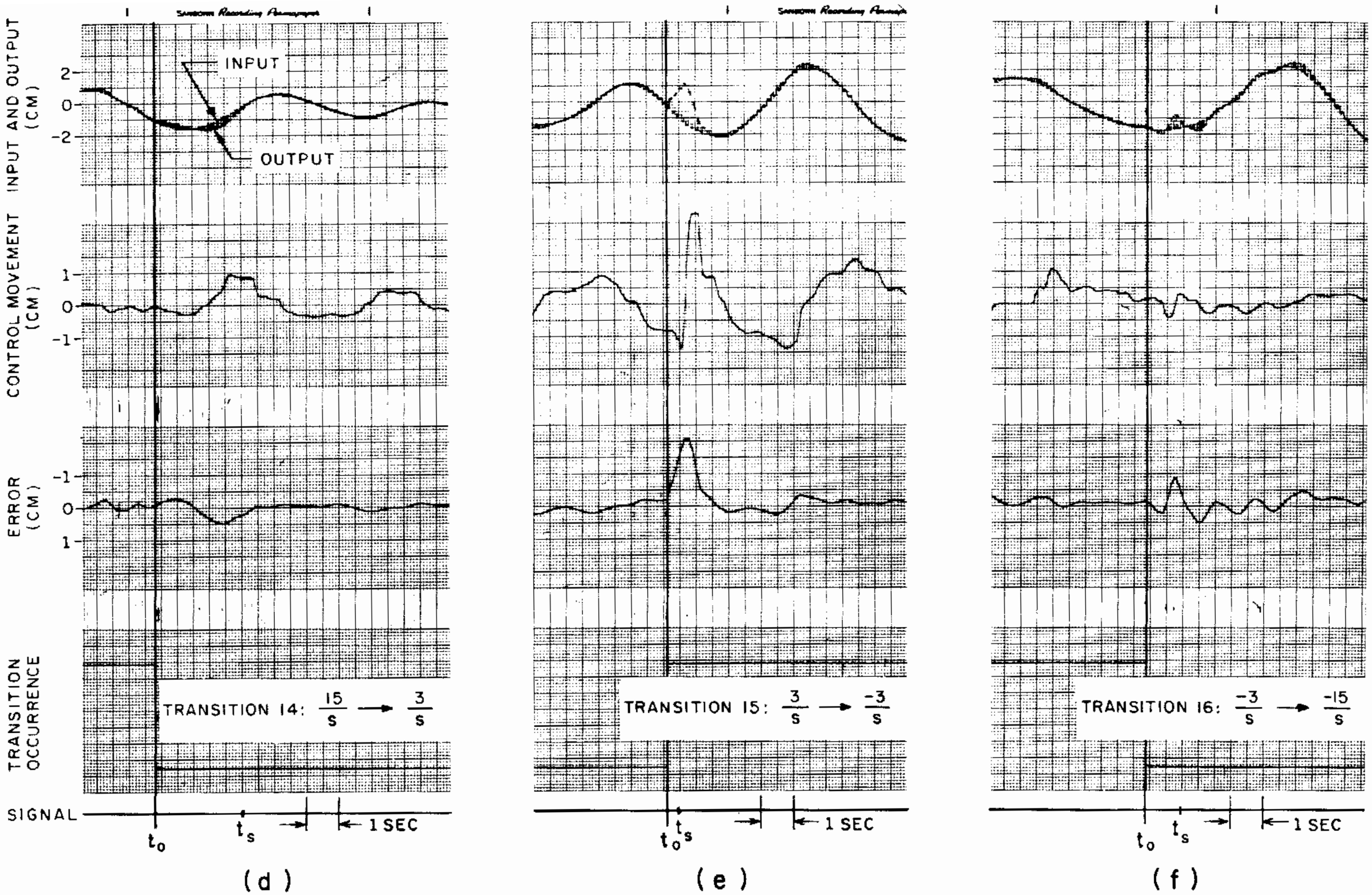

Fig. 9. (Contd) Typical time histories from Miller's experiment. Samples from each of the six types of transitions are shown. The transition times and the signaling times are denoted by t_0 and t_s, respectively.

input where the input rate was high and where the control displacement was large. As a result, the transitions were accompanied by large changes in error rate. These transitions were detected almost immediately.

Transitions 9 and 14 in Figs. 9c and 9d were gain decreases. For these transitions the control displacement was small at the time of transition, largely because the pre-transition gain was high. As a consequence, the change in error rate at transition was small and detection usually occurred .5 to 2 sec after the transition. Transition 16 in Fig. 9f was a gain increase which occurred during a quiescent region of the input signal. Because of this, the control displacement and movements were small and there was only a small change in error rate at transition. Here too, detection did not occur until about 1 sec after the transition.

Table 7 gives, for each type of transition, the average time interval between the transition and the button release. We call this interval the signaling time. The standard deviations of the signaling times are also given in the Table. For transitions 7, 8 and 15, which were accompanied by large changes of the error rate at the transition, the signal followed the transition by .3 to .5 sec. For these transitions the subject evidently was able to detect the transition in the control interval in which it occurred. The signaling times are in the range expected for a reaction time to a complex visual stimulus.

The longer detection times observed with transitions 9, 14 and 16 indicate that detection could not be made in the CI in which the transition occurred. Several CI's were needed before the subject reported a transition. The results with these transitions should provide a good basis for testing certain aspects of the monitoring model of Chapter III. In particular note the large difference between the signaling times for the two gain decrease transitions (Transitions No. 9 and 14). Transition 14 occurred during a quiescent region of the input. Since little control activity was required following the transition, the detection of the transition was postponed. It will be interesting to see how well the monitoring model accounts for this effect.

Table 7

SIGNALING TIMES

Subject:		RGT		RBT		JCV	
No.	Transition Type	Mean (sec)	Stand. Devia. (sec)	Mean (sec)	Stand. Devia. (sec)	Mean (sec)	Stand. Devia. (sec)
7	+3/s → +15/s	.44	.10	.38	.04	.36	.05
8	+15/s → -15/s	.36	.10	.32	.04	.30	.09
15	+3/s → -3/s	.43	.04	.36	.10	.32	.04
9	-15/s → -3/s	.84	.30	.88	.30	1.06	.53
14	+15/s → +3/s	2.52	.15	2.32	.17	2.40	.36
16	-3/s → -15/s	.88	.35	1.20	.32	.70	.19

4. Validation of Monitoring Model

a. Restatement of the Model

The most basic form of the monitoring model states that a transition should be reported if

$$(V_{\bar{0}\bar{0}} - V_{0\bar{0}})P(\bar{C}_0|D_n) \geq (V_{00} - V_{\bar{0}0})P(C_0|D_n) \tag{6.1}$$

If we assume that $(V_{\bar{0}\bar{0}} - V_{0\bar{0}})$ is equal to $(V_{00} - V_{\bar{0}0})$, as we did in Chapter III, and that D_n is $(\Delta\dot{e},c)$, Eq. (6.1) becomes

$$\begin{aligned} P(\bar{C}_0|\Delta\dot{e},c;n) &\geq P(C_0|\Delta\dot{e},c;n) \\ &\geq .5 \end{aligned} \tag{6.2}$$

We want to expand the posterior probabilities in this equation in terms of the likelihood functions and the prior probabilities as we did in Chapter III. For the case of K/s controlled dynamics this expansion must take into account the fact that a transition introduces a discontinuity in $\Delta\dot{o}(t)$. Thus, for a CI in which no transition occurs,

$$\Delta\dot{o}(t) = K_1\Delta c(t) \tag{6.3}$$

whereas, for a CI in which a transition does occur,

$$\Delta\dot{o}(t) = K_1\Delta c(t) + \Delta K_1 c(t-T) \tag{6.4}$$

In this last expression ΔK_1 is $(K_1 - K_0)$.

Thus the expansion of $P(\bar{C}_0|\Delta\dot{e},c;n)$ must be the sum of two terms: (1) the likelihood and prior probability that a transition occurred during the n^{th} CI and (2) the likelihood and prior probability that a transition occurred before the n^{th}

CI. In Miller's experiment transitions could be made only to two alternative dynamics which we label $C_1(s)$ and $C_2(s)$. The probability of a transition to either of these is q/2. In all cases we will use $C_1(s)$ to represent the dynamics to which the transition was actually made and $C_2(s)$ to represent the other dynamics.

With these considerations in mind, we can write

$$P(\bar{C}_0|\Delta\dot{e},c;n) = P(C_1|\Delta\dot{e},c;n) + P(C_2|\Delta\dot{e},c;n) \tag{6.5}$$

where

$$P(C_1|\Delta\dot{e},c;n) \doteq \frac{p(\Delta\dot{e}|C_{1t},c;n)\ P(C_0;n)q}{2p(\Delta\dot{e}|c)} + \frac{p(\Delta\dot{e}|C_1,c;n)P(C_1;n)}{p(\Delta\dot{e}|c)} \tag{6.6}$$

A similar expression can be written for $P(C_2|\Delta\dot{e},c;n)$. The term $p(\Delta\dot{e}|C_{1t},c;n)$ is the likelihood that the observed $\Delta\dot{e}$ resulted from a transition from $C_0(s)$ to $C_1(s)$ during the present, that is, the n^{th}, CI. Since $P(C_0;n)$ is equal to the probability that the dynamics were $C_0(s)$ at the end of the previous CI, and since q/2 is the probability of a transition to $C_1(s)$ at the beginning of the CI, q/2 $P(C_0;n)$ is the prior probability of a transition at the beginning of the n^{th} CI. The numerator of the second term of Eq. (6.6) is composed of the likelihood that the observed $\Delta\dot{e}$ resulted from $C_1(s)$ (and no transition occurred during the n^{th} CI) and the prior probability of $C_1(s)$. Because q is assumed to be very small, $P(C_1;n)$ is approximately the posterior probability of $C_1(s)$ at the end of the $(n-1)^{st}$ CI.

The expression for $P(C_0|\Delta\dot{e},c;n)$ is considerably simpler since we need consider only the likelihood that the observed $\Delta\dot{e}$ came from $C_0(s)$ without any transitions having occurred. Thus

$$P(C_0|\Delta\dot{e},c;n) \doteq \frac{p(\Delta\dot{e}|C_0,c;n)P(C_0;n)}{p(\Delta\dot{e}|c)} \quad (6.7)$$

where $P(C_0;n)$ is approximately equal to the posterior probability of $C_0(s)$ at the end of the $(n-1)^{\underline{st}}$ CI.

Since the decision processes for monitoring are sequential and since the prior probabilities for one CI are derived from the posterior probabilities for the previous interval which in turn depend upon the likelihoods for that interval, it is clear that the key elements of the model are the likelihood functions. Given values for these likelihood functions for several successive CI and approximate values for the prior probabilities for the first CI, we can compute the posterior probabilities for the $n^{\underline{th}}$ CI. These posterior probabilities will, in general, not depend very much upon the initial values used for the prior probabilities provided n is large.

In Chapter III we showed that the likelihood functions could be assumed to be normal and in particular that $p(\Delta\dot{e}|C_i,c;n)$ could be written

$$p(\Delta\dot{e}|C_i,c;n) = \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{1}{2}\left(\frac{\delta\dot{e}_i}{\sigma_i}\right)^2} \quad (6.8)$$

where $\delta\dot{e}_i$ is $\Delta\dot{e} - \mu_i$ and μ_i and σ_i are the mean and standard deviation of the distribution of $\Delta\dot{e}$ conditioned upon $C_i(s)$ and $c(t)$ for the $n^{\underline{th}}$ CI. We will make the same assumption of normality for $p(\Delta\dot{e}|C_{it},c;n)$ so that

$$p(\Delta\dot{e}|C_{it},c;n) = \frac{1}{\sqrt{2\pi}\sigma_{it}} e^{-\frac{1}{2}\left(\frac{\delta e_{it}}{\sigma_{it}}\right)^2} \tag{6.9}$$

where $\delta\dot{e}_{it}$ is $\Delta\dot{e} - \mu_{it}$ and μ_{it} and σ_{it} are the mean and standard deviation.

b. Parameters of the Model

In order to implement and test the monitoring model we must know the mean and standard deviation of each of the likelihood densities and the value of q. The procedure followed in Miller's experiment determines q and its value is easy to estimate. We will consider it first. The arguments advanced in Chapter III, together with some of Miller's results can be used to estimate the means and standard deviations, but these estimates are more difficult to obtain, and we will consider them second.

In Miller's experiment the average interval between transitions was about 18 sec. If we assume that the duration of a CI is 0.2 sec, then on the average a transition occurred in 1/90 of the control intervals. If we assumed that transitions could occur with equal probability in any CI, q, the probability of a transition during a CI would be about .01. However, in fact transitions could not occur with equal probability during all CI, and in particular the shortest interval between transitions was at least 10 sec. Thus, the longer the elapsed time from the previous transition, the greater became the probability of a transition. Thus we should use a value of q higher than .01. A value of q equal to .02 seems reasonable and in the testing of the model we used this value as well as several other values greater and less than .02.

Now consider μ_i and σ_i, the mean and standard deviation of $p(\Delta\dot{e}|C_i,c;n)$ For K/s dynamics and CI in which no transition occurs

$$\mu_i = -K_i\Delta c \tag{6.10}$$

and

$$\sigma_i^{\,2} = \sigma_{\Delta\dot{d}}^{\,2} + \mu_i^{\,2}\sigma_\eta^{\,2} \tag{6.11}$$

μ_i is just the value of $-\Delta\dot{o}(t)$ expected from $C_i(s)$ when Δc is the control movement. Given the values of the gain K_i of the allowable controlled dynamics and $\Delta c(t)$, μ_i can be computed for each $C_i(s)$ for every CI.

Of the two components of $\sigma_i^{\,2}$, the one due to changes in input disturbance rate during a CI ($\sigma_{\Delta\dot{d}}^{\,2}$) can be calculated from knowledge of the input and also can be estimated from measurements made on the input. $\sigma_{\Delta\dot{d}}^{\,2}$ is the variance of $\Delta\dot{d}$. But $\Delta\dot{d}$ is approximately equal to T times the second derivative of d(t). Therefore,

$$\sigma_{\Delta\dot{d}} \doteq T\sigma_{\ddot{d}}$$

$\sigma_{\ddot{d}}$ is the standard derivation of the second derivative of d(t).

In Miller's experiments the power density spectrum of d(t) was approximately rectangular and, therefore,

$$\sigma_{\ddot{d}}^{2} = \frac{1}{2\pi}\int_{0}^{\omega_{co}} S_{dd}\,\omega^{4}\,d\omega = \frac{\omega_{co}^{4}}{5}\,\sigma_{d}^{2} \tag{6.13}$$

where ω_{co} is the cutoff frequency which was 1.5 rad/sec, S_{dd} is the power density spectrum of the input and σ_d is the standard deviation of d(t). In Miller's experiments σ_d was 2.1 cm. Using these values in Eq. (6.13) we find that

$$\sigma_{\Delta\ddot{d}} \doteq .42 \tag{6.14}$$

Direct measurment of $\sigma_{\Delta\ddot{d}}$ using recorded values of d(t) gave the result

$$\sigma_{\Delta\ddot{d}} \doteq .5 \tag{6.15}$$

When we tested the monitoring models with both values of $\sigma_{\Delta\ddot{d}}$, we found that setting $\sigma_{\Delta\ddot{d}}$ equal to 0.5 gave a somewhat better match to the human controller's detection performance than did the smaller value.

We will now use some of the results that Miller obtained with transition 14, a gain decrease in which the detection signal did not occur until approximately 2.5 sec after the transition, to verify the assumption that the second component of σ_i^2 is proportional to μ_i^2 and to obtain bounds of the proportionality constant σ_η^2. For large values of μ_i, σ_i in Eq. (6.11) should be approximately $|\mu_i|\sigma_\eta$. The likelihood function of

Eq. (6.8) should be approximately

$$p(\Delta\dot{e}|C_1,c;n) \doteq \frac{1}{\sqrt{2\pi}|\mu_1|\sigma_\eta} e^{-\frac{1}{2}\left(\frac{\delta\dot{e}_1}{\mu_1\sigma_\eta}\right)^2} \tag{6.16}$$

Since for transition 14, detection did not occur until long after transition, the subjects never made incorrect transitions, and the variability of the signaling time was relatively small; it seems reasonable to assume that $P(C_2|\Delta\dot{e},c)$, the posterior probability of the incorrect post-transition dynamics, will be negligible near detection and that $p(\Delta\dot{e}|C_{1t},c)$ will also be small. Thus, in the neighborhood of the signaling time it should be sufficient to expand $P(C_1|\Delta\dot{e},c)$ only in terms of $p(\Delta\dot{e}|C_1,c)$ and the prior probability associated with it in Eq. (6.6).

With these simplifying assumptions in mind, the inequality of Eq. (6.2) upon which the monitoring decision is based becomes

$$P(C_1|\Delta\dot{e},c;n) \geq P(C_0|\Delta\dot{e},c;n) \tag{6.17}$$

We expand both sides in terms of the likelihoods and the prior probabilities

$$p(\Delta\dot{e}|C_1,c;n)P(C_1;n) \geq p(\Delta\dot{e}|C_0,c;n)P(C_0;n) \tag{6.18}$$

We substitute Eq. (6.16) for $p(\Delta\dot{e}|C_1,c;n)$ in this expression, take the logarithms of both sides, and rearrange terms to obtain

$$\begin{aligned} -\left(\frac{\delta\dot{e}_1}{\mu_1}\right)^2 + \left(\frac{\delta\dot{e}_0}{\mu_0}\right)^2 &\geq 2\sigma_\eta{}^2 \ln\left[\left|\frac{\mu_1}{\mu_0}\right| \frac{P(C_0;n)}{P(C_1;n)}\right] \\ (\delta\dot{e}_0)^2 - \left(\frac{\mu_0}{\mu_1}\delta\dot{e}_1\right)^2 &\geq 2\sigma_\eta{}^2 \ln\left[\left|\frac{\mu_1}{\mu_0}\right| \frac{P(C_0;n)}{P(C_1;n)}\right] \mu_o{}^2 \end{aligned} \tag{6.19}$$

To simplify notation let D equal $\left[(\delta\dot{e}_0)^2 - (\mu_0\delta\dot{e}_1/\mu_1)^2\right]$. Thus we would expect the human controller to decide that a transition had occurred whenever

$$D \geq 2\sigma_\eta^{\,2} \, \ell n\left[\left|\frac{\mu_1}{\mu_0}\right| \frac{P(C_0;n)}{P(C_1;n)}\right]\mu_0^{\,2} \tag{6.20}$$

The values of D observed at the time of detection should be proportional to μ_0^2. This proportionality is a direct consequence of the hypothesis that σ_1 is proportional to $|\mu_1|$ for large values of μ_1.

In order to verify whether or not D is proportional to μ_0^2 at detection, we must first determine when detection actually occurred. The results from transitions 8 and 15, the polarity reversals, can be used to estimate the latency between the actual detection and the detection signal. For these transitions, the $\Delta\dot{e}$ at the transition was very large and detection should have been immediate. The average time between the transition and the detection signal for these transitions was about 0.4 sec. We have used this 0.4 sec latency to estimate the time of detection and have computed D for transition 14 at times approximately 0.4 sec prior to the detection signal. The values of logD at these assumed detection times are plotted in Fig. 10 versus log μ_0. $\delta\dot{e}_1$ and μ_1 were obtained assuming K_1 was 3, the correct post-transition gain. The results for all three subjects are shown in Fig. 10. Each point represents one transition. For subject RGT, data from two transitions in which μ_0 was very small were omitted.

The line drawn on the graph has a slope of two which indicates that D is approximately proportional to μ_0^2, as was postulated. We can use this line to obtain an estimate of σ_η^2. From Fig.

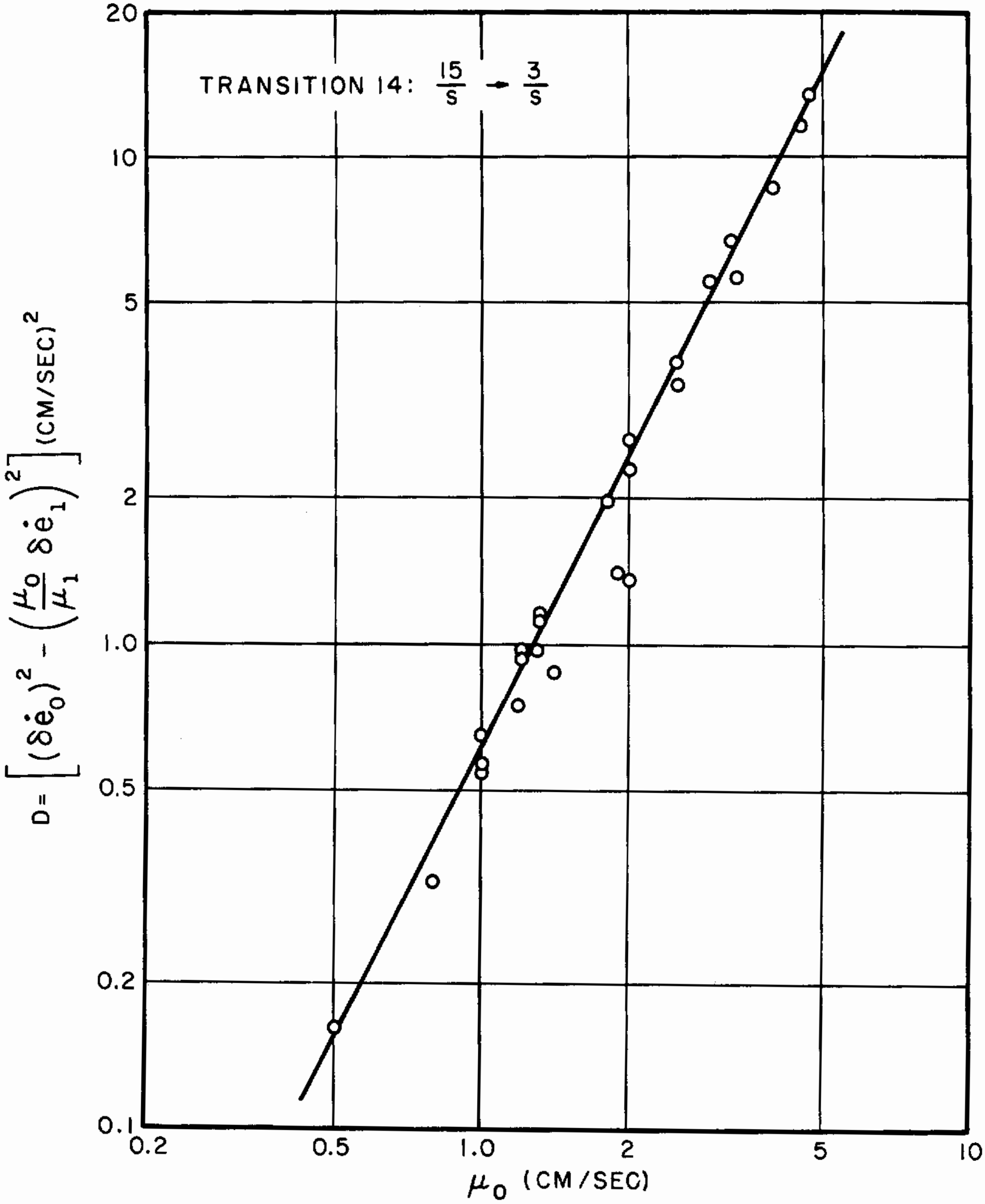

Fig. 10. Dependence of $D = \delta\dot{e}_0^{\,2} - (\mu_0/\mu_1 \delta\dot{e}_1)^2$ upon μ_0 for transition 14. Values taken 0.4 sec prior to detection signal.

10 we find that the coefficient of $\mu_0^{\,2}$ in Eq. (6.20),

$$2\sigma_\eta^{\,2} \ \ln \left[\left|\frac{\mu_1}{\mu_0}\right| \ \frac{P(C_0;n)}{P(C_1;n)} \right] \pm \ .6 \qquad (6.21)$$

$P(C_0;n)$ and $P(C_1;n)$, the prior probabilities for the CI in which detection is made, are essentially equal to the posterior probabilities for the previous interval. If the ratio of these posterior probabilities were less than one, the human controller would presumably have detected during this previous CI. However, if we set $P(C_0;n)/P(C_1;n)$ equal to one, to obtain an upper bound on $\sigma_\eta^{\,2}$, we find from Eq. (6.21) that $\sigma_\eta^{\,2}$ must be negative, an obvious impossibility. The negative value results because the ratio μ_1/μ_0 is just equal to K_1/K_0, which for this transition was .2. We can resolve this difficulty by taking advantage of the fact that the button release times showed considerable consistency. This suggests that the ratio of the prior probabilities was much greater than the lower bound of unity. It does not seem unreasonable to assume that $P(C_0;n)/P(C_1;n)$ is between 10 or 100, which would lead to a value of σ_η between 0.66 and 0.32. We found that setting $\sigma_\eta = .3$ gave the best match to our data.

Finally, let us consider μ_{it} and σ_{it}, the mean and standard deviation of $p(\Delta\dot{e}|C_{it},c;n)$. As before, it seems reasonable to assume that μ_{it} is equal to the value of $-\Delta\dot{o}(t)$ expected from the system given complete knowledge of the control displacement and the dynamics. From Eq. (6.4) we have

$$\mu_{it} = -\ [K_i \Delta c(t) + \Delta K_i c(t-T)] \qquad (6.22)$$

$$\mu_{it} = \mu_i - \Delta K_i c(t-T) \qquad (6.23)$$

We will make somewhat different assumptions in developing an expression for σ_{it} than we did for σ_i. To predict the $\Delta\hat{e}$ for a CI in which a transition occurs the human controller must know c(t) as well as $\Delta c(t)$. Whereas we assumed that the component of σ_i^2 due to $K_i \Delta c(t)$ was proportional to $(\Delta c)^2$ and K_i^2, we will assume that the component of σ_{it}^2 due to $\Delta K_i c(t-T)$ is proportional to $(\Delta K_i)^2$ and to σ_c^2, the variance of c(t). By making this assumption we are in effect saying that the human controller's knowledge of c(t) is poor and he cannot use c(t) to reduce his uncertainty of prediction of $\Delta\hat{e}$.

More precisely, we may write

$$\begin{aligned} \sigma_{it}^2 &= \text{Var}\,[-K_i \Delta c(t) - \Delta K_i c(t-T)] \\ &= \text{Var}\,[\ K_i \Delta c(t)] + \text{Var}[\Delta K_i c(t-T)] \qquad (6.24) \\ &= \sigma_i^2 + (\Delta K_i)^2 \sigma_c^2 \end{aligned}$$

For Miller's experiments in which the input bandwidth was relatively low and in which the controlled dynamics were K/s, the control movement is proportional to the output rate which in turn is approximately equal to the input rate. The variance of the input rate is simple to compute

$$c(t) = \frac{\dot{o}(t)}{K_i} \doteq \frac{\dot{d}(t)}{K_i} \qquad (6.25)$$

Thus

$$\sigma_c^2 \doteq \frac{\sigma_{\dot{d}}^2}{K_i^2}$$

$$= \frac{1}{K_i^2} \frac{1}{2\pi} \int_0^{\omega_{co}} S_{dd}(\omega)\, \omega^2\, d\omega \qquad (6.26)$$

$$= \frac{\omega_{co}^2 \sigma_d^2}{3K_i^2}$$

For the parameters of Miller's experiment $\sigma_c^2 \doteq (1.8/K_i)^2$. When we tested the monitoring model we found that using a coefficient of 1.8 in the expression for σ_c^2 did not give as close a match to the human controller's monitoring performance as did a coefficient of 1.0. Using this latter value the complete expression for σ_{it}^2 is

$$\sigma_{it}^2 \doteq \sigma_i^2 + \left(\frac{\Delta K_i}{K_i}\right)^2$$

$$\doteq \sigma_{\Delta\dot{d}}^2 + \mu_i^2 \sigma_\eta^2 + \left(\frac{\Delta K}{K_i}\right)^2 \qquad (6.27)$$

c. Computer Simulation of Monitoring Model

A computer program was written to test the monitoring model. The data used by the program were values of $\dot{e}(t)$ and $c(t)$ taken from recordings obtained by Miller. Samples of these quantities were taken every 0.2 sec in the region about each transition starting at .4 to 1.0 sec before the transition and ending at the detection signal. The program first computed $\Delta\dot{e}$ and Δc and then μ_i, σ_i, μ_{it} and σ_{it}. From these quantities the values of the likelihoods were determined for each CI for all of the allowable dynamics and transitions.

Then the posterior probabilities of the three possible dynamics, $C_0(s)$, $C_1(s)$ and $C_2(s)$, were computed for each CI by using Eq. (6.7) for $C_0(s)$ and Eq. (6.6) for $C_1(s)$ and $C_2(s)$. To start this computation values for the prior probabilities for the first CI had to be assumed. The program would report a transition whenever $P(C_0)$ became less than 0.5.

The computer program has six parameters that must be assigned values. In the previous subsection we derived approximate values for, or bounds on, four of these: q, $\sigma_{\Delta d}$, σ_η and σ_c. The other two are the initial prior probabilities $P(C_0;0)$ and $P(C_1;0)$ used to start the computational process.* Rather arbitrarily we set $P(C_1;0)$ and $P(C_2;0)$ to 1-q/2. The model's performance is very insensitive to this initial choice and we need not be concerned with it further.

When we used the derived values for the four principal parameters, the computer model did not match the human controller's detection performance as well as we had hoped. We then investigated the effects of the four principal parameters upon the performance of the model and, in effect, sought to tune the model by finding the best set of parameter values. The tuning was a fairly extensive searching process in which we made changes one at a time to each of the parameters and compared performance of the model and the human controller.

*Specifying two of these priors specifies the third, since they must sum to one.

First, the results from the gain decrease transitions were used to establish the best values for q, $\sigma_{\Delta\dot{d}}$ and σ_η. For these transitions, detection occurred some time after transition and the model's behavior was relatively insensitive to variations in σ_c. Then the best value of σ_c was determined by investigating the polarity reversal transitions for which $p(\Delta\dot{e}|C_{it},c)$ was the dominant likelihood and for which σ_c was the principal parameter affecting performance. In the evaluation of the model's performance, we considered the ability of the model to predict both the monitoring and the identification behavior of the human controller.

In Table 8 are shown the parameters of the model, the derived values, or bounds, on these parameters, and the values that gave the best match between model and human controller. These "optimum" values are the ones used to obtain all of the results that we present in the discussion of the model's performance which follows.

Table 8

PARAMETERS OF MODEL

Parameter	Derived Value	Value Giving Best Match
q	.01	.02
$\sigma_{\Delta\dot{d}}$	.42 to .5	.5
σ_η	reasonable value between .32 and .66	.3
σ_c	$1.8/K_1$	$1/K_1$
$P(C_0;0)$		.98
$P(C_1;0)$		.01
$P(C_2;0)$		.01

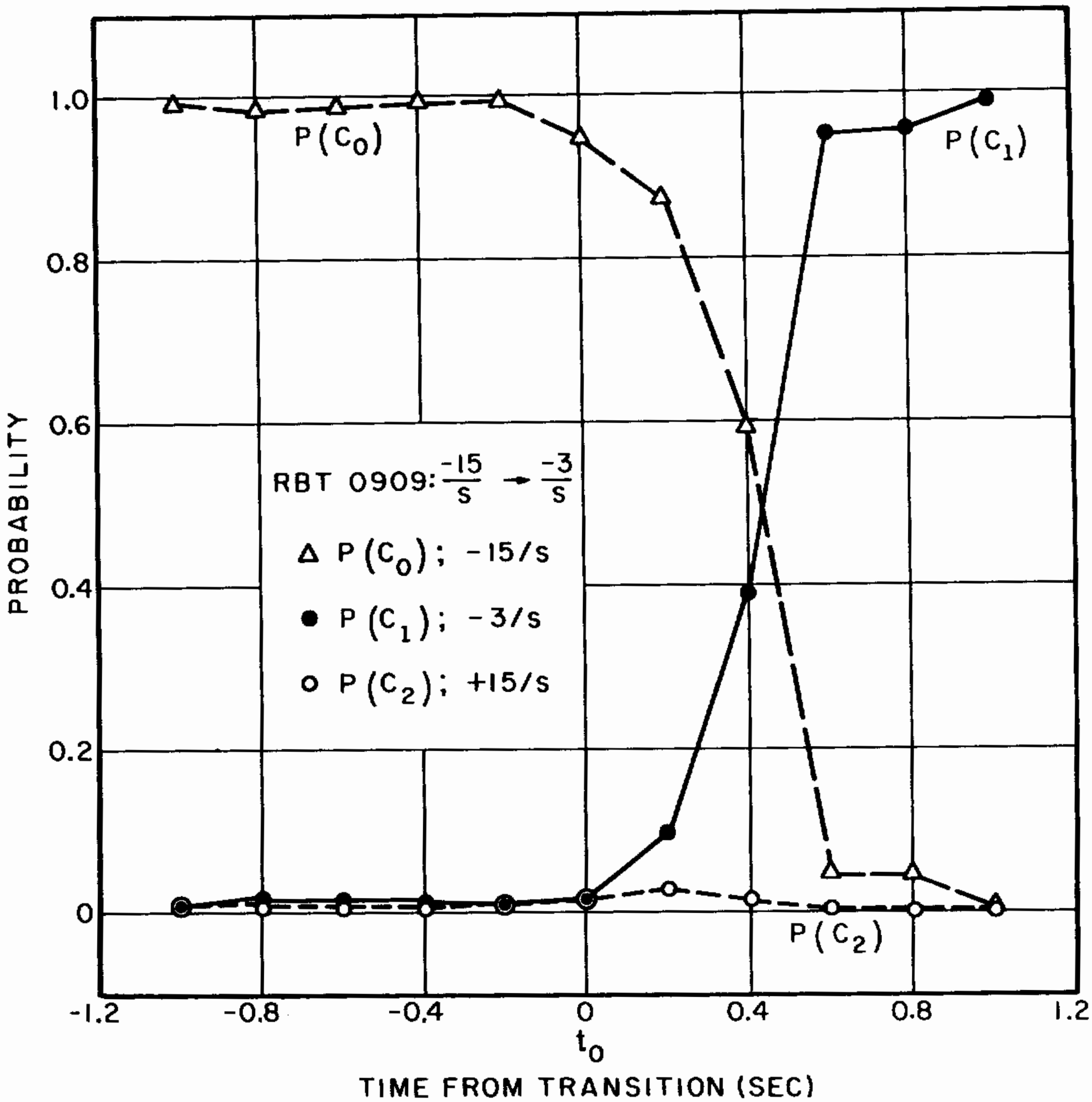

Fig. 11. Typical time history of the posterior probabilities of C_0, C_1, and C_2 for a gain decrease transition (transition 9 with subject RBT).

The best values shown in Table 8 are very close to the derived values. q, $\sigma_{\Delta\dot{d}}$ and σ_c are within a factor of two of the derived values. For σ_η we could derive only a reasonable range of values and the optimum value is at the low end of this range. The difference between the best and derived values of q is easily attributed to the fact that the probability of a transition during a CI was not constant for all CI, but increased as the time from the previous transition increased. The derived and optimum values for $\sigma_{\Delta\dot{d}}$ are very nearly equal and, in fact, the optimum value is equal to the estimate of $\sigma_{\Delta\dot{d}}$ obtained from measurements of $\dot{d}(t)$. The fact that the optimum value of σ_η was .3 (the low end of the derived range) implies that in Eq. (6.20) the ratio of the prior probabilities $P(C_0;n)/P(C_1;n)$ for the CI in which detection took place was approximately 100. This value is not inconsistent with those observed when the model was actually run with a wide variety of transitions. The best value of σ_c was slightly more than one-half the derived value which implies that the controller used some knowledge of c(t) to reduce the variance of his prediction of $\Delta\dot{e}$. Considering the large number of the assumptions involved in the derivation of these parameters, it is gratifying to find that the best values are so close to the derived values.

d. Comparison of Model and Human Controller Performance

In Fig. 11 is a typical time history of the posterior probabilities of the three controlled dynamics for one of the gain decrease transitions (transition 9 with subject RBT). $P(C_0)$ remains fairly close to unity until time t_o when it starts to decrease. At about .5 sec, $P(C_0)$ falls below 0.5 and $P(C_1)$ rises above 0.5, and shortly thereafter approaches 1.0. Detection should have occurred at this point

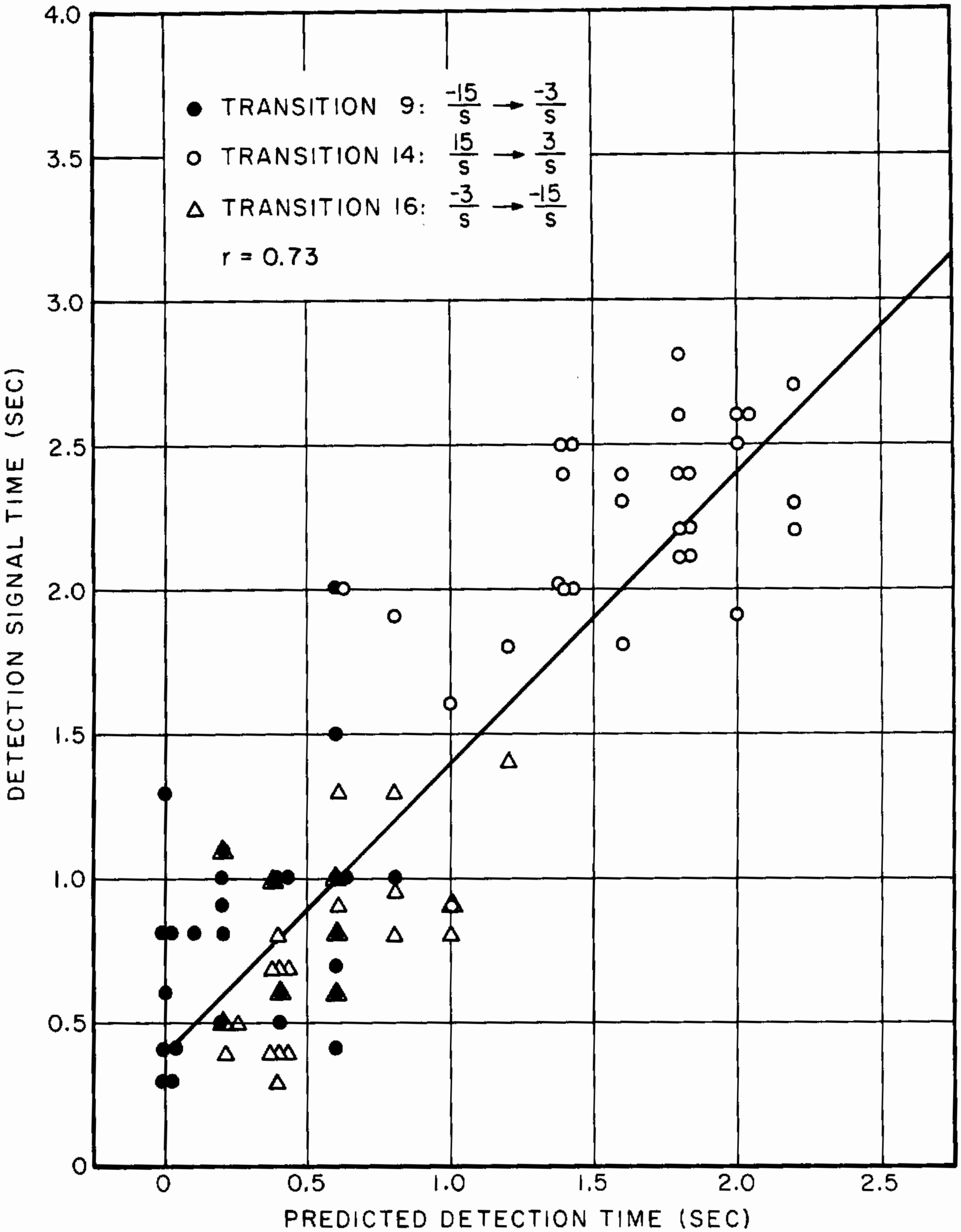

Fig. 12. Comparison of the detection signal times with the detection times predicted by the model.

in time. The detection signal was at 1.0 sec. A latency of 0.5 sec between detection and detection signal is not unreasonably large.

The time histories of the posterior probabilities are typically well behaved in the manner shown in Fig. 11. Prior to a transition, $P(C_0)$ is usually fairly stable near unity and the other probabilities are near zero. After the transition $P(C_0)$ will begin to decrease usually monotonically toward zero and the point in time at which $P(C_0)$ falls below 0.5 will be well defined. Once $P(C_0)$ was below 0.5, it rarely rose above this critical value and thus rarely would the model have to retract a detection decision.

The major purpose of the monitoring model, of course, is to predict the time of detection. It is interesting, therefore, to compare the detection times predicted by the model with times at which the human controller signalled detection. In Fig. 12 the detection signal times are plotted against the detection times predicted by the model for each of the three slow transitions. In Table 9 are the mean and standard deviations of the signal times and the predicted detection times for the three fast transitions and also for the three slow transitions.

Figure 12 provides a critical test of the model. The line drawn in the figure has unity slope and intersects the ordinate at 0.4 sec, the expected latency of the detection signal. We would expect the points to be distributed about this line, as, in fact they are. There is a fairly large scatter in the points about the line of prediction, but this should not be surprising. The model is a probabilistic representation of a

Table 9

MEAN SIGNALING TIMES AND MEAN PREDICTED DETECTION TIMES

Transition		Signal Time Mean (sec)	Signal Time Standard Deviation (sec)	Predicted Detection Time Mean (sec)	Predicted Detection Time Standard Dev. (sec)
7	3/s → 15/s	.39	.07	.05	.14
8	15/s → -15/s	.33	.09	.09	.18
15	3/s → -3/s	.37	.08	.05	.1
9	-15/s → -3/s	.93	.44	.34	.32
14	15/s → 3/s	2.41	.25	1.61	.16
16	-3/s → -15/s	.93	.36	.54	.30

human detection process and we can not expect a point-to-point match between model and human behavior. We can expect that the model will predict the statistical characteristics of the human controller's monitoring behavior. Table 9 shows that mean predicted detection times and the mean signaling times vary with transition number in very much the same way. The correlation between the mean signaling times and the mean predicted detection times shown in the table is .99. The differences between these two sets of means range from about .15 sec to about .8 sec. The larger value is about twice the expected value of the detection signal latency that we expected. The smaller value is about .25 sec less than the expected latency. The standard deviations of the predicted and observed detections are in very close agreement. The correlation between the two sets of standard deviations is .85. Also, Fig. 12 shows that not only is the model able to predict the mean detection signal times for each transition reasonably well, but it is also able to account for much of the variation in the signaling times for each transition. The correlation between the predicted and observed times plotted in the figure is .73.

5. Validation of Identification Model

a. Restatement of Model

The identification model states that the $C_i(s)$ whose expected value is maximum should be selected as the best estimate of the post-transition dynamics. If we assume that the values associated with correct identification decisions are all equal, and those associated with incorrect identifications are also equal, the decision rule simplifies to the selection of the $C_i(s)$ whose posterior probability is the largest. If we make the same assumptions as before about the nature of the data that is used for the decision, the model requires that we find the $C_i(s)$ for which

$$P(C_i|\Delta\dot{e},c;n) \geq P(C_K|\Delta\dot{e},c;n) \quad \text{for all } k\neq i \tag{6.28}$$

The posterior probabilities in this expression can be computed from Eqs. (6.6) and (6.7), which were derived for the monitoring model. It is reasonable to assume that the likelihood functions have the same means and standard deviations as they did for the monitoring process. By making this assumption, the computer program used to simulate the monitoring model can also be used to simulate and test the identification model. Recall that this program computes the posterior probabilities of the three possible dynamics for each CI. When the probability of C_0 falls below 0.5 it declares that a transition occurs. To apply this program to identification, a second decision process must be added which finds the $C_1(s)$ having the highest posterior probability and declares it to be the new controlled dynamics.

b. Comparison of Model and Human Controller Identification Performance

The identification model can be tested by comparing its identification decisions with those of the human controller. At the very least we would hope that the model would make approximately the same number of correct and incorrect identification decisions. A still better confirmation of the model would be obtained if each of the model's identification decisions matched each of the human controller's decisions. Finally, we would hope that the times of identification by the model would correspond to the times of identification by the human controller.

Control movement and error rate data for each transition were the source data for the computer simulation of the model and the posterior probabilities of each $C_1(s)$ were computed for each CI. For each transition the identification decision required choosing one of three dynamics: $C_0(s)$, the initial dynamics, $C_1(s)$, the correct dynamics; and $C_2(s)$ the possible alternative, but incorrect dynamics. The subjects knew that if a transition occurred the post-transition dynamics could be one of only two possible dynamics and they were trained long enough so that they should have known what the alternatives were. Most of the time the subjects identified correctly, but occasionally with transition 9 ($-15/s \rightarrow -3/s$) they made incorrect identifications.

The relative values of the posterior probability of $C_1(s)$ and of $C_2(s)$ in the first CI in which either of these probabilities is larger than the probability of $C_0(s)$ determines the model's identification decision. For two of the fast transitions (7: $+3/s \rightarrow +15/s$ and 15: $3/s \rightarrow -3/s$), the probability of $C_1(s)$, $P(C_1)$, was almost always very near unity and $P(C_2)$ and $P(C_0)$ were almost always very near zero in this critical CI. For the other four types of transitions evidence in favor of $C_1(s)$ was not so overwhelming. In Figs. 13a through 13d are plotted $P(C_1)$ against $P(C_2)$ for each of these four types of transitions. Each point in the figures represents a single trial.

If the points lie above the criterion line of unity slope drawn on the graphs, the model would have chosen $C_1(s)$, the correct dynamics. If they lie below the line, the model would have chosen $C_2(s)$, the incorrect dynamics. The filled points in the figure represent transitions that the subjects identified correctly. The open points represent transitions that the subjects identified incorrectly.

Figure 13a shows the results for transition 16: $-3/s \rightarrow -15/s$. All of these transitions were identified correctly by the model and by the subjects. The points in the figure are all clustered in a region where $P(C_1)$ is high and $P(C_2)$ is low. Thus the identification decision is unambiguous.

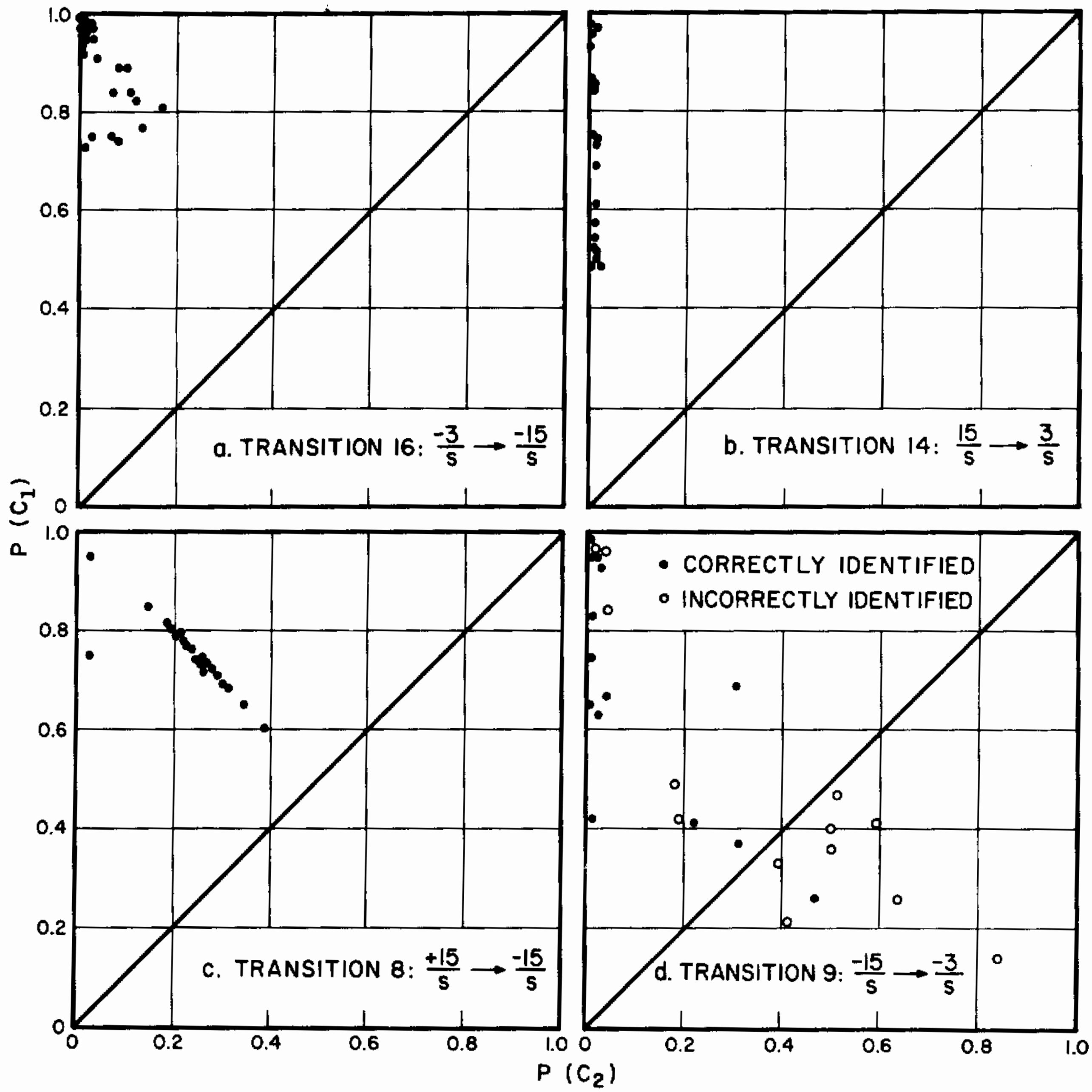

Fig. 13. Posterior probabilities of $C_1(s)$ and $C_2(s)$ for the CI in which identification is made by the model. Results from four types of transitions are shown.

Figure 13b shows the results for transition 14: 15/s → 3/s. The points all lie in a region of low $P(C_2)$, and although $P(C_1)$ varies over a wide range, all of the points are well above the criterion line. Hence, the model always identified correctly, as did the subjects.

The results for transition 8: 15/s → -15/s, are in Fig. 13c. Most of the points lie about a line of slope -1 and span a large range of values of $P(C_1)$ and $P(C_2)$. None of the points lie below the criterion line shown in the graph and, therefore, would have been identified correctly by the model. All the transitions appeared to have been correctly identified by the subjects.

The results of transition 9: -15/s → -3/s, shown in Fig. 13d are the most interesting. The points are widely dispersed indicating a very large variation in $P(C_1)$ and $P(C_2)$. Nine of the points lie below the criterion line and were identified by model to be polarity reversals, and eighteen above it. Thirteen of the transitions were incorrectly identified by the subjects to be polarity reversals rather than gain decreases. Of these thirteen, eight were also identified by the model to be polarity reversals, and five were identified to be gain decreases. One transition was identified by the model to be a reversal whereas the subject identified it correctly to be a gain decrease. Thus, in only six of the twenty-seven transitions plotted the model did not reproduce the decisions made by the subject. Half of these lie close to the criterion line.

Because the model is probabilistic, we should not expect it to reproduce the subject's behavior exactly and match every decision. The fact the model does reproduce almost 80 per cent of the transition 9 decisions and shows essentially the same probability of error as did the subjects is very encouraging.

Now let us compare the timing of the identification decisions of the model and the subjects. In Table 10 are the average times of detection and identification by the model for each type of transition. The human controller's average signaling times are also given. We did not obtain an explicit indication of the subject's identification time from the experiment, but for the polarity reversal transitions we can infer the identification time from the subject's control movements. It is reasonable to expect that the subjects will make an identification decision before they modify their control strategy. The beginning of modification is sharply delineated in the time histories of the control movements following the polarity reversal transitions. In Table 10 we are given the average times at which modification began for the two sets of polarity reversal transitions. Note that for these transitions modification begins only a few hundredths of a second later than the detection signaling response. The model's identification decisions either occur simultaneously with detection or follow detection by about the same amount of time.

Table 10

AVERAGE TIMES OF DETECTION AND IDENTIFICATION

Transition		Model Detection Time(sec)	Model Indenti. Time (sec)	Subjects Signaling Time(sec)	Subjects Est. Identi. Time (sec)
7	3/s → 15/s	.05	.05	.39	
8	15/s → -15/s	.09	.09	.33	.37
15	3/s → -3/s	.05	.05	.37	.44
9	-15/s → -3/s	.34	.39	.93	
14	15/s → 3/s	1.61	1.62	2.41	
16	-3/s → -15/s	.54	.55	.93	

B. EXPERIMENT IIIB: STUDY OF K/s^2 DYNAMICS

1. Experimental Conditions

We have examined the extent to which the model represents the decision behavior of the human controller in control situations in which the controlled dynamics were $C(s) = K/s$. Clearly, if the model is to be useful, it must be capable of representing the decision processes with other controlled dynamics as well. We have tested the model against results obtained in a small experiment (Experiment IIIB) with K/s^2 dynamics. One highly-trained subject, the most proficient of all of our subjects, was used in this experiment. The apparatus was the same as that used in Experiment II. The post-transition dynamics were not completely predictable. Starting from a base, or pre-transition, dynamics of $8/s^2$, transitions could be made to any of seven post-transition dynamics: $16/s^2$, $4/s^2$, $2/s^2$, $-16/s^2$, $-8/s^2$, $-4/s^2$ and $-2/s^2$. There was no signaling device so the time of identification could only be inferred from a change in tracking behavior.

2. Validation of the Decision Model

The form of the decision model for K/s^2 dynamics is slightly different from that for K/s dynamics. A change of gain with K/s^2 does not lead to a discontinuity of the output rate as it does with K/s dynamics and we do not need a second set of likelihood functions to account for the hypothesis that the transition occurred during the current CI.

The expression for the posterior probability of $C_i(s)$ is

$$P(C_i|\Delta\dot{e},c;n) \doteq \frac{p(\Delta\dot{e}|C_i,c;n)}{p(\Delta\dot{e}|c)} \left[P(C_0;n)\frac{q}{K} + P(C_i;n) \right] \qquad (6.29)$$

for $i \neq 0$

where q is the probability of a transition during the $n^{\underline{th}}$ CI and K is the number of possible alternative dynamics. For $C_0(s)$ we have

$$P(C_0|\Delta\dot{e},c;n) \doteq \frac{p(\Delta\dot{e}|C_0,c;n)}{p(\Delta\dot{e}|c)} P(C_0;n) \qquad (6.30)$$

We assume q is very small so that we do not have to consider multiple transitions.

The monitoring model was tested with data from a set of eight time histories obtained from this experiment. From these records of the system behavior, the output and error rates, $\dot{o}(t)$ and $\dot{e}(t)$, at points 0.2 sec apart were computed. The changes in these quantities, $\Delta\dot{o}(t)$ and $\Delta\dot{e}(t)$, during 0.2 sec long control intervals were then computed and used to calculate the likelihood functions of Eqs. (6.29) and (6.30). For the calculation we made essentially the same assumptions about the means and standard deviations of the likelihood functions as we did for K/s dynamics. We assumed that the mean μ_i was unbiased and equal to the $-\Delta\dot{o}$ observed, properly scaled by the ratio of the actual gain to the gain associated with $C_i(s)$, K_i. We assumed that ${\sigma_i}^2$ was the sum of $\sigma^2_{\Delta\dot{d}}$ and $(\mu_i)^2\ \sigma^2_\eta$, as before, and chose the same values for $\sigma^2_{\Delta\dot{d}}$ and σ^2_η as we did for the K/s case.

Typically, $P(C_i;n)$ for $i \neq 0$ remained small until some time after transition when at least one of the $P(C_i;n)$ would increase and $P(C_0;n)$ would decrease from its initial value near unity. The time at which $P(C_0;n)$ fell below 0.5 was chosen as the detection time. The time at which one of the $P(C_i;n)$ exceeded $P(C_0;n)$ was chosen as the identification time. There was no signaling device in this experiment. Therefore we can not check the detection performance of the model against the subject's. However, we can obtain from the tracking records estimates of the subject's identification times. In Fig. 14 are plotted the times at which the controller first started to change his tracking behavior (our best estimate of his identification time) versus the model's identification times. The line drawn in the figure has unity slope and an intercept of 0.4 sec, the assumed modification response latency. The points lie about this line, a result that is consistent with that obtained with $C(s) = K/s$.

In Table 11 are given, for each transition, the probabilities of each of the $C_i(s)$ for the CI in which the model made an identification. The highest probabilities, the ones corresponding to the $C_i(s)$ chosen by the model, are underlined in the table. The correct $C_i(s)$ are in brackets. It is evident from the table that the highest probability usually corresponds to the correct gain, and that when it does not, it is off by a factor of 2 at the most. Thus, the model accurately accounts for the identification of transitions made by changing the gain of K/s^2 controlled-element dynamics as well as transitions with K/s dynamics. The good results obtained with both of these dynamics suggest that the model has a good chance of predicting behavior with other dynamics as well.

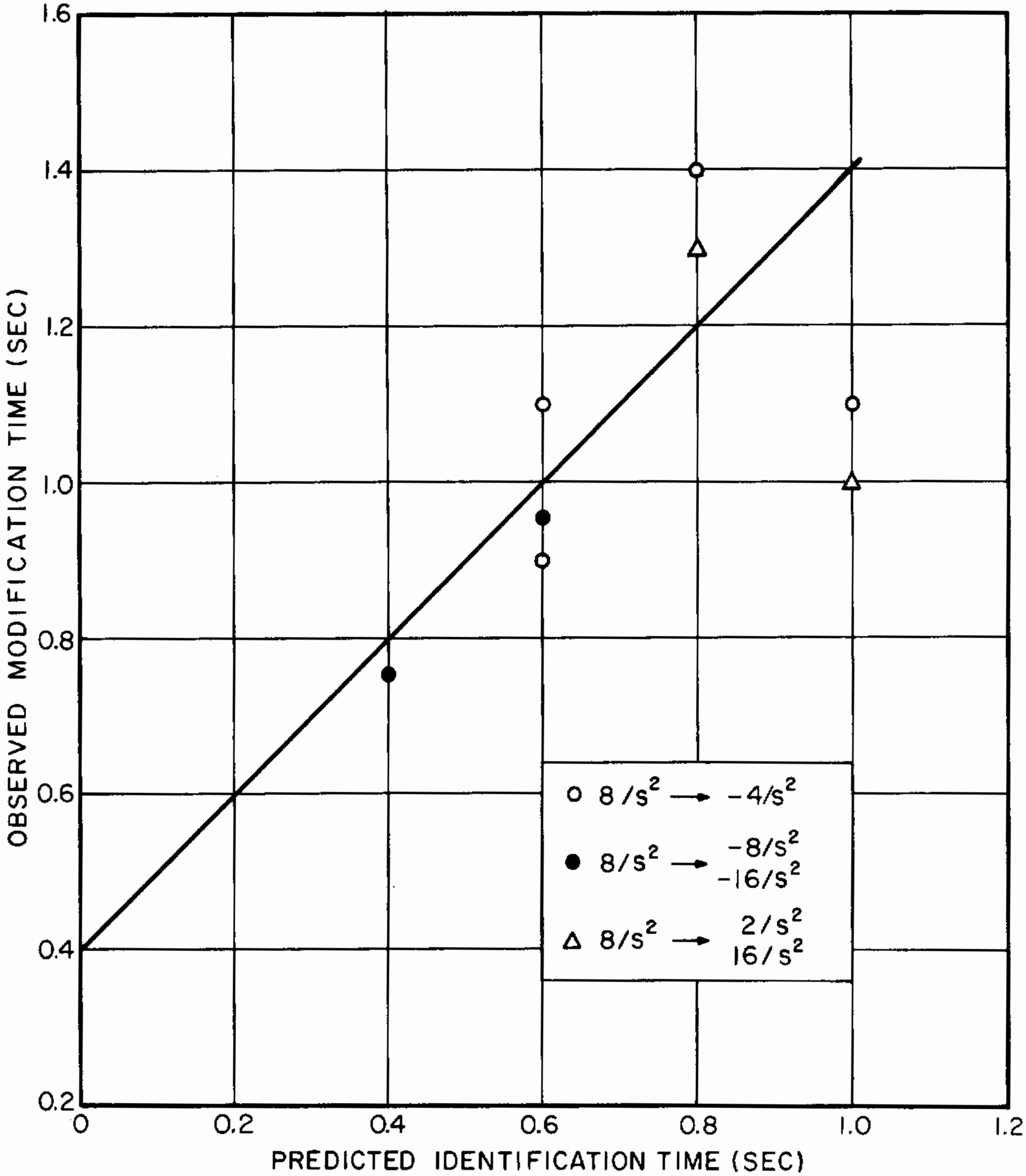

Fig. 14. Comparison of observed modification times with predicted identification times for K/s^2 dynamics.

Table 11

$P(C_i)$ FOR K/s^2 TRANSITIONS FOR IDENTIFICATION CI

Transition		$C_i(s)$							
		$-16/s^2$	$-8/s^2$	$-4/s^2$	$-2/s^2$	$2/s^2$	$4/s^2$	$8/s^2$	$16/s^2$
$8/s^2$	$-4/s^2$	.04	.33	[.35]	.22	.03	.006	.02	0
	$-4/s^2$	.002	.04	[.29]	.44	.10	.02	.09	.0001
	$-4/s^2$	.02	.19	[.31]	.25	.06	.02	.16	.0003
	$-4/s^2$	.03	.13	[.43]	.12	.06	.03	.0003	.0009
$8/s^2$	$-8/s^2$	.65	[.20]	.06	.03	.006	.003	.05	0
$8/s^2$	$-16/s^2$	[.75]	.19	.04	.01	.002	.0005	.006	0
$8/s^2$	$16/s^2$	0	0	0	0	0	.0001	.36	[.64]
$8/s^2$	$2/s^2$	0	0	.02	.12	[.56]	.17	.13	0

CHAPTER VII

STUDIES OF CONTROL MODEL

In this chapter we present and discuss the results obtained from several studies of the control processes of adaptation. In one set of studies we sought to determine the average characteristics of the human controller during the post-transition retention phase and during the transient tracking phase. Ensemble average error curves in the post-transition period were measured using techniques described in Chapter IV. These results were obtained using data from Experiments IA and IB and cover a wide variety of transitions, including gain increases and decreases, polarity reversals and order changes. In another set of studies we investigated the detailed structure of the modification phase. Ensemble average techniques were used to compute the human controller's gain as a function of time immediately before and after a transition. The methods described in Chapter IV were used to obtain these results. Data from Experiment IIIA were analyzed in this fashion. Although we did not study in detail the vernier adjustment and continuous tracking processes, some information relevant to these phases can be obtained from the average error and time-varying gain results.

Table 12

TRANSITIONS USED IN EXPERIMENTS IA and IB

GAIN CHANGES

Pre-Transition Dynamics $C_0(s)$	Post-Transition Dynamics $C_1(s)$				
	$C_1(s)/C_0(s)$				
	4	1/4	-1/4	-1	-4
2	8			-2	-8
8		2			
-8			2		
4/s	16/s			-4/s	-16/s
16/s		4/s			
-16/s			4/s		
$4/s^2$	$16/s^2$			$-4/s^2$	$-16/s^2$
$16/s^2$		$4/s^2$			
$-16/s^2$			$4/s^2$		

ORDER INCREASES

	$C_1(s)/C_0(s)$			
	2/s	1/2s	-1/2s	-2/s
2	4/s			-4/s
8		4/s		
-8			4/s	
4/s	$8/s^2$			$-8/s^2$
16/s		$8/s^2$		
-16/s			$8/s^2$	

ORDER DECREASES

	$C_1(s)/C_0(s)$			
	2s	s/2	-s/2	-2s
4/s	8	2		no data
-4/s			2	
$8/s^2$	16/s	4/s		-16/s
$-8/s^2$			4/s	

A. EXPERIMENTAL CONDITIONS

1. Experiments IA and IB

In these experiments we investigated adaptation in situations in which transitions were made between only two controlled-element dynamics at more or less unpredictable times. A variety of different dynamics were employed to determine the effects of gain and order changes. The two subjects were well-trained to control all of the dynamics and to make all of the transitions they would encounter. This combination of highly-trained subjects and known dynamics provided adaptive response results that should represent lower bounds on the time required for the human controller's adaptation and on the errors resulting from the transition. If the subject had less complete knowledge of the nature of the transition and less training, we would expect the time for adaptation and the errors to increase.

Two sets of experiments were performed. In Experiment IA the transitions involved changes of the magnitude and polarity of the gain, but not of the order of the controlled dynamics. Controlled dynamics of the forms $C(s) = K, K/s, K/s^2$ were used. This experiment was a continuation and extension of the work of Young et al[7] in which the transitions were changes of the polarity and of the gain with $C(s) = K$. In fact, we have used his results for this condition rather than repeat his experiment. The pre- and post-transition dynamics employed in Experiment IA, including values for the gains, are given in Table 12. The experimental conditions and apparatus for this experiment are discussed in more detail in Chapter IV, and are summarized in Table 1.

In Experiment IB, the transitions involved changes of the order of the controlled-element dynamics as well as of the polarity and magnitude of the gain. The order changes included increases of system order from position to velocity dynamics and from velocity to acceleration dynamics and decreases of system order from acceleration to velocity dynamics, and from velocity to position dynamics. Table 12 also gives detailed information about these transitions.

As may be seen from Table 12, the same kinds of changes of dynamics were made for position, velocity, and acceleration pre-transition dynamics. Thus, in Experiment IA, the ratios of post-transition to pre-transition gain were 4, 1/4, -1/4, -1, and -4 with each of the three types of controlled dynamics: position, velocity and acceleration dynamics. Similarly, in Experiment IB, the order increases and decreases were accompanied by the same kinds of changes in the polarity and magnitude of the gain for each type of pre-transition dynamics. For example, for the increases of order, transitions in which $C_1(s)/C_0(s)$ was equal to 2/s, 1/2s, -1/2s, and -2/s were studied for both position and velocity pre-transition dynamics.* By keeping the changes in dynamics invariant with system order we were able to determine the effects of order on the adaptive process.

2. Experiments Relevant to the Modification Phase

We have used the data from Miller's experiment with K/s dynamics, Experiment IIIA, to obtain information about the nature

*Unfortunately, data could not be analyzed for one of the order decrease transitions with K_o/s pre-transition dynamics. This was the transition for which $C_1(s)/C_0(s)$ = -2s.

of the modification process. These results are much less complete than some of the others discussed in this report, but nevertheless they provide at least a partial verification of the model for the modification phase discussed in Chapter III. It will be recalled that for K/s dynamics, the human controller's characteristics can be represented approximately by a gain and a time delay. In a time-varying control situation one can consider a gain to be time-varying. The ensemble averaging technique discussed in Chapter IV (Eqs. (4.4) through (4.7)) can be used to determine the controller's gain as a function of time. We have performed this analysis using data from two sets of the polarity reversal transitions and one set of the gain decrease transitions. The results obtained are discussed in Section E of this chapter.

Table 13

ROOTS OF POST-TRANSITION DYNAMICS

$C_1(s)/C_0(s)$	$C_0(s)$		
	K_0	K_0/s	K_0/s^2
GAIN CHANGES			
4	5.2 ± 13j	5.0 ± 11j	3.8 ± 7.1j
1	-2.5 ± 6.7j	-1.9 ± 5.9j	-.96 ± 3.9j
1/4	-6.9, -1.9	-5.4, -1.8	-2.9, -1.4
-1/4	-12, 1.0	-10, .92	-6.6, .63
-1	-18, 2.8	-15, 2.5	-10, 1.6
-4	-36, 5.7	-32, 4.9	-21, 3.1
ORDER INCREASES			
2/s	-11.6,.82±2.9j	-10,.84±2.6j	
1/2s	-10.5,.24±1.5j	-8.8,.25±1.5j	
-1/2s	-5.1,-7.1,2.2	-3.7,-6.5,1.9	
-2/s	2.5,-6.2±1.5j	2.3,-5.3±2.3j	
ORDER DECREASES			
2s		9.4 ± 13j	4.8 ± 8.1j
s/2		3.5 ± 13j	1.2 ± 8.1j
-s/2		3.5	1.2
-2s		9.4	4.8

B. APPLICATION OF CONTROL MODEL

1. Post-Transition Retention Phase Tracking

Prior to transition we approximate the open-loop system by the crossover model of Eq. (3.45). After transition and before modification the open systems can be approximated by the crossover model multiplied by $C_i(s)/C_o(s)$, the ratio of the post-transition to the pre-transition dynamics. The error signal is related to the disturbance input by the transfer function

$$\frac{E(s)}{D(s)} = \frac{1}{1 + \frac{C_i(s)}{C_o(s)} \frac{\omega_{co} e^{-\tau_o s}}{s}} \qquad (7.1)$$

The roots of the denominator of this expression are the closed-loop roots of the system and they determine the system's behavior.

In Figs. 15a, b, c are plotted the locus of the closed-loop roots as a function of the ratio of post- to pre-transition gain for the three sets of transitions in which only the gain changed. In Figs. 16a, b are root locus plots for the order increase transitions. The root locus plots for the order decrease transitions are in Figs. 16 c, d. The parameter for all of these plots is the ratio of the post-transition gain to the pre-transition gain. The loci were computed using the values for ω_c and τ given in Table 1, values taken from McRuer et al.[1] A first-order Padé approximation to the time delay in Eq. (7.1) was used in the computation of the roots for all transitions except those in which the order of the dynamics decreased. For the order decrease transitions the time delay was used without approximation and the principal roots were found. The post-transition closed-loop roots for each transition investigated in these studies are shown on the loci and also are tabulated in Table 13.

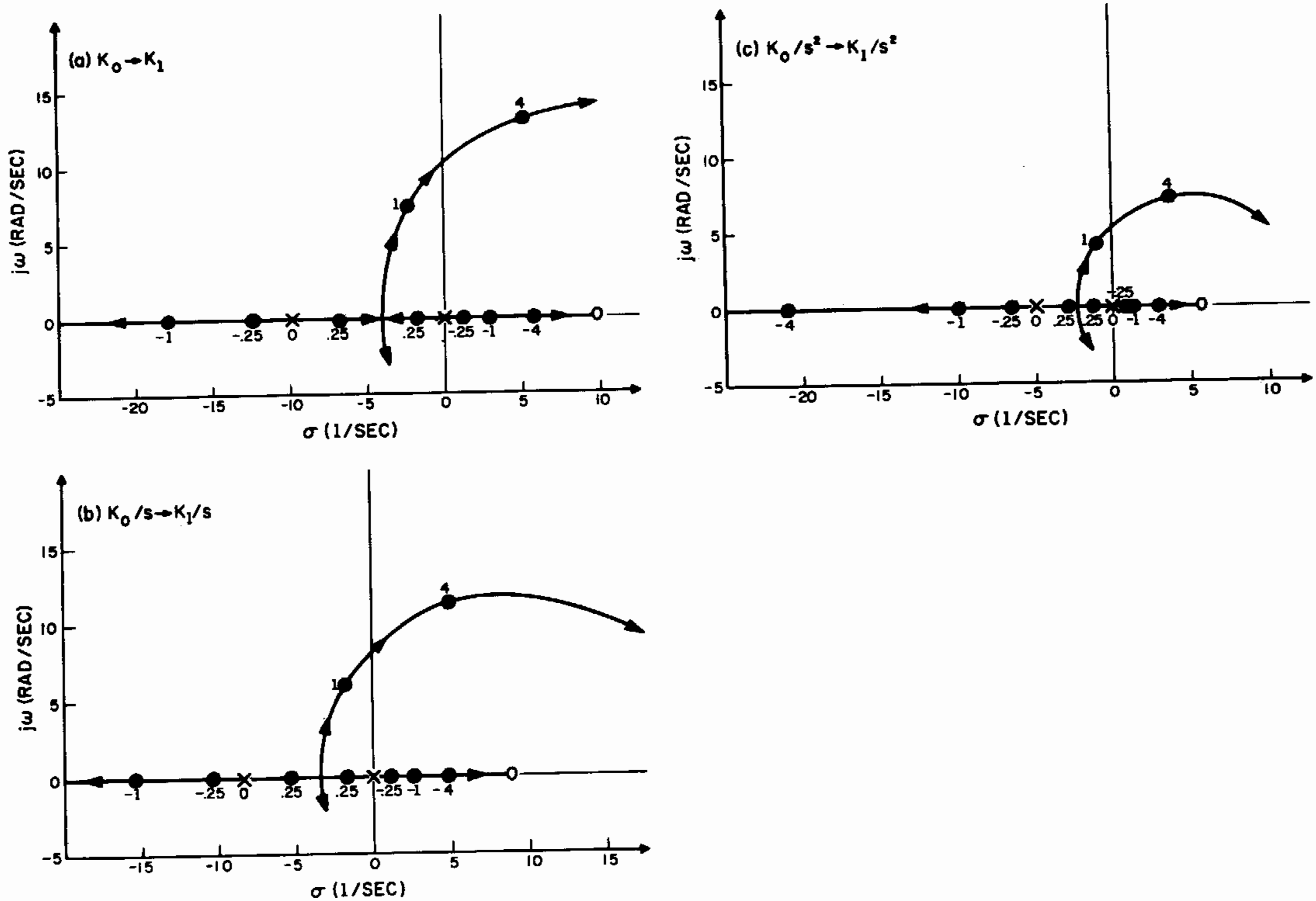

Fig. 15. Locus of closed-loop roots as a function of K_{10}, the ratio of post- to pre-transition gain for gain change transitions.

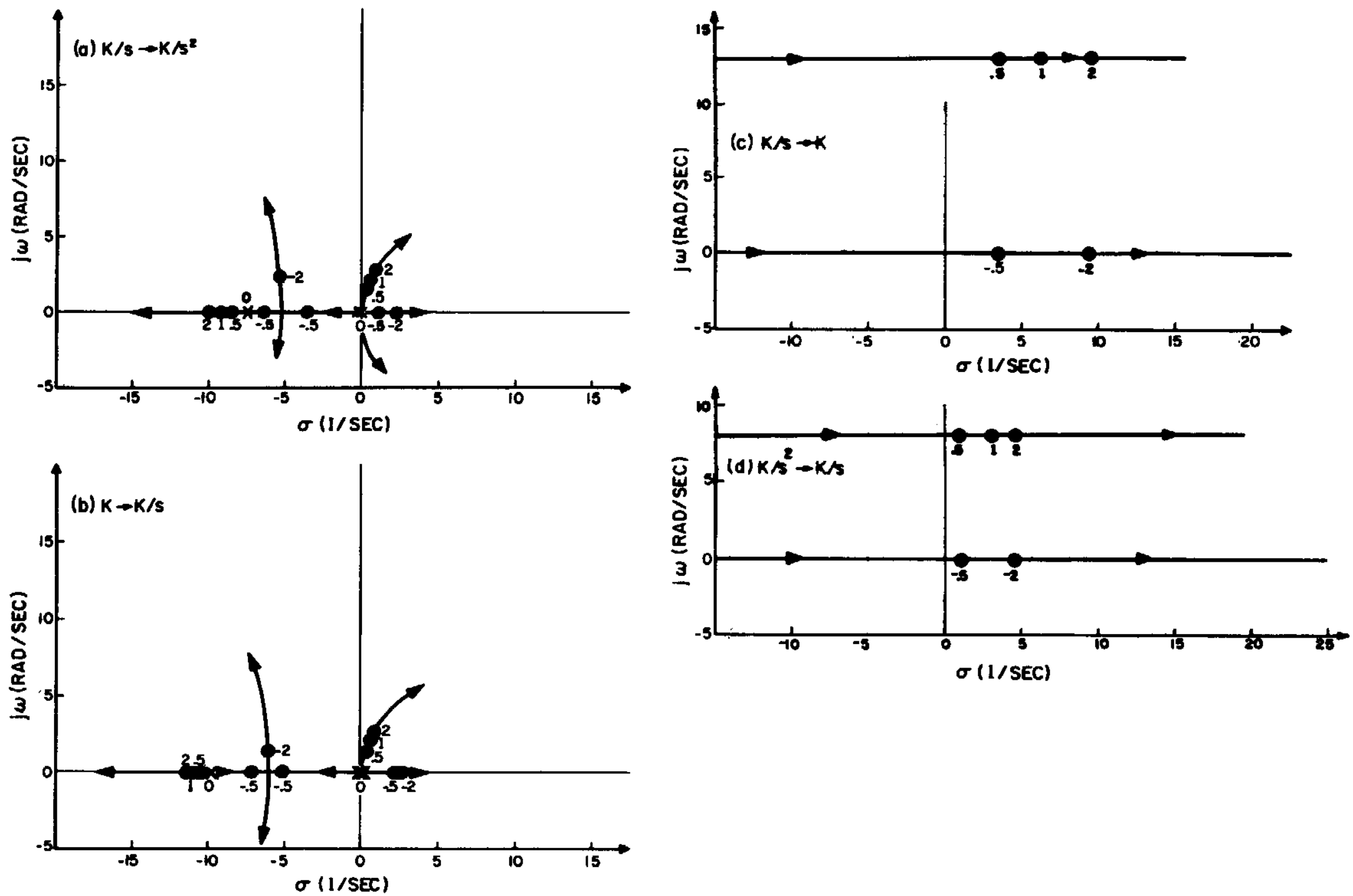

Fig. 16. Locus of closed-loop roots as a function of K_{10}, the ratio of post- to pre-transition gain for order increase (a,b) and order decrease (c,d) transitions.

For the pure gain change transitions of Experiment IA we see from Table 13 that the closed-loop roots indicate an oscillatory divergence for the gain increase transitions and a non-oscillatory divergence for the transitions in which the polarity was reversed. The rate of divergence increases as the post-transition gain increases. It also increases as the system order decreases.

For the transitions of Experiment IB in which the order of the controlled dynamics increased, we see that the post-transition system has a slightly divergent oscillatory characteristic if there is no change of polarity and is non-oscillatory divergent if there is a polarity change. The divergence results from the fact that immediately after transition HC(s) has a double pole at the origin which moves into the right-half plane with either positive or negative values of gain. Figures 16a and b show this effect.

For the order decrease transitions, the post-transition closed-loop system has only one pole. It lies in the right-half plane for all gain values used in the experiment (see Figs. 16c and d). This pole results from the time delay τ_o. Unless the post-transition loop gain is very low (less than 0.5 for the conditions used in these experiments) a divergence will result.

Given the closed-loop poles and zeroes, the initial conditions on the error at the time of transition, and the input disturbance, we can compute the error transient that should result from a transition by using standard Laplace transform techniques. The computed error responses can then be compared with the observed average error curves.

All of the transitions in this experiment occurred at a point in time at which the output was zero and at which the input had a high velocity. The input disturbance in the vicinity of a transition can be approximated by a ramp function. Typically, just prior to a transition the error was significant, but the error rate was approximately zero. For the computation of the system's response, we have assumed that the initial condition on the error rate is zero.

We have determined the form of the error response to a gain change transition given a ramp disturbance and an initial error. If the closed-loop roots of the system are complex with

$$s_1, s_2 = \sigma \pm j\omega$$

we find that

$$e(t) = \frac{2\dot{d}_o}{\omega_o^2 \tau_o} + \left(\frac{2\dot{d}_o}{\tau_o \omega_o \omega} - \frac{e_o \omega_o}{\omega} \right) e^{\sigma t} \sin(\omega t - \psi) \qquad (7.2)$$

where

τ_o = effective time delay for pre-transition dynamics.

$\omega_o^2 = \sigma^2 + \omega^2$

$\psi = \tan^{-1}(\frac{\omega}{\sigma})$

$\dot{d}_o$ = slope of ramp disturbance.

e_o = initial error at transition.

If the closed-loop roots are real so that

$$s_1 = \sigma_1$$
$$s_2 = \sigma_2$$

the error becomes

$$e(t) = \frac{2\dot{d}_o}{\tau_o\sigma_1\sigma_2} + \left(\frac{2\dot{d}_o}{\tau_o\sigma_1\sigma_2} - e_o\right)\left(\frac{\sigma_2 e^{\sigma_1 t} - \sigma_1 e^{\sigma_2 t}}{\sigma_1 - \sigma_2}\right) \qquad (7.3)$$

For transitions in which one of the roots, say σ_2, is positive and the other root, σ_1, is negative, the error determined by Eq. (7.3) will be dominated by the exponential term containing σ_2. If, moreover, $|\sigma_1|>>|\sigma_2|$, Eq. (7.3) can be approximated by

$$e(t) = \frac{2\dot{d}_o}{\tau_o\sigma_1\sigma_2} - \left(\frac{2\dot{d}_o}{\tau_o\sigma_1\sigma_2} - e_o\right) e^{\sigma_2 t} \qquad (7.4)$$

For large values of $\sigma_2 t$, this becomes approximately

$$e(t) \doteq - \left(\frac{2\dot{d}_o}{\tau_o\sigma_1\sigma_2} - e_o\right) e^{\sigma_2 t} \qquad (7.5)$$

We can use Eq. (7.5) to determine approximate values of σ_2 from the average error response curves obtained with the gain change transitions. This can be done by estimating the time required for the average error to increase by some factor, for example, a factor of two. By comparing these experimental values with those given in Table 13, we can obtain a check on the accuracy with which the crossover model predicts the post-transition behavior of the system.

If the roots are complex and ω is much larger than σ, Eq. (7.2) can be approximated by the relation:

$$\varepsilon(t) \doteq \frac{2d_o}{\omega_o^2 \tau_o} - \left(\frac{2\dot{d}_o}{\tau_o \omega_o \omega} - \frac{e_o \omega_o}{\omega} \right) e^{\sigma t} \cos \omega t \qquad (7.6)$$

The first peak of the average responses should occur at a time that is approximately equal to π/ω. Thus, from a measurement of time of the first peak of the average error curve we can determine the approximate value of the imaginary part of the closed-loop roots.

2. Modification

The model presented in Chapter III postulates that modification consists essentially of a switching of control strategies from the one appropriate to the pre-transition dynamics to the one appropriate to the dynamics selected by the identification process. The switching time was assumed to be equal to τ, the duration of a CI. An additional τ sec was also assumed to be required to retrieve the appropriate control strategy from memory once the identification had been made. Thus we postulated that modification would be completed approximately 2τ sec after an identification is made. For K and K/s dynamics 2τ will be about 0.4 sec. For K/s^2 it will be about 0.8 sec.

3. Transient Tracking

Following modification we postulated that the controller will enter a transient tracking mode. His characteristics in this mode were approximated by the crossover model:

$$HC(s) = \frac{\omega_{c_{opt}} e^{-\tau s}}{s} \qquad (7.7)$$

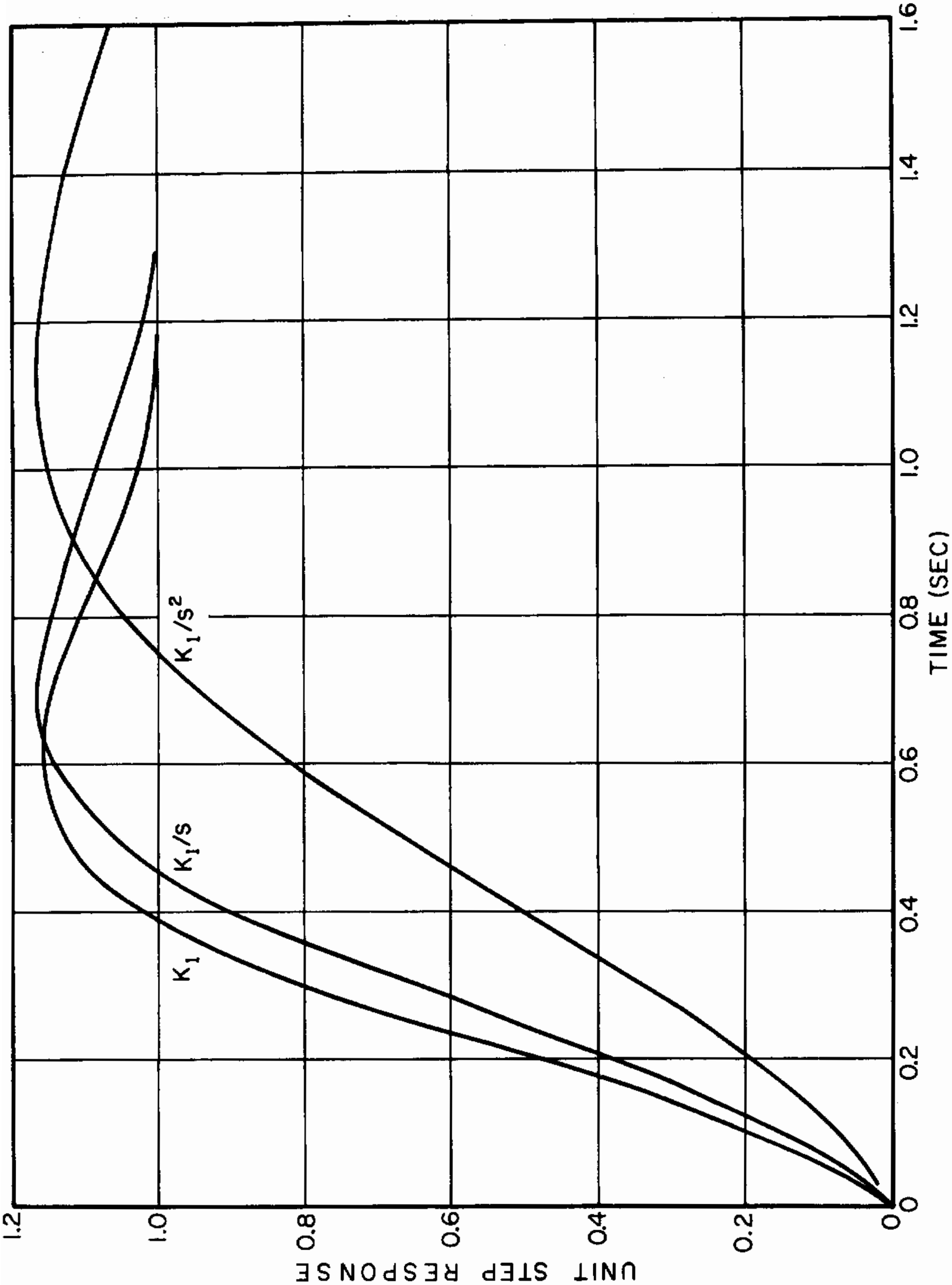

Fig. 17. Step response of the transient tracking model.

where $\omega_{c_{opt}}$ is chosen so that the mean-squared error will be minimum. Values for $\omega_{c_{opt}}$ for the three types of dynamics investigated in these experiments are given in Table 2.

If we represent the delay in Eq. (7.7) by a first order Padé approximation, the closed-loop dynamics can be approximated by a quadratic function of frequency in which the damping factor is about 0.5 and the natural frequency is about $1.2/\tau$. In Fig. 17 are shown the step responses of the transient tracking model for K, K/s, and K/s^2 post-transition dynamics. If at the beginning of the transient tracking phase there is an initial error displacement, Fig. 17 shows that this transient error will be reduced to a small value in about .8 sec for K dynamics, 1.0 sec for K/s dynamics, and 1.6 sec for K/s^2 dynamics.

4. Vernier Adjustment

After the transient resulting from the transition has been nulled, we postulated that there would be a gradual adjustment of the gain until it becomes approximately equal to the value appropriate to steady-state tracking with the post-transition dynamics. In terms of the crossover model the gain is ω_c. Thus, during the vernier adjustment phase we postulate that ω_c should go from $\omega_{c_{opt}}$ to the steady-state value of ω_c, values for which are given in Table 2. The values of $\omega_{c_{opt}}$ for transient tracking are between 65 and 80 percent of the values of ω_c for steady-state tracking. The higher the system order, the greater is the difference between the transient and steady-state values of ω_c. For all of the systems we have studied, we would expect to see an increase in ω_c in the vernier adjustment phase.

C. ENSEMBLE AVERAGE ERROR RESULTS

In Figs. 18 and 19 are the average error results for the simple gain changes for each of the two subjects, respectively. The results for the order increases are in Fig. 20 and those for the order decreases are in Fig. 21. When we arranged the average error responses for these figures we took advantage of the fact that for all transitions of a given type the velocity of the input disturbance was of the same sign, although the velocity was positive for certain types of transitions and negative for others. We have standardized all of the average error curves, reversing their polarity where necessary, so that they represent the error response of the system to an input whose velocity is negative. Thus, in the discussion of these average error curves we will be concerned with the response of the system to a negative ramp-like disturbance input. For such an input disturbance, the error at transition was usually negative, and, as was pointed out earlier, the error rate was nearly zero. Note that there is a discontinuity at the beginning of each of the average error curves. It results from the fact that there was a delay of either 76 or 152 msec (depending upon the time scale used) in the digital computer program between the occurrence of a transition and the first point averaged in the average error curves.

1. Experiment IA - Gain Changes with System Order Constant (Figs. 18 and 19)

Gain Increase. As shown in Table 13, a gain increase by a factor of four leads to an oscillatory instability. This oscillation is more evident in the average response for the subject RB than for GK. It is also more evident in many of the individual time histories of the transition response than in the average response curves. The first peak of the error curves for K dynamics occurred .2 to .3 sec after the transition, for K/s dynamics it occurred between .3 and .4 sec and for K/s^2 dynamics between .3 and .6 sec.* These values are consistent with the results of Table 13 that show the imaginary part of the closed-loop roots decreases with increasing system order.

Of particular interest in these figures is the observation that in none of the cases is the subject's second error peak of higher amplitude than the first one. Thus, some modification must have taken place before the second peak is reached, and, undoubtedly, before the time the error has crossed the base line. For both subjects the average error crosses the base line at approximately .6 sec for K dynamics, at approximately .8 sec for K/s dynamics and at approximately 1 to 1.3 sec for K/s^2 dynamics. Modification must have been started before these times.

Note that subject RB's error curve for K/s^2 dynamics is oscillatory, but does not actually cross the base line before the first complete oscillation. This subject appeared to be more conservative in making gross responses with these dynamics and tended to bias his responses so that they would not overshoot.

The error curves approach their asymptotic levels within about 1 sec for K dynamics, 1.1 sec for K/s dynamics, and 3 sec for K/s^2 dynamics.

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*Descriptive measures of the average curves are in Table 14 and are discussed on page 162

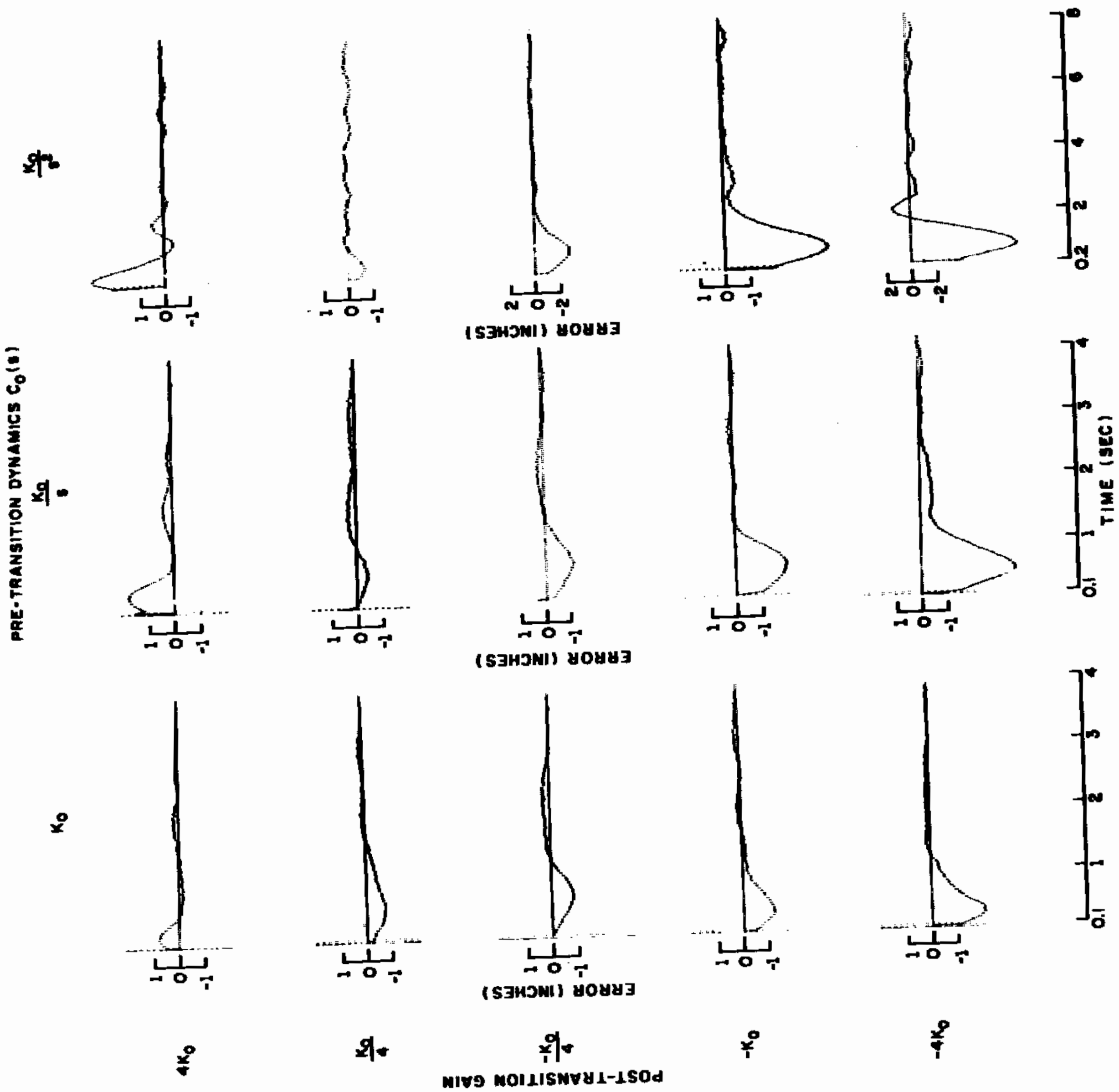

Fig. 18. Average error responses for gain change transitions with subject GK.

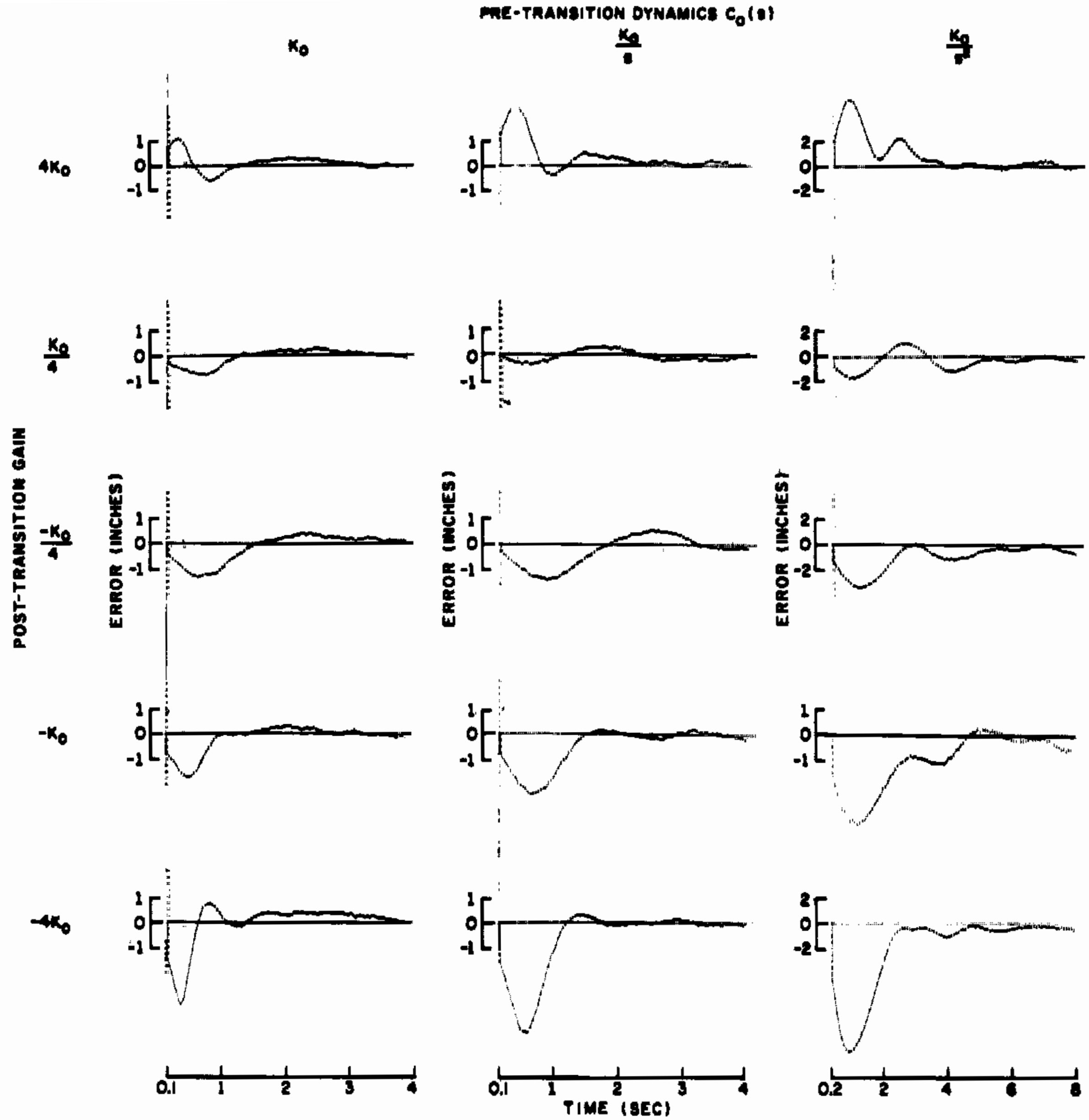

Fig. 19. Average error responses for gain change transitions with subject RB.

Gain Decrease. A decrease in the gain of the controlled-element by a factor of four does not lead to instability in the closed-loop performance, but simply lowers the open-loop gain to a point where the system response becomes sluggish. Such a transition produces very small initial error when the input signal is of low-frequency and does not have serious effects on system performance if adaptation does not take place rapidly. The average error curves slowly increase negatively reaching a rather flat peak at about .7 sec, and then gradually returning toward zero. They cross the axis between 1.0 and 2.0 sec for all cases.

Polarity Reversal and Gain Decrease. From Table 13 we see that one of the post-transition roots is positive real and the system should be divergent. The average error curves increase slowly in a negative direction for a fairly long time and have a rather broad peak indicating a large variation in the time at which modification is made. The times of the first peak are about .7 sec for K dynamics, .8 sec for K/s dynamics, and about .9 sec for K/s^2 dynamics. Following this peak, the average error curves decrease toward zero. For the subject RB, and to some extent for the subject GK, there appears to be evidence of a long, low overshoot extending from about 2 to 5 sec after the transition, indicating that the subjects had not completely compensated for the gain decrease.

Polarity Reversal. A simple polarity reversal will result in a positive, real root that will lead to a more rapidly divergent response than in the case of a polarity reversal accompanied by a gain decrease. The average error curves increase rapidly in a negative direction following the polarity reversal as each attempt to decrease the error merely increases it further. The error reaches a peak at about .5 sec for K dynamics, .7 sec for K/s dynamics, and .9 sec for K/s^2 dynamics. The peak error can occur only after the subjects have changed their control strategy by reversing the polarity of their control movements. Thus, the major modification required for this type of transition must have been completed in a time less than the peak error time.

Following the change of the polarity of the controller's movements, the error decreased rapidly to a low level usually without significant overshoot. The times required for GK to reach the asymptote are 1 sec for K dynamics, 1.2 to 1.5 sec for K/s dynamics, and about 2 to 4 sec for K/s^2 dynamics. Once again we see timidity on the part of subject RB with K/s^2 dynamics, and he does not manage to cancel out the accumulated errors until about 4 sec after the transition.

Polarity Reversal and Gain Increase. For this case, the system is even more highly divergent. To correct for the change of dynamics the controller must reverse the polarity of his response and also decrease his gain. We see from the average error curves that the polarity reversal occurs at a slightly earlier time than it did for a simple change of polarity of the controlled dynamics. The first peak of the error curves occur at about .4 sec for K dynamics, and about .5 sec for K/s dynamics, and about .7 to .9 sec for K/s^2 dynamics. These times are also slightly shorter than the times of the first peak for a simple gain increase. For several of the transitions there is evidence of a second peak or overshoot such as was observed for the case of the simple gain increase. When an overshoot occurs it is of lower amplitude than the initial peak, indicating that an adjustment of gain had been made probably prior to the first crossing of the base line. The time for the first base line crossing tends to be longer in the case of the combined gain increase and polarity reversal than it was for the case of the simple gain increase.

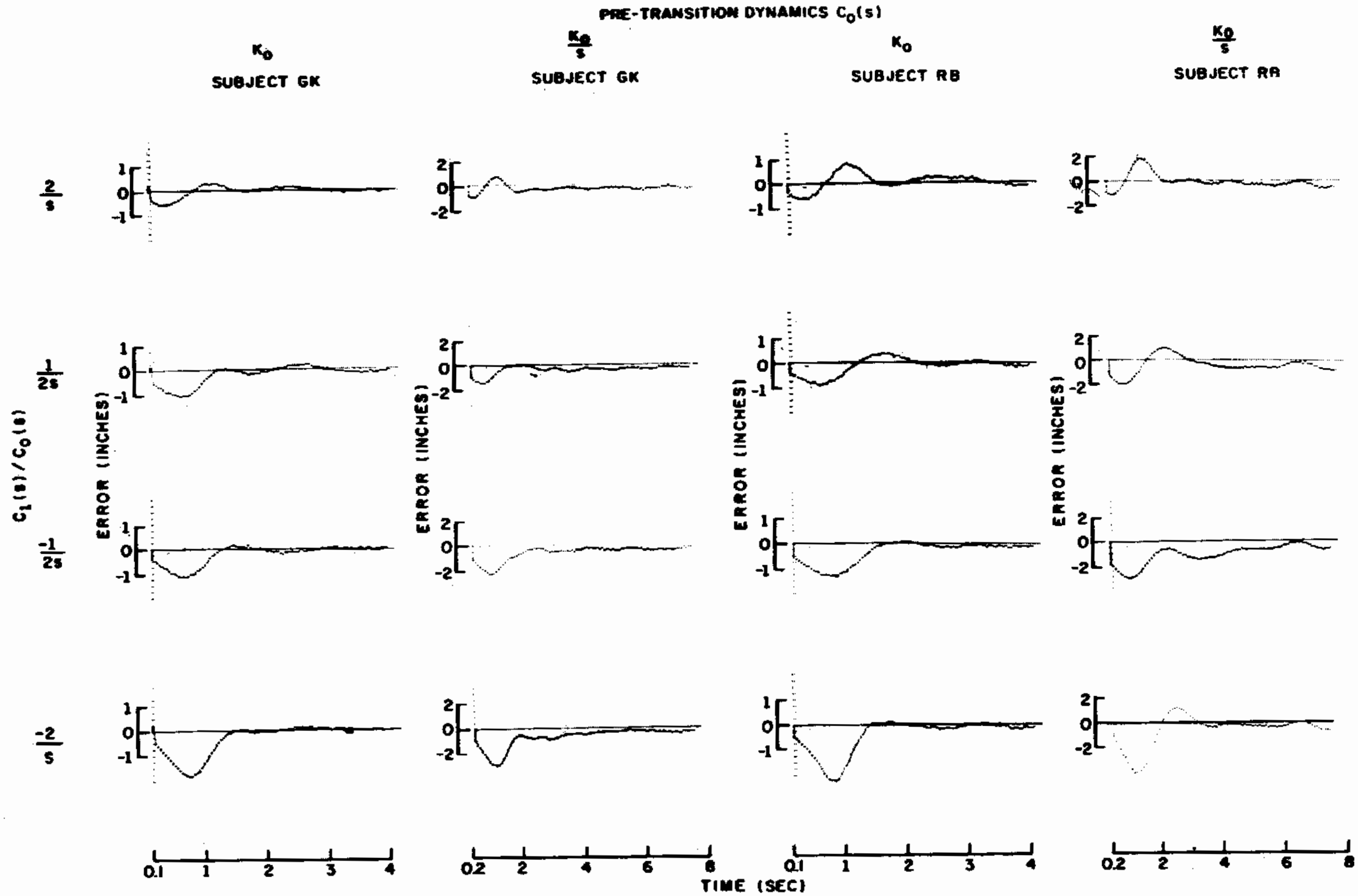

Fig. 20. Average error responses for order increase transitions.

2. Experiment IB - Gain and Order Changes

Order Increases. (Fig. 20). The human controller adjusts his characteristics so that the total open-loop characteristics are approximately the same in the critical region around gain crossover for all the dynamics that we have employed in this experiment. Thus, we would expect that the response of the system to a change in controlled-element dynamics would depend primarily upon the nature of the change and secondarily upon the pre-transition dynamics. From Table 13 we see that a doubling of the gain and increasing of the plant order from K_0 to $2K_0/s$ leads to about the same closed-loop roots as a transition from a K_0/s to a $2K_0/s^2$ plant. The average error curves for the K_0 to the K_1/s transitions are very similar to those for the K_0/s to K_1/s^2 transitions. Note the difference in time scales for these two sets of transitions. When the plant order is increased without a change of polarity, the post-transition system will tend to have an oscillatory divergence. This is clearly seen in Fig. 20 for the case of a gain doubling and order increase. The effect is less evident when the plant gain is decreased as the order is increased. Of course, when there is a reversal of the polarity of the gain the post-transition system will show a non-oscillatory divergence. This is also evident in Fig. 20.

Order Decreases. (Fig. 21). A sudden decrease in the order of the system will leave the post-transition dynamics with too little lag compensation, and unless the gain is very low, a rapid divergence will result. From the plots of the locus of the principal roots of Figs. 16c and 16d, we see that the system will be oscillatory if K_{10} is positive and non-oscillatory if K_{10} is negative. The location of the post-transition, closed-loop roots is a sensitive function of the gain and we would expect, therefore, large trial-to-trial variations in the human controller's and in the system's response to transitions of this type. Large variations were, in fact, observed in the time histories. In

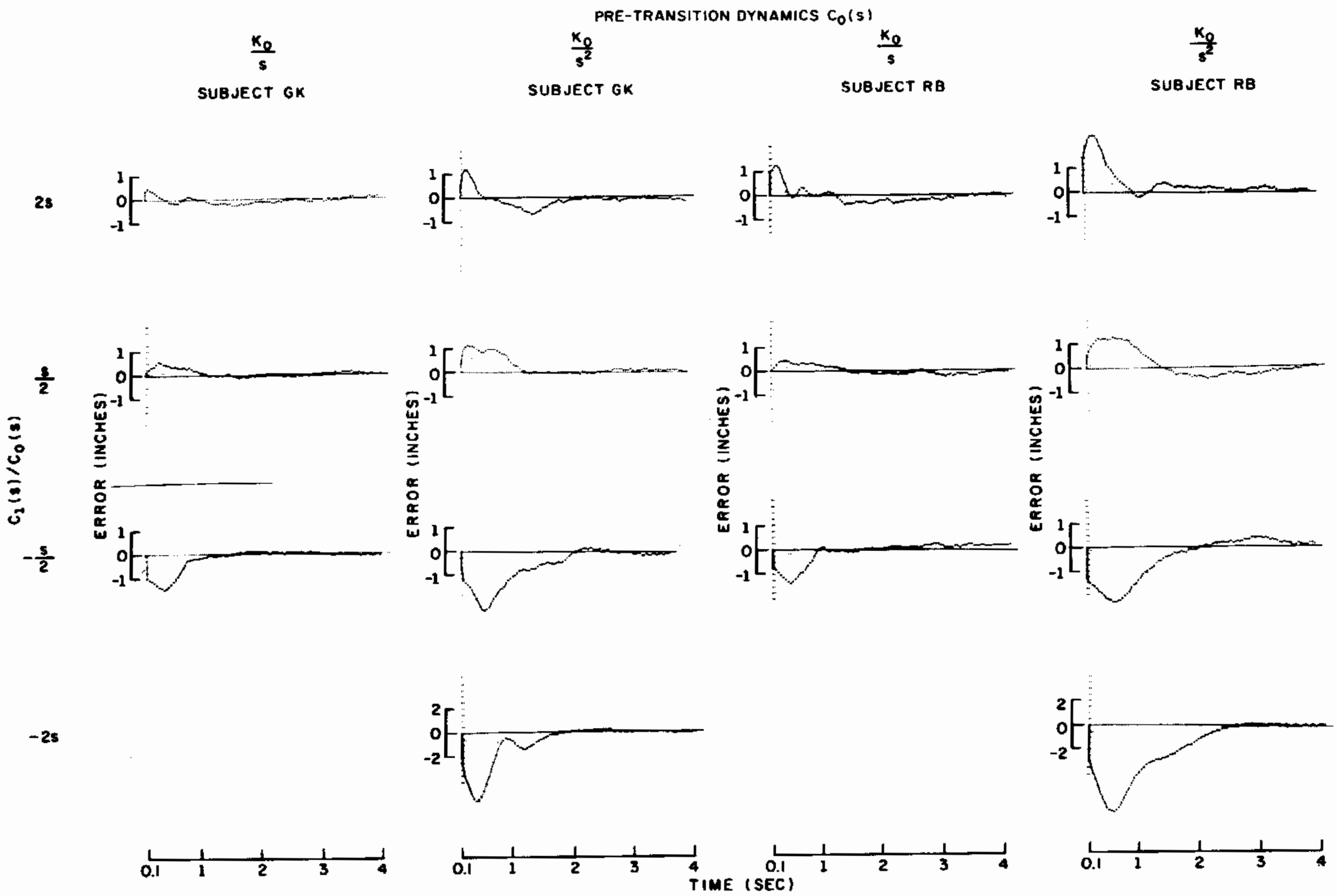

Fig. 21. Average error responses for order decrease transitions.

spite of this large variability there is still considerable similarity between the transitions from K_0/s to K_1 and those from K_0/s^2 to K_1/s. The time histories for the order decrease transitions indicate that modification frequently was not accomplished in a single switching from the pre-transition control strategy to the correct post-transition strategy. Rather, the controller appeared to modify his control strategy in sequential fashion, correcting separately for different aspects of the change in controlled dynamics resulting from the transition. Fig. 22 shows a sample time history of a transition from K_0/s^2 to $-2K_0/s$. Note that the correction for the polarity reversal occurs first. But after this modification, the gain remains too high, as is evident from the oscillatory character of the error. This indicates that modification had not yet been completed. It is likely that the subject had compensated for the order decrease before the first zero crossing because the error is oscillatory. The root loci of Fig. 16d indicate a non-oscillatory divergence for negative values of gain if there is no compensation for the change in system order.

The fact that modification appears to be a sequential process in these transitions does not necessarily contradict the modification model which postulates a simple switching of control strategies. It is possible that the controller was not able to make a complete identification of the post-transition dynamics, and that, initially, he identified correctly the polarity and order of new dynamics, but did not identify the gain. After he initiated modifications to correct for the polarity and order change, he may then have revised his identification decision and correctly compensated for the gain change. We cannot determine from our present results whether or not the sequential modification process can be explained by the identification model and this is a subject that should be pursued in further research.

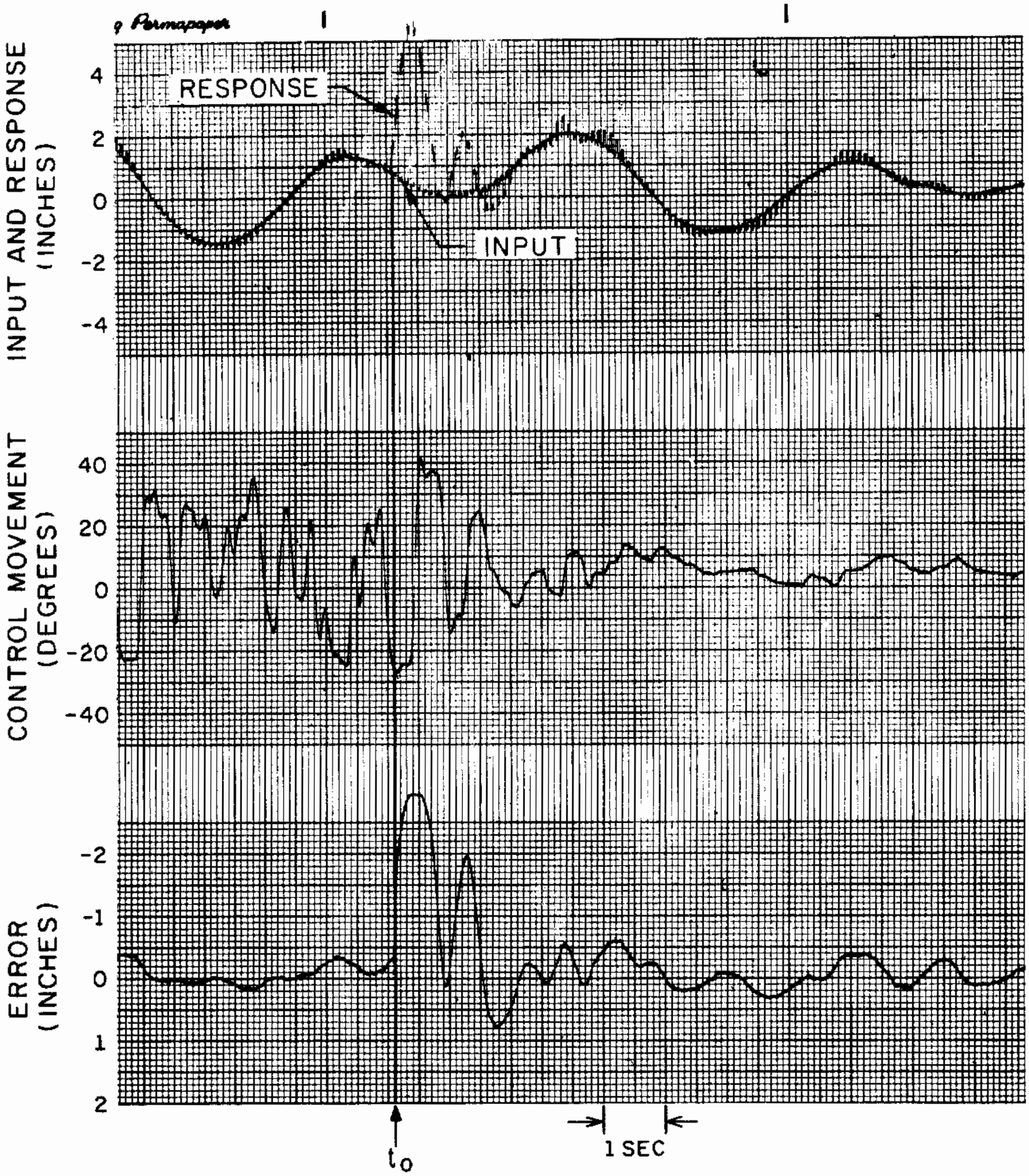

Fig. 22. Sample time history of an order decrease transition: K_0/s^2 to $-2K_0/s$.

D. DESCRIPTIVE MEASURES FROM THE AVERAGE RESPONSES

In Table 14 are descriptive measures of the adaptive process for gain changes. These measures were taken from the average error curves. The peak error, time of peak error, time to first zero crossing, and time for the average error curve to reach asymptote are given in the table. The entries are the averages of the measures for the two subjects. The times to reach asymptote are rough estimates of the time required for the error to reach a small relatively steady value.

The peak error times provide an estimate of, usually a lower bound on, the modification time. The increase in peak error times with system order reflects two factors. First, the post-transition system is more divergent with low order dynamics than with high order dynamics. Therefore, the error builds up more slowly with the higher-order dynamics, and detection and identification can be expected to occur later. Second, because of the lower bandwidth of the higher order systems, which results from the longer time delay associated with such systems, corrective actions to reverse the growth of the error will take longer to be effective. In the case of the transitions with polarity reversal, the more divergent the system, that is, the higher the gain, the shorter is the time to peak error.

The peak error magnitudes also increase with system order. This result is expected for the same two reasons. The longer detection and identification times for the higher-order dynamics should result in larger errors. The lower the order of the dynamics the more rapidly can the subject reverse the divergent course of the error. A single movement with K dynamics will change the error displacement, whereas,

Table 14

DESCRIPTIVE MEASURES FROM AVERAGE ERROR CURVES

(Average of Two Subjects)

$C_1(s)/C_0(s)$	Controlled Dynamics	Peak Average Error (in.)	Time of Peak (sec)	Time of 1st zero (sec)	Time to Asymptote (sec)
4	K	1.1	.3	.6	1.0
	K/s	2.4	.4	.8	1.1
	K/s^2	4.5	.5	1.3	2.8
1/4	K	.8	.7	1.5	1.5
	K/s	.6	.7	1.1	1.1
	K/s^2	1.1	.7	1.5	3.1
-1/4	K	1.3	.7	1.5	1.5
	K/s	1.4	.8	1.7	2.3
	K/s^2	3.3	.9	-	2.3
-1	K	1.7	.5	-	1.1
	K/s	2.4	.7	-	1.4
	K/s^2	4.1	.9	-	3.5
-4	K	2.9	.4	.9	1.2
	K/s	4.3	.5	1.2*	1.6
	K/s^2	10.0	.8	1.8*	2.2

*Only one subject had a zero crossing. Entry is the time for that subject.

a similar movement with K/s^2 will merely provide a correcting acceleration of the error. Some time must elapse before this acceleration is reflected in a decrease of the error.

We also find that the times of the first zero crossing and the times to reach asymptote generally increase with order. Much of the increase in these times can be attributed to the longer transient response times associated with the higher order dynamics. This effect is evident in the step responses of Fig. 17 which show the postulated characteristics of the system during the transient tracking phase.

In Table 15 are given approximate values for the dominant root in the post-transition retention period for the gain change transitions in which there was a polarity reversal. Also given are approximate values for the imaginary part of the roots for the gain increase transition without polarity reversal. These were determined from measurements made on the average error response curves. For the transitions, in which there was a polarity reversal, the roots were determined by using Eq. (7.5) and measurements of the time required for the error to double. For the gain increase transitions, in which the system was oscillatory divergent, the estimates of the frequency of oscillation, ω, were based upon Eq. (7.6) and measurements of the time to the first peak of the average error curves. The values shown in the table are the averages of the estimates of the roots for the two subjects.

Comparison of the measured roots given in Table 15 with the corresponding predicted roots, which are also given in Table 15, shows that except for the transition in which $K_1 = -K_0/4$, the measured roots are generally within a factor of two of the predicted roots. This agreement is not too bad, when we take into account the approximations made in deriving Eqs. (7.5) and

Table 15

MEASURED AND PREDICTED POST-TRANSITION ROOTS

$C_1(s)/C_0(s)$	Pre-Transition Dynamics $C_0(s)$: K_0	$K_0(s)$	K_0/s^2
	Measured Roots		
4	ω = 10.5	ω = 7.9	ω = 6.3
-1/4	σ_2 = 3.2	σ_2 = 2.8	σ_2 = 1.6
-1	σ_2 = 4.2	σ_2 = 2.9	σ_2 = 1.6
-4	σ_2 = 4.9	σ_2 = 3.5	σ_2 = 2.2
	Predicted Roots		
4	ω = 13	ω = 11	ω = 7.1
-1/4	σ_2 = 1	σ_2 = .92	σ_2 = .63
-1	σ_2 = 2.8	σ_2 = 2.5	σ_2 = 1.6
-4	σ_2 = 5.7	σ_2 = 4.9	σ_2 = 3.1

(7.6) and the difficulty of determining the roots accurately from the average error curves. In particular, we can not be certain that modification had not started during part of the interval used to determine the time for the error to double (for the divergent cases) or for the error to reach peak (for the oscillatory cases). Thus we conclude that the crossover model provides a reasonably good prediction of the controller's behavior in the post-transition retention phase.

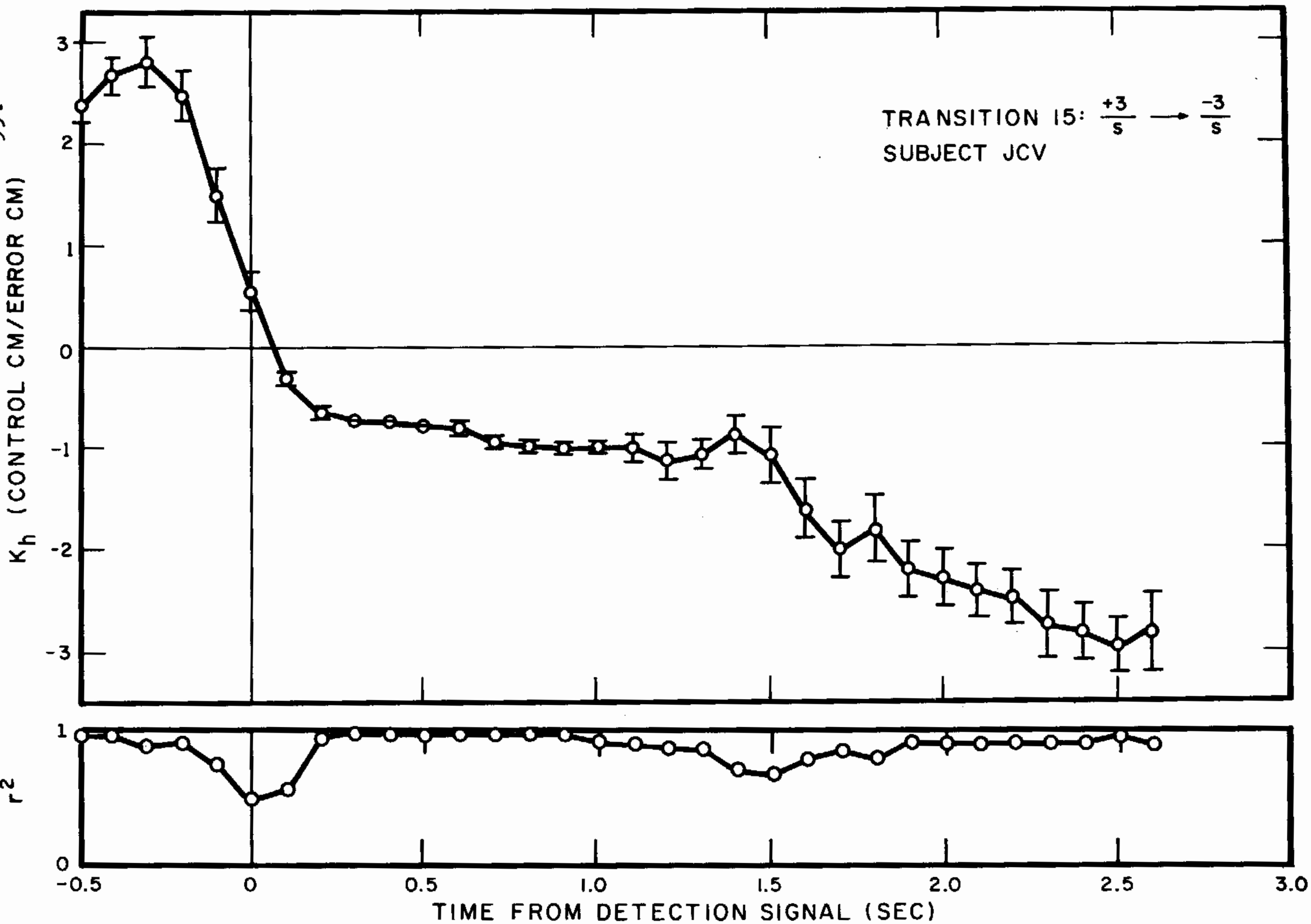

Fig. 23. Time histories of the ensemble average of $K_h(t)$ and $r^2(t)$ for polarity reversal transitions with subject JCV.

E. TIME-VARYING GAIN MEASUREMENTS

In Figs. 23 through 26 are presented the human controller's gain as a function of time for the three types of transitions investigated in Miller's experiment: a polarity reversal, a gain decrease and a gain increase with K/s controlled dynamics. The time histories of the gain were determined by averaging over an ensemble of 8 to 10 transitions using the techniques described in Chapter IV, Eq. (4.4), (4.5) and (4.7). For several of the transitions estimates of the standard deviations of the measured K_h are shown by the brackets around the K_h points. Values of r^2, the squared correlation between $e(t-\tau)$ and $c(t)$ as a function of time, are also shown in the figures. The stick and error signals were synchronized with the detection signaling time for computation of $K_h(t)$ and $r^2(t)$.

The time history of Fig. 23, a polarity reversal with one subject, was determined over a particularly long time span. We will discuss it in detail first. K_h starts to change about 0.1 to 0.2 sec before the detection signal and changes polarity 0.1 sec after the signal. For the next 1.5 sec, K_h remains fairly constant at about 1/3 its pre-transition magnitude. Then, the gain gradually increases over the next 1.5 sec period from about -1 to about -3. A gain of -3 is approximately equal in magnitude (but opposite in sign) to the pre-transition gain of about 3. The gain appears to approach -3 approximately exponentially with a time constant of about 0.4 sec.

Note that the standard deviations of the estimate of K_h are small which indicates that the estimates are reliable. Note also that the r^2 in Fig. 23 is generally close to unity which indicates that the simple linear model of Eq. (4.3), namely that

$$c_{\lambda}(t) = K_h(t)e(t-\tau) + n(t), \qquad (7.8)$$

accounts for most of the human controller's control movements and that the remnant n(t) is small.

The $K_h(t)$ behavior correlates well with that which we would expect from the control model. From the identification results of Chapter VI, we would expect that for this transition identification would be completed about 0.3 to 0.4 sec prior to the detection signal. In Fig. 23 we see that K_h begins to change somewhere between 0.1 and 0.2 sec prior to the detection signal. The time of this change in gain corresponds to the beginning of modification and occurs about τ sec after identification as was postulated in the model. The gain reverses polarity at about 0.1 sec and is near -1 at about 0.2 sec. Since it remains near -1 for the next 1.5 sec, we may conclude that modification is complete by 0.1 to 0.2 sec after the detection signal, or about 0.3 sec after the modification begins. This is just slightly more than the time period of τ sec that we postulated would be required for the human controller to modify his control strategy. The gain remains relatively constant until 1.6 sec when it begins to increase. If we take 1.6 sec as the beginning of the vernier adjustment phase, then the transient tracking phase must extend from about 0.2 sec to 1.6 sec, an interval of 1.4 sec. From Fig. 17 we note that the step response for transient tracking with K/s dynamics reaches asymptote in about 1 to 1.2 sec, a duration that is reasonably consistent with that observed from the K_h results of Fig. 23. K_h during transient tracking is about 37 percent of that observed for steady-state tracking. From Table 2 we would have predicted K_h for transient tracking to be about 75 percent of the steady-state K_h. Thus, the observed gain differs from the predicted gain by a factor of 2.

In Fig. 24 are shown somewhat shorter segments of the time histories of $K_h(t)$ for the same polarity reversal transition as is in Fig. 23 for all three subjects. For all subjects modification appears to begin 0.1 to 0.2 sec prior to the detection signal. For two of the subjects, modification appears to be largely completed about .2 sec after the signal. For these two subjects the gain remains fairly constant with a magnitude that is about 1/2 to 1/3 its pre-transition value for the remainder of the time shown in the figure. For the third subject, RBT, modification is a more gradual process and about 1 sec is required for the gain magnitude to reach 1/3 of its pre-transition value. During the time period in which the gain is changing, his r^2 is very low, which indicates that the model is not successfully accounting for the human controller's behavior. These low correlations result primarily from run-to-run variations in the controller's behavior. For several of the transitions for this subject, the polarity of the subject's control movements was not changed until 0.3 to 0.4 sec after the detection signal. These delayed reversals cause the average gain $K_h(t)$ to change gradually.

The significant characteristics of the time histories of $K_h(s)$ for the gain decrease transitions in Fig. 25 resemble those for the polarity reversal transitions of Fig. 24. For subjects RBT and RGT, $K_H(s)$ starts to increase about 0.2 to 0.3 sec prior to the detection signal from its pre-transition value of about 0.5. Subject JCV does not appear to increase his gain until shortly after the time of signal. For all three subjects, the gain reaches approximately 1.3 within 0.2 sec after the signal. This value of gain is about 2/3 of the expected steady-state gain and thus there should be a gradual increase in the gain magnitude later in the post-transition period, probably beginning at a time greater than that plotted in Fig. 25.

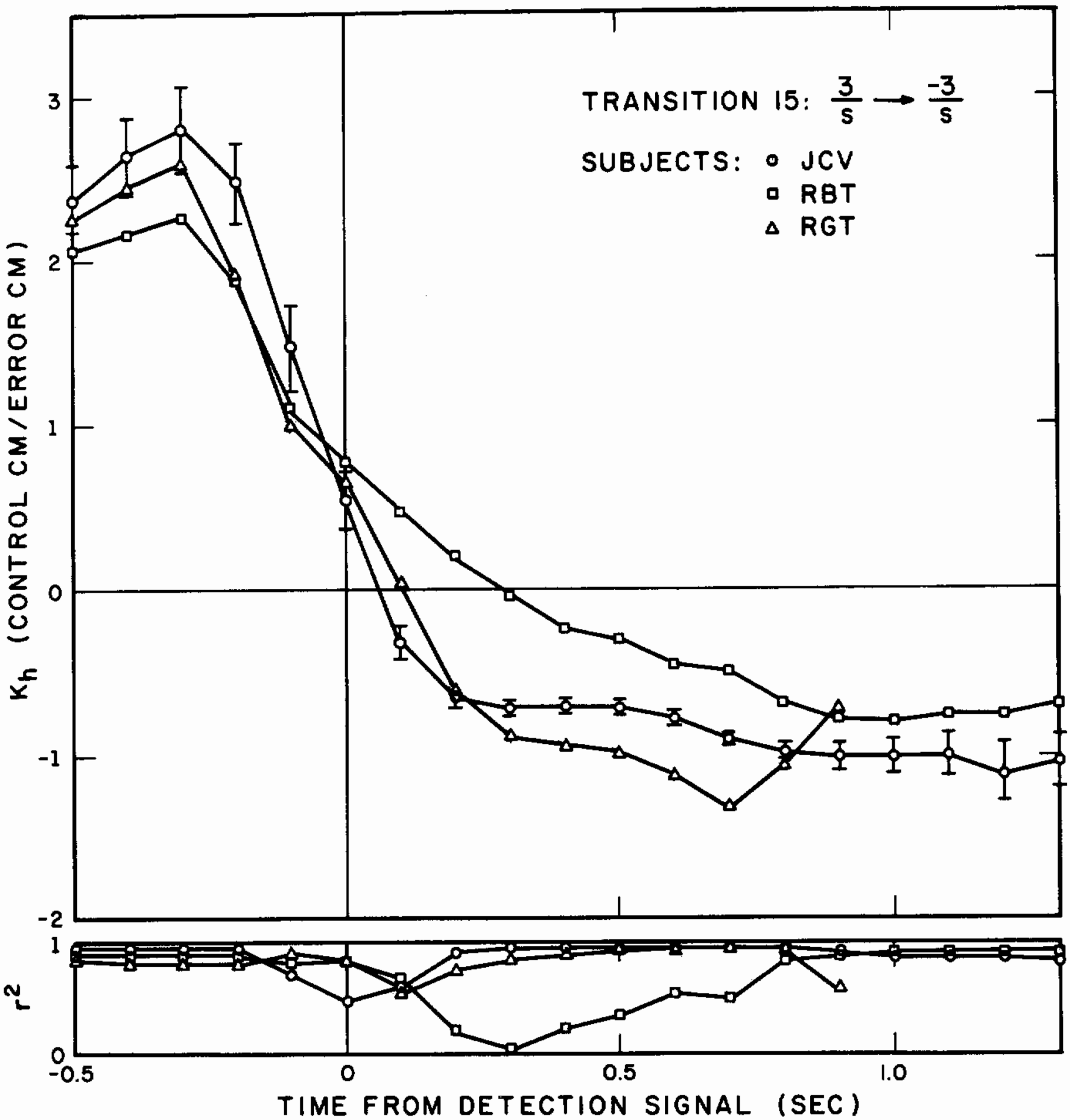

Fig. 24. Time histories of the ensemble average of $K_h(t)$ and $r^2(t)$ for polarity reversal transitions with three subjects.

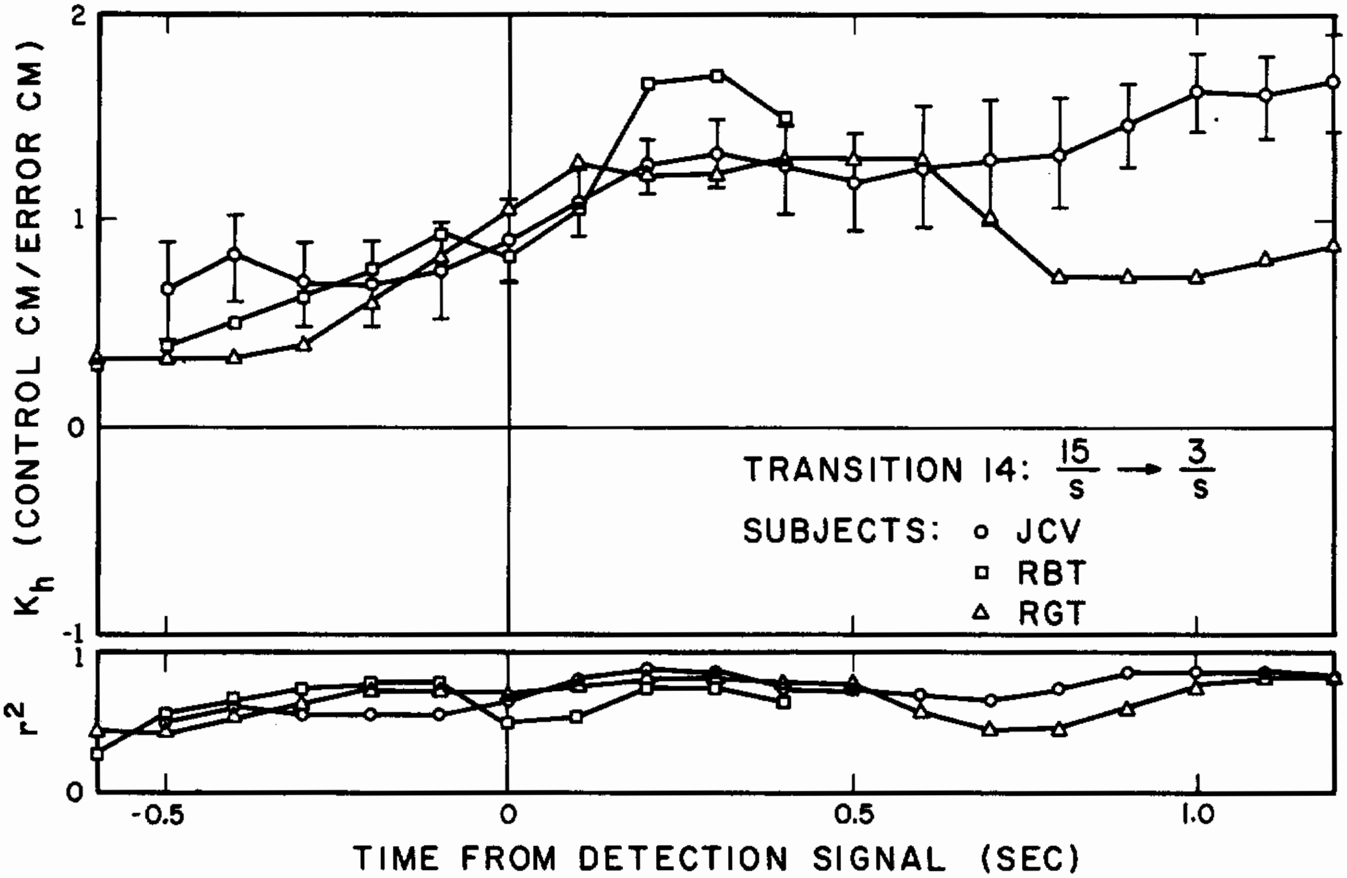

Fig. 25. Time histories of the ensemble average of $K_h(t)$ and $r^2(t)$ for gain decrease transitions with three subjects.

The results for the gain increase transition are in Fig. 26. The gain K_h decreases suddenly, and in some cases actually goes negative, 0.3 to 0.2 sec prior to the detection signal. This change in gain is in part an artifact that results from the fact that the system has an oscillatory divergence following the increase in controlled-element gain. As a result, there is a considerable run-to-run variability in relation between control movement and error. Note that for all three subjects r^2 is very small in this period, which is further evidence of large variability.

By about 0.2 sec after the detection signal the gain appears to stabilize at a value of about 0.2, which is about 1/15 of the nominal pre-transition value of about 3. For this transition the gain of the controlled dynamics increased by a factor of five and we would expect to see the steady-state post-transition gain of the human controller be 1/5 of his pre-transition gain. Thus once again we see that the controller's gain during the transient tracking phase is about 1/3 his steady-state gain.

$K_h(t)$ is not given for large enough values of time to clearly show the vernier adjustment phase. However, there is some evidence in the results for subject RGT that the vernier adjustment begins at about 1.0 sec. At this time the magnitude of the gain begins to increase toward .5, a value that would be appropriate for steady-state tracking.

The significant features of the time histories of $K_h(t)$ of Figs. 23 through 26 for the three types of transitions are in good agreement with each other and substantiate the model for modification and transient tracking. The timing of the modification process is observed to correspond closely with that posulated in the model. The gain during transition tracking,

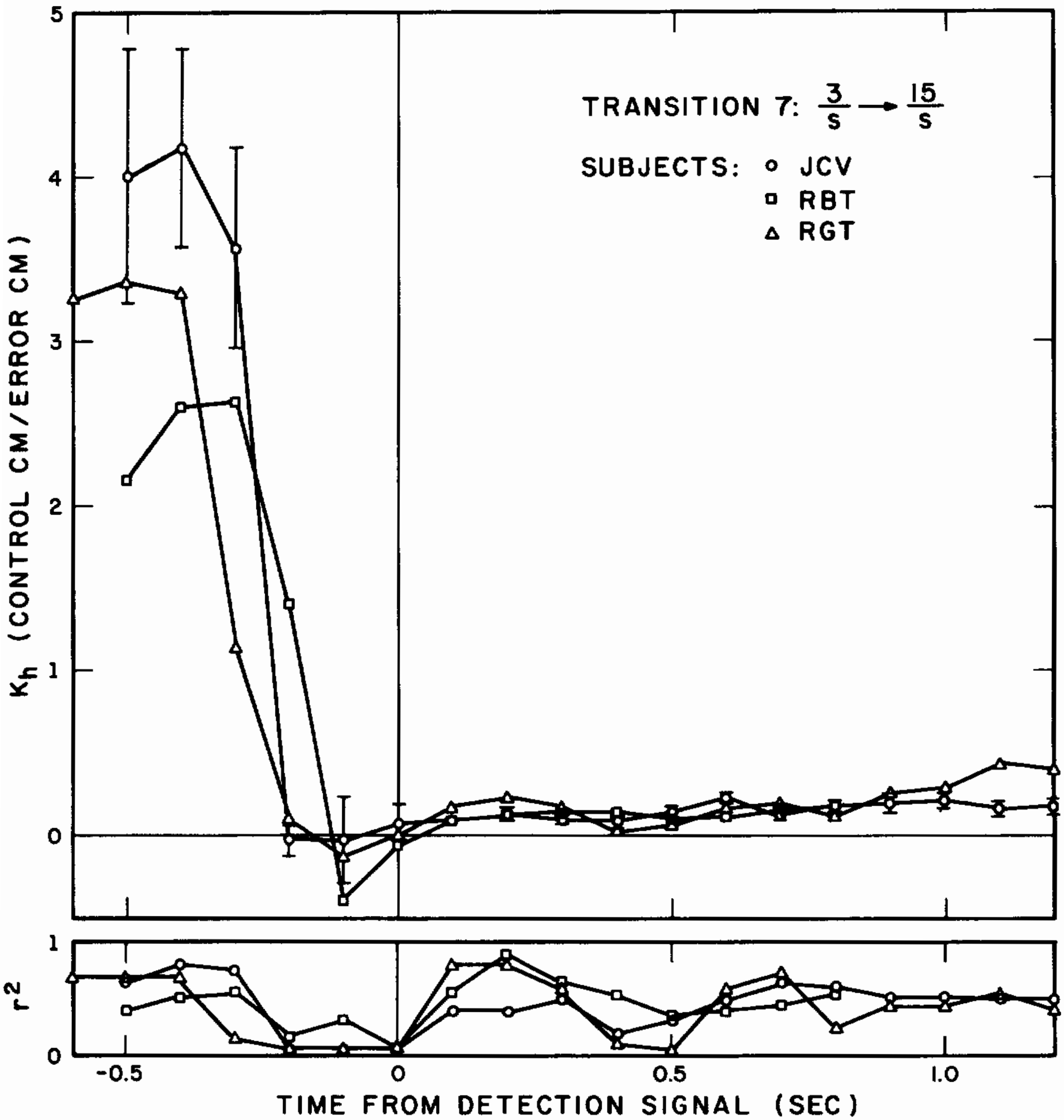

Fig. 26. Time histories of the ensemble average of $K_h(t)$ and $r^2(t)$ for gain increase transitions with three subjects.

however, is lower than predicted. The few results that we obtained for large values of time indicate that vernier adjustment begins at a time that is consistent with that predicted by the transient tracking model.

CHAPTER VIII

CONCLUSIONS

We have proposed a set of models for the decision and control processes involved in the adaptation by the human controller to sudden changes (transitions) in the dynamics that they are controlling. The decision processes postulated by the model are detection of a transition, identification of the new dynamics, and selection of the appropriate new control strategy. The control processes postulated are steady-state tracking, modification, transient tracking, vernier adjustment and then steady-state tracking with the new dynamics.

A statistical decision theory model is proposed for the detection and identification processes. This model has been tested in detail in experiments with controlled dynamics of the form K/s and K/s^2. The agreement between the detection and identification performance of the model and that obtained in the experiments is very good. The selection process is postulated to be a simple retrieval from memory of the appropriate control strategy. Retrieval is assumed to take a fixed time τ. A few data supporting this hypothesis were obtained.

Steady-state tracking performance is represented by the simple crossover model proposed by McRuer et al.[1] Average error response data obtained with K, K/s and K/s^2 dynamics showed that this model predicts with fair accuracy the system behavior in the post-transition retention period during which the controller retains his pre-transition control strategy, but the controlled dynamics have changed. Modification was postulated to be a simple switching of control strategies, the switching time taking τ sec. Ensemble average measurements of the human controller's time-varying gain with K/s dynamics support this hypothesis. For transient tracking we postulated a crossover model with the gain adjusted to give minimum mean-squared error. The average error curves and the time-varying gain measurements indicate that the time to correct for the transient errors predicted by such a model is approximately correct. However, the controller's gain during transient tracking predicted by the model is about twice that actually observed in the time-varying gain measurements. Evidence for a vernier adjustment of the controller's gain is presented, but detailed studies of this part of the adaptive process were not made.

The results of this study that are best substantiated and most significant are the decision theory models for detection and identification of a transition. These models cast the detection and identification processes in a framework that is intuitively satisfying and amenable to application to realistic control situations in which transition may occur. To apply the models, we must know the probability of a transition and the dynamics that are possible if a transition occurs. These are quantities that usually are known, or can be estimated, at least roughly in, for example, a reliability analysis. We must also know the human controller's subjective likelihood functions that a given error behavior will result from the control movements that the subject makes. These functions, in effect, summarize the human controller's state of knowledge or his ability to predict the response of the possible controlled dynamics. Although it is not clear how to estimate these likelihood functions, if we make the assumption that they have a normal distribution, they can be characterized by a few parameters that probably can be measured in some simple experiments. In any case, the models delineate those aspects of the detection and identification process that are a function of the physical situation and those that are a function of the human controller's knowledge of the physical situation. Such a delineation is helpful for the analysis and puts in focus a number of theoretical and empirical questions that can only be answered by further studies.

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13. ABSTRACT

The studies described in this report are directed toward the development of models for one aspect of human controller adaptation--adaptation by trained controllers to sudden changes in controlled-element dynamics of single-axis compensatory control systems. The multi-phase model proposed consists of detection, identification, modification, and optimization phases. Detailed models for all four phases of the adaptive process are presented and substantiated by experimental results.

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