A DESIGN FOR IMPROVING THE STRUCTURAL DAMPING PROPERTIES OF AXIAL MEMBERS

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ABSTRACT

In this work a laminated composite construction is studied for the purposes of improving the damping properties of axially loaded structural members. The lamination consists of an anisotropic arrangement of stiffness and damping materials. Under vibratory loading the anisotropy produces stress coupling effects that shear the damping materials, thus leading to a high dissipation of vibrational energies. A structural theory is formulated for examining this construction and is applied to the analysis of a three layered member. Parametric studies are performed for a specific design case. The results demonstrate the effectiveness of the new design for controlling the resonant response of simple harmonic oscillators.

1.0 INTRODUCTION

Although traditionally treated as a secondary design variable, damping is being increasingly relied upon to improve the performance of sophisticated structures. A popular design method for increasing structural damping is the combination of viscoelastic materials and stiffness materials into composite constructions. Structural components based on this approach possess stiffness, strength and a high damping capacity [1-2].

A well-established composite construction is the constrained layer treatment which consists of laminations of stiffness and damping layers. The study of this design is now new; such designs have a technological history approximately as old as that of modern composite materials. Over this period, theories and practice methods have arisen to address beam and plate components [3-9]. Constrained layer treatments are now the preferred method of construction in many designs that require high structural damping in order to control bending modes of vibration.

In spite of their success, damping methods have an inherent limitation. Damping materials typically have a high shear viscosity but a low bulk viscosity. Thus, vibrational energies are not dissipated unless high frequency shear modes are excited. For some structural components, such as axially loaded members, relying on such a driving mechanism can render a damping design virtually ineffective.

To rectify this shortcoming, a new design for axial members is proposed. This design makes use of the stress coupling of anisotropic constructions which can be used to generate local and global shear deformations. In a composite construction these deformations can be employed to shear layers of damping materials.

The pioneering work in the use of stress coupling to assist the control of dynamic responses examined the performance of sandwich plates [10]. The results of this study were limited since the analytical model was based on displacement fields that were too elementary to fully characterize the deformational response of composite constructions. This shortcoming was rectified in a later work [11] which examined laminated sandwich constructions.

In the present work a structural theory is devised which characterizes the behavior of an axially loaded composite construction. The axial member consists of concentric cylindrical layers of stiffness and damping materials. Stress coupling can be introduced into this design through the use of off-axis orthotropic materials and/or orthotropic constructions (surface waffling, spiral stiffeners, surface scoring, etc.). The equations of the theory are solved for the steady state damped response of a simple harmonic oscillator.

2.0 THEORY OF LAMINATED ANISOTROPIC AXIAL COMPOSITE CONSTRUCTION

2.1 Analytical Model

The axial composite construction is a cylinder of successive concentric layers whose central axis is oriented along the axial coordinate line of a right handed cylindrical coordinate system (Figure 1). Although other constructions are possible, this work assumes that each damping cylinder is bounded by stiffness cylinders. The thickness of an individual layer is designated h_n while the radial location of the layer's midsurface is R_n. The symbol n, used as a subscript and a superscript in this text, refers to the layer of interest (the numbering proceeds from inside to outside). The overall length of the cylinder is ℓ.

The mechanical response of the axial composite construction is examined through it's role as a support for a lumped mass single degree of freedom oscillator subjected to a steady state harmonic axial excitation. The equation of motion for such a system can be derived directly from elementary equilibrium considerations. It remains then to characterize the viscoelastic spring stiffness of the support. In doing so the elastic spring stiffness is initially computed with the viscosity introduced later through the use of the Correspondence Principle.

In determining the elastic stiffness, the following assumptions are made:

1. The stiffness cylinders can be treated with thin wall theory.

2. The moduli of the damping cylinders are negligible in comparison to the moduli of the stiffness cylinders.

3. The cylinders are perfectly bonded together.

Assumption 1 implies that in each stiffness cylinder the membrane stresses are independent of the radial coordinate and that the non-membrane stresses are negligible. From Assumption 2, and the purely axial loading,, it is seen that all of the stresses in the damping cylinders are negligible except for the radial-tangential shear stress. Assumption 3 implies that the displacements are continuous across the interfaces. From the geometry of the construction and the type of applied loading, it follows that all of the field variables are independent of the circumferential coordinate.

The method of solution will be to develop the equilibrium equations for the individual cylinders in terms of the local displacements and tractions. By applying interfacial equilibrium the tractions are eliminated from these equations yielding a set of linear simultaneous differential equations. The solution to these displacement equilibrium equations can be used to compute the stiffness characteristics of the composite construction.

2.2 Stiffness Cylinders

From the assumptions it is seen that the midsurface displacements in the stiffness cylinders reduce to

$$U_Z^n(Z, \theta, R) \longrightarrow U_Z^n(Z) \tag{1}$$

$$U_\theta^n(Z, \theta, R) \longrightarrow U_\theta^n(Z) \tag{2}$$

$$U_R^n(Z, \theta, R) \longrightarrow U_R^n(Z) \tag{3}$$

Then using the stress-strain relations of an orthotropic system, the stress-displacement equations become:

$$\sigma_{ZZ}^n = C_{11}^n U_Z^n{}_{,Z} + C_{12}^n R_n^{-1} U_R^n + C_{16}^n U_\theta^n{}_{,Z} \tag{4}$$

$$\sigma_{\theta\theta}^n = C_{12}^n U_Z^n{}_{,Z} + C_{22}^n R_n^{-1} U_R^n + C_{26}^n U_\theta^n{}_{,Z} \tag{5}$$

$$\sigma_{Z\theta}^n = C_{16}^n U_Z^n{}_{,Z} + C_{26}^n R_n^{-1} U_R^n + C_{66}^n U_\theta^n{}_{,Z} \tag{6}$$

where the C_{ij}^n are the elastic stiffnesses and a comma denotes

differentiation with respect to the listed variable. Imposing equilibrium and eliminating the radial displacement yields the following equations:

$$\left(C_{11}^n - \frac{C_{12}^n C_{12}^n}{C_{22}^n}\right) U_Z^n{}_{,ZZ} + \left(C_{16}^n - \frac{C_{12}^n C_{26}^n}{C_{22}^n}\right) U_\theta^n{}_{,ZZ} = 0 \tag{7}$$

$$\left(C_{16}^n - \frac{C_{26}^n C_{12}^n}{C_{22}^n}\right) U_Z^n{}_{,ZZ} + \left(C_{66}^n - \frac{C_{26}^n C_{26}^n}{C_{22}^n}\right) U_\theta^n{}_{,ZZ} = -\frac{\left({}^sT_\theta^n\, b_n b_n + {}^sT_\theta^{n+1}\, b_{n+1} b_{n+1}\right)}{R_n R_n h_n} \tag{8}$$

where the b_n are the radial location of the cylinders lateral faces and ${}^sT_\theta^n$ (innermost surface) and ${}^sT_\theta^{n+1}$ (outermost surface) are the circumferential tractions acting on these lateral faces.

2.3 Damping Cylinders

For the damping cylinders the stress-displacement relation for the radial-tangential shear stress is:

$$\sigma_{R\theta}^n(R, Z) = G^n\left(U_\theta^n{}_{,R} - R^{-1} U_\theta^n \right) \tag{9}$$

where G^n is the shear modulus. By imposing circumferential equilibrium the circumferential displacements can be determined in terms of the displacements on the lateral boundaries which can be directly related to the displacements of the adjacent stiffness cylinders. The shear stress then becomes:

$$\sigma_{R\theta}^n(Z) = -2G^n h_2^n R^{-2} \tag{10}$$

where

$$h_2^n = \frac{b_n^2 b_{n+1}^2 (R_{n+1} U_\theta^{n-1} - R_{n-1} U_\theta^{n+1})}{R_{n-1} R_{n+1} (b_{n+1}^2 - b_{n+1}^2)} \qquad (11)$$

From equation (10) the tractions on the inner ${}^{D}T_\theta^{\,n}$ and the outer ${}^{D}T_\theta^{\,n+1}$ boundaries can be computed.

2.4 Interfacial Equilibrium

From interfacial equilibrium the tractions between adjacent damping and stiffness cylinders are related as

$$\overset{s}{T}_\theta^{\,n+1} = -\overset{D}{T}_\theta^{\,n+1} \qquad (12)$$

$$\overset{s}{T}_\theta^{\,n} = -\overset{D}{T}_\theta^{\,n} \qquad (13)$$

3.0 APPLICATION TO THREE LAYERED AXIAL MEMBER

As an application of the theory consider a composite construction of three cylinders. The outer and inner cylinders are stiffness layers while the middle cylinder is a damping layer. Solving the governing equations yields the following displacements in the stiffness cylinders.

$$U_\theta^1(Z) = d_1 + d_2 Z + d_3 e^{m_3 Z} + d_4 e^{m_4 Z} \qquad (14)$$

$$U_\theta^3(Z) = d_5 + d_6 Z + d_7 e^{m_3 Z} + d_8 e^{m_4 Z} \qquad (15)$$

$$U_Z^1(Z) = -\frac{A_{12}^1}{A_{11}^1}(d_3 e^{m_3 Z} + d_4 e^{m_4 Z}) + d_9 Z + d_{10} \qquad (16)$$

$$U_Z^3(Z) = -\frac{A_{12}^3}{A_{11}^1}\frac{B_4}{B_2}(d_3 e^{m_3 Z} + d_4 e^{m_4 Z}) + d_{11} Z + d_{12} \qquad (17)$$

where the B_i and m_i are constants [12] and the d_i are constants of integration (only eight of which are independent). The stresses are then computed as:

$$\sigma_{zz}^1(Z) = V_1 d_2 + V_3 d_9 \qquad (18)$$

$$\sigma_{zz}^3(Z) = V_4 d_2 + V_6 d_{11} \qquad (19)$$

$$\sigma_{z\theta}^1(Z) = W_1 d_2 + W_2 (d_3 e^{m_3 Z} - d_4 e^{m_4 Z}) + W_3 d_9 \qquad (20)$$

$$\sigma_{z\theta}^3(Z) = W_4 d_2 + W_5 (d_3 e^{m_3 Z} - d_4 e^{m_4 Z}) + W_6 d_{11} \qquad (21)$$

where the V_i and W_i are constants [12].

The solutions for the displacement fields contain rigid body, constant strain and exponential terms. These last terms embody the effects of stress-coupling on the composite construction. Since these terms are exponential it is concluded that the contribution of the middle layer to component damping is primarily based on end effects. This observation has implications for an optimum design configuration (i.e., designs based on segmented constrained layer treatments or on variable stiffness cylinders.) The present work will not seek to treat optimum designs, which involve greater analytical difficulties, but will concentrate on examining the basic structural theory.

4.0 DESIGN STUDY

4.1 Description

The displacement solutions contain eight unknown constants which are determined from the displacement and traction boundary conditions acting on the ends of the stiffness cylinders. Various sets of boundary conditions have practical interest. For the purposes of this study the following boundary conditions are imposed:

$$U_z^1(0) = 0 \qquad \text{(Fixed)} \tag{22}$$

$$U_z^3(0) = 0 \qquad \text{(Fixed)} \tag{23}$$

$$U_z^1(\ell) = U_z^3(\ell) \qquad \text{(Rigid Link)} \tag{24}$$

$$\sigma_{zz}^1(\ell) A_1 + \sigma_{zz}^3(\ell) A_3 = P \tag{25}$$

$$U_\theta^1(0) = 0 \qquad \text{(Fixed)} \tag{26}$$

$$U_\theta^3(0) = 0 \qquad \text{(Fixed)} \tag{27}$$

$$\sigma_{z\theta}^1(\ell) = 0 \qquad \text{(Free)} \tag{28}$$

$$\sigma_{z\theta}^3(\ell) = 0 \qquad \text{(Free)} \tag{29}$$

where the A_i are the cross sectional areas of the stiffness cylinders and P is the applied axial load.

Applying the boundary conditions to the general solution leads to a set of linear simultaneous algebraic equations the solution of which are the constants of integration. Once these constants are determined the solution to any field variable can be found. By applying a unit load to the column and solving for the field variables, influence coefficients and hence stiffness can be determined for any point of interest. The viscosity of the support is introduced by invoking the Correspondence Principle.

In the following example a composite construction is examined that combines a fiber reinforced material with a centrally positioned damping material (Table 1). The anisotropy of the construction is varied via a $-\theta^{o}/\theta^{o}$ (inside cylinder/outside cylinder) off-axis fiber orientation. In addition the response of a conventional cylindrical design is also computed and used as a baseline from which to judge the performance of the new design. The baseline design [13] contains no damping layers in its section and is constructed from the same stiffness material as the composite construction. The principal material direction of the baseline design is always oriented parallel to the central axis of the cylinder (maximum static stiffness). The structural damping of the baseline design is equal to the material damping of the stiffness materials. In the design study the cross sectional area of the baseline and composite constructions are equal so that the analysis examines the effect of replacing stiffness material with damping material.

The shear modulus (.024 GPa) and the loss factor (1.0) of the damping material are taken as constants thus ignoring the frequency and temperature dependency of these properties. Since the purpose of the design study is to make structural comparisons this simplification is not critical. However for actual designs the effects of the operational environment must be considered in more detail.

The axial member to be examined here has a length of 25.4 cm and a supported mass of 232 Kg. The mass of the support, which can be treated in the analysis through a lumped mass approach, is much less than the supported mass. Therefore in this problem the frequency response is practically independent of the structural mass. Nevertheless it should be noted that viscoelastic materials are less massive than reinforced epoxies so that composite construction produces lighter structure.

4.2 Results

The results of the analyses are presented as the amplitudes of the frequency response and are normalized with respect to the response of the baseline design (indicated by the superscript B on the plots).

The dynamic response of the composite construction is shown in Figure 2 where it is seen that for small offset angles the response actually increases. However moving to greater offset angles results in a dramatic decrease in resonant response. Figure 3 shows that this trend continues until further increases in structural damping (Figure 4) can no longer counter the effects of lost stiffness (Figure 5) so that it is seen that there is an optimum balance of these properties for minimizing the displacement response. Figure 3 also shows the effect of off-axis orientations on the circumferential displacements. These displacements, which drive the shearing of the damping layer, must be managed in order to produce a feasible design. Figure 6 shows that the resonant axial stresses continue to decrease even after the axial displacement begins to rebuild. This drop in stress can be directly related to the increasing loss factor. Figure 6 also shows the maximum shear and interfacial stresses which are generated in the new design. Finally, Figure 7 plots the shift in resonant frequency as the structural stiffness is lowered.

5.0 DISCUSSION AND CONCLUSION

The design studies explored the use of stress coupling and lamination in an axial composite construction, a new structural component, that is intended for use in vibration sensitive structure. Incorporating these components into structures will affect the structural response through several ways including: energy dissipation, the reduction of stiffness, the reduction of mass and the introduction of anisotropic effects.

Energy dissipation is the prime motivation for using the new design approach and the design study shows that by combining diverse materials (stiffness and damping) into a composite construction, components can be produced with superior structural properties. The reduction of resonant stress and displacements is directly attributable to the increase in damping in these components. That there is a trade-off in these components between static stiffness and damping simply introduces a new variable to the design process. Since in the anticipated applications the overriding design goal will be to reduce resonant responses the sacrifice of stiffness for energy dissipation presents design opportunities.

The introduction of anisotropic effects leads to the most formidable difficulties for application. Anisotropy results in shear stresses and circumferential displacements in the axial composite construction, mechanical responses that do not exist in an axially loaded baseline design. The presence of shear stresses do not necessarily represent a design penalty, especially since the axial stresses are significantly reduced. However, depending on the material of construction, shear stresses may cause new modes of failure which must be considered in the design process. The occurrence of shear stresses may also require a redesign of the structural connections. Such redesigns will be even more difficult when allowing for the circumferential displacements that occur at the ends of members. Some critical applications may justify the effort of accommodating these displacements but for the design to achieve widespread use a way of eliminating these displacements altogether must be found. If the construction had a variable axial stiffness (stepped thicknesses, variable angled filament winding, etc.) components could be produced for which the circumferential displacements vanish at the element ends. In such designs the circumferential displacements, which are necessary for shearing the damping layers, can be confined to the central portions of the component so that fixed circumferential boundary conditions can be applied while still achieving a drive for damping.

An additional advantage of the axial composite construction is its resistance to all modes of vibration. For flexural modes the component acts similar to damped sandwich constructions since the damping layers will be sheared by the displacements that occur in beam bending. Torsional modes are resisted by the same stress coupling effects that occur under axial vibrations with the difference being that the damping layers are now sheared by differential axial displacements.

The design studies were based on viscoelastic properties that assumed a peak value for the loss factor. However, both the stiffness and loss factor of these materials are frequency and temperature dependent with peak values occurring over limited ranges. In order to extend the capability of the axial composite construction, dissimilar viscoelastic materials, each with a distinct range of peak effectiveness, could be incorporated in the component. Design approaches that may be taken include a four layer cylinder with adjacent interior damping layers or a five layer cylinder with interior stiffness and damping layers.

The work presented here demonstrates that stress coupling and lamination can be used to great advantage in increasing the damping of axial structural components. Further research in this area will investigate generalizing the basic approach to other types of structural components.

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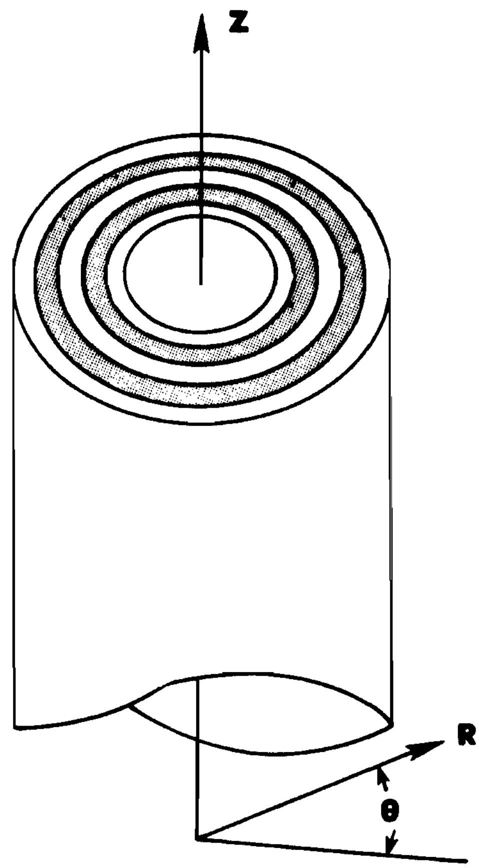


FIGURE 1 AXIAL COMPOSITE CONSTRUCTION

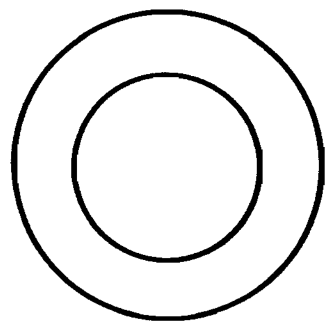
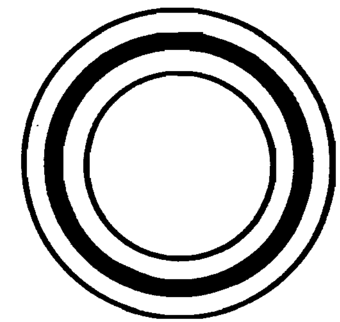
DESIGN CASE	 BASELINE	 COMPOSITE CONSTRUCTION
CONSTRUCTION	MONOLITHIC	THREE CONCENTRIC CYLINDERS
DIAMETER	2.54 cm	2.54 cm
WALL THICKNESS	.152 cm	.152 cm (.0699/.0127/.0699 cm)
MATERIALS	HMF GRAPHITE/EPOXY	HMF GRAPHITE/EPOXY PLUS DAMPING MATERIAL
FIBER ORIENTATION	0°	VARIABLE

TABLE 1 Design Description

N1808-GA-89-00000

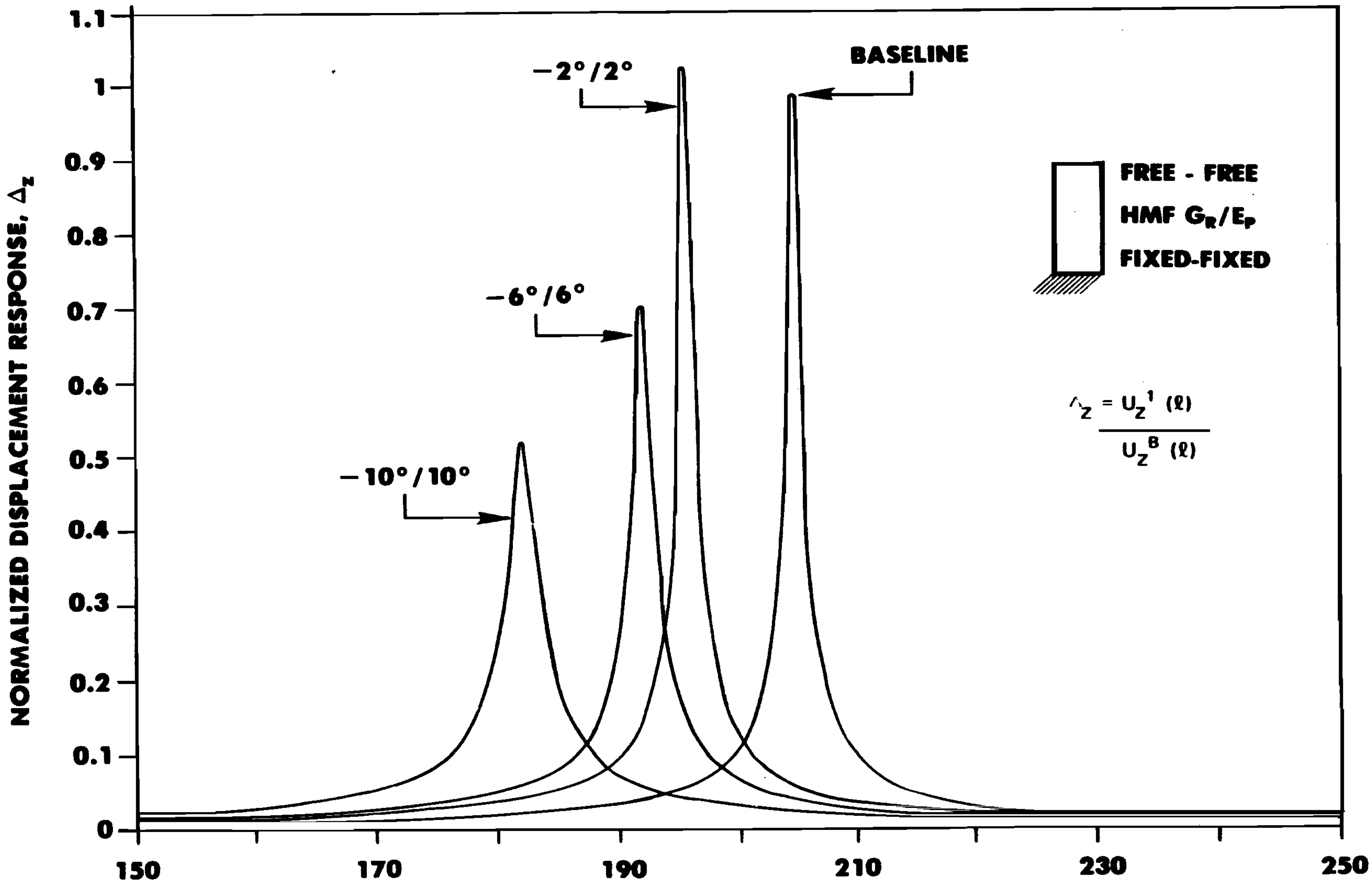
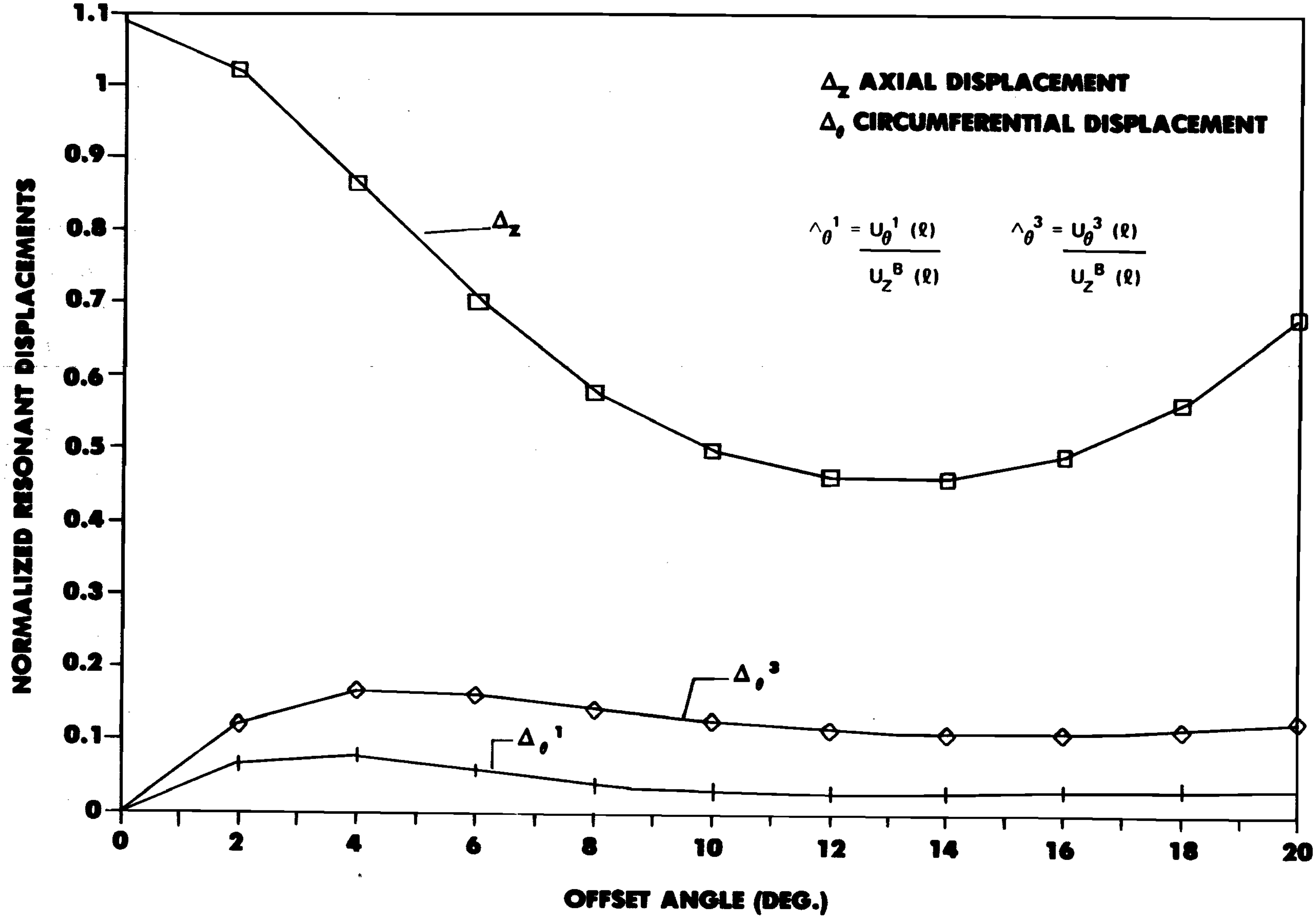


FIGURE 2 NORMALIZED DIPLACEMENT RESPONSE SPECTRUM, COMPOSITE CONSTRUCTION #1



$$\Delta_\theta^1 = \frac{U_\theta^1(\ell)}{U_z^B(\ell)} \qquad \Delta_\theta^3 = \frac{U_\theta^3(\ell)}{U_z^B(\ell)}$$

FIGURE 3 NORMALIZED RESONANT DISPLACEMENT VS OFFSET ANGLE, COMPOSITE CONSTRUCTION #1

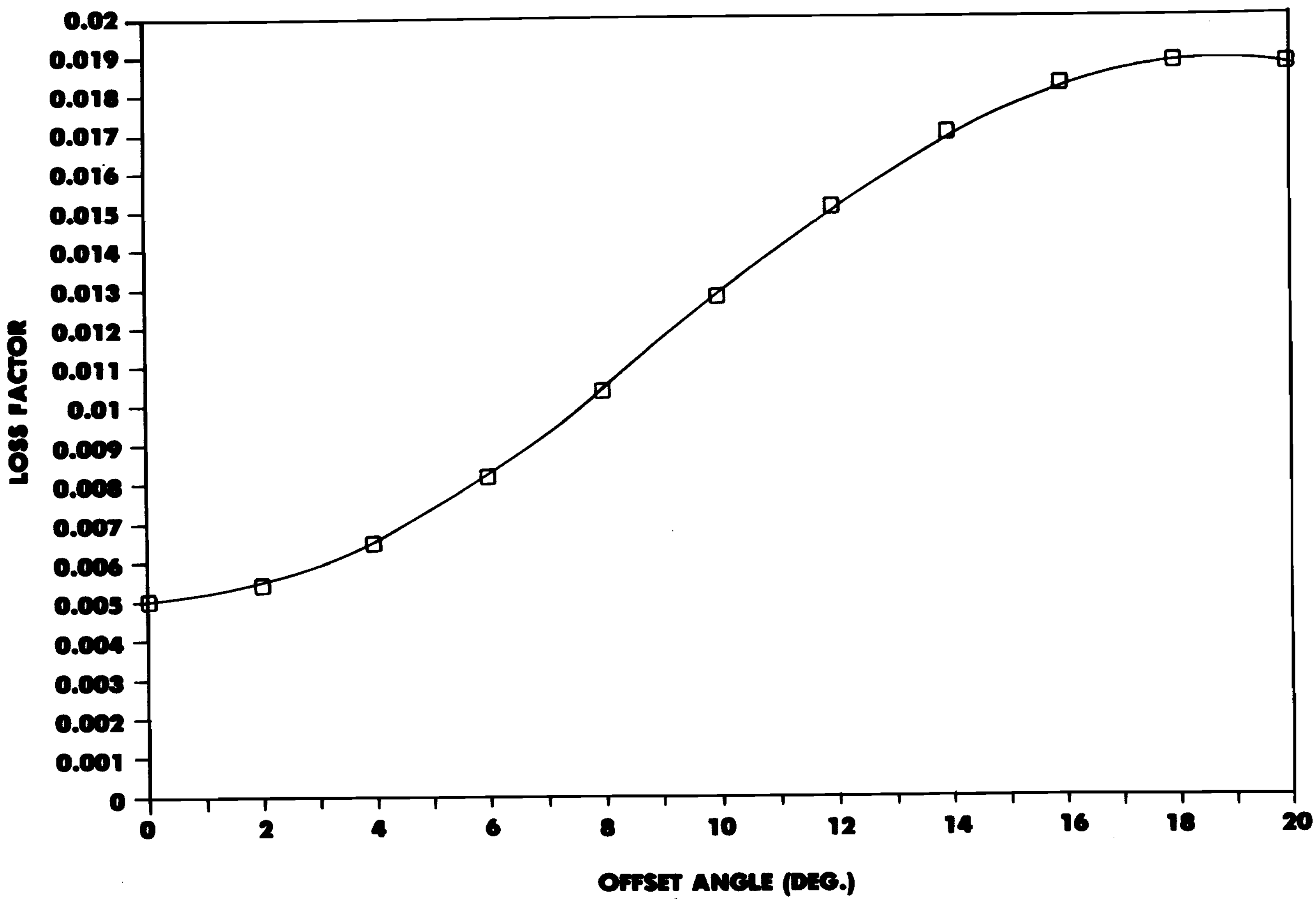


FIGURE 4 LOSS FACTOR VS. OFFSET ANGLE, COMPOSITE CONSTRUCTION #1

HCB-16

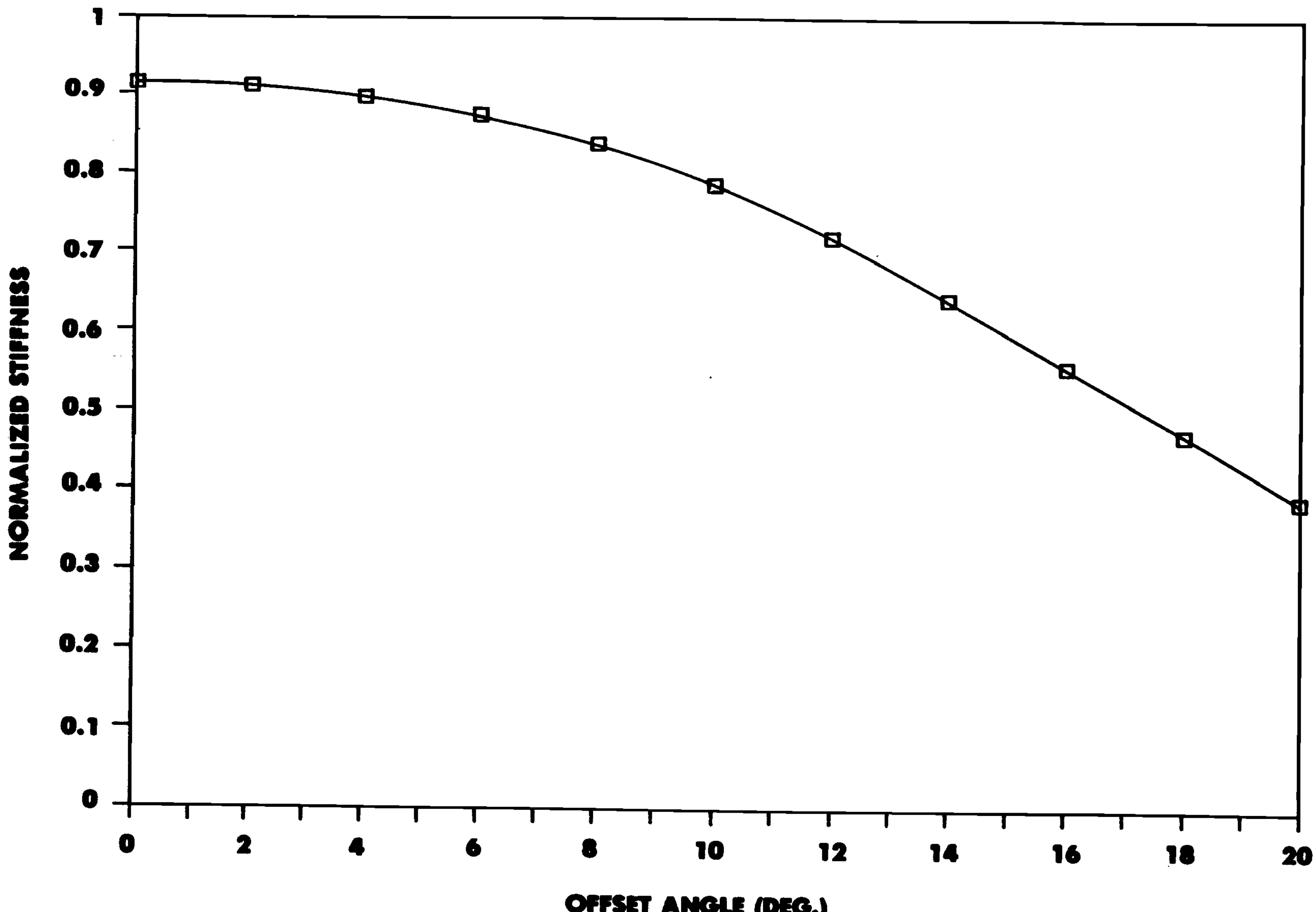


FIGURE 5 NORMALIZED STIFFNESS VS. OFFSET ANGLE, COMPOSITE CONSTRUCTION #1

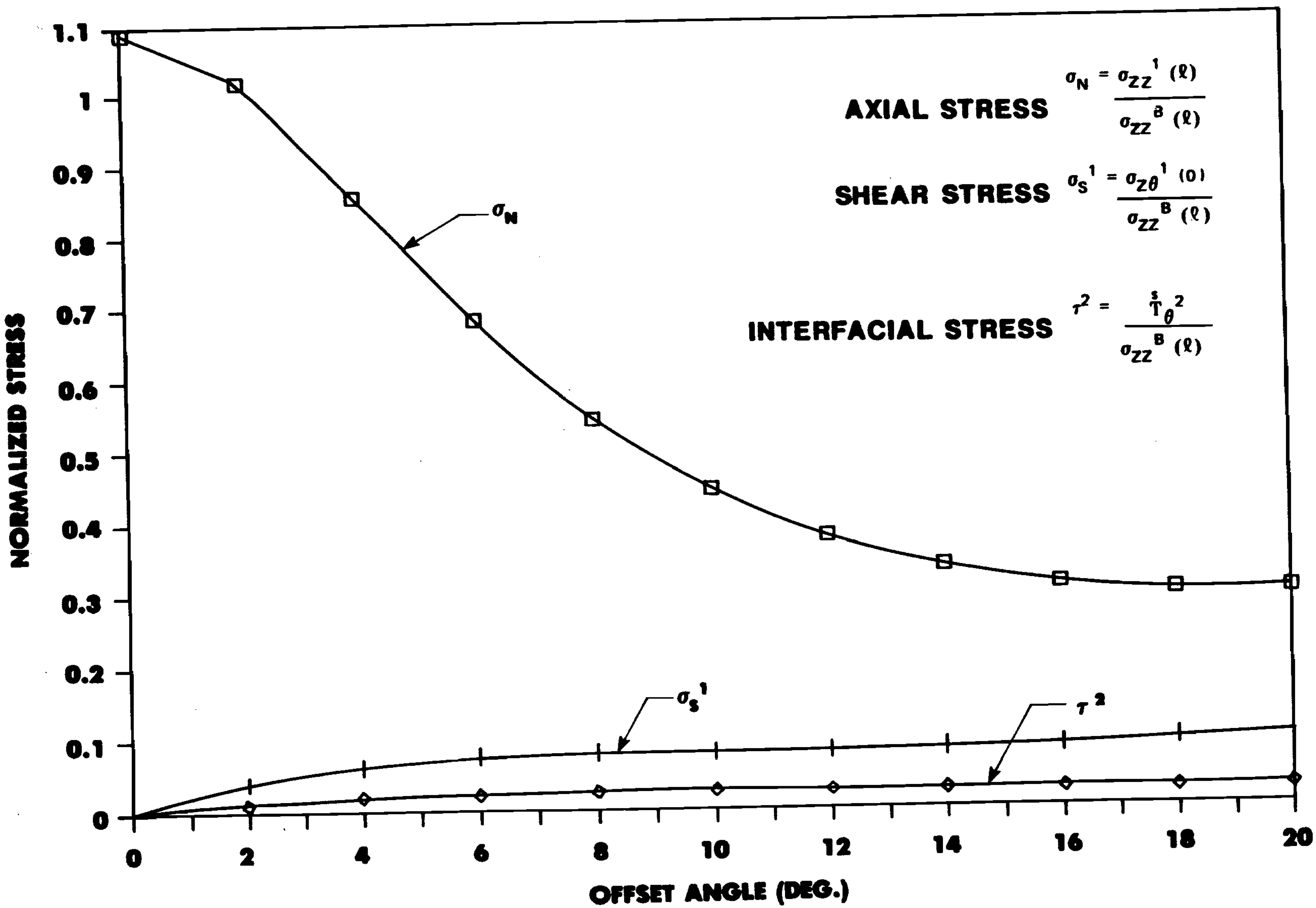


FIGURE 6 NORMALIZED RESONANT STRESSES VS. OFFSET ANGLE, COMPOSITE CONSTRUCTION #1

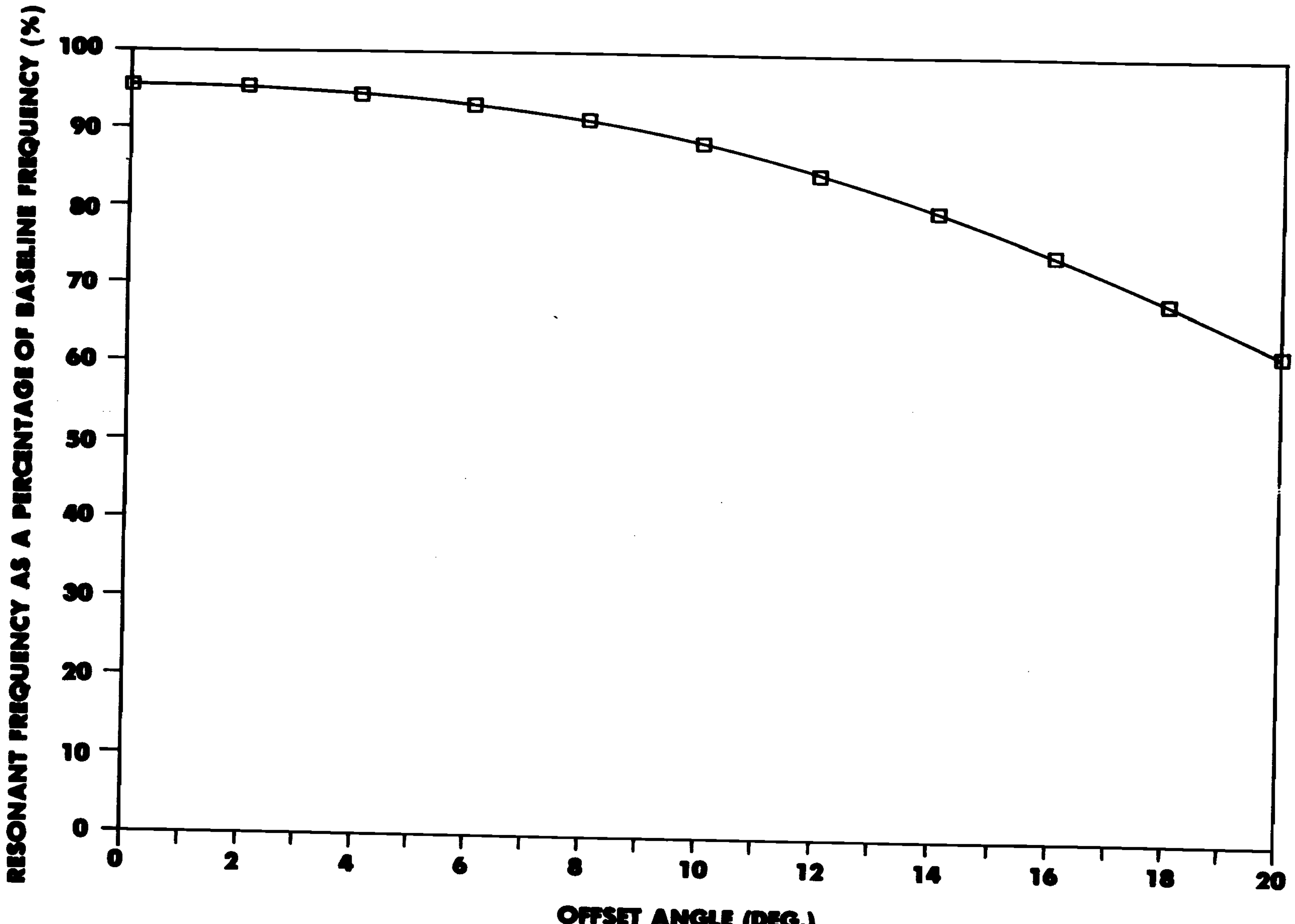


FIGURE 7 RESONANT FREQUENCY VS. OFFSET ANGLE, COMPOSITE CONSTRUCTION #1