FOREWORD

This report covers the research conducted from 1 April 1963 to 15 November 1964 by the Space and Information Systems Division of North American Aviation, Inc., Downey, California, for the Aerospace Dynamics Branch, Vehicle Dynamics Division, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, under Contract No. AF 33(657)-10399.

The work was performed to advance the state of the art of flutter prediction for flight vehicles as part of the Research and Technology Division, Air Force Systems Command's exploratory development program. This research was conducted under Project No. 1370 "Dynamic Problems in Flight Vehicles," and Task No. 137003 "Prediction and Prevention of Aerothermoelastic Instabilities." Mr. James Olsen of the Vehicle Dynamics Division, Air Force Flight Dynamics Laboratory, was the Task Engineer.

Mr. L.V. Andrew was the Program Manager for North American Aviation. Mr. M.T. Moore formulated the approach based on an outline provided by Professor H. Ashley who acted as consultant during the development of this report and on previous work by L.V. Andrew. Mr. Moore also directed and contributed to the writing of the program for the IBM 7094 computer. He was assisted by Mr. J.B. O'Neill. Dr. E.R. Rodemich contributed many enlightening discussions.

The contractor's designation of this report is SID 64-1512-4.

This report has been reviewed and approved.

Walter J. Mykytow
Walter J. Mykytow
Asst. for Research & Technology
Vehicle Dynamics Division

ABSTRACT

In this volume, Ashley's approach to mutual interference theory by source superposition methods has been applied to the prediction of supersonic air loads on intersecting thin lifting surfaces in steady or oscillatory motion. Steady loading is regarded as the special case of zero frequency of oscillation. Each surface may be oscillating in a mode of rigid or elastic vibration or linear combinations thereof. Evvard's diaphragm concept has been extended to treat the out-of-plane interference problem. As a result, any leading or side edge on any of the intersecting surfaces may be subsonic. The study reported herein has lead to the formulation of a method by which diaphragm regions can be selected that eliminate the need for calculating out-of-plane velocity potentials. Based on mutual interference theory, the method requires only the calculation of out-of-plane velocities so that tangential flow conditions may be met.

The Mach-box method has been used to obtain an approximate solution to the problem. In following the aerodynamic influence coefficient procedure of Zartarian and Hsu, each surface and diaphragm is overlaid with a grid of rectangular Mach boxes, the diagonals of which are parallel to the Mach lines. For purposes of calculating induced velocities and velocity potentials, the source strength over the area of each box is assumed to be constant. Out-of-plane velocity influence coefficients are calculated for the center of each box in the interference region, and the tangential flow condition is satisfied there. For purposes of calculating generalized forces, the resulting velocity potential over the area of each box is also assumed to be constant and equal to the value at the center of the box.

A computer program that calculates a matrix of generalized forces for the special case of a wing with symmetrically folded tips is also presented. The generalized forces are those due to motion of the surface which may consist of a linear combination of as many as ten modes of motion. By proper specification of the modes and frequencies, the user may obtain lift coefficients, longitudinal stability derivatives, loads due to symmetric sinusoidal gusts, drag due to surface warpage, or an array of generalized forces which may be used to calculate flutter speeds. If the surface is free from flutter, the same arrays may be used in a Fourier series approach to calculate the responses to random or discrete gusts.

Favorable comparisons of the results have been obtained for delta wings with folded tips and without folded tips. Results for rectangular and delta wings in both steady and oscillatory motion are presented and compare

favorably with theoretical forces calculated from both exact and other approximate numerical methods. A smooth decrease in the steady state lift and moment on a delta wing is calculated when the tips are folded down to the vertical position. These results agree with the trends that have been observed in recent experiments.

CONTENTS

ILLUSTRATIONS

SYMBOLS

Symbol	Definition
a_∞	Free-stream speed of sound
b	Box length
b/β	Box width
C_p	Surface pressure coefficient
$\frac{D}{Dt}$	Linearized substantial derivative
f_i	Nondimensional displacement mode of lifting surface
G	Strength of unit source
H	Source strength per unit area
k	Reduced frequency based on box length, $\omega b/U_\infty$
$\bar{k}$	Modified reduced frequency, $k\ M_\infty^2/\beta^2$
k_r	Reduced frequency based on root chord, $\omega c/U_\infty$
M_∞	Free-stream Mach number, U_∞/a_∞
N	Normal velocity influence coefficient
n, m, ℓ	Location of receiving box center in x_1, y_1, z_1 coordinate system
p	Local pressure at the airfoil surface
p_∞	Free-stream pressure
Q_i	Generalized force
$\vec{q}$	Unsteady perturbation velocity
R	$\sqrt{(x-\xi)^2 - (y-\eta)^2 - (z-\zeta)^2}$
R_1	$\sqrt{\bar{\xi}_1^{\,2} - \bar{\eta}_1^{\,2} - \bar{\zeta}_1^{\,2}}$

List of Symbols continued on next page.

Symbol	Definition
S	Reference area
U_∞	Free-stream velocity
U, L	Superscripts referring to upper or lower source sheet, respectively
u, v, w	Components of $\vec{q}$ in the x, y, z directions, respectively
$\bar{u}, \bar{v}, \bar{w}$	Time-independent factors of u, v, w
V	Horizontal velocity influence coefficient
W	Vertical velocity influence coefficient
X, Y, Z $\tilde{\xi}, \tilde{\eta}, \tilde{\zeta}$	Cartesian coordinate system (applicable in Section 2 only)
x, y, z ξ, η, ζ	Cartesian coordinate system transformed to $M = \sqrt{2}$
x_1, y_1, z_1 ξ_1, η_1, ζ_1	Cartesian coordinate system transformed to $M = \sqrt{2}$ and nondimensionalized by box length
$\bar{Z}$	Dimensional deflection mode of lifting surface
β	$\sqrt{M_\infty^2 - 1}$
δC_w	Incremental virtual work coefficient
δr	Nondimensional virtual displacement
δq_i	Virtual displacement in the generalized coordinate, q_i
ρ_∞	Free-stream density
ω	Oscillatory frequency - $2\pi f$
ϕ	Unsteady perturbation velocity potential

List of Symbols continued on next page.

Symbol	Definition
$\bar{\phi}$	Time-independent factor of ϕ
Φ	Velocity potential influence coefficient
ν, u, ℓ	Location of sending box center in x_1, y_1, z_1 coordinate system
$\bar{\nu}, \bar{u}, \ell$	Fixed values of $\bar{\xi}$, $\bar{\eta}$, $\bar{\zeta}$ respectively
$\bar{\xi}_1$, $\bar{\eta}_1$, $\bar{\zeta}_1$	Relative position coordinates

I. INTRODUCTION

Garrick and Rubinow (Reference 1) have shown that every solution of the boundary-value problem describing small disturbances in a parallel supersonic flow can be built up by superposing elementary solutions. One of these elementary solutions is that of a pulsating sound source fixed in the flow; however, exact numerical evaluations of the integral equations resulting from the source superposition method are laborious, if not impossible, to obtain.

Pines, Dugundji, and Neuringer (Reference 2) published the first source superposition method to successfully approximate the aerodynamic forces on an oscillating thin planar surface in supersonic flow. They employed Evvard's diaphragm concept (Reference 3) to handle subsonic leading edges and overlaid the surface and diaphragm with a grid of square boxes. For purposes of calculating pressures, they assumed that the source strength over the area of each box is a constant value which satisfies the condition of tangential flow at the center of the box. Thus, they established a method for calculating aerodynamic influence coefficients similar to structural stiffness influence coefficients.

Li (Reference 4) then published the Mach-box formulation, that greatly simplified calculation of the aerodynamic influence coefficients. Li's formulation was fully developed for planar surfaces by Zartarian and Hsu (Reference 5). The Mach-box procedure is basically the same as Pines' method, differing only in that the surface and diaphragm is overlaid with a grid of rectangular boxes the diagonals of which are parallel to the Mach lines.

Zartarian and Hsu were the first to establish the minimum number of boxes on a swept, low-aspect-ratio lifting surface that can be expected to yield reasonable results when the leading edge of the surface is represented by the jagged pattern of box leading edges. They were also the first to use velocity potential influence coefficients (VPIC's). Because the velocity potential is a better behaved function, near the leading edge, than the pressure, the VPIC's yield more accurate generalized forces than pressure influence coefficients for the same grid. The Mach-box approach, using VPIC's, evolved as the most efficient way to calculate generalized forces on planar lifting surfaces in supersonic flow.

The Mach-box approach to the source superposition method is currently accepted as a standard procedure for determining the generalized

Manuscript released by authors February 1964 for publication as an RTD technical documentary report.

forces on planar lifting surfaces in either steady or oscillatory motion at supersonic speeds. Ashley (Reference 6) has shown how the source superposition method may be applied to two or more planar surfaces flying in the disturbance field of each other. He also outlined application of the method to two intersecting planar surfaces when the line of intersection is parallel to the free-stream. The development contained in this report is the outgrowth of several iterations on the procedure and several consultations with Professor Ashley.

The extension of the source superposition method to intersecting lifting surfaces creates some difficulty in understanding the physics as well as the mathematics of the problem. This is primarily due to the artificiality of the method of using sheets of symmetric disturbances to obtain antisymmetric air loads.

An attempt was made to reduce the diaphragm area to a minimum in the expectation that the number of numerical operations would thereby be reduced to a minimum. Another desirable property of this arrangement of disturbance sheets is that the flow region is separated into a single upper and a single lower region. Further study revealed that this arrangement does not lead to a minimum number of operations because of the more complicated geometry.

A simpler geometric arrangement can be achieved by constructing the Mach envelopes of each of the separate components of the lifting surface and then removing redundant diaphragm regions. By this means the flow region is separated into a minimum of three regions. Removal of the redundant diaphragms results in the necessity to define separate sending regions for upper and lower parts of each sheet of disturbances in the interference region. The computer program contained in this report is based on this geometric arrangement.

The Mach-box method is used in the computer program to obtain aerodynamic forces on a wing with non-coplanar tips. In the computer program, discussed in Section 4, the velocity potential distribution over all surfaces is calculated and then integrated to obtain generalized forces. These quantities may be used to obtain (1) steady-state air loads on each surface for any small angle of attack and camber distribution, (2) stability derivatives for 20 different frequencies of rigid pitch or plunge motion, or (3) matrices of generalized forces for up to 10 modes of rigid or elastic vibration. For each mode, the distribution of velocity potentials, containing all the interference effects, may then be smoothed by fitting to the point values a polynomial that has the proper edge conditions. Operating on the resulting polynomial with the linearized substantial derivative provides an expression for the distribution of lifting pressure over the entire surface.

After the computer program was written it was discovered that a more elegant approach is one in which the redundant diaphragms are retained. In this approach (1) the definition of sending surfaces is general and simple, (2) satisfaction of tangential flow conditions on each of the surfaces and the continuity conditions on diaphragms is a straightforward sequential process, (3) the necessity for calculating out-of-plane velocity potentials disappears (thus reducing the number of numerical operations), and (4) insight is provided for further extension. A discussion of this approach is contained in Section 6 of this report.

2. THIN AIRFOIL THEORY IN SUPERSONIC FLOW

LINEARIZED EQUATIONS

Thin airfoil or small perturbation theory is applied to describe the flow patterns that result when small disturbances are superposed on parallel uniform flow. The small perturbation methods used to linearize the second-order partial differential equation that characterizes compressible fluid flow are completely described for steady flow by Sears (Reference 7) and for unsteady flow by Ashley (Reference 8). These methods are summarized in Part I of this report and presented in a form consistent with the following application to a slightly perturbed, uniform, parallel, supersonic flow.

Consider such a perturbed supersonic flow in the direction of the positive X-axis of a Cartesian coordinate system (Figure 1). The linearized partial differential equation describing this flow may be expressed in the form

$$\phi_{XX} + \phi_{YY} + \phi_{ZZ} = \frac{1}{a_\infty^2}\left(U_\infty^2 \phi_{XX} + 2U_\infty \phi_{XT} + \phi_{TT}\right) \tag{1}$$

where ϕ is the perturbation velocity potential, a_∞ is the fixed acoustic speed far upstream, and U_∞ is the uniform flow velocity. By collecting terms and introducing the free-stream Mach number

$$M_\infty = U_\infty / a_\infty$$

Equation 1 can be cast in the form of a hyperbolic differential equation

$$\left(M_\infty^2 - 1\right)\phi_{XX} - \phi_{YY} - \phi_{ZZ} = -\frac{1}{a_\infty^2}\left(2U_\infty \phi_{XT} + \phi_{TT}\right) \tag{2}$$

which, for the supersonic case, is satisfied only within a characteristic region. This region is the downstream Mach cone; its semivertex angle is equal to the Mach angle and its axis is parallel to the X-axis.

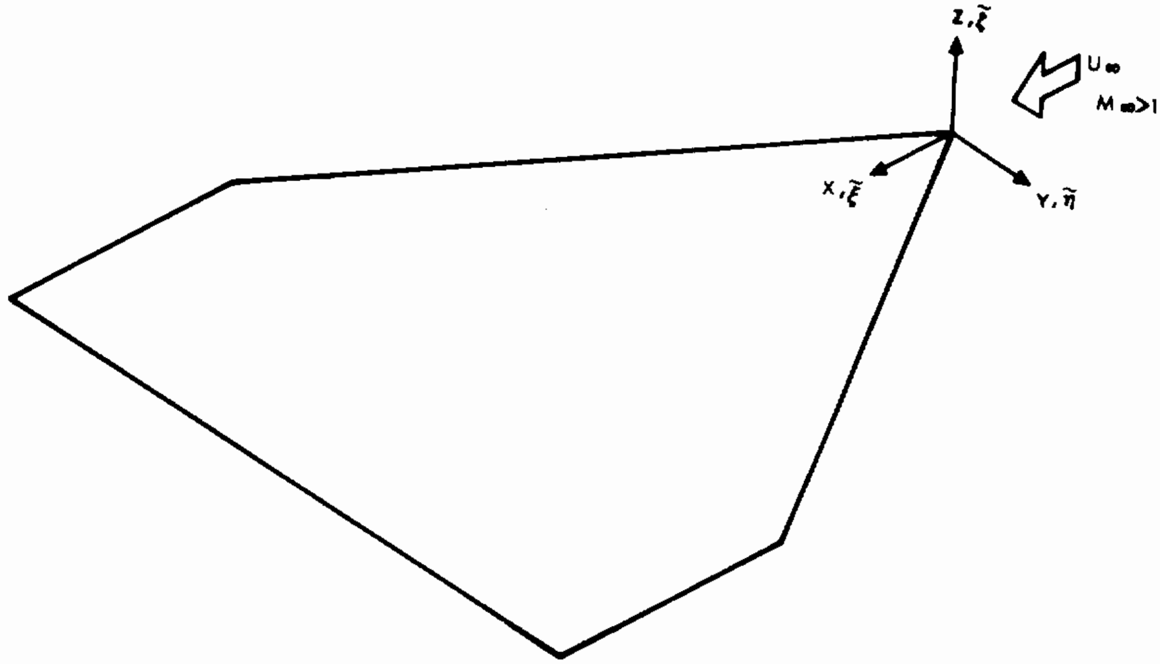


Figure 1. Lifting Surface in Z = 0 Plane in Parallel Supersonic Flow

The small disturbance (Figure 2) has a velocity given by the expression

$$\vec{q} = u\vec{i} + v\vec{j} + w\vec{k} \tag{3}$$

where,

$$u = \frac{\partial\phi}{\partial X}\ ,\quad v = \frac{\partial\phi}{\partial Y}\ ,\quad w = \frac{\partial\phi}{\partial Z}$$

and is placed in a uniform parallel flow at the point $(\tilde{\xi}, \tilde{\eta}, \tilde{\zeta})$. The signals from the disturbance can only be received at the points (X, Y, Z) within the downstream Mach cone which has its vertex at the sending point $(\tilde{\xi}, \tilde{\eta}, \tilde{\zeta})$. Conversely, a given receiving point feels only those signals transmitted by sending points in the upstream Mach cone which has its vertex at the receiving point.

BOUNDARY CONDITIONS

Because the fluid is unbounded, the solution to Equation 2 must satisfy boundary conditions at infinity as well as at the surface of the immersed airfoil. The conditions at infinity specify: (1) the fluid must be in uniform motion with no disturbances ahead of the downstream Mach lines from the

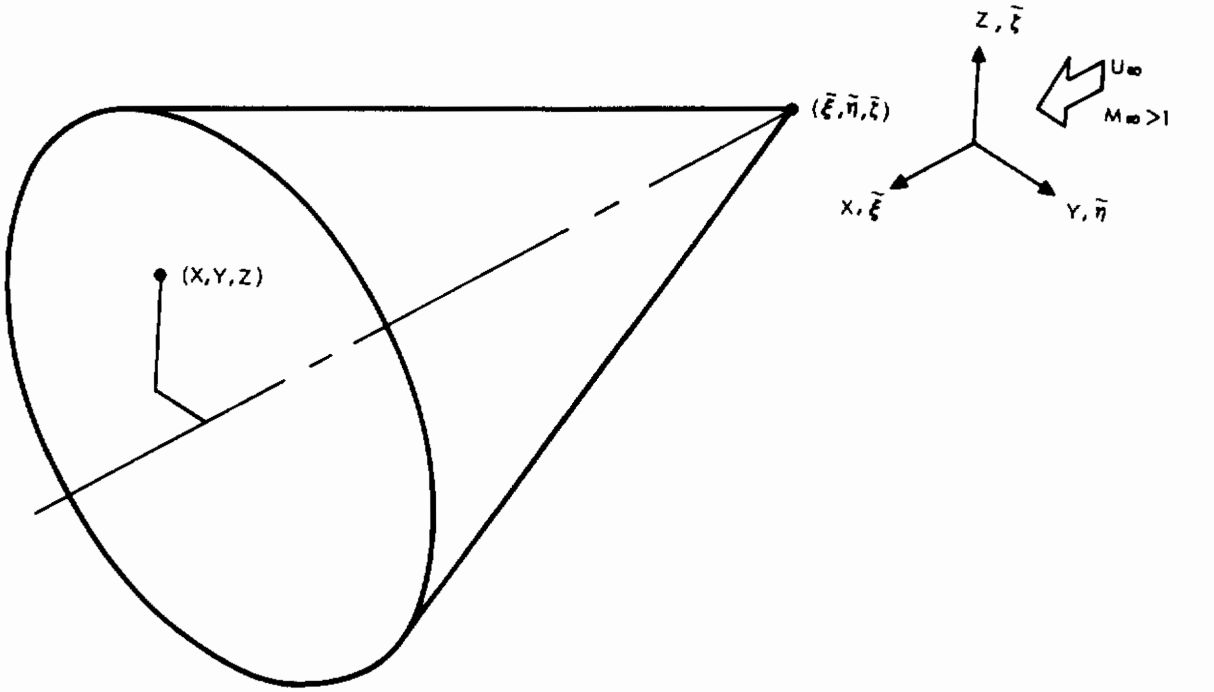


Figure 2. Small Disturbance and its Zone of Influence

leading edge of the surface, and (2) the disturbances behind these downstream Mach lines must be directed outwardly from their sources. The conditions at the surface of the thin airfoil require the flow to be tangent at all points on the surface. Setting the fluid velocity component normal to the surface equal to the normal velocity of the surface ensures this condition. The linearized expression of this condition for an airfoil lying close to the XY plane is

$$w(X, Y, Z, T) = \left[U_\infty \frac{\partial}{\partial X} + \frac{\partial}{\partial T}\right] Z(X, Y, T) \tag{4}$$

where Z(X, Y, T) represents the instantaneous position of the mean surface moving normal to the XY plane. The condition in Equation 4 is applicable only when the slopes over the airfoil surface are small.

THE PRESSURE RELATION

The pressure relation, applicable to this problem and consistent with the assumptions of small perturbation theory, is given by Ashley (Reference 8) in the form

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty U_\infty^2} = -\frac{2}{U_\infty}\phi_X - \frac{2}{U_\infty^2}\phi_T \tag{5}$$

where the pressure coefficient, C_p, is the ratio of the difference between the free-stream pressure and the local pressure, $(p - p_\infty)$, to the free-stream dynamic pressure, $\left(\frac{1}{2}\rho_\infty U_\infty^2\right)$. The difference between the pressure coefficient on the upper side and the lower side of the surface is obtained by replacing the velocity potential in Equation 5 with the change in the potential across the surface.

$$\Delta Cp = \frac{p^U - p^L}{\frac{1}{2}\rho_\infty U_\infty^2} = -\frac{2}{U_\infty}\Delta(\phi_X) - \frac{2}{U_\infty^2}\Delta(\phi_T) \tag{6}$$

This pressure difference coefficient is zero everywhere except at the airfoil surface.

GENERALIZED FORCES

If a point on the airfoil surface undergoes a small virtual displacement (δr) in the direction of the pressure difference across the surface, virtual work will be done on the system.

$$\delta C_W = \Delta C_p \, \delta r \tag{7}$$

The nondimensional displacement δr, can be expressed in terms of the N independent generalized coordinates, q, as

$$\delta r = \sum_{i=1}^{n} \frac{\partial r}{\partial q_i} \delta q_i \tag{8}$$

For the case of the airfoil surface

$$\frac{\partial r}{\partial q_i} \equiv f_i$$

where f_i is the displacement normal to the surface in the i^{th} vibration mode. (See Section 1.4 of Reference 9 for a complete discussion of generalized coordinates and generalized forces.) The total virtual work done by all the applied pressures on the airfoil surface is then

$$\delta C_W = \frac{1}{S}\iint_S \Delta C_p \, \delta r \, dX \, dY \tag{9}$$

which becomes, upon substitution of Equation 8 and 8a for δr

$$\delta C_W = \sum_{i=1}^{n} Q_i \, \delta q_i \tag{10}$$

The symbol Q_i refers to the generalized force which results from integrating the pressure times the displacements in the i^{th} vibration mode.

$$Q_i = \frac{1}{S} \iint_S \Delta C_p \, f_i \, dX \, dY \tag{11}$$

where S is the area of the airfoil surface.

SIMPLE HARMONIC MOTION

The time dependence of the motion, if small in amplitude, can be completely arbitrary, since the solutions to Equation 2 are valid for any small motion; however, simple harmonic motion is assumed because of the wide applicability of the results. These results may be used in flutter analyses and in determination of transient response to externally applied loads such as gusts where the response can be expressed as a Fourier series consisting of simple harmonic terms.

Simple harmonic motion of the airfoil is expressible as

$$Z(X, Y, T) = \overline{Z}(X, Y) \, e^{i\omega T} \tag{12}$$

or in terms of generalized coordinates,

$$Z(X, Y, T) = \sum_{j=1}^{n} f_j \, \overline{q}_j \, e^{i\omega T} \tag{12a}$$

where $i = \sqrt{-1}$ and ω is the frequency of motion. Since the boundary conditions relate the disturbance velocity to the airfoil motion, substitution of Equation 12 into Equation 4 gives the following relation for the perturbation velocity

$$\overline{w}(X, Y) \, e^{i\omega T} = \left[U_\infty \frac{\partial}{\partial X} + i\omega \right] \overline{Z}(X, Y) \, e^{i\omega T} \tag{13}$$

where the operator

$$U_\infty \frac{\partial}{\partial X} + i\omega = \frac{D}{DT} \tag{14}$$

is the linearized substantial derivative for simple harmonic motion and, therefore, $\overline{w}(X, Y)$ is complex.

The velocity potential at a receiving point is solely determined by the free-stream velocity and the disturbances within the fore Mach cone from that point. Since the only disturbances are those resulting from the harmonically oscillating airfoil, the variation of the linearized perturbation velocity potential is also simple harmonic and may be written in the form:

$$\phi(X, Y, T) = \overline{\phi}(X, Y)\, e^{i\omega T} \tag{15}$$

Substitution of this expression for the velocity potential into Equation 2 results in

$$\beta^2 \overline{\phi}_{XX} - \overline{\phi}_{YY} - \overline{\phi}_{ZZ} = -\frac{1}{a_\infty^2}\left(2i\omega U_\infty \overline{\phi}_X - \omega^2 \overline{\phi}\right) \tag{16}$$

where

$$\beta = \sqrt{M_\infty^2 - 1}$$

The equation is now expressed in terms of the complex velocity potential, $\overline{\phi}(X, Y)$.

Substituting Equation 15 into the pressure-difference velocity-potential relation, Equation 6, reduces the expression for the change in pressure coefficient across the surface to:

$$\Delta\overline{C}_p = -\frac{2}{U_\infty}\Delta\left(\overline{\phi}_X\right) - \frac{2i\omega}{U_\infty^2}\Delta(\overline{\phi}) \tag{17}$$

where $\Delta\overline{C}_p$ is the complex pressure difference coefficient.

THE EQUIVALENT PROBLEM AT MACH NO. $\sqrt{2}$

As the final step in the development, a transformation of the Cartesian coordinates is introduced

$$x = X, \quad y = \beta Y, \quad z = \beta Z, \quad t = T \tag{18}$$

This step transforms all supersonic flow problems to equivalent problems at $M_\infty = \sqrt{2}$. Upon substitution of Equation 18, Equation 16 becomes

$$\bar{\phi}_{xx} - \bar{\phi}_{yy} - \bar{\phi}_{zz} = - \frac{1}{\beta^2 a_\infty^2} \left(2 i \omega U_\infty \bar{\phi}_x - \omega^2 \bar{\phi} \right) \tag{19}$$

Equation 19 is satisfied within a 45-degree Mach cone downstream from the point disturbance. Substitution of Equations 18 and 17 into the expression for the generalized force in Equation 11 gives an expression in terms of the frequency of motion (ω)

$$Q_i = \frac{-2}{\beta U_\infty S} \iint_S \left[\Delta(\bar{\phi}_x) + \frac{i\omega}{U_\infty} \Delta(\bar{\phi}) \right] f_i \, dx \, dy \tag{20}$$

Now that the linearized equations describing the fluid motion are complete, there remains the problem of obtaining solutions to Equation 19 which satisfy the boundary conditions at infinity and at the surface of planar airfoils in supersonic flow. All equations and expressions in the ensuing development will be functions of the transformed coordinates (x, y, z), (ξ, η, ζ) unless otherwise noted.

3. SOURCE SUPERPOSITION METHOD

Consider a pulsating sound source to be fixed in an otherwise undisturbed supersonic flow that has been transformed to $M_\infty = \sqrt{2}$. Each spherical pulse is emitted $t = t_i$ and expands with radius, $a_\infty (t - t_i)$, while its center moves downstream at the free-stream velocity, U_∞. The boundary of the positions of the sphere is the familiar downstream Mach cone.

The basic solution for spherical waves emitted from a stationary sound source is

$$\phi_s (x, y, z, t) = \frac{H(\xi, \eta, \zeta)}{R} g(t)$$

where

$$R = \sqrt{(x - \xi)^2 - (y - \eta)^2 - (z - \zeta)^2}$$

Superposition of a parallel supersonic flow on the sound source transforms the time coordinate, and the solution becomes

$$\phi_s (x, y, z, t) = \frac{H(\xi, \eta, \zeta)}{R} \left[g\left(t - \frac{R}{a_\infty}\right) + g\left(t + \frac{R}{a_\infty}\right) \right] \tag{21}$$

The source is fixed at the point (ξ, η, ζ) with strength $H(\xi, \eta, \zeta)\, g(t)$. The two time functions, $g(t - R/a_\infty)$ and $g(t + R/a_\infty)$, represent the receding and advancing portions of the waves, respectively. The explicit form of the fundamental solution at (x, y, z) corresponding to the amplitude of a harmonically pulsating source at (ξ, η, ζ) in a parallel supersonic stream is given by Stewart (Reference 10) as,

$$\overline{\phi} (x, y, z, \xi, \eta, \zeta) = H(\xi, \eta, \zeta)\, G(x - \xi, y - \eta, z - \zeta) \tag{22}$$

where

$$G = -\frac{1}{\pi R} \exp\left[\frac{-i\omega U_\infty}{a_\infty^2 \beta^2} (x - \xi) \right] \cos\left(\frac{\omega}{a_\infty \beta^2} R \right)$$

which is parametrically dependent upon the free-stream conditions and the frequency of oscillation.

H is the space variation of the source strength; $\bar{\phi}$ is assigned the value zero everywhere outside the downstream Mach cone from (ξ, η, ζ) and is singular all over the conical surface because the signals from a concentrated source reinforce one another there to an infinite degree. The elementary source, Equation 22, may be integrated with respect to any of the variables ξ, η, ζ to give extended solutions to the transformed linearized equation of fluid motion, Equation 19.

SINGLE PLANAR SURFACES

For the purpose of solving the thin planar lifting surface problem in supersonic flow, the mean surface of the airfoil is considered to lie nearly in the $\zeta = 0$ plane. It can then be replaced by two sheets of the discontinuities given by Equation 22. One of the two sheets is placed on the upper surface of the airfoil and the other on the lower surface. The strength distributions of the source sheets are determined by the boundary conditions at the upper and lower sides of the surface, respectively.

The condition of tangential flow to the harmonically oscillating surface suggests that the strength of each source sheet is proportional to the normal velocity of the surface. Stewart (Reference 10) has shown that for a single planar surface lying in the $\zeta = 0$ plane, the value of the strength of the upper source sheet is

$$H^U(\xi, \eta) = \bar{w}(\xi, \eta) \tag{23}$$

where $\bar{w}(\xi, \eta)$ is the local normal component of the perturbation velocity. The antisymmetry of the normal velocity at any instant in time, with respect to the outward pointing normal on the upper and lower sides of the surface, implies that the lower source sheet is of opposite strength to the upper sheet, i.e.,

$$H^L(\xi, \eta) = -\bar{w}(\xi, \eta) \tag{24}$$

Keep in mind that a single source sheet causes disturbances in the flow which are symmetric relative to the plane of the source sheet. Therefore, it is essential to the source solution of the problem that we consider only those disturbances in the upper half space due to the upper source sheet and only those disturbances in the lower half space due to the lower source sheet. By placing two antisymmetric sheets of sources on the $\zeta = 0$ plane, it is possible to obtain the proper lifting antisymmetry of the disturbance due to a thin airfoil oscillating in a supersonic main stream. It follows that the velocity potentials will also be antisymmetric.

If any portion of the edges of the airfoil is subsonic (i.e., the component of the free-stream velocity normal to some point on the edge is less than the acoustic velocity) the source superposition method seems to be inadequate. The underlying assumption that the upper and lower source sheets be isolated or not feel each other is violated in the region near a subsonic edge.

To apply the source sheet simulation to airfoils at all supersonic speeds Evvard (Reference 3) suggested extending the source sheets to completely cover that portion of the $\zeta = 0$ plane within the Mach envelope from the leading and trailing edges of the surface. The edges of the source sheets then become either sonic or supersonic, and disturbances from the upper and lower sheets are confined to the upper and lower half spaces, respectively, of the Mach envelope. The regions of the source sheet between a subsonic edge of the lifting surface and the Mach envelope are referred to as diaphragms and are placed there to satisfy conditions of continuity of pressure and velocity and to prevent signals from being propagated from one half space to the other.

Since there is no surface motion in the diaphragm region, the perturbation velocity must be determined from other boundary conditions. These conditions are that the diaphragm causes no discontinuities in either the pressure,

$$\Delta \overline{p} = \overline{p}(\xi, \eta, 0+) - \overline{p}(\xi, \eta, 0-) = 0 \tag{25}$$

or the normal component of perturbation velocity

$$\Delta \overline{w} = \overline{w}(\xi, \eta, 0+) - \overline{w}(\xi, \eta, 0-) = 0 \tag{26}$$

The diaphragm normal velocity distribution must preserve the slopes which the streamlines would have if flow passed through the diaphragm regions. This condition is satisfied by making the source strength distribution proportional to the normal velocity distribution.

On the part of the diaphragm that is neither in the wake of its associated lifting surface nor in the wake of another body or surface upstream, an even stronger condition than that in Equation 25 has been formulated by Evvard (Reference 3). This stronger condition, which then replaces Equation 25, specifies that there can be no jump in velocity potential

$$\Delta \overline{\phi} = \overline{\phi}(\xi, \eta, 0+) - \overline{\phi}(\xi, \eta, 0-) = 0 \tag{27}$$

at any point in a diaphragm not in a wake region.

With the $\zeta = 0$ plane of discontinuities dividing the Mach envelope into two half spaces, it is now possible to write the complete solution to Equation 19 for the velocity potential at a point (x, y, z) due to the presence of an oscillating lifting surface in a supersonic flow. The solution at any receiving point may be obtained by integrating both sides of Equation 22 over those portions of the airfoil and associated diaphragms that are within the zone of influence for that point. The velocity potential is given by

$$\overline{\phi}^{U,L}(x, y, z) = \frac{1}{\beta} \iint_A H^{U,L}(\xi, \eta)\, G(\overline{\xi}, \overline{\eta}, \overline{\zeta})\, d\eta\, d\xi \tag{28}$$

where G is given in Equation 22 with $\bar{\xi} = x - \xi$, $\bar{\eta} = y - \eta$, $\bar{\zeta} = z - \zeta$. The sending area, A, is the region of the $\zeta = 0$ plane bounded by the Mach envelope and the hyperbola $\bar{\xi}^2 - \bar{\eta}^2 - \bar{\zeta}^2 = 0$. Figure 3 shows a typical area on the upper side of the $\zeta = 0$ plane that influences the point (x, y, z) in the upper half space. It is emphasized that the upper source sheet (with strength = $\bar{w}(\xi, \eta)$) defines the velocity potential in the upper half space ($\bar{\phi}^U$) and the lower source sheet (with strength = $-\bar{w}(\xi, \eta)$) defines the potential in the lower half space ($\bar{\phi}^L$).

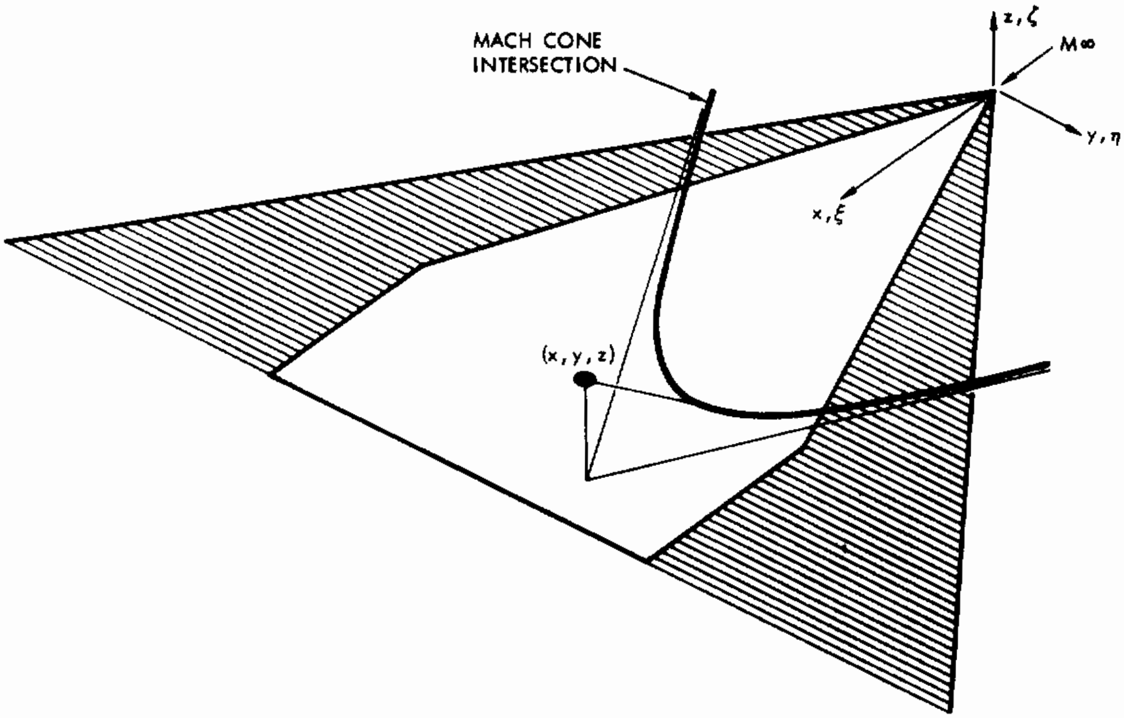


Figure 3. Source Sheet and Area of Influence for (x, y, z)

Before the velocity potential can be calculated, using Equation 28, a complete description of the strength distribution over the source sheet will be necessary. The source strengths in the region that replaces the lifting surface result from substituting the tangential flow boundary condition (Equation 13) into Equation 23 for the upper surface

$$H^U(\xi, \eta) = \frac{D}{Dt} \bar{Z}(\xi, \eta) \tag{29}$$

and into Equation 24 for the lower surface

$$H^L(\xi, \eta) = -\frac{D}{Dt} \bar{Z}(\xi, \eta) \tag{30}$$

The source-sheet strength distribution in the diaphragm regions can be obtained by applying to the pressure difference coefficient the general condition of no pressure jump across the region. However, if we allow only subsonic leading and side edges, Equation 28 can be substituted directly into Equation 27. For a receiving point (x, y, 0) ahead of or beside the airfoil surface, but within the Mach envelope, Equation 27 becomes

$$0 = \iint_A \Delta H(\xi, \eta)\, G(\overline{\xi}, \overline{\eta}, \overline{\zeta})\, d\eta\, d\xi \tag{31}$$

where the area of integration, A, includes portions of both the airfoil surface and its associated diaphragms within the fore-Mach triangle from the point (x, y, 0). Separation of the integral into a sum of integrals over the airfoil and diaphragms, respectively, and substitution of the strength-per-unit area in the region representing the airfoil, produces an integral equation,

$$\iint_{A_D} \Delta H(\xi, \eta)\, G(\overline{\xi}, \overline{\eta}, \overline{\zeta})\, d\eta\, d\xi = -2 \iint_{A_S} \frac{D}{Dt}\, \overline{Z}(\xi, \eta)\, G(\overline{\xi}, \overline{\eta}, \overline{\zeta})\, d\eta\, d\xi \tag{32}$$

for the source-sheet strength per unit area in the diaphragm region. Areas of integration, A_D and A_S, for a typical lifting surface are shown in Figure 4. For the single-planar lifting surface, separate integral equations for the upper and lower diaphragm strength distributions can be obtained. Applying the condition of antisymmetry to the normal component of perturbation velocity, Equation 32 becomes

$$\iint_{A_D} \overset{U,L}{H}(\xi, \eta)\, G(\overline{\xi}, \overline{\eta}, \overline{\zeta})\, d\eta\, d\xi = \mp \iint_{A_S} \frac{D}{Dt}\, \overline{Z}(\xi, \eta)\, G(\overline{\xi}, \overline{\eta}, \overline{\zeta})\, d\eta\, d\xi \tag{33}$$

where the plus sign (+) goes with the lower source sheet and the minus sign (-) goes with the upper source sheet.

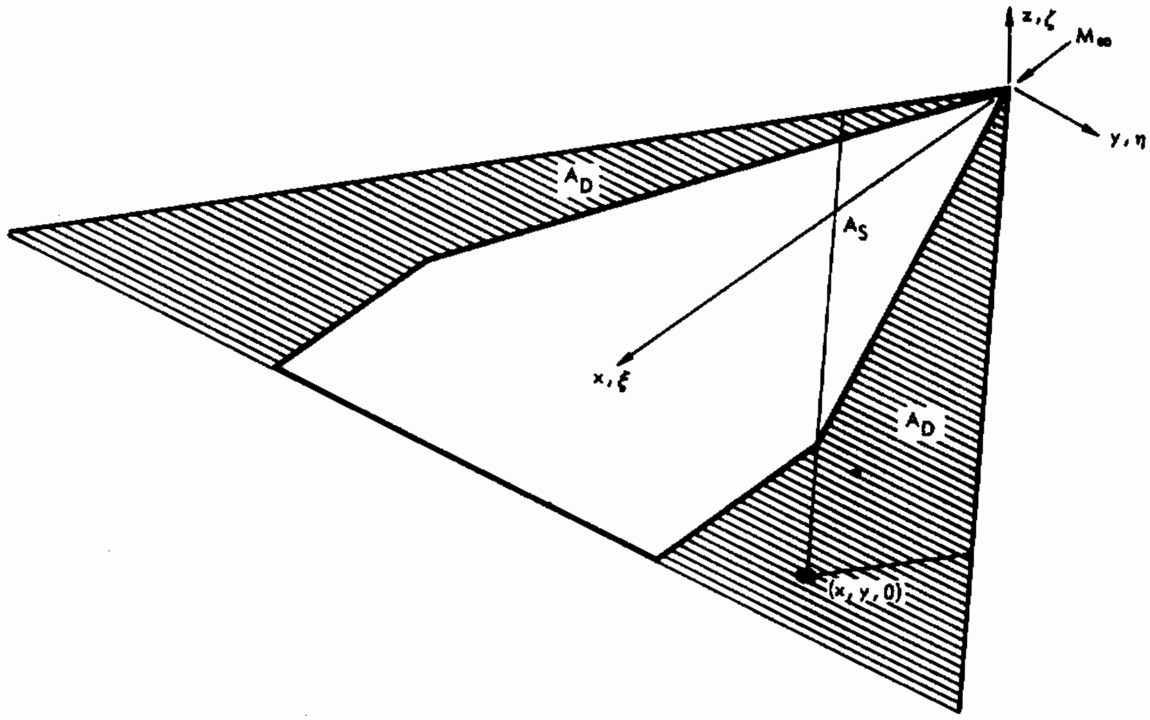


Figure 4. Source Sheet and Area of Influence for (x, y, 0)

It is emphasized that the foregoing development of the source-sheet solution to Equation 19 for a single-planar lifting surface at any supersonic speed provides the proper lifting antisymmetry only through proper use of the diaphragm in the $\zeta = 0$ plane between a subsonic edge and the Mach envelope. The diaphragm must isolate the upper and lower airfoil surface source sheets while ensuring that no pressure loading acts on regions that are incapable of sustaining it.

The following review of the Mach-box scheme outlines the method for determining planar aerodynamic influence coefficients for velocity potentials and applying them to obtain lifting pressures and generalized forces.

If we let the wing and diaphragm in the $\zeta = 0$ plane (Figure 4) be covered with a grid of boxes of length b in the flow direction and width b/β, their diagonals are then parallel to the Mach lines (Figure 5). The coordinate system has been nondimensionalized by the box length b so that the integral equation for the perturbation potential, Equation 28, at the point (x_1, y_1, z_1) becomes

$$\bar{\phi}^{U,L}(x_1, y_1, z_1) = \frac{b}{\beta} \iint_A H^{U,L}(\xi_1, \eta_i)\, G(\bar{\xi}_1, \bar{\eta}_1, \bar{\zeta}_1)\, d\eta_1,\ d\xi_1 \qquad (34)$$

where

$$G(\bar{\xi}_1, \bar{\eta}_1, \bar{\zeta}_1) = \frac{-1}{\pi R_1} \exp\left[-i\bar{k}\bar{\xi}_1\right] \cos\left(\frac{\bar{k}}{M_\infty} R_1\right)$$

$$R_1 = \sqrt{\bar{\xi}_1^2 - \bar{\eta}_1^2 - \bar{\zeta}_1^2}$$

$$\bar{k} = \frac{\omega b}{U_\infty} \frac{M_\infty^2}{\beta^2}$$

and the new coordinates are

$$x_1, \xi_1 = \frac{x, \xi}{b} \quad ; \qquad y_1, \eta_1 = \frac{y, \eta}{b} \quad ; \qquad z_1, \zeta_1 = \frac{z, \zeta}{b}$$

$$\bar{\xi}_1 = x_1 - \xi_1 \quad ; \qquad \bar{\eta}_1 = y_1 - \eta_1 \quad ; \qquad \bar{\zeta}_1 = z_1 - \zeta_1$$

Now, assume that the strength distribution over each rectangular portion of the superposed source sheets is constant and equal to the value at the box center, and consider a sending area or Mach box to be centered at

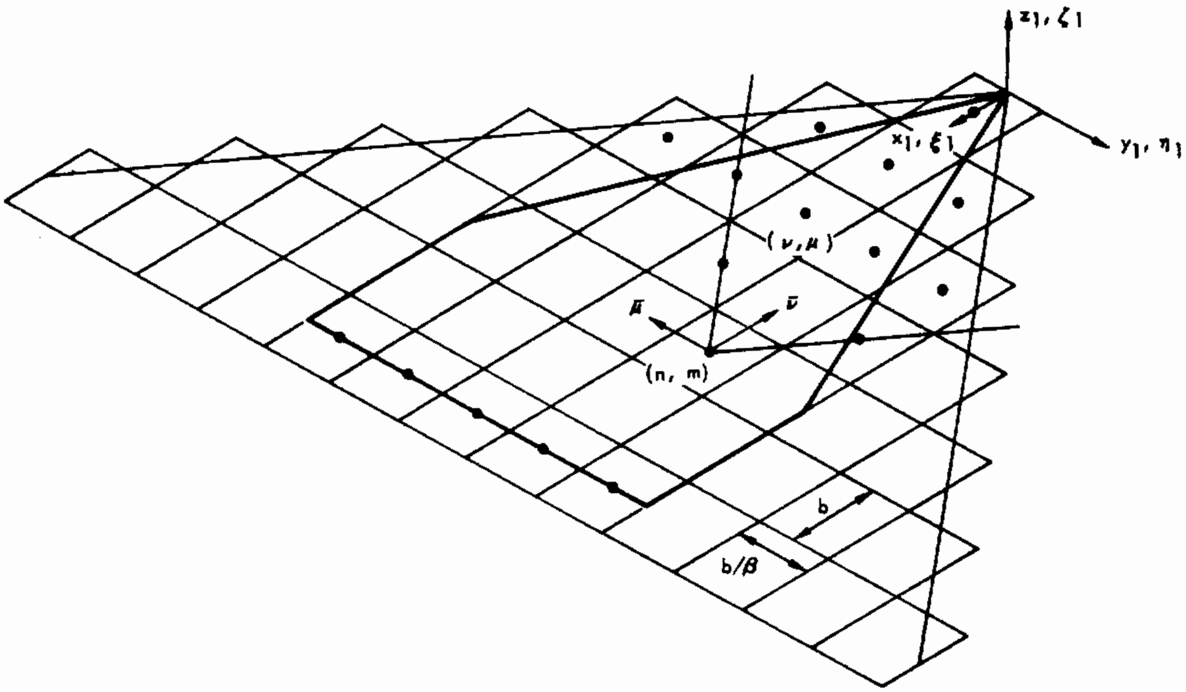


Figure 5. Boxes on Figure 4

the point $\xi_1 = \nu$, $\eta_1 = \mu$, $\zeta_1 = 0$, while a receiving point is located at the position $x_1 = n$, $y_1 = m$, $z_1 = 0$. When the source strength, $H^{U,L}(\nu, \mu)$, is constant, the velocity potential at the receiving point is

$$\bar{\phi}^{U,L}(n, m, 0) = \frac{b}{\beta} \sum H^{U,L}(\nu, \mu)\, \Phi(\bar{\nu}, \bar{\mu}, 0) \tag{35}$$

where $\Phi(\bar{\nu}, \bar{\mu}, 0)$ is the planar velocity potential influence coefficient (VPIC) at the receiving point, due to a source box of unit strength with the center located $\bar{\nu} = n - \nu$ box lengths ahead and $\bar{\mu} = m - \mu$ box widths to the left. The VPIC is dependent only on the relative distances $\bar{\nu}$ and $\bar{\mu}$, and the Mach number and reduced frequency parameters. The summation in Equation 35 is extended over all boxes wholly or partially within the fore Mach cone (Figure 5). This then divides the area of integration in Equation 34 into several subareas over which the planar VPIC may be evaluated using the equation

$$\Phi(\bar{\nu}, \bar{\mu}, \ell) = \iint_{A_{box}} G(\bar{\xi}_1, \bar{\eta}_1, \ell)\, d\bar{\eta}_1, d\bar{\xi}_1 \tag{36}$$

where $\ell = 0$ and where $\bar{\xi}_1 = n - \xi_1$ and $\bar{\eta}_1 = m - \eta_1$. When a point lies just above

When a point lies just above a source sheet, the only perturbation velocity there is due to the outwardly directed strength at the adjacent point on the sheet.

$$H^U(n, m) = \frac{D}{Dt}\, \bar{Z}(n, m) \tag{37}$$

and

$$H^L(n, m) = -\frac{D}{Dt}\, Z(n, m) \tag{38}$$

On the other hand, the strength distribution over a diaphragm box not in a wake must be determined by application of the condition that the velocity potential is continuous across the diaphragm. This condition is satisfied by summing Equation 35 over all upper and lower boxes with known source strengths in the fore cone of the receiving point and over the upper and lower sides of the box of unknown source strength at the receiving point and setting the total equal to zero. Solution for the unknown source strength gives:

$$\Delta H(n, m) = -\Sigma\, \Delta H(\nu, \mu)\, \Phi(\bar{\nu}, \bar{\mu}, 0)/\Phi(0, 0, 0) \tag{39}$$

because continuity of velocity through the diaphragm means that the upper and lower source strengths are equal in magnitude and opposite in sign, Equation 39 can be written

$$H^{U,L}(n, m) = \mp \Sigma\, H^{U,L}(\nu, \mu)\, \Phi(\bar{\nu}, \bar{\mu}, 0)/\Phi(0, 0, 0) \tag{40}$$

where the minus or plus signs of Equation 40 are to be used with the superscripts U or L, respectively.

Calculation of the velocity potential discontinuity across the airfoil surface may now be performed by applying to the appropriate boxes the relationships given in Equations 35, 37, 38, and 40. To eliminate the need for solving sets of simultaneous equations for the unknown strength distribution over the diaphragm, the following rules for calculation are followed:

1. Starting at the foremost centerline box, calculate the change in velocity potential across the surface at all box centers in the first row

2. Return to the centerline box in the next row; calculate the velocity potential change at all surface box centers in that row and then the source strengths at box centers on the diaphragm

3. Repeat procedure 2 for each subsequent row of boxes

The resulting velocity potential distribution over the surface can then be substituted into Equation 17 to obtain the pressure difference coefficient, or into Equation 20 to obtain the generalized force.

INTERSECTING PLANAR SURFACES

The following development parallels that of Ashley's (References 6, 11, and 12) and presents the results in a form amenable to high-speed computational techniques.

The success of the extension of the source superposition method to intersecting surfaces depends heavily on careful application of the diaphragm concept to regions between subsonic edges and Mach envelopes. Before developing the mathematical expressions for the necessary source strength distributions and resulting velocity potentials over these surfaces, the following general statements are made concerning the construction of diaphragm regions:

1. A diaphragm is a device used to ensure continuity of pressure and velocity between every pair of adjacent points in the flow field except those on opposite sides of lifting surfaces. When adjacent points are not on opposite sides of a surface, or the wake of a surface, the stronger condition of continuity of velocity potential replaces the condition of continuity of pressure. In particular, the diaphragm is placed between all pairs of adjacent points that receive signals from some pair of opposite half-envelopes.

2. There is no unique set of diaphragms for a particular configuration and Mach number. Any set which provides the required continuity of pressure and stream lines is acceptable. Diaphragms do not necessarily have to be coplanar with any lifting surface to meet this requirement; furthermore, diaphragms do not even have to be planar surfaces.

3. The concept is easiest to envision and much easier to apply if diaphragms are parallel to the free-stream direction.

Also, the following general statements and definitions are made concerning interference regions:

1. A portion of a surface is in an interference region when it is within the Mach envelope of another surface.

2. Mutual interference exists between two surfaces when a portion of each surface lies within the Mach envelope of the other surface.

The basic difference between application of the source superposition method to isolated planar lifting surfaces and to a collection of intersecting lifting surfaces lies in the way in which the strength distribution of the various superposed source sheets is determined. In both cases, the strength is determined from either the tangential flow or continuous flow boundary conditions.

When a point that lies just above a source sheet is not in a region of interference, the only perturbation velocity there is due to the outwardly directed strength at the adjacent point in the sheet; however, if the point is in an interference region it will not only be within the half-envelope associated with its source sheet, but also within the upper or lower half-envelopes associated with one or more of the other surfaces. In addition to the velocity due to the point source, velocities are induced there by outwardly directed disturbances on portions of the other source sheets. The portions of these source sheets referred to will be those areas within the Mach hyperbola formed on each sheet by its intersection with the fore Mach cone from the receiving point.

The strength of the local source sheet is still unknown and is designated $H^U(x, y)$. On the other hand, the strengths of the other source sheets are presumed to be known and, therefore, the induced normal velocity at the receiving point, $\partial\overline{\phi}/\partial z$, may be determined. The induced velocity potential is $\overline{\phi}$. The sum of all the perturbation velocities in the direction of the outwardly directed normal to the upper surface may be written as

$$\overline{w}^U(x, y) = H^U(x, y) + \left(\frac{\partial\overline{\phi}}{\partial z}\right)^U \tag{41}$$

The outwardly directed normal on the lower side of the surface is opposite in sense to that on the upper side. Since the two sides move together the velocity of the lower side, in the direction of the outwardly directed normal on that side, is the negative of its upper counterpart. Thus, $\bar{w}^L(x, y) = -\bar{w}^U(x, y)$. The source strength at the lower point is still unknown and not necessarily proportional to its upper counterpart. To complete the description of the normal velocity at the lower point, the induced velocities in the direction of the lower normal are $\partial\bar{\phi}^L/\partial n^L = -\partial\bar{\phi}^L/\partial z$. Thus, the expression for the sum of all the perturbation velocities normal to the lower surface becomes, after multiplying both sides by -1.0,

$$\bar{w}^U(x, y) = -H^L(x, y) + \left(\frac{\partial\bar{\phi}}{\partial z}\right)^L \tag{42}$$

The superscripts on w in Equations 41 and 42 may now be dropped and the unknown local source strengths on the upper and lower parts of the surface may be determined separately.

All portions of each surface, whether or not they are in the interference region, will have flow tangent to both upper and lower sides if the perturbation velocity at the surface in the positive Z_i direction is set equal to the velocity of the mean surface in that direction,

$$\bar{w}(x_i, y_i) = \frac{D}{Dt}\bar{Z}_i(x_i, y_i) \tag{43}$$

where the coordinates x_i, y_i, z_i* are placed so that the ith surface lies near the $z_i = 0$ plane and Z_i represents its variation from that plane. For the region covering the ith intersecting lifting surface, the ith upper source-sheet strength is given by substitution of Equation 41 into Equation 43,

$$\frac{D}{Dt}\bar{Z}_i(x_i, y_i) = H^U(x_i, y_i) + \left(\frac{\partial\bar{\phi}}{\partial z_i}\right)^U \tag{44}$$

and the ith lower source-sheet strength is given by substitution of Equation 44 into Equation 43

$$\frac{D}{Dt}\bar{Z}_i(x_i, y_i) = -H^L(x_i, y_i) + \left(\frac{\partial\bar{\phi}}{\partial z_i}\right)^L \tag{45}$$

The velocity potential, $\bar{\phi}$, at any point may be determined by adding contributions induced there by the various source sheets associated with the Mach half-envelopes within which the point lies. For a point on the ith upper

*The subscripts i, j are a convenient way to distinguish the ith receiving point or plane from the j th sending point or plane and are not to be confused with the subscripts used for normalization of coordinates.

surface, the velocity potential is due to all the disturbances on the ith upper source sheet within the fore Mach triangle as well as those disturbances from upstream out-of-plane source sheets, i.e.,

$$\bar{\phi}^U(x_i,\ y_i) = \frac{1}{\beta} \iint_{A_i} H^U(\xi_i,\ \eta_i)\ G\ (\bar{\xi}_i,\ \bar{\eta}_i,\ \bar{\zeta}_i)\ d\eta_i\ d\xi_i + \left[\frac{1}{\beta} \sum_j \iint_{A_j} H^{U,L}(\xi_j,\ \eta_j)\ G\ (\bar{\xi}_j,\ \bar{\eta}_j,\ \bar{\zeta}_j)\ d\eta_j d\xi_j\right]^U \tag{46}$$

where $H^{U,L}(\xi_j,\ \eta_j)$ is the jth upper or lower source sheet strength distribution within the area, A_j, that affects the point $(x_i,\ y_i)$. G is the velocity potential at the point $(x_i,\ y_i)$ due to a source of unit strength located at a sending point $(\xi_i,\ \eta_i,\ \zeta_i)$ upstream. In addition to its parametric dependence upon the free-stream Mach number and frequency of oscillation, G is dependent only on the relative positions of the receiving and sending points. The velocity potential at the point $(x_i,\ y_i)$ on the ith lower source sheet is given by

$$\bar{\phi}^L(x_i,\ y_i) = \frac{1}{\beta} \iint_{A_i} H^L(\xi_i,\ \eta_i)\ G\ (\bar{\xi}_i,\ \bar{\eta}_i,\ \bar{\zeta}_i)\ d\eta_i d\xi_i + \left[\frac{1}{\beta} \sum_j \iint_{A_j} H^{U,L}(\xi_j,\ \eta_j)\ G\ (\bar{\xi}_j,\ \bar{\eta}_j,\ \bar{\zeta}_j)\ d\eta_j d\xi_j\right]^L \tag{47}$$

where the area, A_i, is the Mach triangle on the ith lower source sheet. The second term on the right side of Equations 46 and 47 represents the velocity potential induced at the point by the upstream disturbances.

For points on diaphragms that are not in the wake of any upstream disturbances, the condition for continuous velocity across the diaphragm is given by Equation 26 which, upon substitution of Equations 41 and 42, becomes

$$0 = H^U(x_i, y_i) + H^L(x_i, y_i) + \left[(\frac{\partial \bar{\phi}}{\partial z})^U - (\frac{\partial \bar{\phi}}{\partial z})^L \right] \tag{48}$$

An additional equation is necessary to solve the unknown upper and lower source strengths at a point on the diaphragm. The condition of continuity of velocity potential across the diaphragm is obtained by substitution of Equations 47 and 46 into Equation 27 to get

$$\iint_{A_i} \Delta H(\xi_i, \eta_i)\, G(\bar{\xi}_i, \bar{\eta}_i)\, d\eta_i d\xi_i = 0$$

$$= -\sum_j \iint_{A_j} \Delta H^{U,L}(\xi_j, \eta_j)\, G(\bar{\xi}_j, \bar{\eta}_j, \bar{\zeta}_j) d\eta_j d\xi_j \tag{49}$$

Once the velocity potential distribution over the surface is obtained, the pressure loading and generalized forces may be obtained by substituting the discontinuity in velocity potential $\Delta\phi = \bar{\phi}^U - \bar{\phi}^L$, across the surface into Equations 17 and 20, respectively.

A Discussion of Possible Approaches

The previous discussion implies that a single diaphragm may be associated with two or more lifting surfaces. As an example, consider a thin wing with foldable tips and subsonic leading edges (Figure 6). As the tips fold from 0 to 90 degrees, the diaphragm might also be folded and then extended to a hyperbolic intersection with the Mach envelope of the wing (Figure 7). The diaphragms that are coplanar with the folded tips actually

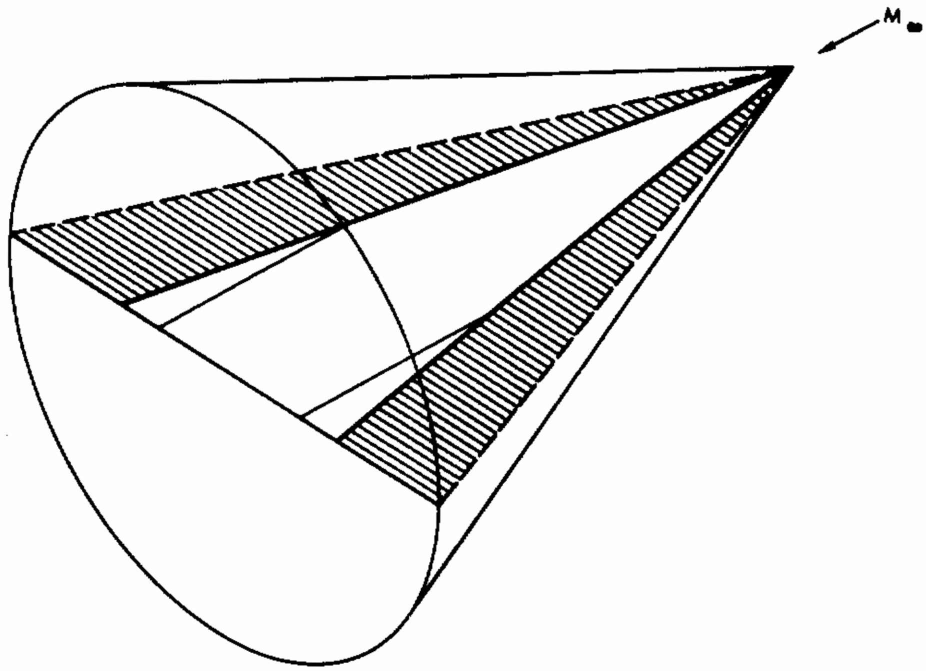


Figure 6. Planar Wing With Tips at 0-Degree Fold

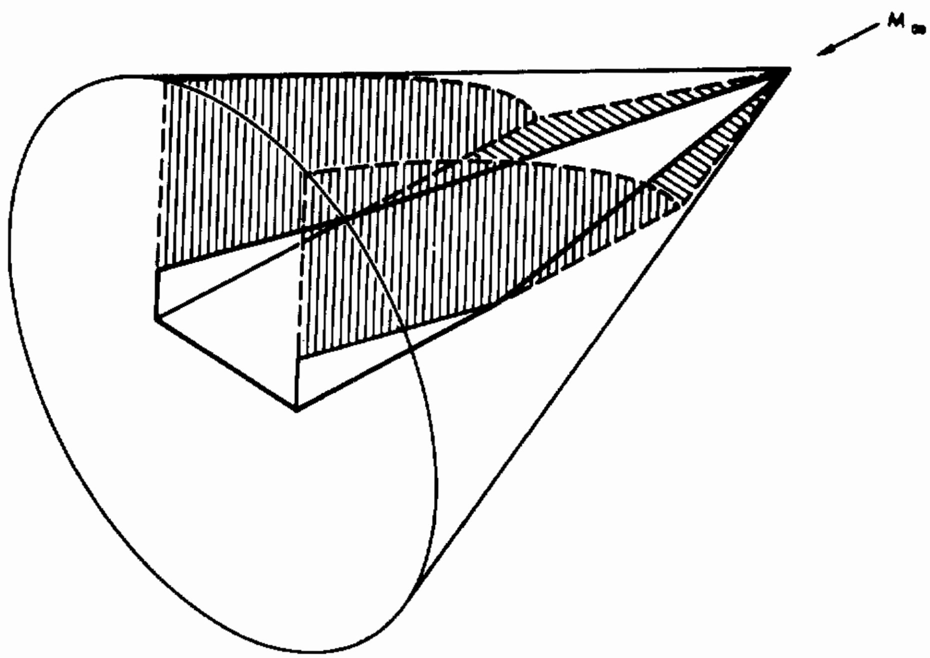


Figure 7. Planar Wing With Tips at 90-Degree Fold

serve to complete isolation of the upper and lower sides of the wing as well as both sides of the tip. The flow is divided into two regions, and points in one region are not influenced by disturbances in the other region except through the diaphragm regions which are now nonplanar and will require three-dimensional calculations. Extending diaphragms until they intersect with the largest Mach envelope not only causes an unnecessarily large interference region between source sheets but also makes the area of the diaphragm dependent upon the angle of intersection between the surfaces.

Another approach is one in which the diaphragm remains fixed in the plane of the wing as the tip folds from zero degrees. To complete the isolation of the upper surface from the lower surface with a small amount of out-of-plane diaphragm, a diaphragm is placed between the leading edge of the tip and the wing diaphragm (Figure 8). This diaphragm is not parallel with the free-stream direction which is permissible as long as the replacement source sheets do not alter the steady-state direction of the free stream. The nonparallel-to-flow diaphragm does complicate the three-dimensional calculations required for out-of-plane source sheet interference. While this diaphragm construction represents an apparent minimum in out-of-plane calculations, its area and angle with the flow are both dependent upon the angle of intersection of the two surfaces. These problems together with the numerical difficulties associated with the three-dimensional construction offset any advantages obtained by this approach.

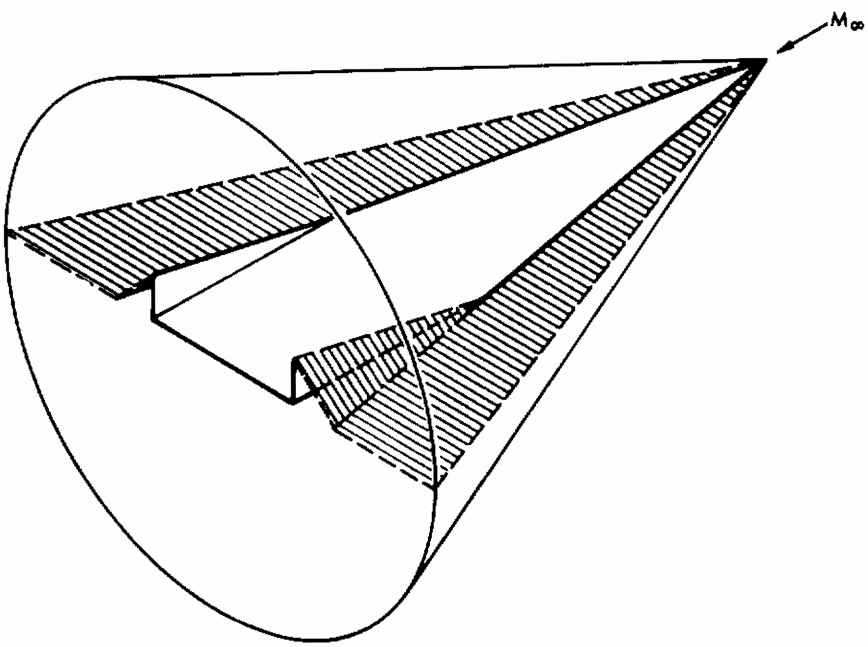


Figure 8. Alternate Diaphragm Construction

Another approach which is also the result of trying to reduce the diaphragm area to a practical minimum is one step removed from the approach suggested for future development. In this approach the several intersecting surfaces are separated into isolated planar regions and a Mach envelope is constructed for each surface. After superposing source sheets on the upper and lower sides of the surface, each sheet is extended to intersect the associated Mach envelope. If the surfaces are actually isolated, the disturbances from each upper or lower source sheet will be confined to the respective half-envelope (Figure 9). The configuration is then reassembled and the relationship between each surface and its Mach envelope is retained. (Figure 10)

At this point a step is performed which is shown later to increase rather than decrease the number of required numerical operations. Since one region may serve to isolate the upper and lower sides of more than one surface, it may be that some of the diaphragms are unnecessary when the surfaces are rejoined. They will be unnecessary and may be removed if continuity of pressure and velocity can be ensured without them.

To demonstrate this approach consider the wing with folded tips to be separated into three isolated surfaces. Figure 9 shows the three surfaces, their Mach envelopes, and their diaphragm regions for a particular Mach number. When the two tips are reattached to the wing, the tip diaphragms in the lower half-envelope of the wing may be folded into the wing diaphragm. This particular set of diaphragms is shown in Figure 11, where the tips are folded to 90 degrees.

With this set of boundaries, a point on the upper surface of the wing receives from the upper parts of the tip source sheets as well as from the upper part of the wing source sheets. The adjacent point on the lower surface of the wing receives only from the lower parts of the wing source sheet. The boundary condition applied on each side of the wing surface is that of tangential flow.

A point on either side of the tip surface receives from the upper part of the wing source sheets as well as from the tip source sheets on its side. Finally, a point on that portion of the upper surface of the wing diaphragm which is in the interference region receives from the lower parts of the tip source sheets as well as from the upper parts of the wing source sheets. Table 1 shows the sending and receiving relationships described above and shown in Figure 12.

This approach, described previously, is used in the computer program. Subsequent development has shown that it is not the most efficient approach but it does serve to demonstrate one of the options.

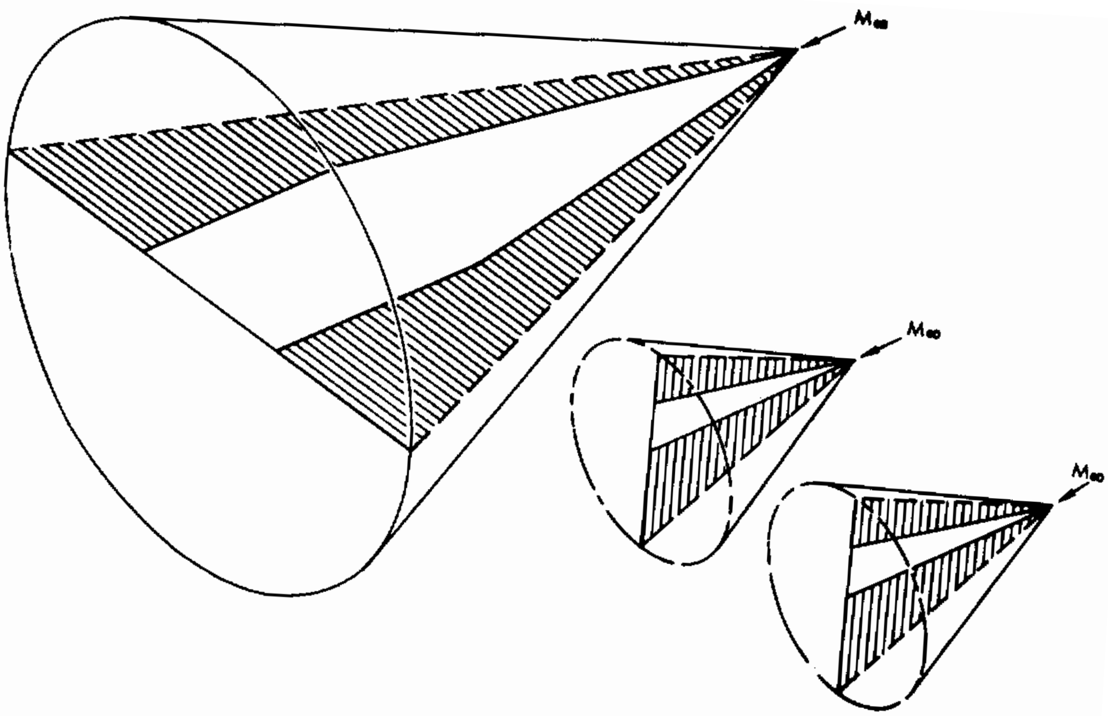


Figure 9. Wing and Tips Isolated

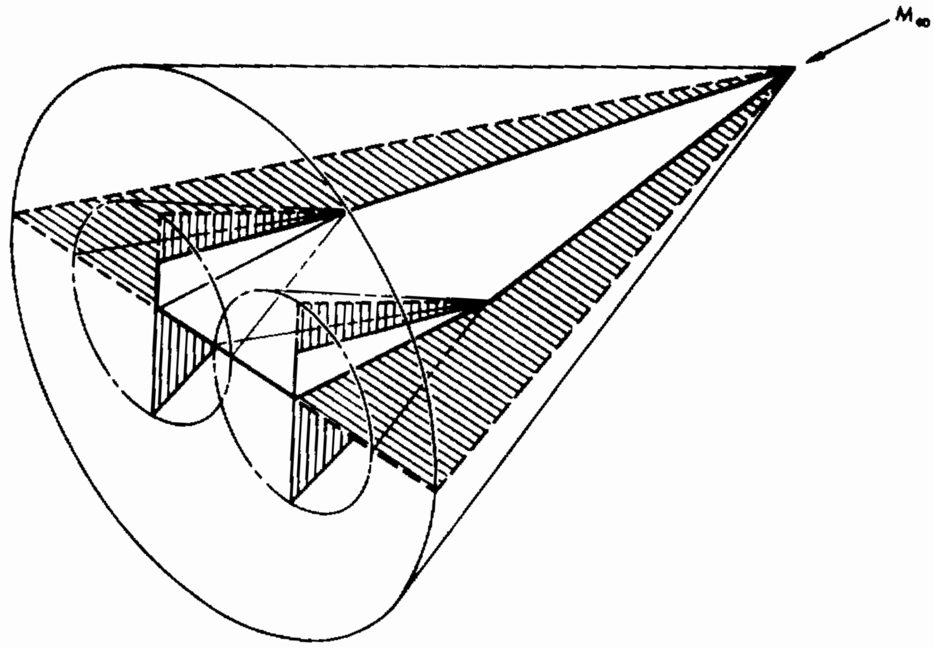


Figure 10. Wing and Tips Reattached

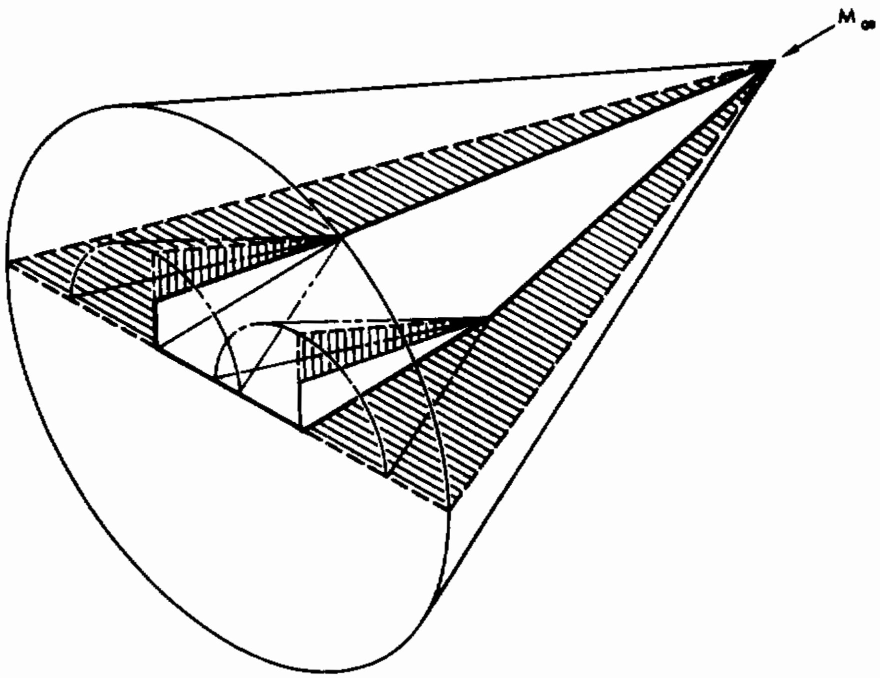


Figure 11. Wing and Tips Reattached - Lower Diaphragm Removed

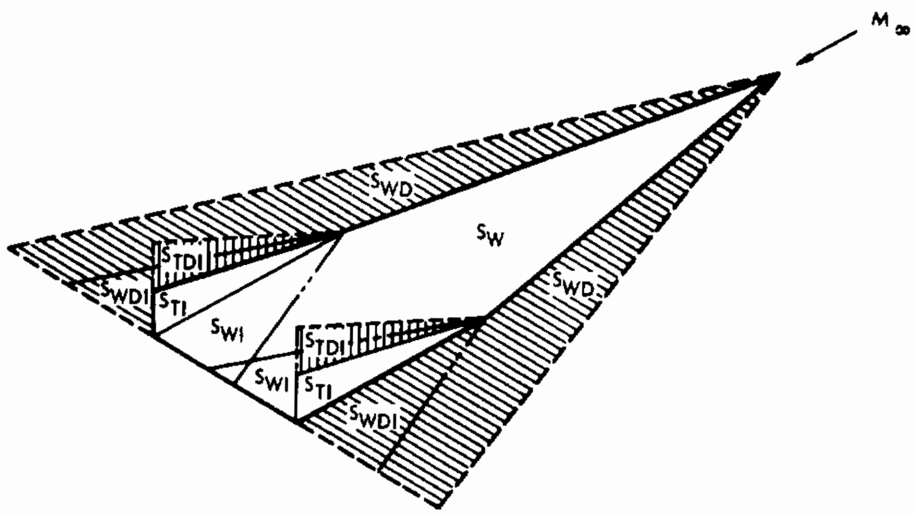


Figure 12. Zones on Figure 11

Table 1. Legend to Determine the Sending and Receiving Relationship Between Regions of the Configuration Shown in Figure 11

i \ j		UPPER				LOWER			
		SW & SWD	SWI	SWDI	STI & STDI	SW & SWD	SWI	SWDI	STI & STDI
UPPER	SW & SWD	0	0	0	0				0
UPPER	SWI & SWDI		0	0	0				0
UPPER	STI & STDI		0		0				
LOWER	SW & SWD					0	0	0	
LOWER	SWI & SWDI						0	0	
LOWER	STI & STDI			0					0

The Aerodynamic Influence Coefficient Method

The wing with symmetrically folded tips is analyzed and presented as an example of one of the more efficient of the possible approaches to the problem of intersecting lifting surfaces. The discussion concerns only one-half of the wing because the configuration has a plane of symmetry through the centerline.

We now overlay a grid of Mach boxes on the wing, tips, and associated diaphragms of the surfaces shown in Figure 12. The result is shown in Figure 13. Note that the grid is arranged so that box centers lie along the centerline and box edges coincide with the line of intersection of the wing and tip.

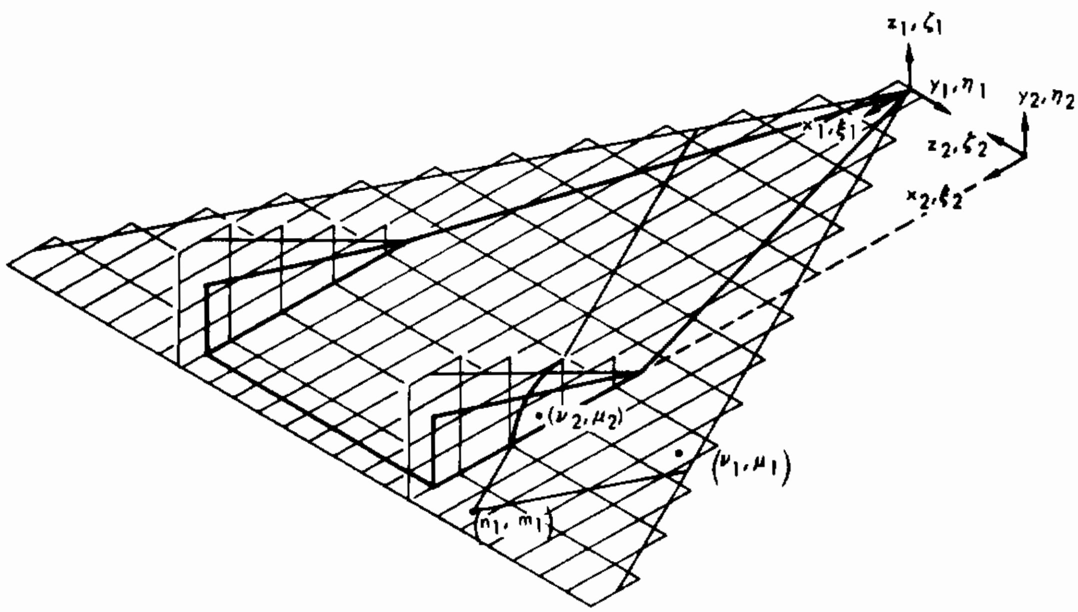


Figure 13. Boxes on Figure 12 and Area of Influence for (ν, μ)

The wing coordinate system (x_1, y_1, z_1) and the tip coordinate system (x_2, y_2, z_1) enable relative distances between sending and receiving boxes to be set up as follows:

Wing boxes (n_1, m_1) receiving from wing boxes (ν_1, μ_1)

$$\bar{\nu}_{11} = n_1 - \nu_1 \tag{50}$$

$$\bar{\mu}_{11} = m_1 - \mu_1$$

$$\ell_{11} = 0$$

Wing boxes (n_1, m_1) receiving from tip boxes (ν_2, μ_2)

$$\bar{\nu}_{12} = n_1 - \nu_2 \tag{51}$$

$$\bar{\mu}_{12} = -(y_{f\ell} - m_1) \cos \gamma - \mu_2$$

$$\ell_{12} = -(y_{f\ell} - m_1) \sin \gamma$$

Tip boxes (n_2, m_2) receiving from wing boxes (ν_1, μ_1)

$$\bar{\nu}_{21} = n_2 - \nu_1 \tag{52}$$

$$\bar{\mu}_{21} = m_2 \cos \gamma + (y_{f\ell} - \mu_1)$$

$$\ell_{21} = m_2 \sin \gamma$$

Tip boxes (n_2, m_2) receiving from tip boxes (ν_2, μ_2)

$$\bar{\nu}_{22} = n_2 - \nu_2 \tag{53}$$

$$\bar{\mu}_{22} = m_2 - \mu_2$$

$$\ell_{22} = 0$$

The fold angle, γ, is measured positive when the tips fold up from the wing plane and $y_{f\ell}$ is the y_1 distance to the fold line or x_2 axis.

The continuous strength distribution over the upper and lower source sheets on both tips and the wing is replaced by the discontinuous distribution that results when the source sheet on each box is assumed to have a constant strength equal to the value computed for its center. With this approximation and the information contained in Table 1, Equations (28), (46), and (47) are written for the upper and lower sides of the wing and tip surfaces as follows:

$$S_W:\ \bar{\phi}^U(n_1, m_1) = \frac{b}{\beta} \sum H^U(\nu_1, \mu_1)\, \Phi\, (\bar{\nu}_{11}, \bar{\mu}_{11}, 0) \tag{54}$$

$$S_W:\ \bar{\phi}^L(n_1, m_1) = \frac{b}{\beta} \sum H^L(\nu_1, \mu_1)\, \Phi\, (\bar{\nu}_{11}, \bar{\mu}_{11}, 0) \tag{55}$$

$$S_{WI}:\ \bar{\phi}^U(n_1, m_1) = \frac{b}{\beta} \sum H^U(\nu_1, \mu_1)\, \Phi\, (\bar{\nu}_{11}, \bar{\mu}_{11}, 0) \tag{56}$$

$$+ \frac{b}{\beta} \sum H^U(\nu_2, \mu_2)\, \Phi\, (\bar{\nu}_{12}, \bar{\mu}_{12}, \ell_{12})$$

$$\bar{\phi}^L(n_1, m_1) = \frac{b}{\beta} \sum H^L(\nu_1, \mu_1)\, \Phi\, (\bar{\nu}_{11}, \bar{\mu}_{11}, 0) \tag{57}$$

$$S_{TI}:\ \bar{\phi}^U(n_2, m_2) = \frac{b}{\beta} \sum H^U(\nu_2, \mu_2)\, \Phi\, (\bar{\nu}_{22}, \bar{\mu}_{22}, 0) \tag{58}$$

$$+ \frac{b}{\beta} \sum H^U(\nu_1, \mu_1)\, \Phi\, (\bar{\nu}_{21}, \bar{\mu}_{21}, \ell_{21})$$

$$\bar{\phi}^L(n_2, m_2) = \frac{b}{\beta} \sum H^L(\nu_2, \mu_2)\, \Phi\, (\bar{\nu}_{22}, \bar{\mu}_{22}, 0) \tag{59}$$

$$+ \frac{b}{\beta} \sum H^U(\nu_1, \mu_1)\, \Phi\, (\bar{\nu}_{21}, \bar{\mu}_{21}, \ell_{21})$$

The sums are over all boxes that influence the receiving point. An example of the sending areas for a typical receiving point on the tip is shown in Figure 13.

The source strengths on those boxes outside the interference region are determined by the relationships given in the preceding section. However, when a box is within the interference region, the strength distribution is modified to account for the interaction with other boxes. In this way, the mutual interaction effects will be properly represented in the evaluation of the velocity potentials.

In anticipation of the need to calculate velocities induced at points above or below the sending plane, we derive here the formulas for velocity components normal and parallel to the plane of disturbances. When the source strengths over the areas of sending boxes are constant, these induced velocities are

$$\underset{\text{induced}}{\overline{w}(n,\ m,\ \ell)} = \sum H^{U,L}(\nu, \mu)\ W\ (\overline{\nu},\ \overline{\mu},\ \ell) \tag{60}$$

and

$$\underset{\text{induced}}{\overline{v}(n,\ m,\ \ell)} = \sum H^{U,L}(\nu,\ \mu)\ V\ (\overline{\nu},\ \overline{\mu},\ \ell) \tag{61}$$

The W and V are derived in Appendix I. The sum is over all the boxes partially or wholly within the forward Mach hyperbola associated with the receiving point (n, m, ℓ) and the strength is on the upper or lower sheet depending on the sign of ℓ.

The source strengths at the box centers on the tip and on the portion of the wing within the interference region are corrected using Equations 44 and 45 in the following forms:

$$S_{wI}: \frac{D}{Dt}\overline{Z}_1\ (n_1, m_1) = H^U(n_1, m_1) + \sum\ H^U(\nu_2,\ \mu_2)\ N\ (\overline{\nu}_{12},\ \overline{\mu}_{12},\ \ell_{12}) \tag{62}$$

$$S_{TI}: \frac{D}{Dt}\overline{Z}_2\ (n_2, m_2) = H^U(n_2, m_2) + \sum\ H^U(\nu_1,\ \mu_1)\ N\ (\overline{\nu}_{21}, \overline{\mu}_{21}, \ell_{21}) \tag{63}$$

$$\frac{D}{Dt}\overline{Z}_2\ (n_2, m_2) = -\ H^L\ (n_2, m_2) + \sum\ H^U(\nu_1, \mu_1)\ N(\overline{\nu}_{21}, \overline{\mu}_{21}, \ell_{21}) \tag{64}$$

where the velocity influence coefficient (VIC) is written

$$N\ (\overline{\nu}, \overline{\mu}, \ell) = W\ (\overline{\nu}, \overline{\mu}, \ell)\ \cos\gamma\ +\ V\ (\overline{\nu}, \overline{\mu}, \ell)\ \sin\gamma \tag{65}$$

The VIC gives the component of velocity normal to a source sheet at a Mach box center. The magnitude of the velocity is that which is induced there by an out-of-plane Mach box $\bar{\nu}$ box lengths ahead, $\bar{\mu}$ box widths to the port side, and ℓ box widths down. The source strength at the sending box is unity and the dihedral angle between the two planes is γ.

The source strengths on boxes on the wing and tip diaphragms that are within the interference region must also be adjusted for interaction effects. This correction has to be done in such a way as to preserve the continuity of the normal velocity at the diaphragm centers. The perturbation velocity at the center of the diaphragm boxes is given by

$$S_{WDI:}\ \bar{w}\,(n_1, m_1) = H^U(n_1, m_1) + \sum H^L(\nu_2, \mu_2)\, N\,(\bar{\nu}_{12}, \bar{\mu}_{12}, \ell_{12}) \tag{66}$$

$$S_{TDI:}\ \bar{w}\,(n_2, m_2) = H^U(n_2, m_2) + \sum H^U(\nu_1, \mu_1)\, N\,(\bar{\nu}_{21}, \bar{\mu}_{21}, \ell_{21}) \tag{67}$$

$$\bar{w}\,(n_2, m_2) = -\,H^L(n_2, m_2) + \sum H^U(\nu_1, \mu_1)\, N\,(\bar{\nu}_{21}, \bar{\mu}_{21}, \ell_{21}) \tag{68}$$

To ensure continuity of the normal velocity across those diaphragm boxes that are in the interference region, substitute Equations 66 and 24 into Equation 26 to get

$$0 = H^u(n_1, m_1) + H^L(n_1, m_1) + \sum H^L(\nu_2, \mu_2)\, N(\nu_{12}, \mu_{12}, \ell_{12}) \tag{69}$$

for the wing diaphragm and substitute Equations 67 and 68 into Equation 26 to get

$$0 = H^U(n_2, m_2) + H^L(n_2, m_2) \tag{70}$$

for the tip diaphragm. There is no net interference effect across the tip diaphragm because it is completely within the wing Mach envelope, and is, therefore, exposed on both sides to the same portions of the wing and wing diaphragm.

The condition of zero jump in velocity potential across the wing and tip diaphragms can be applied to the portions in the interference region because neither is in the wake of the other. This condition can be easily written by applying the approximation of constant strength at box centers on the wing diaphragm to Equation 49. After separating out the n_1, m_1 terms we get

$$\Delta H(n_1, m_1) = -\sum \Delta H(\nu_1, \mu_1)\ \Phi\ (\bar{\nu}_{11}, \mu_{11}) / \ \Phi\ (0,0)$$

$$- \sum H^L(\nu_2, \mu_2)\ \Phi\ (\bar{\nu}_{12}, \mu_{12}, \ell_{12}) / \Phi\ (0, 0) \qquad (71)$$

A similar relationship can be obtained for the tip diaphragm by approximating Equation 49 and isolating the n_2, m_2 terms,

$$\Delta H(n_2, m_2) = -\sum \Delta H(\nu_2, \mu_2)\ \Phi\ (\bar{\nu}_{22}, \bar{\mu}_{22}) / \Phi\ (0, 0) \qquad (72)$$

Here again, there is no net interference effect across the tip diaphragm. This result is consistent with the result observed in Equation 70.

4. COMPUTER PROGRAM

USE AND LIMITATIONS

An IBM 7094 Fortran IV language computer program is presented which calculates supersonic unsteady aerodynamic forces on a wide variety of wings with symmetrical folded tips. The calculation procedure is based on the source superposition method that has been extended to account for the interference effects between intersecting lifting surfaces. The Mach box approximation is employed to reduce the integral equations to sums of constant values of source strength at box centers times certain integrals dependent upon relative position, Mach number, and reduced frequency. These integrals are the aerodynamic influence coefficients which express the velocity potential influence coefficient (VPIC) or velocity influence coefficient (VIC) induced at a receiving box center by a unit strength source sheet covering the area of influence of a sending box. The VPIC's and VIC's are developed and presented in the form used for programming in Appendix I.

The configuration to be analyzed must have a plane of symmetry, wing and tip leading edges that are not swept forward, and supersonic trailing edges. The surfaces may have any small angle of attack or camber distribution or may be oscillating in an arbitrary mode of rigid or elastic vibration. Each tip may have a side edge, subsonic leading edge, and fore or aft swept trailing edge. The wing may also have a subsonic leading edge and fore or aft swept trailing edge.

The program has been specifically designed to calculate generalized forces to be used in determination of steady-state lift and moment coefficients, oscillatory stability derivatives, gust loads and aeroelastic stability. Also computed are source strength distributions corrected for interference effects and velocity potential distributions over the entire surface. The velocity potential values are used to determine the generalized forces and may also be employed to calculate surface pressure or pressure difference distributions over the configuration. For one run, these quantities are calculated for one supersonic Mach number, up to 20 reduced frequencies (steady-state loadings result when the oscillatory frequency is zero for a nonzero mode shape) and up to 10 modes of deflection.

The Mach box approximation is achieved by overlaying the surfaces and diaphragms with a grid of rectangular boxes with chordwise length b and spanwise width, b/β, which makes the diagonals parallel to Mach lines.

The program will subdivide the source sheets into as many as 20 chordwise and 30 spanwise boxes with the condition that box edges lie along the fold line. Past experience indicates that 12 chordwise boxes adequately defines the motion at low Mach numbers and as few as 8 boxes along the chord will in some cases describe an arbitrary vibration mode.

The method of isolating the upper and lower sides with diaphragms used in the computer program is shown in Figure 11. The program was written for a set of diaphragms that are a practical minimum in area and it was not discovered until after completion that the more general diaphragm construction shown in Figure 10 greatly simplifies the algebra and logic at only a slight increase in diaphragm area. Consequently, the equations used in the program are more complicated and will be listed here for completeness along with the pictures of the actual diaphragm regions used.

The folded tips wing and diaphragms actually used in the program are shown in Figure 14 with the zones of influence identified. The portions of the wing diaphragms outboard of the tips in the zero-degree fold position do not affect any portion of the surfaces and are, therefore, eliminated. The Mach box overlay for this configuration is shown in Figure 15. The following relative position coordinates will be used in the influence coefficients

$$\bar{\nu}_{11} = n_1 - \nu_1;\ \bar{\mu}_{11} = m_1 - \mu_1;\ \ell_{11} = 0$$

$$\bar{\nu}_{12} = n_1 - \nu_2;\ \bar{\mu}_{12} = (m_1 - y_{f\ell}) \cos \gamma - \mu_2;\ \ell_{12} = (m_1 - y_{f\ell}) \sin \gamma$$

$$\bar{\nu}_{21} = n_2 - \nu_1;\ \bar{\mu}_{21} = m_2 \cos \gamma - (\mu_1 - y_{f\ell});\ \ell_{21} = - m_2 \sin \gamma$$

$$\bar{\nu}_{22} = n_2 - \nu_2;\ \bar{\mu}_{22} = m_2 - \mu_2;\ \ell_{22} = 0$$

where γ is the fold angle positive tips up and $y_{f\ell}$ is the distance from the wing centerline to the tip foldline.

The equations for velocity potential difference are

$$S_W:\ \Delta\phi(n_1, m_1) = \frac{b}{\beta} \sum \Delta H(\nu_1, \mu_1)\ \Phi\ (\bar{\nu}_{11}, \bar{\mu}_{11}) \tag{73}$$

$$S_{WI}:\ \Delta\phi(n_1, m_1) = \frac{b}{\beta} \sum \Delta H(\nu_1, \mu_1)\ \Phi\ (\bar{\nu}_{11}, \bar{\mu}_{11}) \tag{74}$$

$$+ \frac{b}{\beta} + \sum H^U(\nu_2, \mu_2)\ \Phi\ (\bar{\nu}_{12}, \bar{\mu}_{12}, \ell_{12})$$

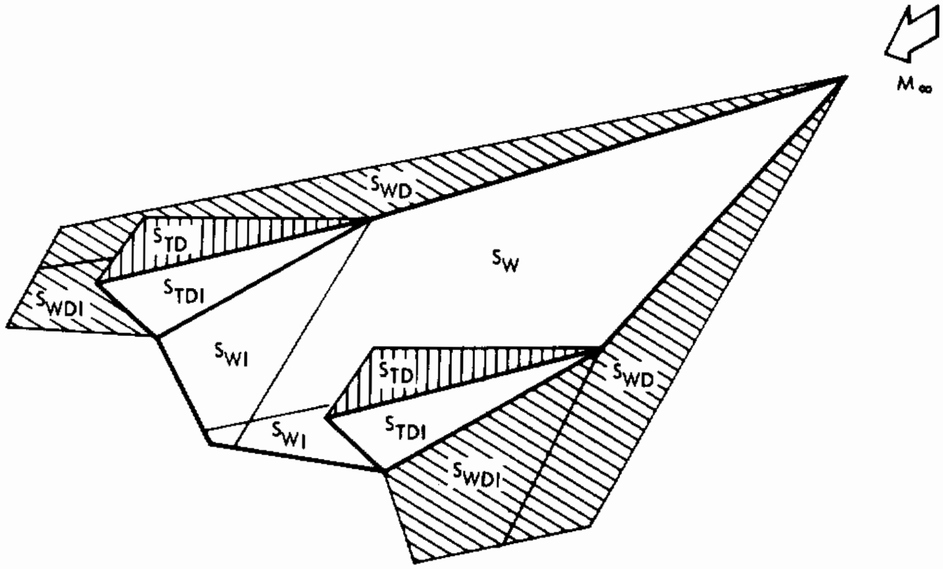


Figure 14. Zones on General Case In Program

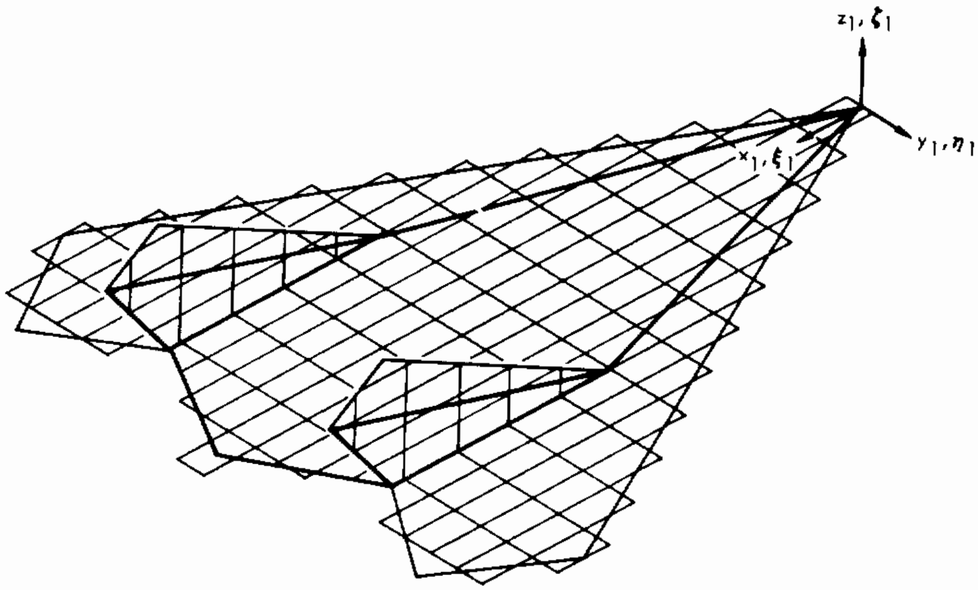


Figure 15. Boxes on General Case In Program

$$S_{TI}: \Delta\phi(n_2, m_2) = \frac{b}{\beta} \sum \Delta H(\nu_2, \mu_2)\ \Phi\ (\bar{\nu}_{22}, \bar{\mu}_{22}) \tag{75}$$

The source strengths at the surface box centers are given by

$$S_w: \frac{D}{Dt} Z\ (n_1, m_1) = H^U\ (n_1, m_1) \tag{76}$$

$$\frac{D}{Dt} Z\ (n_1, m_1) = -\ H^L(n_1, m_1) \tag{77}$$

$$S_{wI}: \frac{D}{Dt}\ Z(n_1, m_1) = H^U(n_1, m_1) + \sum H^U(\nu_2,\ \mu_2)\ N\ (\bar{\nu}_{12},\ \bar{\mu}_{12},\ \ell_{12}) \tag{78}$$

$$\frac{D}{Dt} Z\ (n_1, m_1) = -\ H^L(n_1, m_1) \tag{79}$$

$$S_{TI}: \frac{D}{Dt} Z\ (n_2, m_2) = H^U(n_2, m_2) + \sum H^U(\nu_1, \mu_1)\ N\ (\bar{\nu}_{21}, \bar{\mu}_{21}, \ell_{21}) \tag{80}$$

$$\frac{D}{Dt}\ Z(n_2, m_2) = -\ H^L(n_2, m_2) + \sum H^U(\nu_1, \mu_1)\ N\ (\bar{\nu}_{21}, \bar{\mu}_{21}, \ell_{21}) \tag{81}$$

where $N(\bar{\nu}, \bar{\mu}, \ell) = W\ (\bar{\nu}, \bar{\mu}, \ell) \cos\gamma + V\ (\bar{\nu}, \bar{\mu}, \ell) \sin\gamma$ and the VIC's W and V are given in Appendix I.

The diaphragm source strength differences are calculated from

$$S_{wD}: \Delta H(n_1, m_1) = -\sum \Delta H(\nu_1, \mu_1)\ \Phi\ (\bar{\nu}_{11}, \bar{\mu}_{11}) / \Phi\ (0,0) \tag{82}$$

$$S_{wDI}: \Delta H(n_1, m_1) = -\sum \Delta H(\nu_1, \mu_1)\ \Phi\ (\bar{\nu}_{11}, \bar{\mu}_{11}) / \Phi\ (0,0)$$

$$-\sum\ H^L(\nu_2, \mu_2)\ \Phi\ (\bar{\nu}_{12}, \bar{\mu}_{12}, \ell_{12}) / \Phi\ (0,0) \tag{83}$$

$$S_{TDI}: \Delta H(n_2, m_2) = -\sum \Delta H(\nu_2, \mu_2)\ \Phi\ (\bar{\nu}_{22}, \bar{\mu}_{22}) / \Phi\ (0,0) \tag{84}$$

These equations reflect the slight differences between the program and the general method. The outline presented below details the logical and computative operations of the program. Also provided is a flow chart and list of sequential operations that parallel the isolated surface Mach box technique. The rules for computation given in Section 2 are for the general case, whereas the outline given below is for the special case of a wing with symmetrical folded tips.

THE LOGICAL FLOW

The computer program, MBX, consists of an executive or main program with several subroutine subprograms. The purpose of some of the subprograms is purely logical or decision making while other subprograms are developed for the many repetitive calculations necessary in this type of program. A flow chart with descriptive statements is shown in Figure 16 and will be used as reference in this outline. A complete set of program listings in Fortran IV language is presented in Appendix IV.

The main program contains all the input and output statements with their associated format statements. Tape 5 is used for input and Tape 6 is used for output in the version of the program included in the listings.

After the data arrays are initialized, the main program, MBX, reads the data describing the flight conditions, the configuration geometry, the number of frequencies and modes, and the number of boxes to be fitted in the chordwise direction. Also determined, at this point, is whether the wing and tip mode shapes are represented by polynomial coefficients or deflection patterns and the various printing options.

The geometry is converted by BØUNDS to the transformed coordinate system, $x = X/b$, $y = \beta Y/b$, $z = \beta Z/b$, and the surfaces and diaphragms are approximated with a grid of boxes based on the input maximum number in the chordwise direction. The number of boxes is adjusted so that box edges will coincide with the fold line. The subroutine then defines the inner and outer wing, tip and outer diaphragm boxes as those boxes with centers just inside the actual planform and diaphragm limits (Figure 15). If either trailing edge is not subsonic, BØUNDS then calculates the exact streamwise location of the trailing edge for each row of box centers.

MBX then writes the heading, flight conditions, and geometry as well as the outer box limits of the wing, tip, and their associated diaphragms. After the x_1, y_1 locations of the wing and tip trailing edges are printed, the list of frequencies and number of fold angle changes for each frequency are read. MBX begins the frequency loop by converting the input frequency to reduced frequencies based on the box length ($k = b\omega/U_\infty$ and $\overline{k} = k\, M_\infty{}^2/\beta^2$) and based on the root chord ($k_r = c\,\omega/U_\infty$).

After MBX sets the value of reduced frequency the in-plane, VPIC's are computed by CAPHI and stored in a table. These coefficients depend

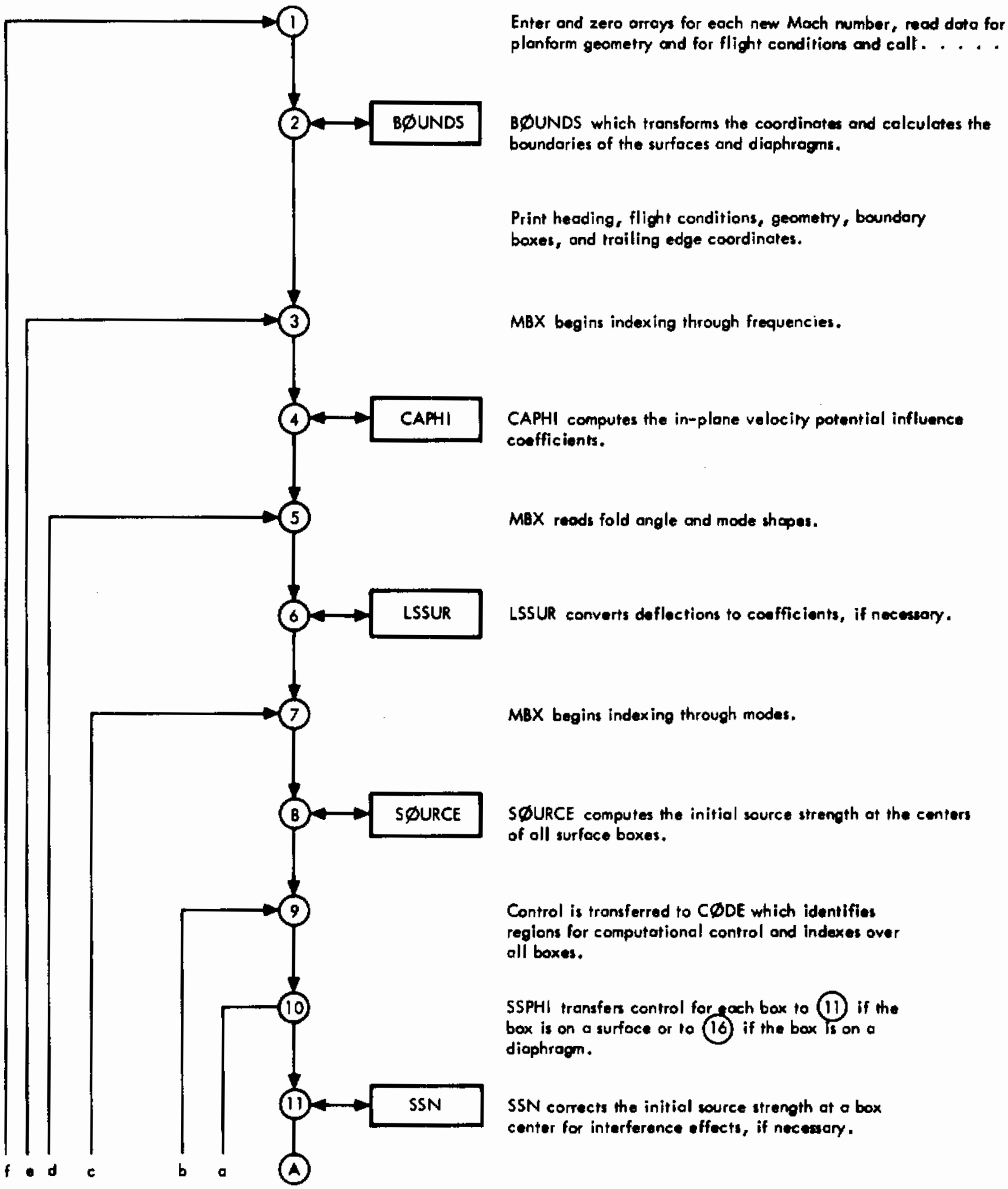


Figure 16. Flow Chart

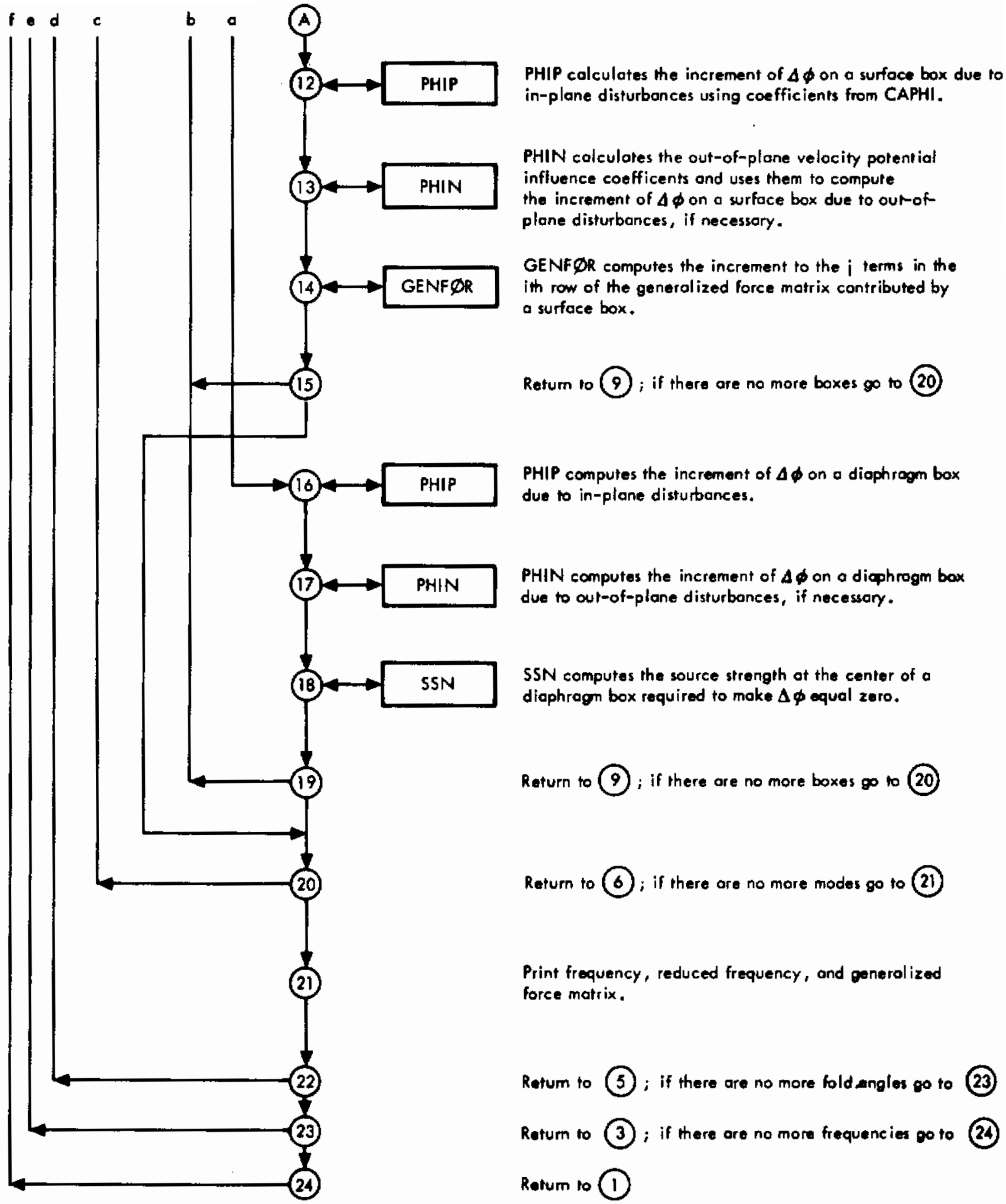


Figure 16. Flow Chart (Cont)

only upon relative position and may be computed for each Mach number-reduced frequency combination and used with all modes. These coefficients are optionally printed out by MBX.

The fold angle (0° to 90°) and up to ten sets of mode shape coefficients or deflections are read into the program by MBX. Previously read indicators for both the wing and tip are used to read the mode shapes before the deflections at the box centers can be determined. If the mode shapes are to be determined from the input deflection coordinates, a smoothing process is used. The input deflection data for all modes is supplied to a least squares polynomial fit subroutine (LSSUR) where the coefficients, A_t, $t = (r^2 + r + 2s + 2)/2$, for a prespecified degree polynomial are determined. The coefficients for the deflection polynomials are

$$\overline{Z}_i(\xi_1, \eta_1) = \sum_{r=0}^{R} \sum_{s=0}^{r} A_t \, \xi_1^{r-s} \eta_1^{s} \quad i=1, \ldots, \text{MØDES} \tag{85}$$

where $0 \le R \le s$ is the input value, for the degree of the wing polynomials or of the tip polynomials. The polynomial coefficients, either input or calculated are then optionally printed by MBX.

MBX sets the mode number and if the velocity potential is to be determined for this mode transfers control to SØURCE. This subroutine calculates the initial source strength values on the upper and lower sides of the surface boxes for the particular mode. This is accomplished by evaluating the polynomials and their derivatives at box centers from the coefficients.

The task of passing over all the boxes from centerline outboard and front to aft is carried out by CØDE. This subroutine assigns a KØDE number to each box which indicates the equations that will be used to determine the quantities at its center. With the KØDE value and location of the box, SSPHI selects the proper computation subroutines depending on whether the box is on a surface or a diaphragm. The surface boxes have the source strength corrected by SSN, if necessary. The in-plane velocity potential is calculated by PHIP and, if necessary the out-of-plane velocity potential is calculated by PHIN. The increment to the mode row in the generalized force matrix due to the change in velocity potential across the box is then calculated by GENFØR.

GENFØR calculates the contribution to the generalized force integral by each box. This is accomplished by forming a separate integral for each of the deflection polynomial coefficients that consists of the potential times the appropriate powers of x and y. (Part II of this report details this method for obtaining generalized forces.) To calculate the generalized forces using the velocity potential and the displacements the trailing edge values must be determined. Since the trailing edge is generally not located along box centers a second order interpolation procedure is employed to calculate the velocity potential at the trailing edge of each column of boxes. These values along with the previously determined x and y coordinates of the trailing edge are added to the integrals. GENFØR also weights the centerline and trailing edge box contributions to the generalized forces by the partial area of the box on the planform.

If the box is a diaphragm box, SSPHI calls PHIP and, if necessary, PHIN to calculate the velocity potential increment due to all boxes upstream that influence the box center. SSN then divides this value by $\Phi(0, 0)$ to calculate the source strength at the diaphragm box.

When the pass over the wing is completed, there will be a set of velocity potential differences and corrected source strengths available for optional printout. Also in common is the completed jth row in the generalized force matrix.

If there are more modes the next pass over the wing will be with the same frequency but with a new mode to obtain the next row in the matrix of generalized forces. When the mode loop is complete the matrix is printed with the reduced frequency based on the root chord.

A new fold angle and modes may be read in at this point, if not, the next frequency is set by MBX and the mode loop restarted so that a new matrix may be computed from the same coefficients still in storage.

The operator must be warned that the maximum number of boxes, modes, and frequencies will take a considerable length of time to run. The surest way to save time is to cut down the number of boxes since the computational time varies with the fourth power of the number of boxes. The large number of boxes is provided so that a single critical mode and frequency may be evaluated with a very dense grid.

Input data sheets with a short explanation of each item are provided in the following text.

INPUT AND OUTPUT

A standard input format of six 12-column fields per card is used in this program. The floating point numbers are to be to the left of the field starting in the second column and the fixed point numbers are to be to the right of the field ending in the twelfth column. Sample standard input sheets are provided in Appendix II.

The first two cards will provide the flight conditions, and planform geometry that is to remain fixed throughout the complete run. The Mach number (EMACH) and speed of sound (AS) are the free stream conditions while the root chord (CRØØT), leading edge sweep angle (SLEW), trailing edge sweep angle (STEW), and distance from centerline to fold line (YFL) determine the wing planform. The tip geometry constants are leading edge sweep angle (SLET), trailing edge sweep angle (STET) and YFL plus the distance from the fold line to the tip line (YTIP). The length units on the speed of sound should agree with those on the chord and span distances.

The next card contains integer constants that specify the number of boxes to be fitted in the chordwise direction (NBØX), the number of frequencies (NFREQ), the number of deflection mode shapes (MØDES), and the print option indicators. The velocity potential influence coefficients (LVPIC), the initial upper and lower source strengths (LSSUL), and the final velocity potential differences with the corrected source strength (LDPHI) can be printed out by simply setting each indicator to a non zero number.

The fourth and fifth cards contain the integers that determine the mode shape input. The indicator for the method of determining the mode shapes for the wing (MDEW) and tip (MDET) is a negative integer for supplying a set of deflection points that will be smoothly fit with the specified degree polynomial or a positive integer for directly supplying the polynomial coefficients in accordance with Equation 85. The degree of the polynomial for the wing (NPØLW) and for the tip (NPØLT) need not be equal but each applies to all modes for the respective surface. MBX either reads in the correct number of coefficients computed from the degree of the polynomial or calculates the coefficients for a polynomial of that degree for each mode. If coefficients are input all modes must have enough coefficients or zeros to satisfy this requirement. If the mode shape polynomial coefficients are to be computed by the program from deflection points the number of points for the wing (NPØW) and for the tip (NPØT) is specified. Finally the coefficients, either input or calculated, can be printed out by setting the indicator for the wing (LCØW) or for the tip (LCØT) to a non zero number.

The program can be used to calculate forces for a single surface with only one leading edge and one trailing edge sweep angle by simply setting the tip indicator (MDET) equal to zero and leaving out all subsequent tip data. The tip line is then coincident with the fold line and the surface has the geometric characteristics described by the wing input data.

The next card or cards contain the lists of frequencies (ØMEGA(I)) and number of fold angles or sets of modes per frequency (ANGS(I)). There are to be two times NFREQ numbers on these cards for MBX to read a pair of numbers for each of NFREQ values. A generalized force matrix will be calculated for each ANGS(I) at the ith frequency (ØMEGA(I)) and it will contain MØDES by MØDES terms. This provision enables the user to repeat the modes or put in new modes at different frequencies without calculating all the meaningless off diagonal terms obtained when all the modes are included in one set. Also the fold angle can be varied at different frequencies and a complete set of reduced frequency, fold angle, and mode shape variations can be performed at the same Mach number during the same run .

The ensuing data cards must be carefully provided to assure successful operation of the program. There is to be one set of fold angle and mode shape data for each of the ANGS(I) and ØMEGA (I) combinations. Each of these sets must have the current value of the fold angle on the first card even if there are no tips to be folded.

Special care is necessary to be sure that the thickness indicator is correctly specified for both wing and tip portions of each mode shape. These numbers tell whether the coefficients are to be used to calculate the velocity potential due to a thickness distribution or due to a deflection distribution. If any of the thickness indicators is a negative number the program will use the input as a symmetric thickness distribution and calculate the potential and generalized forces accordingly.

If any of the wing thickness indicators has a zero value there will be no velocity potential calculate for either the wing or tip but the mode will be used to obtain generalized forces due to velocity potentials in other modes.

The thickness indicators for the wing and tip must be either zero or positive for all vibratory mode shapes. However, if the input frequency is zero the wing or tip may have thickness to determine the various steady state thickness effects.

If the mode shapes are to be given as a set of deflections the card or cards after the fold angle card will contain the x-y coordinates of the deflections on the wing, 3 points or 6 numbers per card until NPØW points have been entered, which will apply to all modes. The dimensions of the coordinates should be compatible with those of the chord and span distances input earlier.

The next cards will contain the values of the deflections at the several x-y points on the wing for all the modes, one mode at a time. Each mode should start on a new card with the thickness indicator, THW(I), as the first number and the deflections in all the following consecutive locations until NPØW numbers have been listed.

If there is a tip (MDET≠O) the x-y coordinates, measured from the wing axis system, of the tip deflections will be on the cards immediately following the last of the wing deflection data. There should be 2 NPØT numbers entered with 6 on each card and 6 or less on the last card.

The tip deflections are to be listed on the next cards with each mode starting on a new card. The first card of the mode will have the thickness indicator, THT(I), as its first number and the first 5 deflection points in the remaining locations. The rest of the deflections will follow, 6 points per card until NPØT points are entered.

If the mode shape coefficients are to be input rather than computed the first cards after the fold angle card will contain the wing coefficients for all the modes, one mode at a time. Each mode should begin on a new card with the first number the wing thickness indicator for that mode. The wing coefficients for each mode are entered into the next 1/2(NPØLW+2)(NPØLW+1) consecutive locations.

The tip coefficients, if applicable are placed on the cards immediately following the wing coefficients for the same number of modes. As before, each mode should begin on a new card with the tip thickness indicator as the first number. There are 1/2(NPØLT+2) (NPØLT+1) coefficients for each mode that are to be entered.

The cards must then be placed in the order presented in this discussion at the back of the program cards. Appendix II provides further explanation of the input data as well as demonstrates the use of the coefficient input option.

The output from the sample data sheets is shown in Appendix III. The first page of the output will always contain the program title, the input flight conditions and geometry, calculated fold-line and tip-line chords and reference area, number of boxes in the chordwise and spanwise directions, and the box length and width.

The next page always depicts the placement of the boxes over the wing, tip, and associated diaphragms. The box number for each row that its center inboard of the wing, tip, tip diaphragm and wing diaphragm outer edge appears as MØBW, MØBT, MØBTD, and MØBWD, respectively. The box number for each row that has its center outboard of the wing and tip inner edge appears as MIBW and MIBT, respectively.

The nondimensionalized x and y coordinates of the wing and tip trailing edge will always be on the page following the box boundaries.

If there is none of the optional printout the ensuing pages will contain the generalized forces as well as the reduced frequency, fold angle, Mach number, and chordwise boxes information.

There will be one set of generalized forces for each frequency-fold angle combination that contains the real and imgainary components as well as the magnitude and phase angle of each generalized force. Each generalized force is the pressure due to displacement in the DPHI mode weighted by the displacement in the DEFL mode.

The optional output, which will precede each table of generalized force, includes any of the following information. The input or calculated polynomial coefficients will be printed out for all modes for the wing if LCØW≠O and then for the tip if LCØT≠O. The initial upper and lower source strengths at all box centers will then be printed for all modes except those that have THW(I)=O if LSSUL≠O. Finally the velocity potential difference distribution and the source strengths with interference included will be printed for each mode with THW(I)≠O if LDPHI≠O. The source strengths and velocity potential differences are printed for each box with row (N) and column (M) locations starting with (1, 1) as the foremost centerline box and proceeding outward and aft. The boxes in the plane of the wing and its diaphragm are numbered (N, M) while the boxes in the intersecting plane of the tip and its diaphragm are numbered (N, M+MMAX) where MMAX equals the most outboard box on the wing or wing diaphragm. When the tip is folded into the plane of the wing the tip surface then replaces a portion of the wing diaphragm and the tip boxes then assume the numbers of the replaced wing diaphragm boxes.

Complete listings for the program and all non-systems subroutines are presented in Appendix IV.

5. RESULTS

SINGLE PLANAR SURFACES

The extension of the source superposition method with Mach box approximations to intersecting surfaces is verified on the basis of results for a single planar surface. The computer program calculates potential distributions and generalized forces that compare well with exact theoretical results, other analytical methods, and experimental results. Figure 17 compares the theoretical lift and moment slopes using the exact expression with the lift curve slope calculated from the MBX program. These data are for a 65-degree delta at Mach numbers from 1.05 to 2.37. Figure 18 shows the effect of reduced frequency on the stability derivatives for a 65-degree delta at M = 2.0.

Data for a rectangular wing calculated by the characteristic box method (Reference 13) are shown in Figure 19 compared with results from the MBX program for pitch and plunge stability derivative variations with Mach number. These data are at a reduced frequency of 0.3 based on a unit root chord.

It is well known that the pressure distribution along the centerline of a delta wing at angle of attack is constant thereby implying that the velocity potential is linear. Figure 20 shows the centerline potential distribution on a 65-degree delta at M = 2.0 as calculated by exact theory compared with the box center values calculated by the MBX program. The computed values oscillate around the exact line due to the jagged leading effects but the lift curve slope, obtained by integrating the velocity potential is seen to agree very well with exact theory. To obtain pressures using the MBX program one would have to fit a curve through the potential distribution by the methods described in Part II of this report. This representative function could then be differentiated to obtain the pressure distribution. Generalized forces are obtained by the MBX program by direct integration of the potential (Equation 20) which cancels the leading-edge induced oscillations.

INTERSECTING PLANAR SURFACES

To demonstrate the effect of wing droop or fold on the stability derivatives, the tips of a 65-degree delta were folded at the 60 percent semispan line. These results are plotted in Figure 21 and 22 for C_{L_α} and C_{M_α} at various supersonic Mach numbers. The coefficients shown are based on a unit root chord and the reference area remains that of the unfolded

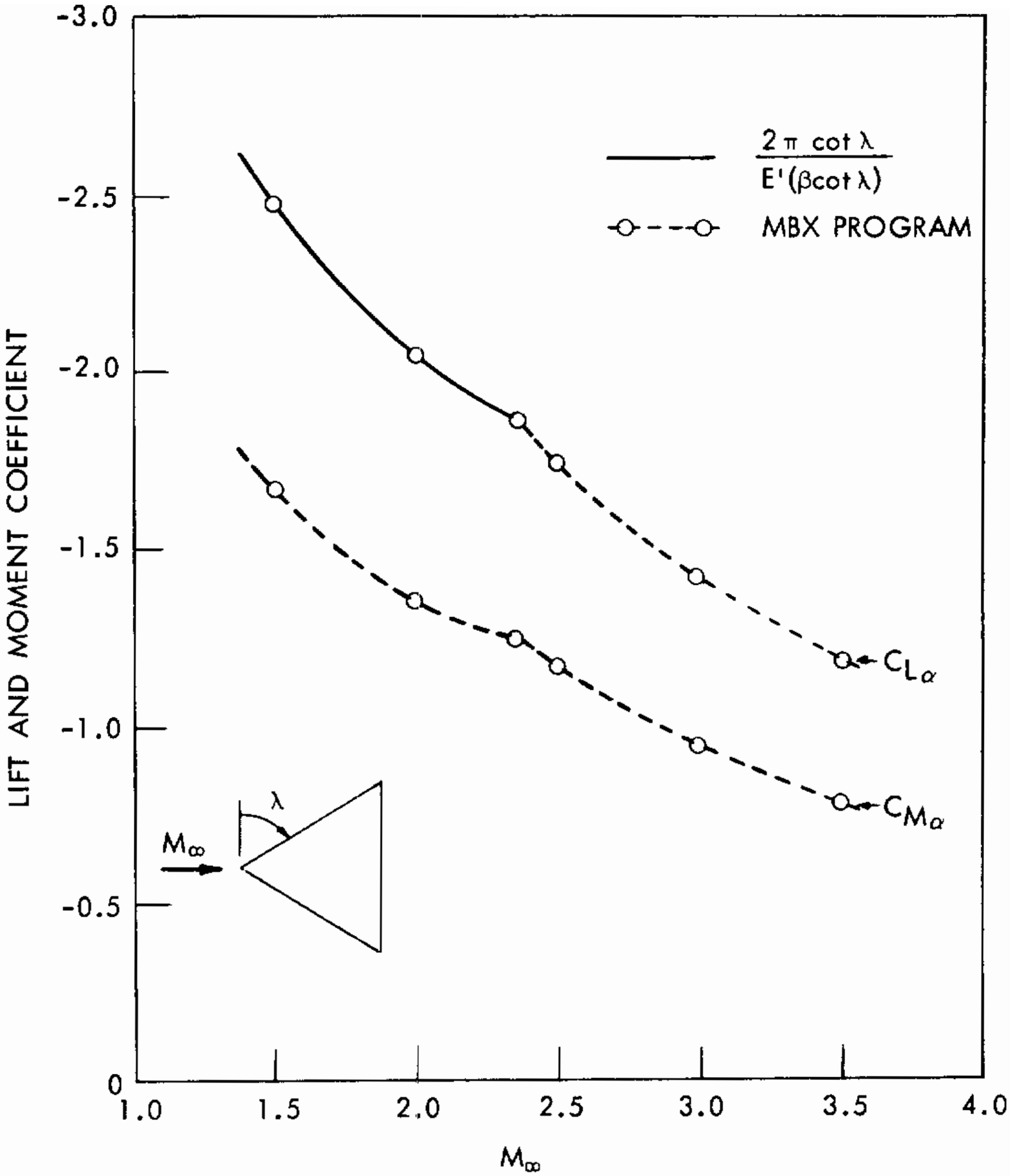


Figure 17. C_{M_α} and C_{L_α} Vs. M for 65° Delta

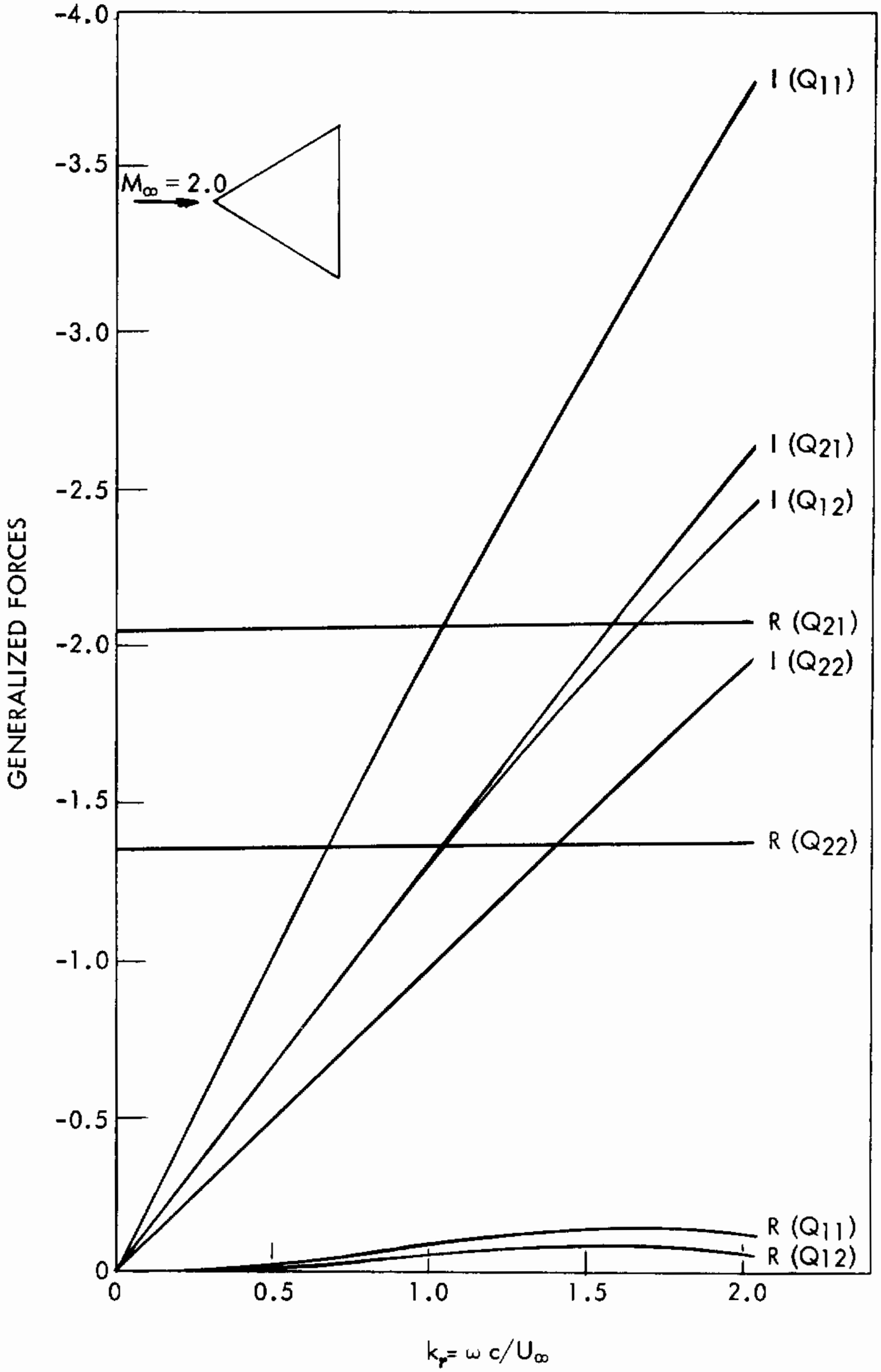


Figure 18. Forces on 65° Delta for $Z_1 = 1.0$, $Z_2 = X$

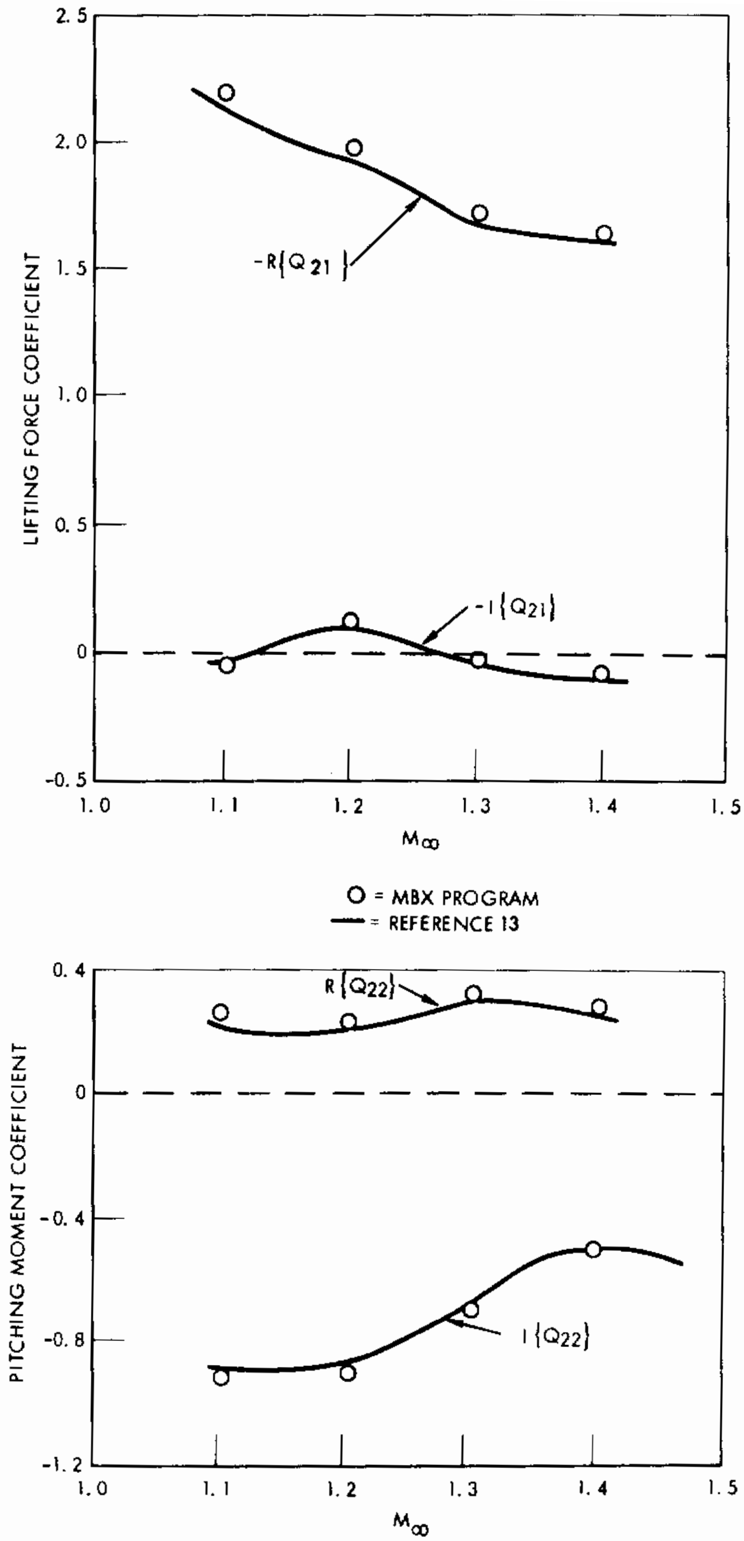


Figure 19. Aerodynamic Coefficients for a Rectangular Æ = 2.0 Wing Oscillating in Pitch About the Midchord With k = 0.3

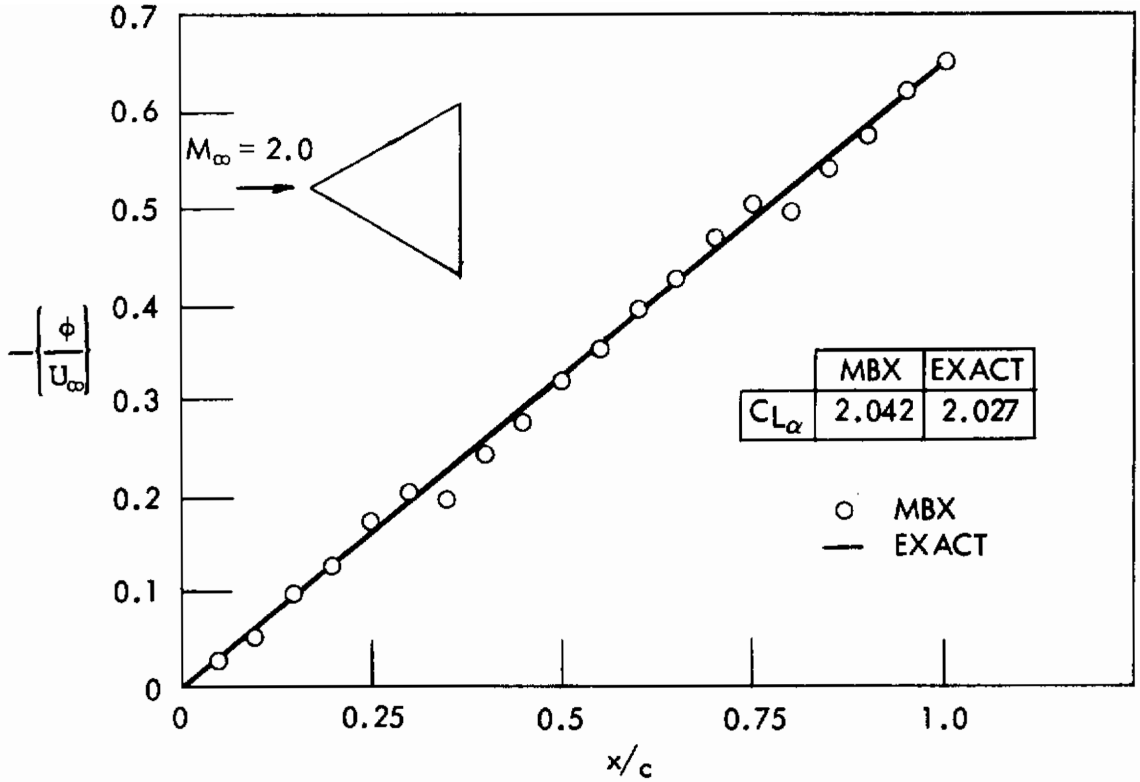


Figure 20. Potential Distribution on ℄ of a 65° Delta at α

65-degree delta when the tips are taken off. Note that even without thickness the configuration has greater lift and pitching moment when the tips are folded to 90 degrees than when they are removed. These trends in lift and moment variation with fold angle are similar to the trends observed experimentally in tests on configurations that use the drooped tips in increase lift to drag ratios.

Figure 23 depicts the variation of lift and moment curve slopes, C_{L_α} and C_{M_α}, respectively, with tip fold angle, γ. The curve was produced by connecting the computed values at 5 degrees and above (up to 90 degrees), with the 0 degree point. The value for zero fold angle agrees with supersonic theory for planar surfaces (c.f. Figure 17). The coefficient values, which are based on the unfolded planar area, increase smoothly as the fold angle decreases from 90 to 0 degrees.

In order to obtain the smooth variation of aerodynamic force coefficients with fold angle using only a moderately dense grid of Mach boxes (up to 20 boxes chordwise), the wing diaphragm source strengths were computed using an equivalent but more accurate form of Equation 83. The expression more accurately accounts for the increasing lag in signals between the wing diaphragm and the lower side of the tip as the fold angle approaches 0 degrees.

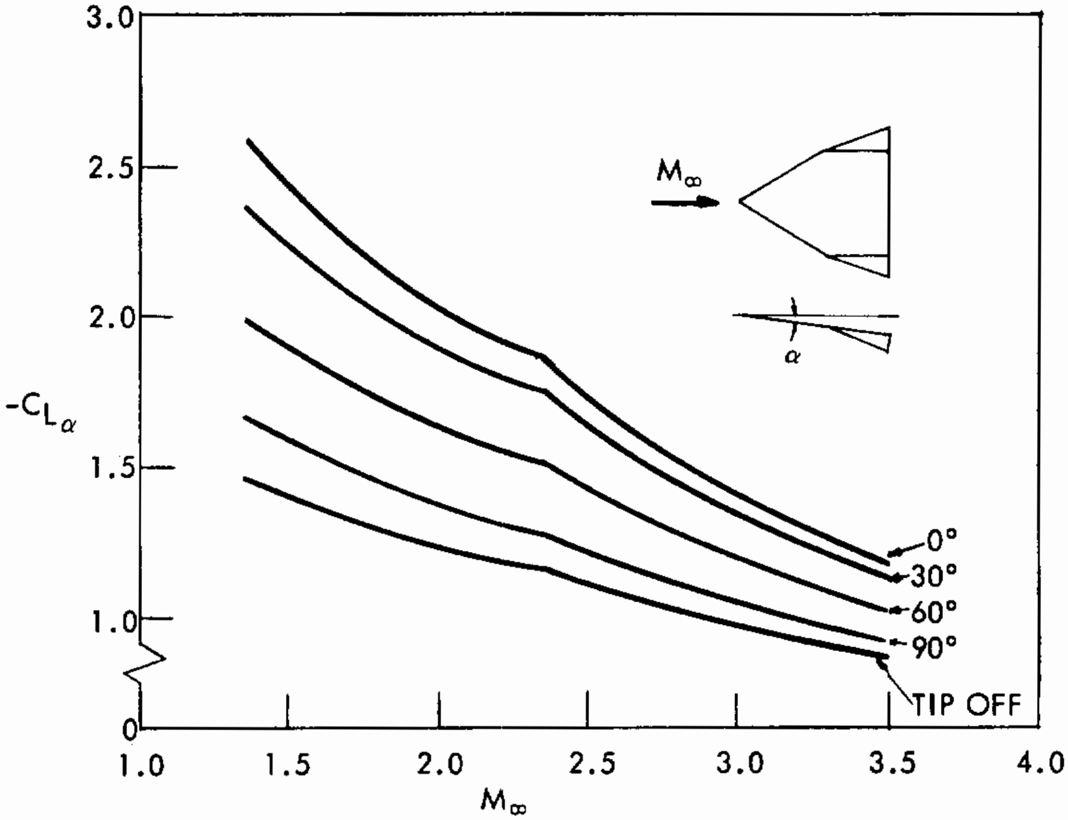


Figure 21. Variation of C_{L_α} on 65° Delta Due to Folding the Tips at the 60% Semispan

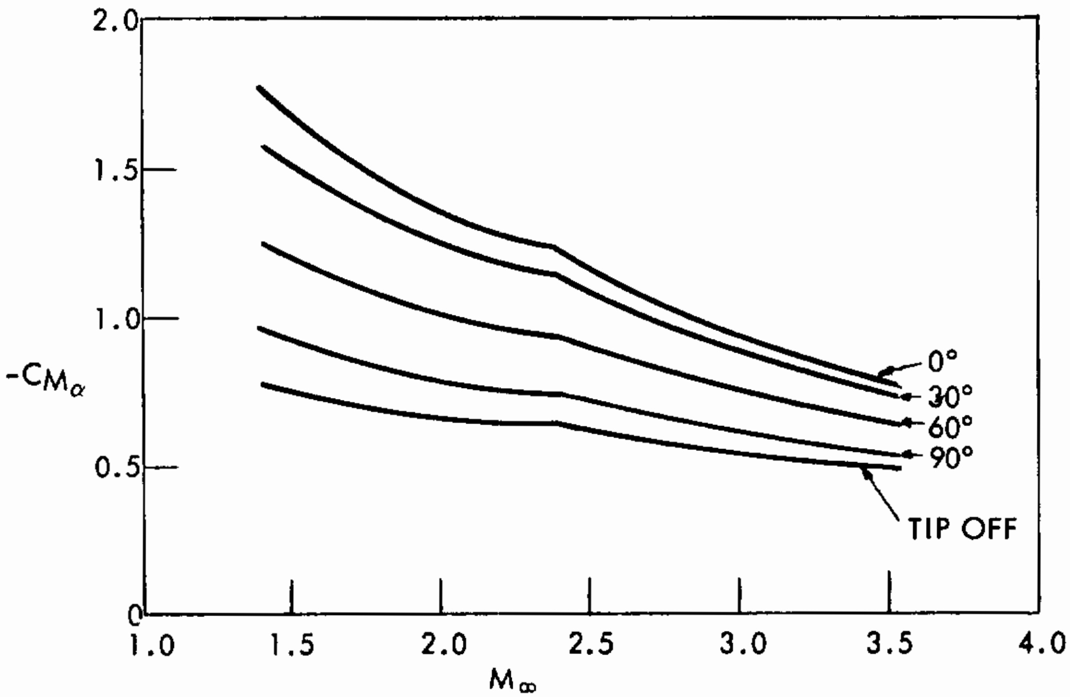


Figure 22. Variation of C_{M_α} on 65° Delta Due to Folding the Tips at the 60% Semispan

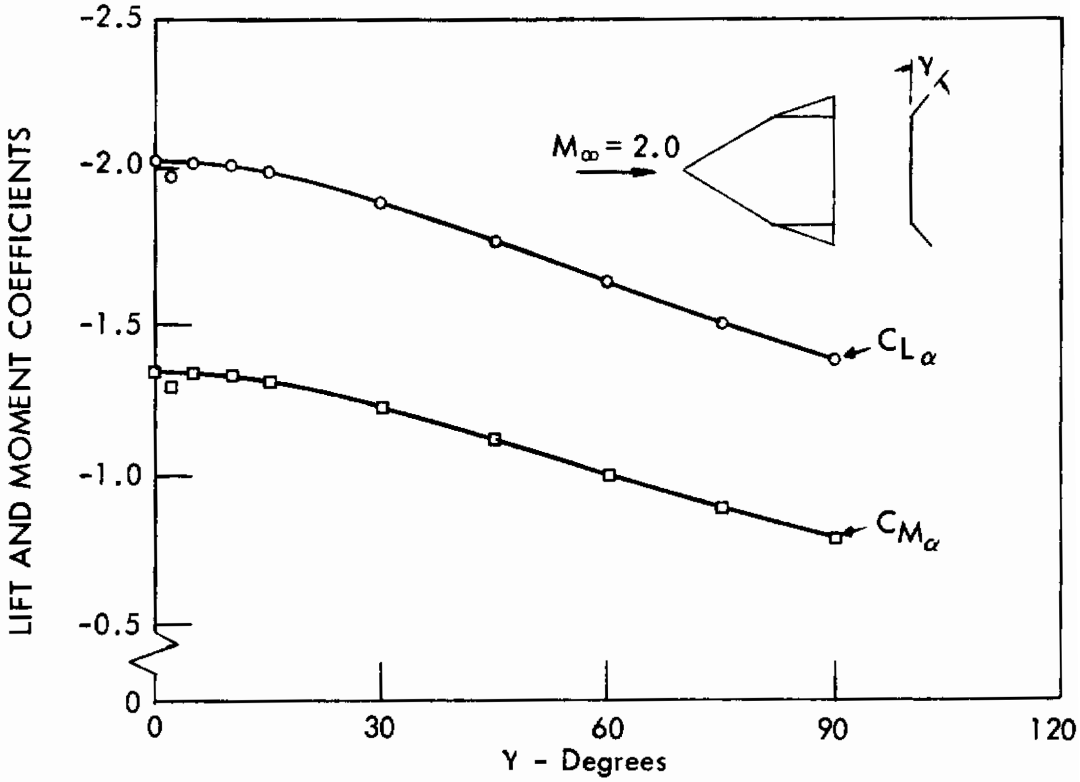


Figure 23. Variation of $C_{L\alpha}$ and $C_{M\alpha}$ on 65° Delta Due to Folding the Tips at the 60% Semispan

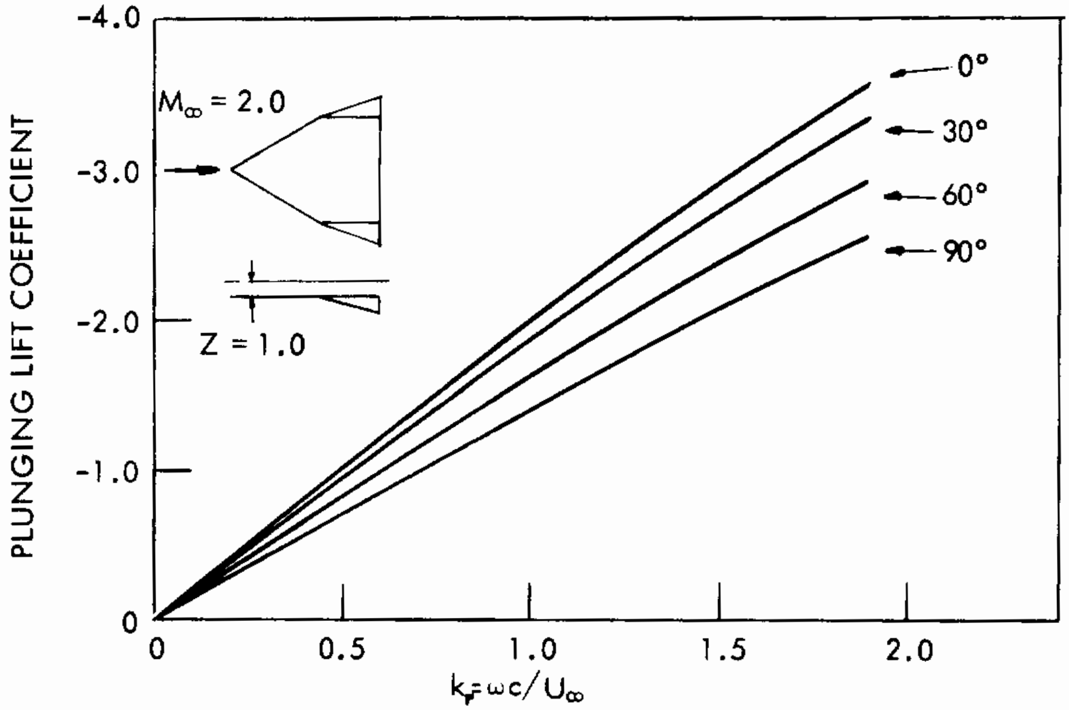


Figure 24. Variation of Plunging Lift on 65° Delta Due to Folding the Tips at the 60% Semispan

The effect of the grid size is apparent if coefficients are computed for fold angles below 5 degrees. Fortunately, one can take advantage of the fact that in the steady state case the values below 5 degrees are within 1 per cent of those for planar surfaces. Because this accuracy is not expected to be significantly different in the unsteady case, a slight modification was made to the computer program to ensure the accuracy of calculations at all values of fold angle. When the fold angle is less than 5 degrees in absolute value the program treats the configuration as if it were planar. The more complicated non-planar logic is used only if the fold angle is 5 degrees or greater in absolute value.

The program listings, Appendix IV, note the above changes at the appropriate places.

6. CONCLUSIONS AND RECOMMENDATIONS

As a result of this study, the practical technique for solving supersonic aerodynamic interference problems of intersecting planar surfaces using the source superposition method is now clear. Mach envelopes should be constructed for each of the separate components of the system as though the other components were nonexistent. Diaphragms should be constructed in the planes of the components to separate the upper and lower half-envelopes (Figure 10). Overlapping Mach envelopes should then be defined as interference regions. A grid of Mach boxes all of the same dimensions may then be overlaid on each of the surfaces and its diaphragm. Starting with the foremost row of boxes (rows of boxes if two or more leading edges intersect at the foremost point), the source sheets should be computed that satisfy the pertinent boundary conditions at the center of the boxes in that row. This process should then be repeated for each successive row (or rows) of boxes until the source strengths on the aftmost boxes on the surfaces have been determined.

This technique has several advantages:

1. It entirely eliminates the need for calculating source strengths on both sides of the surfaces that are in the interference region. Source strengths on one side are equal in magnitude and opposite in sense to those on the other.

2. When only the difference between upper and lower pressures on the surfaces is of concern, it eliminates the need for calculating out-of-plane velocity potentials. In most aeroelastic problems, the upper surface of an airfoil does not move relative to the lower surface, therefore, only the differences between pressures are needed.

3. The concept is simple and, therefore, provides insight which makes the technique more readily extendable. For instance, the extension that would be required to handle T-tails, V-tails, and top-mounted vertical tails is immediately apparent. Also, the extension to handle the wing-body interference problem should be a fairly simple one.

The following recommendations are made:

1. Perform an experimental study to verify the theoretical results

2. Modify the computer program to incorporate the pressure smoothing technique used in the transonic box method described in Volume II of this report

3. Modify the computer program to handle T-tails, V-tails, and top-mounted vertical tails as well as symmetrically folded tips; a single efficient computer program could probably handle all these configurations.

4. Modify the computer program to handle trailing edge control surfaces. This would involve only subdividing the boxes at the edges of the control surfaces and providing separate modal displacement functions to be used for the tangential flow conditions of those boxes on the control surfaces.

5. Apply the technique to other aerodynamic interference problems such as wing-body interference and wing-empennage interference problems

6. Apply the technique to very thick lifting surfaces

REFERENCES

1. Garrick, I. E., and S. I. Rubinow. Theoretical Study of Air Forces on an Oscillating or Steady Thin Wing in a Supersonic Main Stream. NACA Report 872 (1948).

2. Pines, S., J. Dugundji, and J. Neuringer. "Aerodynamic Flutter Derivatives for a Flexible Wing with Supersonic and Subsonic Edges," Journal of Aeronautical Sciences, Vol. 22, No. 10 (October 1955) pp. 693-700.

3. Evvard, J. C. Use of Source Superposition Distributions for Evaluating Theoretical Aerodynamics of Thin Finite Wings at Supersonic Speeds. NACA Report 951 (1950).

4. Li, T. C. Aerodynamic Influence Coefficients for an Oscillating Finite Thin Wing. Chance Vought Aircraft, Inc. Report (June 1954).

5. Zartarian, G., and P. T. Hsu. Theoretical Studies on the Prediction of Unsteady Supersonic Airloads on Elastic Wings, Parts I & II. Wright Air Development Center Technical Report 56-97 (1955-1956).

6. Ashley, H. Supersonic Airloads on Interfering Lifting Surfaces by Aerodynamic Influence Coefficient Theory. The Boeing Co. Report No. D2-22067 (November 1962).

7. Sears, W. R. "Small Perturbation Theory," Sec. C, Vol. VI, General Theory of High Speed Aerodynamics. Princeton Series on High Speed Aerodynamics and Jet Propulsion. Princeton University Press (1957).

8. Bisplinghoff, R. L., H. Ashley, and R. L. Halfman. Aeroelasticity, Cambridge: Addison-Wesley Publishing Co., Inc. (1953).

9. Goldstein, H. Classical Mechanics. Cambridge: Addison-Wesley Publishing Co., Inc. (1953).

10. Stewart, H. J. "A Review of Source Superposition and Conical Flow Methods in Supersonic Wing Theory," Journal of Aeronautical Sciences, Vol. 23, No. 5 (May 1956) pp 507, 516.

11. Ashley, H. Some Recent Developments in Interference Theory for Aeronautical Applications, MIT Fluid Dynamics Research Laboratory Report No. 63-3 (July 1963).

12. Ashley, H. Linearized Time Dependent Loading of Intersecting Lifting Surfaces. NAA S&ID, SID 63-1020 (August 1963).

13. Stark, V. J. E. Calculation of Aerodynamic Forces on Two Oscillating Finite Wings at Low Supersonic Mach Numbers, Saab Technical Notes TN 53. Svenska Aeroplan Aktiebolaget (Saab Aircraft Company), Linkoping Sweden (February 1964).

APPENDIX I. AERODYNAMIC INFLUENCE COEFFICIENTS

The velocity potential influence coefficient (VPIC) defines the velocity potential at a point in space due to that portion of a rectangular unit strength source sheet that lies within the upstream zone of influence from the point. The velocity induced at the point, by the rectangular sheet, is determined by differentiating the VPIC, with respect to the direction of the velocity, to obtain the velocity influence coefficient (VIC). The rectangular source sheet or box can always be positioned in the $\zeta = 0$ plane with its length, b, parallel to the flow direction. The width of the box, b/β, is set so that its diagonals are parallel to Mach lines in the flow. The VPIC's and VIC's are dependent only upon the relative position of the sending area and the receiving point and the Mach number and reduced frequency parameters.

Consider a point (n, m, ℓ) placed above a unit strength source sheet located in the $\zeta = 0$ plane (Figure 25). The source sheet has been divided into Mach boxes whose centers lie at the points (ν, μ). For a typical Mach number $(M > 1)$, the portion of the source sheet within the Mach hyperbola will influence the point (n, m, ℓ). This results in VPIC's and VIC's for both full and partial box areas sending to the point.

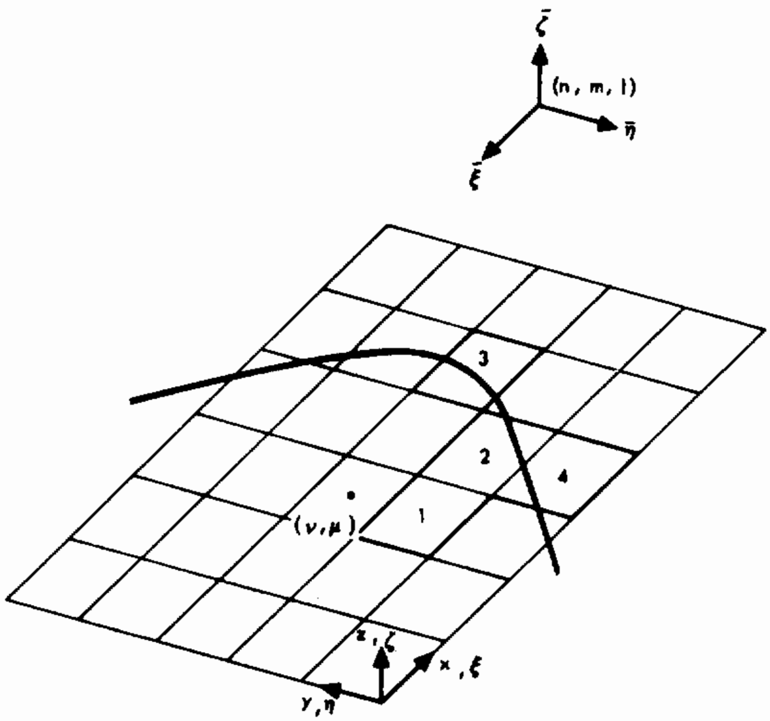


Figure 25. Boxes That Influence an Out-of-Plane Point

The expression for the VPIC is

$$\Phi(\bar{\nu},\bar{\mu},\ell) = -\frac{1}{\pi}\int_{\bar{\xi}_L}^{\bar{\xi}_U}\int_{\bar{\eta}_L}^{\bar{\eta}_U} \frac{e^{-i\bar{k}\bar{\xi}}\cos\left(\frac{\bar{k}}{M}\sqrt{\bar{\xi}^2-\bar{\eta}^2-\ell^2}\right)}{\sqrt{\bar{\xi}^2-\bar{\eta}^2-\ell^2}}\, d\bar{\eta}\, d\bar{\xi} \tag{86}$$

where

$$\bar{k} = \frac{\omega b}{U_\infty}\frac{{M_\infty}^2}{\beta^2}\ ;\ \beta^2 = M_\infty^2 - 1$$

The limits of integration correspond to the area of the box that is within the Mach hyperbola.

Since the integrand is singular along the Mach hyperbola, $\bar{\xi} = \sqrt{\bar{\eta}^2+\ell^2}$ the integration of Equation 86 for the three types of intersected boxes (Figure 25) must be performed after the singularity has been removed. This may be accomplished by using the following identity

$$\frac{\cos\left(\frac{\bar{k}}{M}\sqrt{\bar{\xi}^2-\bar{\eta}^2-\ell^2}\right)}{\sqrt{\bar{\xi}^2-\bar{\eta}^2-\ell^2}} = -\frac{\partial}{\partial\bar{\eta}}\,\psi\left(\sqrt{\bar{\xi}^2-\ell^2}\ ,\bar{\eta}\right) \tag{87}$$

where

$$\psi(\Omega,\bar{\eta}) = J_0\left(\frac{\bar{k}}{M}\Omega\right)\sin^{-1}\left(\frac{\bar{\eta}}{\Omega}\right) + \sum_{r=1}^{\infty}\frac{(-1)^r}{r}J_{2r}\left(\frac{\bar{k}}{M}\Omega\right)\sin\left(2r\sin^{-1}\left(\frac{\bar{\eta}}{\Omega}\right)\right)$$

and $\Omega = \sqrt{\xi^2-\ell^2}$

Upon substitution of Equation 87 into the VPIC expression the inner integration may be performed to obtain

$$\Phi = -\frac{1}{\pi}\int_{\bar{\xi}_L}^{\bar{\xi}_U} e^{-i\bar{k}\bar{\xi}}\left[\psi(\Omega,\bar{\eta}_U) - \psi(\Omega,\bar{\eta}_L)\right]d\bar{\xi} \tag{88}$$

The integral is now in a form suitable for numerical evaluation and the values of the function, ψ, will correspond to the area within the Mach hyperbola on cut boxes if the following interpretation is adhered to,

$$-\frac{\pi}{2} < \sin^{-1}\frac{\bar{\eta}}{\Omega} < \frac{\pi}{2} \qquad -\Omega < \bar{\eta} < \Omega$$

$$\sin\frac{\bar{\eta}}{\Omega} = \frac{\pi}{2} \qquad \bar{\eta} \geq \Omega \tag{89}$$

$$\sin\frac{\bar{\eta}}{\Omega} = -\frac{\pi}{2} \qquad \bar{\eta} \leq -\Omega$$

The VIC in the ℓ direction is given by

$$W = -\frac{1}{\pi}\frac{\partial}{\partial \ell}\int_{\bar{\xi}_L}^{\bar{\xi}_U}\int_{\bar{\eta}_L}^{\bar{\eta}_U}\frac{e^{-i\bar{k}\bar{\xi}}\cos\left(\frac{\bar{k}}{M}\sqrt{\bar{\xi}^2 - \bar{\eta}^2 - \ell^2}\right)}{\sqrt{\bar{\xi}^2 - \bar{\eta}^2 - \ell^2}}\, d\bar{\eta}\, d\bar{\xi} \tag{90}$$

Substitution of the identity (Equation 87), performing the inner integration, and interchanging the differentiation with the outer integration results in

$$W = -\frac{1}{\pi}\int_{\bar{\xi}_L}^{\bar{\xi}_U} e^{-i\bar{k}\bar{\xi}}\frac{\partial}{\partial \ell}\left[\psi(\Omega, \bar{\eta}_U) - \psi(\Omega, \bar{\eta}_L)\right] d\bar{\xi} \tag{91}$$

Since ℓ only appears in the parameter $\Omega = \sqrt{\bar{\xi}^2 - \ell^2}$, the differentiation variable may be changed, i.e.,

$$\frac{\partial}{\partial \ell}\Omega = -\frac{\ell}{\bar{\xi}}\frac{\partial}{\partial \bar{\xi}}\Omega$$

The expression for the ℓ direction VIC may be integrated by parts after the variable change with the following relationships

$$u = \frac{1}{\bar{\xi}}\, e^{-i\bar{k}\bar{\xi}} \qquad du = \frac{1 + i\bar{k}\bar{\xi}}{\bar{\xi}^2}\, e^{-i\bar{k}\bar{\xi}}\, d\bar{\xi}$$

$$dv = \frac{\partial}{\partial \bar{\xi}}\psi\, d\bar{\xi} \qquad v = \psi$$

to obtain:

$$W = \frac{\ell}{\pi}\left\{ \left(\frac{1}{\bar{\xi}} e^{-i\bar{k}\bar{\xi}} \left[\psi(\Omega, \bar{\eta}_U) - \psi(\Omega, \bar{\eta}_L) \right] \right)_{\bar{\xi}_L}^{\bar{\xi}_U} + \int_{\bar{\xi}_L}^{\bar{\xi}_U} \frac{1 + i\bar{\xi}\bar{k}}{\bar{\xi}^2} e^{-i\bar{k}\bar{\xi}} \left[\psi(\Omega, \bar{\eta}_U) - \psi(\Omega, \bar{\eta}_L) \right] d\bar{\xi} \right\} \tag{92}$$

The relationships in Equation 89 also apply to the function, ψ , in the ℓ-direction VIC.

The unsteady VIC in the $\bar{\eta}$-direction is given by

$$V = -\frac{1}{\pi}\frac{\partial}{\partial\bar{\eta}} \int_{\bar{\xi}_L}^{\bar{\xi}_U}\int_{\bar{\eta}_L}^{\bar{\eta}_U} \frac{e^{-i\bar{k}\bar{\xi}} \cos\left(\frac{\bar{k}}{M}\sqrt{\bar{\xi}^2 - \bar{\eta}^2 - \ell^2} \right)}{\sqrt{\bar{\xi}^2 - \bar{\eta}^2 - \ell^2}} \, d\bar{\eta}\, d\bar{\xi} \tag{93}$$

where the integral becomes singular when $\bar{\xi} = \sqrt{\bar{\eta}^2 + \ell^2}$ and,

$$F(\bar{\xi}, \bar{\eta}, \ell) = \frac{\cos\left(\frac{\bar{k}}{M}\sqrt{\bar{\xi}^2 - \bar{\eta}^2 - \ell^2} \right)}{\sqrt{\bar{\xi}^2 - \bar{\eta}^2 - \ell^2}}$$

$$V = -\frac{1}{\pi}\frac{\partial}{\partial\bar{\eta}} \int_{\bar{\xi}_L}^{\bar{\xi}_U}\int_{\bar{\eta}_L}^{\bar{\eta}_U} e^{-i\bar{k}\bar{\xi}} \, F(\bar{\xi}, \bar{\eta}, \ell)\, d\bar{\eta}\, d\bar{\xi} \tag{94}$$

The limits are not functions of $\bar{\eta}$ therefore the differentiation can be performed inside the integral and then the inner integration performed to obtain

$$V = -\frac{1}{\pi} \int_{\bar{\xi}_L}^{\bar{\xi}_U} e^{-i\bar{k}\bar{\xi}} \left[F(\bar{\xi}, \bar{\eta}_U, \ell) - F(\bar{\xi}, \bar{\eta}_L, \ell) \right] d\bar{\xi} \tag{95}$$

Integration by parts may be performed with the following relationships

$$u = \frac{e^{-i\bar{k}\bar{\xi}}}{\bar{\xi}} \qquad du = \frac{1 + i\bar{\xi}\bar{k}}{\bar{\xi}^2} e^{-i\bar{k}\bar{\xi}} d\bar{\xi}$$

$$dv = \frac{\bar{\xi} \cos\left(\frac{\bar{k}}{M} \sqrt{\bar{\xi}^2 - \bar{\eta}^2 - \ell^2}\right)}{\sqrt{\bar{\xi}^2 - \bar{\eta}^2 - \ell^2}} d\bar{\xi} \qquad v = \frac{M}{\bar{k}} \sin\left(\frac{\bar{k}}{M} \sqrt{\bar{\xi}^2 - \bar{\eta}^2 - \ell^2}\right)$$

$$v = \frac{M}{\bar{k}} E(\bar{\eta})$$

to obtain for the VIC,

$$V = -\frac{M}{\pi\bar{k}} \left\{ \left(\frac{e^{-i\bar{k}\bar{\xi}}}{\bar{\xi}} \left[E(\eta_U) - E(\eta_L) \right] \right)_{\bar{\xi}_L}^{\bar{\xi}_U} + \int_{\bar{\xi}_L}^{\bar{\xi}_U} \frac{1 + i\bar{k}\bar{\xi}}{\bar{\xi}^2} e^{-i\bar{k}\bar{\xi}} \left[E(\bar{\eta}_U) - E(\bar{\eta}_L) \right] d\bar{\xi} \right\} \tag{96}$$

To ensure proper values for cut boxes adopt the interpretation $\sin(a) = 0$, if a is imaginary.

For steady flow the $\bar{\eta}$-direction VIC is given by

$$V^S = -\frac{1}{\pi} \frac{\partial}{\partial\bar{\eta}} \int_{\bar{\xi}_L}^{\bar{\xi}_U} \int_{\bar{\eta}_L}^{\bar{\eta}_U} \frac{d\bar{\eta}\, d\bar{\xi}}{\sqrt{\bar{\xi}^2 - \bar{\eta}^2 - \ell^2}} \tag{97}$$

Interchanging the order of differentiation and integration yields

$$V^S = -\frac{1}{\pi}\int_{\bar{\xi}_L}^{\bar{\xi}_U}\left[\frac{1}{\sqrt{\bar{\xi}^2 - \bar{\eta}_U^{\,2} - \ell^2}} - \frac{1}{\sqrt{\bar{\xi}^2 - \bar{\eta}_L^{\,2} - \ell^2}}\right] d\bar{\xi} \tag{98}$$

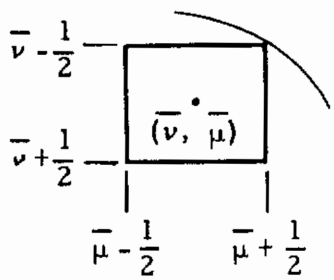
which may be further integrated to obtain

$$\begin{aligned} V^S = -\frac{1}{\pi}\Bigg[&\cosh^{-1}\frac{\bar{\xi}_U}{\sqrt{\left(\bar{\mu}+\frac{1}{2}\right)^2 + \ell^2}} - \cosh^{-1}\frac{\bar{\xi}_L}{\sqrt{\left(\bar{\mu}+\frac{1}{2}\right)^2 + \ell^2}} \\ &- \cosh^{-1}\frac{\bar{\xi}_U}{\sqrt{\left(\bar{\mu}-\frac{1}{2}\right)^2 + \ell^2}} + \cosh^{-1}\frac{\bar{\xi}_L}{\sqrt{\left(\bar{\mu}-\frac{1}{2}\right)^2 + \ell^2}}\Bigg] \end{aligned} \tag{99}$$

The proper values can be obtained from Equation 99 for the steady-state VIC for cut boxes if the following interpretation is adopted.

$$\cosh^{-1}\frac{a}{b} = 0 \text{ for } \left|\frac{a}{b}\right| \leq 1.0$$

In summary, the influence coefficients are written in their final form for an uncut box.



UNCUT BOX

$$\Omega = \sqrt{\bar{\xi}^2 - \ell^2}$$

$$\bar{\xi}_L = \bar{\nu} - \frac{1}{2}$$

$$\Phi = -\frac{1}{\pi}\int_{\bar{\xi}_L}^{\bar{\nu}+1/2} e^{-i\bar{k}\bar{\xi}}\left[\psi\left(\Omega, \bar{\mu}+\frac{1}{2}\right) - \psi\left(\Omega, \bar{\mu}-\frac{1}{2}\right)\right] d\bar{\xi} \tag{100}$$

$$W = + \frac{\ell}{\pi} \left\{ \left(\frac{e^{-i\bar{k}\bar{\xi}}}{\bar{\xi}} \left[\psi\left(\Omega, \bar{\mu} + \frac{1}{2}\right) - \psi\left(\Omega, \bar{\mu} - \frac{1}{2}\right) \right] \right)_{\bar{\xi}_L}^{\bar{\nu} + \frac{1}{2}} \right.$$

$$\left. + \int_{\bar{\xi}_L}^{\bar{\nu}+1/2} \frac{1 - i\bar{k}\bar{\xi}}{\bar{\xi}^2} e^{-i\bar{k}\bar{\xi}} \left[\psi\left(\Omega, \bar{\mu} + \frac{1}{2}\right) - \psi\left(\Omega, \bar{\mu} - \frac{1}{2}\right) \right] d\bar{\xi} \right\} \tag{101}$$

$$V = - \frac{M}{\pi \bar{k}} \left\{ \left(\frac{e^{-i\bar{k}\bar{\xi}}}{\bar{\xi}} \left[E\left(\bar{\mu} + \frac{1}{2}\right) - E\left(\bar{\mu} - \frac{1}{2}\right) \right] \right)_{\bar{\xi}_L}^{\bar{\nu} + \frac{1}{2}} \right.$$

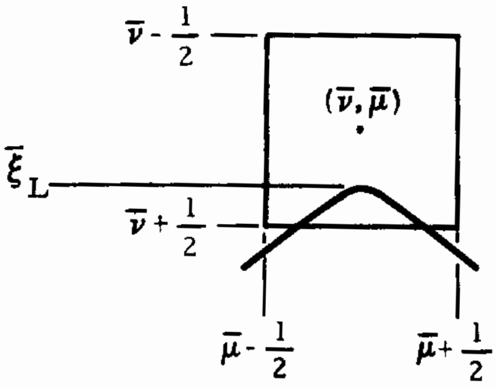
$$\left. + \int_{\bar{\xi}_L}^{\bar{\nu}+1/2} \frac{1 + i\bar{k}\bar{\xi}}{\bar{\xi}^2} e^{-i\bar{k}\bar{\xi}} \left[E\left(\bar{\mu} + \frac{1}{2}\right) - E\left(\bar{\mu} - \frac{1}{2}\right) \right] d\bar{\xi} \right\} \tag{102}$$

$$V^S = - \frac{1}{\pi} \left[\cosh^{-1} \frac{\bar{\xi}}{\sqrt{\left(\bar{\mu} + \frac{1}{2}\right)^2 + \ell^2}} - \cosh^{-1} \frac{\bar{\xi}}{\sqrt{\left(\bar{\mu} + \frac{1}{2}\right)^2 + \ell^2}} \right]_{\bar{\xi}_L}^{\bar{\nu} + \frac{1}{2}} \tag{103}$$

Equations 100, 101, 102, and 103 apply to all possible influence coefficients relating the velocity potential and its vertical and horizontal derivatives at a receiving point due to the area of a unit strength source box within the fore Mach cone from that point. The majority of those boxes have either no cut or at least one of the streamwise sides cut.

When one of the boxes contains the apex point of the Mach hyperbola, the lower limit of integration in Equations 100, 101 and 102 becomes the relative height, ℓ, instead of either the box trailing edge or the most aft intersection point. The special treatment of the transendental functions $\sin(a)$, $\sin^{-1}(a)$, and $\cosh^{-1}(a)$ when a is an invalid argument still applies to the apex box.

The influence coefficients for this box are somewhat simplified by the fact that the hyperbola does not intersect either streamwise side. The box and its influence coefficients are given below.



$$\bar{\xi}_L = \ell$$

$$\Phi = -\int_{\ell}^{\bar{\nu}+1/2} e^{-i\bar{k}\bar{\xi}} J_o\left(\frac{\bar{k}}{M}\sqrt{\bar{\xi}^2 - \ell^2}\right) d\bar{\xi} \tag{104}$$

$$W = \ell \left\{ \frac{e^{-i\bar{k}(\bar{\nu}+1/2)}}{\bar{\nu}+1/2} J_o\left(\frac{\bar{k}}{M}\sqrt{(\bar{\nu}+1/2)^2 - \ell^2}\right) \right. \tag{105}$$

$$\left. + \int_{\ell}^{\bar{\nu}+1/2} \frac{1+i\bar{k}\bar{\xi}}{\bar{\xi}^2} e^{-i\bar{k}\bar{\xi}} J_o\left(\frac{\bar{k}}{M}\sqrt{\bar{\xi}^2 - \ell^2}\right) d\bar{\xi} \right\}$$

$$V = O \tag{106}$$

In the limit as $\ell \rightarrow 0$ these influence coefficients become those for the planar case of a box affecting itself. The coefficients are:

$$\Phi\ (0,\ 0) = -\int_{o}^{1/2} e^{-i\bar{k}\bar{\xi}} J_o\left(\frac{\bar{k}}{M}\bar{\xi}\right) d\bar{\xi} \tag{107}$$

$$W\ (0,\ 0) = 1.0 \tag{108}$$

$$V\ (0,\ 0) = 0 \tag{109}$$

The expressions for the other planar influence coefficients can be obtained by setting $\ell = 0$ in Equations 100, 101, 102, and 103.

The influence coefficients are evaluated numerically by a 5-point Gaussian quadrature technique similar to the method described in Part II of this report.

APPENDIX II. SAMPLE INPUT FOR MACH BOX INTERFERENCE PROGRAM

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE ____ of ____ JOB NO. ______

Col.	NUMBER	IDENTIFICATION	DESCRIPTION	DO NOT KEY PUNCH	
1	2.0		EMACH	- MACH NUMBER	$1.0 < M_\infty \leq 3.5$
13	628.318		AS	- SPEED OF SOUND	L/T
25	1.0		CRØØT	- ROOT CHORD	L
37					
49		73 ... 80			
61					
1	65.0		SLEW	- WING L.E. SWEEP ANGLE	$0° \leq \Lambda < 90°$
13	0.0		STEW	- WING T.E. SWEEP ANGLE	$-\cos^{-1}\frac{1}{M} \leq \Lambda \leq \cos^{-1}\frac{1}{M}$
25	0.279786		YFL	- ROOT CHORD TO FOLD LINE - L	
37	65.0		SLET	- TIP L.E. SWEEP ANGLE	$0° \leq \Lambda \leq 90°$
49	0.0	73 ... 80	STET	- TIP T.E. SWEEP ANGLE	$-\cos^{-1}\frac{1}{M} \leq \Lambda \leq \cos^{-1}\frac{1}{M}$
61	0.46631		YTIP	- ROOT CHORD TO TIP LINE - L	
1	10		NBØX	- NUMBER OF CHORDWISE BOXES	MAX 20
13	2		NFREQ	- NUMBER OF INPUT FREQUENCIES	MAX 20
25	2		MØDES	- NUMBER OF MODE SHAPES	MAX 10
37	0		LVPIC	- PRINT VPIC'S	0 ~ NO 1 ~ YES
49	0	73 ... 80	LSSUL	- PRINT INITIAL H^U, H^L	0 ~ NO 1 ~ YES
61	1		LDPHI	- PRINT FINAL $\Delta\phi$'S AND H^U, H^L	0 ~ NO 1 ~ YES
1	1		MDEW	- WING MODE SHAPE INPUT	-1 ~ DEFLECTIONS 1 ~ COEFFICIENTS
13	1		NPØLW	- INPUT OR FITTED POLYNOMIAL DEGREE	MAX 5
25	0		NPØW	- NUMBER OF DEFLECTION POINTS	MAX 100
37	0		LCØW	- PRINT POLYNOMIAL COEFFICIENTS	0 ~ NO 1 ~ YES
49		73 ... 80			
61					

FORM 114-C-17 REV. 7-58 - VELLUM

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ________ PROGRAMMER ____________ DATE ________ PAGE ____ of ____ JOB NO. ________

Col.	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	1	73 … 80	MDET - TIP MODE SHAPE INPUT -1 ≈ DEFLECTIONS 1 ≈ COEFFICIENTS; 0 FOR NO TIP
13	1		NPØLT - INPUT OR FITTED POLYNOMIAL DEGREE MAX 5
25	0		NPØW - NUMBER OF DEFLECTION POINTS MAX 100
37	0		LCØT - PRINT POLYNOMIAL COEFFICIENTS 0 ≈ NO 1 ≈ YES
49			
61			
1	0 . 0	73 … 80	ØMEGA (1) ith FREQUENCY 1≤i≤NFREQ - CPS
13	2 . 0		ANGS (1) NUMBER OF FOLD ANGLES/ith FREQUENCY
25	1 0 . 0		ØMEGA (I)
37	1 . 0		ANGS (I)
49			
61			
1	0 . 0	73 … 80	FANG FOLD ANGLE FOR SET 1 0°≤Y≤90°
13			CORRESPONDING TO OMEGA (1)
25			AND FIRST OF ANGS (1) VALUES.
37			
49			*This card must precede mode shape data even if
61			single planar surface is being run.
1	0 . 0	73 … 80	THW(1) -1.0 (THICKNESS MODE), 0.0 (NO DPHI), 1.0 (NONTHICKNESS)
13	1 . 0		CØW (1, 1) COEFFICIENTS FOR WING MODE 1
25	0 . 0		CØW (1, 2)
37	0 . 0		CØW (1, 3)
49			
61			

FORM 114-C-17 REV. 7-58 - VELLUM

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE ____ of ____ JOB NO. ______

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	1.0		THW (I) -1.0 (THICKNESS MODE), 0.0 (NO DPHI), 1.0 (NONTHICKNESS)
13	0.0		CØW (2,1) COEFFICIENTS FOR WING MODE I
25	1.0		CØW (2,2)
37	0.0		CØW (I,J)
49		73 80	
61			
1	1.0		THT (1) -1.0(THICKNESS), 0.0 OR 1.0 (NONTHICKNESS MODE)
13	1.0		CØT (1,1) COEFFICIENTS FOR TIP MODE 1
25	0.0		CØT (1,2)
37	0.0		CØT (1,3)
49		73 80	
61			
1	1.0		THT (I) -1.0 (THICKNESS), 0.0 OR 1.0 (NONTHICKNESS MODE)
13	0.0		CØT (2,1) COEFFICIENTS FOR TIP MODE I
25	1.0		CØT (2,2)
37	0.0		CØT (I,J)
49		73 80	
61			
1	30.0		FANG FOLD ANGLE FOR SET I $0° \leq \gamma \leq 90°$
13			CORRESPONDING TO ØMEGA (I)
25			AND NEXT OF ANGS (I) VALUES.
37			
49		73 80	
61			

FORM 114-C-17 REV. 7-58-VELLUM

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ____________ DATE ______ PAGE ____ of ____ JOB NO. ________

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	0 . 0		THW (1)
13	1 . 0		CØW (1, 1)
25	0 . 0		CØW (1, 2)
37	0 . 0		CØW (1, 3)
49		73 80	
61			
1	1 . 0		THW (I)
13	0 . 0		CØW (2, 1)
25	1 . 0		CØW (2, 2)
37	0 . 0		CØW (I, J)
49		73 80	
61			
1	0 . 0		THT (1)
13	0 . 8 6 6 3		CØT (1, 1)
25	0 . 0		CØT (1, 2)
37	0 . 0		CØT (1, 3)
49		73 80	
61			
1	1 . 0		THT (I)
13	0 . 0		CØT (2, 1)
25	0 8 6 6 3		CØT (2, 2)
37	0 0		CØT (I, J)
49		73 80	
61			

FORM 114-C-17 REV. 7-58-VELLUM

FORTRAN FIXED 1C DIGIT DECIMAL DATA

DECK NO. ________ PROGRAMMER ________________ DATE ________ PAGE ______ of ______ JOB NO. ____________

Col.	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	30.0		FANG
13			
25			
37			
49		73 … 80	
61			
1	1.0		THW (1)
13	1.0		CØW (1, 1)
25	0.0		CØW (1, 2)
37	0.0		CØW (1, 3)
49		73 … 80	
61			
1	1.0		THW (I)
13	0.0		CØW (2, 1)
25	1.0		CØW (2, 2)
37	0.0		CØW (I, J)
49		73 … 80	
61			
1	1.0		THT (1)
13	0.8663		CØT (1, 1)
25	0.0		CØT (1, 2)
37	0.0		CØT (1, 3)
49		73 … 80	
61			

FORM 114-C-17 REV. 7-58- VELLUM

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE ____ of ____ JOB NO. ______

Col.	NUMBER	IDENTIFICATION (73–80)	DESCRIPTION DO NOT KEY PUNCH
1	1 . 0		THT (I)
13	0 . 0		CØT (2, 1)
25	0 . 8 6 6 3		CØT (2, 2)
37	0 . 0		CØT (I, J)
49			
61			
1			
13			
25			
37			
49			
61			
1			
13			
25			
37			
49			
61			
1			
13			
25			
37			
49			
61			

FORM 114-C-17 REV. 7-58 - VELLUM

APPENDIX III. SAMPLE OUTPUT FOR MACH BOX INTERFERENCE PROGRAM

FØRTRAN IV PRØGRAM FØR CALCULATING AERØDYNAMICS ØF INTERSECTING
SUPERSØNIC LIFTING SURFACES BY THE SØURCE SUPERPØSITIØN METHØD.
THE MACH BØX TECHNIQUE IS APPLIED TØ A WING WITH FØLDED TIPS.

FLIGHT CØNDITIØNS AND GEØMETRY

MACH NUMBER	SPEED ØF SØUND	RØØT CHØRD	REFERENCE AREA
2.CCC00	628.318 L/T	1.C00 L	0.466 L**2
WING L.E. SWEEP	WING T.E. SWEEP	FØLD LINE SPAN	FØLD LINE CHØRD
65.CC0 DEG	0. DEG	0.280 L	0.400 L
TIP L.E. SWEEP	TIP T.E. SWEEP	TIP LINE SPAN	TIP LINE CHØRD
65.000 DEG	0. DEG	C.466 L	-0.000 L
CHØRDWISE BØXES	BØX CHØRD	SPANWISE BØXES	BØX SPAN
9	0.1077 L	7	0.0622 L

INNER AND OUTER WING AND TIP BOUNDARY BOXES

MIBW	MOBW	MIBT	MOBT	MOBTD	MOBWD
1	1	0	0	0	1
1	2	0	0	0	2
1	3	0	0	0	3
1	3	0	0	0	4
1	4	0	0	0	5
1	5	0	0	0	6
1	5	6	6	6	7
1	5	6	7	7	8
1	5	6	7	7	7

TRAILING EDGE COORDINATES

X/BOXL	Y/BOXW
9.286	0.
9.286	1.000
9.286	2.000
9.286	3.000
9.286	4.000
9.286	5.000
9.286	6.000

VELOCITY POTENTIAL DIFFERENCES (PHI UPPER - PHI LOWER) AND
SOURCE STRENGTHS WITH INTERFERENCE EFFECTS FOR MODE 2

N	M	R DPHI(N,M)	I DPHI(N,M)	R SSU(N,M)	I SSU(N,M)	R SSL(N,M)	I SSL(N,M)
1	1	-6.2175E-02	-0.	1.0000E 00	0.	-1.0000E 00	-0.
2	1	-1.1002E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
2	2	-1.0042E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
3	1	-2.0699E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
3	2	-1.7366E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
3	3	-1.2699E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
4	1	-2.7113E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
4	2	-2.7322E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
4	3	-2.1979E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
4	4	0.	0.	-1.3901E 00	-0.	1.3901E 00	-0.
5	1	-3.6280E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
5	2	-3.4188E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
5	3	-2.3463E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
5	4	-1.4357E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
5	5	0.	0.	-2.2053E-01	-0.	2.2053E-01	-0.
6	1	-4.3055E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
6	2	-3.7060E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
6	3	-3.3971E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
6	4	-2.7576E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
6	5	-1.7214E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
6	6	0.	0.	-1.8792E-01	-0.	1.8792E-01	-0.
7	1	-4.1659E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
7	2	-4.5710E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
7	3	-4.2856E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
7	4	-3.8843E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
7	5	-3.0709E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
7	6	-1.8649E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
7	7	0.	0.	-1.2195E-01	-0.	1.2195E-01	-0.
7	8	0.	0.	0.	0.	0.	0.
7	9	0.	0.	0.	0.	0.	0.

VELOCITY POTENTIAL DIFFERENCES (PHI UPPER - PHI LOWER) AND
SOURCE STRENGTHS WITH INTERFERENCE EFFECTS FOR MODE 2

N	M	R DPHI(N,M)	I DPHI(N,M)	R SSU(N,M)	I SSU(N,M)	R SSL(N,M)	I SSL(N,M)
8	1	-5.1052E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
8	2	-4.9452E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
8	3	-5.1930E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
8	4	-4.7656E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
8	5	-4.2609E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
8	6	-3.3582E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
8	7	-2.0125E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
8	8	0.	0.	-9.1364E-02	-0.	9.1364E-02	-0.
8	9	0.	0.	0.	0.	0.	0.
8	10	0.	0.	0.	0.	0.	0.
9	1	-5.8374E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
9	2	-5.8841E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
9	3	-5.5946E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
9	4	-5.7048E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
9	5	-5.2035E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
9	6	-4.6143E-01	-0.	1.0000E 00	0.	-1.0000E 0C	-0.
9	7	-3.6219E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
9	8	0.	0.	0.	0.	0.	0.
9	9	0.	0.	0.	0.	0.	0.
9	10	0.	0.	0.	0.	0.	0.

MACH BOX INTERFERENCE PROGRAM OUTPUT

OSCILLATORY FREQUENCY (CPS)	0.		9 BOXES IN CHORD DIRECTION	
REDUCED FREQUENCY (BOX LENGTH)	0.		FREE STREAM MACH NUMBER	2.000
REDUCED FREQUENCY (ROOT CHORD)	0.		TIP FOLD ANGLE (DEGREES)	0.

GENERALIZED FORCES

DPHI	DEFL	REAL PART	IMAG PART	ABS VALUE	PHASE ANGLE
1	1	-0.	-0.	0.	0. DEG
1	2	-0.	-0.	0.	0. DEG
2	1	-2.02671E 00	-0.	2.02671E 00	180.000 DEG
2	2	-1.30922E 00	-0.	1.30922E 00	180.000 DEG

VELOCITY POTENTIAL DIFFERENCES (PHI UPPER - PHI LOWER) AND
SOURCE STRENGTHS WITH INTERFERENCE EFFECTS FOR MODE 2

N	M	R DPHI(N,M)	I DPHI(N,M)	R SSU(N,M)	I SSU(N,M)	R SSL(N,M)	I SSL(N,M)
1	1	-6.2175E-02	-0.	1.0000E 00	0.	-1.0000E 00	-0.
2	1	-1.1002E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
2	2	-1.0042E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
3	1	-2.0699E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
3	2	-1.7366E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
3	3	-1.2699E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
4	1	-2.7113E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
4	2	-2.7322E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
4	3	-2.1979E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
4	4	0.	0.	-1.3901E 00	-0.	1.3901E 00	-0.
5	1	-3.6280E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
5	2	-3.4188E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
5	3	-2.3463E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
5	4	-1.4357E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
5	5	0.	0.	-2.2053E-01	-0.	2.2053E-01	-0.
6	1	-4.3055E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
6	2	-3.7060E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
6	3	-3.3971E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
6	4	-2.7576E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
6	5	-1.7214E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
6	6	0.	0.	-1.8792E-01	-0.	1.8792E-01	-0.
7	1	-4.1659E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
7	2	-4.5710E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
7	3	-4.2856E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
7	4	-3.8843E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
7	5	-3.0709E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
7	6	0.	0.	-1.9995E 00	-0.	-6.2407E-01	-0.
7	7	0.	0.	-1.2195E-01	-0.	1.2195E-01	-0.
7	8	0.	0.	0.	0.	0.	0.
7	9	-1.8835E-01	-0.	3.0294E 00	0.	-3.0294E 00	-0.

VELOCITY POTENTIAL DIFFERENCES (PHI UPPER - PHI LOWER) AND
SOURCE STRENGTHS WITH INTERFERENCE EFFECTS FOR MODE 2

N	M	R DPHI(N,M)	I DPHI(N,M)	R SSU(N,M)	I SSU(N,M)	R SSL(N,M)	I SSL(N,M)
8	1	-5.1052E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
8	2	-4.9452E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
8	3	-5.1930E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
8	4	-4.7656E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
8	5	-4.3328E-01	-0.	1.3477E 00	0.	-1.0000E 00	-0.
8	6	0.	0.	-2.0928E 00	-0.	3.9366E-01	-0.
8	7	0.	0.	-3.9155E-01	-0.	4.0362E-02	-0.
8	8	0.	0.	-9.1364E-02	-0.	9.1364E-02	-0.
8	9	-3.1177E-01	-0.	2.6830E 00	0.	-2.6830E 00	-0.
8	10	-2.0306E-01	-0.	1.4022E 00	0.	-1.4022E 00	0.
9	1	-5.8374E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
9	2	-5.8841E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
9	3	-5.5946E-01	-0.	1.0000E 00	0.	-1.0000E 00	-0.
9	4	-5.7903E-01	-0.	1.1121E 00	0.	-1.0000E 00	-0.
9	5	-4.9924E-01	-0.	1.2430E 00	0.	-1.0000E 00	0.
9	6	0.	0.	-2.1980E 00	-0.	-1.4318E-01	-0.
9	7	0.	0.	-6.2728E-01	-0.	-2.0469E-01	0.
9	8	0.	0.	0.	0.	0.	0.
9	9	-4.1930E-01	-0.	2.8195E 00	0.	-2.8195E 00	-0.
9	10	-3.3742E-01	-0.	1.4604E 00	0.	-1.4604E 00	0.

MACH BØX INTERFERENCE PRØGRAM ØUTPUT

ØSCILLATØRY FREQUENCY (CPS)	0.	9 BØXES IN CHØRD DIRECTIØN	
REDUCED FREQUENCY (BØX LENGTH)	0.	FREE STREAM MACH NUMBER	2.000
REDUCED FREQUENCY (RØØT CHØRD)	0.	TIP FØLD ANGLE (DEGREES)	30.000

GENERALIZED FØRCES

DPHI	DEFL	REAL PART	IMAG PART	ABS VALUE	PHASE ANGLE
1	1	-0.	-0.	0.	0. DEG
1	2	-0.	-0.	0.	0. DEG
2	1	-1.87613E 00	0.	1.87613E 00	180.000 DEG
2	2	-1.17817E 00	0.	1.17817E 00	180.000 DEG

VELOCITY POTENTIAL DIFFERENCES (PHI UPPER - PHI LOWER) AND
SOURCE STRENGTHS WITH INTERFERENCE EFFECTS FOR MODE 1

N	M	R DPHI(N,M)	I DPHI(N,M)	R SSU(N,M)	I SSU(N,M)	R SSL(N,M)	I SSL(N,M)
1	1	-5.5791E-04	-3.1081E-02	0.	5.0000E-01	-0.	-5.0000E-01
2	1	-2.0870E-03	-5.4939E-02	0.	5.0000E-01	-0.	-5.0000E-01
2	2	-2.0219E-03	-5.0141E-02	0.	5.0000E-01	-0.	-5.0000E-01
3	1	-6.4435E-03	-1.0316E-01	0.	5.0000E-01	-0.	-5.0000E-01
3	2	-5.2994E-03	-8.6545E-02	0.	5.0000E-01	-0.	-5.0000E-01
3	3	-4.0007E-03	-6.3264E-02	0.	5.0000E-01	-0.	-5.0000E-01
4	1	-1.1349E-02	-1.3476E-01	0.	5.0000E-01	-0.	-5.0000E-01
4	2	-1.1673E-02	-1.3578E-01	0.	5.0000E-01	-0.	-5.0000E-01
4	3	-9.3184E-03	-1.0921E-01	0.	5.0000E-01	-0.	-5.0000E-01
4	4	0.	0.	-8.1243E-02	-6.8868E-01	8.1243E-02	6.8868E-01
5	1	-1.9702E-02	-1.7963E-01	0.	5.0000E-01	-0.	-5.0000E-01
5	2	-1.8309E-02	-1.6927E-01	0.	5.0000E-01	-0.	-5.0000E-01
5	3	-1.1136E-02	-1.1632E-01	0.	5.0000E-01	-0.	-5.0000E-01
5	4	-6.4766E-03	-7.1176E-02	0.	5.0000E-01	-0.	-5.0000E-01
5	5	0.	0.	-2.9390E-02	-1.0643E-01	2.9390E-02	1.0643E-01
6	1	-2.8031E-02	-2.1219E-01	0.	5.0000E-01	-0.	-5.0000E-01
6	2	-2.1745E-02	-1.8302E-01	0.	5.0000E-01	-0.	-5.0000E-01
6	3	-1.9912E-02	-1.6773E-01	0.	5.0000E-01	-0.	-5.0000E-01
6	4	-1.6520E-02	-1.3609E-01	0.	5.0000E-01	-0.	-5.0000E-01
6	5	-9.7556E-03	-8.4994E-02	0.	5.0000E-01	-0.	-5.0000E-01
6	6	0.	0.	-2.6957E-02	-8.9705E-02	2.6957E-02	8.9705E-02
7	1	-2.5458E-02	-2.0545E-01	0.	5.0000E-01	-0.	-5.0000E-01
7	2	-3.2195E-02	-2.2456E-01	0.	5.0000E-01	-0.	-5.0000E-01
7	3	-3.0388E-02	-2.1050E-01	0.	5.0000E-01	-0.	-5.0000E-01
7	4	-2.7375E-02	-1.9077E-01	0.	5.0000E-01	-0.	-5.0000E-01
7	5	-2.2021E-02	-1.5070E-01	0.	5.0000E-01	-0.	-5.0000E-01
7	6	0.	0.	-1.7538E-01	-9.7694E-01	-2.1059E-02	-3.0891E-01
7	7	0.	0.	-2.2307E-02	-5.6588E-02	2.2307E-02	5.6588E-02
7	8	0.	0.	0.	0.	0.	0.
7	9	-1.4103E-02	-9.2313E-02	2.0016E-01	1.4887E 00	-2.0016E-01	-1.4887E 00

VELOCITY POTENTIAL DIFFERENCES (PHI UPPER - PHI LOWER) AND
SOURCE STRENGTHS WITH INTERFERENCE EFFECTS FOR MODE 1

N	M	R DPHI(N,M)	I DPHI(N,M)	R SSU(N,M)	I SSU(N,M)	R SSL(N,M)	I SSL(N,M)
8	1	-3.8351E-02	-2.5017E-01	0.	5.0000E-01	-0.	-5.0000E-01
8	2	-3.6414E-02	-2.4249E-01	0.	5.0000E-01	-0.	-5.0000E-01
8	3	-4.2690E-02	-2.5353E-01	0.	5.0000E-01	-0.	-5.0000E-01
8	4	-3.9303E-02	-2.3258E-01	0.	5.0000E-01	-0.	-5.0000E-01
8	5	-3.6795E-02	-2.1119E-01	3.3287E-02	6.6974E-01	-0.	-5.0000E-01
8	6	0.	0.	-2.1562E-01	-1.0124E 00	8.4990E-02	1.8635E-01
8	7	0.	0.	-6.3473E-02	-1.8435E-01	3.2249E-02	1.3231E-02
8	8	0.	0.	-1.9658E-02	-4.1084E-02	1.9658E-02	4.1084E-02
8	9	-2.7680E-02	-1.5158E-01	1.9491E-01	1.3092E 00	-1.9491E-01	-1.3092E 00
8	10	-1.6987E-02	-9.8980E-02	6.8009E-02	6.9002E-01	-6.8009E-02	-6.9002E-01
9	1	-5.0137E-02	-2.8434E-01	0.	5.0000E-01	-0.	-5.0000E-01
9	2	-5.1254E-02	-2.8642E-01	0.	5.0000E-01	-0.	-5.0000E-01
9	3	-4.7701E-02	-2.7254E-01	0.	5.0000E-01	-0.	-5.0000E-01
9	4	-5.5190E-02	-2.8038E-01	1.4519E-02	5.5368E-01	-0.	-5.0000E-01
9	5	-4.7753E-02	-2.4151E-01	2.2102E-02	6.1819E-01	-0.	-5.0000E-01
9	6	0.	0.	-2.2390E-01	-1.0615E 00	4.8558E-02	-8.0207E-02
9	7	0.	0.	-9.7709E-02	-2.9466E-01	3.8760E-02	-1.1195E-01
9	8	0.	0.	0.	0.	0.	0.
9	9	-4.0363E-02	-2.0277E-01	2.0292E-01	1.3756E 00	-2.0292E-01	-1.3756E 00
9	10	-3.2927E-02	-1.6301E-01	8.4994E-02	7.1431E-01	-8.4994E-02	-7.1431E-01

VELØCITY PØTENTIAL DIFFERENCES (PHI UPPER - PHI LØWER) AND
SØURCE STRENGTHS WITH INTERFERENCE EFFECTS FØR MØDE 2

N	M	R DPHI(N,M)	I DPHI(N,M)	R SSU(N,M)	I SSU(N,M)	R SSL(N,M)	I SSL(N,M)
1	1	-6.2191E-02	-5.5772E-04	1.0000E 00	2.6922E-02	-1.0000E 00	-2.6922E-02
2	1	-1.1005E-01	-2.1313E-03	1.0000E 00	8.0767E-02	-1.0000E 00	-8.0767E-02
2	2	-1.0045E-01	-2.0031E-03	1.0000E 00	8.0767E-02	-1.0000E 00	-8.0767E-02
3	1	-2.0727E-01	-6.0365E-03	1.0000E 00	1.3461E-01	-1.0000E 00	-1.3461E-01
3	2	-1.7382E-01	-5.3773E-03	1.0000E 00	1.3461E-01	-1.0000E 00	-1.3461E-01
3	3	-1.2702E-01	-4.1519E-03	1.0000E 00	1.3461E-01	-1.0000E 00	-1.3461E-01
4	1	-2.7179E-01	-1.1739E-02	1.0000E 00	1.8846E-01	-1.0000E 00	-1.8846E-01
4	2	-2.7388E-01	-1.1207E-02	1.0000E 00	1.8846E-01	-1.0000E 00	-1.8846E-01
4	3	-2.2015E-01	-9.2931E-03	1.0000E 00	1.8846E-01	-1.0000E 00	-1.8846E-01
4	4	0.	0.	-1.3886E 00	3.6504E-02	1.3886E 00	-3.6504E-02
5	1	-3.6439E-01	-1.9110E-02	1.0000E 00	2.4230E-01	-1.0000E 00	-2.4230E-01
5	2	-3.4323E-01	-1.8790E-02	1.0000E 00	2.4230E-01	-1.0000E 00	-2.4230E-01
5	3	-2.3567E-01	-1.8492E-02	1.0000E 00	2.4230E-01	-1.0000E 00	-2.4230E-01
5	4	-1.4409E-01	-1.3895E-02	1.0000E 00	2.4230E-01	-1.0000E 00	-2.4230E-01
5	5	0.	0.	-2.1681E-01	4.3481E-02	2.1681E-01	-4.3481E-02
6	1	-4.3326E-01	-2.8715E-02	1.0000E 00	2.9615E-01	-1.0000E 00	-2.9615E-01
6	2	-3.7351E-01	-3.3454E-02	1.0000E 00	2.9615E-01	-1.0000E 00	-2.9615E-01
6	3	-3.4209E-01	-3.1188E-02	1.0000E 00	2.9615E-01	-1.0000E 00	-2.9615E-01
6	4	-2.7734E-01	-2.3899E-02	1.0000E 00	2.9615E-01	-1.0000E 00	-2.9615E-01
6	5	-1.7295E-01	-1.7733E-02	1.0000E 00	2.9615E-01	-1.0000E 00	-2.9615E-01
6	6	0.	0.	-1.8313E-01	3.9905E-02	1.8313E-01	-3.9905E-02
7	1	-4.2254E-01	-5.5862E-02	1.0000E 00	3.4999E-01	-1.0000E 00	-3.4999E-01
7	2	-4.6203E-01	-4.6288E-02	1.0000E 00	3.4999E-01	-1.0000E 00	-3.4999E-01
7	3	-4.3280E-01	-4.2439E-02	1.0000E 00	3.4999E-01	-1.0000E 00	-3.4999E-01
7	4	-3.9182E-01	-3.9292E-02	1.0000E 00	3.4999E-01	-1.0000E 00	-3.4999E-01
7	5	-3.0925E-01	-2.9960E-02	1.0000E 00	3.4999E-01	-1.0000E 00	-3.4999E-01
7	6	0.	0.	-2.0108E 00	-5.9613E-02	-6.3261E-01	-2.0052E-01
7	7	0.	0.	-1.1625E-01	3.6067E-02	1.1625E-01	-3.6067E-02
7	8	0.	0.	0.	0.	0.	0.
7	9	-1.8978E-01	-1.8675E-02	3.0466E 00	3.5512E-01	-3.0466E 00	-3.5512E-01

VELOCITY POTENTIAL DIFFERENCES (PHI UPPER - PHI LOWER) AND
SOURCE STRENGTHS WITH INTERFERENCE EFFECTS FOR MODE 2

N	M	R DPHI(N,M)	I DPHI(N,M)	R SSU(N,M)	I SSU(N,M)	R SSL(N,M)	I SSL(N,M)
8	1	-5.1958E-01	-7.0037E-02	1.0000E 00	4.0384E-01	-1.0000E 00	-4.0384E-01
8	2	-5.0334E-01	-6.9919E-02	1.0000E 00	4.0384E-01	-1.0000E 00	-4.0384E-01
8	3	-5.2620E-01	-5.7545E-02	1.0000E 00	4.0384E-01	-1.0000E 00	-4.0384E-01
8	4	-4.8228E-01	-5.2450E-02	1.0000E 00	4.0384E-01	-1.0000E 00	-4.0384E-01
8	5	-4.3801E-01	-4.5725E-02	1.3527E 00	4.2362E-01	-1.0000E 00	-4.0384E-01
8	6	0.	0.	-2.1076E 00	-7.1716E-02	3.9800E-01	-2.0703E-01
8	7	0.	0.	-3.8929E-01	6.0494E-02	3.4936E-02	-8.5057E-02
8	8	0.	0.	-8.4881E-02	3.3157E-02	8.4881E-02	-3.3157E-02
8	9	-3.1456E-01	-3.0362E-02	2.6968E 00	4.1413E-01	-2.6968E 00	-4.1413E-01
8	10	-2.0470E-01	-2.2627E-02	1.4023E 00	3.1490E-01	-1.4023E 00	-3.1490E-01
9	1	-5.9661E-01	-8.7007E-02	1.0000E 00	4.5768E-01	-1.0000E 00	-4.5768E-01
9	2	-6.0111E-01	-8.5535E-02	1.0000E 00	4.5768E-01	-1.0000E 00	-4.5768E-01
9	3	-5.7153E-01	-8.4815E-02	1.0000E 00	4.5768E-01	-1.0000E 00	-4.5768E-01
9	4	-5.8817E-01	-6.7913E-02	1.1135E 00	4.5604E-01	-1.0000E 00	-4.5768E-01
9	5	-5.0639F-01	-5.9909E-02	1.2473E 00	4.9233E-01	-1.0000E 00	-4.5768E-01
9	6	0.	0.	-2.2299E 00	-1.8398E-01	-1.4943E-01	-2.9759E-01
9	7	0.	0.	-6.2593E-01	6.8099E-02	-2.1256E-01	-2.1521E-01
9	8	0.	0.	0.	0.	0.	0.
9	9	-4.2509E-01	-5.0567E-02	2.8561E 00	5.7705E-01	-2.8561E 00	-5.7705E-01
9	10	-3.4082E-01	-3.7447E-02	1.4614E 00	3.5701E-01	-1.4614E 00	-3.5701E-01

MACH BØX INTERFERENCE PRØGRAM ØUTPUT

ØSCILLATØRY FREQUENCY (CPS) 100.00000

REDUCED FREQUENCY (BØX LENGTH) 0.05384

REDUCED FREQUENCY (RØØT CHØRD) 0.50000

9 BØXES IN CHØRD DIRECTIØN

FREE STREAM MACH NUMBER 2.000

TIP FØLD ANGLE (DEGREES) 30.000

GENERALIZED FØRCES

DPHI	DEFL	REAL PART	IMAG PART	ABS VALUE	PHASE ANGLE
1	1	-1.68557E-02	-9.30777E-01	9.30929E-01	-91.037 DEG
1	2	-1.23360E-02	-5.83169E-01	5.83299E-01	-91.212 DEG
2	1	-1.88000E 00	-6.25115E-01	1.98121E 00	-161.608 DEG
2	2	-1.18191E 00	-4.56494E-01	1.26700E 00	-158.882 DEG

APPENDIX IV. PROGRAM LISTINGS

MACH BOX INTERFERENCE PROGRAM

```
      MBOX       MAIN PROGRAM
      COMMON/A/DPHI(20,60,2),SSU(20,60,2),SSL(20,60,2),THW(10),THT(10)   MBX00020
      COMMON/B/MIBW(20),MOBW(20),MIBT(20),MOBT(20),MOBTD(20),MOBWD(20)   MBX00030
      COMMON/C/BARNU,BARMU,EL,CFR,CFI,CNR,CNI,EKBAR,EKM,P(10),W(10)      MBX00040
      COMMON/D/XTE(40),YTE(40),QR(21,2),QI(21,2),COW(10,21),COT(10,21)   MBX00050
      COMMON/E/BOXL,BOXW,EMACH,EK,YNFL,FANG,CFA,SFA,NBOX,CAPPHI(20,20,2)MBX00060
      COMMON/F/MODE,KODE,NP,MP,MFL,NMAX,NBW,NBT,NPINT,NPFLET,NPOLW,NPOLTMBX00070
      COMMON/G/SLEW,STEW,YFL,SLET,STET,YTIP,CROOT,CFL,CTIP,MDET,MTIP     MBX00080
                                DIMENSION                                MBX00090
     1XINW(100),YINW(100),ZINW(10,100),XINT(100),YINT(100),ZINT(10,100),MBX00100
     2OMEGA(20),ANGS(20),Q(10,10,2)                                      MBX00110
                               EQUIVALENCE                               MBX00120
     1(DPHI,XINW,XINT),(DPHI(101),YINW,YINT),(DPHI(201),ZINW,ZINT)       MBX00130
 2345 READ (5,31) EMACH,AS,CROOT                                         MBX00140
      IF(EMACH.GT.1.0.AND.EMACH.LE.3.5) GO TO 2348                       MBX00150
      WRITE (6,2347)                                                     MBX00160
 2347 FORMAT(1H1,20X,41HMACH NUMBER OUT OF LIMITS,PROGRAM STOPPED)       MBX00170
      CALL EXIT                                                          MBX00180
 2348 READ (5,31) SLEW,STEW,YFL,SLET,STET,YTIP                           MBX00190
   31 FORMAT(6E12.5)                                                     MBX00200
      READ (5,10) NBOX,NFREQ,MODES,LVPIC,LSSJL,LDPHI                     MBX00210
      READ (5,10) MDEW,NPOLW,NPOW,LCOW                                   MBX00220
      READ (5,10) MDET,NPOLT,NPOT,LCOT                                   MBX00230
   10 FORMAT(6I12)                                                       MBX00240
      CALL BOUNDS(ERROR)                                                 MBX00250
      IF(ERROR)63,98,63                                                  MBX00260
   63 WRITE (6,74) EMACH                                                 MBX00270
   74 FORMAT(51H1  YOUR CONFIGURATION HAD A SJBSONIC TRAILING EDGE,/ 2X,MBX00280
     161HTO MAKE THAT EDGE SONIC THE MACH NUMBER HAS BEEN INCREASED TO//MBX00290
     220X, 8HEMACH = , F8.5)                                             MBX00300
   98 CONTINUE                                                           MBX00310
      AREA = YFL*(CROOT - CTIP) + YTIP*(CFL + CTIP)                      MBX00320
      QF = -4.0 * BOXL * BOXW / AREA                                     MBX00330
      WRITE (6,309)                                                      MBX00340
  309 FORMAT(1H1//////        21X,63HFORTRAN IV PROGRAM FOR CALCULATING MBX00350
     1AERODYNAMICS OF INTERSECTING /1H0,20X,63HSUPERSONIC LIFTING SURFACMBX00360
     2ES BY THE SOURCE SUPERPOSITION METHOD. /1H0,20X,61HTHE MACH BOX TEMBX00370
     3CHNIQUE IS APPLIED TO A WING WITH FOLDED TIPS.  )                  MBX00380
      WRITE (6,310) EMACH,AS,CROOT,AREA,SLEW,STEW,YFL,CFL,               MBX00390
```

MBØX   MAIN PRØGRAM

```
     1SLET,STET,YTIP,CTIP,NBØX,BØXL,MTIP,BØXW                              MBX00400
 310 FØRMAT(1H-,/          ,37X,30HFLIGHT CØNDITIØNS AND GEØMETRY,/1H-/ MBX00410
    1 11X,11HMACH NUMBER,11X,14HSPEED ØF SØUND,13X,10HRØØT CHØRD,12X,    MBX00420
    2 14HREFERENCE AREA//F20.5,F22.3,4H L/T,F21.3,2H L,F21.3,5H L**2/    MBX00430
    3 1H-,8X,15HWING L.E. SWEEP,9X,15HWING T.E. SWEEP ,9X ,14HFØLD LINEMBX00440
    4 SPAN,10X,15HFØLD LINE CHØRD//F18.3,4H DEG,F20.3,4H DEG,F21.3,      MBX00450
    5 2H L,F21.3,2H L/1H-,9X,14HTIP L.E. SWEEP,10X,14HTIP T.E. SWEEP,    MBX00460
    6 10X,13HTIP LINE SPAN,11X,14HTIP LINE CHØRD//F18.3,4H DEG,F20.3,    MBX00470
    7 4H DEG,F21.3,2H L,F21.3,2H L/1H-,8X,15HCHØRDWISE BØXES,12X, 9HBØXMBX00480
    8 CHØRD,12X,14HSPANWISE BØXES,14X,6HBØX SPAN//I17,F26.4,2H L,I19,    MBX00490
    9 F27.4,2H L)                                                        MBX00500
      WRITE (6,311) (MIBW(I),MØBW(I),MIBT(I),MØBT(I),MØBTD(I),           MBX00510
     1MØBWD(I),I = 1,NBØX)                                               MBX00520
 311 FØRMAT(1H1,31X,43HINNER AND ØUTER WING AND TIP BØUNDARY BØXES/1H-/MBX00530
    112X,4HMIBW,11X,4HMØBW,11X,4HMIBT,11X,4HMØBT,11X,5HMØBTD,10X,        MBX00540
    25HMØBWD /1H0/(6I15))                                                MBX00550
      WRITE (6,333) (XTE(I),YTE(I),I = 1,MTIP)                           MBX00560
 333 FØRMAT(1H1,39X,25HTRAILING EDGE CØØRDINATES/1H-/43X, 6HX/BØXL, 8X,MBX00570
    1 6HY/BØXW /1H0/(35X,2F14.3))                                        MBX00580
      NMAX = 1                                                           MBX00590
      DØ 123 I = 1,NBØX                                                  MBX00600
 123  NMAX = MAX0(MØBWD(I),NMAX)                                         MBX00610
      READ (5,31) (ØMEGA(I),ANGS(I),I=1,NFREQ)                           MBX00620
      DØ 820 L=1,NFREQ                                                   MBX00630
      EK = ØMEGA(L) * BØXL /(EMACH * AS) * 6.2831853                     MBX00640
      EKBAR = EK *(EMACH**2)/((EMACH**2)-1.0)                            MBX00650
      EKREF = EK * CRØØT/BØXL                                            MBX00660
      EKM = EKBAR/EMACH                                                  MBX00670
      FAN = ANGS(L)                                                      MBX00680
      DØ 140 I1 = 1,20                                                   MBX00690
      DØ 140 I2 = 1,20                                                   MBX00700
      CAPPHI(I1,I2,1) = 0.0                                              MBX00710
 140  CAPPHI(I1,I2,2) = 0.0                                              MBX00720
      CALL CAPHI                                                         MBX00730
      IF(LVPIC)198,202,198                                               MBX00740
 198  WRITE (6,111)                                                      MBX00750
 111  FØRMAT(1H1,27X,48HPLANAR VELØCITY PØTENTIAL INFLUENCE CØEFFICIENTSMBX00760
```

MBOX   MAIN PROGRAM

```
    1/1H-,9X,1HI,5X,1HJ,10X,5HBARNU,5X,5HBARMU,10X,16HREAL  CAPPHI(I,J) MBX00770
    2,6X,16HIMAG CAPPHI(I,J),/1HO)                                      MBX00780
     DO 200 I = 1,NBOX                                                  MBX00790
     DO 200 J = 1,I                                                     MBX00800
     BARNU = I - 1                                                      MBX00810
     BARMU = J - 1                                                      MBX00820
 200 WRITE (6,5) I,J,BARNU,BARMU,CAPPHI(I,J,1),CAPPHI(I,J,2)            MBX00830
   5 FORMAT( 5X,2I6,4X,2F10.1,4X,1P1E20.4,1P1E22.4)                     MBX00840
 202 IF(L.GT.1 .AND. ANGS(L).LT.1.0) GO TO 95                           MBX00850
  20 READ (5,31) FANG                                                   MBX00860
     CFA  = COSD(FANG)                                                  MBX00870
     SFA = SIND(ABS(FANG))                                              MBX00880
     IF(MDEW)30,60,60                                                   MBX00890
  30 DO 36 IX = 1,100                                                   MBX00900
     XINW(IX) = 0.0                                                     MBX00910
     YINW(IX) = 0.0                                                     MBX00920
     DO 36 IZ = 1,10                                                    MBX00930
  36 ZINW(IZ,IX) = 0.0                                                  MBX00940
     READ (5,31) (XINW(J),YINW(J),J=1,NPOW)                             MBX00950
     DO 50 I = 1,MODES                                                  MBX00960
  50 READ (5,31) THW(I),(ZINW(I,J),J=1,NPOW)                            MBX00970
     CALL LSSUR(XINW,YINW,ZINW,NPOW,MODES,NPOLW+1,NCW,COW)              MBX00980
     GO TO 81                                                           MBX00990
  60 NCW =((NPOLW + 3)*NPOLW + 2)/2                                     MBX01000
  70 DO 80 I = 1,MODES                                                  MBX01010
  80 READ (5,31) THW(I),(COW(I,J),J=1,NCW)                              MBX01020
  81 IF(LCOW)84,85,84                                                   MBX01030
  84 WRITE (6,770)                                                      MBX01040
 770 FORMAT(1H1,32X,39HWING DEFLECTION POLYNOMIAL COEFFICIENTS/1HO,11X,MBX01050
    138HZ(MODE) = SUM(A(MODE,T)*X**(R-S)*Y**S) /16X,                    MBX01060
    232HWHERE T = (R**2 + R + 2*S + 2)/2 /1H-, 4X, 4HMODE, 3X,25HA(MODEMBX01070
    3,T),T = 1,2,---,NCW )                                              MBX01080
     DO 771 I = 1,MODES                                                 MBX01090
 771 WRITE (6,772) I,(COW(I,J),J=1,NCW)                                 MBX01100
 772 FORMAT(1HO, I7,1P6E15.4 /( 8X,1P6E15.4))                           MBX01110
  85 IF(MDET)90,108,110                                                 MBX01120
 108 NPOLT = -1                                                         MBX01130
```

MBOX MAIN PROGRAM

```
      GO TO 95                                                          MBX01140
   90 DO 96 IX = 1,100                                                  MBX01150
      XINT(IX) = 0.0                                                    MBX01160
      YINT(IX) = 0.0                                                    MBX01170
      DO 96 IZ = 1,10                                                   MBX01180
   96 ZINT(IZ,IX) = 0.0                                                 MBX01190
      READ (5,31) (XINT(J),YINT(J),J=1,NPOT)                            MBX01200
      DO 105 I = 1,MODES                                                MBX01210
  105 READ (5,31) THT(I),(ZINT(I,J),J=1,NPOT)                           MBX01220
      CALL LSSUR(XINT,YINT,ZINT,NPOT,MODES,NPOLT+1,NCT,COT)             MBX01230
      GO TO 800                                                         MBX01240
  110 NCT =((NPOLT + 3)*NPOLT + 2)/2                                    MBX01250
  120 DO 130 I = 1,MODES                                                MBX01260
  130 READ (5,31) THT(I),(COT(I,J),J=1,NCT)                             MBX01270
  800 IF(LCOT)94,95,94                                                  MBX01280
   94 WRITE (6,780)                                                     MBX01290
  780 FORMAT(1H1,33X,38HTIP DEFLECTION POLYNOMIAL COEFFICIENTS /1H0,11X,MBX01300
     138HZ(MODE) = SUM(A(MODE,T)*X**(R-S)*Y**S) /16X,                   MBX01310
     232HWHERE T = (R**2 + R + 2*S + 2)/2 /1H-, 4X, 4HMODE, 3X,25HA(MODEMBX01320
     3,T),T = 1,2,---,NCT )                                             MBX01330
      DO 781 I = 1,MODES                                                MBX01340
  781 WRITE (6,772) I,(COT(I,J),J=1,NCT)                                MBX01350
   95 DO 801 I1 = 1,10                                                  MBX01360
      DO 801 I2=1,10                                                    MBX01370
      Q(I1,I2,1)=0.0                                                    MBX01380
801   Q(I1,I2,2)=0.0                                                    MBX01390
      DO 810 MODE=1,MODES                                               MBX01400
      IF(THW(MODE))807,810,807                                          MBX01410
  807 MODE = MODE                                                       MBX01420
      DO 2 L1=1,20                                                      MBX01430
      DO 2 L2=1,60                                                      MBX01440
      DO 2 L3=1,2                                                       MBX01450
      DPHI(L1,L2,L3) = 0.0                                              MBX01460
      SSU(L1,L2,L3)=0.0                                                 MBX01470
2     SSL(L1,L2,L3)=0.0                                                 MBX01480
      CALL SOURCE                                                       MBX01490
      IF(LSSUL)150,160,150                                              MBX01500
```

MBØX   MAIN PRØGRAM

```
150 WRITE (6,4) MØDE                                                          MBX01510
  4 FØRMAT(1H1,32X,34HINITIAL SØURCE STRENGTHS FØR MØDE ,I2 / 1H-,9X,  MBX01520
   1 1HN, 5X,1HM, 6X,13HREAL SSU(N,M) 5X,13HIMAG SSU(N,M), 5X,         MBX01530
   2 13HREAL SSL(N,M), 5X,13HIMAG SSL(N,M) /1H0)                       MBX01540
    DØ 100 N = 1,NBØX                                                  MBX01550
    MPEND = MØBWD(N)                                                   MBX01560
    IF(N.GE.NPFLET) MPEND = NMAX+MØBTD(N)-MFL                          MBX01570
    DØ 100 M = 1,MPEND                                                 MBX01580
100 WRITE (6,19) N,M,SSU(N,M,1),SSU(N,M,2),SSL(N,M,1),SSL(N,M,2)       MBX01590
 19 FØRMAT( 5X,2I6,    1P4E18.4 )                                      MBX01600
160 DØ 1605 I2 = 1,2                                                   MBX01610
    DØ 1605 I1 = 1,21                                                  MBX01620
    QR(I1,I2) = 0.0                                                    MBX01630
1605 QI(I1,I2) = 0.0                                                   MBX01640
    CALL CØDE                                                          MBX01650
    IF(LDPHI)73,83,73                                                  MBX01660
 73 WRITE (6,76) MØDE                                                  MBX01670
 76 FØRMAT(1H1,23X,58HVELØCITY PØTENTIAL DIFFERENCES (PHI UPPER - PHI  MBX01680
   1LØWER) AND /26X,51HSØURCE STRENGTHS WITH INTERFERENCE EFFECTS FØR  MBX01690
   2MØDE,I3/1H-,6X,1HN,3X,1HM,5X,11HR DPHI(N,M),3X,11HI DPHI(N,M),4X,  MBX01700
   3 10HR SSU(N,M),4X,10HI SSU(N,M),4X,10HR SSL(N,M),4X,10HI SSL(N,M)  MBX01710
   4 /1H0)                                                             MBX01720
    DØ  75 N = 1,NBØX                                                  MBX01730
    MPEND = MØBWD(N)                                                   MBX01740
    IF(N.GE.NPFLET) MPEND = NMAX+MØBTD(N)-MFL                          MBX01750
    DØ  75 M = 1,MPEND                                                 MBX01760
 75 WRITE (6,72) N,M,DPHI(N,M,1),DPHI(N,M,2),SSU(N,M,1),               MBX01770
   1 SSU(N,M,2),SSL(N,M,1),SSL(N,M,2)                                  MBX01780
 72 FØRMAT( 4X,2I4,2X,1P6E14.4)                                        MBX01790
 83 CØNTINUE                                                           MBX01800
    NC = MAX0(NCW,NCT)                                                 MBX01810
    DØ 750 J = 1,MØDES                                                 MBX01820
    DØ 750 N = 1,NC                                                    MBX01830
    Q(MØDE,J,1) = Q(MØDE,J,1) + QR(N,1)*CØW(J,N) + QR(N,2)*CØT(J,N)    MBX01840
750 Q(MØDE,J,2) = Q(MØDE,J,2) + QI(N,1)*CØW(J,N) + QI(N,2)*CØT(J,N)    MBX01850
810  CØNTINUE                                                          MBX01860
    WRITE (6,2001) ØMEGA(L),NBØX,EK,EMACH,EKREF,FANG                   MBX01870
```

```
      MBØX       MAIN PRØGRAM

 2001 FØRMAT(1H1,33X,37HMACH BØX INTERFERENCE PRØGRAM ØUTPUT ,///          MBX01880
     1 1H0,8X,27HØSCILLATØRY FREQUENCY (CPS),F12.5,14X,I2,   25H BØXES INMBX01890
     2 CHØRD DIRECTIØN /1H0,8X,30HREDUCED FREQUENCY (BØX LENGTH),F9.5,    MBX01900
     3 14X,23HFREE STREAM MACH NUMBER, F9.3,/1H0,8X,30HREDUCED FREQUENCYMBX01910
     4 (RØØT CHØRD),F9.5,14X,24HTIP FØLD ANGLE (DEGREES), F8.3 /1H-/42X,MBX01920
     5 18HGENERALIZED FØRCES /1H-,6X,4HDPHI,3X,4HDEFL,10X,9HREAL PART,   MBX01930
     6 10X,9HIMAG PART,10X,9HABS VALUE,10X,11HPHASE ANGLE //)            MBX01940
      DØ 2010 I = 1,MØDES                                                MBX01950
      DØ 2010 J = 1,MØDES                                                MBX01960
      QREAL =  QF * Q(I,J,1)                                             MBX01970
      QIMAG =  QF * Q(I,J,2)                                             MBX01980
      QABS = SQRT(QREAL**2 + QIMAG**2)                                   MBX01990
      QANGLE = 0.0                                                       MBX02000
      IF(QABS.NE.0.0) QANGLE = ATAND(QIMAG,QREAL)                        MBX02010
 2010 WRITE (6,2007) I,J,QREAL,QIMAG,QABS,QANGLE                         MBX02020
 2007 FØRMAT(1H0,2X,2I7,2X,1P3E19.5,0P1F16.3,4H DEG)                     MBX02030
      IF(FAN - 1.0)820,820,1211                                          MBX02040
 1211 FAN = FAN - 1.0                                                    MBX02050
      GØ TØ 20                                                           MBX02060
 820  CØNTINUE                                                           MBX02070
      GØ TØ 2345                                                         MBX02080
      END                                                                MBX02090
```

BOUND   WING AND TIP BOUNDARYS

```
      SUBROUTINE BOUNDS(ERROR)                                          MBX02110
      COMMON/B/MIBW(20),MOBW(20),MIBT(20),MOBT(20),MOBTD(20),MOBWD(20)  MBX02120
      COMMON/D/XTE(40),YTE(40),QR(21,2),QI(21,2),COW(10,21),COT(10,21)  MBX02130
      COMMON/E/BOXL,BOXW,EMACH,EK,YNFL,FANG,CFA,SFA,NBOX,CAPPHI(20,20,2)MBX02140
      COMMON/F/MODE,KODE,NP,MP,MFL,NMAX,NBW,NBT,NPINT,NPFLET,NPOLW,NPOLTMBX02150
      COMMON/G/SLEW,STEW,YFL,SLET,STET,YTIP,CROOT,CFL,CTIP,MDET,MTIP    MBX02160
      TANDF(A) = SIND(A)/COSD(A)                                        MBX02170
      ERROR = 0.0                                                       MBX02180
      DO 1070 I = 1,40                                                  MBX02190
      XTE(I) = 0.0                                                      MBX02200
 1070 YTE(I) = 0.0                                                      MBX02210
      DO 1080 I=1,20                                                    MBX02220
      MIBW(I)=0                                                         MBX02230
      MOBW(I)=0                                                         MBX02240
      MOBWD(I)=0                                                        MBX02250
      MIBT(I)=0                                                         MBX02260
      MOBT(I)=0                                                         MBX02270
 1080 MOBTD(I)=0                                                        MBX02280
    5 BETA = SQRT(EMACH**2 - 1.0)                                       MBX02290
      PGWL = TANDF(SLEW) / BETA                                         MBX02300
      PGWT = TANDF(STEW) / BETA                                         MBX02310
      IF(1.0 - ABS(PGWT))10,20,20                                       MBX02320
   10 EMACH = SQRT(1.0 + (PGWT * BETA)**2)                              MBX02330
      ERROR = 1.0                                                       MBX02340
      GO TO 5                                                           MBX02350
   20 PGTL = TANDF(SLET) / BETA                                         MBX02360
      PGTT = TANDF(STET) / BETA                                         MBX02370
      IF(1.0 - ABS(PGTT))30,40,40                                       MBX02380
   30 EMACH = SQRT(1.0 + (PGTT * BETA**2))                              MBX02390
      ERROR = 1.0                                                       MBX02400
      GO TO 5                                                           MBX02410
   40 IF(MDET.EQ. 0 ) YTIP = YFL                                        MBX02420
      DY = YTIP - YFL                                                   MBX02430
      XFL = YFL * PGWL * BETA                                           MBX02440
      XFT = YFL * PGWT * BETA + CROOT                                   MBX02450
      CFL = XFT - XFL                                                   MBX02460
      XTL = XFL + DY * PGTL * BETA                                      MBX02470
```

MACH BØX INTERFERENCE PRØGRAM

```
      BØUND      WING AND TIP BØUNDARYS

      XTT = XFT + DY * PGTT * BETA                                      MBX02480
      CTIP = XTT - XTL                                                  MBX02490
      CREF = AMAX1(CRØØT,XFT,XTT)                                       MBX02500
   70 BØXL = CREF / FLØAT(NBØX)                                         MBX02510
      BØXW = BØXL / BETA                                                MBX02520
      MFL = 1.0 + YFL / BØXW                                            MBX02530
      YNFL = FLØAT(MFL) - 0.5                                           MBX02540
      BØXW = YFL / YNFL                                                 MBX02550
      BØXL = BØXW * BETA                                                MBX02560
      IF(20.4*BØXL - CREF)80,90,90                                      MBX02570
   80 NBØX = NBØX - 1                                                   MBX02580
      GØ TØ 70                                                          MBX02590
   90 MTIP = 1.0 + YTIP/BØXW                                            MBX02600
      IF(60-2*MTIP+MFL-2-IFIX (CTIP/BØXL))80,100,100                    MBX02610
  100 XNRT = CRØØT / BØXL                                               MBX02620
      XNFL = XFL / BØXL                                                 MBX02630
      XNFT = XFT / BØXL                                                 MBX02640
      NBØX = CREF/BØXL + 0.5001                                         MBX02650
      NPRØØT = XNRT + 0.5001                                            MBX02660
      NPFLEW = XNFL - PGWL/2.0 + 1.5001                                 MBX02670
      NPFTEW = XNFT - PGWT/2.0 + 0.5001                                 MBX02680
      IF(NPFTEW - NPFLEW)110,120,120                                    MBX02690
  110 NPFTEW = NPFLEW                                                   MBX02700
  120 NPFLET = XNFL + PGTL/2.0 + 1.5001                                 MBX02710
      NPFTET = XNFT + PGTT/2.0 + 0.5001                                 MBX02720
  130 NPTPLE = XNFL + PGTL*(FLØAT(MTIP-1) - YNFL) + 1.5001              MBX02730
      NPTPTE = XNFT + PGTT*(FLØAT(MTIP-1) - YNFL) + 0.5001              MBX02740
      IF(NPTPTE - NPTPLE)140,150,150                                    MBX02750
  140 MTIP = MTIP - 1                                                   MBX02760
      GØ TØ 130                                                         MBX02770
  150 NBW = MAX0(NPRØØT,NPFTEW)                                         MBX02780
      NBT = MAX0(NPFTET,NPTPTE)                                         MBX02790
      MFLT = MFL + 1                                                    MBX02800
      DØ 220 NP = 1,NBW                                                 MBX02810
      MIBW(NP) = 1                                                      MBX02820
      IF(NP - NPRØØT)170,170,160                                        MBX02830
  160 MIBW(NP) = (FLØAT(NP) - 0.5 - XNRT)/PGWT + 2.0001                 MBX02840
```

```
      BOUND      WING AND TIP BOUNDARYS

170 IF(NP - NPFTEW)190,190,180                                  MBX02850
180 MOBW(NP) = (FLOAT(NP) - 0.5 - XNRT)/PGWT + 1.0001           MBX02860
    GO TO 220                                                   MBX02870
190 MOBW(NP) = MFL                                              MBX02880
    MOBWD(NP) = MAX0(NP,NP+MFL-NPFLEW)                          MBX02890
    IF(NP - NPFLEW)200,210,210                                  MBX02900
200 MOBW(NP) = (FLOAT(NP) - 0.5)/PGWL + 1.0001                  MBX02910
    MOBWD(NP) = MAX0(NP,MOBW(NP))                               MBX02920
210 MP = MFL + NPFTEW - NP                                      MBX02930
    IF(MDET)215,220,215                                         MBX02940
215 MP = MTIP + NPTPTE - NP                                     MBX02950
220 MOBWD(NP) = MIN0(MOBWD(NP),MP)                              MBX02960
    DO 225 MP = 1,MFL                                           MBX02970
    YTE(MP) = MP - 1                                            MBX02980
225 XTE(MP) = XNRT + FLOAT(MP-1)*PGWT                           MBX02990
    IF(MDET)230,320,230                                         MBX03000
230 DO 300 NP = NPFLET,NBT                                      MBX03010
    MIBT(NP) = MFLT                                             MBX03020
    IF(NP - NPFTET)250,250,240                                  MBX03030
240 MIBT(NP) = YNFL +(FLOAT(NP) - XNFT - 0.5)/PGTT + 2.0001     MBX03040
250 IF(NP - NPTPTE)270,270,260                                  MBX03050
260 MOBT(NP) = YNFL +(FLOAT(NP) - XNFT - 0.5)/PGTT + 1.0001     MBX03060
    GO TO 300                                                   MBX03070
270 MOBT(NP) = MTIP                                             MBX03080
    MP = NP - NPFLET + MFLT                                     MBX03090
    MOBTD(NP) = MAX0(MP,NP+MTIP-NPTPLE)                         MBX03100
    IF(NP - NPTPLE)280,290,290                                  MBX03110
280 MOBT(NP) = YNFL +(FLOAT(NP) - XNFL - 0.5)/PGTL + 1.0001     MBX03120
    MOBTD(NP) = MAX0(MP,MOBT(NP))                               MBX03130
290 MOBTD(NP) = MIN0(MOBTD(NP),MTIP+NPTPTE - NP)                MBX03140
    MOBWD(NP) = MAX0(MOBTD(NP),MOBWD(NP))                       MBX03150
300 CONTINUE                                                    MBX03160
    DO 310 MP = MFLT,MTIP                                       MBX03170
    YTE(MP) = MP - 1                                            MBX03180
310 XTE(MP) = XNFT + (FLOAT(MP-MFL) -0.5)*PGTT                  MBX03190
320 RETURN                                                      MBX03200
    END                                                         MBX03210
```

CAPHI   IN-PLANE VPICS

```
C     INPLANE VELØCITY PØTENTIAL INFLUENCE CØEFFICIENTS.                    MBX03230
      SUBRØUTINE CAPHI                                                      MBX03240
      CØMMØN/C/BARNU,BARMU,EL,CFR,CFI,CNR,CNI,EKBAR,EKM,P(10),W(10)         MBX03250
      CØMMØN/E/BØXL,BØXW,EMACH,EK,YNFL,FANG,CFA,SFA,NBØX,CAPPHI(20,20,2)MBX03260
      IF(EKBAR)99,20,30                                                     MBX03270
   20 CAPPHI(1,1,1) = -0.5                                                  MBX03280
      GØ TØ 50                                                              MBX03290
   30 DØ 40 I = 1,5                                                         MBX03300
      J = 6 - I                                                             MBX03310
      ARG = EKBAR * P(J) / 2.0                                              MBX03320
      ARGM =    ARG /  EMACH                                                MBX03330
      BSL = ZJ(ARGM)                                                        MBX03340
      CAPPHI(1,1,1) = CAPPHI(1,1,1) - CØS(ARG)*BSL*W(I)/2.0                 MBX03350
      CAPPHI(1,1,2) = CAPPHI(1,1,2) + SIN(ARG)*BSL*W(I)/2.0                 MBX03360
   40 CØNTINUE                                                              MBX03370
C     CAPPHI(1,1,1) IS NØW CØMPLETE                                         MBX03380
   50 EL = 0.0                                                              MBX03390
      DØ 60 NP = 2,NBØX                                                     MBX03400
      BARNU=NP-1                                                            MBX03410
      DØ 60 MP = 1,NP                                                       MBX03420
      BARMU=MP-1                                                            MBX03430
      CALL VPIC                                                             MBX03440
      CAPPHI(NP,MP,1) = CFR                                                 MBX03450
      CAPPHI(NP,MP,2) = CFI                                                 MBX03460
   60 CØNTINUE                                                              MBX03470
   99 RETURN                                                                MBX03480
      END                                                                   MBX03490
```

SØURCE   INITIAL SØURCE STRENGTHS

```
C     SETS UP ØRIGINAL SØURCE STRENGTH  ARRAY FØR ØNE MØDE
C     WITH SSU(REAL)= DZDX = - SSL(REAL) ØN SURFACE AND ZERØ ØN DIAFRAM
C          SSU(IMAG)= EK*Z = - SSL(IMAG) ØN SURFACE AND ZERØ ØN DIAFRAM
      SUBRØUTINE SØURCE
      CØMMØN/A/DPHI(20,60,2),SSU(20,60,2),SSL(20,60,2),THW(10),THT(10)
      CØMMØN/B/MIBW(20),MØBW(20),MIBT(20),MØBT(20),MØBTD(20),MØBWD(20)
      CØMMØN/D/XTE(40),YTE(40),QR(21,2),QI(21,2),CØW(10,21),CØT(10,21)
      CØMMØN/E/BØXL,BØXW,EMACH,EK,YNFL,FANG,CFA,SFA,NBØX,CAPPHI(20,20,2)
      CØMMØN/F/MØDE,KØDE,NP,MP,MFL,NMAX,NBW,NBT,NPINT,NPFLET,NPØLW,NPØLT
      DØ  90 NP = 1,NBW
      MPL = MIBW(NP)
      MPR = MØBW(NP)
      DØ 90 MP = MPL,MPR
      X =(FLØAT(NP) - 0.5)* BØXL
      Y =(FLØAT(MP-1))*BØXW
      Z = CØW(MØDE,1)
      DZDX = 0.0
      I = 1
      J = 0
      DØ 70 N = 1,NPØLW
      DØ 70 M = J,N
      NM = N-M
      FN = NM
      I = I + 1
      YM = CØW(MØDE,I)
      IF(M)30,40,30
   30 YM = Y**M * YM
   40 XM = 1.0
      IF(NM)50,60,50
   50 XM = X**NM
      DZDX = DZDX + FN * XM/X * YM
   60 Z = Z + XM * YM
   70 CONTINUE
      SSU(NP,MP,1) = DZDX
      SSL(NP,MP,1) = -DZDX * SIGN(1.0,THW(MØDE))
      SSU(NP,MP,2) = EK * Z / BØXL
      SSL(NP,MP,2) = -EK * Z * SIGN(1.0,THW(MØDE))/BØXL
```

SØURCE   INITIAL SØURCE STRENGTHS

```
   90 CØNTINUE
      IF(NPØLT + 1)1000,1000,110
  110 DØ 200 NP = NPFLET, NBT
      MPL = MIBT(NP)
      MPR = MØBT(NP)
      DØ 200 MP = MPL,MPR
      X = (FLØAT(NP) - 0.5)*BØXL
      Y = (FLØAT(MP-1))*BØXW
      MPN = MP + NMAX - MFL
      IF(ABS(FANG).LE. 5.0) MPN = MP
      Z = CØT(MØDE,1)
      DZDX = 0.0
      I = 1
      DØ 170 N = 1,NPØLT
      DØ 170 M = J,N
      NM = N - M
      FN = NM
      I = I + 1
      YM = CØT(MØDE,I)
      IF(M)130,140,130
  130 YM = Y**M * YM
  140 XM = 1.0
      IF(NM)150,160,150
  150 XM = X**NM
      DZDX = DZDX + FN * XM/X * YM
  160 Z = Z + XM * YM
  170 CØNTINUE
      SSU(NP,MPN,1) = DZDX
      SSL(NP,MPN,1) = -DZDX * SIGN(1.0,THT(MØDE))
      SSU(NP,MPN,2) = EK * Z / BØXL
      SSL(NP,MPN,2) = -EK * Z * SIGN(1.0,THT(MØDE)) / BØXL
  200 CØNTINUE
 1000 RETURN
      END
```

CØDE BØX CØDE

```
C     IDENTIFIES BØX TYPE FØR LØGICAL PURPØSES WITH VARIØUS KØDE VALUES   MBX04230
C       KØDE = 1 , WING IN NØN-INTERFERENCE REGIØN                        MBX04240
C       KØDE = 2 , WING DIAFRAM IN NØN-INTEFERENCE REGIØN                 MBX04250
C       KØDE = 3 , WING IN INTERFERENCE REGIØN                            MBX04260
C       KØDE = 4 , TIP IN INTERFERENCE REGIØN                             MBX04270
C       KØDE = 5 , TIP DIAFRAM IN INTERFERENCE REGIØN                     MBX04280
C       KØDE = 6 , WING DIAFRAM IN INTERFERENCE REGIØN                    MBX04290
        SUBRØUTINE CØDE                                                   MBX04300
        CØMMØN/B/MIBW(20),MØBW(20),MIBT(20),MØBT(20),MØBTD(20),MØBWD(20)  MBX04310
        CØMMØN/E/BØXL,BØXW,EMACH,EK,YNFL,FANG,CFA,SFA,NBØX,CAPPHI(20,20,2)MBX04320
        CØMMØN/F/MØDE,KØDE,NP,MP,MFL,NMAX,NBW,NBT,NPINT,NPFLET,NPØLW,NPØLTMBX04330
        DØ 1000 NP = 1,NBØX                                               MBX04340
        NP = NP                                                           MBX04350
        MIW = MIBW(NP)                                                    MBX04360
        MØW = MØBW(NP)                                                    MBX04370
        MIT = MIBT(NP)                                                    MBX04380
        MØT = MØBT(NP)                                                    MBX04390
        MTD = MØBTD(NP)                                                   MBX04400
        MWD = MØBWD(NP)                                                   MBX04410
        NPINT = NP - NPFLET + 1                                           MBX04420
        IF(NPINT)10,10,30                                                 MBX04430
     10 MPL = MIW                                                         MBX04440
        MPR = MØW                                                         MBX04450
        KØDE = 1                                                          MBX04460
        CALL SSPHI(MPL,MPR)                                               MBX04470
        MPL = MPR + 1                                                     MBX04480
        IF(MPL - MWD)20,20,1000                                           MBX04490
     20 MPR = MWD                                                         MBX04500
        KØDE = 2                                                          MBX04510
        CALL SSPHI(MPL,MPR)                                               MBX04520
        GØ TØ 1000                                                        MBX04530
     30 IF(NPINT - MFL)40,40,50                                           MBX04540
     40 MPL = MIW                                                         MBX04550
        MPR = MFL - NPINT + 1                                             MBX04560
        KØDE = 1                                                          MBX04570
        CALL SSPHI(MPL,MPR)                                               MBX04580
        IF(NPINT - 1)1000,60,50                                           MBX04590
```

MACH BØX INTERFERENCE PRØGRAM

```
      CØDE       BØX CØDE

   50 MPL = MFL - NPINT + 2
      MPL =  MAXO (MPL,MIW)
      MPR = MØW
      KØDE = 3
      IF(ABS(FANG).LE. 5.0) KØDE = 1
   55 CALL SSPHI(MPL,MPR)
   60 IF(MFL + 1 - MIT)70,70,80
   70 MPL = MIT
      MPR = MØT
      KØDE = 1
      IF(ABS(FANG).LE. 5.0) GØ TØ 75
   72 KØDE = 6
      CALL SSPHI(MPL,MPR)
      KØDE = 4
   75 CALL SSPHI(MPL,MPR)
   80 IF(MØT - MTD)90,100,100
   90 MPL = MØT + 1
      MPR = MTD
      KØDE = 2
      IF(ABS(FANG).LE. 5.0) GØ TØ 95
   92 KØDE = 6
      CALL SSPHI(MPL,MPR)
      KØDE = 5
   95 CALL SSPHI(MPL,MPR)
  100 IF(MTD - MWD)110,1000,1000
  110 MPL = MTD + 1
      MPR = MWD
      KØDE = 2
      CALL SSPHI(MPL,MPR)
 1000 CØNTINUE
      RETURN
      END
```

MACH BOX INTERFERENCE PROGRAM

SSPHI LOGICAL FLOW

```
C      TO ACCOUNT FOR LAG IN SIGNALS BETWEEN PLANES THAT INTERSECT AT AN MBX04930
C      ACUTE ANGLE THAT RESULTS FROM THE FINITE GRID SIZE,CERTAIN NOTED  MBX04940
C      MODIFICATIONS TO THE EQUATIONS FOR THE WING DIAFRAM INTERFERENCE  MBX04950
C      REGION (KODE = 6) WERE MADE TO SSN,PHIP,AND PHIN.                 MBX04960
       SUBROUTINE SSPHI(MPL,MPR)                                         MBX04970
       COMMON/A/DPHI(20,60,2),SSU(20,60,2),SSL(20,60,2),THW(10),THT(10)  MBX04980
       COMMON/E/BOXL,BOXW,EMACH,EK,YNFL,FANG,CFA,SFA,NBOX,CAPPHI(20,20,2)MBX04990
       COMMON/F/MODE,KODE,NP,MP,MFL,NMAX,NBW,NBT,NPINT,NPFLET,NPOLW,NPOLTMBX05000
       DO 100 MP = MPL,MPR                                               MBX05010
       MP = MP                                                           MBX05020
       MM = MP                                                           MBX05030
       GO TO (10,20,10, 5,15,20),KODE                                    MBX05040
     5 MM = MP + NMAX - MFL                                              MBX05050
    10 IF(DPHI(NP,MM,1))100,25,100                                       MBX05060
    15 MM = MP + NMAX -MFL                                               MBX05070
    20 IF(SSU(NP,MM,1))100,25,100                                        MBX05080
    25 GO TO (55,55,50,50,55,55),KODE                                    MBX05090
    50 CALL SSN                                                          MBX05100
    55 CALL PHIP                                                         MBX05110
       GO TO (70,65,60,70,65,65),KODE                                    MBX05120
    60 CALL PHIN                                                         MBX05130
       GO TO (70,65,70,70,65,65),KODE                                    MBX05140
    65 CALL SSN                                                          MBX05150
       GO TO 100                                                         MBX05160
    70 DPHI(NP,MM,1) = DPHI(NP,MM,1) * BOXW                              MBX05170
       DPHI(NP,MM,2) = DPHI(NP,MM,2) * BOXW                              MBX05180
       CALL GENFOR                                                       MBX05190
   100 CONTINUE                                                          MBX05200
       RETURN                                                            MBX05210
       END                                                               MBX05220
```

MACH BØX INTERFERENCE PRØGRAM

SSN   CØRRECTS THE DIAFRAM AND INTERFERENCE SØURCE STRENGTHS

```
      SUBRØUTINE SSN                                                    MBX05240
      CØMMØN/A/DPHI(20,60,2),SSU(20,60,2),SSL(20,60,2),THW(10),THT(10)  MBX05250
      CØMMØN/B/MIBW(20),MØBW(20),MIBT(20),MØBT(20),MØBTD(20),MØBWD(20)  MBX05260
      CØMMØN/C/BARNU,BARMU,EL,CFR,CFI,CNR,CNI,EKBAR,EKM,P(10),W(10)     MBX05270
      CØMMØN/E/BØXL,BØXW,EMACH,EK,YNFL,FANG,CFA,SFA,NBØX,CAPPHI(20,20,2)MBX05280
      CØMMØN/F/MØDE,KØDE,NP,MP,MFL,NMAX,NBW,NBT,NPINT,NPFLET,NPØLW,NPØLTMBX05290
      DSSR = 0.0                                                        MBX05300
      DSSI = 0.0                                                        MBX05310
      GØ TØ (100,90,30,10,90,30),KØDE                                   MBX05320
C     CALCULATE INDUCED VELØCITY SUM ØN TIP PLANE FRØM WING PLANE       MBX05330
 10   DØ 23 IC=2,NP                                                     MBX05340
      BARNU = IC - 1                                                    MBX05350
      NU = NP - IC + 1                                                  MBX05360
      YMTFL = FLØAT(MP-1) - YNFL                                        MBX05370
      XCLB = FLØAT(MFL) + YMTFL * CFA                                   MBX05380
      IND = 1                                                           MBX05390
      IF(XCLB -(BARNU + 0.5))11,12,12                                   MBX05400
   11 IND = 2                                                           MBX05410
   12 JCL = 1                                                           MBX05420
      JCR = MØBWD(NU)                                                   MBX05430
      DØ 23 JC=JCL,JCR                                                  MBX05440
      YMWFL = YNFL - FLØAT(JC-1)                                        MBX05450
      BARMU = YMTFL * CFA       + YMWFL                                 MBX05460
      EL =   YMTFL * SFA                                                MBX05470
      DØ 19 K = 1,IND                                                   MBX05480
      IF((BARNU+0.5)**2 - (ABS(BARMU)-0.5)**2 - EL**2 - 1.E-6)20,20,15  MBX05490
   20 IF(ABS(BARMU).GE.0.5 .ØR.ABS(EL).GE.BARNU+0.5) GØ TØ 22           MBX05500
   15 CALL VIC                                                          MBX05510
      CALL KØMPLX(DSSR ,DSSI ,CNR  ,CNI  ,SSU(NU,JC,1),SSU(NU,JC,2))    MBX05520
   22 IF(JC - 1)23,23,19                                                MBX05530
   19 BARMU = XCLB - 0.5 + FLØAT(JC-1)                                  MBX05540
 23   CØNTINUE                                                          MBX05550
      GØ TØ 70                                                          MBX05560
C     CALCULATE INDUCED VELØCITY SUM ØN WING PLANE FRØM TIP PLANE       MBX05570
 30   IF(NPINT-1) 50, 50,31                                             MBX05580
   31 NPXCL = NPINT - MFL                                               MBX05590
      IND = 1                                                           MBX05600
```

SSN   CØRRECTS THE DIAFRAM AND INTERFERENCE SØURCE STRENGTHS

```
      IF(MP - NPXCL)33,33,34                                          MBX05610
   33 IND= 2                                                          MBX05620
   34 DØ 43 IC = 2,NPINT                                              MBX05630
      BARNU = IC - 1                                                  MBX05640
      NU = NP - IC + 1                                                MBX05650
      JCL = MFL + 1                                                   MBX05660
      JCR = MØBTD(NU)                                                 MBX05670
      DØ 43 JC=JCL,JCR                                                MBX05680
      YMWFL =-FLØAT(MP-1) + YNFL                                      MBX05690
      YMTFL =-YNFL + FLØAT(JC-1)                                      MBX05700
      TRE = 1 - KØDE/6                                                MBX05710
      SIX = KØDE/3 - 1                                                MBX05720
      MPN = JC + NMAX - MFL                                           MBX05730
      DØ 43 K = 1,IND                                                 MBX05740
      BARMU = YMWFL * CFA        + YMTFL                              MBX05750
      EL = YMWFL * SFA                                                MBX05760
      IF((BARNU+0.5)**2 - (ABS(BARMU)-0.5)**2 - EL**2 - 1.E-6)40,40,35 MBX05770
   40 IF(ABS(BARMU).GE.0.5 .ØR.ABS(EL).GE.BARNU+0.5) GØ TØ 43         MBX05780
   35 CALL VIC                                                        MBX05790
      SSR = TRE * SSU(NU,MPN,1) + SIX * SSL(NU,MPN,1)                 MBX05800
      SSI = TRE * SSU(NU,MPN,2) + SIX * SSL(NU,MPN,2)                 MBX05810
      CALL KØMPLX(DSSR,DSSI,CNR  ,CNI  ,SSR,SSI)                      MBX05820
   43 YMWFL = FLØAT(MP-1) + YNFL                                      MBX05830
   50 GØ TØ (100,90,70,70,90,90),KØDE                                 MBX05840
C     CØRRECT SURFACE SØURCE STRENGTH FØR  INTERFERENCE EFFECTS       MBX05850
   70 TRE = 1 - KØDE/4                                                MBX05860
      SIX = KØDE/2 - 1                                                MBX05870
      MPN = MP + NMAX - MFL                                           MBX05880
      MTM = IFIX(TRE+0.5)*MP + IFIX(SIX+0.5)*MPN                      MBX05890
      SSU(NP,MTM,1) = SSU(NP,MTM,1) - DSSR                            MBX05900
      SSU(NP,MTM,2) = SSU(NP,MTM,2) - DSSI                            MBX05910
      SSL(NP,MTM,1) = SSL(NP,MTM,1) + SIX * DSSR                      MBX05920
      SSL(NP,MTM,2) = SSL(NP,MTM,2) + SIX * DSSI                      MBX05930
      IF(KØDE.NE.4) GØ TØ 100                                         MBX05940
      YT = FLØAT(MP - MFL) - 0.5                                      MBX05950
      EL = YT*SFA                                                     MBX05960
      IF(ABS(EL).GE.0.5) GØ TØ 100                                    MBX05970
```

```
      SSN        CORRECTS THE DIAFRAM AND INTERFERENCE SOURCE STRENGTHS

      BARNU = 0.0                                                       MBX05980
      BARMU = YT*CFA - YT                                               MBX05990
      CALL COREC                                                        MBX06000
      GO TO 100                                                         MBX06010
C     CALCULATE THE DIAFRAM SOURCE STRENGTH                             MBX06020
   90 TRE = 1 - KODE/6   - 2/KODE                                       MBX06030
      SIX = KODE/3 - 1 + 4/KODE                                         MBX06040
      MPN = MP + NMAX - MFL                                             MBX06050
      MTM = IFIX(SIX+0.5)*MP + IFIX(TRE+0.5)*MPN                        MBX06060
      DEN = CAPPHI(1,1,1)**2 + CAPPHI(1,1,2)**2                         MBX06070
      DELSSR = 0.0                                                      MBX06080
      DELSSI = 0.0                                                      MBX06090
      BR = 0.5 * DPHI(NP,MTM,1)                                         MBX06100
      BI = 0.5 * DPHI(NP,MTM,2)                                         MBX06110
      CR = CAPPHI(1,1,1)/DEN                                            MBX06120
      CI =-CAPPHI(1,1,2)/DEN                                            MBX06130
      CALL KOMPLX(DELSSR,DELSSI,BR,BI,CR,CI)                            MBX06140
C     INTERFERENCE CORRECTIONS TO WING DIAFRAM BOXES ARE MADE BY ADDING MBX06150
C     THE INDUCED VELOCITY TO THE LOWER SIDE WHEN SATISFYING  DPHI = 0. MBX06160
C     THIS IS EQUIVALENT TO EQ 83 WHEN THE DPHI SUM IS MODIFIED         MBX06170
   80 SSU(NP,MTM,1) = -DELSSR                                           MBX06180
      SSU(NP,MTM,2) = -DELSSI                                           MBX06190
      SSL(NP,MTM,1) =  DELSSR - FLOAT(KODE/6) * DSSR                    MBX06200
      SSL(NP,MTM,2) =  DELSSI - FLOAT(KODE/6) * DSSI                    MBX06210
      IF(KODE.NE.5) GO TO 99                                            MBX06220
      YT = FLOAT(MP - MFL) - 0.5                                        MBX06230
      EL = YT*SFA                                                       MBX06240
      IF(ABS(EL).GE.0.5) GO TO 99                                       MBX06250
      BARNU = 0.0                                                       MBX06260
      BARMU = YT*CFA - YT                                               MBX06270
      CALL COREC                                                        MBX06280
   99 DPHI(NP,MTM,1) = 0.0                                              MBX06290
      DPHI(NP,MTM,2) = 0.0                                              MBX06300
  100 RETURN                                                            MBX06310
      END                                                               MBX06320
```

PHIP   CALCULATES PLANAR PORTION OF THE VELOCITY POTENTIAL

```
      SUBROUTINE PHIP                                                   MBX06340
      COMMON/A/DPHI(20,60,2),SSU(20,60,2),SSL(20,60,2),THW(10),THT(10)  MBX06350
      COMMON/B/MIBW(20),MOBW(20),MIBT(20),MOBT(20),MOBTD(20),MOBWD(20)  MBX06360
      COMMON/E/BOXL,BOXW,EMACH,EK,YNFL,FANG,CFA,SFA,NBOX,CAPPHI(20,20,2)MBX06370
      COMMON/F/MODE,KODE,NP,MP,MFL,NMAX,NBW,NBT,NPINT,NPFLET,NPOLW,NPOLTMBX06380
      IT = KODE/4 - KODE/6                                              MBX06390
      IW = 1 - IT                                                       MBX06400
      N = NP*IW + NPINT*IT                                              MBX06410
      M = MP + (NMAX-MFL)*IT                                            MBX06420
      ML = 1 + MFL*IT                                                   MBX06430
      DO 20 IC = 1,N                                                    MBX06440
      NU = NP - IC + 1                                                  MBX06450
      MR = MOBWD(NU)*IW + MOBTD(NU)*IT                                  MBX06460
      JCL = MAX0(ML,MP-IC+1)                                            MBX06470
      JCR = MIN0(MR,MP+IC-1)                                            MBX06480
      DO 20 JC = JCL,JCR                                                MBX06490
      MU = JC + (NMAX-MFL)*IT                                           MBX06500
      MUBAR = IABS(MP-JC) + 1                                           MBX06510
      DSSR = SSU(NU,MU,1) - SSL(NU,MU,1)                                MBX06520
      DSSI = SSU(NU,MU,2) - SSL(NU,MU,2)                                MBX06530
      IF(KODE.NE.6) GO TO 15                                            MBX06540
C     THE LOWER WING AND UPPER WING DIAFRAM BOXES ARE USED TO MODIFY    MBX06550
C     THE PLANAR PORTION OF DPHI FOR THE WING DIAFRAM BOXES.            MBX06560
      DSSR = -2.0*SSL(NU,MU,1)                                          MBX06570
      DSSI = -2.0*SSL(NU,MU,2)                                          MBX06580
      IF(JC.LE.MFL)GO TO 15                                             MBX06590
      DSSR =  2.0*SSU(NU,MU,1)                                          MBX06600
      DSSI =  2.0*SSU(NU,MU,2)                                          MBX06610
   15 DPR = CAPPHI(IC,MUBAR,1)                                          MBX06620
      DPI = CAPPHI(IC,MUBAR,2)                                          MBX06630
      IF(JC.EQ.1.OR .JC.GT.1+IC-MP) GO TO 20                            MBX06640
      MUBAR = MP + JC - 1                                               MBX06650
      DPR = CAPPHI(IC,MUBAR,1) + DPR                                    MBX06660
      DPI = CAPPHI(IC,MUBAR,2) + DPI                                    MBX06670
   20 CALL KOMPLX(DPHI(NP,M,1),DPHI(NP,M,2),DPR,DPI,DSSR,DSSI)          MBX06680
  100 RETURN                                                            MBX06690
      END                                                               MBX06700
```

MACH BOX INTERFERENCE PROGRAM

PHIN CALCULATES NONPLANAR PORTION OF THE VELOCITY POTENTIAL

```
      SUBROUTINE PHIN                                                   MBX06720
      COMMON/A/DPHI(20,60,2),SSU(20,60,2),SSL(20,60,2),THW(10),THT(10)  MBX06730
      COMMON/B/MIBW(20),MOBW(20),MIBT(20),MOBT(20),MOBTD(20),MOBWD(20)  MBX06740
      COMMON/C/BARNU,BARMU,EL,CFR,CFI,CNR,CNI,EKBAR,EKM,P(10),W(10)     MBX06750
      COMMON/E/BOXL,BOXW,EMACH,EK,YNFL,FANG,CFA,SFA,NBOX,CAPPHI(20,20,2)MBX06760
      COMMON/F/MODE,KODE,NP,MP,MFL,NMAX,NBW,NBT,NPINT,NPFLET,NPOLW,NPOLTMBX06770
C     THE MODIFIED WING DIAFRAM DPHI SUM CONTAINS NO OUT-OF-PLANE TERMS MBX06780
      IF(NPINT.LE.1)RETURN                                              MBX06790
      NPXCL = NPINT - MFL                                               MBX06800
      IND = 1                                                           MBX06810
      IF(MP.LE.NPXCL) IND = 2                                           MBX06820
      DO 80 IC = 2,NPINT                                                MBX06830
      BARNU = IC - 1                                                    MBX06840
      NU = NP - IC + 1                                                  MBX06850
      JCL = MFL + 1                                                     MBX06860
      JCR = MOBTD(NU)                                                   MBX06870
      DO 80 JC = JCL,JCR                                                MBX06880
      YMWFL = YNFL - FLOAT(MP-1)                                        MBX06890
      YMTFL = FLOAT(JC-1) - YNFL                                        MBX06900
      MM = JC + NMAX - MFL                                              MBX06910
      DO 80 K = 1,IND                                                   MBX06920
      BARMU = YMWFL * CFA + YMTFL                                       MBX06930
      EL = YMWFL * SFA                                                  MBX06940
      IF((BARNU+0.5)**2 - (ABS(BARMU)-0.5)**2 - EL**2.LE.1.E-6)GO TO 80 MBX06950
      CALL VPIC                                                         MBX06960
      CALL KOMPLX(DPHI(NP,MP,1),DPHI(NP,MP,2),CFR,CFI,SSU(NU,MM,1),     MBX06970
     1 SSU(NU,MM,2))                                                    MBX06980
   80 YMWFL = YNFL + FLOAT(MP-1)                                        MBX06990
  100 RETURN                                                            MBX07000
      END                                                               MBX07010
```

MACH BØX INTERFERENCE PRØGRAM

GFØR GENERALIZED FØRCES

```
      SUBRØUTINE GENFØR                                                 MBX07030
      CØMMØN/A/DPHI(20,60,2),SSU(20,60,2),SSL(20,60,2),THW(10),THT(10)  MBX07040
      CØMMØN/D/XTE(40),YTE(40),QR(21,2),QI(21,2),CØW(10,21),CØT(10,21)  MBX07050
      CØMMØN/E/BØXL,BØXW,EMACH,EK,YNFL,FANG,CFA,SFA,NBØX,CAPPHI(20,20,2)MBX07060
      CØMMØN/F/MØDE,KØDE,NP,MP,MFL,NMAX,NBW,NBT,NPINT,NPFLET,NPØLW,NPØLTMBX07070
      CØN = 1.0                                                         MBX07080
      K = 1                                                             MBX07090
      IF(MP-1) 20,10,20                                                 MBX07100
   10 CØN = 0.5                                                         MBX07110
   20 WGT = XTE(MP) - FLØAT(NP-1)                                       MBX07120
      IF(WGT -1.5)30,40,40                                              MBX07130
   30 CØN = CØN * WGT                                                   MBX07140
      K = 2                                                             MBX07150
   40 NS = 1                                                            MBX07160
      MM = MP                                                           MBX07170
      NPØL = NPØLW                                                      MBX07180
      GØ TØ (60,200,60,50,200,200),KØDE                                 MBX07190
   50 NS = 2                                                            MBX07200
      MM = MP + NMAX - MFL                                              MBX07210
      NPØL = NPØLT                                                      MBX07220
   60 CØNTINUE                                                          MBX07230
      X = (FLØAT(NP) - 0.5)*BØXL                                        MBX07240
      Y =  FLØAT(MP-1)* BØXW                                            MBX07250
      DPHIR = DPHI(NP,MM,1)                                             MBX07260
      DPHII = DPHI(NP,MM,2)                                             MBX07270
      I = 0                                                             MBX07280
      DZ = 0.0                                                          MBX07290
      J = 0                                                             MBX07300
      DØ 80 N = J,NPØL                                                  MBX07310
      DØ 80 M = J,N                                                     MBX07320
      NM = N - M                                                        MBX07330
      FN = NM                                                           MBX07340
      I = I + 1                                                         MBX07350
      YM = 1.0                                                          MBX07360
      IF(M)62,64,62                                                     MBX07370
   62 YM = Y**M                                                         MBX07380
   64 XM = 1.0                                                          MBX07390
```

```
GFØR       GENERALIZED FØRCES

      IF(NM)66,68,66                                                   MBX07400
   66 XM = X**NM                                                       MBX07410
      DZ = FN * XM/X * YM                                              MBX07420
   68 Z = XM * YM                                                      MBX07430
      CALL KØMPLX(QR(I,NS),QI(I,NS),DPHIR*CØN,DPHII*CØN,DZ,-EK*Z/BØXL) MBX07440
   80 CØNTINUE                                                         MBX07450
      GØ TØ (200,90),K                                                 MBX07460
   90 CØN = 1.0                                                        MBX07470
      IF(MP - 1) 110,100,110                                           MBX07480
  100 CØN = 0.5                                                        MBX07490
  110 DX = XTE(MP) + 2.5 - FLØAT(NP)                                   MBX07500
      X = XTE(MP) *BØXL                                                MBX07510
      Y = YTE(MP) *BØXW                                                MBX07520
      A1 = DPHI(NP-2,MM,1)                                             MBX07530
      IF(A1.EQ.0.0) GØ TØ 190                                          MBX07540
      A2 = -1.5*A1 + 2.0*DPHI(NP-1,MM,1) - 0.5*DPHIR                   MBX07550
      A3 = 0.5*A1 - DPHI(NP-1,MM,1) + 0.5*DPHIR                        MBX07560
      DPHIR = A1 + A2*DX + A3*DX*DX                                    MBX07570
      A1 = DPHI(NP-2,MM,2)                                             MBX07580
      A2 = -1.5*A1 + 2.0*DPHI(NP-1,MM,2) - 0.5*DPHII                   MBX07590
      A3 = 0.5*A1 - DPHI(NP-1,MM,2) + 0.5*DPHII                        MBX07600
      DPHII = A1 + A2*DX + A3*DX*DX                                    MBX07610
  190 I = 0                                                            MBX07620
      DØ 180 N = J,NPØL                                                MBX07630
      DØ 180 M = J,N                                                   MBX07640
      NM = N - M                                                       MBX07650
      I = I+1                                                          MBX07660
      YM = 1.0                                                         MBX07670
      IF(M)120,130,120                                                 MBX07680
  120 YM = Y**M                                                        MBX07690
  130 Z = X**NM * YM                                                   MBX07700
      QR(I,NS) = QR(I,NS) - DPHIR * Z * CØN / BØXL                     MBX07710
      QI(I,NS) = QI(I,NS) - DPHII * Z * CØN / BØXL                     MBX07720
  180 CØNTINUE                                                         MBX07730
  200 CØNTINUE                                                         MBX07740
      RETURN                                                           MBX07750
      END                                                              MBX07760
```

VPIC   VELOCITY POTENTIAL INFLUENCE COEFFS.

```
C     VELOCITY POTENTIAL INFLUENCE COEFFICIENT AT A POINT              MBX07780
C     DUE TO A RECTANGULAR SOURCE SHEET OF UNIT STRENGTH THAT LIES     MBX07790
C     AT LEAST PARTIALLY WITHIN THE FORE MACH CONE FROM THAT POINT.    MBX07800
C     THE VPIC MAY BE FOR A POINT THAT IS EITHER COPLANAR OR           MBX07810
C     NONCOPLANAR WITH THE SOURCE SHEET.                               MBX07820
C                                                                      MBX07830
      SUBROUTINE VPIC                                                  MBX07840
      COMMON/C/BARNU,BARMU,EL,CFR,CFI,CNR,CNI,EKBAR,EKM,P(10),W(10)    MBX07850
      CFR = 0.0                                                        MBX07860
      CFI = 0.0                                                        MBX07870
      ELS = EL**2                                                      MBX07880
      ETAR = ABS(BARMU) - 0.5                                          MBX07890
      ETAL = ETAR + 1.0                                                MBX07900
      XR = SQRT(ETAR**2 + ELS)                                         MBX07910
      XL = SQRT(ETAL**2 + ELS)                                         MBX07920
      XU = BARNU + 0.5                                                 MBX07930
      XB = BARNU - 0.5                                                 MBX07940
      XIR = AMAX1(XB,AMIN1(XU,XR))                                     MBX07950
      XIL = AMAX1(XB,AMIN1(XU,XL))                                     MBX07960
      XILO = XIR                                                       MBX07970
      IF(ETAR.LT.0.0) XILO = AMAX1(ABS(EL),XB)                         MBX07980
      DO 42 K = 1,3                                                    MBX07990
      GO TO (6,8,10),K                                                 MBX08000
    6 XI = XILO                                                        MBX08010
      DX = XIR - XILO                                                  MBX08020
      IF(DX)42,42,12                                                   MBX08030
    8 XI = XIR                                                         MBX08040
      DX = XIL - XIR                                                   MBX08050
      IF(DX)42,42,12                                                   MBX08060
   10 XI = XIL                                                         MBX08070
      DX = XU - XIL                                                    MBX08080
      IF(DX)42,42,12                                                   MBX08090
   12 DO 40 I = 1,5                                                    MBX08100
      X = XI + P(I)*DX                                                 MBX08110
      GC = DX*W(I)                                                     MBX08120
      Y = SQRT(ABS(X**2 - ELS))                                        MBX08130
      WR = EKBAR * X                                                   MBX08140
```

```
 VPIC       VELOCITY POTENTIAL INFLUENCE COEFFS.

    IF(K.EQ.1) GO TO 16                                          MBX08150
    DV = GC * GO(Y,ETAR,ETAL,EKM)/3.14159265                     MBX08160
    GO TO 18                                                     MBX08170
 16 DV = - GC * ZJ(Y*EKM)                                        MBX08180
 18 CFR = CFR + COS(WR)*DV                                       MBX08190
 40 CFI = CFI - SIN(WR)*DV                                       MBX08200
 42 CONTINUE                                                     MBX08210
    RETURN                                                       MBX08220
    END                                                          MBX08230
```

VVIC VELOCITY INFLUENCE COEFFS.

```
C     NORMAL,HORIZONTAL,AND VERTICAL VELOCITY INFLUENCE COEFFICIENTS    MBX08250
C     AT A POINT DUE TO AN OUT-OF-PLANE UNIT STRENGTH RECTANGULAR       MBX08260
C     SOURCE SHEET THAT LIES AT LEAST PARTIALLY WITHIN THE FORE MACH    MBX08270
C     CONE FROM THAT POINT.                                             MBX08280
C                                                                       MBX08290
      SUBROUTINE VIC                                                    MBX08300
      COMMON/C/BARNU,BARMU,EL,CFR,CFI,CNR,CNI,EKBAR,EKM,P(10),W(10)     MBX08310
      COMMON/E/BOXL,BOXW,EMACH,EK,YNFL,FANG,CFA,SFA,NBOX,CAPPHI(20,20,2)MBX08320
      VR = 0.0                                                          MBX08330
      VI = 0.0                                                          MBX08340
      HR = 0.0                                                          MBX08350
      HI = 0.0                                                          MBX08360
      B = SIGN(1.0,BARMU)                                               MBX08370
      ELS = EL**2                                                       MBX08380
      ETAR = ABS(BARMU) - 0.5                                           MBX08390
      ETAL = ETAR + 1.0                                                 MBX08400
      ETRS = ETAR**2                                                    MBX08410
      ETLS = ETAL**2                                                    MBX08420
      XR = SQRT(ETRS + ELS)                                             MBX08430
      XL = SQRT(ETLS + ELS)                                             MBX08440
      XU = BARNU + 0.5                                                  MBX08450
      XB = BARNU - 0.5                                                  MBX08460
      XIR = AMAX1(XB,AMIN1(XU,XR))                                      MBX08470
      XIL = AMAX1(XB,AMIN1(XU,XL))                                      MBX08480
      XILO = XIR                                                        MBX08490
      IF(ETAR.LT.0.0) XILO = AMAX1(ABS(EL),XB)                          MBX08500
      XI = XU                                                           MBX08510
      WR = - 1.0                                                        MBX08520
      DO 10 I = 1,2                                                     MBX08530
      YS = ABS(XI**2 - ELS)                                             MBX08540
      Y = SQRT(YS)                                                      MBX08550
      S = EKBAR * XI                                                    MBX08560
      C = COS(S)                                                        MBX08570
      S = SIN(S)                                                        MBX08580
      DV = GO(Y,ETAR,ETAL,EKM)/XI                                       MBX08590
      VR = VR + WR*C*DV                                                 MBX08600
      VI = VI - WR*S*DV                                                 MBX08610
```

MACH BØX INTERFERENCE PRØGRAM

```
      NVIC        VELØCITY INFLUENCE CØEFFS.

      DH = FØ(YS,ETRS,ETLS,EKM)/XI                                      MBX08620
      HR = HR + WR*C*DH                                                 MBX08630
      HI = HI - WR*S*DH                                                 MBX08640
      XI = XILØ                                                         MBX08650
      IF(XI.NE.XB) GØ TØ 12                                             MBX08660
   10 WR= -WR                                                           MBX08670
   12 DØ 28 J = 1,3                                                     MBX08680
      GØ TØ(14,16,18),J                                                 MBX08690
   14 DX = XIR - XI                                                     MBX08700
      IF(DX)28,28,20                                                    MBX08710
   16 XI = XIR                                                          MBX08720
      DX = XIL - XI                                                     MBX08730
      IF(DX)28,28,20                                                    MBX08740
   18 XI = XIL                                                          MBX08750
      DX = XU  - XI                                                     MBX08760
      IF(DX)28,28,20                                                    MBX08770
   20 DØ 26 I = 1,5                                                     MBX08780
      X = XI + DX*P(I)                                                  MBX08790
      GC = DX*W(I)/X**2                                                 MBX08800
      YS = ABS(X**2 - ELS)                                              MBX08810
      Y = SQRT(YS)                                                      MBX08820
      WR = X*EKBAR                                                      MBX08830
      C = CØS(WR)                                                       MBX08840
      S = SIN(WR)                                                       MBX08850
      IF(J.EQ.1) GØ TØ 22                                               MBX08860
      DV = GC*GØ(Y,ETAR,ETAL,EKM)                                       MBX08870
      DH = GC*FØ(YS,ETRS,ETLS,EKM)                                      MBX08880
      HR = HR - (C + WR*S)*DH                                           MBX08890
      HI = HI + (S - WR*C)*DH                                           MBX08900
      GØ TØ 24                                                          MBX08910
   22 DV = -3.14159265 *GC*ZJ(EKM*Y)                                    MBX08920
   24 VR = VR - (C + WR*S)*DV                                           MBX08930
   26 VI = VI + (S - WR*C)*DV                                           MBX08940
   28 CØNTINUE                                                          MBX08950
      VR = VR*EL                                                        MBX08960
      VI = VI*EL                                                        MBX08970
      IF(EKM.EQ.0.0)GØ TØ 30                                            MBX08980
```

MACH BØX INTERFERENCE PRØGRAM

```
 NVIC      VELØCITY INFLUENCE CØEFFS.

      HR = HR/EKM                                                          MBX08990
      HI = HI/EKM                                                          MBX09000
      GØ TØ 36                                                             MBX09010
   30 FTØP = 1.0                                                           MBX09020
      FBØT = 1.0                                                           MBX09030
      IF( XL.LT.XU) FTØP =(XU/ XL) + SQRT((XU/ XL)**2 - 1.0)               MBX09040
      IF( XR.LT.XU) FBØT =(XU/ XR) + SQRT((XU/ XR)**2 - 1.0)               MBX09050
      IF( XL.LT.XB) FBØT = FBØT*((XB/ XL) + SQRT((XB/ XL)**2 - 1.0))       MBX09060
      IF( XR.LT.XB) FTØP = FTØP*((XB/ XR) + SQRT((XB/ XR)**2 - 1.0))       MBX09070
      HR = ALØG(FTØP/FBØT)                                                 MBX09080
   36 CNR = (VR*CFA + B*HR*SFA)/3.1415926                                  MBX09090
      CNI = (VI*CFA + B*HI*SFA)/3.1415926                                  MBX09100
      RETURN                                                               MBX09110
      END                                                                  MBX09120
```

MACH BØX INTERFERENCE PRØGRAM

```
VVEL       VERTICAL VELØCITY AND VELØCITY PØTENTIAL INTEGRAND

      FUNCTIØN GØ(R,ETAR,ETAL,EKM)                                    MBX09140
      DIMENSIØN BSL(23),S(2),C(2),AS(2),SO(2)                         MBX09150
      S(1)=ETAR                                                       MBX09160
      S(2)=ETAL                                                       MBX09170
      DØ 6 I=1,2                                                      MBX09180
      IF (ABS(S(I)).GE.R   ) GØ TØ 4                                  MBX09190
      S(I) = S(I)/R                                                   MBX09200
      C(I)=SQRT(1.0-S(I)**2)                                          MBX09210
      AS(I)=2.0*ATAN(S(I)/(1.0+C(I)))                                 MBX09220
      S(I)=2.0*S(I)*C(I)                                              MBX09230
      C(I)=2.0*C(I)**2-1.0                                            MBX09240
      GØ TØ 6                                                         MBX09250
    4 AS(I)=SIGN(1.57079633,S(I))                                     MBX09260
      S(I)=0.0                                                        MBX09270
    6 SO(I)=0.0                                                       MBX09280
      GØ=AS(1)-AS(2)                                                  MBX09290
      IF (ABS(GØ).LE.1.E-5) GØ TØ 86                                  MBX09300
      ARG=EKM*R                                                       MBX09310
      IF (ARG.EQ.0.0) RETURN                                          MBX09320
      CALL BSLS(BSL,ARG,N)                                            MBX09330
      GØ=BSL(1)*GØ                                                    MBX09340
      F=1.0                                                           MBX09350
      FI=1.0                                                          MBX09360
      DØ 100 J=1,N                                                    MBX09370
      GØ=BSL(J+1)*(S(1)-S(2))/FI-GØ                                   MBX09380
C      S(I) IS SIN(2*J*AS(I))                                         MBX09390
      DØ 90 I=1,2                                                     MBX09400
      S4=2.0*S(I)*C(I)-SO(I)                                          MBX09410
      SO(I)=S(I)                                                      MBX09420
   90 S(I)=S4                                                         MBX09430
      F=-F                                                            MBX09440
  100 FI=FI+1.0                                                       MBX09450
      IF (F.LT.0.0) GØ=-GØ                                            MBX09460
      RETURN                                                          MBX09470
   86 GØ=0.0                                                          MBX09480
      RETURN                                                          MBX09490
      END                                                             MBX09500
```

MACH BØX INTERFERENCE PRØGRAM

HVEL   HØRIZØNTAL VELØCITY INTEGRAND

```
C     HØRIZØNTAL VELØCITY INTEGRAND FUNCTIØN                           MBX09520
C                                                                      MBX09530
      FUNCTIØN FØ(RS,ETRS,ETLS,EKM)                                    MBX09540
      FØ = 0.0                                                         MBX09550
      IF(EKM.LE.0.0) RETURN                                            MBX09560
      FR = RS - ETRS                                                   MBX09570
      FL = RS - ETLS                                                   MBX09580
      IF(FR.GT.0.0) FØ = SIN(EKM*SQRT(FR))                             MBX09590
      IF(FL.GT.0.0) FØ = FØ - SIN(EKM*SQRT(FL))                        MBX09600
      RETURN                                                           MBX09610
      END                                                              MBX09620
```

ZJ   ZERØ ØRDER BESSEL FUNCTIØN

```
      FUNCTIØN ZJ(ARG)                                              MBX09640
      A = -(ARG/2.0)**2                                             MBX09650
      ZJ = 1.0                                                      MBX09660
      PF = 1.0                                                      MBX09670
      AN = 1.0                                                      MBX09680
      DØ 30 K = 1,20                                                MBX09690
      AN = AN * A/PF**2                                             MBX09700
      PF = PF + 1.0                                                 MBX09710
      IF(ABS(AN)- 1.E- 5)40,40,30                                   MBX09720
   30 ZJ = ZJ + AN                                                  MBX09730
   40 RETURN                                                        MBX09740
      END                                                           MBX09750
```

MACH BØX INTERFERENCE PRØGRAM

```
BSLS      EVEN BESSEL FUNCTIØNS

      SUBRØUTINE BSLS(BSL,ARG,N)                                        MBX09770
      DIMENSIØN BSL(23)                                                 MBX09780
      DØ 1000 I=1,23                                                    MBX09790
 1000 BSL(I)=0.0                                                        MBX09800
      IF(ABS(ARG) - 0.10)70,5,5                                         MBX09810
    5 N = MIN1(20.0,(ARG + 10.0))                                       MBX09820
      ARGS = ARG ** 2                                                   MBX09830
   20 AN = 2 * N + 4                                                    MBX09840
      BSL(N+2) = (4.0 * AN * (AN-1.0)/ARGS - (AN-1.0)/AN) * 0.3E-30     MBX09850
      PF = 0.0                                                          MBX09860
      DØ 50 I=0,N                                                       MBX09870
      M = N-I+1                                                         MBX09880
      AN = 2*M+1                                                        MBX09890
      BSL(M) =(4.0*(AN-1.0)*(AN-2.0)/ARGS-(AN-2.0)/AN-1.0)*BSL(M+1)     MBX09900
     X -(AN-2.0)/AN*BSL(M+2)                                            MBX09910
   50 PF = PF + 2.0*BSL(M+1)                                            MBX09920
      PF = PF + BSL(1)                                                  MBX09930
      AN = 0.0                                                          MBX09940
      IF(ABS(PF).GT.1.0) AN = ABS(PF)*1.E-10                            MBX09950
      M = N+2                                                           MBX09960
      DØ 60 I = 1,M                                                     MBX09970
      IF(AN - ABS(BSL(I)))54,56,56                                      MBX09980
   54 BSL(I) = BSL(I) / PF                                              MBX09990
      GØ TØ 60                                                          MBX10000
   56 BSL(I) = 0.0                                                      MBX10010
   60 CØNTINUE                                                          MBX10020
      DØ 180 I = 1,M                                                    MBX10030
      J = M - I + 1                                                     MBX10040
      IF(ABS(BSL(J)) - 1.E-7)180,180,120                                MBX10050
  180 N = J - 1                                                         MBX10060
      GØ TØ 120                                                         MBX10070
   70 BSL(2) = 0.125 * ARG**2                                           MBX10080
      BSL(1) = 1.0 - 2.0*BSL(2)                                         MBX10090
      N = 2                                                             MBX10100
  120 RETURN                                                            MBX10110
      END                                                               MBX10120
```

MACH BOX INTERFERENCE PROGRAM

COREC   CORRECTS TIP OR WING DIAFRAM BOX FOR MUTUAL INTERFERENCE

```
C     WHEN A TIP BOX FEELS A PORTION OF ITS WING DIAFRAM COUNTERPART     MBX10140
C     THE FOLLOWING CORRECTIONS ARE MADE TO BOTH SIDES OF BOTH BOXES     MBX10150
C     THESE EQUATIONS ARE CONSISTENT WITH THE MODIFICATIONS TO THE      MBX10160
C     WING DIAFRAM BOX SOURCE STRENGTH CALCULATIONS                     MBX10170
      SUBROUTINE COREC                                                   MBX10180
      COMMON/A/DPHI(20,60,2),SSU(20,60,2),SSL(20,60,2),THW(10),THT(10)   MBX10190
      COMMON/C/BARNU,BARMU,EL,CFR,CFI,CNR,CNI,EKBAR,EKM,P(10),W(10)      MBX10200
      COMMON/F/MODE,KODE,NP,MP,MFL,NMAX,NBW,NBT,NPINT,NPFLET,NPOLW,NPOLTMBX10210
      MM = MP + NMAX - MFL                                               MBX10220
      CALL VIC                                                           MBX10230
      SUDR = SSU(NP,MP,1)                                                MBX10240
      SUDI = SSU(NP,MP,2)                                                MBX10250
      SLTR = SSL(NP,MM,1) +  CNR*SUDR - CNI*SUDI                         MBX10260
      SLTI = SSL(NP,MM,2) +  CNR*SUDI + CNI*SUDR                         MBX10270
      SSL(NP,MP,1) = SSL(NP,MP,1) + CNR*SLTR - CNI*SLTI                  MBX10280
      SSL(NP,MP,2) = SSL(NP,MP,2) + CNR*SLTI + CNI*SLTR                  MBX10290
      SSL(NP,MM,1) = SLTR                                                MBX10300
      SSL(NP,MM,2) = SLTI                                                MBX10310
      SSU(NP,MM,1) = -SLTR                                               MBX10320
      SSU(NP,MM,2) = -SLTI                                               MBX10330
      RETURN                                                             MBX10340
      END                                                                MBX10350
```

MACH BOX INTERFERENCE PROGRAM

GAUSS   GAUSSIAN INTEGRATION CONSTANTS

```
      BLOCK DATA                                                        MBX10370
      COMMON/C/BARNU,BARMU,EL,CFR,CFI,CNR,CNI,EKBAR,EKM,P(10),W(10)     MBX10380
C     P(I) IS = 0.5 + ITH ZERO OF LEGENDRE POLYNOMIAL                   MBX10390
      DATA P/ 0.953089923,0.769234655,0.5,0.230765345,0.046910077,      MBX10400
     1        0.0,0.0,0.0,0.0,0.0 /                                     MBX10410
C     W(I) IS THE ITH WEIGHT COEFFICIENT FOR GAUSSIAN QUADRATURE        MBX10420
      DATA W/ 0.118463442,0.239314335,0.284444444,0.239314335,          MBX10430
     1        0.118463442,0.0,0.0,0.0,0.0,0.0 /                         MBX10440
      END                                                               MBX10450
```

MACH BOX INTERFERENCE PROGRAM

KPLX   COMPLEX MULTIPLICATION

```
      SUBROUTINE KOMPLX(AR,AI,BR,BI,CR,CI)                              MBX10470
      AR=AR+BR*CR-BI*CI                                                 MBX10480
      AI=AI+BR*CI+BI*CR                                                 MBX10490
      RETURN                                                            MBX10500
      END                                                               MBX10510
```

LSSUR   LEAST SQUARES SURFACE FIT

```
      SUBRØUTINE LSSUR(X2,Y2,Z2,N,NZ,ND,NG,A)                           MBX10760
      CØMMØN/A/X2(100),Y2(100),Z2(10,100),C(21,100),P1(100,21),         MBX10770
     1         X1F1(21,21),PTP(21,21),PTPINV(21,21),F1(21)              MBX10780
      DIMENSIØN A(10,21),X1F1C(21,100),X1M(21,21)                       MBX10790
      EQUIVALENCE (P1,X1F1C),(PTP,X1M)                                  MBX10800
C     FIND CENTRØID X1,Y1                                               MBX10810
      SUM1=0.0                                                          MBX10820
      SUM2=0.0                                                          MBX10830
      DØ 5 I=1,N                                                        MBX10840
      SUM1=SUM1+X2(I)                                                   MBX10850
    5 SUM2=SUM2+Y2(I)                                                   MBX10860
      FN=N                                                              MBX10870
      X1=SUM1/FN                                                        MBX10880
      Y1=SUM2/FN                                                        MBX10890
C     SHIFT TØ X1,Y1                                                    MBX10900
      DØ 7 I=1,N                                                        MBX10910
      X2(I)=X2(I)-X1                                                    MBX10920
      Y2(I)=Y2(I)-Y1                                                    MBX10930
    7 CØNTINUE                                                          MBX10940
C     NØRMALIZE ØN LARGEST LENGTH                                       MBX10950
      BIG=0.0                                                           MBX10960
      DØ 9 I=1,N                                                        MBX10970
    9 BIG = AMAX1(BIG,ABS(X2(I)),ABS (Y2(I)))                           MBX10980
   15 CØNTINUE                                                          MBX10990
      DØ 16 I=1,N                                                       MBX11000
      X2(I)=X2(I)/BIG                                                   MBX11010
      Y2(I)=Y2(I)/BIG                                                   MBX11020
   16 CØNTINUE                                                          MBX11030
C     CØMPUTE P MATRIX                                                  MBX11040
      DØ 50 L=1,N                                                       MBX11050
      NG=1                                                              MBX11060
      P1(L,1)=1.0                                                       MBX11070
      IF(NG - 2) 60,20,20                                               MBX11080
   20 CØNTINUE                                                          MBX11090
      DØ 40 I=2,ND                                                      MBX11100
      IM1=I-1                                                           MBX11110
      DØ 30 K=1,IM1                                                     MBX11120
```

```
      LSSUR     LEAST SQUARES SURFACE FIT

      NG=NG+1                                                     MBX11130
      NGMI=NG+1-I                                                 MBX11140
      P1(L,NG)=P1(L,NGM1)*X2(L)                                   MBX11150
   30 CØNTINUF                                                    MBX11160
      NG=NG+1                                                     MBX11170
      P1(L,NG)=P1(L,NGM1)*Y2(L)                                   MBX11180
   40 CØNTINUE                                                    MBX11190
   50 CØNTINUE                                                    MBX11200
   6C CØNTINUE                                                    MBX11210
      DØ 250 I=1,NG                                               MBX11220
      DØ 250 J=1,NG                                               MBX11230
      PTP(I,J)=0.0                                                MBX11240
      DØ 230 K=1,N                                                MBX11250
      PTP(I,J)=PTP(I,J)+P1(K,I)*P1(K,J)                           MBX11260
  230 CØNTINUE                                                    MBX11270
  250 CØNTINUE                                                    MBX11280
      CALL MATINV(PTP,PTPINV,NG)                                  MBX11290
C     C=(PTPINV)(PT)                                              MBX11300
      DØ 130 I=1,NG                                               MBX11310
      DØ 130 J=1,N                                                MBX11320
      C(I,J)=0.0                                                  MBX11330
      DØ 130 K=1,NG                                               MBX11340
      C(I,J)=C(I,J)+PTPINV(I,K)*P1(J,K)                           MBX11350
  130 CØNTINUE                                                    MBX11360
C     X1 MATRIX                                                   MBX11370
C     INITIALIZE ZERØS AND ØNES ØN DIAGØNAL                       MBX11380
      DØ 610 I=1,21                                               MBX11390
      DØ 610 J=1,21                                               MBX11400
  610 X1M(I,J)=0.0                                                MBX11410
      DØ 620 I=1,21                                               MBX11420
  620 X1M(I,I)=1.0                                                MBX11430
C     INITIALIZE CØLUMN INDEX AND SIGN                            MBX11440
      NG=1                                                        MBX11450
      SIGNR=1.0                                                   MBX11460
C     INDEX DEGREE ØF FIRST RØW                                   MBX11470
      DØ 760 L1=1,ND                                              MBX11480
      SIGNR=-SIGNR                                                MBX11490
```

LSSUR   LEAST SQUARES SURFACE FIT

```
      L1P1=L1+1                                                         MBX11500
      DØ 750 L2=1,L1P1                                                  MBX11510
      NG=NG+1                                                           MBX11520
      NEXPYC=L2-1                                                       MBX11530
      NEXPXC=L1-NEXPYC                                                  MBX11540
      FEXPXC=NEXPXC                                                     MBX11550
      FEXPYC=NEXPYC                                                     MBX11560
      X1M(1,NG)=X1**NEXPXC*Y1**NEXPYC*SIGNR                             MBX11570
C     INDEX DØWN CØLUMN THRU (L1-1)DEGREE                               MBX11580
      L1M1=L1-1                                                         MBX11590
      NG1=1                                                             MBX11600
      SIGNC=1.0                                                         MBX11610
      DØ 740 K1=1,L1M1                                                  MBX11620
      SIGNC=-SIGNC                                                      MBX11630
C     INDEX TERMS ØF DEGREE(IN CØLUMN)                                  MBX11640
      K1P1=K1+1                                                         MBX11650
      DØ 730 K2=1,K1P1                                                  MBX11660
      NG1=NG1+1                                                         MBX11670
      NEXPYR=K2-1                                                       MBX11680
      NEXPXR=K1-NEXPYR                                                  MBX11690
C     SUBTRACT EXPØNENTS                                                MBX11700
      NXMNX=NEXPXC-NEXPXR                                               MBX11710
      NYMNY=NEXPYC-NEXPYR                                               MBX11720
      IF(NXMNX)730,660,660                                              MBX11730
660   IF(NYMNY)730,670,670                                              MBX11740
670   FX=1.0                                                            MBX11750
      FY=1.0                                                            MBX11760
      IF(NXMNX)700,700,680                                              MBX11770
680   FXMNX=NXMNX                                                       MBX11780
      DØ 690 K3=1,NXMNX                                                 MBX11790
      FK3=K3                                                            MBX11800
      FX=FX*(FEXPXC-(FK3-1.0))/FK3                                      MBX11810
690   CONTINUE                                                          MBX11820
700   IF(NYMNY)720,720,705                                              MBX11830
705   FYMNY=NYMNY                                                       MBX11840
      DØ 710 K3=1,NYMNY                                                 MBX11850
      FK3=K3                                                            MBX11860
```

LSSUR LEAST SQUARES SURFACE FIT

```
      FY=FY*(FEXPYC-(FK3-1.0))/FK3
 710  CONTINUE
 720  X1M(NG1,NG)=X1**NXMNX*Y1**NYMNY*FX*FY*SIGNR*SIGNC
 730  CONTINUE
 740  CØNTINUE
 750  CONTINUE
 760  CONTINUE
C     F1 MATRIX
      F1(1)=1.0
      L=1
      DØ 4010 I=1,ND
      IP1=I+1
      FACTØR=1.0/BIG **I
      DØ 4010 K=1,IP1
      L=L+1
      F1(L)=FACTØR
4010  CØNTINUE
C     X1F1=(X1M)*(F1)
      DØ 4020 J=1,NG
      DØ 4020 I=1,NG
      X1F1(I,J)=X1M(I,J)*F1(J)
4020  CØNTINUE
C     X1F1C=(X1F1)(C)
      DØ 4030 I=1,NG
      DØ 4030 J=1,N
      X1F1C(I,J)=0.0
      DØ 4030 K=1,NG
4030  X1F1C(I,J)=X1F1C(I,J)+X1F1(I,K)*C(K,J)
C     A=(X1F1C)(Z)
      DØ 4040 I=1,NG
      DØ 4040 J=1,NZ
      A(J,I) = 0.0
      DØ 4040 K=1,N
4040  A(J,I) = A(J,I) + X1F1C(I,K)*Z2(K,J)
      RETURN
      END
```

MINV   MATRIX INVERSION

```
      SUBROUTINE MATINV(PTP,PTPINV,NG)                                  MBX12240
C     MATRIX INVERSION BY GAUSSIAN ELIMINATION                          MBX12250
      DIMENSION PTP(NG,NG),PTPINV(NG,NG)                                MBX12260
C     SET-UP IDENTITY MATRIX                                            MBX12270
      DO 310 I=1,NG                                                     MBX12280
      DO 310 J=1,NG                                                     MBX12290
  310 PTPINV(I,J)=0.0                                                   MBX12300
      DO 320 I=1,NG                                                     MBX12310
  320 PTPINV(I,I)=1.0                                                   MBX12320
C     ENTER OVERALL ELIMINATION LOOP                                    MBX12330
      DO 460 I=1,NG                                                     MBX12340
      IP1=I+1                                                           MBX12350
C     CHECK FOR ZERO IN PIVOTAL ELEMENT                                 MBX12360
      IF(PTP(I,I))355,325,355                                           MBX12370
  325 IF(IP1-NG)326,326,331                                             MBX12380
C     SEARCH FOR NON-ZERO IN SAME COLUMN                                MBX12390
  326 DO 330 K = IP1,NG                                                 MBX12400
      IF(PTP(K,I))335,330,335                                           MBX12410
  330 CONTINUE                                                          MBX12420
C     NON-ZERO PIVOT CANNOT BE FOUND                                    MBX12430
  331 WRITE (6,332)                                                     MBX12440
  332 FORMAT(1HX,23HDETERMINANT IS SINGULAR)                            MBX12450
      CALL EXIT                                                         MBX12460
  335 CONTINUE                                                          MBX12470
C     INTERCHANGE I AND K ROWS IN BOTH MATRICES                         MBX12480
      DO 340 L = 1,NG                                                   MBX12490
      HOLD=PTP(I,L)                                                     MBX12500
      PTP(I,L)=PTP(K,L)                                                 MBX12510
      PTP(K,L)=HOLD                                                     MBX12520
  340 CONTINUE                                                          MBX12530
      DO 350 L=1,NG                                                     MBX12540
      HOLD=PTPINV(I,L)                                                  MBX12550
      PTPINV(I,L)=PTPINV(K,L)                                           MBX12560
      PTPINV(K,L)=HOLD                                                  MBX12570
  350 CONTINUE                                                          MBX12580
  355 CONTINUE                                                          MBX12590
C     DIVIDE ROW BY PIVOT                                               MBX12600
```

```
        MINV        MATRIX INVERSION

          PIVOT=PTP(I,I)                                                MBX12610
C         PTP MATRIX                                                    MBX12620
          DO 360 L=I,NG                                                 MBX12630
   360    PTP(I,L)=PTP(I,L)/PIVOT                                       MBX12640
C         PTPINV MATRIX                                                 MBX12650
          DO 370 L=1,NG                                                 MBX12660
   370    PTPINV(I,L)=PTPINV(I,L)/PIVOT                                 MBX12670
C         ARE WE AT LAST ELEMENT                                        MBX12680
          IF(I-NG)375,470,470                                           MBX12690
   375    CONTINUE                                                      MBX12700
C         ZERO COLUMN AND ROW OF PTP SIMULTANEOUSLY                     MBX12710
          DO 450 K=IP1,NG                                               MBX12720
C         FIND ZEROING FACTOR FOR COLUMN                                MBX12730
          CO=PTP(K,I)                                                   MBX12740
          PTP(K,I)=0.0                                                  MBX12750
C         MULTIPLY ROW BY ZEROING FACTOR                                MBX12760
          DO 390 L=IP1,NG                                               MBX12770
   390    PTP(K,L)=PTP(K,L)-CO*PTP(I,L)                                 MBX12780
          DO 400 L=1,NG                                                 MBX12790
   400    PTPINV(K,L)=PTPINV(K,L)-CO*PTPINV(I,L)                        MBX12800
   450    CONTINUE                                                      MBX12810
   460    CONTINUE                                                      MBX12820
   470    CONTINUE                                                      MBX12830
          J1=NG+1                                                       MBX12840
   472    J1=J1-1                                                       MBX12850
          IF(J1-1)488,488,474                                           MBX12860
   474    J1M1=J1-1                                                     MBX12870
          DO 477 I1=1,J1M1                                              MBX12880
          CO=PTP(I1,J1)                                                 MBX12890
          DO 477 J2=1,NG                                                MBX12900
          PTP(I1,J2)=PTP(I1,J2)-CO*PTP(J1,J2)                           MBX12910
          PTPINV(I1,J2)=PTPINV(I1,J2)-CO*PTPINV(J1,J2)                  MBX12920
   477    CONTINUE                                                      MBX12930
          GO TO 472                                                     MBX12940
   488    CONTINUE                                                      MBX12950
          RETURN                                                        MBX12960
          END                                                           MBX12970
```

UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* North American Aviation, Inc. Space and Information Systems Division Downey, California	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP N/A

3. REPORT TITLE

Unsteady Aerodynamics for Advanced Configurations, Part IV - Application of the Supersonic Mach Box Method to Intersecting Planar Surfaces

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Final Report

5. AUTHOR(S) *(Last name, first name, initial)*

Moore, Michael T.
Andrew, Lowell V.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
May 1965	135	13

8a. CONTRACT OR GRANT NO. AF33(657)-10399	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO. 1370	FDL-TDR-64-152, Part IV
c. Task 137003	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
d.	SID 64-1512-4

10. AVAILABILITY/LIMITATION NOTICES

None

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Air Force Flight Dynamics Laboratory (FDDS) Wright-Patterson AFB, Ohio 45433

13. ABSTRACT Ashley's approach to mutual interference theory by source superposition methods has been applied to the prediction of supersonic air loads on intersecting thin lifting surfaces in steady of oscillatory motion. Steady loading is regarded as the special case of zero frequency of oscillation. Each surface may be oscillating in a mode of rigid or elastic vibration or linear combinations thereof. Evvard's diaphragm concept has been extended to treat the out-of-plane interference problem. As a result, any leading or side edge on any of the intersecting surfaces may be subsonic. The study reported herein has lead to the formulation of a method by which diaphragm regions can be selected that eliminate the need for calculating out-of-plane velocity potentials. Based on mutual interference theory, the method requires only the calculation of out-of-plane velocities so that tangential flow conditions may be met.

The Mach-box method has been used to obtain an approximate solution to the problem. In following the aerodynamic influence coefficient procedure of Zartarian and Hsu, each surface and diaphragm is overlaid with a grid of rectangular Mach boxes, the diagonals of which are parallel to the Mach lines. For purposes of calculating induced velocities and velocity potentials, the source strength over the area of each box is assumed to be constant. Out-of-plane velocity influence coefficients are calculated for the center of each box in the interference region, and the tangential flow condition is satisfied there. For purposes of calculating generalized forces, the resulting velocity potential over the area of each box is also assumed to be constant and equal to the value at the center of the box.

DD FORM 1 JAN 64 1473

UNCLASSIFIED
Security Classification

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization *(corporate author)* issuing the report.

2*a*. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2*b*. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7*a*. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7*b*. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8*a*. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8*b*, 8*c*, & 8*d*. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9*a*. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9*b*. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers *(either by the originator or by the sponsor)*, also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through
____."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through
____."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through
____."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS)*, *(S)*, *(C)*, or *(U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.

Security Classification