HARDENING BY SPINODAL DECOMPOSITION

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ABSTRACT

The effect of the internal stresses produced by spinodal decomposition on dislocation behavior is investigated for several slip systems in cubic materials.

INTRODUCTION

There has been much work on the effect of precipitation on mechanical properties[1-4]. Almost all of the theoretical work has been concerned with discrete particles, either coherent or incoherent. There exist, however, a number of precipitating systems in which the early stages of precipitation resemble long-range coherent composition fluctuations with no abrupt changes in composition. This kind of precipitation results from a thermodynamic instability called the spinodal. The theory of spinodal decomposition has been developed for both isotropic[5, 6] and cubic[7] materials. There is good reason to believe that a number of important age hardening alloys[8] belong to this class, among them the nickel base Inconel 80 and Nimonics. It is the purpose of this paper to explore theoretically how the mechanical properties of a cubic crystal should be affected by the long-range coherent composition fluctuations resulting from spinodal decomposition.

At a first glance a structure of such composition fluctuations might seem too vague to give any hope of calculating much about its properties. Actually just the reverse is true. There is a great deal of regularity in this structure. In cubic crystal the composition fluctuations are best described as interpenetrating stationary (non-propagating) {100} plane waves. This has been derived theoretically[7] and the sharp directionality has long been known[8] experimentally from the streaking of X-ray spots. We will therefore assume that we are dealing with fluctuations all of whose Fourier components have <100> wave vectors.

For some of the calculations it will also be assumed that the structure consists of only three perpendicular {100} waves all having the same wavelength. Again there is good experimental and theoretical justification for choosing a single wavelength to describe the structure in the early stages of decomposition. Such a structure consists of two interpenetrating simple cubic arrays (a large CsCl structure) of pseudo particles (maxima and minima in composition), connected by an interpenetrating simple cubic lattice work of <100> pseudo rods.

The experimental variables at our disposal are amplitude of the Fourier components and wavelength of the most prominent ones. Temperature of aging primarily affects the wavelength, while time, at temperature, primarily affects amplitude. Thus we can change the spacing and resistance to deformation of our pseudo particles. The spacing ranges from 10Å to 10μ, and the amplitude is usually limited by the loss of coherence which occurs when the lattice parameter variations due to the composition variations result in strains of the order of percents.

In this paper we will discuss the force on a single dislocation due to the internal stresses and composition gradients in the undeformed structure.

THE INTERNAL STRESSES

Consider first a single Fourier component of the composition fluctuation whose wave vector β is in the [001] direction (z direction) and whose amplitude is $A(\beta)$

$$c - c_o = A(\beta) \cos \beta z \tag{1}$$

in a cubic alloy of average composition c_o. Let $\eta = \frac{\partial \ln a}{\partial c}$ describe the compositional variation of stress-free lattice parameter a. The internal stress σ produced by this plane wave[7] is given by

$$\begin{aligned} \sigma_{xx} &= \sigma_{yy} = (\eta \Delta c Y) = A\eta\, Y(100) \cos \beta z \\ \sigma_{zz} &= \sigma_{xz} = \sigma_{yz} = \sigma_{xy} = 0 \end{aligned} \tag{2}$$

where $Y(100) = (C_{11} + 2C_{12})(C_{11} - C_{12})/C_{11}$.

Consider next the superpositioning of all Fourier components whose wave vectors are in the [001] direction

$$\begin{aligned} \sigma_{xx} &= \sigma_{yy} = S_3(z) \\ \sigma_{zz} &= \sigma_{xz} = \sigma_{yz} = \sigma_{xy} = 0 \end{aligned} \tag{3}$$

where $S_3(z)$ is given by integrating equation 2 over all Fourier components in the z direction

$$S_3(z) = \eta Y \int A(\beta) e^{i\beta z}\, d\beta \,.$$

For the [100] and [010] components we may define similar quantities for the internal stress, $S_1(x)$ and $S_2(y)$ respectively. Thus the internal stress resulting from all Fourier components is

$$\sigma = \begin{pmatrix} S_2(y) + S_3(z) & 0 & 0 \\ 0 & S_3(z) + S_1(x) & 0 \\ 0 & 0 & S_1(x) + S_2(y) \end{pmatrix} \quad (4)$$

For the particular case of three equal perpendicular waves of the same wavelength $2\pi/\beta$

$$S_1(x) = A\eta Y(100)\ \cos \beta x$$

$$S_2(y) = A\eta Y(100)\ \cos \beta y \quad (3')$$

$$S_3(z) = A\eta Y(100)\ \cos \beta z\ .$$

The stress pattern is a tessellated stress in the literal sense. Although the literature on tessellated stresses is voluminous[9], there appear to be only a few papers[(10-12)] which consider internal stresses that vary sinusoidally.

THE FORCE ON DISLOCATIONS DUE TO THE INTERNAL STRESSES

In this section we shall derive the forces exerted on a dislocation by the internal stress field. These forces are superimposed on those resulting from an externally applied stress. We shall follow the sign convention of Nabarro[4] throughout, and thereby eliminate ambiguity regarding the sign of the force. For simpler stresses it is sometimes convenient to rotate axes so that the Burgers vector is parallel to x and the slip plane is the xy plane and then determine the magnitude of the force from $b\sigma_{xz}$ and the sign from physical intuition. For the present case intuition is not quite as easy to apply, and Nabarro's convention applied to the Peach and Koehler formulation will be used since it gives, in a quite straightforward manner, not only the magnitude of the force but also its direction.

The force $\bar{F}$ on a dislocation per unit length produced by a stress field σ is

$$\bar{F} = (b \cdot \sigma) x \zeta \quad (5)$$

where b is the Burgers vector and ζ the unit tangent to the dislocation line. The sign of b depends on the direction in which the Burgers circuit is taken which in turn is related by convention to the direction of ζ so that the sign of $\bar{F}$ depends only on the physical situation and not on the direction of the Burgers circuit. Reversing ζ also reverses b.

The force on a dislocation is always at right angles to ζ, and for edge dislocations consists of both climb and glide components. The glide component F_g is given by

$$F_g = (n x \zeta)(b \sigma n) \tag{6}$$

where n is the normal to the glide plane. Reversing the sign of n leaves F_g unaltered. The product $b\sigma n$ is $b\sigma_{xz}$ if the coordinate axes are rotated so that b coincides with the x axis and n with the z (slip on the xy plane). The vector product $(n x \zeta)$ is a unit vector in the slip plane at right angles to the dislocation. According to equation 6 the magnitude of the force on a dislocation is independent of the direction of the dislocation line. The product of resolved shear stress $(1/|b|)(b\sigma n)$ and the Burgers vector determines the magnitude of the force, and the remainder of the expression serves to determine its direction.

Equation 6 applies equally well to screw dislocations but since these can glide on a number of planes it is sometimes convenient to use equation 5 which for screw dislocations becomes

$$\bar{F} = \frac{1}{|b|}(b \cdot \sigma x b) \tag{7}$$

This gives the force on the dislocation and its direction, rather than the force resolved in some plane n.

Let us now apply these equations to the internal stress given by equation 4 to a dislocation whose Burgers vector has components (b_1, b_2, b_3) and which slips on a glide plane whose normal has components (n_1, n_2, n_3). The scalar force $b \cdot \sigma \cdot n$ is

$$b \cdot \sigma \cdot n = n_1 b_1 (S_2 + S_3) + n_2 b_2 (S_1 + S_3) + n_3 b_3 (S_1 + S_2) \tag{8}$$

Bearing in mind that b is perpendicular to n

$$b_1 n_1 + b_2 n_2 + b_3 n_3 = 0 \tag{9}$$

we may rewrite equation 8

$$b \cdot \sigma \cdot n = -(n_1 b_1 S_1 + n_2 b_2 S_2 + n_3 b_3 S_3) \tag{10}$$

For screw dislocations we have from equation 7

$$F = \frac{1}{|b|} \{ib_2b_3(S_3 - S_2) + jb_1b_3(S_1 - S_3) + kb_1b_2(S_2 - S_1)\} \qquad (11)$$

where i, j and k are unit vectors along [100], [010] and [001] respectively. Equation 11 may be rearranged as the sum of three vectors, each one depending only on waves in a single direction.

$$F = b_1S_1 \left(0 + \frac{jb_3}{|b|} - \frac{kb_2}{|b|}\right) + b_2S_2 \left(- \frac{ib_3}{|b|} + 0 + \frac{kb_1}{|b|}\right) + b_3S_3 \left(\frac{ib_2}{|b|} - \frac{jb_1}{|b|} + 0\right) . \qquad (12)$$

Here $b_1/|b|$, $b_2/|b|$ and $b_3/|b|$ are simply the cosines of the angle that b makes with the three <100> directions. Each Fourier component produces an alternating force on the screw dislocation which is both perpendicular to the Burgers vector and the wave vector, and whose magnitude is zero, if the Burgers vector and wave vector are either perpendicular or parallel to each other.

OTHER FORCES ON THE DISLOCATIONS

In these inhomogeneous alloys there exist several other sources of forces on dislocations that are not present in a homogeneous alloy of the same composition. These relate mainly to the composition gradients. For very small gradients the self energy γ of a dislocation is approximately given by

$$\gamma \sim Gb^2 \sim Yb^2 \qquad (13)$$

where G is the shear modulus. If γ varies with position there will be a force on the dislocation [13] given by

$$\bar{F}_c = tx(\text{grad } \gamma \text{ } xt) = (\gamma(\frac{d\ln G}{dc} + 2\eta)(\nabla cxt)xt . \qquad (14)$$

The corresponding glide force in a plane whose normal is n

$$\bar{F}_{cg} = \gamma(\frac{\partial \ln G}{\partial c} + 2\eta)(\nabla cxt \cdot n)(txn) \qquad (15)$$

As before the last factor just expresses the direction; the magnitude is given by the remaining factors.

For the three equal perpendicular waves in composition

$$c - c_o = A(\cos \beta x + \cos \beta y + \cos \beta z) \qquad (16)$$

equation (15) gives for the magnitude of F_{cg}

$$|F_{cg}| = A\gamma_\beta\left(\frac{\partial \ln G}{\partial c} + 2\eta\right)[\sin\beta x(t_2n_3 - t_3n_2) + \sin\beta y(t_3n_1 - t_1n_3)$$

$$+ \sin\beta z(t_1n_3 - t_3n_1)] \, . \tag{17}$$

Comparison with equations 11 or 12 and 3' shows that this force is on the average smaller than the force due to the internal stresses by a factor of $b\beta$, but that it differs in phase and direction. It is therefore negligible except where, because of symmetry, the forces due to internal stresses are zero.

Another source of resistive forces in a composition gradient comes from the fact that after slip through an inhomogeneous region, the two opposing faces of a slip plane now differ in composition by a

$$\Delta c = b \cdot \nabla c. \tag{18}$$

Such an interface requires an additional energy and gives rise to a force proportional to $(\Delta c)^2$. It is the analogue of chemical hardening model suggested by Kelly and Fine[15]. Since it is quadratic in amplitude A it is negligible whenever there are dominant terms of lower power in A.

DISCUSSION OF SEVERAL SLIP SYSTEMS

(100) Slip Plane

Equation 10 reveals that if either the slip plane normal or the Burgers vector are either parallel or perpendicular to the wave vector of a Fourier component, the internal stresses resulting from that Fourier component will exert no glide force on that dislocation. (This result is not restricted to <100> wave vectors but holds true for all orientations). Thus slip involving either <100>b or {100} slip planes will be unaffected by the internal stress field. This result is somewhat unexpected, if one bears in mind that we have here an assembly of psuedo particles and large internal stresses resulting from them. Consider, for example, the {100} slip plane.

If we examine the special case of three perpendicular waves of the same wavelength one sees that one of the (100) slip planes could have encountered a square array of the peaks in composition (pseudo particles of one phase). A parallel plane half a wavelength down would have encountered all the lows in composition (pseudo particles of the other plane). Another parallel plane, a quarter wavelength down, would have encountered no extremes in composition. One might have expected large differences in forces, but there are none. A simple insight into why this is so is provided by equation 4, which shows that the cube axes are principal stress axes and therefore there is no shear on any cube plane.

For such a slip system we are left only with the forces due to the variations in line tension due to the composition changes (equations 14-17). These tend to pin the dislocations along either the maxima or minima in composition, that is inside one of the pseudo phases.

(110) Slip Plane

Consider next a (110) slip plane which is often a primary slip plane in B.C.C. Let the equation of the slip plane be

$$x + y = \sqrt{2}d, \tag{19}$$

where d is the distance of the plane from the origin. Let $\sqrt{2}x' = x - y$ and $z' = z$ be the coordinates in the plane (Fig. 1). All shear stresses in the plane are along the x' direction and alternate in sign. For the special case of 3 equal perpendicular waves of the same wavelength we have for the force on a $[11\ell]$ dislocation from equation 10

$$b\cdot\sigma\cdot n = -\frac{|b|}{(4 + 2\ell)^{1/2}}(S_1 - S_2)$$

which by equation 3' becomes

$$b\cdot\sigma\cdot n = \frac{A\eta Y|b|}{(4 + 2\ell)^{1/2}} (\cos \beta x - \cos \beta y) \tag{20}$$

$$= \frac{A\eta Y|b|}{(1 + \ell^2/2)^{1/2}} \sin(1/\sqrt{2}\beta d) \sin(1/\sqrt{2}\beta x') .$$

Dislocations in the various slip planes encounter no forces if $(1/\sqrt{2})\beta d$ is an integer and sinusoidal forces if $(1/\sqrt{2})\beta d$ is not an integer. The former correspond to slip planes through the centers of the pseudo particles. The strongest forces are encountered in slip planes that avoid the pseudo particles.

Let us next consider how these forces should affect the shape of the dislocation line. In the absence of an applied stress, the dislocation must curve and at equilibrium, assuming a string model for the dislocation, it must obey the following differential equation

$$b\cdot\sigma\cdot n = \gamma \frac{\frac{d^2z'}{dx'^2}}{[1 + (\frac{dz'}{dx'})^2]^{3/2}} . \tag{21}$$

Letting $w = \frac{dz'}{dx'}$, $b\cdot\sigma\cdot n = A'\sin 1/\sqrt{2}\, \beta x'$ where

$$A' = \frac{A\eta Y|b|}{(1 + \ell^2/2)^{1/2}} \sin \frac{\beta d}{\sqrt{2}}$$

we obtain

$$\frac{A'}{\gamma} \sin \frac{1}{\sqrt{2}} \beta x' = \frac{\frac{dw}{dx}}{(1+w^2)^{3/2}}$$

which can be integrated to give

$$\text{const.} - \frac{\sqrt{2}A'}{\beta\gamma} \cos \frac{1}{\sqrt{2}} \beta x' = \frac{w}{(1 + w^2)^{1/2}} \quad . \qquad (22)$$

Since the right-hand side is bounded by ±1 a stable dislocation configuration straddling a band of force can exist only if

$$\frac{\sqrt{2}A'}{\beta\gamma} \leq 1 \ . \qquad (23)$$

If it exceeds 1, the internal stresses will align all dislocations along [001] in the stable zero force positions. Every mobile and flexible dislocation will be forced into such configurations. Using equation 13 this condition becomes

$$A\eta \sin \frac{1}{\sqrt{2}} \beta d \leq \beta b \ . \qquad (23')$$

For example, the aligning of dislocations begins at wavelengths of 1000Å when the lattice parameter amplitude reaches 1%.

In considering slip in such a crystal one must carefully distinguish dislocations that already straddle the pseudo particles and those that do not. The former can move easily in the presence of an applied stress very much like a kink straddling a Peierls-Nabarro barrier can move sideways. On the other hand, formation of new kinks by having a section of a dislocation slip across a pseudo particle is very much more difficult and varies from plane to plane. At low stress levels only dislocations going through the centers of the pseudo particles can move. With increasing stress the planes on which slip is possible form an ever increasingly thick band about the centers.

The equations for a number of dislocations piling up on such a sinusoidal barrier have recently been derived[12].

The (111)[1$\bar{1}$0] Slip System

The (111)[1$\bar{1}$0] slip system is another one which interacts with only two of the three fluctuation components and therefore should behave as if the sample contained an assembly of pseudo rods along [001]. In this case the slip plane cuts across the rods and all parallel (111) planes encounter the same obstacles as do the $(11\bar{1})$ cross slip planes. Therefore cross slip would be of no help in avoiding the resistance imposed by the internal stresses.

Let the equation of the slip plane be

$$x + y + z = \sqrt{3}d \ . \tag{24}$$

The resolved force on a $[1\bar{1}0]$ dislocation is

$$b\cdot\sigma\cdot n = -\frac{|b|}{\sqrt{6}}(S_1 - S_2) = \frac{A\eta|b|Y}{\sqrt{6}}(\cos\beta x - \cos\beta y)$$

$$= \sqrt{\frac{2}{3}}\, A\eta|b|Y \sin[1/2\ \beta(x+y)] \sin[1/2\ \beta(x-y)] \ .$$

Rotating the coordinate system such that x' is along $[1\bar{1}0]$ and y' along $[11\bar{2}]$ in the slip plane

$$\begin{aligned} \sqrt{2}x' &= x - y \\ \sqrt{6}y' &= x + y - 2z + 2\sqrt{3}d \ . \end{aligned} \tag{25}$$

By virtue of equation 24

$$\sqrt{6}y' = 3x + 3y$$

so that

$$b\cdot\sigma\cdot n = \sqrt{\frac{2}{3}}\, A\eta|b|Y \sin\left(\frac{1}{\sqrt{6}}\beta y'\right) \sin\left(\frac{1}{\sqrt{2}}\beta x'\right) \ . \tag{26}$$

The (111) slip plane thus resembles a rectangular checker board of alternating forces, as shown in Fig. 2. The sides of the rectangles are the locus of zero force positions and within each rectangle the force rises or falls to an extremum in the center.

Using the string model for a mobile dislocation one obtains an equation for the dislocation time at rest in the presence of such a force field.

$$\frac{\gamma \dfrac{d^2y'}{dx'^2}}{\left(1 + \left(\dfrac{dy'}{dx'}\right)^c\right)^{3/2}} = \sqrt{\frac{2}{3}}\, A\eta|b|Y \sin\left(\frac{1}{\sqrt{6}}\beta y'\right)\left(\sin\frac{1}{\sqrt{2}}\beta x'\right) + \sigma_a|b| \tag{27}$$

where we have added a term to account for an applied stress, whose resolved shear stress is σ_a.

Equations of this type are known as Duffing's equation and occur in forced vibrations.[16] One method of solution is the Galerkin-Ritz variational method in which one assumes a solution to the equation and adjusts parameters so that certain Fourier components of the error vanish.

As in all problems of this type there are two convenient extremes. If $A\eta|b|Y/\gamma\beta < 1$ the dislocation line is almost straight, the wavelength or amplitude of the internal stresses being too small to cause much bending. The other extreme is when $A\eta|b|Y/\gamma\beta > 1$. In this case the dislocation curves around all obstacles.

In the first case, assuming a sinusoidal form for the dislocation line

$$y' = C_1 + C_2 \sin \frac{\beta x'}{\sqrt{2}} \quad \text{for screw dislocation}$$

and

$$x' = B_1 + B_2 \sin \frac{\beta y'}{\sqrt{6}} \quad \text{for edge dislocations} \tag{28}$$

we obtain, using the Galerkin method

$$\cos C_1 = \frac{3\sqrt{3}\,\beta\gamma}{4\sqrt{2}\,A^2\eta^2Y^2|b|}\,\sigma_a$$

$$\cos B_1 = \frac{\beta\gamma}{4\sqrt{2}\,A^2\eta^2Y^2|b|}\,\sigma_a \tag{29}$$

$$C_2^2 = \frac{2}{9}\frac{A^2\eta^2Y^2b^2}{\beta^2\gamma^2}\left[1 + \sqrt{1 - 54\frac{\sigma_a^2\beta^2\gamma^2}{A^4\eta^4Y^4b^2}}\right]$$

$$B_2^2 = 6\,\frac{A^2\eta^2Y^2b^2}{\beta^2\gamma^2}\left[1 + \sqrt{1 - 2\frac{\sigma_a^2\beta^2\gamma^2}{A^4\eta^4Y^4b^2}}\right]$$

At small values of the applied stress the dislocation reaches equilibrium straddling the peaks in the internal stress which occur at $C_1 = B_1 = \pi/2$. There is an additional unstable solution at $C_1 = B_1 = 0$ where there are zero forces on the dislocation. However, certain small fluctuations in position cause the dislocation to move off this position.

At applied stresses exceeding

$$\sigma^*_a = \frac{A^2\eta^2 Y^2 b}{3\sqrt{6}\,\beta\gamma} \quad \text{for screws}$$

or (30)

$$\sigma^*_a = \frac{A^2\eta^2 Y^2 b}{\sqrt{2}\,\beta\gamma} \quad \text{for edges}$$

no solution exists and the dislocation should move continuously through the structure. This corresponds to the Mott and Nabarro yield stress for this structure. As in the Mott and Nabarro relation, this equation reflects the averaging of large forces that alternate in sign. The yield stress is therefore smaller by orders of magnitude than the internal stress maxima, and depend entirely on what little flexing the dislocation can do. Because of the longer apparent wavelength encountered by edges, they can flex more, and as a result are much harder to move than the screws. For the same reason the yield stress increases linearly with wavelength.

The stress is also quadratic in amplitude. This brings it to the same degree in amplitude as the chemical term arising from equation 18. Since such a chemical term is always a retarding force, it survives averaging without a change in degree in amplitude. Whether it is an important term will depend on how its coefficient compares with those given in equation 30. For a 1% lattice parameter difference ($A\eta = 1.6 \times 10^{-3}$) a $\beta = 10^6\ cm^{-1}$ (600Å wavelength), $Y = 10^{12}$ ergs/cc, $b = 10^{-8}$ cm we obtain $\sigma_a = 10^8\ dynes/cm^2$. This stress is close to the upper limit of the assumption of a straight dislocation because $A\eta Y|b|/\gamma\beta = .16$.

The other extreme, $A\eta Y|b|/\gamma\beta > 1$ corresponds to a dislocation that can curl around obstacles. At zero applied stress, we have a checker board of obstacles that touch each other at their corners to form a continuous obstacle. However, the forces on the dislocations at these junctions are very small and as a stress is applied these junctions separate (Fig. 3). The effective distance between obstacles increases with increasing stress and may be found by plotting the contours of zero net force on the dislocation

$$\sqrt{2/3}\,A\eta Y \sin\left(\frac{1}{\sqrt{6}}\beta y'\right) \sin\left(\frac{1}{\sqrt{2}}\beta x'\right) + \sigma_a = 0 \,. \qquad (31)$$

Since one such junction is at $x' = y' = 0$ we may expand the sine terms in this equation to obtain

$$x'y' = -3\sqrt{2}\,\sigma_a/(A\eta Y\beta^2) \,.$$

The separation L between obstacles is thus

$$L = 4.6\,\sigma^{1/2}/[(A\eta Y)^{1/2}\beta] \,. \qquad (32)$$

The dislocation can slip through this opening if

$$\sigma_a b > 2\gamma/L$$

or

$$\sigma_a > .57\ (A\eta Y)^{1/3} \left(\frac{\gamma\beta}{b}\right)^{2/3} . \qquad (33)$$

This then is the analogue of Orowan[17] yielding in this structure. The stress depends on the reciprocal 2/3 power of the wavelength rather than on the reciprocal first power, because the effective distance between these soft particles is a function of the applied stress. Each dislocation that slips through will leave a loop around the obstacles, and should lead to the analogue of Fisher, Hart, Pry[18] hardening, including the possibility that the stress of successive concentric loops will cause the inner loop to cut through the obstacle.

Other Slip Systems

Only if there are no zeros in the Miller index designation of slip plane and slip direction will the structure appear as discrete particles to the dislocation. Thus, for instance, if the <110> Burgers vector in FCC splits into two <112> partials, each of the partials will be able to avoid obstacles by cross slip of the recombined dislocation, but what is an easy glide plane for one partial will abound in obstacles for the other, since the sum of the forces on the partials does not vary from plane to plane.

In BCC the {112} and {123} slip planes will behave as if the structure is particulate and slip bands should occur on the easy planes which will be spaced periodically.

SUMMARY AND DISCUSSION

A dislocation in a spinodally decomposed structure experiences a force from the internal stresses and composition gradients. The former are expected to be the more important, except for special slip systems where the internal stresses exert no force (either $\bar{b}$ or $\bar{n}$ along <100>). The details for the interaction with the internal stress have been worked out for several slip systems, and these give rise to a number of concepts analogous to those encountered in precipitation hardening by discrete particles, except that the details are sometimes quite different. Thus the (111)[1$\bar{1}$0] slip system exhibits the analogues of both Mott and Nabarro and Orowan hardening, at small spacings a hardening linear in spacing (equation 30) and at large spacings hardening proportional to the reciprocal 2/3rd power of spacing (equation 33). The (1$\bar{1}$0)[111] slip system resembles a Peierls-Nabarro trough with an extremely long period, so that thermal activation is almost certainly negligible.

In many respects spinodally decomposing materials should prove to be convenient for experimental studies of hardening. The two variables, wavelength and amplitude, can be varied independently to investigate separately the effects of each. They correspond in precipitation hardening to particle spacing and particle hardness, and certainly the latter is not normally an experimental variable.

There are numerous unexplored areas remaining. The subject of many dislocations on a slip plane is one. For the [111]($1\bar{1}0$) slip system the recent paper by Chou is pertinent. A similar treatment for the [$1\bar{1}0$] (111) should not be too difficult. The analogue of Fisher, Hart and Pry hardening should be investigated for long wavelengths. Still another subject is the mechanism and slip system operating in the loss of coherence. Since loss of coherence is usually not observed until the internal strains are of the order of a percent, the stresses approach the theoretical limit of strength.

Only cubic materials for which $C_{44} > \frac{C_{11} - C_{12}}{2}$ were discussed in this paper. These give rise to {100} plane waves and all known examples of spinodal decomposition belong in this class. There are a number of systems for which $C_{44} < \frac{C_{11} - C_{12}}{2}$ that could give rise to spinodal decomposition. These would give {111} plane waves, which would have quite different effects on dislocations.

Acknowledgement

I am indebted to R. L. Fleischer, E. W. Hart, G. Horvay, J. D. Livingston and H. W. Schadler for stimulating criticism and suggestions about various aspects of this paper. I am especially grateful to J. D. Livingston for a critical review of the manuscript.

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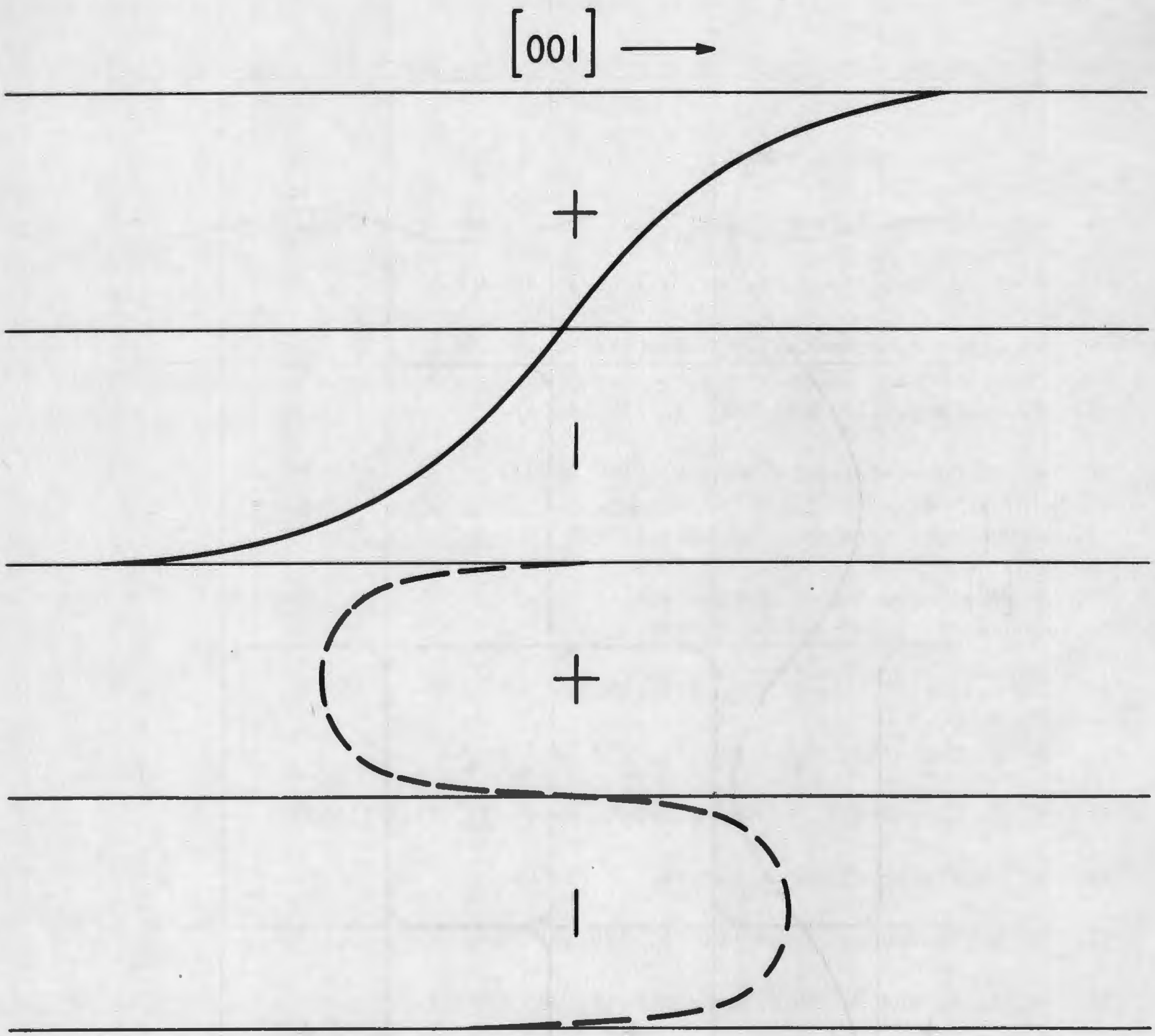

Fig. 1 - The (110) slip plane showing the forces on a dislocation and the resulting dislocation configuration. The plus and minus signs refer to the sign of the force within each region and therefore to the curvature of the dislocation line there. Left-hand dislocation is for low force amplitude (A' in equation 23) for that wavelength. Right-hand dislocation is for large A', near the limit beyond which all dislocations are forced to line up along [001].

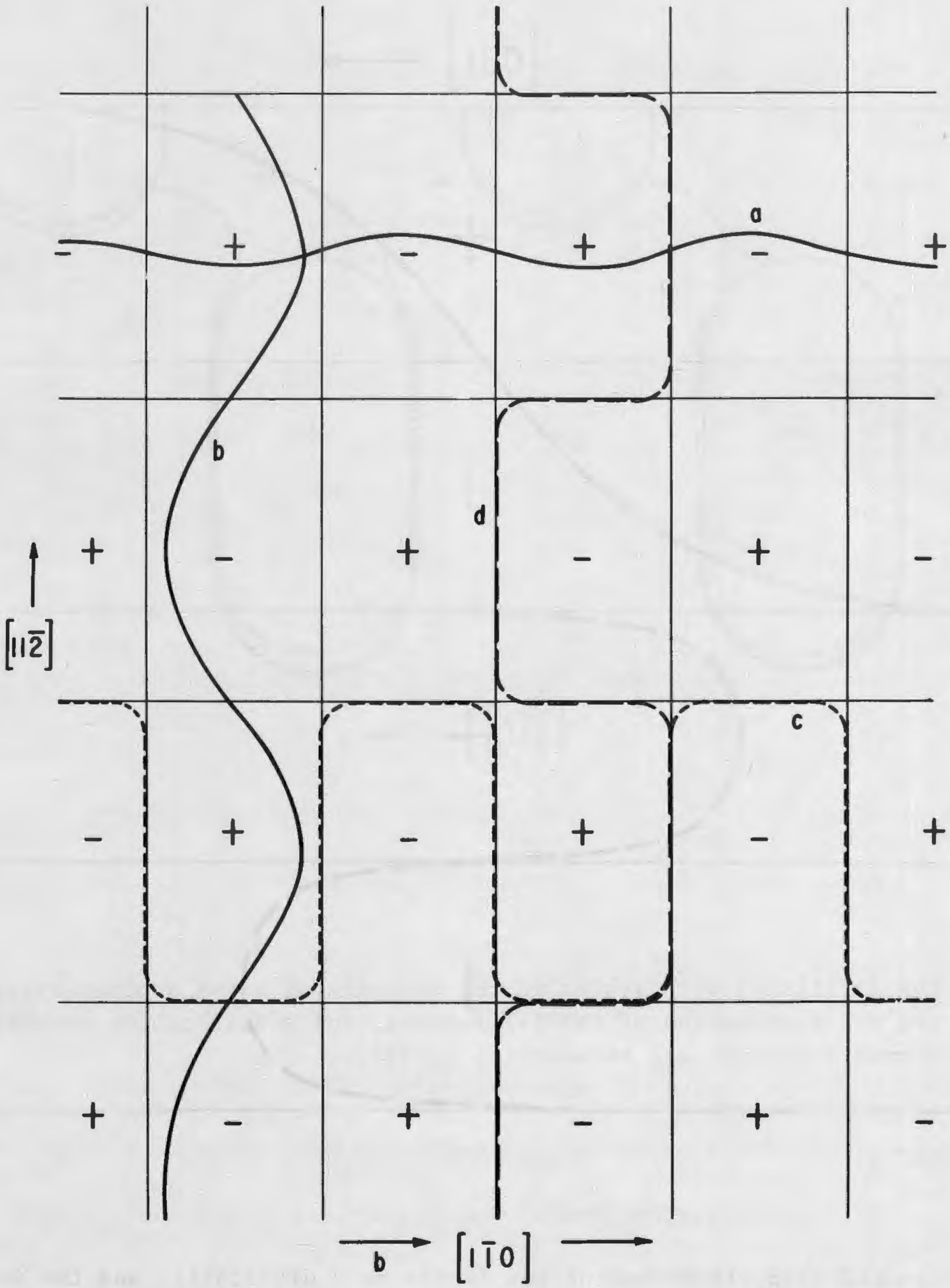

Fig. 2 - The (111)[1$\bar{1}$0] slip system in the absence of applied stress showing the forces on the dislocation and the resulting configurations. (a) screw and (b) edge at small amplitude for that wavelength; (c) "screw" and (d) "edge" at large amplitude.

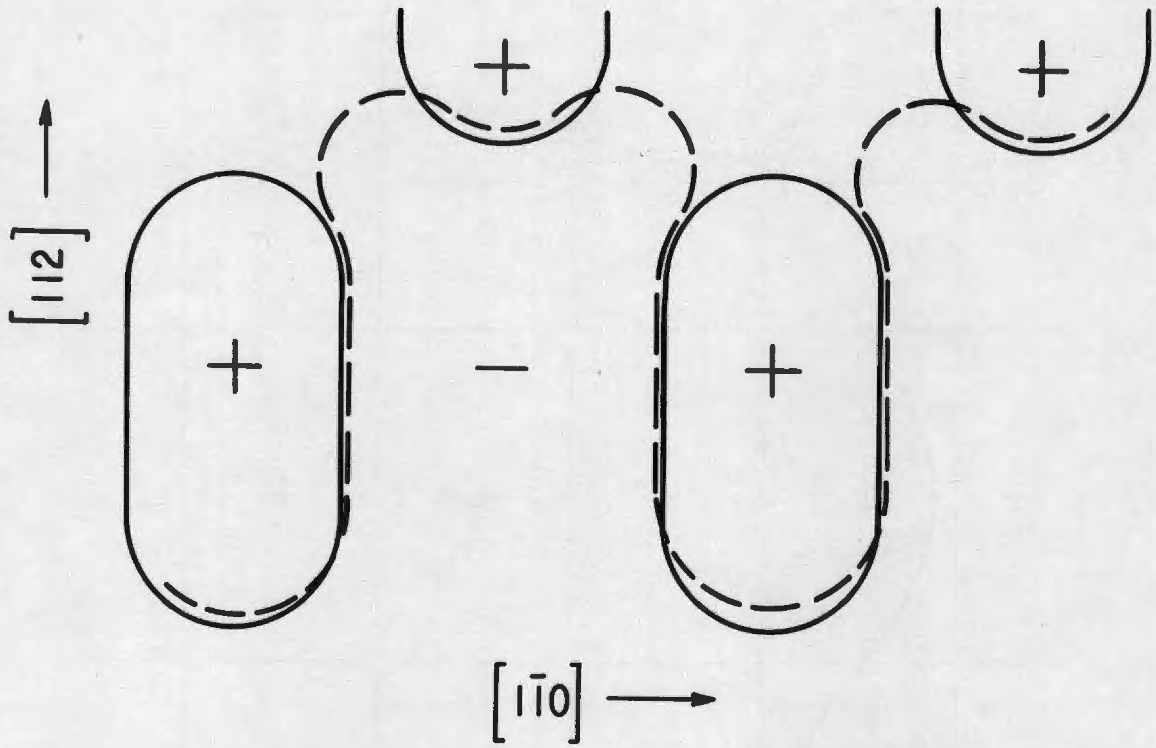

Fig. 3 - The (111)[1$\bar{1}$0] slip system in the presence of a (-) applied stress, showing the contraction of the (+) regions, and a dislocation straddling the stress dependent gap between (+) barriers.