

**EFFECT OF INERTIAL FORCES ON DAMPING IN A DRY FRICTION JOINT**

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**Abstract**

In a press fit joint a rod is considered as being flexible while a bush is rigid. The rod is modelled as a discrete mass system subjected at one end to a harmonic force. Each mass is moving in the presence of dry friction forces of constant intensity. A computer simulation technique is used to investigate the effect of inertial forces on the microslip in a press fit joint and then on the frictional losses during one cycle of loading. The numerical results show that the effect of inertial forces becomes significant at ultrasound frequencies. Similarity criteria are given to extend the results of numerical simulation presented for a particular case and to show the qualitative relationship between the dynamic losses and system parameters.

## 1. Introduction

The effect of friction at the interface of structural members on the damping properties of a structure was always of interest to structural engineers. One aspect of this general problem is associated with the process which takes place in a friction joint subjected to a cyclic loading, namely, the cyclic microslip and corresponding losses. Panovko formulated and solved this problem for a press fit joint subjected to a quasi-static cyclic loading in 1958 [1].

Interest to energy dissipation in a simple joint due to friction was maintained over the past two decades [f.e. 2, 3, 4] and even became stronger in view of more rigid requirements to vibration control in space applications. In analyzing losses in a frictional joint one parameter, namely, the inertial forces, has always been ignored, while the limits of this assumption has never, to the best of author's knowledge, been studied.

The problem for a frictional joint in a dynamic formulation does not have a closed form solution analagous to that obtained by Panovko. In fact, when investigating the dynamics of a frictionally constrained rod the author preferred numerical simulation [5] to any approximate analytical approach because of the uncertainty in the dynamic behaviour which inertial forces introduce. In [5] a simulation technique was used to solve an initial value problem for a discrete system of masses subjected to a periodic load and moving in the presence of friction. However, the question of energy losses was not addressed in [5] and was only partially discussed in [6].

In this paper a more systematic numerical analysis of dynamic vs. static losses is given for a particular case. Also, the effect of such parameters as coefficient of friction and amplitude of the harmonic force is investigated. To generalize the numerical results the similarity criteria are given.

## 2. Mathematical Model and Simulation Procedure

In this paper a simple press fit joint is considered and it is assumed that the rod is flexible while the bush is rigid. The friction forces are assumed to be of constant intensity (dry friction). The rod is acted upon by a harmonic force applied at one of the ends.

A mathematical model describing the outlined problem is intrinsically non-linear due to dry friction. During each half-cycle of the loading every cross-section of the rod might be stationary over some time interval (inter-locking). These stationary cross-sections in fact make-up a stationary region (segment). At any moment in time there is a number of stationary and moving segments. The number of such segments, their length and sense of motion is not known a priori. Besides, the picture may differ from one cycle to another and it is not clear in advance whether a process would converge to a steady state one for a specific set of problem parameters. To investigate the dynamic behaviour of such a system a computer simulation technique was developed

by the author [5]. Here only a brief description of the simulation procedure is given for the sake of clarity. A rod is modelled as a discrete system of equal masses, springs and dashpots (Fig. 1). The simulation process starts during the first half-cycle of the loading when the external force becomes equal to the friction force associated with the first mass. The equation of motion of the first mass is generated and an analytical solution is obtained. This solution is valid until either the second mass starts moving or the first mass stops. At this moment it is either the one-mass system becomes a two-mass system or the whole rod becomes stationary.

In a developed process there can be masses moving in a positive direction, negative direction or remaining stationary. Motion of the entire system is represented by the motion of its subsystems. The system remains invariable if the number of sub-systems and the number of masses in each sub-system remains the same. The invariable system is governed by the linear differential equations. However, the moment in time when the status of the physical system changes cannot be determined analytically and this fact constitutes a need for simulation. A time-stepping procedure is utilized to identify the moment when the system changes its status. Between such two moments in time (events) the analytical solution is used to describe the motion.

### 3. Numerical Results

Numerical examples are given for a steel pin which is 0.025 m long and has a radius of 0.005 m. The pin is subjected to axial harmonic forces of various amplitudes and frequencies  $P = P_1 \sin(\omega t)$ . The total friction force,  $F_t$ , is used as a measure of friction. The pin is modelled as a ten-mass system, the number of discrete masses found to be sufficiently accurate for this type of a problem [6]. The material damping properties are taken in the same form as in [5].

#### 3.1 Discontinuity of Motion

Examples of the displacement of masses in time during the first few cycles of the load are shown in Figs. 2 and 3. The discontinuity of motion is clearly seen: the flat parts on the graphs indicate the length of time during which each mass remains stationary. Also it is seen that various masses start or stop moving not simultaneously. Both shift in phase and the length of the stationary period are dependent on the force frequency and amplitudes. At some frequencies and amplitudes motion becomes continuous. Note also, that it requires a few cycles before the process becomes steady.

#### 3.2 Hysteresis

Since work done by the friction forces during the cycle is equal to the work done by the external force during the same cycle the hysteretic losses were measured by considering the external force-displacement relationship during the cycle and measuring the corresponding hysteretic loop area. Two examples of hysteretic loops are shown in Figs. 4 and 5. It is seen that at  $f = 250$  Hz the system behaves as perfectly quasi-static one with the clearly identified discontinuities in stiffness,

whereas at  $f = 2\text{kHz}$  the hysteretic loop becomes more smooth which is an indication of the effect of inertial forces. However, the maximum displacement of the first mass in both cases is practically unaffected ( $U_{\text{max}} = 2.7 \times 10^{-4} \text{ mm}$ ). It is worth mentioning that exact analytical solution according to [1] gives for given loads and pin parameters maximum displacement during the cycle as  $U_{\text{max}} = 2.9 \times 10^{-4} \text{ mm}$ . The discrepancy is associated with the discretization [6]. A vertical part of the loop in Figs. 4 and 5 indicates a range of force over which the first mass remains stationary.

### 3.3. Cyclic Losses

The effect of frequency and nondimensional amplitude of the external force on dynamic losses is shown in Fig. 6, in which Fig. 6b provides a close-up look of curves in Fig. 6a at low frequencies. The static losses were calculated according to [1]

$$\psi = \frac{2}{3} \frac{P_1^3 L}{F_t \cdot EA} \quad (1)$$

where  $P_1$  is the amplitude of dynamic force,  $L$  is the length of the pin-hub interface,  $F_t$  is the total friction force at the interface,  $E$  is the modulus of elasticity, and  $A$  is the cross-section area of the pin.

It is seen from Fig. 6b that for practically important range of frequencies (up to 60 Hz) the effect of inertial forces on hysteretic losses is negligibly small. However, Fig. 6a indicates that at ultrasound frequencies the losses due to dry friction may grow many folds. This increase is associated with the phenomenon of macroslip of the entire pin at the interface at these frequencies [5]. The effect of inertial forces on losses at ultrasound frequencies can also be seen in Fig. 7 where the relationship between the losses and the amplitude of the dynamic force is shown for a 15 kHz frequency. A cubic parabola indicates a quasi-static relationship according to formula Eq. 1. It is seen that dynamic losses grow faster with the amplitude of the force.

### 4. Similarity Criteria

Because of a number of parameters effecting the losses in a dry friction joint it seems that computer simulation is the only tool available to investigate the phenomenon. The results of computer simulation can be extended to dynamically similar situations. The following two similarity criteria can be derived for the problem under consideration

$$\frac{s^2 s_l^2 s_\rho}{\omega l \rho} = 1 \quad (2)$$

and

$$\frac{S_F}{S_\omega^2 S_E} = 1 \quad (3)$$

where  $S_\omega$  is the ratio of frequencies,  $S_\ell$  is the ratio of geometries,  $S_\rho$  is the ratio of densities,  $S_E$  is the ratio of moduli of elasticities, and  $S_F$  is the ratio of total friction forces in two dynamically similar dry friction joints. If material properties are the same in both joints than  $S_\rho = 1$  and  $S_E = 1$  and Eqs. 2 and 3 are reduced to

$$S_\omega S_\ell = 1 \quad (4)$$

and

$$\frac{S_F}{S_\ell^2} = 1 \quad (5)$$

Equation 4 means that if, for example, the geometry of the joint increased twice,  $S_\ell = 2$ , then the frequency should decrease twice,  $S_\omega = 0.5$ , in order to ensure the similarity. At the same time, as it follows from Eq. 5, the friction forces should be four times larger. Since friction forces are proportional to the surface area, the condition Eq. 5 is satisfied if the coefficient of friction and the contact pressure are maintained the same in both joints. Note that the external force - total friction force ratio is supposed to be maintained the same. Since losses are proportional to

$$S_L \sim S_F S_\ell \quad (6)$$

for the case of identical materials Eq. 6 is reduced to

$$S_L = S_\ell^3 \quad (7)$$

where  $S_L$  is the ratio of losses in two similar systems.

## 5. Conclusions

A computer simulation approach allowed to assess the effect of inertial forces on losses in a simple press fit joint. The results of simulation show that inertial forces become significant at ultrasound frequencies for a specific example considered. The similarity criteria derived indicate that the same losses may take place at lower frequencies for a scaled-up joint. The significance of inertial forces in dry friction joints on structural losses can be assessed in each specific application using the developed methodology [5].

## Acknowledgements

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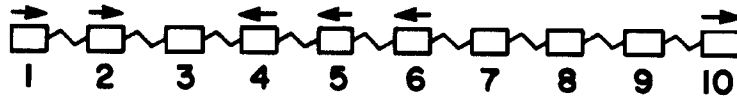


Fig. 1 Schematic representation of discrete model

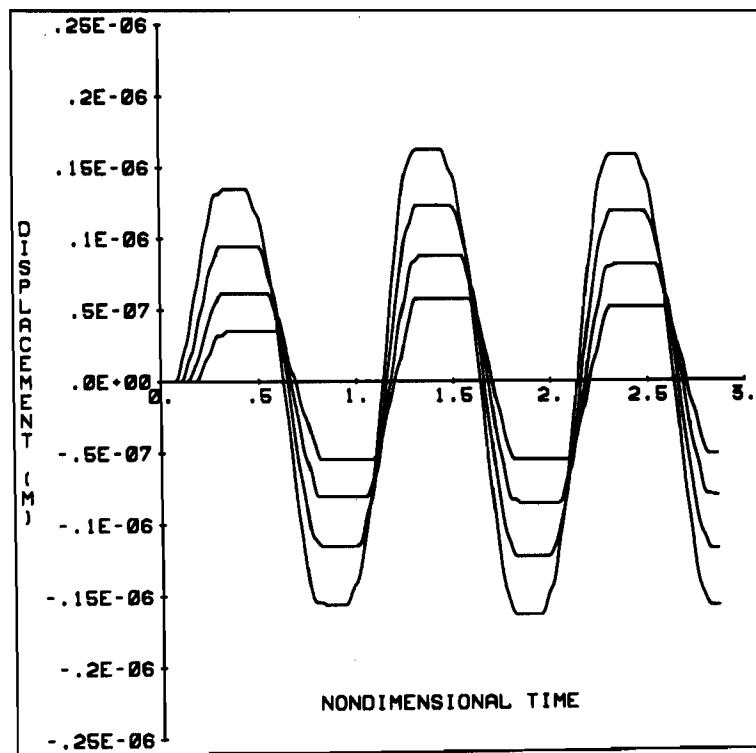


Fig. 2 Displacement of masses in a ten-mass model during first few cycles ( $F_t = 1010 \text{ N}$ ,  $P_1 = 606 \text{ N}$ ,  $f = 15 \text{ kHz}$ )

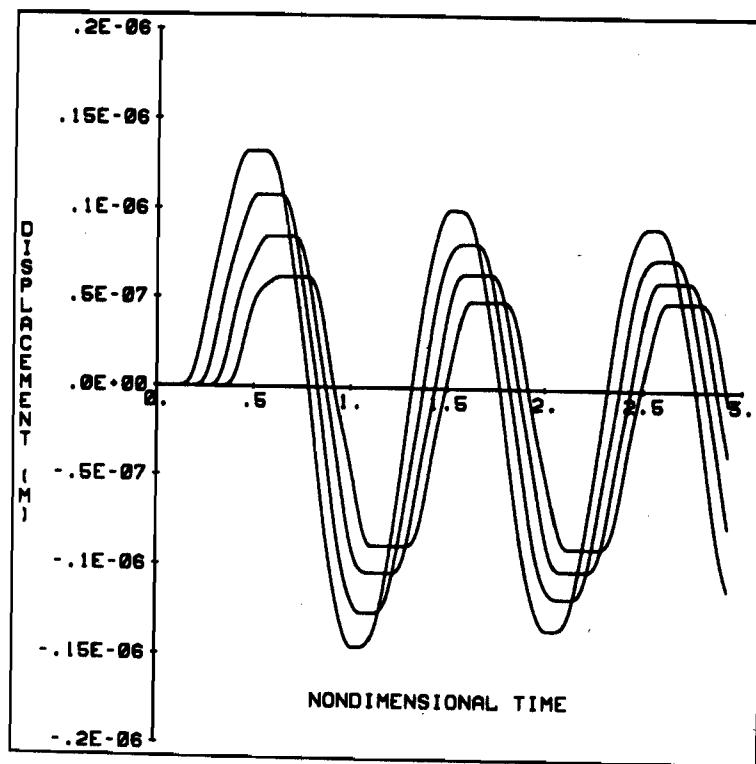


Fig. 3 Displacement of masses in a ten-mass model during first few cycles ( $F_t = 1010$  N,  $P_1 = 606$  N,  $f = 50$  kHz)

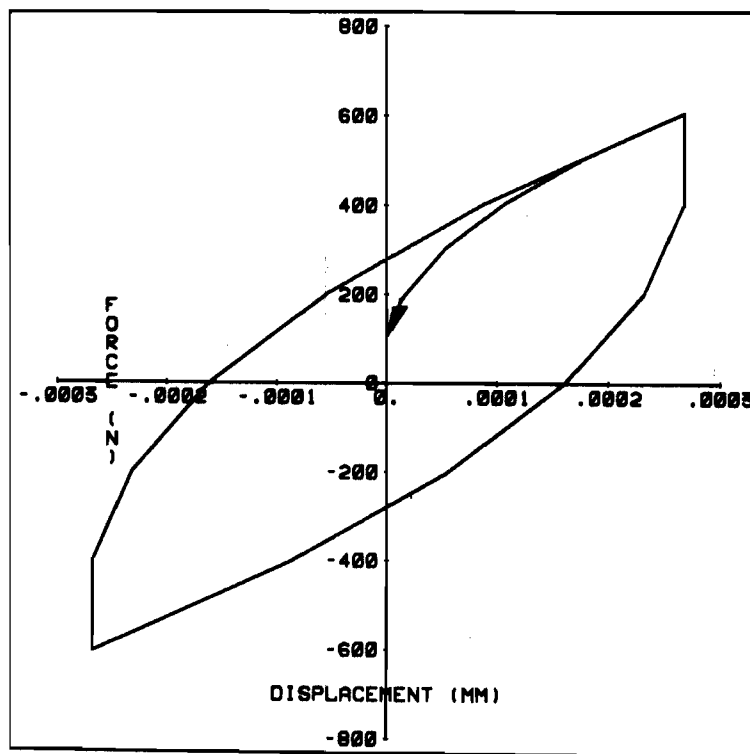


Fig. 4 Hysteretic loop ( $F_t = 1010$  N,  $P_1 = 606$  N,  $f = 0.25$  kHz)



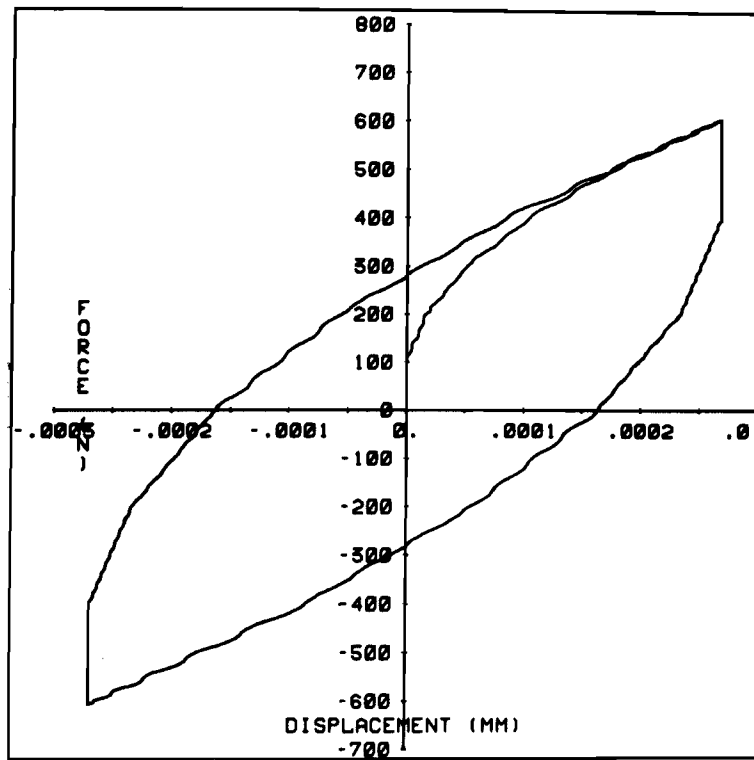


Fig. 5 Hysteretic loop ( $F_t = 1010$  N,  $P_1 = 606$  N,  $f = 2$  kHz)

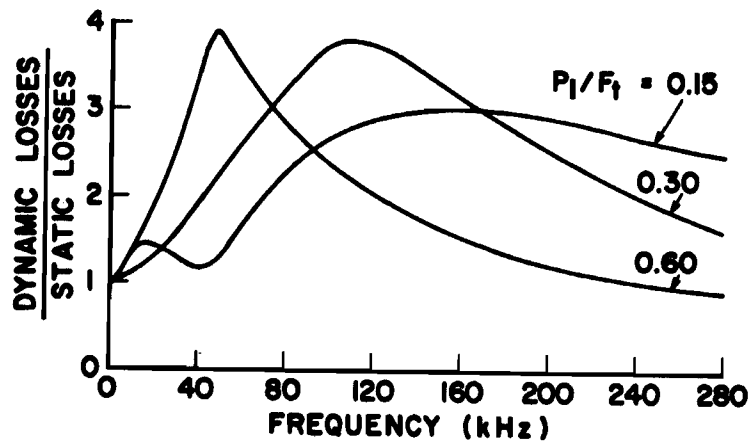


Fig. 6a Effect of frequency and amplitude of dynamic force on damping losses

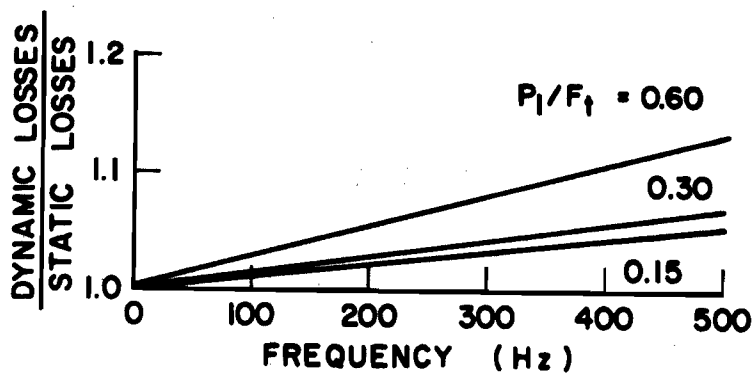


Fig. 6b Effect of frequency and amplitude of dynamic force on damping losses

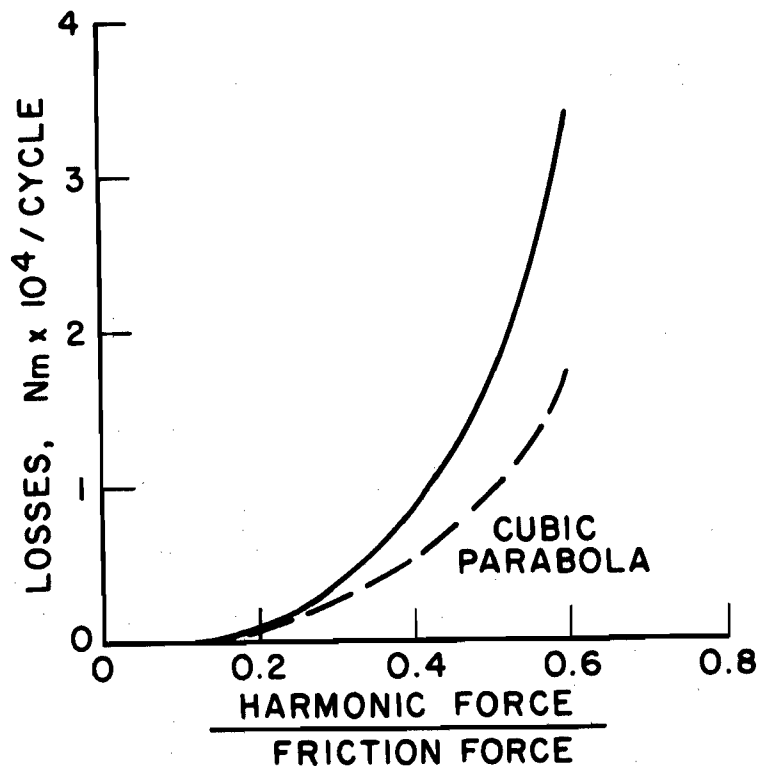


Fig. 7 Effect of the amplitude of dynamic force on damping losses (broken line indicates quasi-static cyclic losses).