Coupled Modal Damping in Transient Solutions

Bruce C. McFarland *
Allen J. Bronowicki†

TRW Space & Technology Group
Redondo Beach CA 90278

March 8, 1989

Abstract

The modal strain energy technique allows one to compute equivalent viscous damping ratios for real normal modes given structural loss factor data. This concept is generalized to include inter-modal coupling effects due to damping forces. Off-diagonal terms in the modal damping matrix are normalized by the geometric mean of the natural frequencies of the coupled modes. Incorporation of the damping model in a transient analysis scheme is described. The model is then demonstrated on a space structure. The effect of damping coupling is shown to be significant when damping is heavy and modal frequencies are closely spaced.

*Staff Engineer, Dynamics Department
†Head, Analytic Methods Section

1 Introduction

The modal strain energy (MSE) technique is a common means of computing equivalent modal damping values for simplified dynamic analysis of systems having structural losses. The method bases the viscous damping ratio assigned to each normal mode on the relative degree of participation in the mode of the various component strain energies, and on the material loss factors in those components. This is a rule of mixtures approach to damping. Apportionment of damping values by mode is useful since it allows one to construct a damping model in which modal damping values do not rise with modal frequency. Generation of physical equivalent viscous damping models have the undesirable effect of producing modal damping values which are proportional to modal frequency.

The technique of basing modal damping values on strain energy participation was first proposed by Ungar [1]. The method was applied to viscoelastic materials using standard finite element codes by Rogers et. al. [2]. It has since seen increasing use in the aerospace industry. The technique allows one to avoid solving a complex eigen–problem by assigning a viscous damping ratio to each real normal mode. The assumption is that damping forces are smaller than elastic forces, and hence do not affect the orthogonality properties of the modes. The standard application of this method thus does not account for coupling between the response of the modes due to the viscous forces. It was shown in Reference 3 that the effects of damping coupling between real normal modes are negligible for lightly damped structures whose modes are well separated in frequency. However, when structures are heavily damped and natural frequencies are not well separated, coupling between modes due to damping forces can be significant. The modal strain energy technique is extended in this work to account for these inter-modal coupling forces. The result is a coupled modal strain energy (CMSE) technique.

2 Damping Models

The following development works toward an equivalent viscous treatment of structural damping which includes inter-modal coupling effects. The CMSE formulation allows the computation of transient response directly, without recourse to transform techniques. An appropriate starting point for the derivation is the set of non-homogeneous equations of equilibrum in the frequency domain.

$$[-\Omega^2[M] + [K] + j\ sgn(\Omega)[K']]\{X(\Omega)\} = \{F(\Omega)\} \tag{1}$$

The system mass, stiffness and structural damping (or loss) matrices, $[M]$, $[K]$ and $[K']$, respectively, are assumed to be real, symmetric and frequency independent. In the case of viscoelastic materials the stiffness and loss matrices actually depend on frequency. The MSE technique often employs material property data near a given natural frequency to obtain equivalent damping values for modes in the vicinity of the assumed frequency. In that case the matrices are assumed to be piecewise frequency independent. For the purposes of this development the assumption of frequency independence is adequate. The sign function is required to ensure that the damping forces always oppose the velocity.

The stiffness and loss matrices are formed from the assembly of a number of elemental stiffness and loss matrices, $[K_i^e]$ and $[K_i^{e\prime}]$. The elemental loss matrices are assumed to be related to their stiffness counterparts through a simple scalar loss factor, g_i. The system stiffness and loss matrices are thus represented:

$$[K] = \sum_i [K_i^e] \tag{2}$$

$$[K'] = \sum_i [K_i^{e\prime}] = \sum_i g_i [K_i^e] \tag{3}$$

The nomenclature "element" may also be taken to mean component or material. The system is merely being subdivided into regions having a constant loss factor.

The objective of the following developments is to find a simple means of computing transient dynamic response which adequately matches the dynamics described by Equation 1. One could simply apply transform techniques to these equations, but the resultant response would be non-causal as discussed by Crandall [4]. A more physical approach which produces the desired behavior without introducing non-causality is desirable.

At a single response frequency, Ω_0, one may compute an equivalent physical viscous damping matrix $[C] \equiv [K']/\Omega_0$ such that the viscous loss forces equal the structural loss forces defined in Equation 1. When the system loss matrix is strictly proportional to the stiffness matrix a set of modal damping ratios may be obtained. In that case these modal damping ratios will be linear with modal frequency. High frequency modes become heavily damped and low frequency modes are lightly damped. This is contrary to the structural damping assumption in which all modes are damped equally when the material is uniform throughout the structure. Thus equivalent viscous damping matrices formed in this simple manner are not realistic.

2.1 The MSE Technique

The MSE technique begins with real normal modes satisfying the system eigenproblem with damping terms discarded:

$$[K][\Phi] = [M][\Phi][\omega^2] \tag{4}$$

The resultant modes are normalized to give a unit diagonal modal mass matrix. The modal transformation $\{x\} = [\Phi]\{\eta\}$ is then applied. Mass-orthonormalized modes are assumed in the ensuing discussion. A truncated set of modes will generally be employed so that the number of modal coordinates, η, will be less than the number of physical coordinates, x. Time domain equations of motion in the modal space may then be defined as follows:

$$[I]\{\ddot{\eta}\} + [c]\{\dot{\eta}\} + [\omega^2]\{\eta(t)\} = [\Phi]^T\{f(t)\} \tag{5}$$

where $[c]$, the viscous damping matrix in modal coordinates, is yet to be assigned. In the MSE method the off-diagonal terms of $[c]$ are zeroed out and viscous damping ratios are assigned to each mode m as follows:

$$2\zeta_m \equiv \frac{\sum_i g_i \{\phi_m\}^T [K_i^{e\prime}] \{\phi_m\}}{\{\phi_m\}^T [K] \{\phi_m\}} = \frac{\sum_i g_i \{\phi_m\}^T [K_i^{e\prime}] \{\phi_m\}}{\omega_m^2} \tag{6}$$

where use has been made of the diagonalization of the stiffness matrix by the mode shapes. Notice that the term in the numerator can be interpreted as the sum of the elemental modal strain energies weighted by the elemental loss factors. In this manner, the modal loss factors are apportioned according to the strain energy participation of the mode in each of the materials. A diagonal modal damping matrix is obtained with non-zero elements

$$c_{mm} = 2\zeta_m \omega_m = \frac{\sum_i g_i \{\phi_m\}^T [K_i^{e\prime}] \{\phi_m\}}{\omega_m} \tag{7}$$

Alternately, one can employ the definition of the system loss matrix, Equation 3, to obtain an expression based on system rather than elemental quantities:

$$c_{mm} = 2\zeta_m \omega_m = \frac{\{\phi_m\}^T [K'] \{\phi_m\}}{\omega_m} \tag{8}$$

The computation using the system loss matrix rather than elemental strain energies is often simpler to perform in practice. It is instructional to view this modal damping matrix as the projection of the loss matrix on the modal space with coupling terms discarded, and then scaled by one power of frequency to reflect the additional time derivative applied to obtain modal velocity from modal displacement. The frequency scale factor allows the loss per cycle to be maintained for sinusoidal motions, assuming each mode responds only at its own natural frequency. Given broad-band excitations and low to moderate levels of damping, each mode will indeed respond primarily at its natural frequency.

When the structure is excited by a narrow-band input, it would be a better approximation to scale the modal loss matrix by a single reference frequency. That reference frequency, Ω_0, is commonly chosen as the half-power frequency of the response power spectral density. This option of computing an equivalent viscous damping matrix, by scaling the projection of the loss matrix on the modal space by one user-defined reference frequency is available in many major finite element codes, such as MSC/Nastran.

2.2 The Coupled MSE Technique

It is proposed here that the modal damping matrix be constructed on the basis of strain energy participation without discarding coupling terms. This may be accomplished by projection on the modal space and application of a diagonal scaling transformation as shown below:

$$[c] \equiv [\omega^{-\frac{1}{2}}][\Phi]^T [K'] [\Phi][\omega^{-\frac{1}{2}}] \tag{9}$$

The diagonal terms which result from the CMSE damping model are identical to those of Equation 7. Coupling terms are scaled by the square root of the product of the natural frequencies of the two modes being coupled, i.e. their geometric mean, as proposed by Rogers et. al. [2]. When modes are closely spaced, and hence coupling terms are important, the geometric mean will be close to the two natural frequencies of interest.

The model is intended to be useful in cases where damping coupling is significant; where the damping forces are considered to be structural, such that the energy loss per cycle is generally independent of response frequency; and where the response is not at a single frequency, disallowing the use of a single reference frequency to scale loss terms. In other words, for want of any better knowledge, the modes are assumed to be responding primarily at their own natural frequencies. The resulting coupled damping matrix will necessitate a further eigen-solution to obtain complex modes if one wishes to diagonalize the equations of motion. Otherwise, coupled solution techniques can be applied in frequency or time domain solutions.

A physical damping matrix, $[C]$, corresponding to the assumed modal damping matrix may now be constructed. We desire a minimum norm matrix which satisfies the relation $\Phi^T C \Phi = c$. Such a matrix may be found through application of the generalized inverse [5], where $\Phi^{\#}$ is defined as $(\Phi^T\Phi)^{-1}\Phi^T$. A computationally efficient substitute for the generalized inverse of the eigenvector matrix for mass-normalized modes was proposed in Reference 6 to be $\Phi^T M$. Applying this quasi-inverse to both sides of the above relation results in the corresponding physical damping matrix

$$[C] \equiv [M][\Phi][c][\Phi]^T[M]^T \tag{10}$$

$\Phi^T M$ satisfies all but one of the sufficient conditions set forth in [5]; $\Phi\Phi^{\#}$ is symmetric, $\Phi\Phi^T M$ is not. C derived from $\Phi^{\#}$ will be the minimum 2-norm physical damping matrix which provides the desired modal damping matrix. Its projection on truncated modes will not be null. C derived $\Phi^T M$ provides the desired modal damping matrix and has null projection on truncated modes. A comparison has shown that transient responses resulting from use of the quasi-inverse vs. those resulting from use of the generalized inverse are within .2 percent.

3 Transient Response Analysis

Having defined an appropriate modal damping matrix, the coupled modal equations of motion may easily be solved by direct integration. It is also possible to solve a complex eigenproblem in the modal space to de-couple the equations prior to integration. If a small number of modes are being used, the coupled integration will not be a computational burden. To recover displacements a mode acceleration approach is more accurate than a mode displacement approach. To do this requires the solution of the equations of equilibrium at each time step given the applied, inertial and viscous loads. The resultant physical displacements are

$$\begin{aligned}\{x\} &= [K]^{-1}(\{F\} - [M][\Phi]\{\ddot{\eta}\} - [C][\Phi]\{\dot{\eta}\}) \\ &= [K]^{-1}(\{F\} - [M][\Phi](\{\ddot{\eta}\} + [c]\{\dot{\eta}\}))\end{aligned} \tag{11}$$

The second, simplified expression given above was obtained by post-multiplying Equation 10 by Φ, and taking advantage of mass-orthonormality to obtain $C\Phi = M\Phi c$. By recovering displacements in this manner, it is not necessary to actually compute a physical damping matrix. The modal viscous loads are used to augment the modal inertial loads, avoiding computation of physical viscous loads. An even more simple form may be found by manipulating the expression for the real eigenproblem, Equation 4. This results in the following simple expression for displacement response due to modal inertial loads [7]:

$$[K]^{-1}[M][\Phi] = [\Phi][\omega^{-2}] \tag{12}$$

Recovery of physical displacements is then obtained in terms of applied physical loads and a summation of modal responses due to modal inertial and viscous loads:

$$\{x\} = [K]^{-1}\{F\} - [\Phi][\omega^{-2}](\{\ddot{\eta}\} + [c]\{\dot{\eta}\}) \tag{13}$$

4 Frequency Response Example

Figure 1 shows a spacecraft truss appendage supporting an optical mount. Damped tripod struts and base joint dampers are modeled with material loss factors. The tip is subjected to a lateral sine wave frequency sweep from 10 Hz to 1000 Hz. This emulates a secondary coolant disturbance. Rotations of the optical mount are monitored for the reference case with coupled imaginary stiffness, Equation 1, and for CMSE and MSE equivalent damping. These responses are shown in plots of response vs. excitation frequency in Figures 2a-2c. The CMSE viscous equivalent case shown in Figure 2b is similar to the imaginary stiffness case in Figure 2a for most the the frequency range. The difference in second mode peak response is about 10 percent. The MSE viscous equivalent case shown in Figure 2c compares well only at specific frequencies. The difference in second mode peak response between the MSE case and the imaginary stiffness reference case is 350 percent.

5 Transient Response Example

Figure 3 shows a spacecraft with solar arrays cantilevered on booms. Active damping is proposed to attenuate oscillations of the solar arrays due to spacecraft slew maneuvers. The active damping is approximated by material loss factors applied to bending strain of the booms. Boom loads recovered using displacements calculated by Equation 13 for CMSE and MSE damping techniques are shown in Figures 4a and 4b. In this example peak boom

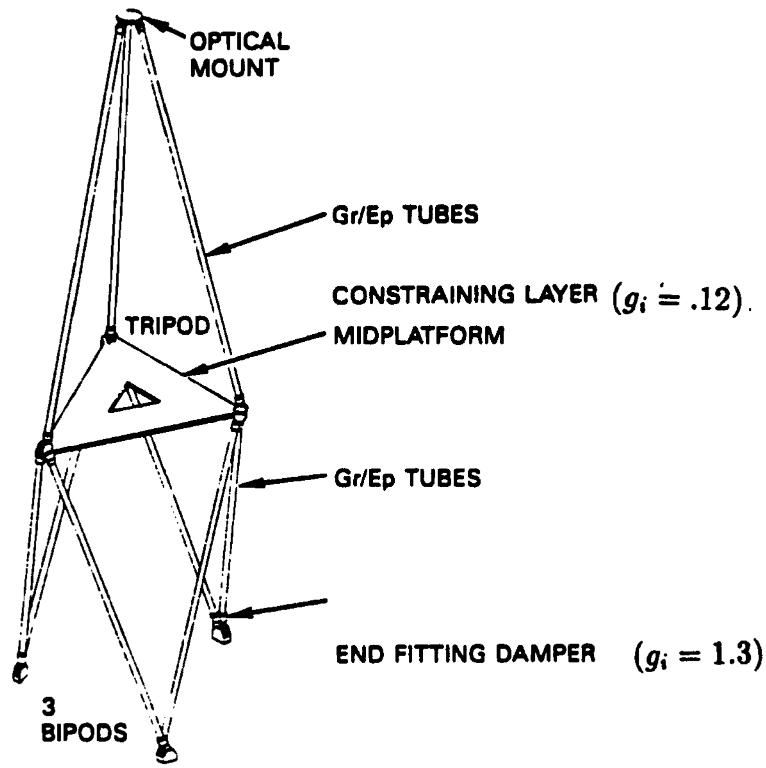


Figure 1: Damped Truss with Optical Mount

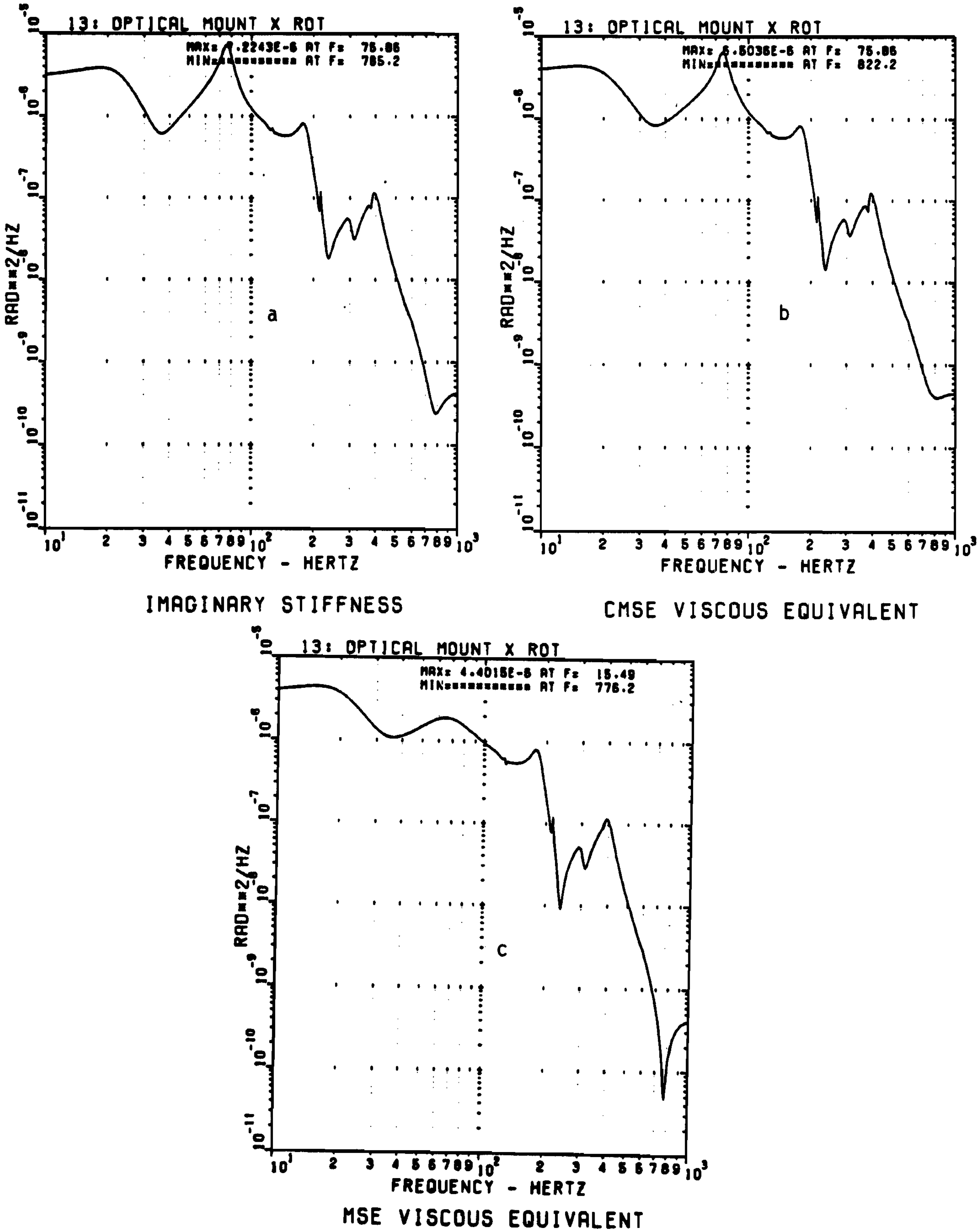


Figure 2: Optical Mount Frequency Response

bending loads are reduced by 20% when the CMSE technique is employed. A 10% increase in positive peak loads is encountered when MSE damping forces are neglected in loads recovery. This can be seen in Figure 4c. In this case neglecting damping forces in recovery improved response as compared with the CMSE reference case, but this may not be true in general. We conclude that in general additional error is incurred by neglecting large damping force corrections in loads recovery.

A more significant result is the amplitude of free vibration after the slew maneuver. At 40 seconds, response in the CMSE case is an order of magnitude greater than response in the MSE case. In this situation neglecting coupling terms in the damping matrix would lead to large errors in prediction of post-slew spacecraft performance.

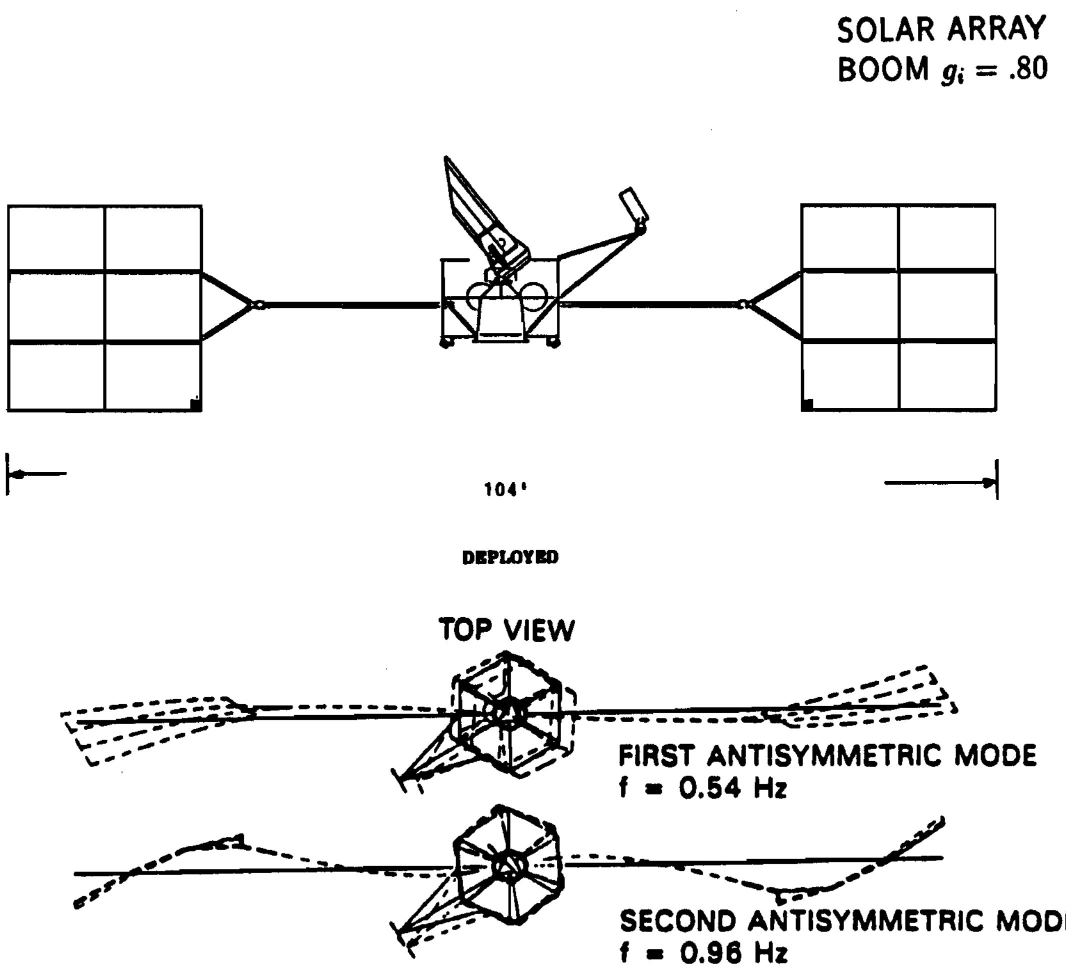
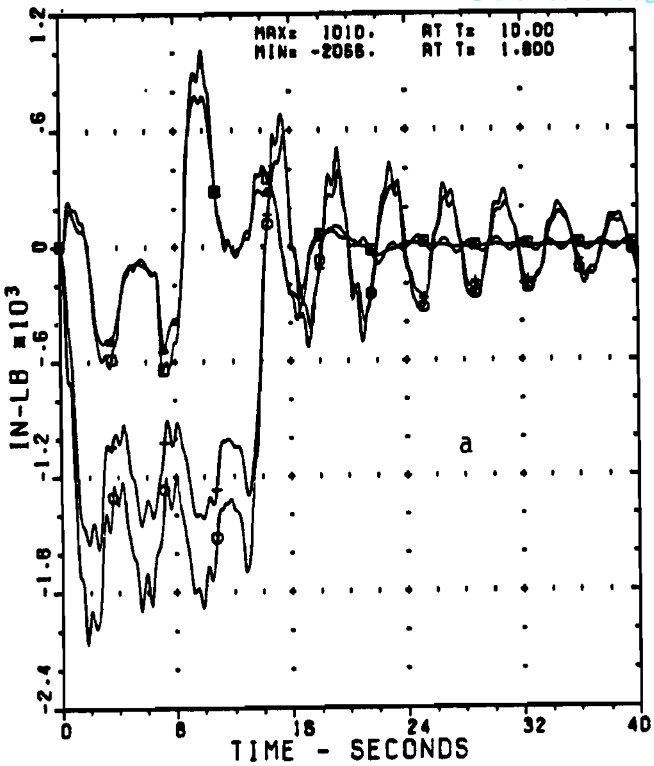
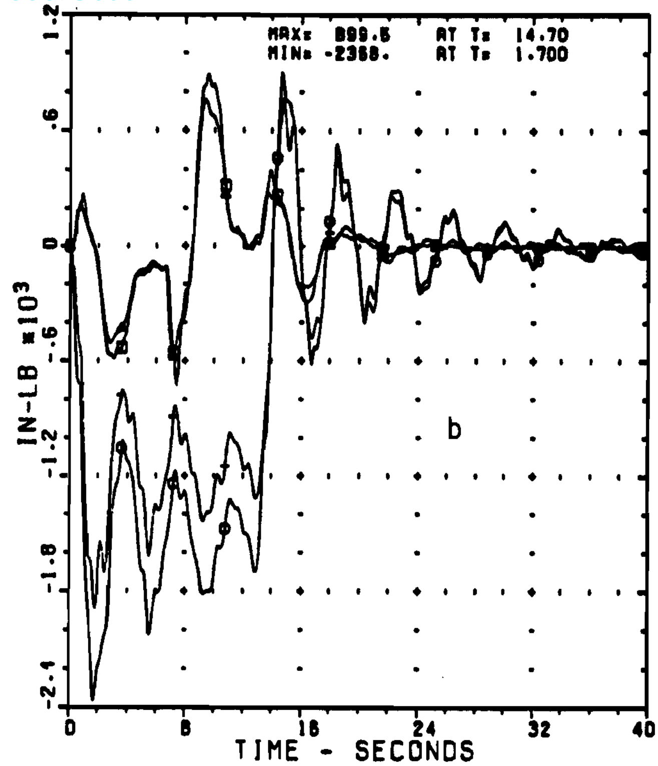


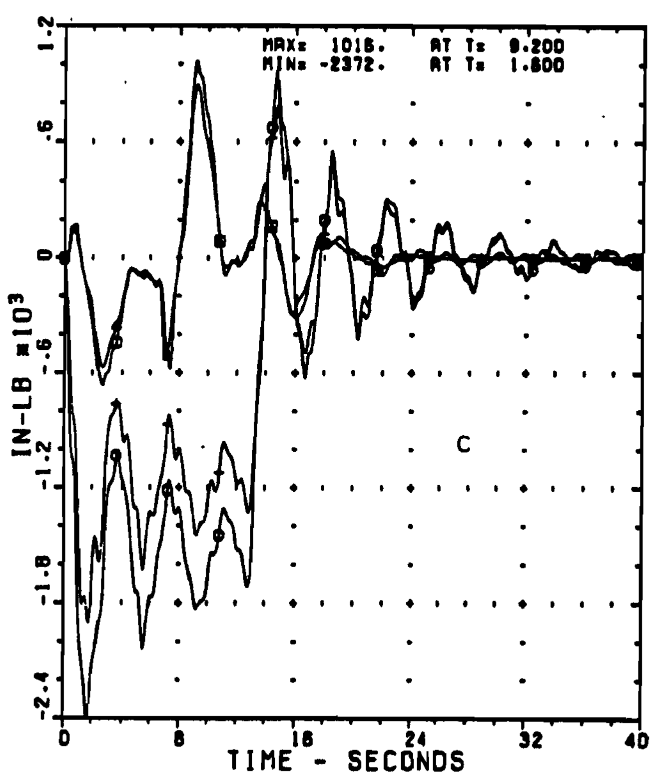
Figure 3: Agile Spacecraft with Damped Solar Array Booms



CMSE, DAMPING IN RECOVERY



MSE, DAMPING IN RECOVERY



MSE, NO DAMPING IN RECOVERY

Figure 4: Solar Array Boom Loads

6 Conclusions

The Coupled Modal Strain Energy technique allows one to build a physically reasonable structural damping model without discarding modal coupling terms. Damping models in both the modal and physical spaces were constructed. Modal coupling terms have previously been shown to be important in the case of large damping and closely spaced modes. The example presented confirms that the retention of modal coupling can have a significant effect on transient response.

References:

1. E. E. Ungar and E. M. Kerwin, "Loss Factors of Viscoelastic Systems in Terms of Energy Concepts", *Journal of the Acoustical Society of America*, Vol. 34, No. 7, pp 954–957, July 1962.

2. C. D. Johnson, D. A. Keinholz and L. C. Rogers, "Finite Element Prediction of Damping in Beams with Constrained Viscoelastic Layers", *Shock and Vibration Bulletin*, No. 51, pp 71–82, May 1981.

3. T. K. Hasselman, "Modal Coupling in Lightly Damped Structures", *AIAA Journal*, Vol. 14, No. 11, pp 1627–1628, November 1976.

4. S. H. Crandall, "The Role of Damping in Vibration Theory," *Journal of Sound and Vibration,* Vol. 11, No. 1, pp 3–18, 1970.

5. B. Noble and J. W. Daniel, *Applied Linear Algebra*, Chapter 9.6, Prentice–Hall, 1977.

6. R. W. Clough and J. Penzien, *Dynamics of Structures*, Chapter 13.3, McGraw–Hill, 1975.

7. R. R. Craig, *Structural Dynamics*, Chapter 15.3, Wiley, 1981.