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**SYSTEMS FOR THE COLLECTION AND
ANALYSIS OF DYNAMIC DATA**

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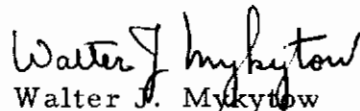
FOREWORD

This report was prepared by the Measurement Analysis Corporation, Los Angeles, California, for the Aero-Acoustics Branch, Vehicle Dynamics Division, AF Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, under Contract AF33(615)-1314. This study covers systems for the collection and analysis of dynamic data in conjunction with optimum utilization of the Sonic Fatigue Facility Data Analysis System. This research is part of a continuing effort to obtain significant information on sound environment simulation and dynamic response to acoustic excitation under the Research and Technology Division, Air Force Systems Command's exploratory development program. The Project Number is 4437, "High Intensity Sound Environment Simulation," and Task Number is 443706, "Advanced Instrumentation Study for Sonic Fatigue Experimental Work." Mr. W. K. Shilling III was the Project Engineer. MAC Report Number is MAC 402-05A.

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ABSTRACT

This report discusses the machine errors of equipment used for the conditioning, collection, recording, editing, and analysis of data from dynamic processes such as acoustics, strain, or vibration. The specific equipment items studied include special features of signal conditioners (e. g., automatic gain control), signal conditioner to tape recorder transmission line, magnetic tape recorders, gross analyzers (e. g., voltmeters), frequency analyzers, amplitude analyzers, and time domain analyzers. (The machine errors associated with transducers and basic signal conditioners are presented in AFFDL-TR-64-171, "Transducers for Sonic Fatigue Measurements.") Where these errors are controllable, such as the smoothing error associated with the rate of scanning an amplitude window for probability density measurements, pertinent equations are presented for determining the magnitude of these errors as a function of the controllable parameter(s). In addition, the report describes practical usage of the analysis equipment for the detection of non-normality, nonstationarity, and periodicity.

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LIST OF SYMBOLS

A	gain
B	bandwidth
c	velocity of light
C	capacitance
e	voltage
f	frequency
G(f)	spectral density
h	delay time resolution
H(f)	frequency response function
$ H(f) $	gain factor
i	current
j	$\sqrt{-1}$
l	length
L	inductance
p(x)	instantaneous amplitude probability density
R	resistance
R _s	scan rate
S ₂	accelerometer voltage sensitivity
(S/N) _{total}	total signal to noise ratio
t	time
T	analysis time duration
w	amplitude window width
\ddot{x}	acceleration
σ	standard deviation
ϵ	standard error
$\theta(f)$	phase factor
λ	wave length
ξ	interchannel time delay error
τ	commutated data sample duration
ϕ	phase error in degrees
ψ_x^2	mean square value

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SYSTEMS FOR THE COLLECTION AND ANALYSIS OF DYNAMIC DATA

1. INTRODUCTION

This report discusses the machine errors associated with systems used for the collection, recording, editing, and analysis of dynamic data. Such equipment items as special features of signal conditioners (e. g., automatic gain control), signal conditioner to tape recorder transmission line, magnetic tape recorders, gross analyzers (e. g., voltmeters), frequency analyzers, amplitude analyzers, and time domain analyzers are examined in detail. The machine errors associated with transducers and signal conditioners employed to measure dynamic environments such as acoustics, strain, and vibration are described in Reference 1.

The machine errors discussed in this report are concentrated primarily on the intrinsic errors of the machine. Environmental and human errors are not covered since these errors can be made negligible. Where appropriate, usage errors are discussed.

While the results presented in this report are quite general and are applicable to any equipment in the categories described, the specific hardware used for illustrative examples is that installed at the RTD Sonic Fatigue Facility of the Aero-Acoustics Branch of the Air Force Flight Dynamics Laboratory at Wright-Patterson Air Force Base.

Section 2 of this report describes certain special features occasionally found in signal conditioners. These special features include time sharing, automatic attenuation, automatic sensitivity normalization, and rapid calibration verification. Section 3 discusses the signal conditioner to tape recorder transmission line. Section 4 concentrates on the magnetic tape recorder errors. In this section, particular emphasis is placed on the phase errors since they so frequently represent the primary limitations on phase,

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impedance, cross-correlation, and cross-spectral analyses. Section 5 describes the gross analysis equipment such as oscillographs, voltmeters, and level recorders. Section 6 is devoted to the frequency analysis equipment. Octave, one-third octave, constant bandwidth frequency analyzers, and ordinary power spectral density analyzers are covered. Section 7 deals with amplitude analyzers covering instantaneous amplitude probability density, peak value probability density, and joint instantaneous amplitude probability density. Section 8 treats the time domain analysis equipment. In addition to discussing the errors associated with the analyzers, Sections 5, 6, 7, and 8 also describe the practical usage of these analyzers to detect non-normality, nonstationarity, nonlinearity, and periodicity.

2. SIGNAL CONDITIONERS

This section discusses the inaccuracies inherent in the special features of the signal conditioners at the RTD Sonic Fatigue Facility. These special features include:

- (1) Time sharing, or commutation
- (2) Automatic attenuation
- (3) Data normalization
- (4) Rapid calibration

2.1 TIME SHARING

The Sonic Fatigue Facility is designed to handle a maximum of three hundred forty-two different transducers with seventy-two signal conditioners. Thirty of the 72 signal conditioners used to measure data from the large test chamber at the Sonic Fatigue Facility have provisions for commutating up to ten channels of transducer outputs into each one of these 30 signal conditioner inputs. This permits the economic use of 30 signal conditioners to monitor as many as 300 transducer outputs. The penalty paid to obtain an increased number of data channels in this manner is a loss in the total monitoring time of any one channel. However, for a typical sonic fatigue test, it is quite reasonable to expect that the total test time is so long that it is economically desirable to record only during a portion of the total test time. Therefore, the above penalty should not be a severe one. Reference 2 discusses the statistical errors due to time sampling and the probabilities of missing significant events.

Figure 1 shows the commutation system. This system has two modes of operation. In the first mode, the output of any one of the ten transducers can be continuously connected to the signal conditioner input (no commutation). In the second mode of operation, the outputs of two to ten transducers are commutated to the input to the signal conditioner. The commutation pattern is restricted in that the commutation always starts

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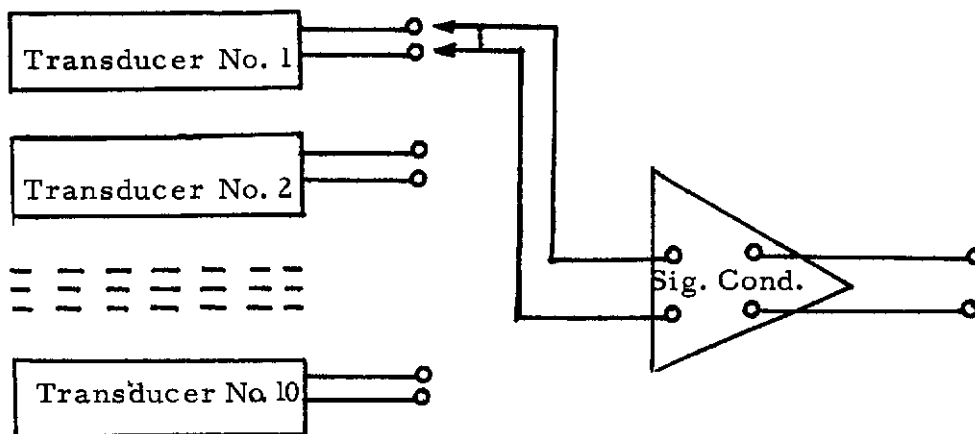


Figure 1. Block Diagram of a Commutated Data Channel

sampling on commutator position number 1, and samples the next $(n - 1)$ positions in series, where (n) is the total desired number of transducers to be sampled per signal conditioner. For example, if it is desired to sample four transducer outputs on a particular signal conditioner, the transducer outputs are connected to commutator positions 1, 2, 3, and 4. It is not possible to use position arrangements such as 3, 4, 5, and 6 or 1, 5, 7, 10 because the commutation must start on position number one and run through consecutive positions.

The commutator is designed so that the time spent in transition between contact positions is very small compared to the time spent on the contact positions. Whenever an electrical pulse is supplied to the coil of the stepping switch, the signal contacts advance one position and remain there until the next electrical pulse is applied. Ten different data sample durations, the time spent on any one commutator position, are available. These data sample durations are:

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1 sec	3 sec	5 sec	8 sec	12 sec
2 sec	4 sec	6 sec	10 sec	15 sec

The time required to make one revolution of the commutator is $n\tau$, where n is the number of commutator positions used and τ is the data sample duration in seconds. Thus, the minimum time for the commutator to make one complete revolution is 2 seconds ($n=2$, $\tau=1$ sec), and similarly the maximum time for the commutator to make one revolution is 150 seconds or 2.5 minutes ($n=10$, $\tau=15$ sec). Determination of the number of commutator positions to be used for any situation is relatively straightforward. In general, use the least possible number of commutator positions per signal conditioner. The reason for this is to maximize (for a given τ) the ratio of the time that the output will be monitored to the time that the output will not be monitored so as to decrease the possibility of missing significant events. For example, consider a case where it is desired to measure the outputs of 83 strain gages on the 30 signal conditioners with commutators. (Assume that the 42 other signal conditioners are required for measurement of some type of other data such as acoustic data.) If all 83 channels are equally important, then connect 23 of the signal conditioners with three transducers and seven signal conditioners with two transducers. Even if some operational condition restricts the above recommendation, it should be applied to the fullest extent possible. For example, it may occur that only 12 tracks of one magnetic tape recorder are available to record the above 83 transducer outputs. Then these outputs should be arranged so that eleven signal conditioners each have seven transducer outputs connected to their input and the one remaining signal conditioner should have six transducer outputs connected to its input.

Defining the manner of selecting the optimum time duration per sample is somewhat more complicated as this "optimum" time duration is a strong function of the characteristics of the data being measured, the type of analysis

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to be performed on the data, and the particular equipment that will be used to analyze the data. For a periodic function, the time duration theoretically needs only be as long as the period of the function. This may be an adequate sample if the data is analyzed by oscillographic means. However, if the rms value of the voltage is measured with a voltmeter, the data sample duration must be long enough for the meter to stabilize on the proper voltage and for the operator to read this voltage.

If the signal is random, and stationary, the time duration must be long enough to minimize statistical sampling fluctuations to the permissible level in addition to satisfying the analysis equipment requirements. To be specific, if the data are random, stationary, normally distributed, and have a spectral density function that is constant over a bandwidth B and zero outside of this bandwidth, the expression for the standard error of the measurement of the mean square value of the data as a function of the time duration of the sample is given by Reference 3 as

$$\epsilon = \frac{\text{s. d. } (\overline{x^2})}{\psi_x^2} = \frac{1}{\sqrt{BT}} \quad (1)$$

where

ϵ = the standard error

$\text{s. d. } (\overline{x^2})$ = the standard deviation in the measurement of the mean square value of the random function $x(t)$

ψ_x^2 = the true (infinite time measure) mean square value of the random process $x(t)$

B = the bandwidth in cps of the random process

T = the time over which the mean square value is measured (also the time that the commutator remains in any one position in the above simplified example)

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This means that for small ϵ , approximately 68% of the samples will have mean square values that will fall within $(1 \pm \epsilon)$ times the true mean square value, and approximately 95% will fall within $(1 + 2\epsilon)$ times the true mean square value. Since the bandwidth is generally dictated by other considerations, one must select the minimum value of τ that is sufficiently long to reduce the sampling fluctuations to the desired level.

For example, assume that it is desired to sample a random signal of 100 cps bandwidth in such a manner that approximately 95% of the samples will deviate less than 6% from the "true" mean square value of the signal. Then

$$(1 + 2\epsilon) \psi_x^2 = (1.06) \psi_x^2$$

$$\epsilon = .03$$

$$\epsilon = \frac{1}{\sqrt{BT}} = \frac{1}{\sqrt{(100) T}}$$

$$\frac{1}{\sqrt{(100) T}} = .03$$

$$T = 11.1 \text{ seconds}$$

Thus, select the 12-second sample duration setting on the commutator (assuming that no additional time is required to meet other data processing needs). Reference 3 discusses the basic considerations in selecting sample record lengths for other types of analyses.

The major intrinsic errors of the time sharing system are:

- (1) Switching transients
- (2) Switching time instability
- (3) Contact noise

Switching transients, the spurious signals at the output of the commutator during transition from one contact position to another, are not a serious

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problem by themselves as this transient portion of the data is not analyzed. However, there are two ways in which these switching transients can cause errors. First, operation of the commutator causes energy to be radiated from it. This energy can be picked up in adjacent signal conditioners as well as in the signal conditioner to which the commutator is attached. Spurious data in the form of "spikes" will appear in the output of these adjacent signal conditioners if they are commutated at a rate different from the rate of the above commutator, or are commutated at the same rate but at different times. The magnitude of the spikes is a function of both the strength of the radiation, and the susceptibility of the signal conditioners. As the commutators wear and oxidize, the radiation, particularly due to arcing, will gradually increase in magnitude. There is no method for correcting the data to account for these spikes. One can analyze only those segments that do not contain spikes. With thirty commutators operating at any of the 90 possible sampling rates (up to nine possible commutated inputs and ten possible sample durations), it may be difficult to find segments long enough to analyze if the radiation problems become severe. Therefore, it is strongly recommended that "dummy" channels be used. (A "dummy" channel is a measuring channel in which the real transducer has been replaced by a passive impedance that simulates the source impedance of the transducer and is insensitive to the environment being measured.) These dummy channels, recommended in Reference 1 for establishment of typical data channel noise levels, should be carefully monitored so that corrective action can be taken if "spikes" from commutator operation appear. Ordinary RFI suppression techniques can be used to remove these spikes. In addition, it is recommended that all commutators be operated at the same rate and at the same time (a common source of timing pulses). If the commutators are operated in this manner, the

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radiation should cause problems only in the uncommutated channels since all the commutators are operating at the same time and the radiation spike will be indistinguishable from the ordinary switching transient. Dummy channels can be used to equalize the number of inputs per signal conditioner.

The second method by which the switching transients of the commutator can cause errors is through saturation of the signal conditioner. If the magnitude of the switching transient occurring on the input to a signal conditioner is great enough to cause the output of the signal conditioner to appreciably exceed its rated full scale voltage or current level, there will be a period of time following the termination of the transient during which the signal conditioner output is not usable. This period of time is known as the recovery time of the signal conditioner. During this period of time, the output of the signal conditioner may saturate, drop to zero, or "motorboat" wildly between positive and negative extremes. The amplifiers used in the signal conditioner at the Sonic Fatigue Facility have recovery times quoted at less than ten microseconds for voltage overloads up to 100% and less than three seconds for "gross" overloads. No figures are quoted when the amplifier is used in conjunction with the impedance converter. Since the signal conditioner has a maximum gain of 10^6 , extreme care must be taken to prevent saturation due to switching transients when the signal conditioners are operated in the high gain positions. Actually, the record/reproduce electronics of the tape recorder are far more likely to saturate than the signal conditioner amplifiers. These signal conditioner amplifiers are capable of linear operation over a ± 50 volt range, and the record/reproduce electronics will be normally adjusted for a maximum input of ± 1 volt. When the signal conditioners are set at maximum gain, ± 1 volt at the input to the tape recorder corresponds to only

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± 1 microvolt at the input to the signal conditioner. Because the commutators are located at the input to the signal conditioners, saturation is most likely to occur when the signal conditioners have high gains. (The gain of the signal conditioners can be varied between 10^6 and 0.01.)

Instability of the switching time should not be much of a problem with the type of commutators used in the RTD Sonic Fatigue Facility because they spend a much greater time dwelling on a position than in transit between positions, and because identification of the commutator position is positive (one bank of a stepping switch is used to commutate the signal input and another bank is used to identify the commutator position). Only when the time instability becomes grossly erroneous, such as causing some of the data samples to be too short for proper analysis, does this become a problem. The sample durations should be periodically checked to assure that they are proper.

The commutators will decrease the signal to noise ratio of the measuring channels by adding contact noise at the input to the signal conditioners. For a maximum output of ± 1 volt from the signal conditioners, the total signal to noise ratio of the signal conditioner is, assuming both amplifiers in each signal conditioner have gains of 100,

$$(S/N)_{\text{total}} = \frac{1}{(100) \sqrt{[(100)(A)(z')]^2 + y^2}} \quad (2)$$

where

$$z' = \sqrt{e_a^2 + e_{b_1}^2 + e_{b_2}^2 + \left(\frac{e_c}{100}\right)^2} \quad (3)$$

$(S/N)_{\text{total}}$ = the ratio of the maximum allowable instantaneous voltage on the output of a signal conditioner to the rms noise voltage on the output of the same signal conditioner.

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A = the total "gain" in the normalizing and automatic attenuators.

y = the equivalent rms input noise voltage of the second amplifier.

e_a = the rms output noise voltage of the impedance converter.

(Note that the magnitude of this term is zero when the impedance converter is not used.)

e_{b_1} = the rms output noise voltage of the commutator.

e_{b_2} = the equivalent rms input noise voltage of the first amplifier.

e_c = the equivalent rms input voltage of the differential isolator.

2.2 AUTOMATIC ATTENUATION

All of the signal conditioners at the RTD Sonic Fatigue Facility are equipped with automatic attenuators. These devices sense the "peak"* output voltage level of the signal conditioners and then either add or subtract attenuation to constrain these output voltage level fluctuations of the signal conditioners to a 20 db range while permitting the full scale voltage level into the signal conditioner to vary over a 60 db range. Thus the signal presented to the tape recorder has a relatively constant voltage level and "optimum" use is made of the dynamic range of the tape recorder. Figure 2 is a block diagram of a typical signal conditioner channel.

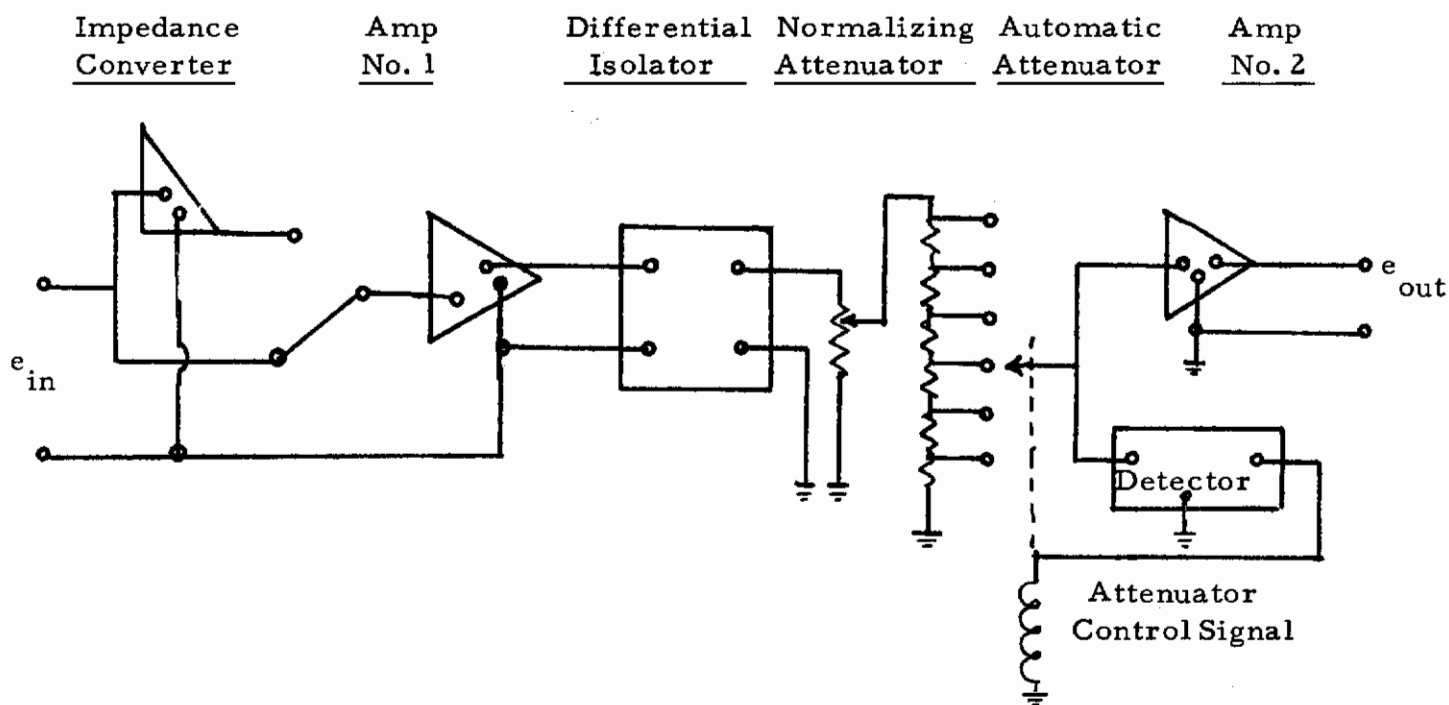


Figure 2. Block Diagram of a Typical Signal Conditioner

* The detectors in the automatic attenuators are not true peak detectors in the sense of measuring and storing the highest peak voltage occurring. The detectors used measure a quantity somewhat proportional to the average peak level. These detectors are discussed in more detail later in this section.

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Each of the six positions on the automatic attenuator represents a 10 db change in the voltage level. Assuming typical operating conditions, as described in Reference 4, the gain of both amplifiers will be 100, and the full scale output of amplifier number 2 will be ± 1 volt. This means that the full scale output of the attenuator will be ± 10 millivolts. The full scale input voltage to the attenuator can then vary from ± 10 mv to $+ 60$ db re ± 10 mv, or ± 10 volts.

This automatic attenuator provides considerable operational convenience as:

- (1) It is not necessary to "crystal-ball" the proper gain settings. The attenuators will adjust themselves to the proper gain.
- (2) Their operation is relatively "fool-proof" as erroneous calibrations will not cause loss of the data.
- (3) Time-varying signal levels can be followed without saturation of tape recorder electronics or disappearance of the signal into the noise level of the tape recorder electronics.

Of course, the above statements are limited to the 60 db full scale range of the attenuator. Because the signal conditioner has an approximately 13 db signal to noise ratio with zero attenuation (impedance converter being used and both amplifier gains equal to 100), the total dynamic range is $60 + 13$ db = 73 db. When the impedance converter is not used, the total dynamic range increases to 78 db. These above figures should be somewhat on the pessimistic side as the highest noise levels quoted in References 5, 6, and 7 were used without regard for bandwidth considerations. Even though the total dynamic range of the tape recorder is quoted as 49 db, it can clearly be seen that the automatic attenuator has substantially increased the usable total signal to noise ratio. Table 1 presents the total dynamic range of the signal conditioner (assuming the impedance converter is being used) as a function of the gain of the two amplifiers. For example, when the gain of the first amplifier is 30 and the

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gain of the second amplifier is 100, the total dynamic range of the signal conditioner is 83 db. Note that at very high gains, the signal to noise ratio is less than 60 db.

Gain of the First Amplifier	Gain of the Second Amplifier				
	10	30	100	300	1000
10	113	103	93	83	73
30	103	93	83	73	63
100	93	83	73	63	53
300	83	73	63	53	43
1000	73	63	53	43	33

Table 1. Signal Conditioner Dynamic Range in db Versus Amplifier Gain Settings

The inaccuracies inherent in the automatic attenuator are quite similar in many respects to those of the time sharing equipment, because both devices use stepping switches. The contact noise and radiated noise problems will be identical in nature. The contact noise should be, in general, less of a problem with the attenuator than with the commutator since there is less gain following the attenuator. This is not true for the radiated noise which depends on the radiation strength and the equipment susceptibility. In addition, the radiated noise from the attenuator is likely to present more of a problem because the operation of the attenuators cannot be synchronized. Hence, the operation of the automatic attenuators of the various channels may be occurring randomly in time. This is another reason why it is a good practice to use "dummy" measuring channels.

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The switching transient in changing from one attenuation setting to another is not a problem in itself since this time is accurately identified by a code signal representing the attenuator position. This code signal is recorded on the same magnetic tape (different track) as the output of the signal conditioner. When the tape is played back for analysis, the code signal is reproduced simultaneously. The data are not analyzed when the code signal indicates that the attenuator position in that channel is changing. If the magnitude of the attenuator switching transient is sufficiently high to cause saturation of the signal conditioner, then data are lost during this saturation period and the following recovery time period as discussed for the commutation equipment.

The two most serious problems in the operation of the automatic attenuator are likely to result in loss of data through failure of the attenuator to track the level change accurately and possible "jitter" in the automatic attenuator. These automatic attenuators have selectable "decision-time" settings of:

0.5 sec 3 sec 10 sec

The decision times refer to the maximum number of seconds required for the automatic attenuator to increase the gain of the signal conditioner by one 10-db step. The time required to decrease the gain of the signal conditioner by 10 db is equal to the time required for the mechanical elements in the decision circuitry (meter and relays) to respond to a +10 db increase in the signal level and is the same for all of the above settings. The decision circuit operates by rectifying the data signal, performing a "peak" averaging, and indicating the "peak" value on a meter containing two preset meter relays. When the indication on the meter increases 10 db above the nominal value, it coincides with the setting of the high level meter relay thereby causing it to actuate and increase the attenuation in the signal conditioner circuit. Likewise, when the signal level decreases 10 db below the nominal value, the low level meter relay is actuated and the attenuation in the signal conditioner circuit is decreased.

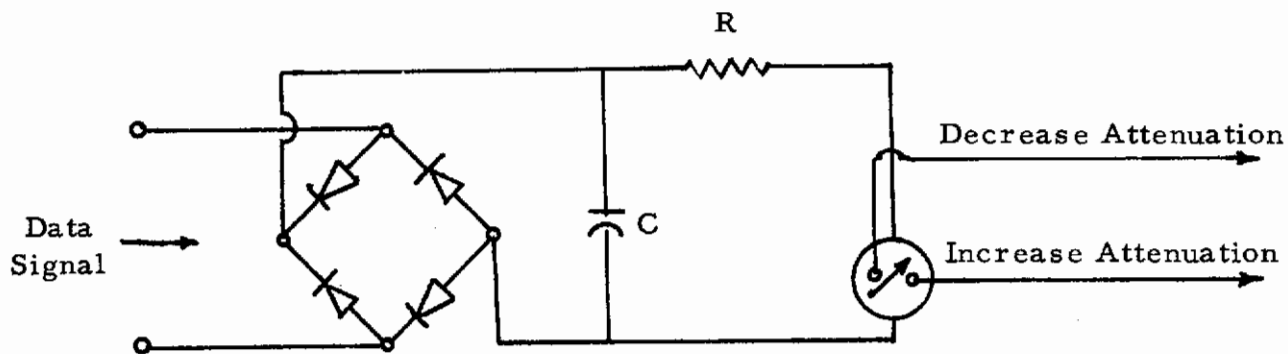


Figure 3. Simplified Schematic Diagram of the Decision Circuit

Figure 3 is a schematic diagram of the decision circuitry. The time required to decrease attenuation is greater than the time required to increase attenuation because the detector is of the "peak", or more properly average peak, variety. When the voltage level of the data signal is increased, the capacitor C is charged through the forward resistance of the diode bridge. Because this forward resistance is low, the charging time is quite small compared to the response time of the mechanical elements (meter and relay). When the voltage level of the data signal is decreased, the capacitor discharges through the parallel combination of the resistor R and the back resistance of the diode bridge. (The meter resistance is a part of the resistance R in Figure 3.) Since the magnitude of the back resistance of the diode bridge is much greater than the magnitude of the resistor R , the current discharged through the diode bridge can be neglected. The total response time then is determined by the product of R and C plus the response time of the mechanical elements. (In the actual attenuator circuit, the decision time is varied by changing the value of C .) The circuit can be redrawn as shown in Figure 4 and can be treated as a linear circuit as long as the polarity of the input does not change.

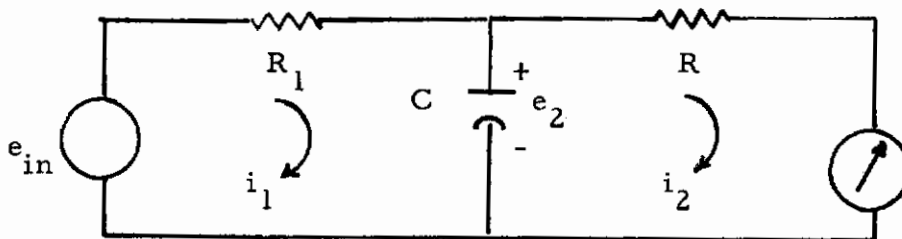


Figure 4. Equivalent Circuit for Figure 3

R_1 = the resistance in ohms across the diode bridge

e_{in} = a DC voltage

e_2 = the initial voltage across the capacitor

The current through the meter in Figure 4 is

$$i_2(t) = \left(\frac{e_{in}}{R + R_1} \right) \left[1 - e^{-\left(\frac{1}{RC} + \frac{1}{R_1 C} \right) t} \right] + \left(\frac{e_2}{R} \right) e^{-\left(\frac{1}{RC} + \frac{1}{R_1 C} \right) t} \quad (4)$$

when e_{in} decreases so that it is less than e_2 , R_1 becomes the back resistance of the diode bridge. In the limit, $R_1 = \infty$ and

$$i_2(t) = \left(\frac{e_2}{R} \right) e^{-\left(\frac{1}{RC} \right) t} \quad (5)$$

Thus, the response time is governed by the combination of R and C . On the other hand, when e_{in} increases appreciably above e_2 , R_1 becomes the forward resistance of the diode bridge. Then in the limit, $R_1 = 0$ and

$$i_2(t) = \frac{e_{in}}{R} \quad (6)$$

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In this case the response is instantaneous. In practice, the forward resistance of the diode bridge will not be zero; hence, the response time will be determined by the combination of R_1 and C, although this response time will be small compared to the response time of the mechanical elements. The actual response time in these automatic attenuators is a function of the level of the signal in addition to the value of the resistance-capacitance combination. The closer the meter reading is to the relay setting, the less time there will be required to actuate that relay. For example, the time required for a 10 db change is 1.15 time constants while the time required for a 20 db change is 2.3 time constants.

A fair amount of judgment must be exercised when selecting a decision time setting. If the decision time is too long, data will be lost when the signal level decreases rapidly. Two examples of cases where this could occur are:

- (1) Measurements where the excitation frequency is swept through a narrow band anti-resonance.
- (2) Measurements where the data are commutated at a rate fast compared to the decision time.

In both cases the advantages of the automatic attenuator are lost. However, no penalty is paid over the operation of a fixed gain amplifier, and one still has the advantage of being able to measure high signal levels without saturation. To the greatest extent possible, one should constrain the signals on any one commutated signal conditioner to those that are expected to have similar voltage levels and similar time histories.

However, if the decision time setting is too low, there are circumstances where the automatic attenuator can become a liability. Specifically, if narrow band random data are being measured, the automatic attenuator may tend to track the envelope of the data. This could result in jitter (non-periodic scillation) in the setting of the automatic attenuator. This jitter should generally only occur with short decision times and very narrow band random noise. (This problem may also occur in those rare cases where the data is

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random and its voltage level coincides almost exactly with one of the meter relay settings.) When the data are narrowband, a rather long sample length is required to perform a statistically significant analysis. If the setting of the attenuator is constantly fluctuating, it will be impossible to obtain a long enough sample for analysis. The minimum requirements on the data signal to prevent jitter should be empirically determined for each of the decision time settings because it is desirable to use the shortest decision time permissible in order to be able to make full use of the automatic attenuator in following the significant fluctuations in the level of the data.

In addition to decision times of 0.5, 3, and 10 seconds, there is a decision time switch position marked "continuous." When this position is selected, a resistor is switched into the circuit between the diode bridge and the "10 second" position circuitry. The resulting circuit diagram is the same as shown in Figure 4 except that R_1 is now the resistance of the diode bridge plus the value of the above resistor. Equations (4) and (5) still apply, but when the input voltage is increased, the current through the meter in Figure 4 is

$$i_2(t) \approx \frac{e_{in}}{R} \left[1 - \left(1 - \frac{e_2}{e_{in}} \right) e^{-\left(\frac{1}{R_1 C} \right) t} \right] \quad (7)$$

since $R_1 \ll R$. Thus, the current into the meter is "smoothed" for increasing signal voltage levels in addition to decreasing levels when the "continuous" setting is selected.

2.3 DATA NORMALIZATION

The signal conditioners at the Sonic Fatigue Facility are designed to minimize the pretest setup time. One of the main time saving features is the servo-controlled sensitivity normalization. This device adjusts the gain of each signal conditioner so that some desired system sensitivity is obtained. A transducer is excited at the desired full scale excitation or some 10 db multiple of this. The gain controls on the two amplifiers and the position of the automatic attenuator of the transducer channel are set to give a predetermined gain. A hand-held servo gun is plugged into the output of the signal conditioner. This servo gun is then held against the shaft of the potentiometer in the normalizing attenuator circuit. (See Figure 2.) The output from the signal conditioner is compared against a predetermined reference voltage. The difference voltage between the signal conditioner output and the reference causes the motor in the servo gun to rotate the shaft of the normalizing potentiometer, thus changing the gain of the signal conditioner, until a null difference voltage is obtained. The channel is then normalized.

To illustrate how this circuitry operates, consider the following example. Assume that it is desired to have a 1 volt rms signal out of the signal conditioner and into the tape recorder when an accelerometer is vibrated at 20 g's rms. Further, assume that it is desired to have the automatic attenuator set at the -30 db position under the above conditions, that only a 2 g rms acceleration calibration is available, and that the following circuit parameters apply:

S_2	= accelerometer voltage sensitivity	= 2.37 $\frac{\text{mvrms}}{\text{grms}}$
C_a	= accelerometer internal capacitance	= 1500 pf
C_i	= signal conditioner input capacitance	= 45 pf
C_c/ft	= accelerometer to signal conditioner cable capacitance per ft.	= 33 pf/ft
l_c	= length of the accelerometer to signal conditioner cable	= 100 ft.

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To perform the sensitivity normalization, the automatic attenuator is set on the -10 db position because the level of the calibration signal is 20 db below the desired 20 g input. (The automatic attenuator is returned to the -30 db position after the normalization has been completed.) The gain of the second amplifier is normally left on the x100 position and will be in this example. To find the gain position of the first amplifier, calculate the total required gain in the signal conditioner. The voltage developed at the input to the first amplifier, e_2 , for a 20 g rms input, (x), is, from Reference 1

$$\begin{aligned} e_2 &= (x)(S_2) \left[\frac{C_a}{C_2 + C_i + C_c} \right] \\ &= (20)(2.37 \times 10^{-3}) \left[\frac{1500 \times 10^{-12}}{1500 \times 10^{-12} + 45 \times 10^{-12} + (100)(33 \times 10^{-12})} \right] \\ &= 14.7 \times 10^{-3} \text{ volts rms} \end{aligned}$$

When the output of the signal conditioner is 1 volt rms and with a 30 db attenuation between the two amplifiers in the signal conditioner, the total gain required is

$$\text{Total Gain} = \left(\frac{1 \text{ volt rms}}{14.7 \times 10^{-3} \text{ volts rms}} \right) (31.62) = 2.15 \times 10^3$$

Because the gain of the second amplifier is 100, the gain of the first amplifier must be $2.15 \times 10^3 / 100 = 21.5$. Therefore, the x100 gain position is selected for the first amplifier. Application of the calibration signal to the accelerometer and the servo gun to the normalizing attenuator will cause the total gain of the signal conditioner to be adjusted to 2.15×10^3 as desired.

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As long as the calibration excitation level is measured accurately and the calibration is recorded on magnetic tape prior to the test, slight misadjustments in the setting of the normalizing potentiometer will be unimportant since a record exists of the true calibration. Assume that in the above example a 2.07 g's rms calibration had been applied and the resulting input to the tape recorder was 0.94 volts rms. The actual sensitivity $\left(\frac{0.94 \text{ volts rms}}{2.07 \text{ g's rms}} = .454 \frac{\text{volts rms}}{\text{g rms}} \right)$ is easily obtained as long as the exact excitation and output levels are recorded. If one assumes that the excitation, output voltage, or both will be some particular value, inaccurate sensitivity calibration can occur. It is strongly recommended that the level of the transducer excitation be accurately measured and recorded in some careful bookkeeping form and that the output voltage representing the calibration be recorded on magnetic tape.

When used with strain gage transducers, the sensitivity normalizing circuitry operates in a slightly different manner than has just been described. Because it is generally not physically possible to subject the strain gages to a calibrated strain level after the gages have been cemented to the test specimen, the calibration is performed artificially by shunting one of the Wheatstone bridge arms with a calibration resistor. (For details on this type of calibration, see Reference 1.) Except for the fact that the artificial calibration produces a DC signal, the normalizing circuit operation is the same as previously described. The one other operational difference in the normalization of strain gage transducer channels is that the channels require balancing of the bridge output. The servo gun also performs this operation. The output of the strain-gage bridge with no load, real or artificial, applied is compared to a predetermined reference level, usually zero. The difference between the output and reference voltage is used to rotate the shaft of the servo gun which is held against the balance potentiometer in the bridge excitation and balance unit. In this manner, the output of the bridge is balanced since the

servo gun will rotate the balance potentiometer until there is a zero difference between the output and reference voltages. As with calibration voltages, the balanced output voltage should be recorded on magnetic tape in case the normalizing circuitry is not operating perfectly or in case there may be defects following the normalizing circuitry.

2.4 RAPID CALIBRATION

The signal conditioners at the Sonic Fatigue Facility have a rapid calibrate mode of operation so that their operational status can be quickly checked. In this mode, the transducers are disconnected from the inputs to the signal conditioner and the normalizing potentiometer is bypassed. A calibrated voltage signal is connected to the inputs of all of the signal conditioners. The output of each signal conditioner is monitored by a GO-NO GO device in the control room. Those signal conditioners whose outputs fall outside predetermined tolerances are pointed out in this manner. This rapid calibration feature provides an extremely convenient way to check the gross operation of all of the signal conditioners just prior to, immediately after, and in some cases even during tests.

3. SIGNAL CONDITIONER TO TAPE RECORDER TRANSMISSION LINE

The cable connecting the outputs of the signal conditioners to the inputs of the tape recorders can contribute measurement errors in three different ways. First, there can be cross-talk between cables. Second, there can be ground loops introduced, and third, there can be a nonconstant frequency response function between the signal applied at the signal conditioner end of the cable and the signal measured at the tape recorder end of the cable. The first two possible error sources should be classified as usage errors because they can be eliminated or reduced to acceptably low values through re-routing of the cables. Since these two errors have been discussed in Section 2.6 of Reference 1, they will not be considered again.

The nonconstant frequency response function is definitely an intrinsic error in the cable. The magnitude of this error is a function of the length of the cable, the impedance characteristics of the cable, and the impedances at both ends of the cable. To determine the possible magnitude of this error, worst case values of impedance, data frequency, and cable lengths reasonable for the Sonic Fatigue Facility are assumed and the magnitude of the gain and phase factors are calculated.

The cable has distributed impedances and considerable simplification in the calculations can be made if these impedances can be treated as lumped parameters. To do this, a quarter cycle of the shortest wave length must be much greater than the length of the cable. The highest frequency of interest is 10KC. Thus, the shortest quarter wave length ($\lambda/4$) is

$$\begin{aligned}\frac{\lambda}{4} &= \frac{c}{4f} & (8) \\ &= \frac{3 \times 10^8 \text{ meters/sec}}{4 \times 10^4 \text{ cps}} = 7500 \text{ m}\end{aligned}$$

$$\approx 25,000 \text{ feet}$$

where c = the velocity of light.

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Since the longest signal conditioner to tape recorder cable length at the Sonic Fatigue Facility is approximately 250 feet, and the shortest quarter wave length is approximately 25,000 ft., the lumped parameter approximation of the cable impedances is reasonable. Figure 5 shows a lumped parameter equivalent circuit of the signal conditioner output, cable, and tape recorder input impedances.

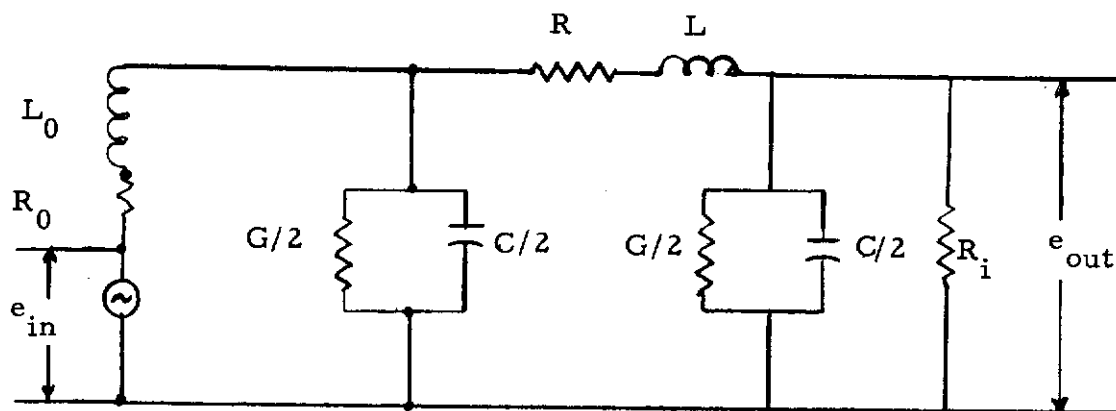


Figure 5. Lumped Parameter Equivalent Circuit of the Signal Conditioner to Tape Recorder Cables

The following impedance values will be assumed and applied to a 250 foot cable.

R_0	= the resistive component of the signal conditioner output impedance	= 0.1Ω
L_0	= the inductive component of the signal conditioner output impedance	= $160 \mu\text{h}$
G	= the shunt conductance of the cable (negligible)	= 0 mhos
C	= the shunt capacitance of the cable (30 pf/ft)	= $.0075 \mu\text{f}$
R	= the series resistance of the cable ($.016 \Omega/\text{ft}$)	= 4Ω
L	= the series inductance of the cable ($0.2 \mu\text{h}/\text{ft}$)	= $50 \mu \text{ henries}$
R_i	= the input resistance of the tape recorder	= $10,000 \Omega$

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The frequency response function, $H(f)$, for the circuit shown in Figure 5 is

$$H(f) = |H(f)| \angle \theta(f) = \frac{e_{\text{out}}}{e_{\text{in}}} = \frac{1}{ab - cd} \quad (9)$$

where

$|H(f)|$ = the gain factor

$\theta(f)$ = the phase factor

$$a = \left[\frac{1}{(R_i) \left(\frac{1}{j\omega C} \right)^2} \right]$$

$$b = \left[\left(R_0 + j\omega L_0 + \frac{2}{j\omega C} \right) \left(R + j\omega L + \frac{4}{j\omega C} \right) \left(R_i + \frac{2}{j\omega C} \right) \right]$$

$$c = \left[\frac{1}{R_i} \right]$$

$$d = \left[R_i + \frac{2}{j\omega C} + R_0 + j\omega L_0 + \frac{2}{j\omega C} \right]$$

For the above circuit impedance values and an operating frequency of 10 KC, the resultant gain factor is

$$|H(f)| = 1.00$$

and the resultant phase factor is

$$\theta(f) = 0^\circ$$

Thus, even with the conservative values assumed, there is negligible signal attenuation or phase shift up to the upper data frequency limit of 10 KC.

4. TAPE RECORDERS

Because of the immense quantities of data that can be measured at the Sonic Fatigue Facility, it is impossible to analyze all of the data in real time during the test. Therefore, it is necessary to record the data during the test and analyze it at a later date. Magnetic tape recorders using a standard FM recording technique, where full scale input voltage to the tape recorder causes a 40% deviation of the carrier frequency, are employed at the Sonic Fatigue Facility for data storage. From the viewpoints of accuracy and operational convenience, this technique is the best presently available. However, there are a number of intrinsic inaccuracies in this type of a recording process. These inaccuracies are due to amplitude nonlinearities, nonconstant gain factor response, nonconstant phase factor response, internal noise, and drift in the sensitivity and DC level.

4.1 AMPLITUDE NONLINEARITY

The magnitude of the frequency response function, $H(f)$, of a tape recorder at any frequency f_0 is the ratio of the rms output voltage of the tape recorder to its rms input voltage.

$$\left| H(f_0) \right| = \frac{e_{\text{out-rms}}}{e_{\text{in-rms}}} \quad (10)$$

Ideally, this ratio should be a constant. Any deviation of this ratio from a constant as a function of the input amplitude is defined as an amplitude non-linearity. The use of the data to frequency modulate a carrier frequency bypasses most of the amplitude nonlinearities inherent in the direct recording of data on to magnetic tape. Amplitude linearity of the carrier is not extremely important since the amplitude of the data is contained in the frequency deviation of the carrier. The main amplitude nonlinearity in a magnetic tape recorder using FM recording occurs in the record electronics.

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Full scale input voltage requires the record electronics to deviate the carrier frequency by 40%. This large deviation of the carrier frequency minimizes the effects of tape speed fluctuations, (these effects are discussed in Section 4.4), but moderately sophisticated design techniques are required to obtain an FM modulator that can linearly deviate its carrier frequency by $\pm 40\%$.

The type of amplitude nonlinearity most common in FM recording is saturation. The record electronics resist deviating the carrier frequency beyond $\pm 40\%$ so that $|H(f_0)|$ decreases as e_{in} increases. This saturation turns into hard clipping as e_{in} is increased. One need only note that at 2.5 times the full scale input voltage, the carrier would be deviated by 100% if the process were linear and one polarity of input voltage would result in the carrier being modulated down to zero frequency. Further increase in this voltage would require the electronics to generate negative frequencies. Obviously, real electronics will limit, or clip, before reaching either of these points. This limiting becomes quite important in the measurement of the extreme values of random phenomena, and must be kept in mind at all times when selecting the sensitivity (% deviation/ e_{in} o-p) of the tape recording process. The amplitude linearity of the tape recorders at the Sonic Fatigue Facility are quoted at less than $\pm 1\%$ up to $\pm 40\%$ deviation of the carrier frequency. See Reference 8.

4.2 GAIN FACTOR RESPONSE

Any variation in the frequency response function of the tape recorder, $H(f)$, as a function of the frequency of the input signal, is defined as a non-constant frequency response. $H(f)$ is a complex quantity that can be separated into two scalar quantities as follows.

$$|H(f)| = H(f) \angle \theta(f) \quad (11)$$

where $|H(f)|$ is called the gain factor and $\theta(f)$ is called the phase factor. The gain factor was defined in Eq. (10) of the preceding section. The variation in the gain factor of a tape recorder with frequency is fairly well-defined and is primarily a function of the design of the tape recorder. Specifically, the gain factor of the lowpass filter in the output section of the reproduce electronics is the main factor causing a nonconstant gain factor in the tape recorder. The gain factor of the tape recorders at the Sonic Fatigue Facility is given as constant within $\pm 1.2\%$ at any data frequency from DC to 0.185 times the carrier frequency (Reference 8).

4.3 PHASE FACTOR RESPONSE

The phase factor, $\theta(f)$, is the difference in phase between the output voltage and the input voltage of the tape recorder as a function of the frequency of the input voltage. Variation in the phase factor as a function of frequency results from two sources. The first is electrical. These electrical phase shifts occur in both the record and reproduce electronics. The primary source of electrical phase shift is in the lowpass filter of the output section of the reproduce electronics. This phase shift is linear with frequency within $\pm 5^\circ$ from DC to 0.185 times the carrier frequency (Reference 8).

The second source of phase shifts is mechanical. These mechanical phase shift sources can be separated into two categories, absolute phase shifts, and relative phase shifts (or, more commonly, intra and interchannel time

delay errors). References 9, 10, and 11 discuss these phase errors in detail. Quantitative values used below are from these references. The intrachannel time delay errors refer to the phase shift between the input and output of any one tape track and the interchannel time delay errors refer to the phase shifts between the outputs of different tape tracks for a common input. Both of these categories of mechanical phase shifts can be further separated into static and dynamic time delay errors.

The static intrachannel time delay errors can result from the spacing between the record and reproduce heads, long term tape speed inaccuracy, and/or tape stretch. The first item is usually properly ignored because it is rare that one is interested in measuring the phase shift between the input to the tape recorder and the output simultaneously. Long term speed inaccuracy results when there is a difference between the average velocity of the tape during recording and the average velocity of the tape during playback. Tape stretch can result from the tape tension, temperature, and relative humidity variations between the time of recording and reproducing the tape. A reasonable value for the total long time speed inaccuracy due to tape speed and tape stretch in a normal laboratory environment would be about 0.3%. This can be reduced by 2 to 3 orders of magnitude by using a tape speed control servo, but at the expense of increasing the short time speed inaccuracies that contribute to the dynamic intrachannel time delay errors.

Short term speed inaccuracies (flutter) are responsible for the dynamic intrachannel time delay errors. These dynamic errors appear as oscillations about the long term error. No flutter specifications are given in Reference 8 for the magnetic tape recorders at the Sonic Fatigue Facility, but values from 0.2% to 1% peak-to-peak flutter are typical (the value decreases as the tape speed increases). Use of a speed control servo can increase these errors by a factor of 2 up to almost two orders of magnitude. This results because the servo system with all of the inertia of the mechanical tape drive is unable to follow rapid changes in the tape speed.

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The intrachannel time delay errors are quite important when it is necessary to compare the output of the tape recorder to absolute time, phase, or frequency, and also from the viewpoints of the noise and the DC voltage offset created. However, in most cases, phase measurements are generally made between data recorded on two or more tracks of the same tape.

(Phase measurements between tapes can be made by recording one common data channel on all tapes.) Thus, interchannel time delay errors are usually of prime importance for phase, cross-correlation, cross-spectral density, etc., measurements.

The static interchannel time delay measurements result from all of the following items:

- (1) head stack scatter
- (2) tape stretch between head stacks
- (3) gap scatter
- (4) mean gap azimuth alignment
- (5) tape curvature
- (6) fixed skew

The first two items above occur only with interleaved heads. Interleaved heads (record or reproduce) are composed of two head stacks. If the tracks are numbered in sequence, all of the even-numbered tracks are contained on one stack and all of the odd-numbered tracks are contained on the other stack. The reason for interleaving the heads is to increase the distance between physically adjacent tracks on the head stack in order to reduce the cross-talk between tracks. Tape recorders built to IRIG specification No. 106-60 (such as the tape recorders at the Sonic Fatigue Facility) have their head stacks separated by 1.500 inches. The tolerance on this dimension is specified at $\pm 1000 \mu$ inches, but many tape recorder manufacturers hold this tolerance to $\pm 500 \mu$ inches. One inch wide magnetic tape with a 1-1/2 mil polyester base will stretch about 0.015% per ounce of tension. Since many instrumentation tape recorders maintain their nominal tension within ± 2 oz. across the

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heads, a tape stretch between the heads of about $\pm 450\mu$ inches is possible due to tension variations. The above tape will stretch about .0015% per $^{\circ}\text{F}$ and about .0011% per % relative humidity. Assuming a temperature variation of $\pm 5^{\circ}\text{F}$ between the recording and playback process, and a $\pm 5\%$ relative humidity variation, the corresponding tape stretch values are about $\pm 113\mu$ inches and $\pm 83\mu$ inches.

Deviation of individual record, or reproduce, head gaps on any one head stack from a straight line drawn through the mean position of the gaps on that head stack is known as gap scatter. The standard tolerance on gap scatter is $\pm 50\mu$ inches. Deviation of this same straight line through the mean gap position of a head stack from perpendicularity to the tape path line can result from mean gap azimuth alignment errors, tape curvature, and fixed tape skew.

The tolerance on mean gap azimuth alignment is ± 1 minute of arc. Tape curvature figures are usually lumped within the above tolerance because it is difficult to measure the separate contributions. The center to center spacing of adjacent tracks on the tape is 70 mils, the center to center spacing of adjacent heads on the same head stack (interleaved heads) is 140 mils, the center to center spacing of outside heads on the same head stack (interleaved heads) is 840 mils, and the center to center spacing between outside tracks for 14 track one-inch wide magnetic tape is 910 mils. The corresponding displacement errors for one minute of arc are $\pm 20\mu$ inches, $\pm 41\mu$ inches, $\pm 245\mu$ inches, and $\pm 265\mu$ inches, respectively.

Fixed skew can cause a static time delay error of 100μ inches per inch of tape width. The corresponding time delay errors for adjacent tape tracks, adjacent heads (interleaved), outside heads on a stack (interleaved), and outside tape tracks (14 channel one inch wide tape) are $\pm 7\mu$ inches, $\pm 14\mu$ inches, $\pm 84\mu$ inches, and $\pm 91\mu$ inches.

These static interchannel time delay errors are listed in Table 2 for comparative purposes.

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Error Source	Error Magnitude
Stack to stack tolerance	\pm 500 μ inches
Tape stretch due to:	
tension	\pm 450 μ inches
temperature	\pm 113 μ inches
humidity	\pm 83 μ inches
Gap scatter tolerance	\pm 50 μ inches
Mean gap azimuth alignment and tape curvature	
adjacent tape tracks	\pm 20 μ inches
adjacent heads on the same stack	\pm 41 μ inches
outside heads on the same stack	\pm 245 μ inches
outside tape tracks	\pm 265 μ inches
Fixed skew	
adjacent tape tracks	\pm 7 μ inches
adjacent heads on the same stack	\pm 14 μ inches
outside heads on the same stack	\pm 84 μ inches
outside tape tracks	\pm 91 μ inches

Table 2. Individual Interchannel Static Time Delay Errors
(One inch tape and interleaved heads)

The above shows that the first two items, stack to stack tolerance and tape stretch, are the major contributors to the static inter-channel time delay errors. It should be noted that the use of in-line heads (all record, or reproduce, heads on one stack) eliminates these two sources of errors (at the expense of increased cross-talk between tracks). These two error sources and the error due to gap scatter are independent of the spacing between the tape tracks; however, the errors due to mean gap azimuth alignment, tape curvature, and fixed skew increase in direct proportion to the center to center spacing between the tape tracks.

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Sources of dynamic interchannel time delay error include dynamic skew, differential flutter, and stack differential flutter. Dynamic skew is defined in Reference 10 as "the oscillatory yawing of the tape in its plane." Differential flutter describes the motion of one tape track relative to another tape track caused by movement of the tape in a direction normal to the plane of the tape. These two errors are usually lumped together because of the difficulty in measuring their separate contributions. An error of $\pm 300 \mu$ inch per inch of tape width is typical. Stack differential flutter occurs only with interleaved heads and contains such things as difference in vibration of the two head stacks. This error is independent of the tape width. A typical value is $\pm 120 \mu$ inches. These dynamic interchannel time delays are listed in Table 3 for comparative purposes.

Error Source	Error Magnitude
Dynamic skew and differential flutter	
adjacent tape tracks	$\pm 21 \mu$ inches
adjacent heads on the same stack	$\pm 42 \mu$ inches
outside heads in the same stack	$\pm 252 \mu$ inches
outside tape tracks	$\pm 273 \mu$ inches
Stack differential flutter	$\pm 120 \mu$ inches

Table 3. Individual Interchannel Dynamic Time Delay Errors
(One inch tape and interleaved heads)

The above shows that the stack differential flutter error is independent of the spacing between tape tracks. This error can also be eliminated by the use of in-line heads. The dynamic skew and differential flutter errors increase in direct proportion to the center to center spacing between tape tracks.

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To compute the total interchannel time delay errors, some assumptions must be made about the values in Tables 2 and 3 and how they combine. To better illustrate the magnitude of the total error, two sets of assumptions will be made. First, a total cumulative error (ξ_c) is computed. To compute the total cumulative error, assume that the \pm values listed in Tables 2 and 3 represent absolute errors, and that these errors sum directly. Second, an rms error (ξ_{rms}) is computed. To compute the total rms error, assume that the \pm values listed in Tables 2 and 3 represent rms (1σ) errors and that the total rms error is equal to the square root of the sum of each individual error squared. (One could treat the \pm error values as 2σ or 3σ values, but without more information the conservative approach of treating them as 1σ values is used.)

When computing the total error for one record/reproduce cycle, be careful about the number of times each error can occur in one complete cycle. All of the errors except those due to gap scatter can occur twice in a complete record/reproduce cycle. Gap scatter errors can occur four times in one complete cycle. For example, assume that during the recording of a tape, channel A is $+50\mu$ inches and channel B is -50μ inches from the mean gap line. If a spike is simultaneously applied to both channels, the spike on channel B will be 100μ inches behind the spike on channel A. If this tape is then reproduced on a recorder where channel A is -50μ inches and channel B is $+50\mu$ inches from the mean gap line, the spike on channel B is delayed another 100μ inches. At the output of the tape recorder the spike on channel B will then be 200μ inches behind the spike on channel A.

To illustrate how the total errors are computed, the total cumulative and rms errors are calculated for the static, dynamic, and composite errors on adjacent tape tracks. The total cumulative static interchannel time delay error for adjacent tape tracks is

$$\begin{aligned}\xi_{c-static} &= [2(500 + 450 + 113 + 83 + 20 + 7) + 4(50)] \times 10^{-6} \text{ inches} \\ &= 2546 \mu \text{ inches}\end{aligned}$$

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The total rms static interchannel time delay error for adjacent tape tracks is

$$\xi_{\text{rms-static}} = \left[2 \left\{ (500)^2 + (450)^2 + (113)^2 + (83)^2 + (20)^2 + (71)^2 \right\} + 4(50)^2 \right]^{1/2} \times 10^{-6} \text{ inches}$$
$$= 977 \mu \text{ inches}$$

The total cumulative dynamic time delay error for adjacent tape tracks is

$$\xi_{\text{c-dynamic}} = 2(21 + 120) \times 10^{-6} \text{ inches}$$
$$= 282 \mu \text{ inches}$$

The total rms dynamic interchannel time delay error for adjacent tape tracks is

$$\xi_{\text{rms-dynamic}} = \left[2 \left\{ (21)^2 + (120)^2 \right\} \right]^{1/2} \times 10^{-6} \text{ inches}$$
$$= 172 \mu \text{ inches}$$

The total cumulative composite interchannel time delay error for adjacent tape tracks is

$$\xi_{\text{c-composite}} = (2546 + 282) \mu \text{ inches}$$
$$= 2828 \mu \text{ inches}$$

The total rms composite interchannel time delay error for adjacent tape tracks is

$$\xi_{\text{rms-composite}} = \left[(977)^2 + (172)^2 \right]^{1/2} \times 10^{-6} \text{ inches}$$
$$= 992 \mu \text{ inches}$$

Table 4 presents the total cumulative and rms versions of the static, dynamic, and composite errors as a function of the location of the data channels on the magnetic tape.

Type of Error and Data Channels	Total Dimensional Error	
	Cumulative	RMS
Static Error		
adjacent tape tracks	2546 μ inches	977 μ inches
adjacent heads on a stack	310 μ inches	117 μ inches
outside heads on a stack	858 μ inches	380 μ inches
outside tape tracks	3204 μ inches	1054 μ inches
Dynamic Error		
adjacent tape tracks	282 μ inches	172 μ inches
adjacent heads on a stack	84 μ inches	59 μ inches
outside heads on a stack	504 μ inches	356 μ inches
outside tape tracks	786 μ inches	422 μ inches
Composite Error		
adjacent tape tracks	2828 μ inches	992 μ inches
adjacent heads on a stack	394 μ inches	131 μ inches
outside heads on a stack	1362 μ inches	521 μ inches
outside tape tracks	3990 μ inches	1135 μ inches

Table 4. Total Interchannel Dimensional Time Delay Error
(One inch tape and interleaved heads)

For analyses such as cross-correlation analyses, where the phase shift between channels becomes important, it is perhaps more meaningful to express the error in terms of an upper frequency limit. That is, the frequency at which the phase shift becomes excessive. To relate dimensional errors to phase errors one must establish a criteria for the maximum allowable phase error. This depends upon the goals of the specific analysis

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performed, but for illustrative purposes in this report, a maximum allowable phase error of 2.5° is assumed as the criteria. The frequency corresponding to ϕ degrees of phase shift is found from Eq. (12) to be

$$f_{\phi} = \left(\frac{v}{\xi}\right) \left(\frac{\phi}{360}\right) \quad (12)$$

where

v = the velocity of the tape in inches per second

ξ = the interchannel time delay error in inches

ϕ = the maximum allowable phase error in degrees

Notice in Eq. (12) that this frequency varies directly with the maximum allowable phase error, the larger the maximum allowable phase error, the higher the upper frequency limit.

For example, the highest frequency that can be recorded on adjacent tape tracks with 2.5° or less phase error due to an rms composite error at a tape speed of 60 inches per second is

$$\begin{aligned} f_{2.5} &= \left(\frac{60}{992 \times 10^{-6}}\right) \left(\frac{2.5}{360}\right) \\ &= 420 \text{ cps} \end{aligned}$$

Table 5 lists this upper frequency limits for 2.5° phase error and various tape speeds for the dimensional errors given in Table 4.

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Type of Error and Data Channel	Upper Frequency in cps for a 2.5° Phase Error					
	Cumulative Error			RMS Error		
	60 ips	30 ips	15 ips	60 ips	30 ips	15 ips
Static Error						
adjacent tape tracks	164	82	41	426	213	107
adjacent heads on a stack	1344	672	336	3561	1781	890
outside heads on a stack	486	243	122	1096	548	274
outside tape tracks	130	65	33	395	198	99
Dynamic Error						
adjacent tape tracks	1478	739	370	2422	1211	606
adjacent heads on a stack	4960	2480	1240	7062	3531	1766
outside heads on a stack	827	413	207	1170	585	292
outside tape tracks	530	265	133	987	494	247
Composite Error						
adjacent tape tracks	147	74	37	420	210	105
adjacent heads on a stack	1058	529	265	3180	1590	795
outside heads on a stack	306	153	76	800	400	200
outside tape tracks	104	52	26	367	184	92

Table 5. Total Interchannel Time Delay Errors (Frequency)
(One inch tape and interleaved heads)

From Table 5, one can summarize a number of very important points:

1. The use of a magnetic tape recorder between the data transducers and analyzers causes serious phase errors. This is a major problem in the performance of cross-correlation and cross-spectral density analyses. The highest frequency based on an rms composite error is only 3180 cps. Even if one considers only rms dynamic errors, the highest frequency is only 7062 cps. This is inadequate for acoustic data, but may be adequate for some acceleration and strain data.

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2. One should use the highest tape speed available when it is necessary to minimize phase errors. For example, the frequency for a 2.5° phase error due to a composite rms error is 3180 cps at a tape speed of 60 inches/sec, 1590 cps at 30 ips, and 795 cps at 15 ips. This is because the errors are due to a dimensional difference between tape tracks. By increasing the speed of the tape, the total distance on the tape for one cycle of data at a given frequency f_1 increases. Because the dimensional error is independent of tape speed its percent of a cycle of f_1 decreases; hence, the phase error decreases as the tape speed increases.
3. Data to be phase analyzed should be recorded on the same head stack. Under the composite rms errors at 60 ips, when the data is recorded on different head stacks (adjacent tape tracks, and outside tape tracks), one obtains an upper frequency limit of between 420 and 367 cps. However, when the data is recorded on the same head stack (adjacent heads on a stack and outside heads on a stack) one obtains an upper frequency limit of between 3180 cps and 800 cps. This is a very significant difference.
4. The most important pair of data for phase analysis should be recorded on adjacent heads of the same head stack. It is shown under the composite rms error at a tape speed of 60 ips column that the upper frequency limit for adjacent heads on a stack is 3180 cps while the upper frequency limit for the outside heads on a stack is only 800 cps. If it is necessary to compare the phase of several channels to a common channel, such as might occur in multiple input-single output measurements, one should record the common channel on the center head of one head stack. The remaining channels should then be recorded on heads on both

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sides of the center head. If it is possible to place a priority on the data channels, the two most important channels should be located on the two heads adjacent (one on each side) to the center heads, the next two most important channels should be located one head removed (on each side) from the center, and so on. By arranging the data channels in this manner one obtains two phase measurements from the common channel that are accurate to 3180 cps (using the above error assumptions), and the upper frequency for phase measurements from the common channel to either of the two outside head channels is almost doubled from the outside head figure of 800 cps. (All of the errors for channels on the same head stack are proportional to the distance between the heads except for the gap scatter error which is constant. By calculating the individual error magnitudes for data channels on the same head, but six tracks apart, one finds the upper frequency limit for a 2.5° phase is 1520 cps - rms composite error at 60 ips.) If it is necessary to record data on both head stacks of a tape recorder, some improvement in phase accuracy can be realized by recording a common channel on the center head of both head stacks. Also, if it is necessary to measure phase data between several tapes, at least one common data channel should be recorded simultaneously on all of the tapes.

The static time delay errors can be compensated for because they produce a linear phase shift with frequency. Insertion of the proper time delay values in all but one of the channels results in zero phase shift between the undelayed channel and the delayed ones. Physical insertion of this time delay may be difficult. However, if the magnitude of this time delay is known accurately, it can be accounted for with appropriate programming whenever digital reduction of the data is desired. Because these static time delays are functions of the tape recorders and of temperature and humidity fluctuations between

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the time of recording and the time of reproduction, it is almost mandatory that some calibration signal adequate for determining the interchannel time delay be recorded on the tape just prior to recording of the data.

Even if the static interchannel time delay errors are completely compensated, the dynamic interchannel time delay errors represent a limit on the accuracy of practical phase analysis of tape recorded data. These errors cannot be compensated for external to the tape recorder. They can only be reduced in magnitude by improved tape handling designs. Table 5 shows 7062 to 1170 cps as the range of upper frequencies one might expect from rms dynamic errors at a tape speed of 60 ips when the data channels are restricted to one head stack.

One other very important point is that whenever a tape is copied, the copy has a reduced upper frequency limit for a given phase error. If one copies the master tape to make a working tape and assumes that the time delay errors (ξ) are cumulative, then Eq. (12) shows that the working tape will have an upper frequency limit (for a given phase error) that is half of the frequency limit for the master tape. If on the other hand one assumes that the errors will combine in an rms manner, then the upper frequency limit on the working tape is $1/\sqrt{2}$ times the frequency limit for the master tape. If one then makes a copy of the working tape for some purpose, such as loop analysis, the upper frequency limit on this tape for a given phase error becomes $1/3$ or $1/\sqrt{3}$ times the upper frequency limit on the master tape. Expressed in equation form

$$f_{\phi} = \frac{1}{(N + 2)} \quad \text{for cumulative errors}$$

and

$$f_{\phi} = \frac{1}{\sqrt{(N + 2)}} \quad \text{for rms errors}$$

where

N = the number of copies between the tape being analyzed and the master tape. (In the above example, N = 0 for the working tape and N = 1 for the copy made from the working tape.)

This means that the values in Table 5 are probably optimistic. When possible, make analyses from the master tape. If copies are required, each copy should be made directly from the master tape. In other words, copies should not be made of copies.

4.4 INTERNAL NOISE

Noise in the recording process reduces the dynamic range of a data measuring system. This noise level establishes a lower bound on the signal level as amplitude nonlinearities establish an upper bound on the signal level. Frequently the dynamic range of the tape recorder is the limiting factor on the dynamic range of a measurement system. However, the automatic attenuator of the Sonic Fatigue Facility essentially eliminates this problem; at least for the measurement of mean square values, by maintaining a relatively constant signal level into the tape recorder. The actual percent of full scale error caused by noise in the recording process becomes a function of the automatic attenuator setting. The noise voltage out of the tape recorder remains constant, but the full scale voltage out of the tape recorder, while actually a constant voltage, represents different percents of full scale. This percentage depends upon the automatic attenuator position, in terms of the variable being measured.

Internal noise in an FM magnetic tape record/reproduce cycle comes from two sources. The first noise source is in the record/reproduce electronics and is a function of the circuit design. The second noise source results from the nonconstant velocity of the tape past the heads, and is termed flutter. Flutter has both a first order, and a second order component. First order flutter refers to modulation of a constant carrier frequency signal by dynamic fluctuations of the tape speed. If one were to short the input to the tape recorder and record only the carrier frequency on a tape, and then play the tape back, a noise voltage would be measured at the output of the reproduce electronics. The difference in level of this voltage and the voltage level

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occurring on the output of the reproduce electronics when the output of the record electronics is connected directly to the input of the reproduce electronics (tape bypassed) is the noise level created by first order flutter. In effect, the flutter acts as an input signal and alters the effective number of cycles of carrier per inch of tape length, and the reproduce electronics cannot discriminate this modulation from an electronic signal modulation.

Second order flutter occurs only when there is a signal modulating the carrier frequency. Then the flutter also causes both amplitude and frequency modulation of the signal. To the author's knowledge, no thorough evaluation of all of the ramifications of second order flutter has yet been published. One example of a second order flutter problem is when either a sinusoidal or narrowband random signal is recorded and then reproduced for analysis through a very narrowband filter. If the second order flutter causes the signal to randomly wander in and out of the pass band of the filter, the results of the analysis will be erroneous. The signal to noise ratio quoted for the tape recorders at the Sonic Fatigue Facility is ≥ 49 db.

4.5 DRIFT

Two types of drifts occur with magnetic tape recorders. The DC level may drift, and the sensitivity may drift. The percentage of sensitivity drift obviously results in an inaccuracy of the same percentage. The DC drift results in an offset error which, when expressed in terms of a full scale error, is equal to the percentage of DC drift in the measurement of DC voltages, and reduces the usable dynamic range of the tape recorder for signals of the same polarity as the DC drift. Both of these errors tend to be time dependent so that minimizing the time required to record and reproduce the data also minimizes these errors. Also, by recording a calibration signal on the tape shortly prior to recording the test data and then using this calibration signal to establish the actual sensitivity at the input to the analyzer, the magnitude of the drift errors can be reduced.

The DC drift error and the sensitivity drift error are each quoted as being less than $\pm 1.5\%$ for an 8-hour period (assuming less than a $\pm 5^{\circ}\text{F}$. temperature variation, constant power line voltage, and a 30-minute initial warm-up period. (See Reference 8.)

5. GROSS ANALYSIS EQUIPMENT

The equipment in this category is used almost exclusively for editing purposes. Only the gross characteristics of the data need be examined during the editing process. Some rational criteria is then applied to this grossly analyzed data and samples are selected for further, more detailed analysis. The gross analysis equipment at the Sonic Fatigue Facility consists of electronic voltmeters with various types of detectors, oscillographs, and graphic level recorders.

The absolute accuracy of this category of equipment is generally not of prime importance since only relative measurements are used in the editing process. However, if the results of the gross analysis are going to be used in the final data presentation or to check on the quality of the detailed analysis, then absolute accuracy does become more important. For example, if the gross analysis is used to select the highest $(M - N)$ channels out of M channels of rms measurements, it is important that the equipment does not distort the rms computations and that all M channels respond in the same amount for a given rms input, but the actual magnitude of the input need not be known accurately.

However, if it is necessary to know something quantitative about the N channels that are not analyzed in detail, then the rms calculations must be performed accurately. Also, to check on the quality of the detailed analysis, it is desirable to have the rms values of the $(M - N)$ channels measured accurately in the gross analysis process. Then the rms value computed during each step in the detailed analysis can be compared to this original value. Significant variations in this rms value with analysis mode indicates poor analysis quality.

5.1 ELECTRONIC VOLTMETERS

The intrinsic inaccuracies of electronic voltmeters are related to:

- the signal amplitude
- the signal frequency
- the signal instantaneous amplitude probability density function

Because the first two factors are so well documented for each individual meter, they will not be discussed further in this report. The effect of the instantaneous amplitude probability density function upon the intrinsic accuracy is a bit more subtle; hence, it will be reviewed. Most electronic voltmeters, (AC), can be separated into three categories by the type of detection used. These categories are:

- 1) true rms
- 2) mean absolute value
- 3) peak

True rms voltmeters employ some type of a square law detector whose output is usually smoothed by an RC circuit, then displayed on a meter. The face of the meter is calibrated to read the square root of the smoothed voltage.

The output of an ideal true mean square detector is described by the following equation.

$$\overline{y^2} = \int_{-\infty}^{\infty} x^2 p(x) dx \quad (13)$$

where

$\overline{y^2}$ = the mean square output voltage of the detector

x = the voltage into the detector

$p(x)$ = the instantaneous amplitude probability density function of the input voltage.

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(Note: It is assumed that the AC voltmeters do not have DC response.

Hence, the mean voltage at the detector input is assumed equal to zero.)

Assume that a sine wave of amplitude A is applied to the input of a true rms detector.

$$x = A \sin \omega t$$

Then,

$$p(x) = \left(\frac{1}{\pi A} \right) \left(\frac{1}{\sqrt{1 - (x/A)^2}} \right) \quad ; \quad -A \leq x \leq A$$
$$= 0 \quad ; \quad \text{elsewhere}$$
(14)

The mean square value is

$$\overline{y^2} = \int_{-A}^A (x^2) \left(\frac{1}{\pi A} \right) \left(\frac{1}{\sqrt{1 - (x/A)^2}} \right) dx = \frac{A^2}{2}$$

The rms value is

$$\sqrt{\overline{y^2}} = 0.707A$$
(15)

which, obviously, is the correct answer.

Suppose that a triangular wave of peak amplitude A is being applied to the input of the square law detector. Here,

$$p(x) = \frac{1}{2A} \quad ; \quad -A \leq x \leq A$$
$$= 0 \quad ; \quad \text{elsewhere}$$
(16)

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Then

$$\begin{aligned}\overline{y^2} &= \int_{-A}^A (x^2) \left(\frac{1}{2A} \right) dx \\ &= \frac{A^2}{3} ; \text{ or the rms value} = .577A\end{aligned}\quad (17)$$

which is also correct.

Consider next the case when a random voltage with a Gaussian instantaneous amplitude probability density function is applied to the input of the true mean square detector. Here

$$p(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-x^2/2\sigma^2}\quad (18)$$

where

σ^2 = the variance of the random signal

Then,

$$\overline{y^2} = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{\infty} (x^2) \left(e^{-x^2/2\sigma^2} \right) dx = \sigma^2\quad (19)$$

For this important case, the correct rms value is obtained, namely,

$$\sqrt{\overline{y^2}} = \sigma\quad (20)$$

These three examples prove that the accuracy of a true rms detector is not effected by the shape of the probability density function.

One limitation of practical true rms detectors that does effect the accuracy of mean square measurements is a finite crest factor capability. (Crest factor is defined as the ratio of the peak value of a signal to its rms value.) Most true rms meters have sufficient range to handle the crest

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factors associated with ordinary periodic signals, but are limited in the case of random signals. The error introduced in the measurement of the mean square value of a signal with a Gaussian instantaneous amplitude probability density function by a detector that has a limited crest factor capability is shown in Appendix B of Reference 12 to be

$$\frac{\overline{y^2}}{x^2} = 2 C_f^2 \int_{C_f}^{\infty} p(z) dz + 2 \int_0^{C_f} p(z) dz - 2 C_f p(C_f) \quad (21)$$

where

$$C_f = z_0 = \frac{x_0}{\sigma_x}$$

$$z = \frac{x}{\sigma_x}$$

$$\sigma_x = \sqrt{\overline{x^2}}$$

The terms

$\overline{x^2}$ = the true mean square value of the voltage at the input to the detector

C_f = the crest factor capability

x_0 = the peak input voltage at which clipping occurs (ideal hard limiting is assumed)

The values of the two integrals in Eq.(21) can be found directly from a table of the Gaussian instantaneous amplitude cumulative distribution function.

The last term in Eq.(21) is simply

$$2 C_f p(C_f) = \left[\frac{2 C_f}{\sigma_x \sqrt{2\pi}} \right] \exp(-C_f^2 / 2\sigma_x^2)$$

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To illustrate the quantitative effects of crest factor limitations on true mean square detectors, the following table is quoted from Reference 12.

Crest Factor Capability C_f	$\overline{y^2} / \overline{x^2}$
0.5	0.185
1.0	0.516
1.5	0.778
2.0	0.921
2.5	0.978
3.0	0.995
3.5	0.999
4.0	1.000 -

Mean absolute voltage detectors employ a linear detector whose output is smoothed by an RC circuit. The output of the RC circuit is usually displayed on a meter that is calibrated to indicate the rms value of a sine wave. The equation that describes the performance of a mean absolute detector in terms of the instantaneous amplitude probability density function of its input is:

$$\overline{|y|} = \int_{-\infty}^0 |x| p(x) dx + \int_0^{\infty} |x| p(x) dx \quad (22a)$$

where

$\overline{|y|}$ = the mean absolute voltage at the output of the detector

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If the density function is symmetric about zero,

$$\overline{|y|} = 2 \int_0^{\infty} |x| p(x) dx \quad (22b)$$

To illustrate the significance of the above equation, consider several typical signals. First, consider a sinusoidal input signal of the form

$$x = A \sin \omega t$$

Then

$$\begin{aligned} \overline{|y|} &= 2 \int_0^A \frac{|x|}{(\pi A) \sqrt{1 - (x/A)^2}} dx \\ &= \frac{2A}{\pi} = 0.636A \end{aligned} \quad (23)$$

Next, assume that a triangular wave of peak amplitude A is applied to the mean absolute value detector. Here

$$\begin{aligned} \overline{|y|} &= 2 \int_0^A \frac{|x|}{2A} dx \\ &= \frac{A}{2} = .500 A \end{aligned} \quad (24)$$

Finally, suppose that a random signal with a Gaussian instantaneous amplitude probability density function of variance σ_x^2 is applied to the input of the mean absolute value detector. Here

$$\begin{aligned} \overline{|y|} &= 2 \int_0^{\infty} \frac{|x|}{\sigma_x \sqrt{2\pi}} \exp(-x^2/2\sigma_x^2) dx \\ &= \frac{\sigma_x}{\sqrt{\pi}} = .798 \sigma_x \end{aligned} \quad (25)$$

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Now if a meter following the mean absolute detector has been calibrated to indicate the rms value of a sine wave and then is used to measure the rms values of other types of signals, erroneous readings are obtained. To illustrate the magnitude of these errors, assume that signals with the above instantaneous amplitude probability density functions are applied. First, the meter calibration factor, K , for a sine wave is

$$\begin{aligned} K &= \frac{\sqrt{\frac{y^2}{2}}}{|y|} \\ &= \frac{.707A}{.636A} = 1.11 \end{aligned}$$

Thus, the meter is calibrated to display a voltage 11% higher than the mean absolute voltage.

Now consider the error resulting in the measurement of the rms value of a triangular wave by a mean absolute detector, followed by a meter that has been calibrated to read the rms value of sine waves. The indicated voltage, e_1 , is

$$\begin{aligned} e_1 &= 1.11 \overline{|y|} \\ &= (1.11)(.500A) \\ &= .555A \end{aligned} \tag{26}$$

Since the true rms value of a triangular wave, from Eq.(17), is .577A, the percent error is

$$\begin{aligned} \% \epsilon &= 100 \left(\frac{.577A - .555A}{.577A} \right) \\ &= 3.8\% \end{aligned}$$

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The corresponding error with a Gaussian input signal is

$$\begin{aligned}\% \epsilon &= 100 \left(\frac{\sigma_x - (1.11)(.798\sigma_x)}{\sigma_x} \right) \\ &= 11.4\%\end{aligned}$$

Hence, when mean absolute detectors are used to measure the rms or mean square value of a signal, the accuracy is a function of the instantaneous amplitude probability density function of both the signal used for calibration and the signal being measured.

Peak detectors perform exactly as their name implies. The maximum value of voltage on the input is measured and held indefinitely. For any signal that has a truncated instantaneous amplitude probability density function (where the ratio of the truncation voltage to the rms voltage is less than the permissible crest factor) these detectors can be used.

Analogous to the mean absolute detector, the accuracy of a peak detector used to measure rms values under the condition of the above paragraph is a function of the instantaneous amplitude probability density function of both the calibrating signal and the signal being analyzed. For example, if the meter is calibrated with a sine wave, the calibration factor is:

$$K = \frac{.707A}{A} = .707$$

When applied to measure the rms value of a triangular wave, the percent error is:

$$\begin{aligned}\% \epsilon &= 100 \left(\frac{.577A - .707A}{.577A} \right) \\ &= 22.5\%\end{aligned}$$

However, when a peak value detector is used to analyze a signal with a probability density function that is continuous out to infinity, or for that

matter, beyond the crest factor capability of the instrument, the reading is useless in terms of rms values.

5.2 OSCILLOGRAPHS

Oscillographic recorders are one of the most valuable tools available in a data editing process. These devices permit visual display of the instantaneous amplitude time history of the data. Through inspection of this type of a display, a talented analyst learns many things about the data. Bad data points are usually most accurately detected in this manner. Also, the most interesting segments of the total record are easily selected for more detailed analyses.

Oscillographs have four major accuracy limitations. The first is one of resolution. The trace recorded has a finite width. By taking precautions such as accurately focusing the galvanometer mirror, adjusting the recording lamp intensity properly, and taking all readings in some prescribed manner such as from lower trace edge to lower trace edge, or by computing the arithmetic average of the outside trace and inside trace readings, the resolution problems can be minimized, but not eliminated.

The other approach to minimize the resolution problem is to increase the deflection of the trace for a given input. This approach is limited by two factors. First, if more than one high frequency channel is recorded, one must usually allow space between data channels to prevent overlap of the channels and blurring of both. The second factor brings up the second accuracy limitation of oscillographs, that of linearity. Since the trace deflection is obtained through rotation of a galvanometer about its vertical axis, the trace deflection tends to be a sine function, not a linear function. In practice, the electrical design properties of the particular galvanometer in use determines the linearity of the trace deflection.

The third accuracy limitation of oscillographs is limited frequency response. This inaccuracy is also primarily a function of the particular

galvanometer that is used. One additional point to be considered on the frequency response of galvanometers is the provision of appropriate damping. The damping may be required in the form of an external shunt resistor, an external series resistor, or may be already incorporated into the galvanometer in the form of fluid damping. The proper damping is required to meet the advertised frequency response range, and is usually selected to give the trace deflection an approximately linear phase relation with the frequency of the input signal.

The fourth accuracy limitation of oscillographs is inaccuracy of the velocity of the recording paper or film. This can be a static and/or dynamic inaccuracy. The use of timing lines helps to minimize the static inaccuracies.

While there are other error sources that must be carefully considered when performing detailed analysis of data from oscillograms, the above four are the major error sources. Even these errors, assuming that a gross error has not been made, are not too important to the editing process since the prime requirement is a qualitative rather than a quantitative analysis.

5.3 LEVEL RECORDERS

The use of a level recorder in conjunction with an oscillograph is an excellent method for selecting short time intervals for detailed analysis from a long record. The graphic level recorder gives a quantitative measure of the magnitude of the data signal as a function of time.

Graphic level recorders are subject to the same sources of errors as the electronic voltmeters. These error sources are amplitude nonlinearities, finite resolution, limited frequency response, and sensitivity of the detector to the instantaneous amplitude probability density function of the signal being measured.

For maximum utility of the graphic level recorder in editing, use the true rms detector to avoid the errors discussed in Section 5.1. The

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maximum benefit of the level recorder to the editing process is in the selection of time intervals for detailed analysis when the data are non-stationary. Otherwise a voltmeter is sufficient for measuring the rms level, and a data sample can be selected from any portion of the record since the process is time invariant.

In a laboratory test situation, there are several causes for nonstationarity. First, there are test start and completion nonstationarities that can frequently be ignored if the test duration is long compared to these times. However, these times should be observed on the graphic level recorder for any anomalies that might occur. Occasionally nonstationary tests are devised to simulate nonstationary environments. Even if the excitation level is held constant, response levels can change with time because of such things as fatigue of a structure or application of a new environment. For example, consider an acoustic test of a typical panel when the over-all sound pressure level is maintained time-invariant, and the panels are intermittently subjected to a transient temperature environment. It is highly likely that the response characteristics of the panel will change during the application of the heat environment.

Unfortunately, there is no explicit practical criterion for the use of a level recorder to analyze nonstationary data. In implicit terms, the averaging time constant should be made as long as possible to minimize uncertainty fluctuations without distorting any underlying time trends in the process being measured. Since these are conflicting requirements, the end result must usually be some compromise. The compromise can usually be best reached by analyzing the data first with a short time constant, and then repeating the analysis several times with increasingly longer time constants until distortion of the time trend is obtained. Then the longest time constant not causing this distortion is used. Of course, a knowledge of the time characteristics of the underlying physical process causing the non-stationarity lends to a more rapid selection of the best averaging time.

5.4 DETECTION OF NON-NORMALITY

As a general rule, the gross analysis equipment is not used to detect normality or non-normality of data since this requires detailed analysis. However, if desired, these equipment items can be used for this purpose within their limitations.

As shown in Section 5.1, true rms detectors can be used to accurately analyze non-normal data. Conversely, if the same data is analyzed with both a true rms detector and a mean absolute detector, the indicated readings should have the following ratio if the data is normal, (assuming that the detectors are calibrated with sine waves)

$$R = \frac{(1.11)(.798\sigma_x)}{\sigma_x} = .886 \quad (27)$$

where

R = the ratio of the indicated rms level measured on a mean absolute detector to the rms level measured on a true rms detector

See Eqs. (20), (25), and (26) for development.

This is not an exclusive test. Departure from the above ratio means that the data are non-normal; but, if the above ratio is measured, there is no guarantee that the signal is Gaussian as theoretically other signals can give this ratio. Thus, the voltmeters and graphic level recorders can be used for a crude detection of non-normality.

The oscillograph is also used to detect the presence of non-normalities. However, the nonlinearities must be rather obvious, such as hard clipping or predominant periodicities.

5.5 DETECTION OF NONSTATIONARITY

One of the primary requisites of the gross analysis equipment is the ability to detect nonstationarities in the data. As explained in Section 5.3, the graphic level recorder is a powerful tool for detecting nonstationarities. When the fluctuations of the rms level become much greater than those expected from uncertainty fluctuations, the data is nonstationary. From Reference 13, the expression for the standardized error ϵ of an rms value is

$$\epsilon = \frac{1}{2\sqrt{BT}} \quad (28a)$$

where

B = the appropriate bandwidth of the data

T = the record length used in the analysis

Since the graphic level recorder uses an RC averager instead of a true integrator, a more useful form of Eq. (28a) is

$$\epsilon = \frac{1}{\sqrt{8BK}} \quad (28b)$$

where

K = the time constant setting of the graphic level recorder

If one makes several simplifying assumptions which are generally appropriate for practical usage, as long as the data are random and have a nearly Gaussian instantaneous amplitude probability density function, then approximately two-thirds of the time the uncertainty fluctuations are less than $\pm (100\epsilon)$ percent of the true rms value.

To illustrate the use of Eq. (28b), assume the graphic level output is shown in Figure 6, the bandwidth of the data is 100 cps and the averaging time constant is 1 second.

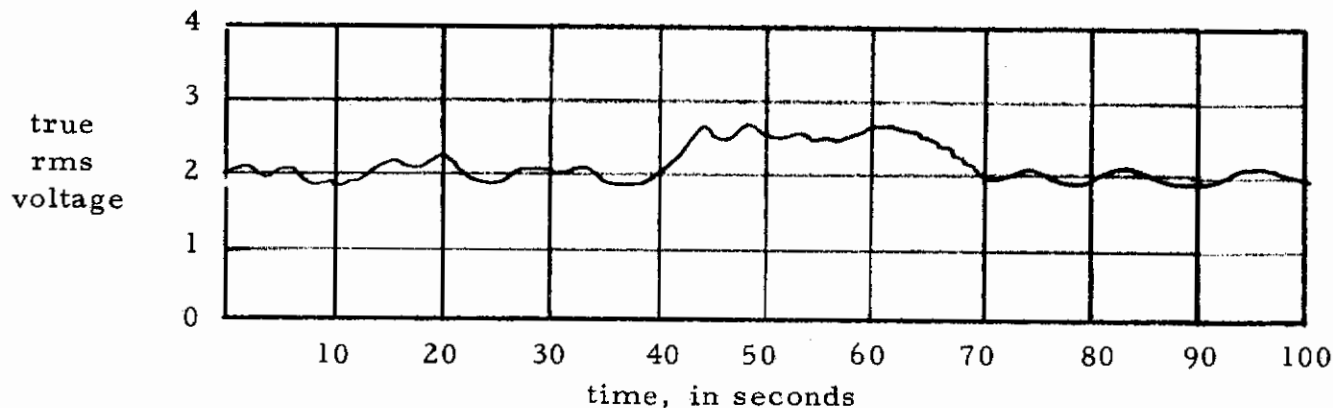


Figure 6. Time History of a True rms Voltage

From Eq. (28b),

$$\begin{aligned}\epsilon &= \frac{1}{\sqrt{(8)(100)(1)}} \\ &= .035 \text{ or } 3.5\%\end{aligned}$$

If one further assumes that the true rms value is 2.0 at the start of the record, then approximately two-thirds of the time the rms level fluctuations should be contained between the rms voltages of 1.9 and 2.1. Obviously the data in Figure 6 are nonstationary between 40 and 70 seconds.

The oscillograph can be effectively used to qualitatively detect nonstationarities and when possible should be used in conjunction with the level recorder. In fact, the oscillographic record may reveal some subtle nonstationarities not observable through a graphic level analysis. For example, a nonstationarity caused by frequency modulation of a stationary spectra would not be revealed in a graphic level analysis since the true rms value is time invariant, but could possibly be detected through an oscillographic analysis.

5.6 DETECTION OF NONLINEARITIES

When discussing the detection of nonlinearities, it must be assumed that at least two or more excitation levels have been applied and recorded and that the corresponding test specimen response levels have been recorded. In other words, the input-output characteristics must be measured to define the linearity or nonlinearity of a system.

If a function H is linear, it must satisfy the following two requirements

$$H(a + b) = H(a) + H(b) \quad (29)$$

and

$$H(Ka) = KH(a) \quad (30)$$

where K is a constant. See Reference 14.

The electronic voltmeters and graphic level recorders can be used to detect nonlinearities through evaluation of the constant K in Eq. (30). Quite simply, if the input (excitation) is increased by a factor K , the output (response) must also increase by a factor of K if the system is linear. Of course, these devices are good only for measuring the effects of amplitude nonlinearities upon the rms values and to properly define most nonlinearities, more detailed analyses are required.

If the data do not satisfy Eq. (30), the system is nonlinear. But if the data do satisfy Eq. (30), this alone is not sufficient to state that the system is linear. It is also necessary that the system satisfy the requirements of Eq. (29). This may be extremely difficult to check in practice. For example, to check if a mechanical system subjected to sonic excitation is linear, Eq. (29) requires that the output of the system be measured three times. First, when the system is excited by one acoustic signal; second, when the system is excited with a second acoustic signal; and third, when the system is excited simultaneously by both of these acoustic signals. If the system is linear, the output obtained by simultaneous application of the acoustic signals will be equal to the sum of the outputs obtained when the

signals are applied separately. It is rare when data are available from normal sonic tests in a form that can be used to perform the linearity test of Eq. (29). Even if the tests have been run in the proper manner to evaluate Eq. (29), one must be careful that the phase characteristics of any device used to measure the data do not introduce severe errors. See Section 4 for a discussion of phase errors.

The oscillographic equipment can be used to detect a few kinds of nonlinearities such as hard clipping of any signal and harmonic and inter-modulation distortion of sinusoidal signals.

5.7 DETECTION OF PERIODICITIES

Only the oscillographic equipment in this category is really useful for detecting periodicities and even then the rms value of the periodicity must be large compared to the rms value of the noise for the periodicity to be visually detectable.

6. FREQUENCY ANALYSIS EQUIPMENT

The Sonic Fatigue Facility has a wide variety of frequency analyzers available for the analysis of data from sonic fatigue tests. These analyzers can be separated into the following categories.

- Constant Bandwidth Frequency Analyzers
- Constant Percentage Bandwidth Frequency Analyzers
- Ordinary Power Spectral Density Analyzers

The constant bandwidth analyzers are of two types -- spectrum analyzers and wave analyzers. Both types operate in basically the same manner. The data signal is heterodyned to ease the design problems in obtaining a narrow band filter and to permit simple adjustment of the center frequency of this filter over the desired range of analysis frequencies. The primary difference between a spectrum analyzer and a wave analyzer is only one of internal display. The spectrum analyzer has a cathode ray tube display and the wave analyzer has a meter.

The constant percentage bandwidth analyzers may be separated into three categories. There are octave, one-third octave, and narrow band analyzers. (The narrow band analyzer has filters with 1.2%, 2%, and 6% bandwidths.)

There are two identical ordinary power spectral density analyzers at the RTD Sonic Fatigue Facility. These analyzers employ constant bandwidth filters in a heterodyne circuit.

The intrinsic accuracy of frequency analyzers is determined by two factors. The first is the distortion of the amplitude reading in a frequency band that is caused by the detector/signal probability density function relation. The second is distortion of the frequency spectrum by the frequency characteristics of the analyzer filter. A third error source occurs when the analyzer filter is continuously scanned across the range of the frequency spectrum.

6.1 DETECTOR CHARACTERISTICS

The effects of the instantaneous amplitude probability density function upon various types of detectors is discussed in some detail in Section 5. 1. All of the readings taken with frequency analyzers are rms or ms values. For that reason use true rms detectors whenever the option is available since they are insensitive to the instantaneous amplitude probability density function of the signal.

6.2 FILTER CHARACTERISTICS

Since the entire objective of performing a frequency analysis is to be able to examine the distribution of the energy of the signal over its frequency range, it is of prime importance that the filter used does not distort the frequency spectrum. Such distortion is manifested in two ways. First, it causes the amplitude of the spectrum to be inaccurate, and second, it causes the bandwidth of spectral peaks to be inaccurate. It has been proposed that a reasonable criterion for proper frequency analysis is (see Reference 15)

$$B \leq \frac{1}{4} B_r \quad (31)$$

where

B = the bandwidth of the analyzer filter in cps

B_r = the bandwidth between half power points of a power spectral density peak in cps

This criterion was checked analytically and experimentally in Reference 16 for the case where the spectral peak results from the excitation of a simple mechanical oscillator with white noise. Reference 16 presents curves for effects of finite filter bandwidth on both the measured bandwidth of spectral density peaks, and the measured peak spectral density value. It further shows that for the criterion in Eq. (31) there is an approximately 3% error in both peak bandwidth and value.

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Clearly, octave and one-third octave frequency analyses will seldom, if ever, have proper frequency resolution. For purposes of convenience and comparison to most past acoustic work, these analyses have their value. It must be kept in mind, however, that these analyses are only crude approximations of the true frequency spectrum and they blur many of the spectral peaks and valleys.

With constant bandwidth analyzers one first selects the required filter bandwidth based on the resolution requirements. Because the minimum bandwidth of constant analyzers is limited, it is not always possible to obtain proper resolution of low frequency spectral peaks. Next, the duration of the sample record length is chosen to satisfy the desired statistical uncertainty fluctuation requirements. The method of selection for spectral density analyses is shown in Reference 3 to be

$$T = \frac{1}{B \epsilon^2} \quad (32)$$

where

- T = the sample record duration in seconds
- B = the bandwidth of the analysis filter in cps
- ϵ = the normalized standard error

As the center frequency of the analyzer filter is increased, the resolution requirements in terms of cps of bandwidth are generally reduced (the Q, center frequency to bandwidth ratio, of mechanical systems tends to be independent of frequency). Therefore, it is desirable to increase the bandwidth of the analyzer filter at the lowest frequency permitted by the resolution criterion. For example, consider a constant bandwidth analyzer having filter bandwidths of 1, 5, 10, and 100 cps. Assume that the maximum Q will be 20. The following equation satisfies the resolution requirement of Eq. (31).

$$B \leq \frac{f_c}{4Q} \quad (33)$$

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where

f_c = the center frequency of the analyzer filter in cps.

Solving Eq. (33) for the available analyzer filter bandwidths and $Q = 20$, it can be seen that the 1 cps filter provides proper resolution when the center frequency of the analyzer filter is above 80 cps, the 5 cps filter does so above 400 cps, the 10 cps filter does so above 800 cps, and the 100 cps filter does so above 8000 cps. Obviously, one must live with the improper frequency resolution below 80 cps.

If a value of $\epsilon = 0.1$ is assumed, the sample record durations required to satisfy Eq. (32) are

$$T (1 \text{ cps filter }) = 100 \text{ seconds}$$

$$T (5 \text{ cps filter }) = 20 \text{ seconds}$$

$$T (10 \text{ cps filter }) = 10 \text{ seconds}$$

$$T (100 \text{ cps filter }) = 1 \text{ second}$$

The total time for the analysis is computed by summing the individual times for each filter. These individual times are found by multiplying the appropriate sample record duration above, with the ratio of the frequency range to be covered to the bandwidth of the analyzer filter. In the present example, assume that it is desired to cover the frequency range from 20 to 10,000 cps. The total analysis time is found as follows.

Time spent analyzing with the 1 cps filter	=	$\frac{400 - 20}{1}$	(100)	=	38,000 seconds
Time spent analyzing with the 5 cps filter	=	$\frac{800 - 400}{5}$	(20)	=	1,600 seconds
Time spent analyzing with the 10 cps filter	=	$\frac{8000 - 800}{10}$	(10)	=	7,200 seconds
Time spent analyzing with the 100 cps filter	=	$\frac{10,000 - 8000}{100}$	(1)	=	20 seconds
Total time spent in analysis				=	46,820 seconds
				≈	130 hours

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This example illustrates that a great deal of analysis time is required to obtain proper resolution. Frequently, in the interests of economy, it is necessary to compromise on the analysis filter bandwidths used. When this step is taken, it is quite important that the end user of the data be made aware of the magnitude of the resolution errors. For example, if the damping of a particular structural resonance is being calculated from the bandwidth of a power spectral density peak, the calculations are not accurate to 1% if the frequency resolution error in the original spectral analysis is 10%. Fortunately, it is frequently possible to perform an analysis with a moderate frequency resolution error, and then reanalyze with higher resolution only selected portions of the entire frequency range.

One other important filter characteristic is the proper definition of B , the analyzer filter bandwidth. Reference 12 has an excellent discussion of the various bandwidths associated with spectral analysis. These discussions are summarized here. The most commonly referred to bandwidth of a band-pass filter is its half-power point, or 3 db, bandwidth (B_r). This is the bandwidth between the lower frequency (f_1) where the signal is attenuated 3 db, and the upper frequency (f_2) where the signal is attenuated 3 db. Mathematically,

$$B_r = f_2 - f_1 ; \text{ where } |H(f_1)|^2 = |H(f_2)|^2 = \frac{1}{2} |H_m|^2 \quad (34)$$

and

$$\begin{aligned} |H(f)| &= \text{the value of the gain factor at frequency } f \\ |H_m| &= \text{the maximum value of the gain factor} \end{aligned}$$

The 3 db bandwidth is easy to measure, but is not of much practical use in spectral density analysis.

The noise bandwidth of an analyzer filter (B_n) is defined as the bandwidth of an ideal rectangularly shaped filter that has the same mean square output as the analyzer filter when both have an equal level of white noise applied to their input. Mathematically,

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$$B_n = \frac{\int_0^{\infty} |H(f)|^2 df}{|H_m|^2} \quad (35)$$

This is the bandwidth required for accurate power spectral density measurements. This bandwidth should be divided into the mean square voltage at the filter output to convert this output into a spectral density.

$$G(f) = \frac{\text{m. s.}}{B_n} \quad (36)$$

where

$G(f)$ = the spectral density (volt²/cps)

m. s. = the mean square voltage at the filter output

Equation (35) shows how this bandwidth is measured. The numerator is evaluated by either measuring the area under the square of the filter transmissibility curve, or the mean square voltage out of the filter when it has white noise applied to its input. (The former method is more accurate and also more time consuming.) The denominator of Eq. (35) is simply the square of the maximum value of the gain factor. Any error in the noise bandwidth is reflected directly into spectral density measurements.

The third bandwidth (B_e) is called the equivalent bandwidth, or the Blackman and Tukey bandwidth, and is developed in Reference 17. This bandwidth is the appropriate one to use in Eq. (32) to determine the uncertainty fluctuations. Mathematically,

$$B_e = \frac{\left[\int_0^{\infty} |H(f)|^2 df \right]^2}{\int_0^{\infty} |H(f)|^4 df} \quad (37)$$

These bandwidths are not equal except for an ideal rectangular filter. Reference 12 shows that for a single-tuned filter

$$\frac{B_e}{2} = B_n = \frac{\pi}{2} B_r$$

6.3 FILTER SCAN CHARACTERISTICS

When the frequency analysis is performed by a continuous scan of a single bandpass filter across the frequency range of interest (as is the case with the spectrum analyzers), another pair of errors must be considered. First, the filter must be swept slowly compared to its bandwidth in order for the filter to respond fully to an abrupt change in the spectral level. The buildup time of a filter is approximately equal to the reciprocal of the bandwidth of the filter. Thus, if the scan rate is restricted to one frequency bandwidth per 10 filter buildup times, one criterion for scan rate is

$$R_s \leq \frac{B^2}{10} \quad (38)$$

The second problem that occurs with continuously scanned analyzers results from the fact that RC averagers are usually used to average the detector output rather than true integrators. The scan rate must be slow enough so that the output of the RC circuit will accurately track the input even when there is an abrupt change in the spectral level. If one assumes a step change in the spectral level and that a typical RC lowpass circuit is being used to perform the averaging, the error in the output voltage is

$$\begin{aligned} \% \text{ error} &= 100 \left\{ \frac{e_{in} - e_{in} [1 - \exp(-t/RC)]}{e_{in}} \right\} \quad (39) \\ &= 100 e^{-t/RC} \end{aligned}$$

where

t = the time in seconds for the filter to be scanned one filter bandwidth

RC = the time constant of the averaging circuit

For a 1% or less error in the indicated output because of scanning through a step change in level of the spectrum, t must be equal to or greater than 4.6 RC. Therefore, this scan criterion becomes

$$R_s \leq \frac{B}{4.6 RC} \quad (40)$$

One should always use the scan criterion that calls for the slowest scan. Generally, it will be the one in Eq. (40).

6.4 DETECTION OF NON-NORMALITY

The frequency analysis equipment is not very useful for the detection of non-normality. If both a true rms and a mean absolute detector are available, measurements of the ratio in Eq. (27) can be made.

6.5 DETECTION OF NONSTATIONARITIES

Generally, the frequency analysis equipment is of no use in detecting nonstationarity since the frequency analysis is based on an ergodicity assumption which excludes nonstationarity. (Time averages of a single record can be substituted for an ensemble average of many records at a single instant of time.) When a sample record is spliced into a loop for repetitive analysis, it is being assumed that the data is stationary. Clearly, if a mean square value is computed by integrating over the entire loop duration, only one value is obtained.

However, frequency analyzers can be used to detect and study time variations of the spectral content of the data under certain conditions. First, one must have a parallel filter analyzer so that the data can be applied to all filters simultaneously, or have the data on tape so that the

input can be reproduced identically for repetitive analysis by a single filter analyzer. The second condition is that the analyzer must use an integrator or averager that has a time constant that is short compared to any of the time trends in the nonstationarity. If true integration is used, some scheme for commutating the integrators must be employed. See Reference 18.

Short time averaging techniques such as those above do have a bias error associated with them (see Reference 19). However, this is the most practical technique available at the present time for performing a detailed analysis on nonstationary data.

6.6 DETECTION OF NONLINEARITIES

This area needs more work to better define the effects of nonlinearities on the spectral content of data. In a simple sense, if it is known that the input spectrum is increased by a constant factor K , and the output spectrum is not changed by a factor of K , then the system is nonlinear. Problems occur in attempting to measure the ratio of two output spectra. First, one has uncertainty fluctuations to account for, and assuming that one could obtain a sample record length long enough to make the uncertainty fluctuations small compared to K , one would anticipate a change in the frequency content of the spectrum due to the nonlinearity. Hence, measurement criteria must be developed.

While considerable work needs to be performed in the area of the detection and measurement of nonlinearities by frequency analysis means, this type of analysis is quite useful for studying the effects of a known nonlinearity upon a given spectra.

6.7 DETECTION OF PERIODICITIES WITH THE FREQUENCY ANALYSIS EQUIPMENT

There is no absolute way of detecting periodicities with the frequency analysis equipment. However, there is one practical method that can be used to give strong indications of periodicities. In this method, spectral peaks are re-analyzed with progressively narrower filters. If the shape of the peak does not change as the filter bandwidth of the analyzer is decreased, then it is not periodic. However, if the shape of the peak does change and essentially is a plot of the gain factor of the filter, then there is a strong indication that this peak in the spectrum is due to a periodicity. This is not an absolute test because the possibility always exists that the peak is due to a random noise that has a narrower bandwidth than the narrowest filter available.

7. AMPLITUDE ANALYSIS EQUIPMENT

The RTD Sonic Fatigue Facility has one two channel amplitude analyzer that is capable of performing the following types of analyses:

- Instantaneous Amplitude Probability Density
- Peak Probability Density of Narrow Band Signals
- Joint Instantaneous Amplitude Probability Density of Two Signals

This analyzer also has a squaring circuit built into each channel so that the analyses can be performed on the square of the signal.

In addition to the statistical uncertainty fluctuations discussed in Reference 3, probability density analyzers of the type used at the RTD Sonic Fatigue Facility have two intrinsic error sources. The first error source is due to the finite amplitude window width used in the analyzer and is quite analogous to the finite filter width situation in frequency analysis. The second error source is from the continuous scan of the amplitude window across the amplitude range.

7.1 AMPLITUDE WINDOW CHARACTERISTICS

The basic assumption is that the probability density function varies linearly across the amplitude window width. Obviously, for any simple density function (such as a uniform density), there is no error because of this approximation. However, except for the uniform density, none of the common density functions have a linear shape. Hence, there is some error owing to the finite window width associated with all probability density measurements. It is also obvious that the smaller the window width, the less the error (the better is the resolution). However, if the window width is very small, unreasonably long sample record lengths are required to attain small uncertainty fluctuations.

The magnitude of this error is determined by comparing the value of a theoretical density function at some amplitude level to the value indicated by the analyzer when its window is centered on the same amplitude

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level. To illustrate the magnitude of this error, two common density function examples will be solved for a sinusoidal signal and a normally distributed random signal.

The probability density function of a sine wave (assuming that its phase angle has a uniform density function) is:

$$p(\alpha) = \frac{1}{\pi} \left(\frac{1}{\sqrt{2 - \alpha^2}} \right) ; \quad |\alpha| \leq \sqrt{2} \quad (41)$$
$$= 0 \quad \text{elsewhere}$$

where

α = a particular value of the normalized amplitude ($\sigma = 1$).

The true value of density can be found by substituting a numerical value for α in Eq. (41). The indicated value, $p_i(\alpha)$, can be found from the following equation:

$$p_i(\alpha) = \frac{1}{w} \int_{\alpha - (w/2)}^{\alpha + (w/2)} p(u) du \quad (42)$$

where

w = the width of the amplitude window.

Using $w = 0.1$, which is the value of the amplitude window width of the analyzer at the Sonic Fatigue Facility, the following values are obtained.

α	$p(\alpha)$	$p_i(\alpha)$
0	.225	.225
.5	.241	.241
1.0	.318	.318
1.2	.425	.428
1.3	.572	.585

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For the normally distributed random signal, the instantaneous amplitude probability density function is

$$p(\alpha) = \frac{1}{\sqrt{2\pi}} e^{-\alpha^2/2} \quad (43)$$

The corresponding theoretical and indicated values are:

α	$p(\alpha)$	$p_i(\alpha)$
0	.399	.399
.5	.352	.352
1.0	.242	.242
1.5	.130	.130
2.0	.054	.054
3.0	.004	.004

The two examples, which are fairly representative of the types of densities to be anticipated in practice, show that with a window width of 0.1, this error is insignificant except near the extreme tails of the sine wave density function.

An approximate expression for this error is developed in Reference 20 and is

$$\text{error} = \frac{w^2 p''(\alpha)}{24} \quad (44)$$

where

$p''(\alpha)$ = the second derivative of the density function evaluated at α

7.2 AMPLITUDE WINDOW SCAN ERROR

This density function analyzer uses a single amplitude window that is scanned across the amplitude range of interest. The output of the window is detected and averaged. Because the amplitude window is scanned, the scan rate must be slow compared to the time constant of the averager so that the output of the averager can follow abrupt changes in the level of the density function. This error is exactly analogous to the one for the frequency analysis equipment. The magnitude of the error is given by Eq. (39).

For a 1% or less error in the indicated density function because of scanning through a step change in level of the density function, t must be equal to or greater than 4.6 RC. Digital computer studies performed for Reference 21 have shown that 2 or more time constants per scan of one window width are required for a 1% of full scale error when the density function is Gaussian.

One other consideration in selecting the scan rate is that the time required to scan through one amplitude window width should be long compared to the period of the lowest frequency component (f_1) of the data.

$$t \gg \frac{1}{f_1} \quad (45)$$

To illustrate the reason for this criterion, consider the case of a single frequency sine wave of f_1 . If $t = 1/f_1$, then there will be exactly one cycle of sine wave for window width scan and there will be no error. However, if $t < 1/f_1$, then in each window scan there will be less than one complete cycle and an incomplete averaging is performed. Thus, the indicated density function is dependent on the phase angle of the sine wave at the start of the scan. Since the analyzer performs on complex signals, there is no way of guaranteeing that the scan rate is correct for all frequency components in the data. Hence, it is necessary that averaging be performed over a large number of cycles of the lowest frequency in the data to minimize the errors occurring from a nonintegral number of cycles.

7.3 OTHER SOURCES OF INACCURACIES

There are several other sources of inaccuracies. First, the data into the analyzer is normalized for operational convenience. This normalization is performed by a variable gain amplifier on the input of the analyzer in such a manner that the rms voltage into the analyzer (out of the variable gain amplifier) is maintained constant. Clearly, errors in the normalizing process result in errors in the density function measurement.

In a very similar manner, errors in the calibration of the amplitude window width result in errors since the analyzer is essentially implementing the following equation.

$$p(\alpha) = \frac{P\left(\alpha + \frac{w}{2}\right) - P\left(\alpha - \frac{w}{2}\right)}{w} \quad (46)$$

where

$P(\alpha)$ = the cumulative probability distribution function of
instantaneous amplitudes

When the data tape is formed into a continuous loop for repetitive analysis, the data must be fed through a high pass filter (the Long Term Averager) before going into the analyzer to remove the spurious periodicity created by formation of the loop. In addition, the Long Term Averager measures the mean value of the data and performs the high pass filtering required to satisfy Eq. (45).

One problem that is unique to the measurement of peak probability is the spectral shape of the signal being analyzed. The basic assumption is that the bandwidth of the data is small compared to the center frequency of the data. Since the analyzer does not actually perform a true peak probability density measurement (count the number of peaks occurring

inside the amplitude window width), there is an error related to the bandwidth to center frequency ratio because of the approximation used. The greater this ratio, the greater the error. (References 2 and 21.) Because envelope detection is used in this particular analyzer, the error may not be exactly the same as discussed in the above references, but it is reasonable to expect it to have a similar form.

7.4 DETECTION OF NON-NORMALITY

The amplitude analysis equipment offers a very powerful method for measuring non-normalities. One simply performs an instantaneous amplitude probability density function analysis and compares the results to the theoretical normal density function. If the record length is long enough to make the uncertainty fluctuations reasonably small, visual comparison to the theoretical normal probability density curve is the best method available. It should never be expected to find that the data are perfectly normal. The analyst will have to make a judgment as to whether the data are normal enough to satisfy his requirements.

When the available record lengths are of limited duration, the uncertainty fluctuations must be accounted for and it becomes necessary to test the measured probability density function for equivalence to the theoretical normal density function. (Reference 22) discusses in detail how to perform this particular type of test.

7.5 DETECTION OF NONSTATIONARITY

The amplitude analysis equipment is not appropriate for the detection of nonstationarities. Methods could be contrived to use it for this purpose, but it is far more practical to use the gross analysis equipment. One might want to employ the amplitude analysis equipment to study if the probability density function changes during nonstationarity of the data. To do this one might assume that the data are stationary over short time intervals and measure a sort of average probability function over each of the time intervals. This should be acceptable for determining if gross changes occur as long as the nonstationarity changes slowly in time compared to the short time interval selected for analysis.

7.6 DETECTION OF NONLINEARITIES

The amplitude analysis equipment is a valuable tool for detecting nonlinearities. For linear systems and normally distributed random signal inputs, there is no difference in shape between the instantaneous amplitude probability density function on the input and the output of the system. Again, as in the detection for non-normality, the best method available if the sample record length is long enough is to visually compare the measured probability density functions on the input and the output. However, if the input is not normal, the output instantaneous amplitude probability density function may differ significantly from that on the input while the system being measured meets the usual definition of linearity. For example, a simple RC circuit is usually considered to be linear. If a triangular wave whose frequency is just below that of the cutoff frequency of the RC lowpass filter is applied to the input of the filter, the instantaneous amplitude probability density function on the input is uniform, while the instantaneous amplitude probability density function on the output is that for a sine wave as defined in Eq. (41) because all but the fundamental harmonic components are filtered out.

Thus the amplitude analysis equipment is more valuable for detecting possible nonlinearities than for detecting linearity. In a practical sense, when the input closely approximates a normal density, the output should also closely approximate a normal density. Nonlinearities usually have a pronounced effect on the density function of the system output.

If only short record lengths are available, a hypothesis test must be performed to test for equivalence of the input and output probability density functions. This test should be performed exactly as discussed in Reference 22, except that the input probability density function is substituted for the theoretical normal density function. (Note that there must be some indication that the input data are approximately normal.)

7.7 DETECTION OF PERIODICITIES

The amplitude analysis equipment is quite useful for detecting periodicities. This is particularly true when this equipment is employed in conjunction with the frequency analysis equipment to determine the randomness/periodicity of peaks in the frequency spectrum. The amplitude analysis equipment can be applied to the over-all signal, but it is then not as powerful a detector of periodicities.

Narrow bandpass filtering enhances the usefulness of the amplitude analysis equipment in two manners.

1. The periodicity is not easily detected if the ratio of the rms noise voltage to the rms periodic voltage is too high. Narrow band filtering decreases this noise voltage by removing all frequency components not in the filter passband. The component of the periodic signal lying inside the passband is not attenuated, thus the noise to periodicity ratio is decreased.
2. The separation of the periodic signal into its Fourier components is of considerable help in the detection of periodicities

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since it means that one need only inspect the data for a characteristic trend toward the familiar bimodal density function of a sine wave. One should remember that the probability density function of the sum of two sine waves is a function of the phase angle between these sine waves. This complicates the inspection of the over-all data signal for any possible periodicity.

8. TIME DOMAIN ANALYSIS EQUIPMENT

The RTD Sonic Fatigue Facility has one instrument capable of performing analyses in the time domain. This instrument can perform autocorrelation analyses on a single signal or cross-correlation analyses of two signals. In this instrument, a single time delay is continuously scanned across the delay range of interest.

With correlation analyses, there is only one major intrinsic error source. That is the error in resolution. If the delay is scanned at an infinitely low rate, an infinite resolution is obtained. When the delay is scanned at a finite rate, the output of the analyzer is not only a function of the correlation function at the instantaneous value of time delay, but also contains a time weighted average of the correlation function at all previous time delay values. Thus, the scan of the delay time must be slow in order to permit the contributions of previous values to become negligibly small. The faster the scan is made, relative to the time constant of the averager, the greater are the contributions of previous values. This means that the correlation function is being smoothed out, or blurred. This is known as the resolution error.

It has been suggested in Reference 23 that a reasonable criterion for resolution of a correlation analysis is:

$$h \leq \frac{1}{4f_m} \quad (47)$$

where

h = the resolution in the delay time

f_m = the maximum frequency of interest in the data

If one thinks of Eq. (47) in terms of a discrete analysis for a moment, it would be interpreted as meaning that if one were analyzing data composed of a single sine wave of frequency f_m , that the autocorrelation function

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would be a cosine wave of frequency f_m and that one would obtain four discrete samples of this cosine wave for every period.

Since a simple RC circuit performs the averaging function in the RTD analyzer, as is the case for most analyzers, the scan must be slow enough for the output of the RC circuit to follow abrupt changes in the correlation function. This problem is an exact analogy to that described in Eq. (39) for the frequency analysis equipment and the same equation applies. For a 1% error to a step change in the correlation function, 4.6 time constants are required. This means that the delay scan rate (R_s) should be

$$R_s \leq \frac{h}{4.6 \text{ sec}} \quad (48)$$

Substituting the value for h from Eq. (47) into Eq. (48), one obtains:

$$R_s \leq \frac{1}{18.4 f_m RC} \quad (49)$$

At this point it should be noted that Eq. (49) is based on several conservative assumptions.

1. The data are all contained at the highest frequency of interest.
2. There will be a step change in the level of the correlation function.
3. Only a 1% error in following the step change is desired.

With all scanned analyses, the optimum scan rate is a function of the quantity being analyzed. If it is possible to analyze the same data several times with different scan rates, then the optimum scan rate is found by determining the scan rate below which the output does not change with further decrease in scan rate. If one cannot do this, then he must use some rational criterion such as Eq. (49), substituting the accuracy values desired.

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One minor error source that deserves some discussion is instability of the drum speed. Since the time delay is obtained by relative motion of a magnetic head to the magnetic drum, the drum speed must remain constant. A static change in speed causes a fixed shift in the apparent width of a time delay interval. This error can be calibrated out by applying a sine wave of accurately known frequency and performing an autocorrelation analysis. The distance between zero crossings on the autocorrelation function will be equal to the period of the sine wave. Dynamic fluctuations in the velocity of the drum increase the resolution error by frequency modulating the data signal.

8.1 DETECTION OF NON-NORMALITY

The time domain analysis equipment is not very useful for detecting any but the most gross non-normalities.

8.2 DETECTION OF NONSTATIONARITY

The time domain analysis equipment is not very appropriate for the detection of nonstationarity. It can be used by making a short time duration stationarity assumption and dividing the total record into a series of short segments. Each of these short segments can then be analyzed and hypothesis tests performed to determine equivalence from segment to segment, etc. However, this is not very practical.

If one knows of a nonstationarity in the data and can select sample records that are short compared to the time variation in the nonstationarity, then it is of value to analyze several short records to see the effects of the nonstationarity on the correlation function.

8.3 DETECTION OF NONLINEARITIES

The time domain analysis equipment is not very useful for the analysis of nonlinearities.

8.4 DETECTION OF PERIODICITIES

The time domain analysis equipment is very useful for the detection of periodicities. In fact, of the three detailed analyses procedures (frequency, amplitude, time) discussed in this report, the time domain analysis procedure is probably the most powerful for the detection of periodicities. This is because the autocorrelation function of random noise approaches zero as the time delay value becomes large, but the autocorrelation function of a periodicity is periodic and will not decay at large time delay values. Thus, when the autocorrelation function of the sum of random noise and a periodic signal is computed, the contributions of the noise tend to die out rapidly while the periodic contribution continues to oscillate as the time delay is increased.

The rate at which the random noise contributions decay is a function of the spectral shape of the random signal. If the data are composed primarily of a very narrow band low frequency random signal, the decay may be so slight over the time delay range of the analyzer that it will not be possible to say if the data is random or periodic. Then, it is necessary to perform an instantaneous amplitude probability density function analysis on the data to make this decision.

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13. ABSTRACT This report discusses the machine errors of equipment used for the conditioning, collection, recording, editing, and analysis of data from dynamic processes such as acoustics, strain, or vibration. The specific equipment items studied include special features of signal conditioners (e. g., automatic gain control), signal conditioner to tape recorder transmission line, magnetic tape recorders, gross analyzers (e. g., voltmeters), frequency analyzers, amplitude analyzers, and time domain analyzers. (The machine errors associated with transducers and basic signal conditioners are presented in AFFDL-TR-64-171, "Transducers for Sonic Fatigue Measurements.") Where these errors are controllable, such as the smoothing error associated with the rate of scanning an amplitude window for probability density measurements, pertinent equations are presented for determining the magnitude of these errors as a function of the controllable parameter(s). In addition, the report describes practical usage of the analysis equipment for the detection of non-normality, nonstationarity, and periodicity.		

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