

FOREWORD

The research work in this report was performed by Armour Research Foundation, Chicago 16, Illinois for the Structural Division, Directorate of Engineering Test, Deputy for Test and Support, Aeronautical Systems Division, Wright-Patterson Air Force Base, Ohio under Contract No. AF 33(616)-8444, Project No. 1347, Task No. 134702. James L. Mullineaux of the Structural Division was the ASD Project Engineer. The research was conducted from June 15, 1961 to July 30, 1962.

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Investigations were made of materials and methods for mechanically transmitting deflections from a structure under radiant heating to a transducer at room temperature. In order to measure structural deflections accurately under static or dynamic loading conditions, it is essential that materials in which thermal expansion and creep are minimized, or properly accounted for, be used for deflection transmission rods or cables.

Transparent fused quartz rods were evaluated for expansion and thermal shock resistance. Rod growth was measured for such rods under several heating rates, with various exposed rod lengths and at temperatures up to 2000°F. The growths enabled fairly accurate monitoring of static and cyclic displacements of a plate to which a transducer was connected through the quartz rod.

High purity recrystallized alumina rods were evaluated for expansion and thermal shock resistance. Rod growth was also measured under several heating rates for a limited exposed length and at temperatures approaching 3000°F. The measured growths were considerably greater than those experienced by the quartz rods but not by a factor equivalent to the ratio of their thermal expansions. A successful means for attaching such rods to alumina structures capable of following static and cyclic displacements was developed.

Rod growths and consequent deflection corrections are dependent upon heat input from radiant heat lamps and reflectors, re-radiated heat from the structure to which rods are attached, other reflected radiation and exposed length of such rods. For this reason corrections must be determined under conditions identical with those under which structural deflections are to be determined in order to be accurate.

PUBLICATION REVIEW

This technical documentary report has been reviewed and is approved.



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I. INTRODUCTION

Structural integrity assessment of flight vehicles requires the accurate determination of component movement with deflection transducers. When such determinations are to be made at elevated temperatures as a result of radiant heating, the thermal growth of the cables employed to mechanically transmit deflections has an adverse effect upon the accuracy. Therefore, to measure actual structural deflections under static or dynamic mechanical loading, it is essential that materials in which thermal expansion and creep are minimized, or properly accounted for, be used for deflection transmission rods or cables.

This program has as its objective the establishment of methods for use in accurately determining structural deflections of flight vehicles subjected to mechanical loading combined with radiant heating. Under room temperature conditions, the accuracy of the deflection measurements is dependent only upon the transducer. However, under elevated temperature conditions, the thermal growth of the cable or rod which mechanically transmits the deflections from the structure to the transducer introduces additional inaccuracies. Under certain conditions, using a rod material whose thermal expansion coefficient is relatively small, the actual expansion which could take place in a deflection cable could be considerably greater than the permissible error for such a measurement. This indicates that correction factors large in comparison with some of the observed deflections will have to be employed. For the resulting net deflections to be accurate, it is essential that the thermal growth, consisting of both creep and thermal expansion, be precisely determined.

Several factors influence such precise determinations. The principle factor is the thermal gradient experienced by the rod when part of the rod is subjected to a high heat flux and part is at room temperature. The precise nature of the thermal gradient as a function of time must be known in order to properly account for the thermal growth. Another factor is the tendency for physical properties of most materials to change once they have been subjected to a very high temperature. Thus, thermal growth characteristics of the materials can be different after one or more thermal cycles.

This report describes both an analytical and an experimental approach to the determination of rod growth correction factors for the measurement of deflections when the structure under test is heated to temperatures as high as 3000°F.

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II. REVIEW OF THE STATE OF THE ART

A. Deflection Transmission Techniques

A limited investigation was made of the current techniques employed for the transmission of deflections from structures under radiant heating to remotely located transducers. The Aeronautical Structures Laboratory of the Naval Air Materials Center has employed quartz rods as deflection transmitters in the structural testing of the Corvus missile at temperatures up to 1000°F. Inconel tips, one of which was pointed, were attached to the ends of the rods with a shrink fit joint. The pointed tip was used to assure alignment. Physical attachment was avoided by using springs to keep the rods under a few grams of compressive force at all times to maintain contact with the structure; the technique was used only for static testing. Only a short segment (about 4 in.) of the quartz rods was exposed to the radiant heat, and, therefore, the errors due to thermal expansion were considered negligible.

At the Aerospace Division of the Boeing Company electrical transducers utilizing potentiometers or strain gages as sensing elements have been used to measure deflections during structural tests to 1700°F. Nilvar wire was used to connect the transducers to the test specimens below 600°F; quartz rods were used in the 600 to 1700°F range. Communication with Boeing also indicated they were conducting tests on electrical and photometric systems developed for use to 3000°F on an experimental Air Force program. Another communication from Boeing indicated that they had been developing and evaluating a cooled quartz deflection transmitting rod, and an extruded, "near zero" thermal expansion coefficient, ceramic rod. Development and evaluation of the cooled quartz system was completed and a report was released to ASD.

At the McDonnell Aircraft Corporation, in conjunction with a refractory metals contract for ASD, Invar wire cable has been used to transmit deflections at the side opposite the heated side and no problems with thermal growth of the wire were experienced. In another structural test application, they employed a 1/8 in. O. D. alumina tube attached to the specimen by means of a wire doubled up inside the tube and bonded with a Sauereisen (No. 63) cement with one end of the wire spot-welded to the structure's surface.

B. Materials

Although thermal properties are fairly well documented for some of the materials that have potential as deflection transmission cables or rods, mechanical properties, especially creep at low stress levels, are not as well documented. This is the case particularly at the higher temperatures of interest in this study. This lack of sufficient physical property data upon which to base selection of potentially satisfactory materials has necessitated some experimentation to assure that the materials chosen have the desired

low expansion and creep characteristics. A discussion of some of the commercial refractory materials and their characteristics which make them potentially suitable for use as deflection transmission cables or rods follows.

1. Characteristics of Refractory Metals

Adequate creep data for materials at stresses as low as 50 psi (5 lb constant tension on a 3/8-in. rod) could not be found in the most recent literature for either the refractory metals or ceramics. It is possible to extrapolate existing data for tantalum* with the assumption that at low stresses the creep rate obeys an Arrhenius temperature dependence. In this way one obtains a value for unalloyed Ta of 7×10^{-3} percent per hour at 3000°F and at 50 psi. It is very likely that both unalloyed columbium and molybdenum will be far inferior to this at 3000°F. However, both the Mo-0.5 Ti alloy and F-48 Cb alloy (Cb-15W-5Mo-1Zr) will support in excess of 15 times the stress, on the basis of creep to rupture behavior, at temperatures near 2000°F.

These materials then, and tungsten (both pure and thorated) offer distinct possibilities for 2 hour service at temperatures up to 3000°F and stresses of 50 psi provided they are coated to minimize oxidation which is catastrophically rapid for all of these metals and alloys at these temperatures (200 mg/cm²/hr at 3000°F), and provided they are fully recrystallized before use. All of the materials cited recrystallize within 2 hours at 3000°F if cold worked and the occurrence of rather serious creep during recrystallization is well known. Both tungsten and the Mo-0.5 Ti alloy are brittle at ambient temperatures after recrystallization, consequently this factor will require consideration in their handling. Neither, for example, will tolerate bending much below 752°F in the recrystallized state.

Linear thermal expansion behavior is of the same order for all of the refractory metals useful here, ranging between $10 \times 10^{-6}/^{\circ}\text{F}$ and $20 \times 10^{-6}/^{\circ}\text{F}$. Exact values for the alloys at 3000°F are not available but they are, as a rule, reasonably close to the values for the base metal where available. The thermal expansion behavior can be allowed for, provided there are no time dependent phase changes occurring during heating or at temperatures up to 3000°F. Tungsten, tantalum, probably Mo-0.5 Ti and possibly F-48 (Cb alloy) will be adequate in this respect.

Coatings are under development to protect all of these materials from oxidation. To the writer's knowledge an entirely satisfactory coating for all temperatures up to 3000°F is not known. There are both proprietary (Chromalloy W-2) and experimental coatings (chiefly base metal aluminides) which are satisfactory for periods up to 100 hours at temperatures as high as 2500°F. These may be satisfactory for 2 hours at 3000°F. Many of the coatings show deterioration at low (1400°F) temperatures but, here again, for an exposure limited to 2 hours they may be satisfactory.

* DMIIC Report No. 133 (1960) July, Battelle Mem. Inst. publ.

2. Characteristics of Refractory Ceramics

Fused quartz is an amorphous material of high purity, produced by the simple fusion of pure silicon dioxide, one of the most inactive of natural substances. It has the lowest linear expansion of any generally available material. The expansion coefficient of $2.5 \times 10^{-7}/^{\circ}\text{F}$ is a fairly good average figure for the range 32 to 1800 $^{\circ}\text{F}$. As a result of its low expansion, fused quartz can tolerate rapid and extreme changes of temperatures. It is also practically free from thermal hysteresis, differing in this respect from most other solids which suffer permanent deformation when subjected to a temperature cycle. It shows no appreciable weight loss on prolonged exposure to temperatures of 1800 $^{\circ}\text{F}$, and is much superior to platinum in this respect.

This material is highly refractory, fusing between 3100 and 3300 $^{\circ}\text{F}$ although this point is not sharply defined. Deformation and losses by volatilization occur at lower temperatures. The life of fused quartz at high temperature can be considerably lengthened if the temperature is not allowed to fall below 600 $^{\circ}\text{F}$ after the first heating.

Aluminum oxide, Al_2O_3 , (alumina) with a melting point of 3660 $^{\circ}\text{F}$, is chemically one of the most stable and mechanically one of the strongest of the refractory oxides. Alumina is very stable in the presence of both oxidizing and highly reducing atmospheres and can be used in either type of atmosphere at temperatures up to about 3500 $^{\circ}\text{F}$ for short periods.

At low temperatures, alumina not only is relatively high in strength but also has high rigidity. At temperatures of about 2200 $^{\circ}\text{F}$ alumina begins to exhibit slight amounts of plasticity, around 0.4×10^{-6} in./in./hr at 2000 $^{\circ}\text{F}$ and a stress of 1200 psi.

The commercial oxides having uniform thermal expansion may be divided into three categories, low (0 to $1 \times 10^{-6}/^{\circ}\text{F}$), medium (1 to $10 \times 10^{-6}/^{\circ}\text{F}$) and high ($10 \times 10^{-6}/^{\circ}\text{F}$ and up). Alumina is considered to have a medium expansion, an average of 9.2×10^{-6} between 75 and 1800 $^{\circ}\text{F}$.

The borides of high-melting metals of the fourth, fifth, and sixth periodic groups have properties which make them of potential value in certain very high temperature applications. These properties include melting points ranging from 3600 to 5400 $^{\circ}\text{F}$, low volatility, high hardness, and high stability. The borides are not particularly resistant to oxidation at high temperatures, being attacked at an appreciable rate at temperatures above 2400 to 2700 $^{\circ}\text{F}$. For this reason, full use of the refractory properties of these borides can only be obtained in inert atmospheres or in vacuums. Experimental work on the borides and carbides indicates that borides are slightly more stable than carbides at high temperatures.

Data on thermal-shock resistance of the borides are not available. However, considering the medium thermal expansion of the borides (zirconium diboride has a slightly lower expansion than alumina at about 2500 $^{\circ}\text{F}$), they should show good resistance to thermal shock, especially at temperatures above 2400 to 2700 $^{\circ}\text{F}$, where some slight degree of plasticity appears to exist.

Carbides are among the most refractory materials known. Many carbides have softening points above 5400°F and the more refractory carbides possess some of the highest melting points measured. A simple refractory carbide is tantalum carbide (TaC) which melts at 7028°F, a more complex carbide is 4TaC:1ZrC which melts at 7128°F.

The carbides vary greatly in their composition, structure, and properties. Some carbides are inert to chemical attack, others decompose readily, even at moderate temperatures. At very high temperatures, all carbides are attacked by oxidizing atmospheres, although many of them have better resistance to oxidation than do the refractory metals, and most are more resistant to oxidation than carbon and graphite. Although consideration must be given to the atmosphere in which carbides are employed, their retention of mechanical properties at high temperature and their high melting points suggest their usefulness as high temperature refractories.

Silicon carbide and titanium carbide have been the most useful commercial refractories. Because of the viscous silica glass formed on its surface, silicon carbide is quite resistant to oxidation at temperatures up to about 2700°F and has useful oxidation resistance for many purposes at temperatures up to 3000°F. Titanium carbide, because of its good mechanical properties at moderately high temperatures (up to 1800°F), has been given considerable interest as a structural material.

Combinations of silicon carbide and silicon nitride have been used in applications at temperatures higher than 2700°F, but where atmospheres may not be strongly oxidizing.

Most carbides have fair thermal conductivity and many of them are quite hard. For most applications, the mechanical properties of the metal bonded carbides are superior to those of the self-bonded carbides.

III. THEORETICAL DETERMINATION OF ROD GROWTH

The analytical determination of thermal expansion corrections to be applied to displacement measurements obtained by deflection probes in thermal environments is complicated by the fact that almost all of the physical and mechanical properties are temperature dependent. Further, it appears that there are no simple integrated effects which would have simplified the specification of thermal correction factors. Indeed, detailed knowledge of the entire thermal history is a prerequisite for reasonable degree of precision.

The model selected for this present study consisted of a probe or rod of fixed length passing through (in a normal direction) a radiating surface of infinite extent (see Fig. 1). While the total length of the push rod is fixed, only a portion of it (L_e) extends through the surface and this portion is assumed to vary with time. The configuration factor for radiant

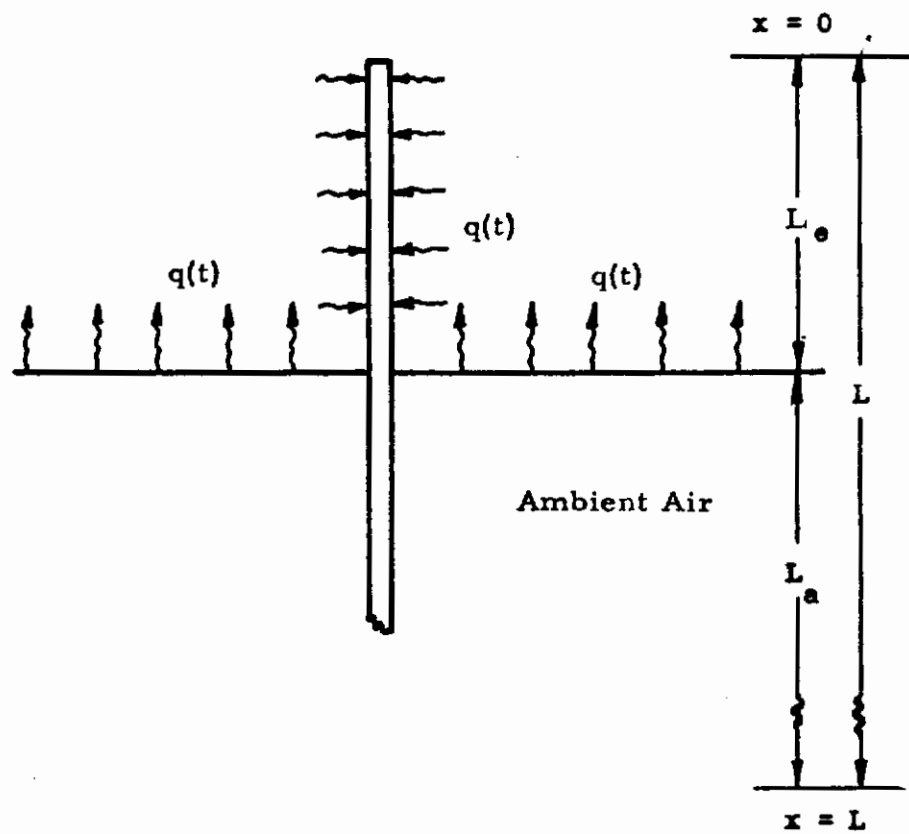


Fig. 1 MATHEMATICAL MODEL OF PUSH ROD AND ENVIRONMENT

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energy transfer between the radiating surface and the portion of the rod in the thermal environment is unity so that the output of the radiating surface and the input on the rod length L_e are assumed identical.

The remaining portion of the rod, L_a , which remains in the ambient air (which is a thermal environment) will reradiate thermal energy in an amount which will depend on the local temperature of the rod and be governed by Newtonian boundary conditions.

The first set of governing equations are the definition of strain and of the coefficient of thermal expansion which we write as

$$\epsilon = \frac{\partial u}{\partial x} \quad \frac{\partial \epsilon}{\partial T} = \beta(T) \quad (1)$$

where ϵ is the strain and β is the local coefficient of expansion. The coordinate along the rod is "x", the local displacement of a point on the rod is "u" and T is the temperature. The total change in the length of the rod is obtained by integrating the last of Eq. 1 for the local value of the strain and then the first of Eq. 1 for the total change in length (i. e. the thermal correction).

Indicating the time-dependent thermal correction by $\delta(t)$ one can write

$$\delta(t) = \int_0^L \int_0^{T(\mu, t)} \beta(\lambda) d\lambda d\mu \quad (2)$$

where we have chosen zero temperature as our reference and initial temperature. Thus, in what follows, ambient temperature is designated to be zero temperature; this is merely a shift in scale to facilitate the subsequent numerical analysis.

Since $\beta(T)$, the coefficient of thermal expansion, admits a polynomial representation over most practical temperature ranges, the first integration in equation can be immediately performed.

Let

$$\beta(T) = \beta_0 + \beta_1 T + \beta_2 T^2 \quad (3)$$

where β_0 , β_1 and β_2 are constants known from the physical properties of the rod material (experimental determinations of these constants are considered elsewhere in this report); then the thermal correction, $\delta(t)$, can be written as

* μ and λ are dummy variables of integration

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$$\delta(t) = \int_0^{L_e} \left[\beta_0 T(\mu, t) + \frac{1}{2} \beta_1 T^2(\mu, t) + \frac{1}{3} \beta_2 T^3(\mu, t) \right] d\mu \quad (4)$$

and it remains to determine the temperature distribution along the rod.

The differential equation which governs this temperature distribution is

$$c\rho \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} (K \frac{\partial T}{\partial x}) + \frac{2\alpha}{R} q(x, t) \quad (5)$$

where c , ρ , K and α are the temperature dependent specific heat, density, conductivity and absorptivity and R is the radius of the rod.

The heat incident on the surface of the rod is $q(x, t)$. In the thermal environment the heat flux is simply the output of the radiant surface; thus, $q(x, t)$ for $0 \leq x \leq L_e$ is $q_e(t)$. On the other hand, we assume the heat is lost from the portion of the rod which extends into the ambient air by convection and radiation so that we write

$$q(x, t) = \begin{cases} q_e(t) & 0 \leq x \leq L_e(t) \\ -h T(x, t) & L_e(t) \leq x \leq L \end{cases} \quad (6)$$

where, in general, the film coefficient, h , is temperature dependent and includes effects of both convection and radiation.

By way of boundary and initial conditions we assume that the ends are insulated, hence

$$\left. \frac{\partial T}{\partial x} \right|_{x=0, L} = 0 \quad (7)$$

and that initially the entire rod is at zero (or ambient) temperature.

In order to complete the description of the governing differential equation the temperature dependence of the various coefficients needs to be formulated. In keeping with the approximate (i. e. one-dimensional nature of Eq. 5) the following simple forms have been used;

for the specific heat, c ,

$$c(T) = C_0 + C_1 T \quad (8)$$

for the conductivity, K ,

$$K = K_0 + K_1 T + K_2 T^2 + K_3 T^3 \quad (9)$$

and for the absorptivity, α ,

$$\alpha = \alpha_0 + \alpha_1 T + \alpha_2 T^2 + \alpha_3 T^3 \quad (10)$$

The seemingly endless list of constants, $\beta_0, \beta_1, \beta_2, C_0, C_1, K_0, K_1, K_2, K_3, \alpha_0, \alpha_1, \alpha_2$ and α_3 represent the minimum number necessary to describe the properties of the material and may not be needed to illustrate the behavior of the entire system. Unfortunately, many calculations would be needed to investigate this question.

The general character of the temperature dependent parameters β, K, c_p, α are sketched in Fig. 2 for the range of temperatures contemplated in this program. It is apparent that temperature dependence precludes the immediate development of a simple linearized theory.

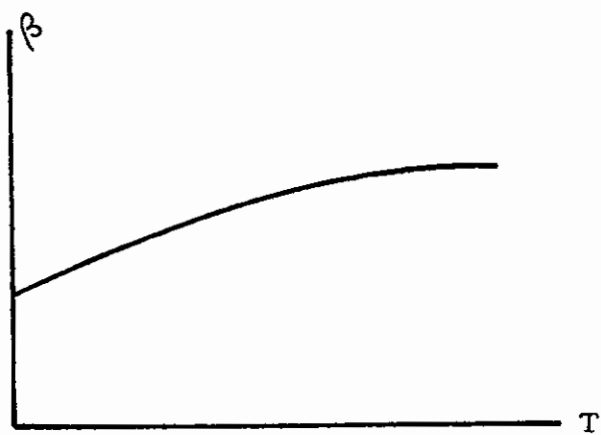
The differential Eq. 5 with the inputs given by Eq. 6 together with the two boundary conditions presented in Eq. 7 and with the implied continuity condition on temperature and thermal gradient at the junction of the rod (or probe) and the radiating plane constitute the mathematical formulation of the present problem.

A direct analytical attack on the problem is not feasible, indeed, there does not exist a classification for the partial differential equation given by Eq. 5. On the other hand, the application of the usual characteristic type of numerical procedures will no doubt lead to serious difficulties regarding a choice of a ratio of space and time increments (in the numerical integration) so as to assure convergence of the numerical process. We have chosen to avoid these convergence questions by applying a mixed-integration procedure in which the space variable(s) and the time variable are treated in a different manner.

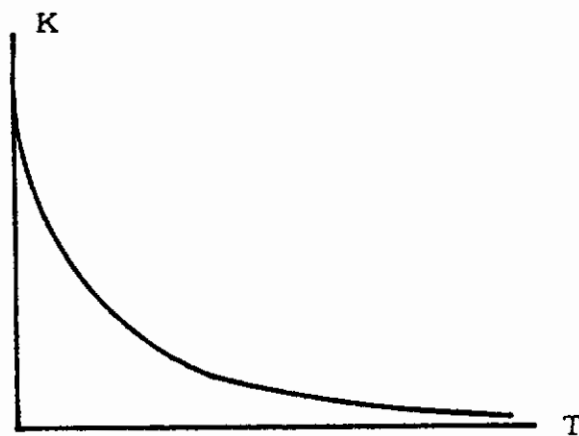
In brief, the numerical integration of Eq. 5, subject to the associated inputs, boundary and initial conditions, is effected by expressing the space derivatives as their finite difference analogs and then performing the time-wise integration by one of the forward integration techniques such as Runge-Kutta. Each of these numerical techniques is completely covered in elementary texts on numerical analysis although the combined usage is seldom, if ever, mentioned either because it hasn't occurred to anyone before or because the application is so obvious that it does not require pointing out.

To some extent it is difficult to ascertain the value of this portion of the project since the programming of numerical process had proceeded only to the point where a working program was obtained. (See Appendix). An internal reapportionment of effort precluded the collection of the several production runs which would be necessary to establish any firm conclusions.

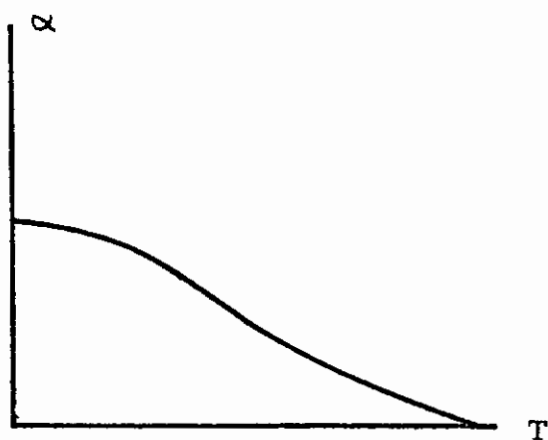
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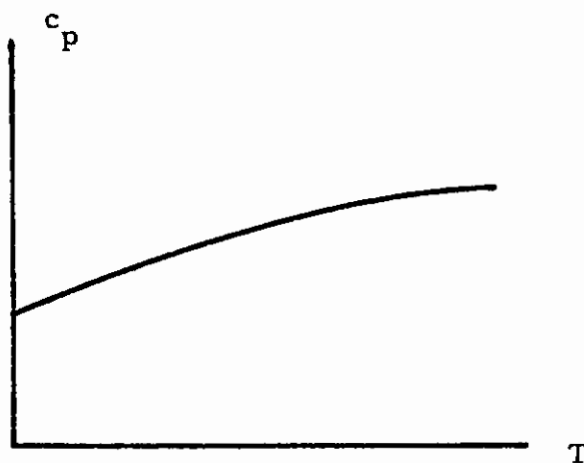
a) coefficient of expansion (Ref. 1)



b) conductivity (Ref. 2)



c) absorbtivity (Ref. 3)



d) specific heat (Ref. 4)

Fig. 2 TEMPERATURE VARIATION OF PHYSICAL PARAMETERS

In some respects this was a fortunate occurrence since the later part of the experimental phase of the program showed that the probe reached thermal equilibrium quite rapidly so that it appears that the transient portion of the thermal correction can be safely neglected. Under these circumstances it would be wasteful to expend additional effort with the present numerical program since the steady state problem obtained by deleting the Left Hand Side of Eq. 5 can be handled more expeditiously by re-programming the entire problem.

IV. PRELIMINARY MATERIAL STUDY

As a result of a review of the available commercial materials described in Part II, it was decided that the refractory metals could be ruled out for applications up to 3000°F where all of them require adequate coating. Instead, the refractory ceramics are considered more satisfactory for the application. Among them fused quartz has the lowest known thermal expansion coefficient over a rather wide temperature range and may be useful for temperatures as high as 2000°F. Another material which has excellent strength even at high temperatures, does not require coating or protection against oxidation, but has a fairly large thermal expansion is aluminum oxide or alumina. It was decided that these two materials be investigated thoroughly to provide more information on their suitability as deflection transmission rod materials and to provide data upon which a theoretical analysis of rod growth may be based.

Samples of 3/8 in. diameter rods were obtained from various manufacturers for further evaluation. These included 3-ft length transparent quartz rods from the General Electric Company, similar rods from Engelhardt Industries, Inc., translucent quartz rods of the same proportions from Thermal American Fused Quartz Co., and alumina rods of the same diameter but only 12 in. long obtained from Morganite, Inc. Some pertinent available data for these materials is given in Table I.

A radiant heating furnace, shown in Fig. 3, was built for making thermal expansion measurements. Experimental trials with the furnace showed it capable of reaching 2200°F in 5 seconds using three G. E. 1000-watt T-3 lamps. The unit has provision for 12 lamps if needed. Temperature measurements were made using a blackened thermocouple junction.

Thermal expansion measurements were made on two specimens of fused quartz rods and on one alumina rod. The resulting linear expansions are given as a function of temperature for G. E. transparent fused quartz in Fig. 4 and for Vitreosil translucent fused quartz in Fig. 5. In both figures, the curve labeled "I" is that portion of the thermal expansion of the first specimen between room temperature and the softening point of fused quartz. Curves labeled II-1, II-2, and II-3, represent three successive expansion and contraction measurements of the second specimen for each material obtained from room temperature to 2200°F and back to room temperature.

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Table I

MANUFACTURER'S DATA FOR PHYSICAL AND MECHANICAL PROPERTIES*

Properties	General Electric Transparent Fused Quartz	Engelhard Industries Transparent Fused Quartz - Amersil	Thermal American Translucent Fused Quartz - Vitreosil	Morganite Alumina
Density	2.2	2.0 - 2.2	2.07 - 2.15	3.78
Thermal Conductivity cal/sec/cm ² /°C	0.0033	0.0033	0.0033	0.045
Thermal Expansion in./in./°F	0.99 x 10 ⁻⁶	0.97 x 10 ⁻⁶	0.97 x 10 ⁻⁶	15.8 x 10 ⁻⁶
Devitrification Temperature, °F	1960	2010	1830 - 2010	2820
Melting Point, °F	3270	3180	3090 - 3270	3540
Compressive Strength, psi	160,000	190,000		400,000
Modulus of Rupture, psi			4000	60,000
Tensile Strength, psi	7,000	7,000	400	25,000
Modulus of Elasticity, psi	10.4 x 10 ⁶		10.0 x 10 ⁶	59.0 x 10 ⁶
Transparency to Infrared	92% (0.7 - 3.2μ)	95% (0.16 - 6.0μ)	Opaque	Opaque

* Room temperature properties except as noted.

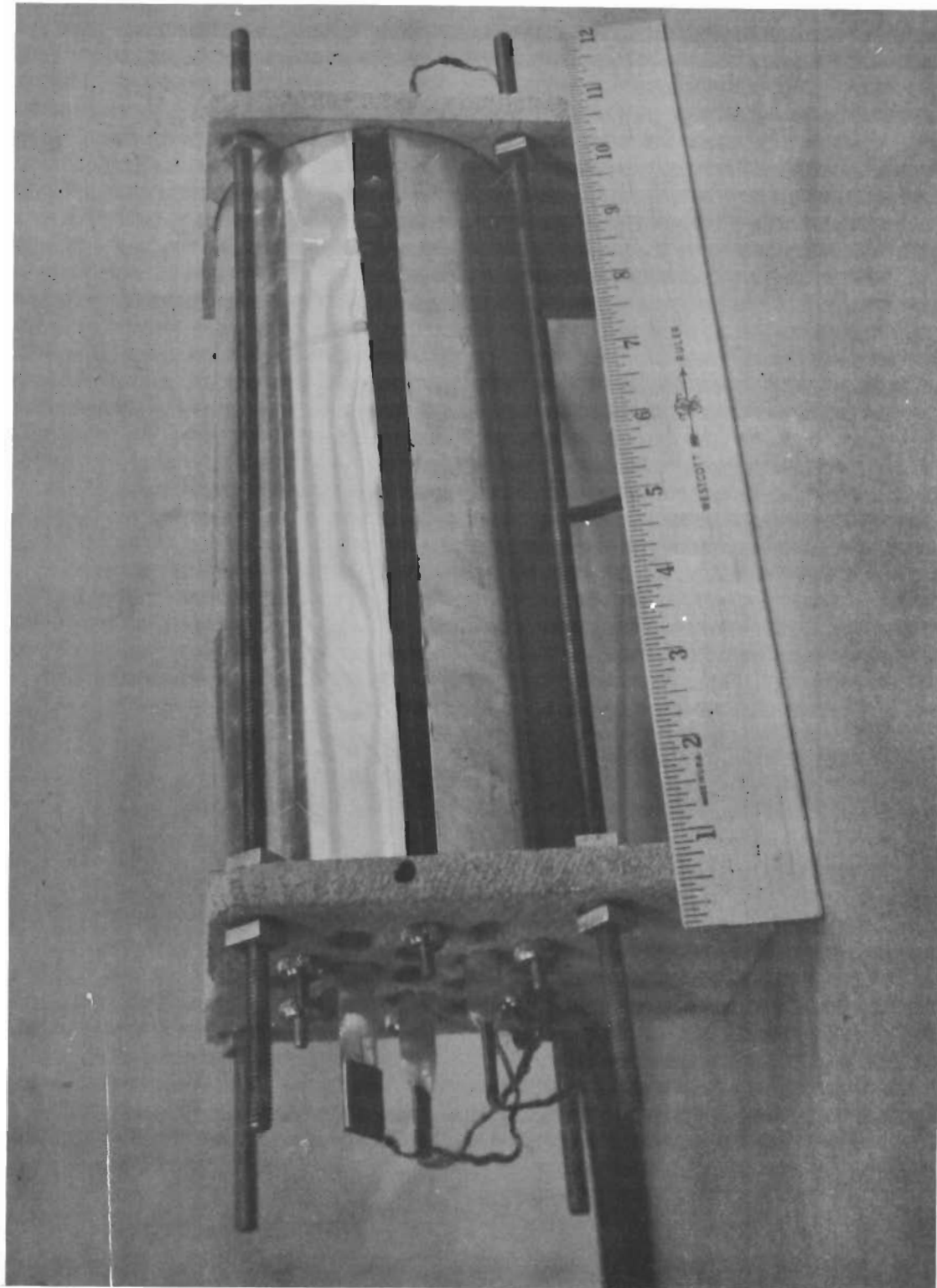


Fig. 3 RADIANT HEATING UNIT FOR DETERMINING THERMAL EXPANSION
CHARACTERISTICS OF RODS

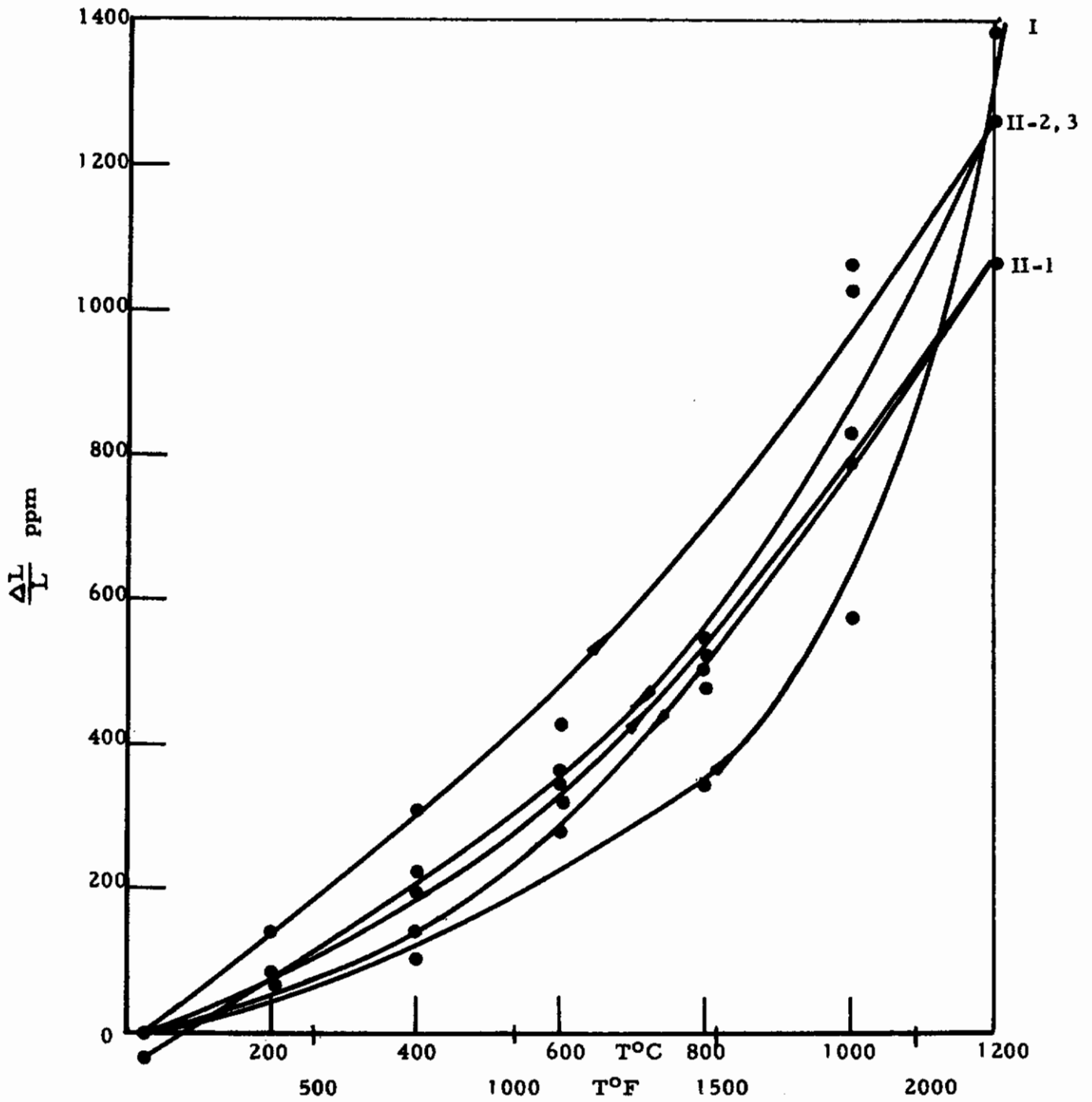


Fig. 4 THERMAL EXPANSION CURVE GE TRANSPARENT FUSED QUARTZ

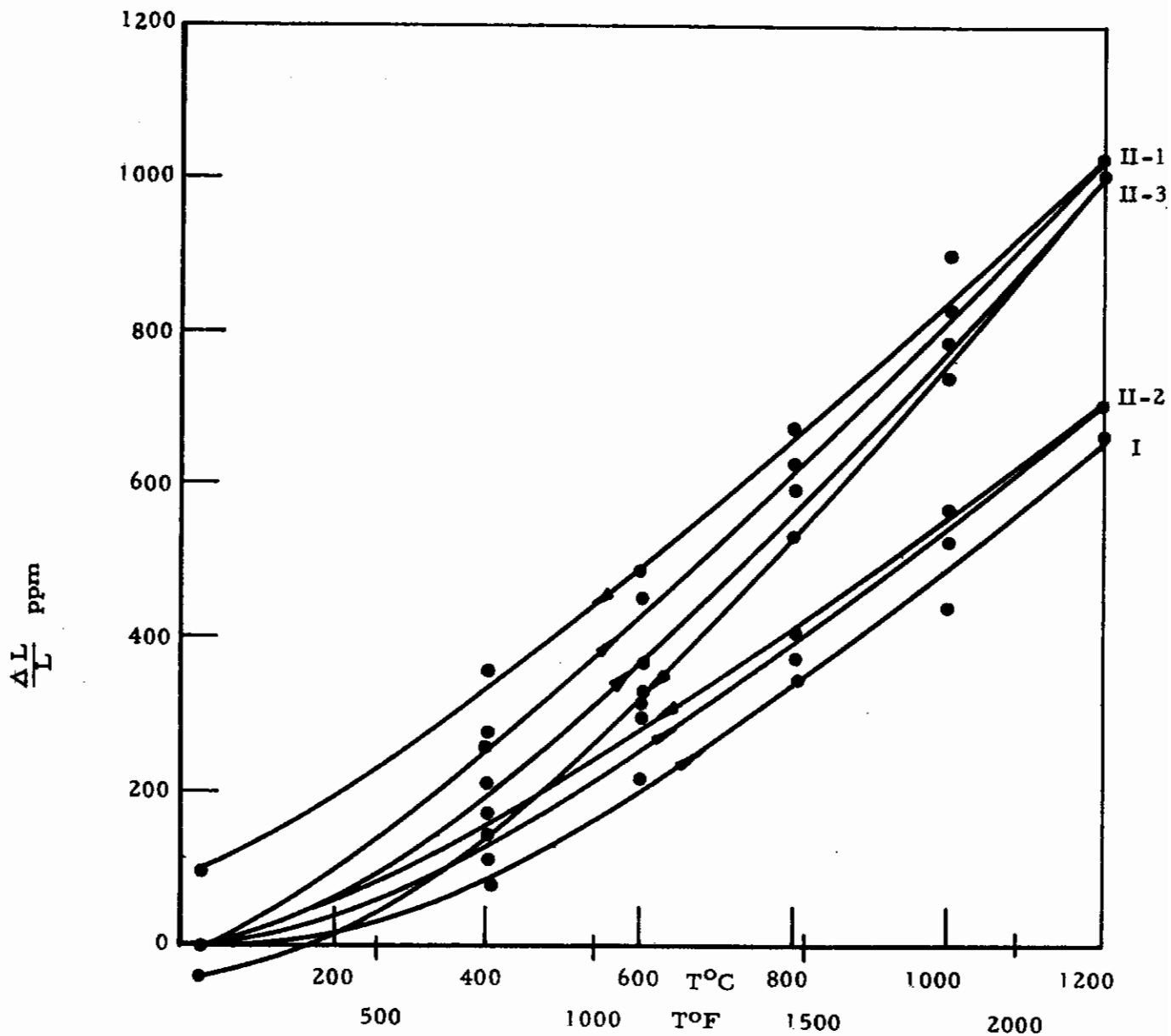


Fig. 5 THERMAL EXPANSION CURVE VITREOSIL TRANSLUCENT FUSED QUARTZ

Continued

The variations among the curves appears to be random. From these curves the thermal expansion coefficient of $0.9 \times 10^{-6}/^{\circ}\text{F}$ was determined for both the transparent and translucent specimens up to about 1800°F . This compares favorably with values of 0.99×10^{-6} and 1.01×10^{-6} , respectively, reported by the manufacturer.

A determination was made of thermal expansion and contraction from room temperature up to 2200°F and down to room temperature for a specimen of Morganite Triangle RR alumina. The result is given in Fig. 6. From the slope of the straight line shown in Fig. 6, the computed coefficient of expansion is $15.5 \times 10^{-6}/^{\circ}\text{F}$ compared with 15.8×10^{-6} reported by the manufacturer.

Additional thermal expansion curves from 68°F to 2200°F were measured for two additional specimens of G. E. transparent fused quartz, Vitreosil translucent fused quartz, Morganite alumina, and Norton zircon rods. These rods were subjected to 10 cycles of rapid heating to 2000°F and rapid cooling to room temperature in an effort to determine their thermal shock resistance and the effect of such exposure on the thermal expansion curves. The surface of the specimens reached 2000°F in less than a minute and cooled to room temperature in about the same length of time. All four specimens withstood this treatment without obvious damage.

The thermal expansion curves for the two quartz specimens are given in Figs. 7 and 8. In both figures, curve A represents the thermal expansion before thermal cycling and curve B after cycling. In both figures the thermal expansion after cycling is less than before cycling especially over the range from room temperature to 1470°F . The thermal expansion of the Morganite alumina duplicated that of previous samples.

The zircon material (zirconium silicate) is one of the best ceramic materials from the standpoint of thermal shock resistance and is usable in an oxidizing atmosphere at temperatures up to 2900°F . Some sample rods of grade RZS 136 body zircon were obtained from the Norton Company. The thermal expansion of this material, as given in the manufacturers literature, is $9.9 \times 10^{-6}/^{\circ}\text{F}$ over a temperature range of 82 to 2370°F . This was confirmed by the thermal expansion determination made and shown in Fig. 9. After the expansion had been measured, the specimen was subjected to 10 cycles of thermal shock and a second expansion measurement was made. The second measurement duplicated the first within the limits of error of measurement.

Other material properties, time dependent properties included, were determined for these materials. These properties were necessary for the theoretical determination of rod growth and, where used, were treated in Part III of this report.

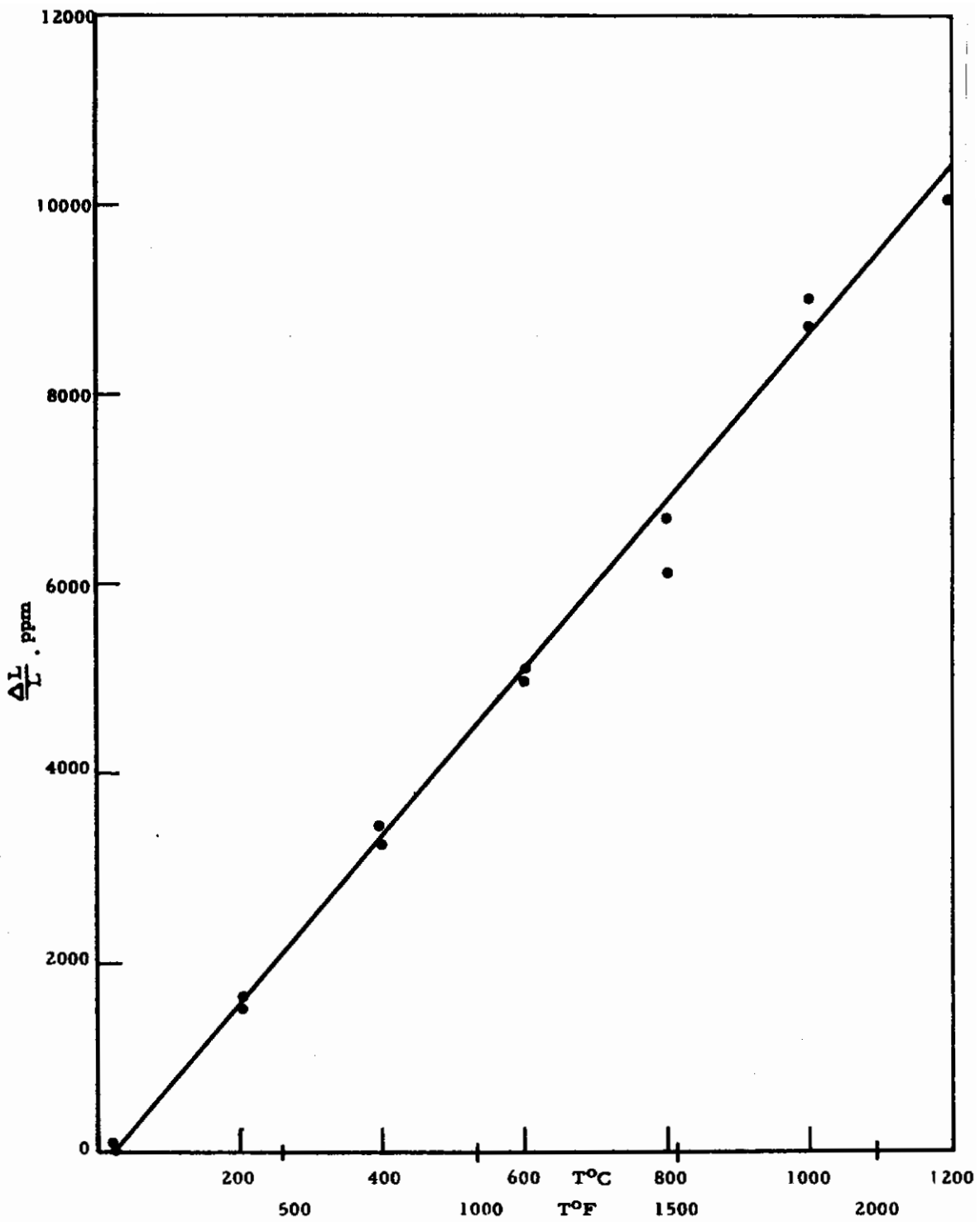


Fig. 6 THERMAL EXPANSION CURVE MORGANITE ALUMINA

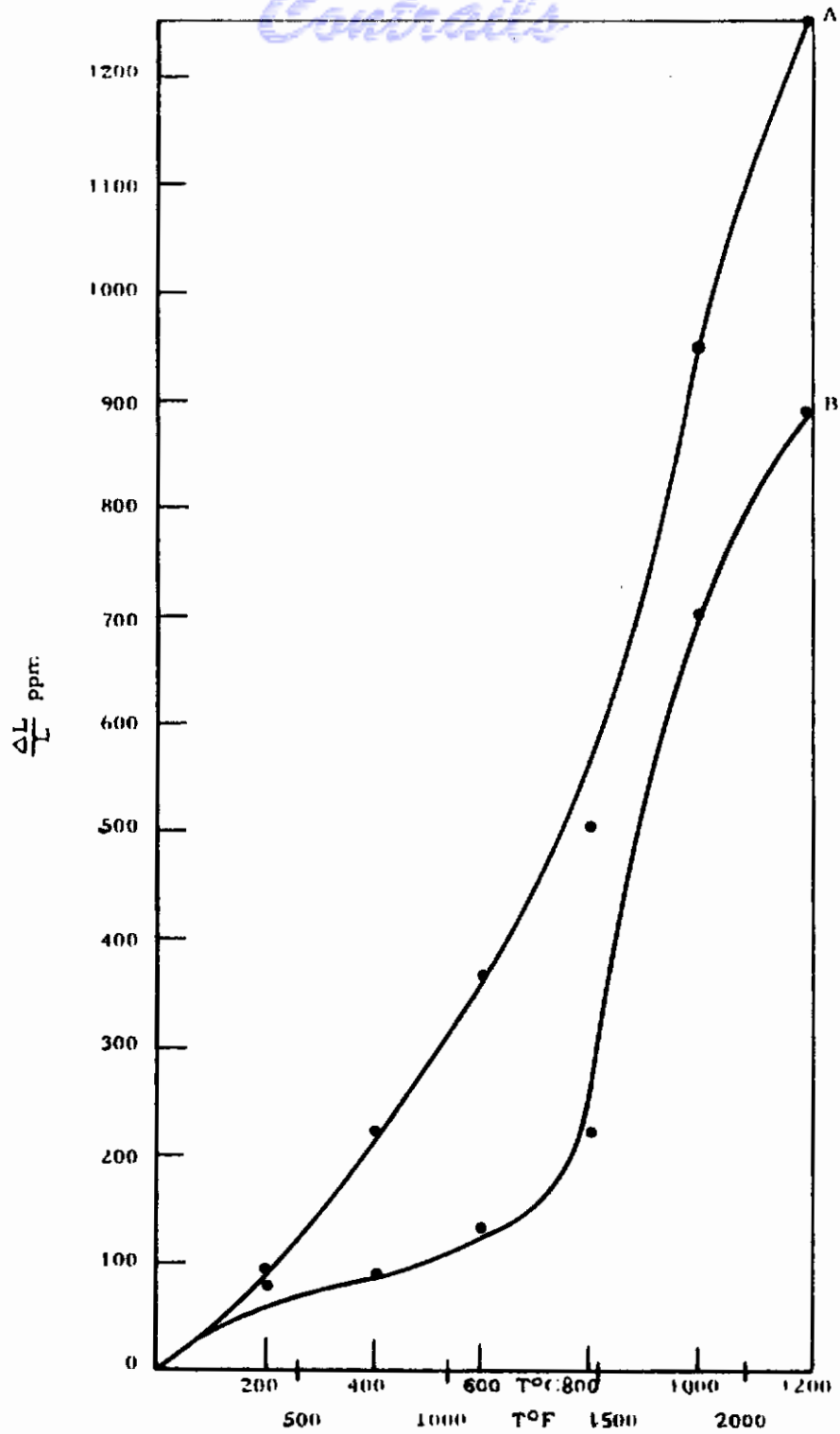


Fig. 7 GE TRANSPARENT QUARTZ

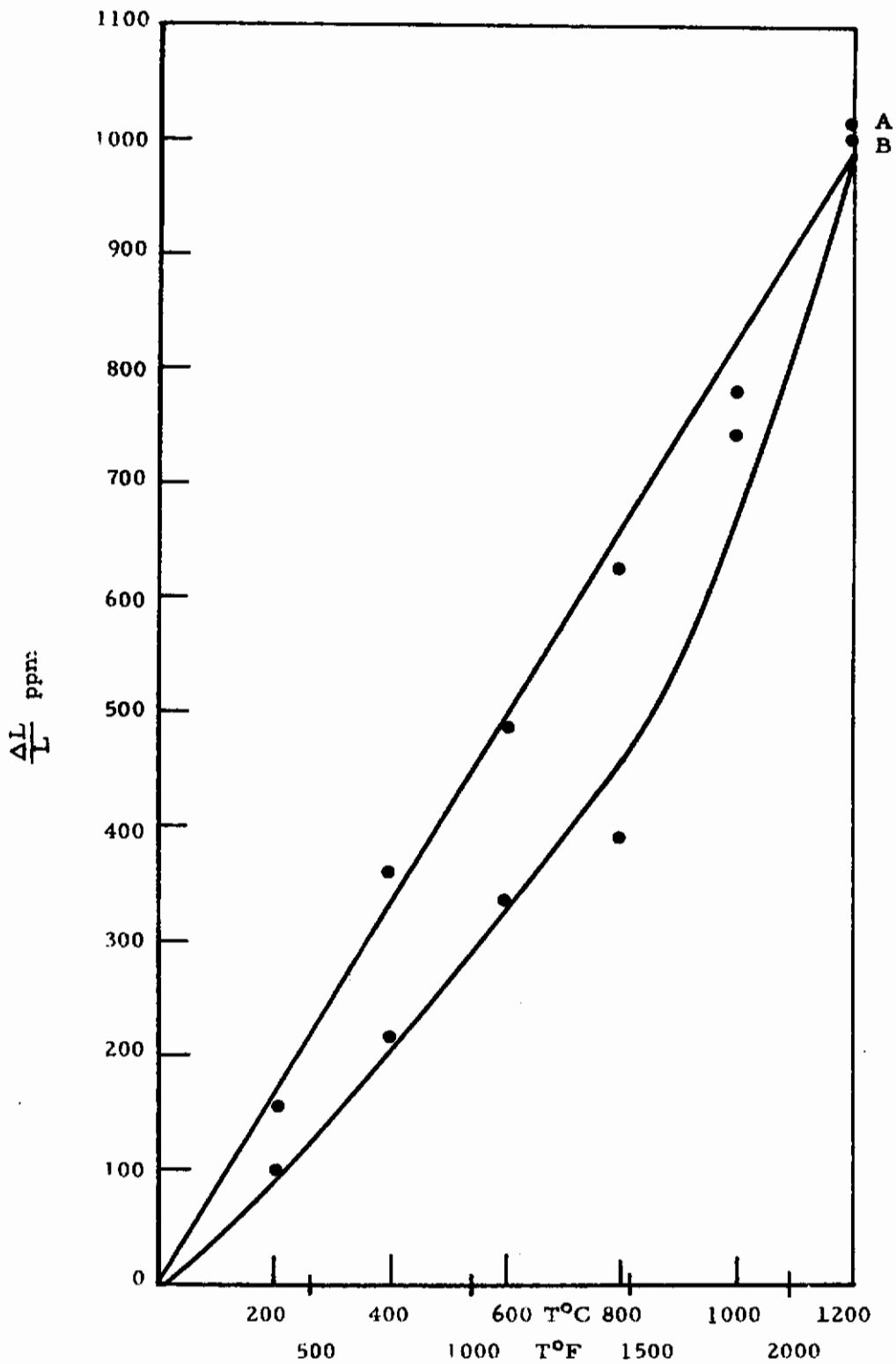


Fig. 8 VITREOSIL TRANSLUCENT QUARTZ

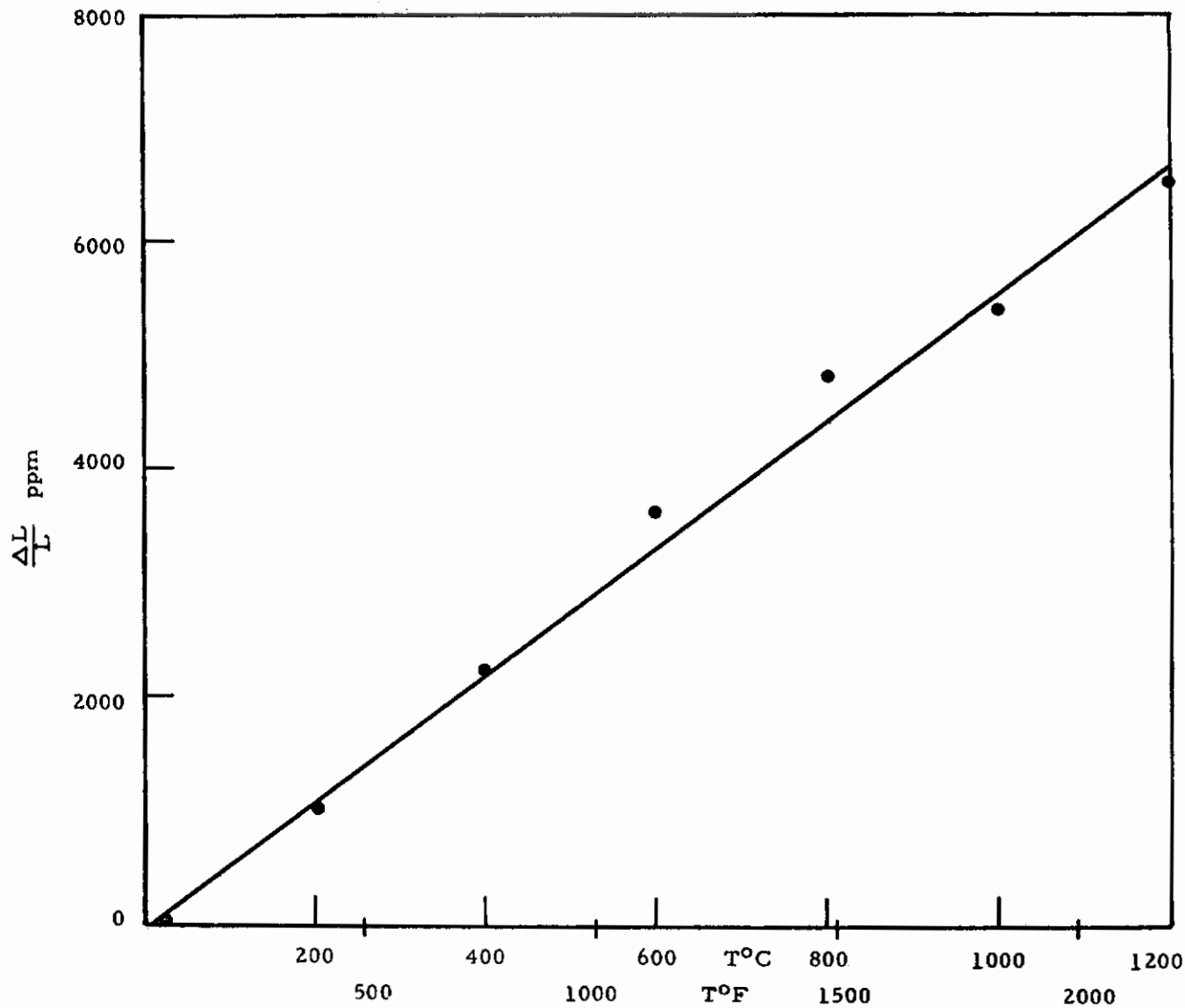


Fig. 9 THERMAL EXPANSION OF NORTON ZIRCON (BEFORE AND AFTER THERMAL CYCLING WERE IDENTICAL)

V. EXPERIMENTAL DETERMINATION OF ROD GROWTH

The analytical approach for determination of rod growth can at best give only an indication of what thermal growth is likely to be experienced by a deflection transmission rod. Theoretically determined rod growth will, of course, be only approximate because of the inability of a theoretical approach to take into account all of the variables. Hence the theoretical approach was supplemented with an experimental investigation. For the experimental study it was decided that fused quartz* would be used for temperatures up to 2000°F because, from the standpoint of thermal expansion and oxidation resistance, it is a much more desirable material than any of the others for the application. At temperatures below 2000°F fused quartz should not encounter creep or dimensional stability problems (attending transformation to crystalline SiO₂). For temperatures from 2000 to 3000°F it was decided that alumina rods would be employed in preference to zircon because of the better strength and considerably lower cost of alumina.

Correction factors determined experimentally for each of the materials include the effects of both thermal expansion and creep and are the factors that must be applied to gross measurements of deflection in order to determine the net deflection of a structure under test. Even with materials of extremely low thermal expansion, the correction factors are sizable especially for conditions in which small deflections are to be measured.

A. Cable and Attachment Design

For the transmission of deflections from a structure under radiant heating conditions to a transducer at essentially room temperature conditions a connecting rod or cable must be employed. For this program a connecting rod of 3/8 in. diameter and 3 ft length was determined as being adequate from the strength and stability standpoint and for transmission through an array of heating lamps. Materials selected for the "specimen" to which the deflection transmission rods would be attached were as follows: (a) for temperatures up to 1000°F a stainless steel plate 1/4 in. thick and 10 x 15 in. in size was selected. (b) For temperatures up to 2000°F, an Inconel X plate 1/4 in. thick and also 10 x 15 in. in size was selected. (c) For temperatures up to 3000°F, a silicon carbide plate 1/2 in. thick and 10 x 12 in. in size was chosen. Later an alumina plate 1/4 in. thick and 6 x 12 in. in size was used.

For attachment of the quartz rod to the stainless steel and the Inconel X specimen the problem of attaching two materials with very dissimilar thermal expansion coefficients for use at elevated temperatures was encountered. Several experimental ceramic adhesives were tested and found to be of little value because of the thermal expansion mismatch and the weakness of such cements under tension. It was thought that possibly a ceramic adhesive may be of some advantage if it were used in shear. Such a bond was tried using an intermediate stainless steel tube attached to the stainless steel plate and bonded in its interior to the exterior of the quartz rod. This attachment was also a failure. Instead a concept was evolved involving a

* General Electric Company transparent 204A fused quartz

** Morganite, Inc. Triangle RR alumina

stainless steel sleeve 1/2 in. long and 1/2 in. outside diameter with an internal diameter of 0.010 in. greater than 3/8 in. The sleeve was threaded 1/2-20 on one end (just a couple of threads), and the stainless steel specimen was blind drilled and tapped 1/2-20 to match the thread on the sleeve for a depth of about 1/8 in. The sleeve was then inserted in the matching hole in the surface of the specimen. Attachment of the rod was accomplished by drilling a hole transversely through the sleeve and through the rod and inserting a stainless steel pin. The pin and hole combination through the quartz rod were chosen so that the pin diameter (nominally 0.123 in.) upon thermal expansion would not cause thermal stresses in the quartz rod (hole size nominally 0.126 in.). Similar sleeve and pin combinations were made for the Inconel X specimen and were fabricated using Inconel X alloy. The threaded attachment was used for the relatively thick specimens used on this program. However for attachment to a relatively thin specimen, the sleeve could have one end closed, like a cartridge, and this end could be spot welded to the specimen. Alternately, one end could be flared out with a flange and this, in turn, spot welded to the specimen.

Tests were performed which demonstrated that the theoretically determined clearances (based upon expansion of the pin material) were adequate. Another test was performed to determine the strength of a 3/8-in. quartz-rod containing a 1/8-in. transverse hole, when loaded in tension using a pin through the hole. The failure load of 73 lbs indicated a stress concentration factor of 6.0 for the hole, based upon a nominal 7000 psi tensile strength for the quartz rod. The strength of the rod with the hole was considered adequate and, therefore, the pin-type attachment was used for the quartz rod-metallic plate specimen combinations.

For attachment of the alumina rod to the silicon carbide specimen a number of ceramic adhesives were evaluated and one was finally selected. The cement is an experimental potassium silicate alumina. In use a small amount of the premixed refractory cement is placed in position on the specimen where the rod is to be attached. The rod is then placed over this cement and held firmly until air drying is accomplished. A period of 24 hours of thorough air drying followed by 4 to 5 hours under infra-red heating up to temperatures of the order of 300 to 400°F is essential for curing. This attachment bond stood up well under several cycles of heating up to 3000°F but was very fragile and sometimes was easily broken by handling. In several tests the bond failed either immediately after or during the test. Therefore the attachment technique was considered a failure.

For subsequent tests with the silicon carbide specimen and alumina rod combination a spring was interposed between the transducer and the upper attachment for the rod to put it under compression and maintain contact with the specimen at all times. The lower end of the rod was sharpened to a radius point to effect contact and alignment with the specimens. In all cases the free end attachment was a clevis fabricated from stainless steel and Inconel X firmly clamping onto the deflection transmission rod. Details of the free end attachment clevis are given in Fig. 10. A photograph showing the attachment of the quartz rod to the Inconel plate through the use of the Inconel sleeve and Inconel pin combination is given in Fig. 11. The

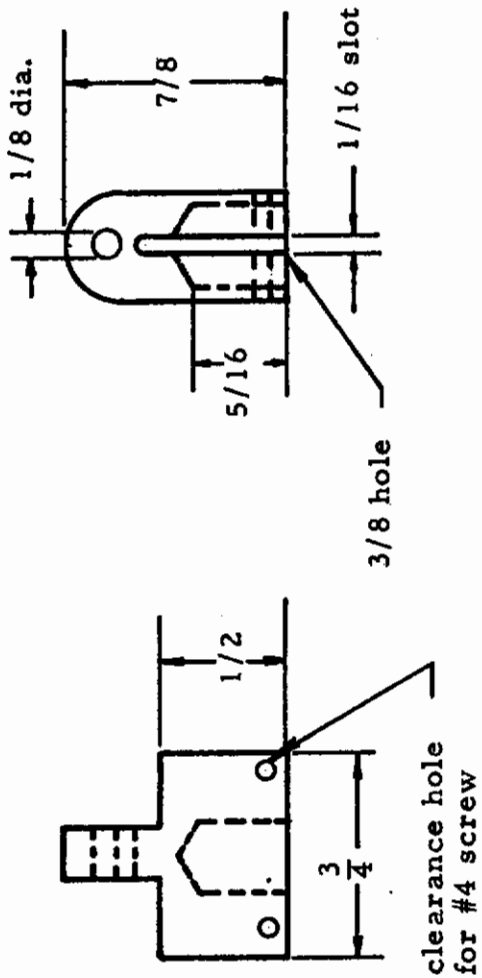


Fig. 10 DETAILS OF CLEVIS FOR FREE END OF ROD

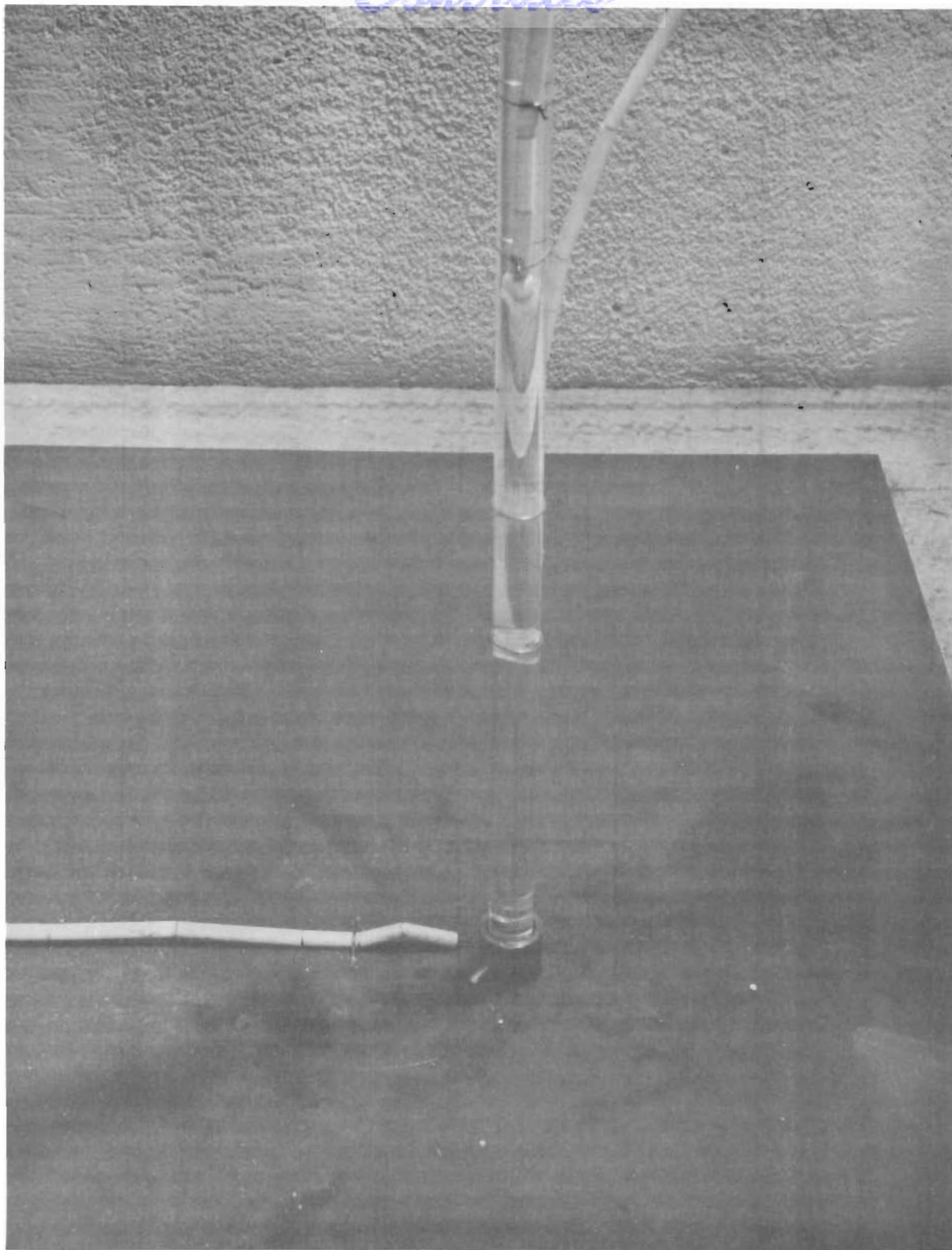


Fig. 11 ATTACHMENT OF QUARTZ ROD TO INCONEL X PLATE

thermocouple at the center of the plate used for determining and monitoring temperature in the vicinity of the attachment is also shown in Fig. 11.

A successful attachment for the alumina rod-alumina plate combination was developed when it was apparent that a positive attachment for the silicon carbide plate could not readily be developed. This attachment consisted of an alumina cup cemented to the alumina plate with a refractory cement prepared from -325 mesh calcined alumina powder and mono-aluminum phosphate, 50 percent solution. After a brief air dry and 24 hours at temperatures up to 400°F, the attachment (plate and cup) was fired in a kiln slowly to 3000°F and cooled slowly to room temperature. The attachment held up well and permitted using the alumina deflection transmission rod to transmit large static and dynamic displacements. Details of the alumina cup and the pin attachment for the alumina rod are given in Fig. 12.

B. Evaluation Equipment

The evaluation equipment employed on this program consisted of a series of T-3 quartz lamp heaters, a suitable power regulator, and a temperature controller-recorder. Instrumentation consisted of deflection transducers, an oscillograph recorder, and thermocouples. For imposing static displacements on a specimen, a motor driven specimen platform was employed and for imposing dynamic deflections a motor driven vibration table was employed. Details of the evaluation equipment follow.

1. Heating Equipment

The heating equipment employed was an array of six Research, Inc. PM-2000 heater units each containing three G. E. 2000 T-3/CL/HT quartz lamps. These lamps are nominally rated at 2000 watts at 230 volts. However, for attainment of temperatures as high as 3000°F it was necessary to use these lamps at voltages up to 480 volts. The six reflector units were arranged parallel to one another so that the total area of approximately one square foot could be covered by the array of 18 lamps. Adjacent reflectors were separated by about 1/8 in. except at the center where the center two reflectors were separated by 1/2 in. to permit a 3/8 in. diameter rod to pass through the array for attachment to the specimen. In this parallel arrangement the heated area was approximately 10 in. x 15 in. The refractory ceramic reflectors and the heat lamps were protected from overheating by an air-cooling system, employing the Bernoulli effect.

A special test stand was designed and fabricated for accommodating the reflectors and the specimen. The rig accommodated six of the reflectors and lamp holders for heating over the approximate one square foot surface area. It also accommodated the specimen with the rod attached. A shielding insulation material at the bottom and sides of the specimen was provided in order to minimize heat losses through the specimen and through edge effects. A special motor driven specimen platform was fabricated incorporating chain and sprocket driven threaded rods to displace the specimen up or down through a total distance of approximately 24 in. The purpose of

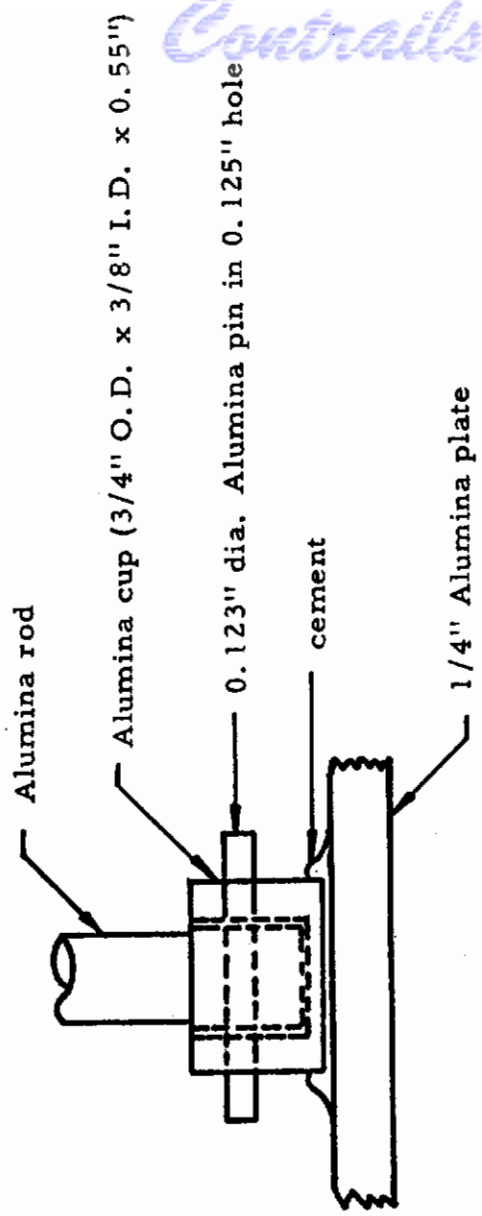


Fig. 12 DETAILS OF ALUMINA PLATE/ALUMINA ROD ATTACHMENT

this device was to provide a means of controlling the placement of the specimen and, thereby, impose a known deflection or displacement (monitored by a deflection transducer from the unheated side) which was to be accurately determined from the heated side of the specimen by means of the deflection transmission rod and associated transducer. A photograph showing the test rig, the motor driven specimen platform and a special frame for mounting the transducers is given in Fig. 13. The quartz lamp heaters are located just below the top of the box-like structure in the center of the photograph. It must be pointed out that the arm hanging down from the topmost frame in the photograph was not used in actual tests for mounting of the transducer. Instead the transducer was mounted at the cross arm (where the vertical arm is attached in the photograph). The horizontal arm to which the transducer was attached was shielded from the radiation by means of aluminum foil.

Shown also at the right of Fig. 13 is the Thyatron 10 channel power regulator (Research, Inc. Model 4079) and the temperature controller-recorder (Research, Inc. Model 4080). Each bank of three lamps in the heating system was connected to a separate channel power regulator; a total of six channels were employed.

Another device not shown in Fig. 13 which was employed in the evaluation of the deflection rod technique was a mechanical vibrator system in which cyclic deflections with amplitudes up to ± 12 in. with a frequency of $2/3$ of a cycle per second could be imposed.

2. Instrumentation

The displacement transducers employed were Research, Inc. Model 4046 with a 0.5 in. range, and Model 4040 with a 6 in. range. It was decided that a 0.1 percent transducer linearity would be required in order to meet the contract requirement of 0.5 percent accuracy for the deflection transmission rods. In most of the tests conducted, two displacement transducers were employed to determine rod growth. One transducer was connected directly to the deflection transmission rod; therefore, this transducer was sensing rod growth and rod movement resulting from deflection of the specimen. The second transducer was attached beneath the specimen on the opposite side of the rod to sense any movements of the specimen.

The output of each transducer was fed to a Consolidated Engineering Corporation Amplifier System D consisting of carrier amplifiers and an oscillator-power supply combination. The output of this system was fed to a recording oscillograph. A trace was thereby obtained indicating the magnitude of the voltage output of each transducer as a function of time, thus the output of each transducer at any instant of time was obtained directly.

By measuring the trace deflection or voltage output of each transducer relative to the "time zero" output and relating this output to a calibration factor, the displacement in inches sensed by each transducer was found. The difference between the two values was a direct measure of rod growth.

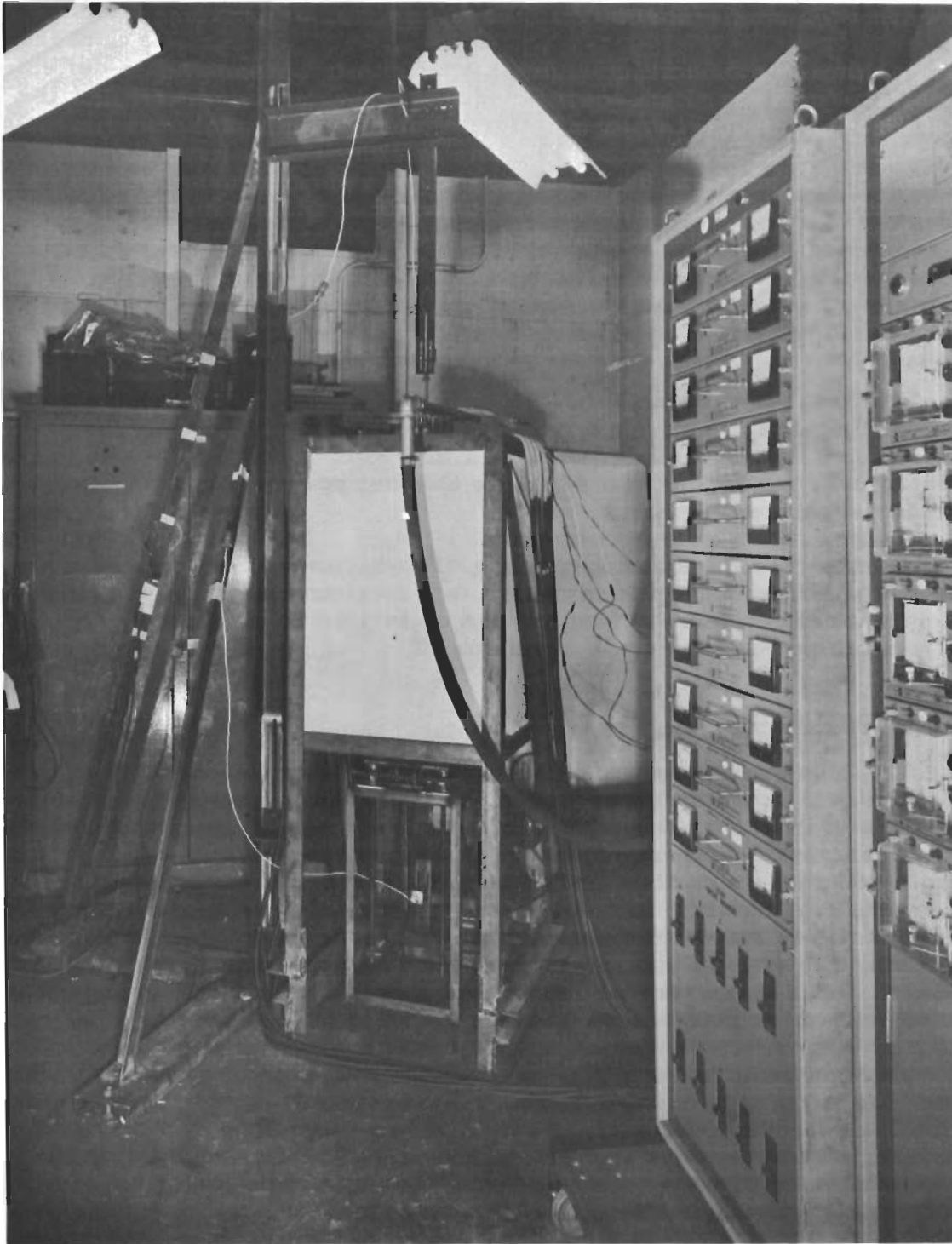


Fig. 13 EVALUATION EQUIPMENT INCLUDING RADIANT HEATING
SETUP, POWER REGULATOR, TEMPERATURE RECORDER-
CONTROLLER

Displacement transducers were calibrated both individually and differentially in pairs in order to determine their resolution and linearity. It was found that the results did not agree with the manufacturers specifications for resolution and linearity. In one of the 1/2 in. range transducers, in which the manufacturer's specification on linearity was + 0.3 percent and resolution + 0.08 percent, it was found that the deviation from the best straight line was of the order of + 1 percent of full range. Using two of these transducers in a differential arrangement could result in + 2 percent deviations. The indications were that the actual resolution for this transducer was closer to the stated value of linearity (+ 0.3 percent). For the 6 in. transducers, employing a wire wound potentiometer, the resolution is determined by the spacing of the wires of the potentiometer. This spacing is of the order of 0.018 to 0.022 in.

For measurement of temperatures up to 2000°F, 24 gage chromel-alumel thermocouples were employed. For temperatures up to 3000°F, 30 gage platinum/platinum-10 percent rhodium thermocouples were employed. Attachment of the chromel-alumel thermocouples to the stainless steel and the Inconel specimen plates was effected by spot welding the parallel thermocouple leads separated by a distance of 1/16 in. for a length of 1/8 in. onto the plates. The thermocouple attachment is seen quite clearly in Fig. 11. The stainless steel and the Inconel X plates had thermocouples attached at the center in the vicinity of the attachment of the deflection transmission rod and at the center of each of two edges, a long edge and a short edge. Chromel-alumel thermocouples with a twisted junction were attached to the quartz rod by means of a ceramic adhesive. However, these thermocouples did not remain attached for long and for subsequent tests were merely wired in place on the quartz rods. Therefore, these thermocouples do not measure actual rod temperatures but at least give an indication of a temperature gradient along the length of the rod at points where the thermocouples were attached. The platinum/platinum-rhodium thermocouples employed on the silicon carbide specimen, on the alumina specimen, and on the alumina rod were of the twisted junction type with attachment effected by means of either the refractory cement or a Sauereisen No. 63 cement. It was found that the Sauereisen cement when completely surrounding the hot junction of the thermocouple will prevent contamination of the platinum/platinum-rhodium thermocouple in the radiant heated environment at temperatures above approximately 2300°F (the temperature at which such contamination usually takes place). This technique permits determination of a temperature for control purposes but does not give a true indication of the surface temperature of the specimen because of the insulating layer of cement between the thermocouple and the specimen.

The complete instrumentation and control flow diagram for the rod calibration and evaluation phases of the program is shown in Fig. 14. The desired temperature environment of the specimen plate was set manually into the recorder-controller. This, in turn, controlled power to the radiating lamps through the Thyatron power regulator. Feedback was obtained from the control thermocouple mounted on the specimen plate. In addition, the several thermocouples mounted on the specimen plate and on the rod were used in conjunction with the recorder-controller to record temperatures at predetermined specimen and rod locations.

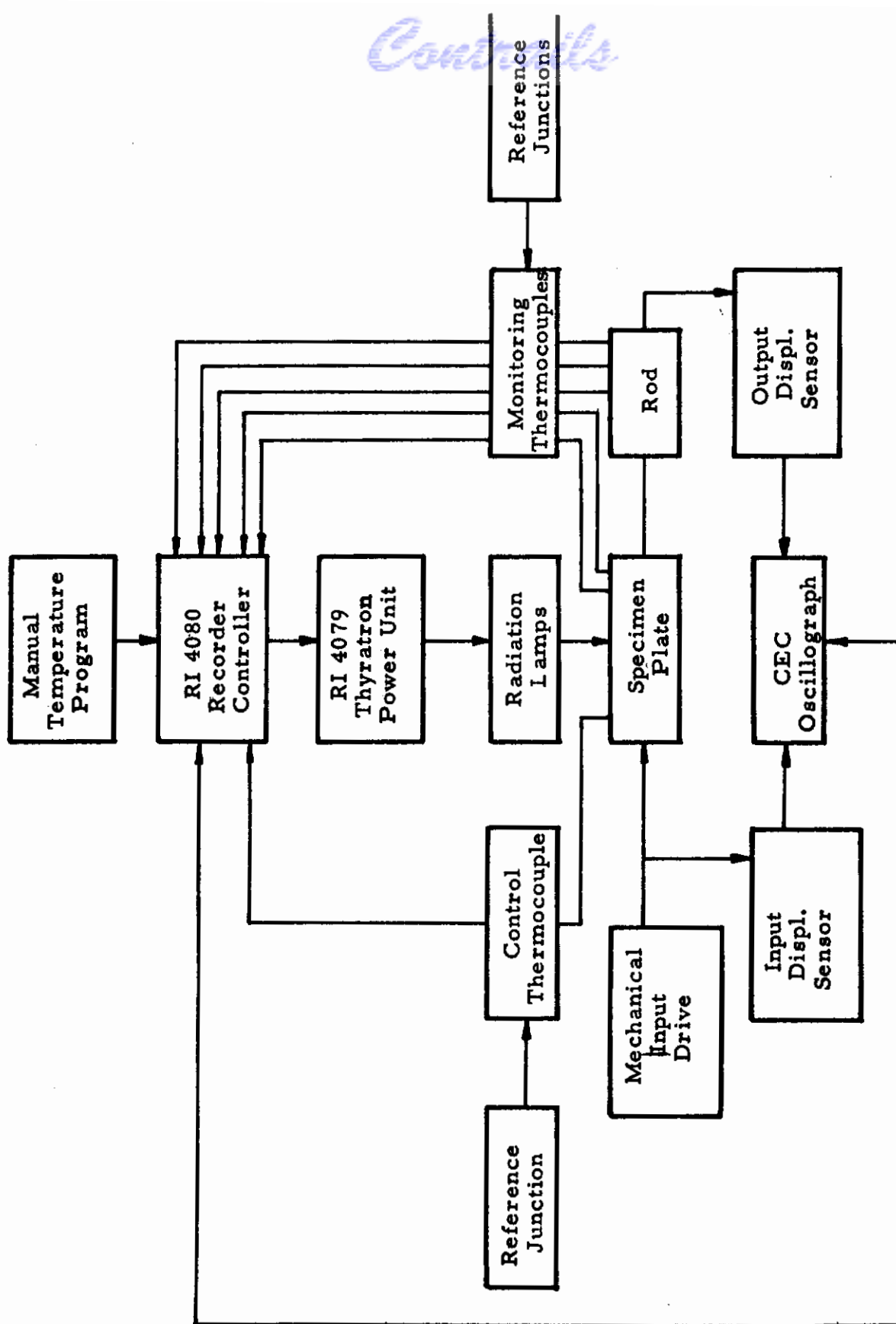


Fig. 14 INSTRUMENTATION AND CONTROL FLOW DIAGRAM

In the static calibration and the determination of rod growth corrections, the specimen plate was clamped and the rod elongation signal from the output sensor was recorded on the oscillograph. For the static and the dynamic evaluations a predetermined displacement analog was programmed into the specimen plate by the input drive mechanism. The input displacement signal was then recorded on the oscillograph. The mechanical input analog and the dynamic elongation signals were recorded simultaneously.

C. Rod Thermal Correction Factor Determination

Thermal correction factors for the rods were determined from three series of experiments. The first series performed at temperatures up to 1000°F utilized a fused quartz rod 3/8 in. in diameter attached to a stainless steel plate. The second series of tests at temperatures up to 2000°F employed also a fused quartz rod attached to an Inconel X plate. The third series of tests at temperatures up to 3000°F employed an alumina rod for which a successful attachment to a silicon carbide plate was never effected. Instead, some thermal growth measurements were made in which the alumina rod was kept into contact with the silicon carbide plate by a spring. In some later tests the alumina rod was attached to a plate of alumina. The test procedure for the three series of tests was as follows: The specimen was adjusted beneath the quartz lamps until the proper distance separating the two was obtained. In the first series of tests this distance was 24 in. whereas in the second series it was 12 and 4 in. and in the third series it was between 2-1/4 to 4 in. With one of the 1/2-in. maximum travel transducers attached to the upper side of the plate through the quartz rod or the alumina rod and the other 1/2-in. transducer attached to the under side of the plate with a cable, or a rod, electrical power was suddenly turned on and the lamps began heating the upper side of the plate. Output of the two transducers was monitored continuously by the oscillograph recorder as heating of the plate and rod combination continued. Outputs of thermocouples on both the plate and on the rod were also recorded. Heating was continued until the maximum temperature of the test was reached or until it was obvious that the temperature could rise no further. At such a time the power to the lamps was suddenly turned off and the specimen was permitted to cool to the ambient temperature. The difference in output from the upper and lower transducers was attributed to thermal growth (including expansion and/or creep) of the rod.

A complete summary of all tests performed including rod growth, static displacement and cycling is given in Table 2. This summary lists test details such as specimen and rod materials, input power, specimen heating rate and maximum temperature, test duration and reasons for any early terminations of tests. A summary of maximum temperatures attained on the specimen plates and the deflection transmission rods under evaluation is given in Table 3. In Table 3, it must be understood that the portion of the rod exposed to radiant heating from the lamps and re-radiation from the plate is given by the distance between lamps and specimen. The temperatures indicated for the specimen are believed to be quite reliable, whereas those indicated for the rod are not actual rod temperatures but merely thermocouple temperatures along the length of the rod because of the

Table 2

SUMMARY OF TEST DETAILS

Test No.	Set point and limiter	Lamp to Specimen Dist., in.	Power Input (start)		Power Input (control or end)		Avg. plate temp. rise rate of/sec	Type of Test	Specimen (plate)	Rod Material	Max temp. attained on plate at control point of	Test Duration min.	Remarks (or reason for early termination of test)
			Average Voltage (6 channels) volts	Total current amps	Average Voltage (6 channels) volts	Total current amps							
1	4.0; 41.7	24	not determined	not determined	not determined	3.4	Rod growth	Stainless Steel	Quartz	850	3.8	upper attachment slipped off rod.	
2	4.0; 41.7	24	not determined	not determined	not determined	4.6	Rod growth	Stainless Steel	Quartz	900	6.2	Successful test.	
3	7.5; 41.7	24	not determined	not determined	not determined	4.1	Rod growth	Stainless Steel	Quartz	960	7.5	Successful test.	
4	4.0; 83.3	12	not determined	not determined	not determined	6.3	Rod growth	Inconel	Quartz	1860	9.1	Inadequate power.	
5	7.5; 88.0	12	not determined	not determined	not determined	7.6	Rod growth	Inconel	Quartz	2000	9.8	Successful test.	
6	10.0; 88.0	12	470	208	not determined	6.2	Rod growth	Inconel	Quartz	2000	16.0	Successful test.	
7	4.0; 88.0	4	437	200	290	15.8	Rod growth	Inconel	Quartz	2000	10.5	Good test but record off chart.	
8	2.0; 83-88	4	298	165	282	6.8	Rod growth	Inconel	Quartz	1950	10.6	Successful test.	
9	10.0; 87-85	4	468	222	288	17.3	Rod growth	Inconel	Quartz	2020	8.8	Successful test.	
10	2-4; 87	12	283	162	418	3.7	Static Defl.	Inconel	Quartz	1910	51.2	Successful test.	
11	10.0; 87	12	478	222	468	7.1	Static Defl.	Inconel	Quartz	2000	20.0	Successful test.	
12	2.0; 87	4	274	150	290	4.6	Static Defl.	Inconel	Quartz	1980	21.8	Successful test.	
13	10.0; 87	4	478	208	295	17.1	Static Defl.	Inconel	Quartz	2000	10.6	Successful test.	
14	2-7; 100	4	266	150	465	2.2	Rod growth	Silicon Carbide	Alumina	2340	39	Lower transducer cable came loose.	
15	10; 100	2-1/2	480	199	478	15.7	Rod growth	Silicon Carbide	Alumina	2460	11.5	Lower transducer cable came loose.	
16	10; 100	1-1/2	480	187	not determined	17.3	Rod growth	Silicon Carbide	Alumina	2580	11.8	Upper rod detached.	
17	10; 100	2-1/2	480	148	not determined	14.6	Rod growth	Silicon Carbide	Alumina(1)	2370	12.9	Inadequate power - 6 lamps out.	
18	10; 100	2-1/2	474	192	474	18.0	Rod growth	Silicon Carbide	Alumina(1)	2800	35.8	Fairly successful test.	
19	10; 84.3	12-1/2	478	223	not determined	7.7	Dynamic Defl.	Inconel	Quartz	1490	3.1	7 lamps out - reflector overheating.	
20	10; 84.3	12-1/2	480	223	466	7.5	Dynamic Defl.	Inconel	Quartz	2020	10.3	Successful test.	
21	10; 100	3	480	222	not determined	17.6	Rod growth	Alumina	Alumina	2360	11.0	Apparently inadequate power(2)	
22	10; 100	2-1/4	480	221	478	40.0	Rod growth	Alumina	Alumina(3)	2920	9.7	Successful but poor record.	
23	10; 100	2-1/4	478	218	not determined	40.0	Static Defl.	Alumina	Alumina(3)	2920	9.7	Successful test.	
24	10; 100	2-1/4	not determined	not determined	not determined	40.0	Dynamic Defl.	Alumina	Alumina(3)	2860	9.7	Successful test.	

Note: (1) Spring loaded upper and lower transducer rods under compression against plate.

(2) It was theorized that the reflectivity of the white alumina plate was too high to permit attaining 3000°F.

(3) Plate coated with chromic oxide - alcohol slurry to increase absorbtivity.

Table 3

MAXIMUM TEMPERATURES ATTAINED DURING TESTS

Test No.	Center of plate	Center of plate (insulated from plate)	Specimen Temperatures, °F		Rod Temperatures, °F								
			Center of short edge	Center of long edge	1" up	2" up	3" up	4" up	5" up	6" up	12" up	18" up	24" up
1	850	-	900	890	-	-	-	-	-	550	650	760	1060
2	900	-	940	890	790	790	-	-	-	-	1080	1080	1200
3	960	-	1000	990	790	790	-	-	-	720	1090	1090	1360
4	1860	-	1800	1760	1400	1400	-	-	-	1380	1010	240	220
5	2000	-	1960	1880	1570	1570	-	-	-	1540	1150	230	200
6	2000	-	1980	1910	1600	1600	-	-	-	1560	1220	160	220
7	2000	-	1800	1840	1290	1290	-	-	-	480	160	160	170
8	1950	-	1850	1700	900	900	-	-	-	380	140	140	140
9	2020	-	1870	1840	1340	1340	-	-	-	670	-	-	160
10	1910	-	1970	1870	1550	1550	-	-	-	1510	-	-	-
11	2000	-	2060	1970	1680	1680	-	-	-	1700	-	-	-
12	1980	-	1970	1710	-	-	-	-	-	-	-	-	-
13	2000	-	-	1780	-	-	-	-	-	-	-	-	-
14	2340	-	1580	1980	-	-	1500	-	360	180	-	-	-
15	2460	2640	1860	1970	-	-	1170	-	180	-	-	-	-
16	-	2580	-	-	-	-	-	-	-	-	-	-	-
17	2370(a)	2190	-	-	-	-	1140	570	140	-	-	-	-
18	2800(a)	2700	-	-	-	-	1500	1000	260	-	-	-	-
19	1490	-	-	-	-	-	-	-	-	-	-	-	-
20	2020	-	-	-	-	-	-	-	-	-	-	-	-
21	2360	-	-	-	-	-	-	-	-	-	-	-	-
22	-	2920(a)	-	-	-	-	-	-	-	-	-	-	-
23	-	2920(a)	-	-	-	-	-	-	-	-	-	-	-
24	-	2860(a)	-	-	-	-	-	-	-	-	-	-	-

(a) Approximately 1-1/2 in. from center of plate directly under center of one lamp bank.

Contrails

difference in radiation absorption characteristics of the thermocouples, cements and rod materials. Even though these may not be true temperatures, they do give an indication of the temperature gradients along the rods both in the exposed and unexposed portions.

An excellent record of outputs from the upper and lower transducers during a rod growth determination in Test No. 18 is given in Fig. 15. This reproduction of the original oscillograph record shows a typical behavior. That is, both upper and lower transducers indicate a rapid bowing or buckling of the plate when first subjected to the one side radiant heat. The plate then settles somewhat and the rod growth continues. For comparison with the plate deflection and early rod growth as depicted in Fig. 15, the specimen temperature rise record during Test No. 18 is given in Fig. 16.

The results of the thermal growth factor determinations from the first two series of tests employing the quartz rod are presented in Fig. 17. At temperatures up to 1000°F with the distance of 24 in. between the lamps and the specimen (24 in. exposed length of the quartz rod) and at heating rates of about 4 to 4.5 deg/sec, total thermal growth of the quartz rod was 0.023 in. (or about 0.0010 in./in.) with a spread of from 0.020 to 0.027 in. At temperatures up to 2000°F with a distance of 12 in. between lamps and specimen and with heating rates of the order of 6 to 7.5 deg/sec the thermal growth correction was 0.029 in. (or about 0.0024 in./in.) with a spread of from 0.027 to 0.033 in. However, with a distance of only 4 in. between specimen and lamps and with heating rates of 6.8 and 17.3 deg/sec the thermal growth experienced was 0.038 in. (or 0.0095 in./in.) with a spread of 0.036 to 0.040 in.

Several attempts were made to determine rod growth with the alumina rod at temperatures up to 3000°F. However, in a total of 7 tests only 2 yielded useful rod growth data and then only up to a temperature of 2720°F. In the other tests, either a transducer cable or the rod came detached during heating and inadequate power prevented attainment of the desired 3000°F temperature.

Results of thermal growth correction factor determinations with the alumina rod in Tests Nos. 18 and 21 are presented in Fig. 18. It must be pointed out that Test No. 18 employed a spring loaded alumina rod with a sharpened tip under compression to maintain contact with the silicon carbide specimen, while Test No. 21 employed the alumina rod attached with a pin to the alumina cup bonded to an alumina specimen. Therefore, the results are not directly comparable. Results indicate a rod growth of 0.064 in. at about 2400°F in Test No. 21 and of 0.074 in. at about 2700°F in Test No. 18. The unit growths are 0.021 in./in. and 0.030 in./in., respectively. The results if extrapolated linearly to a temperature of 3000°F would be about 0.110 in. and 0.100 in. or unit growths of 0.044 in./in. and 0.033 in./in., respectively.

An interesting comparison of specimen temperature and indicated rod temperatures at three locations along its length during Test No. 18 is given in Fig. 19.

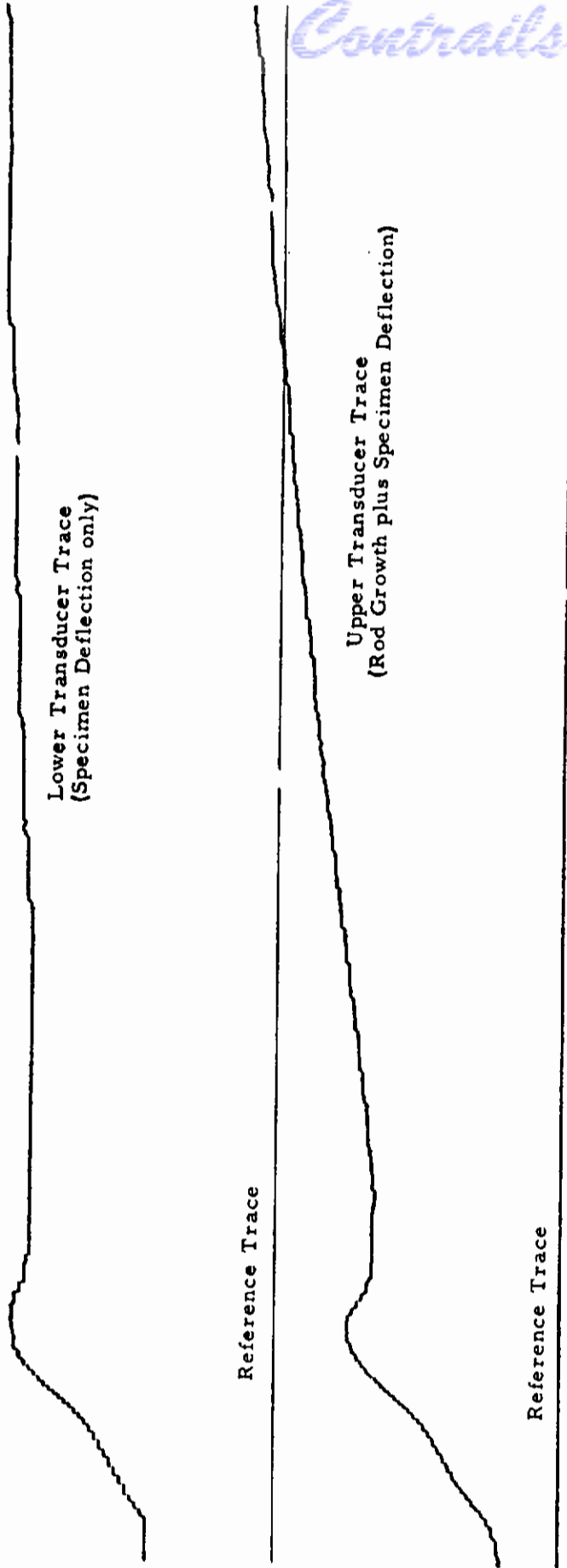


Fig. 15 TEST NO. 18, ROD GROWTH AND SPECIMEN DEFLECTION RECORDS OBTAINED FROM
UPPER AND LOWER TRANSDUCERS (RECORD NO. 26690) (Chart Speed = 9.4 in. /min)

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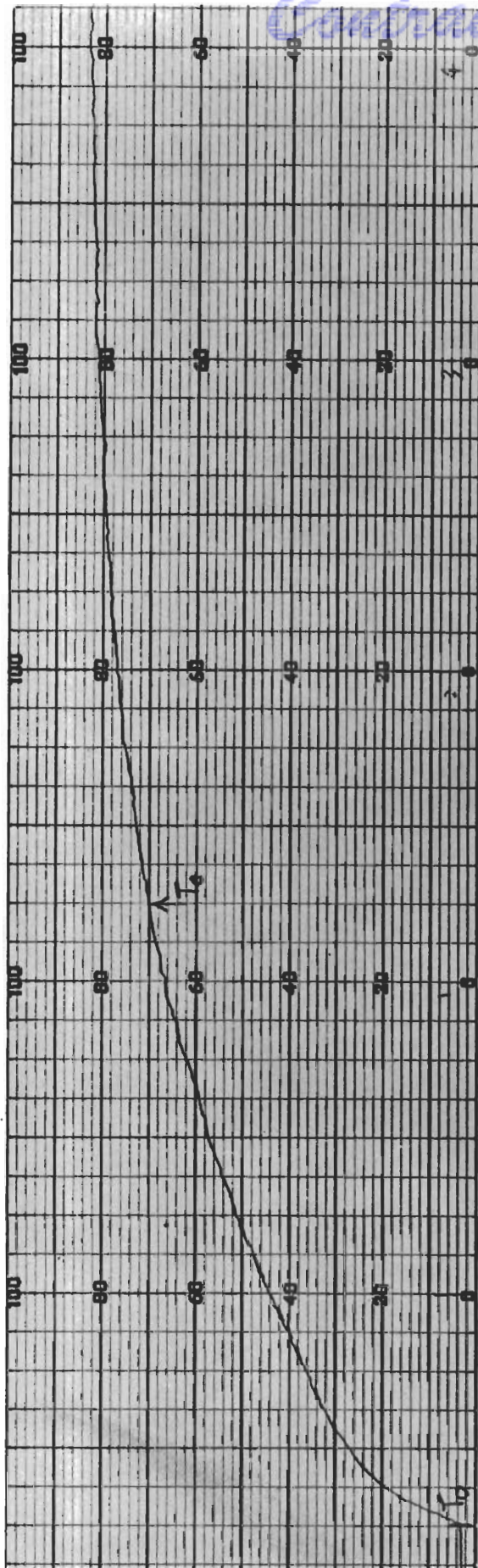


Fig. 16 TEST NO. 18, PLATE TEMPERATURE RISE (Chart Speed = 2 in. / min)
(Rise Rate, T_0 to $T_e = 18^\circ\text{F}/\text{sec}$)

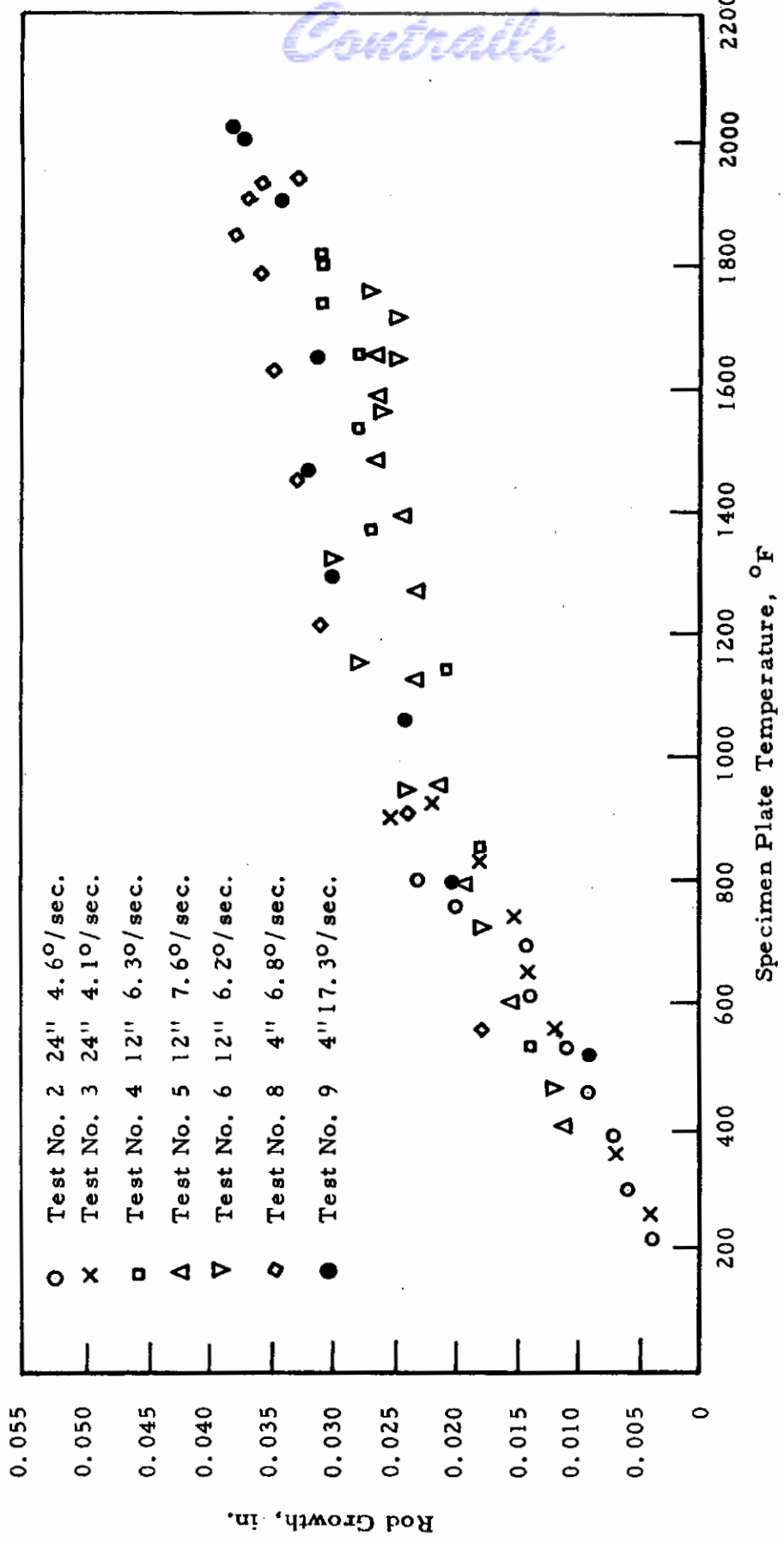


Fig. 17 ROD GROWTH FOR QUARTZ ROD AS A FUNCTION OF TEMPERATURE

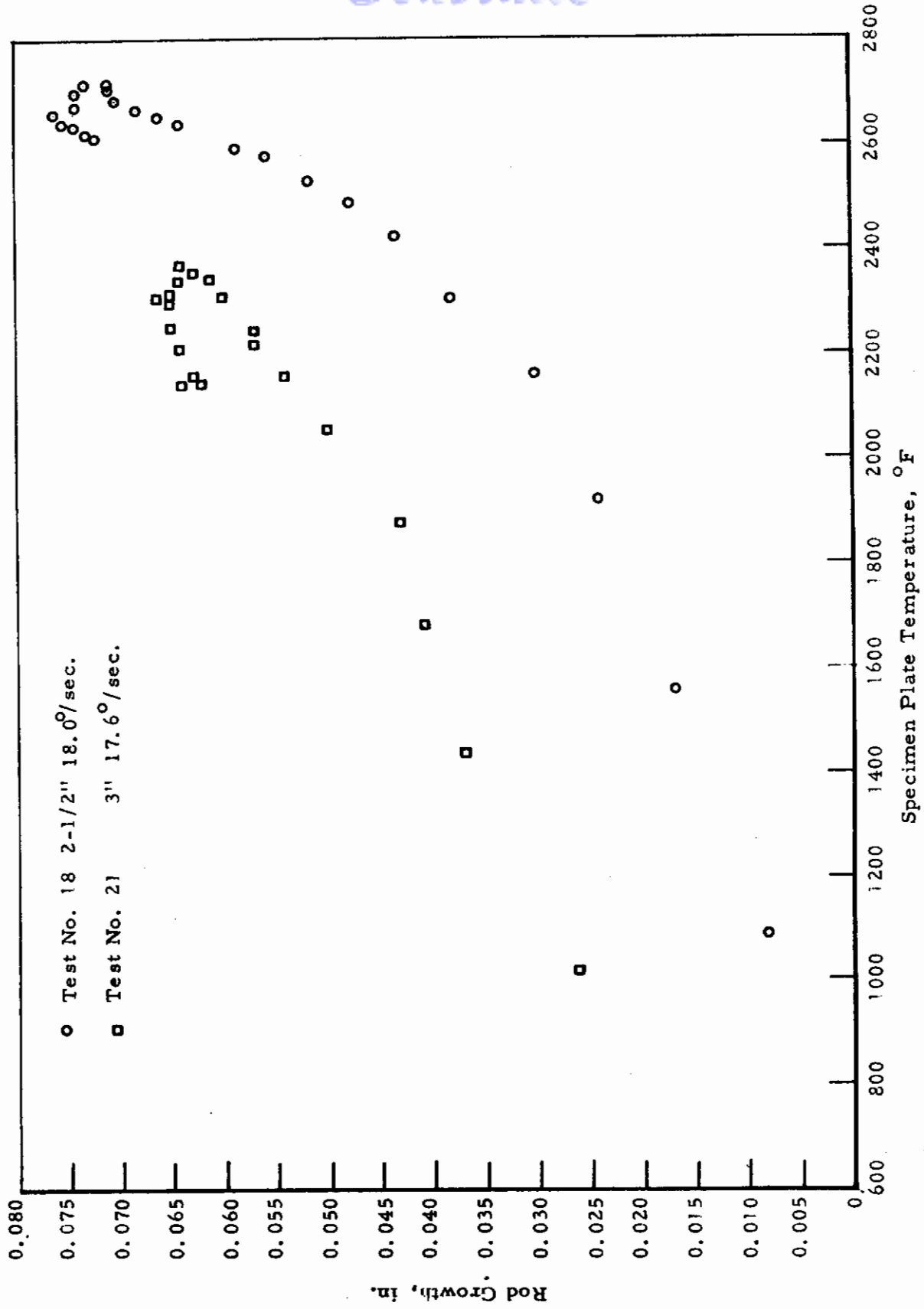


Fig. 18 ROD GROWTH FOR ALUMINA ROD AS A FUNCTION OF TEMPERATURE

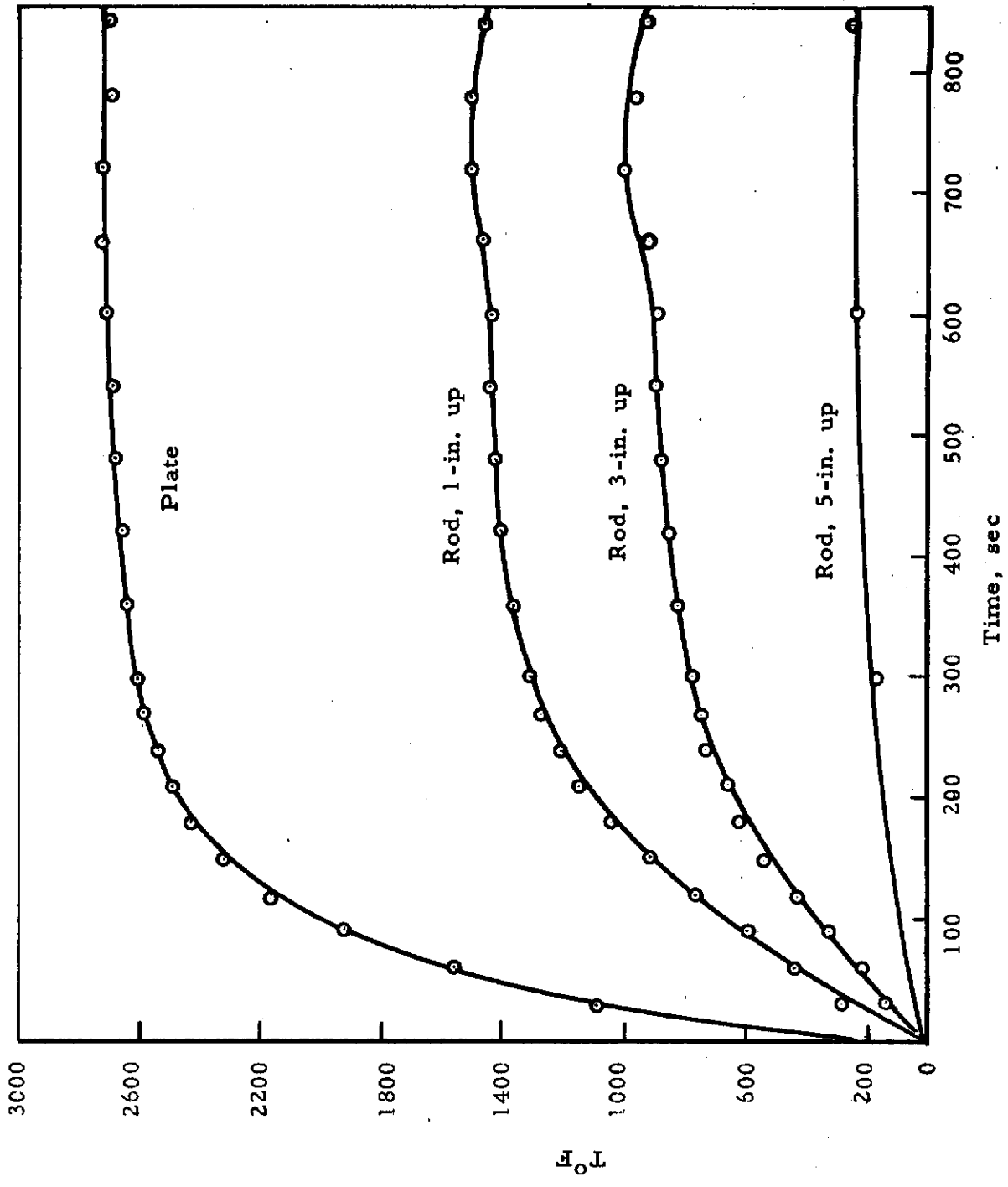


Fig. 19 PLATE AND INDICATED ROD TEMPERATURES IN TEST NO. 18

Contrails

An attempt was made to determine rod growth to 3000°F in Test No. 22, in which a 6-in. maximum displacement transducer was used to monitor rod growth. In this test, either the transducer was not sensitive enough to detect the rod growth and plate movement, or the rod growth itself was negligibly small during the brief period of time required to attain the actual maximum temperature of 2920°F. The attainment of this temperature and in a very brief period of time (a heating rate of about 40°F/sec) was attributed to a coating of chromic oxide (in the form of a slurry of chromic oxide powder and alcohol) placed on the surface of the alumina. This considerably increased the absorptivity of the alumina and permitted rapid heating. This is evident from a study of the initial portion (first 90 sec) of the oscillograph traces produced by outputs from the lower (1/2 in. maximum displacement) and upper transducers in Fig. 20. The measured displacements at 180 sec (2880°F) were 0.040 in. and 0.038 in., respectively. If, however, it is assumed that the potential divider in the 6-in. transducer was on the verge of moving from one winding in the potentiometer to the next at the start of the test and again at the time when the maximum temperature was approached, and if the transducer sensitivity of about 0.020 in. is added twice to the rod growth and specimen deflection recorded in Fig. 20, this would result in a total displacement sensed by the upper transducer of 0.078 in. When the plate movement of 0.040 in. is subtracted a net rod growth of 0.038 in. results. This growth is reasonable when compared with the rod growths determined in Tests Nos. 18 and 21 presented as functions of time in Fig. 21. Unfortunately, no further growth after the initial 3 minutes was detected because at that time the lower transducer was detached and the gain on the oscillograph recorder for the upper transducer was considerably reduced in preparation for the static displacement and cycling of Tests Nos. 23 and 24, which were conducted during the test run following Test No. 22.

VI. STATIC AND DYNAMIC EVALUATION OF DEFLECTION TRANSMISSION METHODS

The developed deflection transmission methods employing the quartz rod and the alumina rod were evaluated in series of static and dynamic tests. The static tests involved static movements of the specimen through displacements of the order of 5-1/2 to 6 in. while the dynamic tests involved dynamic movements with displacements also of the order of 5-1/2 to 6 in. but at a much greater frequency (a frequency of the order of 2/3 cps was employed). The static drive unit and lower transducer (1/2 in.) are shown in place in Fig. 22, while the dynamic drive unit is shown in Fig. 23. The ability of the deflection transmission rods to follow such static and dynamic movements comprised the evaluations.

A. Static Movement

Static displacement evaluations were performed with the Inconel X and quartz rod combination at temperatures up to 2000°F. The procedure followed was identical with that of the rod growth experiments except that

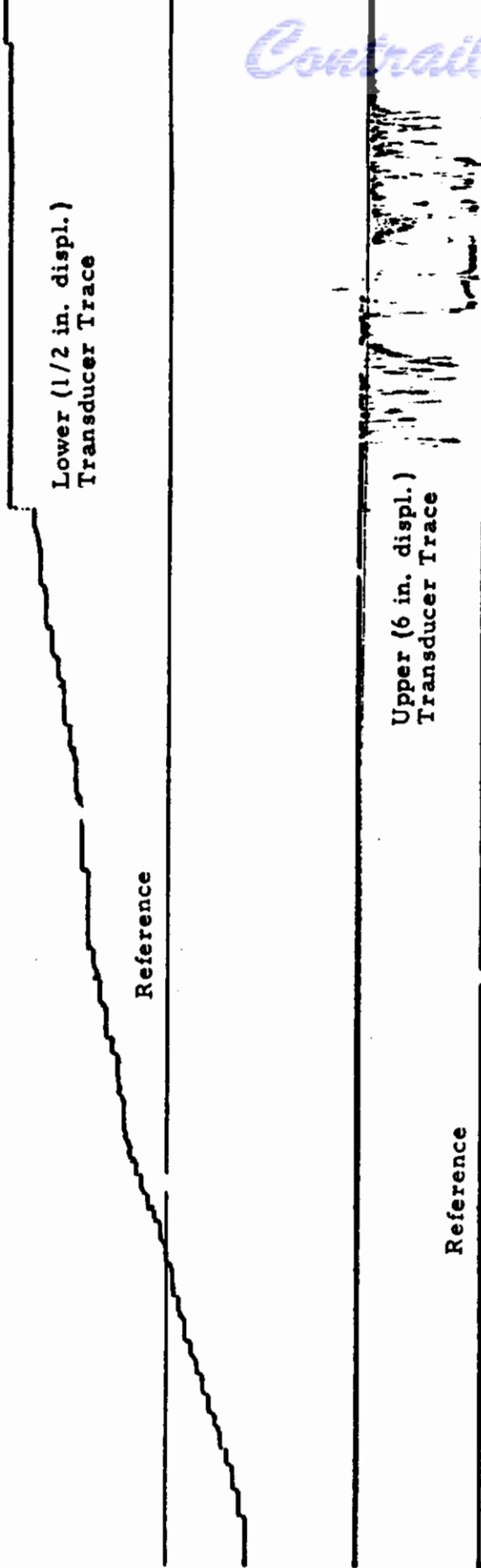


Fig. 20 TEST NO. 22, ROD GROWTH AND SPECIMEN DEFLECTION RECORDS OBTAINED FROM UPPER AND LOWER TRANSDUCERS (RECORD NO. 26765)

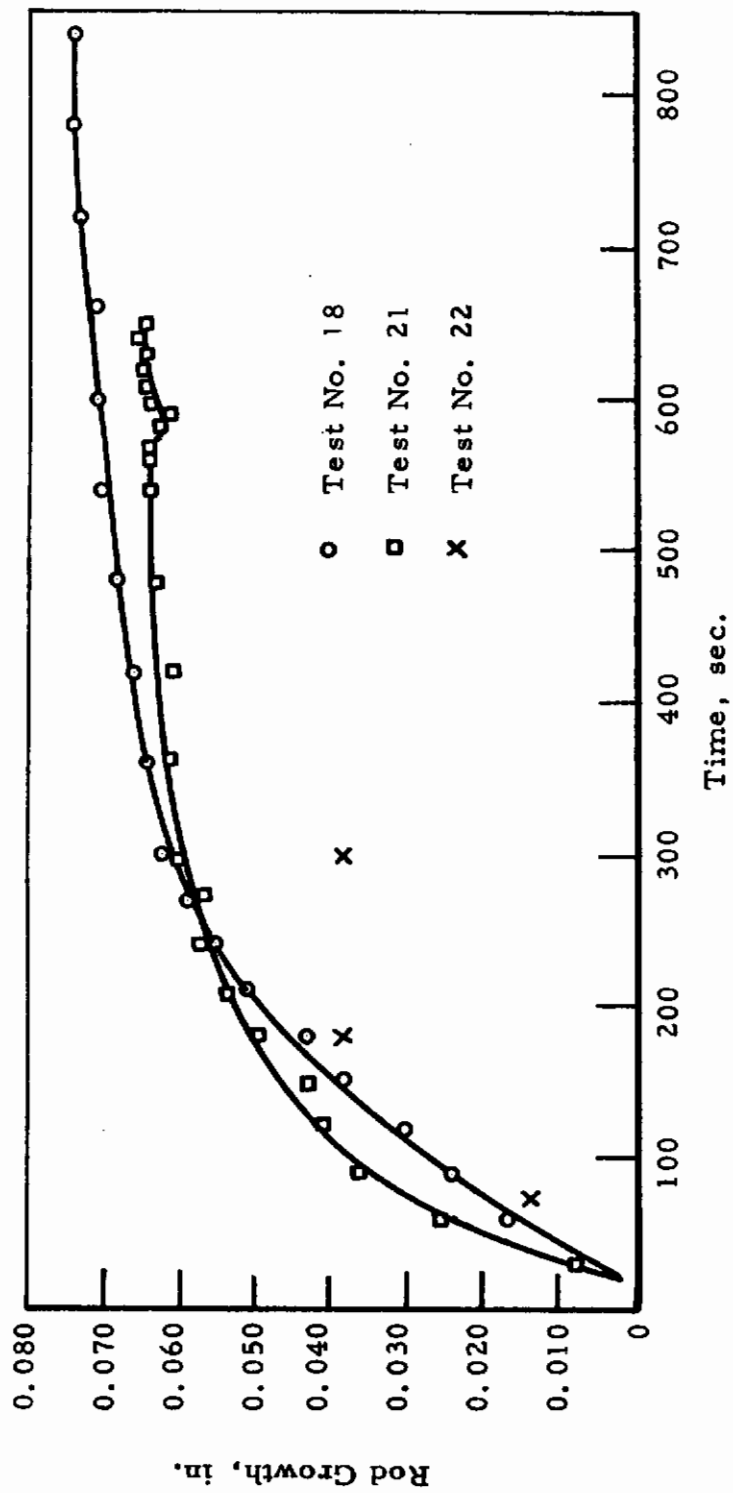


Fig. 21 ALUMINA ROD GROWTH AS FUNCTION OF TIME IN 3 TESTS

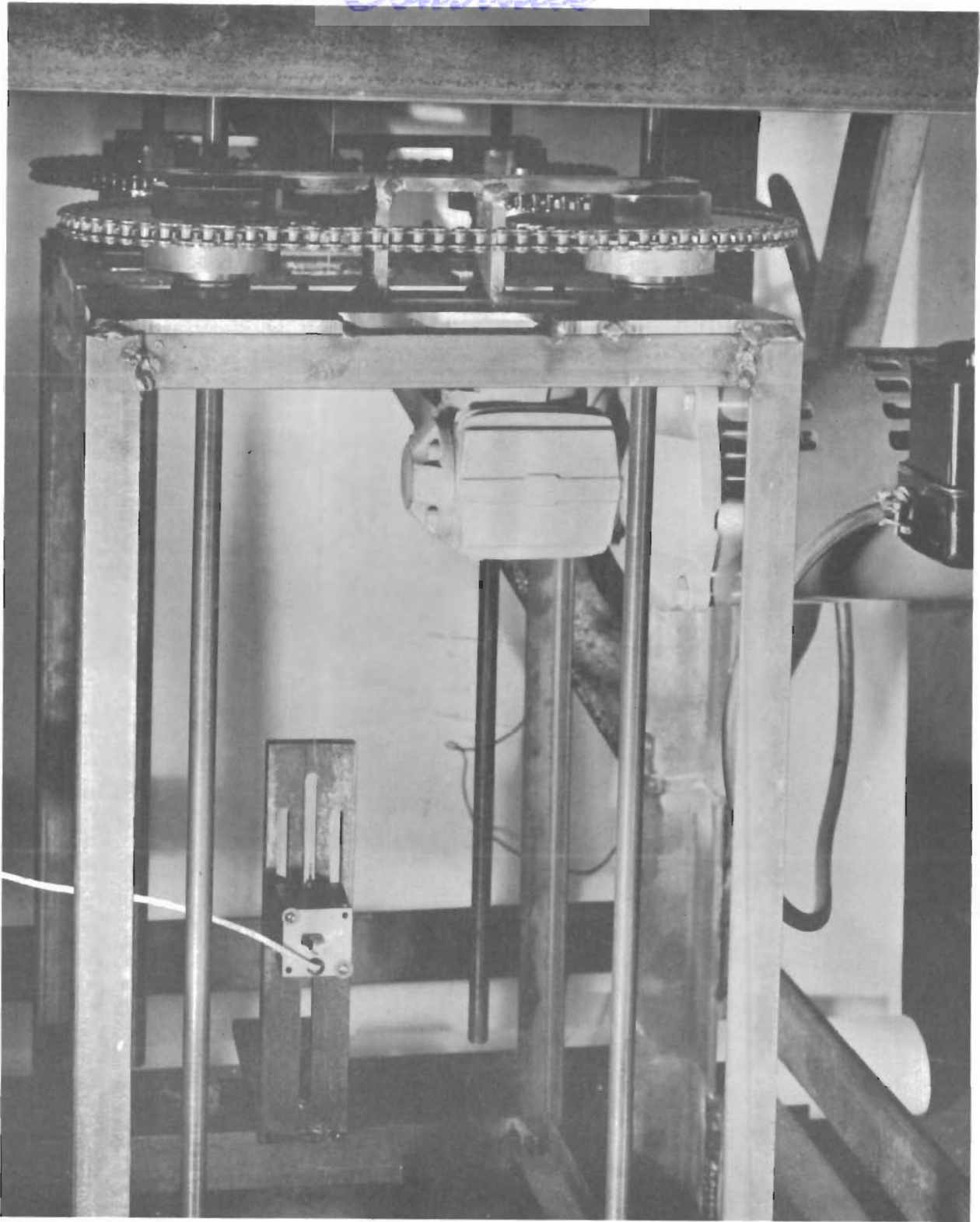


Fig. 22 CLOSE-UP OF MOTOR DRIVEN UNIT FOR IMPOSING STATIC DISPLACEMENTS ON SPECIMEN

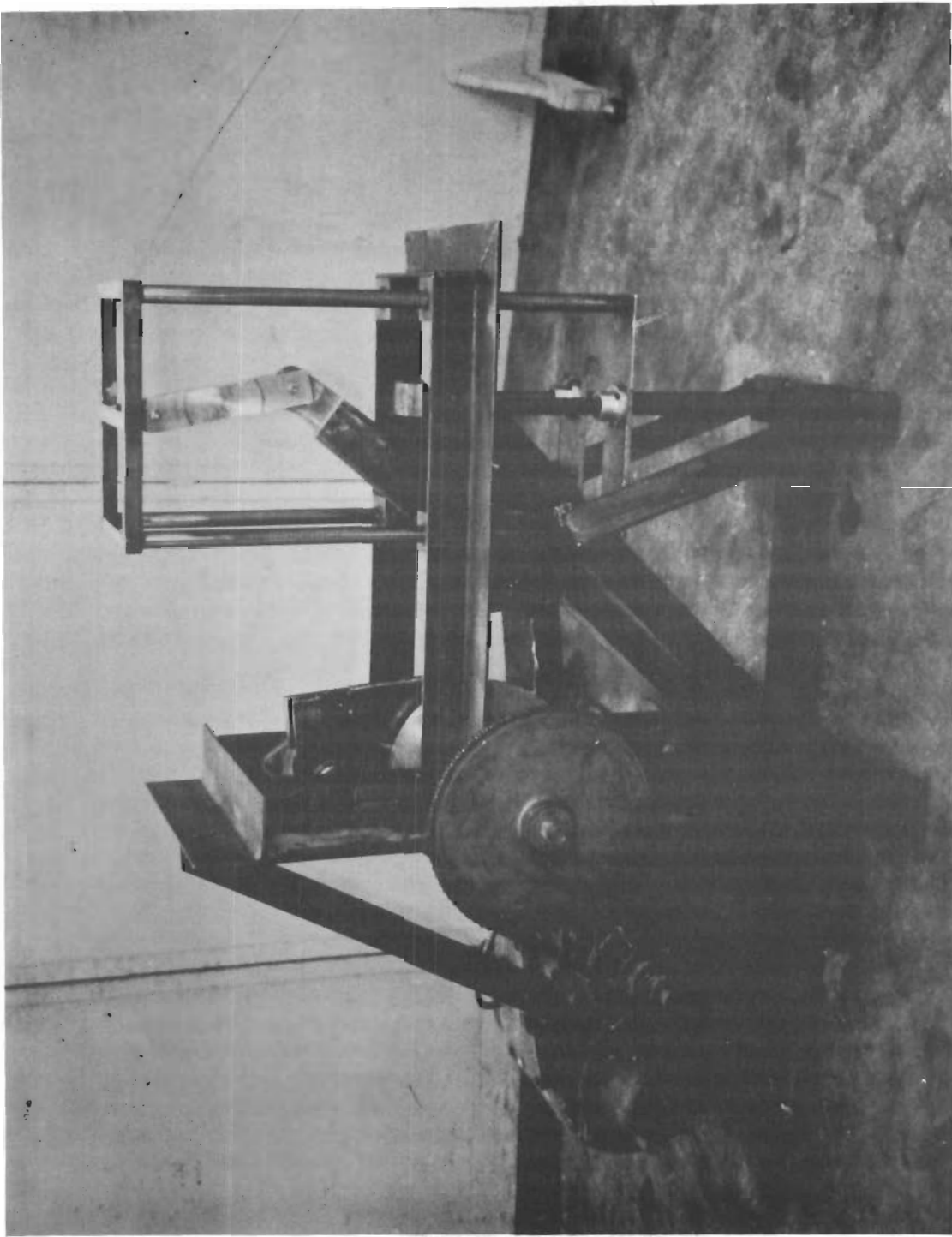


Fig. 23 MOTOR DRIVEN VIBRATION UNIT FOR IMPOSING DYNAMIC DISPLACEMENTS ON SPECIMEN

Continued

after the maximum temperature was attained or when it was apparent a higher temperature could not be achieved, the specimen was displaced through a distance of about 5-1/2 to 6 in. in 1 in. increments either toward or away from the lamps and returned again to its original position. Displacement was followed by means of displacement transducers with a 6 in. maximum travel. Specifically, in a pair of tests with the Inconel plate and the quartz rod (Test Nos. 10 and 11) the initial distance between lamps and specimen was 12 in. The specimen was then raised 5-3/4 in. in 1 in. increments and then it was returned to its initial position, a distance of 12 in. from the lamps. For another pair of tests with the Inconel X plate and the quartz rod (Test Nos. 12 and 13) the initial distance between lamps and plate was 4 in. The specimen was then raised 2 in., lowered approximately 6 in. and then it was finally raised again to its original position 4 in. away from the lamps. A typical displacement record during two 1-in. displacements is given in Fig. 24, while the plate temperature rise and stability during another static displacement test are shown in Figs. 25 and 26.

Static movements were not determined with the stainless steel plate-quartz rod combination. Results of the static displacement tests for the Inconel plate-quartz rod combination are summarized in Table 4. The results of these evaluations did not meet the contract requirement of 0.5 percent accuracy for displacement. This is perhaps partly as a result of the relatively poor sensitivity of the 6-in. transducers with which these displacements were measured.

Because of the inability to produce a satisfactory attachment for the alumina rod to the silicon carbide plate, no static movement evaluations were performed for this combination. Instead, as was noted earlier, a satisfactory attachment was evolved for the alumina rod to an alumina plate. Thus the alumina plate/alumina rod combination was employed in static displacement tests. For this combination the initial distance between lamps and plate was set at 2-1/4 in. After the maximum temperature was attained (the temperature rise preceding this test is shown in Fig. 27), the plate was driven down approximately 6 in. and then raised to its original position about 2-1/4 in. away from the lamps. The temperature record during this displacement is given in Fig. 28.

The record of the static displacement test with the alumina plate/alumina rod combination is given in Fig. 29. No determination of accuracy can be made since only the upper transducer was used to follow the motion. Since the rod attachment permitted the rod to follow the plate movement, at a temperature of 2920°F, the evaluation was considered a success.

B. Dynamic Movement

Two evaluations were conducted to determine the capability of the rods and attachment methods to follow cyclic displacements. One test was performed with the Inconel X plate/quartz rod combination. The second of two attempts to perform this test was successful; the first attempt was terminated prematurely because of many lamps burning out during the test and because a gold plated stainless steel reflector was overheating from inadequate

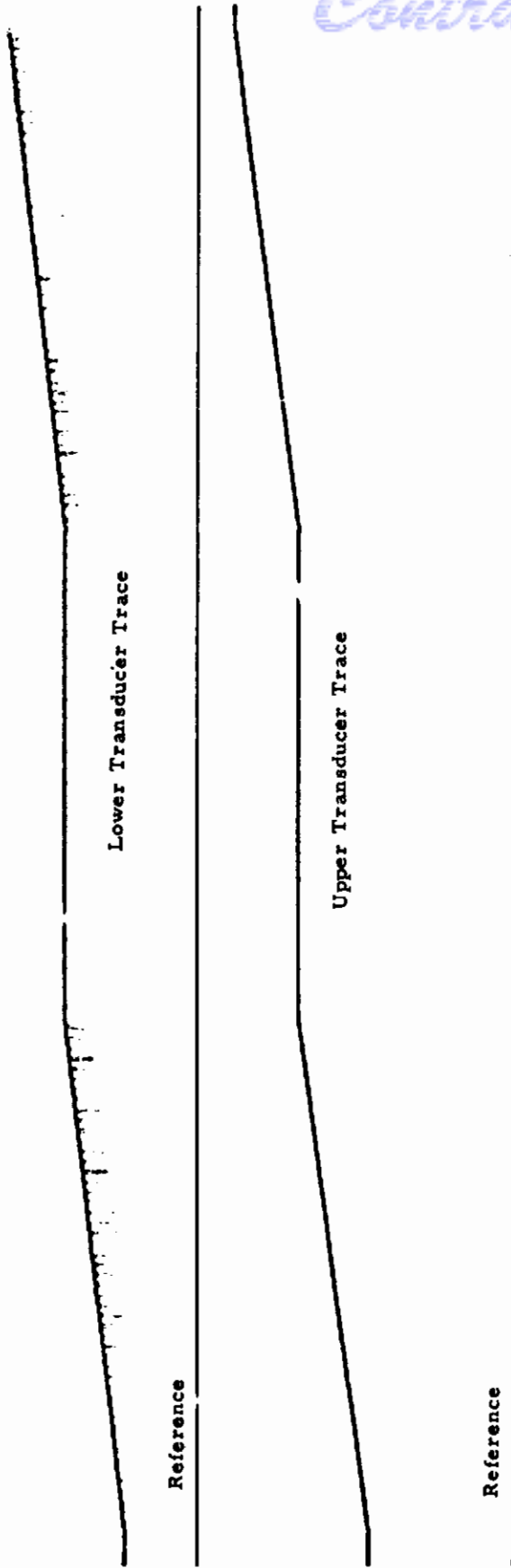


Fig. 24 TEST NO. 10, SPECIMEN DISPLACEMENT RECORDS DURING TWO SUCCESSIVE ONE-INCH DISPLACEMENTS (RECORD NO. 26567)

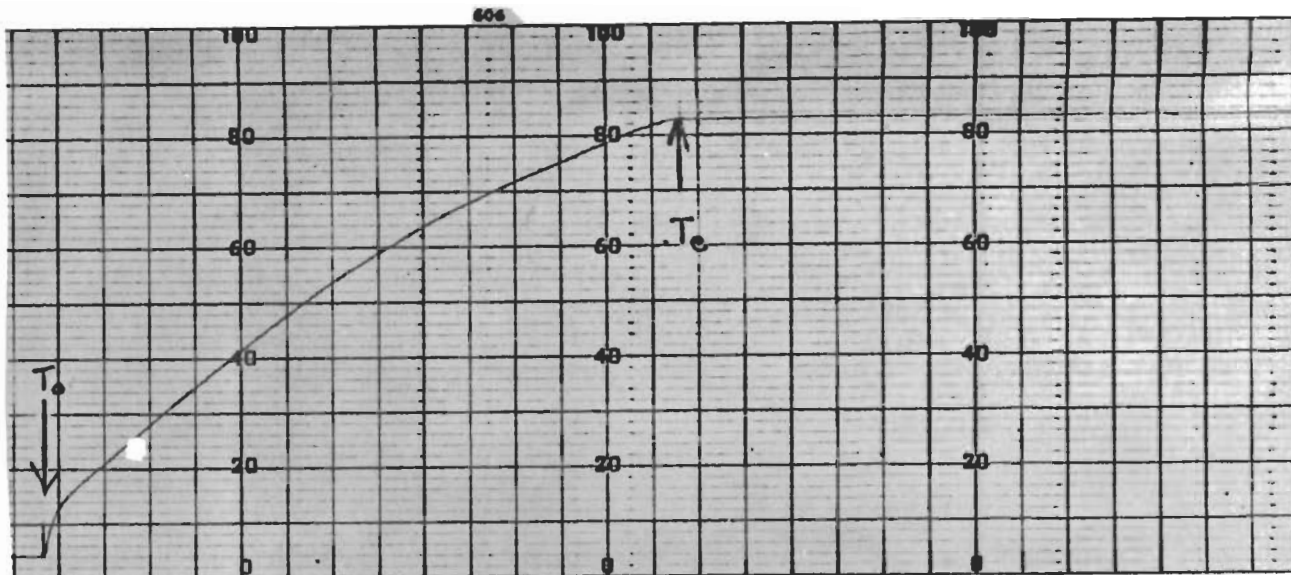


Fig. 25 TEST NO. 13, PLATE TEMPERATURE RISE TO 2000°F
(Rise Rate T_0 to $T_e = 17.1^\circ\text{F}/\text{sec}$)

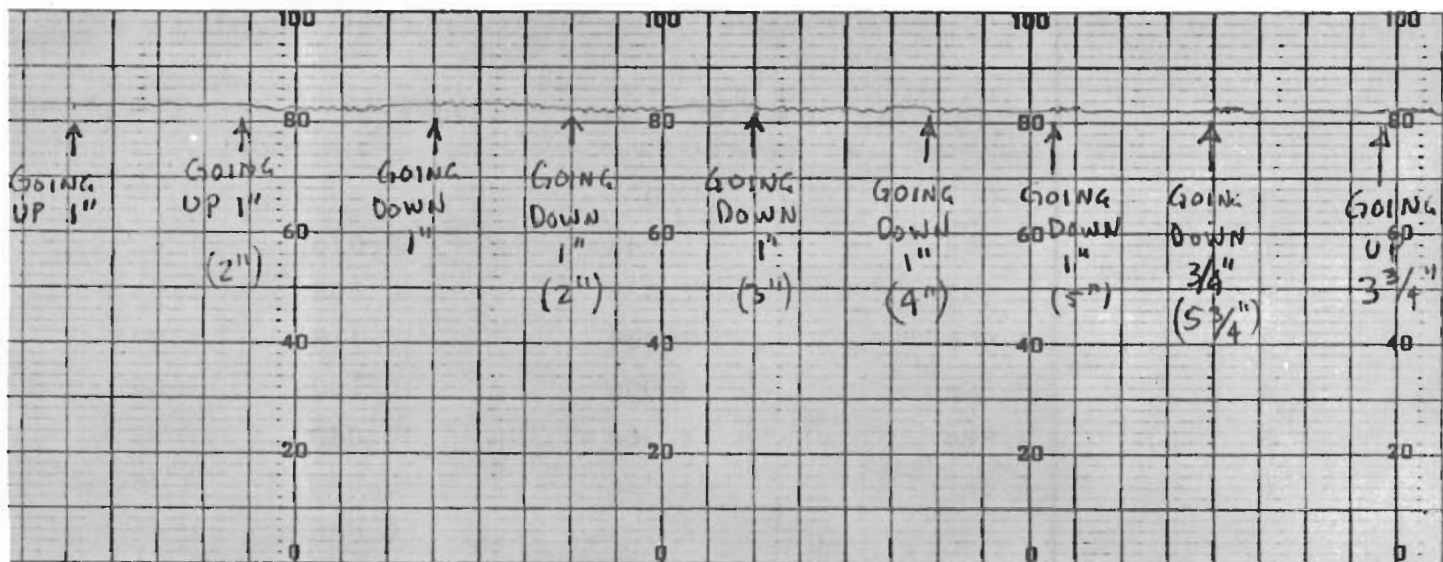


Fig. 26 TEST NO. 13, PLATE TEMPERATURE RECORD DURING
STATIC DISPLACEMENTS AT 2000°F

Table 4EVALUATION OF STATIC DEFLECTION TRANSMISSION
CAPABILITY, TEST NO. 10, TEMP AT START: 1900°F

Position	Actual Displacement in.	Transmitted Displacement in.	Error in.	%
0	0	0	-	
1	0.988	1.039	+0.051	0.85
2	1.998	2.040	+0.042	0.70
3	2.986	3.061	+0.075	1.25
4	3.996	4.044	+0.048	0.80
5	4.962	4.971	+0.009	0.15
6	5.511	5.472	-0.039	0.65
7	0.571	0.612	+0.042	0.70
		avg.	0.044	0.73

TEST NO. 11, TEMP AT START: 1970°F

0	0	0	-	
1	0.969	1.045	+0.076	1.27
2	1.937	2.035	+0.098	1.63
3	2.906	3.024	+0.118	1.97
4	3.896	3.996	+0.100	1.67
5	4.886	4.968	+0.080	1.33
6	5.381	5.426	+0.045	0.75
		avg.	0.086	1.43

Contrails

Table 4 (Cont.)

TEST NO. 12, TEMP AT START: 1970°F

Position	Actual Displacement in.	Transmitted Displacement in.	Error in.	%
0	0	0	-	
1	0.990	0.976	-0.014	0.23
2	2.002	1.934	-0.068	1.13
3	1.012	1.051	+0.039	0.65
4	0	0.075	+0.075	0.92
5	-1.012	-0.957	+0.055	0.92
6	-2.002	-1.971	+0.041	0.68
7	-3.035	-3.023	+0.012	0.20
8	-3.551	-3.567	-0.016	0.27
9	-0.022	-0.038	-0.016	0.27
		avg.	0.037	0.62

TEST NO. 13, TEMP AT START: 2000°F

0	0	0	-	
1	0.994	0.971	-0.023	0.38
2	1.967	1.872	-0.095	1.58
3	0.994	0.953	-0.041	0.68
4	-0.041	-0.035	+0.006	0.10
5	-1.015	-1.024	-0.009	0.15
6	-2.009	-2.031	-0.022	0.37
7	-3.044	-3.090	-0.046	0.77
8	-3.769	-3.849	-0.080	1.33
9	-0.104	-0.071	+0.033	0.55
		avg.	0.039	0.65

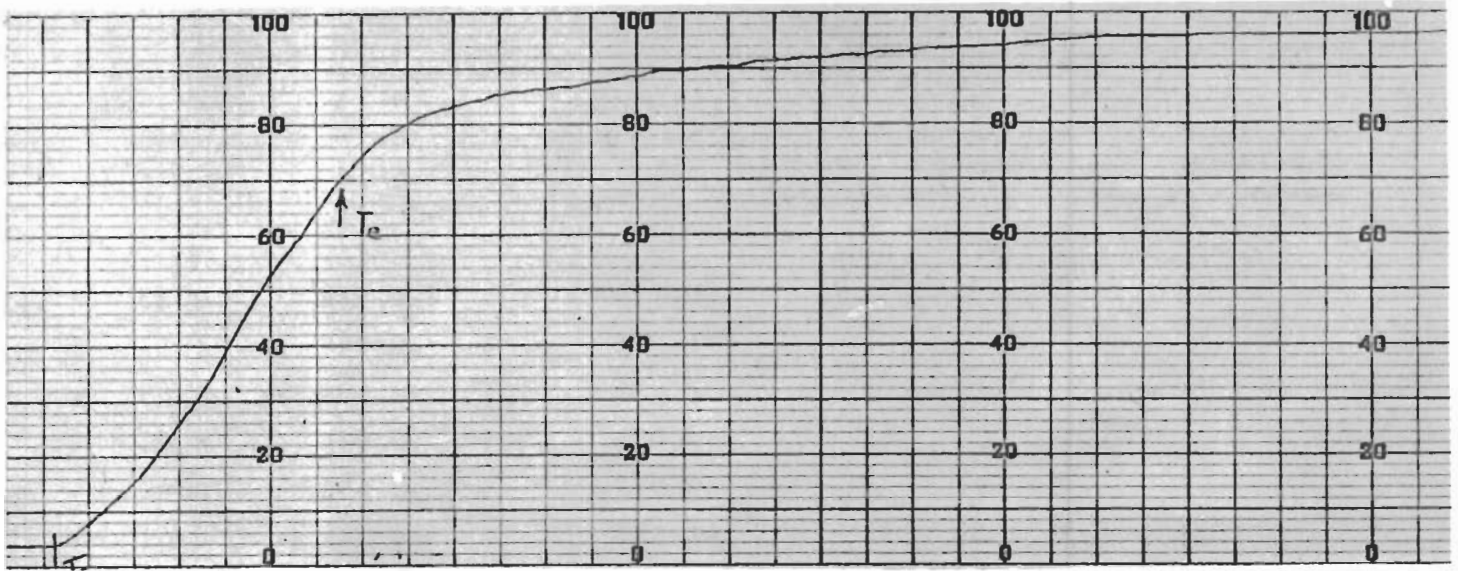


Fig. 27 TEST NO. 22, PLATE TEMPERATURE RISE TO 2920°F
(Rise T_o to $T_e = 40^\circ\text{F}/\text{sec}$)

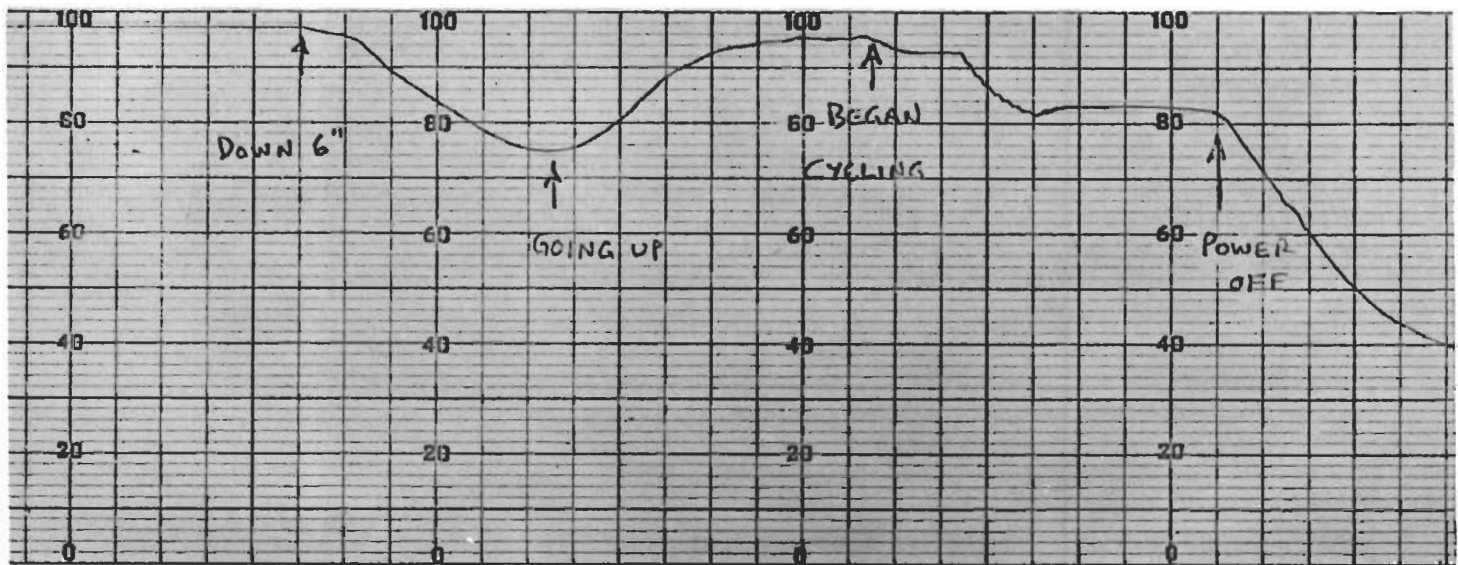


Fig. 28 TESTS NOS. 23 AND 24, PLATE TEMPERATURE RECORD
DURING STATIC DISPLACEMENT AND CYCLING

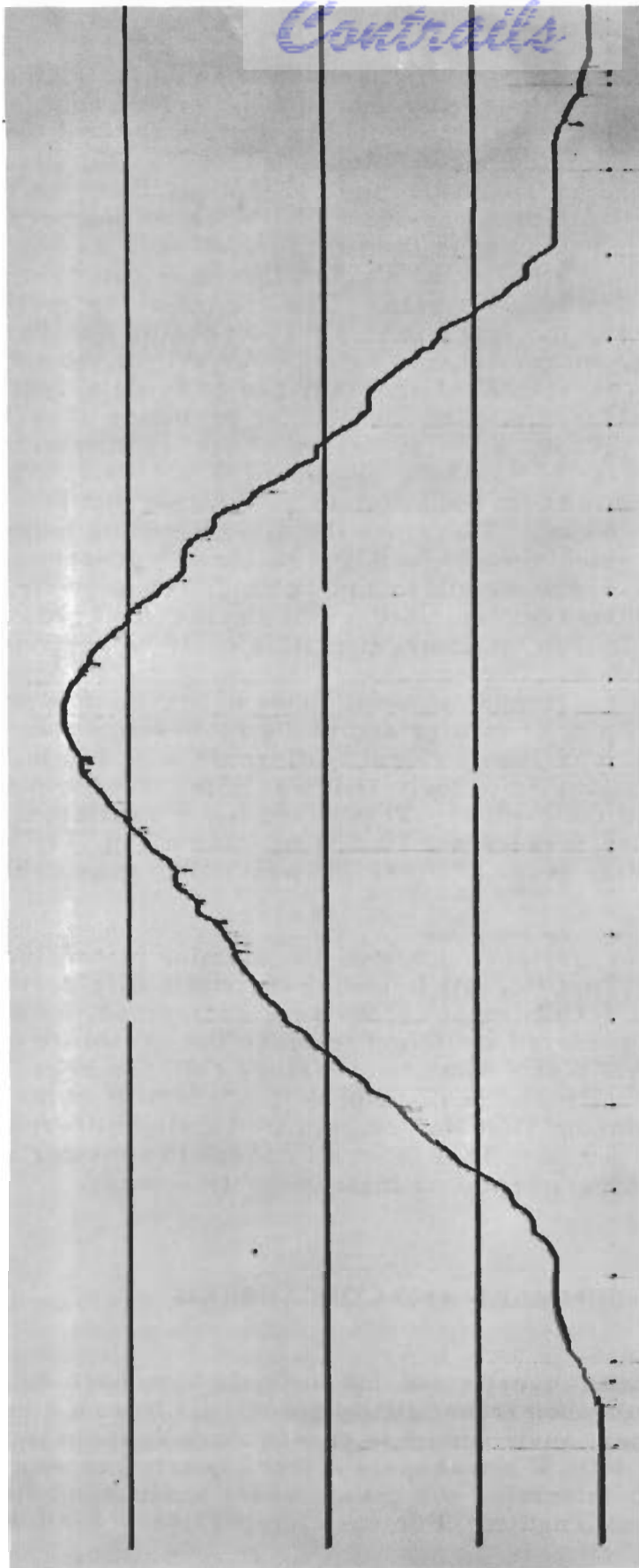


Fig. 29 TEST NO. 23, SPECIMEN DISPLACEMENT RECORD DURING A 6-INCH
DOWNWARD DISPLACEMENT AND RETURN (RECORD NO. 26765) .

Contrails

cooling. (The gold plated reflector was intended to enable attainment of the 3000°F temperature for the alumina rod growth and displacement tests. A cooling system devised for this reflector could not properly cool the quartz lamps and the reflector.) In the second attempt (Test No. 20) the ceramic reflectors were again used and, with the plate 12-1/2 in. from the lamps, the specimen temperature attained was 2020°F. When the temperature was stabilized, the specimen, beginning at the top of its stroke, was cycled downward through approximately a 5-3/4 in. displacement and returned at a rate of 2/3 cps. This continued for 29 cycles. The record of the cyclic displacement as transmitted to the upper (6 in.) transducer by the quartz rod is given in Fig. 30. The movement was not monitored by a second transducer. Therefore, no assessment of accuracy can be made except to note that the peak to peak measured displacement at the beginning of cycling (as shown in Fig. 30) was 5.853 in. while at the end of cycling it was 5.916 in. This represents an increase of 0.063 in. or about 1.1 percent. The cyclic displacement measured in a room temperature calibration just before heating the specimen was 5.789 in. Therefore the disagreements between hot and cold cycling measurements would be 0.064 in. or 1.1 percent and 0.127 in. or 2.2 percent at the beginning and end of cycling, respectively. These differences, however, cannot be explained on the basis of rod growth. Rather, something like drift in the instrumentation could be responsible.

On the other hand, a comparison was made of maximum downward displacement in the beginning of cycling and in the room temperature cycling calibration. This comparison yielded a difference of 0.145 in. for this measurement. Upon conclusion of the test it was noted that the plate had bowed upward 0.10 in. at the center. This would leave a difference of 0.045 in. If a rod growth correction of 0.035 in., from Fig. 17, were applied, this would yield an error of 0.010 in. well within acceptable measurements.

In the other cyclic displacement test, the alumina plate/alumina rod combination was cycled (Test No. 24) following the static displacement evaluation (Test No. 23). Only eight cycles were performed, as shown in Fig. 31, but this was considered sufficient to prove the capability of the rod and attachment. For this test a room temperature calibration could not be made. However, computation of peak-to-peak displacement at the beginning and end of cycling during Test No. 24 indicated a slight decrease from 5.559 to 5.541 in. or 0.018 in. This is small enough to consider as negligible or attributable to experimental or instrumentation error.

VII. SUMMARY AND CONCLUSIONS

Materials have been investigated and methods have been developed for accounting for rod growth when transmitting deflections from a structure under test in a radiant heat environment to transducers at room temperature. For temperatures up to 2000°F a transparent fused quartz rod was selected and subjected to tests to determine rod growth under various conditions of heating rates and exposed lengths. For such temperatures and for use with

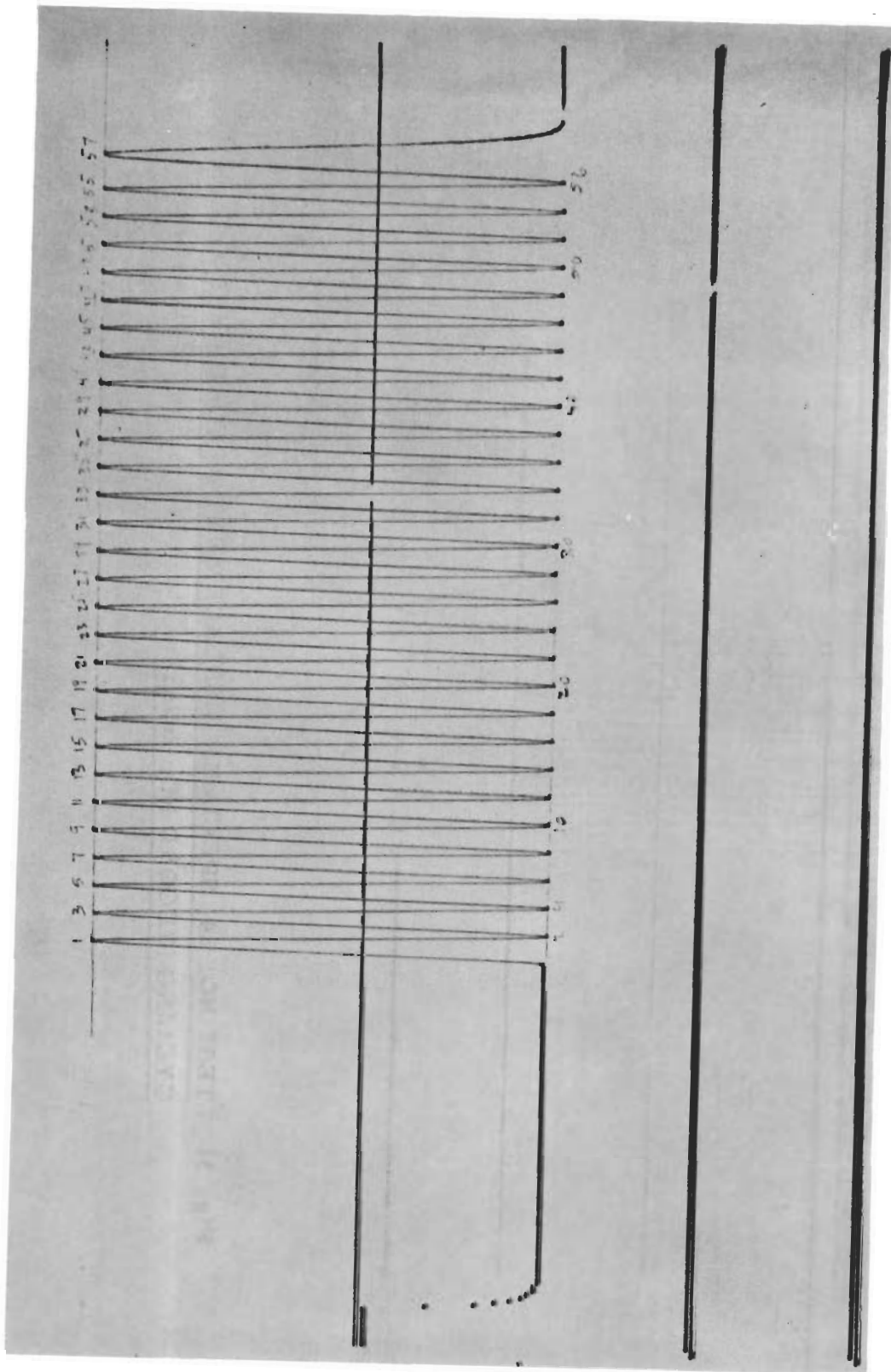


Fig. 30 TEST NO. 20, SPECIMEN DISPLACEMENT RECORD DURING CYCLING
(RECORD NO. 26748)

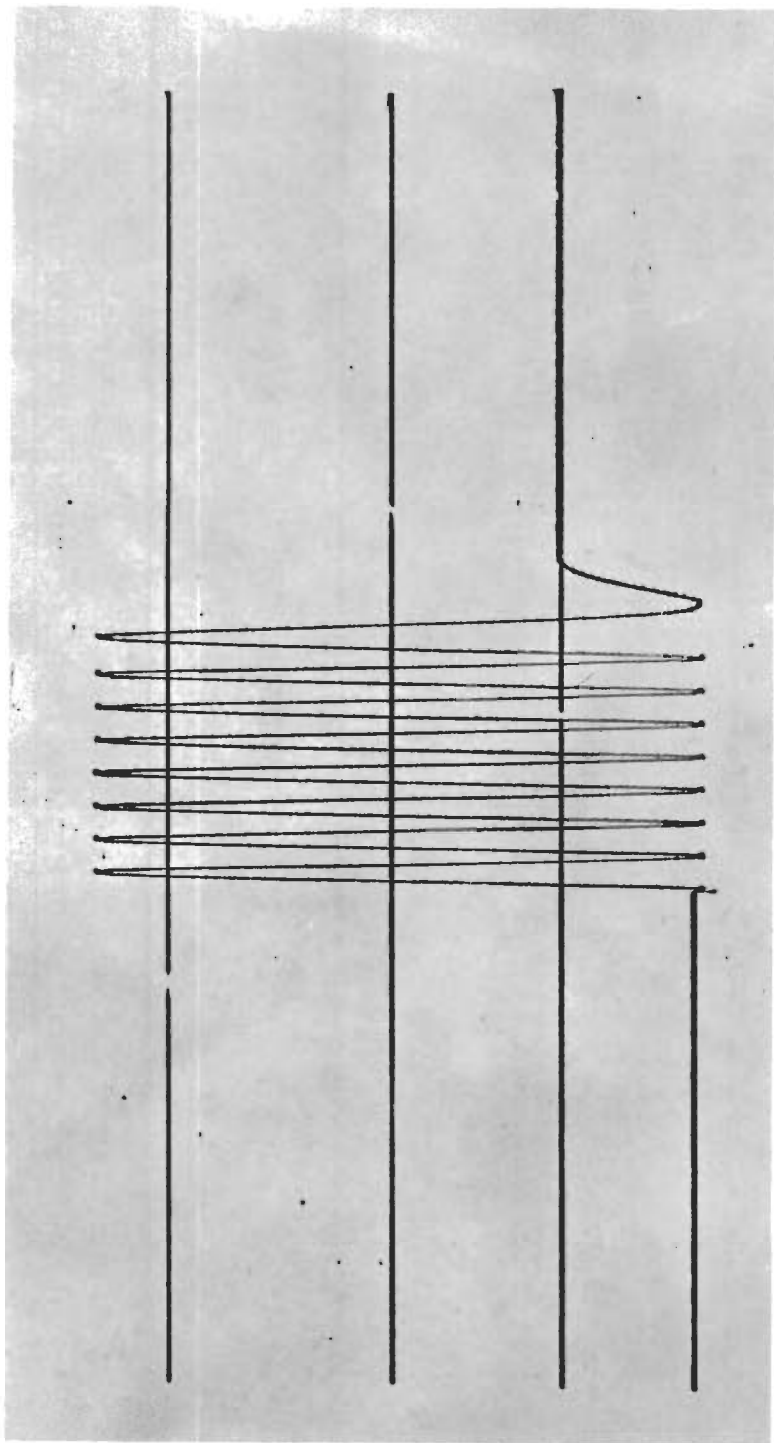


Fig. 31 TEST NO. 24, SPECIMEN DISPLACEMENT RECORD DURING
CYCLING (RECORD NO. 26765)

Continued

potentiometer transducers employing negator springs, which place the deflection transmitting rods under slight tension, rod growths of 0.022 to 0.027 in. may be experienced at a temperature of 1000°F with a 24 in. exposed length of rod under a specimen heating rate of about 5°F/sec. Rod growths of 0.030 to 0.035 in. may be experienced at a temperature of 2000°F with a 12 in. exposed length under heating rates of 6 to 8°F/sec and rod growths of 0.035 to 0.038 in. may be experienced at a temperature of 2000°F with a 4 in. exposed length under heating rates from about 7 to 17°F/sec.

A successful attachment for the quartz deflection rods consisted of an Inconel X sleeve (stainless steel is adequate for temperatures up to 1000°F) threaded into the plate and the rod attached to the sleeve through an Inconel X pin. For thin structures the sleeve may be spot welded with a flange or bottom to the structure. The hole in the quartz rod must be a few thousandths greater than the pin diameter to permit the pin to expand at temperatures without cracking the rod. Also the rod must be handled carefully especially when attached since the fused quartz material is brittle.

The growths experienced by the quartz rods appears to be primarily due to thermal expansion with very little if any creep occurring because of the extremely low tensile stresses to which the rods are subjected. When the rods are used to transmit relatively large deflections, the accuracy of such measurements will be improved by making corrections for rod growth but will depend greatly upon the accuracy of the transducer employed. The developed rod and attachment methods are capable of following both static and dynamic deflections with a frequency response equal to that of the transducers.

For the measurement of deflections at temperatures as high as 3000°F or slightly higher, a high purity recrystallized alumina rod was selected and subjected to tests to determine rod growth. Because of difficulties with attainment of the required temperature and with rod attachments only a limited amount of rod growth information for the alumina rod was obtained. Such information indicates that growth for an alumina rod with about 2-1/2 to 3 in. of exposed length at specimen heating rates of 17 to 18°F/sec will amount to about 0.065 in. at 2400°F, 0.074 in. at 2700°F and, if extrapolated linearly, from about 0.100 to 0.110 in. at 3000°F. Rod growth appears to be a function of the absorptivity and reflectivity of the structural material to which it is attached as well as to the power input of the heating lamps, especially for a rod material such as alumina which is a fairly good reflector of radiation and yet a good heat conductor. A structural material with a high absorptivity will itself heat rapidly and perhaps reflect little radiation with which to heat the rod. Under such circumstances the rod will heat slowly and the rate of rod growth will be small. Large or rapid rod growth may be expected under conditions when the structural material has a high reflectivity and more heat is thus available to heat the rod.

Attempts to provide a positive attachment for alumina rod and silicon carbide plate combinations resulted in failure. Some rod growth measurements were made with a compression spring loaded rod to keep it in contact with the plate. However, for monitoring large static deflections and dynamic displacements, the spring forces required to maintain contact may be

Contrails

excessive. Therefore the development of a positive attachment was mandatory. The substitution of an alumina plate for the silicon carbide plate permitted this development because the problem of thermal mismatch was eliminated. The developed attachment involves an alumina cup bonded to the alumina plate with a refractory alumina cement; the rod is pin connected to the cup. The attachment method proved capable of reliably maintaining the connection between rod and plate under static and dynamic displacement evaluations at temperatures approaching 3000°F.

Under the heating and testing conditions employed in this investigation it may be concluded that quartz rods may be used for accurately transmitting deflections at temperatures up to 2000°F and alumina rods may be used for accurately transmitting deflections at temperatures up to 3000°F or higher since the rod material is capable of slightly higher temperatures and since the rod does not appear to heat as readily as the structure to which it is attached.

During the course of the investigations successful means were found for protecting platinum/platinum-rhodium thermocouples from contamination and consequently permitting accurate determinations of temperatures in the region of 3000°F. This consisted of an application of Sauereisen No. 63 cement entirely around the hot junction. Another application of this cement was made when the hot junction was in place on the plate. Surrounding the hot junction and separating it from the plate by a layer of cement prevents determination of the true plate heating rate but another uninsulated thermocouple may be used for that purpose at least up to the temperature at which contamination takes place.

In conclusion it must be observed that the results of this investigation are applicable only to testing conditions identical with or closely similar to those conditions described herein. If structural testing is to be performed under different conditions, it is essential that rod growth correction factors be determined under such conditions.

Continental
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2. D. A. Ditmars and D. C. Ginnings, "Thermal Conductivity of Beryllium Oxide from 40° to 750°C", J. of Res., Nat. Bur. of Stats., 59, Aug. 1957.
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APPENDIX

NUMERICAL INTEGRATION - TRANSIENT THERMAL DEFLECTIONS

THE FOLLOWING IS THE ALGEBRAIC PROGRAM IN UNIVAC 1105 IT COMPILER LANGUAGE FOR THE TRANSIENT THERMAL DEFLECTION PROBLEM DESCRIBED BY EQUATIONS 1 THROUGH 10 IN THE MAIN BODY OF THIS REPORT. FOR CONVENIENCE THE TRANSLATION FROM COMPILER LANGUAGE TO MATHEMATICAL LANGUAGE IS LISTED BELOW.

c0	t (time)	c107	ρ	v100	T_0
c100	K_0	c117	R	v101	T_1
c101	K_1	c127	L	v102	T_2
c102	K_2	c108	L_c	...	
c103	K_3	c118	N		
c110	α_0	c128	h	v(100+N)	T_N
c111	α_1	c130	β_0		
c112	α_2	c131	β_1	...	
c113	α_3	c132	β_2		
c120	C_0	c200	q_0	v(100+N)	T_N
c121	C_1				

i21
v800
c200
s27
r10

```

READ F
i20=c118 F
i21=c108/c127 F
19 20,i1,0,1,i20, F
18 v(200+i1)=v(100+i1) F
v(300+i1)=c100+v(200+i1)*(c101+v(200+i1)*(c102+
v(200+i1)*c103)) F
v(400+i1)=c110+v(200+i1)*(c111+v(200+i1)*(c112+
v(200+i1)*c113)) F
    
```

Contrails

20	$y(500+i1)=c120 +y(200+i1)*c121$	F
	$c140=c127/c118$	F
	$y600=0$	F
	21, i1, 1, 1, i20-1,	F
21	$y(600+i1)=(y(200+i1+1)-y(200+i1-1))/2*c140$	F
	$y(600+i20)=0$	F
	22, i1, 0, 1, i20,	F
22	$y(600+i1)=y(600+i1)*y(300+i1)$	F
	$y700=(-3*y600)+(4*y601)-y602)/2*c140$	F
	23, i1, 1, 1, i20-1,	F
23	$y(700+i1)=(y(600+i1+1)-y(600+i1-1))/2*c140$	F
	$y(700+i20)=((3*y(600+i20))+(-4*y(600+i20-1)))+$ $y(600+i20-2))/2*c140$	F
	24, i1, 0, 1, i21,	F
17	$y(600+i1)=y(700+i1)+(2*y(400+i1)*c200/c117)$	F
24	$y(600+i1)=y(600+i1)/y(500+i1)*c107$	F
	25, i1, i21+1, 1, i20,	F
16	$y(600+i1)=y(700+i1)+(-c120*y(200+i1))$	F
25	$y(600+i1)=y(600+i1)/y(500+i1)*c107$	F
	a1	F RUNGE-KUTTA
15	26, i1, 0, 1, i20	F
26	$y(200+i1)=(c1130+y(100+i1)*((0.5*c131)+y(100+i1)*$ $(c132/3)))*y(100+i1)$	F
	$y200=(y200+y(200+i20))/2$	F
	27, i1, 1, 1, i20-1,	F
27	$y(200+i1)=y(200+i1)+y(200+i1-1)$	F
	$c150=y0(200+i20-1)*c140$	F DELTA
	rc0 rc150	F
	a19	FF

Contrails