A Study of a Vibration Absorber to Control the Vibration of Rectangular Plate

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ABSTRACT

A vibration absorber was studied to control the dynamic behavior of a rectangular plate. The absorber consists of a vibration damping composite steel beam and an additive mass. By evaluating the loss factor and the bending rigidity of the composite steel beam using the Ross-Kerwin-Ungar model, the length and the thickness of the composite steel beam and the additive mass were determined in order to tune the resonance frequency of the absorber to any resonance frequency of the rectangular plate. The dynamic behavior of the rectangular plate with the absorber was measured and compared with the calculation. The close agreement achieved suggests that the present method is sufficiently reliable to predict the dynamic behavior of the vibration absorber consisting of the vibration damping composite steel beam.

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INTRODUCTION

A vibration absorber is an effective method to solve vibration and noise problems in several industrial structures and machines. The absorber is a passive damping device to reduce the amplitude of vibration at resonance. The optimum design method for the single of degree system was already derived [(1)]. Though many investigations of vibration absorbers have been reported [(2)-(5)], there is few reports to study the absorbers using a vibration damping composite steel beam as a spring and damping element.

In this paper, we propose a method to design the absorber consisting of the composite steel beam and an additive mass. By evaluating the loss factor and the bending rigidity of the beam using the Ross-Kerwin-Ungar model [(6),(7)], the thicknesses of the steel and viscoelastic resin layers and the length of the beam are determined to tune a resonance frequency and of a vibrating main body to be damped.

The vibration absorber was designed to control the first vibration mode of the rectangular aluminum plate using the above method. The frequency response curve of inertance of the plate with the absorber is calculated to be compared with the experimental results. The close agreement achieved suggests that this method is sufficiently reliable to predict the dynamic behavior of the absorber consisting of the composite steel beam.

1. CALCULATION METHOD OF DYNAMIC BEHAVIOR OF THE ABSORBER

1.1 CALCULATION MODEL

The vibration absorber is shown in Fig. 1. It consists of the vibration damping composite steel beam and the additive mass placed on the both ends. The beam is supported at the center and attached to the vibrating main body (a rectangular aluminum plate in this paper) to be controlled. As the shape of the absorber is symmetric, the calculation model is assumed to be the cantilever with an additive mass at the free end as depicted in Fig. 2.

1.2 CALCULATION OF THE DYNAMIC BEHAVIOR OF THE ABSORBER

The vibration damping composite steel beam is considered to be equivalent to the homogeneous beam with the structural damping. To incorporate the damping into the beam, it is necessary to replace a bending rigidity EI of the beam by a complex bending rigidity $EI(1+j\eta)$. The complex bending rigidity $EI(1+j\eta)$ can be calculated by using the Ross-Kerwin-Ungar model(referred to hereafter the RKU model) [(8)]. Bending wave equation for the vibration absorber is given by

$$\rho A \frac{\partial^2 w}{\partial t^2} + EI(1+j\eta)\frac{\partial^4 w}{\partial x^4} = 0 \tag{1}$$

where w, ρ, A, and I are the displacement in Y-axis, the mass per unit length, the cross-sectional area, and the area moment of inertia of the composite steel beam, respectively. η is the loss factor. The general solution to Eq.(1) is given by

$$w = W\exp(j\omega t) \tag{2}$$

$$W = A_1\exp(-jk_bx) + A_2\exp(jk_bx) + A_3\exp(-k_bx) + A_4\exp(k_bx) \tag{3}$$

where W is the amplitude of displacement in Y-axis, A1-A4 are undetermined constants, and k_b is a complex wave number defined by

$$k_b = \left[\frac{\rho A \omega^2}{EI(1+\eta^2)^{1/2}} \right]^{1/4} (\alpha - j\beta) \tag{4}$$

$$\alpha = \cos\left(\frac{1}{4}\tan^{-1}\eta\right) \tag{5}$$

$$\beta = \sin\left(\frac{1}{4}\tan^{-1}\eta\right) \tag{6}$$

The boundary conditions at the clamped end(X=0) and the free end(X=L) are given by

Clamped end(X=0) $\quad W = 0 \quad$ (7)

$$\frac{dW}{dx} = 0 \tag{8}$$

Free end(X=L) $\quad \dfrac{F_0}{EI(1+j\eta)} + \dfrac{m\omega^2 W}{EI(1+j\eta)} = \dfrac{d^3W}{dX^3} \quad$ (9)

$$\frac{d^2W}{dx^2} = 0 \tag{10}$$

where m is the additive mass and ω is the circular frequency. Substituting Eqs.(3)-(6) into Eqs.(7)-(10), we obtain

$$A_1 = j(\cos k_bL - \sin k_bL + \cosh k_bL - \sinh k_bL)F/D \tag{11}$$

$$A_2 = j(\cos k_bL + \sin k_bL + \cosh k_bL + \sinh k_bL)F/D \tag{12}$$

$$A_3 = [-\sin k_bL - \sinh k_bL - j(\cos k_bL + \cosh k_bL)]\ F/D \tag{13}$$

$$A_4 = [\ \sin k_bL + \sinh k_bL - j(\cos k_bL + \cosh k_bL)]\ F/D \tag{14}$$

where

$$F = F_0 k_b / \rho A \omega^2 \tag{15}$$

$$D = 4\,[1+\cos k_b L \cosh k_b L + H(\cos k_b L \sinh k_b L - \sin k_b L \cosh k_b L)] \tag{16}$$

$$H = m k_b L / \rho A \tag{17}$$

Substituting Eqs.(11)-(17) into Eq.(2),(3), we can calculate the amplitude of the displacement. The frequency response curve of inertance of the absorber is found by defferentiating the displacement W exp($j\omega$t) twice with respect to time t and divided by the sinusoidal force F0 exp($j\omega$t) which acts on the free end.

1.3 RKU MODEL OF THE VIBRATION DAMPING COMPOSITE STEEL BEAM

The vibrating damping composite steel beam has three layers as shown in Fig. 3. The complex bending rigidity EI(1+jη) of the beam can be calculated by substituting Young's moduli E_1, E_3 and the thicknesses t_1, t_3 of the steel, the complex shear modulus G_1+jG_2 and the thickness t_2 of the viscoelastic layer into Eqs.(18)-(20).

The value of G_1+jG_2 used in this calculation is the reduced data of the modulus over the frequency range 10.0Hz to 2.0kHz at +24°C obtained from the measured data over the frequency range 0.03 to 80.0 Hz and the temperature range -30 to +50°C by using the temperature-frequency superposition principle[7].

$$EI(1+j\eta) = \frac{E_1 b t_1^3}{12} + \frac{E_3 b t_3^3}{12} + D E_1 b t_1 \left(\frac{t_1+t_3}{2} + t_2\right) \tag{18}$$

$$D = \frac{g E_3 t_3}{E_1 t_1 + g(E_1 t_1 + E_3 t_3)} \left(\frac{t_1+t_3}{2} + t_2\right) \tag{19}$$

$$g = \frac{G_1 + jG_2}{\omega E_3 t_3 t_2} \left[\frac{EI(1+j\eta)}{\rho A}\right]^{1/2} \tag{20}$$

2. OPTIMUM DESIGN OF THE ABSORBER

2.1 DETERMINATION OF THE OPTIMUM DIMENSIONS OF THE BEAM

The vibration absorber is applied to control the first mode of the rectangular aluminum plate(1000x1000x4mm) in this section. After measuring the resonance frequency f_0 and the equivalent mass M for the plate, The optimum values of the resonance frequency f_{opt} and loss factor η_{opt} of the absorber can be calculated using Eqs.(21)-(23)[1].

$$\mu = 2m/M \tag{21}$$

$$f_{opt} = \frac{1}{1+\mu} f_0 \quad (22)$$

$$\eta_{opt} = [3\mu/2(1+\mu)]^{1/2} \quad (23)$$

The values of f_0 and M can be evaluated by applying the half-power bandwidth method[9] to the frequency response curve of inertance, measured at the center of the plate where the maximum amplitude of the first vibration mode occurs. The frequency response curve measured is shown in Fig. 4. We obtain f_0=24.5Hz, M=2.88 kg, and η =0.0171. Assuming the additive mass m=110g (μ =3.82%), we obtain f_{opt}=23.7Hz, η_{opt}=0.235.

To achieve these values, the frequency response curve of inertance for several dimensions of the absorber were calculated. The resonance frequency f and the loss factor η of the absorber were estimated by applying the half-power bandwidth method to the frequency response curve of inertance calculated by using Eqs.(3)-(20). The calculation of the frequency response curves of inertance was carried out for the cases of t_2=30,50,70,100 μm and L=90,100,110,120 mm under the condition that the both of t_1 and t_3 are fixed at 1.6mm. The calculated results for t_2=70μm, L=100,110,120 mm are shown in Fig. 5. The optimum values of t_2 and L are determined by choosing their optimum combination so that the calculated results of f and η equal f_{opt} and η_{opt}, respectively. The relation between f and η of the absorber for each values of t_2 and L is shown in Fig. 6. This shows that f=24.9Hz , η =0.245 can be achieved by setting t_2=70μm, L=110mm.

2.2 CALCULATION OF THE REDUCTION OF VIBRATION AMPLITUDE

The frequency response curve of inertance of the rectangular aluminum plate with the absorber can be calculated from the values of the equivalent mass M , the spring constant K , and the loss factor η of the first vibration mode of the plate and that of the absorber as shown in Table 1.

The calculated results of the frequency response curve are shown in Fig. 7. The solid line represents a calculated result without the absorber. The dashed line represents a calculated one with the absorber setting t_2=70μm. The broken line represents a calculated one with the absorber setting t_2=30 μm. The former is tuned to the optimum value, and it reduces the vibration amplitude by about 20 dB. On the other hand, the loss factor of the latter is about half of the optimum value, and it causes 3 dB inferiority in the reduction in the vibration amplitude of the absorber.

3. COMPARISON OF CALCULATIONS WITH EXPERIMENT

3.1 THE INERTANCE TRANSFER FUNCTION

The measuring system of the dynamic behavior of the absorber is shown in Fig. 8 (the temperature of the thermostatic oven is set at +24°C). An absorber is made to realize the optimum dimension and the frequency response curve of inertance of the absorber is measured. The result is shown in Fig. 9. The values of the equivalent mass m, the spring constant k, and the loss factor η of the absorber can be obtained from the measured frequency response curve, and compared with the calculated result as shown in Table 2. These results show the close agreement, and it can be concluded that the present method is sufficiently accurate to predict the dynamic behavior of the absorber.

3.2 THE EFFECT OF THE VIBRATION ABSORBER

Figure 10 shows the frequency response curve of inertance at a center of the aluminum plate with the absorber. The solid line represents the calculated result and the broken line represents the measured result. The close agreement of the measured and calculated results suggests that the present method is sufficiently reliable to predict the reduction in the vibration amplitude at resonance.

CONCLUSION

In this paper, the effective method to design the vibration absorber using the vibration damping composite steel beam was proposed to tune the resonance frequency and the loss factor of the absorber to the optimum value. The following results were obtained;

(1) We proposed a method to design the vibration absorber consisting of the composite steel beam and the additive mass. The optimum values of the thicknesses of the steel and viscoelastic layers and the length of the beam can be determined by using this method.

(2) The dynamic behavior of the rectangular aluminum plate with the absorber can be predicted with a practical accuracy by using this method.

(3) The remarkable reduction of 25dB in the amplitude of vibration can be achieved by applying the absorber to the rectangular aluminum plate.

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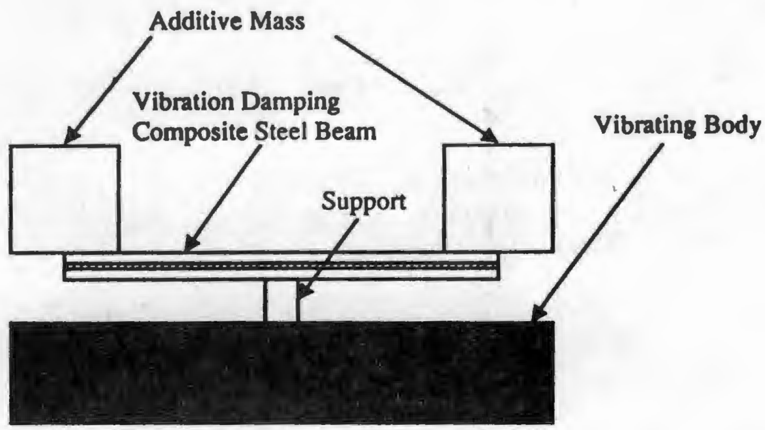

Fig. 1 Structure of Vibration Absorber

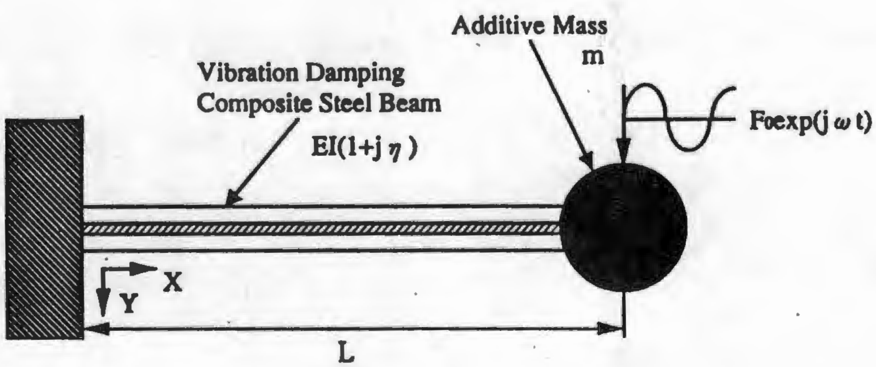

Fig. 2 Calculation Model of Absorber

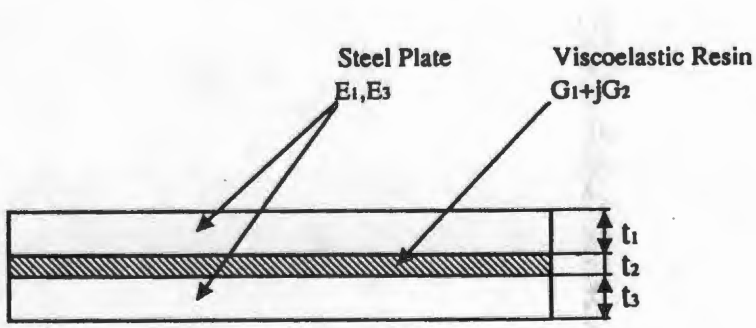

Fig. 3 Structure of Vibration Damping

Composite Steel Beam

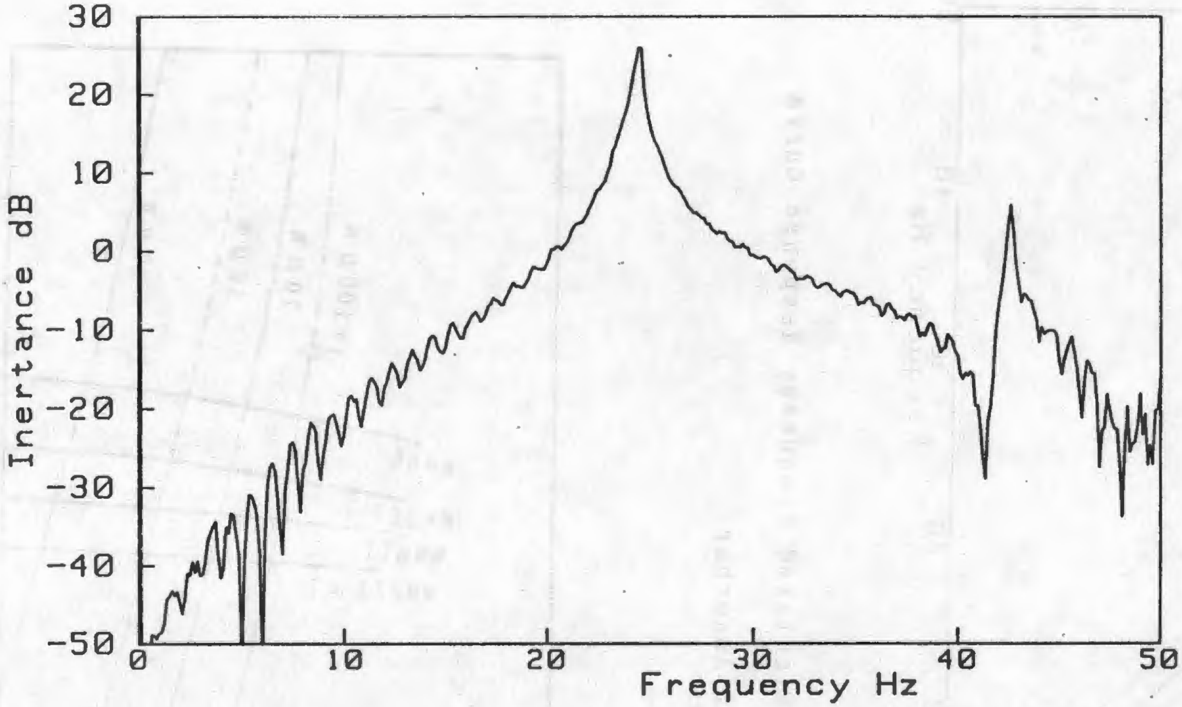

Fig.4 Measured Frequency Response Curve of Aluminum Plate

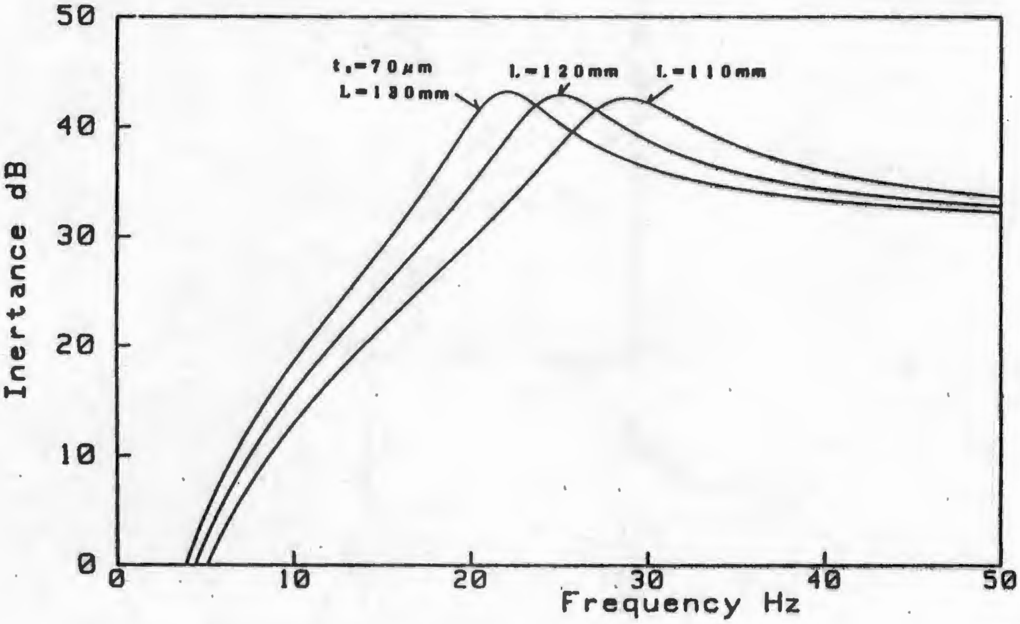

Fig. 5 Calculated Frequency Response Curve of Absorber

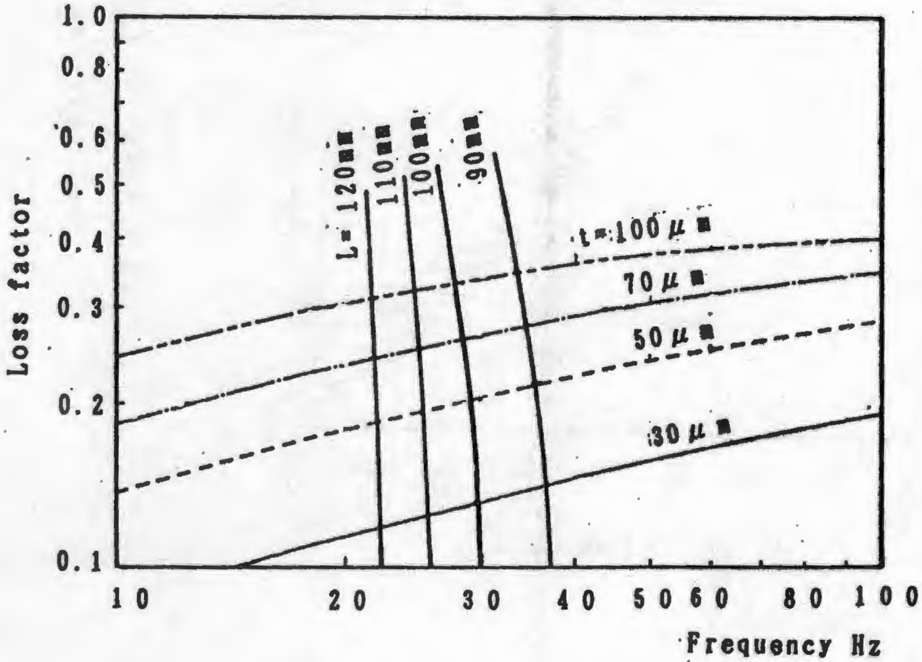

Fig. 6 Relation between Resonance Frequency and Loss Factor of Absorber

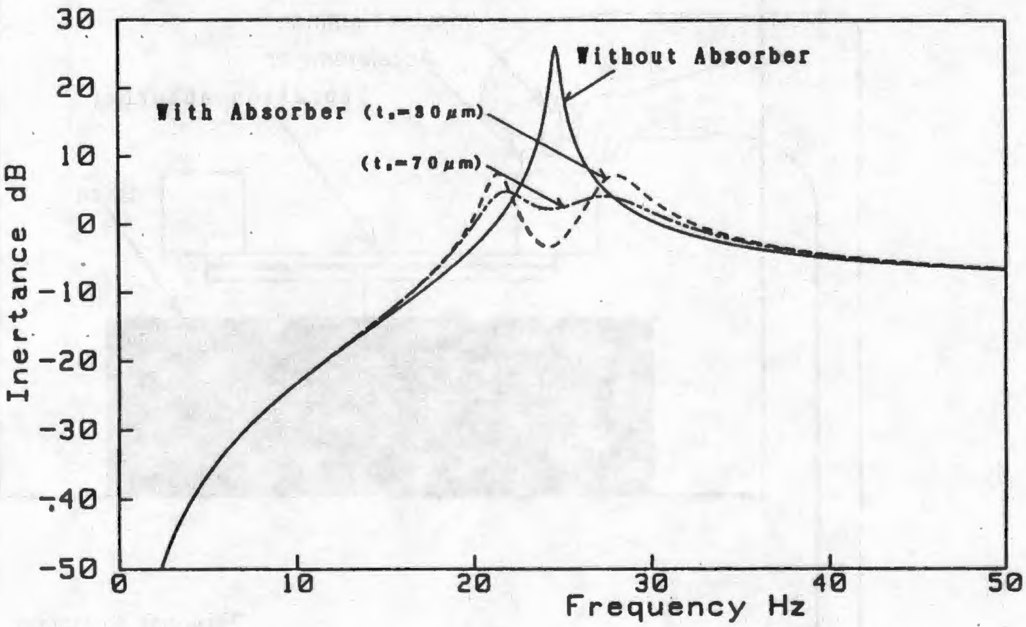

Fig. 7 Prediction of Reduction in Vibration by Absorber

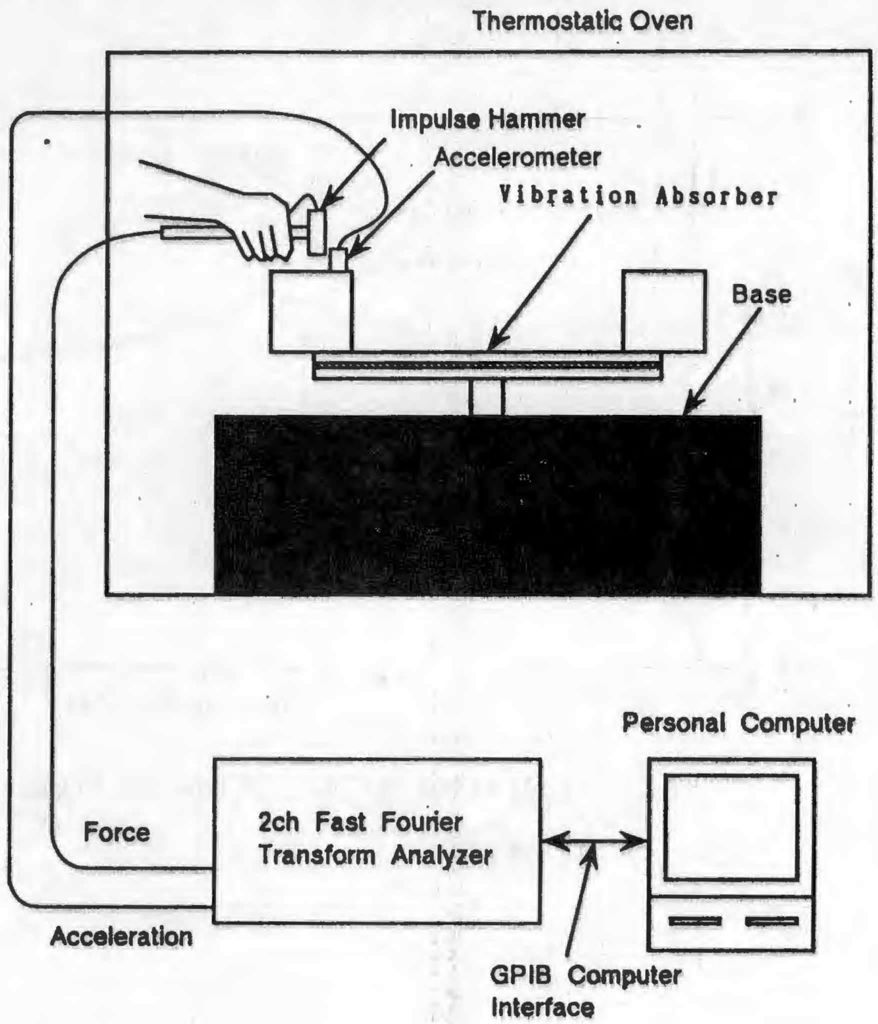

Fig. 8 Measuring System of Dynamic Behavior of Absorber

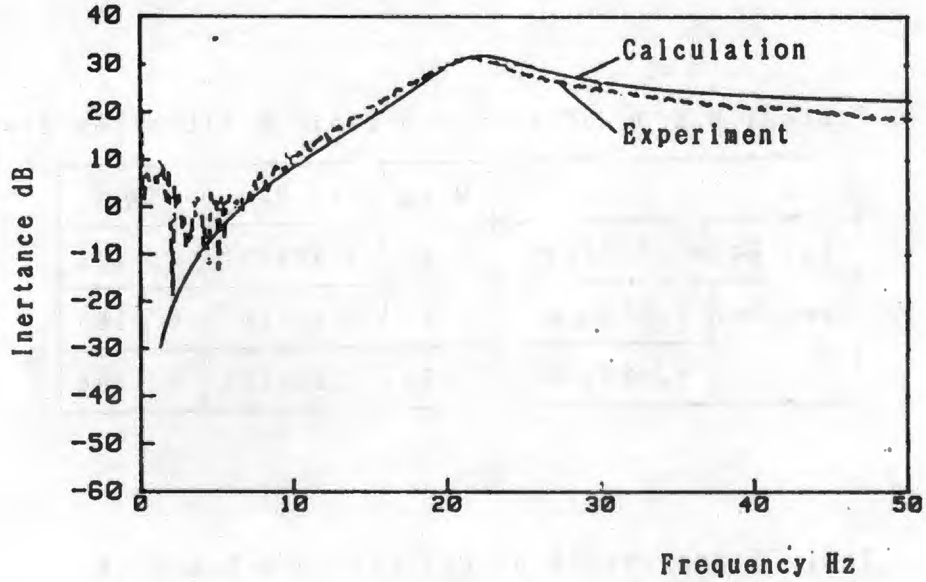

Fig. 9 Frequency Response of Vibration Absorber

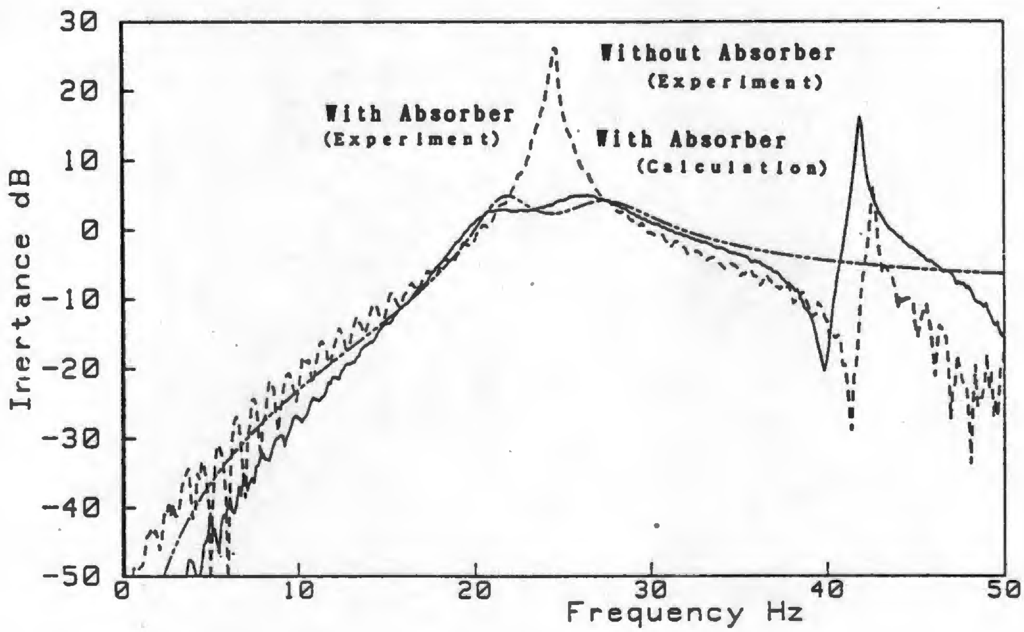

Fig. 10 Frequency Response of Aluminum Plate

Table. 1 M, K, η of Aluminum Plate & Vibration Absorber

		M kg	K N/m	η
1st Mode of Plate		2.88	6.90×10^4	0.0171
Absorber	$t_2 = 70\,\mu m$	.109	2.56×10^3	0.2450
	$t_2 = 30\,\mu m$	.108	2.54×10^3	0.1204

Table. 2 Comparison of Calculated & Measured Values of m, k, η of Absorber

	m kg	k N/m	η
Calculated	1.09×10^{-1}	2.56×10^3	0.2450
Measured	1.12×10^{-1}	2.74×10^3	0.2521