ANALYTIC APPROXIMATIONS TO DYNAMIC PRESSURE AND IMPULSE AND OTHER FITS FOR NUCLEAR BLASTS

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ABSTRACT

Recent fits to data and calculational results provide simple analytic approximations to the overpressure and dynamic pressure from nuclear explosions. The analytic forms provide fits as a function of ground range and height-of-burst for time of arrival, peak overpressure, peak dynamic pressure, onset of Mach reflection, duration of positive phase for both dynamic and overpressure, time histories for overpressure and dynamic pressure, dynamic pressure impulse and overpressure impulse (in the positive phase), and other blast parameters.

INTRODUCTION

Recently Kaman AviDyne (KA) produced for the Defense Nuclear Agency (DNA) 3 two-dimensional hydrodynamic calculations of the nuclear blast wave over an ideal reflecting plane at burst heights of 200, 400 and 700 ft/kt$^{1/3}$ (1). These solutions, together with the DNA kiloton standard (2) (using 2M) for a surface burst allow some definition of both overpressure and dynamic pressure as a function of time, burst height, range and yield.

These KA solutions are being studied further to determine their accuracy and limits, and other DNA sponsored calculations are under way. For the present, these KA results appear to provide the best dynamic pressure height-of-burst information. Any subsequent confirmation by independent solutions or other improvements due to experiments or recalculation are likely to be many months in coming. In this note, relatively simple analytic forms which approximate the KA results are presented and compared with calculations.

In the absence of sufficiently detailed calculations and in view of a paucity of relevant measurements, Brode and Speicher (3) invented an analytic approximation to the dynamic pressure from height-of-burst blasts. The first approximation was almost immediately improved (4). The time dependence was based on one-dimensional calculations appropriate for free air or surface nuclear bursts (5) and on various analytic approximations for time-of-arrival and overpressure-time HOB behavior as provided in earlier fits (6,7). An improved description of the height of burst dependence of overpressure, based on both HE data and calculations, was published in late 1981 (8,9).

A "quick fix" analytic approximation for the dynamic pressure, based on the recent KA calculations, was offered in a memo from S. J. Speicher in December 1982 (10). This quick fix is cast in terms of the previous fit to the overpressure as a function of time, burst height, ground range and yield (8).

DYNAMIC PRESSURE AND IMPULSE FITS

In this report, the peak dynamic pressure and the total dynamic impulse in the positive phase are approximated by analytic forms and simple fits to match the KA and 1 KT Standard results. In the regular reflection region, (of

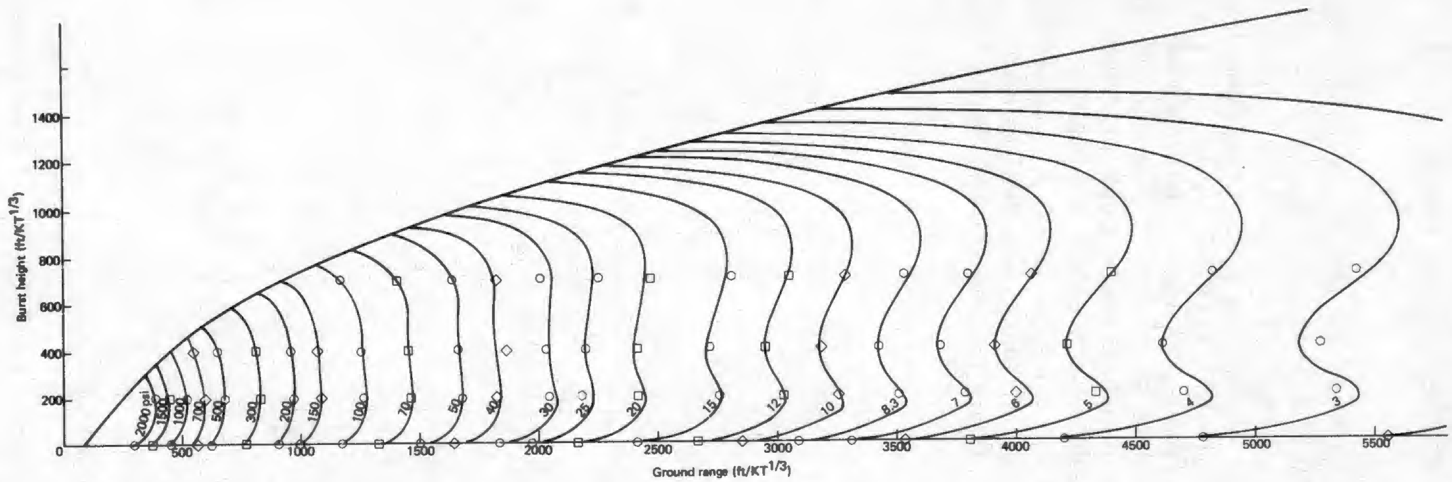

Figure 1--Height of burst fit for peak dynamic pressure

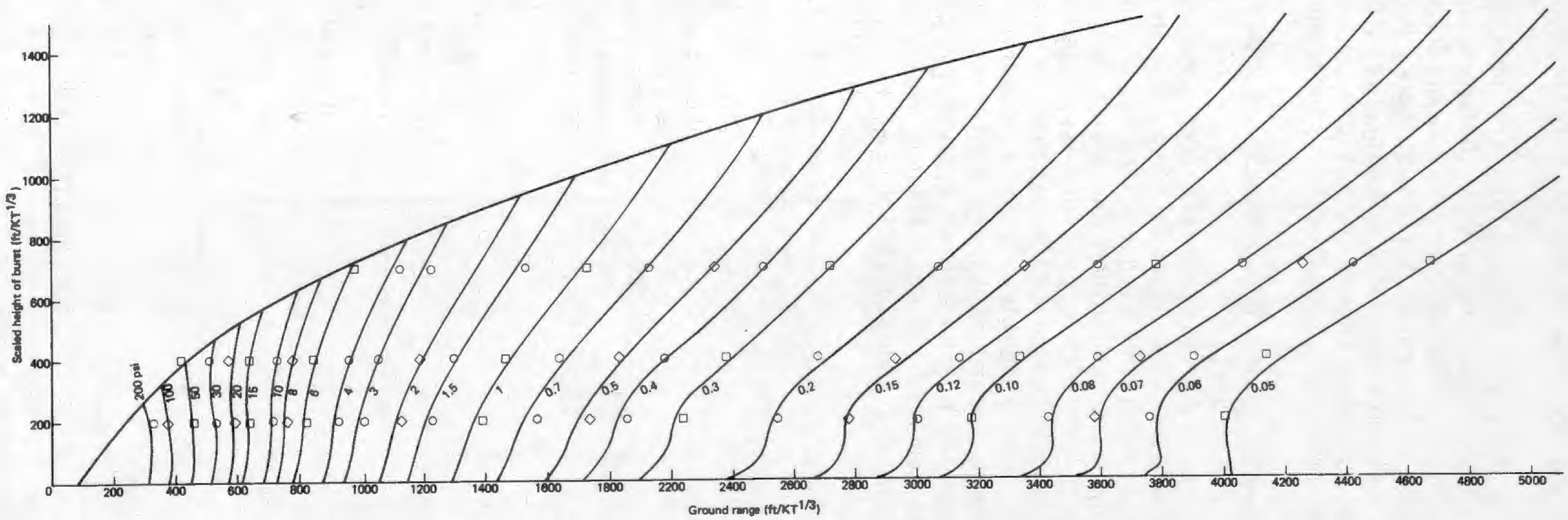

Figure 2--Dynamic impulse fit versus range and burst height

less interest for targeting or damage assessment) the fits are inappropriate, and should be used in the Mach reflection region only.

The fits and comparisons are illustrated in Figures 1 and 2. Tables 1 and 2 compare the differences between the values provided by the fits and the values for 0, 200, 400 and 700 scaled feet HOB from the calculation. In all cases the differences are less than a few percent at variance. As more and better calculations become available, these fits may be amended or replaced, but for now they represent a simple description of the dynamic pressure and dynamic impulse versus height, range and yield.

In evaluating the goodness of these approximations or the accuracy of the detailed calculations, it is well to keep in mind the basic variability of blast data. A review of the peak overpressures as measured on nuclear tests (11) shows scatter of more than ±15% in range for any given peak overpressure. Figure 3 shows one such collection of "data" for 15 psi (scaled to 1 KT) as a function of ground range and burst height. Far fewer and less accurate measurements of dynamic pressure exist, although, these pressures are expected to follow shock (Hugoniot) relations in most of the non-precursed peak pressure regions. An exception is in a portion of the double Mach shock reflection region where second peaks are dominant. In precursor or dust laden blast waves, peak dynamic pressures can exceed the classical shock values by appreciable factors (as much or more than a factor of two). Observed durations and time behaviors are equally variable.

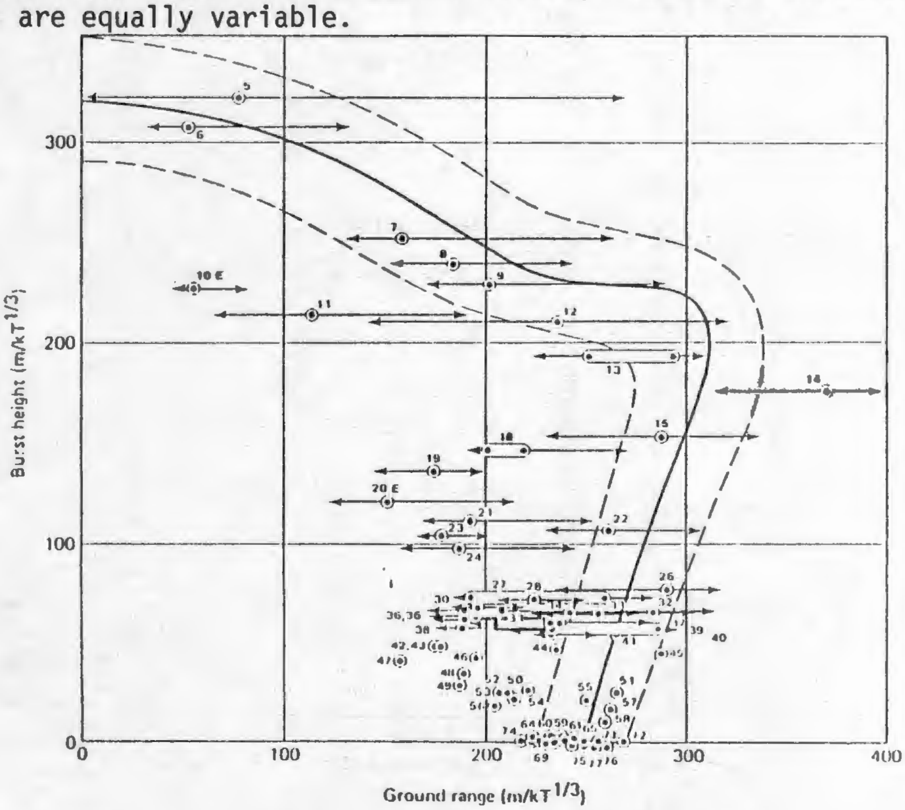

Figure 3 Fifteen psi (103 kPa) nuclear test data versus HOB and range scaled to 1 kT.

SOME ADDED USEFUL ANALYTIC APPROXIMATIONS

For shock waves in air (at sea level) all of the conditions at the shock front can be expressed in terms of a single variable, such as peak overpressure. The non-ideal gas behavior of air at high temperatures and pressures

Table 1: Peak Dynamic Pressure, Q_s (psi)

Scaled Ground Range ≡ R ($KFT/KT^{1/3}$), Scaled Burst Height ≡ H ($KFT/KT^{1/3}$)

$$Q_s = \frac{A}{R^B + .025R^{4.5}} + \frac{C}{R + DR^4} \text{ psi; R,H in } KFT/KT^{1/3}$$

$$A = 2.28 + 12.5H^2/(1 + 1.292H),\ B = 3 + (0.86 + 2.47H)/(1 + 114H^3)$$

$$C = (.21 + 2.2H)/(1 + 200H^3),\ D = (.008 + .24H)/(1 + 260H^3)$$

$$\text{Valid for : } R > \frac{170(H + .09)}{1 + 337(H + .09)^{1/4}} + 0.914(H + .09)^{2.5}$$

Comparison with Kaman Avidyne Calculations (scaled to 1 KT)

HOB (ft)	Max Difference %	Max Difference Q_s (psi)	Average Difference	Net Difference
0	-4	200	0.3 %	-0.3 %
200	+5	200	1.6 %	+1.4 %
400	-2	30	1.1 %	-1.1 %
700	+2	4	0.6 %	+0.5 %

$$\% \text{ Difference} = \frac{(KA) - (FIT)}{(KA)} \times 100$$

Table 2: Dynamic Impulse, I_u^+ (psi-ms)

Scaled Ground Range ≡ R ($ft/KT^{1/3}$), Scaled Burst Height ≡ H ($ft/KT^{1/3}$)

$$I_u^+ = \left[\frac{ER}{F + R^{3.61}} + \frac{G}{1 + 0.22R^2}\right] W^{1/3} \text{ psi-ms; H,R in } KFT/KT^{1/3} \text{ W in KT}$$

$$E = \frac{183(H^2 + .00182)}{(H^2 + .00222)},\ F = 0.00058 \exp(9.5H) + 0.0117 \exp(-22H)$$

$$G = 2.3 + \frac{29H}{(1 + 1760H^5)} + \frac{25H^4}{(1 + 3.76H^6)}$$

$$\text{Valid for: } R > \frac{170(H + .09)}{1 + 337(H + .09)^{1/4}} + 0.914(H + .09)^{2.5}$$

Goodness of Fit: Comparison with Kaman AviDyne Calculations

HOB (ft)	Max Difference %	Max Difference I_u (psi)	Average Difference	Net Difference
0	+7	1000 & 700	2	-1/4
200	+5	2000	1	-1/4
400	+5	2	1/2	+1/4
700	-3	30	2/3	-1/8

$$\% \text{ Difference} = \frac{(KA) - (FIT)}{(KA)} \times 100$$

precludes an ideal gas formulation, but, since the effective specific heat ratio (γ) changes slowly with peak overpressure or shock strengths, empirical fits can be found which follow fairly simple forms. Some such approximate forms for Peak Dynamic Pressure (Q_S), shock velocity (U_S), peak particle velocity (u_S), peak density (ρ_S), shock temperature (T_S) and normal reflection factor (RF) (all shock front quantities that can be described as function of peak overpressure (ΔP_S)) are offered here.

Peak Dynamic Pressure for an ideal gas:

$$Q_s(\Delta P_s, P_0, \gamma) \simeq \Delta P_s^{\,2}/[2\gamma P_0 + (\gamma - 1)\Delta P_s] \text{ psi} \tag{1}$$

$P_0 \simeq 14.7$ psi at sea level; $\gamma \equiv$ effective specific heat ratio ($1.08 \leqslant \gamma \leqslant 1.67$ For $P_0 = 14.7$, $\gamma \simeq 1.4$ for $\Delta P_s < 300$ psi)

$$Q_s(\Delta P_s) \simeq \Delta P_s^{\,2}\left[1 + \frac{1}{2}\left(\frac{\Delta P_s}{1000}\right)^2\right]\cdot\left[41 + 0.4\Delta P_s + \frac{1}{10}\left(\frac{\Delta P_s}{100}\right)\right]^{-3} \text{ psi} \tag{2}$$

(Eq. (2) accurate to < 5% for $2 \leqslant \Delta P_s \leqslant 100{,}000$ psi)

Shock Velocity for an ideal gas:

$$U_s(\Delta P_s, C_0, P_0, \gamma) \simeq C_0[1 + (\gamma + 1)\Delta P_s/2\gamma P_0]^{1/2} \tag{3}$$

C_0 is sound speed ahead of shock, ($C_0 \simeq 1.1$ Kft/sec)

for air ($\gamma = 1.4$):

$$U_s(\Delta P_s, C_0, P_0) = C_0\{(.857 + .006\xi/(1 + .2\xi))\xi + .143\}^{1/2} \tag{4}$$

$$\xi \equiv (\Delta P_s + P_0)/P_0 \tag{5}$$

(Eq. (4) accurate to < 8% for $\Delta P_s < 100{,}000$ psi)

Peak Particle Velocity for an ideal gas:

$$u_s(\Delta P_s, C_0, P_0, \gamma) = C_0(\Delta P_s/\gamma P_0)(1 + (\gamma + 1)\Delta P_s/2\gamma P_0)^{-1/2} \tag{6}$$

(for air, $\gamma = 1.4$, Eq. (6) accurate to < 4% for $1 \leqslant \Delta P_s \leqslant 500{,}000$ psi)

Shock Temperature:

$$\frac{\Delta T_s(^\circ C)}{T_0} = \xi\left(\frac{R_0}{R_s}\right)\left(\frac{(\gamma - 1)_s\xi + (\gamma + 1)_0}{(\gamma + 1)_s\xi + (\gamma - 1)_0}\right) - 1 \tag{7}$$

$$R_i = P_i/(\rho_i T_i) \tag{8}$$

for air (T_0 = 273°K, P_0 = 14.7 psi):

$$\Delta T_s/T_0 = (\xi^2 - 1)/[7 + B(\xi - 1)] \tag{9}$$

$$B = 6 + 1.76\xi^{2.5}/[10^5 + 4.38\xi^2] \tag{10}$$

Normal Reflection factor (RF):

$$RF \equiv \frac{\Delta P_r}{\Delta P_s} = \frac{2 + \left(\frac{3\gamma - 1}{2\gamma}\right)\left(\frac{\Delta P_s}{P_0}\right)}{1 + \left(\frac{\gamma - 1}{2\gamma}\right)\left(\frac{\Delta P_s}{P_0}\right)} \tag{11}$$

ΔP_r is reflected peak overpressure from a normally incident shock of peak overpressure ΔP_s in an ambient atmosphere of pressure P_0.

For air at sea level and ΔP_s in psi (6)

$$RF \simeq 2 + \frac{.002655\Delta P_s}{1 + .0001728\Delta P_s + 1.921 \times 10^{-9}\Delta P_s^{\,2}}$$

$$+ \frac{.004218 + .04834\Delta P_s + 6.856 \times 10^{-6}\Delta P_s^{\,2}}{1 + .007997\Delta P_s + 3.844 \times 10^{-6}\Delta P_s^{\,2}} \tag{12}$$

SUMMARY

Simple analytic fits have been developed to describe the results of recent Kaman AviDyne hydrocode calculations. Algebraic relations for the peak dynamic pressure and peak dynamic impulse are presented and compared to the hydrocode results. The simple functions are easily programmable on a hand-held calculator and agree well with the KA calculations. Analytical and empirical relations describing blast wave characteristics are presented and their accuracy noted.

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