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PART VII

UNSTEADY AERODYNAMICS FOR ADVANCED CONFIGURATIONS

PART VII — VELOCITY POTENTIALS IN NON-UNIFORM TRANSONIC FLOW OVER A THIN WING

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FOREWORD

This report covers a portion of the research conducted by the Los Angeles Division of North American Rockwell Corporation, Los Angeles, California, for the Aerospace Dynamics Branch, Vehicle Dynamics Division, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, under Contract No. AF33(615)-2896.

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Mr. H. Hoge was the Program Manager for North American Rockwell. Mr. L. V. Andrew and Mr. T. E. Stenton were Principal Investigators. The basic approach was outlined by Dr. M. T. Landahl of the Massachusetts Institute of Technology. The calculus of variations approach was suggested by Mr. James Olsen.

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ABSTRACT

Two methods have been outlined in detail, and one of them has been mechanized, for calculating acoustic ray paths emanating from any point in a non-uniform transonic flow field surrounding a wing. It gives the ray path, and the time, for the minimum time of travel from the acoustic source point to the field point. The resulting velocity potential is also computed.

It was necessary to establish an accurate representation of the flow characteristics in the field surrounding the wing. Some ray lines travel over the planform and into the surrounding flow field. It was established that once off the planform they do not return.

Available methods predict phase lags based on the assumption that acoustic rays travel in straight lines. The results of this study show this to be a very poor approximation at transonic speeds. Therefore, it is recommended that the method presented in this report be fully developed for the purpose of calculating generalized forces on wings in harmonic motion at transonic speeds. A computer program that would predict these phase lags with reasonable accuracy, and the corresponding flutter characteristics and unsteady aerodynamic loads on a wing responding to externally applied forces, such as gusts, would fill an important gap in the available technology.

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SYMBOLS

c	chord
C	speed of sound
g	time of travel of an acoustic signal
M	Mach number
r	Slope in the y-direction, dx/dy
δR	Increment in radius vector
s	Distance along a ray path, span
t	Time
U	Free-stream velocity
V	Velocity
x, y, z	Location of a field point
x_o, y_o, z_o	Location of a source or doublet point
X, Y	$x/\beta s$, y/s
X^*, Y^*	Linear transformation of coordinates X, Y
$\hat{i}', \hat{j}', \hat{k}'$	Unit vectors along x', y', z' axes
$\mathbb{R}$	Radius vector
∇	Vector gradient operator, $\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$
β	$\sqrt{1-M^2}$, $\sqrt{r^2 + 1-M^2}$
δ	$\sqrt{5 + M^2}$, an increment
ϕ	Velocity potential
Λ	Ray Angle
τ	Thickness ratio

Subscripts

a	Advancing
ℓ	Local, lower

SYMBOLS (Continued)

r	Receding
u	Upper
x, y	Partial derivatives with respect to x, y
σ	Sonic line
∞	Infinity
Superscripts	
•	Derivative with respect to time
'	Derivative with respect to the independent variable

INTRODUCTION

When an airfoil travels through the air at speeds near the speed of sound, the local speed of flow varies from subsonic near the forward edges to supersonic near the trailing edges. These wide variations of speed from that of the free-stream characterize the non-uniform transonic flow. This non-uniformity of the flow field must be accounted for in accurate calculations of unsteady pressures and forces; particularly their phase lags.

In order to determine an unsteady transonic flow field one requires solutions for singularities immersed in a non-uniform steady flow, (Reference 1). Source solutions for a mean flow that varied in the x-direction only were given in the high-frequency limit by Landahl (Reference 2). Rodemich (Reference 3) presented a "box" solution, based on pulsating doublets, which assumes a uniform mean flow at Mach number 1.0. No exact solutions for the case of a mean flow with arbitrary spatial variations have been found, thus far, but Landahl proposed the basic form of a solution which removes most of the limitations and restrictions of these approximate solutions. The method focuses attention on the time of transmission of an acoustic signal from a pulsating sending source to a distant receiving point. The signal travels through a nearly sonic flow field where the Mach number varies in a prescribed manner.

This report contains a difference equation approach, and differential equation approach to computing the paths and the transmission times for acoustic signals. The independent variable in the latter approach is a spatial rather than a time variable. A procedure that could be used to calculate the velocity potentials and generalized forces on an oscillating surface is described.

POTENTIAL OF A UNIT SOURCE

The basic expressions proposed by Landahl for the velocity potential at the point (x,y,z) due to a pulsating source at (x_o,y_oz_o) are:

(a) for a source in a locally subsonic flow region

$$\phi = \frac{-1}{4\pi\bar{R}} \exp \{i\omega[t-g(x,y,z,x_o,y_o,z_o)]\} \tag{1}$$

where

$$\bar{R} = \sqrt{(x-x_o)^2 + [1-M^2(x,y,z)][(y-y_o)^2 + (z-z_o)^2]}$$

M = Local Mach Number

x_o,y_o,z_o = Location of source point

$g(x,y,z,x_o,y_o,z_o)$ = Time required for a disturbance to travel from (x_o,y_o,z_o) to (x,y,z).

(b) for a source in a locally supersonic flow region

$$\phi = \frac{-1}{4\pi\bar{R}} \{\exp[i\omega(t-g_a)] + \exp[i\omega(t-g_r)]\} \tag{2}$$

where

$g_{a,r} = g_{a,r}(x,y,z,x_o,y_o,z_o)$ = Time required for the advancing, receding wave to travel from (x_o,y_o,z_o) to (x,y,z)

It is likely that good accuracy may be obtained with use of the value of g_a for uniform flow (in the supersonic case, and also for the advancing wave portion in the subsonic case). However, our purpose is to produce a general solution for g which applies to both the advancing and the receding portions of the wave and compare values with those for uniform flow.

Since the primary interest is in wing flows, we consider that both the source and receiver points lie in the x, y-plane, so that $z = z_o = 0$. Furthermore, we consider that signals do not return to the plane once they leave. The problem is thus simplified to one in two spatial dimensions. Its solution should be applicable to a wide variety of nearly planar lifting surfaces.

Consider a signal emanating from a source at the point (x_o,y_o) on a wing. A second point past which the signal travels is located an incremental distance (dx, dy) away. There are two components of velocity of the signal, a radial component, C, where C is the local speed of sound and an x-component, U, where U is the local speed of flow over the wing. Λ is the angle the radial component makes with the negative extension of the x-axis. The path of this wavefront point will be referred to as a "ray". The shape of any ray depends on the initial choice of Λ; for a given Λ, dx and dy are components of the first element of this particular ray emanating from (x_o,y_o). The situation depicted is general in that it applies not only at the source, but at any point on the ray path. Thus, the velocity at any point on the path is a function of three spatial parameters which vary with position, U, C, and Λ. From the sketch, it is clear that

$$dx = [U(x, y) - C(x, y) \cos \Lambda]\, dt \qquad (3)$$
$$dy = C(x, y)\, dt \sin \Lambda$$

Equations were developed for two methods of tracing the ray path to establish the magnitude and the phase relationship at field points to a unit source. These methods are: (1) a difference equation method, and (2) a non-linear differential equation method.

Difference Equation Method

In this method, time is the independent variable. Equations (3) are two of the three equations needed to establish the variation of x, y, and Λ with time. The third equation is obtained by considering the acceleration of the ray in the non-uniform flow field (see Figure 1).

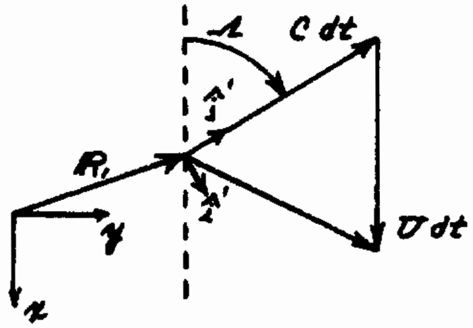


Figure 1. Velocity Components of a Sonic Ray Line In A Moving Airstream

In terms of components in the directions of the rotating unit vectors $\hat{i}'$ and $\hat{j}'$

$$\dot{R}_1 = (U \sin \Lambda)\hat{i}' + (C - U \cos \Lambda)\hat{j}' \qquad (4)$$

and

$$\ddot{R}_1 = (\dot{U} \sin \Lambda + C\dot{\Lambda})\hat{i}' + (\dot{C} - \dot{U} \cos \Lambda)\hat{j}'$$

It is necessary to express the angular velocity $\dot{\Lambda}$ in terms of space variables. To do this, consider that at time t a second ray point is located at $R_2 = R_1 + \delta R\, \hat{i}'$, where δR is small, and it's direction of travel is $\dot{R}_2 = \dot{R}_1 + \delta\dot{R}$. Let the superscripts (o) and (1) denote times t_o and $t_1 (= t_o + \Delta t)$. Then at time t_1

$$\mathbb{R}_1^{(1)} = \mathbb{R}_1^{(0)} + \dot{\mathbb{R}}_1^{(0)} \Delta t$$

and $$\mathbb{R}_2^{(1)} = \mathbb{R}_2^{(0)} + \dot{\mathbb{R}}_2^{(0)} \Delta t$$

Subtracting the first equation from the second

$$\delta \mathbb{R}^{(1)} = \delta \mathbb{R}^{(0)} + \delta \dot{\mathbb{R}}^{(0)} \Delta t \tag{5}$$

where $$\delta \mathbb{R} = \mathbb{R}_2 - \mathbb{R}_1$$

Recalling that the cross product of two vectors is a vector normal to the plane defined by the two vectors, and has a magnitude equal to the product of the two magnitudes times the sine of the angle between them, then

$$\delta \mathbb{R}^{(0)} \times \delta \mathbb{R}^{(1)} = \hat{k}' \left(- \delta R^{(0)} \delta R^{(1)} \sin \Delta \Lambda \right) \tag{6}$$

which has the correct sense. When ΔΛ is small, and when Equation (5) is substituted into the left side of Equation (6), we get

$$\delta \mathbb{R}^{(0)} \times \delta \dot{\mathbb{R}}^{(0)} \Delta t = \hat{k}' \left(- \delta R^{(0)} \delta R^{(1)} \Delta \Lambda \right)$$

This may be rewritten as

$$\frac{\Delta \Lambda}{\Delta t} = - \frac{\delta (C - U \cos \Lambda)}{\delta R^{(1)}}$$

and in the limit as Δt → 0

$$\dot{\Lambda} = - \hat{\lambda}' \cdot \nabla (C - U \cos \Lambda) \tag{7}$$

where the operator $\hat{\lambda}' \cdot \nabla$ is

$$\hat{\lambda}' \cdot \nabla = \left(\sin \Lambda \frac{\partial}{\partial x} + \cos \Lambda \frac{\partial}{\partial y} \right)$$

and operates only on C and U.

Equation (7) has a revealing physical interpretation. From Figure 4 we see that the gradient of the speed of sound C, on forward portions of the wing, is a vector pointing forward and slightly outward from the centerline; whereas, from Figure 3 we see that the gradient of the local flow speed U is nearly in the opposite direction. Although it is not apparent from the figures because they are plotted to different scales, the magnitude of the gradient of U is about five times that of the gradient of C. From the energy equation $C^2 + \frac{\gamma - 1}{2} U^2 = \text{constant}$, $\nabla U = -5.0 \nabla C$. The local Mach number is increasing in the downstream direction. Figure 2 shows that, under these conditions there are only two stable ray angles; those for which the gradient of C - U cos Λ is zero. As the ray propagates through the flow field it will always tend towards one of these two orientations.

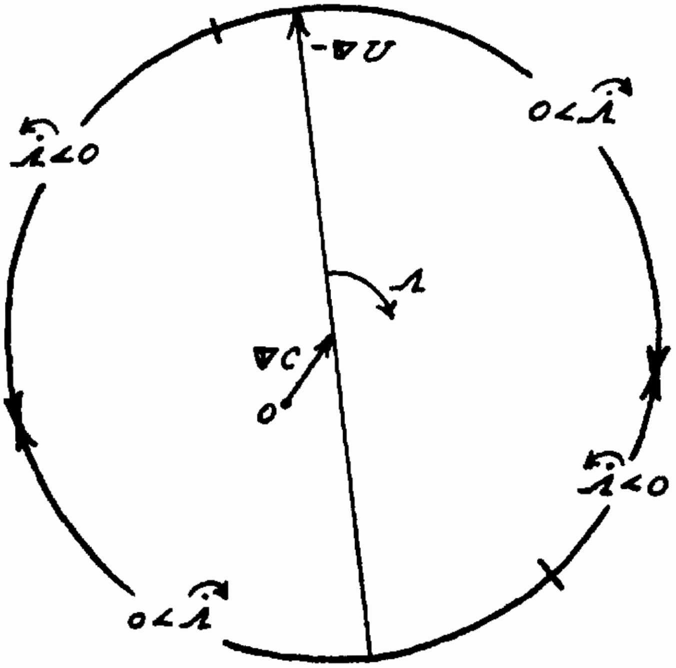


Figure 2. Stability of Ray Angles When The Gradient of Local Flow Speed Exceeds the Gradient of Local Speed of Sound.

We now write Equations (3) and (7) in difference form

$$\Delta x = \left[U - C \cos \Lambda\right] \Delta g \tag{8-a}$$

$$\Delta y = \left[C \sin \Lambda\right] \Delta g \tag{8-b}$$

and

$$\Delta \Lambda = - \left[\sin \Lambda \left(\frac{\partial C}{\partial x} - \cos \Lambda \frac{\partial U}{\partial x}\right) + \cos \Lambda \left(\frac{\partial C}{\partial y} - \cos \Lambda \frac{\partial U}{\partial y}\right)\right] \Delta g \tag{8-c}$$

where Δg represents an increment in disturbance travel time g, defined previously. To determine $\varphi(x, y, 0, x_0, y_0, 0)$ it is necessary to know a steady state distribution of $C(x, y)$, $U(x, y)$, and their derivatives at any point in the flow field over the wing and in the surrounding flow field in the plane of the wing. A means for establishing these is given in Section 5. Assume they are known. Then the procedure used is as follows:

1. Select any source point, on or off the wing, (x_o, y_o).

2. Select a series of initial ray angles, Λ_i, i = 1, 2, ----.

3. Select an initial increment in disturbance travel time, Δg_0.

4. For each of the ray angles store $x^{(i)}$, $y^{(i)}$, $\sin \Lambda^{(i)}$, $\cos \Lambda^{(i)}$, and $\Delta g^{(i)}$, $i = 1, 2,$ ----.
    a. At $x^{(i)}$, $y^{(i)}$ compute and store $x^{(i)} = x^{(i)} + \Delta x^{(i)}/2$ and $y^{(i)} + y^{(i)} + \Delta y^{(i)}/2$, holding Λ constant.
    b. Iterate on $x_2{}^{(i)} = x^{(i)} + \Delta x^{(i)}/2$, $y_2{}^{(i)} = y^{(i)} + \Delta y^{(i)}/2$, and $\Delta\Lambda(x_2{}^{(i)}, y_2{}^{(i)})$ until they converge or exceed ten trials. In the latter case replace $\Delta g^{(i)}$ by $\Delta g^{(i)}/2$ and repeat the iteration. If they converge in three trials or less, replace $\Delta g^{(i)}$ by $2\Delta g^{(i)}$.
    c. Replace $x^{(i)}$ by $x_2{}^{(i)}$, $y^{(i)}$ by $y_2{}^{(i)}$, and return to a.

The solutions presented above are believed to be good approximations to the exact solutions for the following reasons:

1. For the case of a uniform flow they reduce to the proper linearized expressions.

2. The phase of the disturbance will be exact, although the amplitude may be slightly in error.

3. In an inner region in the immediate neighborhood of the source location (x_o, y_o, z_o) they approach the correct solution.

4. For a one-dimensional mean flow with M_ℓ approaching unity they reduce to Landahl's earlier solution (Reference 2).

5. In the limit of steady flow ($\omega = 0$), the solutions give results equivalent to the local linearization method of Spreiter and Alksne (Reference 4). This has been demonstrated by Rubbert (Reference 5).

6. Inasmuch as the proposed approximation only affects the receding part of the solution, the proper limiting solution for high frequencies (Reference 1), should always be obtained since then receding-wave effects are largely cancelled out due to the rapid phase variations.

This method gives reasonable results, i.e., reasonable based on a comparison with results obtained from the differential equation method. However, the ray paths did not conclusively show the existence of the focal point that the second method revealed.

Non-Linear Differential Equation Method

From Equations (3) we may write the slope of the ray path

$$\frac{dx}{dy} = \frac{M - \cos \Lambda}{\sin \Lambda} \tag{9}$$

and solving this equation for cos Λ, we get

$$\cos \Lambda = \frac{M \pm r\sqrt{r^2+1-M^2}}{1+r^2} \tag{10}$$

where $r = \frac{dx}{dy}$

The transmission time from source to receiving point is given by

$$\mathfrak{z} = \int \frac{ds}{V} \tag{11}$$

where the integration is taken along the path and

$$ds = \sqrt{1 + r^2}\ dy \tag{12}$$

The velocity along the path is obtained from the vector sum of the two velocity components

$$V = C\sqrt{M^2 + 1 - 2M \cos \Lambda} \tag{13}$$

Substituting equations (12), (13), and (10) into equation (11) we have:

$$\mathfrak{z} = \int \frac{\sqrt{1+r^2}\, dy}{C\sqrt{M^2+1-2M\left[\frac{M \pm r\sqrt{r^2+1-M^2}}{1+r^2}\right]}}$$

which reduces to

$$\mathfrak{z} = \int \frac{(1+r^2)\, dy}{C\sqrt{M^2r^2 \mp 2Mr\sqrt{r^2+1-M^2}+r^2+1-M^2}} \tag{14}$$

The radicand in the denominator is a perfect square. Thus,

$$\mathfrak{z} = \int \frac{(1+r^2)\, dy}{C\left[Mr \mp \sqrt{r^2+1-M^2}\right]}$$

which reduces to

$$\mathfrak{z} = \int \frac{Mr \pm \sqrt{r^2+1-M^2}}{C\,(M^2-1)}\, dy \tag{15}$$

At this point we relate the local acoustic velocity, $C = C(x, y)$, to the local Mach number by imposing the condition of conservation of energy. For non-viscous flow, the total temperature is conserved. It is easily verified, that under this condition

$$\frac{C^2}{C_\infty^2} = \frac{5+M^2}{5+M_\infty^2} \tag{16}$$

where $\gamma = 1.4$, for a diatomic gas, has been used. Substituting Equation (16) into Equation (15), we get

$$g = \frac{1}{C_\infty\sqrt{5+M_\infty^2}}\int \frac{\sqrt{5+M^2}\left[Mr \pm \sqrt{r^2+1-M^2}\right]}{(M^2-1)}\,dy \tag{17}$$

where the upper sign applies to receding waves and the lower sign to advancing waves. Equation (17) contains all the elements for the solution. However, the integrand is a function of x, y, and dx/dy. This equation may be written in symbolic form

$$g = \int_{y_0}^{y_1} F\left(x, y, \frac{dx}{dy}\right) dy$$

which suggests the use of Euler's equation to find the minimum time g, for the disturbance to travel to a field point (x_1, y_1)

$$\frac{d}{dy}\frac{\partial F}{\partial r} - \frac{\partial F}{\partial x} = 0 \tag{18}$$

In order to simplify the notation, we set

$$F = \frac{\delta\,(Mr \pm \beta)}{M^2-1}$$

where $\delta = \delta(x, y) = \sqrt{5 + M^2}$

$\beta = \beta(x, y, r) = \sqrt{r^2 + 1 - M^2}$

and r has been previously defined. We will need

$$\frac{\partial F}{\partial x} = \frac{\delta}{M^2-1}\left[rM_x \mp \frac{MM_x}{\beta}\right] + \frac{Mr \pm \beta}{(M^2-1)^2}\left[(M^2-1)\frac{MM_x}{\delta} - 2\delta MM_x\right]$$

$$\frac{d}{dy}\left(\frac{\partial F}{\partial r}\right) = \frac{\delta}{M^2-1}\left[\frac{dM}{dy} \pm \frac{\beta\frac{dr}{dy} - r\frac{d\beta}{dy}}{\beta^2}\right] + \left(M \pm \frac{r}{\beta}\right)\left[\frac{(M^2-1)\frac{d\delta}{dy} - 2\delta M\frac{dM}{dy}}{(M^2-1)^2}\right]$$

Then, making use of the relationships

$$\frac{dM}{dy} = rM_x + M_y$$

$$\frac{d\beta}{dy} = \frac{1}{\beta}\left[r\frac{dr}{dy} - rMM_x - MM_y\right]$$

$$\frac{d\delta}{dy} = \frac{1}{\delta}\left[rMM_x + MM_y\right],$$

solving for dr/dy, and combining terms, we get

$$\frac{dr}{dy} = \frac{1}{\delta^2(M^2-1)}\left\{\left[\frac{-M(M^2+1)r}{M^2-1} \mp \frac{\beta(7M^2+5)}{M^2-1}\right]r^2 + \left[2M(M^2+8)r \pm \beta(7M^2+5)\right]\right\}M_y + \left\{\frac{M}{\delta^2}(r^2+6)\right\}M_x \tag{19}$$

Equation (19) is a second order, second degree differential equation of the form

$$\frac{d^2x}{dy^2} = f\left(x, y, \frac{dx}{dy}\right)$$

It is second degree because β represents a radical. However, it can be solved numerically by any of the standard repetitive processes. We employed a fourth order Runge-Kutta procedure.

There are certain difficulties that arise in the numerical evaluation of Equation (19). These are first listed and interpreted and then equations used to surmount them are presented.

(1) Along some ray paths dx/dy becomes infinite even when the Mach number is not equal to one.

(2) Equation (19) is singular at Mach number = 1.0.

(3) In the supersonic region, signals sometimes become trapped on the local Mach line. This happens when cos Λ = 1/M. Signals tend to gravitate to this condition. Such trapped signals cannot then cross the sonic line. They approach the sonic line as a limit, and are cancelled out there.

To overcome the difficulty listed in Item (1), it is necessary to use x instead of y as the independent variable. This is done by applying the equation

$$\frac{d^2y}{dx^2} = \frac{-1}{\left(\frac{dx}{dy}\right)^3}\,\frac{d^2x}{dy^2} \tag{20}$$

It is convenient here to introduce some new notation. Re-write equation (19) in the form

$$x'' = \frac{1}{AB}\left\{-\frac{M}{B}(M^2+11)x'^3+2M(M^2+8)x' \mp (7M^2+5)\frac{R_1^3}{B}\right\}M_y + \frac{M}{A}\left\{x'^2+6\right\}M_x \tag{21}$$

where the new notation, together with some other notation which will be used later, is defined as follows

$$\begin{array}{lll} x' = \dfrac{dx}{dy} & A = M^2+5 & M_x = \dfrac{\partial M}{\partial x} \\ y' = \dfrac{dy}{dx} & B = M^2-1 & M_y = \dfrac{\partial M}{\partial y} \\ R_1 = \sqrt{x'^2-(M^2-1)} & \beta = \sqrt{B} & \\ R_2 = \sqrt{1-y'^2(M^2-1)} & E = c_\infty\sqrt{5+M_\infty^2} & \end{array} \tag{22}$$

Substituting Equation (20) into Equation (21), we get

$$y'' = \frac{1}{AB}\left\{\frac{M}{B}(M^2+11)-2M(M^2+8)y'^2 \pm \frac{R_2^3}{B}(7M^2+5)\right\}M_y - \frac{My'}{A}(6y'^2+1)M_x \quad ; \quad 0 \leq y' \tag{23-a}$$

$$y'' = \frac{1}{AB}\left\{\frac{M}{B}(M^2+11)-2M(M^2+8)y'^2 \mp \frac{R_2^3}{B}(7M^2+5)\right\}M_y - \frac{My'}{A}(6y'^2+1)M_x \quad ; \quad y' < 0 \tag{23-b}$$

The limiting form of Equation (20) at M = 1 is:

$$x''\Big|_{M=1.0} = \frac{1}{2A}\left\{2x'^3+x'+\frac{9}{x'}\right\}M_y + \frac{1}{A}(x'^2+6)M_x$$

In the supersonic region, when the signal is trapped on the local Mach line, and

$$\cos\mathcal{A} = \frac{1}{M}, \quad \sin\mathcal{A} = \frac{\sqrt{1-M^2}}{M}, \quad \text{and} \quad |x'| = \beta$$

equation (20) reduces to

$$x'' = M\left(\frac{M_y}{x'} + M_x\right)$$

A complete set of equations, together with their areas of applicability, will now be outlined.

Complete Set of Equations where Y is the Independent Variable

$$x'' = \frac{1}{AB}\left\{\frac{-M}{B}(M^2+11)x'^3+2M(M^2+8)x' \mp (7M^2+5)\frac{R_1^3}{B}\right\}M_y + \frac{M}{A}(x'^2+6)M_x \tag{24}$$

$$\frac{dt}{dy} = \frac{1}{E}\,\frac{\sqrt{5+M^2}\,(Mx' \pm R_1)}{M^2-1} \tag{25}$$

$$x''\Big|_{M=1.0} = \frac{1}{2A}\left\{2x'^3+x'+\frac{9}{x'}\right\}M_y + \frac{M}{A}(x'^2+6) \tag{26}$$

$$\frac{dt}{dy}\Big|_{M=1.0} = \frac{\sqrt{6}}{2E}\left(x'+\frac{1}{x'}\right) \tag{27}$$

$$x''\Big|_{|x'|=\beta} = M\left(\frac{M_y}{x'}+M_x\right) \tag{28}$$

$$\frac{dt}{dy}\Big|_{|x'|=\beta} = \frac{M\sqrt{5+M^2}}{E\,x'} \tag{29}$$

A complete set of equations were also developed using x as the independent variable. However, for the sake of brevity, and since they are obtained by a simple change of variable, they will not be listed here. Equations (26) and (27) apply where an advancing ray path crosses the sonic line, and equations (28), (29) apply where a ray path, in the supersonic region, becomes trapped on the local Mach line. It remains to describe the regions of applicability of the upper and lower signs of equations (24) and (25). In what follows, "right branch" will be specified where $(0 < \alpha < \pi)$ and left branch will be specified if $(-\pi < \alpha < 0)$. Here α is the local value along the ray path. The end points are not specified because for these points we use x as the independent variable.

The upper sign is used for

(1) Subsonic, left branch

(2) Supersonic, receding, right branch

(3) Supersonic, advancing, left branch

The lower sign is used for

(1) Subsonic, right branch

(2) Supersonic, receding, left branch

(3) Supersonic, advancing, right branch

THE NON-UNIFORM FLOW FIELD

In the application of each of the methods contained in this report, it is necessary to know certain of the properties of the transonic flow field on, and in the neighborhood of, the wing. Figures 3 and 4 show the distributions of local flow speeds and sonic speeds over a 65° delta wing model in a wind tunnel in which the Mach number was 1.04 (taken from Reference 6). Speeds were computed from steady state pressure data at 27 points on the wing. The figures are intended only to show the general characteristics of the flow, such as: (1) The local sonic line shifts aft with distance from the centerline but crosses the leading edge inboard of the tip, (2) Mach number variations in both the streamwise and spanwise directions must be considered and cannot be considered to be linear, and (3) Separated flow is indicated over the aft and inboard portion of the wing. To consider the last of these characteristics is beyond the scope of this study. However, the first two are amenable to analysis using available theories and techniques.

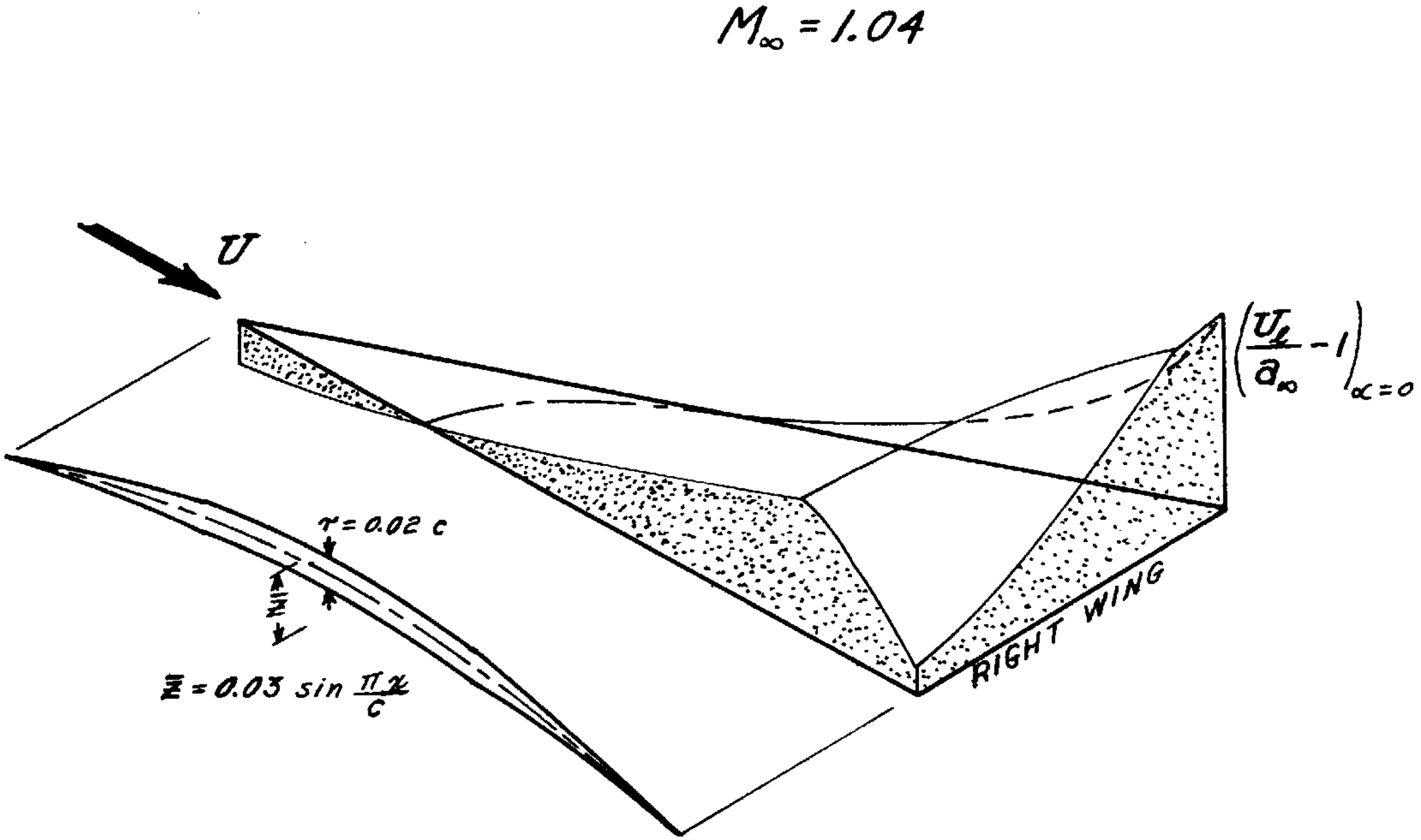


Figure 3. Local Flow Distribution on a 65° Δ at a Transonic Speed

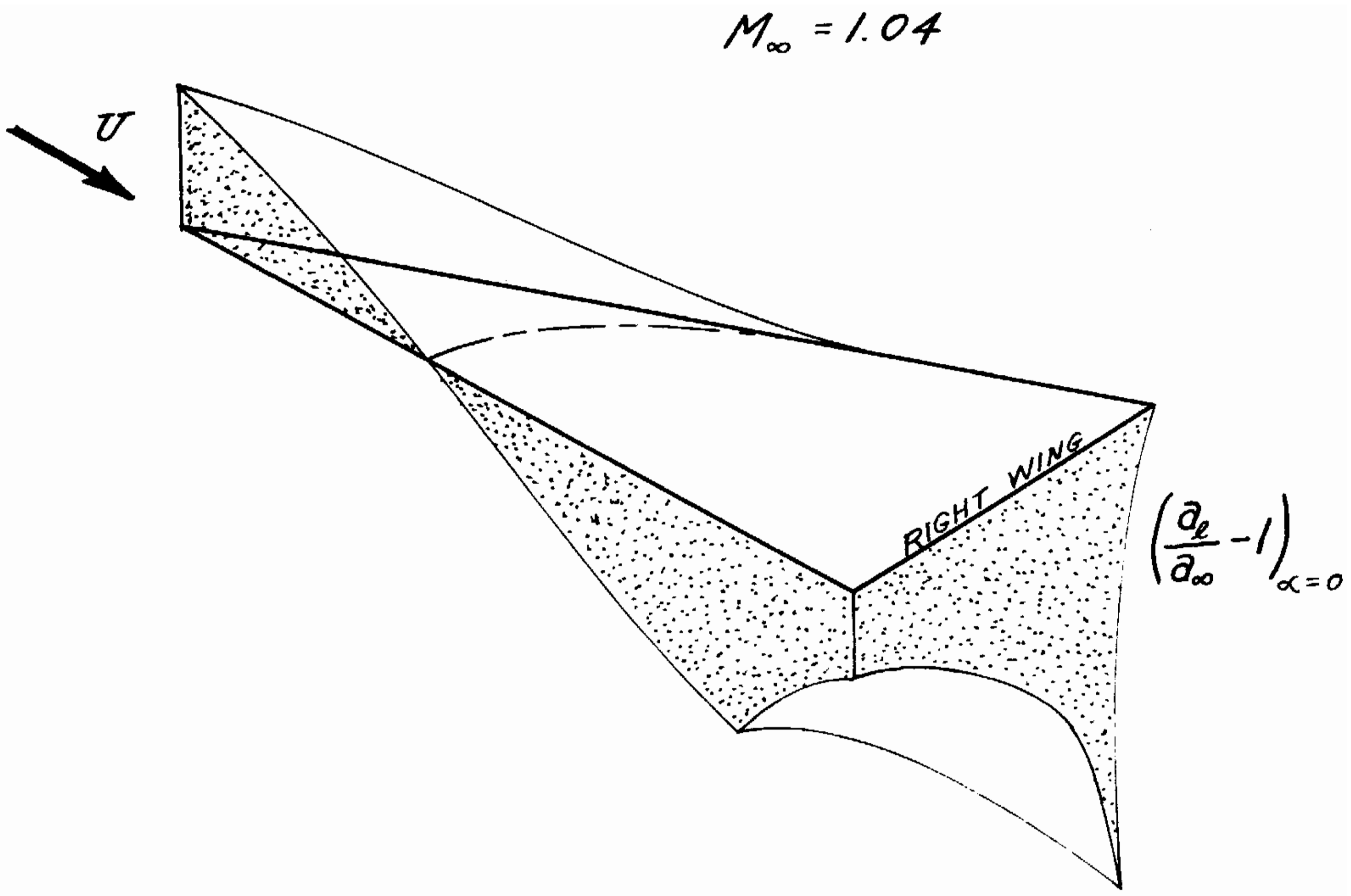


Figure 4. Sonic Speed Distribution on a 65° Δ at a Transonic Speed

Mach number distributions over areas off the wing were computed from an approximate theoretical solution of the flow field that matched pressure distributions on the wing. In order to avoid a discontinuity at the juncture of the two regions, a small transition region was defined over which the two functions were joined by a numerical smoothing technique.

Let:

$M_L = M_L(x,y) =$ Mach number

$\varphi = \varphi(x,y) =$ Perturbation potential

$\tau = T/C(x,y) =$ Thickness ratio

For a steady-state, non-lifting flow

$$(1-M_L^2)\varphi_{xx} + \varphi_{yy} + \varphi_{zz} = 0 \tag{30}$$

and

$$\varphi_z(x,y,0^+) = \pm \tau f_x(x,y) \tag{31}$$

Where $f(x,y)$ is a function describing the variation of the surface from the mean.

Using parametric differentiation with respect to τ, (Reference 5),

$$g = g(x,y) = \frac{\partial \varphi}{\partial \tau}$$

Equation (30) becomes:

$$\frac{\partial}{\partial x}\left\{\left[1-M_L^2\right] g_x\right\} + g_{yy} + g_{zz} = 0 \tag{32}$$

$$g_z(x,y,0^+) = \pm f_x(x,y)$$

After having obtained the solution of equation (32), the local Mach number distribution is obtained by relating local Mach number to the coefficient of pressure, (C_p). Starting with the following basic relations:

Let $u = \dfrac{U_L - U_\infty}{U_\infty}$

then

$$u = \frac{1}{U_\infty}\frac{\partial \varphi}{\partial x} = -C_p/2 \tag{33}$$

$$a^2 + \frac{1}{2}(\gamma-1) q^2 = \text{constant} \tag{34}$$

where $q = U_\infty$ at infinity

$q = U_\infty(1+u)$ elsewhere

$a =$ speed of sound

We have: $$a_\infty^2 + \tfrac{1}{2}(\gamma-1)v_\infty^2 = a_L^2 + \tfrac{1}{2}(\gamma-1)v_\infty^2(1+u)^2$$

$$v_\infty^2(1+u)^2 \cong v_\infty^2(1+2u)$$

$$a_L^2 \cong a_\infty^2 - (\gamma-1)v_\infty^2 u$$

using equation (33)

$$a_L^2 \cong a_\infty^2\left[1 + \tfrac{1}{2}(\gamma-1)M_\infty^2 C_p\right]$$

The coefficient of pressure, C_p is of order (.1), and M is O(1.). Therefore, to sufficient accuracy.

$$a_L \cong a_\infty\left[1 + \tfrac{1}{4}(\gamma-1)M_\infty^2 C_p\right]$$

$$V_L = v_\infty(1+u) = v_\infty(1 - \tfrac{1}{2}C_p)$$

and from these relations:

$$M_L \cong \frac{M_\infty(1 - \frac{1}{2}C_p)}{1 + \frac{1}{4}(\gamma-1)M_\infty^2 C_p}$$

Noting again the order of M_∞ and C_p, to sufficient accuracy.

$$M_L \cong M_\infty(1 - \tfrac{1}{2}C_p)\left[1 - \tfrac{1}{4}(\gamma-1)M_\infty^2 C_p\right]$$

or $$M_L \cong M_\infty\left[1 - \frac{\gamma+1}{4}C_p\right] \tag{35}$$

Equation (35) is the expression that was used to relate local Mach number to C_p on regions off the wing.

A solution of equation (32), using the results of equation (35) was worked out for a special configuration. The special wing configuration is depicted in figure (5).

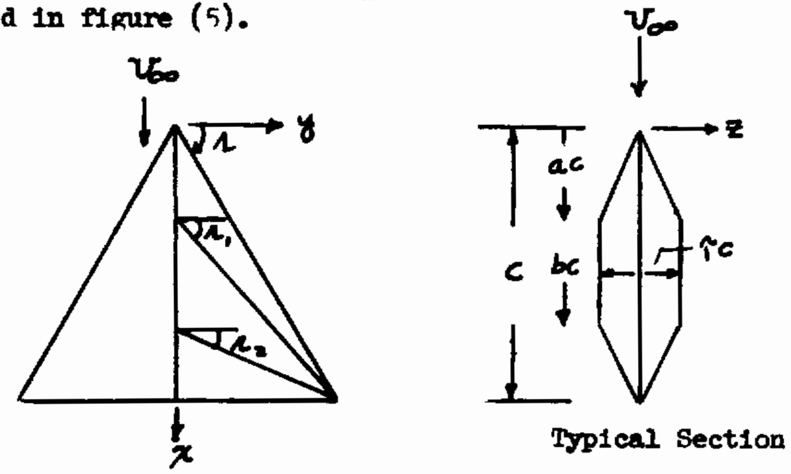


Fig. 5. A Thin Wing In Rectilinear Flight

The solution is:

$$\begin{aligned} C_p(x,y) - C_p(x,0) = &-2f\left[|y-s|^{\varepsilon} + |y+s|^{\varepsilon} - 2|s|^{\varepsilon}\right] \\ &-2f_1\left[|y-s_1|^{-\varepsilon_1} + |y+s_1|^{-\varepsilon_1} - 2|s_1|^{-\varepsilon_1}\right]H(x-a) \\ &-2f_2\left[|y-s_2|^{-\varepsilon_2} + |y+s_2|^{-\varepsilon_2} - 2|s_2|^{-\varepsilon_2}\right]H(x-b) \end{aligned} \tag{36}$$

where $H(x)$ is a step function.

$$\frac{\partial \Delta C_p}{\partial x} = -\frac{2f\varepsilon}{\tan\Lambda}\left[-|y-s|^{\varepsilon-1} + |y+s|^{\varepsilon-1} - 2|s|^{\varepsilon-1}\right.$$
$$+\frac{2f_1\varepsilon_1}{\tan\Lambda_1}\left[-|y-s_1|^{-\varepsilon_1-1} + |y+s_1|^{-\varepsilon_1-1} - 2|s_1|^{-\varepsilon_1-1}\right]H(x-a)$$
$$+\frac{2f_2\varepsilon_2}{\tan\Lambda_2}\left[-|y-s_2|^{-\varepsilon_2-1} + |y+s_2|^{-\varepsilon_2-1} - 2|s_2|^{-\varepsilon_2-1}\right]H(x-b) \qquad (37)$$

$$\frac{\partial \Delta C_p}{\partial y} = -2f\varepsilon\left[|y-s|^{\varepsilon-1} + |y+s|^{\varepsilon-1}\right]$$
$$+2f_1\varepsilon_1\left[|y-s_1|^{-\varepsilon_1-1} + |y+s_1|^{-\varepsilon_1-1}\right]H(x-a)$$
$$+2f_2\varepsilon_2\left[|y-s_2|^{-\varepsilon_2-1} + |y+s_2|^{-\varepsilon_2-1}\right]H(x-b) \qquad (38)$$

WHERE:

$$f = \frac{\cos^2\Lambda}{\sin\Lambda}, \quad \varepsilon = \frac{r}{2\pi a\cos\Lambda}, \quad s = x/\tan\Lambda$$

$$f_1 = \frac{\cos^2\Lambda_1}{\sin\Lambda_1}, \quad \varepsilon_1 = \frac{r}{2\pi a\cos\Lambda_1}, \quad s_1 = (x-a)/(1-a)\tan\Lambda_1$$

$$f_2 = \frac{\cos^2\Lambda_2}{\sin\Lambda_2}, \quad \varepsilon_2 = \frac{r}{2\pi a\cos\Lambda_2}, \quad s_2 = (x-b)/(1-b)\tan\Lambda_2$$

$$\tan\Lambda_1 = (1-a)\tan\Lambda, \quad \tan\Lambda_2 = (1-b)\tan\Lambda$$

After determining a distribution of C_p and its derivatives from equations (36), (37), and (38), the Mach number distribution, with its derivatives, is computed from equation (35).

DESCRIPTION OF THE COMPUTER PROGRAM

The equations for the ray paths are solved in the following manner: Let the independent variable be y and

$$V_1 = \frac{dx}{dy}$$

$$V_2 = x$$

$$V_3 = t$$

Then

$$\frac{dV_1}{dy} = f_1(V_1, V_2, y)$$

$$\frac{dV_2}{dy} = V_1$$

$$\frac{dV_3}{dy} = f_3(V_1, V_2, y)$$

These three simultaneous differential equations are solved in a step-by-step manner by use of a standard "SHARE" subroutine which is based on the Runge Kutta method. When d_x/d_y becomes greater than one, a variable change takes place in the program, and x becomes the independent variable.

A signal (in the supersonic region) is considered "trapped" on the local Mach line when

$$\left| x'^2 - (M^2-1) \right| \leq E1$$

When, for this trapped signal, (M-1) < E2, the integration stops and a new ray line is started. This logical flow is shown in the chart on page 21.

The values of λ_0 used in the program are determined by the parameter (NLA). If (NLA) is an odd integer, it will be rounded down in the program to an even integer. Values of λ_0 vary from zero to π and from zero to $-\pi$ in an arithmetic progression.

Computation of a ray path (other than for a "trapped signal") ceases under the following conditions:

$$x \leq 0$$

$$1 \leq x$$

$$|YMAX| \leq |y|$$

$$NMAX \leq NCNT$$

where NCNT is the number of points on the ray path already computed. This logical flow is shown in the chart on page 22.

Subroutine DERIV computes the appropriate derivatives.

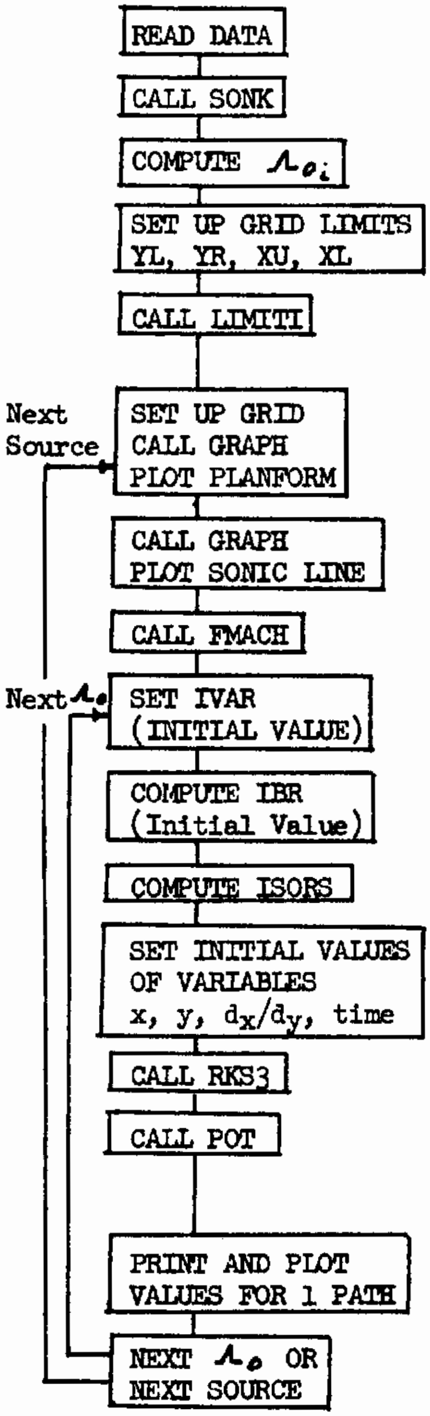
Subroutine CNTRL accomplishes variable changes, stores local values in appropriate locations for later printing, and performs exit tests.

Subroutine FMACH computes the local Mach number and the partial derivatives of the Mach number.

Subroutine SONK computes coordinates on the planform where M = 1.

Sample data sheets with numbers which have been used in a computer run are in Appendix II. The output sheets are included. The output format is self-explanatory, with the exceptions of certain test words that are printed out at the beginning of the plots for each ray-path. Definitions for these words can be found in the comment statements at the beginning of the listing in Appendix I. The values listed for these test words apply to the last point plotted for the ray-path.

MAIN PROGRAM



Subroutine SONK Computes Sonic Line

Subroutine LIMITI Sets Plotting Grid Limits

Subroutine GRAPH Produces Cathode Ray Tube Plots

Subroutine FMACH Computes Mach No.

Determines Whether X or Y is Independent

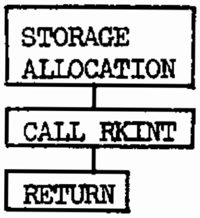
Determines Left or Right Branch

Determines Type of Source

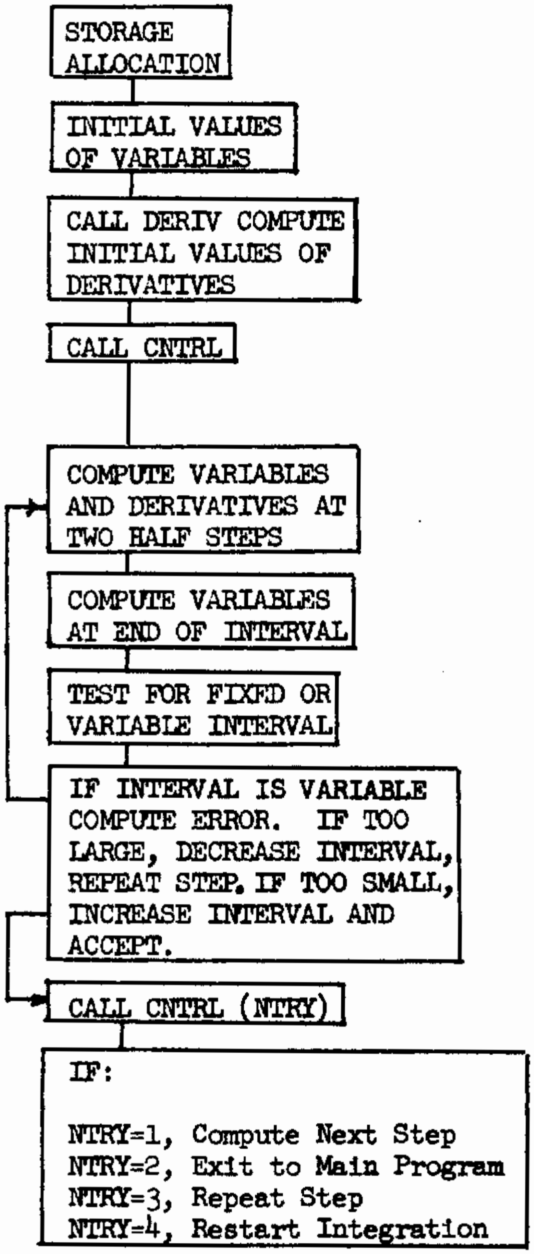
Runge Kutta Integrating Subroutine

Subroutine POT Computes Velocity Potential Along Path due to Source at (X_o, Y_o)

SUBROUTINE RKS3



SUBROUTINE RKINT



Subroutine DERIV Computes Derivatives

Subroutine CNTRL Executes Variable Changes, Stores Current Values, Executes Exit Tests

This Loop Calls DERIV 8 Times

NTRY is Re-set in CNTRL

DISCUSSION OF RESULTS

This report contains two methods for calculating the velocity potential along sonic ray lines emanating from any point in a non-uniform flow field, i.e., one that varies from locally subsonic to supersonic speeds. Both methods apply to pulses emitted by sources or doublets. It has been demonstrated that both methods yield nearly identical ray paths and times of transmission. Those presented were obtained using the second method.

Figures 6 through 13 show ray paths of acoustic signals emanating from various points in a non-uniform transonic flow field. The reader may want to try his hand at tracing one of the ray paths in a region of interest such as near a leading edge. If so, it should be helpful to recall the discussion starting with Equation (7), through the difference equations of the path, Equation (8), and to the end of that section. An analysis of the differential equation of the path, Equation (24) should also be helpful. These show, for instance, that where the Mach number is constant the curvature of the ray path is zero; for a given Mach number and slope of ray path the curvature is proportional to rate of change of Mach number along the path. Figures 6, 7, 9, and 10 conclusively show that when the variation in Mach number is parabolic in the chordwise and spanwise directions focal pointsexist, both in subsonic and supersonic portions of the flow. None of the present theories accounts for the corresponding multiple crossings of the acoustic wave front. Figures 9 and 12 show acoustic signals traveling from regions of supersonic flow to regions of subsonic flow. This can occur, of course, only when the sonic line is swept downstream. Figures 9 and 12 also show rays that have been trapped on the Mach wave, travel outward to the sonic line where the spanwise slope of the ray path becomes zero, and are cancelled there. A study of the ray paths that cross the leading edge shows that in practical applications it is correct to assume they do not return.

These results permit the formulation of a numerical procedure. A box method is outlined in Appendix III. It establishes velocity potentials at all box centers on an aerodynamic surface and the corresponding generalized forces.

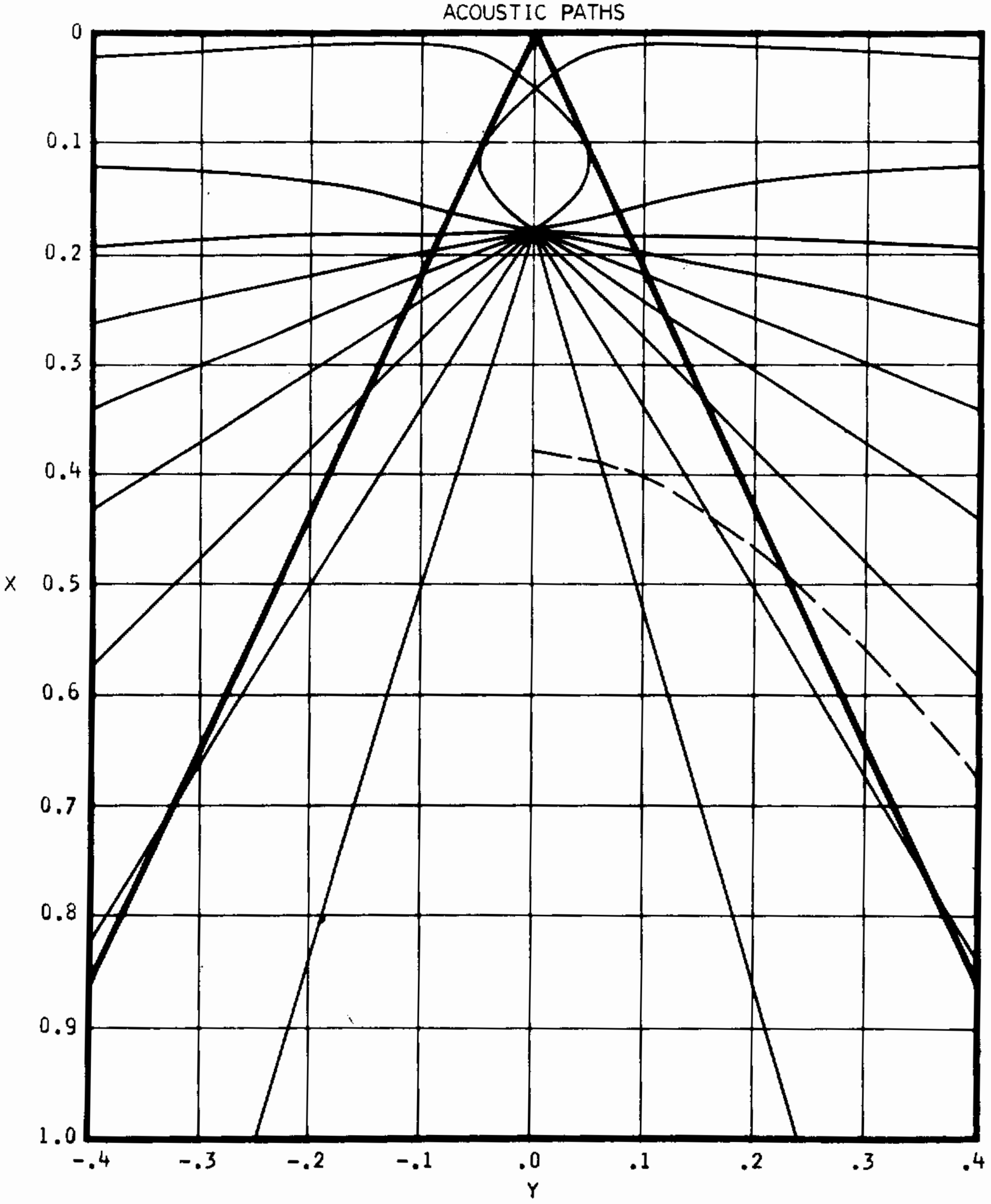


Figure 6. Ray Paths for a Source or Doublet at (0.18c, 0.0)

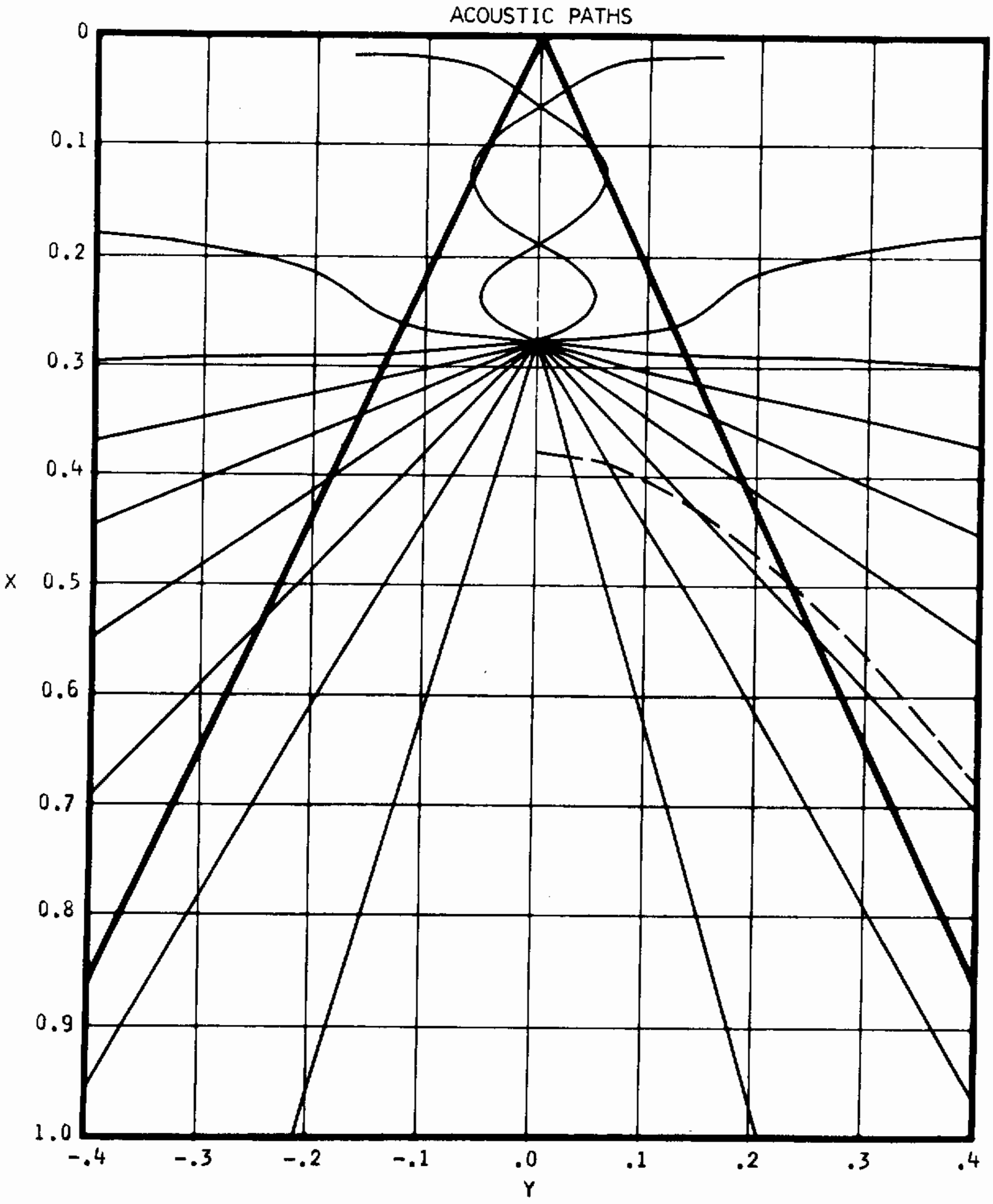


Figure 7. Ray Paths for a Source or Doublet at (0.28c, 0.0)

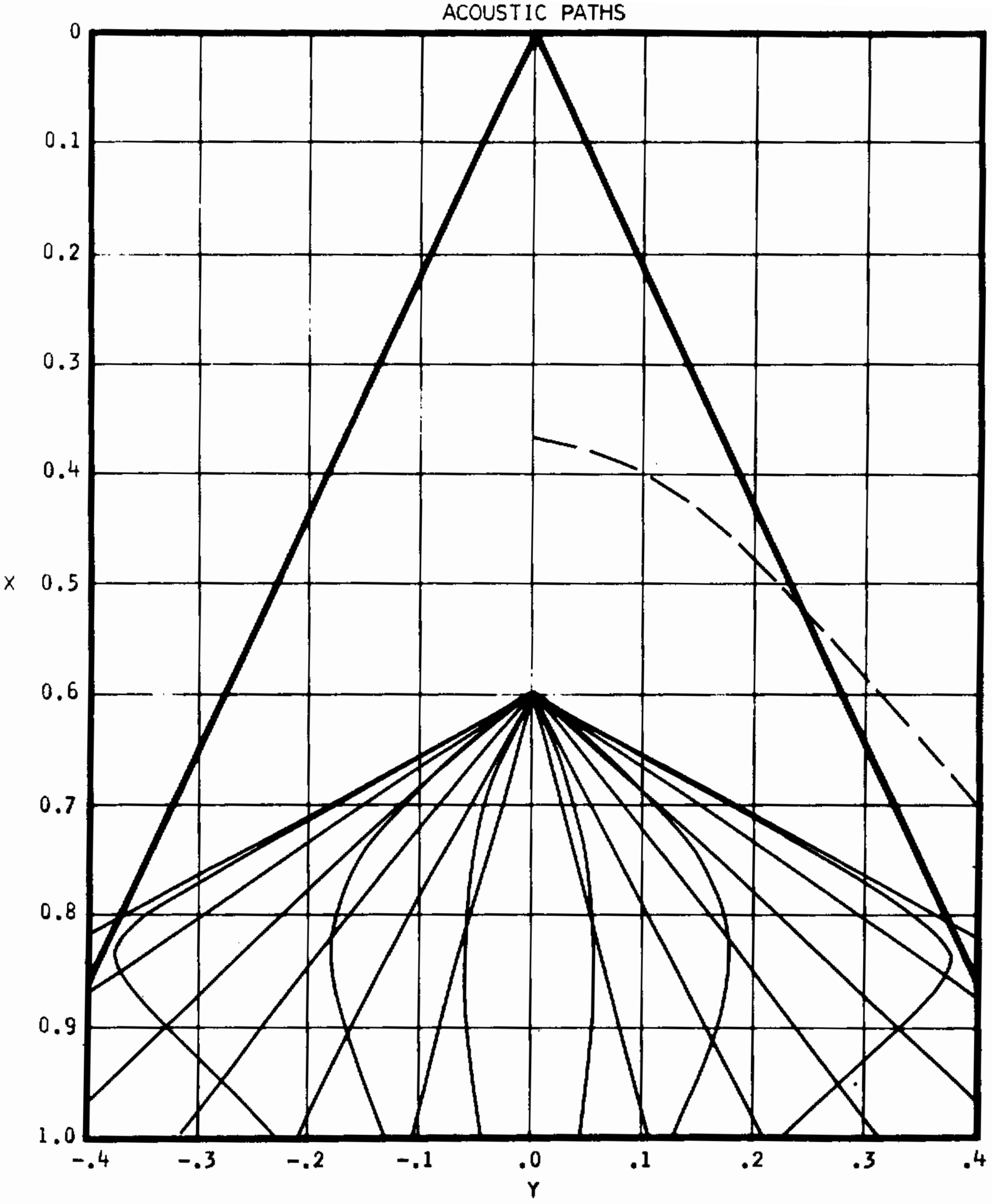


Figure 8. Ray Paths for a Source or Doublet at (0.6c, 0)

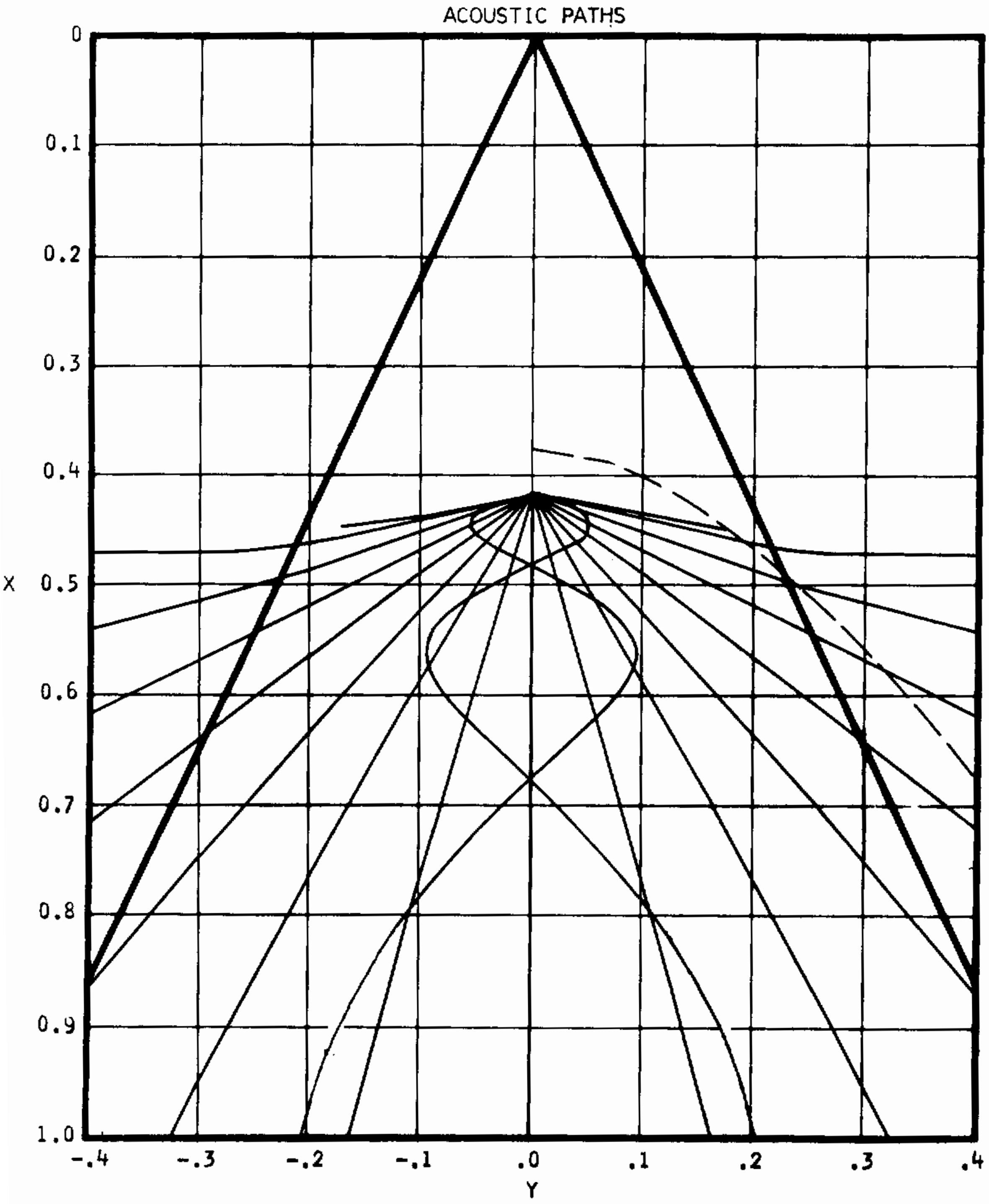


Figure 9. Ray Paths for a Source or Doublet at (0.42c, 0.0)

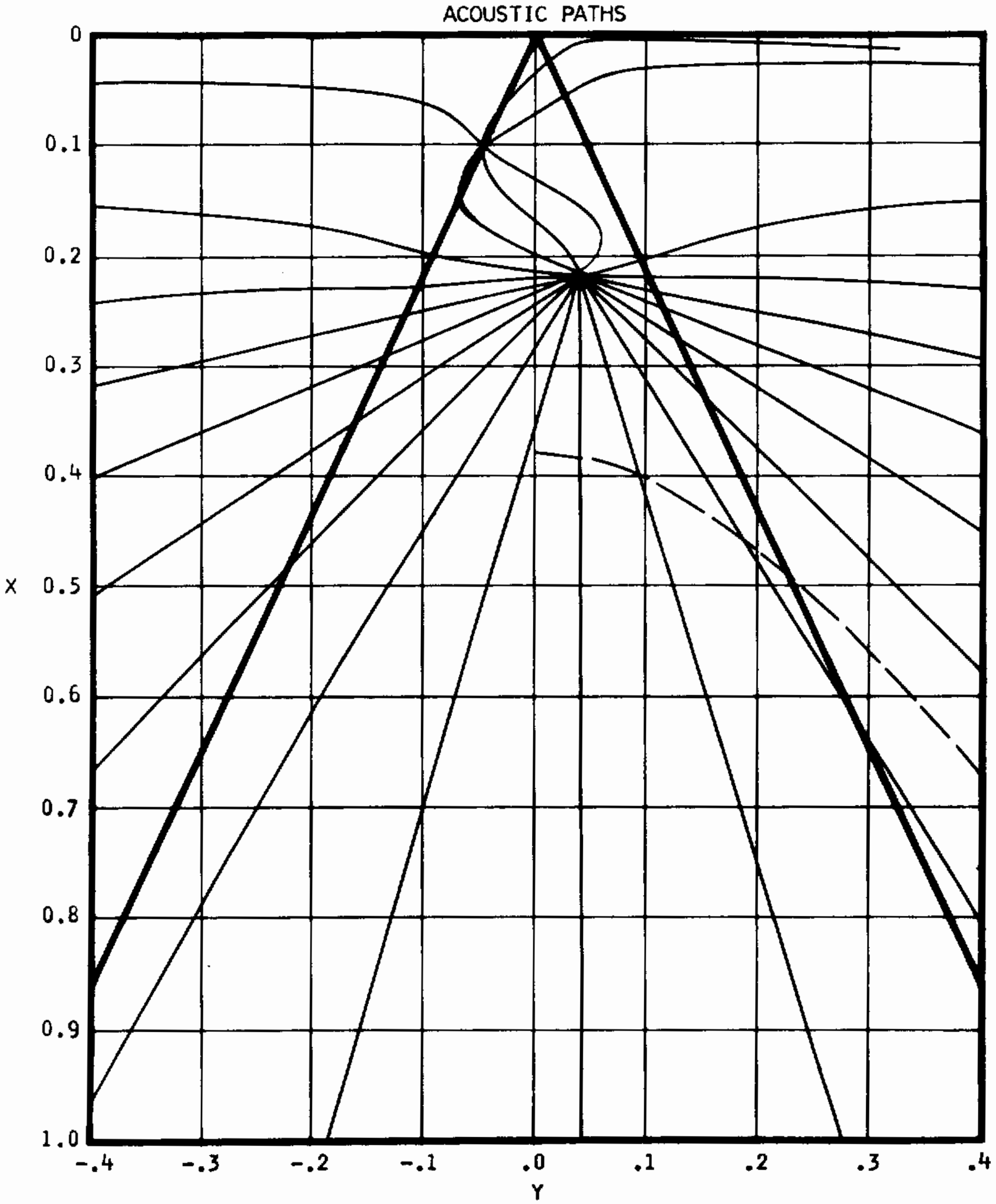


Figure 10. Ray Paths for a Source or Doublet at (0.22c, 0.04c)

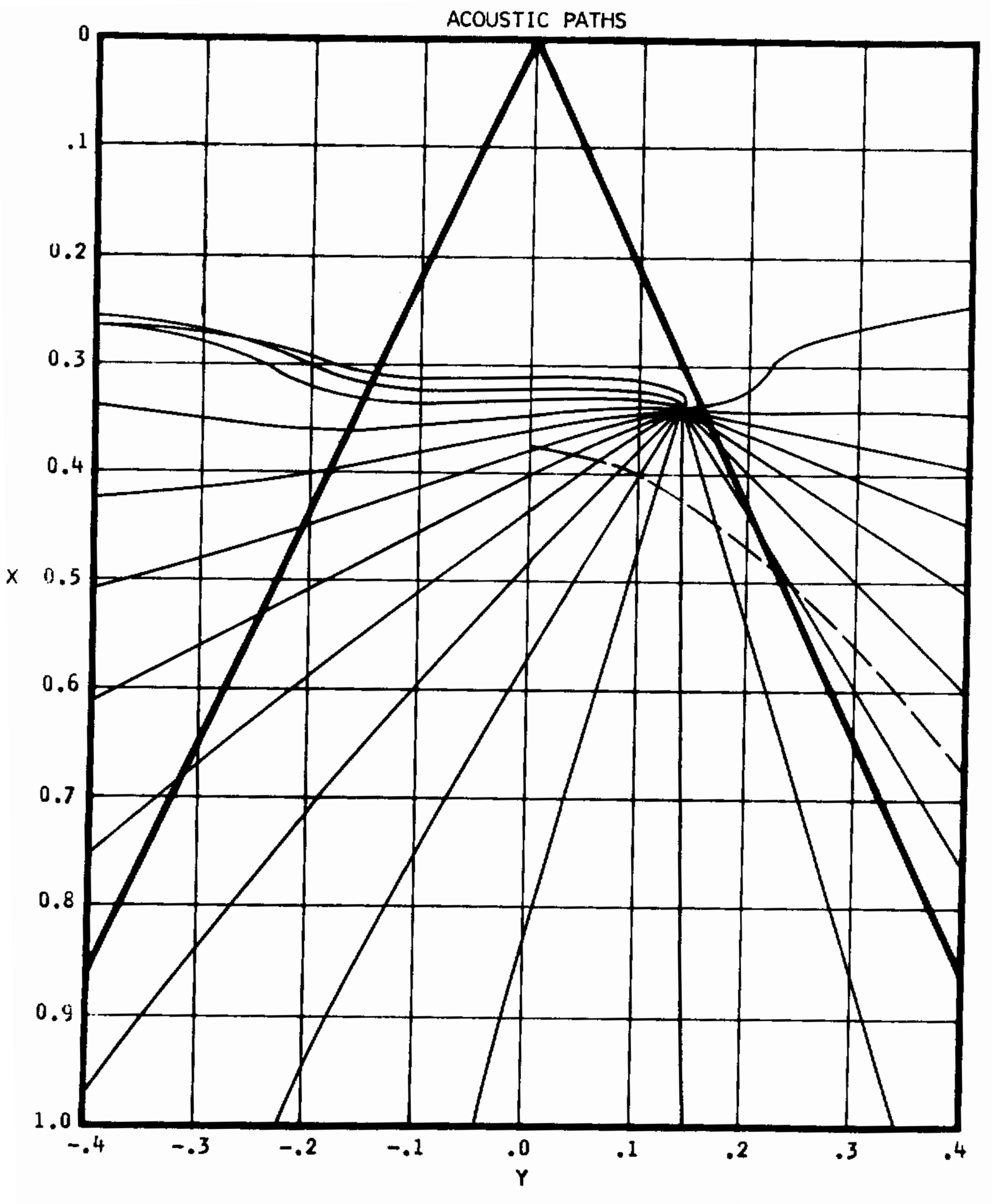


Figure 11. Ray Paths for a Source or Doublet at (0.34c, 0.14c)

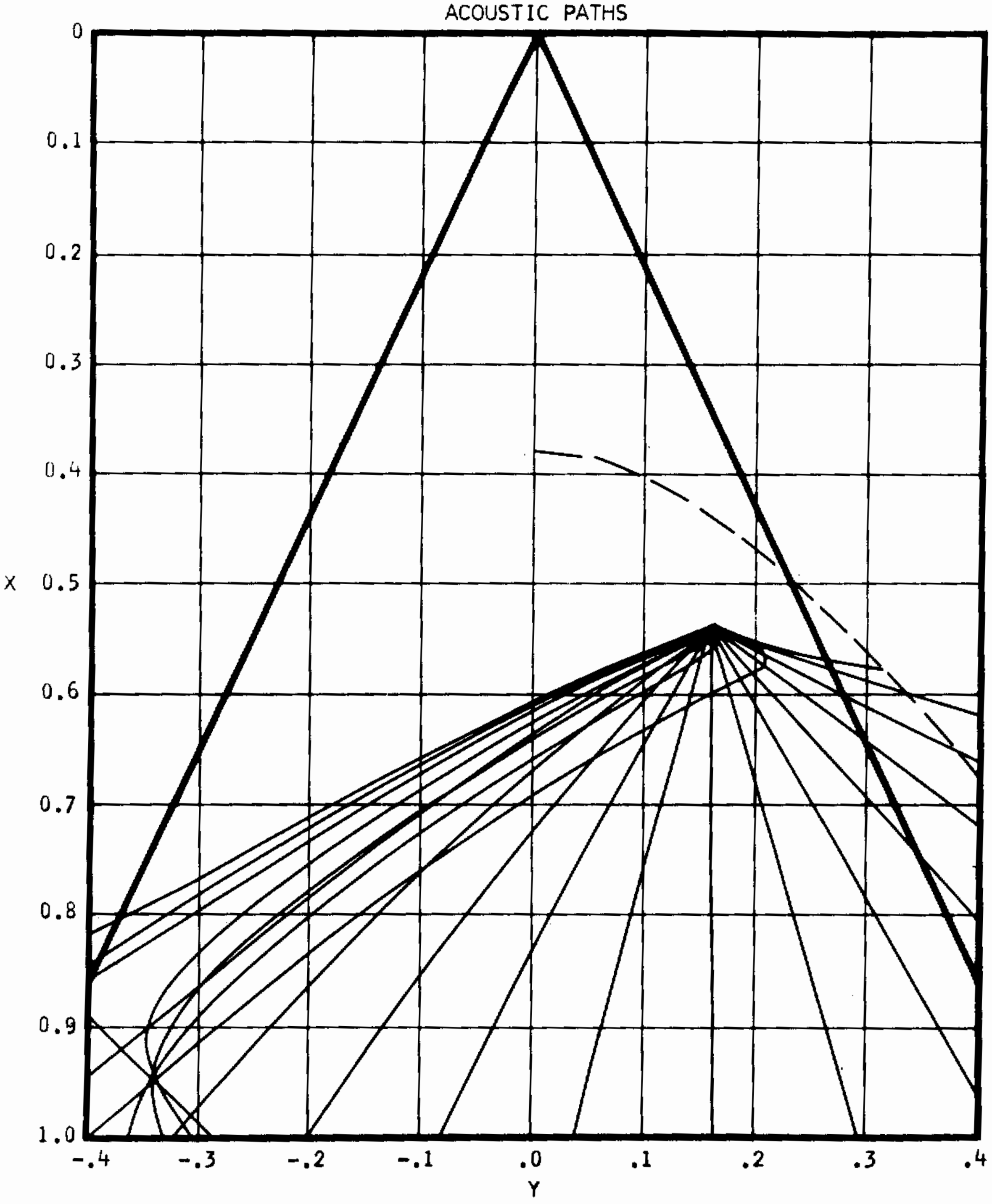


Figure 12. Ray Paths for a Source or Doublet at (0.54c, 0.16c)

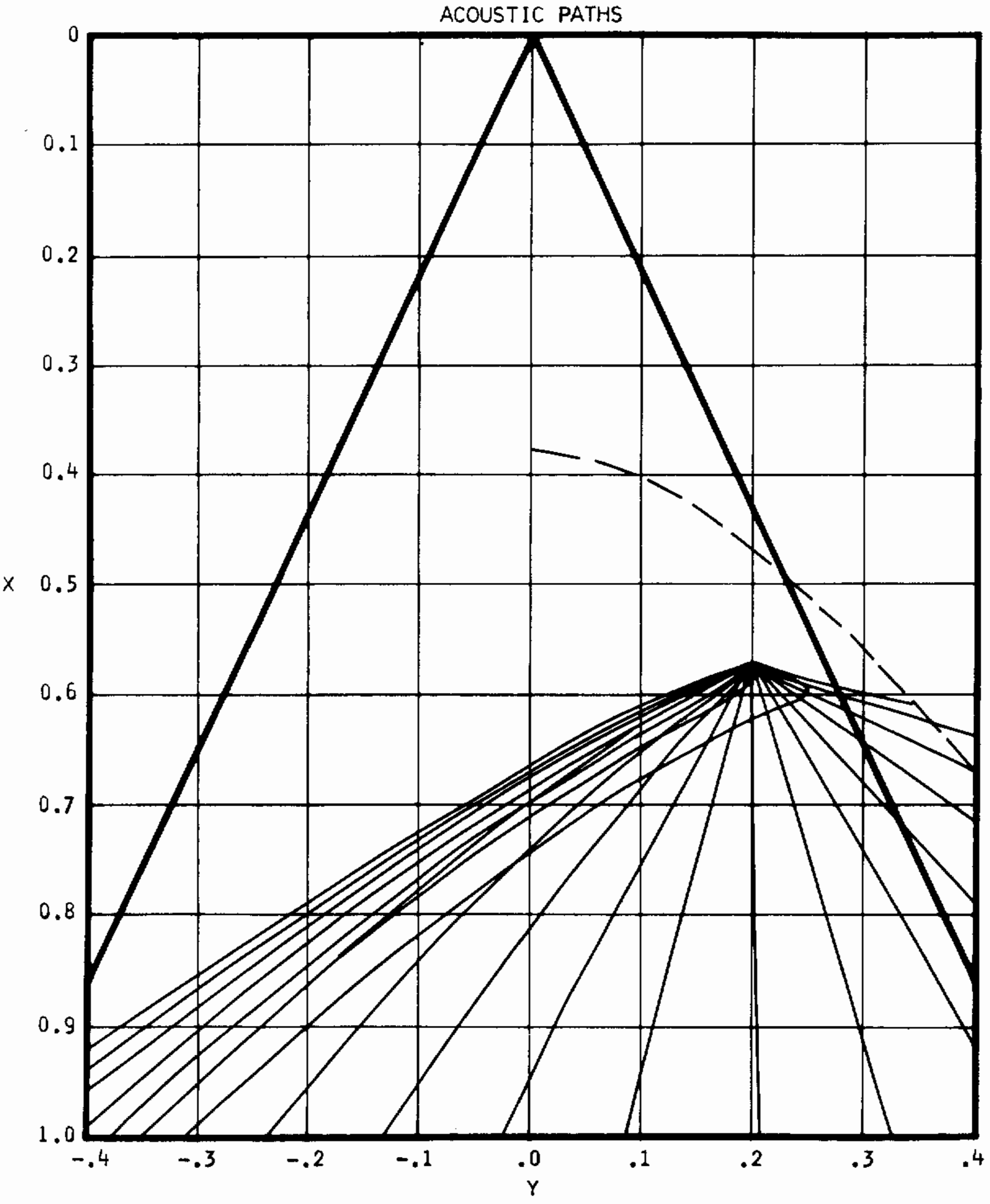


Figure 13. Ray Paths for a Source or Doublet at (0.57c, 0.20c)

CONCLUSIONS AND RECOMMENDATIONS

Two methods have been outlined in detail, and one of them has been completely mechanized for calculating the velocity potentials along acoustic ray paths emanating from any point in a non-uniform transonic flow field over a lifting surface. The one mechanized gives the ray path and velocity potential for the minimum time of travel from the source point to the field point.

To calculate pressures over the planform and generalized forces, it will be necessary to develop a procedure for calculating the velocity potential at an arbitrary point due to a sheet of sources, covering the wing surface, and the flow field in the plane of the wing out to a distance of several wing spans in the y-direction, or due to a sheet of doublets covering the wing surface. The latter is recommended for economy reasons.

The computer program in this report may be used to refine the doublet box method of Rodemich (3) in such a way as to include the (possibly very important) influence of wing thickness distribution on transonic airloads. A doublet box method similar to the one Rodemich developed (Reference 3) is recommended. The procedure is heuristically described in Appendix III. For each of a selected set of points in a sending box, the distribution of velocity potentials along ray lines throughout the zone of influence can be determined. An interpolation scheme will yield from these the velocity potentials at box centers and a numerical integration procedure will yield a velocity potential influence coefficient for each of the box centers. It will be necessary to solve a set of simultaneous equations to establish the strengths of doublets required to satisfy the tangential flow condition in the subsonic flow region. The order of the set will be equal to the number of box centers in the subsonic region on the wing. In the supersonic region the doublet strengths can be established sequentially. The use of doublets to solve unsteady supersonic flow problems has been outlined by Ashley in Reference 7.

It is recommended that this method be fully developed for the purpose of calculating generalized forces on wings in harmonic motion at transonic speeds. A computer program that would predict, with reasonable accuracy, the flutter characteristics and unsteady aerodynamic loads on a wing responding to externally applied forces, such as gusts, would fill an important gap in available technology.

REFERENCES

1. Landahl, M. Uniformly Valid Approximate Solution for an Oscillating Source in a Transonic Flow Field. Unpublished paper, (1965).

2. Landahl, M. Approximate Solution for an Oscillating Source in a Non Uniform Transonic Stream. NAA, SID 63-1194 (August 1963)

3. Rodemich, E. R. and L. V. Andrew Unsteady Aerodynamics for Advanced Configurations, Part II - A Transonic Box Method for Planar, Lifting Surfaces. FDL-TDR-64-152, (May 1965)

4. Spreiter, J. R., and A. Y. Alksne, Thin Airfoil Theory Based on Approximate Solution of the Transonic Flow Equation, NACA Report 1359.

5. Rubbert, Paul E., Analysis of Transonic Flow by Means of Parametric Differentiation AFOSR 64-1932, MIT Fluid Dynamics Research Laboratory Report No. 65-2 (1965).

6. Wind Tunnel Tests of Four Reflection Plane Mounted .024 Scale Models Simulating The YB-70 Wing to Investigate the Effect of Camber on the Chordwise Pressure Distribution at Mach Numbers from 0.4 to 3.0. NA-61-55, TWT-50, (Unpublished)

7. Ashley, Holt, Further Studies on Aerodynamic Influence Coefficients For Supersonic Wings - Use of Doublet Sheets. NAA, SID 66-373 (July 1964).

8. Andrew, L. V. and Stenton, T. E., Unsteady Aerodynamic Forces on a Thin Wing Oscillating in Transonic Flow. AIAA Paper No. 67-16, January 23-26, 1967.

APPENDIX I. Program Listings

```
$IBFTC MAIN    SOD                                                      SNIC0005
C     FORTRAN PROGRAM TO COMPUTE (AND PLOT) THE PATHS OF ACOUSTIC SIG - SNIC0010
C     NALS (AND TRANSMISSION TIMES) ON AN AIRFOIL IN A SONIC FLOW FIELD,SNIC0015
C     ACCOUNTING FOR VARIATION IN LOCAL MACH NUMBER.                    SNIC0020
C     CM = COEFFICIENTS OF MACH EQUATION. (SEE SUBROUTINE FMACH )       SNIC0025
C     PLX AND PLY ARE CONSTANTS DESCRIBING THE PLANFORM GEOMETRY.       SNIC0030
C     THE PROGRAM ALLOWS FOR EITHER X OR Y TO BE THE INDEPENDENT VARIA- SNIC0035
C     BLE, DEPENDING ON THE CURRENT VALUE OF X-PRIME, WHICH SETS IVAR.  SNIC0040
C            IF IVAR = 1,                      IF IVAR = 2,             SNIC0045
C      YY = CURRENT VALUE OF X          YY = CURRENT VALUE OF Y         SNIC0050
C      DYY= CURRENT VALUE OF DX         DYY= CURRENT VALUE OF DY        SNIC0055
C      XX(1)= CURRENT VALUE OF Y-PRIME  XX(1)= CURRENT VALUE OF X-PRIME SNIC0060
C      XX(2)  = CURRENT VALUE OF Y      XX(2)  = CURRENT VALUE OF X     SNIC0065
C      XX(3)  = CURRENT VALUE OF TIME   XX(3)  = CURRENT VALUE OF TIME  SNIC0070
C      XX(4)  = CURRENT VALUE OF R-BAR  XX(4)  = CURRENT VALUE OF R-BAR SNIC0075
C      DXX(1)= Y-DOUBLE PRIME           DXX(1)= X-DOUBLE PRIME          SNIC0080
C      DXX(2)= CURR. VALUE OF Y-PRIME   DXX(2)= CURR. VALUE OF X-PRIME  SNIC0085
C      DXX(3)= CURR. VALUE OF DT/DX     DXX(3)= CURR. VALUE OF DT/DY    SNIC0090
C      DXX(4)= CURRENT VALUE OF DR/DX   DXX(4)= CURRENT VALUE OF DR/DY  SNIC0095
C                                                                       SNIC0100
C      IVAR IS ORIGINALLY SET IN MAIN  PROGRAM, AND THEN RESET ON EACH  SNIC0105
C     PASS THROUGH SUBROUTINE CNTRL.                                    SNIC0110
C                                                                       SNIC0115
C     WORK = WORKING AREA FOR SUBROUTINE RKS3 .                         SNIC0120
C     IFVD = FALSE AND IBKP= TRUE FOR VARIABLE INTERVAL.                SNIC0125
C     IFVD = TRUE FOR FIXED INTERVAL.                                   SNIC0130
C                                                                       SNIC0135
C      SX = VECTOR CONTAINING COMPUTED  X- VALUES.                      SNIC0140
C      SXP = VECTOR CONTAINING COMPUTED X-PRIME VALUES.                 SNIC0145
C      SY CONTAINS COMPUTED Y VALUES                                    SNIC0150
C      SYP CONTAINS COMPUTED R-BAR VALUES                               SNIC0155
C      TIM CONTAINS TRANSMISSION TIMES.                                 SNIC0160
C      FM = CURRENT MACH NUMBER                                         SNIC0165
C      ISORS = -1 DEFINES A SUPERSONIC SOURCE, RECEDING PATH.           SNIC0170
C      ISROS = 0 DEFINES A SUPERSONIC SOURCE, ADVANCING PATH.           SNIC0175
C      ISORS = 1 DEFINES A SUBSONIC SOURCE.                             SNIC0180
C      IBR = 1 FOR RIGHT BRANCH, 2 FOR LEFT                             SNIC0185
C      NCNT IS THE COUNTER FOR THE VECTORS SX,SY,SXP,SYP,TIM.  WHEN NCNTSNIC0190
C      = NMAX, INTEGRATION STOPS, AND THE FLOW PASSES TO NEXT PATH      SNIC0195
C      ITRAP =1 INDICATES SIGNAL IS TRAPPED ON THE LOCAL MACH CONE.     SNIC0200
C      DZ IS INITIAL VALUE OF INCREMENT.                                SNIC0205
C      CINF = REMOTE SPEED OF SOUND IN ROOT CHORDS PER SECOND.          SNIC0210
C      FMINF= REMOTE MACH NUMBER                                        SNIC0215
C      POTE - THE POTE MATRIX CONTAINS THE VELOCITY POTENTIALS ALONG A  SNIC0220
C             RAY PATH, NORMALIZED ON B0 .                              SNIC0225
C      FREQ =ASSUMED FREQUENCIES IN RADIANS PER SECOND.                 SNIC0230
C                                                                       SNIC0235
      EXTERNAL DERIV, CNTRL                                             SNIC0240
      COMMON                                                            SNIC0245
     */WORK/ WORK(50)                                                   SNIC0250
```

```
     */XYZ/ SX(101),SXP(101),SY(101),SYP(101),AL(41),TIM(101)           SNIC0255
     */XDX/ XX(4),DXX(4),YY,DYY,DZ                                      SNIC0260
     */CM/ CM(6)                                                        SNIC0265
     */TABLE/ ATABL(4),RTABL(4)                                         SNIC0270
     */PL/  PLX(8),PLY(8)                                               SNIC0275
     */ICNT/ IVAR,NCNT,ISORS,IBR,ITRAP,NMAX                             SNIC0280
     */SOURCE/ XO(20),YO(20)                                            SNIC0285
     */EPS/ E1,E2,FM,YMAX                                               SNIC0290
     */NNN/ NSS,NLCS,NLLS                                               SNIC0295
     */ECM/ ECM                                                         SNIC0300
     */C4/ CM2(7)                                                       SNIC0305
C                                                                       SNIC0310
 1000 FORMAT(2L12 )                                                     SNIC0315
 1010 FORMAT(6E12.8)                                                    SNIC0320
 1020 FORMAT(6I12 )                                                     SNIC0325
    3 READ (5,1020) NSORCE,NLA,NPL,NMAX,NF                              SNIC0330
      READ (5,1000) IFVD,IBKP                                           SNIC0335
      READ (5,1010) (XO(I),YO(I),I=1,NSORCE)                            SNIC0340
      READ (5,1010)  (CM(I),I=1,6 )                                     SNIC0345
      READ (5,1010) DZ,E1,E2,YMAX                                       SNIC0350
      READ(5,1010)(ATABL(I),I=1,4),(RTABL(I),I=1,4 )                    SNIC0355
      READ (5,1010) (PLX(I),PLY(I),I=1,NPL )                            SNIC0360
      READ (5,1010) CINF,FMINF,TAU,TSAA                                 SNIC0365
C                                                                       SNIC0370
C     TAU=MAX. (T/C), TSAA = TANGENT OF SEMI-APEX ANGLE                 SNIC0375
      DIMENSION FREQ(10), POTE(101,2,10 )                               SNIC0380
      READ (5,1010) (FREQ(I),I=1,NF )                                   SNIC0385
C                                                                       SNIC0390
      DIMENSION  XSO(40),YSO(40)                                        SNIC0395
      ECM = CINF*SQRT(5.0+FMINF**2)                                     SNIC0400
      ECM=1.0/ECM                                                       SNIC0405
      CALL SONK(40,NXY,YMAX,YSO,XSO,IER )                               SNIC0410
 2000 FORMAT(49H0 ERROR IN SUBROUTINE SONIC. CHECK MACH CONSTANTS )     SNIC0415
      GO TO (1,2), IER                                                  SNIC0420
    2 WRITE (6,2000)                                                    SNIC0425
    1 CONTINUE                                                          SNIC0430
      NVAR =4                                                           SNIC0435
C     NVAR IS THE NUMBER OF VARIABLES                                   SNIC0440
      CM2(1) =0.3                                                       SNIC0445
      CM2(2) =0.7                                                       SNIC0450
      CM2(3) =ATAN(1./TSAA)                                             SNIC0455
      CM2(4) =TAU                                                       SNIC0460
      CM2(5) =1.18*TSAA                                                 SNIC0465
      CM2(6) =.04                                                       SNIC0470
      CM2(7)=FMINF                                                      SNIC0472
C     DEVELOP LAMDAS                                                    SNIC0475
      NL=2*(NLA/2)                                                      SNIC0480
C     THERE WILL ACTUALLY BE NL VALUES. IF NLA IS EVEN,NL=NLA. BUT NL=  SNIC0485
C     NLA - 1  IF NLA IS ODD.                                           SNIC0490
      NL1=NL-1                                                          SNIC0495
```

```
      NL2 =NL/2                                                         SNIC0500
      XN= NL2*(NL2+1)                                                   SNIC0505
      DG= 6.28318/XN                                                    SNIC0510
      AL(1)=0.                                                          SNIC0515
      DO 10 J=3,NL1,2                                                   SNIC0520
      XJ = (J-1)/2                                                      SNIC0525
      J1 = J-1                                                          SNIC0530
      AL(J)=AL(J-2)+XJ*DG                                               SNIC0535
   10 AL(J1)=-AL(J)                                                     SNIC0540
      AL(NL)= 3.14159                                                   SNIC0545
C     SET UP GRID LIMITS                                                SNIC0550
      XU=0.                                                             SNIC0555
      XL=1.                                                             SNIC0560
      YL=-YMAX                                                          SNIC0565
      YR = YMAX                                                         SNIC0570
      CALL LIMITI(YL,YR,XL,XU)                                          SNIC0575
      DO 600 NS=1,NSORCE                                                SNIC0580
      NSS = NS                                                          SNIC0585
      CALL GRAPH(1,42,-NPL,PLY,PLX,2H Y,2H X,15H ACOUSTIC PATHS )       SNIC0590
      XOF=XO(NS)                                                        SNIC0595
      YOF=YO(NS)                                                        SNIC0600
      CALL GRAPH(0,42,-NXY,YSO,XSO )                                    SNIC0605
      NLLS = NL                                                         SNIC0610
      DO 500 NLC=1,NL                                                   SNIC0615
      NLCS = NLC                                                        SNIC0620
      ITRAP = 0                                                         SNIC0625
      CALL FMACH (XOF,YOF,FM,FMX,FMY )                                  SNIC0630
      TEST1 =FM - COS(AL(NLC))                                          SNIC0635
      TEST2 = SIN(AL(NLC))                                              SNIC0640
      IF(NLC .NE. NL ) GO TO 11                                         SNIC0645
      IF (YOF .GT. 0. ) GO TO 11                                        SNIC0650
      TEST2 = -TEST2                                                    SNIC0655
   11 IF (NLC-1) 14,12,14                                               SNIC0660
   12 IVAR=1                                                            SNIC0665
      GO TO 30                                                          SNIC0670
   14 IF(NLC-NL) 18,12,18                                               SNIC0675
   18 IF(TEST1) 22,20,22                                                SNIC0680
   20 IVAR=2                                                            SNIC0685
      GO TO 30                                                          SNIC0690
   22 TEST = TEST1/TEST2                                                SNIC0695
      ART = ABS(TEST)                                                   SNIC0700
      IF(ART-1.0) 20,12,12                                              SNIC0705
   30 CONTINUE                                                          SNIC0710
C     SET IBR                                                           SNIC0715
      FL=AL(NLC)                                                        SNIC0720
      IF(NLC-1) 32,31,32                                                SNIC0725
   31 IF(YOF) 41,41,42                                                  SNIC0730
   32 IF(NLC-NL) 36,34,36                                               SNIC0735
   34 IF(YOF) 42,41,41                                                  SNIC0740
   36 IF(FL) 42,42,41                                                   SNIC0745
```

```
  41 IBR=1                                                              SNIC0750
     GO TO 50                                                           SNIC0755
  42 IBR=2                                                              SNIC0760
  50 CONTINUE                                                           SNIC0765
C    SET ISORS                                                          SNIC0770
     CSL =COS(FL)                                                       SNIC0775
     RM=1.0/FM                                                          SNIC0780
     IF(FM-1.0) 60,51,51                                                SNIC0785
  51 IF((FM-1.0)-E2) 52,52,58                                           SNIC0790
  52 GO TO (53,54),IVAR                                                 SNIC0795
  53 YPR =TEST2/TEST1                                                   SNIC0800
     TST= 1.0-YPR**2*(FM**2 - 1.0)                                      SNIC0805
  55 IF(TST-E1) 500,500,58                                              SNIC0810
  54 XPR = TEST1/TEST2                                                  SNIC0815
     TST= XPR**2-(FM**2-1.0)                                            SNIC0820
     GO TO 55                                                           SNIC0825
  58 IF(CSL-RM) 68,68,64                                                SNIC0830
  60 ISORS=1                                                            SNIC0835
     GO TO 70                                                           SNIC0840
  64 ISORS= -1                                                          SNIC0845
     GO TO 70                                                           SNIC0850
  68 ISORS = 0                                                          SNIC0855
C                                                                       SNIC0860
  70 NCNT=1                                                             SNIC0865
     GO TO (80,90),IVAR                                                 SNIC0870
C    IF IVAR=1,X IS THE INDEPENDENT VARIABLE.                           SNIC0875
C                                                                       SNIC0880
  80 YY = XOF                                                           SNIC0885
     IF(TEST1) 81,81,82                                                 SNIC0890
  81 DYY=-DZ                                                            SNIC0895
     GO TO 83                                                           SNIC0900
  82 DYY = DZ                                                           SNIC0905
  83 XX(1) = TEST2/TEST1                                                SNIC0910
     XX(2)=YOF                                                          SNIC0915
     XX(3) = 0.                                                         SNIC0920
     XX(4) = 0.                                                         SNIC0925
     GO TO 100                                                          SNIC0930
C    IF IVAR=2,Y IS THE INDEPENDENT VARIABLE.                           SNIC0935
  90 YY = YOF                                                           SNIC0940
     GO TO (91,92),IBR                                                  SNIC0945
  91 DYY = DZ                                                           SNIC0950
     GO TO 93                                                           SNIC0955
  92 DYY = -DZ                                                          SNIC0960
  93 XX(1) = TEST1/TEST2                                                SNIC0965
     XX(2)= XOF                                                         SNIC0970
     XX(3) = 0.                                                         SNIC0975
     XX(4) = 0.                                                         SNIC0980
 100 CALL RKS3 (DERIV,CNTRL,XX,DXX,ATABL,RTABL,WORK,YY,DYY,NVAR,IFVD,IBSNIC0985
    1KP,NTRY,IERR  )                                                    SNIC0990
1070 FORMAT(1H1,30X,43H IVAR     NCNT     ISORS    IBR     ITRAP    NLCS = )SNIC0995
```

```
1080 FORMAT(1H0,27X, 6I7 )                                               SNIC1000
     WRITE (6,1070)                                                      SNIC1005
     WRITE (6,1080) IVAR,NCNT,ISORS,IBR,ITRAP,NLCS                       SNIC1010
C                                                                        SNIC1015
1060 FORMAT(22H ERROR IN RKS3, IERR = I4   )                             SNIC1020
     IF(IERR) 103,140,103                                                SNIC1025
 103 WRITE(6,1060) IERR                                                  SNIC1030
     GO TO 500                                                           SNIC1035
1050 FORMAT(1H-,42X,4HXO = E16.8/ 43X,4HYO = E16.8/ 43X,10HMACH NO. = ESNIC1040
    116.8// 29X,31H ACOUSTIC RAY PATH FOR LAMBDA = E16.8///17X,1HX,17X,SNIC1045
    11HY,14X,7HX-PRIME,11X,7HR-BAR   ,12X,4HTIME// )                     SNIC1050
1040 FORMAT(1H 7X,5E18.8)                                                SNIC1055
 140 WRITE(6,1050) XO(NS),YO(NS),FM,FL                                   SNIC1060
     WRITE(6,1040) (SX(I),SY(I),SXP(I),SYP(I),TIM(I),I=1,NCNT )          SNIC1065
C                                                                        SNIC1070
     CALL GRAPH (0,NLC,-NCNT,SY,SX )                                     SNIC1075
     CALL POT (NF,FREQ,POTE )                                            SNIC1080
C                                                                        SNIC1085
1100 FORMAT(1H1,25X,54H VELOCITY POTENTIALS ALONG A RAY PATH FOR A SOURSNIC1090
    1CE AT  )                                                            SNIC1095
1110 FORMAT(1H-,42X,4HXO = E16.8/43X, 4HYO = E16.8/ 43X, 8HLAMBDA = E16SNIC1100
    1.8 //39X,30HALTERNATING REAL AND IMAGINARY  )                       SNIC1105
1120 FORMAT (1H-,6X,7HOMEGA =E16.8// )                                   SNIC1110
1090 FORMAT(1H 6X,6E16.6)                                                SNIC1115
C                                                                        SNIC1120
     DO 300 N=1,NF                                                       SNIC1125
     IF(N .NE. 1 ) GO TO 200                                             SNIC1130
     WRITE (6,1100)                                                      SNIC1135
     WRITE (6,1110) XO(NS),YO(NS),FL                                     SNIC1140
 200 WRITE (6,1120 ) FREQ(N)                                             SNIC1145
     WRITE(6,1090)(( POTE(I,K,N),K=1,2 ),I=1,NCNT )                      SNIC1150
 300 CONTINUE                                                            SNIC1155
 500 CONTINUE                                                            SNIC1160
 600 CONTINUE                                                            SNIC1165
     GO TO 3                                                             SNIC1170
     END                                                                 SNIC1175
```

```
$IBFTC DERI     SDD                                                     SNIC1180
      SUBROUTINE DERIV                                                  SNIC1185
      COMMON                                                            SNIC1190
     */XDX/ XX(4),DXX(4),YY,DYY,DZ                                      SNIC1195
     */CM/ CM(6)                                                        SNIC1200
     */ICNT/ IVAR,NCNT,ISORS,IBR,ITRAP,NMAX                             SNIC1205
     */EPS/ E1,E2,FM,YMAX                                               SNIC1210
     */NNN/ NSS,NLCS,NLLS                                               SNIC1215
     */ECM/ ECM                                                         SNIC1220
C                                                                       SNIC1225
      GO TO(10,50),IVAR                                                 SNIC1230
C     X IS THE INDEPENDENT VARIABLE                                     SNIC1235
   10 CALL FMACH (YY,XX(2),FM,FMX,FMY )                                 SNIC1240
      R=XX(1)                                                           SNIC1245
      DXX(2)=R                                                          SNIC1250
      B =FM*FM -1.0                                                     SNIC1255
      TSI = 1.0-R*R*B                                                   SNIC1260
      A =FM*FM + 5.0                                                    SNIC1265
      SA = SQRT(A)                                                      SNIC1270
      IF(B) 103,103,101                                                 SNIC1275
  101 BETA = SQRT(B)                                                    SNIC1280
      IF(ITRAP .EQ. 1) GO TO 104                                        SNIC1285
      IF(ISORS .EQ. 1) GO TO 103                                        SNIC1290
      IF(TSI .GT. E1) GO TO 103                                         SNIC1295
      ITRAP = 1                                                         SNIC1300
      GO TO  104                                                        SNIC1305
  103 IF(TSI .GE. 0. ) GO TO 215                                        SNIC1310
      ITRAP = 2                                                         SNIC1315
      TSI = 0.                                                          SNIC1320
  215 RAD = SQRT (TSI)                                                  SNIC1325
      DXX(4) = RAD                                                      SNIC1330
      RAB= 1.0/(A*B)                                                    SNIC1335
      TM1=FM*(FM**2 + 11.0)/B                                           SNIC1340
      TM2= 2.0*FM*(FM**2+8.0)*R**2                                      SNIC1345
      TM3=((RAD**3)/B)*(7.0*FM**2+5.0)                                  SNIC1350
      TM4=(FM/A)*R*(6.0*R**2 +1.0)                                      SNIC1355
      GO TO 105                                                         SNIC1360
  104 RBB = 1.0/(B**2)                                                  SNIC1365
      DXX(4) = 0.                                                       SNIC1370
  105 IF(ISORS) 11,15,18                                                SNIC1375
   11 GO TO (12,13),IBR                                                 SNIC1380
   12 IF(ITRAP) 91,91,7                                                 SNIC1385
   13 IF(ITRAP) 92,92,8                                                 SNIC1390
   15 GO TO(16,17),IBR                                                  SNIC1395
   16 IF(ITRAP) 92,92,7                                                 SNIC1400
   17 IF(ITRAP) 91,91,8                                                 SNIC1405
   18 GO TO (92,91),IBR                                                 SNIC1410
   91 IF(R) 4,3,3                                                       SNIC1415
   92 IF(R) 3,3,4                                                       SNIC1420
C     Y IS THE INDEPENDENT VARIABLE                                     SNIC1425
```

```
   50 CALL FMACH (XX(2),YY,FM,FMX,FMY )
      R = XX(1)
      DXX(2) = R
      B = FM*FM-1.0
      TSI = R*R-B
C
      A= 5.0+FM*FM
      SA = SQRT(A)
      IF(B .LT. 0. )  GO TO  108
  106 BETA = SQRT(B)
      IF(ITRAP .EQ. 1) GO TO 109
      IF(ISORS .EQ. 1) GO TO 108
      IF(TSI .GT. E1) GO TO 108
      ITRAP = 1
      GO TO 109
  108 IF(TSI .GE. 0.) GO TO 107
      TSI=0.
      ITRAP = 2
  107 RAD = SQRT(TSI)
      DXX(4) = RAD
      RAB = 1.0/(A*B)
      TM1=(FM/B)*(FM**2+11.0)*R**3
      TM2= 2.0*FM*(FM**2+8.0)*R
      TM3=(RAD**3/B)*(7.0*FM**2+5.0)
      TM4 =(FM/A)*(R**2+6.0)
      GO TO 110
  109 DXX(4) = 0.
  110 IF(ISORS) 52,60,68
   52 GO TO (54,56),IBR
   54 IF(ITRAP ) 1,1,5
   56 IF(ITRAP ) 2,2,6
   60 GO TO (62,64),IBR
   62 IF(ITRAP ) 2,2,5
   64 IF(ITRAP ) 1,1,6
   68 GO TO (2,1),IBR
C     FORMULAS FOR THE SECOND DERIVS FOLLOW
C
    1 IF(ABS(B) .LE. 1.E-03) GO TO 220
      DXX(1)=RAB*(-TM1 + TM2 - TM3)*FMY + TM4 *FMX
      DXX(3)=(SA*ECM/B)*(FM*XX(1)+RAD)
      GO TO 100
    2 IF(ABS(B) .GT. 1.E-03 ) GO TO 209
  220 DXX(1)=(.5/A)*(2.*R**3+R+9./R)*FMY + (FM/A)*(R**2+6.)*FMX
      DXX(3)=(1.22475*ECM)*(R+(1./R))
      GO TO 100
  209 DXX(1)= RAB*(-TM1+TM2+TM3)*FMY+TM4 * FMX
      DXX(3)=(SA*ECM/B)*(FM*XX(1)-RAD)
      GO TO 100
    3 IF(NLCS .EQ. NLLS) GO TO 4
      DXX(1)=RAB*(TM1-TM2+TM3)*FMY -TM4* FMX
```

```
      DXX(3)=(SA*ECM/B)*(FM + RAD)
      GO TO 100
    4 IF(ABS(B) .GT. 1.E-03 ) GO TO 205
  204 DXX(1)=-(.5/A)*(9.*R**4+R**2+2.)*FMY - (R/A)*(6.*R**2+1.)*FMX
      DXX(3)=(1.22475*ECM)*(1.+R*R)
      GO TO 100
  205 DXX(1)= RAB*(TM1-TM2-TM3)*FMY-TM4* FMX
      DXX(3)=(SA*ECM/B)*(FM - RAD)
      GO TO 100
    5 DXX(1)=FM*((FMY/BETA) +FMX)
      DXX(3)=(SA*ECM/B)* FM * XX(1)
      GO TO 100
    6 DXX(1)=FM*((-FMY/BETA) +FMX )
      DXX(3)= (SA*ECM/B)*FM*XX(1)
      GO TO 100
    7 DXX(1)= -(FM*RBB)*(FMY+BETA*FMX )
      DXX(3)= (SA*ECM/B)*FM
      GO TO 100
    8 DXX(1)=FM*RBB*(-FMY+BETA*FMX)
      DXX(3)= (SA*ECM/B)*FM
  100 IF(DYY .LT. 0. )  GO TO 31
      DXX(3) = ABS(DXX(3))
      GO TO 32
   31 DXX(3) = - 1.0*(ABS(DXX(3)))
      DXX(4) = -1.0*(ABS(DXX(4)))
   32 RETURN
      END
```

```
$IBFTC CONT     SDD                                                     SNIC1815
      SUBROUTINE CNTRL(NTRY)                                            SNIC1820
      COMMON                                                            SNIC1825
     */XYZ/ SX(101),SXP(101),SY(101),SYP(101),AL(41),TIM(101)           SNIC1830
     */XDX/ XX(4),DXX(4),YY,DYY,DZ                                      SNIC1835
     */CM/ CM(6)                                                        SNIC1840
     */ICNT/ IVAR,NCNT,ISORS,IBR,ITRAP,NMAX                             SNIC1845
     */EPS/ E1,E2,FM,YMAX                                               SNIC1850
     */NNN/ NSS,NLCS,NLLS                                               SNIC1855
      IF(NCNT .NE. 1 ) GO TO 6                                          SNIC1860
      NCO = 1                                                           SNIC1865
      IF (NR .EQ. 1) GO TO 6                                            SNIC1870
      NR = 1                                                            SNIC1875
      IF(ABS(DXX(1)*DYY ) .LE. .25) GO TO 6                             SNIC1880
    4 DYY = .5*DYY                                                      SNIC1885
      IF(ABS(DXX(1)*DYY ) .LE. .25) GO TO 7                             SNIC1890
      GO TO 4                                                           SNIC1895
    7 NTRY = 4                                                          SNIC1900
      RETURN                                                            SNIC1905
    6 IF(ABS(XX(1)).LT. 1.0 ) GO TO 20                                  SNIC1910
    1 NTRY =4                                                           SNIC1915
      GO TO (2,3),IVAR                                                  SNIC1920
    2 IVAR=2                                                            SNIC1925
      GO TO 5                                                           SNIC1930
    3 IVAR=1                                                            SNIC1935
C       SWITCH VARIABLES,SET NEW INITIAL CONDITIONS                     SNIC1940
    5 SAV =YY                                                           SNIC1945
      DYY = DYY*XX(1)                                                   SNIC1950
   10 YY = XX(2)                                                        SNIC1955
      XX(1)=1.0/XX(1)                                                   SNIC1960
      XX(2)=SAV                                                         SNIC1965
      RETURN                                                            SNIC1970
   20 GO TO (25,35),IVAR                                                SNIC1975
C     STORE CURRENT VALUES WHERE X IS INDEPENDENT VARIABLE.             SNIC1980
   25 SX(NCNT) = YY                                                     SNIC1985
C     CHANGE IBR WHEN Y-PRIM PASSES THROUGH ZERO                        SNIC1990
      IF(ABS(XX(1)) .GT. 1.0 E-02) GO TO 15                             SNIC1995
      IF((DXX(1)*DYY*XX(1)) .GE. 0.0) GO TO 15                          SNIC2000
      IF(NCO .EQ. 2) GO TO 15                                           SNIC2005
      NCO = 2                                                           SNIC2010
      XX(1)=-XX(1)                                                      SNIC2015
      NTRY = 4                                                          SNIC2020
      GO TO (11,12), IBR                                                SNIC2025
   11 IBR =2                                                            SNIC2030
      GO TO 19                                                          SNIC2035
   12 IBR = 1                                                           SNIC2040
      GO TO 19                                                          SNIC2045
   15 IF(NCO .NE. 2 ) GO TO 19                                          SNIC2050
      IF(ABS(XX(1)) .LT. 1.0 E-01) GO TO 19                             SNIC2055
      NCO = 1                                                           SNIC2060
```

```
   19 IF(XX(1)  .NE.  0.0 )  GO  TO  27                                      SNIC2065
   26 SXP(NCNT)=  UNDEF                                                      SNIC2070
      GO  TO  28                                                             SNIC2075
   27 SXP(NCNT)=  1.0/XX(1)                                                  SNIC2080
   28 SY(NCNT)  =  XX(2)                                                     SNIC2085
      SYP(NCNT)  =XX(4)                                                      SNIC2090
      TIM(NCNT)  =  XX(3)                                                    SNIC2095
      GO  TO  50                                                             SNIC2100
   35 SX(NCNT)=XX(2)                                                         SNIC2105
      TIM(NCNT)  =  XX(3)                                                    SNIC2110
      SXP(NCNT)=XX(1)                                                        SNIC2115
      SY(NCNT)=YY                                                            SNIC2120
      SYP(NCNT)  =  XX(4)                                                    SNIC2125
   50 CONTINUE                                                               SNIC2130
C     NOW  TEST  FOR  EXIT  CONDITIONS                                       SNIC2135
      IF(ITRAP  .NE.  2)  GO  TO  51                                         SNIC2140
      ITRAP  =  0                                                            SNIC2145
      NCNT  =  NCNT  -  1                                                    SNIC2150
      GO  TO  100                                                            SNIC2155
   51 IF(ITRAP)  60,60,52                                                    SNIC2160
   52 TEST  =FM-1.0                                                          SNIC2165
      IF(TEST)  100,100,53                                                   SNIC2170
   53 IF  (TEST-E2 )  100,100,60                                             SNIC2175
   60 IF  (SX(NCNT))  100,70,70                                              SNIC2180
   70 IF  (SX(NCNT)-1.0)  80,100,100                                         SNIC2185
   80 AY  =ABS(SY(NCNT))                                                     SNIC2190
      IF(AY-YMAX)  105,100,100                                               SNIC2195
  105 IF(NCNT-NMAX)  110,100,100                                             SNIC2200
  100 NTRY  =  2                                                             SNIC2205
      NR  =  0                                                               SNIC2210
      RETURN                                                                 SNIC2215
  110 NCNT  =  NCNT  +  1                                                    SNIC2220
      RETURN                                                                 SNIC2225
      END                                                                    SNIC2230
```

```
$IBFTC MACH                                                             SNIC2235
C     MASTER SUBR., M, MX, MY                                           SNIC2245
      SUBROUTINE FMACH(FX,FY,FMS,FMXS,FMYS)                             SNIC2240
      COMMON                                                            SNIC2250
     */C4/ CM2(7)                                                       SNIC2255
      EQUIVALENCE (A,CM2(1)),(B,CM2(2)), (AL,CM2(3)), (TAU,CM2(4)),(AK, SNIC2260
     * CM2(5)), (R1,CM2(6)) ,(FMINF,CM2(7))                             SNIC2265
      AY=ABS(FY)                                                        SNIC2275
      AYY = ABS(AK*FX)                                                  SNIC2280
      IF(AY .LE. AYY) GO TO 200                                         SNIC2285
      SK = 1./ (SQRT(1.+AK*AK))                                         SNIC2290
      T =(AY-AYY)*SK                                                    SNIC2295
  100 CALL FMAC1 (FX,AYY,FMS,FMXS,FMYS)                                 SNIC2300
      CALL FMAC2 (FX,AY,A,B,AL,TAU,D1FM,D1MX,D1MY )                     SNIC2305
      CALL FMAC2 (FX,AYY,A,B,AL,TAU,D2FM,D2MX,D2MY)                     SNIC2310
C                                                                       SNIC2315
      FMS = FMS -0.6*FMINF*(D1FM-D2FM)                                  SNIC2320
      FMXS= FMXS+FMYS*AK-0.6*FMINF*(D1MX-D2MX-AK*D2MY)                  SNIC2325
      FMYS = -0.6*FMINF*D1MY*(AY/FY)                                    SNIC2330
      IF (T .GE. R1) GO TO 300                                          SNIC2335
  120 CALL FMAC1 (FX,FY,SM,SMX,SMY )                                    SNIC2340
      ARG =1.57079*T/R1                                                 SNIC2345
      SI = SIN(ARG)                                                     SNIC2350
      SMO = SI*SI                                                       SNIC2355
      FMS=(FMS-SM)*SMO + SM                                             SNIC2360
      FMXS=(FMXS-SMX)*SMO + SMX                                         SNIC2365
      FMYS=(FMYS-SMY)*SMO +SMY                                          SNIC2370
      GO TO 300                                                         SNIC2375
  200 CALL FMAC1(FX,FY,FMS,FMXS,FMYS)                                   SNIC2380
  300 CONTINUE                                                          SNIC2385
      RETURN                                                            SNIC2390
      END                                                               SNIC2395
```

```
$IBFTC MAC2     SDD
      SUBROUTINE FMAC2(X,Y,A,B,AL,TAU,DELCP,DDXCP,DDYCP)
C
C     SUBROUTINE COMPUTES DELTA CP
C
      CS= COS(AL)
      CS1=1./(SQRT(1.+((1.-A)**2)*(CS**2)))
      CS2=1./(SQRT(1.+((1.-B)**2)*(CS**2)))
      TA = SIN(AL)/CS
      TA1=(1.-A)*TA
      TA2=(1.-B)*TA
      EPS=TAU/(2.*3.1415927*A*CS)
      EPS1= EPS*CS/CS1
      EPS2 = EPS*A*CS/((1.-B)*CS2)
      EDS = 1.0 - EPS
      EDS1 = EPS1 + 1.0
      EDS2 = EPS2 + 1.0
      S = ABS(X/TA)
      S1=(X-A)/TA1
      S2=(X-B)/TA2
      Q1=ABS(Y-S)
      Q2=ABS(Y+S)
      Q3=ABS(Y-S1)
      Q4=ABS(Y+S1)
      Q5 =ABS(Y-S2)
      Q6 =ABS(Y+S2)
      FAC =2.*CS/TA
      FAC1=2.*CS1/TA1
      FAC2=2.*CS2/TA2
      DEL =-FAC*(Q1**EPS+Q2**EPS-2.*S**EPS)
      DDX=-FAC*(-1./(Q1**EDS)+1./(Q2**EDS)-2./(S**EDS)) * EPS /TA
      DDY =-FAC*(1./(Q1**EDS)+1./(Q2**EDS))             * EPS
      IF (S1) 10,10,5
   10 DELCP= DEL
      DDXCP= DDX
      DDYCP= DDY
      GO TO 50
    5 DEL1=-FAC1*(1./(Q3**EPS1)+1./(Q4**EPS1)-2./(S1**EPS1))
      DDX1=FAC1*(-1./(Q3**EDS1)+1./(Q4**EDS1)-2./(S1**EDS1)) * EPS1 /TA1
      DDY1= FAC1*(1./(Q3**EDS1)+1./(Q4**EDS1))              * EPS1
      IF (S2) 20,20,30
   20 DELCP= DEL+ DEL1
      DDXCP= DDX+ DDX1
      DDYCP= DDY+ DDY1
      GO TO 50
   30 DEL2=-FAC2*(1./(Q5**EPS2)+1./(Q6**EPS2)-2./(S2**EPS2))
      DDX2=FAC2*(-1./(Q5**EDS2)+1./(Q6**EDS2)-2./(S1**EDS2)) * EPS2 /TA2
      DDY2= FAC2*(1./(Q5**EDS2)+1./(Q6**EDS2))              * EPS2
      DELCP = DEL+ DEL1 + DEL2
      DDXCP= DDX+ DDX1 + DDX2
```

```
      DDYCP= DDY+ DDY1  + DDY2                                        SNIC2650
   50 RETURN                                                          SNIC2655
      END                                                             SNIC2660
```

```
$IBFTC MAC1    SDD                                                      SNIC2665
      SUBROUTINE FMAC1(FX,FY,FMS,FMXS,FMYS)                             SNIC2670
C                                                                       SNIC2675
C     SUBROUTINE COMPUTES MACH NO, MX, MY.                              SNIC2680
C          FX = X                           FY = Y                      SNIC2685
C          FMS = MACH NO.                   FMXS= PARTIAL M W/RESP TO X SNIC2690
C          FMYS= PARTIAL M W/RESP TO Y                                  SNIC2695
C       EQ. FOR MACH IS M=CM(2)+EXP(-CM(1)*Y**2/X)*(CM(3)*X+CM(4)*X**2+ SNIC2700
C       CM(5)*Y**2+CM(6)*Y**4 )                                         SNIC2705
      COMMON                                                            SNIC2710
     */CM/ CM(6)                                                        SNIC2715
C                                                                       SNIC2720
      EQUIVALENCE                                                       SNIC2725
     1     ( C ,CM(1)),   ( FMO,CM(2)),     (A1 ,CM(3)),   ( A2 ,CM(4)),SNIC2730
     2     (A3 ,CM(5)),   ( A4 ,CM(6))                                  SNIC2735
      IF(FX .EQ. 0.) GO TO 5                                            SNIC2740
      ARG1 =(-C*FY**2)/FX                                               SNIC2745
      ARG1 = - ABS(ARG1)                                                SNIC2750
      IF (ABS(ARG1) .GE. 50.) GO TO 5                                   SNIC2755
      ARG2 = A1*FX+A2*FX**2 +A3*FY**2 +A4*FY**4                         SNIC2760
C                                                                       SNIC2765
      ARG3 = A1+ 2. * A2*FX                                             SNIC2770
      ARG4 = 2.*A3*FY +4.*A4*FY**3                                      SNIC2775
      EX = EXP(ARG1)                                                    SNIC2780
      GO TO 10                                                          SNIC2785
    5 FMS = FMO                                                         SNIC2790
      FMXS = 0.                                                         SNIC2795
      FMYS = 0.                                                         SNIC2800
      RETURN                                                            SNIC2805
   10 FMS = FMO +EX* ARG2                                               SNIC2810
      FMXS= EX*((-ARG1/FX)* ARG2 +ARG3)                                 SNIC2815
      PAUL= -2.*C*FY/FX                                                 SNIC2820
      FMYS= EX*( PAUL*ARG2 + ARG4)                                      SNIC2825
      RETURN                                                            SNIC2830
      END                                                               SNIC2835
```

```
$IBFTC SONI     SDD                                                     SNIC2840
      SUBROUTINE SONK(NM,NCR,YM,FY,FX,IER  )                            SNIC2845
C                                                                       SNIC2850
C     NM = MAX NO OF X,Y ALLOWED. MUST EQUAL DIMENSION OF X,Y, IN MAIN  SNIC2855
C     NCR = NO OF X,Y ACTUALLY COMPUTED                                 SNIC2860
C     YM = MAX. ALLOWABLE VALUE OF Y                                    SNIC2865
C     FX = X-VALUES                                                     SNIC2870
C     FY = Y-VALUES                                                     SNIC2875
C     IER = 1 IS NORMAL RETURN                                          SNIC2880
C     IER = 2 INDICATES AN ERROR                                        SNIC2885
C     CM= MACH CONSTANTS IN THE EQUATION  M=EXP(-CM(1)*Y**2/X)*(CM(3)*X SNIC2890
C     +CM(4)*X*X+CM(5)*Y*Y+CM(6)*Y**4)  +CM(2).                         SNIC2895
C     THE SUBROUTINE COMPUTES A SET OF X AND Y VALUES ON THE WING WHERE SNIC2900
C     M= 1                                                              SNIC2905
C                                                                       SNIC2910
C                                                                       SNIC2915
      COMMON                                                            SNIC2920
     */CM/ CM(6)                                                        SNIC2925
      DIMENSION FX(1),FY(1)                                             SNIC2930
      IER =1                                                            SNIC2935
      C=CM(1)                                                           SNIC2940
      FMO=CM(2)                                                         SNIC2945
      A1 =CM(3)                                                         SNIC2950
      A2 =CM(4)                                                         SNIC2955
      A3 =CM(5)                                                         SNIC2960
      A4 =CM(6)                                                         SNIC2965
C     FIRST COMPUTE X WHEN Y=0                                          SNIC2970
      ARG = A1**2 -4.*A2*(FMO-1.)                                       SNIC2975
      IF(ARG .GE. 0.0) GO TO 2                                          SNIC2980
    1 IER = 2                                                           SNIC2985
      RETURN                                                            SNIC2990
    2 FX(1) =(.5/A2)*(-A1+SQRT(ARG))                                    SNIC2995
      FY(1) = 0.                                                        SNIC3000
      IF(FX(1) .LT. 0.0) GO TO 1                                        SNIC3005
      IF(FX(1) .LT. 1.0) GO TO 4                                        SNIC3010
      FX(1)=(.5/A2)*(-A1-SQRT(ARG))                                     SNIC3015
      IF(FX(1) .LT. 0.0) GO TO 1                                        SNIC3020
      IF(FX(1) .GE. 1.0) GO TO 1                                        SNIC3025
    4 NCR = 2                                                           SNIC3030
   10 NC1= NCR - 1                                                      SNIC3035
      FX(NCR)= FX(NC1)+.01                                              SNIC3040
      X= FX(NCR)                                                        SNIC3045
      R = C/X                                                           SNIC3050
      B = X*(A1+A2*X)                                                   SNIC3055
      TO = FY(NC1)**2                                                   SNIC3060
      TM1 = A3-R*B                                                      SNIC3065
      TM2 = 2.*A4-R*A3                                                  SNIC3070
      TM3 = R*A4                                                        SNIC3075
      TM4 = 2.*A4+R*(R*B-2.*A3)                                         SNIC3080
      TM5 = R*(R*A3-4.*A4)                                              SNIC3085
```

```
      TM6 = R*R*A4                                                      SNIC3090
      IMAX =1                                                           SNIC3095
   12 ET =EXP(-R*TO)                                                    SNIC3100
      FT= ET*(B+A3*TO+A4*TO*TO)+FMO-1.                                  SNIC3105
      FPT =ET*(TM1+TM2*TO-TM3*TO**2)                                    SNIC3110
      FPPT =ET*(TM4+TM5*TO+TM6*TO**2)                                   SNIC3115
      HO = -FT/FPT                                                      SNIC3120
      IF((FT*FPPT) .GE. 0.0) GO TO 14                                   SNIC3125
      HO = .75*HO                                                       SNIC3130
   14 TO =TO+HO                                                         SNIC3135
      IMAX =IMAX +1                                                     SNIC3140
 1000 FORMAT(52H0 COMPUTATION FOR SONIC LINE WILL NOT CONVERGE, HO = E  SNIC3145
     116.8  )                                                           SNIC3150
      IF(IMAX .LT. 10 ) GO TO 18                                        SNIC3155
      WRITE (6,1000) HO                                                 SNIC3160
      GO TO 1                                                           SNIC3165
   18 IF(HO .GT. .0001) GO TO 12                                        SNIC3170
      FY(NCR)= SQRT(TO)                                                 SNIC3175
      IF(NCR .GE. NM ) GO TO 20                                         SNIC3180
      IF(FY(NCR) .GE. YM) GO TO 20                                      SNIC3185
      IF(FX(NCR) .GE. 1.0) GO TO 20                                     SNIC3190
      NCR = NCR +1                                                      SNIC3195
      GO TO 10                                                          SNIC3200
   20 RETURN                                                            SNIC3205
      END                                                               SNIC3210
```

```
$IBFTC POTE                                                             SNIC3215
      SUBROUTINE POT (NFR,FR,P)                                         SNIC3220
      COMMON                                                            SNIC3225
     */XYZ/ SX(101),SXP(101),SY(101),SYP(101),AL(41),TIM(101)           SNIC3230
     */CM/ CM(6)                                                        SNIC3235
     */ICNT/ IVAR,NCNT,ISORS,IBR,ITRAP,NMAX                             SNIC3240
     */SOURCE/ XO(20),YO(20)                                            SNIC3245
     */EPS/ E1,E2,FM,YMAX                                               SNIC3250
     */NNN/ NSS,NLCS,NLLS                                               SNIC3255
C                                                                       SNIC3260
      DIMENSION  FR(10),P(101,2,10)                                     SNIC3265
      CON=-.25/3.14159                                                  SNIC3270
      XS =XO(NSS)                                                       SNIC3275
      YS =YO(NSS)                                                       SNIC3280
      DO 100 N=1,NCNT                                                   SNIC3285
      X=SX(N)                                                           SNIC3290
      Y=SY(N)                                                           SNIC3295
      T = TIM(N)                                                        SNIC3300
      RBAR = SYP(N)                                                     SNIC3305
   10 DO 30 NF=1,NFR                                                    SNIC3310
      IF(RBAR) 12,14,16                                                 SNIC3315
   12 P(N,1,NF)=0.                                                      SNIC3320
      P(N,2,NF)=0.                                                      SNIC3325
      GO TO 30                                                          SNIC3330
   14 P(N,1,NF)=UNDEF                                                   SNIC3335
      P(N,2,NF)=UNDEF                                                   SNIC3340
      GO TO 30                                                          SNIC3345
   16 IF(RBAR .LE. 1.E-9) GO TO 14                                      SNIC3350
      FACT = CON/RBAR                                                   SNIC3355
      ARG = FR(NF)*T                                                    SNIC3360
      CO = COS(ARG)                                                     SNIC3365
      SI = SIN(ARG)                                                     SNIC3370
      P(N,1,NF)= CO*FACT                                                SNIC3375
      P(N,2,NF)= -SI*FACT                                               SNIC3380
   30 CONTINUE                                                          SNIC3385
  100 CONTINUE                                                          SNIC3390
      RETURN                                                            SNIC3395
      END                                                               SNIC3400
```

```
$IBFTC RKS3*    RUNGE-KUTTA, FORTRAN IV, VERSION 13, SHARE D2*ATFRKS3   SNIC3405
      SUBROUTINE RKS3 (DERIV,CNTRL,Y,DY,ATABL,RTABL,WORK,X,DX,N,IFVD    SNIC3410
     1                 ,IBKP,NTRY,IERR)                                  SNIC3415
      EXTERNAL DERIV,CNTRL                                              SNIC3420
      INTEGER N,NTRY,IERR                                               SNIC3425
      LOGICAL IFVD,IBKP                                                 SNIC3430
      REAL Y,DY,ATABL,RTABL,X,DX                                        SNIC3435
      DIMENSION Y(N),DY(N),ATABL(N),RTABL(N)                            SNIC3440
      DIMENSION WORK(1)                                                 SNIC3445
C     DIMENSION WORK(9*N+8)                                             SNIC3450
      CALL RKINT (DERIV,CNTRL,Y,DY,ATABL,RTABL,WORK(1),WORK(3),WORK(5)  SNIC3455
     1           ,WORK(7),WORK(9),WORK(2*N+9),WORK(4*N+9),WORK(6*N+9)   SNIC3460
     2           ,WORK(7*N+9),WORK(8*N+9),X,DX,N,IFVD,IBKP,NTRY,IERR)   SNIC3465
      RETURN                                                            SNIC3470
      END                                                               SNIC3475
$IBFTC RKINT*   CALLED BY RKS3, RUNGE-KUTTA, F 4, V13, SHARE D2*ATFRKS3 SNIC3480
      SUBROUTINE RKINT (DERIV,CNTRL,REALY,DY,ATABL,RTABL,DELTAX,X,XHALF SNIC3485
     1                 ,XZERO,Y,YHALF,YZERO,DYHALF,DYZERO,DELTAY,REALX  SNIC3490
     2                 ,DX,N,IFVD,IBKP,NTRY,IERR)                        SNIC3495
      EXTERNAL DERIV,CNTRL                                              SNIC3500
      INTEGER N,NTRY,IERR                                               SNIC3505
      LOGICAL IFVD,IBKP                                                 SNIC3510
      REAL REALY,DY,ATABL,RTABL,DELTAX,DYHALF,DYZERO,DELTAY,REALX,DX    SNIC3515
      DOUBLE PRECISION X,XHALF,XZERO,Y,YHALF,YZERO                      SNIC3520
      DIMENSION REALY(N),DY(N),ATABL(N),RTABL(N),Y(N),YHALF(N),YZERO(N) SNIC3525
     1          ,DYHALF(N),DYZERO(N),DELTAY(N)                          SNIC3530
      IERR = 0                                                          SNIC3535
   10 DELTAX = DX                                                       SNIC3540
      X = REALX                                                         SNIC3545
      DO 20 I=1,N                                                       SNIC3550
   20 Y(I) = REALY(I)                                                   SNIC3555
      CALL DERIV                                                        SNIC3560
      GO TO 200                                                         SNIC3565
   30 IF (DX .EQ. 0.) GO TO 230                                         SNIC3570
      DELTAX = DX                                                       SNIC3575
      DX2 = DX/2.                                                       SNIC3580
      DX4 = DX/4.                                                       SNIC3585
      XZERO = X                                                         SNIC3590
      DO 40 I=1,N                                                       SNIC3595
      YZERO(I) = Y(I)                                                   SNIC3600
   40 DYZERO(I) = DY(I)                                                 SNIC3605
      DO 110 J=1,2                                                      SNIC3610
      XHALF = X                                                         SNIC3615
      X = X+DX4                                                         SNIC3620
      REALX = X                                                         SNIC3625
      DO 50 I=1,N                                                       SNIC3630
      DELTAY(I) = DY(I)*DX4                                             SNIC3635
      YHALF(I) = Y(I)                                                   SNIC3640
      Y(I) = Y(I)+DELTAY(I)                                             SNIC3645
   50 REALY(I) = Y(I)                                                   SNIC3650
```

```
      CALL DERIV                                                        SNIC3655
      DO 60 I=1,N                                                       SNIC3660
      DELTAY(I) = DELTAY(I)+DY(I)*DX2                                   SNIC3665
      Y(I) = YHALF(I)+DY(I)*DX4                                         SNIC3670
   60 REALY(I) = Y(I)                                                   SNIC3675
      CALL DERIV                                                        SNIC3680
      X = XHALF+DX2                                                     SNIC3685
      REALX = X                                                         SNIC3690
      DO 70 I=1,N                                                       SNIC3695
      DELTAY(I) = DELTAY(I)+DY(I)*DX2                                   SNIC3700
      Y(I) = YHALF(I)+DY(I)*DX2                                         SNIC3705
   70 REALY(I) = Y(I)                                                   SNIC3710
      CALL DERIV                                                        SNIC3715
      DO 80 I=1,N                                                       SNIC3720
      DELTAY(I) = (DELTAY(I)+DY(I)*DX4)/3.                              SNIC3725
      Y(I) = YHALF(I)+DELTAY(I)                                         SNIC3730
   80 REALY(I) = Y(I)                                                   SNIC3735
      CALL DERIV                                                        SNIC3740
      GO TO (90,110),J                                                  SNIC3745
   90 DO 100 I=1,N                                                      SNIC3750
  100 DYHALF(I) = DY(I)                                                 SNIC3755
  110 CONTINUE                                                          SNIC3760
      IF (IFVD) GO TO 200                                               SNIC3765
      ERRMAX = 0                                                        SNIC3770
      DO 120 I=1,N                                                      SNIC3775
      ERR = ATABL(I)+ABS(RTABL(I)*REALY(I))                             SNIC3780
      IF (ERR .EQ. 0.) GO TO 220                                        SNIC3785
      SR = (DYZERO(I)+4.*DYHALF(I)+DY(I))/3.*DX2                        SNIC3790
  120 ERRMAX = AMAX1(ERRMAX,ABS (SR-(REALY(I)-SNGL(YZERO(I))))/ERR)     SNIC3795
      IF (ERRMAX-1.) 130,170,160                                        SNIC3800
  130 IF (ERRMAX-.75) 140,200,170                                       SNIC3805
  140 IF (ERRMAX-.075) 150,200,200                                      SNIC3810
  150 DX = DX*1.5845932                                                 SNIC3815
      GO TO 200                                                         SNIC3820
  160 DX = DX/1.5848932                                                 SNIC3825
      IF (.NOT. IBKP) GO TO 180                                         SNIC3830
      ERRMAX = ERRMAX/10.                                               SNIC3835
      IF (ERRMAX .GT. 1.) GO TO 160                                     SNIC3840
      GO TO 180                                                         SNIC3845
  170 DX = DX/1.5848932                                                 SNIC3850
      GO TO 200                                                         SNIC3855
  180 X = XZERO                                                         SNIC3860
      DO 190 I=1,N                                                      SNIC3865
      Y(I) = YZERO(I)                                                   SNIC3870
  190 DY(I) = DYZERO(I)                                                 SNIC3875
      GO TO 30                                                          SNIC3880
  200 NTRY = 1                                                          SNIC3885
      CALL CNTRL (NTRY)                                                 SNIC3890
      GO TO (30,210,180,10),NTRY                                        SNIC3895
  210 RETURN                                                            SNIC3900
```

```
220 IERR = 1                                                      SNIC3905
    RETURN                                                        SNIC3910
230 IERR = -1                                                     SNIC3915
    RETURN                                                        SNIC3920
    END                                                           SNIC3925
```

APPENDIX II. Sample Input and Output

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE ____ of ____ JOB NO. ___

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	6		NSOURCE (NUMBER OF SOURCE POINTS, 20 MAXIMUM)
13	20		NLA (NUMBER OF A PER SOURCE, 40 MAXIMUM)
25	5		NPL (NUMBER OF PLANFORM COORDINATES, 8 MAXIMUM)
37	70		NMAX (LIMIT NUMBER OF POINTS PER PLOT, 100 MAX)
49	3	73 80	NF (NUMBER OF ASSUMED FREQUENCIES, 10 MAXIMUM)
61		1	
1	FALSE		IFVD LOGICAL WORDS - VARIABLE INTERVAL MODE
13	TRUE		IBKP IF IFVD = FALSE AND IBKP = TRUE THIS CHOICE
25			IS RECOMMENDED. FIXED INTERVAL IF
37			IFVD = TRUE.
49		73 80	
61		2	
1			
13			
25			
37			
49		73 80	
61			

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO.______ PROGRAMMER__________ DATE______ PAGE____ of____ JOB NO._

	NUMBER		IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	1 6	+ 00		XO(1) COORDINATES OF SOURCE POINTS.
13	0	+ 00		YO(1) (ALL GEOMETRY IS NORMALIZED ON b_o, THE
25	2 2	+ 00		XO(2) DISTANCE FROM MOST FORWARD TO MOST AFT
37	0. 4	+ 00		YO(2) PORTION OF THE WING.)
49	3 8	+ 00	73 80	
61	1 4	+ 00	3	
1	4 8	+ 00		
13	1 8	+ 00		
25	5	+ 00		
37	0	+ 00		
49	6	+ 00	73 80	XO(NSØRCE)
61	2	+ 00	4	YO(NSØRCE)
1				
13				
25				
37				
49			73 80	
61				

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO.______ PROGRAMMER______ DATE______ PAGE____ of____ JOB NO.___

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	1 + 0 1		CM(1) COEFFICIENTS IN MACH NUMBER EQUATION
13	9 5 7 9 2 5 + 0 0		CM(2) $M = CM(2) + EXP \{-CM(1)(Y^2/X)\}.$
25	1 2 6 6 7 3 + 0 0		$\{CM(3)x + CM(4)\ x^2 + CM(5)Y^2$
37	6 3 0 5 9 + 0 0		$+ CM(6)\ Y^4\}$
49	7 9 0 1 5 5 + 0 0	73 80	THE COEFFICIENTS WERE DETERMINED BY A
61	8 9 1 5 6 9 + 0 0	5	CM(6) LEAST-SQUARE PROCEDURE.
1	2 - 0 1		DZ INITIAL VALUE OF INCREMENT
13	1 - 0 1		E1 TEST WORD FOR TRAPPING SIGNAL ON LOCAL M.L.
25	1 - 0 2		E2 TEST WORD FOR STOPPING TRAPPED SIGNAL ON SONIC LINE
37	4 6 6 3 1 + 0 0		YMAX NORMALIZED SEMI-SPAN
49		73 80	
61		6	
1			
13			
25			
37			
49		73 80	
61			

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO.______ PROGRAMMER ____________ DATE ______ PAGE ____ of ____ JOB NO.

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	1 -04		ATABL(1) ATABL AND RTABL DETERMINE THE ACCURACY
13	1 -04		ATABL(2) REQUIREMENTS FOR DECREASING OR IN-
25	1 -04		ATABL(3) CREASING THE INTERVAL IN THE VARIABLE
37	1 -04		ATABL(4) INTERVAL MODE.
49	1 -03	73 80	RTABL(1)
61	1 -03	7	RTABL(2)
1	1 -03		RTABL(3)
13	1 -03		RTABL(4)
25			
37			
49		73 80	
61		8	
1	1 +01		PLX(1) PLANFORM COORDINATES. LIST ALL CORNERS
13	-46631 +00		PLY(1) STARTING FROM LEFT ALONG LEADING EDGE
25	0 +00		PLX(2) TOWARD THE RIGHT, AND AGAIN STARTING
37	0 +00		PLY(2) AT LEFT, ALONG TRAILING EDGE TOWARD
49	1 +01	73 80	PLX(3) THE RIGHT. ALL GEOMETRY IS NORMALIZED
61	46631 +00	9	PLY(3) on b_o.
1			

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE ____ of ____ JOB NO. ____

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	1 +01		PLX(4)
13	-46631 +00		PLY(4)
25	1 +01		PLX(5)
37	46631 +00		PLY(5)
49		73 80	
61		10	
1	94853 +01		CINF REMOTE SPD. OF SND. IN bo UNITS/SEC.
13	1 +01		FMINF REMOTE MACH NUMBER
25	02 +00		TAU MAXIMUM THICKNESS RATIO (T/C)
37	46631 +00		TSAH TANGENT SEMI-APEX ANGLE
49		73 80	
61		11	
1	1 +02		FREQ(1) FREQUENCIES, FOR COMPUTATION
13	2 +02		FREQ(2) OF POTENTIALS, (RADIANS/SEC)
25	0 +00		FREQ(NF)
37			
49		73 80	
61		12	

```
             IVAR    NCNT    ISORS    IBR    ITRAP    NLCS =

               2      12       1       1       0       9


                          XO =   0.18000000E 00
                          YO =   0.00000000E-38
                          MACH NO. =   0.97311907E 00

                 ACOUSTIC RAY PATH FOR LAMBDA =   0.57119817E 00

        X                 Y                X-PRIME            R-BAR              TIME

  0.18000000E 00    0.00000000E-38    0.21141810E 00    0.00000000E-38    0.00000000E-38
  0.18424259E 00    0.20000000E-01    0.21257251E 00    0.72580551E-02    0.38412277E-02
  0.19096695E 00    0.51691864E-01    0.21112035E 00    0.18791161E-01    0.99172437E-02
  0.20135082E 00    0.10191058E 00    0.20083363E 00    0.37255190E-01    0.19553514E-01
  0.20530157E 00    0.12189922E 00    0.19445186E 00    0.44702658E-01    0.23404072E-01
  0.20915295E 00    0.14188787E 00    0.19213422E 00    0.52104825E-01    0.27296191E-01
  0.21302884E 00    0.16187273E 00    0.19647354E 00    0.59213898E-01    0.31281433E-01
  0.21700866E 00    0.18185759E 00    0.20163166E 00    0.66174245E-01    0.35289611E-01
  0.22350139E 00    0.21352546E 00    0.20816299E 00    0.77123095E-01    0.41616165E-01
  0.23415963E 00    0.26370616E 00    0.21631019E 00    0.94319344E-01    0.51590675E-01
  0.25178554E 00    0.34322216E 00    0.22669061E 00    0.12128799E 00    0.67289645E-01
  0.28126384E 00    0.46922266E 00    0.24104052E 00    0.16353054E 00    0.91917480E-01
```

```
                    VELOCITY POTENTIALS ALONG A RAY PATH FOR A SOURCE AT

                                   XO =   0.18000000E 00
                                   YO =   0.00000000E-38
                                   LAMBDA =   0.57119817E 00

                                ALTERNATING REAL AND IMAGINARY

 OMEGA =  0.10000000E 02

          ......          ......   -0.109559E 02   0.421050E 00   -0.421403E 01   0.419291E 00
   -0.209531E 01   0.415009E 00   -0.173162E 01   0.412835E 00   -0.147071E 01   0.411726E 00
   -0.127868E 01   0.413568E 00   -0.112844E 01   0.415620E 00   -0.943756E 00   0.417118E 00
   -0.733892E 00   0.416219E 00   -0.513086E 00   0.408919E 00   -0.295125E 00   0.386914E 00

 OMEGA =  0.20000000E 02

          ......          ......   -0.109317E 02   0.841478E 00   -0.415181E 01   0.834462E 00
   -0.197475E 01   0.814201E 00   -0.158867E 01   0.803160E 00   -0.130527E 01   0.792965E 00
   -0.108936E 01   0.786997E 00   -0.915255E 00   0.780015E 00   -0.694583E 00   0.763032E 00
   -0.433042E 00   0.724092E 00   -0.146383E 00   0.639566E 00    0.128649E 00   0.469308E 00

 OMEGA =  0.00000000E-38

          ......          ......   -0.109640E 02   0.000000E-38   -0.423484E 01   0.000000E-38
   -0.213601E 01   0.000000E-38   -0.178015E 01   0.000000E-38   -0.152726E 01   0.000000E-38
   -0.134390E 01   0.000000E-38   -0.120255E 01   0.000000E-38   -0.103182E 01   0.000000E-38
   -0.843703E 00   0.000000E-38   -0.656104E 00   0.000000E-38   -0.486622E 00   0.000000E-38
```

APPENDIX III. Application to the Boundary Value Problem

A procedure that may be used to match the tangential flow condition on a wing surface is, in principle, the same as that employed by Rodemich in the box method for uniform sonic flow (Reference 3). The velocity potential at a field point (x,y,z) due to a doublet sheet in its zone of influence, is

$$\bar{\phi}(x,y,z) = \frac{\partial}{\partial z}\iint\limits_{S+W} \Delta\bar{\phi}(\xi,\eta)\,\phi_o(x-\xi,y-\eta,z)\,d\xi\,d\eta \tag{39}$$

where $\Delta\phi(\xi,\eta)$ is the velocity potential discontinuity through the doublet sheet over the region S + W (the surface and its wake), and

$$\phi_o(x-\xi,y-\eta,z) = \frac{-1}{2\pi\bar{R}}\sum_{n=1}^{N} e^{-i\omega g_n} \tag{40}$$

where $\bar{R} = \sqrt{(x-\xi)^2 + [1-M_L^2(x,y,z)]\,[(y-\eta)^2 + z^2]}$

and where N represents the number of times the wave front passes the field point. In uniform subsonic flow N equals one, in uniform supersonic flow it equals two, and in the limiting case of uniform sonic flow it equals one. As discussed previously, in uniform sonic flow the stationary portion of the perturbation wave front is not augmented by high frequency signals that follow it; instead, the pressure discontinuity is dissipated by them.

When the local flow in a non-uniform flow field is sonic the wave front gradually becomes stationary and is dissipated. Rays of this type are shown in Figures 9, 12, and 13. In certain regions of non-uniform flow a wave front may pass field points more than twice as shown in Figures 6, 7, 9, 10, and 12. These regions may be in the region of subsonic flow or in supersonic flow. Multiple crossings normally occur on receding portions of the wave front. Ray lines on advancing portions normally pass over the trailing edge before they cross. In these regions of multiple crossings of the wave front, care must be taken to establish an accurate value of N, and of each of the corresponding g_n's, $n = 1, 2, \ldots N$. A computer program that may be used to do this is contained herein. Figures 11 and 13 show that in some regions of both subsonic and supersonic flow even the receding ray lines do not cross. All of Figures 6 through 13 show that once a ray crosses the transition region at the edge of the planform it does not return to the wing region. This characteristic is important because when a doublet solution is employed a ray trace can be ignored once it reaches an edge that is not adjacent to the wake.

The next step in the procedure is to define a grid of square boxes over the region S + W, and assume that $\Delta\phi(\xi,\eta)$ is constant over the area of each box. For this to be a valid assumption as many as 50 boxes along the root chord may be required. The upwash adjacent to the upper surface may be written

$$\overline{W}(x,y,0+) = \lim_{z \to 0+} \frac{\phi(x,y,z)}{z}$$

or,

$$\overline{W}(x_i,y_i,0+) = \sum_{i'j'} \overline{\Delta\phi}_{i'j'} \iint_{B_{i'j'}} \psi(x_i-\xi,y_i-\eta)\,d\xi\,d\eta \qquad (41)$$

i.e., the upwash at (x_i,y_i) equals the summation (over all boxes $B_{i'j'}$ that influence it), of products of the constant velocity potential discontinuities and their downwash influence coefficients. The latter are represented by the double integral of the kernel ψ over the areas of the boxes. The limits of integration and $\overline{\Delta\phi}$ of Equation (39) are not functions of z, so from Equation (40) we get

$$\psi(x_i-\xi,y_j-\eta) = \frac{-1}{2\pi}\lim_{z \to 0+} \frac{1}{z}\frac{\partial}{\partial z} \frac{\Sigma\, e^{-i\omega g_n}}{\overline{R}} \qquad (42)$$

At this point it is theorized that for non-uniform flow around a nearly planar surface the variation in signal transmission time with distance normal to the surface is approximately equal to the variation in uniform flow, i.e.,

$$\frac{\partial g_n}{\partial z} = \frac{\partial}{\partial z} \frac{M(x-\xi) \mp \overline{R}}{C(M^2-1)}$$

or, performing the differentiation

$$\frac{\partial g_n}{\partial z} = \frac{\pm z}{C\overline{R}} \qquad (43)$$

where the upper sign refers to the advancing portion of the wave front and the lower sign to the receding portion. C is the speed of sound. Making use of equation (43) when taking the derivative in equation (42).

$$\psi(x_i-\xi,y_i-\eta) = \frac{-1}{2\pi} \frac{\beta^2 C \pm i\omega M\overline{R}}{C\overline{R}^3} \sum_n e^{-i\omega g_n} \qquad (44)$$

The g_n's are those obtained by tracing ray paths through the non-uniform flow field.

One way in which Equation (44) may be evaluated and integrated is as follows: Say for nine values of (ξ,η) on each sending box, the values of the kernel at the center of the receiving box (x_i,y_j) are evaluated. Since the ray paths are not known in advance, each of these values must be interpolated from values in its neighborhood. It is then necessary to evaluate the integral in Equation (41) given the values of the integrand at nine points in the region of integration.

The unknowns in Equation (41) are the $\overline{\Delta\phi}_{i'j'}$'s. When the center of a receiving box (x_i,y_j) lies in the subsonic flow region it lies in the zone of influence of every other point in the subsonic region and may lie in the zone of influence of a small portion of the supersonic region (Figure 9). All velocity potentials in zones of mutual influence must be determined simultaneously. Once velocity potentials have been established that meet the tangential flow conditions on the surface and the zero pressure difference condition on the wake they may be fitted with analytical expressions that have the proper edge behavior. Using these expressions, local oscillatory pressures and generalized forces may be obtained in the way outlined in Reference 3.

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13. ABSTRACT

Two methods have been outlined in detail, and one of them has been mechanized, for calculating acoustic ray paths emanating from any point in a non-uniform transonic flow field surrounding a wing. It gives the ray path, and the time, for the minimum time of travel from the acoustic source point to the field point. The resulting velocity potential is also computed.

It was necessary to establish an accurate representation of the flow characteristics in the field surrounding the wing. Some ray lines travel over the planform and into the surrounding flow field. It was established that once off the planform they do not return.

Available methods predict phase lags based on the assumption that acoustic rays travel in straight lines. The results of this study show this to be a very poor approximation at transonic speeds. Therefore, it is recommended that the method presented in this report be fully developed for the purpose of calculating generalized forces on wings in harmonic motion at transonic speeds. A computer program that would predict these phase lags with reasonable accuracy, and the corresponding flutter characteristics and unsteady aerodynamic loads on a wing responding to externally applied forces, such as gusts, would fill an important gap in the available technology.

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