OPTIMUM SELECTION OF DAMPERS FOR FREELY VIBRATING MULTIDEGREE OF FREEDOM SYSTEMS

by

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Abstract

A numerical method is presented for the optimum selection of the magnitude(s) and location(s) of from 1 to N viscous dampers, out of $(N^2 + N)/2$ possible absolute and relative dampers, for a freely vibrating discrete N degree of freedom systems consisting of masses and springs. The optimization algorithm combines iterative application of a pattern search in damper space over all possible damper configurations, along with numerical integration of the coupled equations of motion, in order to minimize an objective function. The objective function used is the integral of time multiplied by the squared error (ITSE), where the squared error is the sum of the squares of the displacements and velocities of the masses. The method does not require the solution of the eigenvalue problem. The method can accommodate constraints on the magnitudes and locations of dampers, as well as the motion of the masses, and is also applicable for optimum selection of damper in order to minimize the total response of driven systems. Results are presented for 2 and 3 degrees of freedom systems.

1. INTRODUCTION

The optimum design of dynamic vibration absorber(s) has received considerable attention since the publication of the analysis of the dynamic vibration absorber by Ormondroyd and Den Hartog[1] and the optimization of dynamic absorber design contained in Den Hartog's[2] classic text. Recent efforts include analytical and/or numerical techniques for the optimum design of linear and nonlinear dynamic absorber(s) for the passive damping of the steady state response of harmonically driven discrete[3-5] and continuous[6-9] mechanical systems.

The optimal design of dynamic vibration absorbers for minimizing the transient response of freely vibrating discrete mechanical systems has received considerably less attention. Nagaev and Stepanov[10] obtained an analytical expression for optimum damper parameters for an undamped primary system, and Rowbottom[11] developed an analytical method for optimum damping of electrical transmission lines. Ebrahimi[12] developed a numerical algorithm for optimizing dynamic absorber design for a damped primary system using two different time-domain objective functions, i.e., the logarithmic decrement of the combined system and the real part of the roots of the characteristic equation.

In this paper a numerical method is presented for the optimum selection of the magnitude(s) and location(s) of from 1 to N viscous dampers, out of $(N^2 + N)/2$ possible absolute and relative dampers, in order to minimize the transient response of a discrete N degree of freedom system consisting of masses and linear springs. The method involves the iterative application of a pattern search in damper space over all possible configurations of dampers, along with direct numerical integration of the coupled equations of motion, in order to minimize an objective function. The method is equally applicable to minimizing the total response of harmonically driven systems and can handle constraints on damper type and magnitude, as well as constraints on the motion of one or more of the masses.

2. DESIGN PROBLEM FORMULATION

A three degree of freedom system containing the maximum number of nonredundant viscous dampers, i.e., three absolute and three relative dampers, is shown in Fig. 1. A double subscript notation is employed for the dampers in order to specify their location. Equal subscripts (d_{11}, d_{22} and d_{33}) signify absolute dampers between masses 1 through 3 and ground, respectively. Unequal subscripts signify relative dampers between the masses identified by each of the subscripts, e.g., d_{23} is a relative damper between m_2 and m_3.

Assuming that the values of the masses and the stiffness of the springs are known, the problem of optimum selection of dampers for such a system will depend on the design constraints, e.g., the number ND of absolute and/or relative dampers permitted, constraints on the magnitude of one or more of the dampers, constraints on the absolute or relative motion of the masses, etc. For example, the design problem can vary from determining the magnitudes of the three absolute dampers which will produce maximum damping, to determining the location(s) and magnitude(s) of one or two dampers which will minimize the motion of the masses. In order to keep the problem general in nature, the approach used in this paper is to iterate over all possible configurations of dampers for the number of dampers ND over the range: $1 \leq \text{ND} \leq \text{N}$.

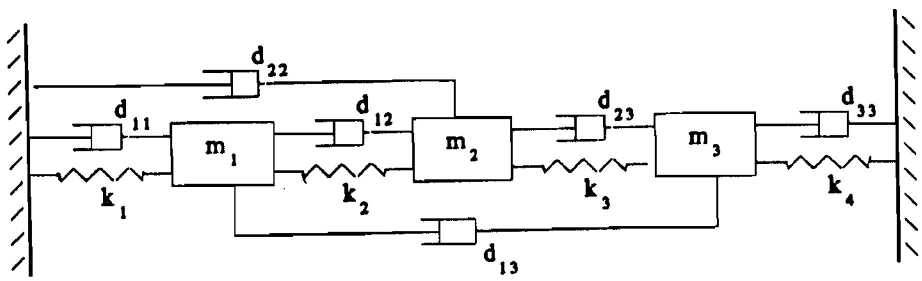

Figure 1. Three degree of freedom system with maximum number of nonredundant dampers.

The complete set of nonredundant dampers for the three degree of freedom system of Fig. 1 can be written as a 3x3 array with zero elements below the diagonal:

$$\begin{bmatrix} d_{11} & d_{12} & d_{13} \\ 0 & d_{22} & d_{23} \\ 0 & 0 & d_{33} \end{bmatrix}$$

Generalizing to an N degree of freedom system, the complete set of nonredundant dampers is given by a N x N array containing $(N^2 + N)/2$ non-zero elements or nonredundant dampers. Alternately, the system damping can be described by an $(N^2 + N)/2$ element vector, which for the system of Fig. 1 is given by:

$$\mathbf{d} = \begin{bmatrix} d_{11} \\ d_{12} \\ d_{13} \\ d_{22} \\ d_{23} \\ d_{33} \end{bmatrix} \tag{1}$$

For ND = 3, there are twenty possible configurations of the three non-zero dampers. The number of combinations C_r^m of r items from m items, where the order of selection is not important, is given by[14]:

$$C_r^m = m!/[(m - r)!\ r!]. \tag{2}$$

For ND = 2 or ND = 1, there are fifteen or six possible combinations of dampers, respectively, for the 3 degree of freedom system.

The equations of motion for the system in Fig. 1. can be written in matrix notation as:

$$M\,\ddot{x}(t) + C\,\dot{x}(t) + K\,x(t) = 0 \tag{3}$$

where

$$M = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}, \quad K = \begin{bmatrix} (k_1 + k_2) & -k_2 & 0 \\ -k_2 & (k_2 + k_3) & -k_3 \\ 0 & -k_3 & (k_3 + k_4) \end{bmatrix},$$

and

$$C = \begin{bmatrix} (d_{11} + d_{12} + d_{13}) & -d_{12} & -d_{13} \\ -d_{12} & (d_{12} + d_{22} + d_{23}) & -d_{23} \\ -d_{13} & -d_{23} & (d_{13} + d_{23} + d_{33}) \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

The x_i's are the displacements of the three masses relative to their equilibrium positions.

The coupled equations of motion of eqn. (3) are given by:

$$\begin{aligned} &m_1\ddot{x}_1 + (d_{11} + d_{12} + d_{13})\dot{x}_1 - d_{12}\dot{x}_2 - d_{13}\dot{x}_3 + (k_1 + k_2)x_1 - k_2x_2 = 0 \\ &m_2\ddot{x}_2 - d_{12}\dot{x}_1 + (d_{12} + d_{22} + d_{23})\dot{x}_2 - d_{23}\dot{x}_3 - k_2x_1 + (k_2 + k_3)x_2 - k_3x_3 = 0 \\ &m_3\ddot{x}_3 - d_{13}\dot{x}_1 - d_{23}\dot{x}_2 + (d_{13} + d_{23} + d_{33})\dot{x}_3 - k_3x_2 + (k_3 + k_4)x_3 = 0. \end{aligned} \tag{4}$$

Given the initial conditions, i.e., $x_1(0)$, $x_2(0)$, $x_3(0)$, $\dot{x}_1(0)$, $\dot{x}_2(0)$ and $x_3(0)$, and the values of the masses, springs and dampers; the coupled equations of motion, eqn. (4), can be numerically integrated to yield the displacements, velocities and accelerations of the masses as a function of time without solving the eigenvalue problem. Application to higher order systems is straight forward. However, the values of the dampers are not know. The optimization process described in the next section is used to determine the optimum damping space vector **d**, i.e., the magnitudes and locations of the ND dampers, which will minimize the transient motion of the system.

3. OPTIMIZATION

The damping optimization problem may be viewed as a determination of the damping vector **d** which gives the coordinates of the extremum of an objective function in damper space. The objective function used to quantify the motion of the system is the integral of time multiplied by the squared error (ITSE)[13], i.e.,

$$OF = \int t\, e^2(t)\, dt = \sum_k \sum_{i=1}^{N} t_k \left[x_i^2(t_k) + \dot{x}_i^2(t_k)\right] \Delta t. \qquad (5)$$

The coordinates in damper space of the minimum of the ITSE objective function give the optimum damper values for the given combination of dampers, i.e., for the non-zero elements of the damping vector **d**.

An advantage of using ITSE as the objective function is that it does not require that the eigenfrequencies of the system be determined. Also, constraints can be placed on the objective function so as to weight the motion of the individual masses, or limits can be imposed on the maximum displacement, velocity or acceleration of one or more of the masses.

The method used to determine the minimum of the objective function in damper space is the Hooke-Jeeves[15] pattern search. A step increment size δ_i, a step reduction factor α and a termination parameter ε are defined. An initial guess is made for the non-zero elements of **d**, the **C** matrix of eqn. (3) is formed and the equations of motion are integrated to determine the value of OF. Next, an exploratory search is conducted by incrementing each non-zero component of **d** by $\pm\ \delta_i$ and a new OF is calculated. If the exploratory search is successful, i.e, the new value of OF is less than the previous value, then the following pattern move is made:

$$\mathbf{d}^{(k+1)} = \mathbf{d}^{(k)} + (\mathbf{d}^{(k)} - \mathbf{d}^{(k-1)}). \qquad (6)$$

The procedure is continued until there is no further reduction in OF and then the step increment size is reduced by dividing it by α. The process continues until the reduction in OF is less than the termination parameter ε. The value of **d** at the conclusion of the search defines the location of the minimum of OF in damper space. Numerical examples are present in the following section.

4. NUMERICAL EXAMPLES

Numerical examples are presented for both two and three degree of freedom systems. In order to keep the design problem general in nature, the results of application of the optimization algorithm to all possible combinations of damper for $1 \leq ND \leq N$ are presented. This iterative approach allows comparison of the effectiveness of optimum damping for all possible damping configurations.

For the two degree of freedom system shown in Fig. 2, the following values are assigned for the masses, springs and initial conditions: $m_1 = 4.$, $m_2 = 1.$, $k_1 = 3.$, $k_2 = 1.$, $x_1 0) = 2.$, $x_2 0) = -2.$ and $\dot{x}_1 0) = \dot{x}_2 0) = 0.$ The undamped time history of the displacements of the masses is shown in Fig. 3.

The results of application of the optimization algorithm to all configurations of dampers for both ND = 2 and ND = 1 are presented in Table 1. For purposes of comparison, the value of the objective function for each configuration is normalized with respect to the minimum value of the objective function for ND = N. Review of Table 1 shows that optimum damping, i.e., OF = 1. and DN = 2, requires two absolute dampers whose values are $d_{11} = 6.33$ and $d_{22} = 2.46$. Application of the optimization algorithm to the over design case of ND = 3 yields optimum damper values of $d_{11} = 6.26$, $d_{12} = 0.05$ and $d_{22} = 2.37$ and an OF which is identical to that of the optimum ND = 2 configuration previously discussed. For the configuration of an absolute damper on m_1 and a relative damper between m_1 and m_2, optimum damping occurs for damper values of $d_{11} = 4.40$ and $d_{12} = 1.37$. This damping configuration yields an OF = 1.07; whereas changing the location of the absolute damper from m_1 to m_2 yields an OF = 17.08 for optimum damper values of d_{12} = 1.67 and d_{22} = 2.56. The time histories of the displacements of the masses for all three ND = 2 cases are shown in Fig. 4.

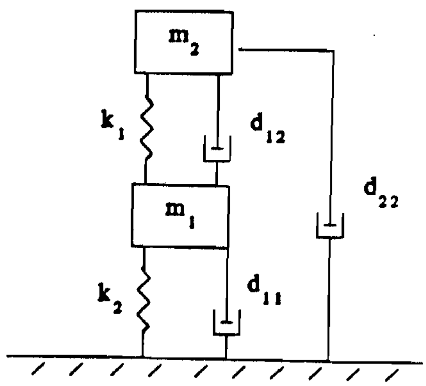

Figure 2. Two degree of freedom system with maximum number of nonredundant dampers.

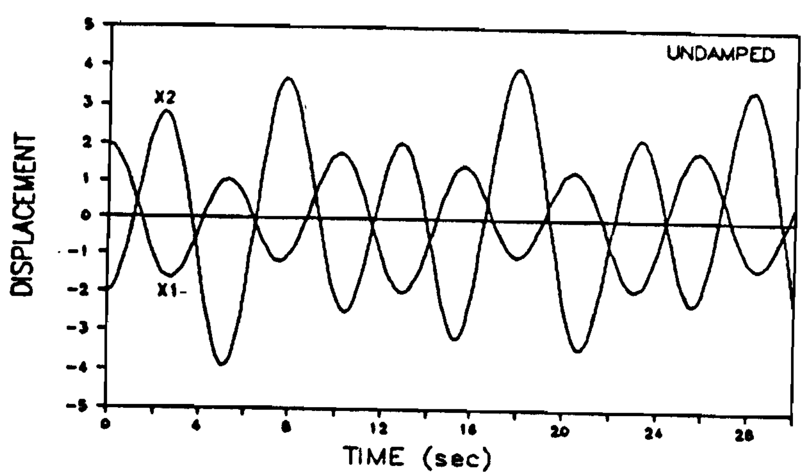

Figure 3. Time history of undamped displacements x_i of masses m_i for two degree of freedom system: m_1 = 4., m_2 = 1.,k_1 = 3. and k_2 = 1.

A contour plot of the objective function in damper space, i.e., OF versus d_{11} and d_{22}, for the optimum case is shown in Fig.5. The minimum value of OF is 7.94 with coordinates d_{11} = 6.33 and d_{22} = 2.46. This minimum value of OF is used to normalize all OF values in Table 1.

The effects on the time histories of the displacements of the masses by alternately changing d_{11} and d_{22} by ± 2.0 from the optimum damping values are shown in Figs. 6 and 7, respectively. Increasing and decreasing d_{11} by 2.0 results in OF values of 1.24 and 1.34, respectively; whereas increasing and decreasing d_{22} by 2.0 results in OF values of 2.41 and 8.53, respectively.

TABLE 1

Results of optimization for two degree of freedom system with two or one dampers; system parameters: $m_1 = 4.$, $m_2 = 1.$, $k_1 = 3.$, and $k_2 = 1.$

NUMBER OF DAMPERS	DAMPER MAGNITUDES			NORMALIZED* OBJECTIVE FUNCTION
	d_{11}	d_{12}	d_{22}	
2	4.40	1.37	-	1.07
2	6.33	-	2.46	1.00
2	-	1.67	2.56	17.08
1	2.78	-	-	51.19
1	-	0.44	-	91.40
1	-	-	0.88	35.65

* Objective function divided by minimum value of objective function with number of dampers equal to the number of degrees of freedom

Using a single damper in the two degree of freedom system under consideration, optimum damping occurs with an absolute damper on m_2, i.e., $d_{22} = 0.88$, with an OF = 35.65. If the location of the absolute damper is changed from m_2 to m_1, the magnitude of the damper required for optimum damping is slightly more than tripled with an OF = 51.19. The least effective ND = 1 damping configuration occurs with a relative damper between m_1 and m_2 of magnitude $d_{12} = 0.44$ with an OF = 91.40. The time histories of the displacements of the masses for these three ND = 1 cases are shown in Fig. 8.

For the three degree of freedom system shown in Fig. 1, the following values are assigned for the masses, springs and initial conditions: $m_1 = 2.$, $m_2 = 1.$, $m_3 = 1.$, $k_1 = 3.$, $k_2 = 2.$, $k_3 = 1.$, $k_4 = 0.$, $x_1(0) = 1.$, $x_2(0) = 0.$, $x_3(0) = -2.$ and $\dot{x}_1(0) = \dot{x}_2(0) = \dot{x}_3(0) = 0$. The undamped time history of the displacements of the masses is shown in Fig. 9.

The results of application of the optimization algorithm to the twenty possible configurations of dampers for ND = 3 are presented in Table 2. For ND = 3, optimum damping is achieved by an absolute damper on m_1 and a pair of relative dampers between m_1 and m_2, and m_2 and m_3, respectively. The optimum values of the dampers are: $d_{11} = 2.83$, $d_{12} = 1.26$ and $d_{23} = 1.76$. The configuration of an absolute damper on each mass yields optimum damper values of: $d_{11} = 4.44$, $d_{22} = 5.37$ and $d_{33} = 1.60$ and an OF = 1.06. Review of Table 2 indicates that there are nine other configurations of optimum dampers that will result in OF values which are less than 2.0. However, three of these configurations require unrealistically large absolute dampers on m_2. The least effective ND = 3 damper configuration is for three relative dampers, i.e., $d_{12} = 0.88$, $d_{13} = 0.0$ and $d_{23} = 1.29$, with an OF value of 21.28. Unfortunately, this configuration of all relative dampers is the configuration available in many space applications. Note that the optimum value for d_{13} is zero. Time histories of the displacements for the most and least effective configurations for ND = 3 are shown in Fig. 10.

The results of application of the optimization algorithm to the fifteen configurations of dampers for ND = 2 and the six configurations for ND = 1 are presented in Table 3. Review of the ND = 2 data in Table 3 indicates five damper configurations which will result in normalized OF values which are less than 5.0. The best ND = 2 case consists of an absolute damper on m_1 and a relative damper between m_2 and m_3. The optimum damper values are d_{11} = 1.61 and d_{23} = 1.34 with OF = 2.59. The least effective ND = 2 configuration consists of an absolute damper on m_3 and a relative damper between m_1 and m_2, i.e., d_{33} = 1.06 and d_{12} = 0.34, with an OF = 49.65. Time histories of the displacement of the masses for the most and least effective ND = 2 damper configurations are shown in Fig. 11.

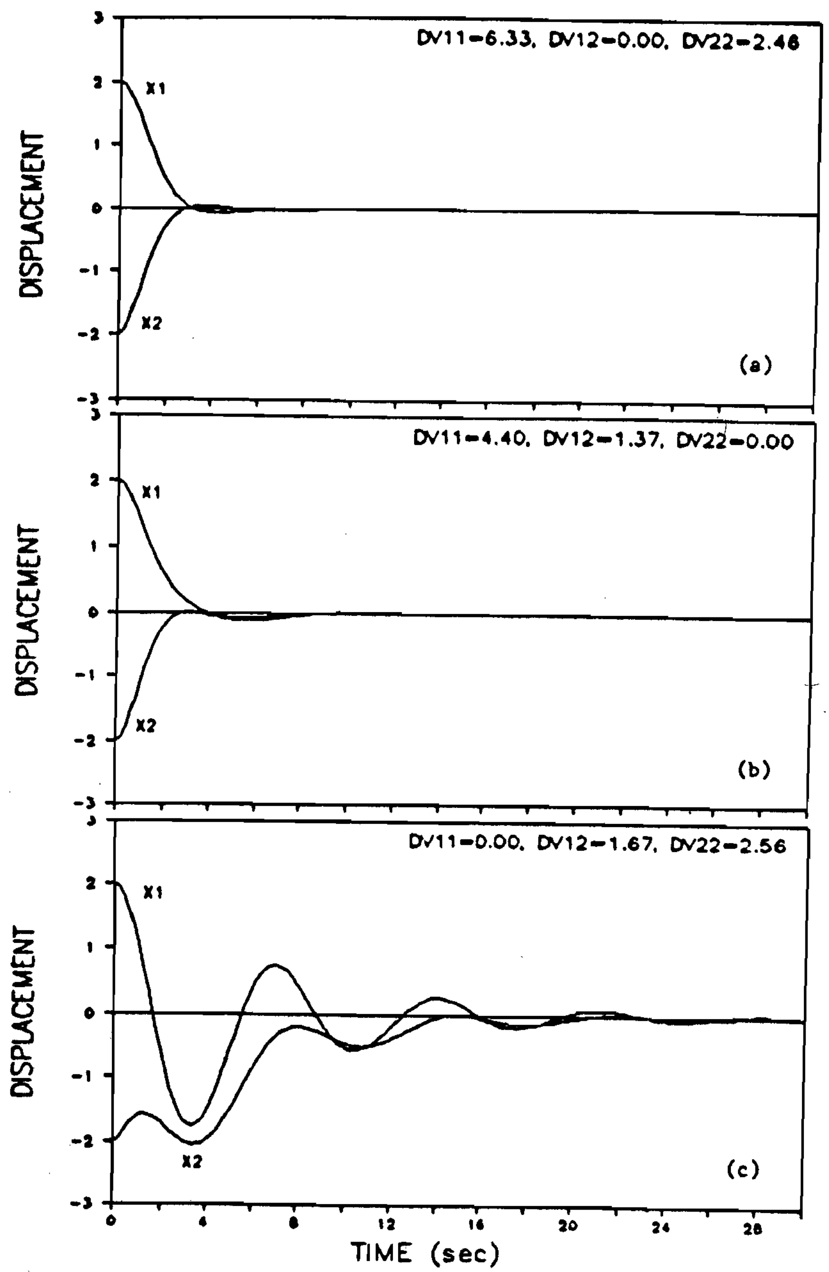

Figure 4. Time histories of the displacements x_i of masses m_i for system of Table 1 with optimum damping configurations: (a) d_{11} = 6.33 and d_{22} =2.46, (b) d_{11} = 4.40 and d_{12} = 1.37, and (c) d_{12} = 1.67 and d_{22} = 2.56.

If a single damper, ND = 1, is used for the three degree of freedom system under consideration, optimum damping occurs with an absolute damper on m_2 of magnitude 2.29. This damping configuration results in an OF = 24.56. If only relative dampers are available, optimum damping occurs with a damper between m_2 and m_3 of magnitude 0.72 with an OF which is 15% higher than the previous case. The least effective single damper configuration is a relative damper between m_1 and m_2 of magnitude 1.98 with an OF which is over forty times greater than the most effective ND = 1 configuration. The time histories of the displacement of the masses for the most and least effective ND = 1 configurations are shown in Fig. 12.

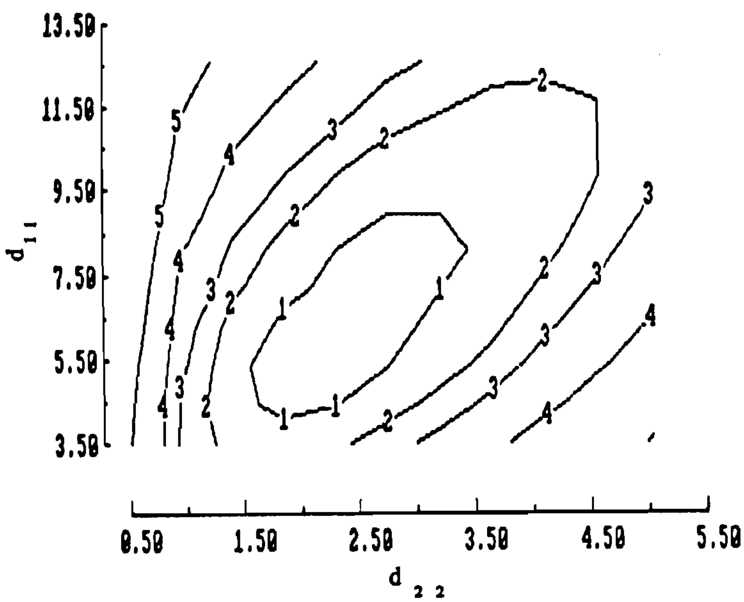

Figure 5. Contour plot of objective function (ITSE) in damper space for system of Table 1. Minimum value of OF = 7.94 at coordinates d_{11} = 6.33 and d_{22} = 2.46. Contour values: 1 = 10, 2 = 15, 3 = 20, 4 = 30 and 5 = 50.

5. CONCLUSIONS

A numerical method is presented for the optimum selection of dampers for minimizing the transient response of discrete N degree of freedom systems. Application of the algorithm in an iterative mode to all damper configurations for N dampers yields the magnitudes and locations of the N dampers, out of (N_2 + N)/2 possible dampers, which will minimize the motion of the system; as well as the minimum value of the objective function. Further application of the algorithm in an iterative mode to all damper configurations for ND dampers, over the range $1 \leq ND \leq (N-1)$, yields the magnitudes and locations of optimum dampers for all possible damping configurations of the system. The effectiveness of each possible damping configuration of the system in minimizing motion can be compared and ranked by comparison of the respective objective functions which are normalized by the minimum objective function for the ND = N configurations.

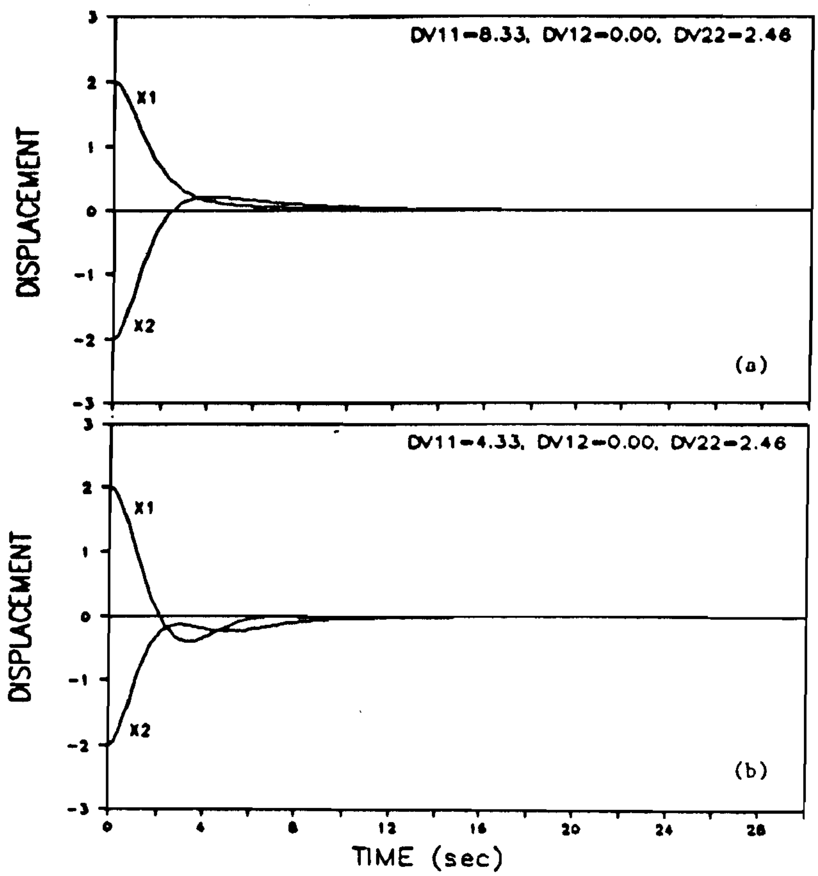

Figure 6. Effects of (a) increasing and (b) decreasing the value of d_{11} by 2.0 on system performance shown in Figure 4.(a).

Numerical examples are presented for both two and three degree of freedom systems. In the following discussion, the objective functions are normalized by the minimum value of the objective function for the ND = N configurations, and the magnitude of the optimum damper values for each configuration are not stated. In the two degree of freedom example, application of the algorithm yields the expected results that optimum damping occurs with absolute dampers on each mass. However, the configuration of an absolute damper on m_1 and a relative damper between m_1 and m_2 results in a objective function of 1.07, whereas changing the location of the absolute damper to m_2 results in an objective function 17.08. If only a single damper is used, the most effective system damping configuration is an absolute damper on m_2 with an objective function of 35.65. An absolute damper on m_1 results in an objective function of 51.19 and the least effective configuration is a relative damper between m_1 and m_2 with an objective function of 91.40. Application to the algorithm to the over design case where three dampers are used also results in an objective function of 1.00.

In the three degree of freedom example, the most effective damping configuration is an absolute damper on m_1 and two relative dampers between m_1 and m_2 and between m_2 and m_3. The configuration of an absolute damper on each mass has an objective function of 1.06. Nine other configurations of three dampers have objective functions which are less than 2.00, but three of these require unrealistically large dampers on m_2. The least effective three damper configuration is three relative dampers with an objective function of

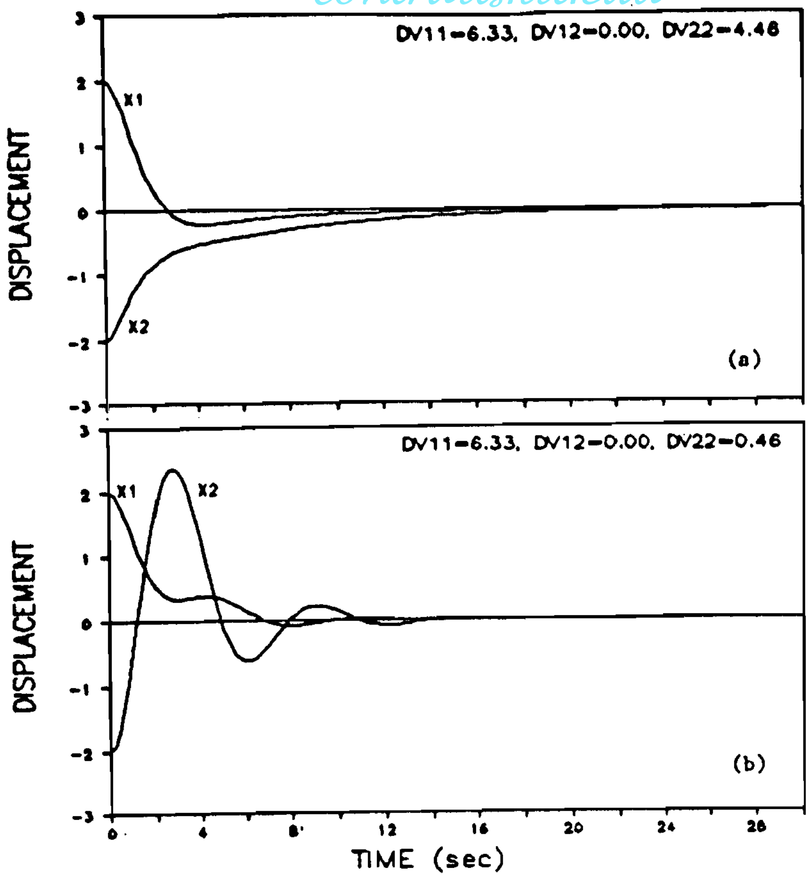

Figure 7. Effects of (a) increasing and (b) decreasing the value of d_{22} by 2.0 on system performance shown in Figure 4.(a).

21.28, however the optimum value of the relative damper between m_1 and m_3 is zero. If two dampers are used, the most effective configuration is an absolute damper on m_1 and a relative damper between m_2 and m_3 with an objective function of 2.59. The least effective two damper configuration is an absolute damper on m_3 and a relative damper between m_1 and m_2 with an objective function of 49.65. Lastly, if only a single damper is used, the most effective configuration is an absolute damper on m_2 and the least effective is relative damper between m_1 and m_2 with objective functions of 24.56 and 1044.51, respectively.

The major advantages of the method are that both the magnitudes and locations of optimum dampers for all possible damping configurations of the system are determined, the evaluation and ranking of alternative damping configurations such as type of dampers (absolute and/or relative) and number or location of dampers is easily accomplished through comparison of the respective objective functions, and the solution of the eigenvalue problem is not required. The method is also applicable for system constraints such as limits on the motion of one or more of the masses. Also, the method can be applied to optimum damper selection for driven systems or systems involving nonlinear springs. The method is general in nature in that other objective functions which depend on the motion of the masses can be used and other optimization algorithms can be used to determine the extremum of the objective function in damper space.

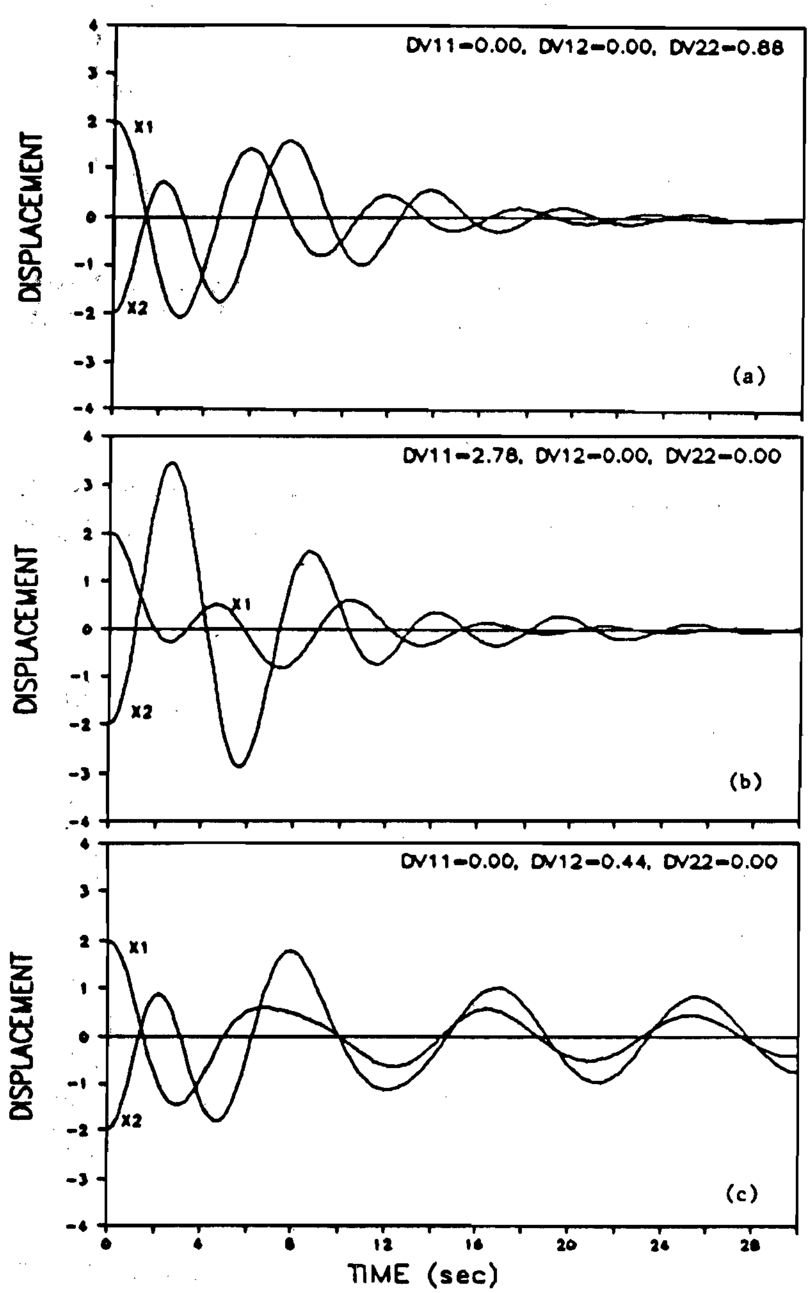

Figure 8. Time histories of the displacements x_i of masses m_i for system of Table 1 with optimum damping configurations: (a) d_{22} = 0.88 (b) d_{11} = 2.78 and (c) d_{12} = 0.44.

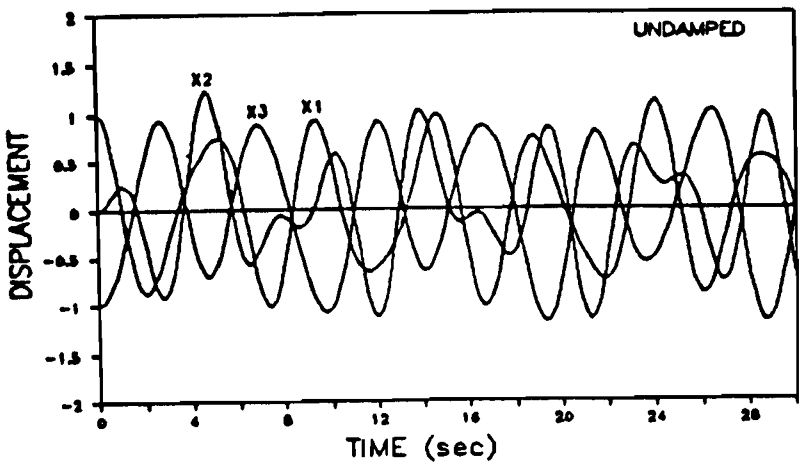

Figure 9. Time history of undamped displacements x_i of masses m_i for three degree of freedom system: $m_1 = 2.$, $m_2 = 1.$, $m_3 = 1.$, $k_1 = 3.$, $k_2 = 2.$ and $k_3 = 1.$

TABLE 2

Results of optimization for three degree of freedom system with three dampers; system parameters: $m_1 = 2.$, $m_2 = 1.$, $m_3 = 1.$, $k_1 = 3.$, $k_2 = 2.$ and $k_3 = 1.$

NUMBER OF DAMPERS	DAMPER MAGNITUDES						NORMALIZED* OBJECTIVE FUNCTION
	d_{11}	d_{12}	d_{13}	d_{22}	d_{23}	d_{33}	
3	4.16	2.11	0.58	-	-	-	1.21
3	3.31	3.16	-	0.00	-	-	4.99
3	2.83	1.26	-	-	1.76	-	1.00
3	5.13	1.28	-	-	-	1.33	1.11
3	3.30	-	0.87	2.06	-	-	1.36
3	3.07	-	0.53	-	0.83	-	1.31
3	5.48	-	0.21	-	-	1.35	2.89
3	3.03	-	-	3.14	2.37	-	1.28
3	4.44	-	-	5.37	-	1.60	1.06
3	4.26	-	-	-	0.76	1.07	1.38
3	-	5.75	0.77	4000.91	-	-	1.32
3	-	0.88	0.00	-	1.29	-	21.28
3	-	0.56	0.47	-	-	0.48	16.34
3	-	5.16	-	3284.19	1.41	-	1.05
3	-	5.16	-	3505.00	-	1.41	1.05
3	-	0.64	-	-	1.72	0.89	9.75
3	-	-	0.66	2.11	0.00	-	4.82
3	-	-	0.66	2.11	-	0.00	4.82
3	-	-	0.42	-	1.53	1.08	12.22
3	-	-	-	0.83	0.39	1.01	11.37

* Objective function divided by minimum objective function with number of dampers equal to the number of degrees of freedom

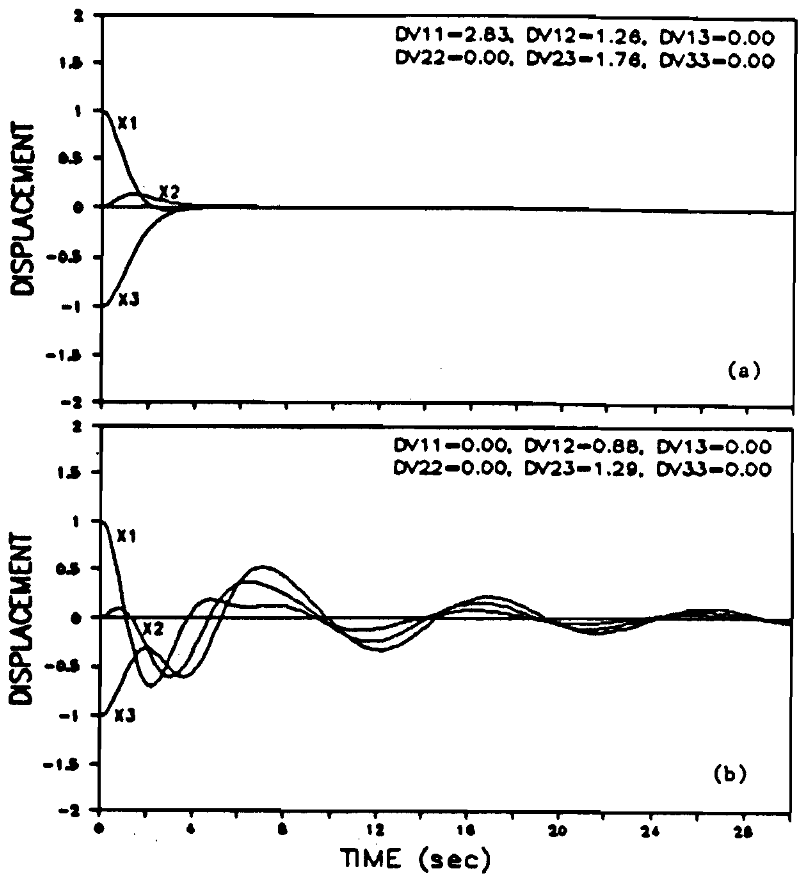

Figure 10. Time histories of the displacements x_i of masses m_i for the system of Table 2 for (a) most effective damping configuration: $d_{11} = 2.83$, $d_{12} = 1.26$ and $d_{23} = 1.76$; and (b) for least effective damping configuration: $d_{12} = 0.88$, $d_{13} = 0.00$ and $d_{23} = 1.29$.

TABLE 3

Results of optimization for three degree of freedom system with one or two dampers; system parameters: $m_1 = 2.$, $m_2 = 1.$, $m_3 = 1.$, $k_1 = 3.$, $k_2 = 2.$ and $k_3 = 1.$

NUMBER OF DAMPERS	DAMPER MAGNITUDES						NORMALIZED* OBJECTIVE FUNCTION
	d_{11}	d_{12}	d_{13}	d_{22}	d_{23}	d_{33}	
2	3.31	3.16	-	-	-	-	4.99
2	6.01	-	1.28	-	-	-	3.60
2	1.31	-	-	1.51	-	-	12.87
2	1.61	-	-	-	1.34	-	2.59
2	5.53	-	-	-	-	1.62	2.91
2	-	0.69	0.54	-	-	-	24.66
2	-	2.41	-	3.14	-	-	11.45
2	-	0.88	-	-	1.29	-	21.28
2	-	0.34	-	-	-	1.06	49.65
2	-	-	0.66	2.11	-	-	4.82
2	-	-	0.28	-	0.66	-	24.19
2	-	-	1.74	-	-	2.62	39.06
2	-	-	-	0.66	0.55	-	14.20
2	-	-	-	1.37	-	1.55	12.39
2	-	-	-	-	0.62	0.54	15.82
1	2.01	-	-	-	-	-	55.87
1	-	1.98	-	-	-	-	1044.51
1	-	-	1.02	-	-	-	307.43
1	-	-	-	2.29	-	-	24.56
1	-	-	-	-	0.72	-	28.29
1	-	-	-	-	-	1.29	212.58

* Objective function divided by minimum objective function with number of dampers equal to the number of degrees of freedom

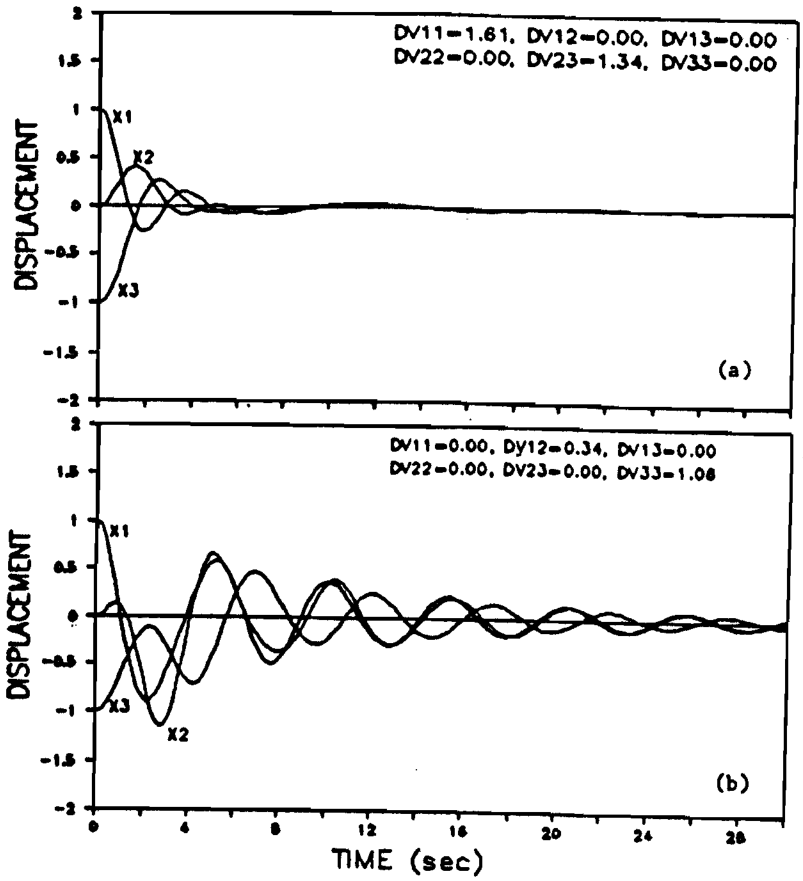

Figure 11. Time histories of the displacements x_i of masses m_i for the system of Table 3 with ND = 2 for (a) most effective damping configuration: d_{11} = 1.61, d_{23} = 1.34; and (b) for least effective damping configuration: d_{12} = 0.34 and d_{33} = 1.06.

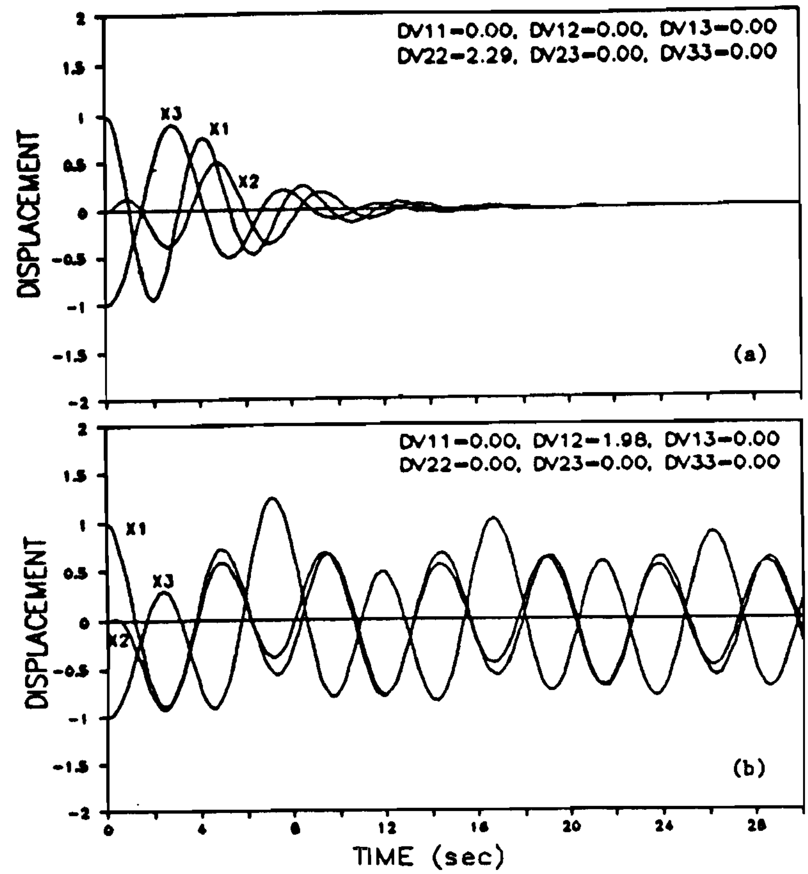

Figure 12. Time histories of the displacements x_i of masses m_i for the system of Table 3 with ND = 1 for (a) most effective damping configuration: d_{22} = 2.29 and (b) for least effective damping configuration: d_{12} = 1.98.

REFERENCES

1. J. ORMONDROYD and J. P. DEN HARTOG 1928 *Transactions of the American Society of Mechanical Engineers* **50**, 9-22. Theory of the dynamic vibration absorber.
2. J. P. DEN HARTOG 1947 *Mechanical Vibrations*. New York: McGraw-Hill.
3. A. G. THOMPSON 1981 *Journal of Sound and Vibration* **77**(3), 403-415. Optimum tuning and damping of a dynamic vibration absorber applied to a forced excited and damped primary system.
4. A. SOOM and MING-SAN LEE 1983 *Transactions of the American Society of Mechanical Engineers Journal of Vibration, Acoustics, Stress, and Reliability in Design* **105**, 112-119. Optimal design of linear and non-linear vibration absorbers for damped systems.
5. I. N. JORDANOV and B. I. CHESHANKOV 1988 *Journal of Sound and Vibration* **123**(1), 157-170. Optimal design of linear and non-linear dynamic vibration absorbers.
6. J. VAN DE VEGTE 1975 *Transactions of the American Society of Mechanical Engineers Journal of Dynamic Systems, Measurement, and Control* **97**, 432-438. Optimal dynamic absorbers for plate vibration control.

7. C. W. DE SILVA 1981 *Journal of Sound and Vibration* **74**(4), 495-502. An algorithm for the optimal design of passive vibration controllers for flexible systems.
8. L. KITIS, B. P. WANG and W. D. PILKEY 1983 *Journal of Sound and Vibration* **89**(4), 559-569. Vibration reduction over a frequency range.
9. H. N. ÖZGÜVEN and B. ÇANDIR 1986 *Journal of Sound and Vibration* **111**(3), 377-390. Suppressing the first and second resonances of beams by dynamic vibration absorbers.
10. R. F. NAGAEV and A. V. STEPANOV 1979 *Mechanics of Solids* **14**, 20-24. Optimization of the damping factor of free oscillations for a two-mass system.
11. M. D. ROWBOTTOM 1981 *Journal of Sound and Vibration* **75**(4), 559-576. The optimization of mechanical dampers to control self-excited galloping oscillations.
12. N. D. EBRAHIMI 1988 *Journal of Sound and Vibration* **120**(3), 445-455. Optimum secondary dampers for free response of underdamped systems.
13. A. F. D'SOUZA 1988 *Design of Control Systems*. New Jersey: Prentice-Hall, Inc.
14. L. L. LAPIN 1983 *Probability and Statistics for Modern Engineers*. Massachusetts: PWS Publishers.
15. G. V. REKLAITIS, A. RAVINDRAN and K. M. RAGSDELL 1983 *Engineering Optimization: Methods and Applications*. New York: John Wiley & Sons.