Response of a Circular Plate with Patch Damping

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Abstract

Originally, we were interested in minimizing, by a damping treatment, the vibration and acoustical response of a plate-like structure (such as a railroad bridge) to a dynamic force applied at a single point. Often, in practice, a damping treatment is applied to the entire surface of such a structure. We were interested in evaluating the effect of providing selected damping treatments on selected surface areas of the plate-like portions of the structure in order to maximize reducing the responses with minimal amounts of selectively located damping treatment. If information of this kind was available, the cost of applying a damping treatment to a large structure could be reduced. Before we proceeded to the practical problem of treating an actual structure, we sought first to obtain information on the responses of a simple structure, such as a circular thin plate, to damping treatments applied selectively to its surface area. We studied the first three axisymmetric modes of vibration of a thin circular plate by a finite-element approach. The elements of the plate were modeled in terms of their mass, loss factor and flexural rigidity. The measured Q factor for an untreated plate vibrating in its third mode was 425. When 15 percent of the surface area of the plate was covered with a damping treatment, the measured Q factor of the treated plate in the same mode was 40. In general, there is good agreement between the computed and experimental frequencies, mode shapes and motion amplitudes of the untreated plate. As the treated area of the plate was increased, the agreement between computed and experimental results deteriorated, particularly with respect to the motion amplitudes of the higher modes at and near the antinodes. Based on the results obtained from these tests, we have concluded that it should be possible to optimize the amount and location of a surface damping treatment on a large plate-like structure in order to obtain reductions in the acoustic and vibration responses that approach those which can be obtained by applying a damping treatment to the entire structure.

Introduction

Structural vibrations cause noise radiation which can be excessive and objectional under certain circumstances. This occurs particularly when a structure goes into resonance. At this condition, the amplitudes of vibration are large, and must be reduced to a safe level.

The most common means of controlling structural vibrations is by applying damping. If a damping treatment is properly applied, it can prevent the structure from failing, perhaps, catastrophically.

Usually, the form of application of the damping treatment depends on the type of the structural system. It is well known that the response of a coupled lumped-mass system at resonance can be reduced significantly, using viscous dampers in the form of dashpots; while a plate vibrating at resonance in more than one mode should be treated with a continuous damping layer.

The analysis of damped structures is relatively simple if the application of damping is proportional, that is, if the damping matrix is a linear combination of the mass and stiffness matrices. This formulation assumes that the structure is fully coated and the coating is uniform. In this study, a viscoelastic material which provides only damping is used, in which case the term characterizing dry friction damping in the inertial matrix, will not appear. The damping matrix is still a linear function of the stiffness matrix, and the undamped modal matrix may be used to uncouple the equations of motion. The damping parameter is introduced only to calculate the response of the structure.

However, the search for light-weight, rigid structures requires that the external damping treatment be minimized. Then, only selected areas must be treated and, since the damping layer is of the free type, it must be applied to regions of large vibration amplitudes where tensile stresses are highest. This will result in a case of nonproportional damping, because the stiffness matrix will contain real and complex elements pertaining to untreated and treated structural elements, respectively. This type of problem is complex and the response of the corresponding system can not be found from the eigensolution containing no damping.

A numerical model of a circular plate clamped at its center is considered. The plastic is partitioned into annular elements, and only axisymmetric modes are allowed.

Formulation of the Problem

Elastic structures can be analyzed by classical mode superposition methods after evaluating their mass, stiffness and damping matrices. The equation of motion of an n-degree of freedom system with hysteretic damping is

$$[M]\{\ddot{q}\} + [[K] + f\,[H]]\{q\} = \{Q\}. \tag{1}$$

The damping matrix [H] can be linearly related to [K], depending on the configuration of the coating. If the entire structure is treated, then

$$[H] = f\,\eta\,[K], \tag{2}$$

where η is a proportionality constant, referred to as the loss factor of that structure. Introducing Eq. (2), Eq. (1) becomes

$$[M]\{\ddot{q}\} + (1 + f\,\eta)\,[K]\,\{q\} = \{Q\}. \tag{3}$$

Assuming a solution in the form $\{q(t)\} = \{q_o\}\sin\omega t$, the classical eigenvalue problem is obtained. The response in the case of a nonproportionally damped system, as expressed by Eq. (1), is [1]

$$\{q_o\} = \sum_{r=1}^{n} \frac{\{q\}\{q\}^T\{Q\}}{\omega_r^2 - \omega^2} \tag{4}$$

where the square of ω_r is the rth eigenvalue in the solution of the eigenproblem corresponding to Eq. (1). Both the eigenvalue and the associated eigenvector are complex. In the case of proportional damping, Eq. (4) takes the form

$$\{q_o\} = \sum_{r=1}^{n} \frac{\{q\}\{q\}^T\{Q\}}{\omega_r^2 (1 + f\ \eta) - \omega^2} \tag{5}$$

where, now, the eigenvalues and eigenvector are real.

In the above, the stiffness matric is a function of the equivalent complex modulus of elasticity of the structure, $E^* = (1 + f\ \eta)\ E$. In turn, the latter is determined experimentally at given frequencies and temperatures. Therefore, the introduction of hysteretic damping in the equations of motion does not necessarily imply that the damping material used has hysteretic properties. Thus, in general, these equations are only valid at the conditions under which the measured quantities are obtained. In this respect, the complexity of frequency dependence of the treatment material is irrelevant, as long as a different eigenvalue problem would have to be solved at each frequency of interest.

The Damping Model

Among the energy-dissipating mechanisms which have been considered for the design of damped structures, hysteretic damping has been the most widely exploited; particularly in structural configurations incorporating viscoelastic materials, capable of dissipating relatively large amounts of energy. The dissipating capacity of a given material is characterized by the loss factor, defined as the ratio of energy dissipated during one cycle to the total energy stored in the system for the duration of that cycle.

Some viscoelastic materials and most metals possess stress-strain characteristics which deviate from the elliptic shape, exhibiting a nonlinear property. In these cases, compromise is necessary and a linear approximation is used, unless the deviation is unacceptably excessive. The best compromise appears to maintain the loop areas and the amplitudes of the stress and strain [2].

Mathematical models developed to evaluate parameters of linear damping have been reviewed by Bert [3]. Because of their complexity, some of these have no immediate implementation in the general sense. Others are limited in their range of application. A comparison of the various models shows that the difference between them lies only in the way the respective loss factor is expressed in terms of the input variables, such as frequency and temperature.

The work done on the dynamics of beams and plates incorporating continuous damping treatments is well documented [1]. In these models, attempts are made to duplicate the dynamic characteristic of the treated plate using an equivalent angle plate, with the loss factor, the mass and the flexural rigidity remaining constant. These conditions may be expressed as follows, see Fig. 1.

$$\eta = \eta_o, \quad (6)$$

$$\rho t = \rho_1 t_1 + \rho_2 t_2 \ , \quad (7)$$

$$EI = E_1 I_1 + E_2 I_2. \quad (8)$$

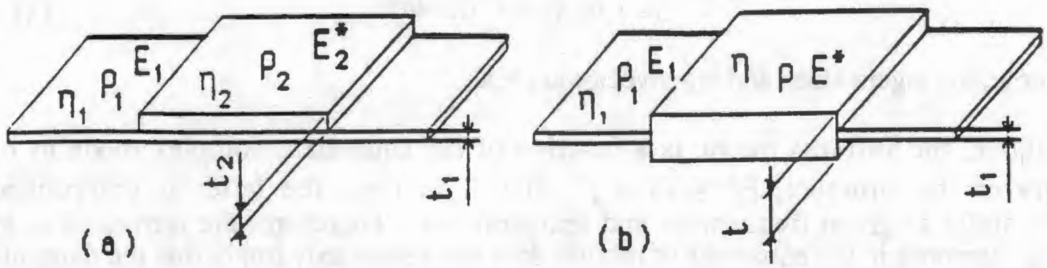

Fig. 1 Transformation of (a) the plate into (b) a singular equivalent plate.

In Eq. (6) it is assumed that the damping of the base plate is negligible. The unknown quantities here being ρ, t, η and E, a fourth equation is required. Since the mass moment of inertia is a function of the material density and the geometry of the composite, it can be used here.

$$J = J_1 + J_2$$

Three of the four unknowns may be calculated using the geometry of the coated plate as shown in Fig. 2, using Eqs. (7) through (9).

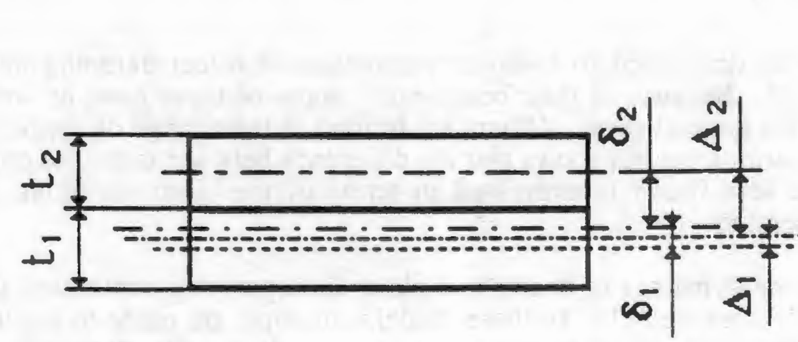

Fig. 2 Cross section of a coated element.

The equivalent loss factor may be determined using different approaches [4,5]. The model due to Cremer *et al*. [4] is used, yielding the following expression for η.

$$\eta = \frac{12\eta_2 E_2 t_2 \Delta_2^2}{E_1 t_1^3 + 12E_2 t_2 \Delta_2^2} \tag{10}$$

Now the four unknowns which define the model of the treated plate completely can be calculated for given a material and thickness of the base plate and the coating. The values of E_2 and η_2 in Eq. (10) are determined experimentally, using the resonant beam method [6]. The inertial and complex matrices of Eqs. (1) and (2) are evaluated from the equivalent parameters obtained for a circular plate clamped at its center. The plate is partitioned into ten annular elements and allowed to vibrate only in its axisymmetric modes.

Experimental Work

The experiments performed in this work may be grouped into two parts. The first part is concerned with the measurement of the loss factor and the storage modulus of the damping material. The resonant-beam technique was used, whereby a carefully machined cantilever beam was coated with a commercial damping material, Type GP-2 supplied by SoundCoat, Inc., New York. The test beam, made of aluminum 66-60, was machined with its root in block to observe the clamped condition.

The coated-beam experiments yielded loss factor values of 0.259 at 76.0 Hz, 0.420 at 136.0 Hz and 0.418 at 758.0 Hz. The two latter frequencies correspond to the second and fourth modes of the composite beam, respectively. According to the recommendations put forth in the measurement method based on its acceptance as ASTM E 756/83, reliable measurements were obtained only if the beam was vibrated at higher modes whose shapes exhibit half wavelengths. The first value of the loss factor was obtained accordingly at the second mode, with a concentrated mass attached to the free end of the beam to lower its resonance frequency to 76.0 Hz, 0.420 at 136.0 Hz, and 0.418 at 758.0 Hz. The two latter frequencies correspond to the second and fourth modes of the composite beam, respectively. According to the recommendations put forth in the measurement method based on its acceptance as ASTM E756/83, reliable measurements are obtained only if the beam is vibrated at higher modes whose shapes exhibit half wavelengths. The first value of the loss factor was obtained accordingly at the second mode, with a cenentrated mass attached to the free end of the beam to lower its resonance frequency to 76.0 Hz. The respective values of the storage modulus were found to be 2.50, 2.74 and 2.82 GNm^{-2}.

Damping due to air resistance was verified by driving the untreated beam into resonance *in vacuo*. The pressure in the vacuum chamber was gradually increased from 18.0μmHg to ambient pressure. At 30.0Hz, the loss factor was found to be 0.0050 at ambient pressure, and 0.0048 at 18 μmHg. At 216.0 Hz these values were 0.00095 and 0.00086, respectively.

The second part of the experiments consisted in measuring the plate response with different damping treatments. The experimental set-up is shown in Fig. 3, showing a plate, 1.2 mm thick and 286.0 mm in diameter, excited by seven electromagnets placed equidistantly around the circumference. The exciting force was kept constant by fixing the value of the current in the coils to 200.0 Ma. This value was chosen to avoid saturation of the circuit cores, and to obtain a response large enough to be measured in the cases of heavy damping. The various positions of the damping patches and their areas as a fraction of the total plate area are shown in Figs. 4 and 5 for the second and third mode, respectively.

The plate was first fully coated, then it was gradually uncovered according to Figs. 3 and 4. In the case of the first mode, the damping material was removed from two elements at a time, starting from the circumference.

Since the damping layer is of the free type, the areas to be treated are those in the neighborhood of a displacement antinode, where strain energy is maximal. If the wave neighborhood of a displacement antinode represents also the point with the smallest radius of curvature, where stress and strain are both high. However, in situations where the deformed shape is not symmetric, the treatment is applied in the area with small radii of curvature.

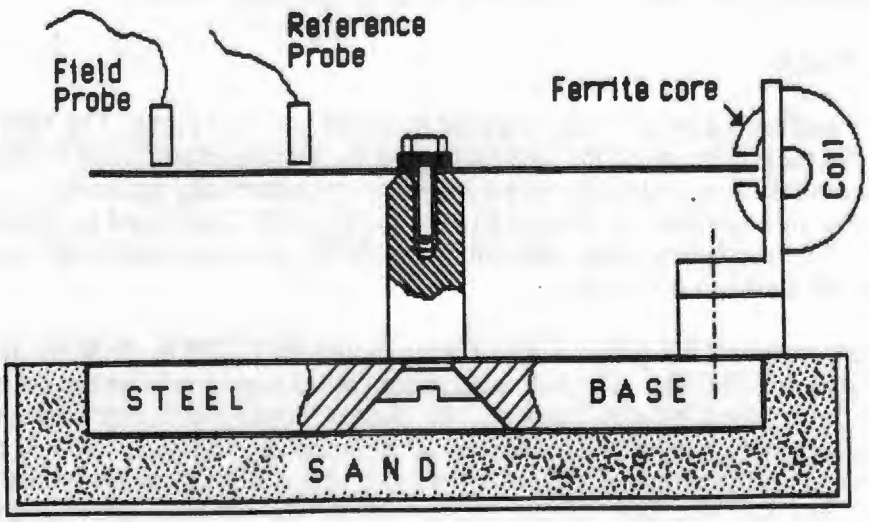

Fig. 3 Experimental set-up for the plate model.

Results and Discussion

The resonance frequencies of the untreated and fully coated plates are given in Table 1.

		Untreated	Fully treated
Mode 1	Calculated	56.3	55.8
	Measured	55.0	54.5
Mode 2	Calculated	329.7	330.7
	Measured	3336.5	3332.0
Mode 3	Calculated	955.9	959.3
	Measured	1016.8	997.7

Table 1 Natural frequencies of the circular plate (Hz).

The measured resonance frequency of the first mode is lower than the calculated value for both the treated and the untreated case. This is consistent with the principle by which the Rayleigh quotient overestimates the natural frequency of a system. In the second and third mode, the measured values are higher. Since the Rayleigh quotient is applicable only to the fundamental mode, it can not be used to justify this result. However, we believe that the numerical model did not predict the proper ratio of the added mass and stiffness. Also, the damping layer in the experimental model is not constrained at the boundaries. The assumption that the coating does not undergo shear deformation, and the invariance of Poisson's ratio in the calculation of the flexural rigidity of the composite plate have contributed to these discrepancies. These will also affect on the amplitudes and mode shapes.

The Q factor of the plate evaluated at element node 4 is shown in Figs. 6 and 7. The measured and the predicted curves are in good agreement. However, this comparison can not be made for the outer region of the plate described above as it will be seen later in the discussion.

The displacement response for the first three modes is shown in Fig. 8 through 13. A curve fit was performed on each set of data for presentation clarity. A detailed comparison was made between the calculated and experimental results for each damping case. The response of the undamped plate was predicted with an error of less than 1%, evaluated at the free edge. A close agreement is also obtained with coverages up to 36% of the plate surface area. The error at 36% coverage is 8%, while that for a full coating is 6%. This discrepancy, which increases as the treated area is reduced, can be attributed to the unconstrained boundary elements. These, being at the edge of the treated area, are not subject to tensile forces, as is assumed in the numerical model. As the treated area is reduced, the surface area of the boundary element becomes relatively high, and its contribution to the damping of the plate appears to be less significant. An error of 32% was recorded in the 4% and 16% coverage cases.

The results for the second mode are presented in Figs. 10 and 11. The damped waveform exhibits two regions of interest. The first is the region within the nodal circle where the measured values are greate than the predictions. This is consistent with the argument on the boundary element as explained above. The error varies from 28% at 27% coverage to 3% for full coating. The second region of concern is that between the nodal circle and the edge, which behaves simply as an annular plate, simply supported at the inner diameter and free at the outer edge, oscillating without undergoing flexural deformation. The induced tensile stresses in this region are small and, as a consequence, the damping treatment has little effect. The vibration amplitudes are controlled by the inertia of the annular region which may have contributed to an observed progressive shift of the nodal circle in the coating cases above 40% coverage of the plate area.

This behavior is more promounced in the third mode, shown in Figs. 12 and 13. The results agree in the area within the first nodal circle, where an error of 22% was recorded for a 15% coverge. This error drops to 8% for the fully coated plate. Although the flexing motion of the region located outside the first nodal circle is pronounced, the local antinode is heavily damped.

Conclusion

The dynamic response of a circular plate incorporating patch damping is investigated using a finite-element approach, with only axisymmetric modes being allowed. The predicted response was in reasonable agreement with our experiments at low frequencies, or in the regions where half wavelengths are described. A flexing motion of the outer part of the plate is observed which the numerical model failed to predict.

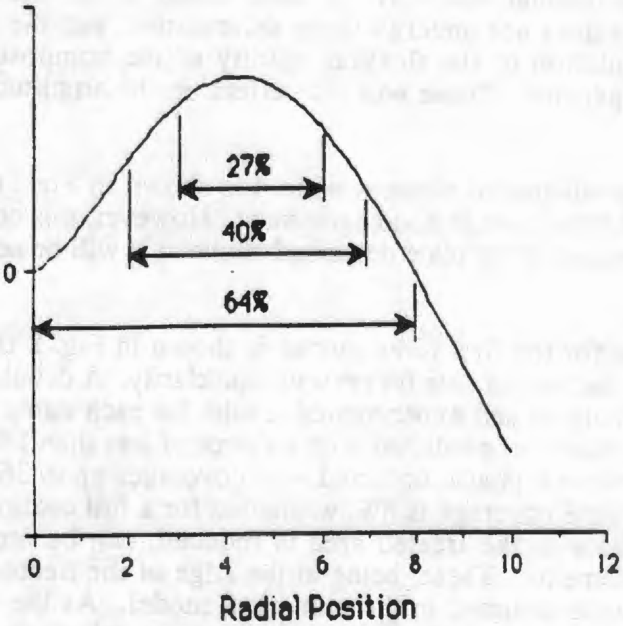

Fig. 4 Different damping treatments in mode 2.

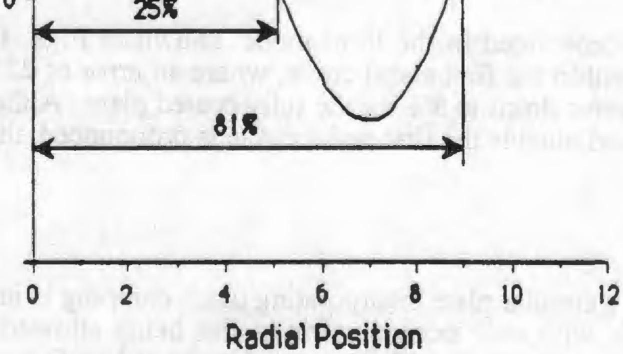

Fig. 5 Different damping treatments in mode 3.

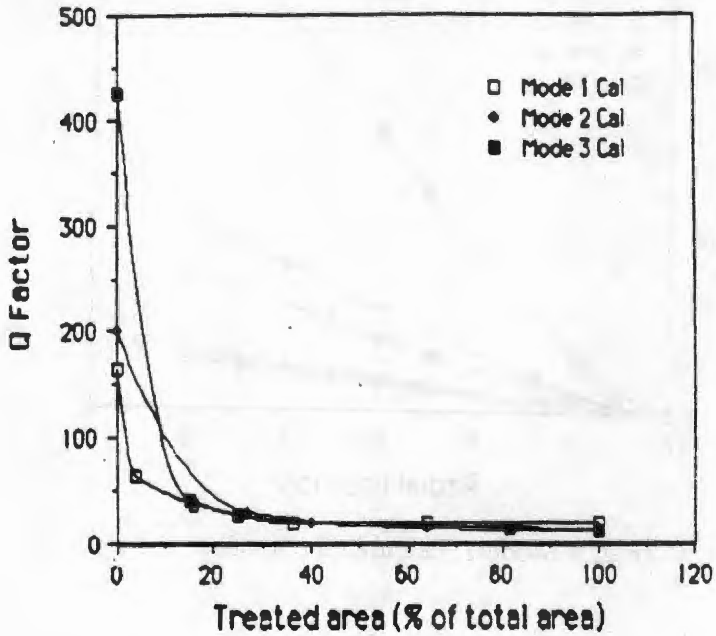

Fig. 6 Calculated Q factor.

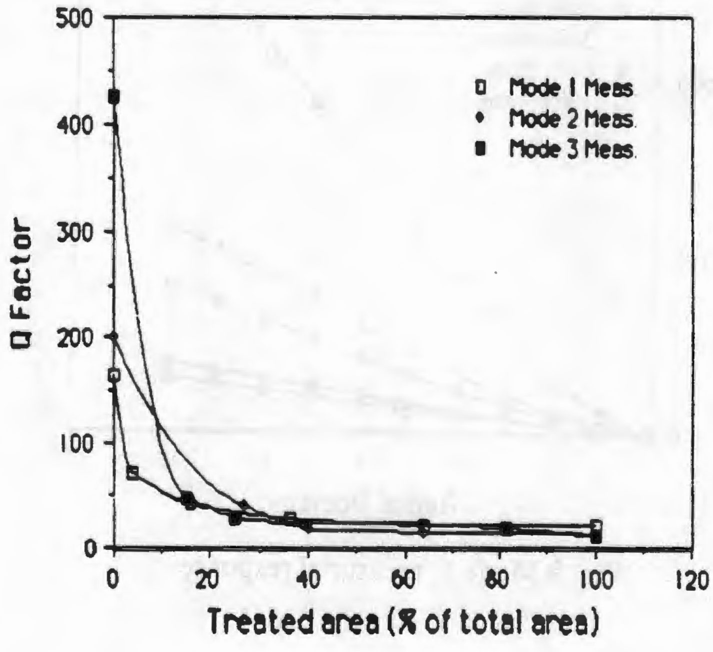

Fig. 7 Measured Q factor

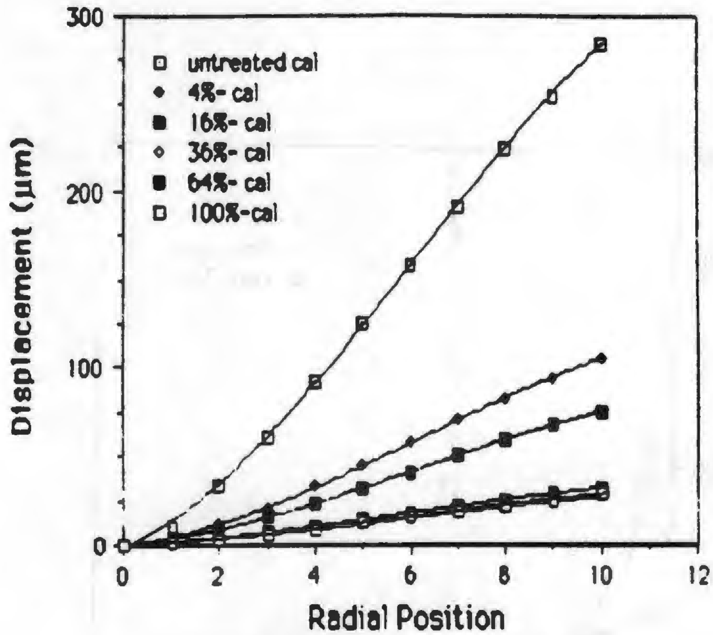

Fig. 8 Mode 1, calculated response.

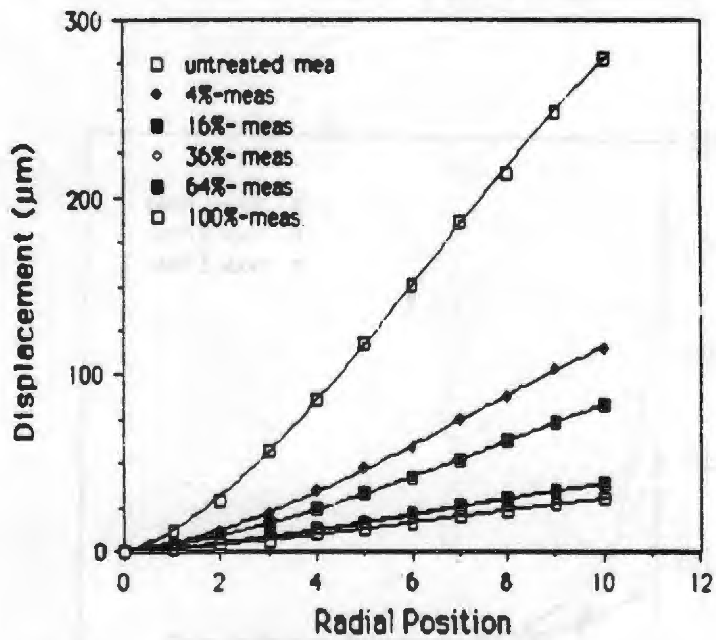

Fig. 9 Mode 1, measured response.

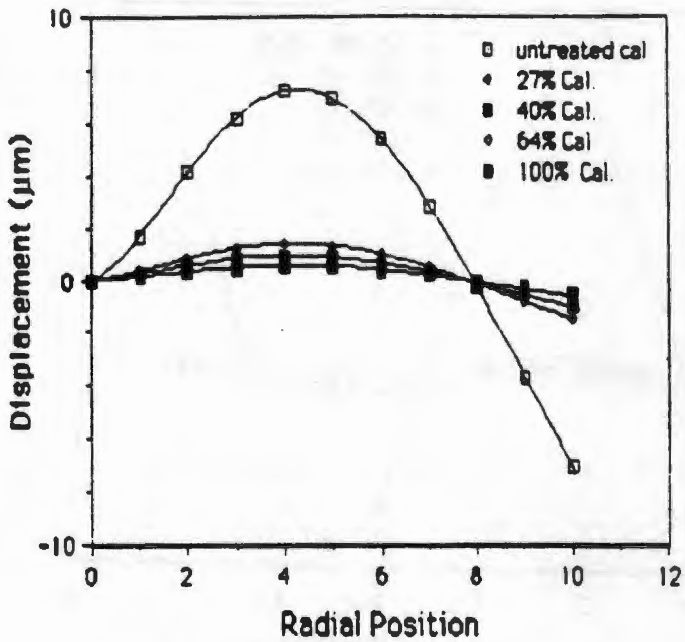

Fig. 10 Mode 2, calculated response.

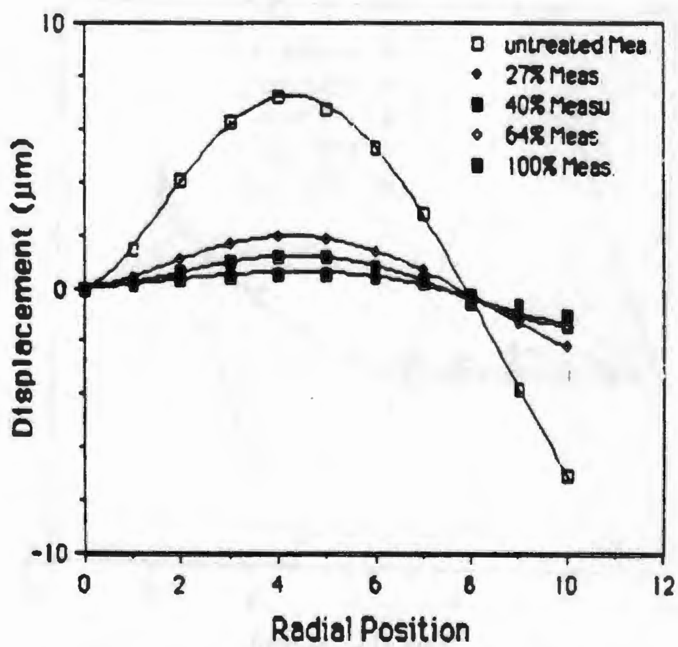

Fig. 11 Mode 2, measured response.

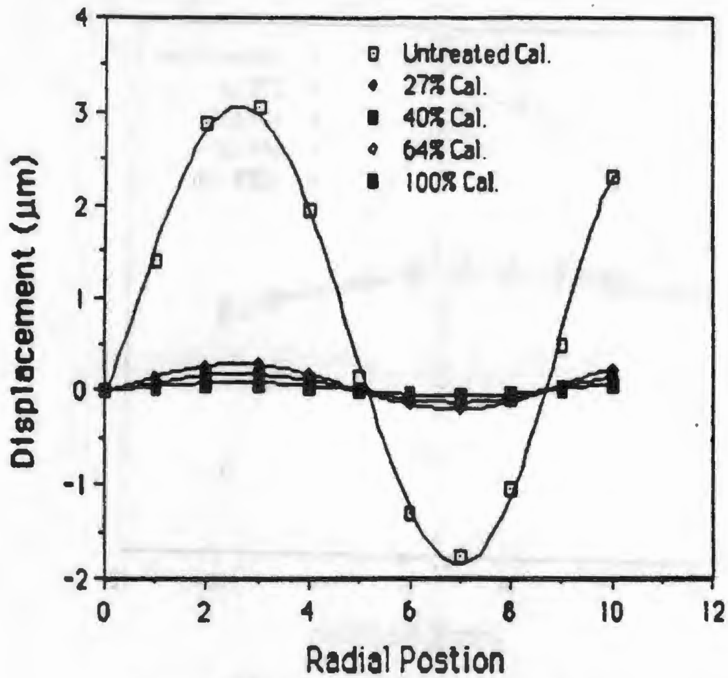

Fig. 12 Mode 3, calculated response.

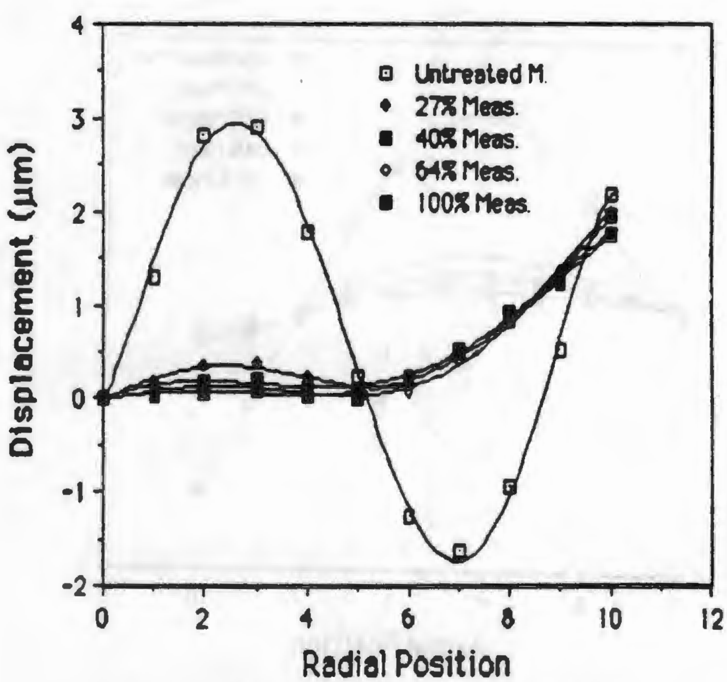

Fig. 13 Mode 3, measured response.

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