FOREWORD

This report covers the research conducted by the Space and Information Systems Division of North American Aviation, Inc., Downey, California, for the Aerospace Dynamics Branch, Vehicle Dynamics Division, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, under Contract No. AF 33(657)-10399.

The work was performed to advance the state of the art of flutter prevention for flight vehicles as part of the Research and Technology Division, Air Force Systems Command's exploratory development program. This research was conducted under Project No. 1370 "Dynamic Problems in Flight Vehicles," and Task No. 137003, "Prediction and Prevention of Dynamic Aerothermoelastic Instabilities." Mr. James Olsen of the Vehicle Dynamics Division, Air Force Flight Dynamics Laboratory was the Project Engineer.

Mr. L. V. Andrew was the Program Manager for North American Aviation. Mr. H. T. Vivian, under the guidance of Dr. H. Ashley, laid out the form of the solution and wrote the computer program. Dr. E. R. Rodemich made many significant contributions to the numerical analysis.

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ABSTRACT

In this part, equations for pressure distributions and generalized aerodynamic forces are derived for a thin nonplanar lifting surface in simple harmonic motion at subsonic speeds. A digital computer program, written in Fortran IV, is also presented herein. The computer program will generate up to a ten by ten matrix of generalized aerodynamic forces when given data for the geometry of a planar lifting surface with a folded planar tip, the flight Mach number, the reduced frequency of motion, and some control constants. Control surface deflections are not accounted for in this study.

The kernel function method given by Watkins, Runyan, and Woolston (Reference 1), which relates the pressure distribution to the downwash on a planar lifting surface, has been extended and applied to a nonplanar lifting surface. Hsu's technique (Reference 4) of employing Gaussian quadrature formulas is used when integrating the product of the kernal function and the lift function over the planform area.

Recommendations are made to extend the method to account for blunted leading edges and the accompanying airfoil thickness and to account for control surface deflections.

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SYMBOLS

a	Local speed of sound (constant in linear theory)
b_o	Root semichord
$b(s)$	Local semichord
C_p	Pressure coefficient
$\overline{C}_p$	Time independent factor of oscillatory part of C_p
i, j, k	Unit vectors parallel to the coordinate axes
i	$\sqrt{-1}$
k	Reduced frequency, $\omega b_o/U_\infty$
k_1	kr_1
K	Kernel function
K_0, K_1, K_2	Modified Bessel functions of the second kind
M	Local Mach number (same as free stream Mach number, M_∞, in linear theory)
n	Coordinate measured normal to wing surface
$n(x, s, t)$	Contribution to n of elastic deformation of the wing
$n_t(x, s)$	Contribution to n of wing thickness
$\overline{n}(x, s)$	Time independent factor of $n(x, s, t)$
p	Pressure
$\vec{q}$	Fluid velocity
R	Gas constant
R	$\sqrt{(x-\xi)^2 + \beta^2 r_1^2}$

List of Symbols continued on next page.

r_1	$\sqrt{(y-\eta)^2 + (z-\zeta)^2}$
s	Curvilinear coordinate on the wing (page 10)
T	Absolute temperature
t	Time
u, ℓ	Subscripts indicating upper and lower wing surfaces
u, v, w	Components of perturbation velocity
U, W	x- and z-components of $\vec{V}$
U_∞	$\lvert\vec{V}\rvert$ (speed at infinity)
$\vec{V}$	Uniform fluid velocity at infinity
$\overline{W}$	Time independent factor of normal velocity
x, y, z	Cartesian coordinates
x_o	$x - \xi$
y_o	$y - \eta$
β	$\sqrt{1-M^2}$
γ	Ratio of specific heats
$\gamma(s)$	Local angle between wing surface and xy-plane
$\Delta\bar{p}$	Time independent factor of pressure difference between wing surfaces, $\bar{p}_\ell - \bar{p}_u$
ξ, η, ζ	Cartesian coordinates
$\tilde{\xi}$	Coordinate on the wing (page 22)
ϕ	Velocity potential
φ	Perturbation velocity potential
$\bar{\varphi}$	Time independent factor of φ
$\bar{\psi}$	Acceleration potential
ρ	Fluid density
ω	Angular frequency (radians per unit time)

I. INTRODUCTION

The first published numerical method for solving the subsonic pressure distribution problem for planar lifting surfaces undergoing simple harmonic motion was developed at NASA's Langley Research Center by Watkins, Runyan, and Woolston (Reference 1). Watkins, et al., presented two methods of handling the numerical integration of the kernel function in the region where high-order singularities exist. Both methods involved a dense concentration of integration points in the neighborhood of the singularity. Using these methods, it is possible to obtain downwash integrals, in terms of the pressure-loading coefficients, at any arbitrary set of points on the surface (e.g., at all the kinematic downwash points known from previously determined vibration mode data). However, in order to reduce the running time on the computer, the downwash integrals were obtained at a selected set of collocation points, such as those at intersections of quarter, half, and three-quarter chord stations and like half-span stations. When downwashes were matched exactly (and thus, boundary conditions) at these collocation points, responsibility was placed upon the user to evaluate the kinematic downwashes there. A least-square error surface fitted to the mode data was commonly used to evaluate them. Furthermore, if the user desired that the boundary conditions be satisfied at a greater number of points, it was necessary that he use a correspondingly greater number of loading functions.

Procedures were then described by Rodden and Revell (Reference 2) and the correct form of the equations were presented by Fromme (Reference 3) for calculating pressure-loading coefficients which match a greater number of kinematic downwashes than coefficients, in the sense that the sum of squares of amplitudes of differences of complex numbers are minimized. Since it was still the responsibility of the user to evaluate the kinematic downwashes at the collocation point, least-square error procedures were used twice: once implicitly and once explicitly.

Hsu (Reference 4) significantly advanced the logical development of the kernel function approach when he established an optimum set of collocation and integration points. He started with the previously established chordwise pressure functions based on steady-state, two-dimensional, incompressible aerodynamics, and with spanwise loading functions, based on steady-state lifting-line theory. He concluded that there is sufficient reason to believe that these functions display the proper characteristics near the edges of lifting surfaces oscillating in a compressible fluid.

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Returning to the two-dimensional case, Hsu established that if the chordwise distribution of modal deflections (and thus downwashes) is accurately represented by a polynomial of degree 2N-1 and is approximated by a polynomial of degree N-1, then the integral for the sectional load is evaluated with zero error by a N-point Gaussian quadrature if the difference between the accurate and the approximate representation of the downwashes is made equal to zero at each of the N points (i.e., the chordwise collocation stations). Conversely, still for the two-dimensional case, Hsu established that if the product of the pressure function and the kernel, divided by the Jacobi-Gauss weight factor (which produces the square root singularity at the leading edge), is accurately represented by a polynomial of degree 2N-1, then the integral for the downwash at any one of the collocation stations is evaluated with zero error by a N-point Gaussian quadrature. These N points are then made the chordwise integration stations.

For the spanwise direction, using lifting-line theory, Hsu similarly established M-spanwise collocation stations and M + 1 interdigitated spanwise integration stations plus the conditions under which the Gaussian quadrature can be used with zero error.

It is important to note that the kernel of the integral equation for the downwashes in unsteady, three-dimensional, compressible flow cannot be accurately represented by a polynomial of finite degree. It is equally important to note, however, that, because of the edge characteristics of the pressure and loading functions, the Gaussian quadratures employed at Hsu's optimum point set evaluate the integrals with the least squared error for a given number of integration points. We have yet to match the boundary conditions using Hsu's method.

The downwash matching problem in Hsu's approach is basically the same as in Watkin's approach; we merely have a more logical choice of points at which to match them. In the examples Hsu used to demonstrate his approach, he chose to use the same number of pressure-loading functions as collocation points. However, the approach is not dependent upon that choice. If a smaller number of pressure-loading functions are used, then the procedures described by Rodden, Revell, and Fromme may be used to compute pressure-loading coefficients which yield a minimum sum of squares of amplitudes of differences in downwashes.

A need has arisen for application of the kernel function method to non-planar lifting surfaces on future aerospace vehicles. Application is also required to more conventional non-planar surfaces such as T-tail, V-tail, and wing-vertical tail combinations.

Professor H. Ashley outlined the application to the folded tip configuration. A computer program based on Ashley's work was developed for steady-state flow by L. Johnson, et al., of the Los Angeles Division of North American Aviation, Inc.

The work reported herein is based on Professor Ashley's outline. However, the expression for the kernel has been greatly simplified by Dr. E. R. Rodemich of North American Aviation, Inc., Space and Information Systems Division.

II. FUNDAMENTAL EQUATIONS OF FLUID MOTION

Consider a body immersed in a compressible, nonviscous, perfect fluid and assume the fluid flow to be isentropic and irrotational. Under these conditions, a velocity potential ϕ, exists:

$$\vec{q} = \nabla\phi \tag{1}$$

where $\vec{q}$ is the velocity vector of a fluid element and ∇ is the gradient operator (See Reference 4.) Also under these conditions, the isentropic (constant entropy) pressure-density relationship is valid. Thus,

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s = \gamma RT \tag{2}$$

Other equations which govern the flow are the continuity equation for conservation of mass

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho\vec{q}) = 0 \tag{3}$$

and Euler's equations for conservation of momentum

$$\frac{D\vec{q}}{Dt} = \frac{-1}{\rho}\nabla p \tag{4}$$

where, $\frac{D}{Dt}$, the substantial derivative, is

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{q} \cdot \nabla \tag{5}$$

These equations may be combined, as described in Reference 5, to yield the nonlinear, unsteady flow equation

$$\nabla^2\phi - \frac{1}{a^2}\left[\frac{\partial^2\phi}{\partial t^2} + \frac{\partial q^2}{\partial t} + (\vec{q} \cdot \nabla)\frac{q^2}{2}\right] = 0 \tag{6}$$

Consider, then, that the fluid motion consists of a perturbation superimposed on a uniform stream velocity $\vec{V} = Ui + Wk$ parallel to the xz-plane of a rectangular Cartesian coordinate system. Then the velocity potential may be expressed as the sum of a uniform part and a perturbation part

$$\phi = Ux + Wz + \varphi \tag{7}$$

and, similarly, the velocity vector becomes

$$\vec{q} = \vec{V} + \nabla\varphi = \nabla\phi \tag{8}$$

The pressure coefficient at any point in an isentropic flow field is

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty U_\infty^2} \tag{9}$$

where $p - p_\infty$ is the difference between local pressure and free-stream pressure, $U_\infty = |\vec{V}|$, and $1/2\ \rho_\infty\ U_\infty^2$ is the free-stream dynamic pressure. From Kelvin's equation (Reference 5) for isentropic flow

$$C_p = \frac{2}{\gamma M_\infty^2}\left\{\left[1 + \frac{\gamma - 1}{2} M_\infty^2 \left(1 - \frac{\vec{q}\cdot\vec{q} + 2\frac{\partial\varphi}{\partial t}}{U_\infty^2}\right)\right]^{\frac{\gamma}{\gamma-1}} - 1\right\} \tag{10}$$

A complete statement of the fundamental problem requires specification of the boundary conditions. The boundary conditions at infinity depend upon the free-stream velocity. When it is less than the speed of sound in the fluid, the disturbances to the flow die out and are not felt at infinity. When it is greater than the sonic speed, then in the region where disturbances are felt, even at infinity, the component of flow due to the disturbance is directed away from the source of disturbance and otherwise the free-stream flow is undisturbed. The boundary conditions at the surface of the body require that the flow be tangent to the surface everywhere on the body. This condition is satisfied by the equation

$$\frac{D}{Dt} B(x, y, z, t) = 0 \tag{11}$$

where

$$B(x, y, z, t) = 0 \tag{12}$$

is the equation for the position of the surface at any time t, and the substantial derivative D/Dt is defined by Equation 5.

III. LINEARIZED EQUATIONS OF MOTION

Linearization of the equations of motion is not dependent upon an explicit form of the body equation, Equation 12, so long as the normal derivatives of the equation are everywhere nearly perpendicular to the free-stream direction. Thin lifting surfaces at small angle of attack satisfy this condition and are treated herein and in Parts 2 and 4 of this report. The special considerations required for thick bodies and high angles of attack are treated in Parts 3 and 5. The following development is, therefore, restricted to thin airfoils.

We first obtain the specialized form of Equation 7 when the uniform stream velocity lies along the x-axis; i. e., $W = 0$ and, therefore, $\vec{V} = Ui$, $U_\infty = U$. The velocity potential is

$$\phi = Ux + \varphi \tag{13}$$

and the velocity vector of a fluid element becomes

$$\vec{q} = (U + u)\, i + vj + wk \tag{14}$$

where

$$u = \frac{\partial \varphi}{\partial x}, \quad v = \frac{\partial \varphi}{\partial y}, \quad \text{and } w = \frac{\partial \varphi}{\partial z}$$

The perturbation velocities u, v, and w are assumed to be much smaller than the free-stream velocity; i. e., $u, v, w \ll U$.

The linearization procedure when applied to Equation 10 yields the fully linearized pressure coefficient

$$Cp = -\frac{2}{U_\infty^{\,2}} \left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \varphi \tag{15}$$

and when applied to Equation 6 yields the fully linearized unsteady flow equation

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} - \frac{1}{a_\infty^2}\left[U^2 \frac{\partial^2 \varphi}{\partial x^2} + 2U\frac{\partial^2 \varphi}{\partial x \partial t} + \frac{\partial^2 \varphi}{\partial t^2}\right] = 0 \qquad (16)$$

Next, we write the body equation for a thin, nonplanar lifting surface (Figure 1), in terms of a curvilinear coordinate

$$s = s(y)$$

s represents the integral of distance along the line of the mean position of the airfoil from the centerline to y,

$$s(y) = \int_0^y ds \qquad (17)$$

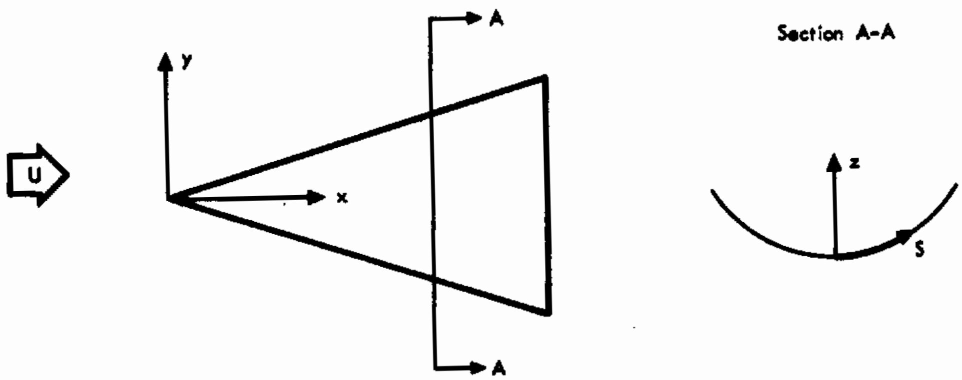


Figure 1. Generalized Curvilinear Planform

The position of the surface in terms of s and n (the normal to S), is separated into two parts; one part for the upper (or inner) surface, and the other for the lower (or outer) surface

$$B_u(x, n, s, t) = n - n_T(x, s) - n(x, s, t) \tag{18a}$$

$$B_\ell(x, n, s, t) = n + n_T(x, s) - n(x, s, t) \tag{18b}$$

where $n_T(x, s)$ represents the thickness of the airfoil and $n(x, s, t)$ represents the elastic deflection of the airfoil. In accordance with Equation 11, we use the operator

$$\nabla = i\frac{\partial}{\partial x} + j'\frac{\partial}{\partial s} + k'\frac{\partial}{\partial n} \tag{19}$$

on Equation 18a to get

$$\nabla B_u(x, n, s, t) = -i\left[\frac{\partial}{\partial x}n_T(x, s) + \frac{\partial}{\partial x}n(x, s, t)\right] - j'\left[\frac{\partial}{\partial s}n_T(x, s) + \frac{\partial}{\partial s}n(x, s, t)\right] + k'$$

Substitution into

$$\frac{D}{Dt}B_u(x, s, n, t) = \frac{\partial B_u}{\partial t} + (Ui + \nabla\varphi)\cdot\nabla B_u = 0 \tag{20}$$

of the equation for the upper surface, after higher order terms have been discarded, gives

$$\frac{\partial\varphi}{\partial n} = \left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)n(x, s, t) + U\frac{\partial}{\partial x}n_T(x, s) \tag{21}$$

The same procedure, for the lower surface, gives

$$\frac{\partial \varphi}{\partial n} = \left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right) n\,(x,\ s,\ t) - U\frac{\partial}{\partial x}\ n_{\tau}(x,\ s) \tag{22}$$

Finally, we restrict the analysis to that class of problems in which the effects of thickness on the time dependent forces can be neglected. By letting

$$n_{\tau}(x,\ s) = 0 \tag{23}$$

we get a single expression for the boundary condition

$$\frac{\partial \varphi}{\partial n} = \left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right) n\,(x,\ s,\ t) \tag{24}$$

It is evident from Equation 24 that, when the motion of the surface is simple harmonic motion,

$$n\,(x,\ s,\ t) = \overline{n}\,(x,\ s)\ e^{i\omega t} \tag{25}$$

then,

$$\varphi\,(x,\ s,\ n,\ t) = \overline{\varphi}\,(x,\ s,\ n)\,e^{i\omega t} \tag{26}$$

Substitution of Equations 25 and 26 into Equations 15, 16, and 24 gives

$$\overline{C}_p = -\frac{2}{U_\infty^{\ 2}} \quad \frac{D\overline{\varphi}}{Dt} \tag{27}$$

$$\nabla^2 \overline{\varphi} = \frac{1}{a_\infty^{\ 2}} \quad \frac{D^2\overline{\varphi}}{Dt^2} \tag{28}$$

$$\frac{\partial \overline{\varphi}}{\partial n} = \frac{D\overline{n}}{Dt} \tag{29}$$

where

$$\frac{D}{Dt} \equiv U \frac{\partial}{\partial x} + i\omega \tag{30}$$

and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial s^2} + \frac{\partial^2}{\partial n^2} \tag{31}$$

To this degree of approximation, $M = M_\infty$ and $a = a_\infty$.

IV. THE ACCELERATION POTENTIAL

PLANAR WINGS

Equation 27 shows that the calculation of pressure requires taking derivatives of the velocity potential, and Equation 29 states the boundary condition which must be satisfied. Watkins, Runyan and Woolston (Reference 1) solved this problem for planar surfaces in terms of a series of pressure functions, or acceleration potential functions ($\overline{\psi}$), where,

$$\overline{\psi}\,(x, y, z) = \frac{D}{Dt}\,\overline{\phi}\,(x, y, z) \tag{32}$$

Integration of Equation 32 gives the general expression for the velocity potential in terms of the acceleration potential:

$$\overline{\phi}\,(x, y, z) = \frac{1}{U}\, e^{-i\omega \frac{x}{U}} \int_{-\infty}^{x} e^{i\omega \frac{\lambda}{U}}\, \overline{\psi}\,(\lambda, y, z)\, d\lambda \tag{33}$$

if the velocity at infinity is $\vec{V} = Ui$. The velocity potential ϕ due to a pulsating doublet satisfies Equation 28 (in the planar case $s = y$ and $n = z$); and, because the order of operators is interchangeable, the acceleration potential ψ also satisfies Equation 28.

A complete discussion of the application of boundary conditions in the planar case is given in Section 6-4 of Reference 5. The application in the nonplanar case is discussed in less detail in the following text.

NONPLANAR WINGS

In the nonplanar case, the acceleration potential at a point (x, s, N) due to a pulsating doublet located at the point (ξ, σ, n), or (ξ, η, ζ) in the direction of n is

$$\overline{\psi}\,(x, s, N) = -A\,\frac{\partial}{\partial n}\left\{\frac{e^{i\omega\left[\frac{M}{a_\infty \beta^2}(x-\xi) - \frac{R}{a_\infty \beta^2}\right]}}{R}\right\} \tag{34}$$

where

$$\frac{\partial}{\partial n} = \cos\gamma(\eta)\frac{\partial}{\partial \zeta} - \sin\gamma(\eta)\frac{\partial}{\partial \eta} \tag{35}$$

$$R = \sqrt{(x-\xi)^2 + \beta^2\left[(y-\eta)^2 + (z-\zeta)^2\right]} \tag{36}$$

and $\gamma(\eta)$ is angle between the wing and the xy-plane at the point (ξ, η, ζ). The perturbation velocity potential may be built up from a distribution of doublets of acceleration potential over the wing. If $\bar{A}$ is the infinitesimal doublet strength at (ξ, σ, η), the contribution to $\bar{\varphi}$ from the doublet at this point, from Equations 33 and 34, is

$$\Delta\bar{\varphi}(x,s,N) = \frac{-\bar{A}}{U}\frac{\partial}{\partial n} e^{-i\omega\frac{x-\xi}{U}} \int_{-\infty}^{x-\xi} \frac{e^{i\omega\left[\frac{\lambda}{U} + \frac{M\lambda}{a_\infty\beta^2} - \frac{R'}{a_\infty\beta^2}\right]}}{R'} d\lambda \tag{37}$$

where

$$R' = \sqrt{\lambda^2 + \beta^2\left[(y-\eta)^2 + (z-\zeta)^2\right]}$$

and the velocity component normal to the surface at (x,s,N) is

$$\Delta\bar{w}(x,s,0) = \lim_{N\to 0}\frac{\partial}{\partial N}\Delta\bar{\varphi}(x,s,N) \tag{38}$$

where

$$\frac{\partial}{\partial N} = \cos\gamma(y)\frac{\partial}{\partial z} - \sin\gamma(y)\frac{\partial}{\partial y} \tag{39}$$

Note that when the operator $\partial/\partial n$ is applied in Equation 37, the partials $\partial/\partial\zeta$ and $\partial/\partial\eta$ may be replaced by $-\partial/\partial z$ and $-\partial/\partial y$, respectively. Substitution of Equation 37 into Equation 38 gives

$$\Delta\bar{w}(x,s,0) = \frac{\bar{A}}{U} e^{-i\omega\frac{x-\xi}{U}} \lim_{N\to 0} P \int_{-\infty}^{x-\xi} \frac{e^{i\omega(\lambda - MR')/U\beta^2}}{R'} d\lambda \tag{40}$$

where the operator P is

$$P = \left[\cos\gamma\,(y)\frac{\partial}{\partial z} - \sin\gamma\,(y)\frac{\partial}{\partial y}\right]\left[\cos\gamma\,(\eta)\frac{\partial}{\partial z} - \sin\gamma\,(\eta)\frac{\partial}{\partial y}\right] \quad (41)$$

and finally

$$A = \frac{\Delta\bar{p}\,d\xi\,d\sigma}{4\pi\rho} \quad (42)$$

and $\Delta\bar{p}$ is the complex amplitude of the difference in pressure on the upper and lower sides of the surface at (ξ, σ),

$$\Delta\bar{p}(\xi, \sigma) = \bar{p}_u(\xi, \sigma) - \bar{p}_\ell(\xi, \sigma) \quad (43)$$

and $d\xi\,d\sigma$ is the incremental area of the doublet sheet.

The normal wash at $(x, s, 0)$ given by Equation 40 is that due to a point pressure doublet at $(\xi, \sigma, n = 0)$. The total normal wash is the integral over the surface of all the pressure doublets,

$$\frac{\bar{w}(x, s, 0)}{U} = \frac{-1}{4\pi\rho U^2} \iint \Delta\bar{p}(\xi, \sigma)\, K(x - \xi, s, \sigma, \omega, M)\, d\xi\, d\sigma \quad (44)$$

where $\int$ denotes the Mangler formula for evaluating infinite integrals (Reference 7), and the kernel of the integral equation is (omitting the arguments ω, M for brevity)

$$K(x_o, s, \sigma) = \lim_{\substack{n \to 0 \\ N \to 0}} \left\{ e^{-i\frac{\omega x_o}{U}}\, P \int_{-\infty}^{x_o} \frac{e^{i\omega(\lambda - MR')/U\beta^2}}{R'}\, d\lambda \right\} \quad (45)$$

where

$$R' = \sqrt{\lambda^2 + \beta^2 r_1^2}$$

$$r_1 = \sqrt{y_o^2 + z_o^2}$$

and

$$x_o = x - \xi, \quad y_o = y - \eta, \text{ and } z_o = z - \zeta$$

Now, by putting $k_1 = \omega r_1/U$ and $v = \lambda/\beta r_1$; then, by putting $u = -\frac{v - M\sqrt{1+v^2}}{\beta}$ the integral in Equation 45 may be written

$$I_o \equiv \int_{-\infty}^{x_o} \frac{e^{i\omega(\lambda - MR')/U\beta^2}}{R'} \, d\lambda = \int_{u_1}^{\infty} \frac{e^{-ik_1 u}}{\sqrt{1+u^2}} \, du \qquad (46)$$

where

$$u_1 = -\frac{x_o - MR}{\beta^2 r_1}$$

By breaking up the interval of integration into three subintervals and in the first two integrals letting $u = w/i$

$$I_o = -i\int_0^1 \frac{e^{-k_1 w}}{\sqrt{1-w^2}} \, dw + \int_1^{\infty} \frac{e^{-k_1 w}}{\sqrt{w^2-1}} \, dw - \int_0^{u_1} \frac{e^{-ik_1 u}}{\sqrt{1+u^2}} \, du$$

or,

$$I_o = K_o(k_1) - i\int_0^1 \frac{e^{-k_1 w}}{\sqrt{1-w^2}} \, dw - \int_0^{u_1} \frac{e^{-ik_1 u}}{\sqrt{1+u^2}} \, du \qquad (47)$$

where $K_o(k_1)$ is the modified Bessel function of the second kind and zeroth order of the argument k_1.

To obtain the analytic form of the kernel, we write the operator P (Equation 41) in a more convenient form, taking advantage of the fact that I_o is a function of only r_1 when x_o is held constant

$$PI_o = \cos\left[\gamma(y) - \gamma(\eta)\right]\left(\frac{1}{r_1}\frac{\partial I_o}{\partial r_1}\right)$$

$$+ \left[z_o\cos\gamma(y) - y_o\sin\gamma(y)\right]\left[z_o\cos\gamma(\eta) - y_o\sin\gamma(\eta)\right]\left(\frac{1}{r_1}\frac{\partial}{\partial r_1}\right)\left(\frac{1}{r_1}\frac{\partial I_o}{\partial r_1}\right)$$

The resulting kernel is

$$\overline{\overline{K}}(x_o, s, \sigma) = s_{TIP}^{2} K(x_o, s, \sigma)$$

$$= e^{-i\frac{\omega x_o}{U}} \left\{ \frac{T_1 K_1(x_o, s, \sigma) + T_2 K_2(x_o, s, \sigma)}{\underline{r}_1^{2}} \right\} \tag{48}$$

where

$$T_1 = \cos\left[\gamma(\underline{s}) - \gamma(\underline{\sigma})\right]$$

$$T_2 = \left[\frac{\underline{z}_o}{\underline{r}_1}\cos\gamma(s) - \frac{\underline{y}_o}{\underline{r}_1}\sin\gamma(\underline{s})\right]\left[\frac{\underline{z}_o}{\underline{r}_1}\cos\gamma(\underline{\sigma}) - \frac{\underline{y}_o}{\underline{r}_1}\sin\gamma(\underline{\sigma})\right] \tag{48a}$$

$$K_1(x_o, s, \sigma) = -k_1K_1(k_1) - \frac{x_o}{R}e^{-k_1u_1} + ik_1\int_0^1 \frac{w\,e^{-k_1w}}{\sqrt{1-w^2}}\,dw$$

$$+ ik_1\int_0^{u_1}\frac{ue^{-ik_1u}}{\sqrt{1+u^2}}\,du$$

$$K_2(x_o, s, \sigma) = k_1^{2}K_2(k_1) + \left(\frac{2x_o}{R} + \frac{\beta^2 r_1^{2} x_o}{R^3} + i\frac{k_1x_o(Mr_1 + Ru_1)}{R^2}\right)e^{-ik_1u_1}$$

$$- ik_1\int_0^1\frac{we^{-k_1w}}{\sqrt{1-w^2}}\,dw - ik_1^{2}\int_0^1\frac{w^2e^{-k_1w}}{\sqrt{1-w^2}}\,dw$$

$$- ik_1\int_0^{u_1}\frac{ue^{-ik_1u}}{\sqrt{1+u^2}}\,du + k_1^{2}\int_0^{u_1}\frac{u^2e^{-ik_1u}}{\sqrt{1+u^2}}\,du \tag{48b}$$

$$r_1 = s_{TIP} \sqrt{\underline{y}_o^{\,2} + \underline{z}_o^{\,2}}$$

$$R = \sqrt{x_o^{\,2} + \beta^2 r_1^{\,2}}$$

$$u_1 = - \frac{x_o - MR}{\beta^2 r_1}$$

$$k_1 = \frac{\omega r_1}{U} \qquad (48c)$$

and $K_1(k_1)$ and $K_2(k_1)$ are modified Bessel functions of the second kind and first and second orders. The sub-bar indicates division by s_{TIP}, e.g., $\underline{r}_1 = r_1/s_{TIP}$.

V. THE BOUNDARY CONDITIONS

The remainder of the problem is to match the boundary conditions; i.e., to find a pressure-loading function $\Delta\bar{p}(\xi, \sigma)$ which, when inserted into Equation 45 and integrated over the surface, yields the kinematic downwashes at selected points on the surface w(x, s, 0).

In subsonic flow, the behavior of the pressure distribution is known in the area of the wing edges from a few of the exact solutions in lifting surface theory. In the neighborhood of the leading edge, the pressure should behave as

$$\lim_{\delta \to 0} \sqrt{\frac{1}{\delta}}$$

In the neighborhood of the trailing edge and all edges parallel to the free-stream direction, the pressure should behave as

$$\lim_{\delta \to 0} \sqrt{\delta}$$

where δ is the distance to the wing edge. Both Hsu and Watkins employ a linear superposition of functions that satisfy these conditions. Hsu's function differs from Watkins only in that for any given number of terms in the series, Hsu's terms are linear combinations of Watkins terms. We use a normalized form of the function given by Watkins:

$$\Delta\bar{p}(\underline{\xi}, \underline{\sigma}) \equiv \frac{\rho U^2/2}{b(\sigma)} \sqrt{1 - \underline{\sigma}^2} \sum_{n=0}^{N} \sum_{m=0}^{M} a_{nm} \underline{\sigma}^m f_n(\tilde{\xi}) \qquad (49)$$

where

$$f_o(\tilde{\xi}) \equiv \sqrt{\frac{1 - \tilde{\xi}}{1 + \tilde{\xi}}}$$

$$f_n(\tilde{\xi}) \equiv \sqrt{1 - \tilde{\xi}^2}\, U_n(\tilde{\xi}); \; 1 \le n$$

$$U_1(\tilde{\xi}) = 1.0$$

$$U_2(\tilde{\xi}) = -2\tilde{\xi}$$

$$U_n(\tilde{\xi}) = -(2\tilde{\xi}\ U_{n-1} + U_{n-2});\ 3 \le n$$

and

$$\tilde{\xi} \equiv \left(\xi - \frac{\xi_{LE} + \xi_{TE}}{2}\right) / b(\sigma)$$

$$b(\underline{\sigma}) = \frac{\xi_{TE} - \xi_{LE}}{2}$$

The a_{nm}'s are unknown pressure coefficients to be determined by matching the kinematic downwashes at the selected points (x_j, s_r) on the surface. Substitution of Equation 49 into Equation 44 leads to the matrix equation given by Rodden and Revell (Reference 2) Equation 39, for the point set x_j, s_r

$$\left\{\frac{\overline{w}_i}{U}\right\} = \left[D^i_{nm}\right] \quad \left\{a_{nm}\right\} \tag{50}$$

where, in this case,

$$D^i_{nm} = \frac{1}{8\pi} \int_{-1}^{1} \sqrt{1 - \underline{\sigma}^2}\ \underline{\sigma}^m \int_{-1}^{1} f_n(\tilde{\xi}) \overline{\overline{K}}(x_j - \xi, \underline{s}_r, \underline{\sigma})\, d\tilde{\xi}\, d\underline{\sigma} \tag{51}$$

We now reexamine the fundamentals of the problem before proceeding to evaluate the integrals in Equation 51 and thence to solve Equation 50.

One of the basic reasons for development of the kernel function method is that pressure distributions over a continuous lifting surface are smooth continuous functions that can be represented with reasonable accuracy by a series of analytic functions. We point out that $f_n(\xi)$ can be written

$$f_o(\tilde{\xi}) = \frac{1 - \tilde{\xi}}{\sqrt{1 - \tilde{\xi}^2}}$$

$$f_n(\tilde{\xi}) = \frac{1 - \tilde{\xi}^2}{\sqrt{1 - \tilde{\xi}^2}}\ U_n(\tilde{\xi});\ 1 \ge n$$

and, therefore, the inner integral in Equation 51 may be written

$$I_1 = \int_{-1}^{1} \frac{P_n(\tilde{\xi})}{\sqrt{1 - \tilde{\xi}^2}} d\tilde{\xi} \tag{52}$$

where

$$P_o(\tilde{\xi}) = (1 - \tilde{\xi})\bar{\bar{K}}(x_j - \xi, \underline{s}_r, \underline{\sigma})$$

$$P_n(\tilde{\xi}) = (1 - \tilde{\xi}^2)U_n(\tilde{\xi})\bar{\bar{K}}(x_j - \xi, \underline{s}_r, \underline{\sigma}); \; 1 \leq n$$

Now, we assume for the moment that the kernel $\bar{\bar{K}}(X_j - \xi, s_r, \sigma)$ can be represented with reasonable accuracy by a polynomial in $\tilde{\xi}$. Then, $P_n(\tilde{\xi})$ is also a polynomial in ξ, and the Chebyshev-Gauss quadrature formula may be used to obtain the exact value of the integral expression (52); i.e.,

$$\int_{-1}^{1} \frac{P_n(\tilde{\xi})}{\sqrt{1 - \tilde{\xi}^2}} d\tilde{\xi} = \sum_{k=1}^{K} \frac{\pi}{K} P_n(\tilde{\xi}_k) + E \tag{52a}$$

where

$$E = \frac{2\pi}{2^{2K}(2K)!} P_n^{(2K)}(\lambda)$$

and,

$$|\lambda| < 1.0$$

The error term E is zero if P_n is a polynomial of degree $\leq 2K - 1$.

A more accurate formula which utilizes the fact that $P_n(1.0) = 0$ is used by Hsu (and by us)

$$f_o(\tilde{\xi}) = \sqrt{\frac{1 - \tilde{\xi}}{1 + \tilde{\xi}}}$$

$$f_n(\tilde{\xi}) = \sqrt{\frac{1 - \tilde{\xi}}{1 + \tilde{\xi}}} (1 + \tilde{\xi}) U_n(\tilde{\xi})$$

Then, the inner integral in Equation 51 may be written

$$I_1 = \int_{-1}^{1} \sqrt{\frac{1-\tilde{\xi}}{1+\tilde{\xi}}}\, F_n(\tilde{\xi})\, d\tilde{\xi} \qquad (53)$$

where

$$F_o(\tilde{\xi}) = \bar{\bar{K}}(x_j - \xi, \underline{s}_r, \underline{\sigma})$$

$$F_n(\tilde{\xi}) = (1+\tilde{\xi})\, U_n(\tilde{\xi})\, \bar{\bar{K}}(x_j - \xi, \underline{s}_r, \underline{\sigma}),\ 1 \le n$$

This is evaluated by the L-point Jacobi-Gauss quadrature with the weight function $\sqrt{(1-\tilde{\xi})/(1+\tilde{\xi})}$ (see Reference 8, Chapter 8). The resulting formula

$$I \cong \sum_{k=1}^{L} W_k F_n(\tilde{\xi}_k) \qquad (54)$$

is exact if $F_n(\tilde{\xi})$ is a polynomial of degree $\le 2L-1$, which corresponds to degree 2L for $P_n(\tilde{\xi})$. Putting $\tilde{\xi} = -\cos\theta$, the polynomials

$$\phi_m(\tilde{\xi}) = \frac{\cos\left(m+\frac{1}{2}\right)\theta}{\cos\frac{1}{2}\theta}$$

are orthogonal with respect to the weight function $\sqrt{(1-\tilde{\xi})/(1+\tilde{\xi})}$

$$\int_{-1}^{1} \phi_m(\tilde{\xi})\phi_n(\tilde{\xi}) \sqrt{\frac{1-\tilde{\xi}}{1+\tilde{\xi}}}\, d\tilde{\xi} = \begin{cases} 0, & m \ne n \\ \pi, & m = n \end{cases}$$

This is easily verified by expressing the integral in terms of θ. Referring to formulas in Reference 8, Chapter 8, Section 8.4, it can be shown, using these polynomials, that Equation 54 takes the form

$$\int_{-1}^{1} \sqrt{\frac{1-\tilde{\xi}}{1+\tilde{\xi}}}\, F_n(\tilde{\xi})\, d\tilde{\xi} = \frac{2\pi}{2L+1} \sum_{k=1}^{L} H_k\, F_n(\tilde{\xi}_k) \qquad (55)$$

where

$$H_k = (1 - \tilde{\xi}_k)$$

and

$$\tilde{\xi}_k = -\cos\left(\frac{2k - 1}{2L + 1} \pi\right)$$

In two-dimensional, steady, incompressible flow, there is an optimum set of chordwise collocation stations $(\tilde{x}_i)$ for the determination of sectional lift, depending upon the order of the polynomial required to adequately represent the downwash distribution

$$\tilde{x}_j = -\cos\left(\frac{2j}{2N + 1} \pi\right), \; j = 1, 2, \ldots . N. \tag{56}$$

Since the behavior of the integrand for the chordwise loading is apt to exhibit similar characteristics near the surface edges, it is inferred that this set should also yield the best approximation in three-dimensional, unsteady, compressible flow. Note that the number of collocation points is not required to be the same as the number of integration points. As will be seen later, it is only necessary that the total number of downwash collocation points be equal to or greater than the number of pressure coefficients a_{nm}. When N chordwise integration stations are used, the quadrature used to evaluate the inner integral of Equation 51 is exact for integrands represented by a polynomial of degree $\leq 2N - 1$.

Hsu shows that an optimum set of interdigitated spanwise collocation stations and integration stations exists for evaluation of the outer integral in Equation 51. By reasoning similar to that used to establish the chordwise collocation stations, it was established that the optimum spanwise collocation stations are

$$\underset{\sim}{s}_r = -\cos \frac{r}{M + 1} \pi, \; r = 1, 2, \ldots M. \tag{57}$$

It was observed that the quadrature for the integral of difference between the actual and polynomial approximation of the spanwise loading is zero when the actual loading is precisely represented by a polynomial of degree $\leq 2M - 1$, and the polynomial approximation is of degree $= N - 1$.

Then, by substitution of Equation 55 into Equation 51,

$$D^{i}_{nm} = \frac{1}{8\pi} \int_{-1}^{1} \frac{\sqrt{1 - \underline{\sigma}^2}}{(\underline{s}_r - \underline{\sigma})^2} G_{nm} (\tilde{x}_j, \underline{s}_r, \underline{\sigma}) \, d\underline{\sigma} \tag{58}$$

where

$$G_{om} = \frac{2\pi}{2L + 1} \sum_{k=1}^{L} \underline{\sigma}^{m} (1 - \tilde{\xi}_k) (\underline{s}_r - \underline{\sigma})^2 \overline{\overline{K}} (x_j - \xi_k, \underline{s}_r, \underline{\sigma})$$

$$G_{nm} = \frac{2\pi}{2L + 1} \sum_{k=1}^{L} \underline{\sigma}^{m} (1 - \tilde{\xi}^2_k) U_n (\tilde{\xi}_k) (\underline{s}_r - \underline{\sigma})^2 \overline{\overline{K}} (x_j - \xi_k, \underline{s}_r, \underline{\sigma}); \; 1 \leq n \tag{59}$$

Hsu established the form of the Gaussian quadrature and the spanwise integration stations. The difficulties of the singularity of the kernel at $\underline{\sigma} = \underline{s}_r$ and the difficulty of differentiation with respect to $\underline{\sigma}$ (he uses the steady-state lifting line formula to derive the form of the quadrature) are avoided by removal of the singularity at $\underline{\sigma} = \underline{s}_r$ and then by an integration by parts.

We first integrate by parts to get

$$D^{i}_{nm} = \frac{1}{8\pi} \int_{-1}^{1} \frac{\frac{\partial}{\partial \underline{\sigma}} \left[\sqrt{1 - \underline{\sigma}^2} \; G_{nm} (\tilde{x}_j, \underline{s}_r - \underline{\sigma}, \omega, M) \right]}{(\underline{s}_r - \underline{\sigma})} \, d\underline{\sigma}$$

which corresponds to Equation 58 in Reference 9. The Gaussian quadrature formula is developed and shows that when the number of integration stations is one greater than the number of collocation stations, and if they are interdigitated in the prescribed way

$$D^{i}_{nm} = \frac{1}{8\pi} \left\{ \sum_{p=1}^{M+1} \frac{\pi}{M+1} \frac{(1-\underline{\sigma}_p^2)\, G_{nm}(\tilde{x}_j, \underline{s}_r, \underline{\sigma}_p)}{(\underline{s}_r - \underline{\sigma}_p)^2} - \pi (M+1)\, G_{nm}(\tilde{x}_j, \underline{s}_r, \underline{s}_r) \right\} \tag{60}$$

where

$$\underline{s}_r = -\cos \frac{r\pi}{M+1}$$

and

$$\underline{\sigma}_p = -\cos \frac{2p-1}{2(M+1)} \pi$$

$$r = 1, 2, \ldots M$$

Evaluation of the second term in the brackets requires the observation that the multiplier of K_2 in Equation 48 goes to zero whenever the collocation point is in the plane of the doublet sheet located at the integration point. Therefore, the finite part of the integral of the K_2 term is zero, and the entire contribution comes from the K_1 term. In this case, $\gamma(\underline{s}) = \gamma(\underline{\sigma})$ and $\underline{r}_1^2 = (\underline{s}_r - \underline{\sigma})^2 = 0$.

It can be shown that

$$K_1(x-\xi, \underline{s}_r, \underline{s}_r) = \begin{cases} -2, & x > \xi \\ 0, & x < \xi . \end{cases}$$

Thus, the chordwise integral which defines $G_{nm}(x_j, \underline{s}_r, \underline{s}_r)$ is

$$G_{nm}(x_j, \underline{s}_r, \underline{s}_r) = -2 \int_{-1}^{\tilde{x}_j} \underline{\sigma}^m f_n(\tilde{\xi})\, e^{-i\frac{\omega}{U}(x_j - \xi)}\, d\tilde{\xi}$$

If the range of integration is extended to $\tilde{\xi} = 1$ by making the integrand zero for $\tilde{\xi} > \tilde{x}_j$, the integral cannot be well approximated by a polynomial because it has a jump discontinuity at $\tilde{\xi} = \tilde{x}_j$. To overcome this difficulty, we write

$$G_{nm}(x_j, \underline{s}_r, \underline{s}_r) = -2 \int_{-1}^{\tilde{x}_j} \underline{\sigma}^m f_n(\tilde{\xi}) \left[e^{-i \frac{\omega}{U}(x_j - \xi)} - 1 \right] d\tilde{\xi} \tag{61a}$$

$$-2 \int_{-1}^{\tilde{x}_j} \underline{\sigma}^m f_n(\tilde{\xi})\, d\tilde{\xi} .$$

The second term here depends on the integrals

$$h_n(\tilde{x}_j) = \int_{-1}^{\tilde{x}_j} f_n(\tilde{\xi})\, d\tilde{\xi}$$

which may be evaluated exactly. We have

$$h_0(\tilde{x}_j) = \int_{-1}^{\tilde{x}_j} \sqrt{\frac{1 - \tilde{\xi}}{1 + \tilde{\xi}}}\, d\tilde{\xi} = \frac{\pi}{2} + \sin^{-1} \tilde{x}_j + \sqrt{1 - \tilde{x}_j^{\,2}}$$

$$h_1(\tilde{x}_j) = \int_{-1}^{\tilde{x}_j} \sqrt{1 - \tilde{\xi}^2}\, d\tilde{\xi} = \frac{1}{2}\left(\frac{\pi}{2} + \sin^{-1} \tilde{x}_j + \tilde{x}_j \sqrt{1 - \tilde{x}_j^{\,2}} \right)$$

This list may be extended as far as it is needed.

The first integral in Equation 61a is considered as an integral over $-1 < \tilde{\xi} < 1$ with zero integrand when $\tilde{\xi} > \tilde{x}_j$, and Equation 55 is applied. The result is

$$G_{om}(x_j, \underline{s}_r, \underline{s}_r) = -2 \cdot \frac{2\pi}{2L + 1} \sum_{\substack{k \geq 1, \\ \tilde{\xi}_k < \tilde{x}_j}} \underline{\sigma}^m (1 - \tilde{\xi}_k) \left[e^{-i \frac{\omega}{U}(x_j - \xi_k)} - 1 \right]$$

$$- 2\, \underline{\sigma}^m g_o(\tilde{x}_j) \tag{61b}$$

$$G_{nm}(x_j, \underline{s}_r, \underline{s}_r) = -2 \frac{2\pi}{2L+1} \sum_{\substack{k \geq 1, \\ \tilde{\xi}_k < \tilde{x}_j}} \underline{\sigma}^m (1 - \tilde{\xi}_k{}^2)\, U_n(\tilde{\xi}_k) \left[e^{-i \frac{\omega}{U}(x_j - \xi_k)} - 1 \right]$$

$$- 2\underline{\sigma}^m g_n(\tilde{x}_j), \quad n \geq 1.$$

Equations 59 and 61 are used to evaluate the D^i_{nm}'s given by Equation 60. Equation 50 is then used to determine the pressure coefficients a_{nm} to match the downwashes (i.e. the $\overline{w}_i/U$'s) at the collocation points (x_j, s_r). Once the pressure coefficients are determined, the generalized forces are computed. A polynomial expression for the i^{th} modal deflections normal to the surface,

$$n^{(i)}(\xi, \underline{\sigma}) = \sum_{\nu=0}^{N} \sum_{\mu=0}^{M} b^{(i)}_{\nu\mu}\, \xi^{\nu} \underline{\sigma}^{\mu}$$

and Equation 49 are substituted into the equation

$$Q_{ij} = \int_{-s_{TIP}}^{s_{TIP}} \int_{x_{LE}}^{x_{TE}} n^{(i)}(\xi, \underline{\sigma})\, \Delta\overline{p}^{(j)}(\tilde{\xi}, \underline{\sigma})\, d\xi\, d\sigma$$

to get

$$Q_{ij} = s^2_{TIP} (1/2\, \rho U^2) \sum_{n=0}^{N} \sum_{m=0}^{M} \sum_{\nu=0}^{N} \sum_{\mu=0}^{M} a^{(j)}_{nm}\, b^{(i)}_{\nu\mu}\, \Delta Q_{nm\nu\mu} \qquad (62)$$

where

$$\Delta Q_{nm\nu\mu} = \int_{-1}^{1} \int_{-1}^{1} \sqrt{1 - \underline{\sigma}^2}\, \underline{\sigma}^{m+u}\, \xi^{\nu} \sqrt{\frac{1 - \tilde{\xi}}{1 + \tilde{\xi}}}\, F_n(\tilde{\xi})\, d\tilde{\xi}\, d\underline{\sigma}$$

The quadrature formula given by Equation 55 with L set equal to N may be used to evaluate the inner integral. For evaluation of the outer integral, a quadrature formula with the weight function $\sqrt{1 - \underline{\sigma}^2}$, analogous to Equation 60 for a nonsingular integral, may be used. This is not practical for a wing with folded tip, for which a different representation should be used on each plane part of the surface. Here, the spanwise integral over one part of the wing may have a square root factor at one end of the interval of integration, or at neither end. Such integrals may best be evaluated by a suitable application of ordinary Gaussian quadrature with a weight function of 1.0. The points and weights for this quadrature method are not given by simple formulas such as Equation 55. They are listed in many places, (e.g. Reference 5).

VI. APPLICATION TO A WING WITH A FOLDED TIP

The planform may be any continuous surface. However, the computer program was developed to treat either a planar or nonplanar planform of the type shown in Figure 2. (Only one-half the planform is shown.) To facilitate modifications of the program, comment cards are placed throughout the program to indicate where changes may be made to handle other nonplanar surfaces like that of the Paraglider.

In the application of the kernel function method to the planform shown in Figure 2, the computer program calculates from the equations of the leading and trailing edges the collocation and integration points for which the integrands of Equation 51 must be evaluated. For demonstration purposes, we calculate the collocation and integration points for values of L = 4, N = 6, and M = 10 and show the results in Figure 3.

$$x_{LE} = s \tan\lambda_{LE}, \quad x_{LETIP} = s_{F_L} \tan\lambda_{LE} + (s - s_{F_L}) \tan\lambda_{LETIP}$$

$$x_{TE} = 2b_o + s \tan\lambda_{TE}, \quad x_{TETIP} = 2b_o + s_{F_L} \tan\Lambda_{TE} + (s - s_{F_L}) \tan\lambda_{TETIP}$$

$$b(s) = 1/2\,(x_{TE} - x_{LE})$$

$$x_m = 1/2\,(x_{LE} + x_{TE})$$

$$x = x_m + b(s)\,\tilde{x}$$

$$\xi = \xi_m + b(\sigma)\,\tilde{\xi}$$

$$\tilde{x}_j = -\cos\frac{2j}{2N+1}\pi, \quad j = 1, 2, \ldots N.$$

$$\tilde{\xi}_k = -\cos\frac{2k-1}{2L+1}\pi, \quad k = 1, 2, \ldots L.$$

$$s = \underset{\sim}{s}\, s_{TIP}$$

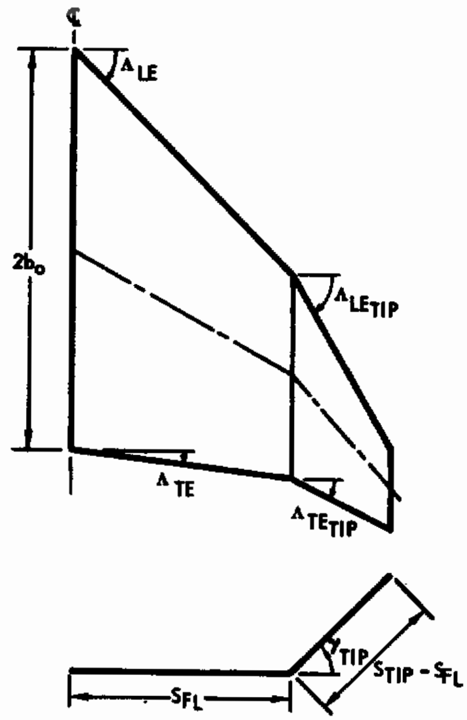


Figure 2. Planar Wing With Planar Symmetrically Folded Tips

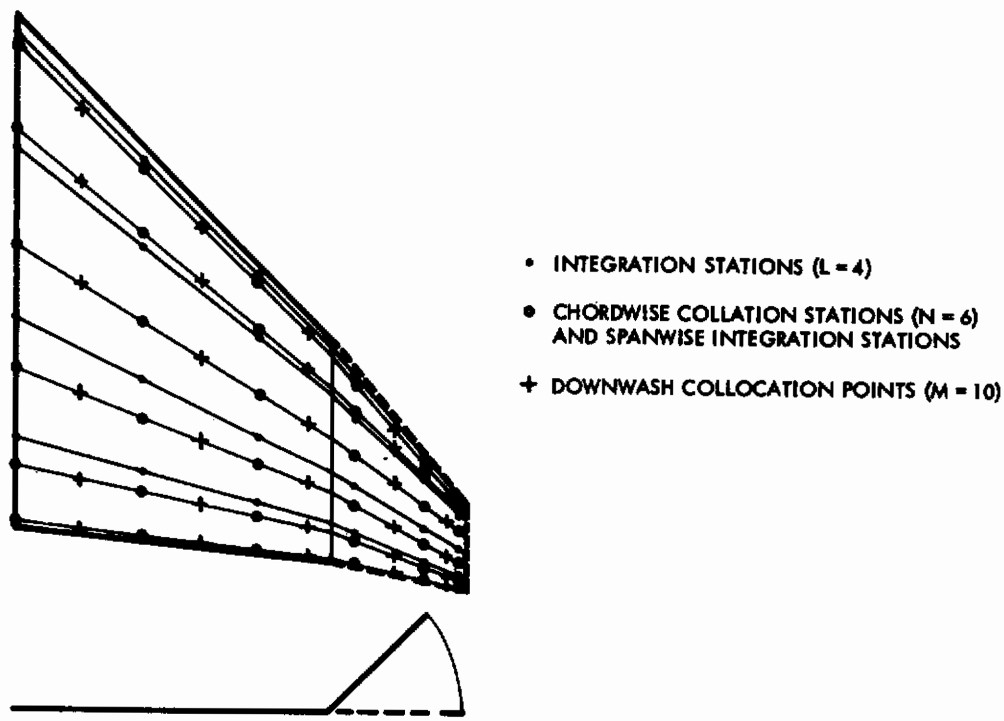


Figure 3. An Optimum Set of Collocation and Integration Points

$$\underline{s}_m = -\cos\frac{m\pi}{M+1}, \quad m = 1, 2, \ldots, M.$$

$$\underline{\sigma}_p = -\cos\frac{2p-1}{2(M+1)}\pi, \quad p = 1, 2, \ldots, (M+1).$$

We have also constructed a table of equations (Table 1), for the functions y_o, z_o, r_1, T_1, and T_2 for use in the expression for the kernel, Equations 48 through 48c. In Table 1, the subscript F_L indicates the fold line.

A difficulty is encountered in the spanwise integration because the kernel function has a finite discontinuity at the fold line. For example, note in Table 1 the change in T_1 and T_2 for receiving or collocation points on the wing as the sending or integration points shift from the port tip to the wing and from the wing to the starboard tip.

If we consider the kernel as a function of $\tilde{\xi}$ and $\underline{\sigma}$ for fixed values of x and $\underline{s}$,

$$q(\tilde{\xi}, \underline{\sigma}) = \bar{\bar{K}}(x - \xi, \underline{s}, \underline{\sigma}),$$

this function may be broken up into a simple discontinuous part g^{**} and a part g^* which is continuous across the fold lines:

$$g(\tilde{\xi}, \underline{\sigma}) = g^*(\tilde{\xi}, \underline{\sigma}) + g^{**}(\tilde{\xi}, \underline{\sigma}) \tag{63}$$

To do this, define

$$g^{**}(\tilde{\xi}, \underline{\sigma}) = \begin{cases} g(\tilde{\xi}, \underline{\sigma}_{F_L}+) - g(\tilde{\xi}, \underline{\sigma}_{F_L}-), & \underline{\sigma} > \underline{\sigma}_{F_L} \\ 0, & -\underline{\sigma}_{F_L} < \underline{\sigma} < \underline{\sigma}_{F_L} \\ g(\tilde{\xi}, -\underline{\sigma}_{F_L}-) - g(\tilde{\xi}, -\underline{\sigma}_{F_L}+), & \underline{\sigma} < -\underline{\sigma}_{F_L} \end{cases} \tag{64}$$

and then define g^* by Equation 63.

More explicitly, for $\underline{\sigma} > \underline{\sigma}_{F_L}$, define

$$\xi_{F_L} = b(\underline{\sigma}_{F_L})\tilde{\xi} + \frac{1}{2}\left[\xi_{LE}(\underline{\sigma}_{F_L}) + \xi_{TE}(\underline{\sigma}_{F_L})\right]$$

Table 1. Spanwise Parameter in the Kernel Function

Functions	Receiving Points (x, s)								
	Port Tip $S < -y_{F_L}$ and $\gamma(S) = -\gamma_{TIP}$			Wing $-y_{F_L} < S < y_{F_L}$ and $\gamma(S) = 0$			Starboard Tip $y_{F_L} < S$ and $\gamma(S) = \gamma_{TIP}$		
	Sending Points (ξ, σ)								
	PT $\gamma(\sigma) = -\gamma_{TIP}$	W $\gamma(\sigma) = 0$	ST $\gamma(\sigma) = \gamma_{TIP}$	PT $\gamma(\sigma) = \gamma_{TIP}$	W $\gamma(\sigma) = 0$	ST $\gamma(\sigma) = \gamma_{TIP}$	PT $\gamma(\sigma) = -\gamma_{TIP}$	W $\gamma(\sigma) = 0$	ST $\gamma(\sigma) = \gamma_{TIP}$
y_o	$s_o \not{c}$	$-\sigma^* + s^* \not{c}$	$(s^* - \sigma_*) \not{c} - 2y_{F_L}$	$s^* - \sigma^* \not{c}$	s_o	$s_* - \sigma_* \not{c}$	$(s_* - \sigma^*) \not{c} + 2y_{F_L}$	$-\sigma_* + s_* \not{c}$	$s_o \not{c}$
z_o	$-s_o \not{s}$	$-s^* \not{s}$	$-(s + \sigma) \not{s}$	$\sigma^* \not{s}$	0	$-\sigma_* \not{s}$	$(s_* + \sigma^*) \not{s}$	$s_* \not{s}$	$s_o \not{s}$
r_1	$\lvert s_o \rvert$	$\sqrt{s^{*2} + \sigma^{*2} - 2 s^* \sigma^* \not{c}}$	$\sqrt{\left[(s^* - \sigma_*)\not{c} - 2y_{F_L}\right]^2 + \left[(s^* + \sigma_*)\not{s}\right]^2}$	$\sqrt{s^{*2} + \sigma^{*2} - 2 s^* \sigma^* \not{c}}$	$\lvert s_o \rvert$	$\sqrt{s_*^2 + \sigma_*^2 - 2 s_* \sigma_* \not{c}}$	$\sqrt{\left[(s_* - \sigma^*)\not{c} + 2y_{F_L}\right]^2 + \left[(s_* + \sigma^*)\not{s}\right]^2}$	$\sqrt{s_*^2 + \sigma_*^2 - 2 s_* \sigma_* \not{c}}$	$\lvert s_o \rvert$
T_1	1.0	$\not{c}$	$\not{c}^2 - \not{s}^2$	$\not{c}$	1.0	$\not{c}$	$\not{c}^2 - \not{s}^2$	$\not{c}$	1 0
T_2	0	$s^* \sigma^* \not{s}^2$	$4(\sigma_* \not{c} + y_{F_L})(s^* \not{c} - y_{F_L}) \not{s}^2$	$s^* \sigma^* \not{s}^2$	0	$s_* \sigma_* \not{s}^2$	$4(\sigma^* \not{c} - y_{F_L})(s_* \not{c} + y_{F_L}) \not{s}^2$	$s_* \sigma_* \not{s}^2$	0

$s^* = s + y_{F_L}$, $\sigma^* = \sigma + y_{F_L}$, $s_* = s - y_{F_L}$, $\sigma_* = \sigma - y_{F_L}$, $s_o = s - \sigma$, $\not{s} = \sin\gamma_{TIP}$, $\not{c} = \cos\gamma_{TIP}$

Then

$$g^{**}(\tilde{\xi},\underline{\sigma}) = \bar{\bar{K}}^{ST}(x - \xi_{F_L}, \underline{s}, \underline{\sigma}_{F_L}) - \bar{\bar{K}}^{W}(x - \xi_{F_L}, \underline{s}, \underline{\sigma}_{F_L})$$

in which

$$\gamma(\underline{\sigma}) = \gamma_{TIP} \text{ in } \bar{\bar{K}}^{ST} \quad (ST \sim \text{starboard tip})$$

$$\gamma(\underline{\sigma}) = 0 \text{ in } \bar{\bar{K}}^{W} \quad (W \sim \text{wing})$$

A similar formula applies when $\underline{\sigma} < -\underline{\sigma}_{F_L}$. g^{**} may be written in a form which indicates that it is independent of σ in each tip region:

$$g^{**}(\tilde{\xi},\underline{\sigma}) = \begin{cases} g^{**}_{ST}(\tilde{\xi}), & \underline{\sigma} > \underline{\sigma}_{F_L} \\ 0, & -\underline{\sigma}_{F_L} < \underline{\sigma} < \underline{\sigma}_{F_L} \\ g^{**}_{PT}(\tilde{\xi}), & \underline{\sigma} < -\underline{\sigma}_{F_L} \end{cases} \tag{65}$$

With the use of Equations 63 and 65, Equation 51 may be rewritten as

$$D^{i}_{mn} = \frac{1}{8\pi} \int_{-1}^{1} \sqrt{1 - \underline{\sigma}^2}\, \underline{\sigma}^m \int_{-1}^{1} f_n(\tilde{\xi})\, g^{*}(\tilde{\xi},\underline{\sigma})\, d\tilde{\xi}\, d\underline{\sigma}$$

$$+ \frac{1}{8\pi} \int_{-1}^{1} f_n(\tilde{\xi})\, g^{**}_{ST}(\tilde{\xi}) \int_{\underline{\sigma}_{F_L}}^{1} \sqrt{1 - \underline{\sigma}^2}\, \underline{\sigma}^m d\underline{\sigma}\, d\tilde{\xi}$$

$$+ \frac{1}{8\pi} \int_{-1}^{1} f_n(\tilde{\xi})\, g^{**}_{PT}(\tilde{\xi}) \int_{-1}^{-\underline{\sigma}_{F_L}} \sqrt{1 - \underline{\sigma}^2}\, \underline{\sigma}^m d\underline{\sigma}\, d\tilde{\xi}$$

The first of these three double integrals is calculated according to Equation 60. In the others, the inner integral may be evaluated exactly. The constants

$$u_m = \int_{\sigma_{F_L}}^{1} \sqrt{1 - \sigma^2}\, \sigma^m \, d\sigma$$

are given by formulas

$$u_0 = \frac{1}{2}\left[\frac{\pi}{2} - \sin^{-1} \sigma_{F_L} + \sigma_{F_L} \sqrt{1 - \sigma_{F_L}^2}\right]$$

$$u_1 = \frac{1}{3}\left(1 - \sigma_{F_L}^2\right)^{3/2}$$

etc.

In terms of these constants

$$D^i_{mn} = \frac{1}{8\pi} \int_{-1}^{1} \sqrt{1 - \sigma^2}\, \sigma^m \int_{-1}^{1} f_n(\tilde{\xi})\, g^*(\tilde{\xi}, \sigma)\, d\tilde{\xi}\, d\sigma$$

$$+ \frac{1}{8\pi} u_m \int_{-1}^{1} f_n(\tilde{\xi}) \left[g^{**}_{ST}(\tilde{\xi}) + (-1)^m g^{**}_{PT}(\tilde{\xi})\right] d\tilde{\xi}$$

The last integral in this formula is evaluated by Equation 55.

In the evaluation of O_{ij}, given by Equation 62 for the plane case, it is assumed that modes numbers i and j are either both symmetric in σ, or both antisymmetric. Then the contribution to the integral for $\sigma < 0$ is the same as the contribution for $\sigma > 0$. Let the deflection in the i^{th} mode be given by

$$n^{(i)}(\xi, \sigma) = \begin{cases} \sum_{\nu=0}^{N} \sum_{\mu=0}^{M} c^{(i)}_{\nu\mu}\, \xi^\nu \sigma^\mu, & 0 < \sigma < \sigma_{F_L} \\ \sum_{\nu=0}^{N} \sum_{\mu=0}^{M} d^{(i)}_{\nu\mu}\, \xi^\nu \sigma^\mu, & \sigma > \sigma_{F_L} \end{cases}$$

Then

$$O_{ij} = 2 \cdot y^2_{TIP} \left(\frac{1}{2} \rho U^2\right) \sum_{n=0}^{N} \sum_{m=0}^{M} \sum_{\nu=0}^{N} \sum_{\mu=0}^{M} a^{(j)}_{nm} \left\{ c^{(i)}_{\nu\mu} I^{(1)}_{nm\nu\mu} + d^{(i)}_{\nu\mu} I^{(2)}_{nm\nu\mu} \right\}$$

in which

$$I^{(1)}_{nm\nu\mu} = \int_0^{\sigma_{F_L}} \sqrt{1 - \sigma^2}\, \sigma^{m+\mu} \int_{-1}^{1} \xi^{\nu} \sqrt{\frac{1 - \tilde{\xi}}{1 + \tilde{\xi}}}\, F_n(\tilde{\xi})\, d\tilde{\xi}\, d\sigma$$

$$I^{(2)}_{nm\nu\mu} = \int_{\sigma_{F_L}}^{1} \sqrt{1 - \sigma^2}\, \sigma^{m+\mu} \int_{-1}^{1} \xi^{\nu} \sqrt{\frac{1 - \tilde{\xi}}{1 + \tilde{\xi}}}\, F_n(\tilde{\xi})\, d\tilde{\xi}\, d\sigma$$

Note that the inner integrals are the integrals of polynomials in $\tilde{\xi}$ multiplied by $\sqrt{(1 - \tilde{\xi})/(1 + \tilde{\xi})}$, by virtue of the relation

$$\xi = b(\sigma)\tilde{\xi} + \frac{1}{2} \left[\xi_{LE}(\sigma) + \xi_{TE}(\sigma) \right]$$

Hence, the inner integral may be evaluated exactly by either Equation 52a or Equation 55 if enough points are used in the formulas. For the limits $\nu \leq 5$, $n \leq 4$ used in the computer program, six points are sufficient for Equation 52a, five points for Equation 55. Equation 52a was used with $K = 6$. (This choice was made arbitrarily; it would be just as good to use Equation 55.)

In the integrations over σ, six-point Gaussian integration with weight function 1.0 was used. The basic formula is

$$\int_0^1 f(v)\, dv = \sum_{\ell=1}^{6} h_\ell f(v_\ell) \tag{66}$$

exact for $f(v)$ a polynomial of degree at most 11. The constants occurring in this formula are given in the subroutine FORCE. They may be derived from a table given by Scarborough (Reference 6, p. 148).

In $I^{(1)}_{nm\nu\mu}$, Equation 66 is applied by putting

$$\sigma = \sigma_{F_L} v$$

The resulting expression is

$$I^{(1)}_{nm\nu\mu} = \frac{\pi}{6} \sigma_{F_L} \sum_{\ell=1}^{6} h_\ell \sqrt{1 - \sigma_\ell^2}\, \sigma_\ell^{m+\mu} \sum_{k=1}^{6} \xi_k(\sigma_\ell)^\nu (1 - \tilde{\xi}_k^2) F_n(\tilde{\xi}_k)$$

in which

$$\sigma_\ell = \sigma_{F_L} v_\ell$$

$$\xi_k(\sigma_\ell) = b(\sigma_\ell)\tilde{\xi}_k + \frac{1}{2}\left[\xi_{LE}(\sigma_\ell) + \xi_{TE}(\sigma_\ell)\right]$$

In $I^{(2)}_{nm\nu\mu}$, the transformation

$$\sigma = 1 - (1 - \sigma_{F_L}) v^2$$

was used. This makes the v-integrand behave like a polynomial at the ends of the interval. The resulting formula is

$$I^{(2)}_{nm\nu\mu} = \frac{\pi}{3} (1 - \sigma_{F_L}) \sum_{\ell=1}^{6} h_\ell v_\ell \sqrt{1 - \sigma_\ell^2}\, \sigma_\ell^{m+\mu} \sum_{k=1}^{6}$$

$$\xi_k(\sigma_\ell)^\nu (1 - \tilde{\xi}_k^2) F_n(\tilde{\xi}_k)$$

in which

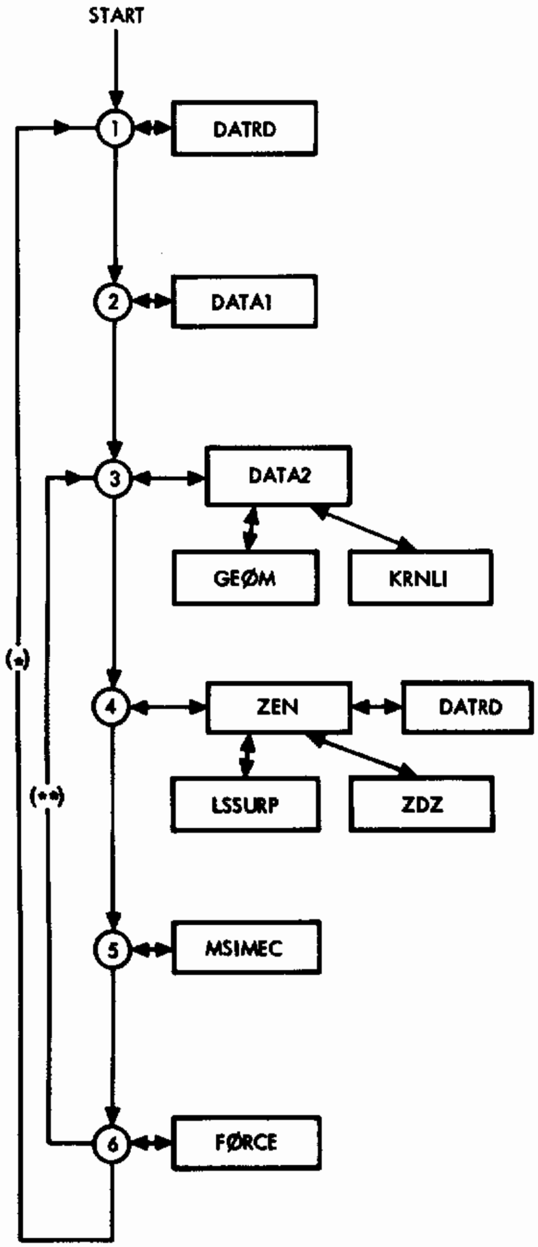
$$\sigma_\ell = 1 - (1 - \sigma_{F_L}) v_\ell^2$$

$$\xi_k(\sigma_\ell) = b(\sigma_\ell)\tilde{\xi}_k + \frac{1}{2}\left[\xi_{LE}(\sigma_\ell) + \xi_{TE}(\sigma_\ell)\right]$$

DESCRIPTION OF THE COMPUTER PROGRAM

A functional diagram of the computer program is given in Figure 4. With the exception of two subroutines named MSIMEC and MSIMER, all of the programs are written in Fortran IV. These two subroutines, written in machine language, are used for complex and real matrix inversion, respectively. There are certain limitations related to the various other subprograms, which are listed below.

Subprogram	Limitations
Subroutine Data	
NCC	The number of chordwise collocation stations must be ≤ 10.
NCS	The number of spanwise collocation stations must be ≤ 9.
NDATA	The number of sets of data must be ≤ 10.
N	The number of chordwise pressure modes ≤ 5.
M	The number of spanwise pressure modes ≤ 5.
Subroutine Zen	
MODES	This is the number of modes used in the calculation of generalized forces, and must be ≤ 10.
NPTS	For a planar wing, this is the number of points at which the deflection is given in the horizontal surface and must be ≤ 66. For a nonplanar wing, there must be ≤ 66 points for the deflections in the horizontal surface and ≤ 66 points for the deflections in the vertical surface.



The first set of data is read. This includes Items 1-14 in the data array, the Mach numbers and frequencies, Item 41, and perhaps some of the mode data that will be needed later.

Subroutine DATA1 performs most of the calculations that are independent of Mach number and frequency, such as setting up the arrays of collocation and integration stations.

This is the beginning of a loop that is gone through for each combination of Mach number and frequency specified. Subroutine DATA2 sets up the matrix (D^i_{nm}) of Equation 50.

In subroutine ZEN, any necessary data for the oscillation modes is read in. The left side of Equation 50 is set up for each mode.

Equation 50 is solved for the pressure coefficients for each mode by MSIMEC.

Subroutine FØRCE computes the generalized force matrix. The program returns to step 3 until the Mach numbers and frequencies are exhausted, then to step 1.

(*) RETURN TO BEGINNING FOR NEXT CASE

(**) LOOP ON NUMBER OF SETS OF MACH NUMBERS AND FREQUENCIES

Figure 4. Functional Flow Diagram—Main Program

USE OF THE COMPUTER PROGRAM

The following rules apply to optimum use of the computer program:

1. Before attempting to use the computer program compute the coefficients of polynomials of minimum order in x and y that will adequately represent the modal deflection distributions. Least-square fitted surfaces are very useful for this purpose, since weighting factors may be used to obtain a better fit in special regions.

2. Set the number of chordwise collocation points M equal to one plus the highest of the orders in x; and, unless there is special reason to reduce the number of chordwise integration points, set L equal to M.

3. Set the number of spanwise collocation points R equal to one plus the highest of the orders in y. This establishes the number of pressure coefficients a_{nm} at M x R. Their values are computed by matching exactly the downwashes at the M x R spanwise collocation points.

The input data is read by the subroutine DATRD. Use of this subroutine requires that, on each data card, the first 72 columns are six fields of width 12, as indicated on the sample date sheets (Figure 5). The first field contains an integer giving the location in the data array in which the number in the second field is to be stored. The numbers in the remaining fields are stored in consecutive locations. If a field is blank, the corresponding location in the data array is unchanged. DATRD reads any number of cards. A minus sign in column 1 indicates the last card to be read; if this minus sign is not present, DATRD continues with the next card. The storage locations of the data on a card are not affected by the sign in column 1. All floating point numbers must be written with decimal points. All integers must be at the right of their fields.

The data array is set up as follows:

1.	N	The number of chordwise pressure modes
2.	M	The number of spanwise pressure modes
3.	NCC	The number of chordwise collocation points
4.	NCS	The number of spanwise collocation points
5.	NDATA	The number of sets of values of Mach number and frequency to be used.
6.	NSYM	Indicator for symmetric (NSYM = +1) or antisymmetric (NSYM = -1) modes of oscillation.

7.	SFOLD	Distance spanwise from wing center line to fold line
8.	STIP	Semispan
9.	BO	One-half of the root chord
10.	ALFA1	The fold angle (in degrees)
11.	λ_{LE}	The sweep angle of the leading edge (in degrees)
12.	λ_{TE}	
13.	$\lambda_{LE_{TIP}}$	
14.	$\lambda_{TE_{TIP}}$	
21-30.	Values of Mach number.	NDATA of these must be entered.
31-40.	Values of reduced frequency, $\omega \cdot BO/U$.	NDATA of these must be entered.
41.	NMOD	The number of modes
42.	JD	Indicator for the type of input data for a mode. If JD=1, the deflections are given at a set of points on the wing (and tip). If JD=2, the coefficients of polynomials for the deflection of the wing and tip are given. If JD=0, the current mode and subsequent modes are not given by data. They are the same as the corresponding modes which were used for the previous frequency and Mach number.
43.	NPTSW	Number of points on the wing at which deflections are given (used if JD=1).
44.	NPTST	Number of points on tip at which deflections are given.
51-71.	Deflection coefficients on the wing.	The coefficients are stored as follows:

$$\overset{51}{a_{00}} + \overset{52}{a_{10}}\,x + \overset{53}{a_{20}}\,x^2 + \overset{54}{a_{30}}\,x^3 + \overset{55}{a_{40}}\,x^4 + \overset{56}{a_{50}}\,x^5 + y(\overset{57}{a_{01}} + \overset{58}{a_{11}}x + \overset{59}{a_{21}}x^2$$

$$+ \overset{60}{a_{31}}\,x^3 + \overset{61}{a_{41}}\,x^4) + y^2(\overset{62}{a_{02}} + \overset{63}{a_{12}}x + \overset{64}{a_{22}}x^2 + \overset{65}{a_{32}}x^3)$$

$$+ y^3(\overset{66}{a_{03}} + \overset{67}{a_{13}}x + \overset{68}{a_{23}}x^2) + y^4(\overset{69}{a_{04}} + \overset{70}{a_{14}}x) + \overset{71}{a_{05}}y^5$$

76-96.	Deflection coefficients on the tip.	Same storage rule as above, except all locations are increased by 25.
98.	Indicator that no more modes are to be read after the present one. (For current and subsequent frequencies and Mach numbers.)	
101-299.	Deflection data at points on the wing, in the order $x_1, s_1, n_1, x_2, s_2, n_2$, etc.	
301-499.	Deflection data at points on the tip.	

Items 1-41 must be read in the first set of data. There may be an additional set of data for each mode, for each Mach number and frequency case. After the indicator DAT(98) has been given a non-zero value, no more deflection data will be read. After JD has been given the value zero, no data will be read for the higher numbered modes.

The data in Figure 5 is for a 60° triangular wing folded at 75 percent semispan, at an angle of 30°. The root chord is 5.0 feet, making BO = 2.5. Three modes: plunge, pitch, and a third nonrigid mode are considered. The Mach number is 0.7, and six frequencies: 10, 20, 30, 40, 50, and 60 cps are used. The speed of sound is taken to be 1000 ft./sec. This gives reduced frequencies of 0.157, 0.314, 0.471, 0.628, 0.785, and 0.942.

Three spanwise and three chordwise pressure modes, six chordwise and eight spanwise collocation stations are specified.

In the set of cards numbered 15 - 30, which give deflection data, some of the cards have been omitted. Otherwise, this is a complete set of data for a computer run.

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE 1 of 5 JOB NO. ______

Col.	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	1		
13	3		N
25	3		M
37	6		NCC
49	8	73 80	NCS
61	6	1	NDATA
1	6		
13	1		NSYM
25	2.165		SFOLD
37	2.887		STIP
49	2.5	73 80	BO
61	30.0	2	Fold Angle
1	11		
13	60.0		Sweep Angles
25	0.0		
37	60.0		
49	0.0	73 80	
61		3	
1	21		
13	0.7		Mach Numbers
25	0.7		
37	0.7		
49	0.7	73 80	
61	0.7	4	

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Figure 5. Sample Data Sheets (Sheet 1 of 5)

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE 2 of 5 JOB NO. ______

Col.	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	26		
13	0.7		Last Mach Number
25			
37			
49		73 80	
61		5	
1	31		
13	0.157		Reduced Frequencies
25	0.314		
37	0.471		
49	0.628	73 80	
61	0.785	6	
1	36		
13	0.942		
25			
37			
49		73 80	
61		7	
1	— 41		Minus sign indicates last card in first set of data
13	3		NMOD
25			
37			
49		73 80	
61		8	

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Figure 5. Sample Data Sheets (Sheet 2 of 5)

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE 3 of 5 JOB NO. ______

Col.	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	4 2		
13	2		JD
25			
37			
49		73 80	
61		9	
1	5 1		
13	1. 0		Coefficients of the deflection polynomial on the wing, which has the
25			constant value 1.0 in this mode.
37			
49		73 80	
61		10	
1	— 7 6		Minus sign indicates last card for first mode.
13	0. 8 6 6		Coefficients of the deflection polynomial on the tip. A vertical
25			deflection of 1.0 causes a normal deflection of cos 30° = 0.866.
37			
49		73 80	
61		11	
1	5 1		
13	0. 0		Coefficients on the wing for the second mode. The polynomial is 0.2X
25	0. 2		
37			
49		73 80	
61		12	

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Figure 5. Sample Data Sheets (Sheet 3 of 5)

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE 4 of 5 JOB NO. ______

Col.	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	–76		
13	0.0		Coefficients on the tip for the second mode,
25	0.1732		
37			
49			
61		13	
1	42		
13	1		JD
25	22		NPTSW
37	6		NPTST
49			
61		14	
1	101		Deflection data on the wing begin.
13	0.9		X1
25	0.4		S1
37	0.314		N1
49	1.6		X2
61	0.4	15	S2
1	106		
13	0.323		N2
25	2.8		X3
37	0.4		S3
49	0.330		N3
61	3.6	16	X4

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Figure 5. Sample Data Sheets (Sheet 4 of 5)

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE 5 of 5 JOB NO. ______

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	1 6 6		End of deflection data on the wing
13	1 2 0 6		N22
25			
37			
49		73 80	
61		2 6	
1	3 0 1		Deflection data on the tip begin
13	4 2 5		X1
25	2 3		S1
37	1 3 8		N1
49	4 5	73 80	X2
61	2 3	2 7	S2
1	3 1 6		End of deflection data on the tip
13	4 8		X6
25	2 7		S6
37	1 4 4		N6
49		73 80	
61		3 0	
1	— 9 8		
13	1		Indicator that modes are not to be read in hereafter
25			
37			
49		73 80	
61		3 1	

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Figure 5. Sample Data Sheets (Sheet 5 of 5)

VII. RESULTS

The computer program was applied to the rigid modes of two wings.

ASPECT RATIO 2.0 RECTANGULAR WING FOLDED AT 80 PERCENT SEMISPAN

C_{L_α} is plotted as a function of Mach number in Figure 6. The dashed curve is a plot of the approximation

$$C_{L_\alpha} = \frac{2\pi A.R.}{2 + \sqrt{4 + (A.R.)^2 (1-M^2)}}$$

where A.R. is the aspect ratio. This is formula (6-31) of Reference 5, for the case of a rectangular wing.

TRIANGULAR WING WITH FOLDED TIPS

The configuration used was a triangular wing with a sweep angle of 65 degrees, folded at 60 percent semispan.

Figures 7 and 8 show C_{L_α} and C_{m_α} as functions of fold angle (at $M = 0.8$), and as functions of Mach number. Figure 9 is a plot of unsteady generalized forces for rigid oscillations of the wing in the pitching mode.

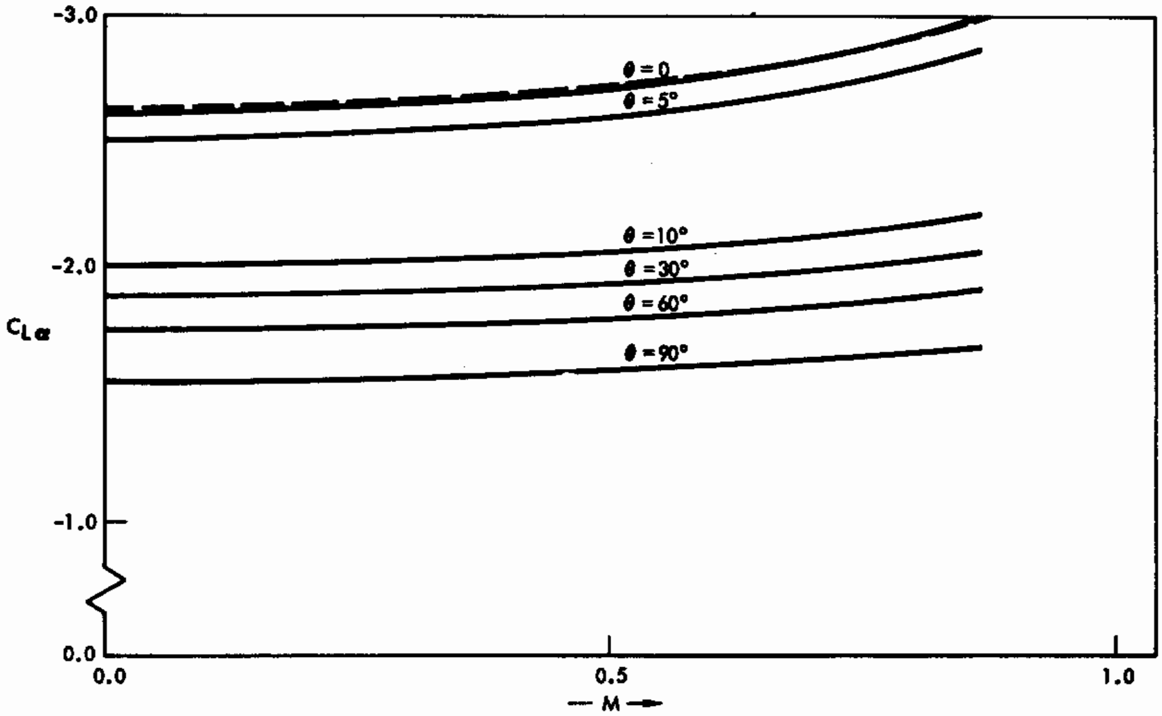


Figure 6. Lift and Moment Coefficients Vs Mach Number for Aspect Ratio 2.0 Rectangular Wing at Various Fold Angles

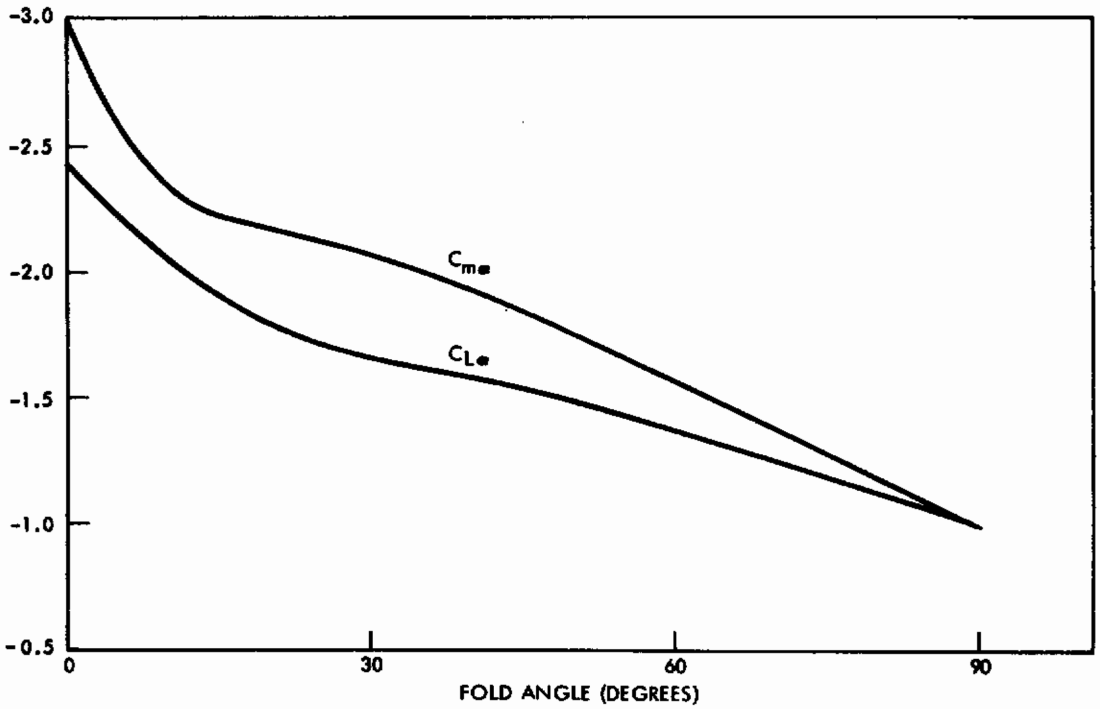


Figure 7. Lift and Moment Coefficients Vs Fold Angle for 65° Triangular Wing at $M_{\infty} = 0.8$

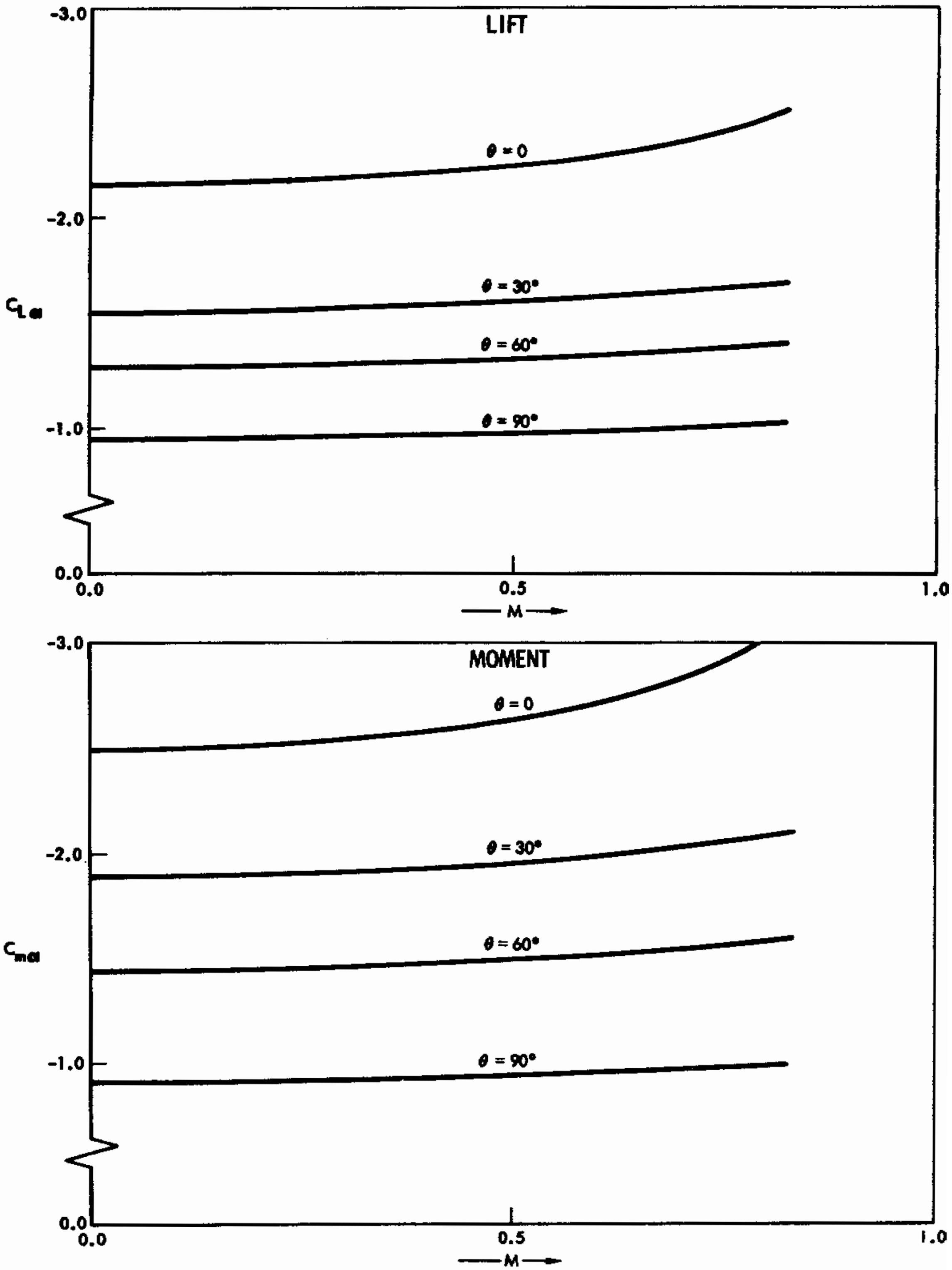
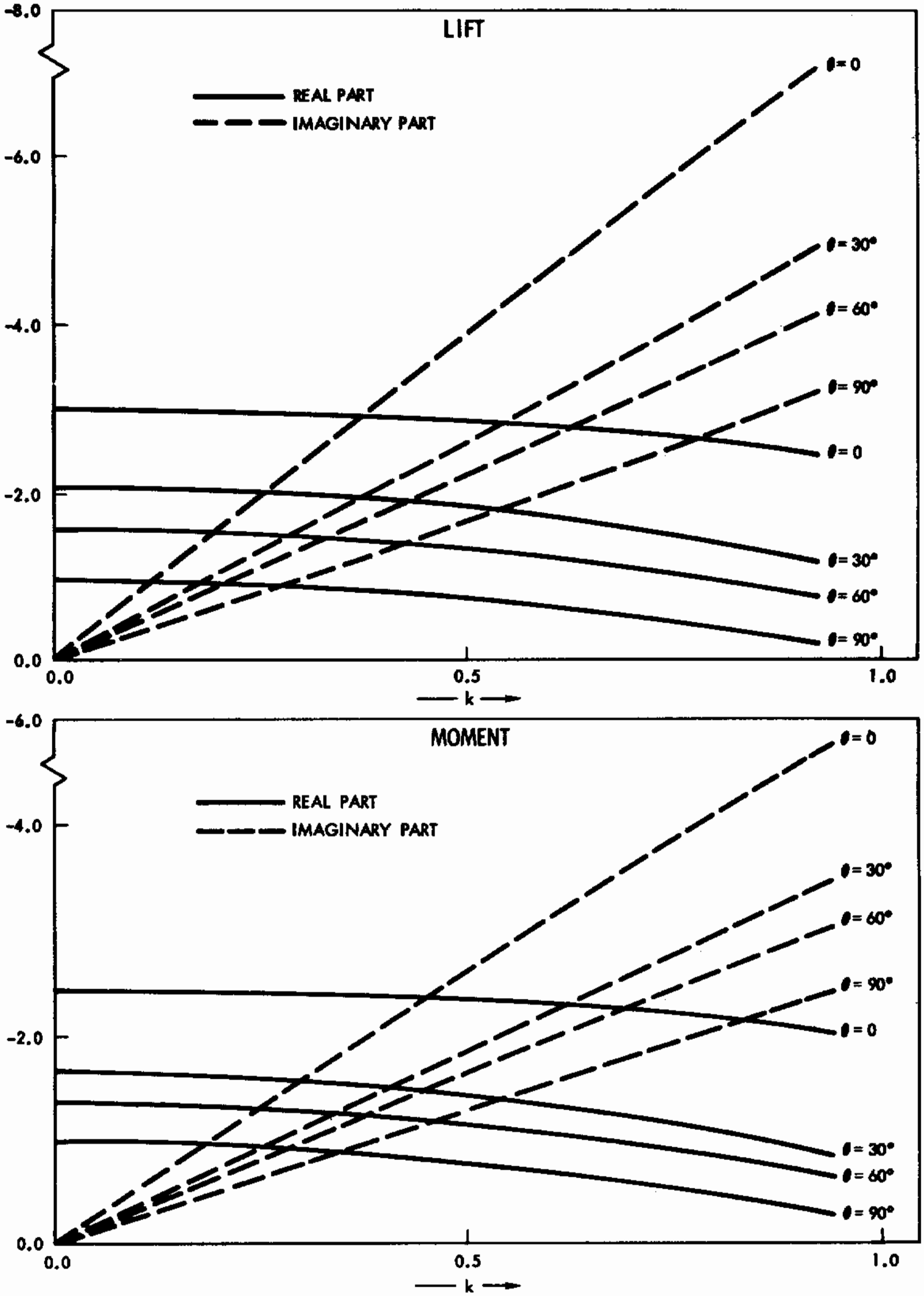


Figure 8. Lift and Moment Coefficients Vs Mach Number for 65° Triangular Wing at Various Fold Angles



Coefficients of Lift and Moment Due to Pitch Vs Reduced Frequency for a 65° Triangular Wing at M_∞ = 0.8 and Various Fold Angles

VIII. CONCLUSIONS AND RECOMMENDATIONS

The results obtained by the nonplanar kernal function method show the expected trend with increasing fold angle, which agrees with the observed experimental trend. For zero fold angle, the method reduces to that already used by Hsu (Reference 4).

Possible extensions of the method include the treatment of more general configurations, or of other specific configurations, such as the T-tail. Also, any generalizations proposed for the planar case should be considered here, such as the problem of a nonplanar wing with a control surface.

The formula that was used for the kernel function (page 19) should be useful in any future developments using the three-dimensional kernel function. The previously available formula was much longer. A special case of that formula is given in Reference 12. The use of the simplified kernel function, together with Hsu's method of integration, results in greatly reduced computer running times. This makes it practical to use the kernel function method as a tool in the preliminary analysis of new wing configurations.

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APPENDIX. COMPUTER PROGRAM LISTINGS

MP

```
      COMMON W(50,25),WI(50,25),A(2,25,25),B(2,25,10),XSV(10,5)
      COMMON X(10),Y(10),PSI(10),ETA(10),ZXY(21,10),ZZZ(21-10)
      COMMON /COM1/ N,M,NR,NC,NDATA,NMOD,NSYM,NDEX,NCC,NCS1,NCST
      COMMON/COM1/FM,FR,BO,SF,ST
      COMMON /CDAT/DAT(500)
      DO 1 I=1,100
    1 DAT(I)=0.0
    2 CALL DATRD(DAT)
      CALL DATA1
      DO 200 ND=1,NDATA
      FR=DAT(ND+30)
      FM=DAT(ND+20)
      CALL DATA2
      CALL ZEN
      DO 35 I=1,NC
      DO 35 J=1,NC
      A(1,I,J)=0.0
      A(2,I,J)=0.0
      DO 35 K=1,NR
      A(1,I,J)=A(1,I,J)+W(K,I)*W(K,J)+WI(K,I)*WI(K,J)
      A(2,I,J)=A(2,I,J)+W(K,I)*WI(K,J)-WI(K,I)*W(K,J)
   35 CONTINUE
      K=MSIMEC(25,NC,NMOD,A,B)
      IF (K.NE.1) GO TO 99
      WRITE (6,650)
  650 FORMAT(1H18X,78HPRESSURE = STIP/B(S) * SUM OF A(N,M)*F(N,PSI)*SQRT
     1(1.0-(S/STIP)**2)*(S/STIP)**)
      IF (NSYM.LT.0) GO TO 10
      WRITE (6,651)
  651 FORMAT(1H+86X,5H(M-1))
      GO TO 15
   10 WRITE (6,652)
  652 FORMAT(1H+86X,1HM)
   15 LINES=1
      LINLIM=35-NC/2
      DO 40 I1=1,NMOD
      WRITE (6,700) I1
```

MP

```
700 FORMAT(1H-20X,41HPRESSURE COEFFICIENTS A(N,M) FOR MODE NO.I2/1H0
   1 2(9X,5HN   M6X,9HREAL PART7X,10HIMAG PART )/1H )
    J1=1
    JCARR=1
    DO 19 NPR=1,N
    DO 19 MPR=1,M
    IF (JCARR.EQ.0) GO TO 17
    JCARR=0
    WRITE (6,705) NPR,MPR,B(1,J1,I1),B(2,J1,I1)
    GO TO 19
 17 JCARR=1
    WRITE (6,706) NPR,MPR,B(1,J1,I1),B(2,J1,I1)
705 FORMAT(I11,I4,1P2E16.5)
706 FORMAT(1H+52X,2I4,1P2E16.5)
 19 J1=J1+1
    LINES=LINES+6+NC/2
    IF (LINES.LE.LINLIM) GO TO 40
    WRITE (6,708)
708 FORMAT(1H1)
    LINES=0
 40 CONTINUE
    IF (LINES.EQ.0) GO TO 24
    LINES=LINES+7+2*NMOD**2
    IF (LINES.LE.40) GO TO 23
    WRITE (6,708)
    GO TO 24
 23 WRITE (6,709)
709 FORMAT(1H0)
 24 WRITE (6,45) FR,FM,DAT(10)
 45 FORMAT(1H010X,26HGENERALIZED FORCES - - K =F7.4,6H,  M =F7.4,
   1 15H,  FOLD ANGLE =F8.4,5H DEG.)
    WRITE (6,46)
 46 FORMAT(1H05X,5HMODES/4X,11HOSC.  DEFL.8X,9HREAL PART10X,9HIMAG PAR
   1T10X,10HABS. VALUE6X,11HPHASE ANGLE)
    CALL FORCE
200 CONTINUE
    GO TO 2
```

```
    MP

99 WRITE (6,98) ND
98 FØRMAT(57H1 PRESSURE CØEFFICIENTS CANNØT BE FØUND FØR DATA CASE NØ
  1.I2)
   GØ TØ 200
   END
```

DATA1

```
      SUBROUTINE DATA1                                                  00000820
      COMMON W(50,25),WI(50,25),A(2,25,25),B(2,25,10),XSV(10,5)         00000830
      COMMON X(10),Y(10),PSI(10),ETA(10),ZXY(21,10),ZZZ(21,10)          00000840
      COMMON /COM1/ N,M,NR,NC,NDATA,NMOD,NSYM,NDEX,NCC,NCS1,NCST        00000850
      COMMON/COM1/FM,FR,BO,SFOLD,STIP                                   00000860
      COMMON /COM2/ Q(5),DELP(5,10),NIC,NIS,ZIS,ZIP,ZCP,SFOST           00000870
      COMMON/COM2/BF,BMF,      COS1,SIN1,CT32P,CT32M,CT54P,CT54M        00000880
      COMMON /CDAT/DAT(500)                                             00000890
      COMMON /CONST/ PI,PIOVR2,RAD,ZI(10)                               00000900
      N=DAT(1)                                                          00000910
      M=DAT(2)                                                          00000920
      NCC=DAT(3)                                                        00000930
      NCS=DAT(4)                                                        00000940
      NDATA=DAT(5)                                                      00000950
      NSYM=DAT(6)                                                       00000960
C     NCS = NO. OF COLLOCATION PTS. SPANWISE                            00000970
C     NCC = NO. OF COLLOCATION PTS. CHORDWISE                           00000980
C     NIS = NO. OF INTEGRATION PTS. SPANWISE                            00000990
C     NIC = NO. OF INTEGRATION PTS. CHORDWISE                           00001000
      NDEX = (1-NSYM)/2                                                 00001010
      NIC = NCC                                                         00001020
      NIS = NCS + 1                                                     00001030
      ZIS = NIS                                                         00001040
      BO=DAT(9)                                                         00001050
      SFOLD=DAT(7)/BO                                                   00001060
      STIP=DAT(8)/BO                                                    00001070
      DO 15 I=1,10                                                      00001080
      X(I)=DOTS                                                         00001090
      Y(I)=DOTS                                                         00001100
      PSI(I)=DOTS                                                       00001110
   15 ETA(I)=DOTS                                                       00001120
      SBO2=STIP**2                                                      00001130
      WRITE (6,20) NCS,NCC,M,N,NDATA,NDATA                              00001140
 20   FORMAT(1H1,54X,10HINPUT DATA/21X,16HCOLLOCATION PTS., 5X,14HPRESSU00001150
     1RE MODES, 5X, 9HMACH NOS., 5X, 6HFREQS./21X,I2,5H SPAN,I3,6H CHORD00001160
     2,4X,I2,5H SPAN,I3,6H CHORD,9X,I2,11X,I2)                          00001170
 13   SFOST = SFOLD/STIP                                                00001180
```

DATA1

```
      ALFA1=DAT(10)/RAD
      ALFA2=(90.0-DAT(11))/RAD
      ALFA3=(90.0-DAT(12))/RAD
      ALFA4=(90.0-DAT(13))/RAD
      ALFA5=(90.0-DAT(14))/RAD
C     EVALUATE TIP INTEGRALS
      F= SQRT(1.0-SFØST**2)
      THE1=ATAN(SFØST/F)
      Q(1)=1.0-(THE1+F*SFØST)/PIØVR2
      F=F**3/PI
      Q(2)=F/3.0
      DØ 1607 IL=3,5
      F=F*SFØST
      Q(IL)=(F+ZI(IL-2)*Q(IL-2))/ZI(IL+1)
 1607 CØNTINUE
C     GENERATE CØLLØCATIØN AND INTEGRATIØN STATIØNS
      ZCP=2.0*PI/FLØAT(2*NCC+1)
      ZIR=SB02*ZCP/8.0
      F=0.5*ZCP
      DØ 27 J=1,NCC
      PSI(J)=-CØS(F)
      W=PSI(J)
      DØ 26 I=1,N
   26 DELP(I,J)=FXI(W,I)*ZIR
      I=NCC+1-J
      X(I)=-W
   27 F=F+ZCP
      ZS=PI/ZIS
      F=-ZS+PIØVR2
      DF=0.5*ZS
      DØ 29 J=1,NCS
      Y(J)=SIN(F)
      W=F+DF
      ETA(J)=SIN(W)
   29 F=F-ZS
      ETA(NIS)=-ETA(1)
      WRITE          ( 6,35)
```

```
      DATA1

 35   FØRMAT(1H0,27X,47HCALCULATED CØLLØCATIØN AND INTEGRATIØN STATIØNS/00001550
     11H ,24X,1HX,15X,3HPSI,15X,1HY,15X,3HETA)                           00001560
      NLIN=MAX0(NIS,NCC)                                                  00001570
      WRITE (6,36) (X(I),PSI(I),Y(I),ETA(I),I=1,NLIN)                     00001580
   36 FØRMAT(13X,4F17.4)                                                  00001590
      WRITE (6,560)                                                       00001610
      WRITE (6,565) (DAT(I+20),DAT(I+30),I=1,NDATA)                       00001620
C     EVALUATE QUANTITIES FØR LATER USE                                   00001625
      CØS1=CØS(ALFA1)                                                     00001630
      SIN1=SIN(ALFA1)                                                     00001640
      NCS1=(NCS+1)/2                                                      00001650
      IF (NSYM.LT.0) NCS1=NCS/2                                           00001660
      CTN2=0.5*CØS(ALFA2)/SIN(ALFA2)                                      00001670
      CTN3=0.5*CØS(ALFA3)/SIN(ALFA3)                                      00001680
      CTN4=0.5*CØS(ALFA4)/SIN(ALFA4)                                      00001690
      CTN5=0.5*CØS(ALFA5)/SIN(ALFA5)                                      00001700
      CT32P=CTN3+CTN2                                                     00001710
      CT32M=CTN3-CTN2                                                     00001720
      CT54P=CTN5+CTN4                                                     00001730
      CT54M=CTN5-CTN4                                                     00001740
      BF =1.0+SFØLD*CT32M                                                 00001750
      BMF=1.0+SFØLD*CT32P                                                 00001760
      NR=NCC*NCS1                                                         00001770
      NC=M*N                                                              00001780
      DØ 82 J=1,10                                                        00001790
      IF (Y(J).LE.SFØST) GØ TØ 84                                         00001800
   82 CØNTINUE                                                            00001810
   84 NCST=J-1                                                            00001820
      ZIP=16.0/SB02                                                       00001830
      RETURN                                                              00001840
  560 FØRMAT(1H0,35X, 9HMACH NØS. ,15X,6HFREQS.)                          00001850
  565 FØRMAT(1H ,36X,F8.4,14X,F8.4)                                       00001860
      END                                                                 00001870
```

DAT2

```
      SUBROUTINE DATA2
      COMMON WASH(50,25),WASHI(50,25),A(2,25,25),B(2,25,10),XSV(10,5)
      COMMON X(10),Y(10),PSI(10),ETA(10),ZXY(21,10),ZZZ(21,10)
      COMMON /COM1/ N,M,NR,NC,NDATA,NMOD,NSYM,NDEX,NCC,NCS1,NCST
      COMMON/COM1/FM,FR,BO,SFOLD,STIP
      COMMON /COM2/ Q(5),DELP(5,10),NIC,NIS,ZIS,ZIP,ZCP,SFOST
      COMMON/COM2/BF,BMF,     COS1,SIN1,CT32P,CT32M,CT54P,CT54M
      COMMON /CDAT/DAT(500)
      COMMON /CONST/ PI,PIOVR2,RAD,ZI(10)
      DIMENSION RA(10),RB(10),RC(5),FS(5),FT(5),FU(5),FV(5),FW(5),FX(5)
      DIMENSION XU(2),XX(2),XY(2),XZ(2)
C     EVALUATION OF MATRIX OF INTEGRALS
      DO 38 J=1,1250
      WASH(J,1) = 0.0
 38   WASHI(J,1) = 0.0
      BETA2=1.0-FM**2
      IU =  0
      DO 1000 IQ=1,NCS1
      CALL GEOM(Y(IQ),XI1,XI2,CO2,SI2,YY,ZY)
      DO 1000 IP = 1,NCC
      XA=X(IP)*XI2+XI1
      XSV(IP,IQ) = XA
      IU = IU+1
      XARG=ZI(IP)*ZCP
      F=SQRT(1.0-X(IP)**2)
      RC(1)=2.0*(F+XARG)
      G=-2.0*X(IP)*F
      RC(2)=XARG-0.5*G
      DO 1604 IC=3,N
      H=-F-2.0*X(IP)*G
      RC(IC)=F/ZI(IC-2)-H/ZI(IC)
      F=G
 1604 G=H
 58   DO 1640 IN = 1,NIS
      YE=ETA(IN)
      CALL GEOM(YE,XI3,XI4,CO4,SI4,YETA,ZETA)
      SY=1.0
```

DAT2

```
      IF (YE.GE.0.0) GØ TØ 54
      SY=SIGN(1.0,NSYM)
      YE=SIGN(YE,NSYM)
   54 CØNTINUE
      YO=YY-YETA
      ZOO=ZY-ZETA
      DØ 1580 IJ=1,N
      FS(IJ) = 0.0
      FT(IJ) = 0.0
      FU(IJ) = 0.0
 1580 FV(IJ)=0.0
      DØ 1600 IL = 1,NIC
      XB=PSI(IL)*XI4+XI3
      XO = XA - XB
   51 CALL KRNLI(FR,XO,YO,ZOO,FM,BETA2,CØ2,SI2,CØ4,SI4,XX)
      IF (SI4.EQ.0.0) GØ TØ 93
C     ETA IS ØUTBØARD ØF FØLDLINE,CALCULATE KPRIME.
 96   IF(ETA(IN))101,101,102
  101 SA=YY+SFØLD
      GØ TØ 103
  102 SA=YY-SFØLD
  103 X1=XA-PSI(IL)*BF-BMF
      CALL KRNLI(FR,X1,SA,ZY,FM,BETA2,CØ2,SI2,CØ4,SI4,XY)
      CALL KRNLI(FR,X1,SA,ZY,FM,BETA2,CØ2,SI2,1.0,0.0,XZ)
C     KPRIME = XX - XY + XZ
      XU(1) = XY(1) - XZ(1)
      XU(2) = XY(2) - XZ(2)
      XX(1) = XX(1) - XU(1)
      XX(2) = XX(2) - XU(2)
      GØ TØ 94
   93 XU(1) = 0.0
      XU(2) = 0.0
   94 DØ 1590 IJ=1,N
      FS(IJ)=FS(IJ)+XX(1)*DELP(IJ,IL)
      FT(IJ)=FT(IJ)+XX(2)*DELP(IJ,IL)
      FU(IJ)=FU(IJ)+XU(1)*DELP(IJ,IL)
 1590 FV(IJ)=FV(IJ)+XU(2)*DELP(IJ,IL)
```

DAT2

```
 1600 CONTINUE
C      CHORDWISE INTEGRATION IS COMPLETE.
       IA = 0
       W=(1.0-YE**2)/ZIS
       IF (NDEX.NE.0) W=W*YE
       DO 1625 IL=1,M
       ILN=IL+NDEX
       DO 1620 IM = 1,N
       IA = 1 + IA
       WASH(IU,IA)=WASH(IU,IA)+FS(IM)*W
       WASHI(IU,IA)=WASHI(IU,IA)+FT(IM)*W
       IF (IN.NE.1.AND .IN.NE.NIS) GO TO 1620
       WASH(IU,IA)=WASH(IU,IA)+FU(IM)*Q(ILN)*SY
       WASHI(IU,IA)=WASHI(IU,IA)+FV(IM)*Q(ILN)*SY
 1620 CONTINUE
 1625 W=YE*W
 1640 CONTINUE
       DO 1601 LO=1,IP
       ARG=FR*(XA-PSI(LO)*XI2-XI1)
       RA(LO)=ZIP*(COS(ARG)-1.0)
       RB(LO)=-ZIP*SIN(ARG)
 1601 CONTINUE
C      RA = REAL, RB = IMAG
       DO 1646 IJ=1,N
       FW(IJ) = 0.0
       FX(IJ) = 0.0
       DO 1646 IK=1,IP
       FW(IJ) = FW(IJ) + DELP(IJ,IK)*RA(IK)
       FX(IJ) = FX(IJ) + DELP(IJ,IK)*RB(IK)
 1646 CONTINUE
       IA = 0
       YP=ZIS/8.0
       IF (NDEX.NE.0) YP=YP*Y(IQ)
       DO 1649 IL = 1,M
       DO 1648 IM = 1,N
       IA = 1+IA
       WASH(IU,IA) = WASH(IU,IA)-(FW(IM)+RC(IM))*YP
```

```
      DAT2

1648 WASHI(IU,IA) = WASHI(IU,IA)-FX(IM)*YP                        00002990
1649 YP=YP*Y(IQ)                                                  00003000
1000 CONTINUE                                                     00003010
1010 WRITE (6,530) FR,FM                                          00003020
     DO 1015 K = 1,IU                                             00003030
     WRITE (6,555)                                                00003040
     WRITE          ( 6,535)(WASH(K,L),WASHI(K,L),L=1,IA)         00003050
1015 CONTINUE                                                     00003060
     RETURN                                                       00003080
 530 FORMAT(1H121X,30HMATRIX FOR REDUCED FREQUENCY =F7.4,         00003090
    1  16H,   MACH NUMBER =F7.4)                                  00003100
535  FORMAT(3(5X,1P2E15.4,1HI))                                   00003110
555  FORMAT(1H )                                                  00003120
     END                                                          00003130
```

GEOM

```
      SUBROUTINE GEOM(Y,X1,X2,CO,SI,YY,ZY)
      COMMON /COM1/ N,M,NR,NC,NDATA,NMOD,NSYM,NDEX,NCC,NCS1,NCST
      COMMON/COM1/FM,FR,BO,SFOLD,STIP
      COMMON /COM2/ Q(5),DELP(5,10),NIC,NIS,ZIS,ZIP,ZCP,SFOST
      COMMON/COM2/BF,BMF,     COS1,SIN1,CT32P,CT32M,CT54P,CT54M
C     EVALUATION OF GEOMETRICAL QUANTITIES AT A GIVEN CHORD
      Y1=STIP*ABS(Y)
      F=Y1-SFOLD
      IF (F) 2,2,4
    2 CO=1.0
      SI=0.0
      YY=Y1
      ZY=0.0
      X1=1.0+Y1*CT32P
      X2=1.0+Y1*CT32M
      GO TO 6
    4 CO=COS1
      SI=SIN1
      YY=SFOLD+F*COS1
      ZY=F*SIN1
      X1=BMF+F*CT54P
      X2=BF+F*CT54M
    6 IF (Y.GE.0.0) RETURN
      SI=-SI
      YY=-YY
      RETURN
      END
```

KRNL

```
      SUBROUTINE KRNLI(CK,X,Y,ZO,CM,B2,C01,SI1,C02,SI2,AKERN)
      DIMENSION Z(6),H(6),AKERN(2)
      DATA H/0.08566225,0.18038079,2*0.23395697,0.18038079,0.08566225/
      DATA Z/0.96623476,0.83060469,0.61930959,0.38069041,0.16939531,
     10.03376524/
      R2 = Y*Y+ZO*ZO
      R =  SQRT(R2)
      CK1 = CK*R
      G1 = 0.0
      G2 = 0.0
      G3 = 0.0
      G4 = 0.0
      G5 = 0.0
      G6 = 0.0
    5 S2 = X*X + B2*R2
      S =  SQRT(S2)
      U1 = (CM*S-X)/(B2*R)
      UK = CK1*U1
      DO 20 I = 1,6
      UZ = U1*Z(I)
      UZ2 = UZ**2
      F = UK*Z(I)
      CO =  COS(F)
      SI =  SIN(F)
      F = H(I)/ SQRT(1.0+UZ2)*UZ*U1
      G3 = G3 + F*CO
      G4 = G4 - F*SI
      F = UZ*F
      G5 = G5 + F*CO
      G6 = G6 - F*SI
      V = 1.0 - Z(I)**2
      F = H(I)*2.0*V* EXP(-CK1*V)/ SQRT(1.0+V)
      G1 = G1 + F
   20 G2 = G2 + V*F
      G7 = G1 + G3
      XS = X/S
      CO =  COS(UK)*XS
```

KRNL

```
      SI =  SIN(UK)*XS
      IF(CK.NE.0.0) GØ TØ 22
      F14 = 1.0
      F15 = 2.0
      GØ TØ 23
22    F14 = CK1*BESEL(CK1,1,3)
      F15 = CK1*CK1*BESEL(CK1,2,3)
23    F11 = -(CK1*G4+F14+CØ)
      F12 = CK1*G7+SI
      F = 2.0+B2*R2/S2
      FP = UK+CK*CM*R2/S
      F21 = CK1*G4+CK1*CK1*G5+F15+CØ*F +SI*FP
      F22 = CK1*(-G7+CK1*(G6-G2))+CØ*FP-SI*F
      XK = CK*X
      CØ =  CØS(XK)
      SI =  SIN(XK)
      F=(CØ1*CØ2+SI1*SI2)/R2
      FP=(ZO*CØ1-Y*SI1)*(ZO*CØ2-Y*SI2)/R2**2
      G1 = F*F11+FP*F21
      G2 = F*F12+FP*F22
      AKERN(1) = -CØ*G1-SI*G2
      AKERN(2) = SI*G1-CØ*G2
      RETURN
      END
```

ZEN

```
      SUBROUTINE ZEN
      COMMON W(50,25),WI(50,25),A(2,25,25),B(2,25,10),XSV(10,5)
      COMMON X(10),Y(10),PSI(10),ETA(10),ZXY(21,10),ZZZ(21,10)
      COMMON /COM1/ N,M,NR,NC,NDATA,NMOD,NSYM,NDEX,NCC,NCS1,NCST
      COMMON/COM1/FM,FR,BO,SF,ST
      COMMON /CDAT/DAT(500)
      DIMENSION XH(66),SH(66),ZH(66),XCH(50),YCH(50),Z(50),DZ(50)
      DIMENSION WD(2,100)
      EQUIVALENCE (XH,A),(SH,A(67)),(ZH,A(133)),(XCH,A(200)),(Z,A(250))
      EQUIVALENCE (YCH,A(300)),(DZ,A(350)),(WD,A(400))
      LINES=50
      NMOD=DAT(41)
      IDEF=0
      DO 50 I=1,NMOD
      IF (DAT(98).NE.0.0) GO TO 20
      IF (IDEF.NE.0) GO TO 20
      CALL DATRD(DAT)
      JD=DAT(42)
      IF (JD-1) 2,4,16
    2 IDEF=1
      GO TO 20
C     READ DEFLECTION VALUES AT POINTS
    4 NPTS=DAT(43)
      L=101
      DO 8 J=1,NPTS
      XH(J)=DAT(L)
      SH(J)=DAT(L+1)
      ZH(J)=DAT(L+2)
    8 L=L+3
      NDH=5
    9 CALL LSSURP(XH,SH,ZH,NPTS,NDH,G,ZXY(1,I),IND)
      IF(IND.EQ.0) GO TO 10
      NDH=NDH-1
      GO TO 9
   10 NPTS=DAT(44)
      L=301
      DO 11 J=1,NPTS
```

ZEN

```
      XH(J)=DAT(L)
      SH(J)=DAT(L+1)
      ZH(J)=DAT(L+2)
   11 L=L+3
      NDV=5
   12 CALL LSSURP(XH,SH,ZH,NPTS,NDV,G,ZZZ(1,I),IND)
      IF (IND.EQ.0) GØ TØ 20
      NDV=NDV-1
      GØ TØ 12
C     READ PØLYNØMIAL CØEFFICIENTS
   16 DØ 17 J=1,21
      ZXY(J,I)=DAT(J+50)
   17 ZZZ(J,I)=DAT(J+75)
C     FIND UPWASHES
   20 K=0
      DØ 24 I2=1,NCST
      DØ 24 J=1,NCC
      K=K+1
      XCH(K)=BO*XSV(J,I2)
   24 YCH(K)=DAT(8)*Y(I2)
      LIND=8+K
      IF (LINES+LIND.LE.40) GØ TØ 25
      WRITE (6,41)
   41 FØRMAT(1H1)
      LINES=0
   25 LINES=LINES+LIND
      WRITE (6,43)
      WRITE(6,42) I,(ZZZ(J,I),J=1,21)
   42 FØRMAT(1H+5X,26HDEFLECTIØN CØEFFICIENTS ØN7X,8HMØDE NØ.I2/1H /
     1 (1P7E14.4))
   43 FØRMAT(1H032X,4HTIP,)
      CALL ZDZ(XCH,YCH,K,ZZZ(1,I),Z,DZ)
      DØ 26 I2=1,K
      WD(1,I2)=DZ(I2)*BO
   26 WD(2,I2)=FR*Z(I2)
      I3=NCST+1
      L=0
```

ZEN

```
      DØ 27 I2=I3,NCS1
      DØ 27 J=1,NCC
      L=L+1
      XCH(L)=BO*XSV(J,I2)
   27 YCH(L)=DAT(8)*Y(I2)
      LIND=8+L
      IF (LINES+LIND.LE.40) GØ TØ 28
      WRITE (6,41)
      LINES=0
   28 LINES=LINES+LIND
      WRITE (6,44)
   44 FØRMAT(1H032X,5HWING,)
      WRITE (6,42) I,(ZXY(J,I),J=1,21)
      CALL ZDZ(XCH,YCH,L,ZXY(1,I),Z,DZ)
      J2=K+1
      DØ 29 I2=1,L
      WD(1,J2)=DZ(I2)*BO
      WD(2,J2)=FR*Z(I2)
   29 J2=J2+1
      DØ 33 J=1,NC
      B(1,J,I)=0.0
      B(2,J,I)=0.0
      DØ 33 K=1,NR
      B(1,J,I)=B(1,J,I)+W(K,J)*WD(1,K)+WI(K,J)*WD(2,K)
   33 B(2,J,I)=B(2,J,I)+W(K,J)*WD(2,K)-WI(K,J)*WD(1,K)
   50 CØNTINUE
      RETURN
      END
```

FORC

```
      SUBROUTINE FORCE
      COMMON W(50,25),WI(50,25),A(2,25,25),AP(2,25,10),XSV(10,5)
      COMMON X(10),Y(10),PSI(10),ETA(10),ZXY(21,10),ZZZ(21,10)
      COMMON /COM1/ N,M,NR,NC,NDATA,NMOD,NSYM,NDEX,NCC,NCS1,NCST
      COMMON/COM1/FM,FR,BO,SF,ST
      COMMON /COM2/ Q(5),DELP(5,10),NIC,NIS,ZIS,ZIP,ZCP,SFOST
      COMMON/COM2/BF,BMF,     COS1,SIN1,CT32P,CT32M,CT54P,CT54M
      COMMON /CONST/ PI,PIOVR2,RAD,ZI(10)
      DIMENSION AW(5,6,10),AT(5,6,10),AS(10)
      EQUIVALENCE (A,AW),(A(301),AT)
      DIMENSION FRC(2)
      DIMENSION U(6),V(6),H(6)
      DATA H/0.085662246,0.18038079,2*0.23395697,0.18038079,0.085662246/
      DATA V/0.96623476,0.83060459,0.61930959,0.38069041,0.16939531,
     10.03376524/
C     WEIGHTS AND POINTS FOR 6 POINT GAUSSIAN QUADRATURE ON (0,1)
      DATA U/-0.965926,-0.707107,-0.258819,0.258819,0.707107,0.965926/
      DATA PI6/0.52359877/
C     POINTS FOR 6 POINT GAUSS QUADRATURE ON (-1,1) WITH WEIGHT FUNCTION
C      1/SQRTF(1-U**2).  WEIGHTS ARE PI/6
      IDEX=NDEX-1
      DO 2 I=1,300
      AT(I,1,1)=0.0
    2 AW(I,1,1)=0.0
    4 DS=ST-SF
      BT=BF+DS*CT54M
      DO 12 L=1,2
    5 DO 12 I=1,6
      GO TO (6,7),L
    6 SG=SF*V(I)
      XM=1.0+SG*CT32P
      B =1.0+SG*CT32M
      RS=SF
      GO TO 8
    7 SG=ST-DS*V(I)**2
      RS=2.0*DS*V(I)
      SG1=SG-SF
```

```
      FØRC
      XM=BMF+SG1*CT54P
      B = BF+SG1*CT54M
    8 RS = RS*PI6*H(I)*SQRT(ST**2-SG**2)
      DØ 9 K=1,10
      AS(K)=RS
    9 RS=SG*RS
      DØ 12 J=1,6
      Z=XM+B*U(J)
C       Z  IS  X
      DØ 12 L1=1,N
      FAC=FXI  (U(J),L1)
      GØ TØ (23,24),L
   23 DØ 11 L2=1,6
      DØ 10 K=1,10
   10 AW(L1,L2,K)=AW(L1,L2,K)+FAC*AS(K)
   11 FAC=Z*FAC
      GØ TØ 12
   24 DØ 21 L2=1,6
      DØ 20 K=1,10
   20 AT(L1,L2,K)=AT(L1,L2,K)+FAC*AS(K)
   21 FAC=Z*FAC
   12 CØNTINUE
      AREA=SF*(1.0+BF)+DS*(BF+BT)
      DØ 30 N1=1,NMØD
      DØ 30 N2=1,NMØD
      FRC(1)=0.0
      FRC(2)=0.0
      I2 = 1
      G0=1.0
      DØ 17 J=1,6
      G=G0
      G0=G0*B0
      JL=7-J
      DØ 17 I = 1,JL
      L3 = 1
      F1=ZXY(I2,N2)*G
      F2=ZZZ(I2,N2)*G
```

```
 FØRC

   G=G*BO
   IF (F1.EQ.0.0 .AND. F2.EQ.0.0) GØ TØ 17
   Z=1.0
   DØ 19 K=1,M
   L2=K+J+IDEX
   DØ 18 L1=1,N
   F=F1*AW(L1,I,L2)+F2*AT(L1,I,L2)
   F=F*Z
   DØ 16 M1=1,2
16 FRC(M1)=FRC(M1)+F*AP(M1,L3,N1)
18 L3=L3+1
19 Z=Z/ST
17 I2 = 1 + I2
   F1=FRC(1)/AREA
   F2=FRC(2)/AREA
   F3=SQRT(F1**2+F2**2)
   F4=DØTS
   IF (F3.NE.0.0) F4=ATAN(F2,F1)*RAD
30 WRITE (6,47) N1,N2,F1,F2,F3,F4
47 FØRMAT(1H02I6,1P3E19.5,0P1F16.4)
   RETURN
   END
```

LSSURP

```
      SUBROUTINE LSSURP(X,Y,Z,N,ND,NG,A,IND)
      DIMENSION X(1),Y(1),Z(1),A(21),C(21,21),B(21)
      DIMENSION G(21),F(21)
      DIMENSION JLIM(6)
C     A POLYNOMIAL WITH THE COEFFICIENTS (A) IS FITTED BY LEAST SQUARES
C      TO THE VALUES (Z) AT THE POINTS (X,Y)
C     DEGREE ND IS GIVEN, MUST BE AT MOST 5
C     NUMBER OF POINTS N IS UNRESTRICTED
C     IND=0 FOR SUCCESSFUL FIT
      IND=0
      ILIM=ND+1
      IF(ILIM.GT.6)ILIM=6
      IJLIM=ILIM+1
      DO 2 I=1,ILIM
    2 JLIM(I)=IJLIM-I
    3 NT=0
      DO 4 I=1,ILIM
    4 NT=NT+JLIM(I)
      DO 5 I=1,NT
      B(I)=0.0
      DO 5 J=1,NT
    5 C(J,I)=0.0
      DO 8 K=1,N
      YP=1.0
      L=1
      DO 7 I=1,ILIM
      XYP=YP
      JL=JLIM(I)
      DO 6 J=1,JL
      G(L)=XYP
      XYP=X(K)*XYP
    6 L=L+1
    7 YP=Y(K)*YP
      DO 8 I=1,NT
      B(I)=B(I)+G(I)*Z(K)
      DO 8 J=1,NT
    8 C(J,I)=C(J,I)+G(J)*G(I)
```

LSSURP

```
      DØ 10 I=1,NT
      F(I)=0.0
      DØ 9 J=1,NT
    9 F(I)=AMAX1(F(I),ABS(C(J,I)))
      DØ 10 J=1,NT
   10 C(J,I)=C(J,I)/F(I)
      K=MSIMER(21,NT,1,C,B)
      IF (K.EQ.1) GØ TØ 15
   11 DØ 12 I=1,ILIM
      IP=ILIM+1-I
      IF (JLIM(IP)+IP.EQ.IJLIM) GØ TØ 13
   12 CØNTINUE
      IJLIM=IJLIM-1
      IF (IJLIM.GT.1) GØ TØ 11
      IND =1
      RETURN
   13 JLIM(IP)=JLIM(IP)-1
      IF (JLIM(IP).EQ.0) ILIM=IP-1
      GØ TØ 3
   15 NG=NT
      K=1
      L=1
      DØ 18 I=1,ILIM
      JL=JLIM(I)
      JM=JL+1
      JN=7-I
      DØ 16 J=1,JL
      A(L)=B(K)/F(K)
      K=K+1
   16 L=L+1
      IF (JN.LE.JL) GØ TØ 18
      DØ 17 J=JM,JN
      A(L)=0.0
   17 L=L+1
   18 CØNTINUE
      IF (L.GT.21) RETURN
      DØ 19 K=L,21
```

```
19 A(K)=0.0
   RETURN
   END
```

ZDZ

```
      SUBROUTINE ZDZ(X,Y,N,A,Z,DZ)
      DIMENSION C(7),X(300),Y(300),Z(300),DZ(300)
      DIMENSION A(21)
      DATA C/0.0,1.0,2.0,3.0,4.0,5.0,6.0/
C     FINDS VALUES OF Z,DZ/DX AT THE COLLOCATION POINTS
      DO 50 IN = 1,N
      Z(IN) = 0.0
      DZ(IN) = 0.0
      YP = 1.0
      NG = 1
      DO 20 I=1,6
      JL=7-I
      XYP = YP
      DO 10 J = 1,JL
      Z(IN) = Z(IN) + A(NG)*XYP
      XYP = X(IN)*XYP
      IF(J.EQ.1) GO TO 5
      DZ(IN) = DZ(IN) + C(J)*A(NG)*XYQ
      XYQ = X(IN)*XYQ
      GO TO 10
    5 XYQ = YP
   10 NG = NG + 1
   20 YP = Y(IN)*YP
   50 CONTINUE
      WRITE (6,101)
  101 FORMAT(1H0,17X,1HX,20X,1HY,20X,1HZ,20X,2HDZ)
      DO 100 I = 1,N
  100 WRITE (6,105) X(I),Y(I),Z(I),DZ(I)
  105 FORMAT(1H ,10X,1PE15.4,5X,1PE15.4,6X,1PE15.4,7X,1PE15.4)
      RETURN
      END
```

```
       CONSTS

      BLOCK DATA                                                        00007110
      COMMON /CONST/ PI,PIOVR2,RAD,ZI(10)                               00007120
C     USEFUL CONSTANTS                                                  00007125
      DATA PI/3.14159265/,PIOVR2/1.57079633/,RAD/57.2957795/            00007130
      DATA ZI/1.,2.,3.,4.,5.,6.,7.,8.,9.,10./                           00007140
      END                                                               00007150
```

FXI

```
      FUNCTION FXI(U1,K)
C     EVALUATES THE KTH CHORDWISE PRESSURE MODE AT U1
      U=U1
      IF (1-K) 2,4,8
    2 FXI=1.0-U*U
      IF (K-5) 3,5,8
    3 IF (K-3) 8,5,7
    4 FXI=1.0-U
      GO TO 8
    5 FXI=-2.0*U*FXI
      IF (3-K) 6,8,8
    6 FXI=FXI*(4.0*U*U-2.0)
      GO TO 8
    7 FXI=FXI*(3.0-4.0*FXI)
    8 RETURN
      END
```

```
        SIMR                   K=MSIMER(N,L,LB,A,B)
*               SIMULTANEOUS EQUATION SUBROUTINE
*       SOLVES THE SYSTEM OF EQUATIONS  A*X=B.
*       TO USE, SET K=MSIMER(N,L,LB,A,B)
*         WHERE N IS THE NUMBER OF ROWS FOR WHICH A IS
*         DIMENSIONED, AND L IS THE NUMBER OF EQUATIONS.
*         LB IS THE NUMBER OF COLUMNS IN B.
*       K=1 DENOTES SUCCESSFUL SOLUTION
*       K=2 FOR A SINGULAR OR ILL-CONDITIONED MATRIX
*       K=3 IF IMPROPER DATA IS GIVEN.
*         TO AVOID THIS SIGNAL, L MUST BE POSITIVE AND AT MOST 100,
*         N MUST NOT BE LESS THAN L, A MUST NOT INCLUDE A ROW
*         OF ZEROS.
*       A IS DESTROYED.
*       IF K=1, THE SOLUTION IS RETURNED IN B
*
*      ENTRY
*
MSIMER SAVE    1,2,3,4,5,6,7
*
*      PROLOGUE
*
        CLA*    3,4
        PAX     0,1
        TXL     E1,1,0
        TXH     E1,1,100
        PCD     0,1
        STD     A4-1
        STD     A6-1
        STD     2A6-1
        STD     A16
        STD     A16+1
        STD     2A21
        STD     A12+1
        STD     2A26
        STD     2A26+1
        STD     2A37
        STD     2A37+1
```

```
       SIMR                 K=MSIMER(N,L,LB,A,B)
       STD     A12+2
       STD     A20
       STD     A22
       STD     A25+1
       STD     A25+2
       STD     A33
       STD     A36+1
       STD     A36+2
       SXD     A52,1
       TXI     *+1,1,1
       SCD     A32,1
       TXI     *+1,1,-2
       SCD     A22+1,1
       CLA*    5,4
       PAX     0,7
       SXA     1A6-1,7
       SXA     A14,7
       SXA     1A21-1,7
       SXA     A26,7
       SXA     A37,7
       CLA*    4,4
       PAX     0,1
       TXL     E1,1,0
A52    TXH     E1,1,**
       SXA     A2,1
       SXA     A5-1,1
       SXD     A9,1
       SXD     A12,1
       TXI     *+1,1,-1
       SXD     A7,1
       SXD     A36,1
       SXD     A18,1
       SXD     A38-1,1
       SXD     A21-1,1
       SXD     A23,1
       SXD     A25,1
       SXD     A31,1
```

```
         SIMR              K=MSIMER(N,L,LB,A,B)
         SCD      A51,1                                  00050750
         SCD      A51+1,1                                00050760
         TXI      *+1,1,1                                00050770
         CLA      6,4                                    00050780
         PAC      0,3                                    00050790
         CLA      7,4                                    00050800
         PAC      0,5                                    00050810
A51      TXI      *+1,3, -L+1                            00050820
         TXI      *+1,5, -L+1                            00050830
*                                                        00050840
*       NORMALIZATION OF ROWS                            00050850
*                                                        00050860
A2       AXT      L,2                                    00050870
         PXA      0,3                                    00050880
         PAX      0,6                                    00050890
         PAX      0,7                                    00050900
         PXA      0,0                                    00050910
A3       LDQ      A,6                                    00050920
         LRS      0                                      00050930
         TLQ      *+2                                    00050940
         XCA                                             00050950
         TNX      A4,2,1                                 00050960
         TXI      A3,6, -N                               00050970
A4       TZE      E1                                     00050980
         STO      T                                      00050990
         CLA      =1.0                                   00051000
         FDP      T                                      00051010
         STQ      T                                      00051020
         AXT      L,2                                    00051030
A5       FMP      A,7                                    00051040
         STO      A,7                                    00051050
         LDQ      T                                      00051060
         TNX      A6,2,1                                 00051070
         TXI      A5,7, -N                               00051080
A6       PXA      0,5                                    00051090
         PAX      0,6                                    00051100
         AXT      LB,4                                   00051110
```

```
        SIMR                     K=MSIMER(N,L,LB,A,B)
1A6     FMP     B,6
        STO     B,6
        LDQ     T
        TNX     2A6,4,1
        TXI     1A6,6, -N
2A6     TNX     A7,1,1
        TXI     *+1,3,1
        TXI     A2,5,1
A7      TXH     B7,2,L -1
*
*      SEARCH FOR MAXIMUM PIVOT IN COLUMN
*
        PXA     0,2
        PAX     0,1
        PXA     0,3
        PAX     0,6
        PXA     0,0
A8      LDQ     A,6
        LRS     0
        TLQ     *+3
        XCA
        SXA     A10,1
        TXI     *+1,1,1
A9      TXH     *+2,1,L
        TXI     A8,6,-1
        LDQ     TOL
        TLQ     *+2
        TRA     E3
A10     AXT     **,1
        SXD     *+1,2
        TNX     A17,1,**
*
*      ROW INTERCHANGE
*
        PXA     0,3
        PAX     0,6
        PAX     0,7
```

```
        SIMR                    K=MSIMER(N,L,LB,A,B)

        SCD     *+1,1                                   00051490
        TXI     *+1,7,**                                00051500
        PXA     0,2                                     00051510
        PAX     0,4                                     00051520
A11     CLA     A,6                                     00051530
        LDQ     A,7                                     00051540
        STO     A,7                                     00051550
        STQ     A,6                                     00051560
        TXI     *+1,4,1                                 00051570
A12     TXH     A13,4,L                                 00051580
        TXI     *+1,6, -N                               00051590
        TXI     A11,7, -N                               00051600
A13     PXA     0,5                                     00051610
        PAX     0,6                                     00051620
        PAX     0,7                                     00051630
A14     AXT     LB,4                                    00051640
        SCD     *+1,1                                   00051650
        TXI     *+1,6,**                                00051660
A15     CLA     B,7                                     00051670
        LDQ     B,6                                     00051680
        STO     B,6                                     00051690
        STQ     B,7                                     00051700
        TNX     A17,4,1                                 00051710
A16     TXI     *+1,6, -N                               00051720
        TXI     A15,7, -N                               00051730
*                                                       00051740
*      DIVISION OF ROW BY PIVOT                         00051750
*                                                       00051760
A17     CLA     =1.0                                    00051770
        FDP     A,3                                     00051780
        STQ     AM                                      00051790
A18     TXH     A21,2,L -1                              00051800
        PXA     0,2                                     00051810
        PAX     0,4                                     00051820
        PXA     0,3                                     00051830
        PAX     0,6                                     00051840
A20     TXI     *+1,6, -N                               00051850
```

```
         SIMR                         K=MSIMER(N,L,LB,A,B)
         LDQ      A,6
         FMP      AM
         STO      A,6
         TXI      *+1,4,1
         TXL      A20,4,L -1
A21      PXA      0,5
         PAX      0,6
         AXT      LB,4
1A21     LDQ      B,6
         FMP      AM
         STO      B,6
         TNX      *+2,4,1
2A21     TXI      1A21,6, -N
*
*       ROW REDUCTION
*
         PXA      0,2
         PAX      0,1
         PAX      0,4
         TNX      A31,1,1
         PXA      0,3
         PAX      0,6
         PAX      0,7
         STA      A29
         SXA      A26+1,5
         SXA      3A26,5
         TXI      *+1,6,1
A22      TXI      *+1,7, -N
         TXI      *+1,3, -N+1
         SXA      A28,7
A23      TXH      A26,2,L -1
         SXA      A27,3
A24      LDQ      A,6
         FMP      A,7
         CHS
         FAD      A,3
         STO      A,3
```

```
        SIMR                              K=MSIMER(N,L,LB,A,B)

        TXI     *+1,4,1                                                   00052230
A25     TXH     A27,4,L -1                                                00052240
        TXI     *+1,3, -N                                                 00052250
        TXI     A24,7, -N                                                 00052260
A27     AXT     **,3                                                      00052270
A26     AXT     LB,4                                                      00052280
        AXT     **,7                                                      00052290
        TXI     *+1,7,1                                                   00052300
        SXA     *-2,7                                                     00052310
1A26    LDQ     A,6                                                       00052320
        FMP     B,5                                                       00052330
        CHS                                                               00052340
        FAD     B,7                                                       00052350
        STO     B,7                                                       00052360
        TNX     3A26,4,1                                                  00052370
2A26    TXI     *+1,5, -N                                                 00052380
        TXI     1A26,7, -N                                                00052390
3A26    AXT     **,5                                                      00052400
        TNX     A29,1,1                                                   00052410
A28     AXT     **,7                                                      00052420
        PXA     0,2                                                       00052430
        PAX     0,4                                                       00052440
        TXI     *+1,3,1                                                   00052450
        TXI     A23,6,1                                                   00052460
A29     AXT     **,3                                                      00052470
A31     TXH     A43,2,L -1                                                00052480
        PXA     0,2                                                       00052490
        PAX     0,1                                                       00052500
        PAX     0,4                                                       00052510
        PXA     0,3                                                       00052520
        PAX     0,6                                                       00052530
        PAX     0,7                                                       00052540
A32     TXI     *+1,3, -N-1                                               00052550
        TXI     *+1,6,-1                                                  00052560
A33     TXI     *+1,7, -N                                                 00052570
        SXA     A37+1,5                                                   00052580
        SXA     A41,5                                                     00052590
```

```
        SIMR            K=MSIMER(N,L,LB,A,B)

        SXA   A40,3
        SXA   A39,7
A34     SXA   A38,3
A35     LDQ   A,6
        FMP   A,7
        CHS
        FAD   A,3
        STØ   A,3
        TXI   *+1,4,1
A36     TXH   A37,4,L -1
        TXI   *+1,3, -N
        TXI   A35,7, -N
A37     AXT   LB,4
        AXT   **,7
        TXI   *+1,7,-1
        SXA   *-2,7
1A37    LDQ   A,6
        FMP   B,5
        CHS
        FAD   B,7
        STØ   B,7
        TNX   A41,4,1
2A37    TXI   *+1,5, -N
        TXI   1A37,7, -N
A41     AXT   **,5
        TXI   *+1,1,1
        TXH   A40,1,L -1
A38     AXT   **,3
A39     AXT   **,7
        PXA   0,2
        PAX   0,4
        TXI   *+1,3,-1
        TXI   A34,6,-1
A40     AXT   **,3
        TXI   *+1,5,-1
        TXI   A7,2,1
A43     CLA   =1
```

```
        SIMR                      K=MSIMER(N,L,LB,A,B)

        TRA     MSIMER+1
*
*       BRANCH FOR LAST ROW
*
B7      CLA     A,3
        SSP
        LDQ     TØL
        TLQ     A17
*
*       ERRØR BRANCHES
*
E3      CLA     =2
        TRA     MSIMER+1
E1      CLA     =3
        TRA     MSIMER+1
*
*       STØRAGE
*
TØL     ØCT     151400000000
AM      EQU     E.1
T       EQU     E.2
A       EQU     0
B       EQU     0
L       EQU     0
N       EQU     0
LB      EQU     0
        END
```

```
         SIMC                   K=MSIMEC(N,L,LB,A,B)
*        COMPLEX SIMULTANEOUS EQUATION SUBROUTINE
*        SOLVES THE SYSTEM OF COMPLEX EQUATIONS A*X=B.
*        TO USE, SET K=MSIMEC(N,L,LB,A,B)
*          WHERE N IS THE NUMBER OF ROWS FOR WHICH A IS
*          DIMENSIONED, AND L IS THE NUMBER OF EQUATIONS.
*          LB IS THE NUMBER OF COLUMNS IN B.
*        K=1 DENOTES SUCCESSFUL SOLUTION
*        K=2 FOR A SINGULAR OR ILL-CONDITIONED MATRIX
*        K=3 IF IMPROPER DATA IS GIVEN.
*          TO AVOID THIS SIGNAL, L MUST BE POSITIVE AND AT MOST 100,
*          N MUST NOT BE LESS THAN L, A MUST NOT INCLUDE A ROW
*          OF ZEROS.
*        A IS DESTROYED.
*        IF K=1, THE SOLUTION IS RETURNED IN B
*
*       ENTRY
*
MSIMEC  SAVE    1,2,3,4,5,6,7
*
*       PROLOGUE
*
        CLA*    3,4
        ALS     1
        PAX     0,1
        TXL     E1,1,1
        TXH     E1,1,200
        PCD     0,1
        STD     A4-1
        STD     A6-1
        STD     2A6-1
        STD     A16
        STD     A16+1
        STD     2A21
        STD     A12+1
        STD     2A26
        STD     2A26+1
        STD     2A37
```

```
        SIMC                      K=MSIMEC(N,L,LB,A,B)

        STD     2A37+1                                        00053620
        STD     A12+2                                         00053630
        STD     A20                                           00053640
        STD     A22                                           00053650
        STD     A25+1                                         00053660
        STD     A25+2                                         00053670
        STD     A33                                           00053680
        STD     A36+1                                         00053690
        STD     A36+2                                         00053700
        SXD     A52,1                                         00053710
        TXI     *+1,1,2                                       00053720
        SCD     A32,1                                         00053730
        TXI     *+1,1,-4                                      00053740
        SCD     A22+1,1                                       00053750
        CLA*    5,4                                           00053760
        PAX     0,7                                           00053770
        SXA     1A6-1,7                                       00053780
        SXA     A14,7                                         00053790
        SXA     1A21-1,7                                      00053800
        SXA     A26,7                                         00053810
        SXA     A37,7                                         00053820
        CLA*    4,4                                           00053830
        ALS     1                                             00053840
        PAX     0,1                                           00053850
        TXL     E1,1,1                                        00053860
A52     TXH     E1,1,**                                       00053870
        SXA     A2,1                                          00053880
        SXA     A5-1,1                                        00053890
        SXD     A9,1                                          00053900
        SXD     A12,1                                         00053910
        TXI     *+1,1,-2                                      00053920
        SXD     A7,1                                          00053930
        SXD     A36,1                                         00053940
        SXD     A18,1                                         00053950
        SXD     A38-1,1                                       00053960
        SXD     A21-1,1                                       00053970
        SXD     A23,1                                         00053980
```

```
        SIMC                  K=MSIMEC(N,L,LB,A,B)

        SXD     A25,1
        SXD     A31,1
        SCD     A51,1
        SCD     A51+1,1
        TXI     *+1,1,2
        CLA     6,4
        PAC     0,3
        CLA     7,4
        PAC     0,5
A51     TXI     *+1,3,2L -2
        TXI     *+1,5,2L -2
*
*      NORMALIZATION OF ROWS
*
A2      AXT     2L,2
        PXA     0,3
        PAX     0,6
        PAX     0,7
        PXA     0,0
A3      LDQ     A,6
        LRS     0
        TLQ     *+2
        XCA
        LDQ     A+1,6
        LRS     0
        TLQ     *+2
        XCA
        TNX     A4,2,2
        TXI     A3,6,2N
A4      TZE     E1
        STO     T
        CLA     =1.0
        FDP     T
        STQ     T
        AXT     2L,2
A5      FMP     A,7
        STO     A,7
```

```
         SIMC                   K=MSIMEC(N,L,LB,A,B)

         LDQ     T                                                      00054360
         FMP     A+1,7                                                  00054370
         STØ     A+1,7                                                  00054380
         LDQ     T                                                      00054390
         TNX     A6,2,2                                                 00054400
         TXI     A5,7,2N                                                00054410
A6       PXA     0,5                                                    00054420
         PAX     0,6                                                    00054430
         AXT     LB,4                                                   00054440
1A6      FMP     B,6                                                    00054450
         STØ     B,6                                                    00054460
         LDQ     T                                                      00054470
         FMP     B+1,6                                                  00054480
         STØ     B+1,6                                                  00054490
         LDQ     T                                                      00054500
         TNX     2A6,4,1                                                00054510
         TXI     1A6,6,2N                                               00054520
2A6      TNX     A7,1,2                                                 00054530
         TXI     *+1,3,2                                                00054540
         TXI     A2,5,2                                                 00054550
A7       TXH     B7,2,2L -2                                             00054560
*                                                                       00054570
*       SEARCH FØR MAXIMUM PIVØT IN CØLUMN                              00054580
*                                                                       00054590
         PXA     0,2                                                    00054600
         PAX     0,1                                                    00054610
         PXA     0,3                                                    00054620
         PAX     0,6                                                    00054630
         PXA     0,0                                                    00054640
A8       LDQ     A,6                                                    00054650
         LRS     0                                                      00054660
         TLQ     *+3                                                    00054670
         XCA                                                            00054680
         SXA     A10,1                                                  00054690
         LDQ     A+1,6                                                  00054700
         LRS     0                                                      00054710
         TLQ     *+3                                                    00054720
```

```
        SIMC                      K=MSIMEC(N,L,LB,A,B)

        XCA
        SXA     A10,1
        TXI     *+1,1,2
A9      TXH     *+2,1,2L
        TXI     A8,6,-2
        LDQ     TØL
        TLQ     *+2
        TRA     E3
A10     AXT     **,1
        SXD     *+1,2
        TNX     A17,1,**
*
*      RØW INTERCHANGE
*
        PXA     0,3
        PAX     0,6
        PAX     0,7
        SCD     *+1,1
        TXI     *+1,7,**
        PXA     0,2
        PAX     0,4
A11     CLA     A,6
        LDQ     A,7
        STØ     A,7
        STQ     A,6
        CLA     A+1,6
        LDQ     A+1,7
        STØ     A+1,7
        STQ     A+1,6
        TXI     *+1,4,2
A12     TXH     A13,4,2L
        TXI     *+1,6,2N
        TXI     A11,7,2N
A13     PXA     0,5
        PAX     0,6
        PAX     0,7
A14     AXT     LB,4
```

```
         SIMC              K=MSIMEC(N,L,LB,A,B)

         SCD     *+1,1                                          00055100
         TXI     *+1,6,**                                       00055110
A15      CLA     B,7                                            00055120
         LDQ     B,6                                            00055130
         STO     B,6                                            00055140
         STQ     B,7                                            00055150
         CLA     B+1,7                                          00055160
         LDQ     B+1,6                                          00055170
         STO     B+1,6                                          00055180
         STQ     B+1,7                                          00055190
         TNX     A17,4,1                                        00055200
A16      TXI     *+1,6,2N                                       00055210
         TXI     A15,7,2N                                       00055220
*                                                               00055230
*       DIVISION OF ROW BY PIVOT                                00055240
*                                                               00055250
A17      NZT     A,3                                            00055260
         TRA     B1                                             00055270
         LDQ     A,3                                            00055280
         FMP     A,3                                            00055290
         STO     T                                              00055300
         LDQ     A+1,3                                          00055310
         FMP     A+1,3                                          00055320
         FAD     T                                              00055330
         STO     T                                              00055340
         CLA     A,3                                            00055350
         FDP     T                                              00055360
         STQ     AM                                             00055370
         CLA     A+1,3                                          00055380
         FDP     T                                              00055390
         STQ     AN                                             00055400
A18      TXH     A21,2,2L -2                                    00055410
         PXA     0,2                                            00055420
         PAX     0,4                                            00055430
         PXA     0,3                                            00055440
         PAX     0,6                                            00055450
A20      TXI     *+1,6,2N                                       00055460
```

```
        SIMC              K=MSIMEC(N,L,LB,A,B)
        LDQ     A,6
        FMP     AN
        STØ     T
        LDQ     A+1,6
        FMP     AM
        FSB     T
        LDQ     A+1,6
        STØ     A+1,6
        FMP     AN
        STØ     T
        LDQ     A,6
        FMP     AM
        FAD     T
        STØ     A,6
        TXI     *+1,4,2
        TXL     A20,4,2L -2
A21     PXA     0,5
        PAX     0,6
        AXT     LB,4
1A21    LDQ     B,6
        FMP     AN
        STØ     T
        LDQ     B+1,6
        FMP     AM
        FSB     T
        LDQ     B+1,6
        STØ     B+1,6
        FMP     AN
        STØ     T
        LDQ     B,6
        FMP     AM
        FAD     T
        STØ     B,6
        TNX     *+2,4,1
2A21    TXI     1A21,6,2N
*
*      RØW REDUCTIØN
```

```
        SIMC                K=MSIMEC(N,L,LB,A,B)
*                                                                00055840
        PXA     0,2                                              00055850
        PAX     0,1                                              00055860
        PAX     0,4                                              00055870
        TNX     A31,1,2                                          00055880
        PXA     0,3                                              00055890
        PAX     0,6                                              00055900
        PAX     0,7                                              00055910
        STA     A29                                              00055920
        SXA     A26+1,5                                          00055930
        SXA     3A26,5                                           00055940
        TXI     *+1,6,2                                          00055950
A22     TXI     *+1,7,2N                                         00055960
        TXI     *+1,3,2N -2                                      00055970
        SXA     A28,7                                            00055980
A23     TXH     A26,2,2L -2                                      00055990
        SXA     A27,3                                            00056000
A24     LDQ     A,6                                              00056010
        FMP     A,7                                              00056020
        STO     T                                                00056030
        LDQ     A+1,6                                            00056040
        FMP     A+1,7                                            00056050
        FSB     T                                                00056060
        FAD     A,3                                              00056070
        STO     A,3                                              00056080
        LDQ     A,6                                              00056090
        FMP     A+1,7                                            00056100
        STO     T                                                00056110
        LDQ     A+1,6                                            00056120
        FMP     A,7                                              00056130
        FAD     T                                                00056140
        CHS                                                      00056150
        FAD     A+1,3                                            00056160
        STO     A+1,3                                            00056170
        TXI     *+1,4,2                                          00056180
A25     TXH     A27,4,2L -2                                      00056190
        TXI     *+1,3,2N                                         00056200
```

```
        SIMC                K=MSIMEC(N,L,LB,A,B)

        TXI     A24,7,2N                                          00056210
A27     AXT     **,3                                              00056220
A26     AXT     LB,4                                              00056230
        AXT     **,7                                              00056240
        TXI     *+1,7,2                                           00056250
        SXA     *-2,7                                             00056260
1A26    LDQ     A,6                                               00056270
        FMP     B,5                                               00056280
        STØ     T                                                 00056290
        LDQ     A+1,6                                             00056300
        FMP     B+1,5                                             00056310
        FSB     T                                                 00056320
        FAD     B,7                                               00056330
        STØ     B,7                                               00056340
        LDQ     A,6                                               00056350
        FMP     B+1,5                                             00056360
        STØ     T                                                 00056370
        LDQ     A+1,6                                             00056380
        FMP     B,5                                               00056390
        FAD     T                                                 00056400
        CHS                                                       00056410
        FAD     B+1,7                                             00056420
        STØ     B+1,7                                             00056430
        TNX     3A26,4,1                                          00056440
2A26    TXI     *+1,5,2N                                          00056450
        TXI     1A26,7,2N                                         00056460
3A26    AXT     **,5                                              00056470
        TNX     A29,1,2                                           00056480
A28     AXT     **,7                                              00056490
        PXA     0,2                                               00056500
        PAX     0,4                                               00056510
        TXI     *+1,3,2                                           00056520
        TXI     A23,6,2                                           00056530
A29     AXT     **,3                                              00056540
A31     TXH     A43,2,2L -2                                       00056550
        PXA     0,2                                               00056560
        PAX     0,1                                               00056570
```

```
        SIMC                K=MSIMEC(N,L,LB,A,B)

        PAX     0,4                                   00056580
        PXA     0,3                                   00056590
        PAX     0,6                                   00056600
        PAX     0,7                                   00056610
A32     TXI     *+1,3,2N +2                           00056620
        TXI     *+1,6,-2                              00056630
A33     TXI     *+1,7,2N                              00056640
        SXA     A37+1,5                               00056650
        SXA     A41,5                                 00056660
        SXA     A40,3                                 C0056670
        SXA     A39,7                                 00056680
A34     SXA     A38,3                                 00056690
A35     LDQ     A,6                                   00056700
        FMP     A,7                                   00056710
        STO     T                                     00056720
        LDQ     A+1,6                                 00056730
        FMP     A+1,7                                 00056740
        FSB     T                                     00056750
        FAD     A,3                                   00056760
        STO     A,3                                   00056770
        LDQ     A,6                                   00056780
        FMP     A+1,7                                 00056790
        STO     T                                     00056800
        LDQ     A+1,6                                 00056810
        FMP     A,7                                   00056820
        FAD     T                                     00056830
        CHS                                           00056840
        FAD     A+1,3                                 00056850
        STO     A+1,3                                 00056860
        TXI     *+1,4,2                               00056870
A36     TXH     A37,4,2L -2                           00056880
        TXI     *+1,3,2N                              00056890
        TXI     A35,7,2N                              00056900
A37     AXT     LB,4                                  00056910
        AXT     **,7                                  00056920
        TXI     *+1,7,-2                              00056930
        SXA     *-2,7                                 00056940
```

```
        SIMC              K=MSIMEC(N,L,LB,A,B)
1A37    LDQ     A,6                                        00056950
        FMP     B,5                                        00056960
        STØ     T                                          00056970
        LDQ     A+1,6                                      00056980
        FMP     B+1,5                                      00056990
        FSB     T                                          00057000
        FAD     B,7                                        00057010
        STØ     B,7                                        00057020
        LDQ     A,6                                        00057030
        FMP     B+1,5                                      00057040
        STØ     T                                          00057050
        LDQ     A+1,6                                      00057060
        FMP     B,5                                        00057070
        FAD     T                                          00057080
        CHS                                                00057090
        FAD     B+I,7                                      00057100
        STØ     B+1,7                                      00057110
        TNX     A41,4,1                                    00057120
2A37    TXI     *+1,5,2N                                   00057130
        TXI     1A37,7,2N                                  00057140
A41     AXT     **,5                                       00057150
        TXI     *+1,1,2                                    00057160
        TXH     A40,1,2L  -2                               00057170
A38     AXT     **,3                                       00057180
A39     AXT     **,7                                       00057190
        PXA     0,2                                        00057200
        PAX     0,4                                        00057210
        TXI     *+1,3,-2                                   00057220
        TXI     A34,6,-2                                   00057230
A40     AXT     **,3                                       00057240
        TXI     *+1,5,-2                                   00057250
        TXI     A7,2,2                                     00057260
A43     CLA     =1                                         00057270
        TRA     MSIMEC+1                                   00057280
B1      STZ     AM                                         00057290
        CLA     =1.0                                       00057300
        FDP     A+1,3                                      00057310
```

```
         SIMC                    K=MSIMEC(N,L,LB,A,B)

         STQ       AN
         TRA       A18
*
*       BRANCH FØR LAST RØW
*
B7       CLA       A,3
         SSP
         LDQ       TØL
         TLQ       A17+2
         CLA       A+1,3
         SSP
         TLQ       A17
*
*       ERRØR BRANCHES
*
E3       CLA       =2
         TRA       MSIMEC+1
E1       CLA       =3
         TRA       MSIMEC+1
*
*       STØRAGE
*
TØL      ØCT       151400000000
AM
AN
T
A        EQU       0
B        EQU       0
2L       EQU       0
2N       EQU       0
LB       EQU       0
         END
```

```
        BESEL   700                 N.A.A. SUBROUTINE LIBRARY
*               BESSEL FUNCTIONS - JNX, INX, YNX, KNX                   BESE0010
*               CALLING SEQUENCE IS BESEL (X,N,P)                       BESE0020
*               N IS ORDER                                              BESE0030
*               P EQUALS 3 FOR KNX                                      BESE0060
 BESEL  SAVE                                                            BESE0090
        CLA*    3,4                                                     BESE0100
        TMI     ERR                 ERROR ON-X                          BESE0110
        STO     FRE                 X TO FRE                            BESE0120
        FDP     FL02                X/2                                 BESE0130
        STQ     FRE+24                                                  BESE0140
        XCA                                                             BESE0150
        TZE     AAA                                                     BESE0160
        SXA     X4,4                                                    BESE0170
        CALL    ALOG(FRE+24)                                            BESE0180
    X4  AXT     **,4                                                    BESE0190
   AAA  STO     FRE+6                                                   BESE0200
        CLA*    4,4                                                     BESE0210
        STO     FRE+14              N AT B35                            BESE0220
        ACL     MAGNM                                                   BESE0230
        FAD     MAGNM                                                   BESE0240
        STO     FRE+1               N IN FL. PT.                        BESE0250
        TRA     K                                                       BESE0320
*               ERROR RETURN                                            BESE0340
   ERR  CALL    .FXEM.(57)                                              BESE0350
*               ESCAPE - NORMAL                                         BESE0370
   ESC  RETURN  BESEL                                                   BESE0380
*               TEST FOR SWITCHOVER, SERIES TO ASYMPTOTIC EXPANSION     BESE2580
 K      CLM                                                             BESE2590
        SLW     FRE+2                                                   BESE2600
        SLW     FRE+16                                                  BESE2610
        SLW     FRE+17                                                  BESE2620
        CLA     FRE                                                     BESE2630
        TZE     ERR                                                     BESE2640
        FSB     KSWO                                                    BESE2650
        TPL     KA                                                      BESE2660
 K1     TSX     A,4                                                     BESE2670
        STO     FRE+18                                                  BESE2680
```

```
         TSX    B,4                 BESE2690
         XCA                        BESE2700
         FMP    FRE+18              BESE2710
         FAD    FRE+16              BESE2720
         STØ    FRE+16              BESE2730
         CAS    FRE+17              BESE2740
         TRA    K2                  BESE2750
         TRA    K3                  BESE2760
K2       STØ    FRE+17              BESE2770
         CLA    FRE+2               BESE2780
         FAD    FLØ1                BESE2790
         STØ    FRE+2               BESE2800
         TRA    K1                  BESE2810
K3       STØ    FRE+18              BESE2820
         CLM                        BESE2830
         SLW    FRE+2               BESE2840
         SLW    FRE+16              BESE2850
         CLA    FLØ1                BESE2860
         STØ    FRE+15              BESE2870
K4       TSX    C,4                 BESE2880
         XCA                        BESE2890
         FMP    FRE+15              BESE2900
         FAD    FRE+16              BESE2910
         STØ    FRE+16              BESE2920
         CLS    FRE+15              BESE2930
         STØ    FRE+15              BESE2940
         CLA    FRE+2               BESE2950
         FAD    FLØ1                BESE2960
         STØ    FRE+2               BESE2970
         FSB    FRE+1               BESE2980
         TMI    K4                  BESE2990
K6       CLA    FRE+14              BESE3000
         LBT                        BESE3010
         TRA    K7                  BESE3020
         CLA    FLØ1                BESE3030
         TRA    K8                  BESE3040
K7       CLS    FLØ1                BESE3050
```

```
     K8 XCA                                             BESE3060
        FMP     FRE+18                                  BESE3070
        FAD     FRE+16                                  BESE3080
        FDP     FL02                                    BESE3090
        STQ     FRE+21                                  BESE3100
        CLA     FRE+21                                  BESE3110
        TRA     ESC                                     BESE3120
*               ASYMPTOTIC EXPANSION OF KNX             BESE3130
 KA     CLS     FRE                                     BESE3140
        FAD     FL088                                   BESE3150
        TMI     KA1                                     BESE3160
        CLS     FRE                                     BESE3170
    KA1 STO     FRE+24                                  BESE3180
        CALL    EXP(FRE+24)                             BESE3190
        STO     FRE+21                                  BESE3200
*               CALCULATE SQUARE RT OF PI/2X            BESE3210
        CLA     FRE                                     BESE3220
        FAD     FRE                                     BESE3230
        STO     FRE+18                                  BESE3240
 KA3    CLA     PI                                      BESE3250
        FDP     FRE+18                                  BESE3260
        STQ     FRE+19                                  BESE3270
        CALL    SQRT(FRE+19)                            BESE3280
        XCA                                             BESE3290
        FMP     FRE+21                                  BESE3300
        STO     FRE+21                                  BESE3310
        CLA     FL01                                    BESE3320
        STO     FRE+3                                   BESE3330
        STO     FRE+9                                   BESE3340
        LDQ     FRE                                     BESE3350
        FMP     FL08                                    BESE3360
        STO     FRE+18                                  BESE3370
        STO     FRE+11                                  BESE3380
        LDQ     FRE+1                                   BESE3390
        FMP     FRE+1                                   BESE3400
        XCA                                             BESE3410
        FMP     FL04                                    BESE3420
```

```
        STO     FRE+7                                     BESE3430
        FSB     FL01                                      BESE3440
        FDP     FRE+11                                    BESE3450
        STQ     FRE+8                                     BESE3460
        CLA     FRE+8                                     BESE3470
KA4     FAD     FRE+16                                    BESE3480
        STO     FRE+16                                    BESE3490
        CAS     FRE+17                                    BESE3500
        TRA     KA5                                       BESE3510
        TRA     KA6                                       BESE3520
KA5     STO     FRE+17                                    BESE3530
        CLA     FRE+3                                     BESE3540
        FAD     FL01                                      BESE3550
        STO     FRE+3                                     BESE3560
        TSX     D,4                                       BESE3570
        TRA     KA4                                       BESE3580
KA6     FAD     FL01                                      BESE3590
        XCA                                               BESE3600
        FMP     FRE+21                                    BESE3610
        STO     FRE+19                                    BESE3620
        CLS     FRE                                       BESE3630
        FAD     FL088                                     BESE3640
        TMI     KA7                                       BESE3650
        CLA     FRE+19                                    BESE3660
        TRA     ESC                                       BESE3670
KA7     LDQ     FRE+19                                    BESE3680
KA8     FMP     EM88                                      BESE3690
        TRA     ESC                                       BESE3700
*               FACTORIAL                                 BESE3710
*               M IN ACC                                  BESE3720
*               LEAVE M/ IN ACC                           BESE3730
FACT    TNZ     FACT+3                                    BESE3740
        CLA     FL01                                      BESE3750
        TRA     1,4                                       BESE3760
        STO     FRE+4                                     BESE3770
        STO     FRE+5                                     BESE3780
        CLA     FRE+4                                     BESE3790
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        FSB     FL01                            BESE3800
        TZE     FACT+12                         BESE3810
        STO     FRE+4                           BESE3820
        LDQ     FRE+5                           BESE3830
        FMP     FRE+4                           BESE3840
        TRA     FACT+4                          BESE3850
        CLA     FRE+5                           BESE3860
        TRA     1,4                             BESE3870
*               SUBR.                           BESE3880
*               X/2,N+2S/S/N+S/                 BESE3890
*               X/2,N AND S IN STORAGE          BESE3900
*               RESULT LEFT IN ACC              BESE3910
 A      SXD     FRE+10,4                        BESE3920
        CLA     FRE+1                           BESE3930
        FAD     FRE+2                           BESE3940
        STO     FRE+7                           BESE3950
        FAD     FRE+2                           BESE3960
        STO     FRE+8                           BESE3970
        CLA     FRE+2                           BESE3980
        TSX     FACT,4          13.1.6          BESE3990
        STO     FRE+9           13.1.7          BESE4000
        CLA     FRE+7           .8              BESE4010
        TSX     FACT,4          .9              BESE4020
        XCA                                     BESE4030
        FMP     FRE+9                           BESE4040
        STO     FRE+9                           BESE4050
        CLA     FRE+8                           BESE4060
        TNZ     A1                              BESE4070
        CLA     FL01                            BESE4080
        TRA     A2                              BESE4090
 A1     LDQ     FRE+6                           BESE4100
        FMP     FRE+8                           BESE4110
        STO     FRE+24                          BESE4120
        CALL    EXP(FRE+24)                     BESE4130
 A2     FDP     FRE+9                           BESE4140
        STQ     FRE+9                           BESE4150
        CLA     FRE+9                           BESE4160
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         LXD     FRE+10,4                                        BESE4170
         TRA     1,4                                             BESE4180
*                PARENTHETICAL SUBR                              BESE4190
*                X/2,N,AND S                                     BESE4200
*                LEAVE RESULT IN ACC.                            BESE4210
 B       SXD     FRE+10,4                                        BESE4220
         CLA     FRE+2                                           BESE4230
         TSX     B1,4                                            BESE4240
         CHS                                                     BESE4250
         STØ     FRE+8                                           BESE4260
         CLA     FRE+1                                           BESE4270
         FAD     FRE+2                                           BESE4280
         TSX     B1,4                                            BESE4290
         CHS                                                     BESE4300
         FAD     FRE+8                                           BESE4310
         FAD     2GAM                                            BESE4320
         STØ     FRE+8                                           BESE4330
         LDQ     FRE+6                                           BESE4340
         FMP     FLØ2                                            BESE4350
         FAD     FRE+8                                           BESE4360
         LXD     FRE+10,4                                        BESE4370
         TRA     1,4                                             BESE4380
*                SUMMATIØN ØF 1/Y FRØM R EQUALS 1 TØ LIMIT       BESE4390
 B1      STØ     FRE+3                                           BESE4400
         TZE     1,4                                             BESE4410
         CLM                                                     BESE4420
         SLW     FRE+9                                           BESE4430
         CLA     FLØ1                                            BESE4440
         FDP     FRE+3                                           BESE4450
         STQ     FRE+11                                          BESE4460
         CLA     FRE+11                                          BESE4470
         FAD     FRE+9                                           BESE4480
         STØ     FRE+9                                           BESE4490
         CLA     FRE+3                                           BESE4500
         FSB     FLØ1                                            BESE4510
         TZE     B1+15                                           BESE4520
         STØ     FRE+3                                           BESE4530
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        TRA    B1+4                                                   BESE4540
        CLA    FRE+9                                                  BESE4550
        TRA    1,4                                                    BESE4560
*              SUBROUTINE FOR LAST TERM OF SERIES OF YN AND KN        BESE4570
*              X/2,N AND S IN STORAGE                                 BESE4580
*              LEAVE RESULT IN ACCUMULATOR                            BESE4590
 C      CLA    FRE+1                                                  BESE4600
        FSB    FL01                                                   BESE4610
        TPL    C3                                                     BESE4620
        ZAC                                                           BESE4630
        TRA    1,4                                                    BESE4640
 C3     SXD    FRE+10,4                                               BESE4650
        CLA    FRE+2                                                  BESE4660
        TSX    FACT,4                                                 BESE4670
        STO    FRE+7                                                  BESE4680
        CLA    FRE+1                                                  BESE4690
        FSB    FRE+2                                                  BESE4700
        FSB    FL01                                                   BESE4710
        TSX    FACT,4                                                 BESE4720
        FDP    FRE+7                                                  BESE4730
        STQ    FRE+7                                                  BESE4740
        CLA    FRE+2                                                  BESE4750
        FAD    FRE+2                                                  BESE4760
        FSB    FRE+1                                                  BESE4770
        TNZ    C1                                                     BESE4780
        LDQ    FL01                                                   BESE4790
        TRA    C2                                                     BESE4800
 C1     STO    FRE+8                                                  BESE4810
        LDQ    FRE+6                                                  BESE4820
        FMP    FRE+8                                                  BESE4830
        STO    FRE+24                                                 BESE4840
        CALL   EXP(FRE+24)                                            BESE4850
        XCA                                                           BESE4860
 C2     FMP    FRE+7                                                  BESE4870
        LXD    FRE+10,4                                               BESE4880
        TRA    1,4                                                    BESE4890
*              LEAVE RESULT IN ACC.                                   BESE4900
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D       LDQ    FRE+3                                    BESE4910
        FMP    FRE+18                                   BESE4920
        STØ    FRE+11                                   BESE4930
        CLA    FRE+9                                    BESE4940
        FAD    FLØ2                                     BESE4950
        STØ    FRE+9                                    BESE4960
        LDQ    FRE+9                                    BESE4970
        FMP    FRE+9                                    BESE4980
        SSM                                             BESE4990
        FAD    FRE+7                                    BESE5000
        FDP    FRE+11                                   BESE5010
        STQ    FRE+2                                    BESE5020
        CLA    FRE+2                                    BESE5030
        ADD    FLØ1                                     BESE5040
        TMI    D1                                       BESE5050
D2      LDQ    FRE+2                                    BESE5060
        FMP    FRE+8                                    BESE5070
        STØ    FRE+8                                    BESE5080
        TRA    1,4                                      BESE5090
D1      LDQ    FRE+2                                    BESE5100
        FMP    FRE+8                                    BESE5110
        FAD    FRE+16                                   BESE5120
        FAD    FRE+16                                   BESE5130
        FDP    FLØ2                                     BESE5140
        STQ    FRE+16                                   BESE5150
        CLA    FRE+16                                   BESE5160
        TRA    2,4                                      BESE5170
*              CØNSTANTS FØR BESSEL FUNCTIØNS           BESE6710
 KSWØ   DEC    3.7                                      BESE6740
 2GAM   DEC    1.154431330                              BESE6760
PI      DEC    3.14159265                               BESE6765
 FLØ1   DEC    1.                                       BESE6840
 FLØ2   DEC    2.                                       BESE6850
 FLØ4   DEC    4.                                       BESE6870
 FLØ8   DEC    8.                                       BESE6890
 FLØ88  DEC    88.                                      BESE6910
 EM88   ØCT    002407555014                             BESE6930
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MAGNM OCT     +233000000000                                   BESE6940
  FRE BSS     25                                              BESE6960
      END                                                     BESE6970
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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT

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Unsteady Aerodynamics for Advanced Configurations, Part I - Application of the Subsonic Kernel Function to Nonplanar Lifting Surfaces

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

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	Air Force Flight Dynamics Laboratory (FDDS) Wright-Patterson AFB, Ohio 45433

13. ABSTRACT

Equations for pressure distributions and generalized aerodynamic forces are derived for a thin nonplanar lifting surface in simple harmonic motion at subsonic speeds. A digital computer program, written in Fortran IV, is also presented. The computer program will generate up to a ten by ten matrix of generalized aerodynamic forces when given data for the geometry of a planar lifting surface with a folded planar tip, the flight Mach number, the reduced frequency of motion, and some control constants. Control surface deflections are not accounted for in this study. The kernel function method given by Watkins, Runyan, and Woolston (Reference 1), which relates the pressure distribution to the downwash on a planar lifting surface, has been extended and applied to a nonplanar lifting surface. Hsu's technique (Reference 4) of employing Gaussian quadrature formulas is used when integrating the product of the kernel function and the lift function over the planform area. Recommendations are made to extend the method to account for blunted leading edges and the accompanying airfoil thickness and to account for control surface deflections.

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