A TECHNIQUE FOR ANALYZING AN ADAPTIVE FLIGHT CONTROL SYSTEM CONTAINING A BI-STABLE ELEMENT

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A highly desirable design objective for an adaptive control system is that the system have the capability for satisfactorily controlling aircraft which exhibit large and rapid changes in performance characteristics in certain flight envelope regions. This should be accomplished along with a significant improvement in the reliability of the flight control system. Minneapolis-Honeywell' s approach to the realization of this objective utilizes nonlinear devices to obtain improvement in reliability by simplifying the basic control loop. The use of a bi-stable controller, as previously mentioned by Mr. Schuck, provides the means for accomplishing this as the possibility exists for combining a major portion of the system into one package. With this type of nonlinear controller the use of any type of mechanical gain scheduling in order to achieve the desired performance characteristics has not been necessary, and if a bi-stable servo actuator is used the adaptive controller and servo can be combined in one unit.

One of the greater difficulties associated with using nonlinear devices appears to be the lack of useful analytical techniques which provide a better understanding of the characteristics of such systems and insight into ways of more effectively utilizing such concepts and improving them. As a result of this difficulty, the development of a better understanding of this type of control system has been dependent, to a great extent, on using an imperical approach utilizing the results of computer studies and observations made during the actual operation of the system.

Realizing the value of mathematical analysis as a tool for synthesizing, evaluating, improving, and more thoroughly understanding various types of control systems, an attempt has been made to utilize nonlinear analysis techniques to gain a better understanding of the Honeywell adaptive concept. The results of this effort have been very encouraging and will be subsequently discussed.

Of the known techniques for analyzing nonlinear systems, the frequency response method proved to be the most promising. This can be attributed to several factors: (1) This method does not become increasingly complicated as the order (or complexity) of the system increases as does the phase space method, (2) the engineer is able to build on a technique commonly used in the analysis of linear systems which readily provides a better physical feel for the operation of the system, and (3) a simple technique for computing the closed loop frequency response of nonlinear systems already exists.

This technique is described in detail in Reference 1.

DISCUSSION

In order to demonstrate how this method can be used, consider the block diagram of the basic F-94C adaptive pitch rate control shown in Figure 1. Since the time response, $\dot{\Theta}(t)$, to a step input, $\dot{\Theta}_c(t)$, closely resembles the time response of the model, $\dot{\Theta}_M(t)$, when the system is operating satisfactorily, an investigation was made to determine if the frequency response of the complete system closely resembled the frequency response of the model over the same range of command inputs and flight conditions. To do this, the frequency response of the closed loop following the model is calculated and combined with the frequency response of the model. The results of this mathematical operation are compared with the model frequency response to determine what conditions must exist for the two functions to be essentially identical at control frequencies.

The successful application of this technique to the analysis of nonlinear systems is based on the validity of the following assumptions:

1. The bi-stable characteristic of the adaptive controller can be adequately described mathematically by an " equivalent gain" for sinusoidal inputs, i.e. the higher harmonics generated at the output of the nonlinear element can be neglected.

2. The bi-stable element is the only significant nonlinearity in the system.

3. Some correlation does exist between the frequency response and transient response of a control system containing a nonlinear element.

Since these assumptions do not generally hold for nonlinear systems, the valid application of this technique must be established by the degree of agreement between theoretical and experimental results. The correlation between experimental and theoretical results for the Honeywell adaptive control system flight tested in an F-94C is sufficiently good to justify the use of this technique as a means for gaining insight into the basic characteristics of the system.

Before discussing the results of this investigation, a brief outline of the approach used will be presented. Details of the basic approach can be found in Reference 1.

In order to conduct a sinusoidal analysis of the pitch rate adaptive control system shown in Figure 1, the bi-stable characteristic of the adaptive controller is represented by a variable gain, $k(\gamma)$, which is defined in Equation 1.

$$k(\gamma) = \frac{\alpha}{\gamma} \qquad (1)$$

where γ is the peak value of any steady state sinusoidal signal which may exist at the input to the bi-stable element, and α is the peak value of the output, which will be approximated as a sinusoid. Since the output of this device has a constant magnitude and the same polarity as the input, the gain, $k(\gamma)$, is inversely proportional to the magnitude of the input, as seen in Figure 2. If a constant amplitude sinusoidal signal is introduced at the input to the model during closed loop operation, another sinusoidal signal, γ, of some undefined magnitude but the same frequency, will exist at the input to the bi-stable element. The magnitude of this signal defines the gain $k(\gamma)$ for a constant α, as seen in Equation 1. If the input signal frequency is changed, γ and thus $k(\gamma)$ will have a new steady state value.

In fact, a unique value of $K(\gamma)$ will exist for each magnitude and frequency of the input sinusoidal signal to the model. If the exact values of this nonlinear gain can be established for specific values of the input signal, then the closed loop frequency can be calculated with linear equations at each frequency by substituting the appropriate value of $K(\gamma)$. The equation used to perform the necessary calculations is shown below and can be derived very simply from Figure 1.

$$\frac{\dot{\Theta}}{\dot{\Theta}_c}\left[j\omega,\ k_n,(\gamma)\right] = \frac{\left[K_m G_m\ (j\omega)\right]\ K_1 G_1 (j\omega)\ \ k_n(\gamma)\ \ K_2 G_2 (j\omega)}{1 + K_1 G_1\ (j\omega)\ \ k_n(\gamma)\ \ K_2 G_2\ (j\omega)} \tag{2}$$

where the subscript n denotes the particular value of $k(\gamma)$ that exists for a specific value of the magnitude and frequency of the input sinusoidal signal. Two distinct operations are necessary to calculate the closed loop frequency response for a given input signal. First, the correct gain of the bi-stable element must be established, and second, the gain and phase of the frequency response function are then determined from Equation 2. These operations must be repeated for as many points as are needed to obtain adequate information. The resulting function will be valid for only one particular amplitude of the input signal because the response of a nonlinear system is amplitude dependent; consequently the procedure must be repeated for each different magnitude of the input signal.

The usefulness of this technique as an engineering tool would be somewhat limited if separate mathematical operations were required to obtain each point on the many frequency response functions that may be needed. The basic method used during this investigation utilizes a graphical technique to greatly reduce the number of necessary calculations.

In order to demonstrate how the closed loop frequency response of the nonlinear portion of the Honeywell adaptive control system is obtained, it is useful to examine a family of linearized frequency response functions of that portion of the system following the model in the block diagram of Figure 1. These curves are calculated using several constant values of the nonlinear gain k(γ). A typical family of these curves, shown in Figure 3, was calculated from the F-94C pitch axis configuration. (The criteria used to select the maximum gain will be discussed later).

If a sufficient number of these curves is plotted over the range of possible values of k(γ), then points on the actual closed loop frequency response for a specific input signal will exist somewhere in this group of curves. These points can be located exactly by establishing the correct value of k(γ) at specific frequencies of the fixed amplitude input signal.

OPEN LOOP FREQUENCY RESPONSE

Before actually determining the frequency response of the closed loop following the model, it is useful to open this loop and consider some of the parameters in the open loop frequency response exclusive of the model. The magnitude of the open loop frequency response at the particular frequency where the open loop phase lag is 180 degrees is a factor of major importance in determining the closed loop response. It is useful to consider the open loop gain at this frequency to be composed of a constant term and a variable term. The later quantity will be the adaptive controller gain, k(γ). Open loop frequency responses of the F-94C pitch rate system of Figure 1 are shown in Figure 4 with all quantities included except the bi-stable element and the model. Aerodynamic data for flight conditions 1, 3, and 10 have been used, representing the landing condition, sea level - Mach .86, and 22,000 ft - Mach .86 respectively. From Equation 1 it is seen that the bi-stable element gain, k(γ), can assume any magnitude from infinity to values approaching zero. In Figure 4, it can be seen that, exclusive of k(γ), the gain margins of the system for the three flight conditions are 28, 36, and 51 db respectively. When this loop is closed the bi-stable element gain will establish itself at some steady state value. It can be shown that the actual steady state gain established in the bi-stable element is exactly equal to the magnitude of the gain margin at each flight condition.

At the frequency where the open loop phase lag is 180 degrees the system will exhibit a stable limit cycle. Since this residual motion always exists, if the adaptive controller characteristics are as shown in Figure 2, the steady state gain of the bi-stable element can never exceed the value which establishes this limit cycle. Consequently, the inclusion of this device provides the system with a variable gain which will always establish itself under steady state conditions at the maximum value that the system can have at any given flight condition.

In Reference 3, Ljungwe shows that the amplitude of the limit cycle motion is directly proportional to the product of bi-stable element output and the surface effectiveness, $M_{\delta e}$. It can further be shown that the limit cycle amplitude decreases as the limit cycle frequency increases if $k(\gamma)_{Max}$ remains constant.

During satisfactory operation of the adaptive system the limit cycle can be superimposed on the response to input sinusoidal excitations so that no further consideration of its existence is needed to calculate the closed loop frequency response other than to establish the maximum value of $k(\gamma)$. This means that the gain characteristics of the bi-stable element used in the F-94C can be accurately represented as shown in Figure 5 for sinusoidal signals less than or equal to twice the cut-off frequency of the model.

Before concluding the discussion of the open loop frequency response it should be noted that the characteristics of the airplane have no effect on the limit cycle frequency in a pitch rate system. This is true because the 90 degrees of phase lag introduced by the airplane at high frequencies is essentially cancelled by the phase lead in the switching logic; consequently, the limit cycle frequency does not change with flight condition, and it is determined primarily by the control lags in the system.

CLOSED LOOP FREQUENCY RESPONSE

As mentioned previously, a graphical technique is used to calculate the closed loop frequency response of the nonlinear part of the Honeywell adaptive system. A detailed discussion of the method will be presented in a forthcoming paper. The actual procedure is as follows:

(1) Calculate and plot families of linearized frequency responses of the closed loop portion of the adaptive control system. These are obtained by replacing the nonlinear gain $k(\gamma)$, by a constant for each curve. One of these should be the largest gain that can exist at each flight condition to be investigated.

(2) Calculate an open loop frequency response from the transfer function $K_2G_2(S)/K_mG_m(S)$, where $K_2G_2(S)$ represent the linear portion of the system between the bi-stable element and the output of the system.

(3) Overlay the open loop frequency response obtained in step 2 on the family of curves obtained in Step 1. The zero db lines for the two graphs are separated by the ratio $\alpha/\dot{\theta}_c$, where α is the output of the bi-stable element and $\dot{\theta}_c$ represents the magnitude of the sinusoidal input.

Typical results for Step 3 are shown in Figure 6 with the dashed lines representing the calculation made in Step 2.

The amplitude of the actual closed loop frequency response is determined by following the maximum $k(\gamma)$ curve until this function is intersected by the open loop function for a specific value α/θ_c. The response then follows the dashed line for higher frequencies as long as this function is less than the amplitude of the $k(\gamma)_{Max}$ curve. Curves for three values of this ratio are shown in Figure 6. Points on the phase curve can be found at each frequency where $k(\gamma)$ is known. This occurs wherever there are intersections between the open loop function and members of the family of linearized closed loop curves. Typical results are shown in Figure 7. To obtain the total response of the adaptive control system, the frequency response of the model is combined with the frequency of the nonlinear system.

RESULTS

The simulation and flight test results of the F-94C system show that the response of the adaptive flight control system closely resembles the model over most of the flight envelope of the airplane. Under certain conditions, however, the response of the airplane differs from the model. Results of the theoretical investigation show that the frequency response of the complete system closely resembles the frequency response of the model under the same conditions that the transient response of the system is essentially identical to the transient response of the model. In Figure 8, typical re sults are shown for 3 values of the ratio α/θ_c. It can be seen that very good correlation exists between the theoretical and experimental results and that both are almost identical with the model. Frequency responses of most flight conditions are essentially identical to the ones shown in Figure 7. Frequency response analyses of two other flight conditions are also included for which the transient response of the system does not closely follow the model for 6 and 12 db ratios of α/θ_c. Frequency response for these two flight conditions are shown in Figures 9 and 10. It should be noted, however, that these frequency responses do closely resemble the model if the ratio α/θ_c is equal to or greater than 20 db.

CORRELATION BETWEEN TRANSIENT AND FREQUENCY RESPONSE OF THE SYSTEM

Transient responses obtained from an analog computer study are shown in Figures 11, 12, and 13. It should be noted that the particular trend in the transient responses for different values of the ratio α/θ_c is also evident in the frequency responses for the same ratios. An attempt was made to calculate the transient response directly from the frequency response of the adaptive control system using the technique described in Reference 2. The results were accurate only when the model dominated the frequency response at all frequencies below the natural frequency of the model. Investigations of the correlation between frequency and transient response of non-linear systems is continuing.

CONCLUSIONS

The use of a bi-stable element in an adaptive control system greatly enhances the possibility of obtaining greater reliability through simplification of the control system. Inclusion of a bi-stable element in a feedback control system provides the system with a variable gain device which will always establish itself at the maximum value that the system can have at any given flight condition. Thus it can be said that the system is self compensating.

With this type of nonlinear device the tight loop required to minimize errors to commands is obtained without the stability problems normally encountered in high gain linear systems. This system exhibits two unique characteristics: for small errors the gain is large, for large error signals the gain is low. This means that the system will operate at maximum bandwidth for small errors (good following of the model) and yet will operate at low bandwidth with sufficient phase margin to provide good response to disturbance inputs such as gusts. In addition wide variations in the airplane static stability and damping have negligible effect on the performance of the system.

Uniform performance characteristics can be obtained for the F-94C system over the complete flight envelope if the ratio of $\frac{\alpha M_{\delta e}}{\dot{\theta}_c}$ is equal to or greater than approximately 30 db, where α is the output of the bi-stable element, $M_{\delta e}$ is the magnitude of the elevator surface effectiveness, and $\dot{\theta}_c$ is the magnitude of the pitch rate command. Since this ratio was not greater than 30 db for all F-94C flight conditions some deteriorations can be expected at flight conditions where the surface effectiveness is low.

Whenever a bi-stable element is included in a feedback loop, a limit cycle will exist. The limit cycle magnitude is directly proportional to the product $\alpha M_{\delta e}$. Since α was held constant in the F-94C system the limit cycle increases in magnitude for high dynamic pressure flight conditions. In fact, the output of the bi-stable was established on the basis of the maximum acceptable limit cycle amplitude. Since this occurs at the highest elevator effectiveness flight conditions, a compromise in performance exists at the flight conditions having low elevator effectiveness.

One possible improvement is to vary the output of the bi-stable element so as to keep the product $\alpha M_{\delta e}$ constant at all flight conditions. If the limit cycle amplitude is kept constant by varying the output of the bi-stable element as the surface effectiveness changes, the necessary value M_{δ_e} can be obtained. Extensive studies have shown a significant improvement in performance with this addition to the system. The principal disadvantage of this approach is that the surface motion required to sustain the limit cycle becomes large at flight conditions where the surface effectiveness is low.

A more desirable modification is to keep the adaptive controller maximum gain at the largest possible value at all flight conditions, with a very small limit cycle at the control surface, but with an adequate value of αM_{δ_e} so that the responses to commands will follow the model. A method for accomplishing this is being included in an adaptive 3 axis automatic flight control system which will be flight tested in the near future. A description of this system is to be presented by Mr. David Mellen of Minneapolis-Honeywell.

REFERENCES

1. A Generalized Method for Determining the Closed Loop Frequency Response of Nonlinear Systems, Luther T. Prince, Jr., AIEE Paper 54-280.

2. A Series Method of Calculating Control-System Transient Response from the Frequency Response, D. V. Stallard, AIEE Paper 55-192.

3. WADC Technical Report 57-349 (P art 3) 10 December 1957.

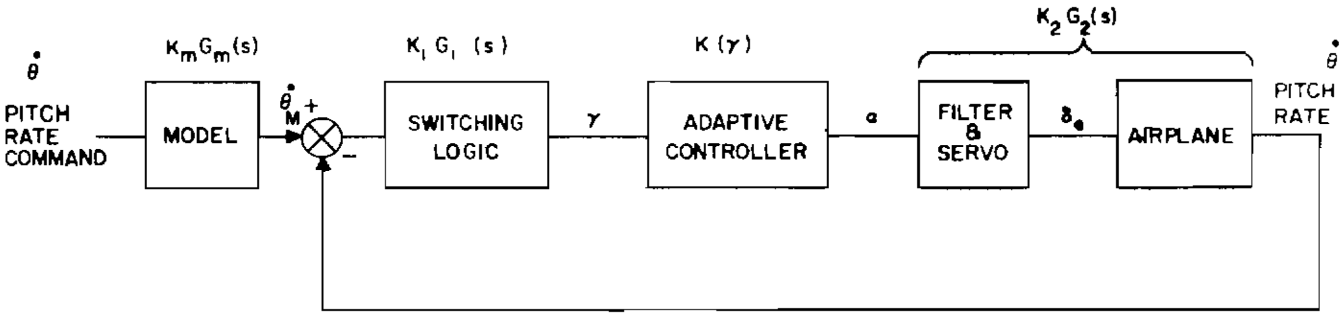

Figure 1. F-94C Adaptive Pitch Rate Control System

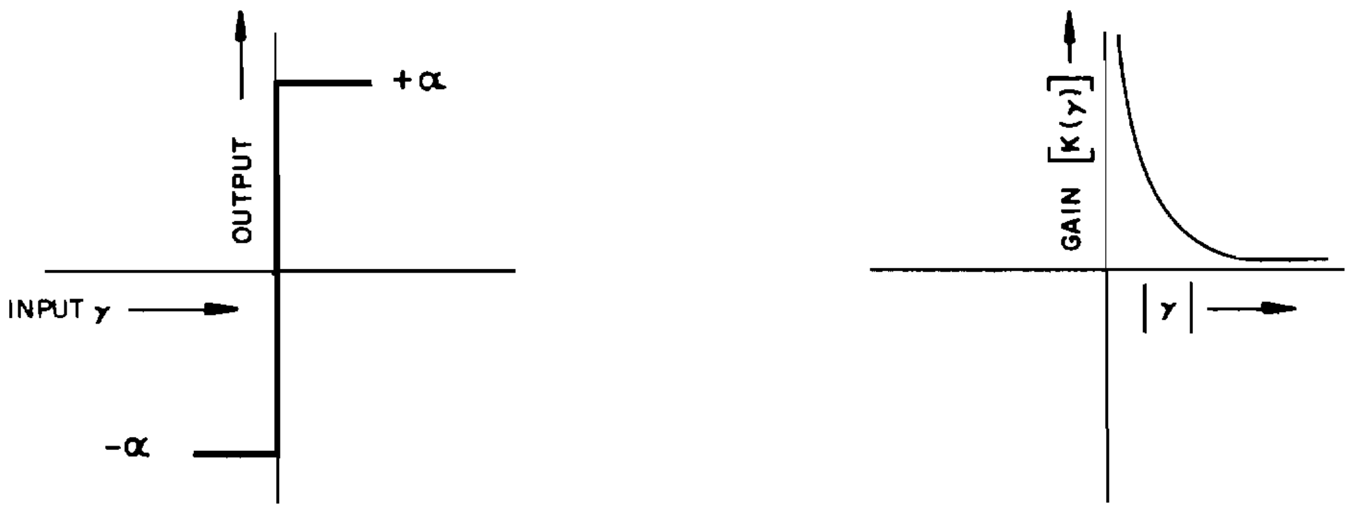

Figure 2. Bistable Element Gain Characteristics

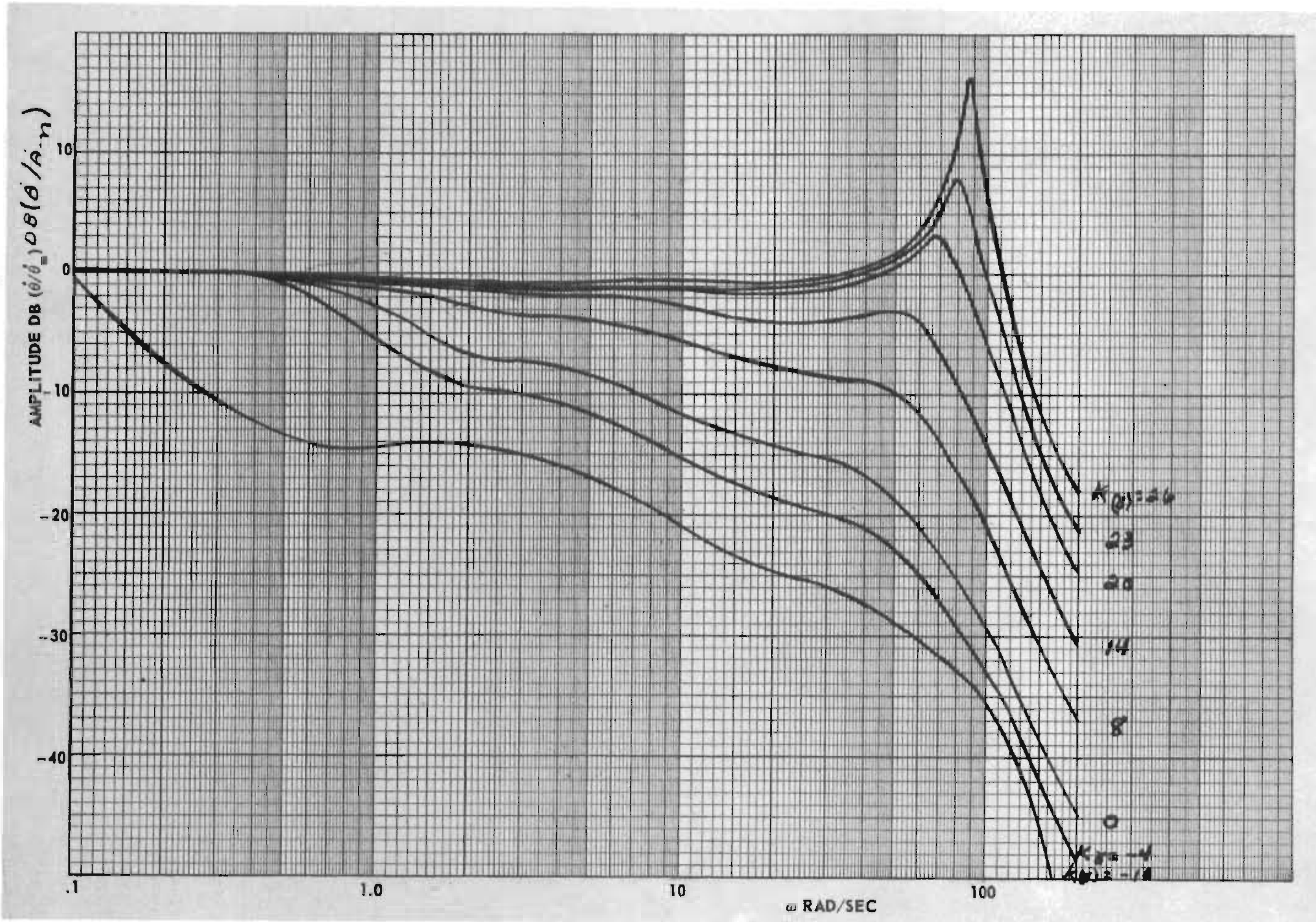

Figure 3a. Typical Family of Linearized Closed-loop Frequency Responses - Amplitude

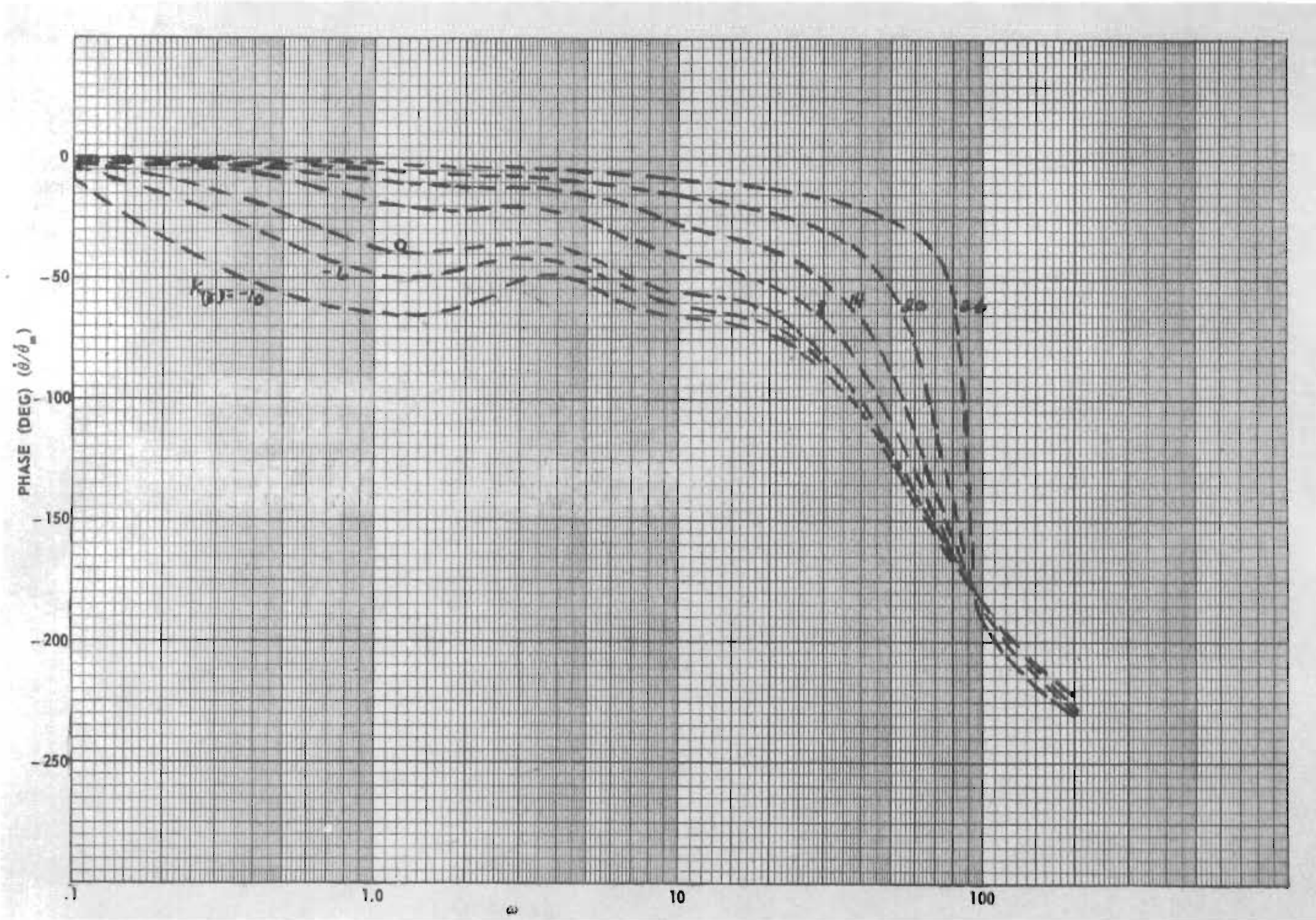

Figure 3b. Typical Family of Linearized Closed-loop Frequency Responses - Phase

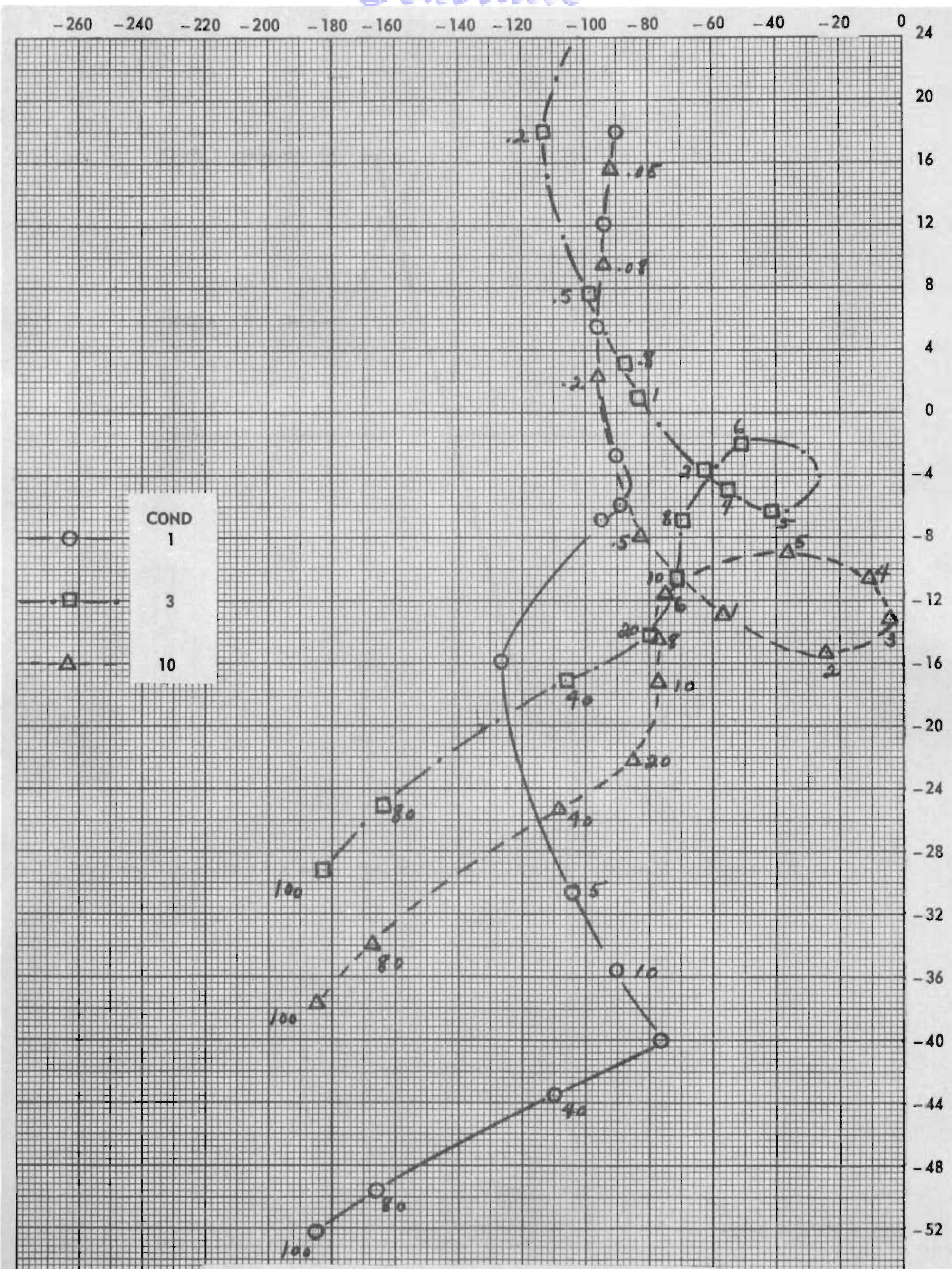

Figure 4. Open-loop Frequency Response

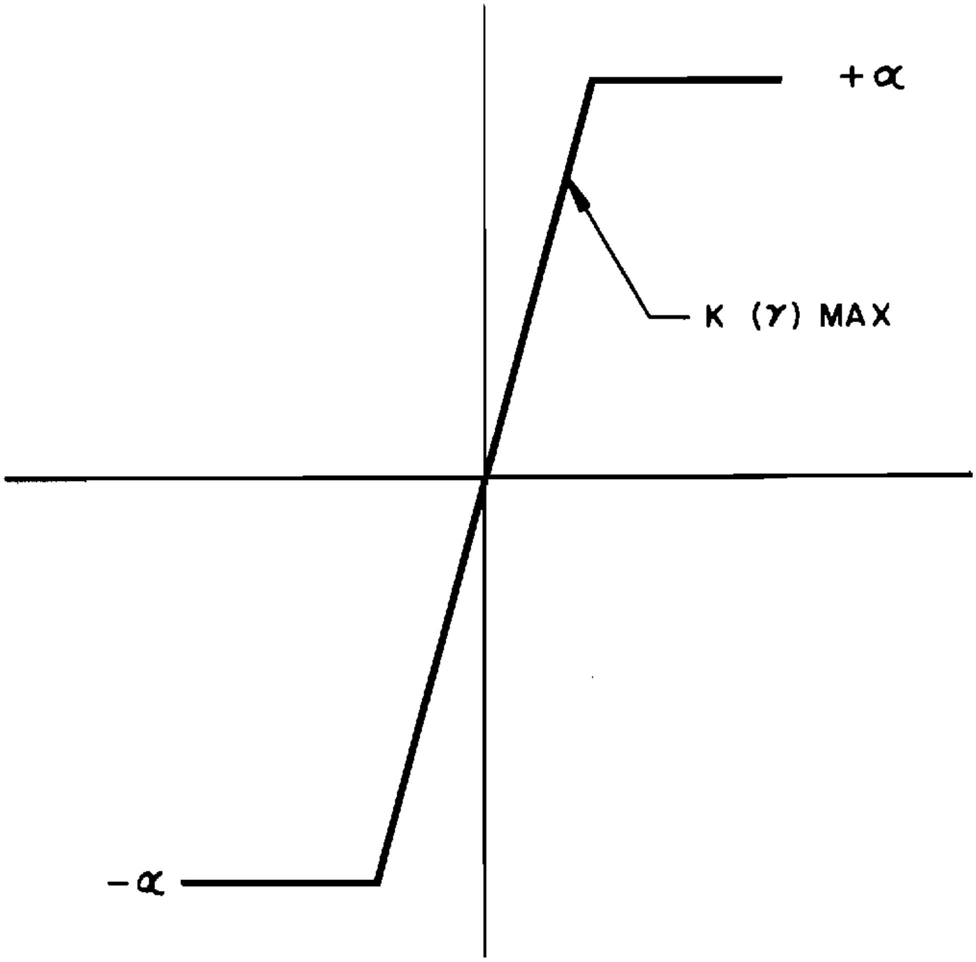

Figure 5. Modified Gain Characteristics of a Bistable Element

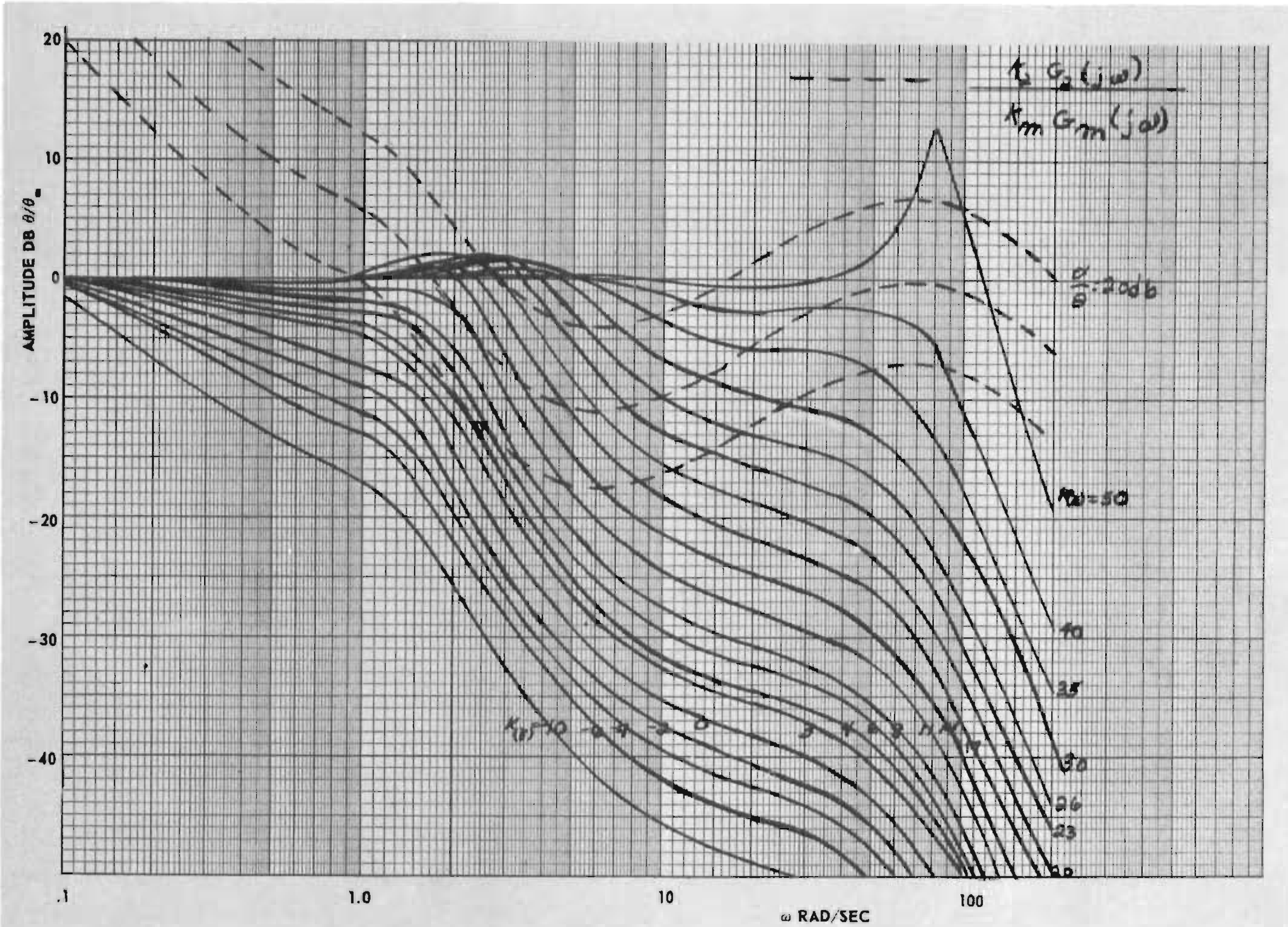

Figure 6. Graphical Solution of Bistable Element Gain during Closed-loop Operation

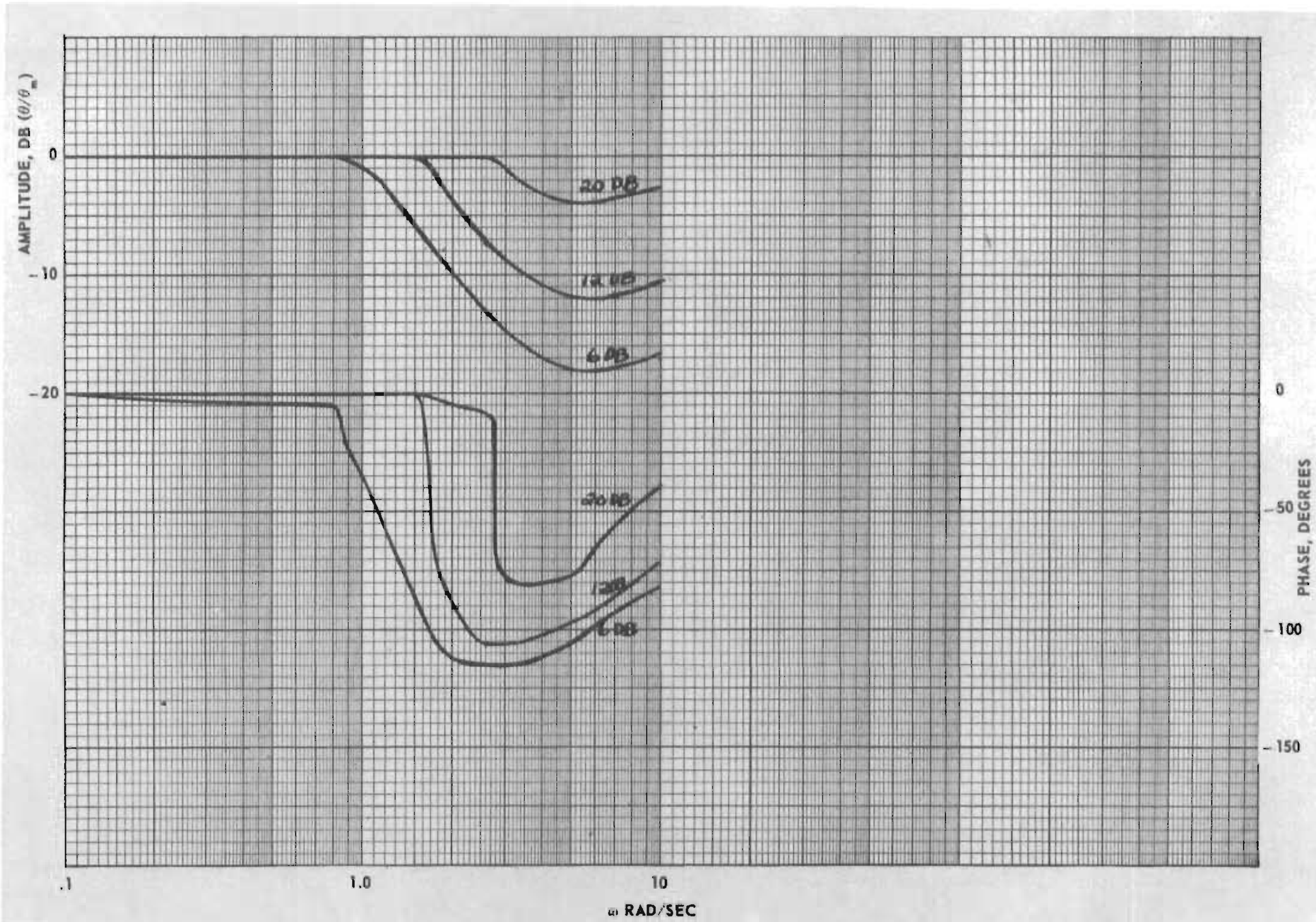

Figure 7. Typical Nonlinear Closed-loop Frequency Responses Excluding the Model

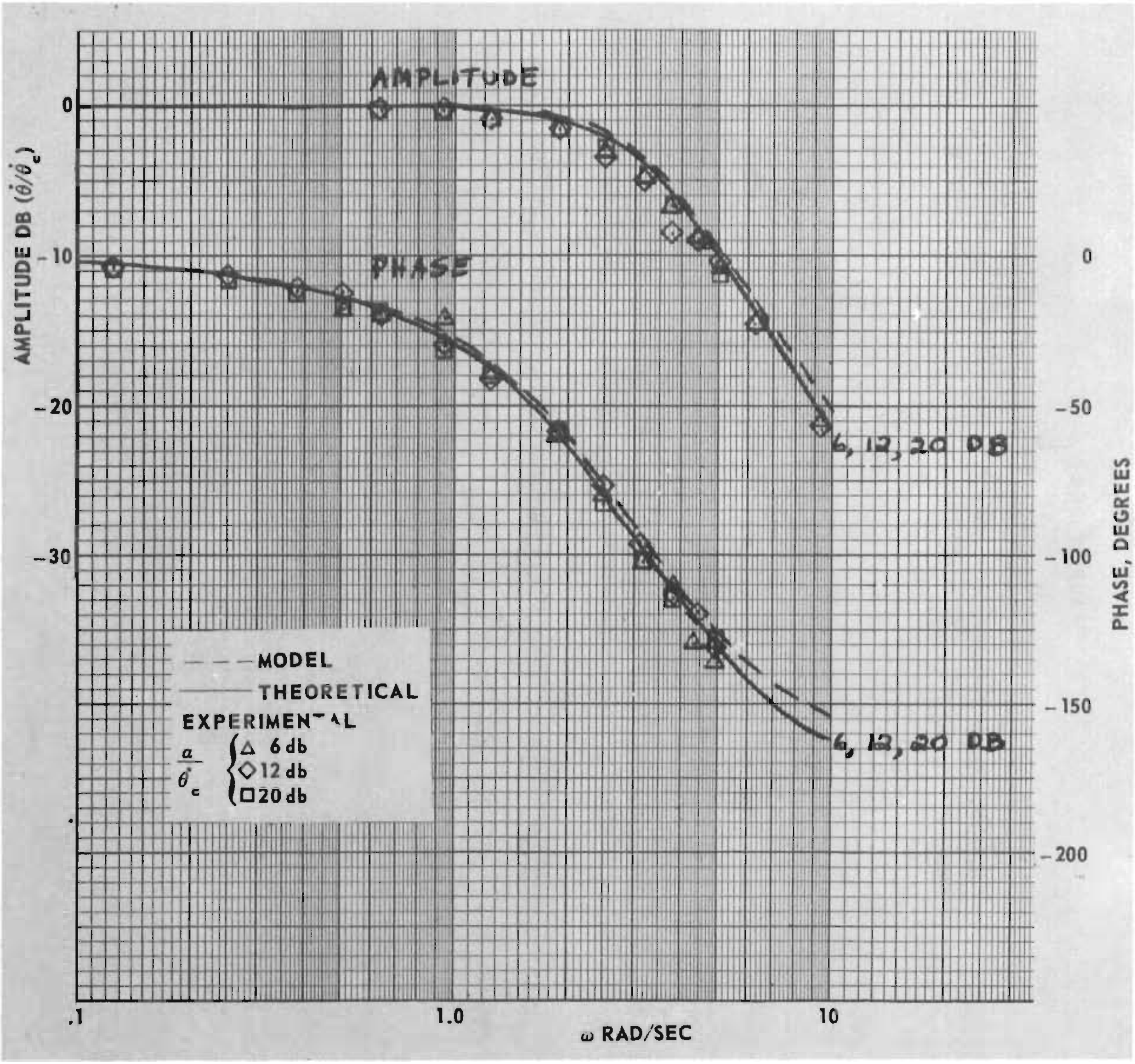

Figure 8. Frequency Response of the Complete Adaptive Control System - Condition 3

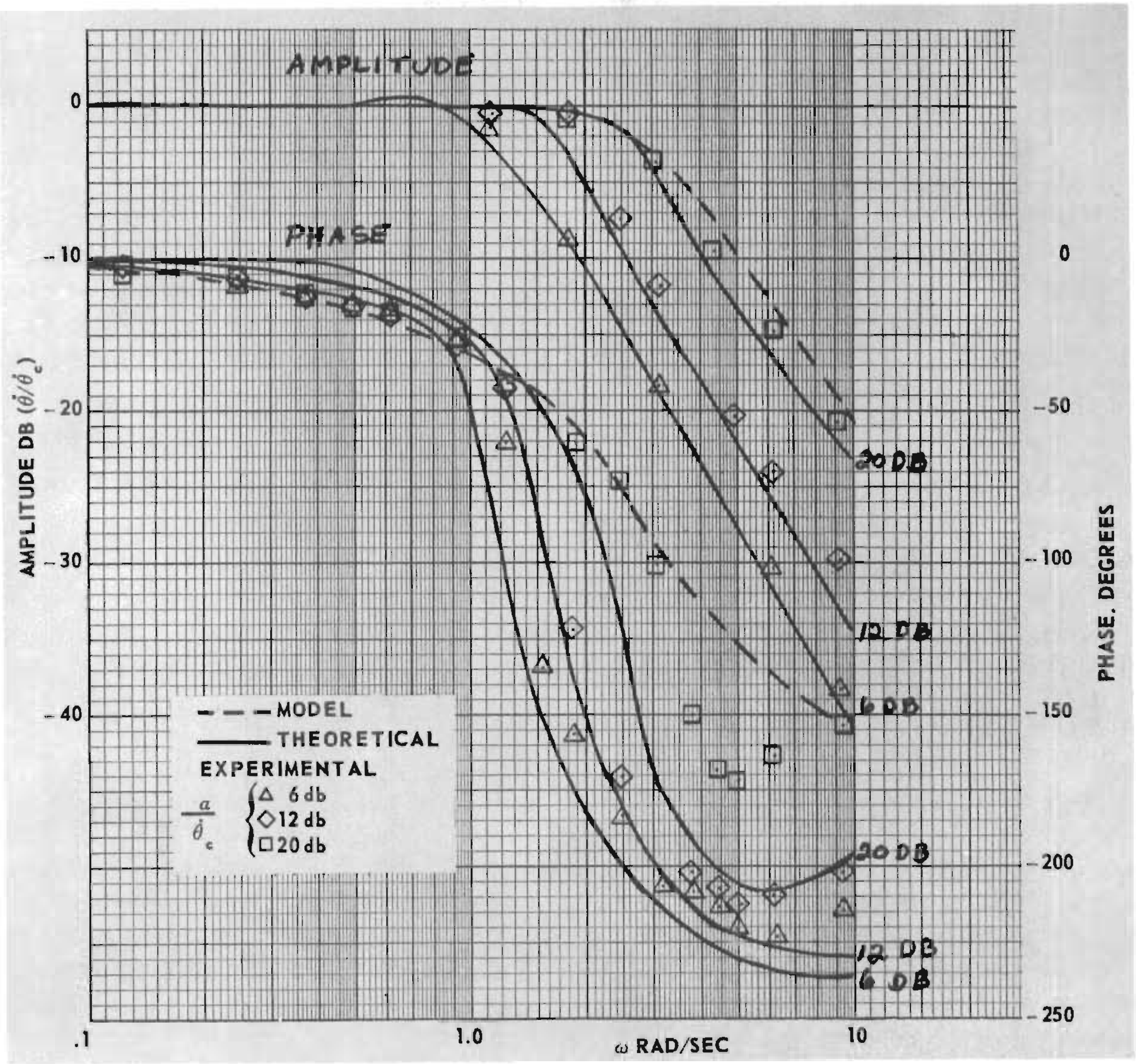

Figure 9. Frequency Response of the Complete Adaptive Control System - Condition 1

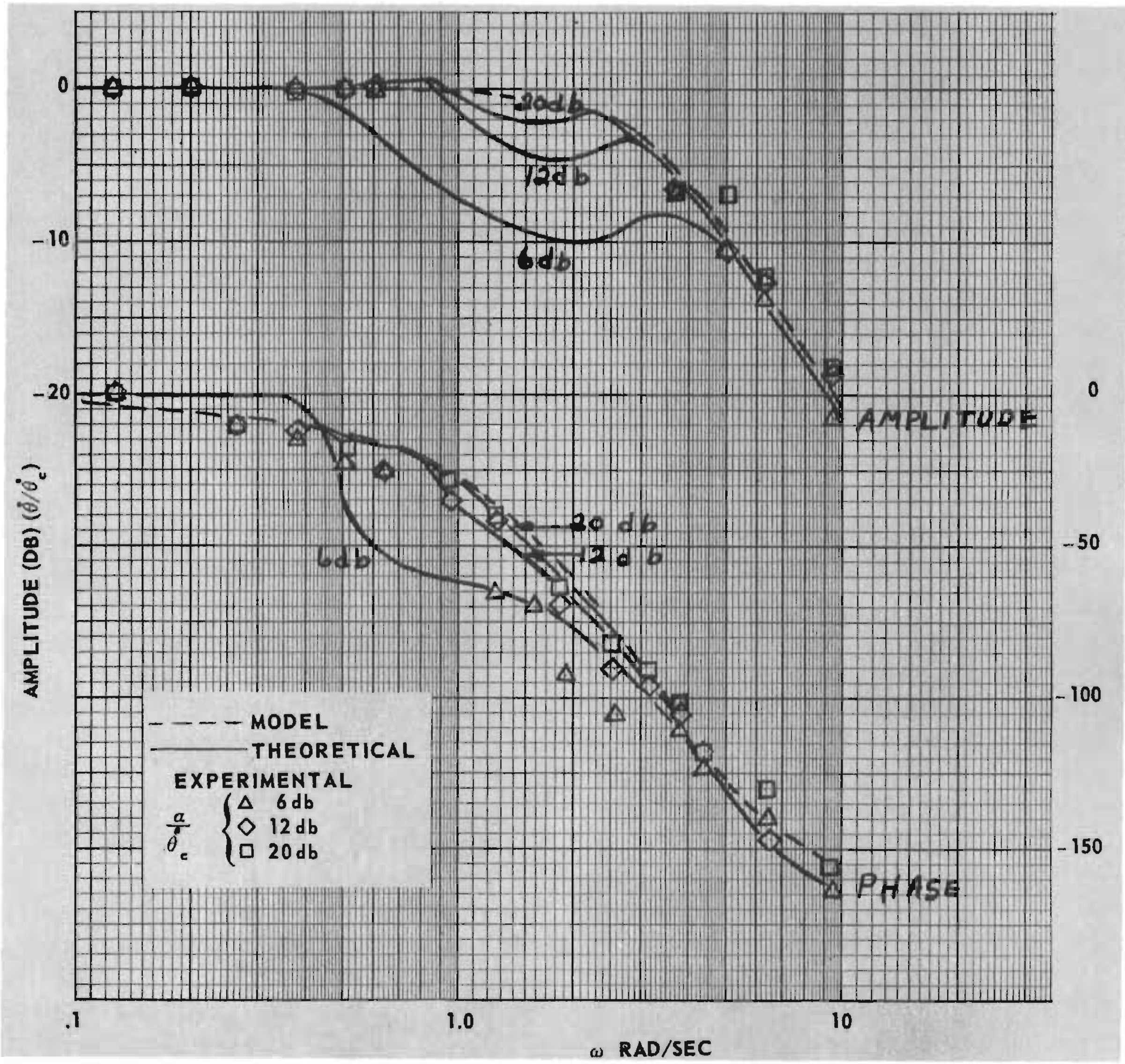

Figure 10. Frequency Response of the Complete Adaptive Control System - Condition 10

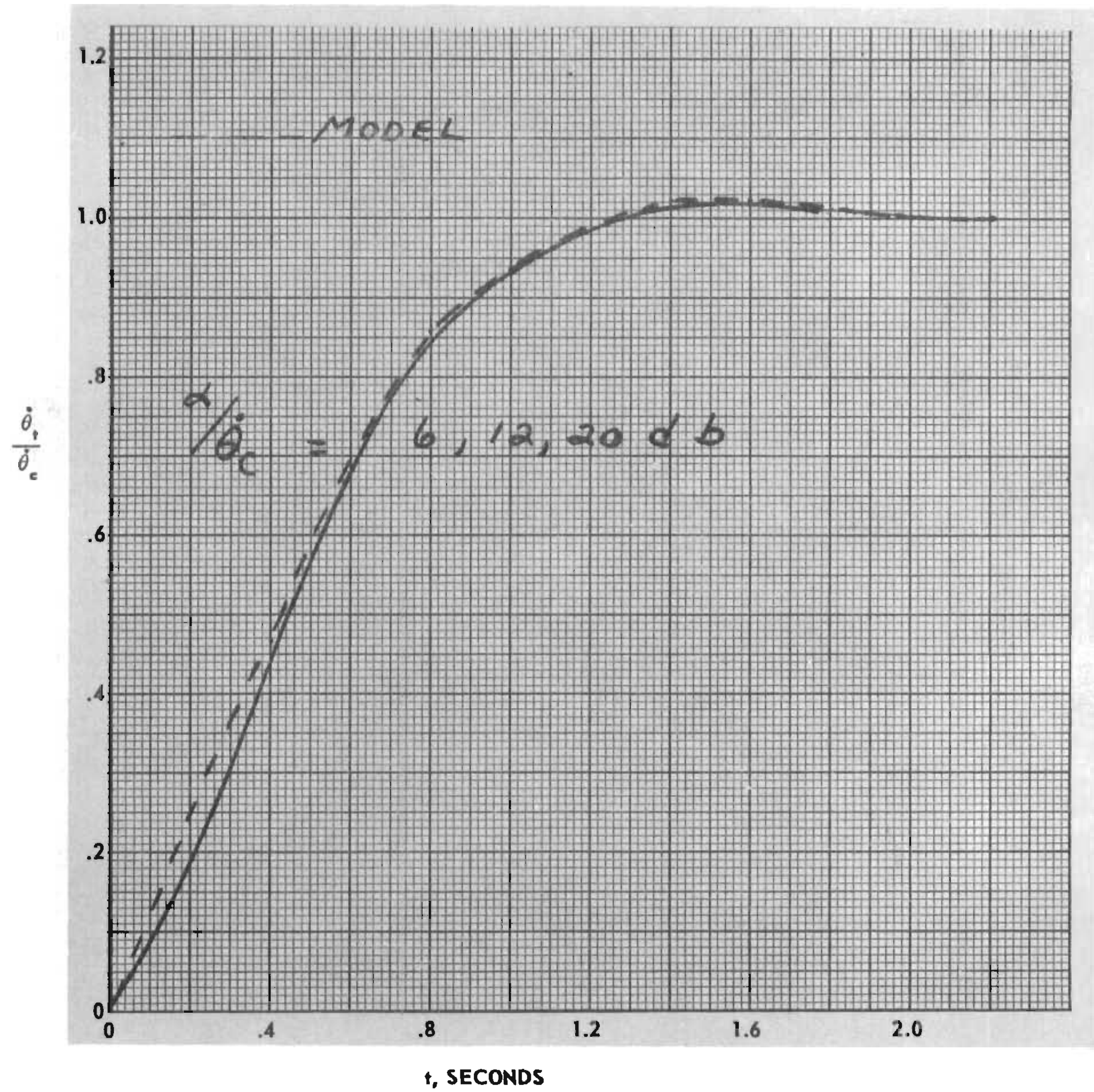

Figure 11. Transient Response of the Complete Adaptive Control System - Condition 3

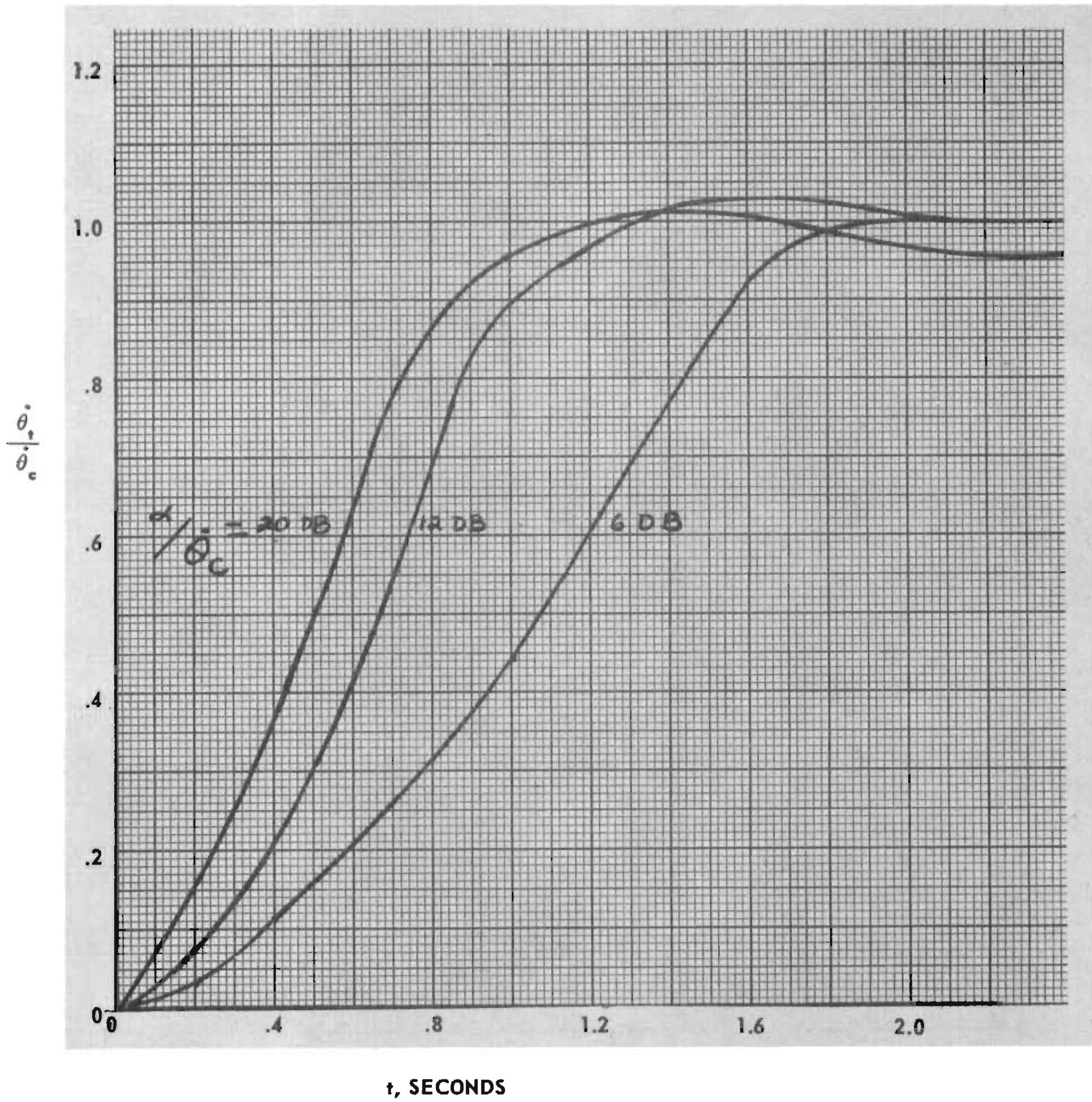

Figure 12. Transient Response of the Complete Adaptive Control System - Condition 1

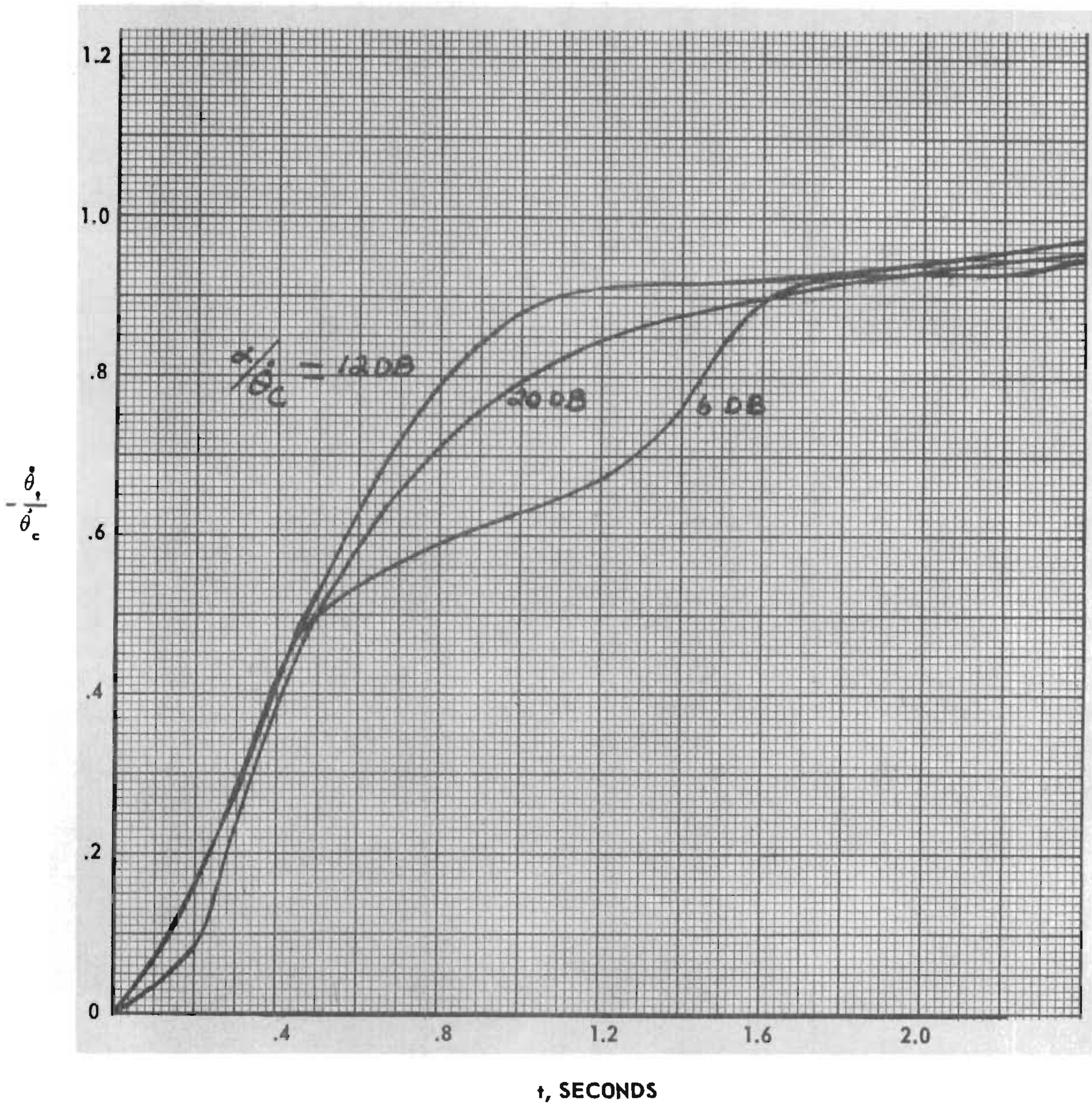

Figure 13. Transient Response of the Complete Adaptive Control System - Condition 10