FOREWORD

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ABSTRACT

Analyses of three nonelectrical methods for maintaining frozen foods between -10°F and +5°F during aerospace missions of 1 to 28 days are presented. The methods considered are: (1) a heat sink, (2) active refrigeration, and (3) radiation cooling. All methods appear feasible. The heat sink method appears to be the simplest and most reliable. A design study of a heat sink system for a 3-man, 14-day mission is included.

PUBLICATION REVIEW

This technical documentary report has been reviewed and is approved.

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SECTION I

INTRODUCTION

The objectives of the work were: (1) to determine feasibility and comparative performance of frozen food storage units for space application; and (2) to make a detailed design for a specific mission. No electrical power was considered for the operation of the food storage device.

There are several reasons for having frozen food available on board a space vehicle. The primary one is to provide a diet with a sufficient quantity of fresh protein. The secondary aim is to avoid a monotonous diet which would be irritating on long missions and might lead to inefficiency of the space vehicle personnel. Frozen food may be packed with minimum weight and it has been suggested that the packaging itself could be of edible material. Unlike dehydrated foods, frozen food is immediately available for consumption upon thawing and does not require additional preparation.

There are many concepts which one might employ to maintain food in a frozen state during a space mission. One approach would be to use a conventional refrigeration system taking into account the various heat loads and the consumption rate of food required. To employ such a conventional approach would require the availability of an adequate supply of electrical power. Electrical power is, however, a precious commodity in a space vehicle and one with which a severe weight penalty is invariably associated. A frozen food storage system which could operate satisfactorily for extended duration missions without the requirement for electrical power would be quite useful. It would, other factors being equal, reduce the take-off weight of the vehicle by reducing the space electrical power requirement; and secondly, it would increase the probability of survival of the crew since an electrical failure would not lead directly to loss of food supply.

The work was divided broadly into two phases. In the first phase, an analytical study was conducted in which several possible techniques were evaluated and one of these selected for a detailed design analysis. The second phase of the program consisted of a detailed design analysis for a food storage system for a typical three-man two-week mission. The results of the investigations are presented in the subsequent portions of this report.

SECTION II

FEASIBILITY STUDY

The first phase of the effort was a three month feasibility study during which several alternative methods for preservation of frozen foods in the space capsule without the use of electrical power were investigated. Many types of food storage systems were considered, and three which appeared most promising were analyzed in sufficient detail to permit a system comparison and the selection of one of the systems for further detailed investigation.

Aside from structural considerations, the principal requirements were that frozen food should be maintained at a temperature between -10°F and +5°F for periods up to one month with sufficient food supply for one to six men. In addition, a food chilling compartment was required which would be suitable for maintaining food between 33°F and 40°F after removing the food from the freezer. Finally, a cabin temperature of between 60°F and 75°F was assumed. The principal problems involved in determining the feasibility of the various systems concerned were that of the heat load imposed on the frozen food due to heat leak from the cabin into the storage unit and potential heat leak from the outside of the vehicle.

DESCRIPTION OF SYSTEMS STUDIED

The three types of systems which were compared in the feasibility study are described briefly below.

The Heat Sink

The first system analyzed was a heat sink system. The concept is that the thermal capacity of the frozen food between -10°F and +5°F can be equated to the heat entering the compartment by heat leakage from the cabin. While the heat capacity of the food is small, the leakage can be reduced to a small value by use of high performance insulation. The so-called superinsulations which have been developed during the last several years for cryogenic tankage applications would be employed. These insulators have a thermal conductivity which is less than 1/1000th of the conductivity of conventional freezer insulation.

Cooling by Liquid Hydrogen

Another means of disposing of part or all of the heat leaking into the food storage unit would be to use a refrigerant. To achieve constant temperature storage, the refrigerant flow rate would be related to the heat leakage rate. Liquid hydrogen has a very large thermal capacity between its boiling point at less than -400°F and the temperatures for which frozen food storage is desired in the present analysis. This heat capacity, which is on the order of 1429 BTU per pound is almost 200 times greater than the thermal capacity of the frozen food between -10°F and +5°F. In addition, hydrogen is an excellent heat exchanger working substance. The liquid hydrogen could be used to absorb all the heat entering the compartment, or it could be used in conjunction with the thermal capacity of the food to reduce the weight of insulation. Liquid hydrogen is a propellant which will be used for many space missions in bipropellant rocket systems and in nuclear propulsion systems. It is reasonable to consider its use in space system applications of the sort considered here.

Other refrigerants can also be considered in addition to hydrogen. However, the best of these, liquid ammonia, offers far less cooling per unit weight, and a much larger weight of coolant would be required. In addition, it is not likely that ammonia would be on board for other reasons. Since an additional logistics problem would be involved in its use, it was not considered further.

Cooling by Radiation

The third concept considered for a nonelectrical food storage system requires an external radiating surface to reject the heat load entering the frozen food compartment by leakage from the cabin. By using suitable coating materials such as aluminium oxide which have high emissivity at low temperature and low absorbtivity to solar radiation, it is possible to maintain an equilibrium external temperature on a radiator panel below the maximum allowable food storage temperature. By suitably insulating the storage unit to minimize the heat leakage, such a radiator can be used to reject the small leakage which does occur from the vehicle.

PROBLEMS CONSIDERED

A list of the various problems considered during the analyses is presented below. These problems are subdivided into two sections: problems common to all systems, and problems unique to individual systems.

Problems Common to All Systems

Several problems are common to all of the systems. These are all soluble and were attacked in the design phase of the contract. They do not affect the relative merits or feasibility of any of the systems. They are:

a. Removal of the food from the freezer to the chiller.

b. Design of the freezer door. (Its insulation should be nearly as good as that of the freezer walls.)

c. Structures internal to the freezer. (These should be lightweight and must be integrated with the design of the removal system.)

d. Temperature monitoring.

e. Design of the chiller and its temperature control, if necessary.

Problems Unique to Individual Systems

These problems are unique to each specific system:

a. Heat Sink

1) The superinsulation requires a high vacuum to be effective; this is a possible weak point due to leakage. However, superinsulation has been used commercially for several years in large mobile tanks successfully.

2) The freezer must be precooled to the food temperature. Otherwise, until equilibrium is established, there is considerable deterioration of insulating properties.

b. Hydrogen Coolant

1) Additional piping and valving are required (it is assumed that the LH_2 is already on board for other purposes).

2) A heat exchanger must be integrated with the internal construction of the freezer. (This causes an increase in weight.)

3) The heat flux into the freezer and the LH_2 plumbing may complicate the heat balance of the cabin. This flux is very small for the heat sink system.

c. Radiator

1) The radiator is exposed to a variable thermal load as it passes from sunlight to shadow during its space voyage. In fact, no entirely passive system was found which would be satisfactory with a single flat plate radiator.

2) The use of an extended surface radiator would result in a satisfactory passive system but would in turn involve complications in terms of integration with the vehicle structure.

COMPARISON OF SYSTEMS CONSIDERED

The detailed thermal and weight analysis of the systems is presented in Appendix I. Some of the results of that analysis are contained in Table I. It may be observed by examination of the table that when superinsulation is used there is no significant difference between the weights of the various systems. In addition, it should be noted that for all systems using superinsulation, the weight of the food storage system (exclusive of hardware and structural considerations) is less than 5% of the total food weight. Weight,

TABLE I

SUMMARY OF THERMAL AND WEIGHT ANALYSIS

System	Weight Comparison						Overall Volume	Complexity	Moving Parts Peculiar to the System	Hazard
	Least Favorable Case			Most Favorable Case						
	$R = \frac{W_S}{W_F}$	No. of Men	No. of Days	R	No. of Men	No. of Days				
Super-Insulation Heat Sink	4.8×10^{-2}	1	28	0.63×10^{-2}	6	7	Most Compact	Simplest	None	Leak in super-insulation
LH_2, Corkboard Insulation	51×10^{-2}	1	28	22×10^{-2}	6	7	Least Compact	More complex (valves, ducting, heat ex-changer)	Valves, possibly a small fan	Hydrogen leak, possible fire or ex-plosion
LH_2, Super-Insulation	2.6×10^{-2}	1	28	1.4×10^{-2}	6	7	Least Compact	More complex (valves, ducting, heat ex-changer)	Valves, possibly a small fan	Hydrogen leak, possible fire or ex-plosion
Radiator, Corkboard Insulation	14×10^{-2}	1	7	4.7×10^{-2}	6	28	Slightly larger than heat sink	Require attitude control or servo con-trolled blinds	Servo	Servo failure
Radiator, Super-Insulation	1.4×10^{-2}	1	7	0.50×10^{-2}	6	28	Slightly larger than heat sink	Require attitude control or servo controlled blinds	Servo	Leak in superinsula-tion servo failure

then, is not a deciding factor in system selection. It should be further noted that each of the systems is feasible and each has special advantages in a given mission. The only significant difference is their relative complexity and flexibility with regard to integration into the space capsule.

It is evident from the system descriptions and from the analytical studies that the heat sink system is to be preferred over the other two approaches. It involves no special logistic, handling, or vehicle integration problems and should have an extremely high reliability.

The liquid hydrogen system combined with superinsulation saves very little in system weight and adds the additional complexity of valving, ducts, heat exchanger, controls, and perhaps even auxiliary hydrogen tankage. It offers no significant improvements in system design or endurance.

The radiator concept combined with superinsulation was the lightest of the systems considered. It has the advantage of unlimited duration capability and deserves special consideration.

The principal problem unique to the radiator cooling system is the presence or absence of solar radiation. With the flat plate radiator, it is possible to maintain the freezer temperature in direct sunlight below $+5^{o}F$, but when the sunlight is removed the temperature will fall below the $-10^{o}F$ minimum allowed.. Such a heat loss can be avoided by means of an internal variable radiation shield which could be thermostatically controlled. This problem would not arise in a vehicle which is attitude controlled with respect to the sun.

An entirely passive thermal radiation cooled food storage system can be devised if some means can be taken to make the radiating area different from the area exposed to normal incident solar radiation. An extended radiating surface projecting from the space capsule or the aft end of a cylindrical space vehicle might provide the necessary ratio of radiating to absorbing surface to keep the temperatures within the specified limits. This, however, involves an extreme degree of integration between the food storage unit and the space capsule design.

In both of the systems considered (passive and combination), it is necessary to have thermal contact with the outside of the vehicle and to specially treat a portion of the vehicle surface.

There may be extended duration applications where these design complications will be warranted, and the unlimited endurance capability of the radiator system will justify its use. This is certainly <u>not</u> the case for the mission duration or number of personnel which were considered during the present investigation.

RECOMMENDATIONS

From the comparisons presented above, the heat sink system was considered to be superior for the proposed application. It was, therefore, recommended at the conclusion of the feasibility study that a detailed design study and thermal analysis of the heat sink system with superinsulation be prepared.

The recommendation was based upon the following advantages of the heat sink system:

1. Low weight.

2. Compactness.

3. Reliability. (No moving parts other than the food removal device; no piping, no valves, no heat exchangers, no system intergration problems, no thermostatic control.)

4. Applicability to all of the missions considered (not limited by availability of liquid hydrogen or feasibility of integration with vehicle surface).

SECTION III

DESIGN STUDY

Upon completion of the feasibility study, a detailed design study of the heat sink frozen food storage unit for a three-man two-week mission was conducted. The result of that design study is described below.

GENERAL DESCRIPTION

The frozen food storage unit, Figure 1, is a cylinder approximately two feet long and one foot in diameter. It is made in two sections, the front half a chiller, the rear a freezer compartment. Each section has an inner and an outer shell.

The chiller assembly has fiberglas wool insulation between its two shells which are of reinforced fiberglas. It is attached to the freezer with a clamp that extends around its circumference.

The door jambs for both the chiller and the freezer are of reinforced fiberglas. The chiller compartment door is insulated with fiberglas wool, and the freezer compartment door is insulated with superinsulation.

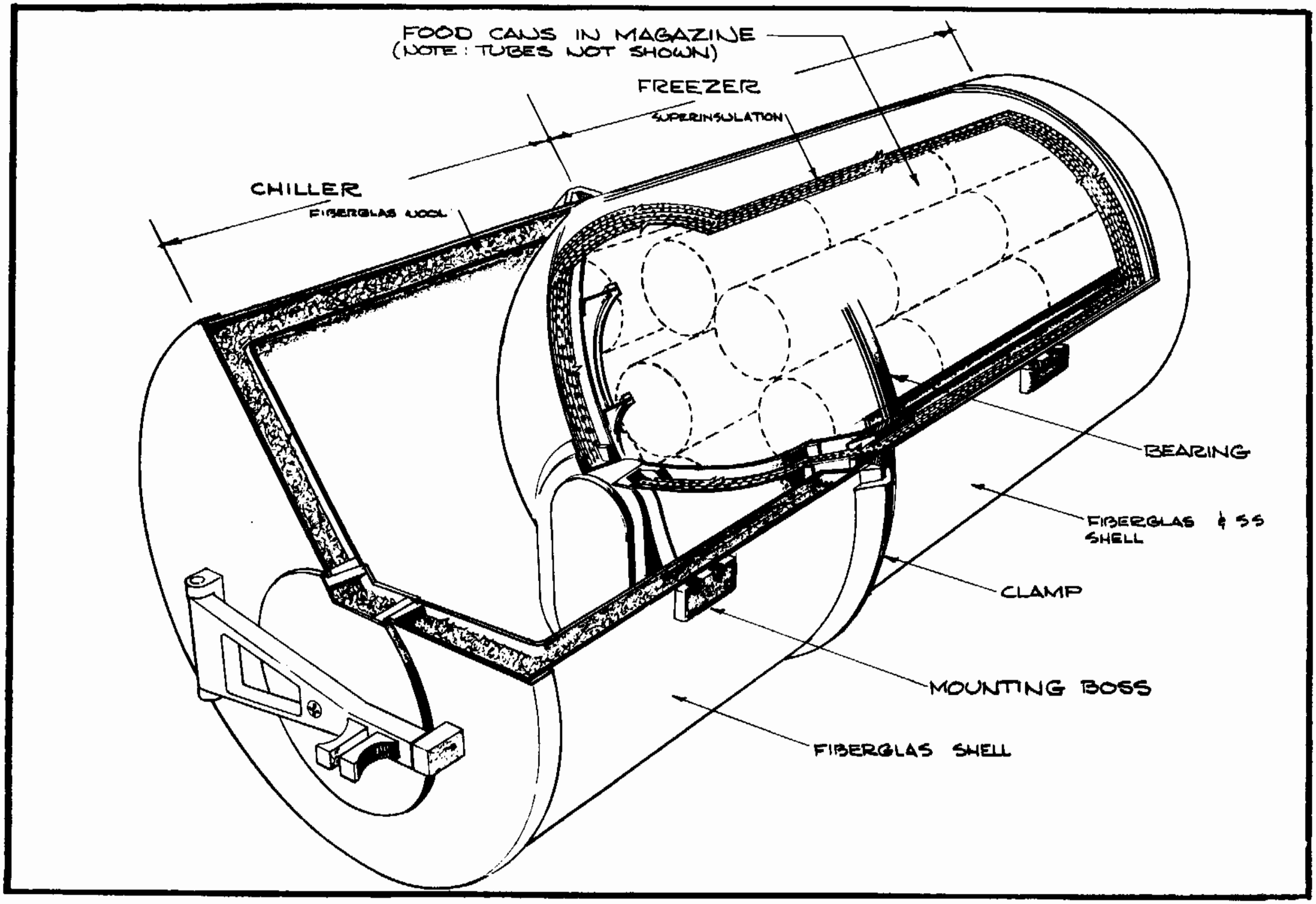
The freezer compartment has superinsulation between its two stainless steel shells. The outside shell is reinforced with a layer of fiberglas. The inner shell is separated from the outer shell by 8 legs of compressed superinsulation. Two large ring bearings which take thrust loads in all directions are installed inside the inner shell. These bearings support the magazine which holds individual tubes for the food cans. The voids in the magazine are filled with foamed plastic. Each tube in the magazine has a spring at the rear, and when the magazine is rotated with the fingers to position a tube in front of the opened freezer door, a string of three connected cans moves forward for easy removal.

A thermocouple is located in both the freezer and chiller, with an output for temperature monitoring located on the face of the chiller.

Attachment points for mounting the complete unit are located at the mid-line, two points on each side.

The unit is designed to withstand a typical space mission environment profile.

A detailed design analysis of the heat sink frozen food storage unit is presented in Appendix II. This analysis includes the following design calculations; determination of minimum freezer wall thickness, and stress analysis of the critical parts of the system.



FROZEN FOOD STORAGE UNIT EMPLOYING SUPERINSULATION

FIGURE 1

SECTION IV

CONCLUSIONS

The analytical and design investigations show that the preservation of frozen foods without the use of electrical power during aerospace missions is feasible and practical both from a fabrication and weight viewpoint. It is evident from the analytical study that there are three feasible techniques for storing frozen food at temperatures between $-10^{\circ}F$ and $+5^{\circ}F$ in comparatively large quantity for durations of up to one month and longer. All of the systems investigated are simple and should be as reliable as any other portion of the space vehicle apparatus. The heat sink superinsulation system in particular is extremely simple and virtually foolproof. The system is entirely self contained and requires no special integration with the vehicle. There are no logistic or special handling problems which arise in connection with this system. It may be that the exercise of greater ingenuity could reduce some of the weights, but it is doubtful that there is any other approach to frozen food storage for medium duration space vehicle applications which is significantly lighter than the heat sink concept presented here.

The two radiation cooled food storage systems considered have the advantage of indefinite endurance capability. One concept involves thermostatically controlled internal radiation shields. The other concept involves extended radiating surfaces which provide more surface for heat rejection than is exposed to solar irradiation. Both systems require a high degree of integration with the space capsule design including special surface treatment and thermal contact between the storage unit and the outside surface. These complications are not warranted for missions of the duration of those studied here (one month maximum). They might be justified, however, for missions of many months duration.

The detailed design of a three-man two-week mission frozen food storage system is presented in Section III of this report. The system utilizes a cylinder 26 inches long and 12 inches in diameter to store 20 pounds of frozen food in a semiautomatic ejection cartridge. It will deliver frozen food as required and will keep it in a chiller compartment for short periods of interim storage at a safe temperature. The design is structurally sound and capable of sustaining both ground handling and launching loads. The detailed thermal analysis of this system indicates that the detailed analysis of local heat losses through structural components is of utmost importance. It is believed that the system as designed is adequate for the stated purpose.

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APPENDIX I

THERMAL ANALYSES OF SEVERAL SPACE FOOD STORAGE SYSTEMS

A. Nomenclature and Values of Physical Constants Used

1. Nomenclature

A_1	Area of the radiating surface
A_2	Area of the remainder of the freezer
A	Total area of the freezer
C_F	Specific heat of the frozen food
C	Constant of integration
C_A	Specific heat of chiller air
d	Thickness of freezer wall
d_s	Thickness of steel shell for freezer wall
H	Heat removal by Hydrogen from its boiling point to average freezer temperature
k	Thermal conductivity of insulation
m	Mass of food in the freezer at any time
m_H	Total mass of Hydrogen required for mission
m_i	Mass of insulation
Δm	Mass of last food packages removed from freezer
$\dot{m}_H$	Mass flow of Hydrogen into the freezer
m_o	Initial mass of frozen food
$\dot{m}$	Rate of removal of frozen food
$\dot{m}_A$	Rate of admittance of air from the chiller into the freezer
N	Number of men on the mission
$\dot{Q}$	Heat flow into the freezer

R	Ratio of weights peculiar to each system to weight of food
T	Temperature of the frozen food at any time
T_A	Ambient temperature in the cabin
T_c	Temperature of chiller air
T_F	Final temperature of frozen food (while still in freezer)
T_o	Initial temperature of frozen food
t	Time
ρ	Density of frozen food
ρ_i	Density of insulation
ρ_s	Density of steel
σ	Stephan-Boltzman constant
τ	Number of days for the mission
α	Absorptivity of a substance
ϵ	Emissivity of a substance
φ	Solar constant

2. Values of Physical Constants Used

$C_F = 0.480 \frac{BTU}{lb\ ^\circ F}$

$C_A = 0.240 \frac{BTU}{lb\ ^\circ F}$

d_s = 10 mils (= 8.35×10^{-4} ft. assumes honeycomb sandwich construction)

$H = 1429 \frac{BTU}{lb}$

k = For corkboard insulator = $2.25 \times 10^{-2} \frac{BTU}{ft\ ^\circ F\ hr}$

For SI-4 (superinsulation) = $2.50 \times 10^{-5} \frac{BTU}{hr\ ft\ ^\circ F}$

$T_A = 70^\circ F$

$T_c = 36^\circ F$

$T_F = 5^\circ F$

$T_o = -10^\circ F$

$\alpha = 0.16$ (for Al_2O_3)

$\epsilon = 0.98$ (for Al_2O_3)

$\rho = 57.2 \frac{lbs}{ft^3}$

$\rho_i =$ For corkboard insulation = 7.0 lbs/ft^3

For SI-4 (superinsulation) = 4.7 lbs/ft^3

$\rho_s = 490\ lbs/ft^3$

$\sigma = 0.1713 \times 10^{-8} \frac{BTU}{ft^2\ hr\ (deg\ R)^4}$

$\varphi = 424 \frac{BTU}{ft^2\ hr}$

3. Notes

ℓn means natural logarithm (base e).

Log means Naperian logarithm (base 10).

B. Physical Assumptions and Analytical Approach

1. On the Specific Systems

a. Heat Sink With Superinsulation

We have assumed the use of SI-4. It is competitive with other insulations of this type, has very low density, some degree of structural rigidity, is commercially available, and its physical properties are well known. *

*Riede, P. M. and D. I-J. Wang, "Characteristics and Applications of Some Superinsulations," D-4, Advances in Cryogenic Engineering, Vol. 5, K. D. Timmerhaus (Editor) Pelnum Press, N. Y., 1960, pp. 209.

b. LH_2 System

The weight of the heat exchange is assumed to be negligible since Hydrogen is a very good heat exchange medium and the state of the art in design of light and compact heat exchangers is well advanced. Weights of external piping and valves were ignored since they will depend on specific mission situations, cabin configuration, and location of the LH_2 tanks. This makes the weight estimates definitely optimistic.

c. Radiator System

The portion of the outside of the hull of which a wall of the freezer is a part, may have a surface with a very low α/ϵ ratio. The best substance examined was lead carbonate ($PbCO_3$) with an α/ϵ of 0.13. There is a possibility of this evaporating in a high vacuum environment. On the other hand, Al_2O_3 (which is quite safe from this point of view) has an α/ϵ ratio of only 0.16.

2. Assumptions Common to All Three Systems

a. A range of missions from one astronaut for one week to six astronauts for 4 weeks are considered.

b. The food is assumed to be at $-10^{\circ}F$ initially, and is not to be allowed to go over $+5^{\circ}F$.

c. Cabin temperature is taken as $70^{\circ}F$.

d. The frozen food is assumed to have the physical properties of ice while in the freezer.

e. One and one-half pounds of food per day per man is consumed.

f. The freezer is assumed to have an adjoining chill compartment where the food is defrosted.

g. The astronauts are assumed to eat three evenly spaced meals every twenty-four hours. This assumption is made to simplify the analysis. However, some work-rest cycles proposed are compatible with evenly spaced meals.

h. Sources of heat transfer are taken as conduction through the insulation, removal of heat with removal of frozen food, and entrance of chiller air into the freezer at this time.

i. The door from the freezer to the chiller is to be mostly of superinsulation. Supporting structure is assumed to be of insulating plastic of negligible outer surface area. This assumption will be reexamined when a detailed design is completed.

j. The food is assumed to have physical contact with the walls of the freezer or with structures connected to them.

k. The freezer is assumed to be cubical in shape.

l. It is assumed that the superinsulation has been cooled to equilibrium temperature prior to lift-off. Otherwise there will be considerable effective deterioration of insulation properties.*

3. Analytical Approach

The exact equation for heat transfer through the walls of the cube is a partial differential equation in three independent variables. General solutions of similar equations are known (however, we have the complication here of a disappearing heat sink).

For simplicity in analysis, several approximations have been made: (1) thermal gradients within the box are small compared to those in the insulation (their conductivities differ by a factor of 4×10^4); (2) the rate of change of temperature of the food is slow; (3) the variations in temperature gradients at the edges of the cube are negligible (this is conservative, since the insulation is thicker there); (4) the discontinuous removal of food and admittance of chiller air is approximated by removal at a constant average rate.

C. Analysis of the Heat Sink with Superinsulation

We wish to determine the thickness, and ultimately the weight, of insulation required in this system.

We equate the rate at which heat is admitted into the freezer with the rate of change of enthalpy in the freezer.

$$\dot{Q} = \frac{kA}{d} (T_A - T) + \dot{m}_A (T_c - T) C_A + \dot{m}_A (T - T_o) C_F$$

$$= \frac{d}{dt} \left[m C_F (T - T_o) \right] \tag{1}$$

*Stoy, S. T., D-5 "Cryogenic Insulation Development," Advances in Cryogenic Engineering, Vol. 5, K. D. Timmerhaus (Editor), Pelnum Press, N. Y. 1960, pp. 216.

This reduces to

$$mC_F \frac{dT}{dt} + T\left(\frac{kA}{d} + \dot{m}_A C_A\right) = \frac{kA}{d} T_A + \dot{m}_A T_c C_A \tag{2}$$

Letting

$$m = m_o - \dot{m}t$$

and taking for brevity

$$\alpha = \frac{1}{\dot{m}C_F}\left(\frac{kA}{d} + \dot{m}_A C_A\right)$$

and

$$\beta = \frac{1}{\dot{m}C_F}\left(\frac{kA\ T_A}{d} + C_A \dot{m}_A T_c\right)$$

$$\frac{dT}{dt} + \frac{\alpha}{\frac{m_o}{\dot{m}} - t} T = \frac{\beta}{\frac{m_o}{\dot{m}} - t} \tag{3}$$

The integrating factor for the above expression is given by

$$\exp\left[\int \frac{\alpha}{\frac{m_o}{\dot{m}} - t} dt\right] = \exp\left[-\alpha \ln\left(\frac{m_o}{\dot{m}} - t\right)\right]$$

$$= \left(\frac{m_o}{\dot{m}} - t\right)^{-\alpha} \tag{4}$$

Multiplying through on both sides by this expression and integrating, we have at once (letting C be the constant of integration),

$$\left(\frac{m_o}{\dot{m}} - t\right)^{-\alpha} T = \beta \int \left(\frac{m_o}{\dot{m}} - t\right)^{-\alpha-1} dt$$

$$= \frac{\beta}{\alpha}\left(\frac{m_o}{\dot{m}} - t\right)^{-\alpha} + C \tag{5}$$

Evaluating the constant at $t = o$,

$$= \left(\frac{m_o}{\dot{m}}\right)^{-\alpha} T_o - \frac{\beta}{\alpha}\left(\frac{m_o}{\dot{m}}\right)^{-\alpha} \tag{6}$$

then

$$T = \frac{\beta}{\alpha} + \left(\frac{m_o}{\dot{m}} - t\right)^{\alpha} \left(\frac{m_o}{\dot{m}}\right)^{-\alpha} \left(T_o - \frac{\beta}{\alpha}\right)$$

or

$$T = \frac{\beta}{\alpha} + \left(1 - \frac{\dot{m}}{m_o} t\right)^{\alpha} \left(T_o - \frac{\beta}{\alpha}\right) \tag{7}$$

The expression is valid until the point where all the food is exhausted. This of course, leads to $T_F \approx T_A$ at the end of the mission since our heat sink has disappeared. Practically, we are interested in the temperature before the removal of the last package. That is, we take our final temperature at

$$t = \frac{m_o - \Delta m}{\dot{m}}$$

where Δm is the weight of the last food package removed.

The term in $\dot{m}_A C_A$ contributes less than 0.7% to the value of α or β. Since it increases the difficutly of calculating d, it was ignored.

Neglecting this term, we have

$$T = T_A + \left(1 - \frac{\dot{m}}{m_o} t\right)^{\alpha} \left(T_o - T_A\right) \tag{8}$$

Evaluating this for the last package,

$$T_F = T_A + \left(\frac{\Delta m}{m_o}\right)^{\alpha} \left(T_o - T_A\right) \tag{9}$$

$$\alpha = \frac{\ln\left(\dfrac{T_A - T_F}{T_A - T_o}\right)}{\ln\left(\dfrac{\Delta m}{m_o}\right)} \tag{10}$$

$$= \frac{kA}{\dot{m} C_F d}$$

Finally,

$$d = \frac{kA}{\dot{m} C_F} \; \frac{\ln\left(\dfrac{\Delta m}{m_o}\right)}{\ln\left(\dfrac{T_A - T_F}{T_A - T_o}\right)} \tag{11}$$

Now we wish to determine the ratio of the weight of insulation and supporting structure to the weight of food. For the supporting structure, we have assumed a honeycomb sandwich of structural steel using a total thickness of 10 mils. Note that the SI-4 itself will carry a moderate load without losing its insulating properties.

$$R = \frac{Ad\rho i}{m_o} + \frac{Ad_s \rho_s}{m_o} \tag{12}$$

Substituting for d,

$$R = \frac{\rho i k A^2 \ln\left(\frac{\Delta m}{m_o}\right)}{m_o \dot{m} C_F \ln\left(\frac{T_A - T_F}{T_A - T_o}\right)} + \frac{Ad_s \rho_s}{m_o} \tag{13}$$

Assuming 1.5 #/day of food per person,

$$m_o = 1.5 \, N\tau \tag{14}$$

$$\dot{m} = 1.5 \, N \tag{15}$$

$$A = 6 \left(\frac{1.5 \, N\tau}{\rho}\right)^{2/3} \tag{16}$$

If we now assume that the last package remains until 8 hours before the end of the flight we can write

$$\frac{\Delta m}{m_o} = \frac{1}{3\tau} \tag{17}$$

This gives us

$$R = 8.20 \times 10^{-3} \, N^{-2/3} \, \tau^{1/3} \log (3\tau) + 2.74 \times 10^{-2} \, (N\tau)^{-1/3} \tag{18}$$

Extreme results (most favorable and least favorable cases) are noted in Table 1 in the main body of the report.

D. Refrigeration With LH_2

In conjunction with the liquid hydrogen refrigeration we have examined two types of insulation, corkboard and SI-4. Corkboard has nearly the lowest thermal conductivity of any conventional insulators in this temperature range, and a very low density. (Chemically treated wood fibers have lower density with about the same conductivity. Supporting structures would be required, however, so that it is an inferior material for this application.) SI-4 is the superinsulator which was used in the heat sink analysis.

Proceeding as in the analysis of the heat sink,

$$\dot{Q} = \frac{kA}{d}(T_A - T) + \dot{m}(T - T_o) - \dot{m}_H H = \frac{d}{dt}\left[m C_F (T - T_o)\right] \quad (19)$$

expanding the derivative,

$$mC_F \dot{T} + \frac{kA}{d} T = \frac{T_A kA}{d} - \dot{m}_H H \quad (20)$$

This is the same basic form as the heat sink equation. We may then write

$$T = \frac{\beta}{\alpha} + \left(1 - \frac{\dot{m}}{m_o} t\right)^{\alpha} \left(T_o - \frac{\beta}{\alpha}\right) \quad (21)$$

where, however, β and α are now defined as

$$\alpha = \frac{kA}{\dot{m} C_F d} \quad (22)$$

and

$$\beta = \frac{1}{\dot{m} C_F}\left(\frac{kA\, T_A}{d} - \dot{m}_H H\right)$$

and we have again taken $m = m_o - \dot{m}t$,

also,

$$T_F = \frac{\beta}{\alpha} + \left(\frac{\Delta m}{m_o}\right)^{\alpha} \left(T_o - \frac{\beta}{\alpha}\right) \quad (23)$$

There is a difference between this case and the heat sink case examined previously. The heat sink disappearance when the last package of food is removed will not result in a sudden rise in ambient temperature since the LH_2 itself is acting as a continual heat sink. We may thus achieve a considerable simplification in the analysis by setting $\Delta m = 0$.

$$T_F = \frac{\beta}{\alpha} = T_A - \frac{\dot{m}_H H d}{kA} \quad (24)$$

solving for $\dot{m}_H$,

$$\dot{m}_H = (T_A - T_F)\,\frac{kA}{Hd} \tag{25}$$

We now define a new ratio similar to the one used in the heat sink analysis.

Let

$$R = \frac{m_H + m_i}{m_o} \tag{26}$$

Using equations 14 and 16 of the heat sink analysis, we obtain at once

$$\frac{m_i}{m_o} = \frac{\rho_i\, dA}{1.5\, N\tau}$$

$$= \frac{0.354\, \rho_i\, d}{(N\tau)^{1/3}} \tag{27}$$

Multiplying 7 by τ (multiplied by 24 to make the units consistent) we have

$$\frac{m_H}{m_o} = \frac{24\; kA}{1.5\; dNH}\,(T_A - T_F) \tag{28}$$

Evaluating A as before, this becomes

$$\frac{m_H}{m_o} = \frac{5.83 \times 10^{-5} k\tau^{2/3}}{d\,N^{1/3}}\,(T_A - T_F) \tag{29}$$

Combining 29 and 27

$$R = \frac{5.83 \times 10^{-3}\, k\tau^{2/3}}{dN^{1/3}}\,(T_A - T_F) + \frac{0.354\,\rho_i\, d}{(N\tau)^{1/3}} \tag{30}$$

We wish to find the optimum thickness (d) which gives lowest R.

Temporarily writing R as

$$R = \frac{a}{d} + bd \tag{31}$$

we differentiate to obtain

$$\frac{dR}{dd} = -\frac{a}{d^2} + b$$

$$= 0 \tag{32}$$

for a stationary value.

Then

$$d = \sqrt{\frac{a}{b}} \tag{33}$$

Taking the second derivative, we find

$$\frac{d^2 R}{dd^2} = \frac{2a}{d^3} > o, \text{ so that the stationary value is a minimum.} \tag{34}$$

Substituting from 33 back into 31,

$$R_{opt} = 2\sqrt{ab} \tag{35}$$

giving the interesting result that the weight of insulation and hydrogen are equal.

Substituting back, we find that

$$R_{opt} = 0.735\sqrt{\frac{k\tau^{1/3}\rho_i}{N^{2/3}}} \tag{36}$$

$$d = 1.035\sqrt{\frac{k\tau}{\rho_i}} \tag{37}$$

Some of these results, for corkboard and SI-4, are in the table in the main body of the report.

E. Refrigeration by a Radiator

The same type of rough calculations made in the LH_2 case showed that insulation is necessary. For the same reasons, we will consider both corkboard and superinsulation. Several subsystems are examined below.

1. Flat Radiator Systems

We first consider the design where a wall of the freezer is part of an Al_2O_3 coated flat surface of the vehicle skin.

a. Passive System, Flat Radiator

Before calculating weight, we wish to show that a completely passive radiation system (with the flat surface as the radiator) will not meet the contract specifications. To demonstrate this, consider a system stabilized at $T_F(-5^oF)$. This is not economical in terms of insulator weight, but will provide an extreme case. To meet this, in full sun, we have

$$A_1 \alpha \varphi + A_2 \frac{k(T_A - T_F)}{d} = \epsilon \sigma T_F{}^4 A_1 \tag{38}$$

When simply radiating into space, we have

$$A_2 \frac{k(T_A - T)}{d} = \epsilon \sigma T^4 A_1 - mC_F \frac{dT}{dt} \tag{39}$$

Let us assume the times involved are short enough so that the variation in m due to food removal may be neglected.

Let us replace $\frac{dT}{dt}$ by $\frac{T_F - T_o}{t}$, where t is the time required to reach the minimum allowable temperature.

Eliminating the term in k between the two equations, we find (setting $T = T_F$ on the left hand side of 2, which is a good approximation),

$$A_1 \alpha \varphi = \epsilon \sigma A_1 (T_F{}^4 - T^4) A_1 + mC_F \frac{(T_F - T_o)}{t} \tag{40}$$

Solving for t, we obtain

$$t = \frac{mC_F (T_F - T_o)}{A_1 \alpha \varphi - \epsilon \sigma A_1 (T_F{}^4 - T^4)} \tag{41}$$

Let us assume that $A_1 = \frac{1}{6} A$ (which says that only one face of the cube radiates), and that T^4 may be replaced by an average value given by

$$\overline{T}^4 = \frac{1}{T_F - T_o} \int_{T_o}^{T_F} T^4 \, dT$$

$$= \frac{1}{5} \left(\frac{T_F^5 - T_o^5}{T_F - T_o} \right) \tag{42}$$

Substituting for m and A from equations 14 and 16 of the heat sink analysis, we have

$$t = 1.93 \frac{N^{1/3}}{\tau^{2/3}} \quad \text{hours} \tag{43}$$

In the worst case, which is one man for 4 weeks, t is 12.5 minutes. It is clear from the sluggish variation of t with N and τ in equation 43 that there are numerous mission situations where t is less than half an hour. This is almost certainly unacceptable. Even the best case, where N = 6 and τ = 7 gives a t of only 57 minutes.

b. Active System, Flat Radiator

The only practicable way out appears to be either attitude control of the capsule, which is probably unacceptable for most missions, or to have a thermostatically controlled blind inside the freezer. This would present a low ϵ surface with no sun, and be wide open to the Al_2O_3 coated hull during full sun. Rough estimates made on the weight of such a device indicate that it may be considerably less than a pound, including the thermostat. It will reduce the compactness of the unit slightly. It probably is not advisable to put the blind outside the hull since monitoring the temperature of the freezer would require some connection through the hull.

We now wish to compute weights using corkboard and superinsulation as in the LH_2 case.

A formal statement of the problem, including heat sink and mass removal, is given by (assuming full sun)

$$A_1 \alpha \varphi + A_2 \frac{k(T_A - T)}{d} - \epsilon \sigma T^4 A_1 + \dot{m} C_F (T - T_o) = \frac{d}{dt} \left(m C_F \left[T - T_o \right] \right) \tag{44}$$

or

$$m C_F \frac{dT}{dt} + \epsilon \sigma T^4_{A_1} - A_1 \alpha \varphi - A_2 \frac{k(T_A - T)}{d} = 0 \tag{45}$$

This highly nonlinear equation cannot be solved in closed form. However, one may choose an optimum case from physical considerations. The lower the temperature at which we stabilize, the more insulation is required. An estimate indicated that, using Al_2O_3 for the radiating surface, the temperature could not be stabilized below -2°F, even with an infinite amount of insulation. Therefore, instead of treating the differential equation rigorously we consider the case of stabilization at the highest specified temperature, +5°F. For this case the first term in equation 45 vanishes and we have,

$$\epsilon \sigma T^4 A_1 - A_1 \alpha \varphi - A_2 \frac{k(T_A - T_F)}{d} = 0 \tag{46}$$

Solving 46 for d , we obtain

$$d = \frac{k(T_A - T_F)^{A_2/A_1}}{\epsilon \sigma T_F^4 - \alpha \varphi} \tag{47}$$

Assuming as before that $A_2/A_1 = 5$, we may now write for R

$$R = \frac{\rho_i dA}{m_o} = \frac{5\rho_i k\,(T_A - T_F)}{1.5\, N\tau} \left(\frac{1.5\, N\tau}{\rho}\right)^{2/3} \frac{1}{\epsilon\sigma T_F^4 - \alpha\varphi} \tag{48}$$

Putting in the α and ϵ values for Al_2O_3,

$$R = \frac{1.641\, k\rho_i}{(N\tau)^{1/3}} \tag{49}$$

Values for corkboard and SI-4 are listed in Table I.

2. Extended Surface Radiator

A possible way to achieve a passive system is to radiate from a curved surface of the vessel, or, more likely, from a section of the freezer protruding from the hull. The principle behind this arrangement is to present different effective areas for absorption and emission of radiation. As an example, if the projection is a hemisphere it will emit from all the surface, $= 2\pi R^2$, where R is the hemispherical radius. When sunlight falls on the surface, the effective area is the cross section which is only half of the surface area.

It is necessary that, either by the geometry of the situation (for instance by placing the freezer so that it extends from the tip of a cigar-shaped hull) or by the use of low emissivity paints on surrounding surfaces, radiation from the vehicle be greatly reduced. The objection to this arrangement is that it is a definite complication in the design of the hull and probably, therefore, will have a limited application.

The proposed concept may be simply analyzed for thermal balance. Taking A_e to be the effective area for emission and A_a that for absorption we have for full sun (stabilizing at T_F).

$$\epsilon \sigma T_F^4 A_e - \alpha A_a \varphi = (T_A - T_F) A_2 \frac{k}{d} \tag{50}$$

In the case of no sun, stabilizing at T_o,

$$\epsilon \sigma T_o^4 A_e = (T_A - T_o) A_2 \frac{k}{d} \tag{51}$$

Eliminating $A_2 \frac{k}{d}$,

$$\frac{\alpha}{\epsilon} \frac{A_a}{A_e} = \frac{\sigma \left[T_F^4 - T_o^4 \left(\frac{T_A - T_F}{T_A - T_{Fo}} \right) \right]}{\varphi} \tag{52}$$

A coating of Al_2O_3 will maintain the food between -10^oF and $+5^oF$ if $\frac{A_a}{A_e} = 0.328$, which should be possible.

The required weight of corkboard insulation may be quite small. This would probably best be determined from structural considerations.

APPENDIX II

DETAILED DESIGN ANALYSIS

A. Insulation

1. Nomenclature

A	Area of the inner shell of the freezer
A_j	Area of the door jamb
A_s	Area of supporting structure
C_F	Specific heat of the frozen food
k	Thermal conductivity of superinsulation
d	Thickness of insulation
k_j	Thermal conductivity of the door jamb
k_s	Thermal conductivity of the supporting structures
ℓ_1	Conducting length through freezer walls and door jamb
ℓ_2	Conducting length through supporting structures
m	Initial weight of frozen food
Δm	Food removed each day
$\dot{m}$	Rate of food removal
T_A	Ambient temperature
T_F	Final temperature of the food
T_o	Initial temperature of the food

2. Analysis

The following values are assumed for superinsulation

$$A = 5.45 \times 10^2 \text{ in}^2$$

$$A_j = 0.427 \text{ in}^2$$

A_s = 0.885 in^2

C_F = 0.480 Btu/lb $^\circ$F

ℓ_1 = 0.700 in.

ℓ_2 = 0.594 in.

k = 1.00×10^{-5} Btu/h ft $^\circ$F (Assumes Linde SI-91)

k_j = 1.84×10^{-2} Btu/h ft $^\circ$F (Assumes a highly resinated fiberglas)

k_s = 4.7×10^{-4} Btu/h ft $^\circ$F (Assumes compressed super-insulation)

m = 16.3 lb. (We assume a three man two week mission)

Δm = 1.248 lbs.

$\dot{m}$ = 1.248 lbs./day

T_A = 70°F

T_F = 5°F

T_o = -10°F

To obtain thickness of insulation for the freezer, the following equation is used, which takes leakage into account.

$$d = \frac{\ln\left(\frac{\Delta m}{m}\right)}{\dot{m}\, C_F \ln\left(\frac{T_A - T_F}{T_A - T_o}\right)} \left[kA + k_j A_j + k_s A_s \frac{\ell_1}{\ell_2} \right] \tag{53}$$

For convenience, substituting our determined quantities into 1,

$$d = 41.0 \left[5.45 \times 10^2\, k + k_j A_j + k_s A_s \times 1.18 \right], \quad \text{where} \tag{54}$$

the areas are in inches^2, d is in inches, and the k's in Btu/ft $^\circ$F hr.

Substituting our tentative values

$$d = 41.0 \left[5.45 \times 10^{-3} + 7.85 \times 10^{-3} + 4.91 \times 10^{-4} \right] \tag{55}$$

The arithmetic was not completed in one step to show the relative importance of the three sources of heat leakage.

The result is d = 0.565".

We have used 0.7 inches to provide a tolerance.

B. Stress

1. Nomenclature

A_c	Area of connection
A_p	Projected area of shell
A_s	Shear area of solder
(a)	Area of section
F	Force
P	Atmospheric pressure
S_s	Shear stress
S_t	Tensile stress
V	Volume
W	Weight
W_m	Weight of magazine

2. Analysis

The following values are assumed.

P	15 psi
W_m	27.5 lbs.

To check the inner freezer shell as a pressure vessel, the following equation is used for the cylindrical portion.

$$S_t = \frac{A_p \times P}{(a)}$$

$$= \frac{(11 \times 9.5)(15)}{(0.020 \times 11 \times 2)}$$

$$= 3550 \text{ psi.} \qquad (56)$$

The result is well below allowable tensile stress; however, the skin thickness of 0.020 is retained for handling strength during fabrication.

The outer shell while strong enough under ideal conditions is reinforced with fiberglas to prevent buckling when a vacuum is pulled on the superinsulation.

To check the loads on the secondary bonds under a 25 G acceleration, and assuming these bonds to be only 50% good, the magazine is assumed to weigh 27.5 lbs. Applying a 25 G load along the main axis from front to rear,

$$S_s = \frac{\text{Bearing Load}}{A_c}$$

$$= \frac{(25 \times 27.5)}{(9.5\ D \times \Pi \times 0.25)}$$

$$= \frac{685}{7.5}$$

$$= 105 \text{ psi} \tag{57}$$

which is well under the 2000 psi shear stress capability of the secondary bond. From (57) above, we also see that the bearing load of 685 lbs, is within the bearing capacity of 3000 lbs.

Taking 105 psi from (57) and adding for the pressure differential

$$S_s = \frac{PA}{(a)}$$

$$= \frac{15 \times (\Pi \times 4.75^2)}{7.5}$$

$$= 142 \text{ psi} \tag{58}$$

Total S_s = 142 + 105 = 247 psi on the solder joint which is approximately 12% of 2000 psi common allowable working stress.

For mounting the unit, 4 of the 8 bolts are assumed to bear the total load. Assuming total weight of the unit at 50 lbs., the shear stress at the bolts under 25 G's is given by

$$S_s = \frac{\text{Load}}{(a)}$$

$$= \frac{50 \times 25}{4 \Pi \times 0.25^2}$$

$$= 3925 \text{ psi} \tag{59}$$

which is well below the 30,000 psi acceptable.

Cross axis calculations have not been made since a comparison of joints shows that all cross axis forces result in lower shear stress values than those calculated above which were all within working limits.

B. Weight Summary

Weight of cans	3.0 lbs.	
Food	17.5	
SI-91	1.6	
Fiberglas	1.0	
Steel Shells	3.1	
Magazine	8.0	
Foam	0.3	
Hinges, Fittings, Bearings, etc.	3.0 (estimated)	
	37.5 lbs.	TOTAL