BUCKLING CRITERIA FOR SANDWICH SHELLS

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Sandwich structures subjected to loadings that cause instability have been observed to fail as a result of either:

a) over-all buckling, where there is no separation of the faces from the core (quasi-Euler mode), or
b) local instability, commonly referred to as wrinkling, where the faces tend to separate from the core.

In this paper, a brief summary of the elastic stability of long circular sandwich cylinders under axial compression, bending, torsion and uniform external pressure, is given. The buckling of conical shells is briefly mentioned and an expression for predicting wrinkling stresses is recommended.

When computing buckling stresses of sandwich structures, over-all buckling as well as wrinkling should be considered, with the lesser of the two determining the mode of buckling.

(I) OVER-ALL BUCKLING

(A) Axial Compressive Buckling of Circular Sandwich Cylinders.

The over-all buckling stress of a "long" (length/diameter >1) sandwich cylinder in axial compression is given approximately by (from ref. (1))

$$(1) \qquad (\sigma_{cr})_f = .4 K_c \frac{E_f (c+t)}{R\sqrt{\lambda_f}} \; {}^{*}$$

where

$$K_c = 1 - .15 H \qquad \text{when} \qquad H \leq .98 \qquad \text{(sandwich strong in shear)}$$

$$K_c = \frac{.834}{H} \qquad \text{when} \qquad H \geq .98 \qquad \text{(sandwich weak in shear)}$$

and the remaining symbols are as defined under Notation.

* In ref. (1), the buckling coefficient K_c was minimized by taking $\lambda_f = .91$ Here λ_f was retained as such in the minimization, which will explain the slight difference in form of eq. (1) and the results reported in ref. (1).

Equation (1) is based on large deflection theory* with the faces assumed to act as membranes, i.e., the bending stiffness about their own middle surface is neglected. The membrane assumption is conservative and is a commonly used one in sandwich structures. The deflection function used to obtain eq. (1) was not complete, only a diamond shape wave pattern was considered. When the results leading to eq. (1) are reduced to the homogeneous, isotropic, thin-walled cylinder, the resulting buckling stress is

$$\sigma_{cr} = .242 \frac{E\bar{t}}{R} \qquad \left(\begin{array}{l} \bar{t} = \text{wall thickness} \\ \nu = .3 \end{array} \right) \tag{2}$$

Kempner, ref. (2), used a more complete deflection function and was able to obtain for the homogeneous, isotropic, thin-walled cylinder the expression

$$\sigma_{cr} = .182 \frac{E\bar{t}}{R} \; ** \tag{3}$$

If the ratio of (3) to (2), which is approximately equal to .75, is applied to eq. (1), the result is

$$(\sigma_{cr})_f = .3 \, K_c \frac{E_f (c+t)}{R\sqrt{\lambda_f}} \tag{1a}$$

It is interesting to note that the experimental buckling stresses reported in ref. (1) are equal to or greater than the theoretical stresses obtained from eq. (1a) (these experiments were performed on curved sandwich plates which were designed in such a way as to include at least one ideal buckle).

Equation (1) applies to an isortopic sandwich cylinder. The corresponding buckling equation for the orthotropic cylinder (like a honeycomb core-glass fiber face sandwich) is much more complicated and has not as yet been minimized. Equation (1) (also 1a), however, may still be used to obtain estimates for the buckling of orthotropic sandwich cylinders, provided the lesser of the two

* The large deflection theory was successfully used for the axial compressive buckling of homogeneous, isotropic, thin-walled cylinders to explain discrepancies between experiments and the classical small deflection theory.

** The classical small deflection theory for the same case yields $\sigma_{cr} = .6 \frac{E\bar{t}}{R}$

transverse core shear moduli is used and the 0° or 90° warp direction of the faces ($E_0 = E_{90}$) is oriented parallel to the cylinder axis. A slight reduction in E_f may be necessary to compensate for the smallness of the face shear modulus (this statement is based on what is already known from the orthotropic flat plate theory).

(B) Pure Bending Buckling of Circular Sandwich Cylinders

A complete theoretical solution for this type of buckling has not been obtained yet. An expression is available for bending buckling of a sandwich cylinder that is weak in shear ($H \geq .9B$). For this case, it was found, ref. (3), that the critical bending stress is equal to the axial compressive buckling stress of a sandwich cylinder weak in shear, namely

$$(\sigma_{cR})_f = \frac{G_c(c+t)^2}{2tc} \tag{4}$$

This expression is eq (1) with $K_c = .834/H$. Physically, this critical stress is associated with shear instability in the core and is characteristic of most sandwich buckling problems when the sandwich construction is exhibiting large shear deformations. This equation says that the critical load is equal to the transverse shear stiffness of the sandwich.

Limited bending buckling tests performed by Gerard, ref. (4), on sandwich cylinders weak in shear (aluminum alloy faces - cellular cellulose acetate core) indicate that the experimental buckling stress is on the average 32 per cent higher that the theoretical value obtained from eq. (4). Tests were also conducted on cylinders having aluminum alloy faces and end grain balsa wood core. The results, however, showed the experimental stresses to be approximately four times less than the theoretical values. This was attributed to either a poor bond or to some other mode of buckling.

(C) Torsional Buckling of Circular Sandwich Cylinders

The over-all buckling stress of a sandwich cylinder in torsion is given by (from ref. (5))

$$(\tau_{cR})_f = K_T \frac{E_f(c+2t)}{R} \tag{5}$$

where K_T for a long, isotropic cylinder, where the faces are considered as membranes, may be obtained from fig. (1). For the buckling coefficients of finite cylinders, and for the case where the faces are not treated as membranes, the curves in ref. (5) may be consulted. In ref. (5), the buckling of the orthotropic cylinder is also treated, however lengthy computations are necessary to minimize K_T.

No experimental confirmation of the theory has as yet been published.

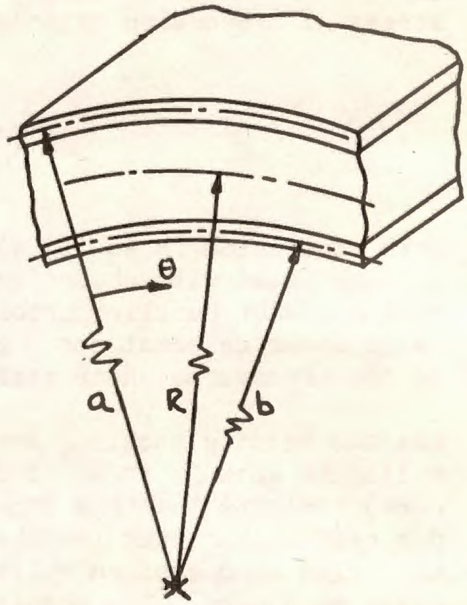
(D) Buckling of Circular Sandwich Cylinders Under Uniform External Pressure

The uniform external pressure under which a long sandwich cylinder will buckle is given by the expression (from ref. (6))

$$q_{CR} = \alpha \frac{3 E_f t (c+t)^2}{2 R^3 \lambda_f} \tag{6}$$

where

$$\alpha = \frac{\left(1+\frac{b}{a}\right)^3}{4\left(1+\frac{b^2}{a^2}\right)\left[1+\frac{2E_f t(c+t)}{G_{R\theta}\, a b \lambda_f}\right]}$$



$G_{R\theta}$ = shear modulus in R-θ plane
a and b are as shown on the sketch

In eq. (6) the faces of the sandwich are assumed to act as membranes.

Some preliminary computations indicate that for long, thin-walled, sandwich cylinders that are strong in shear, the factor α is very close to unity.

If $\alpha = 1$, eq. (6) is in the same form as the buckling expression of a long homogeneous cylinder ($I_s = t(c+t)^2/2$ for a sandwich construction).

For the effects of finite length and the non-membrane action of the faces on the buckling pressure of sandwich cylinders, refs. (7) and (8) may be consulted.

No experimental results are available to substantiate the theory.

(E) Buckling of Circular Cylinders Under Combined Axial Compression, Bending and Torsion

The buckling of isotropic sandwich cylinders under combined axial compression, bending and torsion loads has been investigated theoretically in ref. (9). Minimizations, however, were carried out only for the special case of sandwich constructions weak in shear (large shear deformations in the core). It was found that, for this case, the interaction equation is of the form

$$(R_T)^2 + R_c + R_B = 1 \tag{7}$$

where the R's are the usual stress ratios. For example, R_c is the ratio of applied axial compressive stress to the buckling axial compressive stress, when all other stresses are absent. Since for sandwich constructions weak in shear the buckling load is equal to the transverse shear stiffness,

$$R_T = \frac{T/2\pi R^2}{(c+t)^2 G_c/c}$$

$$R_c = \frac{N}{(c+t)^2 G_c/c} \tag{8}$$

$$R_B = \frac{2MRt/I}{(c+t)^2 G_c/c}$$

where

T = applied torsional moment, in-lb.
M = applied bending moment, in-lb.
N = applied axial compressive load, lb/in.
I = moment of inertia of sandwich cylinder with respect to its diameter, $in.^4$

(F) Buckling of Conical Shells Under External Pressure

No work of this nature has as yet been done for sandwich cones. Recently, Bijlaard, ref. (10), suggested an approximate method for computing critical pressures of homogeneous, isotropic, conical shells. The method consists essentially in replacing the cone by an equivalent cylinder of radius R (see fig. (2)) and a length

$$\ell_{eq.} = \left(\frac{r_1 + 1.2 r_2}{2.2 r_2}\right) \ell \tag{9}$$

where r_1, r_2 and ℓ are as shown in fig. (2).

As a first approximation, the same approach may be used for sandwich cones. It should be noted here, however, that the above reasoning could not be applied to eq. (6), since that equation is for the infinitely long cylinder. Instead, the results of ref. (8), where a finite sandwich cylinder is considered, should be used.

(II) WRINKLING OF SANDWICH FACES

The wrinkling type of instability of the sandwich faces has received considerable attention, both experimentally and theoretically. The solution that seems most promising at the present, and one that is fairly simple to use, is that of Goodier and Hsu, ref (11). By assuming nonsinusoidal modes of buckling, they were able to obtain wrinkling stresses of half the magnitude obtained in previous investigations. The critical wrinkling stress in the sandwich faces is given by

$$(\sigma_{w,cr})_f = \frac{E_f E_c t^2}{2 G_c \lambda_f c^2}\left(-1 + \sqrt{1 + \frac{2 G_c^2 \lambda_f c^3}{E_f E_c t^3}}\right) \tag{10}$$

where E_c is the Young's modulus normal to the plane of the sandwich, and the other symbols are as defined under Notation.

Equation (10) was obtained considering a flat plate in edgewise compression. As an estimate to wrinkling stresses of sandwich cylinders it will be assumed that eq. (10) applies to curved surfaces under compressive or torsion loads.

(III) NOTATION

c = core thickness, in.
t = thickness of one face, in.
R = radius of cylinder to mid-plane of sandwich, in.
E_f = Young's modulus of face material, p.s.i.
G_c = transverse shear modulus of core, p.s.i.
ν_f = Poisson's ratio of face material
$\lambda_f = 1 - \nu_f^2$

$$S = \frac{ctE_f}{2\lambda_f(c+2t)RG_c}$$

$$H = \frac{2ctE_f}{3\sqrt{\lambda_f}(c+t)RG_c} \left(= \frac{4\sqrt{\lambda_f}(c+2t)}{3(c+t)} S \right)$$

Other symbols are defined as they appear in the report. The subscript f is here used to identify quantities associated with the faces of the sandwich.

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FIG. 1

BUCKLING COEFFICIENT OF A LONG, ISOTROPIC, SANDWICH CYLINDER IN TORSION.
FACES ARE ASSUMED TO ACT AS MEMBRANES.

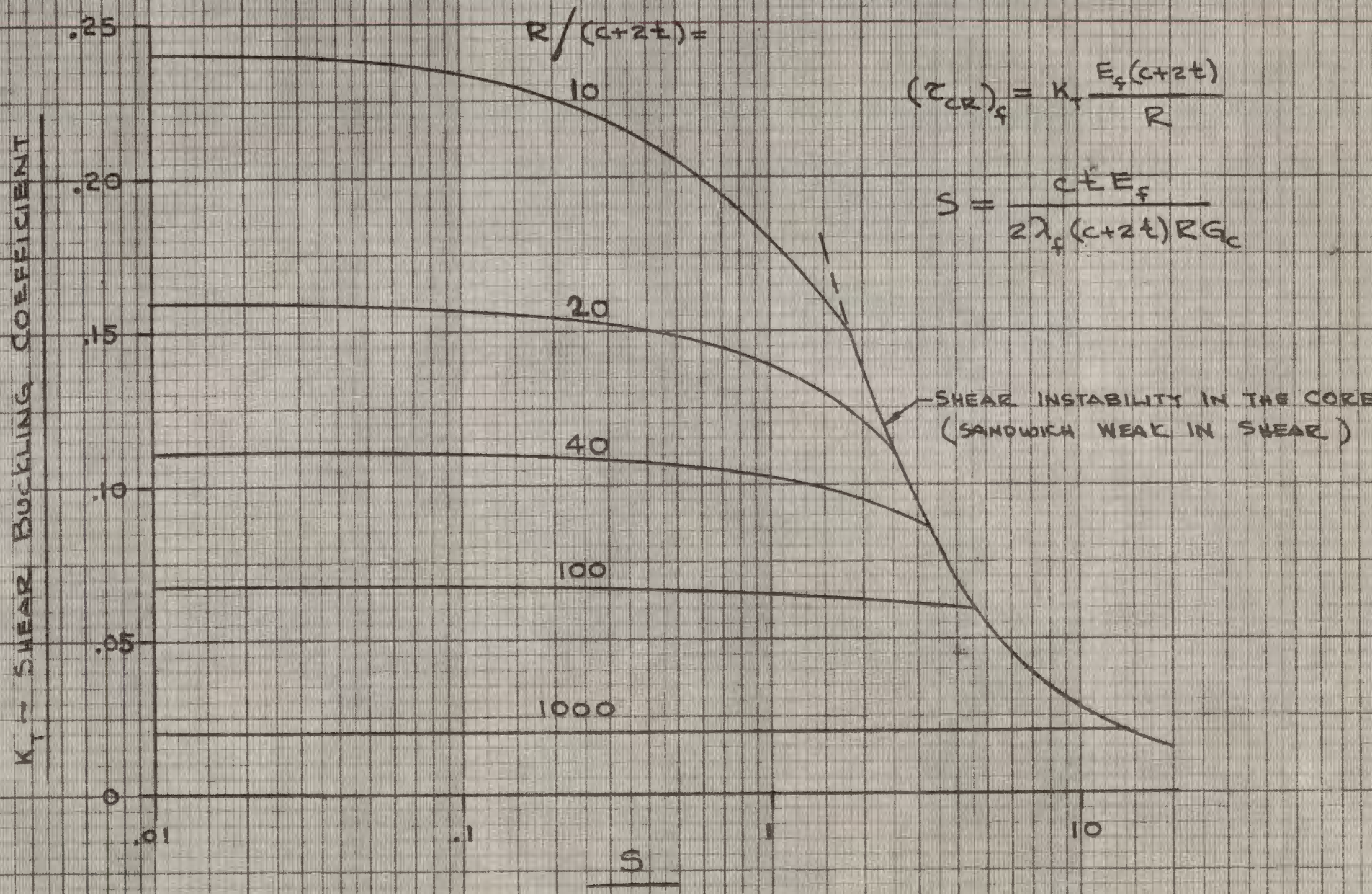

FIG. 2