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A NEW STANDARD ATMOSPHERE:
THE WADC 1952 MODEL ATMOSPHERE

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FOREWORD

This report first appeared as an introduction to a set of airspeed tables computed by Battelle Memorial Institute at Columbus, Ohio, under the sponsorship of the Instruments Branch, Equipment Laboratory (now the Aircraft Laboratory), at Wright Air Development Center (WADC). The title of the complete tables is "Tables and Data for Computing Airspeeds, Altitudes, and Mach Numbers, Based on the WADC 1952 Model Atmosphere".

The work was undertaken early in 1952. The constants used in defining the WADC 1952 Model Atmosphere were selected basically from the NACA standard atmosphere (DI 1)(1) and agreed upon by the International Civil Aviation Organization (ICAO) (IC 2). The choices, modified as required in conformity with the objectives of the work, were presented for criticism to a number of recognized authorities in the field, as well as to interested Government agencies. Dr. W. G. Brombacher of the National Bureau of Standards and Mr. L. P. Harrison of the U. S. Weather Bureau were especially helpful, and the final form of the 1952 Model Atmosphere was worked out in conformity with their suggestions. Agencies consulted included the Bureau of Aeronautics of the Navy Department, the NACA, and the Ordnance Department, Ballistics Research Laboratory, at Aberdeen, Maryland.

Recent researches (RP 1) on the characteristics of the atmosphere by use of rockets reveal substantial discrepancies between the previous proposed standard-atmosphere temperatures and those directly measured, both in the stratosphere and in the troposphere. These doubtless originate in part from the fact that the NACA standard atmosphere is designed to approximate the actual temperature variation at 45° N latitude, whereas the rocket tests have been made at about 33° N latitude. However, any standard atmosphere is an abstraction that must not be easily influenced by new information. In the course of time it may be found desirable to change the altitude tables in the direction of more complete agreement with experimental observation. Pending such an eventuality, the present tables are presented in the hope that they will serve a useful purpose for some years to come.

The present report has essentially three authors. It was written at Battelle Memorial Institute by D. T. Williams, who was the primary agent in constructing the standard atmosphere, and by J. C. Bell, who planned and coordinated the execution of the over-all project. The project was initiated and monitored by W. F. Nash of the Wright Air Development Center; he also constructed the basic outline of the tables. Also included among the authors of the complete tables is Jack Belzer of Battelle Memorial Institute, who was the primary computer.

In addition to the authors mentioned, several other people aided substantially in selecting this standard atmosphere. At Wright Air Development Center, G. H. Purcell provided useful advice. At Battelle Memorial Institute, A. J. Ness assisted in selecting the standard atmosphere, H. W. Russell contributed advice on physical constants, and H. R. Nelson supervised the work.

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(1) References may be found in the Bibliography.

ABSTRACT

A new standard atmosphere, the WADC 1952 Model Atmosphere, has been developed under the sponsorship of the Aircraft Laboratory of the Wright Air Development Center. The new atmosphere is designed to be self-consistent, and is in essential agreement with the new ICAO standard atmosphere; however, this newer atmosphere extends higher, up to 140,000 feet of altitude. Close coordination with recognized authorities in the field has been carried out in the hope that the atmosphere may receive general acceptance. Properties of the atmosphere above 82,000 feet are proposed for use pending designation of an official standard by the Committee on the Standard Atmosphere convened by the Air Force Cambridge Research Center.

Formulas for airspeed as a function of altitude and dynamic pressure, or the equivalent, are set up in standard form in this report. These formulas are the basis for an extensive set of airspeed tables which have been computed for WADC but are not included in this report.

PUBLICATION REVIEW

The publication of this report does not constitute approval by the Air Force of the findings or the conclusions contained therein. It is published only for the exchange and stimulation of ideas.

FOR THE COMMANDER:

David [illegible], Maj

for Daniel D. McKee
Colonel, USAF
Chief, Aircraft Laboratory
Directorate of Laboratories

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Introduction

Aircraft technology has shown in the last few years an advance, and rate of advance, that can be characterized as nothing short of spectacular. Since World War II, each year has seen new records of aircraft performance set, sometimes to be broken in a few short months. Jet aircraft speeds, once comfortably subsonic, now tend to crowd or surpass the transonic range; service ceilings, once well down in the troposphere, threaten to move up into the stratosphere. And rocket interceptors, in a class by themselves, are capable of speeds comparable to those of rifle bullets, with maximum altitudes of ascent limited only by the amount of fuel the craft can carry. Progress in the same direction, at a rate as spectacular, would seem likely in future years; new power plants of ever greater thrust are being built; methods of testing aircraft of extreme speeds, using computers and telemetered data, are only now being perfected to make further advances less risky and expensive.

As often happens .in technology, the rapid increase in aircraft performance limits have not always been matched by equal progress in techniques and instrumentation of flight. Flight instruments are normally modified or devised to meet the requirements of a new aircraft, and they can be built for installation in the first experimental craft to fly. However, this provision of a new or modified instrument is often far from satisfactory in meeting the new demands imposed by increase in aircraft performance capabilities.

Not least among the deficiencies revealed in connection with the delivery of new aircraft models is the lack of complete standards for the calibration of flight instruments. At the present time, an altimeter can be calibrated up to 80,000 feet of pressure altitude, by use of the familiar NACA standard-altitude tables (DI 1). However, aircraft have been revealed to fly much higher than that; if they are to be instrumented for such flight, obviously some standard for use in calibration must be supplied.

Tables for the use of manufacturers, and for use in standardizing and calibrating flight instruments, are a great convenience in the laboratory. This convenience verges on necessity when the equipment is for military use and hence must meet a standard specification. Accordingly, a radical extension of the standard-atmosphere tables and formulation of other suitable tables for convenience in calibration of flight instruments have been prepared and are presented in the following pages.

Parallel formulation of a standard atmosphere is being made by the International Civil Aviation Organization. This body has agreed upon the constants defining a standard atmosphere up to 20 kilometers. A standard atmosphere based on the constants is being computed by the National Advisory Committee for Aeronautics in their laboratory at Langley Field; it will be published for use both inside the country and around the world. The constants on which the present tables are based should agree with those of the ICAO-NACA standard atmosphere up to 20 kilometers, insofar as the requirement of self-consistency will permit. Values of sonic velocity in these tables will apparently differ from those of the ICAO-NACA; all other values will be in agreement.

The Need for Tables for Instrument Calibration

Flight instruments of interest in the present connection are the altimeter and the airspeed indicator.

The altimeter is simply an aneroid barometer calibrated directly in feet of altitude and with a convenient zero adjustment. This adjustment is used either for correcting for barometric variation due to the weather, or on occasion for making the altitude readings correspond to altitude above a particular landing field rather than above sea level. The calibration consists of placing the instrument in a chamber whose pressure can be set precisely to any desired value, corresponding to an assigned pressure altitude. The reading of the instrument, properly zeroed, must then be the number of feet or meters corresponding to the standard altitude, as given in a table showing altitude as a function of static pressure, within specified tolerances.

The airspeed indicator is likewise based on an air-pressure measuring device. In its simplest form, the pressure gauge is a capsule similar in shape to an aneroid capsule, however, the pressure measured is the dynamic pressure difference as taken from the total and static pressure taps on a pitot-static tube. Obviously, the instrument must be quite sensitive, in view of the small pressure differentials.

The scale of the airspeed indicator shows the indicated airspeed, which is the same as the calibrated airspeed except for irregularities of the instrument and its installation. Originally, this quantity was sufficient as a flight parameter, for the calibrated airspeed will tell when an aircraft is about to stall. By definition(1), the calibrated airspeed is equal to the true airspeed only under standard sea-level conditions, but stalling occurs at a specific calibrated airspeed rather than true airspeed at all altitudes. Therefore, indicated or calibrated airspeed provides a better indication of the approach to stall than does true airspeed.

Modern aircraft ought still to have some stall indication on the instrument panel; however, other dangerous speeds are known. Thus transonic aircraft may tend to go into a buffeting regime when they fly near the speed of sound. Furthermore, in certain jet aircraft, flight at maximum aerodynamic efficiency can be most easily controlled by flying at a given Mach number. Accordingly new aircraft require an improved airspeed indicator that reads both the calibrated airspeed and the Mach number.

A Mach meter uses a differential-air-pressure capsule and with it an aneroid capsule detecting static pressure. The movements are combined by a system of levers, and the scale is calibrated to give a Mach-number indication at all altitudes.

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(1) See table of definitions.

Indication of another flight parameter called "equivalent airspeed" and closely related to true airspeed, has been found necessary as a "safety indication" in some new aircraft designed for transonic and supersonic speeds. These new craft are built for transition through Mach 1 and either do not buffet or else can withstand the stresses. However, at some high equivalent airspeed the normal aerodynamic forces become too high for the aircraft structure. Equivalent airspeed is related to the true airspeed by the equation

$$V_{\text{equiv.}} = V_{\text{true}} \sqrt{\frac{\rho}{\rho_o}}$$

where ρ is ambient air density, and the subscript o indicates sea-level conditions. An indication of at least the limiting value of equivalent airspeed would be desirable in certain aircraft; the airspeed indicator would require some modification for this indication.

For purposes of navigation by dead reckoning, the true airspeed may be used together with a correction for wind drift. If the true airspeed is to be measured by an airspeed-indicator-type instrument, three data are necessary: dynamic pressure, static pressure, and air temperature. In present instruments, the total temperature may be used as measured by a special thermocouple mounted in an appropriate stagnation region. Modifications of this arrangement are conceivable, and would be required whenever the manner of temperature measurement is changed.

It is now proposed to discuss how the tables of standard atmosphere and airspeed are computed in practice, how the parameters are chosen, and any justification as to assumptions that may appear to be called for.

The Concept of a Standard Atmosphere in Relation to the Earth's Atmosphere

The practical method of use of a standard-atmosphere table indicates in a very specific way just what it is: an arbitrary relation between static air pressure and altitude. This pressure-altitude relation ought to be sufficiently accurate that an aviator can trust his altimeter in flying over mountains of known height. Beyond that requirement, however, accuracy as relating to altitude as a function of pressure is not of overriding importance at present.

When this has been said, it is proper to add that the standard atmosphere should correspond, if convenient, to reality. Some considerable agreement between the atmosphere and any standard is indeed to be expected. The atmosphere is an ocean of air, free to flow wherever gravitation causes it to flow, subject only to the universal laws of nature. Accordingly, one might expect, in analogy with common engineering experience with other fluids, that static pressure at all points at the same altitude above some datum plane would be the same. At any rate, if small pressure differences exist at a given altitude, it is certain that the wind must blow in such fashion as to tend to remove them.

That the wind does blow is evidence of deviations from true equilibrium. As a matter of fact, the changeable nature of weather, and the unpredictability, in the larger sense, of the winds, is proverbial. Most of the changes of air pressure with time and place at a given altitude are due to uneven heating of the earth by the sun. Variations in temperature, and hence in air pressure, are therefore expected to depend on whether the land is bare or covered with water, snow, or vegetation; the latitude; the presence of mountains; variations in gravitational acceleration; and clouds. Each condition has its effect. Accordingly, it would appear that a standard atmosphere that everywhere closely approximates reality might tend to be unpleasantly complicated to compute and use.

This type of dilemma is so common in science--the complexity of nature superposed on the fundamental simplicity of natural law--that the solution is altogether expected. The standard atmosphere is an abstraction, constructed on the basis of a minimum number of simple assumptions, resembling reality as closely as convenience permits. It is now proposed to discuss the earth's atmosphere in order to understand what assumptions may be safely made and which ones may lead to serious error.

General Characteristics of the Earth's Atmosphere

The earth is a flattened sphere spinning around on its axis of symmetry and heated on one side by the sun. The amount of heat which the earth absorbs is greater in the regions where the sun's rays are more nearly vertical; consequently, the earth's surface temperature is high near the equator and low at the poles.

The air which covers the earth is nearly transparent to the sun's radiation, but it is heated by contact with the earth's surface. Consequently, air is heated in the warmer equatorial regions. It rises and carries heat in convection currents from the tropics toward the poles at high levels, and cold air flows from the poles toward the equator at low altitudes.

This convection pattern is not by any means a simple one. In the first place, air may flow down a pressure gradient in equatorial regions, but the rotation of the earth causes it to flow, instead, more nearly at right angles to the pressure gradient in the temperate regions. As a result, the transfer of cold air southward from the poles takes place, not by simple and direct winds, but instead through the relatively slow southward drift of large air masses, each surrounded by its whirlpool of winds.

These air masses bring the weather and influence the succession of temperature changes that compose a climate. Superposed on the zonal drift of air masses, and often masking their effects, are local high- and low-pressure areas. These may be caused by the unequal heating of the continents and the oceans and by temperature changes, as between day and night. The resulting winds may be deflected by mountains; they carry moist air from oceans to land, bring clouds and rain, and, in general, are part of a large-scale turbulence that is much studied and little understood.

Obviously, diurnal and short term variations in air pressure and temperature must be neglected in a standard altitude, since permanence is the principal virtue of the concept. Furthermore, seasonal variations are neglected.

In general, the air in the lower atmosphere tends to decrease in temperature with increase in altitude, the rate of change being the "lapse rate". At a level called the "tropopause", the lapse rate becomes zero; the layer above the tropopause is the "stratosphere". At a certain level called the "stratopause", the temperature is found to rise with altitude; the region above this level is the "chemosphere", which ends eventually at the "chemopause". Our interest does not, in these tables, go beyond the chemopause.

General Characteristics of the WADC 1952 Model Atmosphere

A number of specific assumptions must be made in constructing the WADC 1952 Model Atmosphere. It is proposed now to discuss some of these in a qualitative way before deriving in more rigorous fashion the basis for the tables. The assumptions made here fix the form of the equations used in deriving the model atmosphere; the values of the parameters used are fixed in a second group of assumptions considered later.

Assumption 1: the atmosphere is a gas, chemically uniform everywhere.

The ceaseless motion of the winds would appear to insure that the air is always well mixed; consequently the assumption that the air is a uniform gas is reasonable. Specifically, the gas constant R is assumed to be the same everywhere. In more detailed fact, of course, air has a highly variable content of water vapor. This impurity is neglected for no reason except that it is convenient to do so. Assumptions involving the presence of water vapor would require some allowance for precipitation, with a train of complicating changes in the properties of the air.

The air also contains small and variable amounts of carbon dioxide and ozone. Both of these gases, as well as water vapor, absorb heat directly from the sun. This has a profound influence on the distribution of temperature in the atmosphere; nevertheless, the quantities of gas are so small that their presence is neglected as far as the densities and gas constant of the air is concerned.

At higher levels in the atmosphere, the molecular oxygen in the air is completely dissociated into atomic oxygen. As a matter of fact, this atomic oxygen forms ozone in reaction with oxygen molecules within regions of present interest. The presence of oxygen atoms is, nevertheless, disregarded up to the level chosen as the limit of this table.

Assumption 2. the atmosphere has the same characteristics at all latitudes.

This assumption refers specifically to the distribution of air temperature with altitude.

The lapse rate at lower levels might be considered reasonably to be the same at all latitudes, on the assumption that this decrease is due to the thermodynamic properties of air. This assumption may be tolerable, but the altitude of the tropopause is known to be substantially lower at the poles than at the equator--25,000 feet and 60,000 feet, respectively--with stratosphere temperatures correspondingly different. If this variation were to be incorporated into the standard atmosphere, the result would be effectively a different standard for each latitude. In preference to this inconvenient choice, the standard is instead fixed to be reasonably correct at some convenient latitude, and the actual deviations from the standard at other latitudes are disregarded. The latitude that has usually been chosen to fix the standard has been in the past 45° North, which corresponds roughly to the latitude of the larger laboratories in our country. Since in more recent years the center of upper atmosphere experimental research has shifted to latitudes of 33° N or thereabout, it would appear that the standard atmosphere of this and future tables may well be a hybrid. The trends at lower levels will be taken from observations at 45° N and those at higher altitudes will agree with observations at 33° N. The only justification for such a procedure is that data are at present not complete for more than a very few latitudes, and any data, no matter how incomplete, are preferable to none at all.

When all is said, the fact remains that pressure as a function of altitude must inevitably be more or less the same over the whole earth in view of the fluid nature of the atmosphere.

Assumption 3: the air temperature varies only with altitude and then in a simple manner.

This assumption is one made obviously for convenience. Specifically, the temperature in the troposphere is assumed to drop linearly with altitude. There is a simple theory that would predict this type of variation, based on the assumption of adiabatic equilibrium of air for change in altitude. This theory is not quantitatively adequate at any altitude, and at higher levels the theory is qualitatively wrong.

At a level where the assumed linear decrease in temperature ceases, the temperature changes in a manner that is quite impossible to describe by any simple theory thus far proposed. Accordingly, from the tropopause upward, the assumed variation of the temperature with the standard altitude is arbitrarily chosen as a series of straight lines approximating the observed trends in fairly rough fashion.

Assumption 4: the gravitational acceleration, g, of the earth is a constant independent of latitude and altitude.

This assumption is in error, as to the latitude variation, by as much as 5 parts in a thousand; in respect to altitude, the error is as much as ten parts per thousand. The error in altitude at given air pressure can be rationalized by giving a special name to the units of length used in measuring altitude when g is assumed constant. Such units as the "geopotential meter", etc., would be unusual units in that they have a length that varies with altitude. In practice the assumption g constant is conservative in the sense that the altimeter reading is always low, as far as the effect of the constant g assumption is concerned. As a result, the pilot that flies high enough by altimeter to clear a given mountain whose altitude is recorded on a map is likely to be on the safe side, for his height will always be a little greater than his altimeter reading.

The assumption g constant is therefore tolerable in practice. In computing the standard altitude, the assumption g constant results in much greater elegance and simplicity than a more accurate but complicated premise.

Other assumptions, less general, have been made for convenience in the rigorous definition of the WADC 1952 Model Atmosphere. These will be mentioned and justified later in the detailed discussion.

Procedures Used in the WADC 1952 Model Atmosphere

In view of the rather significant uncertainties in the actual physical properties of the atmosphere, even under the most ideal imaginable conditions, the fixing of a standard becomes a task of negotiation rather than determination in the usual experimental sense. That is, the standard is chosen to be acceptable to as large a number of interested agencies as possible, regardless of detailed deviation of altitudes at given pressures from the latest experimental results.

Accordingly, the procedure that was followed may be outlined in the following steps.

(a) A rigorous analytical procedure was set up by means of which the model atmosphere could be defined on the basis of a minimum number of assumed constants. Actually such a procedure has been used in the past in defining other standard atmospheres. At times in the past, errors in consistency have been made inadvertently, or because more constants were defined than are rigorously required. This error has been avoided here. Insofar as possible, however, the previously used procedures have been adopted.

(b) The constants required in fixing the model atmosphere were chosen to conform with the most widely acceptable relation between altitude and air temperature. These constants were taken from the older NACA standard atmosphere and other sources of more recent date. Especially useful were WA 1 and IC 2. In these references the constants have in general followed practice in a large number of sciences, so that the choice of constants is likely to be quite generally acceptable.

Where new information has tended to disagree with previous assumptions, or where exact duplication of previous assumptions is impossible or inconvenient, some small departure from previous recommendations has been made in the fundamental assumptions. In particular, the report of the Rocket Panel (RP 1) contains some marked divergences from WA 1, which modified the choice of constants.

(c) The procedure and table of fundamental constants were submitted to the agencies most likely to have an interest in the new tables. The particular agencies, and the scientists who have ably assisted the work are listed in the Foreword.

It is now proposed to define the WADC 1952 Model Atmosphere within the framework of the general assumptions reviewed above. In this derivation, a minimum number of constants are chosen so as to insure complete self-consistency.

In such tables as here presented, one is dealing not with observable quantities *per se*, but instead with mathematical relations between variables. The basic constants are numbers, chosen for convenience or for other reasons, but not at all subject to experimental error. Accordingly, the number of significant figures used in the tables need not be limited by any experimental precision. The choice of six or seven significant figures is therefore logically sound; any errors found are strictly due to rounding off, and hence are limited to one or two units in the last place. Such detailed precision requires corresponding precision, of course, in the basic constants.

To proceed with the derivation, the following assumptions are made:

(a) Assumptions as to air temperature:

(1) ***At sea level***, the air temperature is T_o, an arbitrary constant value

(2) ***In the troposphere*** with constant lapse rate a the temperature T falls linearly with altitude Z. That is, in the troposphere, for $0<Z<Z_a$,

$$T = T_o - aZ, \tag{1a}$$

where Z is the altitude above sea level, and Z_a is the level of the tropopause.

(3) ***In the stratosphere*** the temperature is constant at T_a. That is, for $Z_a<Z<Z_b$,

$$T = T_a. \tag{1b}$$

The level Z_b is the stratopause.

(4) ***In the chemosphere***, the temperature rises with constant b. That is, for $Z_b<Z$,

$$T = T_a + bZ. \tag{1c}$$

The tables terminate at the general level of the "chemopause", an altitude which is not specified in this work.

(b) Assumptions of static equilibrium: For this assumed state of affairs, there will be a difference in pressure dP between two levels dZ apart just equal to the weight of a column of air having unit cross section and height dZ. If pressure is measured in force units, this assumption is expressed

$$-dP = \rho g_s dZ, \tag{2}$$

where ρ is the air density, and g_s is the acceleration due to the earth's gravitation, assumed constant as previously discussed. Altitude Z is measured upward from sea level as the datum plane.

(c) Assumption as to properties of the air: Air is assumed to be dry, and to act as a perfect gas with constant R, based on unit mass of air. Specifically, the pressure P is related to the air density ρ and absolute temperature T by the gas law

$$P = R\rho T. \tag{3}$$

The density ρ can be eliminated from Equations (2) and (3) to yield the general differential equation of the atmosphere:

$$-dP = g_s \frac{P}{RT(Z)} \; dZ$$

or

$$-\frac{dP}{P} = \frac{g_s}{R} \frac{dZ}{T(Z)} \; . \tag{2a}$$

Here, T(Z) is temperature expressed as a function of altitude Z. The solution of this equation is

$$\ln P = -\frac{g_s}{R} \int \frac{dZ}{T(Z)} + \text{constant}. \tag{2b}$$

There will be a different function T(Z) within each region of the atmosphere, corresponding to the assumed temperature variations. The integrals are in all cases readily found:

(a) In the troposphere $T = T_o - aZ$,

$$\ln P = \frac{g_s}{aR} \ln (T_o - aZ) + \text{constant}.$$

If the pressure is P_o at altitude $Z = 0$, the constant can be determined and the result may be expressed

$$P = P_o \left[1 - \frac{aZ}{T_o} \right]^{\frac{g_s}{aR}} = P_o \left[1 - \frac{aZ}{T_o} \right]^{n} . \tag{4}$$

Here, $n \equiv g_s/(aR)$.

This equation is used to establish the pressure P_a at the tropopause by use of Equation (4) and the altitude Z_a of the top of the troposphere:

$$P_a = P_o \left[1 - \frac{aZ_a}{T_o} \right]^{\frac{g_s}{aR}} . \tag{4a}$$

(b) In the stratosphere, the same differential equation leads to the expression

$$\ln P = - \frac{g_s}{RT_a} Z + \text{constant.}$$

At $Z = Z_a$, the tropopause, the pressure must be equal to P_a as given in Equation (4a). Accordingly the constant can be found to yield the equation for pressure as a function of the altitude Z in the stratosphere. This may be written:

$$P = P_a e^{- \frac{g_s}{RT_a} (Z - Z_a)} \tag{5}$$

The pressure P_b at the stratopause can be fixed by Equation (5) when Z is set arbitrarily at the altitude Z_b. That is,

$$P_b = P_a e^{- \frac{g_s}{RT_a} (Z_b - Z_a)} \tag{6}$$

(c) In the chemosphere the altitude equation is similar to that in the troposphere. That is,

$$\ln P = - \frac{g_s}{R} \int \frac{dZ}{T_a + bZ}$$

$$= - \frac{g_s}{Rb} \ln (T_a + bZ) + \text{constant.}$$

If the constant is fixed by setting $P = P_b$ at $Z = Z_b$ as before, the result may be written

$$P = P_b \left[\frac{T_a}{T_a + b(Z - Z_b)} \right]^{\frac{g_s}{bR}} \equiv P_b \left[\frac{T_a}{T_a + b(Z - Z_b)} \right]^{k} \tag{7}$$

Again, $k \equiv g_s/(bR)$.

The three equations of pressure as a function of altitude, Equations (4), (5), and (7), are valid in the troposphere, the stratosphere, and the chemosphere, respectively; they have been derived in order to clearly identify the number of constants required to make a rigidly self-consistent standard atmosphere. There are, in all, eleven constants. Of these, eight are independent, and are fixed as later to be discussed in detail: standard sea-level pressure and temperature P_o and T_o, standard gravitational acceleration g_s, gas constant R, temperature gradients a and b, and temperature of stratosphere and altitude of stratopause, T_a and Z_b, respectively. The altitude of the tropopause, Z_a, is fixed by Equation (1a) and the chosen value of T_a. The pressures P_a and P_b at tropopause and stratopause are fixed by Equations (4a) and (6) respectively.

When the eight arbitrary constants are chosen, they serve to define, with Equations (4), (5), and (7), a truly self-consistent atmosphere. Alternative constants might be chosen, but no more than eight can be fixed in the framework reviewed without jeopardizing self-consistency. The precision of the relation between pressure and altitude can be just as great as is desired, limited only by the precision of the constants chosen and the reliability of the computing machine.

It is clear that some quantities such as P_o, T_o, Z_b, b and g_s actually vary so much in nature that no "standard" value exists except by fairly arbitrary definition. On the other hand, the gas constant for air, R, the relation between Celsius and Kelvin temperature scales, or between the Fahrenheit and Rankine scales, the relation between different units used in measuring pressure, etc., are numbers widely used in technology. Their values cannot be established or changed without introducing possible inconsistencies with practice in other sciences.

In this present work, it is necessary to choose between two horns of a dilemma. On the one hand, the constants required for defining the standard atmosphere may be fixed by arbitrary choice to a convenient high exactness. In this case, whenever a number is changed as a result of research in another science, the tables loses its consistency with that science. On the other hand, the constants might be chosen with exactness no greater than that dictated by the precision with which the least precise has been measured experimentally. Thus, the relation between Celsius (centigrade) and Kelvin temperature is known only to five significant figures. If such a choice were made, the standard atmosphere would be defined only to a limited precision somewhat less than is now desired.

The choice made in this work is the former; that is, constants are chosen as nearly accurate as possible and further fixed by arbitrary choice to yield a standard atmosphere of high exactness. Pressures and altitudes required for the calculations are listed to eight figures, since six figures are required in many tables, while one figure is often lost in calculations and another should be allowed for rounding off. None of these figures is in doubt by any amount other than the uncertainty incident to rounding off in the calculation. At the same time, the constants are in agreement as far as possible with values chosen for other scientific purposes.

Airspeed as a Function of Dynamic Pressure and Stagnation Temperature

The equations used in calculating the velocity in terms of dynamic pressure are derived directly from the thermodynamic equations for isentropic changes in flowing air. Thus, air slowed from a subsonic speed V_t and Mach number M to rest, rises in pressure from P to P + ΔP. These quantities are related by the equation

$$\frac{\Delta P}{P} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} - 1 \tag{8}$$

for subsonic flow. The constant γ is the ratio of specific heats for air. If velocity V_t is desired, it may be computed from the Mach number M by the relation

$$V_t = MC_s. \tag{9}$$

Here, C_s is the velocity of sound given in terms of ambient temperature by

$$C_s^2 = \gamma RT. \tag{10}$$

It is clear that C_s the sonic speed, is fixed by R, the gas constant of the air--always assumed that of the Model Atmosphere--and the ambient temperature. This temperature T cannot be assumed in general equal to that specified by the standard-atmosphere-table at given altitude without an error that is unreasonably large. This matter is discussed further below.

The calibrated airspeed V_c is defined in the subsonic range by the equation

$$\Delta P \equiv \left(1 + FV_c^2\right)^{\frac{\gamma}{\gamma-1}} - 1 \tag{8a}$$

for values of V_c less than C_{so}, the sonic velocity at standard sea level, where

$$F \equiv \frac{\gamma-1}{2} \frac{1}{\gamma RT_o}$$

Here, P_o and T_o are pressure and temperature at standard sea level, respectively.

For supersonic speeds, the relation between dynamic pressure ΔP and Mach number is derived by analysis of the aerothermodynamic processes that occur in a normal shock, as well as of the isentropic compression of the air as it is brought to rest behind the shock in a pitot tube. This relation will not be derived here; it is presented in Liepmann and Puckett *Aerodynamics of a Compressible Fluid* (John Wiley and Sons, Inc., New York, 1947). It is

$$\frac{\Delta P}{P} = \frac{(\gamma+1)^{\frac{\gamma+1}{\gamma-1}}}{(\gamma-1)^{\frac{1}{\gamma-1}}\,2^{\frac{\gamma}{\gamma-1}}} \left\{ \frac{M^{\frac{2\gamma}{\gamma-1}}}{\left(\frac{2\gamma}{\gamma-1}M^{2}-1\right)^{\frac{1}{\gamma-1}}} \right\} - 1 \,. \tag{11}$$

The calibrated airspeed in the supersonic range is given, in analogy with Equation (8a) by the relation

$$\frac{\Delta P}{P_o} = \frac{(\gamma+1)^{\frac{\gamma+1}{\gamma-1}}}{(\gamma-1)^{\frac{1}{\gamma-1}}\,2^{\frac{\gamma}{\gamma-1}}} \left\{ \frac{\left(\frac{V_c}{C_{so}}\right)^{\frac{2\gamma}{\gamma-1}}}{\left[\frac{2\gamma}{\gamma-1}\left(\frac{V_c{}^{2}}{C_{so}}\right)-1\right]^{\frac{1}{\gamma-1}}} \right\} - 1 \,. \tag{11a}$$

As before, C_{so} and P_o are sonic speed and pressure, respectively, at standard sea level.

According to Equations (8) and (11), the ratio of dynamic to static pressure fixes the Mach number when γ is known. In case the true airspeed is to be computed, the Mach number, as determined by the observation of dynamic and static pressures, will yield the true airspeed if only the velocity of sound in the ambient air is known. This is presumed known, in turn, if only the static temperature is known, for the sonic speed is fixed by the assumed composition of the standard atmosphere and the temperature.

Determination of Ambient Temperature

In general practice, the ambient temperature cannot be directly observed. Instead, a temperature is measured at some stagnation region which will lead in principle to the static temperature. Specifically, the so-called indicated temperature is related to the Mach number for both subsonic and supersonic airspeeds by the equation

$$\frac{T_i}{T} = 1 + \frac{\gamma-1}{2} KM^{2}. \tag{12}$$

Here, T_i is the indicated temperature, T is the ambient static temperature, M is Mach number, and K is the so-called recovery factor, which has a value lying somewhere between 0 and 1. With thermocouples located with flush temperature elements in the free air stream, K is often nearly equal to 0.8. The ambient temperature is then given by the equation

$$T = \frac{T_i}{1 + \frac{\gamma-1}{2} KM^{2}} \quad , \tag{13}$$

both T and T_i being measured on an absolute temperature scale.

Maximum and Minimum Temperature

In the tables showing true airspeeds as a function of Mach number at various altitudes, the temperature of the air is used as a parameter essential to the use of the table. In general, this temperature will not be equal to the standard temperature at a given pressure altitude, so a range of possible temperatures must be provided in the table. This range should include all values likely to be observed in practice, and hence should be bounded by some reasonable estimate of maximum and minimum temperatures.

Obviously, the making of an estimate of maximum and minimum temperatures is a highly arbitrary procedure at best. The data on which the estimate is based cannot possibly be complete in any absolute sense, so that any figure chosen is only a guess. The choice for these tables is defined in detail in a later paragraph.

Values of Parameters

In the following pages, a detailed presentation is made of the values of constants recommended for use in calculating the WADC 1952 Model Atmosphere. The reasons for making the choice are indicated, and a complete table is included with all numerical values required for computation.

The fundamental values chosen are in agreement with those suggested by the International Civil Aviation Organization and the National Advisory Committee for Aeronautics. Wherever possible, the chosen values also agree with the values used in industrial practice.

This part of the report is divided into four sections. Section 1 is concerned with the constants which define the standard atmosphere. The limits of application of these arbitrary values are given.

Section 2 deals with physical constants necessary for calculation of the data. These constants were checked with the National Bureau of Standards for the most recently accepted values. The Bureau also confirmed the conversion values listed in Section 3.

For convenience in computation, it will be assumed that the basic values given in the first three sections are exact.

Section 4 presents a step-by-step computation of derived constants and equivalent values.

Section 1. The WADC 1952 Model

1.1 Gas constant for standard air, R.

$$R = 2.8704 \times 10^2 \ m^2/sec^2K \ (exact)$$

The value of R is obtained from a recent choice of the universal gas constant, R^*, and the arbitrary value of the gram molecular weight of standard air as accepted by ICAO and the International Meteorological Organization.

The Subcommittee on Fundamental Constants of the Committee on Physical Chemistry of National Research Council (RO 2) has selected the value $R^* = 8.31439 \times 10^7$ erg/mole degree (chemical scale). This value has been chosen for the present calculation.

Standard air is assumed to be a perfect dry gas with a gram molecular weight M_{air} = 28.966 g/mole. This value was determined by Paneth from a direct chemical investigation (PA 1). This assumed composition deviates from that of the real atmosphere, of course, as the amount of moisture in the air is too unpredictable to be considered. Hence, the ratio of R^* to M_{air} yields the above value of R to five significant digits. For the purpose of the present computation, this value of R is assumed to be exact.

The composition of air below 80 kilometers in the daytime and 105 kilometers at night is constant (PA 1). Since our concern is limited to altitudes below 140,000 feet or 42.672 kilometers, the composition of the standard atmosphere is assumed independent of altitude. This assumption disregards any gravitational separation of light and heavy gases, such as is claimed by Paneth to begin between 20 and 30 kilometers (PA 1).

1.2 Temperature at sea level, T_o, is defined to be 15 C (exact) (59 F) (DI 1, IC 2). This is the commonly accepted value.

1.3 Pressure at sea level, P_o, is defined to be 1,013,250 dynes/cm^2 (exact) in accordance with the recommendation of the Subcommittee on Fundamental Constants of the Committee on Physical Chemistry of the National Research Council (RO 2). In addition, they have chosen to define the standard millimeter-of-mercury pressure as 1/760 (exact) of standard sea-level pressure. In this definition of 1 mm Hg pressure we concur. It is interesting to note that this definition, combined with a choice of the value for the acceleration due to gravity at sea level, is, in effect, the definition of a standard density for "mercury", or, more precisely, the specification of the density of a standard fluid for pressure measurements.

1.4 Standard gravitational acceleration, g_s, is defined to be 980.665 cm/sec^2 (exact) (GU 1, DI 1, ST 1). It was adopted by the International Committee on Weights and Measures in 1901, and corresponds nearly to g as measured at latitude 45° and sea level. It is assumed, herein, that g_s is constant with altitude. This assumption of a constant g_s leads to reasonably large discrepancies between true and pressure altitudes. Such errors are strictly of no importance as long as the conventional altimeter, using a measurement of static pressure, is the only altimeter used. Normally, altitudes of aircraft are given in feet only as a means of visualizing a measurement that is actually a pressure measurement. If, in the future, a new and independent means of altitude measurement becomes widely used, tables such as the ones here will be found in error. Their use will also be somewhat reduced. In any case, until such new means of altitude measurement are common, the use of a g_s constant with altitude is considered to have an important advantage in simplicity. This same assumption has been made in other standard atmospheres.

1.5 The temperature-lapse rate, a, from sea level to the tropopause is assumed to be constant at 0.0065 C/m (exact) (IC 2). This is an arbitrary assumption that has been used for many years. It corresponds reasonably well with direct observation at 45° North latitude, as indicated in a reference quoted in (IC 2).

1.6 Constant stratosphere temperature, T_a, extending from the calculated tropopause altitude up to 32 kilometers is defined as -56.5 C (exact). This value was set up by the International Congress on Air Navigation in 1924, and was accepted by the participating countries except the United States. The present successor of the International Congress is the International Civil Aviation Organization; it has accepted the same value for stratosphere temperature. The NACA participated in this choice through a consultant L. P. Harrison, of the Scientific Services of the U. S. Weather Bureau. The official U. S. representative is Rear Admiral Paul A. Smith.

1.7 The altitude of the top of the stratosphere, Z_b, is chosen to be 32,000 meters (exact). This value is the one proposed by Warfield (WA 1); it is accepted by us in view of the fact that no alternative proposal has been found.

1.8 Temperature-rise rate in the chemosphere is b = 0.0074 degrees Celsius per meter (exact). This value is nearly, but not exactly, equal to the value proposed by Warfield, which turns out to be 0.007333. The recommendation here has been made as more convenient than Warfield's, without introducing any significant difference in his proposed chemosphere. Such changes as would result involve small alterations in the proposed height of his second isothermal region, or its proposed temperature, neither of which value has interest in the present connection. In any case, Warfield's proposals differ slightly from our recommendations in a number of points which cannot be eliminated by any reasonable adjustment of our values.

1.9 Maximum and minimum temperatures at altitude. The standard atmosphere is based on a variation of temperature with altitude, chosen in an arbitrary and convenient fashion, but designed to conform roughly to mean values at 45° North latitude. The maximum and minimum temperatures, T_{max} and T_{min}, chosen here are the same as those proposed by Warfield (WA 1). They fall on a series of straight lines in the same general fashion as the standard temperature, but with changes in slope so chosen that the standard temperature lies approximately midway between the two limits at all altitudes. This means that the altitudes of slope change of T_{max} and T_{min} are not the same as the altitudes of slope change in the model-temperature variation with altitude.

The values chosen are given in a series of linear equations for T_{min} and T_{max}, respectively, as functions of altitude Z. These equations will be shown for T in degrees Kelvin and Z in kilometers.

Minimum temperatures, T_{min}

$$T_{min} = 225 - \frac{45}{17} Z, \text{ if } 0<Z<17 \text{ km}$$

$$= 180 + \frac{4}{7} (Z-17), \text{ if } 17 \text{ km}<Z< \text{limit of tables}$$

Maximum temperatures, T_{max}

$$T_{max} = 320 - \frac{70}{11} Z, \text{ if } 0<Z<11 \text{ km}$$

$$= 250 + \frac{5}{14}(Z-11), \text{ if } 11 \text{ km}<Z<25 \text{ km}$$

$$= 255 + \frac{25}{4} (Z-25), \text{ if } 25 \text{ km}<Z<\text{limit of tables}$$

It will be evident to the reader that the range of temperatures chosen is less than have been reported from observations at weather stations on the ground. It is believed that the choice made is adequate in view of the fact that temperatures at ground level show wider fluctuation than those in the free air, because of the effect of heat gain or loss by radiation.

A recent paper by Cox (CO 1) points out that the temperatures in the upper atmosphere show a different trend than that proposed by Warfield. For our work, this would involve a somewhat reduced increase in temperature above 42 kilometers. However, the data are not very complete, and the proposal of Warfield is accepted as reasonable within the altitudes of this table, that is, up to 42.672 kilometers.

Section 2. Physical Constants

The following physical constants necessary for calculation of the tabulated constants have been verified by the National Bureau of Standards as agreed upon by international concurrence. Their values will be assumed for computational convenience to be exact as written. Particularly, these values are the ones which were agreed upon at the Ninth General Conference on Weights and Measures, Paris, 1948. Supporting references are also given.

2.1 Absolute temperature at the melting point of ice under a pressure of one atmosphere, T_b.

$$T_b = 273.16 \text{ K (exact)}.$$

The history of the value of T_b is summarized in Reference (ST 1). The value 273.15 K ± .02 for the temperature of the ice point was originally proposed by the Advisory Committee on Thermometry in 1939, and, in 1948, the Committee did not consider that enough new knowledge had been gained to change this value. However, some uncertainty was felt about this action, as indicated by a clause in Resolution 2 of the Ninth General Conference:

> "The Advisory Committee recognizes the principle of an absolute thermodynamic scale requiring only one fixed point which would now be the triple point of pure water for which the absolute temperature will be chosen later. In the USA, the value 273.16 K for the temperature of the ice point has been so generally accepted that its use prevails and will continue to be used until a more accurate one has been agreed upon."

An advisory committee met in June, 1952, in Paris to discuss the value of T_b. The official meeting in 1953 may agree to a value of T_b = 273.15 K instead of 273.16 K. We retain the value 273.16 K as the present standard.

2.2 Density of water at 25 C, ρ_{H_2O}.

$$\rho_{H_2O} = 0.9970751 \text{ g/ml (exact).}$$

The specific volume of water at 4 C is 1.000000 ml/g (= 1.000028 cm^3/g) when under a pressure of one standard atmosphere. Various values of the density at 25 C are given in the following tabulation:

Reference	Density, g/ml at 25 C
TI 1	0.9970751
CH 2	0.9970770
TH 1	0.9970708
IC 2	0.9970739 [the average of References TH 1 and CH 2]

Until the discovery of isotypes of hydrogen and oxygen, water as commonly purified by careful distillation was regarded as a perfectly definite, homogeneous substance, the same the world over. Since the discovery of isotopes, we now know that water is a mixture in which the relative amounts of the several constituents vary with the source and with the manner of purification. In those cases in which extreme precision of measurement has been obtained -- in which errors from other sources do not exceed one part in a million or thereabouts -- it is necessary to consider whether differences in the composition of different specimens of "pure water" may cause significant differences in the property being studied.

Over the range 0 to 40 C, values of the density of water are published to one part in ten million. But there are as yet no data that enable one to say with certainty whether the density of "pure water" commonly used in such work is definite to that precision.

If a sample of water contained one part D_2 to 6500 parts H_2 (the average ratio), then removing the D_2 would decrease its density by about 17 parts in a million. In the fractional distillation of tap water, the first and last fractions have differed as much as 20.0 parts in a million.

The value chosen here for the density of water at 25 C, 0.9970751 g/ml is the one published by Tilton (TI 1). Using an equation first suggested by Thiesen, he has recomputed the values of Chappuis (CH 1). This value is far more reliable than the mean value given in the International Critical Tables, 1928, although it differs from it only by one part in a million.

2.3 Ratio of specific heats of air, γ,

$$\gamma = 1.4 \text{ (exact)}.$$

The universally accepted value of $\gamma = 1.4$ is in rather good agreement with experiment for adiabatic changes at low temperatures and pressures. It is furthermore required by the kinetic theory for diatomic gases. Under actual conditions of high-speed flight, and because of the presence of CO_2 and the noble gases in air, the figure $\gamma = 1.4$ is probably not ever exactly right. On the other hand, there is no agreement on what value it should have, or even whether a value of γ exists that is independent of the size of the pitot tube used for measuring high airspeeds. Furthermore, values of γ different from 1.4 would introduce a great deal of complexity in the calculations of airspeed. Accordingly, the value of $\gamma = 1.4$ exactly is used in agreement with common practice.

Section 3. Conversion Units

3.1 Length.

1 inch = 25.4 millimeters (exact).

In 1933, a relation between the yard and meter was adopted by the American Standards Association (ASA B 48.1 - 1933) and by similar organizations in 15 other countries and has been widely accepted in industrial practice. This relation, used in these tables, is

1 inch = 25.4 millimeters (exact)

1 foot = 0.3048 meter (exact)

1 centimeter = 0.393700787 inch.

The adoption of this relation by industry, for use in making conversions between inches and millimeters, did not change the official definition of the yard or meter as enacted by the U. S. Congress, July 28, 1866, which legally fixed the conversion of length to

1 centimeter = 0.3937 inch (exact)

1 foot = 0.3048006096 meter

1 inch = 2.5400508 centimeters.

It is clear that there is a disagreement between the American official value and the ASA value. In discussions concerning this disagreement, the point has been made that any change in the official value will be repugnant to map makers, even though in engineering practice the change has already been accepted.

One purpose of the projected tables is to determine distance traveled on the basis of a pitot-tube reading. The use made here involves, in essence, a small change in map distances as measured in miles on most present American charts. Such a change is small--of the order of a part per million. It is already somewhat overdue and may be extended in the course of time from its present region of acceptance to include geographical distances.

3.2 Mass.

Since 1893, the avoirdupois pound has been defined in terms of the United States Prototype Kilogram 20 and in accordance with the International Bureau of Weights and Measures. This relation has been recently modified by agreement to

1 avoirdupois pound = 0.4535923000 kilogram (exact)

so that

1 kilogram = 2.20462296 avoirdupois pounds.

These values are those agreed on by ICAO and NACA (BR 2).

3.3 Volume.

1 milliliter = 1.000028 cubic centimeters.

In "Miscellaneous Publication M121", National Bureau of Standards, it is stated that they adopted in 1946 the relation 1 liter = 1.000028 cubic decimeters. For reasons for the change from the relation 1 liter = 1.000027 cubic decimeters, see Reference GU 1.

3.4 Temperature-units conversion.

1 K = 1.8 R (exact).

3.5 Nautical mile to feet.

1 nautical mile = 6080.20 feet (exact).

In U. S. Coast and Geodetic Survey, Publication No. 5, "Tables for a Polyconic Projection of Maps", 2nd Edition, 1900, a nautical mile is defined as "A minute of arc of a great circle of a sphere whose surface equals that of the Clarke representative spheroid of 1866", and the value given is 1853.25 meters or 6080.20 feet. The latter value is obtained from the former when the relation 1 meter = 39.3700 inches is used. The value 6080.20 feet is also given in the "Smithsonian Meterological Tables", 1939, page 24.

However, when the length of the nautical mile in meters is desired, the conversion will be made from 6080.20 feet, using the relation 0.3048 meters/foot (exact), which is the ASA conversion factor.

Our use of 6080.20 feet is in agreement with the practice in the U. S. Navy.

Section 4. Computed Constants and Equivalent Values

4.1 Absolute temperature at sea level, T_o.

$$T_o = T_b + t_o = 273.16 + 15 = 288.16 \text{ K.}$$

4.2 Absolute temperature at the tropopause, T_a.

$$T_a = T_b + t_a = 273.16 - 56.5 = 216.66 \text{ K.}$$

4.3 Altitude of the tropopause, Z_a.

$$Z_a = \frac{T_o - T_a}{a} = \frac{288.16 - 216.66}{0.0065} = 11{,}000.00 \text{ m.}$$

$$Z_a = (Z_a,\ m)\,(ft/m) = (11{,}000)\,(1/0.3048) = 36{,}089.2388 \text{ ft.}$$

4.4 Altitude of the stratopause, Z_b.

$$Z_b = 32{,}000 \text{ meters}$$

$$Z_b = (Z_b,\ m)\,(ft/m) = (32{,}000)\,(1/0.3048) = 104{,}986.88 \text{ ft.}$$

4.5 Equivalent values of the troposphere temperature gradient, a.

$$a = (a,\ C/m)\,(m,\ ft)$$

$$= (0.0065)\,(0.3048) = 0.0019812 \text{ C/ft (exact).}$$

4.6 Equivalent values of the chemosphere temperature gradient, b.

$$b = 0.0074 \text{ C/m (exact)}$$

$$b = (b,\ C/m)\,(m/ft)$$

$$= (0.0074)\,(0.3048) = 0.00225552 \text{ C/ft (exact).}$$

4.7 Equivalent values of the gas constant for standard air, R.

$$R = 2.8704 \times 10^6 \text{ cm}^2/\text{sec}^2 \text{ K (exact) as defined in Section 1.1.}$$

$$R = (R, \text{ cm}^2/\text{sec}^2 \text{ K}) (\text{ft/cm})^2 (\text{K}/{}^\circ\text{R})$$

$$= (2.8704 \times 10^6)(1/30.48)^2(1/1.8)$$

$$= 1716.4849 \text{ ft}^2/\text{sec}^2 \, {}^\circ\text{R}.$$

4.8 Equivalent value of the standard gravitational acceleration, g_s.

$$g_s = 9.80665 \text{ m/sec}^2 \text{ as defined for the standard atmosphere.}$$

$$g_s = (g_s, \text{ m/sec}^2)(\text{ft/m})$$

$$= (9.80665)(1/0.3048) = 32.17404856 \text{ ft/sec}^2.$$

4.9 Constant, $n = g_s/aR$.

$$n = \frac{(g_s, \text{ cm/sec}^2)}{(a, \text{ K/cm})(R, \text{ cm}^2/\text{sec}^2 \text{ K})}$$

$$n = \frac{(980.665)}{(6.5 \times 10^{-5})(2.8704 \times 10^6)} = 5.2561155.$$

4.10 Constant, $k = g_s/bR$.

$$k = \frac{(g_s, \text{ cm/sec}^2)}{(b, \text{ K/cm})(R, \text{ cm}^2/\text{sec}^2 \text{ K})}$$

$$k = \frac{980.665}{(7.4 \times 10^{-5})(2.8704 \times 10^6)} = 4.6168582.$$

4.11 Equivalent values of standard sea-level pressure, P_o.

$$P_o = 760 \text{ mm Hg (exact)}$$

$$= 1013.250 \times 10^3 \text{ dynes/cm}^2 \text{ (exact)}$$

$$= 1013.250 \text{ mb (exact).}$$

$$P_o = (P_o, \text{ cm Hg})(\text{in./cm})$$

$$= (76)(1/2.54) = 29.921260 \text{ inches Hg.}$$

$$P_o = (P_o, \text{ dynes/cm}^2)(1/g_s, \text{ sec}^2/\text{cm})(\text{kg cm}^2/\text{g m}^2)$$

$$= (1013.250 \times 10^3)(1/980.665)(10)$$

$$= 10332.275 \text{ kg/m}^2.$$

$$P_o = (P_o, \text{ kg/m}^2)(\text{lb/kg})(\text{m/ft}^2)$$

$$= (10332.275)(2.20462296)(0.3048)^2$$

$$= 2116.2170 \text{ lb/ft}^2.$$

4.12 Pressure at the tropopause, P_a.

$$P_a = P_o (T_a/T_o)^n$$

$$= (1013.25 \text{ mb})(216.66/288.16)^{5.2561155}$$

$$= (1013.25 \text{ mb})(0.22335930) = 226.31881 \text{ mb.}$$

$$P_a = (760 \text{ mm Hg})(0.22335930) = 169.75307 \text{ mm Hg.}$$

$$P_a = (10332.275 \text{ kg/m}^2)(0.22335930) = 2307.8096 \text{ kg/m}^2.$$

$$P_a = (29.921260 \text{ in. Hg})(0.22335930) = 6.6831917 \text{ in. Hg.}$$

$$P_a = (2116.2170 \text{ lb/ft}^2)(0.22335930) = 472.67674 \text{ lb/ft}^2.$$

4.13 Pressure at the stratopause, P_b

$$P_b = P_a e^{-\frac{g_s}{RT_a}(Z_b - Z_a)} = P_o (T_a/T_o)^{n} e^{-\frac{g_s}{RT_a}(Z_b - Z_a)}$$

$$= (1013.250 \text{ mb}) (0.22335930)$$

$$\times \exp \frac{-(980.665 \text{ cm/sec}^2)(3.2 \times 10^6 \text{ cm} - 1.1 \times 10^6 \text{ cm})}{(2.8704 \times 10^6 \text{ cm}^2/\text{sec}^2 \text{ K})(216.66 \text{ K})}$$

$$= (1013.250 \text{ mb}) (0.22335930) (0.036463095)$$

$$= (1013.250 \text{ mb}) (0.0081443714) = 8.2422843 \text{ mb}.$$

$$P_b = (760 \text{ mm Hg}) (0.0081443714) = 6.1897223 \text{ mm Hg}.$$

$$P_b = (10332.275 \text{ kg/m}^2) (0.0081443714) = 84.149881 \text{ kg m}^2.$$

$$P_b = (29.921260 \text{ in. Hg}) (0.0081443714) = 0.24368985 \text{ in. Hg}.$$

$$P_b = (2116.2170 \text{ lb/ft}^2) (0.0081443714) = 17.235257 \text{ lb/ft}^2.$$

4.14 Standard density at sea level, ρ_o.

$$\rho_o = \frac{(P_o, \text{ dynes/cm}^2)}{(T_o, \text{ K}) (R, \text{ cm}^2/\text{sec}^2 \text{ K})}$$

$$= \frac{(1.013250 \times 10^6)}{(288.16)(2.8704 \times 10^6)} = 1.2250124 \times 10^{-3} \text{ g/cm}^3.$$

$$\rho_o = (\rho_o, \text{ kg/m}^3) (1/g_s, \text{ sec}^2/\text{m})$$

$$= (1.2250124)(1/980.665) = 0.12491650 \text{ kg sec}^2/\text{m}^4.$$

$$\rho_o = (\rho_o, \text{ kg sec}^2/\text{m}^4) (\text{lb/kg}) (\text{m/ft})^4$$

$$= (0.12491650) (2.20462296) (0.3048)^4$$

$$= 0.0023769170 \text{ lb sec}^2/\text{ft}^4.$$

4.15 Speed of sound at sea level, C_{so}.

$$C_{so} = \left[\gamma (R,\ m^2/sec^2\,K)(T_o,\ K)\right]^{1/2}$$

$$= \left[(1.4)(287.04)(288.16)\right]^{1/2} = 340.29226\ m/sec$$

$$C_{so} = (C_{so},\ m/sec)(ft/m)$$

$$= (340.29229)(1/0.3048) = 1116.4444\ ft/sec$$

$$C_{so} = (C_{so},\ ft/sec)(sec/hour)(mile/ft)$$

$$= (1116.4444)(3600)(1/5280) = 761.21212\ mph.$$

$$C_{so} = (C_{so},\ ft/sec)(sec/hour)(nautical\ mile/ft)$$

$$= (1116.4444)(3600)(1/6080.2) = 661.03088\ knots.$$

4.16 Constant, F.

$$F = \frac{\gamma - 1}{2\gamma} \cdot \frac{m^2/ft^2}{(R,\ m^2/sec^2\,K)(T_o,\ K)}$$

$$= \frac{7/5 - 1}{14/5} \cdot \frac{(0.3048)^2}{(287.04)(288.16)}$$

$$= 1.6045593 \times 10^{-7}\ sec^2/ft^2.$$

$$F = (F,\ sec^2/ft^2)(ft/mile)^2 (hr/sec)^2$$

$$= (1.6045593 \times 10^{-7})(5280)^2 (1/3600)^2$$

$$= 0.34515853 \times 10^{-6}\ mph^{-2}.$$

$$F = (F,\ sec^2/ft^2)(ft/nautical\ mile)^2 (hr/sec)^2$$

$$= (1.6045593 \times 10^{-7})(6080.20)^2 (1/3600)^2$$

$$= 0.45770589 \times 10^{-6}\ knots^{-2}.$$

4.17 For supersonic speeds, equations to calculate pitot static pressure differential require a value of the constant

$$2^{\frac{-\gamma}{\gamma - 1}} (\gamma + 1)^{\frac{\gamma + 1}{\gamma - 1}} (\gamma - 1)^{\frac{-1}{\gamma - 1}} = 166.9215801.$$

4.18 Height of water column at 25 C, τ_{H_2O}.

$$\tau_{H_2O} = \frac{(1/g_s,\ \sec^2/\text{cm})(P_o,\ \text{dynes/cm}^2)(\text{in./cm})(\text{cm}^3/\text{ml})}{(\rho_{H_2O},\ \text{g/ml})}$$

$$= \frac{(1/980.665)(1013.250 \times 10^3)(1/2.54)(1.000028)}{(0.9970751)}$$

$$= 407.98717 \text{ in.}$$

4.19 Equivalent length of nautical mile in meters.

$$\text{Nautical mile} = (\text{ft/nautical mile})(\text{m/ft})$$

$$= (6080.20)(0.3048) = 1853.244960 \text{ m (exact).}$$

Tables and Curves of the WADC 1952 Model Atmosphere

Five tables are provided here to summarize the foundations and properties of the WADC 1952 Model Atmosphere. Definitions of the symbols, as well as of certain terms commonly used in the discussion, appear in Table A. Values chosen for the various parameters are presented in Table B. Equations fundamental to the derivation and computation of the numbers in the various tables of the model atmosphere and to the computation of airspeeds are listed for easy reference in Table C.

Tables D and E show several of the physical properties of the model atmosphere as functions of altitude. Table D presents generally English units of measurement, while Table E presents metric units. The absolute temperatures T and pressures P at altitude Z were computed from the equations of the first two sections of Table C. The density ρ was derived from

$$\rho = \rho_o T_o P / T P_o,$$

and from it were found the specific weight $g_s \rho$ and the density ratio ρ/ρ_o. The speed of sound C_s was found from the relation

$$C_s = C_{so}\sqrt{T/T_o}.$$

The dimensionless quantities P/P_o and C_s/C_{so} complete these tables.

TABLE A. DEFINITIONS OF SYMBOLS AND TERMS

Symbol or Term	Definition
a	Standard temperature gradient, or lapse rate, in the troposphere. Its value is 0.0065 C drop in temperature per meter increase in altitude.
b	Temperature gradient of the chemosphere, introduced in Equation (1c), equal to 0.0074 C/meter.
C_s	Speed of sound, equal at any given absolute temperature T to $C_s = \sqrt{\gamma R T}$ Here γ is the ratio of specific heats = 1.4 R = gas constant = 287.04 m^2/sec^2K T = absolute temperature, K.
C_{so}	Speed of sound at standard sea level; temperature, 288.16 K; equal to 340.29226 m/sec.
F	A constant [see Equation (8a)] in the expression for calibrated air speed. By definition $F = \frac{\gamma-1}{2\gamma R T_o} = \frac{\gamma-1}{2C_{so}^2}$ where C_{so} is the sonic speed at standard sea level. It has the value $0.45770589 \times 10^{-6}$ knots^{-2}.
g_s	Standard gravitational acceleration, equal to 9.80665 m/sec^2.
K	Recovery factor, the proportion of the dynamic temperature of moving air that is recovered, to be observed by a thermocouple or other detector.
k	A constant in Equation (7) equal to g_s/bR. It is the exponent in the expression for pressure as a function of altitude in the chemosphere. Its value is 4.6168582.
n	A constant defined in Equation (4), $n = g_s/aR$. It is the exponent in the expression for pressure as a function of altitude in the troposphere. Its value is 5.2561155.
P	Static pressure in the ambient air.

TABLE A. (Continued)

Symbol or Term	Definition
P_o	Standard sea-level pressure equal to 1013.250 millibars or 760 mm Hg.
P_a	Pressure at the tropopause, at 11,000-meters altitude, equal to 226.31881 mb.
P_b	Pressure at top of stratosphere, at 32,000-meters altitude, equal to 8.2522843 mb.
R	Gas constant for WADC 1952 Model Atmosphere. Its value is 287.04 m^2/sec^2K.
T	Absolute temperature of the ambient air.
T_o	Absolute temperature at sea level, 288.16 K.
T_a	Absolute temperature at the tropopause, 216.66 K.
T_b	Absolute temperature at the melting point of ice under one atmosphere of pressure, equal to 273.16 K.
T_i	Indicated temperature, observed by an instrument like a thermocouple, in a stagnation region. The Equation (13) defines T_i for given ambient temperature T; it falls between the total temperature, T_i for K = 1; and the ambient temperature, T_i for K = 0. Normally, K falls in the general neighborhood of K = 0.8.
Z_a	Altitude of the tropopause, 11,000 meters.
Z_b	Altitude of the stratopause, 32,000 meters.
γ	Ratio of specific heats for standard air, 1.4 (exact).
ρ	Density of the ambient air.
ρ_o	Density of standard air at standard sea level, 0.12491650 kg sec^2/m^4.
τ_{H_2O}	Height of water column at 25 C balancing the air pressure corresponding to standard sea level, 407.98717 inches.
Altimeter correction	The difference between the height above mean sea level of any point in the atmosphere and the pressure altitude at that point.

TABLE A. (Continued)

Symbol or Term	Definition
Altimeter setting	The pressure scale of an altimeter is reset in flight near the terminal landing field so that the altimeter will read the airport elevation above sea level upon landing. This "altimeter setting" is equal to the barometric pressure at the airport, as corrected to sea level.
Calibrated airspeed	The airspeed computed from the true dynamic pressure by the equations for idealized flow, Equations (8a) and (11a) for subsonic and supersonic calibrated airspeed, respectively. This quantity should not be confused with the indicated airspeed.
Indicated airspeed	Airspeed as indicated on an airspeed indicator. An errorless indicator will read the calibrated airspeed if it is properly installed; normally instrument errors will introduce some deviation.
Pressure altitude	This quantity represents the quantitative value of the pressure at a given point by giving the height above a given standard altitude at which that pressure occurs in a standard atmosphere.
Zero pressure altitude	The altitude at which the pressure is 760 mm Hg.

TABLE B. VALUES OF CONSTANTS

Element		
Description	Symbol	Values
Standard troposphere-temperature gradient	a	0.0065 C/m*
Standard chemosphere-temperature gradient	b	0.0074 C/m* 0.00225552 C/ft*
Speed of sound at Standard sea level	C_{so}	340.29226 m/sec 1116.4444 ft/sec 761.21212 mph 661.03088 knots
Constant	F	$0.45770589 \times 10^{-6}$ knots^{-2} $0.34515853 \times 10^{-6}$ mph^{-2}
English unit of length	ft	0.3048 m*
Standard gravitational acceleration	g_s	9.80665 m/sec^{2}* 32.17404856 ft/sec^{2}
Constant $[= g_s/(bR)]$	k	4.6168582
Unit of length	mm	0.0393700787 in.
Constant $[= g_s/(aR)]$	n	5.2561155
Standard sea-level pressure	P_o	1013.250 mb* 760 mm Hg* 10,332.275 kg/m^{2} 29.921260 in. Hg 2116.2170 lb/ft^{2}
Pressure at the tropopause	P_a	226.31881 mb 169.75307 mm Hg 2307.8096 kg/m^{2} 6.6831917 in. Hg 472.67674 lb/ft^{2}
Pressure at the stratopause	P_b	8.2522843 mb 6.1897223 mm Hg 84.149881 Kg/m^{2} 0.24368985 in. Hg 17.235257 lb/ft^{2}

TABLE B. (Continued)

Element		Values
Description	Symbol	
Gas constant for standard air	R	287.04 m^2/sec^2 K* 1716.4849 ft^2/sec^2 R
Absolute temperature at sea level	T_o	288.16 K* 518.688 R*
Absolute temperature at the tropopause	T_a	216.66 K* 389.988 R*
Absolute temperature at the melting point of ice under a pressure of one atmosphere	T_b	273.16 K* 491.688 R*
Altitude of the tropopause	Z_a	11,000 m* 36,089.2388 ft
Altitude at the stratopause	Z_b	32,000 meters* 104,986.88 ft
Ratio of specific heats of air	γ	1.400*
Density of standard air at sea level	ρ_o	0.12491650 kg sec^2/m^4 0.0023769170 lb sec^2/ft^4 1.2250124 x 10^{-3} g/cm^3
Height of water column at 25 C	τ_{H_2O}	407.98717 in.
Length of nautical mile		6080.20 ft* 1853.244960 m*

*Indicates a quantity either arbitrarily fixed or else determined exactly by arbitrarily fixed quantities. These numbers are exact. Nonstarred quantities have been rounded off to eight significant digits; recomputation is necessary if more significant figures are desired.

TABLE C. TABLE OF EQUATIONS

Condition	Equation
1. Temperature T in the WADC 1952 Model Atmosphere in degrees K, as a function of altitude Z above sea level in meters.	
a. In the troposphere, $0 < Z < 11,000$	$T = 288.16 - 0.0065Z$
b. In the stratosphere, $11,000 < Z < 32,000$	$T = 216.66$
c. In the chemosphere, $32,000 < Z$	$T = 216.66 + 0.0074\ (Z - 32,000)$
2. Pressure P in the WADC 1952 Model Atmosphere in millimeters Hg, as a function of altitude above sea level Z in meters	
a. In the troposphere, $0 < Z\ 11,000$	$P = 760\ (1 - \frac{0.0065}{288.16} Z)^{5.2561155}$
b. In the stratosphere, $11,000 < Z < 32,000$	$P = 169.75307\, e^{-1.57688316 \times 10^{-4}(Z-11,000)}$
c. In the chemosphere, $32,000 < Z$	$P = 6.1897223 \left\{\frac{216.66}{216.66 + 0.0074(Z-32,000)}\right\}^{4.6168582}$
3. Calibrated airspeed V_c in knots, as a function of dynamic pressure ΔP in inches Hg, in a WADC 1952 Model Atmosphere	
a. Subsonic, $V_c < 661.03088$	$V_c = 661.03088 \sqrt{5[(\frac{\Delta P}{29.921260} + 1)^{2/7} - 1]}$
b. Supersonic, $661.03088 < V_c$	$\frac{\Delta P}{29.921260} = 166.921580 \left\{\frac{(\frac{V_c}{661.03088})^7}{[7(\frac{V_c}{661.03088})^2 - 1]^{5/2}}\right\} - 1$

TABLE C. (Continued)

Condition	Equation
4. True airspeed V_t as a function of dynamic pressure ΔP, ambient pressure P, and velocity of sound C_s in the ambient air, in a WADC 1952 Model Atmosphere	
a. Subsonic flow, $V_t < C_s$	$V_t = C_s \sqrt{5[\frac{\Delta P}{P} + 1)^{2/7} - 1]}$
b. Supersonic flow, $V_t > C_s$	$\frac{\Delta P}{P} = 166.921580 \left[\frac{(\frac{V}{C_s})^7}{[7(\frac{V_c}{C_s})^2 - 1]^{5/2}} \right] - 1$
5. Sonic velocity in knots as a function of absolute temperature in degrees Kelvin in the WADC 1952 Model Atmosphere	$C_s = 38.940803 \sqrt{T}$
6. Equivalent airspeed V_e as a function of the true airspeed V_t of the aircraft and the square root of the ambient-air-density ratio	$V_e = V_t \sqrt{\frac{\rho}{\rho_o}}$ ρ = ambient-air density ρ_o = density of standard sea-level air
7. Mach number M as a function of the ratio of true airspeed to the speed of sound in the ambient air, in a WADC 1952 Model Atmosphere	$M = \frac{V_t}{C_s}$
8. Indicated temperature T_i as a function of ambient temperature T, recovery factor K, and Mach number M	$T_i = T(1 + 0.2KM^2)$
9. Ambient static temperature T in terms of indicated temperature T_i, recovery factor K, and Mach number M	$T = \frac{T_i}{1 + 0.2KM^2}$

TABLE D. PHYSICAL PROPERTIES OF MODEL ATMOSPHERE, ENGLISH UNITS

Altitude, Z, feet	Temperature, T		Pressure, P			
	deg K	deg R	in. Hg	lb/ft^2	mb	P/P_o
0	288.160	518.688	29.9213	2116.22	1013.25	1.00000
5,000	278.254	500.857	24.8959	1760.79	843.07	0.83205
10,000	268.348	483.026	20.5769	1455.33	696.81	0.68770
15,000	258.442	465.196	16.8857	1194.26	571.82	0.56434
20,000	248.536	447.365	13.7500	972.49	465.63	0.45954
25,000	238.630	429.534	11.1035	785.31	376.01	0.37109
30,000	228.724	411.703	8.8854	628.43	300.89	0.29696
35,000	218.818	393.872	7.0406	497.95	238.42	0.23530
40,000	216.660	389.988	5.5380	391.68	187.54	0.18509
45,000	216.660	389.988	4.3550	308.01	147.48	0.14555
50,000	216.660	389.988	3.4246	242.21	115.97	0.11446
55,000	216.660	389.988	2.6931	190.47	91.20	0.09001
60,000	216.660	389.988	2.1178	149.78	71.72	0.07078
65,000	216.660	389.988	1.6654	117.79	56.40	0.05566
70,000	216.660	389.988	1.3096	92.62	44.35	0.04377
75,000	216.660	389.988	1.0298	72.84	34.87	0.03442
80,000	216.660	389.988	0.8099	57.28	27.42	0.02707
85,000	216.660	389.988	0.6368	45.04	21.57	0.02128
90,000	216.660	389.988	0.5008	35.42	16.96	0.01674
95,000	216.660	389.988	0.3938	27.85	13.34	0.01316
100,000	216.660	389.988	0.3097	21.90	10.49	0.01035
105,000	216.690	390.041	0.2435	17.22	8.25	0.00814
110,000	227.967	410.341	0.1927	13.63	6.52	0.00644
115,000	239.245	430.641	0.1542	10.90	5.22	0.00515
120,000	250.522	450.940	0.1246	8.82	4.22	0.00417
125,000	261.800	471.240	0.1017	7.19	3.44	0.00340
130,000	273.078	491.540	0.0837	5.92	2.83	0.00280
135,000	284.355	511.839	0.0694	4.91	2.35	0.00232
140,000	295.633	532.139	0.0580	4.10	1.97	0.00194

TABLE D. (Continued)

Altitude, Z, feet	Density, ρ		Spec. wt., $g_s \rho$	Speed of Sound, Cs		
	10^{-3} slug/ft^3	ρ/ρ_o	lb/ft^3	knots	ft/sec	Cs/Cso
0	2.37692	1.00000	0.076475	661.03	1116.44	1.00000
5,000	2.04811	0.86167	0.065896	649.57	1097.09	0.98266
10,000	1.75529	0.73848	0.056475	637.90	1077.38	0.96501
15,000	1.49563	0.62923	0.048121	626.02	1057.31	0.94703
20,000	1.26643	0.53281	0.040746	613.90	1036.85	0.92871
25,000	1.06513	0.44811	0.034269	601.54	1015.97	0.91001
30,000	0.88927	0.37413	0.028611	588.93	994.66	0.89092
35,000	0.73653	0.30987	0.023697	576.03	972.89	0.87141
40,000	0.58511	0.24617	0.018825	573.18	968.08	0.86711
45,000	0.46012	0.19358	0.014804	573.18	968.08	0.86711
50,000	0.36183	0.15223	0.011642	573.18	968.08	0.86711
55,000	0.28454	0.11971	0.009155	573.18	968.08	0.86711
60,000	0.22375	0.09414	0.007199	573.18	968.08	0.86711
65,000	0.17595	0.07403	0.005661	573.18	968.08	0.86711
70,000	0.13837	0.05821	0.004452	573.18	968.08	0.86711
75,000	0.10881	0.04578	0.003501	573.18	968.08	0.86711
80,000	0.08556	0.03600	0.002753	573.18	968.08	0.86711
85,000	0.06729	0.02831	0.002165	573.18	968.08	0.86711
90,000	0.05291	0.02226	0.001702	573.18	968.08	0.86711
95,000	0.04161	0.01751	0.001339	573.18	968.08	0.86711
100,000	0.03272	0.01377	0.001053	573.18	968.08	0.86711
105,000	0.02573	0.01082	0.000828	573.22	968.14	0.86717
110,000	0.01935	0.00814	0.000623	587.95	993.02	0.88945
115,000	0.01475	0.00621	0.000475	602.32	1017.28	0.91118
120,000	0.01139	0.00479	0.000366	616.35	1040.98	0.93241
125,000	0.00889	0.00374	0.000286	630.07	1064.16	0.95316
130,000	0.00702	0.00295	0.000226	643.50	1086.83	0.97348
135,000	0.00559	0.00235	0.000180	656.65	1109.05	0.99338
140,000	0.00449	0.00189	0.000145	669.55	1130.83	1.01288

TABLE E. PHYSICAL PROPERTIES OF MODEL ATMOSPHERE, METRIC UNITS

Altitude, Z, m	Temperature, T,		Pressure, P		
	deg K	deg C	mm Hg	mb	P/P_o
0	288.16	15.00	760.00	1013.25	1.00000
1,000	281.66	8.50	674.11	898.74	0.88699
2,000	275.16	2.00	596.26	794.95	0.78455
3,000	268.66	- 4.50	525.86	701.08	0.69192
4,000	262.16	-11.00	462.34	616.40	0.60834
5,000	255.66	-17.50	405.18	540.20	0.53313
6,000	249.16	-24.00	353.88	471.81	0.46564
7,000	242.66	-30.50	307.98	410.60	0.40524
8,000	236.16	-37.00	267.02	356.00	0.35134
9,000	229.66	-43.50	230.59	307.42	0.30340
10,000	223.16	-50.00	198.29	264.36	0.26090
11,000	216.66	-56.50	169.75	226.32	0.22336
12,000	216.66	-56.50	144.99	193.30	0.19077
13,000	216.66	-56.50	123.84	165.10	0.16294
14,000	216.66	-56.50	105.77	141.02	0.13917
15,000	216.66	-56.50	90.34	120.44	0.11887
16,000	216.66	-56.50	77.16	102.87	0.10153
17,000	216.66	-56.50	65.91	87.87	0.08672
18,000	216.66	-56.50	56.29	75.05	0.07407
19,000	216.66	-56.50	48.08	64.10	0.06326
20,000	216.66	-56.50	41.06	54.75	0.05403
21,000	216.66	-56.50	35.07	46.76	0.04615
22,000	216.66	-56.50	29.96	39.94	0.03942
23,000	216.66	-56.50	25.59	34.11	0.03367
24,000	216.66	-56.50	21.85	29.14	0.02876
25,000	216.66	-56.50	18.67	24.89	0.02456
26,000	216.66	-56.50	15.94	21.26	0.02098
27,000	216.66	-56.50	13.62	18.15	0.01792
28,000	216.66	-56.50	11.63	15.51	0.01530
29,000	216.66	-56.50	9.93	13.24	0.01307
30,000	216.66	-56.50	8.48	11.31	0.01116
31,000	216.66	-56.50	7.25	9.66	0.00954
32,000	216.66	-56.50	6.19	8.25	0.00814
33,000	224.06	-49.10	5.30	7.07	0.00697
34,000	231.46	-41.70	4.56	6.08	0.00600
35,000	238.86	-34.30	3.95	5.26	0.00519
36,000	246.26	-26.90	3.43	4.57	0.00451
37,000	253.66	-19.50	2.99	3.99	0.00393
38,000	261.06	-12.10	2.62	3.49	0.00344
39,000	268.46	- 4.70	2.30	3.07	0.00303
40,000	275.86	2.70	2.03	2.71	0.00267
41,000	283.26	10.10	1.80	2.39	0.00236
42,000	290.66	17.50	1.59	2.13	0.00210

TABLE E. (Continued)

Altitude, Z, m	Density, ρ		Spec. wt., $g_s \rho$	Speed of Sound, C_s	
	kg sec^2/m^4	ρ/ρ_o	g/cm^3	m/sec	C_s/C_{so}
0	0.124917	1.00000	1.22501	340.292	1.00000
1,000	0.113357	0.90746	1.11165	336.432	0.98866
2,000	0.102634	0.82162	1.00650	332.528	0.97718
3,000	0.092705	0.74214	0.90913	328.577	0.96557
4,000	0.083528	0.66867	0.81913	324.578	0.95382
5,000	0.075063	0.60091	0.73612	320.528	0.94192
6,000	0.067271	0.53852	0.65970	316.428	0.92987
7,000	0.060112	0.48122	0.58950	312.273	0.91766
8,000	0.053552	0.42870	0.52516	308.062	0.90529
9,000	0.047554	0.38069	0.46635	303.793	0.89274
10,000	0.042084	0.33690	0.41270	299.463	0.88002
11,000	0.037109	0.29707	0.36391	295.070	0.86711
12,000	0.031695	0.25373	0.31083	295.070	0.86711
13,000	0.027072	0.21672	0.26548	295.070	0.86711
14,000	0.023122	0.18510	0.22675	295.070	0.86711
15,000	0.019749	0.15810	0.19367	295.070	0.86711
16,000	0.016868	0.13503	0.16542	295.070	0.86711
17,000	0.014407	0.11533	0.14129	295.070	0.86711
18,000	0.012305	0.09851	0.12067	295.070	0.86711
19,000	0.010510	0.08414	0.10307	295.070	0.86711
20,000	0.008977	0.07186	0.08803	295.070	0.86711
21,000	0.007667	0.06138	0.07519	295.070	0.86711
22,000	0.006549	0.05243	0.06422	295.070	0.86711
23,000	0.005593	0.04478	0.05485	295.070	0.86711
24,000	0.004777	0.03825	0.04685	295.070	0.86711
25,000	0.004081	0.03267	0.04002	295.070	0.86711
26,000	0.003485	0.02790	0.03418	295.070	0.86711
27,000	0.002977	0.02383	0.02919	295.070	0.86711
28,000	0.002543	0.02035	0.02493	295.070	0.86711
29,000	0.002172	0.01738	0.02130	295.070	0.86711
30,000	0.001855	0.01485	0.01819	295.070	0.86711
31,000	0.001584	0.01268	0.01554	295.070	0.86711
32,000	0.001353	0.01083	0.01327	295.070	0.86711
33,000	0.001120	0.00897	0.01099	300.066	0.88179
34,000	0.000934	0.00747	0.00916	304.981	0.89623
35,000	0.000782	0.00626	0.00767	309.818	0.91045
36,000	0.000659	0.00528	0.00646	314.581	0.92444
37,000	0.000558	0.00447	0.00547	319.272	0.93823
38,000	0.000475	0.00380	0.00466	323.896	0.95182
39,000	0.000406	0.00325	0.00398	328.454	0.96521
40,000	0.000348	0.00279	0.00342	332.950	0.97842
41,000	0.000300	0.00240	0.00294	337.387	0.99146
42,000	0.000260	0.00208	0.00255	341.765	1.00433

Figures 1 through 5 are graphs of eight of the quantities tabulated in Table D, again as functions of the altitude in feet. These curves will be found convenient in determining the properties of the model atmosphere when extreme accuracy is not required. The curves were plotted from points either contained in Table D or else computed in the same way as those points. The functions shown in the various graphs are:

Figure 1 The absolute temperature in degrees Kelvin

Figure 2 The static pressure P in inches of mercury and the dimensionless pressure ratio P/P_o

Figure 3 The air density ρ in lb sec^2/ft^4 or slugs/ft^3, the specific weight $g_s\rho$ in lb/ft^3 and the density ratio ρ/ρ_o

Figure 4 The sonic speed C_s in knots

Figure 5 The dimensionless sonic speed ratio C_s/C_{so}.

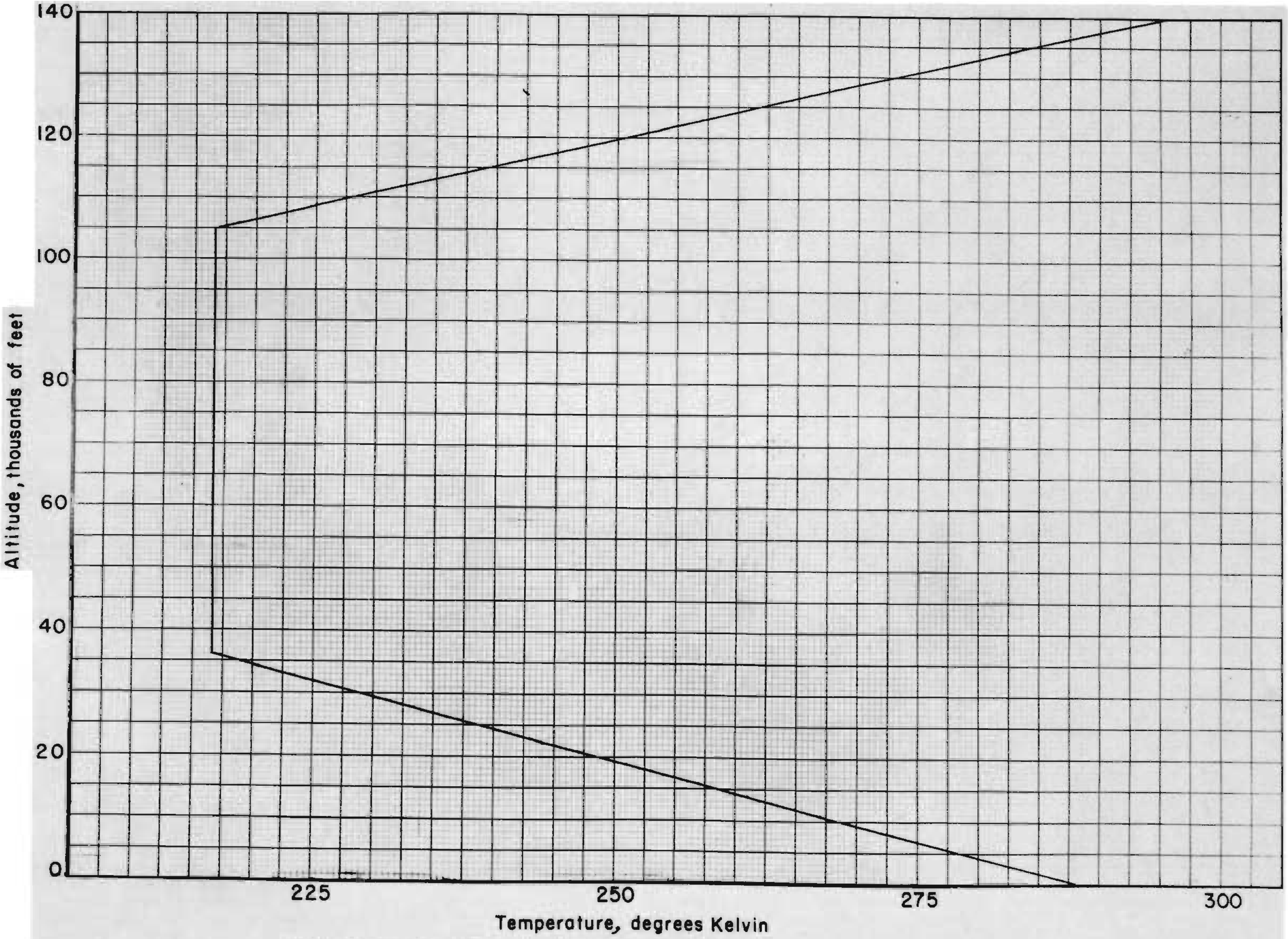

FIGURE I. VARIATION OF TEMPERATURE WITH ALTITUDE

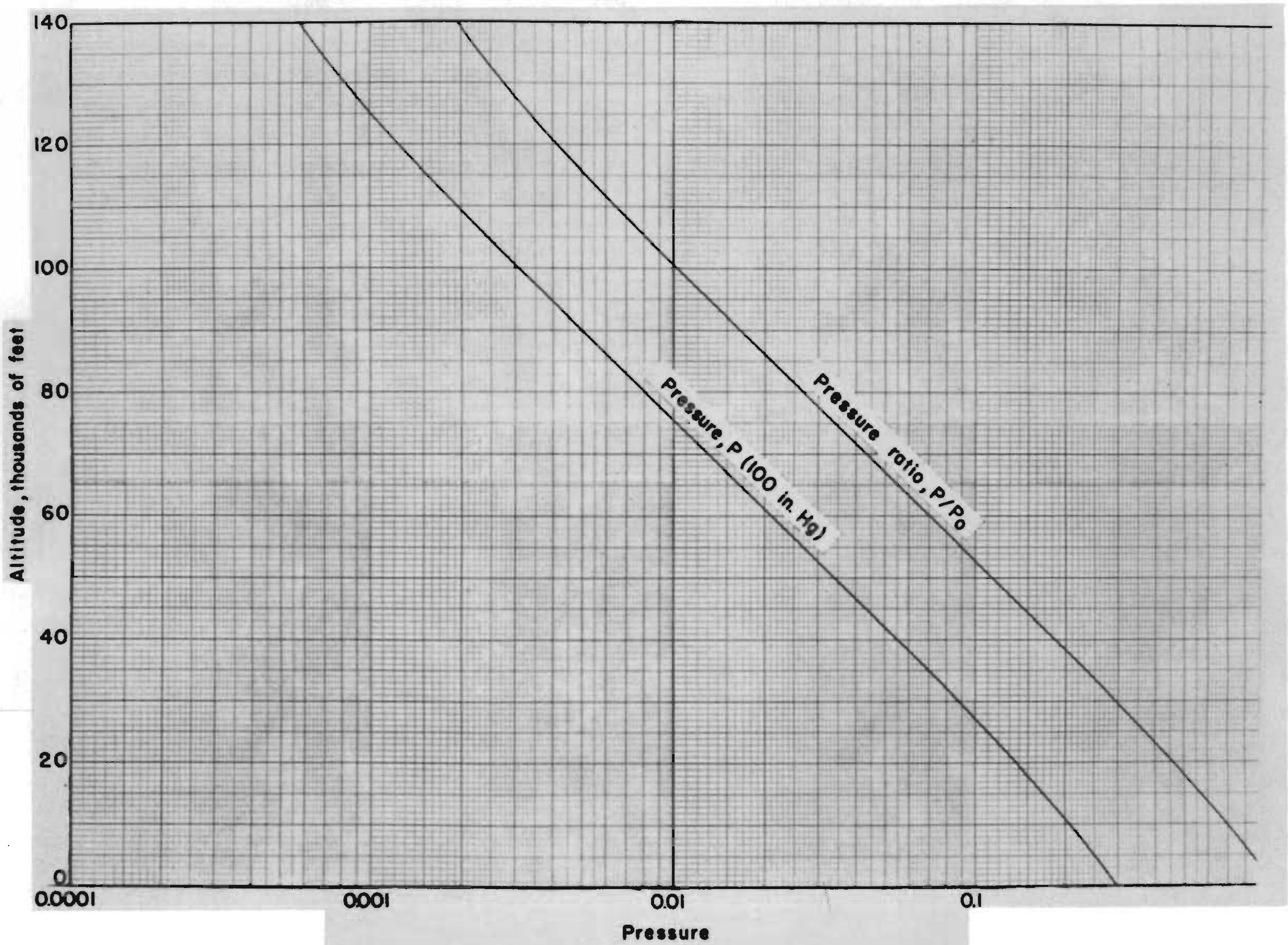

FIGURE 2. VARIATION OF PRESSURE WITH ALTITUDE

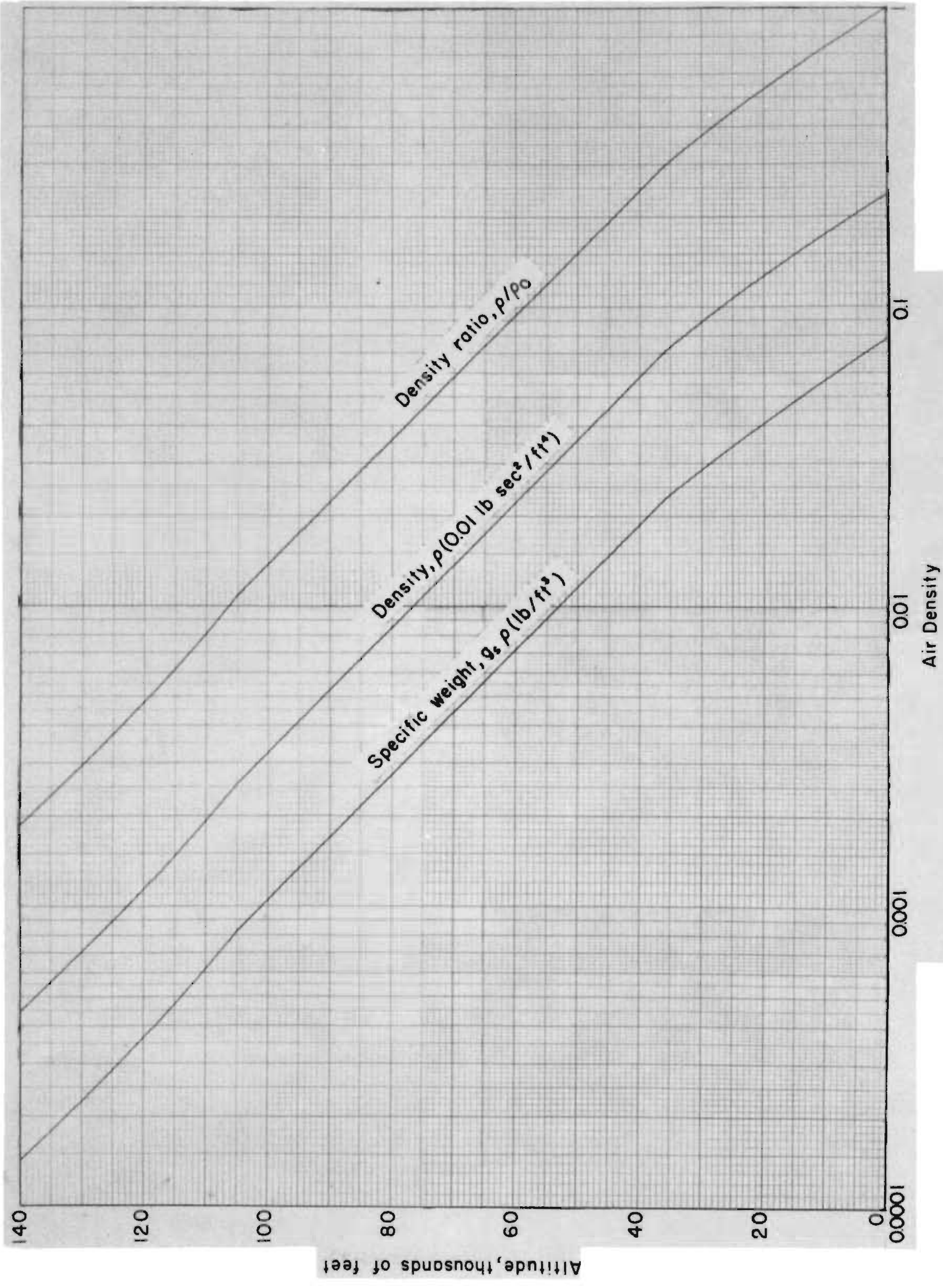

FIGURE 3. VARIATION OF AIR DENSITY WITH ALTITUDE

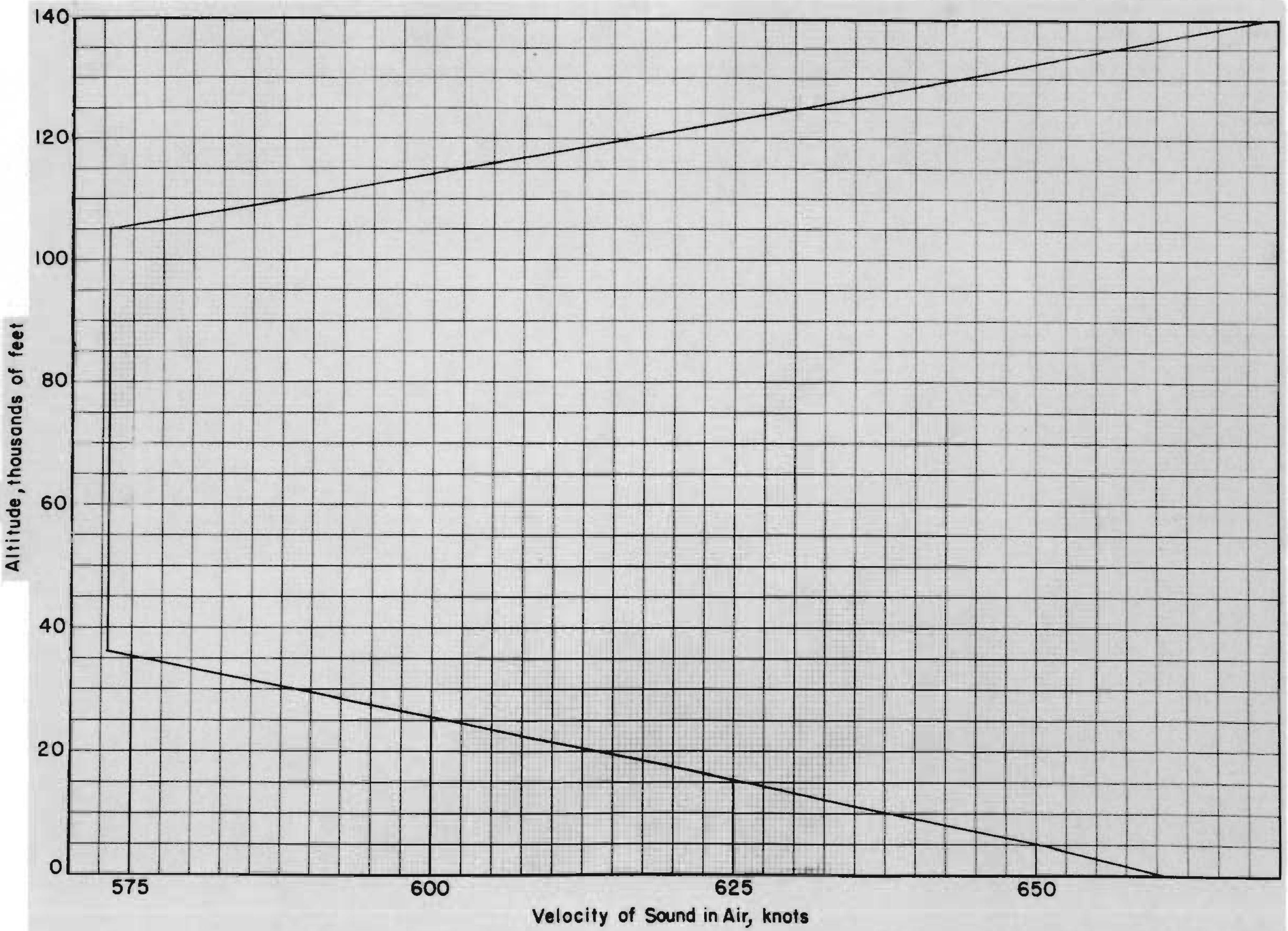

FIGURE 4. VARIATION OF THE VELOCITY OF SOUND WITH ALTITUDE

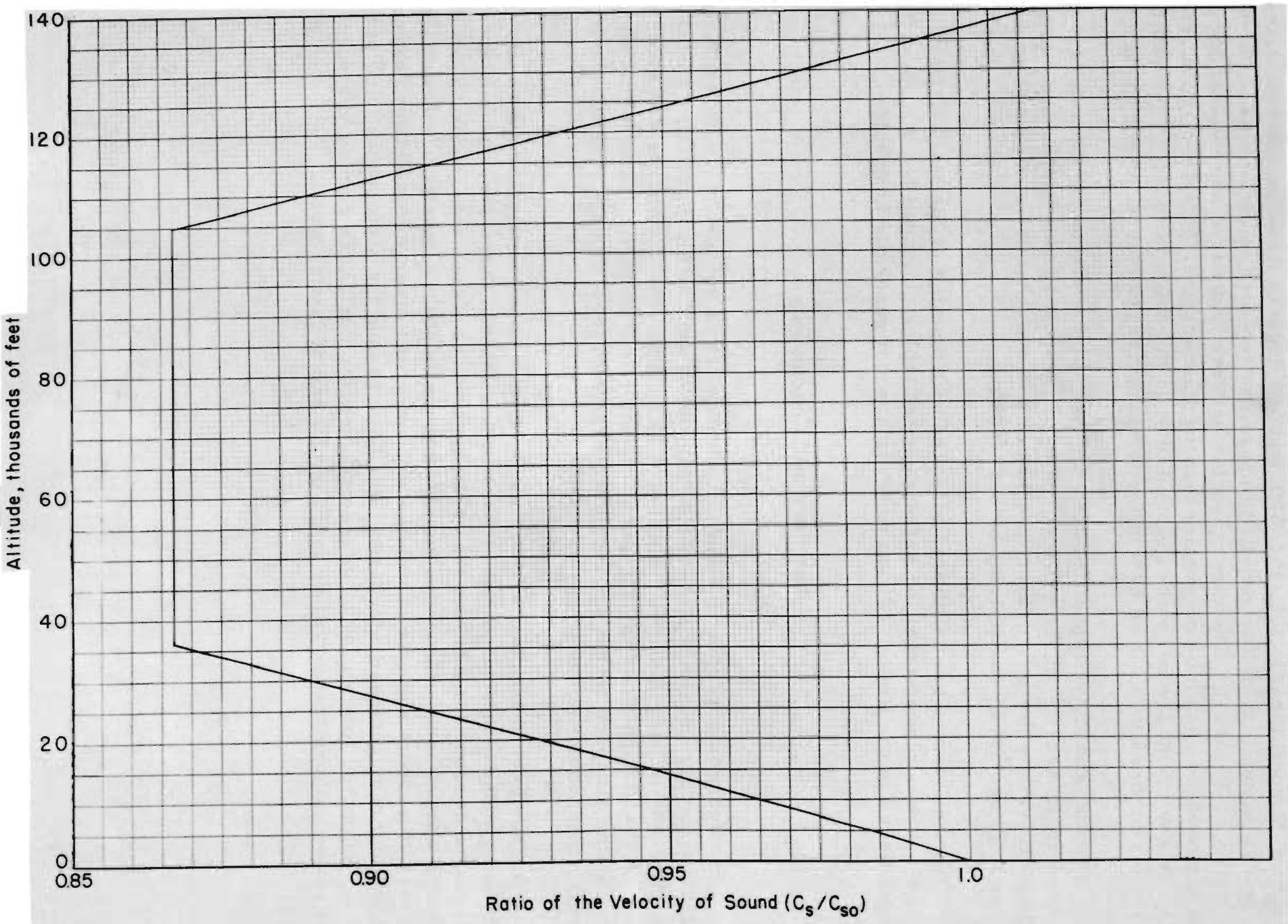

FIGURE 5. RATIO OF THE VELOCITY OF SOUND FOR VARIOUS ALTITUDES TO THAT OF SEA LEVEL

Figure 6 is a plot of the viscosity of air which may be associated with the model atmosphere at various altitudes. The data used for the curve are found in (KE 1), where the values of viscosity in the range of the model-atmosphere temperatures are given at only four temperatures: 400, 450, 500, and 550 degrees Rankine. These values were plotted as a function of the Kelvin temperature in order to obtain values for plotting the curve of Figure 6. The discrepancies between this curve and the viscosity of the real atmosphere should be no greater than the discrepancies contained in the reference data. In any event, the viscosities plotted here do not define a model in the same sense that the equations do as used herein for temperature, pressure, density, and so forth.

Keenan and Kaye used the unit lb/(sec ft) for viscosity; the values were expressed in lb sec/ft^2 in the present curve as computed from Keenan and Kaye by dividing all values by g_s. The curve in units lb sec^{2}knot/ft^3 was computed from the values in lb sec/ft^2 by multiplying by 3600/6080.2.

Figure 7 shows the calibrated airspeed as a function of the Mach number and the altitude. It was plotted from the set of airspeed tables computed for WADC at Battelle and published in the limited edition which is now not generally available.

Summary and Conclusions

The foregoing proposed standard atmosphere is unusual in its characteristic of self-consistency; previous standard atmospheres tend to disregard this virtue. This new proposal is also unusually extensive in height. This extension was made on account of the present need for airspeed tables for high altitudes. The formulas necessary for the airspeed tables are combined here with those for the atmosphere in order to make that tabulation possible.

As has been mentioned in the Foreword, new upper atmosphere data are continually being added to the fund of knowledge now available. There are, and will always be, persuasive reasons for altering the arbitrary choices of constants, basic to any official standard atmosphere. There can hardly be more than the authority of custom and convenience to support any given choice -- short of gross divergences between experiment and the current accepted standard. For this reason the atmosphere here presented will be subjected to scrutiny of interested agencies during the coming years. It is the hope of the authors that it will be found useful and of lasting worth.

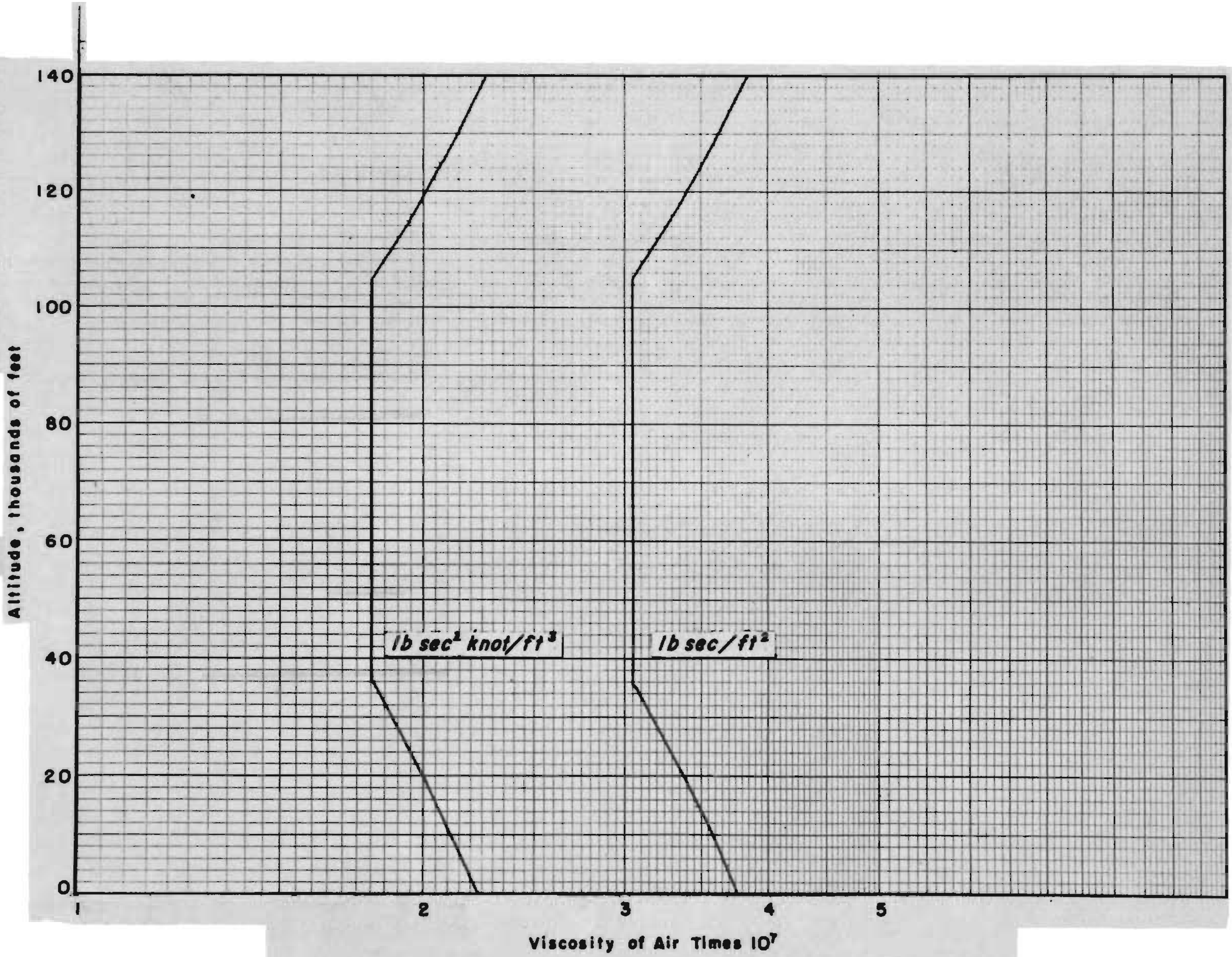

FIGURE 6. VARIATION OF VISCOSITY OF AIR WITH ALTITUDE

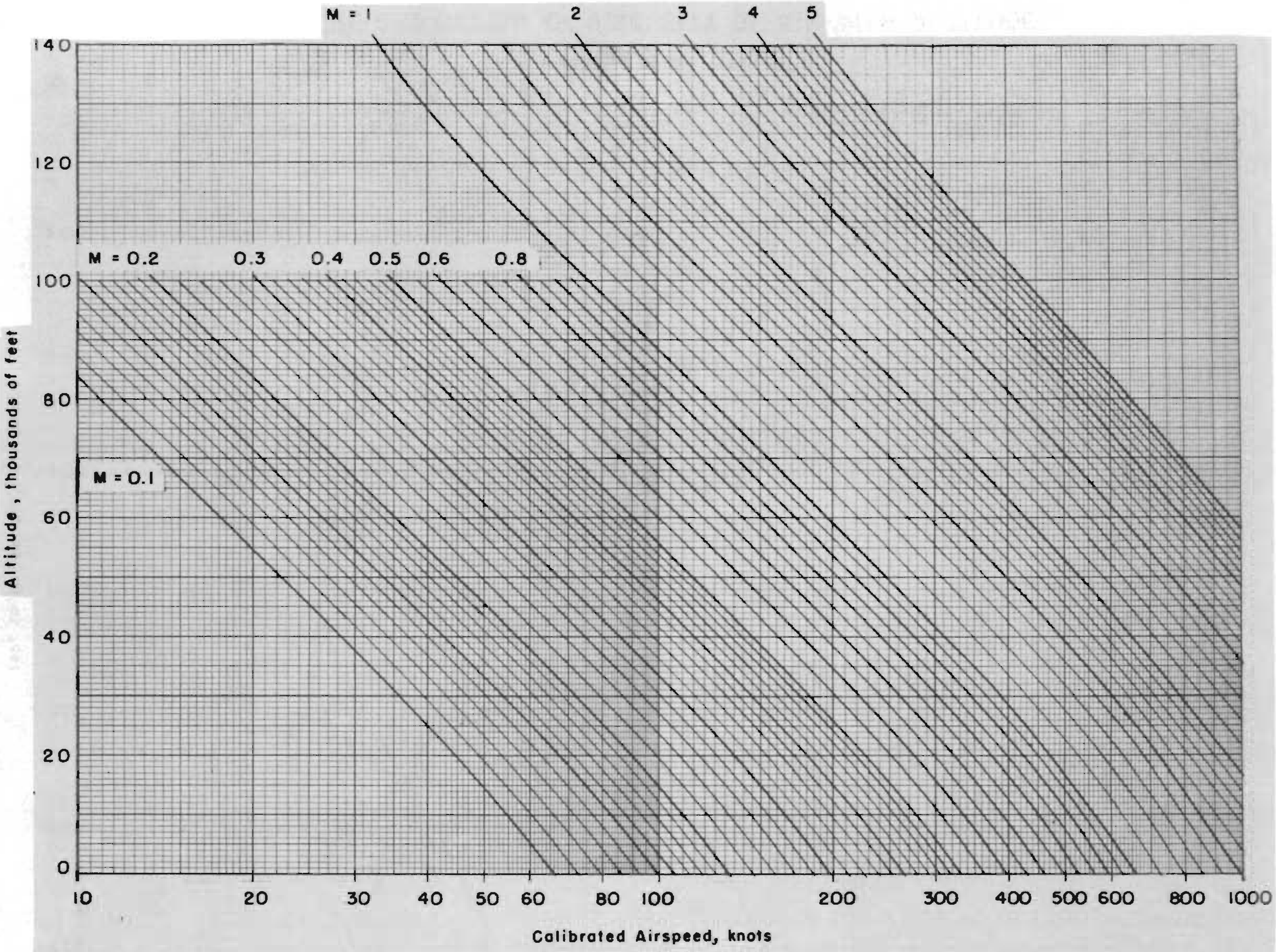

FIGURE 7. VARIATION OF CALIBRATED AIRSPEED WITH ALTITUDE FOR VARIOUS MACH NUMBERS

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