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AN ITERATIVE METHOD FOR THE ANALYSIS OF LARGE STRUCTURAL SYSTEMS

V. B. VENKAYYA

FOREWORD

This report is the result of an in-house effort under Project 1467, "Structural Analysis Methods," Task 146705, "Computer Methods." The work was carried out in the Advanced Theory Group of the Theoretical Mechanics Branch, Structures Division, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio. Mr. V. B. Venkayya, FDTR, was project engineer.

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This technical report has been reviewed and is approved.

FRANCIS J. JANIK, Jr.
Chief, Theoretical Mechanics Branch
Air Force Flight Dynamics Laboratory

ABSTRACT

An iterative method to analyze large structural systems subjected to static and thermal loadings is developed. This method is essentially the displacement method of analysis in which the finite elements are grouped into convenient size substructures. The analysis proceeds with one substructure at a time, and a process of iteration establishes the continuity of the system. The speed of convergence depends on the nature of the stiffness matrices of the substructures. To illustrate the method, two frame structures are analyzed. One is an 1800 degree of freedom system and the other a smaller system. The purpose of the larger one is to estimate the computational time involved. The results of the smaller system (by iterative analysis) are compared with those obtained from direct analysis, and the results are nearly the same. The main advantages of the iteration method are that the entire computational scheme can be carried out without exceeding the core capacity of the computer, and the method is more economical with respect to computational time than the direct analysis.

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INTRODUCTION

In recent years there has been an upsurge in the development of methods to analyze large structures. Now it is possible with the aid of digital computers to analyze quite large structural systems. The displacement and force methods, in conjunction with the finite element approach, are firmly established as convenient methods of structural analysis, but as structural systems become larger and more complex, the discretization errors tend to be more prominent making the finite element analysis ineffective. This situation can be remedied, to some extent, by deriving an adequate element representation (Reference 1). At present, three types of elements are in use, each serving a different category of structures. A straight or curved element is suitable for frame structures, the plate elements are used for two-dimensional problems, and the tetrahedron element is proposed for the analysis of three-dimensional problems.

In any discrete representation of a continuous system the accuracy of the analysis improves as the representative mesh becomes finer. However, the finer mesh rapidly increases the number of degrees of freedom of the system. The large systems, handled as a whole, will strain the computational capabilities of even the largest computers. This is particularly so in the analysis of shell structures. It is possible to handle such large systems using auxiliary storage devices such as magnetic tape. Since the retrieval time when using such devices is excessive, the cost of computation may often be prohibitive unless storage devices are used with discretion. The proposed iterative method alleviates some of these problems by analyzing a part of the structure at a time. The method is simple, readily amenable for modification, and ideally suited for automated analysis. It can be used with equal facility for one-, two-, and three-dimensional problems if a suitable element stiffness matrix is derived. It is particularly convenient when the analysis has to be repeated a number of times with different loading conditions and minor configuration changes in some sections of the structure.

The structure is divided into a number of substructures of convenient size. The analysis proceeds, sequentially, from the first substructure to the last. The compatibility of displacements and equilibrium conditions between the substructures are established by a process of iteration.

There are a number of methods available in literature for the analysis of large structural systems (References 2 through 9). A brief review of some of these methods is presented in Section II and a detailed discussion of the existing methods is in Reference 2. The theoretical basis and details of the proposed iterative method are discussed in Sections III through V. To illustrate the effectiveness of the iterative method, a 5-bay, 100-story building frame and a 10-bay gable frame are analyzed. The results are presented in Section VI.

SECTION II

REVIEW OF THE EXISTING METHODS

1. Analysis by Substructures

In a finite element analysis, a structure is viewed as an assembly of simple structural elements joined together at nodal points or joints. A simple element is one whose force-displacement properties can be determined, at least to a degree of approximation, by the known methods of analysis. The element stiffness and the stiffness of the structure with respect to its joint displacements are related by Equation 1 (Reference 10):

$$K = \sum_{i=1}^{m} a_i^t \; k_i \; a_i \tag{1}$$

in which K and k_i are the stiffness matrices of the structure and the i^{th} element, respectively; a_i is the matrix that relates the i^{th} element and structure displacements; and m is the total number of elements in the structure. The relation between the joint forces and their displacements is given by:

$$R = Kr \tag{2}$$

R and r are matrices of joint forces and their displacements, respectively.

Since the size of the K matrix is directly proportional to the number of joints, the finer division of the structure results in a larger K matrix. In a substructure analysis (Reference 3), the simple elements are grouped into a fewer number of larger elements called substructures. The relation between the structure and substructure stiffnesses is given by Equation 1, except that k is now the stiffness of the substructure instead of the element, and now the size of K is smaller because of the fewer joints. With the aid of simple elements, the force-displacement properties of the substructure can be determined prior to the assembly of K. By using substructures, the sizes of the matrices to be inverted can be reduced to be as small as desired; however, this reduction in size is normally accompanied by an increase in arithmetic operations on smaller matrices. As the size of the structures becomes large, the computational errors (round-off errors) in these matrix operations tend to be cumulative.

2. Tridiagonalization and Recursion

In a finite element analysis it is necessary to number the system coordinates and the element coordinates. By a proper numbering of the system coordinates, one can obtain a banded stiffness matrix. Each row of this banded matrix usually consists of three elements or submatrices placed symmetrically close to the main diagonal. The remaining elements are all zero. A banded matrix can be inverted by the method of recursion or partitioning, in which the entire matrix need not be handled at one time. One limitation of the tridiagonalization method is that not all structures lend themselves for convenient numbering to obtain a banded matrix.

3. Iteration

The moment distribution method developed by Hardy Cross (Reference 11) was the most commonly used iterative method before the advent of computers. A similar method, but in a form more suitable for computer use, was originally developed by Kani (Reference 5) and subsequently improved by others (References 2, 6, and 7). Kani's method is presented in excellent detail in Reference 2. Reference 9 presents another iterative method that is convenient for the analysis of rectangular frames. The method presented in this report is also an iterative method, but it is more general in that it is not limited to frame structures.

SECTION III

SUBSTRUCTURES AND ITERATION

As in any discrete element analysis, the total structural system is divided into a finite number of simple elements, joined together at the nodal points. These elements are further grouped into substructures, each substructure consisting of an arbitrary number of simple elements (see Figure 1). For convenience, the following restrictions are imposed on substructuring:

1. A given sub-structure has common joints with at most two other substructures.

2. The substructures are numbered in sequence so that an i^{th} substructure shares joints with $(i-1)^{th}$ and $(i+1)^{th}$ substructures only (see Figure 1).

3. The total number of joints in the first substructure should be more than the sum of the joints shared with the second substructure and the joints on the natural boundary. In this report, the unyielding supports are referred to as the natural boundary.

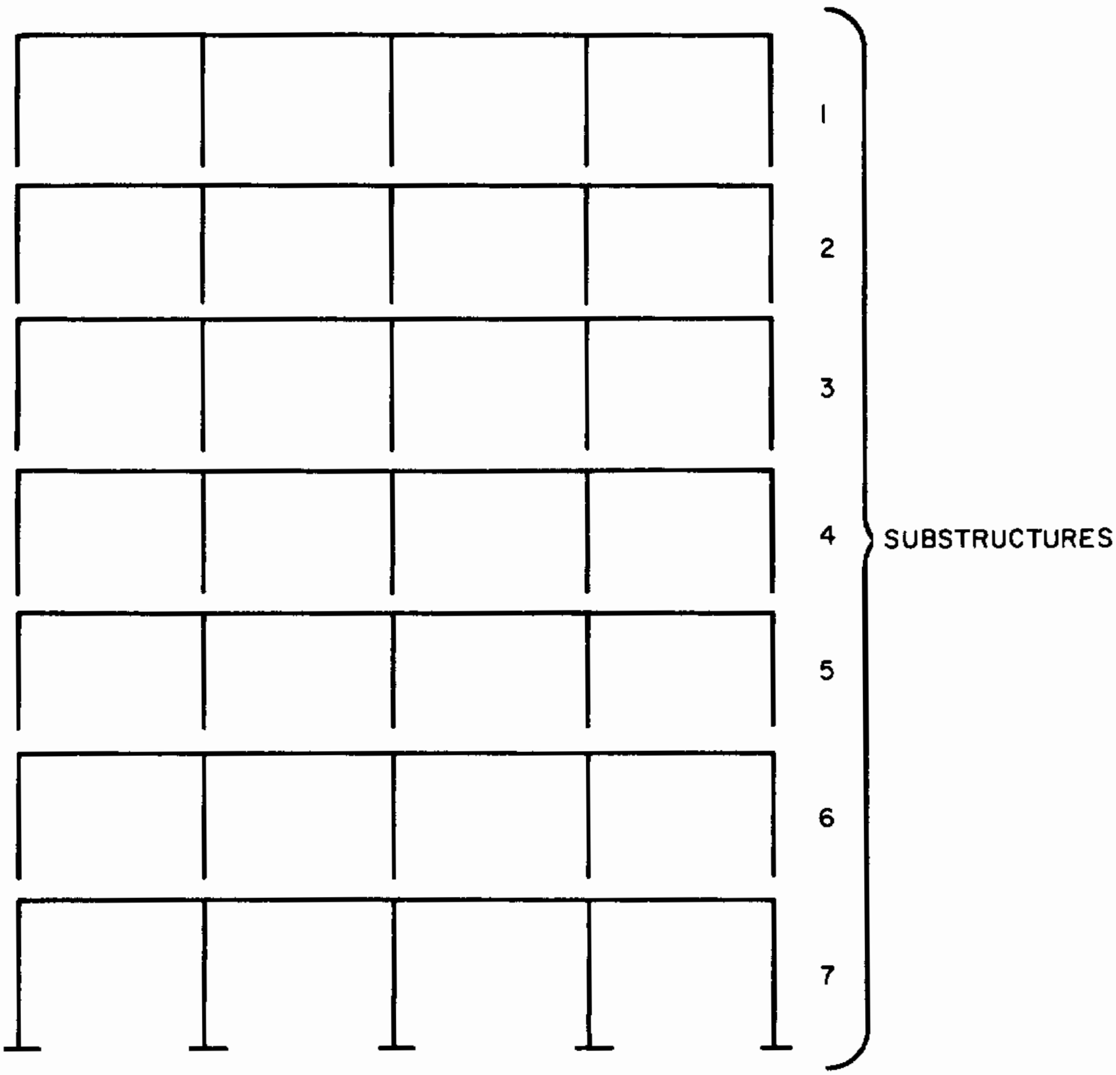
4. The last sub-structure should have an adequate number of joints on the natural boundary to prevent rigid body motion.

The first two restrictions are not essential to the method but they help in orderly presentation and simplification of programming.

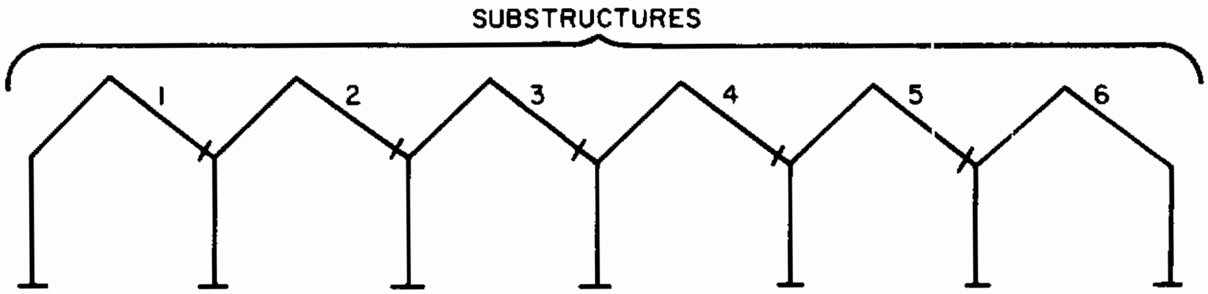
The force-displacement relations for the i^{th} substructure can be written in the form:

$$R^{(i)} = K^{(i)} r^{(i)} \tag{3}$$

$R^{(i)}$ and $r^{(i)}$ are the column matrices of joint forces and displacements, respectively, of the i^{th} substructure, and $K^{(i)}$ is its stiffness matrix.

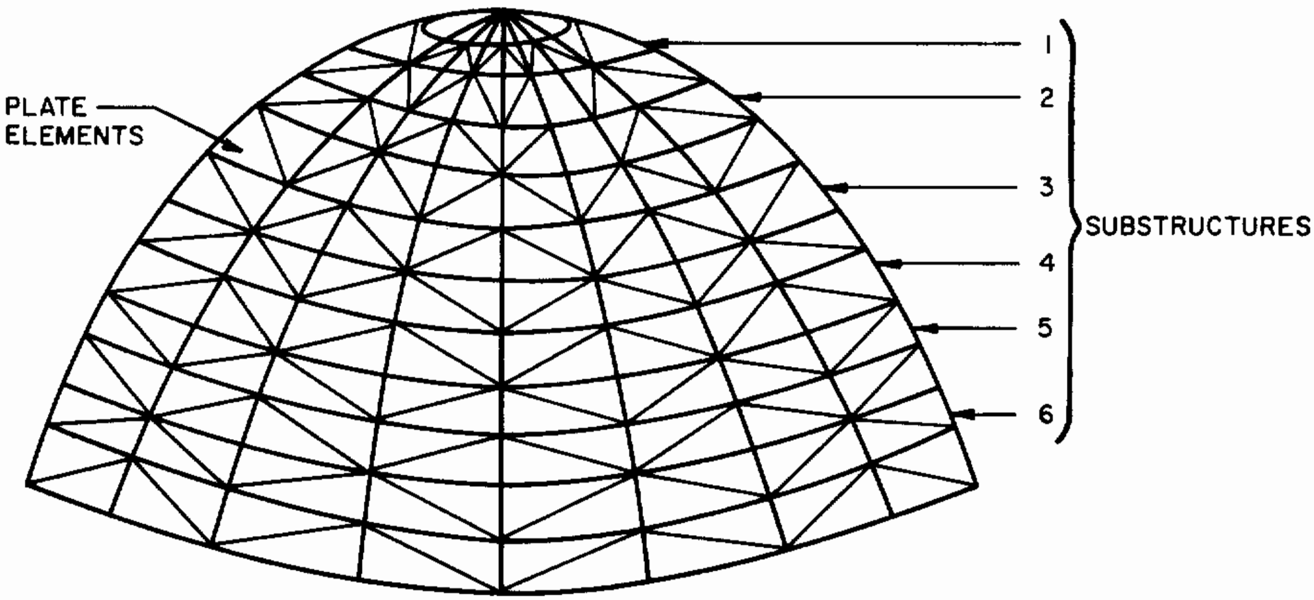


a. Multistory Frame

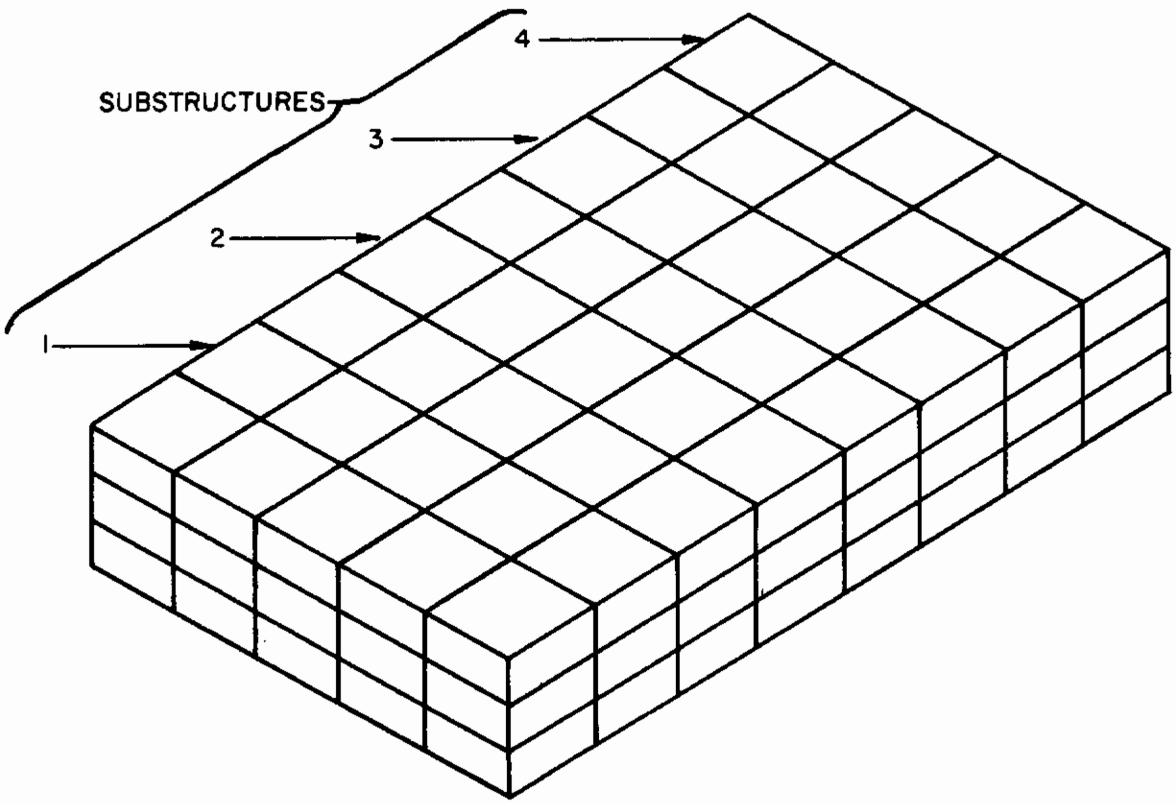


b. Multibay Gable Frame

Figure 1. Typical Substructuring Schemes



c. Spherical Dome



d. Thick Plate

Figure 1. (Contd)

The joints of a substructure are classified into four categories. For instance an i^{th} substructure may have:

1. Joints sharing with $(i-1)^{th}$ substructure
2. Joints belonging to i^{th} substructure alone
3. Joints sharing with $(i+1)^{th}$ substructure
4. Joints on the natural boundary

Since the displacements of the joints on the natural boundary are zero, they are not included in Equation 3.

Equation 3, partitioned to reflect this classification, takes the form:

$$\begin{bmatrix} (R)_1 \\ \hline (R)_2 \\ \hline (R)_3 \end{bmatrix}^{(i)} = \left[\begin{array}{c|c|c} (K)_{11} & (K)_{12} & (K)_{13} \\ \hline (K)_{21} & (K)_{22} & (K)_{23} \\ \hline (K)_{31} & (K)_{32} & (K)_{33} \end{array}\right]^{(i)} \begin{bmatrix} (r)_1 \\ \hline (r)_2 \\ \hline (r)_3 \end{bmatrix}^{(i)} \tag{4}$$

The superscript refers to the number of the substructure. The parentheses ( ) indicate that the quantities are submatrices. The subscript refers to the three-fold partitioning indicated. The submatrix $(r)_1^{(i)}$ is the displacement matrix of the joints common to $(i-1)^{th}$ and i^{th} substructures. Similarly $(r)_3^{(i)}$ is the displacement matrix of the joints common to i^{th} and $(i+1)^{th}$ substructures. $(r)_2^{(i)}$ is the displacement matrix of the joints belonging to i^{th} substructure alone. Equation 4 is rewritten as three separate equations:

$$(R)_1^{(i)} = (K)_{11}^{(i)}(r)_1^{(i)} + (K)_{12}^{(i)}(r)_2^{(i)} + (K)_{13}^{(i)}(r)_3^{(i)} \tag{5a}$$

$$(R)_2^{(i)} = (K)_{21}^{(i)}(r)_1^{(i)} + (K)_{22}^{(i)}(r)_2^{(i)} + (K)_{23}^{(i)}(r)_3^{(i)} \tag{5b}$$

$$(R)_3^{(i)} = (K)_{31}^{(i)}(r)_1^{(i)} + (K)_{32}^{(i)}(r)_2^{(i)} + (K)_{33}^{(i)}(r)_3^{(i)} \tag{5c}$$

The forces $(R)_1^{(i)}$, $(R)_2^{(i)}$, and $(R)_3^{(i)}$ are given the following interpretation: the $(R)_1^{(i)}$ forces are the sum of the external forces applied at the respective joints and the reactions $(R)_3^{(i-1)}$ transmitted from $(i-1)^{th}$ substructure. The submatrix $(R)_2^{(i)}$ is simply the applied forces at the respective joints. The forces $(R)_3^{(i)}$ are the reactions from the $(i+1)^{th}$ substructure to the i^{th} substructure. $(R)_3^{(i)}$ does not include the applied forces at the respective joints. With this interpretation, Equations 5a and 5b can be written in the form:

$$\left[\begin{array}{c|c} (K)_{11}^{(i)} & (K)_{12}^{(i)} \\ \hline (K)_{21}^{(i)} & (K)_{22}^{(i)} \end{array}\right] \left[\begin{array}{c} (r)_1^{(i)} \\ \hline (r)_2^{(i)} \end{array}\right] = \left[\begin{array}{c} (F)_1^{(i)} \\ \hline (F)_2^{(i)} \end{array}\right] - \left[\begin{array}{c} (R)_3^{(i-1)} \\ \hline 0 \end{array}\right] - \left[\begin{array}{c} (K)_{13}^{(i)} \\ \hline (K)_{23}^{(i)} \end{array}\right] (r)_3^{(i)} \tag{6}$$

$F_1^{(i)}$ and $F_2^{(i)}$ are external forces applied at the respective joints.

From Equation 5c, $(R)_3^{(i-1)}$ can be written as:

$$(R)_3^{(i-1)} = (K)_{31}^{(i-1)} (r)_1^{(i-1)} + (K)_{32}^{(i-1)} (r)_2^{(i-1)} + (K)_{33}^{(i-1)} (r)_3^{(i-1)} \tag{7}$$

The substitution of Equation 7 in Equation 6 yields:

$$r^{(i)} = C_1^{(i)} - C_2^{(i)} r^{(i-1)} - C_3^{(i)} r^{(i)} - C_4^{(i)} r^{(i+1)} \tag{8}$$

The notation in Equation 8 is as follows:

$$r^{(i)} = \left[\begin{array}{c} (r)_1^{(i)} \\ \hline (r)_2^{(i)} \end{array}\right]; \quad r^{(i-1)} = \left[\begin{array}{c} (r)_1^{(i-1)} \\ \hline (r)_2^{(i-1)} \end{array}\right]; \quad r^{(i+1)} = \left[\begin{array}{c} (r)_1^{(i+1)} \\ \hline (r)_2^{(i+1)} \end{array}\right]$$

and

$$C_1^{(i)} = \left[\begin{array}{c|c} (K)_{11}^{(i)} & (K)_{12}^{(i)} \\ \hline (K)_{21}^{(i)} & (K)_{22}^{(i)} \end{array}\right]^{-1} \left[\begin{array}{c} (F)_1^{(i)} \\ \hline (F)_2^{(i)} \end{array}\right]; \quad C_2^{(i)} = \left[\begin{array}{c|c} (K)_{11}^{(i)} & (K)_{12}^{(i)} \\ \hline (K)_{21}^{(i)} & (K)_{22}^{(i)} \end{array}\right]^{-1} \left[\begin{array}{c|c} (K)_{31}^{(i-1)} & (K)_{32}^{(i-1)} \\ \hline 0 & 0 \end{array}\right]$$

$$C_3^{(i)} = \left[\begin{array}{c|c} (K)_{11}^{(i)} & (K)_{12}^{(i)} \\ \hline (K)_{21}^{(i)} & (K)_{22}^{(i)} \end{array}\right]^{-1} \left[\begin{array}{c|c} (K)_{33}^{(i-1)} & 0 \\ \hline 0 & 0 \end{array}\right]; \quad C_4^{(i)} = \left[\begin{array}{c|c} (K)_{11}^{(i)} & (K)_{12}^{(i)} \\ \hline (K)_{21}^{(i)} & (K)_{22}^{(i)} \end{array}\right]^{-1} \left[\begin{array}{c|c} (K)_{13}^{(i)} & 0 \\ \hline (K)_{23}^{(i)} & 0 \end{array}\right]$$

In deriving Equation 8 from Equations 6 and 7, the displacement matrices $(r)_3^{(i-1)}$ and $(r)_3^{(i)}$ are replaced by $(r)_1^{(i)}$ and $(r)_1^{(i+1)}$, respectively. This is acceptable because they represent the common joint displacements. Also some of the matrices are expanded by adding rows and columns of zeros without altering the meaning of the equations. For a given structure and loading, the matrices $C_1^{(i)}$, $C_2^{(i)}$, $C_3^{(i)}$, and $C_4^{(i)}$ are constant. Actually, the latter three matrices do not depend on the loading.

Equation 8 forms the basis for writing the recurrence formula for iteration. According to this equation, to calculate the i^{th} substructure displacements, the displacements of the substructure before, the substructure after, and its own displacements are required. Since it is assumed that in each cycle, calculations start from the first substructure and proceed to the last, $(i-1)^{th}$ substructure displacements are known at the time the i^{th} substructure displacements are calculated. The displacements $r^{(i)}$ and $r^{(i+1)}$, on the right side, are obtained from the previous cycle. Then the recurrence formula for iteration takes the form:

$$r_{k+1}^{(i)} = C_1^{(i)} - C_2^{(i)} r_{k+1}^{(i-1)} - C_3^{(i)} r_k^{(i)} - C_4^{(i)} r_k^{(i+1)} \qquad (9)$$

The subscripts on the displacement matrices refer to the cycle of iteration, i.e., to calculate $(k+1)^{th}$ cycle values of $r^{(i)}$, the $(k+1)^{th}$ cycle values of $r^{(i-1)}$ and the k^{th} cycle values of $r^{(i)}$ and $r^{(i+1)}$ are used.

Equation 9 forms the mathematical basis for iteration. For the first substructure, the second term on the right is always zero. Similarly the fourth term does not exist for the last substructure. The actual method can be outlined by referring to Equation 9 or Equations 6 and 7 which are all derived from Equation 5. The method using Equation 9 has the following steps:

1. The iteration starts with the assumption of zero displacements for all joints. This starting cycle may be considered as the zero cycle.

2. The first cycle displacements, $r_1^{(1)}$, $r_1^{(2)}$- - - - - - - - - - - - -$r_1^{(n)}$, are calculated, successively, with the aid of Equation 9. Since the displacements of the previous cycle are all zeros, the last two terms of Equation 9 are zeros for all substructures in this cycle. The old values of the displacements are replaced by the displacements calculated in the present cycle.

3. The displacements of the subsequent cycles are calculated by repeating Step 2.

The iteration may be continued until the absolute difference between the displacements calculated in two successive cycles is sufficiently small. Convergence of the method depends on certain conditions discussed in Section V.

Equations 6 and 7 can also be used as the iterative approach to computer programming. The iteration starts with the first substructure:

1. Assuming the displacements of the common joints between first and second substructures to be zero, the remaining displacements of the first substructure are calculated by Equation 6.

2. The reactions on the second substructure are calculated by substituting the displacements determined in the previous step in Equation 7.

3. The reactions and external forces on the second substructure are combined.

4. Steps 1 through 3 are repeated for each substructure, always replacing the old displacements by the newly calculated displacements. One cycle of iteration is completed when the procedure is repeated on all the substructures.

5. For subsequent cycles, Steps 1 through 4 are repeated except the displacements calculated in the previous cycle are used for common joint displacements instead of zeros.

Iteration is continued until the difference between the displacements calculated in two successive cycles is sufficiently small.

The author found the latter approach to be more convenient for computer programming than direct use of Equation 9. In Section IV, the method is presented in detail for computer use. The presentation includes determination of element forces from the joint displacements obtained by iteration.

SECTION IV

EXPLANATION OF THE PROPOSED METHOD

The method is briefly outlined in Section III and is further explained here in a suitable form for computer use. The flow diagram shown in Figure 2 summarizes the basic steps involved in iteration and each step is explained in the following paragraphs:

a. Input

Input information is similar to any discrete element analysis but includes information pertaining to substructuring.

(1) Structure Information

Give the total number of elements, joints, substructures, natural boundaries, etc.

(2) Substructure Information

List the number of elements and joints in each substructure, the joints shared with the neighboring substructures, and the joints on the natural boundary. In addition, list the first and last elements and joints for each substructure.

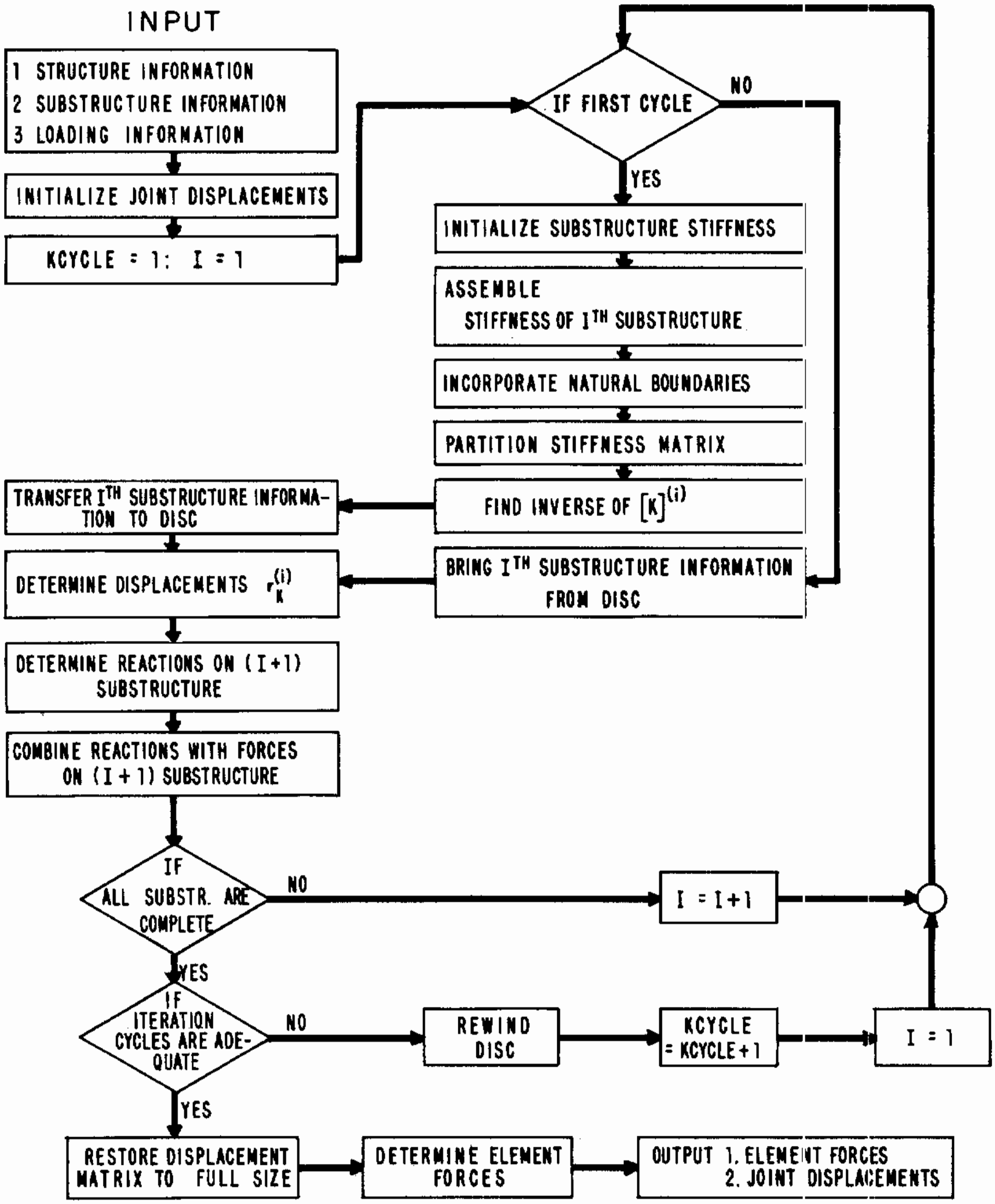


Figure 2. Flow Diagram for Iteration on Substructures

(3) Joint Coordinates

Give the coordinates of each joint (two in case of plane frames and three otherwise) with reference to a convenient set of coordinate axis.

(4) Element Information

For frame members give the relative values of EI and AE and for plate elements, give E, ν , and thickness. E and ν are elastic constants and I and A are the moment of inertia and cross-sectional area, respectively.

(5) Loading Information

Give joint forces if they can be determined directly or make provision to determine the joint forces. A subroutine can be written to determine the joint forces.

b. Initialize Joint Displacements

For the starting cycle, assume the displacements of all joints to be zero. This cycle may be regarded as zero cycle.

c. Initialize Substructure Stiffness Matrix

Allot a block of memory space, large enough to accommodate the stiffness matrix of the largest (in terms of number of joints) substructure. Initialize this memory space to zeros before starting the assembly of the stiffness matrix.

d. Assemble Substructure Stiffness Matrix

Assemble the substructure stiffness by adding successively the stiffness of each element till all the elements of that substructure are completed. This step constitutes the important part of finite element analysis and is explained in two parts.

(1) Element Stiffness Matrix

First, determines the element stiffness matrix with respect to its own coordinate system. The procedure for determining stiffness of standard elements such as frame members and triangular plate elements can be found in many references on matrix structural analysis (References 2, 10 and 12). As an example, the stiffness matrix of a plane frame member with constant section is given in Equation 10. The numbering of the element coordinate system is shown in Figure 3.

$$[k] = \begin{bmatrix} AE/L & & & & & \\ 0 & 12EI/L^3 & & \text{Symmetric} & & \\ 0 & 6EI/L^2 & 4EI/L & & & \\ -AE/L & 0 & 0 & AE/L & & \\ 0 & -12EI/L^3 & -6EI/L^2 & 0 & 12EI/L^3 & \\ 0 & 6EI/L^2 & 2EI/L & 0 & -6EI/L^2 & 4EI/L \end{bmatrix} \quad (10)$$

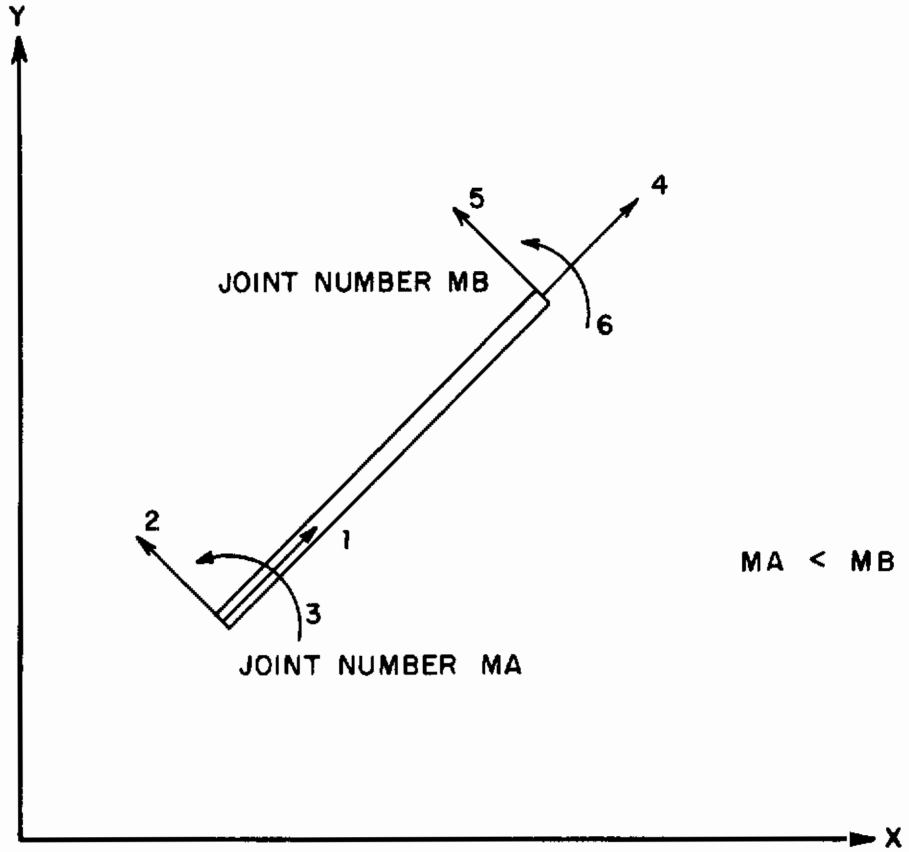


Figure 3. Direction of Positive Member Forces

(2) Transformation and Addition

Transform the element stiffness matrix from its own coordinate system to a general coordinate system before adding to the structure stiffness. Both these operations are accomplished by addition of matrix ψ_j to the structure stiffness. ψ_j is given by:

$$\Psi_j = a_j^t \, k_j \, a_j \tag{11}$$

k_j is the j^{th} element stiffness matrix with respect to its own coordinate system. The displacements of the j^{th} element and the structure are related by the a_j matrix. The a_j matrix is of the size pxq, where p and q are the dimensions of element and structure stiffness matrices, respectively. Since q is usually large, a_j is also large but very sparsely populated. To avoid storing the entire a_j matrix, the transformation and addition are carried out separately in the following manner:

First, the element stiffness matrix is transformed by the relation

$$\Psi_j = \mu_j^t \, k_j \, \mu_j \tag{12}$$

Now, ψ_j is the transformed element stiffness matrix and is the same size as the original matrix. The matrix μ_j for a plane frame member is given by:

$$\mu_j = \left[\begin{array}{c|c} \lambda_j & 0 \\ \hline 0 & \lambda_j \end{array}\right] \tag{13}$$

λ_j is the direction cosine matrix of the element coordinate system with respect to structure coordinates. For a plane frame member, λ_j is given by:

$$\lambda_j = \begin{bmatrix} \cos\theta_j & \sin\theta_j & 0 \\ -\sin\theta_j & \cos\theta_j & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{14}$$

θ_j is the inclination of the member measured from the x-axis of the system coordinates.

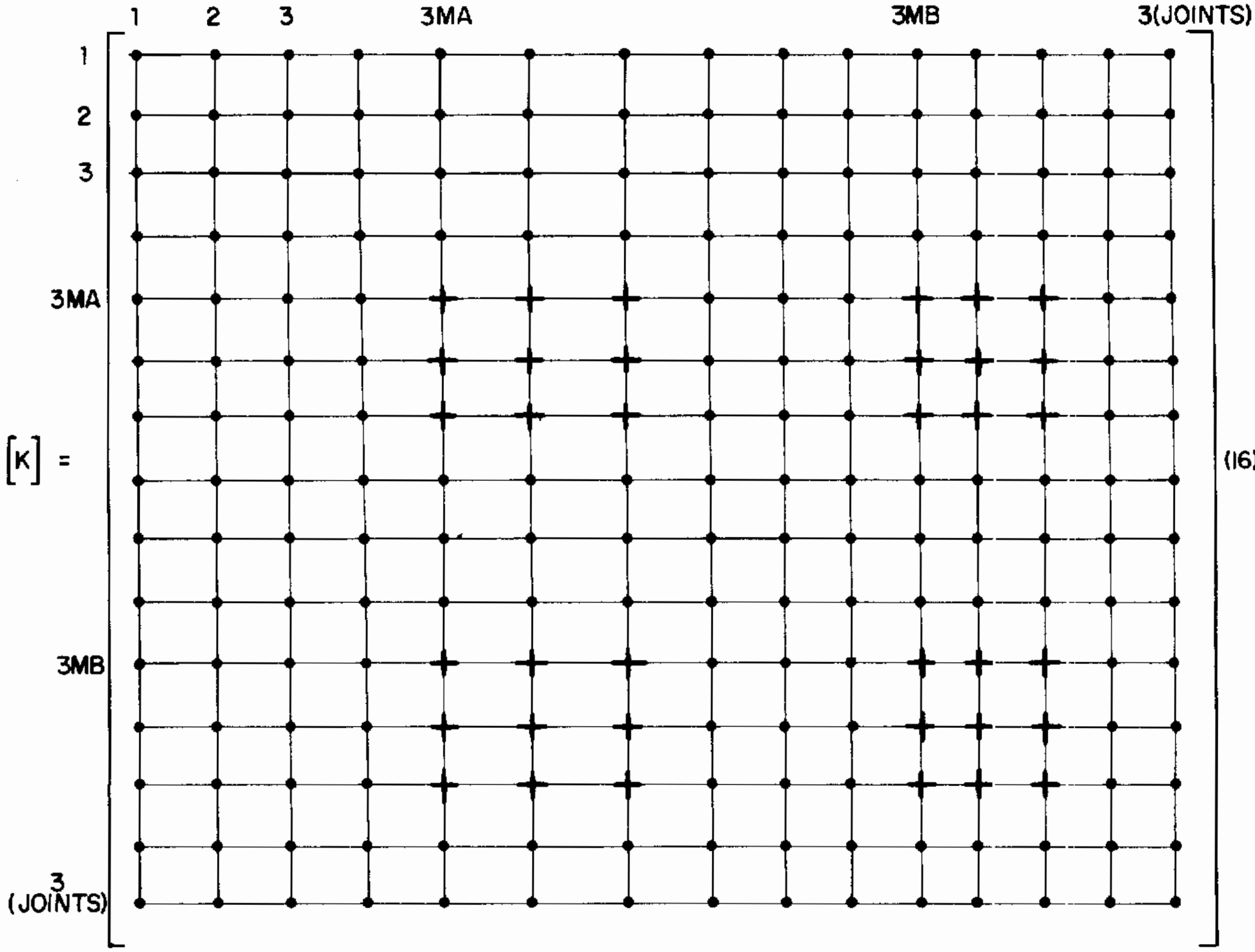
The transformed matrix ψ_j (Equation 11) is partitioned into four quadrants (plane frame) as shown in Equation 15.

$$\Psi_j = \left[\begin{array}{c|c} \Psi_{11} & \Psi_{12} \\ \hline \Psi_{21} & \Psi_{22} \end{array}\right]_j \tag{15}$$

If the plane frame member connects the joints MA and MB (Figure 3), the first quadrant elements of ψ_j are added to the structure stiffness elements, starting with address (3MA, 3 MA). The second, third, and fourth quadrants are added to the elements starting with the addresses (3MA, 3MB), (3MB, 3MA), and (3MB, 3MB), respectively. Equation 16 shows the location of ψ_j in relation to the general stiffness matrix. The cross marks (+) indicate the locations to which ψ_j is added. [K] is the structure or substructure stiffness matrix being assembled.

e. Incorporate Natural Boundaries

Introduce the natural boundaries by eliminating the rows and columns of the stiffness matrix corresponding to the displacements on the natural boundary. The resulting stiffness matrix is a condensed matrix in relation to the one assembled in the previous step. Another procedure to introduce natural boundaries is to substitute zeros in place of the off diagonal elements of the rows and columns corresponding to the boundaries, and ones for the diagonal elements. This way, the size of the matrix remains the same and time saved in condensing the matrix is expended in operating with larger matrices.



(16)

f. Determine Displacements $r_k^{(i)}$

$r_k^{(i)}$ is the displacement matrix of the i^{th} substructure in the k^{th} cycle of iteration. Determine it using Equation 6. $r_k^{(i)}$ does not include the displacements of the common joints between i and (i+1) substructures. However, to determine $r_k^{(i)}$, the displacements of the common joints are necessary (see Equation 6). The common joint displacements are calculated, in the $(k-1)^{th}$ cycle, as part of $(i+1)^{th}$ substructure displacements, and they are used to determine $r_k^{(i)}$. It is important to replace $r_{k-1}^{(i)}$ by $r_k^{(i)}$ so that for the next cycle the latest values are available.

g. Determine the Reactions on $(i+1)^{th}$ Substructure and Combine Them With the Applied Forces

Substitute the displacements $r_k^{(i)}$ and the common joint displacements from the previous cycle in Equation 5c to obtain the reactions on i^{th} substructure from $(i+1)^{th}$ substructure. The same reactions, but in opposite direction, are transmitted from i to (i+1) substructure. These reactions are combined with the external forces applied on (i+1) substructure.

Steps c through g are repeated on all the substructures. At the end of last substructure one cycle of iteration is completed. The procedure is repeated until the difference between the displacement matrices calculated in two successive cycles is relatively small.

The criteria for convergence and some modifications to the iteration process are discussed in Section V.

At the end of iteration only the structure displacements are available. Often it is necessary to determine element forces in addition to the displacements. The next two steps indicate the procedure for calculating element forces.

h. Restore the Displacement Matrix to Full Size

The displacement matrix determined by iteration is a condensed matrix because it does not include the joints on the natural boundary. These joints have zero displacements. Expand the condensed displacement matrix by introducing zeros in place of natural boundary displacements. The enlarged displacement matrix is used to determine element forces in the next step.

i. Element Forces

Determine the i^{th} element forces by substitution of the displacement matrix in the equation:

$$S_i = k_i \, a_i \, r \tag{17}$$

The matrices k_i and a_i are the same as defined in Section II. S_i is the matrix of element end forces and r is the full displacement matrix. Storage of a large a_i matrix can be avoided by using Equation 18 in place of Equation 17:

$$S_i = k_i \, \mu_i \, r' \tag{18}$$

μ_i is previously defined in this Section. The matrix r' consists of only the displacements of the joints connected by the element. Equation 18 gives the element forces due to joint displacements only. If there are forces acting directly on the element, S_i is given by:

$$S_i = k_i \, \mu_i \, r' - S_i^o \tag{18a}$$

S_i^o is the matrix of fixed end forces directly acting on the element.

j. Output

The type of output depends on the problem and, in general, includes the element end forces and/or stresses, the joint displacements, and the applied forces.

SECTION V

CRITERIA FOR CONVERGENCE

In Sections III and IV it was tacitly assumed that iteration using Equation 9 (or 6 and 7) converges. However, convergence depends on the nature of matrices $C_1^{(i)}$, $C_2^{(i)}$, $C_3^{(i)}$, and $C_4^{(i)}$. The matrix $C_1^{(i)}$ is a function of the properties of the structure and the loading, while $C_2^{(i)}$, $C_3^{(i)}$, and $C_4^{(i)}$ depend only on the properties of the structure. Each of these matrices is a product of two matrices. The first matrix, in all four cases, is the inverse of the stiffness matrix with respect to the displacements $r^{(i)}$ (see Equation 6). Since the elements of this inverse contain the term "EI" in the denominator, they are small compared to 1. For stable elastic structures, it is reasonable to assume that the elements of $C_1^{(i)}$, $C_2^{(i)}$, $C_3^{(i)}$, and $C_4^{(i)}$ are small in magnitude, compared to unity. These comments on the nature of the elements of "C" matrices help in arriving at certain conclusions regarding the nature of convergence.

In this discussion, the elements of a given structure are grouped into three substructures. This restriction is necessary only to simplify the discussion. The conclusions reached with this assumption are applicable when there are more than three substructures.

The k^{th} cycle displacement matrix of the i^{th} substructure, according to Equation 9, is as follows:

$$r_k^{(i)} = C_1^{(i)} - C_2^{(i)} r_k^{(i-1)} - C_3^{(i)} r_{k-1}^{(i)} - C_4^{(i)} r_{k-1}^{(i+1)} \tag{9}$$

Applying this relation to the first substructure, the k^{th} cycle displacement matrix is written as:

$$r_k^{(1)} = r_1^{(1)} - C_4^{(1)} r_{k-1}^{(2)} \tag{9a}$$

The constants C_2 and C_3 are zero for the first substructure. For the second substructure the displacement matrix takes the form:

$$\begin{aligned} r_k^{(2)} = {} & \left[I - x^{(2)} + x^{(2)^2} - x^{(2)^3} + ---- (-1)^{k-1} x^{(2)^{k-1}} \right] r_1^{(2)} \\ & -(-1)^{k-2} x^{(2)^{k-2}} C_4^{(2)} r_1^{(3)} - (-1)^{k-3} x^{(2)^{k-3}} C_4^{(2)} r_2^{(3)} ---- - C_4^{(2)} r_{k-1}^{(3)} \end{aligned} \tag{9b}$$

where

$$x^{(2)} = C_3^{(2)} - C_2^{(2)} C_4^{(1)}$$

In the expression for $r_k^{(2)}$, only $r_1^{(2)}$ and the displacement matrices of the third substructure are retained. If the elements of "C" matrices are small compared to unity then the elements of their products would be even smaller. In such a case, it is reasonable to neglect the terms containing higher products. For convenience the expression for $r_k^{(2)}$ is rewritten in the form:

$$r_k^{(2)} = \left[I - x^{(2)} + x^{(2)^2} - x^{(2)^3} + h(x^{(2)^3})\right] r_1^{(2)} + h\,(x^{(2)^3}) \left[r_1^{(3)} + r_2^{(3)} ---- + r_{k-4}^{(3)} \right] + x^{(2)^2} C_4^{(2)}\, r_{k-3}^{(3)} + x^{(2)}\, C_4^{(2)}\, r_{k-2}^{(3)} + C_4^{(2)}\, r_{k-1}^{(3)} \quad (9c)$$

where $h(x^{(2)^3})$ = products of the order greater than $x^{(2)^3}$. With a similar notation the third substructure (last) displacement matrix is written as:

$$r_k^{(3)} = \left[I - x^{(3)} + x^{(3)^2} - x^{(3)^3} - C_2^{(3)} x^{(2)} C_4^{(2)} + h\,(x^{(3)^3})\right] r_1^{(3)} + \left[C_2^{(3)} (x^{(2)} - x^{(2)^2}) - x^{(3)} C_2^{(3)} x^{(2)} + h\,(x^{(3)^3}) \right] r_1^{(2)} \quad (9d)$$

where

$$x^{(3)} = C_3^{(3)} - C_2^{(3)} C_4^{(2)}$$

If the products of the order $x^{(3)^3}$ and higher are neglected, the displacement matrices $r_4^{(3)}$ and $r_3^{(3)}$ would be the same, to the assumed order of approximation. Similarly, if the products of the order $x^{(2)^3}$ and higher are neglected, it can be shown, from Equation 9c, that $r_6^{(2)}$ and $r_5^{(2)}$ would be the same. If the same reasoning is extended to the first substructure, the displacements $r_7^{(1)}$ and $r_6^{(1)}$ are approximately equal. The conclusion from this discussion is that if three cycles are necessary to obtain the approximate displacement matrix for the last substructure then at least n + 3 cycles are necessary to obtain similar approximation for the first substructure displacements (n is the number of substructures).

As an example to illustrate the mode of convergence, the displacements of the last substructure are plotted against the number of cycles in Figure 4. The iteration approaches the true value quite rapidly in the beginning (within 10% of the true values in less than four cycles), then the rate of approach slows down considerably. It is evident from Figure 4, that the iterated values do not even remain on one side of the true value. Instead, they oscillate about the true values with decreasing amplitude. Figure 4 is a qualitative representation of the iterative process; the details vary with the structure.

It is evident, therefore, that convergence starts from the last substructure and proceeds to the first sequentially. To take advantage of this property, a modification is proposed to the iteration scheme. In the original scheme each cycle of iteration starts from the first substructure and proceeds to the last successively. In the modified scheme, however, the cycles of iteration are classified as full cycles and truncated cycles, i.e., (a) a full cycle of iteration starts from the first substructure and proceeds to the last as in the original scheme, and (b) the truncated cycle consists of "n" subcycles. The first subcycle is a full cycle. The second subcycle ends at (n-1)th substructure, the third subcycle stops at (n-2)th substructure,

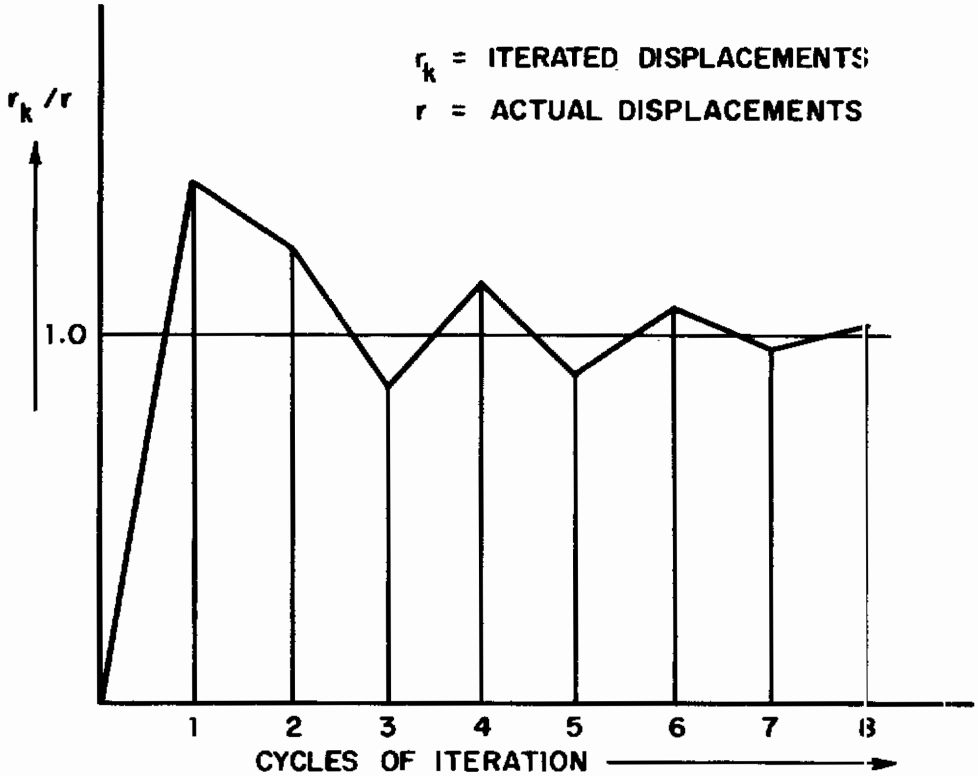


Figure 4. Nature of Convergence of Last Substructure Displacements

and so on until the n^{th} subcycle ends at first substructure. After three or four full cycles, the displacements of the last substructure have attained a measure of stability. Now a truncated cycle is introduced. At the end of the truncated cycle, the displacements of all the substructures are within 10 to 15% of the actual values. Then, the iteration is continued with full cycles. The latter step smooths out the iteration process after the truncated cycle.

In the modified scheme, the displacement matrix obtained in every cycle after truncated cycle is a reasonable approximation. To obtain a similar approximation, at least 2n cycles are necessary in the original scheme. The author adopted the modified scheme in which the first three are full cycles, the fourth is truncated, and the subsequent ones are full cycles.

SECTION VI

RESULTS AND CONCLUSIONS

Two frame structures were selected to illustrate the iterative method. The first structure is a 5-bay, 100-story frame shown in Figure 5 and analyzed for the following three different loading conditions:

1. Lateral loads (in the direction of positive x-axis) on the joints 1, 7, 13, 20, ... 595.

2. Moments, on all the joints, in the clockwise direction.

3. Combination of the first two loads.

Table Ia gives the displacements of the joints on floors 100, 50, 10, and 1. For each joint there are three lines of displacements resulting from the three loading conditions. Table Ib gives the internal forces of the members on and under floors 100, 50, 10, and 1. Once again each member has three lines of internal forces resulting from three loading conditions. The total computational time for the three loading conditions is approximately 22 minutes on IBM 7094. During this time, in addition to the displacements of the 600 joints, internal forces of the 1100 members are calculated. The estimated time for each additional loading condition is less than a minute.

The second structure is a 10-bay gable frame shown in Figure 6 and also analyzed for the three different loading conditions defined in Figure 6. Tables IIa and IIb give the joint displacements and the member forces, respectively, for this frame. The computational time, excluding time of compilation, is about 30 seconds. The increase in time for an additional loading condition is almost too insignificant to report. For the purpose of comparison, the results of direct analysis for this frame are presented in Tables IIIa and IIIb. It is evident from these tables (II and III) that results of iteration are not too different from those obtained from direct analysis. The computational time for direct analysis is about 22 seconds, which is less time than for iteration. However, for larger structures this difference in computational time becomes increasingly in favor of iteration. The following two examples attest to this fact. The computational times for a 20-bay gable frame (analyzed for 3 loading conditions) are 1 minute and 58 seconds and 2 minutes and 10 seconds, respectively, for iteration and direct analysis. A more significant difference is shown in the analysis of a 5-bay, 10-story frame (180 degrees of freedom). The direct analysis of this frame for three loading conditions requires 5 minutes and 30 seconds. The same frame is analyzed by iteration in 1 minute 10 seconds.

In developing utilitarian programs, the computational times can be reduced further with increased effort in programming. Stiffness matrices are, in general, sparsely populated and the computational time can be reduced by avoiding operations with zero elements. If two or more substructures are similar, the time can be reduced by avoiding repetition in assembly of matrices wherever possible. The author has not taken advantage of these properties in analyzing the two frames. A more significant change in computational time can be effected by balancing the size and number of substructures. The time for inversion of matrices increases rapidly with the size of substructures while the number of cycles of iteration are proportional to the number of substructures. By relaxing the first two restrictions on substructuring (see Section III), the substructures can be reduced to the convenient size. Now $r^{(i-1)}$ may be interpreted as the displacement matrix of the substructures

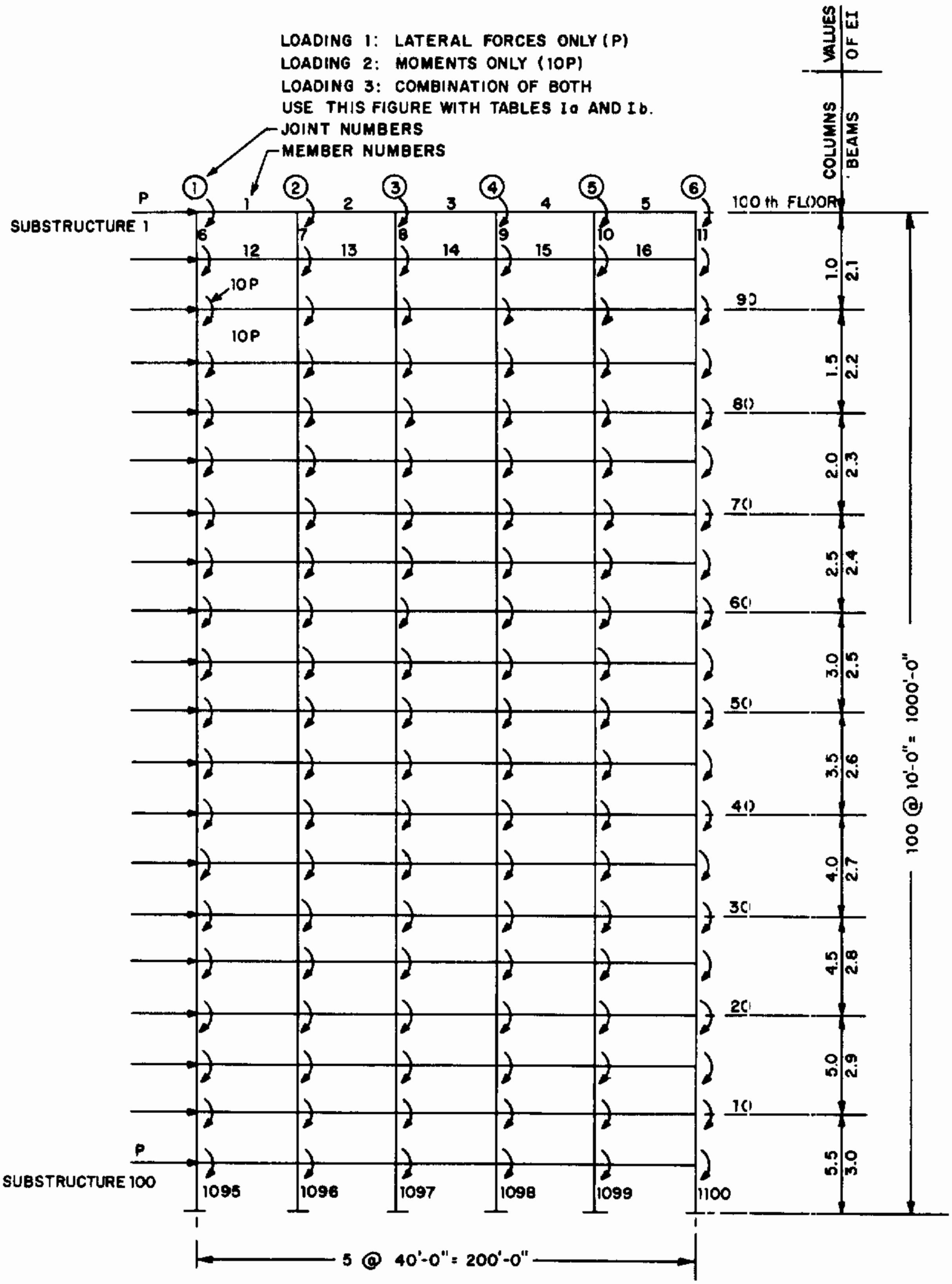


Figure 5. Five-Bay, 100-Story Frame

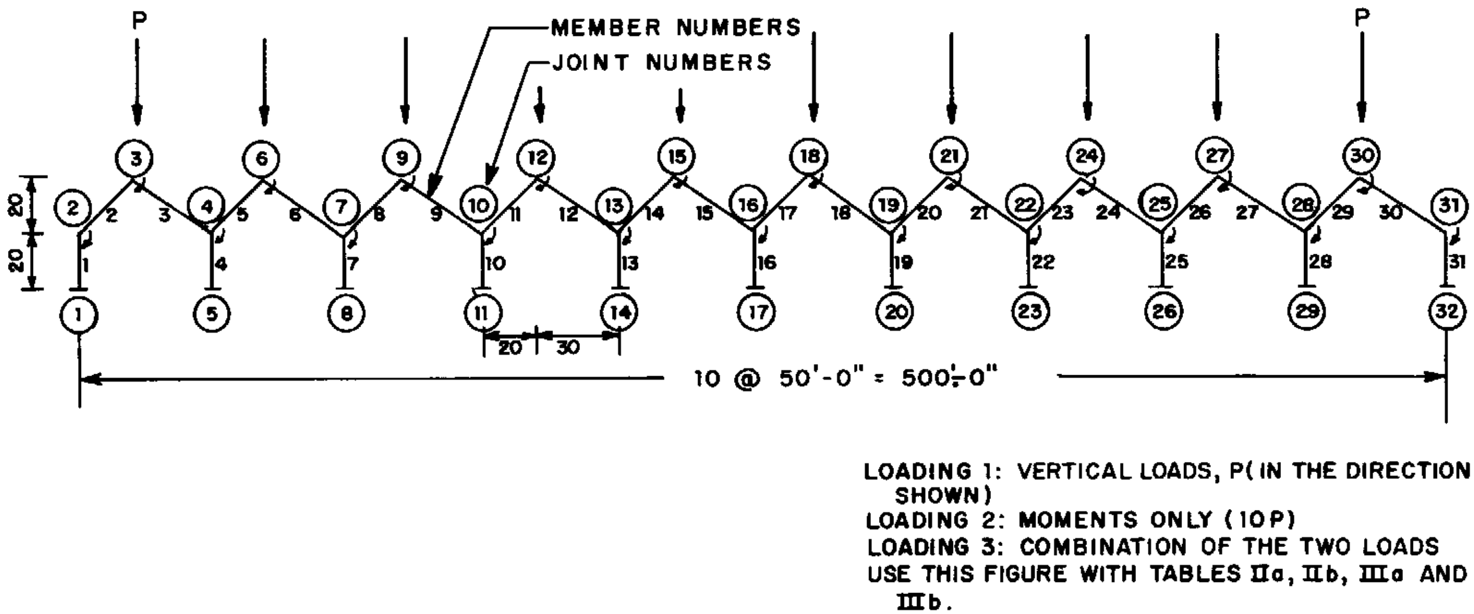


Figure 6. Ten-Bay Gable Frame

around and before i^{th} substructure. Similarly $r^{(i+1)}$ is the displacement matrix of the substructures around and after i^{th} substructure. The word "around" refers to the substructures sharing joints with i^{th} substructure. "Before" and "after" refer to the sequence of analysis.

In conclusion, the advantages of the iteration method are:

1. From the flow chart (Figure 2), it is evident that the method is simple and demands little sophistication in programming. All computations can be carried out without exceeding the core capacity.

2. Since the recurrence formula (Equation 9) was derived for a general structure, the method is applicable to plate and shell structures as well as frames.

3. Since an approximate result is available at any stage of iteration after the truncated cycle, the method can be used to great advantage in preliminary design.

4. The increase in computational time for additional loading conditions is a small fraction of the time required for one loading.

5. The effects of minor configuration changes can be localized without drastically altering the total computational scheme.

Because of these advantages, the method is particularly attractive for use with structural synthesis programs. The current limitation is that only static and thermal loadings can be handled. Another method with similar capabilities to handle dynamic loading is therefore necessary for the analysis of large structural systems.

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LEGEND FOR ACCOMPANYING TABLES

All frames are analyzed for I/A = 20/144 FT^2

TABLES Ia, IIa, IIIa

1. The first column refers to joint numbers shown in the applicable figure (Figure 5 or Figure 6).

2. The second and third columns refer to joint coordinates in feet. Reference axes are shown in the applicable figure (Figure 5 or Figure 6).

3. Columns 4, 5, and 6 refer to applied joint forces and moments in kips (or lbs) and ft-kips (ft-lbs), respectively, in terms of quantity P.

4. Columns 7, 8, and 9 are the joint displacements in terms of quantity P/EI. The units are kips (or lbs) and feet.

   EI is the lowest flexural rigidity.

TABLES Ib, IIb, IIIb

1. The first column refers to the member number shown in the applicable figure (Figure 5 or Figure 6).

2. The last two columns are the joint numbers to which the member is connected. (Figures 5 or 6)

3. Column 8 gives the relative values of the flexural rigidity EI.

4. Columns 2 through 4 are the member forces at the end MA. These forces are in terms of P. For interpretation of signs, see Figure 3. Similarly, columns 5 through 7 are the member forces at the end MB. (Note: MA is always less than MB.)

TABLE Ia

FIVE-BAY, 100 STORY FRAME: JOINT COORDINATES, FORCES, AND DISPLACEMENTS
(ITERATION)

	JOINT	-X	-Y	FORCE-X	FORCE-Y	MOMENT	DISPL-X	DISPL-Y	ROTATION
100th Floor	1	0.	0.	1.000	0.	0.	1.6156E 05	2.4403E 03	-3.7692E 01
				0.	0.	-10.000	1.9029E 04	5.1999E 02	-3.3673E 01
				1.000	0.	-10.000	1.8058E 05	2.9603E 03	-7.1365E 01
	2	40.000	0.	0.	0.	0.	1.6155E 05	5.8371E 02	-3.0695E 01
				0.	0.	-10.000	1.9027E 04	1.1015E 02	-2.2726E 01
				0.	0.	-10.000	1.8058E 05	6.9386E 02	-5.3420E 01
	3	80.000	0.	0.	0.	0.	1.6155E 05	5.5557E 01	-1.6391E 01
				0.	0.	-10.000	1.9026E 04	1.2354E 01	-2.0723E 01
				0.	0.	-10.000	1.8057E 05	6.7911E 01	-3.7114E 01
	4	120.000	0.	0.	0.	0.	1.6154E 05	-5.5537E 01	-1.6391E 01
				0.	0.	-10.000	1.9026E 04	-1.2355E 01	-2.0723E 01
				0.	0.	-10.000	1.8057E 05	-6.7892E 01	-3.7113E 01
	5	160.000	0.	0.	0.	0.	1.6155E 05	-5.8367E 02	-3.0698E 01
				0.	0.	-10.000	1.9027E 04	-1.1016E 02	-2.2727E 01
				0.	0.	-10.000	1.8057E 05	-6.9383E 02	-5.3424E 01
	6	200.000	0.	0.	0.	0.	1.6155E 05	-2.4407E 03	-3.7697E 01
				0.	0.	-10.000	1.9029E 04	-5.2004E 02	-3.3673E 01
				0.	0.	-10.000	1.8058E 05	-2.9607E 03	-7.1369E 01
50th. Floor	301	0.	-500.000	1.000	0.	0.	1.0519E 05	2.0134E 03	-1.6230E 02
				0.	0.	-10.000	7.9127E 03	3.1320E 02	-2.0103E 01
				1.000	0.	-10.000	1.1310E 05	2.3266E 03	-1.8238E 02
	302	40.000	-500.000	0.	0.	0.	1.0518E 05	3.0054E 02	-1.4844E 02
				0.	0.	-10.000	7.9126E 03	5.4802E 01	-1.8441E 01
				0.	0.	-10.000	1.1309E 05	3.5534E 02	-1.6687E 02
	303	80.000	-500.000	0.	0.	0.	1.0518E 05	3.0350E 01	-1.4528E 02
				0.	0.	-10.000	7.9126E 03	5.8986E 00	-1.7941E 01
				0.	0.	-10.000	1.1309E 05	3.6249E 01	-1.6320E 02
	304	120.000	-500.000	0.	0.	0.	1.0518E 05	-3.0328E 01	-1.4542E 02
				0.	0.	-10.000	7.9126E 03	-5.8996E 00	-1.7941E 01
				0.	0.	-10.000	1.1309E 05	-3.6228E 01	-1.6335E 02
	305	160.000	-500.000	0.	0.	0.	1.0518E 05	-3.0052E 02	-1.4879E 02
				0.	0.	-10.000	7.9126E 03	-5.4808E 01	-1.8441E 01
				0.	0.	-10.000	1.1309E 05	-3.5533E 02	-1.6722E 02
	306	200.000	-500.000	0.	0.	0.	1.0518E 05	-2.0137E 03	-1.6239E 02
				0.	0.	-10.000	7.9127E 03	-3.1323E 02	-2.0103E 01
				0.	0.	-10.000	1.1309E 05	-2.3269E 03	-1.8249E 02

TABLE Ia (Contd)

	JOINT	-X	-Y	FORCE-X	FORCE-Y	MOMENT	DISPL-X	DISPL-Y	ROTATION
	541	0.	-900.000	1.000	0.	0.	2.1806E 04	5.0986E 02	-2.2169E 02
				0.	0.	-10.000	1.2382E 03	6.1961E 01	-1.4749E 01
				1.000	0.	-10.000	2.3043E 04	5.7179E 02	-2.3643E 02
10th. Floor	542	40.000	-900.000	0.	0.	0.	2.1802E 04	9.9368E 01	-2.0661E 02
				0.	0.	-10.000	1.2382E 03	1.5797E 01	-1.3851E 01
				0.	0.	-10.000	2.3039E 04	1.1516E 02	-2.2045E 02
	543	80.000	-900.000	0.	0.	0.	2.1799E 04	1.6751E 01	-2.0445E 02
				0.	0.	-10.000	1.2382E 03	3.0096E 00	-1.3760E 01
				0.	0.	-10.000	2.3036E 04	1.9761E 01	-2.1819E 02
	544	120.000	-900.000	0.	0.	0.	2.1797E 04	-1.6745E 01	-2.0324E 02
				0.	0.	-10.000	1.2382E 03	-3.0102E 00	-1.3760E 01
				0.	0.	-10.000	2.3034E 04	-1.9755E 01	-2.1698E 02
	545	160.000	-900.000	0.	0.	0.	2.1796E 04	-9.9410E 01	-2.0316E 02
				0.	0.	-10.000	1.2382E 03	-1.5800E 01	-1.3851E 01
				0.	0.	-10.000	2.3033E 04	-1.1521E 02	-2.1700E 02
	546	200.000	-900.000	0.	0.	0.	2.1796E 04	-5.0991E 02	-2.1481E 02
				0.	0.	-10.000	1.2382E 03	-6.1970E 01	-1.4750E 01
				0.	0.	-10.000	2.3033E 04	-5.7184E 02	-2.2954E 02
	595	0.	-990.000	1.000	0.	0.	9.9397E 02	5.6205E 01	-1.5645E 02
				0.	0.	-10.000	4.4790E 01	6.5328E 00	-9.4547E 00
				1.000	0.	-10.000	1.0387E 03	6.2733E 01	-1.6589E 02
	596	40.000	-990.000	0.	0.	0.	9.9703E 02	9.4852E 00	-1.4532E 02
				0.	0.	-10.000	4.5073E 01	1.5332E 00	-8.7956E 00
				0.	0.	-10.000	1.0420E 03	1.1018E 01	-1.5411E 02
1st. Floor	597	80.000	-990.000	0.	0.	0.	9.9758E 02	1.6786E 00	-1.4521E 02
				0.	0.	-10.000	4.5200E 01	2.9474E-01	-8.8072E 00
				0.	0.	-10.000	1.0427E 03	1.9733E 00	-1.5401E 02
	598	120.000	-990.000	0.	0.	0.	9.9626E 02	-1.6751E 00	-1.4487E 02
				0.	0.	-10.000	4.5200E 01	-2.9480E-01	-8.8073E 00
				0.	0.	-10.000	1.0414E 03	-1.9699E 00	-1.5366E 02
	599	160.000	-990.000	0.	0.	0.	9.9324E 02	-9.4847E 00	-1.4430E 02
				0.	0.	-10.000	4.5073E 01	-1.5335E 00	-8.7956E 00
				0.	0.	-10.000	1.0382E 03	-1.1018E 01	-1.5309E 02
	600	200.000	-990.000	0.	0.	0.	9.8823E 02	-5.6218E 01	-1.5510E 02
				0.	0.	-10.000	4.4790E 01	-6.5336E 00	-9.4548E 00
				0.	0.	-10.000	1.0330E 03	-6.2747E 01	-1.6454E 02
	601	0.	-1000.000	0.	0.	0.	0.	0.	0.
				0.	0.	0.	0.	0.	0.
				0.	0.	0.	0.	0.	0.

TABLE Ib

FIVE-BAY, 100 STORY FRAME: MEMBER FORCES (ITERATION)

100th. Floor

MEMBER	AXIAL FORCE	SHEAR	MOMENT	AXIAL FORCE	SHEAR	MOMENT	EI	MA	MB
1	1.6392E 00	1.9247E-01	3.4821E 00	-1.6392E 00	-1.9247E-01	4.2168E 00	21.000	1	2
	6.4386E-01	-2.8277E-01	-6.2300E 00	-6.4386E-01	2.8277E-01	-5.0806E C0			
	2.2832E 00	-9.0290E-02	-2.7479E 00	-2.2832E 00	9.0290E-02	-8.6371E-C1			
2	1.7007E 00	-1.6284E-01	-4.0078E 00	-1.7007E 00	1.6284E-01	-2.5059E 00	21.000	2	3
	4.2249E-01	-3.0365E-01	-6.1783E 00	-4.2249E-01	3.0365E-01	-5.9679E C0			
	2.1250E 00	-4.6649E-01	-1.0186E 01	-2.1250E 00	4.6649E-01	-8.4737E C0			
3	4.9551E-01	-2.1441E-01	-4.2883E 00	-4.9951E-01	2.1441E-01	-4.2882E C0	21.000	3	4
	-1.8311E-04	-3.1666E-01	-6.3331E 00	1.8311E-04	3.1666E-01	-6.3331E C0			
	4.9707E-01	-5.3106E-01	-1.0621E 01	-4.9707E-01	5.3106E-01	-1.0621E 01			
4	-7.0117E-01	-1.6287E-01	-2.5062E 00	7.0117E-01	1.6287E-01	-4.0085E C0	21.000	4	5
	-4.2255E-01	-3.0365E-01	-5.9679E 00	4.2255E-01	3.0365E-01	-6.1783E C0			
	-1.1250E 00	-4.6651E-01	-8.4739E 00	1.1250E 00	4.6651E-01	-1.0187E C1			
5	-6.4063E-01	1.9258E-01	4.2191E 00	6.4063E-01	-1.9258E-01	3.4842E C0	21.000	5	6
	-6.4410E-01	-2.8276E-01	-5.0805E 00	6.4410E-01	2.8276E-01	-6.2298E C0			
	-1.2832E 00	-9.0167E-02	-8.6119E-01	1.2832E 00	9.0167E-02	-2.7455E C0			
6	1.9435E-01	-6.3842E-01	-3.4756E 00	-1.9435E-01	6.3842E-01	-2.9086E 00	10.000	1	7
	-2.6876E-01	-6.4442E-01	-3.7723E 00	2.6876E-01	6.4442E-01	-2.6719E C0			
	-7.4615E-02	-1.2821E 00	-7.2440E 00	7.4615E-02	1.2821E 00	-5.5764E C0			
7	-4.5015E-01	-6.1198E-02	-2.0372E-01	4.5015E-01	6.1198E-02	-4.0817E-C1	10.000	2	8
	-5.4225E-02	2.2100E-01	1.2566E 00	5.4225E-02	-2.2100E-01	9.5339E-01			
	-5.0415E-01	1.6067E-01	1.0573E 00	5.0415E-01	-1.6067E-01	5.4967E-C1			
8	2.1087E-01	1.2018E 00	6.7994E 00	-2.1087E-01	-1.2018E 00	5.2186E C0	10.000	3	9
	1.6209E-02	4.2186E-01	2.2987E 00	-1.6209E-02	-4.2186E-01	1.9200E C0			
	2.2704E-01	1.6245E 00	9.1021E 00	-2.2704E-01	-1.6245E 00	7.1426E C0			
9	-2.1079E-01	1.2018E 00	6.7997E 00	2.1079E-01	-1.2018E 00	5.2186E C0	10.000	4	10
	-1.6216E-02	4.2186E-01	2.2987E 00	1.6216E-02	-4.2186E-01	1.9200E 00			
	-2.2703E-01	1.6245E 00	9.1026E 00	2.2703E-01	-1.6245E 00	7.1429E C0			
10	4.4870E-01	-6.1432E-02	-2.0509E-01	-4.4870E-01	6.1432E-02	-4.0920E-01	10.000	5	11
	5.4226E-02	2.2097E-01	1.2564E 00	-5.4226E-02	-2.2097E-01	9.5528E-01			
	5.0291E-01	1.6044E-01	1.0559E 00	-5.0291E-01	-1.6044E-01	5.4863E-C1			
11	-1.9347E-01	-6.3870E-01	-3.4774E 00	1.9347E-01	6.3870E-01	-2.9099E C0	10.000	6	12
	2.6878E-C1	-6.4443E-01	-3.7724E 00	-2.6978E-01	6.4443E-01	-2.6719E C0			
	7.5806E-02	-1.2824E 00	-7.2459E 00	-7.5806E-02	1.2824E 00	-5.5779E C0			

TABLE Ib (Contd)

MEMBER	AXIAL FORCE	SHEAR	MOMENT	AXIAL FORCE	SHEAR	MOMENT	EI	MA	MB
551	1.0762E 00	-2.1947E 00	-4.4795E 01	-1.0762E 00	2.1947E 00	-4.2994E 01	26.000	301	302
	3.0609E-02	-2.4983E-01	-5.1046E 00	-3.0609E-02	2.4983E-01	-4.8885E 00			
	1.1069E 00	-2.4442E 00	-4.9893E 01	-1.1069E 00	2.4442E 00	-4.7877E 01			
552	7.2900E-01	-2.7321E 00	-5.4847E 01	-7.2900E-01	2.7321E 00	-5.4435E 01	26.000	302	303
	2.1423E-02	-3.3088E-01	-6.6501E 00	-2.1423E-02	3.3088E-01	-6.5851E 00			
	7.4951E-01	-3.0627E 00	-6.1492E 01	-7.4951E-01	3.0627E 00	-6.1015E 01			
553	3.4082E-01	-2.8047E 00	-5.6085E 01	-3.4082E-01	2.8047E 00	-5.6103E 01	26.000	303	304
	-6.1035E-05	-3.4410E-01	-6.8820E 00	6.1035E-05	3.4410E-01	-6.8820E 00			
	3.4229E-01	-3.1485E 00	-6.2961E 01	-3.4229E-01	3.1485E 00	-6.2980E 01			
554	-6.3477E-03	-2.7368E 00	-5.4518E 01	6.3477E-03	2.7368E 00	-5.4956E 01	26.000	304	305
	-2.1423E-02	-3.3088E-01	-6.5851E 00	2.1423E-02	3.3088E-01	-6.6501E 00			
	-2.8320E-02	-3.0675E 00	-6.1098E 01	2.8320E-02	3.0675E 00	-6.1601E 01			
555	-2.1777E-01	-2.1989E 00	-4.3094E 01	2.1777E-01	2.1989E 00	-4.4862E 01	26.000	305	306
	-3.0701E-02	-2.4982E-01	-4.8884E 00	3.0701E-02	2.4982E-01	-5.1045E 00			
	-2.5000E-01	-2.4485E 00	-4.7978E 01	2.5000E-01	2.4485E 00	-4.9963E 01			
556	-5.6780E 01	4.8328E 00	2.4587E 01	5.6780E 01	-4.8328E 00	2.3740E 01	35.000	301	307
	-1.3483E 01	-4.5615E-01	-2.3788E 00	1.3483E 01	4.5615E-01	-2.1826E 00			
	-7.0270E 01	4.3848E 00	2.2249E 01	7.0270E 01	-4.3848E 00	2.1598E 01			
557	-1.3609E 01	1.0410E 01	5.2834E 01	1.3609E 01	-1.0410E 01	5.1267E 01	35.000	302	308
	-2.9604E 00	2.0336E-01	9.7587E-01	2.9604E 00	-2.0336E-01	1.0577E 00			
	-1.6570E 01	1.0621E 01	5.3846E 01	1.6570E 01	-1.0621E 01	5.2361E 01			
558	-3.7411E-01	1.1748E 01	5.9496E 01	3.7411E-01	-1.1748E 01	5.7987E 01	35.000	303	309
	-1.4835E-01	4.0104E-01	1.9796E 00	1.4835E-01	-4.0104E-01	2.0308E 00			
	-5.2244E-01	1.2156E 01	6.1508E 01	5.2244E-01	-1.2156E 01	6.0051E 01			
559	3.8240E-01	1.1771E 01	5.9492E 01	-3.8240E-01	-1.1771E 01	5.8215E 01	35.000	304	310
	1.4835E-01	4.0100E-01	1.9794E 00	-1.4835E-01	-4.0100E-01	2.0307E 00			
	5.3068E-01	1.2178E 01	6.1501E 01	-5.3068E-01	-1.2178E 01	6.0275E 01			
560	1.3626E 01	1.0516E 01	5.3005E 01	-1.3626E 01	-1.0516E 01	5.2150E 01	35.000	305	311
	2.9607E 00	2.0331E-01	9.7558E-01	-2.9607E 00	-2.0331E-01	1.0575E 00			
	1.6587E 01	1.0724E 01	5.4009E 01	-1.6587E 01	-1.0724E 01	5.3235E 01			
561	5.6762E 01	5.1048E 00	2.5524E 01	-5.6762E 01	-5.1048E 00	2.5525E 01	35.000	306	312
	1.3484E 01	-4.5614E-01	-2.3789E 00	-1.3484E 01	4.5614E-01	-2.1825E 00			
	7.0250E 01	4.6541E 00	2.3171E 01	-7.0250E 01	-4.6541E 00	2.3370E 01			
562	8.0322E-01	-2.2367E 00	-4.5567E 01	-8.0322E-01	2.2367E 00	-4.3900E 01	26.000	307	308
	-1.7670E-02	-2.4799E-01	-5.0573E 00	1.7670E-02	2.4799E-01	-4.8624E 00			
	7.8369E-01	-2.4844E 00	-5.0619E 01	-7.8369E-01	2.4844E 00	-4.8757E 01			

50th. Floor

TABLE Ib (Contd)

10th. Floor

MEMBER	AXIAL FORCE	SHEAR	MOMENT	AXIAL FORCE	SHEAR	MOMENT	EI	MA	MB
991	1.9655E 00	-4.5875E 00	-9.2882E 01	-1.9655E 00	4.5875E 00	-9.0619E 01	30.000	541	542
	3.0884E-02	-2.9578E-01	-5.9830E 00	-3.0884E-02	2.9578E-01	-5.8482E 00			
	1.9977E 00	-4.8830E 00	-9.8859E 01	-1.9977E 00	4.8830E 00	-9.6462E 01			
992	1.5199E 00	-4.5779E 00	-9.1720E 01	-1.5199E 00	4.5779E 00	-9.1395E 01	30.000	542	543
	1.3062E-02	-3.0343E-01	-6.0754E 00	-1.3062E-02	3.0343E-01	-6.0618E 00			
	1.5366E 00	-4.8810E 00	-9.7790E 01	-1.5366E 00	4.8810E 00	-9.7452E 01			
993	1.0962E 00	-4.5676E 00	-9.1443E 01	-1.0962E 00	4.5676E 00	-9.1262E 01	30.000	543	544
	-2.7466E-04	-3.0622E-01	-6.1244E 00	2.7466E-04	3.0622E-01	-6.1245E 00			
	1.1001E 00	-4.8735E 00	-9.7561E 01	-1.1001E 00	4.8735E 00	-9.7380E 01			
994	6.8665E-01	-4.5255E 00	-9.0516E 01	-6.8665E-01	4.5255E 00	-9.0505E 01	30.000	544	545
	-1.3519E-02	-3.0343E-01	-6.0619E 00	1.3519E-02	3.0343E-01	-6.0755E 00			
	6.7676E-01	-4.8286E 00	-9.6571E 01	-6.7676E-01	4.8286E 00	-9.6573E 01			
995	2.0728E-01	-4.4713E 00	-8.8553E 01	-2.0728E-01	4.4713E 00	-9.0300E 01	30.000	545	546
	-3.1143E-02	-2.9579E-01	-5.8484E 00	3.1143E-02	2.9579E-01	-5.9832E 00			
	1.7773E-01	-4.7667E 00	-9.4394E 01	-1.7773E-01	4.7667E 00	-9.6276E 01			
996	-1.8018E 02	9.1668E 00	4.4010E 01	1.8018E 02	-9.1668E 00	4.7657E 01	55.000	541	547
	-2.3111E 01	-2.9270E-01	-1.6070E 00	2.3111E 01	2.9270E-01	-1.3200E 00			
	-2.0328E 02	8.8842E 00	4.2448E 01	2.0328E 02	-8.8842E 00	4.6393E 01			
997	-4.0891E 01	1.7527E 01	8.8376E 01	4.0891E 01	-1.7527E 01	8.6891E 01	55.000	542	548
	-6.4160E 00	2.3430E-01	1.1280E 00	6.4160E 00	-2.3430E-01	1.2150E 00			
	-4.7307E 01	1.7768E 01	8.9535E 01	4.7307E 01	-1.7768E 01	8.8145E 01			
998	-6.6279E 00	1.9110E 01	9.6218E 01	6.6279E 00	-1.9110E 01	9.4886E 01	55.000	543	549
	-1.2152E 00	2.9463E-01	1.4249E 00	1.2152E 00	-2.9463E-01	1.5214E 00			
	-7.8431E 00	1.9411E 01	9.7671E 01	7.8431E 00	-1.9411E 01	9.6438E 01			
999	6.5759E 00	2.0072E 01	1.0105E 02	-6.5759E 00	-2.0072E 01	9.9667E 01	55.000	544	550
	1.2154E 00	2.9465E-01	1.4249E 00	-1.2154E 00	-2.9465E-01	1.5216E 00			
	7.7912E 00	2.0371E 01	1.0250E 02	-7.7912E 00	-2.0371E 01	1.0121E 02			
1000	4.0885E 01	2.0224E 01	1.0194E 02	-4.0885E 01	-2.0224E 01	1.0030E 02	55.000	545	551
	6.4171E 00	2.3435E-01	1.1279E 00	-6.4171E 00	-2.3435E-01	1.2156E 00			
	4.7303E 01	2.0462E 01	1.0309E 02	-4.7303E 01	-2.0462E 01	1.0153E 02			
1001	1.8027E 02	1.2878E 01	6.4853E 01	-1.8027E 02	-1.2878E 01	6.3924E 01	55.000	546	552
	2.3114E 01	-2.9256E-01	-1.6068E 00	-2.3114E 01	2.9256E-01	-1.3188E 00			
	2.0337E 02	1.2587E 01	6.3262E 01	-2.0337E 02	-1.2587E 01	6.2611E 01			
1002	1.4989E 00	-4.5852E 00	-9.2485E 01	-1.4989E 00	4.5852E 00	-9.0922E 01	30.000	547	548
	-1.8677E-02	-2.9433E-01	-5.9403E 00	1.8677E-02	2.9433E-01	-5.8328E 00			
	1.4784E 00	-4.8790E 00	-9.8416E 01	-1.4784E 00	4.8790E 00	-9.6746E 01			

TABLE Ib (Contd)

1st. Floor

MEMBER	AXIAL FORCE	SHEAR	MOMENT	AXIAL FORCE	SHEAR	MOMENT	EI	MA	MB
1089	2.1915E 02	1.2051E 01	3.3369E 01	-2.1915E 02	-1.2051E 01	8.7137E 01	55.000	594	600
	2.5653E 01	-3.0506E-01	-3.2396E 00	-2.5653E 01	3.0506E-01	1.8893E-01			
	2.4479E 02	1.1748E 01	3.0144E 01	-2.4479E 02	-1.1748E 01	8.7339E 01			
1090	-1.6518E 00	-3.3687E 00	-6.8208E 01	1.6518E 00	3.3687E 00	-6.6539E 01	30.000	595	596
	-1.5254E-01	-2.0250E-01	-4.0995E 00	1.5254E-01	2.0250E-01	-4.0006E 00			
	-1.8045E 00	-3.5709E 00	-7.2303E 01	1.8045E 00	3.5709E 00	-7.0535E 01			
1091	-2.9666E-01	-3.2642E 00	-6.5292E 01	2.9666E-01	3.2642E 00	-6.5275E 01	30.000	596	597
	-6.8827E-02	-1.9734E-01	-3.9458E 00	6.8827E-02	1.9734E-01	-3.9476E 00			
	-3.6584E-01	-3.4613E 00	-6.9233E 01	3.6584E-01	3.4613E 00	-6.9218E 01			
1092	7.1330E-01	-3.2615E 00	-6.5256E 01	-7.1330E-01	3.2615E 00	-6.5204E 01	30.000	597	598
	-3.3379E-05	-1.9783E-01	-3.9566E 00	3.3379E-05	1.9783E-01	-3.9566E 00			
	7.1281E-01	-3.4591E 00	-6.9208E 01	-7.1281E-01	3.4591E 00	-6.9156E 01			
1093	1.6320E 00	-3.2487E 00	-6.5017E 01	-1.6320E 00	3.2487E 00	-6.4932E 01	30.000	598	599
	6.8771E-02	-1.9734E-01	-3.9476E 00	-6.8771E-02	1.9734E-01	-3.9458E 00			
	1.7005E 00	-3.4458E 00	-6.8960E 01	-1.7005E 00	3.4458E 00	-6.8874E 01			
1094	2.7052E 00	-3.3419E 00	-6.6029E 01	-2.7052E 00	3.3419E 00	-6.7649E 01	30.000	599	600
	1.5251E-01	-2.0250E-01	-4.0007E 00	-1.5251E-01	2.0250E-01	-4.0995E 00			
	2.8575E 00	-3.5442E 00	-7.0025E 01	-2.8575E 00	3.5442E 00	-7.1743E 01			
1095	-2.2255E 02	1.3974E 01	-1.6180E 01	2.2255E 02	-1.3974E 01	1.5592E 02	55.000	595	601
	-2.5868E 01	-1.6391E-01	-6.0196E 00	2.5868E 01	1.6391E-01	4.3806E 00			
	-2.4840E 02	1.3809E 01	-2.2196E 01	2.4840E 02	-1.3809E 01	1.6029E 02			
1096	-3.7558E 01	1.7847E 01	9.3076E 00	3.7558E 01	-1.7847E 01	1.6916E 02	55.000	596	602
	-6.0711E 00	7.2263E-02	-4.4763E 00	6.0711E 00	-7.2263E-02	5.1989E 00			
	-4.3629E 01	1.7918E 01	4.8315E 00	4.3629E 01	-1.7918E 01	1.7435E 02			
1097	-6.6468E 00	1.7920E 01	9.7326E 00	6.6468E 00	-1.7920E 01	1.6947E 02	55.000	597	603
	-1.1671E 00	7.6824E-02	-4.4599E 00	1.1671E 00	-7.6824E-02	5.2281E 00			
	-7.8138E 00	1.7996E 01	5.2729E 00	7.8138E 00	-1.7996E 01	1.7468E 02			
1098	6.6327E 00	1.7947E 01	1.0061E 01	-6.6327E 00	-1.7947E 01	1.6941E 02	55.000	598	604
	1.1673E 00	7.6824E-02	-4.4599E 00	-1.1673E 00	-7.6824E-02	5.2281E 00			
	7.8000E 00	1.8023E 01	5.6005E 00	-7.8000E 00	-1.8023E 01	1.7463E 02			
1099	3.7556E 01	1.7934E 01	1.0306E 01	-3.7556E 01	-1.7934E 01	1.6904E 02	55.000	599	605
	6.0722E 00	7.2257E-02	-4.4763E 00	-6.0722E 00	-7.2257E-02	5.1989E 00			
	4.3628E 01	1.8005E 01	5.8296E 00	-4.3628E 01	-1.8005E 01	1.7423E 02			
1100	2.2260E 02	1.4041E 01	-1.5098E 01	-2.2260E 02	-1.4041E 01	1.5551E 02	55.000	600	606
	2.5871E 01	-1.6391E-01	-6.0197E 00	-2.5871E 01	1.6391E-01	4.3806E 00			
	2.4846E 02	1.3876E 01	-2.1116E 01	-2.4846E 02	-1.3876E 01	1.5988E 02			

TABLE IIa

TEN-BAY GABLE FRAME: JOINT COORDINATES, FORCES, AND DISPLACEMENTS
(ITERATION)

JOINT	-X	-Y	FORCE-X	FORCE-Y	MOMENT	DISPL-X	DISPL-Y	ROTATION
1	0.	0.	0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
2	0.	20.000	0.	0.	0.	-4.2786E 02	-1.8163E 00	1.5483E 01
			0.	0.	-10.000	2.0365E 02	1.1291E 00	-3.3636E 01
			0.	0.	-10.000	-2.2420E 02	-6.8712E-01	-1.8154E 01
3	20.000	40.000	0.	-1.000	0.	-2.6076E 02	-1.7309E 02	-9.3721E 00
			0.	0.	-10.000	2.2888E 02	-2.3281E 01	-2.5935E 01
			0.	-1.000	-10.000	-3.1882E 01	-1.9637E 02	-3.5307E 01
4	50.000	20.000	0.	0.	0.	-1.5037E 02	-2.6933E 00	7.4434E 00
			0.	0.	-10.000	2.4199E 02	-6.5633E-02	-2.1940E 01
			0.	0.	-10.000	9.1617E 01	-2.7589E 00	-1.4497E 01
5	50.000	0.	0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
6	70.000	40.000	0.	-1.000	0.	-9.4391E 01	-6.3176E 01	-3.8517E 00
			0.	0.	-10.000	2.3065E 02	1.2132E 01	-2.8548E 01
			0.	-1.000	-10.000	1.3625E 02	-5.1042E 01	-3.2400E 01
7	100.000	20.000	0.	0.	0.	-5.7977E 01	-2.7377E 00	2.7651E 00
			0.	0.	-10.000	2.2046E 02	-5.4550E-03	-2.1491E 01
			0.	0.	-10.000	1.6248E 02	-2.7431E 00	-1.8726E 01
8	100.000	0.	0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
9	120.000	40.000	0.	-1.000	0.	-3.5491E 01	-2.9850E 01	-1.4880E 00
			0.	0.	-10.000	2.1817E 02	3.1678E 00	-2.8929E 01
			0.	-1.000	-10.000	1.8268E 02	-2.6684E 01	-3.0417E 01
10	150.000	20.000	0.	0.	0.	-2.1589E 01	-2.7626E 00	1.0200E 00
			0.	0.	-10.000	2.1398E 02	-2.7456E-03	-2.1155E 01
			0.	0.	-10.000	1.9239E 02	-2.7653E 00	-2.0135E 01
11	150.000	0.	0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
12	170.000	40.000	0.	-1.000	0.	-1.2071E 01	-1.6952E 01	-5.8219E-01
			0.	0.	-10.000	2.1438E 02	4.8049E-01	-2.9097E 01
			0.	-1.000	-10.000	2.0231E 02	-1.6472E 01	-2.9679E 01
13	200.000	20.000	0.	0.	0.	-6.8756E 00	-2.7714E 00	3.0320E-01
			0.	0.	-10.000	2.1205E 02	-4.4093E-04	-2.1071E 01
			0.	0.	-10.000	2.0518E 02	-2.7718E 00	-2.0768E 01

TABLE IIa (Contd)

JOINT	-X	-Y	FORCE-X	FORCE-Y	MOMENT	DISPL-X	DISPL-Y	ROTATION
14	200.000	0.	0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
15	220.000	40.000	0.	-1.000	0.	-1.5859E 00	-1.2747E 01	-2.1129E-01
			0.	0.	-10.000	2.1413E 02	-1.1774E 00	-2.9141E 01
			0.	-1.000	-10.000	2.1255E 02	-1.3925E 01	-2.9352E 01
16	250.000	20.000	0.	0.	0.	7.6684E-01	-2.7737E 00	-8.5669E-02
			0.	0.	-10.000	2.1287E 02	1.2291E-03	-2.1126E 01
			0.	0.	-10.000	2.1365E 02	-2.7725E 00	-2.1212E 01
17	250.000	0.	0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
18	270.000	40.000	0.	-1.000	0.	6.3478E 00	-1.3035E 01	-1.0201E-02
			0.	0.	-10.000	2.1711E 02	-3.3220E 00	-2.9112E 01
			0.	-1.000	-10.000	2.2346E 02	-1.6359E 01	-2.9122E 01
19	300.000	20.000	0.	0.	0.	8.8869E 00	-2.7717E 00	-5.3120E-01
			0.	0.	-10.000	2.1732E 02	4.1397E-03	-2.1384E 01
			0.	0.	-10.000	2.2620E 02	-2.7676E 00	-2.1915E 01
20	300.000	0.	0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	C.
			0.	0.	0.	0.	0.	0.
21	320.000	40.000	0.	-1.000	0.	1.9565E 01	-1.8100E 01	2.2000E-01
			0.	0.	-10.000	2.2634E 02	-8.0790E 00	-2.8979E 01
			0.	-1.000	-10.000	2.4591E 02	-2.6186E 01	-2.8760E 01
22	350.000	20.000	0.	0.	0.	2.5508E 01	-2.7636E 00	-1.4742E 00
			0.	0.	-10.000	2.2975E 02	1.1240E-02	-2.2094E 01
			0.	0.	-10.000	2.5525E 02	-2.7524E 00	-2.3568E 01
23	350.000	0.	0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
24	370.000	40.000	0.	-1.000	0.	5.1171E 01	-5.2994E 01	7.0360E-01
			0.	0.	-10.000	2.5084E 02	-2.0082E 01	-2.8607E 01
			0.	-1.000	-10.000	3.0201E 02	-5.3076E 01	-2.7904E 01
25	400.000	20.000	0.	0.	0.	6.7119E 01	-2.7400E 00	-3.8347E 00
			0.	0.	-10.000	2.6231E 02	2.7024E-02	-2.3980E 01
			0.	0.	-10.000	3.2943E 02	-2.7130E 00	-2.7815E 01
26	400.000	0.	0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.

TABLE IIa (Contd)

JOINT	-X	-Y	FORCE-X	FORCE-Y	MOMENT	DISPL-X	DISPL-Y	ROTATION
27	420.000	40.000	0.	-1.000	0.	1.3141E 02	-7.1373E 01	1.9934E 00
			0.	0.	-10.000	3.1662E 02	-5.3109E 01	-2.7766E 01
			0.	-1.000	-10.000	4.4804E 02	-1.2449E 02	-2.5772E 01
28	450.000	20.000	0.	0.	0.	1.7314E 02	-2.7089E 00	-1.0210E 01
			0.	0.	-10.000	3.5031E 02	1.2450E-01	-2.8421E 01
			0.	0.	-10.000	5.2345E 02	-2.5845E 00	-3.8631E 01
29	450.000	0.	0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
30	470.000	40.000	0.	-1.000	0.	3.6209E 02	-1.9548E 02	3.6337E 00
			0.	0.	-10.000	4.4729E 02	-9.5122E 01	-2.2438E 01
			0.	-1.000	-10.000	8.0938E 02	-2.9060E 02	-1.8804E 01
31	500.000	20.000	0.	0.	0.	4.8799E 02	-1.2407E 00	-2.0754E 01
			0.	0.	-10.000	5.0843E 02	-1.2228E 00	-5.1022E 01
			0.	0.	-10.000	9.9642E 02	-2.4635E 00	-7.1776E 01
32	500.000	0.	0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.

TABLE IIb

TEN-BAY GABLE FRAME; MEMBER FORCES (ITERATION)

MEMBER	AXIAL FORCE	SHEAR	MOMENT	AXIAL FORCE	SHEAR	MOMENT	EI	MA	MB
1	6.5380E-01	-4.0955E-01	-4.8696E 00	-6.5380E-01	4.0955E-01	-3.3213E 00	10.000	1	2
	-4.0646E-01	-1.9906E-01	-3.0883E-01	4.0646E-01	1.9906E-01	-3.6725E 00			
	2.4734E-01	-6.0861E-01	-5.1784E 00	-2.4734E-01	6.0861E-01	-6.9938E 00			
2	7.5192E-01	1.7272E-01	3.3213E 00	-7.5192E-01	-1.7272E-01	1.5638E 00	10.000	2	3
	-1.4667E-01	-4.2817E-01	-6.3275E 00	1.4667E-01	4.2817E-01	-5.7830E 00			
	6.0524E-01	-2.5545E-01	-3.0062E 00	-6.0524E-01	2.5545E-01	-4.2191E 00			
3	5.3266E-01	-6.0876E-02	-1.5638E 00	-5.3266E-01	6.0876E-02	-6.3108E-01	10.000	3	4
	3.9342E-01	-2.2777E-01	-4.2169E 00	-3.9342E-01	2.2777E-01	-3.9954E 00			
	9.2630E-01	-2.8864E-01	-5.7808E 00	-9.2630E-01	2.8864E-01	-4.6264E 00			
4	9.6951E-01	-1.1390E-01	-7.6685E-01	-9.6951E-01	1.1390E-01	-1.5112E 00	10.000	4	5
	2.3626E-02	3.3880E-02	-7.5821E-01	-2.3626E-02	-3.3880E-02	1.4358E 00			
	9.9314E-01	-8.0023E-02	-1.5251E 00	-9.9314E-01	8.0023E-02	-7.5405E-02			
5	8.1089E-01	7.0611E-02	1.3979E 00	-8.1089E-01	-7.0611E-02	5.9925E-01	10.000	4	6
	-1.5393E-01	-3.8749E-01	-5.2463E 00	1.5393E-01	3.8749E-01	-5.7135E 00			
	6.5698E-01	-3.1688E-01	-3.8484E 00	-6.5698E-01	3.1688E-01	-5.1143E 00			
6	6.4436E-01	-2.3061E-02	-5.9925E-01	-6.4436E-01	2.3061E-02	-2.3222E-01	10.000	6	7
	3.4737E-01	-2.2692E-01	-4.2865E 00	-3.4737E-01	2.2692E-01	-3.8951E 00			
	9.9126E-01	-2.4998E-01	-4.8858E 00	-9.9126E-01	2.4998E-01	-4.1273E 00			
7	9.8549E-01	-4.5490E-02	-3.1664E-01	-9.8549E-01	4.5490E-02	-5.9315E-01	10.000	7	8
	1.9637E-03	8.3331E-03	-9.9121E-01	-1.9637E-03	-8.3331E-03	1.1579E 00			
	9.8744E-01	-3.7163E-02	-1.3079E 00	-9.8744E-01	3.7163E-02	5.6466E-01			
8	8.3279E-01	2.8177E-02	5.4886E-01	-8.3279E-01	-2.8177E-02	2.4812E-01	10.000	7	9
	-1.5842E-01	-3.8020E-01	-5.1139E 00	1.5842E-01	3.8020E-01	-5.6399E 00			
	6.7436E-01	-3.5202E-01	-4.5650E 00	-6.7436E-01	3.5202E-01	-5.3917E 00			
9	6.9042E-01	-9.9044E-03	-2.4812E-01	-6.9042E-01	9.9044E-03	-1.0899E-01	10.000	9	10
	3.4557E-01	-2.2989E-01	-4.3600E 00	-3.4557E-01	2.2989E-01	-3.9287E 00			
	1.0360E 00	-2.3979E-01	-4.6081E 00	-1.0360E 00	2.3979E-01	-4.0377E 00			
10	9.9445E-01	-1.7082E-02	-1.1982E-01	-9.9445E-01	1.7082E-02	-2.2182E-01	10.000	10	11
	9.8832E-04	3.6387E-03	-1.0214E 00	-9.8832E-04	-3.6387E-03	1.0941E 00			
	9.9544E-01	-1.3444E-02	-1.1412E 00	-9.9544E-01	1.3444E-02	8.7231E-01			
11	8.4095E-01	1.2174E-02	2.2881E-01	-8.4095E-01	-1.2174E-02	1.1552E-01	10.000	10	12
	-1.6030E-01	-3.7692E-01	-5.0496E 00	1.6030E-01	3.7692E-01	-5.6112E 00			
	6.8064E-01	-3.6474E-01	-4.8208E 00	-6.8064E-01	3.6474E-01	-5.4957E 00			
12	7.0744E-01	-5.0462E-03	-1.1553E-01	-7.0744E-01	5.0462E-03	-6.6416E-02	10.000	12	13
	3.3432E-01	-2.3111E-01	-4.3890E 00	-3.3432E-01	2.3111E-01	-3.9438E 00			
	1.0408E 00	-2.3616E-01	-4.5045E 00	-1.0408E 00	2.3616E-01	-4.0102E 00			
13	9.9762E-01	-5.7654E-03	-4.2494E-02	-9.9762E-01	5.7654E-03	-7.2814E-02	10.000	13	14
	1.5872E-04	2.0174E-03	-1.0334E 00	-1.5872E-04	-2.0174E-03	1.0737E 00			
	9.9778E-01	-3.7451E-03	-1.0758E 00	-9.9778E-01	3.7451E-03	1.0009E 00			

TABLE IIb (Contd)

MEMBER	AXIAL FORCE	SHEAR	MOMENT	AXIAL FORCE	SHEAR	MOMENT	EI	MA	MB
14	8.4334E-01	6.4138E-03	1.0889E-01	-8.4334E-01	-6.4138E-03	7.2514E-02	10.000	13	15
	-1.6163E-01	-3.7537E-01	-5.0232E 00	1.6163E-01	3.7537E-01	-5.5938E 00			
	6.8170E-01	-3.6895E-01	-4.9143E 00	-6.8170E-01	3.6895E-01	-5.5213E 00			
15	7.1374E-01	-3.8291E-03	-7.2515E-02	-7.1374E-01	3.8291E-03	-6.5546E-02	10.000	15	16
	3.3855E-01	-2.3208E-01	-4.4061E 00	-3.3855E-01	2.3208E-01	-3.9615E 00			
	1.0526E 00	-2.3590E-01	-4.4786E 00	-1.0526E 00	2.3590E-01	-4.0271E 00			
16	9.9845E-01	-1.3477E-04	-5.6312E-03	-9.9845E-01	1.3477E-04	2.9357E-03	10.000	16	17
	-4.4245E-04	2.4178E-03	-1.0321E 00	4.4245E-04	-2.4178E-03	1.0805E 00			
	9.9801E-01	2.2873E-03	-1.0377E 00	-9.9801E-01	-2.2873E-03	1.0835E 00			
17	8.4234E-01	5.2217E-03	7.1178E-02	-8.4234E-01	-5.2217E-03	7.6514E-02	10.000	16	18
	-1.6365E-01	-3.7396E-01	-5.0062E 00	1.6365E-01	3.7396E-01	-5.5709E 00			
	6.7870E-01	-3.6873E-01	-4.9350E 00	-6.7870E-01	3.6873E-01	-5.4943E 00			
18	7.1488E-01	-5.0455E-03	-7.6510E-02	-7.1488E-01	5.0455E-03	-1.0541E-01	10.000	18	19
	3.3261E-01	-2.3380E-01	-4.4292E 00	-3.3261E-01	2.3380E-01	-4.0005E 00			
	1.0496E 00	-2.3884E-01	-4.5057E 00	-1.0496E 00	2.3884E-01	-4.1059E 00			
19	9.9774E-01	5.3623E-03	2.7063E-02	-9.9774E-01	-5.3623E-03	8.0183E-02	10.000	19	20
	-1.4902E-03	5.2220E-03	-1.0170E 00	1.4902E-03	-5.2220E-03	1.1214E 00			
	9.9625E-01	1.0580E-02	-9.8996E-01	-9.9625E-01	-1.0580E-02	1.2016E 00			
20	8.3695E-01	7.4184E-03	7.8353E-02	-8.3695E-01	-7.4184E-03	1.3147E-01	10.000	19	21
	-1.6840E-01	-3.7131E-01	-4.9826E 00	1.6840E-01	3.7131E-01	-5.5197E 00			
	6.6852E-01	-3.6389E-01	-4.9042E 00	-6.6852E-01	3.6389E-01	-5.3881E 00			
21	7.1138E-01	-9.8996E-03	-1.3148E-01	-7.1138E-01	9.8996E-03	-2.2546E-01	10.000	21	22
	3.3002E-01	-2.3793E-01	-4.4803E 00	-3.3002E-01	2.3793E-01	-4.0984E 00			
	1.0442E 00	-2.4783E-01	-4.6118E 00	-1.0442E 00	2.4783E-01	-4.3238E 00			
22	9.9483E-01	1.6148E-02	8.7764E-02	-9.9483E-01	-1.6148E-02	2.3519E-01	10.000	22	23
	-4.0460E-03	1.3208E-02	-9.7263E-01	4.0460E-03	-1.3208E-02	1.2368E 00			
	9.9078E-01	2.9353E-02	-8.8486E-01	-9.9078E-01	-2.9353E-02	1.4719E 00			
23	8.2187E-01	1.5180E-02	1.3768E-01	-8.2187E-01	-1.5180E-02	2.9168E-01	10.000	22	24
	-1.8063E-01	-3.6482E-01	-4.9290E 00	1.8063E-01	3.6482E-01	-5.3895E 00			
	6.4125E-01	-3.4963E-01	-4.7912E 00	-6.4125E-01	3.4963E-01	-5.0979E 00			
24	7.0129E-01	-2.3161E-02	-2.9166E-01	-7.0129E-01	2.3161E-02	-5.4340E-01	10.000	24	25
	3.2224E-01	-2.4862E-01	-4.6105E 00	-3.2224E-01	2.4862E-01	-4.3538E 00			
	1.0217E 00	-2.7179E-01	-4.9022E 00	-1.0217E 00	2.7179E-01	-4.8973E 00			
25	9.8633E-01	4.3158E-02	2.3985E-01	-9.8633E-01	-4.3158E-02	6.2332E-01	10.000	25	26
	-9.7278E-03	3.3763E-02	-8.6138E-01	9.7278E-03	-3.3763E-02	1.5366E 00			
	9.7660E-01	7.6920E-02	-6.2156E-01	-9.7660E-01	-7.6920E-02	2.1600E 00			
26	7.8168E-01	3.6037E-02	3.0358E-01	-7.8168E-01	-3.6037E-02	7.1569E-01	10.000	25	27
	-2.1138E-01	-3.4780E-01	-4.7848E 00	2.1138E-01	3.4780E-01	-5.0525E 00			
	5.7032E-01	-3.1176E-01	-4.4813E 00	-5.7032E-01	3.1176E-01	-4.3368E 00			

TABLE IIb (Contd)

MEMBER	AXIAL FORCE	SHEAR	MOMENT	AXIAL FORCE	SHEAR	MOMENT	EI	MA	MB
27	6.7184E-01	-5.8476E-02	-7.1572E-01	-6.7184E-01	5.8476E-02	-1.3926E 00	10.000	27	28
	2.9979E-01	-2.7545E-01	-4.9475E 00	-2.9979E-01	2.7545E-01	-4.9839E 00			
	9.7264E-01	-3.3392E-01	-5.6632E 00	-9.7264E-01	3.3392E-01	-6.3765E 00			
28	9.7514E-01	1.0656E-01	5.5515E-01	-9.7514E-01	-1.0656E-01	1.5761E 00	10.000	28	29
	-4.4817E-02	9.9141E-02	-4.2964E-01	4.4817E-02	-9.9141E-02	2.4125E 00			
	9.3033E-01	2.0571E-01	1.2557E-01	-9.3033E-01	-2.0571E-01	3.9887E 00			
29	6.8875E-01	9.3825E-02	8.3744E-01	-6.8875E-01	-9.3825E-02	1.8163E 00	10.000	28	30
	-3.1314E-01	-3.0936E-01	-4.5865E 00	3.1314E-01	3.0936E-01	-4.1634E 00			
	3.7561E-01	-2.1553E-01	-3.7491E 00	-3.7561E-01	2.1553E-01	-2.3471E 00			
30	5.9777E-01	-1.3827E-01	-1.8163E 00	-5.9777E-01	1.3827E-01	-3.1691E 00	10.000	30	31
	2.4193E-01	-3.6773E-01	-5.8366E 00	-2.4193E-01	3.6773E-01	-7.4221E 00			
	8.3970E-01	-5.0600E-01	-7.6529E 00	-8.3970E-01	5.0600E-01	-1.0591E 01			
31	4.4663E-01	4.2068E-01	3.1691E 00	-4.4663E-01	-4.2068E-01	5.2445E 00	10.000	31	32
	4.4017E-01	-2.6770E-03	-2.5779E 00	-4.4017E-01	2.6770E-03	2.5243E 00			
	8.8680E-01	4.1800E-01	5.9123E-01	-8.8680E-01	-4.1800E-01	7.7688E 00			

TABLE IIIa

TEN-BAY GABLE FRAME: JOINT COORDINATES, FORCES, AND DISPLACEMENTS
(DIRECT ANALYSIS)

JOINT	-X	-Y	FORCE-X	FORCE-Y	MOMENT	DISPL-X	DISPL-Y	ROTATION
1	0.	0.	0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
2	0.	20.000	0.	-0.000	-0.000	-4.2786E 02	-1.8163E 00	1.5483E 01
			0.	-0.000	-10.000	2.0365E 02	1.1291E 00	-3.3636E 01
			0.	-0.000	-10.000	-2.2420E 02	-6.8712E-01	-1.8154E 01
3	20.000	40.000	-0.000	-1.000	-0.000	-2.6076E 02	-1.7309E 02	-9.3722E 00
			-0.000	-0.000	-10.000	2.2888E 02	-2.3283E 01	-2.5935E 01
			-0.000	-1.000	-10.000	-3.1882E 01	-1.9637E 02	-3.5307E 01
4	50.000	20.000	-0.000	-0.000	-0.000	-1.5037E 02	-2.6933E 00	7.4435E 00
			-0.000	-0.000	-10.000	2.4201E 02	-6.5654E-02	-2.1941E 01
			-0.000	-0.000	-10.000	9.1632E 01	-2.7589E 00	-1.4497E 01
5	50.000	0.	0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
6	70.000	40.000	-0.000	-1.000	-0.000	-9.4394E 01	-6.3177E 01	-3.8517E 00
			-0.000	-0.000	-10.000	2.3066E 02	1.2132E 01	-2.8548E 01
			-0.000	-1.000	-10.000	1.3627E 02	-5.1045E 01	-3.2399E 01
7	100.000	20.000	-0.000	-0.000	-0.000	-5.7980E 01	-2.7377E 00	2.7652E 00
			-0.000	-0.000	-10.000	2.2047E 02	-5.4750E-03	-2.1491E 01
			-0.000	-0.000	-10.000	1.6248E 02	-2.7431E 00	-1.8725E 01
8	100.000	0.	0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
9	120.000	40.000	-0.000	-1.000	-0.000	-3.5493E 01	-2.9852E 01	-1.4881E 00
			-0.000	-0.000	-10.000	2.1817E 02	3.1695E 00	-2.8930E 01
			-0.000	-1.000	-10.000	1.8268E 02	-2.6682E 01	-3.0418E 01
10	150.000	20.000	-0.000	-0.000	-0.000	-2.1590E 01	-2.7626E 00	1.0201E 00
			-0.000	-0.000	-10.000	2.1400E 02	-2.7465E-03	-2.1156E 01
			-0.000	-0.000	-10.000	1.9241E 02	-2.7653E 00	-2.0136E 01
11	150.000	0.	0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
12	170.000	40.000	-0.000	-1.000	-0.000	-1.2071E 01	-1.6953E 01	-5.8220E-01
			-0.000	-0.000	-10.000	2.1441E 02	4.7800E-01	-2.9096E 01
			-0.000	-1.000	-10.000	2.0234E 02	-1.6475E 01	-2.9678E 01
13	200.000	20.000	-0.000	-0.000	-0.000	-6.8760E 00	-2.7714E 00	3.0324E-01
			-0.000	-0.000	-10.000	2.1205E 02	-4.7873E-04	-2.1070E 01
			-0.000	-0.000	-10.000	2.0518E 02	-2.7719E 00	-2.0767E 01

TABLE IIIa (Contd)

JOINT	-X	-Y	FORCE-X	FORCE-Y	MOMENT	DISPL-X	DISPL-Y	ROTATION
14	200.000	0.	0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
15	220.000	40.000	-0.000	-1.000	-0.000	-1.5855E 00	-1.2747E 01	-2.1131E-01
			-0.000	-0.000	-10.000	2.1413E 02	-1.1782E 00	-2.9141E 01
			-0.000	-1.000	-10.000	2.1254E 02	-1.3926E 01	-2.9352E 01
16	250.000	20.000	-0.000	-0.000	-0.000	7.6729E-01	-2.7737E 00	-8.5689E-02
			-0.000	-0.000	-10.000	2.1289E 02	1.2217E-03	-2.1127E 01
			-0.000	-0.000	-10.000	2.1366E 02	-2.7725E 00	-2.1213E 01
17	250.000	0.	0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
18	270.000	40.000	-0.000	-1.000	-0.000	6.3488E 00	-1.3035E 01	-1.0195E-02
			-0.000	-0.000	-10.000	2.1712E 02	-3.3232E 00	-2.9112E 01
			-0.000	-1.000	-10.000	2.2347E 02	-1.6358E 01	-2.9122E 01
19	300.000	20.000	-0.000	-0.000	-0.000	8.8889E 00	-2.7717E 00	-5.3130E-01
			-0.000	-0.000	-10.000	2.1733E 02	4.1254E-03	-2.1384E 01
			-0.000	-0.000	-10.000	2.2622E 02	-2.7676E 00	-2.1916E 01
20	300.000	0.	0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
21	320.000	40.000	-0.000	-1.000	-0.000	1.9568E 01	-1.8101E 01	2.2005E-01
			-0.000	-0.000	-10.000	2.2635E 02	-8.0853E 00	-2.8979E 01
			-0.000	-1.000	-10.000	2.4592E 02	-2.6186E 01	-2.8759E 01
22	350.000	20.000	-0.000	-0.000	-0.000	2.5510E 01	-2.7636E 00	-1.4743E 00
			-0.000	-0.000	-10.000	2.2976E 02	1.1212E-02	-2.2095E 01
			-0.000	-0.000	-10.000	2.5527E 02	-2.7524E 00	-2.3569E 01
23	350.000	0.	0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
24	370.000	40.000	-0.000	-1.000	-0.000	5.1175E 01	-3.2995E 01	7.0363E-01
			-0.000	-0.000	-10.000	2.5086E 02	-2.0084E 01	-2.8607E 01
			-0.000	-1.000	-10.000	3.0203E 02	-5.3079E 01	-2.7904E 01
25	400.000	20.000	-0.000	-0.000	-0.000	6.7125E 01	-2.7400E 00	-3.8350E 00
			-0.000	-0.000	-10.000	2.6233E 02	2.6997E-02	-2.3981E 01
			-0.000	-0.000	-10.000	3.2945E 02	-2.7130E 00	-2.7816E 01
26	400.000	0.	0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.

TABLE IIIa (Contd)

JOINT	-X	-Y	FORCE-X	FORCE-Y	MOMENT	DISPL-X	DISPL-Y	ROTATION
27	420.000	40.000	-0.000	-1.000	-0.000	1.3142E 02	-7.1377E 01	1.9935E 00
			-0.000	-0.000	-10.000	3.1664E 02	-5.3111E 01	-2.7765E 01
			-0.000	-1.000	-10.000	4.4806E 02	-1.2449E 02	-2.5772E 01
28	450.000	20.000	-0.000	-0.000	-0.000	1.7315E 02	-2.7090E 00	-1.0210E 01
			-0.000	-0.000	-10.000	3.5032E 02	1.2445E-01	-2.8422E 01
			-0.000	-0.000	-10.000	5.2347E 02	-2.5845E 00	-3.8632E 01
29	450.000	0.	0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
30	470.000	40.000	-0.000	-1.000	-0.000	3.6209E 02	-1.9548E 02	3.6338E 00
			-0.000	-0.000	-10.000	4.4730E 02	-9.5116E 01	-2.2438E 01
			-0.000	-1.000	-10.000	8.0940E 02	-2.9060E 02	-1.8804E 01
31	500.000	20.000	-0.000	-0.000	-0.000	4.8799E 02	-1.2407E 00	-2.0754E 01
			-0.000	-0.000	-10.000	5.0844E 02	-1.2228E 00	-5.1023E 01
			-0.000	-0.000	-10.000	9.9643E 02	-2.4636E 00	-7.1776E 01
32	500.000	0.	0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.
			0.	0.	0.	0.	0.	0.

TABLE IIIb

TEN-BAY GABLE FRAME: MEMBER FORCES
(DIRECT ANALYSIS)

MEMBER	AXIAL FORCE	SHEAR	MOMENT	AXIAL FORCE	SHEAR	MOMENT	EI	MA	MB
1	6.5381E-01	-4.0955E-01	-4.8696E 00	-6.5381E-01	4.0955E-01	-3.3213E 00	10.000	1	2
	-4.0646E-01	-1.9906E-01	-3.0882E-01	4.0646E-01	1.9906E-01	-3.6725E 00			
	2.4734E-01	-6.0861E-01	-5.1784E 00	-2.4734E-01	6.0861E-01	-6.9938E 00			
2	7.5190E-01	1.7272E-01	3.3213E 00	-7.5190E-01	-1.7272E-01	1.5638E 00	10.000	2	3
	-1.4665E-01	-4.2817E-01	-6.3275E 00	1.4665E-01	4.2817E-01	-5.7830E 00			
	6.0525E-01	-2.5545E-01	-3.0062E 00	-6.0525E-01	2.5545E-01	-4.2191E 00			
3	5.3280E-01	-6.0875E-02	-1.5638E 00	-5.3280E-01	6.0875E-02	-6.3106E-01	10.000	3	4
	3.9109E-01	-2.2777E-01	-4.2170E 00	-3.9109E-01	2.2777E-01	-3.9955E 00			
	9.2389E-01	-2.8865E-01	-5.7809E 00	-9.2389E-01	2.8865E-01	-4.6266E 00			
4	9.6951E-01	-1.1391E-01	-7.6689E-01	-9.6951E-01	1.1391E-01	-1.5112E 00	10.000	4	5
	2.3634E-02	3.3894E-02	-7.5811E-01	-2.3634E-02	-3.3894E-02	1.4360E 00			
	9.9314E-01	-8.0013E-02	-1.5250E 00	-9.9314E-01	8.0013E-02	-7.5259E-02			
5	8.1089E-01	7.0612E-02	1.3980E 00	-8.1089E-01	-7.0612E-02	5.9926E-01	10.000	4	6
	-1.5391E-01	-3.8749E-01	-5.2464E 00	1.5391E-01	3.8749E-01	-5.7136E 00			
	6.5698E-01	-3.1688E-01	-3.8484E 00	-6.5698E-01	3.1688E-01	-5.1143E 00			
6	6.4449E-01	-2.3061E-02	-5.9926E-01	-6.4449E-01	2.3061E-02	-2.3221E-01	10.000	6	7
	3.4978E-01	-2.2691E-01	-4.2864E 00	-3.4978E-01	2.2691E-01	-3.8950E 00			
	9.9427E-01	-2.4997E-01	-4.8857E 00	-9.9427E-01	2.4997E-01	-4.1272E 00			
7	9.8548E-01	-4.5493E-02	-3.1667E-01	-9.8548E-01	4.5493E-02	-5.9319E-01	10.000	7	8
	1.9709E-03	8.3388E-03	-9.9114E-01	-1.9709E-03	-8.3388E-03	1.1579E 00			
	9.8745E-01	-3.7154E-02	-1.3078E 00	-9.8745E-01	3.7154E-02	5.6473E-01			
8	8.3279E-01	2.8179E-02	5.4888E-01	-8.3279E-01	-2.8179E-02	2.4813E-01	10.000	7	9
	-1.5841E-01	-3.8020E-01	-5.1139E 00	1.5841E-01	3.8020E-01	-5.6399E 00			
	6.7438E-01	-3.5202E-01	-4.5650E 00	-6.7438E-01	3.5202E-01	-5.3917E 00			
9	6.9039E-01	-9.9050E-03	-2.4813E-01	-6.9039E-01	9.9050E-03	-1.0900E-01	10.000	9	10
	3.4175E-01	-2.2990E-01	-4.3601E 00	-3.4175E-01	2.2990E-01	-3.9290E 00			
	1.0321E 00	-2.3980E-01	-4.6083E 00	-1.0321E 00	2.3980E-01	-4.0380E 00			
10	9.9445E-01	-1.7083E-02	-1.1983E-01	-9.9445E-01	1.7083E-02	-2.2184E-01	10.000	10	11
	9.8864E-04	3.6520E-03	-1.0213E 00	-9.8864E-04	-3.6520E-03	1.0943E 00			
	9.9544E-01	-1.3431E-02	-1.1411E 00	-9.9544E-01	1.3431E-02	8.7250E-01			
11	8.4095E-01	1.2175E-02	2.2883E-01	-8.4095E-01	-1.2175E-02	1.1553E-01	10.000	10	12
	-1.6030E-01	-3.7692E-01	-5.0497E 00	1.6030E-01	3.7692E-01	-5.6112E 00			
	6.8065E-01	-3.6475E-01	-4.8209E 00	-6.8065E-01	3.6475E-01	-5.4956E 00			
12	7.0768E-01	-5.0463E-03	-1.1553E-01	-7.0768E-01	5.0463E-03	-6.6415E-02	10.000	12	13
	3.3816E-01	-2.3110E-01	-4.3888E 00	-3.3816E-01	2.3110E-01	-3.9436E 00			
	1.0458E 00	-2.3615E-01	-4.5044E 00	-1.0458E 00	2.3615E-01	-4.0101E 00			
13	9.9762E-01	-5.7654E-03	-4.2491E-02	-9.9762E-01	5.7654E-03	-7.2816E-02	10.000	13	14
	1.7233E-04	2.0269E-03	-1.0332E 00	-1.7233E-04	-2.0269E-03	1.0738E 00			
	9.9779E-01	-3.7384E-03	-1.0757E 00	-9.9779E-01	3.7384E-03	1.0010E 00			

TABLE IIIb (Contd)

MEMBER	AXIAL FORCE	SHEAR	MOMENT	AXIAL FORCE	SHEAR	MOMENT	EI	MA	MB
14	8.4334E-01	6.4144E-03	1.0891E-01	-8.4334E-01	-6.4144E-03	7.2522E-02	10.000	13	15
	-1.6161E-01	-3.7536E-01	-5.0231E 00	1.6161E-01	3.7536E-01	-5.5938E 00			
	6.8173E-01	-3.6895E-01	-4.9142E 00	-6.8173E-01	3.6895E-01	-5.5213E 00			
15	7.1380E-01	-3.8295E-03	-7.2522E-02	-7.1380E-01	3.8295E-03	-6.5553E-02	10.000	15	16
	3.3638E-01	-2.3208E-01	-4.4062E 00	-3.3638E-01	2.3208E-01	-3.9617E 00			
	1.0502E 00	-2.3591E-01	-4.4787E 00	-1.0502E 00	2.3591E-01	-4.0272E 00			
16	9.9845E-01	-1.3439E-04	-5.6283E-03	-9.9845E-01	1.3439E-04	2.9405E-03	10.000	16	17
	-4.3977E-04	2.4291E-03	-1.0321E 00	4.3977E-04	-2.4291E-03	1.0807E 00			
	9.9801E-01	2.2947E-03	-1.0377E 00	-9.9801E-01	-2.2947E-03	1.0836E 00			
17	8.4234E-01	5.2220E-03	7.1182E-02	-8.4234E-01	-5.2220E-03	7.6520E-02	10.000	16	18
	-1.6364E-01	-3.7396E-01	-5.0063E 00	1.6364E-01	3.7396E-01	-5.5709E 00			
	6.7870E-01	-3.6874E-01	-4.9351E 00	-6.7870E-01	3.6874E-01	-5.4943E 00			
18	7.1477E-01	-5.0463E-03	-7.6520E-02	-7.1477E-01	5.0463E-03	-1.0543E-01	10.000	18	19
	3.3460E-01	-2.3380E-01	-4.4291E 00	-3.3460E-01	2.3380E-01	-4.0005E 00			
	1.0494E 00	-2.3884E-01	-4.5057E 00	-1.0494E 00	2.3884E-01	-4.1059E 00			
19	9.9774E-01	5.3639E-03	2.7074E-02	-9.9774E-01	-5.3639E-03	8.0204E-02	10.000	19	20
	-1.4850E-03	5.2297E-03	-1.0169E 00	1.4850E-03	-5.2297E-03	1.1215E 00			
	9.9626E-01	1.0594E-02	-9.8984E-01	-9.9626E-01	-1.0594E-02	1.2017E 00			
20	8.3695E-01	7.4187E-03	7.8352E-02	-8.3695E-01	-7.4187E-03	1.3148E-01	10.000	19	21
	-1.6838E-01	-3.7131E-01	-4.9826E 00	1.6838E-01	3.7131E-01	-5.5196E 00			
	6.6857E-01	-3.6389E-01	-4.9042E 00	-6.6857E-01	3.6389E-01	-5.3882E 00			
21	7.1156E-01	-9.8999E-03	-1.3148E-01	-7.1156E-01	9.8999E-03	-2.2547E-01	10.000	21	22
	3.3108E-01	-2.3793E-01	-4.4804E 00	-3.3108E-01	2.3793E-01	-4.0985E 00			
	1.0426E 00	-2.4783E-01	-4.6118E 00	-1.0426E 00	2.4783E-01	-4.3239E 00			
22	9.9483E-01	1.6150E-02	8.7783E-02	-9.9483E-01	-1.6150E-02	2.3522E-01	10.000	22	23
	-4.0360E-03	1.3220E-02	-9.7253E-01	4.0360E-03	-1.3220E-02	1.2369E 00			
	9.9079E-01	2.9370E-02	-8.8474E-01	-9.9079E-01	-2.9370E-02	1.4721E 00			
23	8.2187E-01	1.5181E-02	1.3768E-01	-8.2187E-01	-1.5181E-02	2.9169E-01	10.000	22	24
	-1.8059E-01	-3.6481E-01	-4.9290E 00	1.8059E-01	3.6481E-01	-5.3895E 00			
	6.4129E-01	-3.4963E-01	-4.7913E 00	-6.4129E-01	3.4963E-01	-5.0978E 00			
24	7.0100E-01	-2.3163E-02	-2.9169E-01	-7.0100E-01	2.3163E-02	-5.4345E-01	10.000	24	25
	3.2231E-01	-2.4863E-01	-4.6105E 00	-3.2231E-01	2.4863E-01	-4.3539E 00			
	1.0233E 00	-2.7179E-01	-4.9022E 00	-1.0233E 00	2.7179E-01	-4.8973E 00			
25	9.8633E-01	4.3162E-02	2.3987E-01	-9.8633E-01	-4.3162E-02	6.2337E-01	10.000	25	26
	-9.7180E-03	3.3776E-02	-8.6128E-01	9.7180E-03	-3.3776E-02	1.5368E 00			
	9.7662E-01	7.6939E-02	-6.2141E-01	-9.7662E-01	-7.6939E-02	2.1602E 00			
26	7.8169E-01	3.6038E-02	3.0358E-01	-7.8169E-01	-3.6038E-02	7.1572E-01	10.000	25	27
	-2.1134E-01	-3.4780E-01	-4.7849E 00	2.1134E-01	3.4780E-01	-5.0525E 00			
	5.7034E-01	-3.1176E-01	-4.4813E 00	-5.7034E-01	3.1176E-01	-4.3368E 00			

TABLE IIIb (Contd)

MEMBER	AXIAL FORCE	SHEAR	MOMENT	AXIAL FORCE	SHEAR	MOMENT	EI	MA	MB
27	6.7266E-01	-5.8476E-02	-7.1572E-01	-6.7266E-01	5.8476E-02	-1.3926E 00	10.000	27	28
	2.9960E-01	-2.7545E-01	-4.9475E 00	-2.9960E-01	2.7545E-01	-4.9839E 00			
	9.7226E-01	-3.3392E-01	-5.6632E 00	-9.7226E-01	3.3392E-01	-6.3766E 00			
28	9.7515E-01	1.0657E-01	5.5524E-01	-9.7515E-01	-1.0657E-01	1.5762E 00	10.000	28	29
	-4.4798E-02	9.9162E-02	-4.2947E-01	4.4798E-02	-9.9162E-02	2.4127E 00			
	9.3035E-01	2.0573E-01	1.2577E-01	-9.3035E-01	-2.0573E-01	3.9889E 00			
29	6.8876E-01	9.3824E-02	8.3741E-01	-6.8876E-01	-9.3824E-02	1.8163E 00	10.000	28	30
	-3.1314E-01	-3.0936E-01	-4.5866E 00	3.1314E-01	3.0936E-01	-4.1635E 00			
	3.7562E-01	-2.1554E-01	-3.7492E 00	-3.7562E-01	2.1554E-01	-2.3471E 00			
30	5.9777E-01	-1.3827E-01	-1.8163E 00	-5.9777E-01	1.3827E-01	-3.1691E 00	10.000	30	31
	2.4194E-01	-3.6773E-01	-5.8365E 00	-2.4194E-01	3.6773E-01	-7.4221E 00			
	8.3971E-01	-5.0600E-01	-7.6529E 00	-8.3971E-01	5.0600E-01	-1.0591E 01			
31	4.4663E-01	4.2068E-01	3.1691E 00	-4.4663E-01	-4.2068E-01	5.2445E 00	10.000	31	32
	4.4018E-01	-2.6726E-03	-2.5779E 00	-4.4018E-01	2.6726E-03	2.5244E 00			
	8.8681E-01	4.1801E-01	5.9125E-01	-8.8681E-01	-4.1801E-01	7.7689E 00			

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13. ABSTRACT

An iterative method to analyze large structural systems subjected to static and thermal loadings is developed. This method is essentially the displacement method of analysis in which the finite elements are grouped into convenient size substructures. The analysis proceeds with one substructure at a time, and a process of iteration establishes the continuity of the system. The speed of convergence depends on the nature of the stiffness matrices of the substructures. To illustrate the method, two frame structures are analyzed One is an 1800 degree of freedom system and the other a smaller system. The purpose of the larger one is to estimate the computational time involved. The results of the smaller system (by iterative analysis) are compared with those obtained from direct analysis, and the results are nearly the same. The main advantages of the iteration method are that the entire computational scheme can be carried out without exceeding the core capacity of the computer, and the method is more economical with respect to computational time than the direct analysis.

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Iteration						
Substructures						
Finite Element Analysis						
Frames, plates, and shells						
Automated Analysis						
Structural Synthesis						

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