THE "PAPER PILOT"—A DIGITAL COMPUTER PROGRAM TO PREDICT PILOT RATING FOR THE HOVER TASK

JAMES D. DILLOW

MAJOR, USAF

FOREWORD

This report describes the results of an in-house investigation into a new approach to specifying flying qualities, with specific application to VTOL hover dynamics. The method of analysis is based directly on a method developed by Ronald Anderson and described in AFFDL-TR-69-120, "A New Approach to the Specification and Evaluation of Flying Qualities." For this reason, this report emphasizes a description of the mathematical model used in the analysis, the digital computer program, and the numerical results. For a detailed justification of the method of analysis, the reader is referred to AFFDL-TR-69-120.

The work was performed under Project 8219, Task 821909, Work Unit 004. This project is a joint Air Force-Navy-Army development of a V/STOL Flying Qualities Specification. The basic material was prepared over the period of June 1969 to June 1970. A portion of the results are reported in draft form as FDCC-TM-69-3 and in a paper presented to the 1970 Joint Automatic Control Conference, entitled "An Application of Pilot Models to VTOL Flying Qualities Evaluation." The digital computer program described in FDCC-TM-69-3 was revised to provide greater flexibility and to decrease computation time; the revised computer program is described in this report, as well as additional results.

Several Air Force Flight Dynamics Laboratory personnel contributed to this effort: Ronald Anderson gave many valuable innovations to this work, along with great encouragement and useful guidance; Captain Bruce Kujawski and Alonzo Connors lent their expertise to the problems of debugging the computer programs. The digital computer subroutines CAL, CALD, and STAB were developed from a digital subroutine developed by Ferit Konar, Gunter Stein, and Michael Ward of Honeywell, Inc.

Copies of the "paper pilot" program deck are available on request from:

"Paper Pilot" (AFFDL/FGC)
Air Force Flight Dynamics Laboratory
Wright-Patterson Air Force Base, Ohio 45433

This report was submitted by the author 19 February 1971.

This technical report has been reviewed and is approved.

C. B. Westbrook

C. B. WESTBROOK
Chief, Control Criteria Branch
Flight Control Division
Air Force Flight Dynamics Laboratory

ABSTRACT

A mathematical model for predicting the pilot rating of the flying qualities of a VTOL aircraft in the precision hover mode is described. The model includes the following elements: (1) the longitudinal equations of motion for the VTOL aircraft in hover; (2) a stochastic gust model which describes disturbances to the aircraft; (3) a fixed form pilot model which has four free parameters; and (4) a cost functional which is made up of measures of aircraft performance and pilot workload. The four free pilot parameters of the pilot model are selected to minimize the cost functional. These parameters are adjusted to ensure a 20% stability margin in pilot gains and then used to compute a "paper pilot" rating of the flying qualities of the VTOL aircraft in the precision hover mode.

The mathematical equations and digital computer program used to exercise the model are described.

The "paper pilot" rating was computed for 79 aircraft configuration/gust intensity combinations. The aircraft configurations considered include cases with control lag, stability augmentation system lag, and limited pitch rate authority in the stability augmentation system. The "paper pilot" ratings are compared to actual pilot ratings obtained in fixed base simulation. The difference between the actual pilot ratings and the "paper pilot" ratings has a mean of .14 and a standard deviation of .63 out of a 10 point rating scale.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

TABLE OF NOTATION

A	A n x n matrix of coefficients used in the state variable form of the differential equations.
c	A constant used in the conversion from radians to degree, c = 57.3.
g	Gravitational constant, 32.2 ft/sec^2
g_i	$\partial J/\partial k_i$
I	The identity matrix
IER	A computer output code (See Table 2)
J	Cost functional used in minimization procedure
K_{p_x}	Pilot gain in displacement, deg/ft
K_{p_θ}	Pilot gain in pitch, in/deg
k	$k = \begin{bmatrix} K_{p_\theta} \\ T_{L_\theta} \\ K_{p_x} \\ T_{L_x} \end{bmatrix}$
M	Pitching moment divided by pitching moment of inertia
M_e	The effective angular acceleration in pitch generated due to the SAS
M_{ra}	Maximum M_e due to limited pitch rate authority of the SAS
M_q	$\partial M/\partial q$
M_{qe}	Effective M_q for the linear equivalent of the nonlinear SAS
M_u	$\partial M/\partial u$

M_δ	$\partial M/\partial \delta$
M_θ	$\partial M/\partial \theta$
n	Order of system differential equations
n_g	A subscript identifying the system state which is used to represent the gust intensity; i.e., $z_{n_g} = u_g$
PERF	Element of J depending on performance
PR	Predicted pilot rating ("Paper Pilot" rating)
q	Pitch rate, deg/sec
R	Covariance matrix for v(t)
R_1	Element of PR depending on performance
R_2	Element of PR and J depending on pilot lead in pitch loop
R_3	Element of PR and J depending on pilot lead in displacement
$R_{1_{max}}$	Maximum value of R_1
rms	Root mean square
SAS	Stability Augmentation System
s	Laplace operator
T_{L_x}	Pilot lead time in longitudinal displacement, sec
T_{L_θ}	Pilot lead time in pitch, sec
t	Time
u	Longitudinal aircraft velocity, ft/sec
u_g	Longitudinal gust velocity, ft/sec
VTOL	Vertical takeoff and landing
v	nth order Gaussian white noise vector process
v_g	Gaussian white noise process used in model of gust disturbances
W_1	Weighting in J on σ_q
W_2	Weighting in J on σ_θ

W_3 Weighting in J on σ_u

W_4 Weighting in J on σ_x

W_5 Weighting in J on $|T_{L_\theta}|$

W_6 Weighting in J on $|T_{L_x}|$

W_7 Constant in J

X Force in the x-direction divided by mass

X_u $\partial X/\partial u$

X_δ $\partial X/\partial \delta$

x Longitudinal displacement

Y_{p_x} Pilot describing function in the displacement loop

Y_{p_θ} Pilot describing function in the pitch loop

y A dummy state used in the pilot reaction time delay representation

Z Steady state covariance matrix for z(t)

z nth order system state vector

ΔK_{p_θ}, ΔT_{L_θ}, ΔK_{p_x}, ΔT_{L_x} The difference between the indicated actual pilot matched parameter and the corresponding "paper pilot" parameter

ΔPR The difference between actual pilot rating and "paper pilot" rating

$\Delta\sigma_\theta$, $\Delta\sigma_q$, $\Delta\sigma_x$, $\Delta\sigma_u$	The difference between the indicated actual pilot performance and the corresponding "paper pilot" performance
δ	Control stick deflection
δ'	Control stick deflection without pilot reaction time delay
$\delta(t-\tau)$	Impulse function
δ_e	The effective longitudinal control due to a control lag
ε	Constant used in test for convergence
θ	Pitch angle, deg
θ_x	Position loop command angle, deg
θ_ε	$\theta_x - \theta$
σ_j	The standard deviation of z_j
σ_g	The standard deviation of the gust velocity
σ_θ	The standard deviation of θ
σ_q	The standard deviation of q
σ_x	The standard deviation of x
σ_u	The standard deviation of u
τ	Pilot reaction time delay
τ_e	Control lag time constant
τ_g	SAS feedback lag time constant

ω_b Spectral break frequency of the gust velocity

Superscripts

$(\cdot)$ Derivative with respect to time, d/dt

$(')$ Matrix transpose

I. INTRODUCTION

1.1 BACKGROUND

In 1966, the Air Force Flight Dynamics Laboratory began work to write a handling qualities specification for Vertical Take Off and Landing (VTOL) Aircraft. It became apparent that handling qualities specifications for conventional aircraft could not be easily extended to the unconvention dynamics associated with VTOL aircraft in certain flight modes. Particular difficulty was encountered in describing concise and meaningful requirements which are compatible with a variety of stability and control augmentation schemes associated with VTOL aircraft in take-off, hover, and landing. An additional problem lies in the fact that the usual handling qualities specifications do not give designers clear guidelines for the design of a control system which will result in good handling qualities.

In June 1969, R. O. Anderson began an investigation into an alternate method of specifying handling qualities for VTOL aircraft. See Reference [1]. Anderson's original work resulted in a model for predicting pilot ratings which is based upon measured performance and matched pilot parameters for a VTOL aircraft flying in the precision hover mode. The model developed by Anderson is based on the concept of a mathematical model of the human operator, or pilot, which includes pilot gains and pilot lead terms (for additional details see section 2.3). It has been generally observed (Reference [2]) that pilot opinion rating is directly related to pilot lead generation, where the rating tends to increase (less desirable) with a corresponding increase in pilot lead generation. It was also shown in Reference [2] that the pilot opinion rating tends to increase with an increase in mean absolute tracking error for a precision tracking task. In Reference [3], the results of a fixed-base flight simulation studies of VTOL aircraft in the hover mode and low speed flight are reported. These results include pilot opinion rating, rms performance data, and matched pilot parameters for a precision hover task in the longitudinal axis. The pilot opinion ratings reported in Reference [3] are based on the Cooper pilot rating scale with numerical ratings from 1 to 10 and where a rating of 1 is excellent and a rating of 10 is catastrophic. See Table I for a more detailed description of the Cooper pilot rating scale. Based on the results of Reference [2] and the data of Reference [3], Anderson developed an expression for pilot rating as a function of measured rms performance and the matched pilot parameters. This expression gave numerical pilot ratings which correlated extremely well with the actual pilot ratings given in Reference [3], for those cases where measured performance data and matched pilot parameters were available.

TABLE I

COOPER PILOT RATING SCALE

Operating Conditions	Adjective Rating	Numerical Rating	Description	Primary Mission Accomplished	Can Be Landed
Normal Operation	Satisfactory	1	Excellent, includes optimum	Yes	Yes
		2	Good, Pleasant to fly	Yes	Yes
		3	Satisfactory, but with some mildly unpleasant characteristics	Yes	Yes
Emergency Operation	Unsatisfactory	4	Acceptable, but with unpleasant characteristics	Yes	Yes
		5	Unacceptable for normal operation	Doubtful	Yes
		6	Acceptable for emergency condition only[1]	Doubtful	Yes
No Operation	Unacceptable	7	Unacceptable even for emergency condition[1]	No	Doubtful
		8	Unacceptable - dangerous	No	No
		9	Unacceptable - uncontrollable	No	No
	Catastrophic	10	Motions possibly violent enough to prevent pilot escape	No	No

1 Failure of a stability augmenter.

This model has a number of advantages over previous handling qualities specifications. First of all, the predicted pilot rating is strongly correlated to an actual pilot evaluation of the handling qualities of the aircraft. Furthermore, the model is suited to a variety of control augmentation schemes, since the form of the pilot rating expression is not dependent on the order of the system of equations describing the hover dynamics. In addition, the model directly relates closed loop aircraft performance to pilot rating and the model is sensitive to a measure of pilot workload in the sense that increased lead generation corresponds to increased work load in the control task.

At this point the model had one essential drawback. That is that the measured pilot parameters are not available without detailed hover simulation. In order to deal with this problem, Anderson hypothesized that the pilot parameters can be selected to minimize the predicted pilot ratings. This premise has the physical analog that the pilot, in his adaptable way, adjusts his "parameters" to minimize his rating of the aircraft handling qualities, where the lower the rating, the better the handling qualities.

The problem was programmed for the analog computer. Several aircraft configurations were examined and the preliminary results indicated the suitability of the optimal hypothesis. The results are reported in Reference [1]. It became clear at this time that a fully automatic scheme was required for the minimization procedure to avoid the manual drudgery of exercising the model on the analog computer. This led to the digital computer program described in this report, dubbed the "Paper Pilot", for predicting pilot rating of a VTOL aircraft in the hover mode.

1.2 OVERVIEW

A mathematical model for predicting pilot rating of the flying qualities of a VTOL aircraft in the hover mode is described. The predicted pilot rating is based on a Cooper rating scale, 1 to 10. See Table I. The model includes the following key elements:

(1) The longitudinal equations of motion for the VTOL aircraft flying in the hover mode.

(2) A stochastic input model representing gust disturbances to the aircraft.

(3) A fixed form pilot model with four free parameters. The parameters include two pilot gains and two pilot leads.

(4) A cost functional which is made up of a measure of hover performance and pilot workload. For a given aircraft configuration and gust intensity, the cost functional is a real valued function of the pilot parameters.

The scheme is to first find the pilot parameters which minimize the cost functional (4). The pilot gains are then adjusted, if necessary, to provide at least a 20% stability margin in the pilot gains. The resulting pilot parameters are then used to compute a "Paper Pilot" rating of the handling qualities of the VTOL aircraft in the hover mode. Except for a slight modification, this expression for the predicted pilot rating is the same as the one given in Reference [1].

The stability test used in the computer program and the steady state covariance equations are described. The gradient of the cost functional is derived. The computer program is briefly described along with the input-output details. The Program Listing is given in Appendix A.

The model was exercised for 79 aircraft configuration/gust intensity combinations. The aircraft configurations exercised include 32 cases with the usual longitudinal hover dynamics, 21 cases with a control lag, 14 cases with a SAS lag, 9 cases with both control and SAS lags, and 3 cases with limited pitch rate authority of the SAS. The "paper pilot" ratings are compared with actual pilot ratings obtained in fixed base simulation of the hover task as reported in References [3] and [4] and the ratings compare quite well. The difference between the actual pilot rating and the "paper pilot" rating for the 79 cases considered has a mean of .14 and a standard deviation of .63 out of a 10 point scale. This is comparable to the deviations in rating from pilot to pilot and even for the same pilot in repeated runs. For two cases run with high longitudinal gust intensities (σ_g = 20.6 ft/sec), the "paper pilot" ratings do not agree well with actual pilot ratings. The same difficulty was noted in Reference [1] and the data indicates that the model, in its present form, is not valid for rms longitudinal gust intensities of greater than 10.3 ft/sec. This does not appear to be a serious restriction inasmuch as an rms gust intensity of 10 ft/sec represents severe turbulence.

There are also 4 cases with both a control lag and SAS lag where the "paper pilot" rating is in poor agreement with the actual pilot ratings. In all of these cases, the "paper pilot" rating was lower than the actual pilot rating. This result suggests a possible deficiency in the model for higher order systems; however, the results are not conclusive.

Based on the computational results for those cases where actual performance data is available, it appears that the model can be adjusted to predict actual performance. This adjustment should also improve the pilot rating predictions, provided the control and SAS lags are less than or equal to .2 sec.

II. MATHEMATICAL MODEL

The hovering task model equations used in the digital simulation were adapted from Appendix B of Reference [3] and Appendix B of Reference [4]. The loop closures with a pilot model were in the manner of Reference [3] as represented in the Figure below.

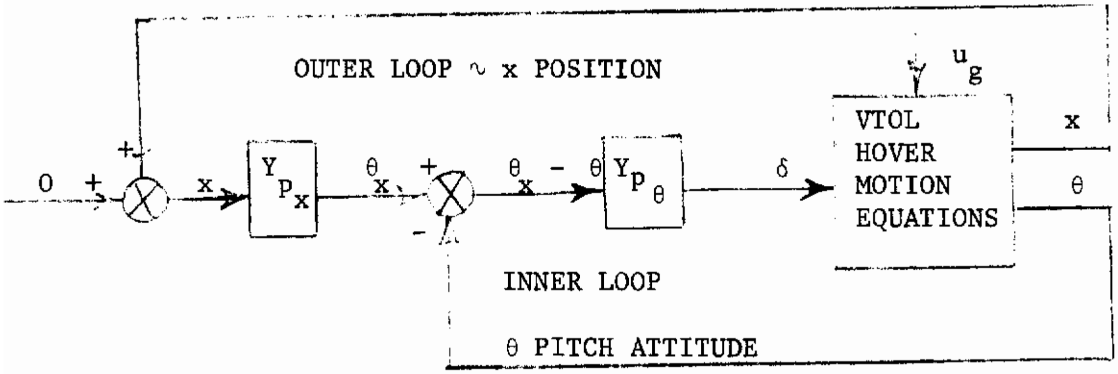

Figure 1

Pilot Model Loop Closures for VTOL Hover Task

Y_{p_x} and Y_{p_θ} are the pilot models for the x and θ loops respectively.

2.1 AIRCRAFT EQUATIONS

2.1.1 GENERAL FORM

The general form of the linearized longitudinal equations of motion which describe the VTOL hover motion in response to control inputs and turbulence is taken from Appendix B of Reference [3] and the equations of motion are

$$\dot{\theta} = q$$

$$\dot{q} = M_\theta \theta + M_q q + M_u u + M_\delta \delta + M_u u_g$$

$$\dot{x} = u$$

$$\dot{u} = -g\theta + X_u u + X_\delta \delta + X_u u_g$$

θ is pitch angle, q is pitch rate, x is forward displacement, u is velocity, δ is stick command, and u_g is the gust velocity in the u direction.

It is assumed that longitudinal translation can be effected only through pitch; consequently, X_δ is taken to be zero. Converting radian measure to degrees, the equations become

$$\dot{\theta} = q$$

$$\dot{q} = M_\theta \Theta + M_q q + 57.3\, M_u u + 57.3\, M_\delta \delta + 57.3\, M_u u_g$$

$$\dot{x} = u$$

$$\dot{u} = -\frac{g}{57.3}\theta + X_u u + X_u u_g$$

This conversion is made to agree with Reference [1].

2.1.2 CONTROL LAG

For the case where the effective control input lags the commanded control input, the appropriate equations of motion as adopted from Appendix B of Reference [4] are

$$\dot{\theta} = q$$

$$\dot{q} = M_\theta \theta + M_q q + 57.3\, M_u u + 57.3\, M_\delta \delta_e + 53.7\, M_u u_g$$

$$\dot{x} = u$$

$$\dot{u} = -\frac{g}{57.3}\theta + X_u u + X_u u_g$$

$$\dot{\delta}_e = -\frac{1}{\tau_e}\delta_e + \frac{1}{\tau_e}\delta$$

where δ_e is the effective control input and τ_e is the control lag time constant.

2.1.3 SAS LAG

For the case where the inner loop dynamics are augmented by a stability augmentation system (SAS) and this feedback includes a lag term, the equations of motion as adopted from Appendix B of Reference [4] are

$$\dot{\theta} = q$$

$$\dot{q} = 57.3\ M_u u + 57.3\ M_\delta \delta + M_e + 57.3\ M_u u_g$$

$$\dot{x} = u$$

$$\dot{u} = \frac{g}{57.3}\theta + X_u u + X_u u_g$$

$$\dot{M}_e = -\frac{1}{\tau_q} M_e + \frac{M_\theta}{\tau_q}\theta + \frac{M_q}{\tau_q} q$$

where M_e is a state representing the effective angular acceleration in pitch generated due to the SAS, τ_q is the SAS feedback lag time constant. In this representation, M_θ and M_q denote SAS parameters and the M_θ and M_q of the bare airframe are considered to be zero.

2.1.4 CONTROL LAG AND SAS LAG

Combining the cases with a control lag, τ_e, and a SAS feedback lag, τ_q, the equations of motion are

$$\dot{\theta} = q$$

$$\dot{q} = 57.3\ M_u u + 57.3\ M_\delta \delta_e + M_e + 57.3\ M_u u_g$$

$$\dot{x} = u$$

$$\dot{u} = -\frac{g}{57.3}\theta + X_u u + X_u u_g$$

$$\dot{\delta}_e = -\frac{1}{\tau_e}\delta_e + \frac{1}{\tau_e}\delta$$

$$\dot{M}_e = -\frac{1}{\tau_q} M_e + \frac{M_\theta}{\tau_q}\theta + \frac{M_q}{\tau_q} q$$

As in the case with a SAS lag, M_θ and M_q are SAS parameters and the M_θ and M_q associated with the bare airframe are assumed to be zero.

2.1.5 LIMITED PITCH RATE AUTHORITY SAS

For the case where the bare airframe dynamics are augmented by a pitch rate SAS with limited authority, the aircraft equations of motion are

$$\dot{\theta} = q$$

$$\dot{q} = M_e + 57.3\, M_u u + 57.3\, M_\delta \delta + 57.3\, M_u u_g$$

$$\dot{x} = u$$

$$\dot{u} = -\frac{g}{57.3}\,\theta + X_u u + X_u u_g$$

where M_e is a function of q as shown in Figure 2. M_{ra} is the maximum

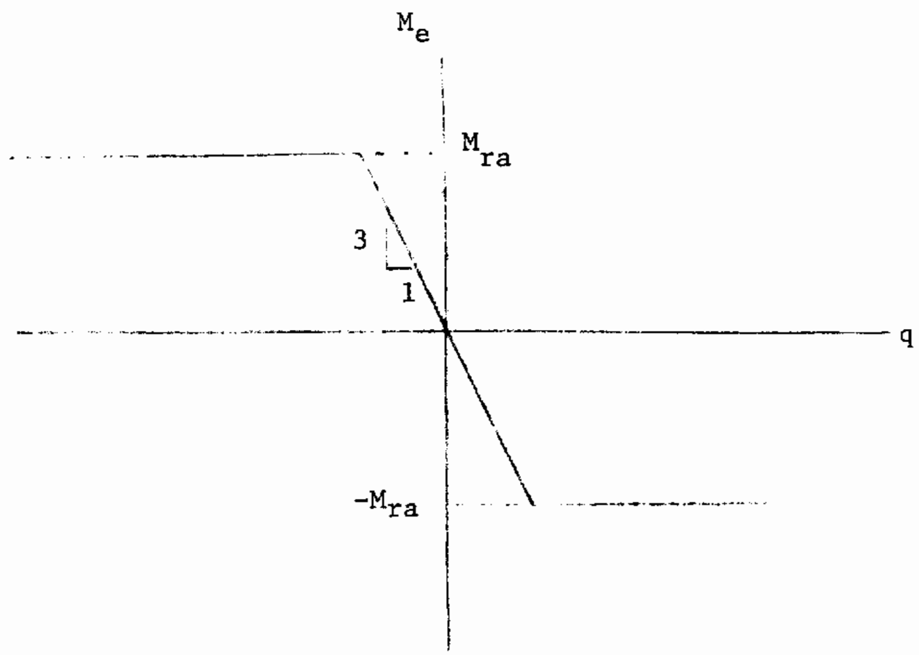

Figure 2

Pitch Rate SAS Non-Linearity

effective angular acceleration in pitch due to the SAS. M_q is a SAS parameter and the M_θ and M_q for the bare airframe are assumed to be zero.

Clearly the aircraft equations of motion for this case are non-linear since M_e is a nonlinear function of pitch rate, q.

2.2 GUST MODEL

Turbulence is represented by a gust velocity, u_g, given by the stochastic differential equation

$$\dot{u}_g = -\omega_b\, u_g + v_g$$

v_g is white Gaussian noise and

$$E\{v_g(t)\} = 0$$

$$\text{cov}[v_g(t_1),\ v_g(t_2)] = 2\omega_b\sigma_g^{\,2}\delta(t_1-t_2)$$

where σ_g is the steady state standard deviation of the gust velocity. The gust break frequency, ω_b, is taken to be 0.314 rad/sec to agree with the gust break frequency used in the fixed base simulation as reported in References [3] and [4].

2.3 PILOT MODEL

The block diagram of the pilot model is shown in Figure 3. K_{p_x} is the position loop pilot gain, T_{L_x} is the position loop pilot

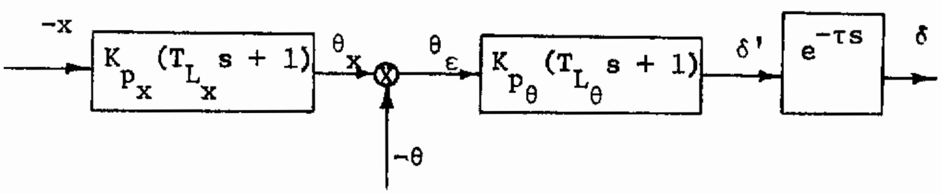

Figure 3

Pilot Model for the VTOL Hover Task

lead, K_{p_θ} is the pitch loop pilot gain, and T_{L_θ} is the pitch loop lead. The $e^{-\tau s}$ is a pure reaction time delay term, where τ is the time delay. The pilot model shown in Figure 3 was taken from Reference [1] and is a simplification of the pilot model used in Reference [3]. A reaction time delay, τ, of .44 sec was used to agree with Reference [1]. This value represents the lump sum of the neuromuscular lag, .35 sec, and the pitch loop reaction time delay, .09 sec, for the pilot model used in Reference [3].

The time delay, $e^{-\tau s}$, is modeled by the Padé approximation,

$$\frac{\delta(s)}{\delta'(s)} = e^{-\tau s} \doteq \frac{-(s - 2/\tau)}{(s + 2/\tau)}$$

With this approximation, the pilot model equation can be written as

$$\dot{y} = \frac{2}{\tau}(\delta' - \delta)$$

$$\delta = y - \delta'$$

$$\delta' = K_{p_\theta} T_{L_\theta} \dot{\theta}_\varepsilon + K_{p_\theta} \theta_\varepsilon$$

$$\theta_\varepsilon = \theta_x - \theta$$

$$\dot{\theta}_\varepsilon = \dot{\theta}_x - \dot{\theta}$$

$$\theta_x = K_{p_x} T_{L_x} u + K_{p_x} x$$

$$\dot{\theta}_x = K_{p_x} T_{L_x} \dot{u} + K_{p_x} u$$

where y is a dummy state variable used in the reaction time delay approximation. In state variable form, the equations for $\dot{y}$ and δ are

$$\dot{y} = -\frac{4}{\tau}(g\, K_{p_\theta} T_{L_\theta} K_{p_x} T_{L_x}/57.3 + K_{p_\theta})\,\theta$$

$$- \frac{4}{\tau} K_{p_\theta} T_{L_\theta} q + \frac{4}{\tau} K_{p_\theta} K_{p_x} x$$

$$+ \frac{4}{\tau}(K_{p_\theta} T_{L_\theta} K_{p_x} T_{L_x} X_u + K_{p_\theta} T_{L_\theta} K_{p_x} + K_{p_\theta} K_{p_x} T_{L_x})u$$

$$+ \frac{4}{\tau} K_{p_\theta} T_{L_\theta} K_{p_x} T_{L_x} X_u u_g - \frac{2}{\tau} y$$

$$\delta = (g\, K_{p_\theta} T_{L_\theta} K_{p_x} T_{L_x}/57.3 + K_{p_\theta})\,\theta$$

$$+ K_{p_\theta} T_{L_\theta} q - K_{p_\theta} K_{p_x} x$$

$$- (K_{p_\theta} T_{L_\theta} K_{p_x} T_{L_x} X_u + K_{p_\theta} T_{L_\theta} K_{p_x} + K_{p_\theta} K_{p_x} T_{L_x})u$$

$$- K_{p_\theta} T_{L_\theta} K_{p_x} T_{L_x} X_u u_g + y$$

2.4 SYSTEM EQUATIONS

The aircraft equations, gust model, and pilot model make up an nth order system which can be represented in state variable form by the differential equation

$$\dot{z} = Az + v$$

(with the exception of the case with the limited pitch rate authority SAS). For the general form of the aircraft equations, the system is 6th order and the states are

$$z_1 = q$$

$$z_2 = \theta$$

$$z_3 = u$$

$$z_4 = x$$

$$z_5 = u_g$$

$$z_6 = y$$

For the case with a control lag, the system is 7th order and δ_e is a system state. For the case with a SAS lag the system is 7th order and M_e is a system state. For the case with both a control lag and SAS lag the system is 8th order and both δ_e and M_e are states of the system. In <u>any</u> case the digital computer program requires that

$$z_1 = q$$

$$z_2 = \theta$$

$$z_3 = u$$

$$z_4 = x$$

This particular ordering is required in the computation of PERF as described in Section 2.5.

The disturbance term, v, is an n-vector white Gaussian process where

$$v_i = \begin{cases} v_g, & i = n_g \\ 0, & i \neq n_g \end{cases}$$

where the n_gth state is the gust intensity, i.e., $z_{n_g} = u_g$. For the general form of the aircraft equations and the states as defined above,

$$v = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v_g \\ 0 \end{bmatrix}$$

The covariance of v is

$$\mathrm{cov}[v(t_1), v(t_2)] = R\delta(t_1 - t_2)$$

where R is a n by n matrix where $R_{n_g n_g} = 2\omega_b \sigma_g{}^2$ and all other entries are zero. For example,

$$R = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\omega_b \sigma_g^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

for the general form of the aircraft equations and the states as defined above.

For convenience, a set of pilot parameters is taken to be a point in Euclidian four-space.

$$k = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} = \begin{bmatrix} K_{P_\theta} \\ T_{L_\theta} \\ K_{P_x} \\ T_{L_x} \end{bmatrix}$$

The coefficient matrix, A, is a function of k (the pilot parameters) and for the general form of the aircraft equations the A matrix is of the form

$$
A(k) = \begin{bmatrix}
M_q + cM_\delta k_1 k_2 & M_\theta + cM_\delta k_1 + gM_\delta k_1 k_2 k_3 k_4 & cM_u - cM_\delta (k_1 k_2 k_3 k_4 X_u + k_1 k_2 k_3 + k_1 k_3 k_4) & -cM_\delta k_1 k_3 & cM_u - cM_\delta k_1 k_2 k_3 k_4 X_u & cM_\delta \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & -g/c & X_u & 0 & X_u & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\omega_b & 0 \\
-\frac{4}{\tau} k_1 k_2 & -\frac{4}{\tau}(g k_1 k_2 k_3 k_4 / c + k_1) & \frac{4}{\tau}(k_1 k_2 k_3 k_4 X_u + k_1 k_2 k_3 + k_1 k_3 k_4) & \frac{4}{\tau} k_1 k_3 & \frac{4}{\tau} k_1 k_2 k_3 k_4 X_u & -\frac{2}{\tau}
\end{bmatrix}
$$

where $c = 57.3$

2.5 COST FUNCTIONAL

The cost functional used in the minimization scheme is

$$J = PERF + R_2 + R_3 + W_7$$

The first term is the "performance" and is given by

$$PERF = \sum_{j=1}^{4} W_j\sigma_j - 1 = W_1\sigma_q + W_2\sigma_\theta + W_3\sigma_u + W_4\sigma_u - 1$$

where W_j is a weighting coefficient and σ_j is the steady state rms value of the jth state; i.e.,

$$\sigma_j = \lim_{T\to\infty} [E\{z_j^2(T)\}]^{1/2}$$

R_2 and R_3 are defined as follows:

$$R_2 = \begin{cases} -W_5 T_{L_\theta}, & T_{L_\theta} < 0 \\ W_5 T_{L_\theta}, & 0 \le T_{L_\theta} \le 1.3 \\ 1.3\ W_5, & T_{L_\theta} > 1.3 \end{cases}$$

$$R_3 = \begin{cases} -W_6 T_{L_x}, & T_{L_x} < 0 \\ W_6 T_{L_x}, & 0 \le T_{L_x} \le 1.2 \\ 1.2\ W_6, & T_{L_x} > 1.2 \end{cases}$$

W_5 and W_6 are weighting coefficients. These functions are shown graphically in Figure 4.

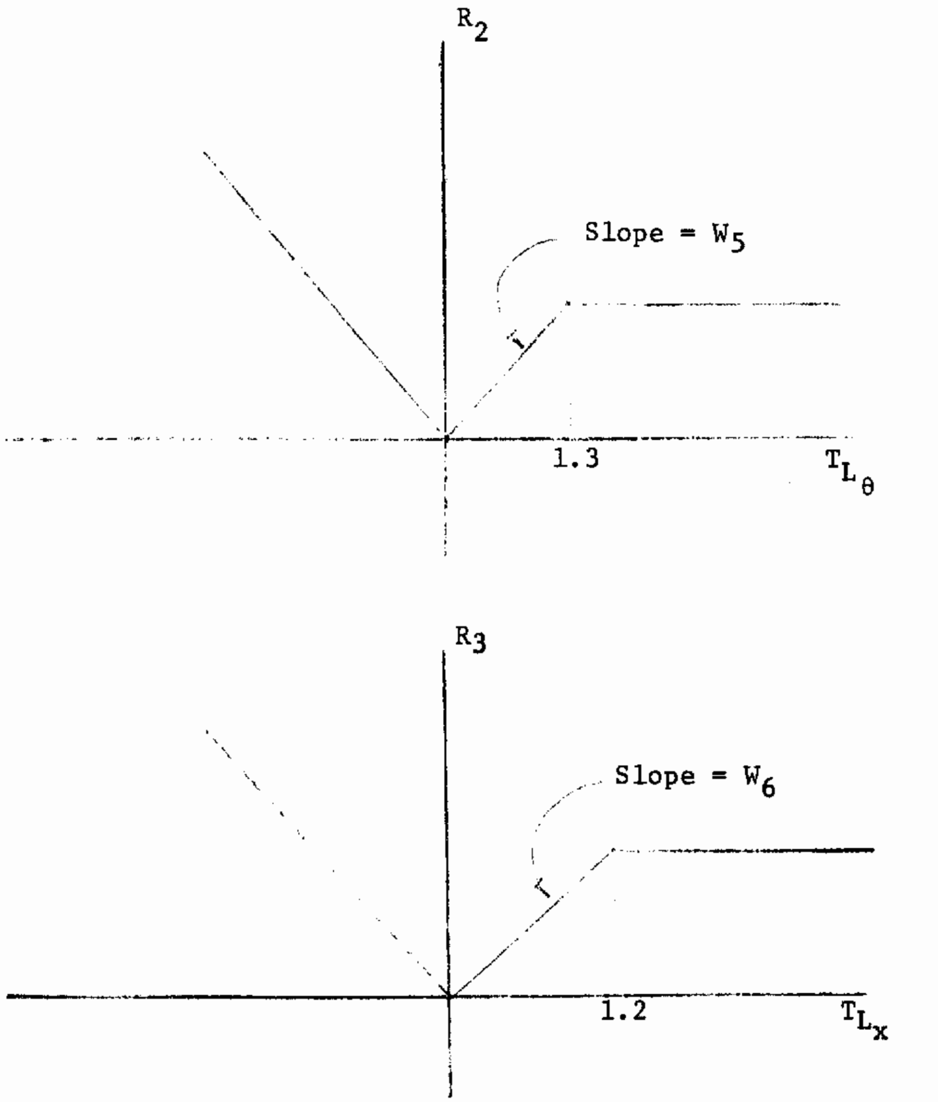

Figure 4

Graphic Description of R_2 and R_3

W_7 is a constant.

For the precision hover task, the coefficients W_1 through W_7 are taken to be

$$W_1 = .218$$

$$W_2 = 0$$

$$W_3 = 0$$

$$W_4 = 1.25$$

$$W_5 = 2.25$$

$$W_6 = 1$$

$$W_7 = 1$$

The rationale for the form of J is deferred to the next section where the "paper pilot" rating scheme and the rating expression are described.

2.6 "PAPER PILOT" RATING SCHEME

The first step in the "paper pilot" rating scheme is to find the pilot parameters, k, which minimize J. The minimization is done in the computer program using a conjugate gradient minimization technique. This process represents the major portion of the digital computer program and computation time in computing a "paper pilot" rating.

Based upon observed data (Reference [5]), $|T_{L_x}|$ and $|T_{L_\theta}|$ should be constrained to be less than or equal to 5. This reflects the fact that the human operator cannot physically generate more than 5 seconds of lead. This constraint was not incorporated into the minimization routine because of the additional computation complexity that results. So far, no cases have been run where either of the minimizing leads have a magnitude greater than 5. The user should be aware, however, that <u>if $|T_{L_x}|$ or $|T_{L_\theta}|$ is greater than 5, the "paper pilot" rating may not be valid</u>.

Once the minimizing pilot parameters are determined, the minimizing pilot gains are simultaneously increased 20% and the system is tested for stability. If the system is asymptotically stable for the increased pilot gains, the minimizing pilot gains are restored and used for the remainder of the procedure. If the system is not asymptotically stable for the increased pilot gains, the pilot gains are decreased so that a simultaneous 20% increase in the pilot gains results in an asymptotically stable system and a 20.03% simultaneous increase in the pilot gains results in a system which is not asymptotically stable.

The requirement for 20% stability margin on pilot gains resulted from two considerations. The first consideration was strictly practical. Preliminary computational results without the stability margin requirement resulted in "paper pilot" ratings that were too low for certain aircraft configurations. It was determined on the analog computer in these cases that the minimizing pilot gains were on the "ragged" edge of instability. It was clear that the "paper pilot" ratings would be increased due to a degradation of performance to some degree by imposing a stability margin on the pilot gains and that a better correlation between the actual and "paper pilot" ratings could be achieved. The second consideration is based on an attempt to represent the actual pilot by the mathematical model. Intuitively, an actual pilot will not operate on the verge of instability in a routine or long term tracking task. It is assumed therefore that the real pilot will adapt to the aircraft configuration in such a way that some stability margin is available. The choice of 20% was made arbitrarily.

The pilot parameters (the minimizing parameters or the adjusted parameters, depending on the stability margin test described above) are then used to compute the performance and hence a "paper pilot" rating using the rating expression given in Reference [1]. That is, the "paper pilot" rating, PR, is given by

$$PR = R_1 + R_2 + R_3 + W_7$$

where

$$R_1 = \begin{cases} 0 & , \ PERF < 0 \\ PERF & , \ 0 \leq PERF \leq R_{1_{max}} \\ R_{1_{max}} & , \ PERF > R_{1_{max}} \end{cases}$$

and

$$PERF = \sum_{j=1}^{4} W_j \sigma_j - 1$$

$$R_{1_{max}} = 7.95 - 1.3\, W_5 - 1.2\, W_6 - W_7$$

R_2, R_3, and W_7 are the same as used in defining J and are described in Section 2.5.

The detail rationale for choosing the form of PR and the values of W_1 through W_7 is given by Anderson in [1]. A brief summary of that rationale is given below to familiarize the reader with the qualitative aspects of the rating expression.

$W_7 = 1$ was included in the rating expression to account for the fact that a pilot rating of 1 on the Cooper pilot rating scale is optimum. Thus, for $R_1 = R_2 = R_3 = 0$, PR = 1, the lowest (optimal) value of PR.

The form of the R_2 and R_3 terms is based on a straight line approximation to observed results for pilot ratings as a function of pilot lead as reported in Reference [2]. The values used in defining R_2 are taken directly from a straight line approximation to the data of Reference [2]. The values used to define R_3 are different from R_2 and were selected to give a good fit of the pilot rating expression to actual pilot ratings given in Table A-VI of Reference [3].

Inasmuch as increased lead generation corresponds to increased pilot attention to rates as derived mentally from observed position, R_2 and R_3 represent a measure of the pilot's "workload" in the tracking task.

The R_1 term reflects the pilot's sensitivity to the pitch rate through σ_q, which is probably a measure of the inner loop performance. The measure of the task performance, the task being precision hover, is reflected in R_1 through σ_x. The weightings used in Reference [1] were basically selected to fit the actual pilot ratings of Reference [3] with the rating expression.

Notice that the only difference in the form of J and PR is in the first term of each expression, PERF and R_1. Furthermore, PERF $\neq$ R_1 only if PERF < 0 or PERF > $R_{1_{max}}$. The decision to find the pilot parameters which minimize J instead of PR is based upon the practical difficulty that PR is not a suitable functional for minimization in accordance with the objectives of the model. Notice, for example, that if PR is the functional to be minimized, by choosing T_{L_θ} and T_{L_x} equal to zero, PR $\leq R_{1_{max}} + W_7 = 3.5$. This is clearly unsuitable in those cases where the predicted pilot rating should be greater than 3.5.

From a physical point of view, the model suggests the following. The pilot in the tracking task adapts his "parameters" to minimize the linear combination of work load and performance as defined by J. Then in determining a rating of the flying qualities, the pilot tends to "wash out" exceedingly poor performance beyond a certain point or exceedingly good performance below a certain point.

Of course, the final "proof of the pudding" is the ability of the model to correlate with actual pilot ratings within the limits of the model.

III. DIGITAL PROGRAM

3.1 MATHEMATICAL FORMULATION

In order to work with steady state conditions for performance, stability of the A matrix for a given set of pilot parameters must be determined. The test used is based upon the following necessary and sufficient condition for stability.

<u>Theorem</u>. The system $\dot{x} = Ax$ is asymptotically stable if, and only if, the Liapunov equation

$$A'X + XA + I = 0$$

has a solution in X which is positive definite, where I is the identity matrix.

Proof of the theorem is given in Reference [6], pages 81-84. If X exists, it is symmetric and the eigenvalues are easily computed using the Jacobi method described in Reference [7]. X is positive definite if and only if all the eigenvalues are greater than zero.

For the system

$$\dot{z} = Az + v$$

where A is asymptotically stable, and v is a vector white Gaussian process with

$$\text{cov}[v(t_1),v(t_2)] = R\delta(t_1-t_2)$$

The steady state covariance matrix, Z, is the solution to the covariance equation

$$AZ + ZA' + R = 0$$

where

$$Z = \lim_{T\to\infty} \text{cov}[z(T),z(T)]$$

(See Reference [8] for the derivation of the covariance equation.) Then

$$\lim_{T\to\infty} E\{z_j^2(T)\} = Z_{jj}$$

and $$\sigma_j = Z_{jj}^{1/2}$$

The gradient of J with respect to the pilot parameters, g(k), is given component wise by

$$g_i(k) = \frac{\partial PERF}{\partial k_i} + \frac{\partial R_2}{\partial k_i} + \frac{\partial R_3}{\partial k_i}$$

Now

$$\frac{\partial PERF}{\partial k_i} = \sum_{j=1}^{4} \frac{W_j}{2\sigma j} \frac{\partial Z_{jj}}{\partial k_i}$$

and $\frac{\partial Z}{\partial k_i}$ is the solution to

$$A\frac{\partial Z}{\partial k_i} + \frac{\partial Z}{\partial k_i} A' + \frac{\partial A}{\partial k_i} Z + Z\frac{\partial A}{\partial k_i} = 0$$

The equation for $\frac{\partial Z}{\partial k_i}$ is analogous to the equation for the steady state covariance, Z. Notice that the gradient is not defined if $\sigma_g = 0$, since for $\sigma_g = 0$, $\sigma_j = 0$. <u>Thus the model requires that $\sigma_g > 0$.</u> It may be possible to consider the zero gust case by taking the limit as σ_g tends to zero.

Since R_2 is dependent only on k_1

$$\frac{\partial R_2}{\partial k_j} = 0, \ i \neq 1$$

and

$$\frac{\partial R_2}{\partial k_1} = \begin{cases} -W_5, & k_1 < 0 \\ W_5, & 0 \le k_1 \le 1.3 \\ 0, & k_1 > 1.3 \end{cases}$$

Similarly

$$\frac{\partial R_3}{\partial k_i} = 0, \ i \neq 3$$

and

$$\frac{\partial R_3}{\partial k_3} = \begin{cases} -W_6 \ , \ k_3 < 0 \\ W_6 \ , \ 0 \leq k_3 \leq 1.2 \\ 0 \ , \ k_3 > 1.2 \end{cases}$$

The fact that the gradient is not continuous may be a point of consternation. The discontinuous gradient could cause difficulty in the search for a minimum in J along the search direction in the conjugate gradient routine. This is done computationally by making a cubic fit to J along the search direction using the function value and the directional derivative at two points along the line of the search direction. Clearly the cubic fit would be of questionable value if the line between the points includes a discontinuity in the directional derivative. So far, however, no difficulty with this aspect of the program has been experienced; and consequently, no attempt has been made to develop a more complicated form of J so that it has a continuous gradient.

An examination of the aircraft equations and the pilot model reveals that the minimum value of J is independent of M_δ. This follows from the fact that an increase (decrease) in M_δ can be compensated by a corresponding decrease (increase) in K_{p_θ}, which does not appear explicitly in the cost functional, J. Thus the model is based on the assumption that M_δ is selected optimally for the given aircraft configuration. The significance of this assumption in predicting pilot ratings is discussed in Section IV, Computational Results.

3.2 PROGRAM DESCRIPTION

The aircraft parameters (M_u, X_u, M_q, M_θ, M_δ, τ_e, τ_q) are read into the computer, along with the gust intensity (σ_g), a lower estimate of J, an epsilon value used to test for convergence in the conjugate gradient routine, and an iteration limit for the conjugate gradient routine. An initial guess of the pilot parameters (K_{p_θ}, T_{L_θ}, K_{p_x}, T_{L_x}) are read into the computer. If these initial pilot parameters result in a system which is not asymptotically stable, computation ceases and the program proceeds to the next set of data. The "paper pilot" cannot rate an aircraft if the initial pilot parameters result in a system which is not asymptotically stable.

If the initial pilot parameters result in an asymptotically stable system, J is minimized using the conjugate gradient method of Fletcher and Reeves [9]. If the convergence is not achieved in the limiting number of iterations, then the minimum value of J is not insured.

The minimizing pilot gains are simultaneously increased 20% and the system is tested for stability. If the system is asymptotically stable the minimizing gains are restored and used for the remainder of the program. If the system is not asymptotically stable, the pilot gains are decreased so that a simultaneous 20% increase in the pilot gains result in an asymptotically stable system and a 20.03% simultaneous increase in the pilot gains results in a system which is not asymptotically stable.

If J $\neq$ PR, PR is computed. The "paper pilot" rating is printed out along with the "paper pilot" parameters and the corresponding performance.

Typically, IBM 7090 computation time for 15 iterations is 70 seconds for a 6th order system.

The Program Listing is given in Appendix A along with a brief description of each routine and how it is used in the program.

3.3 INPUT-OUTPUT DESCRIPTION

For each aircraft configuration and gust intensity, four data cards are used. Table II tabulates each entry by card number, column numbers, format, and description.

Card	Column	Entry	Format	Description
1	1-72	Title	A72	An alpha-numeric title used as a title and/or case number identifier in the printout
2	1-10	M_u	F10.5	1/ft-sec
2	11-20	X_u	F10.5	1/sec
2	21-30	M_q	F10.5	1/sec
2	31-40	M_θ	F10.5	1/sec^2
2	41-50	M_δ	F10.5	(rad/sec^2)/in
2	51-60	τ_e	F10.5	sec
2	61-70	τ_q	F10.5	sec
3	1-10	σ_g	F10.5	ft/sec
3	11-20	Estimate	F10.5	A lower estimate of the minimum value of J. Note, $J \geq 0$
3	21-30	ε	F10.5	A test value used to test for convergence. Convergence is assumed when the norm of the gradient squared < ε. A suggested value is .2

TABLE II
INPUT DESCRIPTION

Card	Column	Entry	Format	Description
3	31-40	Limit	I5	The limit on the number of iterations in the conjugate gradient routine
4	1-10	K_{p_θ}	F10.5	in/deg, initial guess
4	11-20	T_{L_θ}	F10.5	sec, initial guess
4	21-30	K_{p_x}	F10.5	deg/ft, initial guess
4	31-40	T_{L_x}	F10.5	sec., initial guess

TABLE II

INPUT DESCRIPTION (Concluded)

Three possible printouts are shown in Figures 5, 6, and 7.

First the input values are printed out in the following order: M_u, X_u, M_q, M_θ, M_δ, t_e, t_q, σ_g, estimate, ε, limit, K_{p_θ}, T_{L_θ}, K_{p_x}, T_{L_x}.
Next a column header is printed out, where

PR - J in the minimization routine
KPTH - K_{p_θ}, in/deg

TLTH - T_{L_θ}, sec

KPX - K_{p_x}, deg/ft

TLX - T_{L_x}, sec

SIGQ - σ_q, deg/sec

SIGTH - σ_θ, deg

SIGU - σ_u, ft/sec

SIGX - σ_x, ft

// G // = $||g||^2 = \sum_{j=1}^{4} g_j^2$

```
CASE -- PH2

MU       =   0.02081  )
XU       =  -0.05000  )
MQ       =  -3.00000  )
MTHETA   =   0.       )  Vehicle Dynamics
MDELTA   =   0.41200  )
TAUE     =   0.       )
TAUQ     =   0.       )

SIG GUST =   5.10000     Gust Intensity

ESTIMATE =   1.00000  )
EPSILON  =   0.20000  )  Iteration Data for Minimization Routine
LIMIT        15       )

KPTH     =   0.44364  )
TLTH     =   0.23451  )  Initial "Guess" for Pilot Parameters
KPX      =   1.85762  )
TLX      =   0.36041  )

J or PR  |------- Pilot Parameters -------|  |-------- RMS Values of States --------|
   PR     KPTH      TLTH      KPX      TLX      SIGQ      SIGTH     SIGU      SIGX        //G//

UNDRFLOW AT 30345 IN AC AND MC.

UNDRFLOW AT 30102 IN MQ

UNDRFLOW AT 30103 IN MQ

UNDRFLOW AT 30103 IN MQ

UNDRFLOW AT 30345 IN AC AND MQ.                                                             Iterations
  2.60093  0.44364  0.23451  1.85762  0.36041  2.77789  1.75930  0.76692  0.83893  7.93312   |
  2.56371  0.46635  0.23142  1.86058  0.37093  2.94982  1.66814  0.69374  0.77695  0.60594   |
  2.55339  0.47496  0.25711  1.87141  0.37285  2.73494  1.55938  0.65919  0.75323  0.16296   |
  2.55138  0.48035  0.25693  1.88089  0.37436  2.80191  1.54370  0.64579  0.73910  0.32333   |
  2.45496  0.47616  0.27185  2.44690  0.33669  3.18757  1.73567  0.63726  0.60300  1.37006   |
  2.45270  0.49071  0.27541  2.51239  0.33971  3.36925  1.68763  0.59078  0.55997  2.39157   |
  2.45801  0.48155  0.28040  2.51219  0.33472  3.20308  1.70686  0.61655  0.57922  0.47421   |
  2.45653  0.48697  0.28290  2.51265  0.33760  3.22652  1.67081  0.59693  0.56664  0.08395   v
EXIT CONJUGATE GRADIENT SEARCH WITH IER = 0
  2.45628  0.48560  0.28383  2.51289  0.33697  3.20256  1.67328  0.60044  0.56927  0.00148
PILOT GAINS ARE ADJUSTED TO PROVIDE 20 PER CENT STABILITY MARGIN.  PILOT RATING IS MODIFIED AS FOLLOWS
  2.57801  0.44260  0.28383  2.29039  0.33697  2.93050  1.85455  0.74592  0.71410  } Final Results
COST REGION CODE IS 111

FOR A GUST DISTURBANCE MODEL WITH A STANDARD DEVIATION OF  5.100,  )
THE PAPER PILOT RATES THIS AIRCRAFT                                )
                                                                   )
          2.57801                                                  )  Final Numerical Rating
                                                                   )
IN THE PRECISION HOVER MODE.                                       )

   FLYING QUALITIES ARE CLEARLY ADEQUATE FOR THE MISSION FLIGHT    )  Verbal Evaluation
   PHASE OF PRECISION HOVER (PH).                                  )

****** PAPER PILOT CONSIDERS SMOKING HAZARDOUS TO HIS HEALTH ******
```

Figure 5. Computer Printout, Convergence Obtained

```
CASE -- PH 18

MU        =    0.03106
XU        =   -0.20000
MQ        =   -1.00000
MTHETA    =    0.
MDELTA    =    0.48330
TAUE      =    0.
TAUQ      =    0.

SIG GUST  =    5.10000

ESTIMATE  =    3.00000
EPSILON   =    0.20000
LIMIT          15

KPTH      =    0.16961
TLTH      =    0.63387
KPX       =    1.87727
TLX       =    0.23971

    PR        KPTH       TLTH       KPX        TLX        SIGQ       SIGTH      SIGU       SIGX       //G//

  6.05078    0.16961    0.63387    1.87727    0.23971    6.25833    4.10228    1.69448    2.28966    82.91484
  5.93885    0.18023    0.63340    1.87823    0.24041    6.48423    3.97004    1.61132    2.20910     1.99338
  5.97059    0.17721    0.60245    1.90349    0.22535    6.61530    4.17854    1.69279    2.23759    37.61227
  5.91863    0.18809    0.55337    1.94519    0.20207    7.40661    4.38445    1.71345    2.17480     2.34222
  5.90953    0.19417    0.55194    1.55572    0.18815    7.59742    4.39113    1.70849    2.14823    27.78931
  5.88317    0.20077    0.54940    2.00621    0.14140    7.59420    4.55502    1.81425    2.17018    15.22851
  5.88263    0.19499    0.54948    2.00701    0.14162    7.37169    4.61689    1.85946    2.20823    13.43670
  5.86037    0.19419    0.55830    2.03335    0.18106    7.65897    4.44927    1.71208    2.09113    14.72834
  5.85517    0.19063    0.55922    2.03481    0.18159    7.50334    4.47752    1.73625    2.11183     1.81520
  5.85449    0.19421    0.56453    2.03851    0.18006    7.59631    4.42557    1.70719    2.08568    16.06308
  5.85323    0.15515    0.57100    2.04329    0.17790    7.57277    4.40517    1.70220    2.07958    26.05256
  5.84950    0.18857    0.57002    2.04288    0.17678    7.31370    4.47253    1.75369    2.12262    11.52574
  5.84453    0.19332    0.57136    2.04653    0.17677    7.45979    4.41950    1.71741    2.09050     4.41797
  5.84284    0.19112    0.57165    2.04756    0.17622    7.36623    4.44375    1.73588    2.10533     1.17186
  5.84187    0.19270    0.57236    2.04954    0.17613    7.42508    4.42593    1.72280    2.09293     1.44055
EXIT CONJUGATE GRADIENT SEARCH WITH IER =  1
  5.83534    0.19083    0.57402    2.05535    0.17500    7.34556    4.44319    1.73910    2.10237     2.32322
CONJUGATE GRADIENT ROUTINE FAILED TO CONVERGE AFTER 15 ITERATIONS.
PILOT GAINS ARE ADJUSTED TO PROVIDE 20 PER CENT STABILITY MARGIN.  PILOT RATING IS MODIFIED AS FOLLOWS
  5.93929    0.18188    0.57402    1.55900    0.17500    6.90965    4.48628    1.82865    2.25835
COST REGION CODE IS 211
MAX PERFORMANCE CONTRIBUTION IS    2.550.  COMPUTED PERFORMANCE =    3.329.  PILOT RATING IS MODIFIED AS FOLLOWS
  5.16005    0.18188    0.57402    1.55900    0.17500    6.90965    4.48628    1.82865    2.25835


FOR A GUST DISTURBANCE MODEL WITH A STANDARD DEVIATION OF  5.100,
THE PAPER PILOT RATES THIS AIRCRAFT

               5.16005

IN THE PRECISION HOVER MODE.

    FLYING QUALITIES ARE ADEQUATE TO ACCOMPLISH THE PRECISION HOVER
    FLIGHT PHASE, BUT SOME INCREASE IN PILOT WORKLOAD OR DEGRADATION
    IN MISSION EFFECTIVENESS, OR BOTH, EXISTS.  IN PARTICULAR
    SYSTEM PERFORMANCE IN TERMS OF STANDARD DEVIATION IN POSITION
    AND PITCH RATE IS POOR, WITH REDUCED OPERATIONAL EFFECTIVENESS
    RESULTING.



 ****** PAPER PILOT CONSIDERS SMOKING HAZARDOUS TO HIS HEALTH ******
```

Figure 6. Computer Printout, Algorithm Failed to Converge

```
CASE -- SAS DESIGN INVESTIGATION NO. 1

MU         =    0.02081
XU         =   -0.10000
MQ         =   -0.50000
MTHETA     =    0.
MDELTA     =    0.36900
TAUE       =    0.05000
TAUQ       =    0.

SIG GUST =      5.10000

ESTIMATE =      2.00000
EPSILON  =      0.20000
LIMIT           15

KPTH       =    0.29000
TLTH       =    0.46000
KPX        =   -2.10000
TLX        =    0.27000

    PR        KPTH       TLTH       KPX       TLX       SIGQ      SIGTH      SIGU      SIGX      //G//

EXIT CONJUGATE GRADIENT SEARCH WITH IER =  3
INITIAL PILOT PARAMETERS RESULT IN UNSTABLE SYSTEM.
PAPER PILOT REQUIRES STABLE INITIAL PARAMETERS.
 ****** PAPER PILOT CONSIDERS SMOKING HAZARDOUS TO HIS HEALTH ******
```

Figure 7. Computer Printout, Unstable Initial Pilot Parameters

The value of J, the pilot parameters, the rms values of q, θ, u, and x, and the norm of the gradient is printed out for each iteration in the conjugate gradient routine. The conjugate gradient routine exit code, IER, is printed out. The interpretation of the code is listed in Table III.

IER	MEANING
0	Convergence was obtained
1	No convergence in the limiting number of iterations
-1	Error in gradient calculation
2	Linear search techniques indicate it is likely that there exist no minimum
3	Initial parameters result in unstable system
4	Cubic interpolation results in unstable system

TABLE III

IER CODE

An appropriate comment is printed if IER $\neq$ 0.

If the conjugate gradient routine exits with IER=3, no iterations were made in the conjugate gradient routine and the program proceeds to the next set of data cards.

If $|T_{L_\theta}|$ or $|T_{L_x}| > 5$, a warning is printed out that the "paper pilot" rating may not be valid.

If the pilot gains are adjusted to give a 20% stability margin, a new value of J, K_{p_θ}, T_{L_θ}, K_{p_x}, T_{L_x}, σ_q, σ_θ, σ_u, and σ_x are printed out. If the parameter adjustment routine fails, the appropriate comment is printed out and the program proceeds to the next set of data cards.

The cost region code is printed out. The explanation of the code is given in Table IV.

Code	Definition
0_{xx}	PERF < 0
1_{xx}	$0 \leq \text{PERF} \leq R_{1_{max}}$
2_{xx}	$\text{PERF} > R_{1_{max}}$
x0x	$T_{L_\theta} < 0$
x1x	$0 \leq T_{L_\theta} \leq 1.3$
x2x	$T_{L_\theta} > 1.3$
xx0	$T_{L_x} < 0$
xx1	$0 \leq T_{L_x} \leq 1.2$
xx2	$T_{L_x} > 1.2$

TABLE IV

COST REGION CODE DEFINITIONS

If $J \neq PR$, the appropriate comment is printed and PR, K_{p_θ}, T_{L_θ}, K_{p_x}, T_{L_x}, σ_q, σ_θ, σ_u, and σ_x are printed out.

The "paper pilot" rating is printed out along with a verbal description of the flying qualities of the aircraft. The verbal description is essentially extracted from MIL-F-008785A (USAF)[10].

Finally a "humorous" comment of the user's choice is printed out for the enjoyment of his friends.

IV. COMPUTATIONAL RESULTS

The "paper pilot" program was used to compute the "paper pilot" rating for 79 aircraft configuration-gust intensity combinations.

Twenty-six of the cases considered were taken from Table A-VI, Appendix A of Reference [3]. These cases are for the general form of the aircraft equations as described in Section 2.1.1. Table A-VI of Reference [3] includes measured data for σ_θ, σ_q, σ_x, and σ_u; and Table A-VII of Reference [3] gives the matched pilot parameters for these 26 cases. These cases are identified as "PH" cases.

Two cases were taken from Table IV of Reference [3] and include a control lag in the aircraft equation. These cases are identified as "PH(L)" cases.

Forty-eight cases were taken from Table A-VI, Appendix A of Reference [4]. These cases include 19 cases with a control lag, 14 cases with a SAS lag, and 9 cases with both control and SAS lags. These cases are identified as "PL" cases.

Finally, 3 cases were taken from Table VI of Reference [3]. The effect of limited pitch rate authority of the SAS is considered in these cases. The aircraft equations are described in Section 2.1.5 and are non-linear. Since the aircraft equations are non-linear, the resulting system equations are non-linear and existing describing function theory (Reference [10]) was used to determine an equivalent linear system which could be used to determine the "paper pilot" rating. This was done as follows.

Corresponding to Table VI of Reference [3], the following parameters were fixed:

$$M_u = .0208$$

$$X_u = -.1$$

$$M_\theta = 0$$

$$\tau_e = 0$$

$$\tau_q = 0$$

$$\sigma_g = 5.1$$

The "paper pilot" rating was then computed for $M_q = -3, -1.5, -1.0$ $-.5, -.25$, and 0, using the linearized aircraft equations. A plot of σ_q as a function of M_q is shown in Figure 8 and a plot of PR as a function of M_q is shown in Figure 9. For each value of M_{ra}, the describing function for the saturation nonlinearity of Figure 2 is determined from Figure 6-9 of Reference [11] and plotted on the graph of Figure 8. The intersection of the describing function and

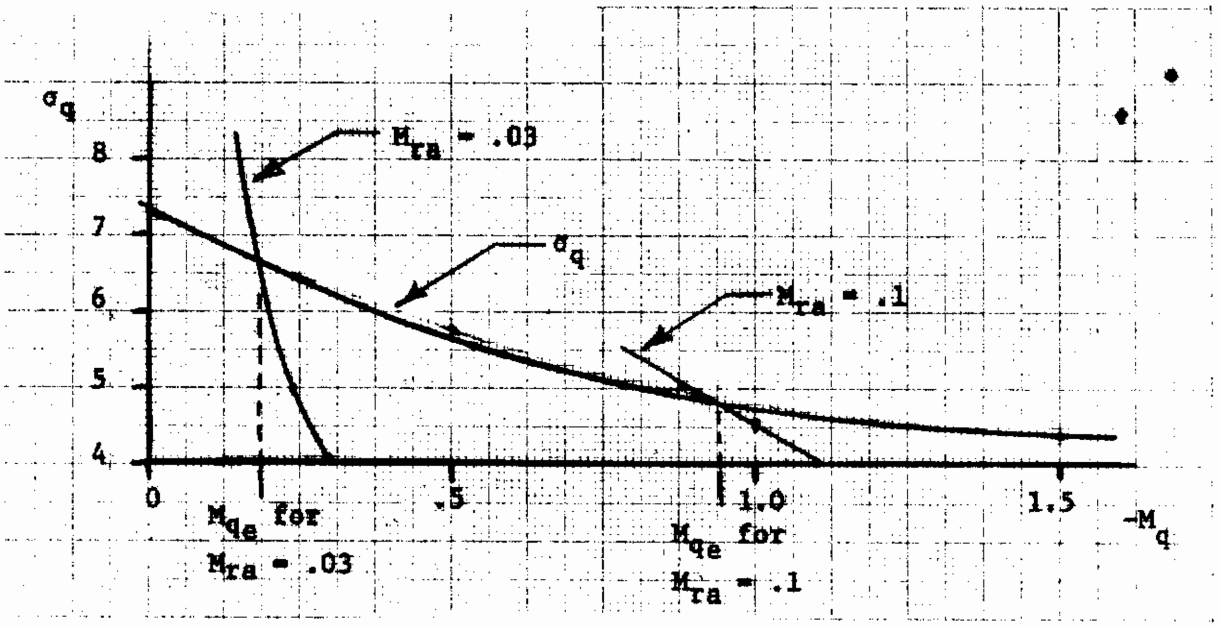

Fig. 8. "Paper Pilot" σ_q and Describing Function as a Function of M_q

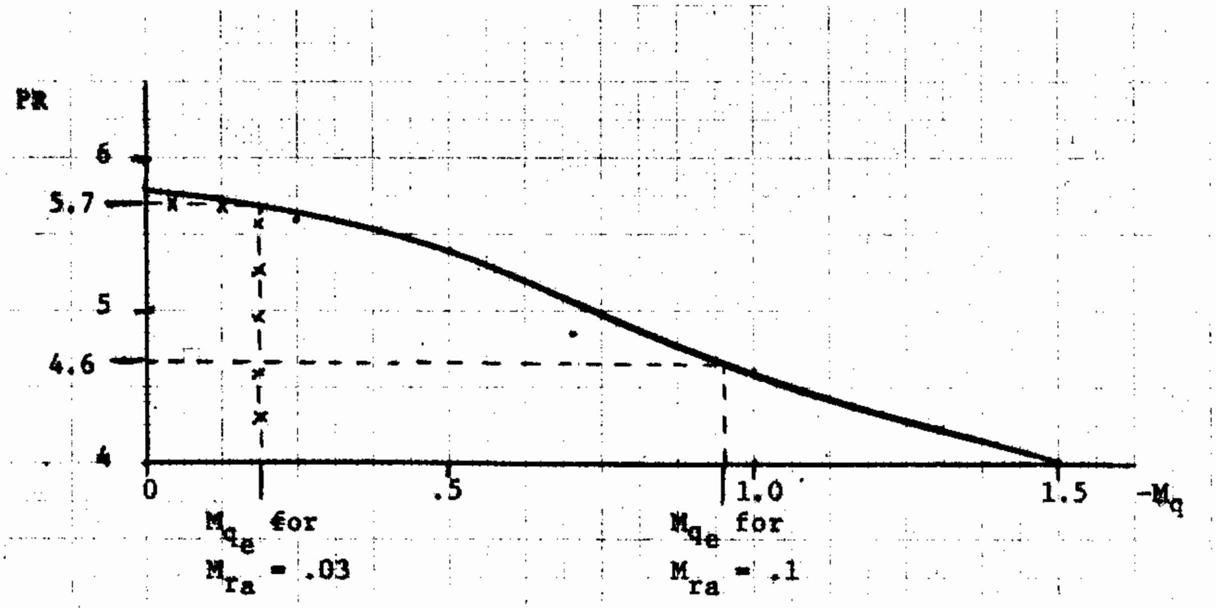

Fig. 9. "Paper Pilot" Rating as a Function of M_q

the "paper pilot" performance curve defines an operating point and a corresponding effective M_q, M_{q_e}, for the equivalent linear system. Then for $M_q = M_{q_e}$, the "paper pilot" rating is taken from Figure 9. These cases are identified as "PN" cases.

In Appendix B, Table VI, the aircraft parameters, gust intensity, actual pilot rating, and "paper pilot" rating are tabulated for each case run. The differences between the actual pilot rating and the "paper pilot" rating are also listed. For those cases where two pilots rated, the average of the two ratings was taken to be the actual pilot rating in computing the difference.

In Table VII (Appendix B), matched pilot parameters and "paper pilot" parameters are tabulated. The matched pilot parameters are taken from Table A-VII of Reference [3] and represent the pilot model parameters (the pilot model in Reference [2] is slightly different from the one described in Section 2.3) which result in a best fit to the actual rms responses. The differences between the matched pilot parameters and the "paper pilot" parameters are also listed in Table VII.

Table VIII (Appendix B) contains a listing of actual pilot performance and "paper pilot" performance for each case. The differences between actual pilot and "paper pilot" performance are also listed in Table VIII.

As it was previously mentioned, the model is based upon the assumption that M_δ is selected optimally for each given aircraft configuration. This assumption is consistent with the results of References [3] and [4]. With the exception of cases PH6, PH7, PH9, PH10, and PH12, the pilots selected the value of M_δ that they considered optimal for each configuration. In cases PH6, PH7, PH9, PH10, and PH12, M_δ was selected from empirically developed curves given in Reference [3] which are used to determine the optimal M_δ for a given aircraft configuration.

A comparison of the "paper pilot" results and actual pilot results is summarized in Table V.

As noted in Table V, the "paper pilot" tends to rate low with increasing σ_g. For the two cases where $\sigma_g \geq 10.3$, PH32 and PH34, the "paper pilot" rates a full rating point under the actual rating. This result indicates that the model, in its present form, is not valid for rms longitudinal gust intensities greater than 10.3 ft/sec. These results are consistent with the results of Reference [1] for high gust intensities. This does not appear to be a serious restriction however, inasmuch as an rms gust intensity of 10 ft/sec represents severe turbulence.

Four of the cases with both a control lag and SAS lag resulted in "paper pilot" ratings that were on the order of one or more rating points lower than the actual pilot. These results suggest a possible deficiency in the model for higher order systems. Since only eight

Data	Trend			Mean of the Difference	Standard Deviation of the Difference*
		τ_e and $\tau_q \leq .2$ sec	τ_e or $\tau_q \geq .3$ sec		
PR	M_q -1	"Paper pilot" PR is too high except for PH32, PH35, PH36, and PL4 (τ_e=.2). (22 cases)	"Paper pilot" PR is too low (1 case)	.14	.63
	M_q -3 -5 -6 -9	"Paper pilot" PR is too low except for PL14 (τ_e=.2) and PL17 (40 cases)	"Paper pilot" PR is too high except for PL16 and PL21. (13 cases)		
	"Paper Pilot" tends to rate too low with increasing σ_g				

*The difference is taken to be (actual pilot data)-("paper pilot" data). In those cases where two actual pilots were used, the average is used in the computation of the differences.

TABLE V

COMPARISON OF "PAPER PILOT" RESULTS TO ACTUAL PILOT RESULTS

Data	Trend	Mean of the Difference*	Standard Deviation of the Difference*
K_{p_θ}	"Paper Pilot" K_{p_θ} is <u>lower</u> except in 2 cases	.18	.13
T_{L_θ}	"Paper pilot" T_{L_θ} is <u>higher</u> except in 2 cases	-.09	.08
K_{p_x}	"Paper pilot" K_{p_x} is <u>higher</u> in every case	-.90	.43
T_{L_x}	"Paper pilot" T_{L_x} is <u>lower</u> in every case	.25	.18
σ_θ	"Paper pilot" σ_θ is <u>greater</u> in every case	-.70	.48
σ_q	"Paper pilot" σ_q is <u>greater</u> except in one case	-1.12	.91
σ_x	"Paper pilot" σ_x is <u>less</u> except in 5 cases	.28	.44
σ_u	"Paper pilot" σ_u <u>agrees well</u> with actual σ_u	-.14	.24

*The difference is taken to be (actual pilot data)-("paper pilot" data). In those cases where two actual pilots were used, the average is used in the computation of the differences.

TABLE V (Cont.)

COMPARISON OF "PAPER PILOT" RESULTS TO ACTUAL PILOT RESULTS cases with both control and SAS lags were considered, the results are not conclusive. In fact, an analysis of the results indicate that this problem might be corrected for those cases with systems lags of less than .2 second by an adjustment to the model.

For τ_e and $\tau_q \leq .2$ seconds, a trend in the "paper pilot" ratings is evident with respect to M_q. For $M_q = -1$, the "paper pilot" ratings tend to be too high and for $M_q < -1$ the "paper pilot" ratings tend to be too low. This trend suggests that a better fit to the actual pilot ratings might be achieved by an increase in the weighting on performance (W_1, W_2, W_3, W_4) and a corresponding decrease in the weighting on T_{L_θ} (W_5), Such a modification may also result in a model which is valid for higher gust intensities. In addition, for those cases where actual performance data was available, the "paper pilot" had poorer performance in rms pitch angle and pitch rate than the actual pilots. Thus an increase in weighting on σ_θ and σ_q might also result in better agreement between "paper pilot" performance and actual performance. This alteration to the model is consistent with the changes suggested for improving the agreement between "paper pilot" ratings and the actual pilot ratings (provided the control and SAS lags are less than .2 seconds).

In those cases with τ_e or $\tau_q \geq .3$, the trend with respect to M_q reverses. In particular, for $M_q \leq -3$, the "paper pilot" ratings tend to be too high.

In general, however, the "paper pilot" ratings agree well with the actual pilot ratings. A standard deviation of .63 in the differences is considered comparable to the variations in pilot ratings from one pilot to another and even to the variations in rating for the same pilot in repeated runs. The actual pilot ratings range from 2.0 to 6.5 on the Cooper pilot rating scale for the 79 cases considered. This is the range of ratings which are of primary interest from a handling qualities point of view.

One final point of concern may be that comparing the "paper pilot" ratings with actual pilot ratings from one and two pilots is not completely statistically sound. The fact remains that although a great deal of pilot rating data is available for aircraft in hover flight, only References [3] amd [4] contain results which are compatible with the "paper pilot" model. This is because the other known sources of pilot rating data contain such inconsistencies as flight test results with incomplete data on the aircraft dynamics, flight test results with no information on the gust environment, simulation runs which included tasks other than precision hover (such as maneuvers, transition), and tests which included in the rating the pilots evaluation of the open loop flying qualities of the aircraft. Furthermore, it is not inconsistent with usual manual control theory practices to develop or validate an analytical model with a statistically limited data base.

V. SUMMARY AND CONCLUSIONS

The following list summarizes the restrictions and limitations in the "paper pilot" program as well as the assumptions made in the development of the model.

1. The model applies to a VTOL aircraft in the precision hover task.

2. Only the longitudinal equations of motion are considered.

3. No plunge is considered. This is based on the assumption that the longitudinal equations and plunge equations are essentially uncoupled.

4. $X_\delta = 0$. That is, it is assumed that translation is accomplished only through pitch.

5. It is assumed that M_δ is chosen optimally by the pilot.

6. $0 < \sigma_g m < 10.3$. The upper limit of 10.3 is based on the fact that this is the highest gust intensity considered for which the "paper pilot" rating is within one rating point of the actual pilot rating.

7. The initial guess for the pilot parameters must result in a stable system or the "paper pilot" program cannot rate the aircraft.

8. The program doesn't limit the magnitudes of T_{L_θ} and T_{L_x}. Thus if $|T_{L_\theta}|$ and/or $|T_{L_x}|$ are greater than 5, the "paper pilot" rating may be invalid.

Within the framework of these restrictions, the method of analysis as implemented by the "paper pilot" computer program provides a capability of "automatically" predicting ratings which correlate well with actual pilot ratings for the VTOL hover task. Thus the program represents a potentially powerful tool for specifying handling qualities. This is true for a number of reasons. For example, the model doesn't require a specific type of augmentation. Also, control and SAS dynamics are easily accounted for in the model and the model accounts for the effect of gust intensity. For a detailed discussion of the advantages of using the model for specifying handling qualities, see Reference [1].

There are a number of possible extensions and applications of the "paper pilot" model. These are --

1. To "fine tune" the model for better agreement with actual pilot rating and performance. This is done by finding a better choice of the coefficients, W_1 through W_7, and by finding a better pilot gain margin requirement (i.e., other than 20%).

2. To include the effect of a non-optimal M_δ in the "paper pilot" model.

3. To investigate the effect of different reaction time delays, τ.

4. To use the "paper pilot" rating as a criteria in the design of the control and augmentation systems.

5. To use the "paper pilot" in conjunction with describing function theory to investigate handling qualities of aircraft with simple nonlinearities.

6. To extend the "paper pilot" concept to other tracking tasks such as landing and precision weapon delivery.

7. To extend the "paper pilot" concept to include additional loops such as height, roll, and yaw.

The ability to achieve the last two extensions is the real test of the concept. The precision hover task may represent an anomaly and the "paper pilot" concept may be of limited value. But on the other hand the "paper pilot" concept may represent the first step of a breakthrough in pilot-vehicle analysis and handling qualities specification. The results reported here seem far too good to represent an "accident."

REFERENCES

[1] R. O. Anderson, A New Approach to the Specification and Evaluation of Flying Qualities, AFFDL-TR-69-120, December 1969.

[2] J. D. McDonnell, Pilot Rating Techniques for the Estimation and Evaluation of Handling Qualities, Systems Technology, Inc., AFFDL-TR-68-76, December 1968.

[3] D. P. Miller and E. W. Vinje, Fixed-Base Flight Simulator Studies of VTOL Aircraft Handling Qualities in Hovering and Low Speed Flight, United Aircraft Research Laboratories, AFFDL-TR-67-152, January 1968.

[4] E. W. Vinje and D. P. Miller, Analytical and Flight Simulator Studies to Develop Design Criteria for VTOL Aircraft Control Systems, United Aircraft Research Laboratories, AFFDL-TR-68-165, April 1969.

[5] D. T. McRuer, et al, "Human Pilot Dynamics in Compensatory Systems", Systems Technology, Inc., AFFDL-TR-65-15, July 1965.

[6] J. LaSalle and S. Lefschetz, Stability by Liapunov's Direct Method, Academic Press, New York, 1961, pp. 81-84.

[7] Mathematical Methods for Digital Computers, Edited by A. Ralston and H. S. Wilf, John Wiley & Sons, New York, 1962, Chapter 7.

[8] A. E. Bryson and Y. Ho, Applied Optimal Control, Blaisdell Publishing Co., Waltham, Mass., 1969, pp. 328-334.

[9] R. Fletcher and C. M. Reeves, Function Minimization by Conjugate Gradients, Computer Journal, Vol. 7, Issue 2, 1964, pp. 149-154.

[10] Military Specification Flying Qualities of Piloted Airplanes, MIL-F-008785A (USAF), 31 October 1968.

[11] D. Graham and D. McRuer, Analysis of Nonlinear Control Systems, John Wiley and Sons, Inc., New York, 1961.

[12] R. A. Smith, Matrix Equation XA + BX = C, SIAM J. Appl. Math. Vol. 16, No. 1, 1968.

[13] D. L. Kleinman, An Iterative Technique for Riccati Equation Computations, Bolt Beranek and Newman, Inc., TM No. DLK-1, June 30, 1970.

APPENDIX A

"PAPER PILOT" PROGRAM LISTING

This appendix contains the program listing of the "paper pilot" digital computer program. The program is written in Fortran IV language and consists of a MAIN program and 12 subroutines, PILOT, FMCG, FPR, AMATRX, STAB, ENVRT, CALD, EIGEN, CAL, DAMRTX, LOGPR, and RITE.

The MAIN program is primarily used to read and write the input data (the input data is described in Table II) and to set up constants that are used in the remainder of the program. Two integer constants must be set by the user. The first integer is N, the order of the system equations. The other integer is NGUST which defines the system state associated with the longitudinal gust equation (i.e., $z(NGUST) = u_g$).

Subroutine PILOT implements the "paper pilot" rating scheme as described in Section 2.6. It is used to call the function minimization routine to find the pilot parameters which minimize J (defined in Section 2.5); to test, and if necessary adjust, the pilot gains for a 20% stability margin; to compute PR (defined in Section 2.6); and to write out a verbal description of the aircraft's handling qualities.

FMCG is an IBM scientific library routine for finding the minimum of a function of several variables by the method of conjugate gradients. The routine has been modified to work with functions which can take on positive infinite values, provided the function is finite valued on a convex set which includes the initial guess of the variables. The routine has also been modified to carry a number of working arrays and constants through the call statement of FMCG and the call statement of the subroutine to call for the function value and the gradient.

Subroutine FPR is used to compute J (defined in Section 2.5) and the gradient of J (described in Section 3.1) as a function of the four pilot parameters. If the pilot parameters result in a system which is not asymptotically stable, the appropriate return code is set and the cost and gradient computation is skipped.

Subroutine AMATRX is a user written subroutine which gives the A matrix as a function of the pilot parameters (for example see Section 2.4).

Subroutine STAB is used to determine whether or not the system $\dot{x} = Ax$ is asymptotically stable. The method used is described in Section 3.1.

ENVRT is a subroutine used to compute the inverse of a matrix using Fedeeva's Algorithm.

Subroutine CALD is used to solve the matrix linear equation X-AXA'-R=0 for X. The method used is described in References [12] and [13].

EIGEN is an IBM scientific library routine used to compute the eigenvalues of a real symmetric matrix. The Jacobi Method described in Reference [7] is used.

Subroutine CAL is used to solve the matrix covariance equation $AX + XA' + Q = 0$ for X. The method used is described in References [12] and [13].

Subroutine DAMTRX is a user written subroutine which gives the tensor, dA/dk, as a function of the pilot parameters. "A" and "k" are defined in Section 2.4.

Subroutine LOGPR is used in two ways depending on an input code. For one input code, LOGPR computes PR given σ_q, σ_θ, σ_u, σ_x, T_{L_θ}, and T_{L_x}. Alternately, LOGPR is called by FPR to account for R1 and R2 in the computation of J and the gradient of J. For the definition of PR see Section 2.6 and for the definition of R1, R2, and J see Section 2.5.

Subroutine RITE is a user written subroutine and is used to write out a concluding alpha-numeric statement of the user's choice, preferably something humorous.

Note that if a different set of system equations is to be used (i.e., to include control lags, SAS lags, etc.), the user must set the values of N and NGUST in the MAIN program and provide the appropriate subroutines AMATRX and DAMTRX. Under any circumstance, the program requires that the first four system states be pitch rate, pitch angle, longitudinal velocity, and longitudinal position in that order. If the dimension of the system equations exceeds 8, then A, Q, B, X, DA, STOR, and QQ must be appropriately redimensioned in the MAIN program.

```
$IBJOB
$IBFTC MAIN     M94,XR7,DECK
C
C    ******************************************************************
C
C
C         ** PAPER PILOT **
C
C
C         A DIGITAL COMPUTER PROGRAM FOR PREDICTING PILOT RATING
C
C         FOR FUNCTIONAL DESCRIPTION, SEE AFFDL TR 70-40
C
C         SUBROUTINE CALLED
C              PILOT(A,Q,B,X,DA,STOR,QQ,EK,G,HC,M,N,NA)
C
      COMMON/PARAM/EMQ,EMTH,EMU,EMDL,XU,WB,GDC,TAUM,TAUE,TAUQ
      COMMON/GUSTW/WG(7),NGUST,SIGUG
      COMMON/MINI/EST,EPS,LIMIT
      DIMENSION A(8,8),Q(8,8),B(36,36),X(36),DA(4,8,8),STOR(8,8),QQ(8,8)
     1,EK(8),G(4),HC(8),TITLE(12)
C
C         SET N=DIMENSION OF SYSTEM EQUATIONS
      N=6
C
C         NGUST IS SET TO CORRESPOND TO THE STATE ASSOCIATED WITH THE
C         LONGITUDINAL GUST EQUATION
      NGUST=5
C
C         SET VALUES OF WEIGHTING COEFFICIENTS FOR PR AND J AS DEFINED IN
C              AFFDL TR 70-40
      WG(1)=.218
      WG(2)=0.
      WG(3)=0.
      WG(4)=1.25
      WG(5)=2.5
      WG(6)=1.
      WG(7)=1.
C
C         READ INPUT DATA PER TABLE II, AFFDL TR 70-40
 1    READ(5,101) TITLE
      READ(5,1020) EMU,XU,EMQ,EMTH,EMDL,TAUE,TAUQ
      READ(5,1021) SIGUG,EST,EPS,LIMIT
      READ(5,103) EKPTH,TLTH,EKPX,TLX
 101  FORMAT(12(A6))
 1020 FORMAT(7(F10.5))
 1021 FORMAT(3(F10.5),I5)
 103  FORMAT(4(F10.5))
C
C         WRITE INPUT DATA
      WRITE(6,104) TITLE
      WRITE(6,1050) EMU,XU,EMQ,EMTH,EMDL,TAUE,TAUQ
      WRITE(6,1051) SIGUG,EST,EPS,LIMIT
      WRITE(6,106) EKPTH,TLTH,EKPX,TLX
 104  FORMAT(1H1,1X,8HCASE -- ,12(A6),//)
```

```
 1050 FORMAT(1X,11HMU          = ,F10.5,/,
     1       1X,11HXU          = ,F10.5,/,
     2       1X,11HMQ          = ,F10.5,/,
     3       1X,11HMTHETA      = ,F10.5,/,
     4       1X,11HMDELTA      = ,F10.5,/,
     5       1X,11HTAUE        = ,F10.5,/,
     6       1X,11HTAUQ        = ,F10.5,//)
 1051 FORMAT(1X,11HSIG GUST    = ,F10.5,//,
     1       1X,11HESTIMATE    = ,F10.5,/,
     2       1X,11HEPSILON     = ,F10.5,/,
     3       1X,11HLIMIT         ,I5,//)
 106  FORMAT(1X,11HKPTH        = ,F10.5,/,1X,11HTLTH       = ,F10.5,/,1X,11HK
     1PX       = ,F10.5,/,1X,11HTLX       = ,F10.5,///)
      WRITE(6,107)
 107  FORMAT(5X,2HPR,9X,4HKPTH,8X,4HTLTH,9X,3HKPX,9X,3HTLX,8X,4HSIGQ,7X,
     15HSIGTH,8X,4HSIGU,8X,4HSIGX,8X,5H//G//,//)
C
C        WB=THE BREAK FREQUENCY OF THE LONGITUDINAL GUST SPECTRUM
      WB=.314
C
C        TAU=THE PILOT REACTION TIME DELAY
      TAU=.44
C
C        M=THE NUMBER OF PILOT PARAMETERS
      M=4
      C=57.3
C
C        GDC=THE CONSTANT OF GRAVITY DIVIDED BY 57.3
      GDC=32.2/C
C
C        TAUM=4.DIVIDED BY THE PILOT REACTION TIME DELAY
      TAUM=4./TAU
      NA=(N*(N+1))/2
      IF(TAUQ.LE.0.) GO TO 5
C
C        TAUQ IS REDEFINED AS 1. DIVIDED BY THE SAS LAG
      TAUQ=1./TAUQ
 5    IF(TAUE.LE.0.) GO TO 6
C
C        TAUE IS REDEFINED AS 1. DIVIDED BY THE CONTROL LAG
      TAUE=1./TAUE
 6    CONTINUE
C
C        THE AIRCRAFT STABILITY DERIVATIVES ARE CONVERTED FROM UNITS OF
C        RADIANS TO DEGREES
      EMDL=C*EMDL
      EMU=C*EMU
C
C        SET EK(1) THRU EK(4) EQUAL TO THE PILOT PARAMETERS
C        EK(5) THRU EK(8) ARE USED TO STORE THE RMS VALUES OF PITCH RATE,
C            PITCH,LONGITUDINAL VELOCIY, AND LONGITUDINAL POSITION,
C            RESPECTIVELY
      EK(1)=EKPTH
      EK(2)=TLTH
      EK(3)=EKPX
```

```
      EK(4)=TLX
C
C        COMPUTE PAPER PILOT RATING AND CLOSED LOOP PERFORMANCE
      CALL PILOT(A,Q,B,X,DA,STOR,QQ,EK,G,HC,M,N,NA)
      GO TO 1
      END
$IBFTC AMX      DECK
C
C     ....................................................................
C
C        SUBROUTINE AMATRX
C
C        PURPOSE
C           TO COMPUTE THE SYSTEM A MATRIX AS A FUNCTION OF THE PILOT
C           PARAMETERS
C
C        USAGE
C           CALL AMATRX(N,A,EK)
C
C        DESCRIPTION OF PARAMETERS
C           N  - THE DIMENSION OF THE SYSTEM EQUATIONS.  THIS IS AN
C                INPUT
C           A  - THE SYSTEM A-MATRIX AS A FUNCTION OF THE PILOT
C                PARAMETERS. THE 1ST ROW MUST CORRESPOND TO THE
C                PITCH RATE EQUATION.  THE 2ND ROW MUST CORRESPOND
C                TO THE PITCH ANGLE EQUATION.  THE 3RD ROW MUST
C                CORRESPOND TO THE LONGITUNAL VELOCITY EQUATION. THE
C                4TH ROW MUST CORRESPOND TO THE LONGITUDINAL POSITION
C                EQUATION.  THE NGUST ROW (SEE THE MAIN PROGRAM) MUST
C                CORRESPOND TO THE LONGITUDINAL GUST EQUATION.
C
C           EK - EK(1) THRU EK(4) ARE USED TO INPUT THE PILOT PARAMETERS
C
      SUBROUTINE AMATRX(N,A,EK)
      DIMENSION A(N,N),EK(1)
      COMMON/PARAM/EMQ,EMTH,EMU,EMDL,XU,WB,GDC,TAUM,TAUE,TAUQ
      ALFA1=EK(1)*EK(2)
      ALFA2=EK(3)*EK(4)
      BETA1=EK(1)
      BETA2=EK(3)
      ALFA=ALFA1*ALFA2
      BETA=BETA1*BETA2
      A1B2=ALFA1*BETA2
      A2B1=ALFA2*BETA1
      DO 1 I=1,N
      DO 1 J=1,N
    1 A(I,J)=0.
      A(2,1)=1.
      A(3,2)=-GDC
      A(3,3)=XU
      A(3,5)=XU
      A(4,3)=1.
      A(5,5)=-WB
      A(6,1)=-ALFA1
      A(6,2)=-ALFA*GDC-BETA1
```

```
      A(6,3)=ALFA*XU+A1B2+A2B1
      A(6,4)=BETA
      A(6,5)=ALFA*XU
      DO 2 I=1,N
      A(1,I)=-EMDL*A(6,I)
 2    A(6,I)=TAUM*A(6,I)
      A(1,1)=A(1,1)+EMQ
      A(1,2)=A(1,2)+EMTH
      A(1,3)=A(1,3)+EMU
      A(1,5)=A(1,5)+EMU
      A(1,6)=EMDL
      A(6,6)=-.5*TAUM
      RETURN
      END
$IBFTC DADK     DECK
C
C     ..................................................................
C
C        SUBROUTINE DAMTRX
C
C        PURPOSE
C           TO COMPUTE THE DERIVATIVE OF THE A-MATRIX WITH RESPECT TO
C           THE PILOT PARAMETERS
C
C        USAGE
C           CALL DAMTRX(N,M,DA,EK)
C
C        DESCRIPTION OF PARAMETERS
C           N   - THE DIMENSION OF THE SYSTEM EQUATIONS.  THIS IS AN
C                 INPUT
C           M   - THE NUMBER OF PILOT PILOT PARAMETERS. THIS IS AN
C                 INPUT
C           DA - M  BY  N BY N  ARRAY WHERE  DA(I,J,K)=THE PARTIAL OF
C                 A(J,K) WITH RESPECT TO EK(I). A IS THE SYSTEM A-MATRIX
C                 AND IS DEFINED IN  SUBROUTINE AMATRX
C           EK    EK(1) THRU EK(4) ARE USED TO INPUT THE PILOT PARAMETERS
C
      SUBROUTINE DAMTRX(N,M,DA,EK)
      DIMENSION DA(M,N,N),EK(1)
      COMMON/PARAM/EMQ,EMTH,EMU,EMDL,XU,WB,GDC,TAUM,TAUE,TAUQ
      ALFA1=EK(1)*EK(2)
      ALFA2=EK(3)*EK(4)
      BETA1=EK(1)
      BETA2=EK(3)
      A123=ALFA1*BETA2
      A124=ALFA1*EK(4)
      A134=ALFA2*BETA1
      A234=ALFA2*EK(2)
      A2P4=EK(2)+EK(4)
      BETA=BETA1*BETA2
      DO 5 K=1,M
      DO 5 I=1,N
      DO 5 J=1,N
 5    DA(K,I,J)=0.
      DA(1,6,1)=-EK(2)
```

```
      DA(1,6,2)=-GDC*A234-1.
      DA(1,6,3)=A234*XU+BETA2*A2P4
      DA(1,6,4)=BETA2
      DA(1,6,5)=A234*XU
      DA(2,6,1)=-BETA1
      DA(2,6,2)=-GDC*A134
      DA(2,6,3)=A134*XU+BETA
      DA(2,6,5)=A134*XU
      DA(3,6,2)=-GDC*A124
      DA(3,6,3)=A124*XU+BETA1*A2P4
      DA(3,6,4)=BETA1
      DA(3,6,5)=A124*XU
      DA(4,6,2)=-GDC*A123
      DA(4,6,3)=A123*XU+BETA
      DA(4,6,5)=A123*XU
      DO 10 I=1,M
      DO 10 J=1,N
      DA(I,1,J)=-EMDL*DA(I,6,J)
 10   DA(I,6,J)=TAUM*DA(I,6,J)
      RETURN
      END
$IBFTC PP       DECK
C
C     ...................................................................
C
C        SUBROUTINE PILOT
C
C        PURPOSE
C           TO IMPLEMENT THE PAPER PILOT RATING SCHEME AS DESCRIBED IN
C           AFFDL TR 70-40
C
C        USAGE
C           CALL PILOT(A,Q,B,X,DA,STOR,QQ,EK,G,HC,M,N,NA)
C
C        DESCRIPTION OF PARAMETERS
C           A      - THE N BY N SYSTEM A-MATRIX. SEE SUBROUTINE AMATRX
C                    FOR A DESCRIPTION.
C           Q      - AN N BY N WORKING MATRIX
C           B      - AN NA BY NA WORKING MATRIX
C           X      - AN NA WORKING VECTOR
C           DA     - AN M BY N BY N ARRAY USED TO STORE THE DERIVATIVE
C                    OF THE SYSTEM A-MATRIX WITH RESPECT TO THE PILOT
C                    PARAMETERS. SEE SUBROUTINE DAMTRX FOR DESCRIPTION.
C           STOR   - AN N BY N WORKING MATRIX
C           QQ     - AN N BY N WORKING MATRIX
C           EK     - EK(1) THRU EK(4) ARE USED TO INPUT THE INITIAL PIOT
C                      PARAMETERS
C                    EK(5) THRU EK(8) ARE USED TO STORE THE RMS VALUES OF
C                      PITCH RATE,PITCH,LONGITUDINAL VELOCITY,AND
C                      LONGITUDINAL POSITION, RESPECTIVELY
C           G      - AN N VECTOR CONTAINING THE GRADIENT OF J (SEE AFFDL
C                    TR 70-40 FOR DEFINITION OF J) ON RETURN FROM
C                    SUBROUTINE FMCG.
C           HC     - WORKING STORAGE OF DIMENSION 2*N
C           M      - NUMBER OF PILOT PARAMETERS
```

```
C             N     - THE DIMENSION OF THE SYSTEM EQUATIONS
C             NA    - N*(N+1)/2
C
C         SUBROCUTINES CALLED
C             AMATRX(N,A,G)
C             FMCG(FDR,M,EK,PR,G,EST,EPS,LIMIT,IER,HC,A,Q,B,X,DA,STOR,QQ,
C                  N,NA)
C             STAB(A,N,NA,Q,B,X,KS)
C             LOGPR(WG,EK,G,PR,KS,R1,R2,R3,M)
C             RITE
C
      SUBROUTINE PILOT(A,Q,B,X,DA,STOR,QQ,EK,G,HC,M,N,NA)
      COMMON/PARAM/WB,TAUM,GDC,XU,EMQ,EMTH,EMDL,TAUE,TAUQ,CTEMU,EMU
      COMMON/GUSTW/WG(7),NGUST,SIGUG
      COMMON/MINI/EST,EPS,LIMIT
      DIMENSION A(N,N),Q(N,N),B(NA,NA),EK(1),G(1),HC(1),X(1)
      EXTERNAL FPR
C
C         FIND  MINIMAL VALUE OF  J, PILOT PARAMETERS, AND PERFORMANCE
C         AS DEFINED IN AFFDL TR 70-40
      CALL FMCG(FPR,M,EK,PR,G,EST,EPS,LIMIT,IER,HC,A,Q,B,X,DA,STOR,QQ,N,
     1NA)
C
C         IDENTIFY IER CODE PER TABLE III OF AFFDL TR 70-40
      WRITE(6,108)IER
 108  FORMAT(1X,42HEXIT CONJUGATE GRADIENT SEARCH WITH IER = ,I2)
C
C         GO TO THE APPROPRIATE WRITE STATEMENT ACCORDING TO VALUE OF IER
      IF(IER.LT.3)GO TO 2
      IF(IER.GT.3) GO TO 8
C
C         IF INITIAL PILOT PARAMETERS RESULT IN UNSTABLE SYSTEM, GO TO
C         NEXT SET OF DATA
      WRITE(6,109)
 109  FORMAT(1X,51HINITIAL PILOT PARAMETERS RESULT IN UNSTABLE SYSTEM.,/
     1,1X,47HPAPER PILOT REQUIRES STABLE INITIAL PARAMETERS.)
      GO TO 50
C
C           COMPUTE THE NORM OF THE GRADIENT SQUARED
 2    GNRM=0.
      DO 7 I=1,M
 7    GNRM=GNRM+G(I)*G(I)
      WRITE(6,113) PR,(EK(I),I=1,8),GNRM
 113  FORMAT(1X,10(F10.5,2X))
      IF(IER.EQ.1)GO TO 3
      IF(IER.EQ.0)GO TO 4
 8    WRITE(6,110)
 110  FORMAT(1X,52HCAUTION.  MINIMIZATION ROUTINE FAILED TO WORK RIGHT.)
      GO TO 4
 3    WRITE(6,111) LIMIT
 111  FORMAT(1X,52HCONJUGATE GRADIENT ROUTINE FAILED TO CONVERGE AFTER ,
     1I2,1X,11HITERATIONS.)
C
C         TEST TO SEE IF ABS(TLTH) OR ABS(TLX) IS GREATER THAN 5
 4    IF(ABS(EK(2)).LE.5.) GO TO 20
```

```
      WRITE(6,125)
 125  FORMAT(1X,67HCAUTION, ABS(TLTH) IS GREATER THAN 5.  PILOT RATING M
     1AY BE INVALID.)
 20   IF(ABS(EK(4)).LE.5.) GO TO 21
      WRITE(6,126)
 126  FORMAT(1X,66HCAUTION, ABS(TLX) IS GREATER THAN 5.  PILOT RATING MA
     1Y BE INVALID.)
C
C       ADJUST PILOT GAIN FOR 20 PERCENT STABILITY MARGIN, IF NECESSARY
 21   DEL=.2
C
C          ADVANCE PILOT GAINS 20 PERCENT
      G(1)=1.2*EK(1)
      G(2)=EK(2)
      G(3)=1.2*EK(3)
      G(4)=EK(4)
      DO 13 I=1,5
C
C          COMPUTE A MATRIX
      CALL AMATRX(N,A,G)
C
C          TEST A MATRIX FOR STABILITY
      CALL STAB(A,N,NA,Q,B,X,KS)
      DEL=.5*DEL
      IF(KS.EQ.0) GO TO 11
C
C          A MATRIX IS UNSTABLE -- DECREASE PILOT GAINS
      G(1)=G(1)-DEL*EK(1)
      G(3)=G(3)-DEL*EK(3)
      GO TO 13
C
C          IF A MATRIX IS STABLE IN FIRST ITERATION, MINIMIZING GAINS
C          HAVE THE REQUIRED STABILITY MARGIN
 11   IF(I.EQ.1) GO TO 6
C
C          A MATRIX IS STABLE -- INCREASE PILOT GAINS
      G(1)=G(1)+DEL*EK(1)
      G(3)=G(3)+DEL*EK(3)
 13   CONTINUE
C
C       IF LAST SET OF PILOT PARAMETERS DO NOT RESULT IN STABLE SYSTEM
C       ADJUST GAINS FOR STABILITY
      IF(KS.EQ.0) GO TO 14
      G(1)=G(1)-2.*DEL*EK(1)
      G(3)=G(3)-2.*DEL*EK(3)
C
C          ADJUST PILOT GAINS FOR 20 PERCENT STABILITY MARGIN
 14   EK(1)=G(1)/1.2
      EK(3)=G(3)/1.2
C
C          COMPUTE  J FOR  ADJUSTED PILOT PARAMETERS
      CALL FPR(M,EK,PR,G,KS,A,Q,B,X,DA,STOR,QQ,N,NA)
      IF(KS.EQ.0)GO TO 15
C
C       IF ROUTINE FOR ADJUSTING PILOT PARAMETERS FAILS TO RESULT IN
```

```
C         20 PERCENT STABILITY MARGIN, GO TO NEXT SET OF DATA.
      WRITE(6,120)
  120 FORMAT(1X,79H*** PILOT PARAMETERS DO NOT RESULT IN 20 PER CENT STA
     1BILITY MARGIN IN GAINS ***,/,1X,70H*** PARAMETER ADJUSTMENT ROUTIN
     2E FAILS TO GIVE STABLE SYSTEM.  PUNT***)
      GO TO 50
   15 WRITE(6,121) PR,(EK(I),I=1,8)
  121 FORMAT(1X,102HPILOT GAINS ARE ADJUSTED TO PROVIDE 20 PER CENT STAB
     1ILITY MARGIN.  PILOT RATING IS MODIFIED AS FOLLOWS,/,1X,9(F10.5,2X
     2))
    6 KS=1
C
C         COMPUTE PILOT RATING AND DETERMINE COST REGION CODE AS DEFINED
C         IN TABLE IV OF AFFDL TR 70-40
      CALL LOGPR(WG,EK,G,PR,KS,R1,R2,R3,M)
      WRITE(6,118) KS
  118 FORMAT(1X,20HCOST REGION CODE IS ,I3)
C
C         IF PERF IS LESS THAN ZERO OR GREATER THAN R1MAX, IDENTIFY THE
C         DIFFERENCE BETWEE PERF AND R1 AS DEFINED IN AFFDL TR 70-40
      IF(KS.LT.100) GO TO 30
      IF(KS.LT.200) GO TO 35
   30 PERF=-1.
      DO 16 I=1,4
      J=I+M
   16 PERF =PERF+EK(J)*WG(I)
      IF(KS.LT.100) GO TO  17
      WRITE(6,117) R1,PERF,PR,(EK(I),I=1,8)
  117 FORMAT(1X,32HMAX PERFORMANCE CONTRIBUTION IS ,F7.3,26H.  COMPUTED
     1PERFORMANCE = ,F8.3,38H.  PILOT RATING IS MODIFIED AS FOLLOWS,/,1X
     2,9(F10.5,2X))
      GO TO 35
   17 WRITE(6,119) R1,PERF,PR,(EK(I),I=1,8)
  119 FORMAT(1X,32HMIN PERFORMANCE CONTRIBUTION IS ,F7.3,26H.  COMPUTED
     1PERFORMANCE = ,F8.3,38H.  PILOT RATING IS MODIFIED AS FOLLOWS,/,1X
     2,9(F10.5,2X))
C
C         WRITE OUT THE GUST INTENSITY AND THE ASSOCIATED PREDICTED PILOT RATING
   35 WRITE(6,201) SIGUG,PR
  201 FORMAT(//,      1X,58HFOR A GUST DISTURBANCE MODEL WITH A STANDARD
     1DEVIATION OF ,F6.3,1H.,/,1X,35HTHE PAPER PILOT RATES THIS AIRCRAFT
     2,//,12X,F10.5,//,1X,28HIN THE PRECISION HOVER MODE.,//)
C
C         DETERMINE A VERBAL DESCRIPTION OF THY FLYING QUALITIES OF THE AIRCRAFT
      IF(PR.GT.3.5)GO TO 36
      WRITE(6,202)
  202 FORMAT(5X,60HFLYING QUALITIES ARE CLEARLY ADEQUATE FOR THE MISSION
     1 FLIGHT,/,5X,30HPHASE OF PRECISION HOVER (PH).,///)
      GO TO 50
   36 IF(PR.LT.6.5)GO TO 37
      WRITE(6,203)
  203 FORMAT(5X,60HFLYING QUALITIES OUTSIDE OF LEVEL 3, AND ARE UNACCEPT
     1ABLE IN,/,5X,30HACCORDANCE WITH CURRENT SPECS.,//,10X,19HBACK TO T
     2HE BOARDS.,///)
      GO TO 50
```

```
 37    IF(PR.LT.5.5)GO TO 38
       WRITE(6,204)
 204   FORMAT(5X61HFLYING QUALITIES ARE SUCH THAT THE AIRCRAFT CAN BE CON
      1TROLLED,/,5X,63HSAFELY,BUT PILOT WORKLOAD IS EXCESSIVE OR MISSION
      2EFFECTIVENESS,/,5X,59HIS INADEQUATE,OR BOTH.  IN PARTICULAR,EVEN W
      3ITH LARGE PILOT,/,5X,65HLEAD GENERATION IN INNER AND OUTER LOOPS,
      4CLOSED-LOOP PERFORMANCE,/,5X,8HIS POOR.,///)
       GO TO 50
 38    WRITE(6,205)
 205   FORMAT(5X,63HFLYING QUALITIES ARE ADEQUATE TO ACCOMPLISH THE PRECI
      1SION HOVER,/,5X,64HFLIGHT PHASE, BUT SOME INCREASE IN PILOT WORKLO
      2AD OR DEGRADATION,/,5X,57HIN MISSION EFFECTIVENESS, OR BOTH, EXIST
      3S.  IN PARTICULAR)
       IF(R1.LT.R2)GO TO 39
       IF(R1.LT.R3)GO TO 39
       WRITE(6,206)
 206   FORMAT(5X,61HSYSTEM PERFORMANCE IN TERMS OF STANDARD DEVIATION IN
      1POSITION,/,5X,62HAND PITCH RATE IS POOR, WITH REDUCED OPERATIONAL
      2EFFECTIVENESS,/,5X,10HRESULTING.,///)
       GO TO 50
 39    WRITE(6,207)
 207   FORMAT(5X,58HPILOT WORKLOAD IN TERMS OF THE REQUIRED LEAD GENERATI
      1ON IS,/,5X 61HEXCESSIVE.  VEHICLE DYNAMICS SHOULD BE IMPROVED IN R
      2ESPECT TO)
       IF(R2.LT.R3)GO TO 40
       WRITE(6,208)
 208   FORMAT(5X,19HPITCH RATE DAMPING.,///)
       GO TO 50
 40    WRITE(6,209)
 209   FORMAT(5X,61HTHE REQUIREMENT FOR LARGE PILOT LEAD GENERATION IN TH
      1E OUTER,/,5X,14HPOSITION LOOP.,//)
C
C        WRITE OUT FINAL COMMENT
 50    CALL RITE
       RETURN
       END
$IBFTC FMCGM   DECK                                                     FMCG0000
C                                                                       FMCG0010
C      ..................................................................FMCG0020
C                                                                       FMCG0030
C         SUBROUTINE FMCG                                               FMCG0040
C                                                                       FMCG0050
C         PURPOSE                                                       FMCG0060
C            TO FIND A LOCAL MINIMUM OF A FUNCTION OF SEVERAL VARIABLES FMCG0070
C            BY THE METHOD OF CONJUGATE GRADIENTS                       FMCG0080
C                                                                       FMCG0090
C         USAGE                                                         FMCG0100
C            CALL FMCG(FUNCT,N,X,F,G,EST,EPS,LIMIT,IER,H,AZY,QZY,BZY,XZY,
C                          DZY,SZY,QQZY,NZY,NAZY)
C                                                                       FMCG0120
C         DESCRIPTION OF PARAMETERS                                     FMCG0130
C            FUNCT  - USER-WRITTEN SUBROUTINE CONCERNING THE FUNCTION TO FMCG0140
C                     BE MINIMIZED. IT MUST BE OF THE FORM              FMCG0150
C                     SUBROUTINE FUNCT(N,ARG,VAL,GRAD,KZAZ,AZY,QZY,BZY,
C                     XZY,DZY,SZY,QQZY,NZY,NAZY)
```

```
C                    AND MUST SERVE THE FOLLOWING PURPOSE                       FMCG0170
C                    FOR EACH N-DIMENSIONAL ARGUMENT VECTOR  ARG.               FMCG0180
C                    FUNCTION VALUE AND GRADIENT VECTOR MUST BE COMPUTEDFMCG0190
C                    AND, ON RETURN, STORED IN VAL AND GRAD RESPECTIVELYFMCG0200
C                    IN ADDITION, IF FUNCTION VALUE IS FINITE, KZAZ
C                    MUST BE SET TO ZERO. OTHERWISE, KZAZ NOT EQUAL
C                    ZERO.
C          N       - NUMBER OF VARIABLES                                        FMCG0210
C          X       - VECTOR OF DIMENSION N CONTAINING THE INITIAL               FMCG0220
C                    ARGUMENT WHERE THE ITERATION STARTS. ON RETURN,            FMCG0230
C                    X HOLDS THE ARGUMENT CORRESPONDING TO THE                  FMCG0240
C                    COMPUTED MINIMUM FUNCTION VALUE                            FMCG0250
C          F       - SINGLE VARIABLE CONTAINING THE MINIMUM FUNCTION            FMCG0260
C                    VALUE ON RETURN, I.E. F=F(X).                              FMCG0270
C          G       - VECTOR OF DIMENSION N CONTAINING THE GRADIENT              FMCG0280
C                    VECTOR CORRESPONDING TO THE MINIMUM ON RETURN.             FMCG0290
C                    I.E. G=G(X).                                               FMCG0300
C          EST     - IS AN ESTIMATE OF THE MINIMUM FUNCTION VALUE.              FMCG0310
C          EPS     - TESTVALUE REPRESENTING THE EXPECTED ABSOLUTE ERROR.FMCG0320
C                    A REASONABLE CHOICE IS 10**(-6), I.E.                      FMCG0330
C                    SOMEWHAT GREATER THAN 10**(-D), WHERE D IS THE             FMCG0340
C                    NUMBER OF SIGNIFICANT DIGITS IN FLOATING POINT             FMCG0350
C                    REPRESENTATION.                                            FMCG0360
C          LIMIT   - MAXIMUM NUMBER OF ITERATIONS.                              FMCG0370
C          IER     - ERROR PARAMETER                                            FMCG0380
C                    IER = 0 MEANS CONVERGENCE WAS OBTAINED                     FMCG0390
C                    IER = 1 MEANS NO CONVERGENCE IN LIMIT ITERATIONS           FMCG0400
C                    IER =-1 MEANS ERRORS IN GRADIENT CALCULATION               FMCG0410
C                    IER = 2 MEANS LINEAR SEARCH TECHNIQUE INDICATES            FMCG0420
C                    IT IS LIKELY THAT THERE EXISTS NO MINIMUM.                 FMCG0430
C                    IER = 3 MEANS INITIAL ARGUMENTS RESULT IN INFINITE
C                    FUNCTION VALUE
C                    IER = 4 MEANS CUBIC INTERPOLATION RESULTS IN
C                    INFINITE FUNCTION VALUE
C          H       - WORKING STORAGE OF DIMENSION 2*N.                          FMCG0440
C          AZY     - AN ARRAY CARRIED IN THE CALL STATEMENT IN ORDER TO
C                    USE A VARIABLE DIMENSION STATEMENT IN SUBROUTINE
C                    FUNCT
C          QZY     - AN ARRAY CARRIED IN THE CALL STATEMENT IN ORDER TO
C                    USE A VARIABLE DIMENSION STATEMENT IN SUBROUTINE
C                    FUNCT
C          BZY     - AN ARRAY CARRIED IN THE CALL STATEMENT IN ORDER TO
C                    USE A VARIABLE DIMENSION STATEMENT IN SUBROUTINE
C                    FUNCT
C          XZY     - AN ARRAY CARRIED IN THE CALL STATEMENT IN ORDER TO
C                    USE A VARIABLE DIMENSION STATEMENT IN SUBROUTINE
C                    FUNCT
C          DZY     - AN ARRAY CARRIED IN THE CALL STATEMENT IN ORDER TO
C                    USE A VARIABLE DIMENSION STATEMENT IN SUBROUTINE
C                    FUNCT
C          SZY     - AN ARRAY CARRIED IN THE CALL STATEMENT IN ORDER TO
C                    USE A VARIABLE DIMENSION STATEMENT IN SUBROUTINE
C                    FUNCT
C          QQZY    - AN ARRAY CARRIED IN THE CALL STATEMENT IN ORDER TO
C                    USE A VARIABLE DIMENSION STATEMENT IN SUBROUTINE
```

```
C                      FUNCT
C           QZY      - AN ARRAY CARRIED IN THE CALL STATEMENT IN ORDER TO
C                      USE A VARIABLE DIMENSION STATEMENT IN SUBROUTINE
C                      FUNCT
C           NZY      - AN ITEGER CARRIED IN THE CALL STATEMENT TO BE USED
C                      IN A VARIABLE DIMENSION STATEMENT IN SUBROUTINE
C                      FUNCT
C           NAZY     - AN ITEGER CARRIED IN THE CALL STATEMENT TO BE USED
C                      IN A VARIABLE DIMENSION STATEMENT IN SUBROUTINE
C                      FUNCT
C                                                                       FMCG0450
C        REMARKS                                                        FMCG0460
C            I) THE SUBROUTINE NAME REPLACING THE DUMMY ARGUMENT  FUNCT FMCG0470
C               MUST BE DECLARED AS EXTERNAL IN THE CALLING PROGRAM.    FMCG0480
C           II) IER IS SET TO 2 IF , STEPPING IN ONE OF THE COMPUTED    FMCG0490
C               DIRECTIONS, THE FUNCTION WILL NEVER INCREASE WITHIN     FMCG0500
C               A TOLERABLE RANGE OF ARGUMENT.                          FMCG0510
C               IER = 2 MAY OCCUR ALSO IF THE INTERVAL WHERE F          FMCG0520
C               INCREASES IS SMALL AND THE INITIAL ARGUMENT WAS         FMCG0530
C               RELATIVELY FAR AWAY FROM THE MINIMUM SUCH THAT THE      FMCG0540
C               MINIMUM WAS OVERLEAPED. THIS IS DUE TO THE SEARCH       FMCG0550
C               TECHNIQUE WHICH DOUBLES THE STEPSIZE UNTIL A POINT      FMCG0560
C               IS FOUND WHERE THE FUNCTION INCREASES.                  FMCG0570
C                                                                       FMCG0580
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                  FMCG0590
C           FUNCT                                                       FMCG0600
C                                                                       FMCG0610
C        METHOD                                                         FMCG0620
C           THE METHOD IS DESCRIBED IN THE FOLLOWING ARTICLE            FMCG0630
C           R.FLETCHER AND C.M.REEVES, FUNCTION MINIMIZATION BY         FMCG0640
C           CONJUGATE GRADIENTS,                                        FMCG0650
C           COMPUTER JOURNAL VOL.7, ISS.2, 1964, PP.149-154.            FMCG0660
C                                                                       FMCG0670
C     ..................................................................FMCG0680
C                                                                       FMCG0690
      SUBROUTINE FMCG(FUNCT,N,X,F,G,EST,EPS,LIMIT,IER,H,AZY,QZY,BZY,XZY,
     1DZY,SZY,QQZY,NZY,NAZY)
C                                                                       FMCG0710
C        DIMENSIONED DUMMY VARIABLES                                    FMCG0720
      DIMENSION X(1),G(1),H(1)                                          FMCG0730
C                                                                       FMCG0740
C                                                                       FMCG0750
C        COMPUTE FUNCTION VALUE AND GRADIENT VECTOR FOR INITIAL ARGUMENTFMCG0760
      CALL FUNCT(N,X,F,G,KZAZ,AZY,QZY,BZY,XZY,DZY,SZY,QQZY,NZY,NAZY)
C
C        IF INITIAL ARGUMENT RESULTS IN INFINITE FUNTION VALUE, SET IER
C        =3 AND RETURN
      IF(KZAZ.EQ.0)GO TO 101
      IER=3
      RETURN
C                                                                       FMCG0780
C        RESET ITERATION COUNTER                                        FMCG0790
  101 KOUNT=0
      IER=0                                                             FMCG0810
      N1=N+1                                                            FMCG0820
```

```
C                                                                       FMCG0830
C        START ITERATION CYCLE FOR EVERY N+1 ITERATIONS                 FMCG0840
    1 DO 43 II=1,N1                                                     FMCG0850
C                                                                       FMCG0860
C        STEP ITERATION COUNTER AND SAVE FUNCTION VALUE                 FMCG0870
      KOUNT=KOUNT+1                                                     FMCG0880
      OLDF=F                                                            FMCG0890
C                                                                       FMCG0900
C        COMPUTE SQUARE OF GRADIENT AND TERMINATE IF ZERO               FMCG0910
      GNRM=0.                                                           FMCG0920
      DO 2 J=1,N                                                        FMCG0930
    2 GNRM=GNRM+G(J)*G(J)                                               FMCG0940
      WRITE(6,500) F,(X(J),J=1,8),GNRM
  500 FORMAT(1X,10(F10.5,2X))
      IF(GNRM)46,46,3                                                   FMCG0950
C                                                                       FMCG0960
C        EACH TIME THE ITERATION LOOP IS EXECUTED , THE FIRST STEP WILL FMCG0970
C        BE IN DIRECTION OF STEEPEST DESCENT                            FMCG0980
    3 IF(II-1)4,4,6                                                     FMCG0990
    4 DO 5 J=1,N                                                        FMCG1000
    5 H(J)=-G(J)                                                        FMCG1010
      GO TO 8                                                           FMCG1020
C                                                                       FMCG1030
C        FURTHER DIRECTION VECTORS H WILL BE CHOOSEN CORRESPONDING      FMCG1040
C        TO THE CONJUGATE GRADIENT METHOD                               FMCG1050
    6 AMBDA=GNRM/OLDG                                                   FMCG1060
      DO 7 J=1,N                                                        FMCG1070
    7 H(J)=AMBDA*H(J)-G(J)                                              FMCG1080
C                                                                       FMCG1090
C        COMPUTE TESTVALUE FOR DIRECTIONAL VECTOR AND DIRECTIONAL       FMCG1100
C        DERIVATIVE                                                     FMCG1110
    8 DY=0.                                                             FMCG1120
      HNRM=0.                                                           FMCG1130
      DO 9 J=1,N                                                        FMCG1140
      K=J+N                                                             FMCG1150
C                                                                       FMCG1160
C        SAVE ARGUMENT VECTOR                                           FMCG1170
      H(K)=X(J)                                                         FMCG1180
      HNRM=HNRM+ABS(H(J))                                               FMCG1190
    9 DY=DY+H(J)*G(J)                                                   FMCG1200
C                                                                       FMCG1210
C        CHECK WHETHER FUNCTION WILL DECREASE STEPPING ALONG H AND      FMCG1220
C        SKIP LINEAR SEARCH ROUTINE IF NOT                              FMCG1230
      IF(DY)10,42,42                                                    FMCG1240
C                                                                       FMCG1250
C        COMPUTE SCALE FACTOR USED IN LINEAR SEARCH SUBROUTINE          FMCG1260
   10 SNRM=1./HNRM                                                      FMCG1270
C                                                                       FMCG1280
C        SEARCH MINIMUM ALONG DIRECTION H                               FMCG1290
C                                                                       FMCG1300
C        SEARCH ALONG H FOR POSITIVE DIRECTIONAL DERIVATIVE             FMCG1310
      FY=F                                                              FMCG1320
      ALFA=2.*(EST-F)/DY                                                FMCG1330
      AMBDA=SNRM                                                        FMCG1340
C                                                                       FMCG1350
```

```
C         USE ESTIMATE FOR STEPSIZE ONLY IF IT IS POSITIVE AND LESS THAN
C         SNRM. OTHERWISE TAKE SNRM AS STEPSIZE.
      IF(ALFA)13,13,11
   11 IF(ALFA-AMBDA)12,13,13
   12 AMBDA=ALFA
   13 ALFA=0.
C
C         SAVE FUNCTION AND DERIVATIVE VALUES FOR OLD ARGUMENT
   14 FX=FY
      DX=DY
C
C         STEP ARGUMENT ALONG H
      DO 15 I=1,N
   15 X(I)=X(I)+AMBDA*H(I)
C
C         COMPUTE FUNCTION VALUE AND GRADIENT FOR NEW ARGUMENT
      CALL FUNCT(N,X,F,G,KZAZ,AZY,QZY,BZY,XZY,DZY,SZY,QQZY,NZY,NAZY)
C
C         IF FUNCTION VALUE IS INFINITE, CONSTRAIN SEARCH BETWEEN NEW
C         ARGUMENT AND OLD ARGUMENT.
      IF(KZAZ.GT.0) GO TO 50
      FY=F
C
C         COMPUTE DIRECTIONAL DERIVATIVE DY FOR NEW ARGUMENT.  TERMINATE
C         SEARCH, IF DY POSITIVE. IF DY IS ZERO THE MINIMUM IS FOUND
      DY=0.
      DO 16 I=1,N
   16 DY=DY+G(I)*H(I)
      IF(DY)17,38,20
C
C         TERMINATE SEARCH ALSO IF THE FUNCTION VALUE INDICATES THAT
C         A MINIMUM HAS BEEN PASSED
   17 IF(FY-FX)18,20,20
C
C         REPEAT SEARCH AND DOUBLE STEPSIZE FOR FURTHER SEARCHES
   18 AMBDA=AMBDA+ALFA
      ALFA=AMBDA
C
C         TERMINATE IF THE CHANGE IN ARGUMENT GETS VERY LARGE
      IF(HNRM*AMBDA-1.E10)14,14,19
C
C         LINEAR SEARCH TECHNIQUE INDICATES THAT NO MINIMUM EXISTS
   19 IER=2
C
C     RESTORE OLD VALUES OF FUNCTION AND ARGUMENTS
      F=OLDF
      DO 100 J=1,N
      G(J)=H(J)
      K=N+J
  100 X(J)=H(K)
      RETURN
C
C         END OF SEARCH LOOP
C
C         INTERPOLATE CUBICALLY IN THE INTERVAL DEFINED BY THE SEARCH
```

```
C          ABOVE AND COMPUTE THE ARGUMENT X FOR WHICH THE INTERPOLATION
C          POLYNOMIAL IS MINIMIZED
C
   20 T=0.
   21 IF(AMBDA)22,38,22
   22 Z=3.*(FX-FY)/AMBDA+DX+DY
      ALFA=AMAX1(ABS(Z),ABS(DX),ABS(DY))
      DALFA=Z/ALFA
      DALFA=DALFA*DALFA-DX/ALFA*DY/ALFA
      IF(DALFA)23,27,27
C
C          RESTORE OLD VALUES OF FUNCTION AND ARGUMENTS
   23 DO 24 J=1,N
      K=N+J
   24 X(J)=H(K)
      CALL FUNCT(N,X,F,G,KZAZ,AZY,QZY,BZY,XZY,DZY,SZY,QQZY,NZY,NAZY)
C
C          IF FUNCTION VALUE IS INFINITE, SET IER=4 AND RETURN.
      IF(KZAZ.EQ.0) GO TO 25
      IER=4
      RETURN
C
C          TEST FOR REPEATED FAILURE OF ITERATION
   25 IF(IER)47,26,47
   26 IER=-1
      GOTO 1
   27 W=ALFA*SQRT(DALFA)
      ALFA=DY-DX+W+W
      IF(ALFA)270,271,270
  270 ALFA=(DY-Z+W)/ALFA
      GO TO 272
  271 ALFA=(Z+DY-W)/(Z+DX+Z+DY)
  272 ALFA=ALFA*AMBDA
      DO 28 I=1,N
   28 X(I)=X(I)+(T-ALFA)*H(I)
C
C          TERMINATE, IF THE VALUE OF THE ACTUAL FUNCTION AT X IS LESS
C          THAN THE FUNCTION VALUES AT THE INTERVAL ENDS. OTHERWISE REDUCE
C          THE INTERVAL BY CHOOSING ONE END-POINT EQUAL TO X AND REPEAT
C          THE INTERPOLATION.  WHICH END-POINT IS CHOOSEN DEPENDS ON THE
C          VALUE OF THE FUNCTION AND ITS GRADIENT AT X
C
      CALL FUNCT(N,X,F,G,KZAZ,AZY,QZY,BZY,XZY,DZY,SZY,QQZY,NZY,NAZY)
C
C          IF FUNCTION VALUE IS INFINITE, RESTORE OLD ARGUMENTS AND SEARCH
C          IN GRADIENT DIRECTION
      IF(KZAZ.GT.0) GO TO 23
      IF(F-FX)29,29,30
   29 IF(F-FY)38,38,30
C
C          COMPUTE DIRECTIONAL DERIVATIVE
   30 DALFA=0.
      DO 31 I=1,N
   31 DALFA=DALFA+G(I)*H(I)
      IF(DALFA)32,35,35
```

```
   32 IF(F-FX)34,33,35
   33 IF(DX-DALFA)34,38,34
   34 FX=F
      DX=DALFA
      T=ALFA
      AMBDA=ALFA
      GO TO 21
   35 IF(FY-F)37,36,37
   36 IF(DY-DALFA)37,38,37
   37 FY=F
      DY=DALFA
      AMBDA=AMBDA-ALFA
      GO TO 20
C
C     TERMINATE IF FUNCTION HAS NOT DECREASED DURING LAST ITERATION
C     OTHERWISE SAVE GRADIENT NORM
 38   IF(OLDF-F+EPS)19,25,39
 39   OLDG=GNRM
C
C     COMPUTE DIFFERENCE OF NEW AND OLD ARGUMENT VECTOR
      T=0.
      DO 40 J=1,N
      K=J+N
      H(K)=X(J)-H(K)
 40   T=T+ABS(H(K))
C
C     TEST LENGTH OF DIFFERENCE VECTOR IF AT LEAST N+1 ITERATIONS
C     HAVE BEEN EXECUTED. TERMINATE IF LENGTH IS LESS THAN EPS
      IF(KOUNT-N1)42,41,41
 41   IF(GNRM-EPS)45,45,42
C
C        TERMINATE, IF NUMBER OF ITERATIONS WOULD EXCEED  LIMIT
 42   IF(KOUNT-LIMIT)43,44,44
   43 IER=0
C        END OF ITERATION CYCLE
C
C        START NEXT ITERATION CYCLE
      GO TO 1
C
C        NO CONVERGENCE AFTER  LIMIT  ITERATIONS
   44 IER=1
      IF(GNRM-EPS)46,46,47
C
C        TEST FOR SUFFICIENTLY SMALL GRADIENT
 45   IF(T-EPS) 46,46,25
   46 IER=0
   47 RETURN
C        SEARCH RESULTED IN UNSTABLE CONDITION.  BACK OFF AND CONSTRAIN
C        SEARCH BETWEEN LAST ARGUMENT AND PREVIOUS ARGUMENT.
 50   AMBDA=.5*AMBDA
      DO 51  J=1,N
 51   X(J)=X(J)-AMBDA*H(J)
C
C        TERMINATE IF CHANGE IN ARGUMENT BECOMES VERY SMALL
 52   IF(HNRM*AMBDA-1.E-10)19,53,53
```

```
C
C         COMPUTE FUNCTION VALUE AND GRADIENT FOR NEW ARGUMENT
 53   CONTINUE
      CALL FUNCT(N,X,F,G,KZAZ,AZY,QZY,BZY,XZY,DZY,SZY,QQZY,NZY,NAZY)
      IF(KZAZ.GT.0)GO TO 50
      FY=F
C
C         COMPUTE DIRECTIONAL DERIVATIVE DY FOR NEW ARGUMENT. TERMINATE
C         SEARCH, IF DY POSITIVE. IF DY IS ZERO THE MINIMUM IS FOUND
      DY=0.
      DO 54 I=1,N
 54   DY=DY+G(I)*H(I)
      IF(DY)55,38,20
C
C         TERMINATE SEARCH ALSO IF FUNCTION VALUE INDICATES THAT THE
C         MINIMUM HAS BEEN PASSED
 55   IF(FY-FX)56,20,20
C
C         HALVE THE STEPSIZE AND CONTINUE TO SEARCH ALONG H
 56   AMBDA=.5*AMBDA
      FX=FY
      DX=DY
      DO 57 I=1,N
 57   X(I)=X(I)+AMBDA*H(I)
      GO TO 52
      END                                                                FMCG2670
$IBFTC FPRA    M94,XR7,DECK
C
C    ....................................................................
C
C         SUBROUTINE FPR
C
C         PURPOSE
C            TO COMPUTE THE FUNCTION VALUE AND GRADIENT OF J AS DEFINED
C            IN AFFDL TR 70-40
C
C         USAGE
C            CALL FPR(M,EK,PR,G,KS,A,Q,B,X,DA,STOR,QQ,N,NA)
C
C         DESCRIPTION OF PARAMETERS
C            M  - THE NUMBER OF PILOT PARAMETERS
C            EK    - EK(1) THRU EK(4) ARE USED TO INPUT THE PILOT
C                      PARAMETERS
C                    EK(5) THRU EK(8) ARE USED TO STORE THE RMS VALUES OF
C                      PITCH RATE,PITCH,LONGITUDINAL VELOCITY,AND
C                      LONGITUDINAL POSITION, RESPECTIVELY
C            PR    - THE VALUE OF J AT EK
C            G     - THE GRADIENT OF J AT EK
C            KS    - A RETURN CODE
C                      KS=0 - THE SYSTEM IS ASYMPTOTICALLY STABLE FOR THE
C                             INPUT PILOT PARAMETERS
C                      KS=1 - THE SYSTEM IS NOT ASYMTOTICALLY STABL FOR
C                             THE INPUT PILOT PARAMETERS
C                      KS=2 - THE SYSTEM IS ASYMPTOTICALLY STABLE FOR THE
C                             INPUT PILOT PARMETERS, BUT THE SOLUTION FOR
```

```
C                           THE COVARIANCE MATRIX FAILED.
C           A      - THE N BY N SYSTEM A-MATRIX. SEE SUBROUTINE AMATRX
C                    FOR A DESCRIPTION.
C           Q      - AN N BY N WORKING MATRIX
C           B      - AN NA BY NA WORKING MATRIX
C           X      - AN NA WORKING VECTOR
C           DA     - AN M BY N BY N ARRAY USED TO STORE THE DERIVATIVE
C                    OF THE SYSTEM A-MATRIX WITH RESPECT TO THE PILOT
C                    PARAMETERS. SEE SUBROUTINE DAMTRX FOR DESCRIPTION.
C           STOR   - AN N BY N WORKING MATRIX
C           QQ     - AN N BY N WORKING MATRIX
C           N      - THE DIMENSION OF THE SYSTEM EQUATIONS
C           NA     - N*(N+1)/2
C
C        SUBROUTINES CALLED
C             AMATRX(N,A,EK)
C             STAB  (A,N,NA,Q,B,X,KST)
C             CAL(AA,QQ,Q  ,N,N,IMAX,2)
C             DAMTRX(N,M,DA,EK)
C             LOGPR(WG,EK,G,PR,KS,R1,R2,R3,M)
C
      SUBROUTINE FPR(M,EK,PR,G,KS,A,Q,B,X,DA,STOR,QQ,N,NA)
      COMMON/PARAM/EMQ,EMTH,EMU,EMDL,XU,WB,GDC,TAUM,TAUE,TAUQ
      COMMON/GUSTW/WG(7),NGUST,SIGUG
      DIMENSION A(N,N),Q(N,N),B(NA,NA),DA(M,N,N),STOR(N,N),QQ(N,N),
     1X(1),SIGST(4),QW(4),EK(1),G(1)
C
C        COMPUTE A MATRIX
      CALL AMATRX(N,A,EK)
C
C        DETERMINE IF A MATRIX IS STABLE
      CALL STAB(A,N,NA,Q,B,X,KST)
C
C        IF A MATRIX IS NOT STABLE, SET KS=1 AND RETURN
      IF(KST.GT.0) GO TO 50
C
C        COMPUTE PERF PER AFFDL TR 70-40
      R=SIGUG*SIGUG*2.*WB
C
C        COMPUTE COVARIENCE MATRIX Q
      DO 60 I=1,N
      DO 60 J=1,N
 60   QQ(I,J)=0.
      QQ(NGUST,NGUST)=R
      IMAX=30
      CALL CAL(A ,QQ,Q,N,N,IMAX,2)
      PR=0.
      DO 4 I=1,4
C
C        IF THE VARIENCE IS NEGATIVE, SET IER=2 AND RETURN
      IF(Q(I,I).LT.0.) GO TO 51
      SIGST(I)=SQRT(Q(I,I))
      J=I+M
C
C        STORE THE STANDARD DEVIATION OF THE STATES IN EK(M+1) THRU
```

```
C          EK(M+4)
      EK(J)=SIGST(I)
C
C          COMPUTE COEFFICIENTS USED IN COMPUTING THE GRADIENT
      QW(I)=WG(I)/(2.*SIGST(I))
 4    PR=PR+WG(I)*SIGST(I)
      PR=PR-1.
C
C          COMPUTE DA/DK
      CALL DAMTRX(N,M,DA,EK)
      DO 30 NG=1,M
C
C          COMPUTE (DA/DK)(COV MATRIX)+(COV MATRIX)(DA/DK TRANSPOSE)
      DO 24 I=1,N
      DO 24 J=1,N
      STOR(I,J)=0.
      DO 24 K=1,N
 24   STOR(I,J)=DA(NG,I,K)*Q(K,J)+STOR(I,J)
C
C          SOLVE FOR D(COV MATRIX)/DK BY SOLVING  A*D(COV MATRIX)/DK+
C          D(COV MATRIX)/DK*A(TRANSPOSE)=-DA/DK*(COV MATRIX)-(COV MATRIX)
C          *DA/DK(TRANPOSE).
      DO 61 I=1,N
      DO 61 J=1,N
 61   QQ(I,J)=STOR(I,J)+STOR(J,I)
      CALL CAL(A ,QQ,STOR,N,N,IMAX,2)
      G(NG)=0.
      DO 30 I=1,4
 30   G(NG)=G(NG)+QW(I)*STOR(I,I)
      KS=0
C
C          COMPUTE J AND GRADIENT OF J TAKING INTO ACCOUNT R2 AND R3 AS
C          DEFINED IN AFFDL TR 70-40
      CALL LOGPR(WG,EK,G,PR,KS,R1,R2,R3,M)
      KS=0
      RETURN
 50   KS=1
      RETURN
 51   KS=2
      RETURN
      END
$IBFTC LOGPRA  DECK
C
C     ....................................................................
C
C          SUBROUTINE LOGPR
C
C          PURPOSE
C             TO COMPUTE J AND THE GRADIENT OF J / OR PR,COST REGION CODE,
C             R1,R2,AND R3, DEPENDING ON INPUT CODE.  J,PR,COST REGION
C             CODE,R1,R2,AND R3 ARE DEFINED IN AFFDL TR 70-40.
C
C          USAGE
C             CALL LOGPR(WG,EK,G,PR,KX,R1,R2,R3,M)
C
```

```
C          DESCRIPTION OF PARAMETERS
C             WG - 7-VECTOR INPUT USED TO DEFINE J AND PR.  SEE AFFDL
C                  TR 70-40
C             EK - AN M+4 VECTOR INPUT
C                  EK(1) THRU EK(4) CONTAIN THE PILOT PARAMETERS
C                  EK(5) THRU EK(8) CONTAIN THE RMS VALUES OF PITCH RATE,
C                        PITCH ANGLE, LONGITUDINAL VELOCITY, AND
C                        LONGITUDINAL POSITION, RESPECTIVELY
C             G  - 4 VECTOR USED TO INPUT PARTIAL OF PERF WITH RESPECT TO
C                  K AND RETURN THE GRADIENT OF J
C             PR - USED TO INPUT PERF AND TO RETURN J OR PR DEPENDING ON
C                  INPUT CODE
C             KX - INPUT - OUTPUT CODE
C                  INPUT
C                    KX=0 - USED TO CALL FOR J AND GRADIENT OF J
C                    KX=1 - USED TO CALL FOR PR,COST REGION CODE,R1,R2,R3
C                  OUTPUT IS COST REGION CODE PER TABLE III OF AFFDL TR
C                  70-40
C             R1 - OUTPUT FUNCTION OF PERFORMANCE
C             R2 - OUTPUT FUNCTION OF TLTH
C             R3 - OUTPUT FUNCTION OF TLX
C             M  - NUMBER OF PILOT PARAMETERS
C
      SUBROUTINE LOGPR(WG,EK,G,PR,KX,R1,R2,R3,M)
      DIMENSION WG(1),EK(1),G(1)
      KS=100
C
C          IF INPUT CODE IS 0, SKIP COMPUTATION OF R1
      IF(KX.EQ.0) GO TO 4
C
C          COMPUTE R1 AND COST REGION CODE FOR R1
      R1=0.
      DO 1 I=1,4
      J=I+M
 1    R1=R1+EK(J)*WG(I)
      R1=R1-1.
      RMAX=8.-WG(5)*1.3-WG(6)*1.2-WG(7)
      IF(R1.GT.RMAX) GO TO 3
      IF(R1.GT.0.) GO TO 5
      KS=0
      R1=0.
      GO TO 5
 3    R1=RMAX
      KS=200
      GO TO 5
C
C          R1 IS SET EQUAL TO PERF
 4    R1=PR
C
C          COMPUTE R2,G(2),AND COST REGION CODE FOR R2
 5    IF(EK(2).GT.1.3) GO TO 10
      IF(EK(2).GT.0.) GO TO 11
      R2=-WG(5)*EK(2)
      G(2)=G(2)-WG(5)
      GO TO 12
```

```
 10    R2=WG(5)*1.3
       KS=KS+20
       GO TO 12
 11    R2=WG(5)*EK(2)
       G(2)=G(2)+WG(5)
       KS=KS+10
C
C          COMPUTE R3,G(4),AND COST REGION CODE FOR R3
 12    IF(EK(4).GT.1.2) GO TO 20
       IF(EK(4).GT.0.) GO TO 21
       R3=-WG(6)*EK(4)
       G(4)=G(4)-WG(6)
       GO TO 22
 20    R3=WG(6)*1.2
       KS=KS+2
       GO TO 22
 21    R3=WG(6)*EK(4)
       G(4)=G(4)+WG(6)
       KS=KS+1
C
C          COMPUTE J OR PR, DEPENDING ON HOW R1 WAS COMPUTED
 22    PR=R1+R2+R3+WG(7)
       KX=KS
       RETURN
       END
$IBFTC WRITER  DECK
C
C    ..................................................................
C
C          SUBROUTINE RITE
C
C          PURPOSE
C             TO WRITE OUT SOME CONCLUDING HUMEROUS STATEMENT
C
C          USAGE
C             CALL RITE
C
       SUBROUTINE RITE
 50    WRITE(6,210)
 210   FORMAT(2X,67H****** PAPER PILOT CONSIDERS SMOKING HAZARDOUS TO HIS
      1 HEALTH ****** )
       RETURN
       END
$IBFTC ENVERT  DECK
C      XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
C          SUBROUTINE ENVRT
C
C          PURPOSE
C              TO FIND THE INVERSE OF THE MATRIX A
C
C          USAGE
C              CALL ENVRT(ICODE,N,A,BB)
C
C          DESCRIPTION OF PARAMETERS
```

```
C              ICODE - A CODE INDICATING EXISTENCE OF A(INVERSE) (OUTPUT)
C                        0 - A (INVERSE) EXISTS
C                        1 - A (INVERSE) DOES NOT EXIST
C              N       - DIMENSION OF INPUT MATRIX A (INPUT)
C              A       - AN NXN MATRIX (INPUT)
C              BB      - AN NXN MATRIX INVERSE OF A  (OUTPUT)
C
C
      SUBROUTINE ENVRT(ICODE,N,A,BB)
      DIMENSION A(30,30),BB(30,30),AB(30,30),C(30),B(30,30)
      DOUBLE PRECISION C,AB,B
      ICODE=0
      DO 2 I=1,N
      C(I)=0.0
      DO 1 J=1,N
    1 B(I,J)=0.0
    2 B(I,I)=1.0
      DO 8 K=1,N
      DO 3 I=1,N
      DO 3 J=1,N
      AB(I,J)=0.0
      DO 3 L=1,N
    3 AB(I,J)=A(I,L)*B(L,J)+AB(I,J)
      DEN=K
      DO 4 I=1,N
    4 C(K)=C(K)+AB(I,I)
      C(K)=(-1.0/DEN)*C(K)
      IF(C(K).NE.0.0) GO TO 5
      ICODE=1
      RETURN
    5 IF(K.EQ.N) GO TO 9
      DO 8 I=1,N
      AB(I,I)=AB(I,I)+C(K)
      DO 8 J=1,N
    8 B(I,J)=AB(I,J)
    9 CONTINUE
      DO 7 I=1,N
      DO 7 J=1,N
    7 BB(I,J)=(-1./C(N))*B(I,J)
      RETURN
      END
$IBFTC CALDIS  DECK
C
C     ..................................................................
C
C        SUBROUTINE CALD
C
C        PURPOSE
C            TO SOLVE THE LINEAR MATRIX EQUATION  X=A(TRANSPOSE)XA+Q,
C            WHERE Q IS SYMMETRIC
C
C        USAGE
C            CALL CALD(A,Q,X,N,ND,EPS,IMAX,NF)
C
C        DESCRIPTION OF PARAMETERS
```

```
C              A     - AN N BY N INPUT MATRIX DESTROYED IN THE COMPUTATION
C              Q     - AN N BY N INPUT SYMMETRIC MATRIX DESTROYED IN THE
C                      COMPUTATION
C              X     - THE SOLUTION TO THE MATRIX LINEAR EQUATION
C                      X=A(TRANSPOSE)XA+Q
C              N -     THE ORDER OF THE MATRIX EQUATION X=A(TRANSPOSE)XA+Q
C              ND    - A,Q,AND X ARE DIMENSIONED ND BY ND IN THE CALLING
C                      ROUTINE
C              EPS   - AN INPUT VALUE USED TO TEST FOR CONVERGENCE
C              IMAX  - AN INPUT INTEGER USED TO INDICATE THE MAXIMUM NUMBER
C                      OF ITERATION
C              NF    - A RETURN INDICATING THE NUMBER OF TERMS OF X THAT
C                      FAILED TO CONVERGE IN THE LIMING NUMBER OF ITERATIONS.
C                      IF NF=0, THE ALGORITHM CONVERGED. IF NF=-1 THE
C                      ALGORITHM DIVERGED
C
      SUBROUTINE CALD(A,Q,X,N,ND,EPS,IMAX,NF)
      DIMENSION A(ND,1),Q(ND,1),X(ND,1)
      DIMENSION XL(30,30)
      IF(N.LE.30) GO TO 1
      WRITE(6,100) N
  100 FORMAT(1X,23HSUBROUTINE CALD -- N = ,I3,24H,  DIMENSION EXCEEDS 30
     1. )
      STOP
    1 ITERC=0
      DO 2 I=1,N
      DO 2 J=1,N
    2 XL(I,J)=Q(I,J)
    3 CONTINUE
      ITERC=ITERC+1
      DO 5 I=1,N
      DO 5 J=1,N
      SUM=0.
      DO 4 K=1,N
    4 SUM=SUM+XL(I,K)*A(K,J)
    5 Q(I,J)=SUM
      DO 7 I=1,N
      DO 7 J=I,N
      SUM=0.
      DO 6 K=1,N
    6 SUM=SUM+A(K,I)*Q(K,J)
      X(I,J)=SUM+XL(I,J)
    7 X(J,I)=X(I,J)
      NF=0
      DO 10 I=1,N
      DO 10 J=I,N
      IF(X(I,J).EQ.0.) GO TO 8
      RAT=1.-(XL(I,J)/X(I,J))
      RAT=ABS(RAT)
      IF(RAT-EPS)10,10,9
    8 RAT=ABS(XL(I,J))
      IF(RAT.LE.EPS) GO TO 10
    9 NF=NF+1
   10 CONTINUE
      IF(NF.LE.0) RETURN
```

```
      IF(ITERC.GE.IMAX) RETURN
      DO 11  I=1,N
      DO 11  J=1,N
      Q(I,J)=A(I,J)
 11   XL(I,J)=X(I,J)
      DO 13  I=1,N
      DO 13  J=1,N
      SUM=0.
      DO 12  K=1,N
 12   SUM=SUM+Q(I,K)*Q(K,J)
      DIVT=ABS(SUM)
      IF(DIVT.GE.1.E10) GO TO 14
 13   A(I,J)=SUM
      GO TO 3
 14   NF=-1
      RETURN
      END
$IBFTC SUB1     DECK
C
C     ...................................................................
C
C        SUBROUTINE CAL
C
C        PURPOSE
C           TO SOLVE THE LINEAR MATRIX EQUATION A*XN+XN*A(TRANSPOSE)
C           +Q=0 OR A(TRANSPOSE)*XN+XN*A+Q=0,DEPENDING ON THE INPUT.
C           Q IS A SYMMETRIC MATRIX.
C
C        USAGE
C           CALL CAL(A,Q,XN,N,NR,IMAX,IT)
C
C        DESCRIPTION OF PARAMETERS
C           A     - AN N BY N INPUT MATRIX
C           Q     - AN N BY N SYMMETRIC MATRIX.  THIS IS AN INPUT
C           XN    - THE SOLUTION OF THE LINEAR MATRIX EQUATION A*XN+
C                   XN*A(TRANSPOSE)+Q=0 OR A(TRANSPOSE)*XN+XN*A+Q=0,
C                   DEPENDING ON THE INPUT CODE.
C           N     - THE ORDER OF THE MATRIX EQUATION
C           NR    - A,Q,AND XN ARE DIMENSIONED NR BY NR IN THE CALLING
C                   ROUTINE
C                   OF ITERATION
C           IMAX  - AN INPUT INTEGER USED TO INDICATE THE MAXIMUM NUMBER
C           IT    - AN INPUT CODE
C                      IT=1 - SOLVE A(TRANSPOSE)*XN+XN*A+Q=0
C                      IT=2 - SOLVE A*XN+XN*A(TRANSPOSE)+Q=0
C
C        SUBROUTINES CALLED
C           CALL ENVRT(ICODE,N,P,P)
C           CALL CALD(P,E,X,N,30,EE,IMAX,NF)
C
      SUBROUTINE CAL(A,Q,XN,N,NR,IMAX,IT)
      DIMENSION A(NR,1),Q(NR,1),XN(NR,1)
      DIMENSION X(30,30),E(30,30),P(30,30),DX(30,30),KWA(30)
      IF(N.GT.30) GO TO 21
      TR=0.
```

```
      DO 300 I=1,N
  300 TR=TR+A(I,I)
      FN=N
      IF(TR)301,23,301
 23   TR=1.
  301 ALF=ABS(TR)/FN
      EE=.001
      NC=N*(N+1)
      NC=NC/2
      DO 60 I=1,N
      DO 63 J=1,N
      GOTO(61,62),IT
   61 P(I,J)=A(I,J)
      X(I,J)=A(I,J)
      GOTO 63
   62 P(I,J)=A(J,I)
      X(I,J)=A(J,I)
   63 CONTINUE
      P(I,I)=P(I,I)-ALF
      X(I,I)=X(I,I)-ALF
   60 CONTINUE
      CALL ENVRT(ICODE,N,P,P)
      IF(ICODE.EQ.1) GO TO 2
      GOTO 3
 21   WRITE(6,22)
 22   FORMAT(1X,54HSUBROUTINE CAL --  DIMENSION OF SYSTEM GREATER THAN 3
     10,/,20X,47HCHANGE DIMENSION STATEMENT FOR X,E,P,DX,AND KWA,/,
     220X,49HREMOVE IF STATEMENT FOLLOWING DIMENSION STATEMENT  )
      STOP
 2    WRITE(6,27)
 27   FORMAT(1X,49HSUBROUTINE CAL -- (TAU*A-I)INVERSE DOES NOT EXIXT)
      STOP
    3 CONTINUE
      DO 4 I=1,N
      DO 4 J=1,N
      E(I,J)=0.
      DO 4 K=1,N
    4 E(I,J)=E(I,J)+P(K,I)*Q(K,J)*2.*ALF
      DO 5 I=1,N
      DO 5 J=1,N
      XN(I,J)=0.
      DO 5 K=1,N
    5 XN(I,J)=XN(I,J)+E(I,K)*P(K,J)
      DO 7 I=1,N
      DO 8 J=1,N
    8 P(I,J)=P(I,J)*2.*ALF
    7 P(I,I)=P(I,I)+1.
      DO 9 I=1,N
      DO 9 J=1,N
 9    E(I,J)=XN(I,J)
      CALL CALD(P,E,X,N,30,EE,IMAX,NF)
      DO 10 I=1,N
      DO 10 J=1,N
 10   XN(I,J)=X(I,J)
      RETURN
```

```
      END
$IBFTC STABCL  DECK
C
C     ..............................................................
C
C        SUBROUTINE STAB
C
C        PURPOSE
C           TO DETERMINE IF THE DIFFERENTIAL EQUATION DX/DT=A*X IS
C           ASYMPTOTICALLY STABLE
C
C        USAGE
C           CALL STAB(A,NR,NC,Q,XN,ZARG,KST)
C
C        DESCRIPTION OF PARAMETERS
C           A     - AN NR BY NR INPUT MATRIX
C           NR    - THE DIMENSION OF THE LINEAR DIFFERENTIAL EQUATION
C           NC    - NC=NR*(NR+1)/2
C           Q     - AN NR BY NR ARRAY OF WORKING STORAGE
C           XN    - AN NR BY NR ARRAY OF WORKING STORAGE
C           ZARG  - WORKING STORAGE OF DIMENSION NC
C           KST   - A RETURN CODE
C                     KST=0 - DX/DT =A*X IS ASYMPTOTICALLY STABLE
C                     KST=1 - DX/DT =A*X IS NOT ASYMPTOTICALLY STABLE
C
C        SUBROUTINES CALLED
C           CALL ENVRT(ICODE,N,P,P)
C           CALD(P,E,X,N,30,EE,IMAX,NF)
C           EIGEN(ZARG,RDUM,N,KST)
C
      SUBROUTINE STAB(A,NR,NC,Q,XN,ZARG,KST)
      DIMENSION A(NR,1),Q(NR,1),XN(NR,1)
      DIMENSION X(30,30),E(30,30),P(30,30),DX(30,30),KWA(30)
      DIMENSION ZARG(1)
      N=NR
      IT=1
      IMAX=20
      IF(N.GT.30) GO TO 21
      DO 12 I=1,N
      DO 11 J=1,N
 11   Q(I,J)=0.
 12   Q(I,I)=1.
      TR=0.
      DO 300 I=1,N
  300 TR=TR+A(I,I)
      FN=N
      IF(TR)301,23,301
 23   TR=1.
  301 ALF=ABS(TR)/FN
      EE=.001
      NC=N*(N+1)
      NC=NC/2
      DO 60 I=1,N
      DO 63 J=1,N
      GOTO(61,62),IT
```

```
   61 P(I,J)=A(I,J)
      X(I,J)=A(I,J)
      GOTO 63
   62 P(I,J)=A(J,I)
      X(I,J)=A(J,I)
   63 CONTINUE
      P(I,I)=P(I,I)-ALF
      X(I,I)=X(I,I)-ALF
   60 CONTINUE
      CALL ENVRT(ICODE,N,P,P)
      IF(ICODE.EQ.1) GO TO 2
      GOTO 3
21    WRITE(6,22)
22    FORMAT(1X,54HSUBROUTINE CAL --  DIMENSION OF SYSTEM GREATER THAN 3
     10,/,20X,47HCHANGE DIMENSION STATEMENT FOR X,E,P,DX,AND KWA,/,
     220X,49HREMOVE IF STATEMENT FOLLOWING DIMENSION STATEMENT   )
      STOP
2     WRITE(6,27)
27    FORMAT(1X,49HSUBROUTINE CAL -- (TAU*A-I)INVERSE DOES NOT EXIXT)
      STOP
    3 CONTINUE
      DO 4 I=1,N
      DO 4 J=1,N
      E(I,J)=0.
      DO 4 K=1,N
    4 E(I,J)=E(I,J)+P(K,I)*Q(K,J)*2.*ALF
      DO 5 I=1,N
      DO 5 J=1,N
      XN(I,J)=0.
      DO 5 K=1,N
    5 XN(I,J)=XN(I,J)+E(I,K)*P(K,J)
      DO 7 I=1,N
      DO 8 J=1,N
    8 P(I,J)=P(I,J)*2.*ALF
    7 P(I,I)=P(I,I)+1.
      DO 9 I=1,N
      DO 9 J=1,N
9     E(I,J)=XN(I,J)
      CALL CALD(P,E,X,N,30,EE,IMAX,NF)
      IF(NF.GE.1) GO TO 16
      IF(NF.LE.-1) GO TO 16
      L=1
      DO 13 I=1,N
      DO 13 J=1,I
      ZARG(L)=X(J,I)
13    L=L+1
      KST=1
      CALL EIGEN(ZARG,RDUM,N,KST)
      L=1
      DO 14 I=1,N
      DO 14 J=1,I
      X(J,I)=ZARG(L)
14    L=L+1
      DO 15 I=1,N
15    IF(X(I,I).LE.0.) GO TO 16
```

```
      KST=0
      RETURN
 16   KST=1
      RETURN
      END
$IBFTC EIGENA  DECK
C                                                                       EIGEN001
C     ..................................................................EIGEN002
C                                                                       EIGEN003
C        SUBROUTINE EIGEN                                               EIGEN004
C                                                                       EIGEN005
C        PURPOSE                                                        EIGEN006
C           COMPUTE EIGENVALUES AND EIGENVECTORS OF A REAL SYMMETRIC    EIGEN007
C           MATRIX                                                      EIGEN008
C                                                                       EIGEN009
C        USAGE                                                          EIGEN010
C           CALL EIGEN(A,R,N,MV)                                        EIGEN011
C                                                                       EIGEN012
C        DESCRIPTION OF PARAMETERS                                      EIGEN013
C           A - ORIGINAL MATRIX (SYMMETRIC), DESTROYED IN COMPUTATION.  EIGEN014
C               RESULTANT EIGENVALUES ARE DEVELOPED IN DIAGONAL OF      EIGEN015
C               MATRIX A IN DESCENDING ORDER.                           EIGEN016
C           R - RESULTANT MATRIX OF EIGENVECTORS (STORED COLUMNWISE,    EIGEN017
C               IN SAME SEQUENCE AS EIGENVALUES)                        EIGEN018
C           N - ORDER OF MATRICES A AND R                               EIGEN019
C           MV- INPUT CODE                                              EIGEN020
C                   0   COMPUTE EIGENVALUES AND EIGENVECTORS            EIGEN021
C                   1   COMPUTE EIGENVALUES ONLY (R NEED NOT BE         EIGEN022
C                       DIMENSIONED BUT MUST STILL APPEAR IN CALLING    EIGEN023
C                       SEQUENCE)                                       EIGEN024
C                                                                       EIGEN025
C        REMARKS                                                        EIGEN026
C           ORIGINAL MATRIX A MUST BE REAL SYMMETRIC (STORAGE MODE=1)   EIGEN027
C           MATRIX A CANNOT BE IN THE SAME LOCATION AS MATRIX R         EIGEN028
C                                                                       EIGEN029
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                  EIGEN030
C           NONE                                                        EIGEN031
C                                                                       EIGEN032
C        METHOD                                                         EIGEN033
C           DIAGONALIZATION METHOD ORIGINATED BY JACOBI AND ADAPTED     EIGEN034
C           BY VON NEUMANN FOR LARGE COMPUTERS AS FOUND IN -MATHEMATICALEIGEN035
C           METHODS FOR DIGITAL COMPUTERS-, EDITED BY A. RALSTON AND    EIGEN036
C           H.S. WILF, JOHN WILEY AND SONS, NEW YORK, 1962, CHAPTER 7   EIGEN037
C                                                                       EIGEN038
C     ..................................................................EIGEN039
C                                                                       EIGEN040
      SUBROUTINE EIGEN(A,R,N,MV)                                        EIGEN041
      DIMENSION A(1),R(1)                                               EIGEN042
C                                                                       EIGEN043
C        ...............................................................EIGEN044
C                                                                       EIGEN045
C        IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE  EIGEN046
C        C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION      EIGEN047
C        STATEMENT WHICH FOLLOWS.                                       EIGEN048
C                                                                       EIGEN049
```

```
C     DOUBLE PRECISION A,R,ANORM,ANRMX,THR,X,Y,SINX,SINX2,COSX,           EIGEN050
C    1                 COSX2,SINCS                                          EIGEN051
C                                                                           EIGEN052
C        THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS        EIGEN053
C        APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS          EIGEN054
C        ROUTINE.                                                           EIGEN055
C                                                                           EIGEN056
C        THE DOUBLE PRECISION VERSION OF THIS SUBROUTINE MUST ALSO          EIGEN057
C        CONTAIN DOUBLE PRECISION FORTRAN FUNCTIONS.  SQRT IN STATEMENTSEIGEN058
C        40, 68, 75, AND 78 MUST BE CHANGED TO DSQRT.  ABS IN STATEMENT EIGEN059
C        62 MUST BE CHANGED TO DABS.                                        EIGEN060
C                                                                           EIGEN061
C        ...............................................................EIGEN062
C                                                                           EIGEN063
C        GENERATE IDENTITY MATRIX                                           EIGEN064
C                                                                           EIGEN065
      IF(MV-1) 10,25,10                                                     EIGEN066
   10 IQ=-N                                                                 EIGEN067
      DO 20 J=1,N                                                           EIGEN068
      IQ=IQ+N                                                               EIGEN069
      DO 20 I=1,N                                                           EIGEN070
      IJ=IQ+I                                                               EIGEN071
      R(IJ)=0.0                                                             EIGEN072
      IF(I-J) 20,15,20                                                      EIGEN073
   15 R(IJ)=1.0                                                             EIGEN074
   20 CONTINUE                                                              EIGEN075
C                                                                           EIGEN076
C        COMPUTE INITIAL AND FINAL NORMS (ANORM AND ANORMX)                 EIGEN077
C                                                                           EIGEN078
   25 ANORM=0.0                                                             EIGEN079
      DO 35 I=1,N                                                           EIGEN080
      DO 35 J=I,N                                                           EIGEN081
      IF(I-J) 30,35,30                                                      EIGEN082
   30 IA=I+(J*J-J)/2                                                        EIGEN083
      ANORM=ANORM+A(IA)*A(IA)                                               EIGEN084
   35 CONTINUE                                                              EIGEN085
      IF(ANORM) 165,165,40                                                  EIGEN086
   40 ANORM=1.414*SQRT(ANORM)                                               EIGEN087
      ANRMX=ANORM*1.0E-6/FLOAT(N)                                           EIGEN088
C                                                                           EIGEN089
C        INITIALIZE INDICATORS AND COMPUTE THRESHOLD, THR                   EIGEN090
C                                                                           EIGEN091
      IND=0                                                                 EIGEN092
      THR=ANORM                                                             EIGEN093
   45 THR=THR/FLOAT(N)                                                      EIGEN094
   50 L=1                                                                   EIGEN095
   55 M=L+1                                                                 EIGEN096
C                                                                           EIGEN097
C        COMPUTE SIN AND COS                                                EIGEN098
C                                                                           EIGEN099
   60 MQ=(M*M-M)/2                                                          EIGEN100
      LQ=(L*L-L)/2                                                          EIGEN101
      LM=L+MQ                                                               EIGEN102
   62 IF( ABS(A(LM))-THR) 130,65,65                                         EIGEN103
   65 IND=1                                                                 EIGEN104
```

```
      LL=L+LQ
      MM=M+MQ
      X=0.5*(A(LL)-A(MM))
   68 Y=-A(LM)/ SQRT(A(LM)*A(LM)+X*X)
      IF(X) 70,75,75
   70 Y=-Y
   75 SINX=Y/ SQRT(2.0*(1.0+( SQRT(1.0-Y*Y))))
      SINX2=SINX*SINX
   78 COSX= SQRT(1.0-SINX2)
      COSX2=COSX*COSX
      SINCS =SINX*COSX
C
C        ROTATE L AND M COLUMNS
C
      ILQ=N*(L-1)
      IMQ=N*(M-1)
      DO 125 I=1,N
      IQ=(I*I-I)/2
      IF(I-L) 80,115,80
   80 IF(I-M) 85,115,90
   85 IM=I+MQ
      GO TO 95
   90 IM=M+IQ
   95 IF(I-L) 100,105,105
  100 IL=I+LQ
      GO TO 110
  105 IL=L+IQ
  110 X=A(IL)*COSX-A(IM)*SINX
      A(IM)=A(IL)*SINX+A(IM)*COSX
      A(IL)=X
  115 IF(MV-1) 120,125,120
  120 ILR=ILQ+I
      IMR=IMQ+I
      X=R(ILR)*COSX-R(IMR)*SINX
      R(IMR)=R(ILR)*SINX+R(IMR)*COSX
      R(ILR)=X
  125 CONTINUE
      X=2.0*A(LM)*SINCS
      Y=A(LL)*COSX2+A(MM)*SINX2-X
      X=A(LL)*SINX2+A(MM)*COSX2+X
      A(LM)=(A(LL)-A(MM))*SINCS+A(LM)*(COSX2-SINX2)
      A(LL)=Y
      A(MM)=X
C
C        TESTS FOR COMPLETION
C
C        TEST FOR M = LAST COLUMN
C
  130 IF(M-N) 135,140,135
  135 M=M+1
      GO TO 60
C
C        TEST FOR L = SECOND FROM LAST COLUMN
C
  140 IF(L-(N-1)) 145,150,145
```

```
  145 L=L+1
      GO TO 55
  150 IF(IND-1) 160,155,160
  155 IND=0
      GO TO 50
C
C        COMPARE THRESHOLD WITH FINAL NORM
C
  160 IF(THR-ANRMX) 165,165,45
C
C        SORT EIGENVALUES AND EIGENVECTORS
C
  165 IQ=-N
      DO 185 I=1,N
      IQ=IQ+N
      LL=I+(I*I-I)/2
      JQ=N*(I-2)
      DO 185 J=I,N
      JQ=JQ+N
      MM=J+(J*J-J)/2
      IF(A(LL)-A(MM)) 170,185,185
  170 X=A(LL)
      A(LL)=A(MM)
      A(MM)=X
      IF(MV-1) 175,185,175
  175 DO 180 K=1,N
      ILR=IQ+K
      IMR=JQ+K
      X=R(ILR)
      R(ILR)=R(IMR)
  180 R(IMR)=X
  185 CONTINUE
      RETURN
      END
```

APPENDIX B

COMPUTATIONAL DATA

COMPARISON OF PAPER PILOT AND ACTUAL PILOT RATING, PILOT PARAMETERS, AND PERFORMANCE

This appendix contains a detailed listing of the computational results and the comparable data for actual pilots performing in fixed-based simulation studies as reported in References [3] and [4].

The aircraft configurations, gust intensities, pilot ratings, and the differences between the actual pilot rating and the "paper pilot" rating are listed in Table VI. In Table VII, the "paper pilot" parameters are listed along with actual pilot matched parameters from Reference [3]. In addition the differences between the actual pilot matched parameters and the "paper pilot" parameters are listed. The actual pilot performances, the "paper pilot" performances, and the differences between the two are listed in Table VIII. The actual measured performance data is from Reference [3].

In those cases where two actual pilots were used, the average data for the actual pilots was used in computing the differences between actual pilot data and "paper pilot" data.

CASE	M_{ug}	X_u	M_q	M_θ	τ_e	τ_q	σ_g	Pilot	M_δ	Pilot Rating	ΔPR*
PH1	.67	0	-3	0	0	0	5.1	A B Paper	.282 .287 .282	2.5 2.5 2.14	.36
PH2	.67	-.05	-3	0	0	0	5.1	A B Paper	.412 .420 .412	2.5 3.0 2.58	.17
PH3	.67	-.1	-3	0	0	0	5.1	A B Paper	.356 .385 .356	3.0 3.25 2.94	.18
PH4	.67	-.2	-3	0	0	0	5.1	A B Paper	.465 .469 .465	4.25 3.75 3.68	.32
PH5	.67	-.3	-3	0	0	0	5.1	A B Paper	.506 .516 .506	5.0 4.5 4.31	.44
PH6	0	-.1	-3	0	0	0	5.1	A B Paper	.300** .300** .300	3.25 3.5 2.25	1.12
PH7	.33	-.1	-3	0	0	0	5.1	A B Paper	.360** .360** .360	2.75 3.25 2.59	.41
PH9	1.00	-.1	-3	0	0	0	5.1	A B Paper	.481** .481** .481	3.75 3.75 3.35	.40

* $\Delta PR = \left(\frac{\text{A pilot rating} + \text{B pilot rating}}{2}\right) - (\text{"paper pilot" rating})$

** These values of M_δ were selected from United Aircraft Research Laboratory contours for selecting optimal control sensitivity.

TABLE VI

Aircraft Configurations, Gust Intensities, and Pilot Ratings

CASE	M_{u_g}	X_u	M_q	M_θ	τ_e	τ_q	σ_g	Pilot	M_δ	Pilot Rating	ΔPR*
PH10	.67	-.1	-1	0	0	0	5.1	A	.369**	4.25	-.37
								B	.369**	4.25	
								Paper	.369	4.62	
PH12	.67	-.1	-5	0	0	0	5.1	A	.493**	3.0	.71
								B	.493**	3.25	
								Paper	.493	2.41	
PH13	.67	-.1	-3	0	0	0	2.6	A	.396	2.5	.42
								B	.396	2.5	.42
								Paper	.396	2.08	
PH16	.67	-.1	-3	0	0	0	7.7	A	.476	4.75	.85
								B	.477	4.75	
								Paper	.476	3.90	
PH17	1.00	-.05	-1	0	0	0	5.1	A	.429	4.25	-.59
								B	.403	4.50	
								Paper	.429	4.96	
PH18	1.0	-.2	-1	0	0	0	5.1	A	.483	5.0	-.34
								P B	.433	5.0	
								Paper	.483	5.34	
PH19	1.0	-.05	-1	-3	0	0	5.1	A	.385	3.75	-.62
								B	.352	3.5	
								Paper	.385	4.17	
PH20	1.0	-.2	-1	-3	0	0	5.1	A	.430	4.25	-.78
								B	.405	4.25	
								Paper	.430	5.03	

* $\Delta PR = \left(\frac{\text{A pilot rating + B pilot rating}}{2}\right)$ - ("paper pilot" rating)

** These values of M_δ were selected from United Aircraft Research Laboratory contours for selecting optimal control sensitivity.

TABLE VI (Cont.)

Aircraft Configurations, Gust Intensities, and Pilot Ratings

CASE	M_{u_g}	X_u	M_q	M_θ	τ_e	τ_q	σ_g	Pilot	M_δ	Pilot Rating	ΔPR*
PH21	1.00	-.05	-1	-5	0	0	5.1	A B Paper	.359 .327 .359	3.25 4.0 4.17	-.55
PH22	1.00	-.2	-1	-5	0	0	5.1	A B Paper	.454 .436 .454	3.75 4.5 4.87	-.75
PH28	1.00	-.05	-5	-8	0	0	5.1 5.1	B Paper	.585	2.0 1.83	.17
PH29	.33	-.05	-3	0	0	0	10.3	B Paper	.394	4.0 3.14	.86
PH30	1.33	-.2	-3	0	0	0	2.6	B Paper	.433	3.5 2.74	.76
PH31	.33	-.1	-1	0	0	0	5.1	B Paper	.277	3.5 3.68	-.18
PH32	.33	-.02	-1	0	0	0	20.6	B Paper	.444	6.5 5.42	1.08
PH34	.33	-.02	-3	0	0	0	20.6	B Paper	.480	5.0 3.91	1.09
PH35	.2	-.09	-1	0	0	0	8.6	B Paper	.305	5.0 4.38	.62
PH36	.8	-.36	-1	0	0	0	2.1	B Paper	.279	3.5 3.47	.03

$$* \quad \Delta PR = \left(\frac{\text{Pilot A rating} + \text{Pilot B rating}}{2}\right) - (\text{"paper pilot" rating})$$

or

$$\Delta PR = (\text{Pilot B rating}) - (\text{"paper pilot" rating})$$

TABLE VI (Cont.)

Aircraft Configurations, Gust Intensities, and Pilot Ratings

CASE	M_u^g	X_u	M_q	M_θ	τ_e	τ_q	σ_g	Pilot	M_δ	Pilot Rating	ΔPR*
PH(L)1	.67	-.1	-3	0	.1	0	5.1	Actual Paper	.356	4.0 3.89	.11
PH(L)2	.67	-.1	-3	0	.5	0	5.1	Actual Paper	.356	6.0 5.52	.48
PL1	.1	-.1	-1	0	0	0	5.1	Actual Paper	.264	3.0 3.11	-.11
PL2	.1	-.1	-1	0	.05	0	5.1	Actual Paper	.276	3.0 3.57	-.57
PL3	.1	-.1	-1	0	.1	0	5.1	Actual Paper	.240	4.0 4.14	-.14
PL4	.1	-.1	-1	0	.2	0	5.1	Actual Paper	.441	5.5 4.97	.53
PL5	.1	-.1	-1	0	0	.1	5.1	Actual Paper	.223	3.0 4.97	-.15
PL6	.1	-.1	-1	0	0	.5	5.1	Actual Paper	.228	3.0 3.98	-.98
PL7	.1	-.1	-1	0	0	1.0	5.1	Actual Paper	.229	3.5 4.24	-.74
PL8	.1	-.1	-1	0	0	2.0	5.1	Actual Paper	.238	4.0 4.22	-.22
PL9	.1	-.1	-1	0	0	4.0	5.1	Actual Paper	.210	4.5 4.14	.36
PL10	.1	-.1	-1	0	.05	.05	5.1	Actual Paper	.240	3.0 3.57	-.57

* ΔPR = (Actual Pilot Rating) - ("Paper Pilot" Rating)

TABLE VI (Continued)

CASE	M_{u_g}	X_u	M_q	M_θ	τ_e	τ_q	σ_g	Pilot	M_δ	Pilot Rating	ΔPR*
PL11	.1	-.1	-1	0	.1	.1	5.1	Actual Paper	.288	4.0 4.05	-.05
PL12	.1	-.1	-1	0	.2	.2	5.1	Actual Paper	.275	4.5 5.04	-.54
PL13	.1	-.1	-3	0	0	0	5.1	Actual Paper	.384	2.5 2.43	.07
PL14	.1	-.1	-3	0	.2	0	5.1	Actual Paper	.403	3.0 3.33	-.33
PL15	.1	-.1	-3	0	.3	0	5.1	Actual Paper	.451	3.5 3.83	-.33
PL16	.1	-.1	-3	0	.4	0	5.1	Actual Paper	.601	4.5 4.12	.38
PL17	.67	-.1	-3	0	0	0	5.1	Actual Paper	.432	3.0 3.01	-.01
PL18	.67	-.1	-3	0	.05	0	5.1	Actual Paper	.433	3.5 3.47	.03
PL19	.67	-.1	-3	0	.1	0	5.1	Actual Paper	.433	4.0 3.87	.13
PL20	.67	-.1	-3	0	.2	0	5.1	Actual Paper	.406	5.0 4.79	.21
PL21	.67	-.1	-3	0	.5	0	5.1	Actual Paper	.405	6.0 5.83	.17
PL22	.67	-.1	-3	0	0	.01	5.1	Actual Paper	.410	3.0 2.99	.01

* ΔPR = (Actual Pilot Rating) - ("Paper Pilot" Rating)

TABLE VI (Continued)

CASE	$M_u g$	X_u	M_q	M_θ	τ_e	τ_q	σ_g	Pilot	M_δ	Pilot Rating	ΔPR*
PL23	.67	-.1	-3	0	0	.1	5.1	Actual Paper	.450	3.5 3.11	.39
PL24	.67	-.1	-3	0	0	.3	5.1	Actual Paper	.435	4.0 4.18	-.18
PL25	.67	-.1	-3	0	0	.5	5.1	Actual Paper	.447	4.5 5.05	-.55
PL26	.67	-.1	-3	0	0	1.0	5.1	Actual Paper	.364	5.0 5.94	-.94
PL27	.67	-.1	-5	0	0	.5	5.1	Actual Paper	.434	4.5 4.66	-.16
PL31	.67	-.1	-3	-3	.05	.05	5.1	Actual Paper	.445	3.5 3.08	.42
PL32	.67	-.1	-3	-3	.1	.1	5.1	Actual Paper	.445	4.5 3.43	1.07
PL33	.67	-.1	-3	-3	.2	.2	5.1	Actual Paper	.462	5.5 4.12	1.38
PL34	.67	-.1	-6	0	0	0	5.1	Actual Paper	.535	2.5 2.34	.16
PL35	.67	-.1	-6	0	.3	0	5.1	Actual Paper	.606	3.0 3.68	-.68
PL36	.67	-.1	-6	0	.5	0	5.1	Actual Paper	.703	3.0 4.16	-1.16
PL37	.67	-.1	-6	0	1.0	0	5.1	Actual Paper	.848	4.0 4.92	-.92

* ΔPR = (Actual Pilot Rating) - ("Paper Pilot" Rating)

TABLE VI (Continued)

CASE	$M_u g$	X_u	M_q	M_θ	τ_e	τ_q	σ_g	Pilot	M_δ	Pilot Rating	ΔPR*
PL38	1.5	-.1	-6	0	0	0	5.1	Actual Pilot	.614	3.5 2.72	.78
PL39	1.5	-.1	-6	0	.05	0	5.1	Actual Pilot	.633	3.5 3.11	.39
PL40	1.5	-.1	-6	0	.12	0	5.1	Actual Pilot	.671	4.0 3.59	.41
PL41	1.5	-.1	-6	0	.2	0	5.1	Actual Pilot	.737	4.5 4.06	.44
PL42	1.5	-.1	-6	0	0	.05	5.1	Actual Pilot	.630	3.5 2.71	.79
PL43	1.5	-.1	-6	0	0	.1	5.1	Actual Pilot	.571	4.0 2.81	1.19
PL44	1.5	-.1	-6	0	0	.2	5.1	Actual Pilot	.596	5.5 4.12	1.38
PL45	1.5	-.1	-6	0	.05	.025	5.1	Actual Pilot	.608	3.5 2.87	.63
PL46	1.5	-.1	-6	0	.05	.05	5.1	Actual Pilot	.620	4.0 3.01	.99
PL47	1.5	-.1	-6	0	.1	.1	5.1	Actual Pilot	.638	5.0 3.31	1.69
PL48	1.5	-.1	-9	0	0	0	5.1	Actual Pilot	.713	2.5 2.32	.18
PL49	1.5	-.1	-9	0	.3	0	5.1	Actual Pilot	.872	3.0 3.65	-.65

* ΔPR = (Actual Pilot Rating) - ("Paper Pilot" Rating)

TABLE VI (Continued)

CASE	$M_u g$	X_u	M_q	M_θ	τ_e	τ_q	σ_g	Pilot	M_δ	Pilot Rating	ΔPR*
PL50	1.5	-.1	-9	0	.5	0	5.1	Actual Paper	.990	4.0 4.17	-.17
PL51	1.5	-.1	-9	0	1.0	0	5.1	Actual Paper	1.442	4.5 4.74	-.24
PN1	.67	-.1	.1**	0	0	0	5.1	Actual Paper	.352	4.0 4.6	-.6
PN2	.67	-.1	.03**	0	0	0	5.1	Actual Paper	.253	5.5 5.7	-.2
PN3	.67	-.1	0**	0	0	0	5.1	Actual Paper	.272	6.0 5.8	.2

* ΔPR = (Actual Pilot Rating) - ("Paper Pilot" Rating)

** Values given are M_{RA}

TABLE VI (Concluded)

CASE	Pilot	K_{p_θ}	T_{L_θ}	K_{p_x}	T_{L_x}	ΔK_{p_θ} *	ΔT_{L_θ} *	ΔK_{p_x} *	ΔT_{L_x} *
PH1	A	.81	.18	.85	.82	.10	-.02	-.77	.52
	B	.69	.27	1.29	.91				
	Paper	.65	.25	1.85	.35				
PH2	A	.54	.19	1.16	.73	.08	-.08	-1.16	.46
	B	.50	.22	1.11	.88				
	Paper	.44	.28	2.29	.34				
PH3	A	.67	.16	1.15	.49	.18	-.14	-1.14	.10
	B	.73	.18	1.54	.39				
	Paper	.52	.31	2.49	.34				
PH4	A	.46	.13	1.05	.66	.23	-.15	-1.12	.18
	B	.80	.16	1.36	.35				
	Paper	.40	.30	2.33	.32				
PH5	A	.38	.14	.88	.84	.15	-.15	-1.44	.24
	B	.65	.20	1.23	.26				
	Paper	.36	.32	2.49	.31				
PH6	A	.46	-.07	.74	.79	0	-.18	-.76	.34
	B	.57	.01	.97	.72				
	Paper	.52	.15	1.62	.41				
PH7	A	.49	.08	.94	.78	.08	-.15	-.96	.28
	B	.67	.10	1.15	.53				
	Paper	.50	.24	2.00	.37				
PH9	A	.41	.23	1.20	.85	.16	-.08	-.80	.28
	B	.64	.20	1.44	.50				
	Paper	.37	.30	2.12	.39				
PH10	A	.41	.43	1.31	.39	.20	-.06	-.28	.16
	B	.45	.50	1.53	.41				
	Paper	.23	.53	1.73	.24				
PH12	A	.67	.03	1.11	.66	.10	-.15	-1.08	.24
	B	.76	.05	1.34	.56				
	Paper	.61	.19	2.30	.37				
PH13	A	.27	.22	.82	1.58	-.12	-.02	-.76	.96
	B	.38	.14	1.02	.94				
	Paper	.45	.20	1.68	.30				
PH16	A	.58	.17	1.14	.55	.32	-.14	-.82	.04
	B	.79	.18	1.66	.31				
	Paper	.37	.32	2.22	.39				
PH17	A	.34	.52	1.41	.37	.14	-.04	-.18	.16
	B	.35	.54	2.01	.36				
	Paper	.20	.57	1.89	.21				
PH18	A	.27	.44	1.35	.34	.16	-.18	-.52	.11
	B	.47	.47	1.75	.28				
	Paper	.17	.64	2.07	.20				

$$* \ \Delta \text{ parameter} = \left(\frac{\text{Pilot A matched parameter} + \text{Pilot B matched parameter}}{2}\right)$$

$$- (\text{"paper pilot" parameter})$$

TABLE VII

Pilot Parameters

CASE	Pilot	K_{p_θ}	T_{L_θ}	K_{p_x}	T_{L_x}	ΔK_{p_θ} *	ΔT_{L_θ} *	ΔK_{p_x} *	ΔT_{L_x} *
	A	.26	.57	2.34	.43				
PH19	B	.41	.56	2.23	.33	.12	.06	.58	.25
	Paper	.21	.51	2.87	.13				
	A	.23	.47	1.83	.43				
PH20	B	.27	.39	2.50	.34	.06	-.12	-1.22	.28
	Paper	.19	.55	3.39	.11				
	A	.26	.55	2.52	.41				
PH21	B	.37	.50	2.73	.33	.12	.06	-.41	.19
	Paper	.19	.47	3.04	.18				
	A	.14	.51	2.88	.50				
PH22	B	.16	.31	3.47	.33	.01	-.09	-1.44	.32
	Paper	.16	.50	4.61	.09				
PH28	B	.86	.07	1.36	.61	.37	-.04	-2.33	.21
	Paper	.49	.11	3.69	.40				
PH29	B	.88	.19	1.23	.42	.42	-.11	-1.13	.04
	Paper	.46	.30	2.36	.38				
PH30	B	.76	.20	1.30	.31	.32	-.07	-.84	.07
	Paper	.44	.27	2.14	.24				
PH31	B	.55	.46	1.07	.51	.24	-.02	-.72	.24
	Paper	.31	.48	1.79	.27				
PH32	B	.38	.60	1.49	.44	.21	-.01	-.48	.09
	Paper	.17	.61	1.97	.35				
PH34	B	.68	.13	.95	.60	.32	-.20	-1.18	.15
	Paper	.36	.33	2.13	.45				
PH35	B	.68	.25	1.38	.51	.41	-.28	-.53	.21
	Paper	.27	.53	1.91	.30				
PH36	B	.55	.40	1.17	.34	.20	-.04	-.70	.31
	Paper	.35	.44	1.87	.03				

$$* \ \Delta \text{ parameter} = \left(\frac{\text{Pilot A matched parameter} + \text{Pilot B matched parameter}}{2}\right)$$

$$- (\text{"paper pilot" parameter})$$

or

Δ parameter = (Pilot B matched parameter) - ("paper pilot" parameter)

TABLE VII (Concluded)

CASE	Pilot	σ_θ	σ_q	σ_x	σ_u	$\Delta\sigma_\theta$*	$\Delta\sigma_q$*	$\Delta\sigma_x$*	$\Delta\sigma_u$*
PH1	A	.73	1.42	.68	.39				
	B	.72	1.38	.47	.32	-.55	-.73	.01	-.21
	Paper	1.27	2.13	.57					
PH2	A	1.23	1.91	.94	.58				
	B	1.15	1.75	.96	.56	-.66	-1.10	.24	-.17
	Paper	1.85	2.93	.71	.74				
PH3	A	1.75	2.48	1.40	.87				
	B	1.86	2.78	1.08	.80	-.52	-.97	.40	-.01
	Paper	2.32	3.60	.84	.85				
PH4	A	2.81	3.47	2.35	1.28				
	B	2.87	3.46	1.97	1.29	-.46	-1.02	.85	.11
	Paper	3.29	4.48	1.31	1.17				
PH5	A	3.47	3.67	3.59	1.60				
	B	3.64	3.93	2.88	1.67	-.69	-1.68	1.63	.25
	Paper	4.25	5.48	1.61	1.39				
PH6	A	1.63	1.88	1.46	.84				
	B	1.62	2.08	1.10	.72	-.11	.21	.42	.03
	Paper	1.73	1.77	.86	.75				
PH7	A	1.71	2.10	1.48	.84				
	B	1.72	2.40	1.26	.80	-.21	-.25	.52	.04
	Paper	1.93	2.50	.85	.78				
PH9	A	1.93	2.87	1.50	.87				
	B	1.90	3.22	1.20	.81	-.48	-.93	.27	-.10
	Paper	2.40	3.97	1.08	.94				
PH10	A	2.14	3.61	1.51	.99				
	B	1.92	3.35	1.17	.80	-1.01	-1.08	-.31	-.46
	Paper	3.04	4.56	1.65	1.36				
PH12	A	1.76	2.42	1.32	.82				
	B	1.74	2.44	1.10	.75	-.21	-.37	.44	.04
	Paper	1.96	2.80	.77	.74				

$$* \ \Delta\sigma = \left(\frac{\sigma \text{ pilot A} + \sigma \text{ pilot B}}{2}\right) - (\sigma \text{ "paper pilot"})$$

TABLE VIII

RMS Aircraft State Deviations

CASE	Pilot	σ_θ	σ_q	σ_x	σ_u	$\Delta\sigma_\theta$*	$\Delta\sigma_q$*	$\Delta\sigma_x$*	$\Delta\sigma_u$*
PH13	A	1.07	1.34	1.08	.54	-.26	-.34	.25	-.11
	B	1.09	1.47	.86	.51				
	Paper	1.34	1.75	.72	.63				
PH16	A	2.59	3.75	2.07	1.21	-.40	-.34	.45	0
	B	2.92	5.02	1.54	1.23				
	Paper	3.16	4.72	1.35	1.22				
PH17	A	1.97	3.52	1.39	.95	-1.42	-1.97	-.46	-.60
	B	2.02	4.11	.97	.80				
	Paper	3.42	5.79	1.64	1.47				
PH18	A	3.55	5.32	2.56	1.58	-.74	-1.19	.33	-.15
	B	3.75	5.71	2.35	1.63				
	Paper	4.39	6.71	2.13	1.75				
PH19	A	1.72	3.29	1.12	.76	-1.08	-1.82	-.34	-.48
	B	1.56	3.14	.96	.69				
	Paper	2.72	5.04	1.38	1.21				
PH20	A	3.03	4.26	2.57	1.42	-.72	-1.86	.60	-.07
	B	3.59	5.36	2.26	1.53				
	Paper	4.03	6.67	1.82	1.54				
PH21	A	1.54	2.86	1.21	.76	-.65	-1.37	-.40	-.34
	B	1.58	3.16	.98	.71				
	Paper	2.21	4.38	1.49	1.08				
PH22	A	2.94	4.10	2.52	1.34	-.65	-1.70	.66	-.05
	B	3.66	5.35	2.22	1.54				
	Paper	3.95	6.42	1.71	1.59				
PH28	B	.93	1.36	.84	.51	-.48	-1.11	.35	-.01
	Paper	1.41	2.47	.49	.52				
PH29	B	1.86	2.38	1.42	.97	-.66	-1.39	.48	0
	Paper	2.52	3.77	.94	.97				

* $\Delta\sigma = \left(\frac{\sigma \text{ pilot A} + \sigma \text{ pilot B}}{2}\right) - (\sigma \text{ "paper pilot"})$ or $\Delta\sigma = (\sigma \text{ pilot B}) - (\sigma \text{ "paper pilot"})$

TABLE VIII (Continued)

CASE	Pilot	σ_θ	σ_q	σ_x	σ_u	$\Delta\sigma_\theta$*	$\Delta\sigma_q$*	$\Delta\sigma_x$*	$\Delta\sigma_u$*
PH30	B	1.54	2.04	1.17	.73	-.52	-1.02	.24	-.09
	Paper	2.06	3.06	.93	.82				
PH31	B	1.51	1.85	1.40	.79	-.91	-1.56	.22	-.24
	Paper	2.42	3.41	1.18	1.03				
PH32	B	2.62	4.42	1.85	1.29	-2.64	-4.09	-.53	-.93
	Paper	5.26	8.51	2.38	2.22				
PH34	B	2.17	3.18	1.51	1.06	-.72	-1.51	.23	-.14
	Paper	2.89	4.69	1.28	1.20				
PH35	B	2.45	2.74	1.97	1.29	-.74	-1.47	.51	-.02
	Paper	3.19	4.21	1.46	1.31				
PH36	B	1.94	2.26	1.51	.90	0.63	-1.16	.23	-.13
	Paper	2.57	3.42	1.28	1.03				

*$\Delta\sigma$ = (σ pilot B) - (σ "paper pilot")

TABLE VIII (Concluded)

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* Air Force Flight Dynamics Laboratory Air Force Systems Command Wright-Patterson Air Force Base, Ohio	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE
The "Paper Pilot"--A Digital Computer Program to Predict Pilot Rating for the Hover Task

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

5. AUTHOR(S) *(First name, middle initial, last name)*
James D. Dillow
Major, USAF

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
March 1971	84	13

8a. CONTRACT OR GRANT NO. b. PROJECT NO. 8219 c. Task No. 821909 d.	9a. ORIGINATOR'S REPORT NUMBER(S) AFFDL-TR-70-40 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

10. DISTRIBUTION STATEMENT
This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Air Force Flight Dynamics Laboratory Wright-Patterson Air Force Base, Ohio

13. ABSTRACT

A mathematical model for predicting the pilot rating of the flying qualities of a VTOL aircraft in the precision hover mode is described. The model includes the following elements: (1) the longitudinal equations of motion for the VTOL aircraft in hover; (2) a stochastic gust model which describes disturbances to the aircraft; (3) a fixed form pilot model which has four free parameters; and (4) a cost functional which is made up of measures of aircraft performance and pilot workload. The four free pilot parameters of the pilot model are selected to minimize the cost functional. These parameters are adjusted to ensure a 20% stability margin in pilot gains and then used to compute a "paper pilot" rating of the flying qualities of the VTOL aircraft in the precision hover mode.

The mathematical equations and digital computer program used to exercise the model are described.

The "paper pilot"rating was computed for 79 aircraft configuration/gust intensity combinations. The aircraft configurations considered include cases with control lag, stability augmentation system lag, and limited pitch rate authority in the stability augmentation system. The "paper pilot" ratings are compared to actual pilot ratings obtained in fixed base simulation. The difference between the actual pilot ratings and the "paper pilot" ratings has a mean of .14 and a standard deviation of .63 out of a 10 point rating scale.

DD FORM 1 NOV 65 1473

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Parameter Optimization						
Flying Qualities						
Human Response						
Manual Control						
Pilot Opinion Rating						
Pilot-Vehicle Analysis						
Hover Dynamics						
VTOL						
Conjugate Gradient Optimization						
Describing Functions						