SESSION 2

GENERAL MATRIX METHODS II

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AUTOMATED STRESS ANALYSIS USING SUBSTRUCTURES

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The concept of subdivision of structures into any number of substructure levels is presented and the requirements for introducing this concept into automated stress analysis are discussed. The principles of substructure analysis are summarized and related to conventional matrix stress analysis techniques. A range of practical applications of substructure analysis is considered and potential benefits in reducing problem formulation and increasing analytical flexibility are described. Applications include partial solution of large structures, reduction of repetitive data preparation, automated idealization and subdivision and systematic modification of structures. An automated stress analysis scheme currently under development is briefly described, with particular attention given to the implementation of substructure analysis facilities. The organization of the scheme is outlined in three parts:—expansion of condensed data, assembly of structural behavior data and solution and backsubstitution. Finally the question of selecting and condensing the voluminous data output is considered.

INTRODUCTION

The fundamental requirement of a modern stress analysis procedure is the ability to analyze real engineering structures with a minimum of problem formulation and specification and with maximum flexibility in computation and handling the results. It is imperative that fully tested and automatic general purpose computer routines should be developed to provide this capability and that the functional requirements for these routines should extend far beyond the manipulation of the formal algebra of the problem.

The bulk of the vast literature now available on the subject of numerical stress analysis is concerned with the formal mathematics of an increasingly wide range of structural problems and the programming of this central mathematics. In practice this established core of the analysis problem is now well understood and extensively programmed and most present day effort in an advanced organization is directed towards providing flexible engineering facilities, input and output data processing and improved engineer-computer communication. One of the best-known examples of this trend is the STRESS system developed by MIT (Reference 1).

This paper deals with certain aspects of the structural analysis and matrix handling schemes at present under development in the British Aircraft Corporation. It is concerned with the provision of facilities to analyze structures in terms of smaller compound units called substructures and with the engineering requirements for these facilities. It also outlines the integration of these facilities into a comprehensive user-oriented stress analysis scheme.

1.0 DEFINITIONS AND PRINCIPLES

A structure is essentially an assembly of individual members arranged to transmit loads between different points in space. The individual structural members are usually physically defined as separately manufactured items, but for the purposes of structural analysis it is

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often necessary to subdivide them into smaller units which we term <u>structural elements</u>. In this paper two types of elements will be considered which are defined as follows:—

<u>Simple elements</u> are the smallest units into which the structure is subdivided for analytical purposes. The internal state of a simple element is completely defined by a finite number of boundary force and deflection functions and the behavior of the element (i.e. the relationship between boundary forces and displacements) is derived directly from the governing assumptions. Internal stress and strain distributions within simple elements are generally a function of these assumptions and we are only concerned with their overall or average values.

<u>Compound elements</u> are the smallest elements which are required for the determination of overall force and displacement distributions. They differ from simple elements in that their behavior is too complex to be defined in terms of simple assumptions and they are therefore further subdivided into sub-elements of simple form in order to determine their boundary behavior. They resemble simple elements in that detailed internal stress and displacement distributions are not normally of interest.

The structural members may be grouped (either physically for manufacturing convenience, or to simplify computation) into compound units which are then joined to other such units or to simple members to form the complete structure. These units are referred to as <u>substructures</u> and are distinguished from compound elements, in that the behavior of their constituent members is of direct concern to the analyst. For complete generality we visualize any substructure as being made up of simple elements, compound elements and further substructures in any combination. This leads to the concept of a structure built up as a "tree" or hierarchy of substructures each of which may contain further substructures down to any level of detail refinement. This concept is illustrated diagrammatically in Figure 1 and given physical meaning in relation to an aircraft structure in Figure 2. To make the definition precise we can refer to the levels of subdivision by numerical labels as shown in Figure 1, and an easy means of identifying any substructure is obtained by generating cumulative reference numbers separated by a spacing symbol (/) in which the last number is the sequential number of the substructure or element within the next numbered substructure, and so on.

Thus $i/j/k$ represents a 4th level element or substructure defined as the kth (4th level) substructure of the jth (3rd level) substructure of the ith (2nd level) substructure of the complete (1st level) structure. This type of identification makes it easy to add or remove members and substructures without affecting the remainder.

If identical elements or substructures are accounted for then a second identification is needed to define the common source of member data in addition to the essential number needed to distinguish between the members.

The fundamental principles of substructure analysis have been clearly outlined by several authors, for example Przemieniecki (Reference 2) using a displacement approach and Argyris and Kelsey (Reference 3) using redundant interaction force concepts. The most compact treatment, and that most generally suitable for automation is the displacement method and this is used in the following condensed theory.

A substructure is visualized as a structurally rigid assembly of members which intersect at points which we term the substructure nodes. Some of these nodes are common to other members of the structure and will be called <u>terminal</u> nodes: the remainder appear only in the one substructure and are called <u>internal</u> nodes. The substructure behavior is defined by the substructure displacements of all the nodes together with any internal forces in its constituent members. The substructure displacements in a linearly elastic structure can be obtained by the superposition of two systems:

(a) The displacements of the internal nodes, under the action of the forces applied to them and the internal stresses in the members, the terminal nodes being held fixed;

(b) The displacements due to release of the terminal nodes under the action of the external forces applied to them less the constraint forces appropriate to system (a).

From the viewpoint of the remainder of the structure the influence of any given substructure is completely defined by the constraint forces due to the internal loads and stresses of system (a) together with the terminal node stiffness matrix with the internal freedoms unconstrained. Thus for the purposes of analyzing the structural assembly the substructure is the exact counterpart of a simple element in the presence of initial strains. The mathematical techniques of solution of structures composed of simple elements, compound elements and substructures can thus be written in the common form applicable to simple structures.

For each member i of a structure there exists a stiffness matrix k_i and a set of internal loadings which result in boundary reactions S_i^*, whether this member be a simple or compound element or a substructure. The resultant boundary forces S_i associated with member displacements u_i are given by:—

$$S_i = k_i u_i + S_i^* \tag{1}$$

If the full set of structural deflections in all relevant degrees of freedom is U_F and the member deflections u_i are selected from U_F by the simple (Boolean type) matrix a_i such that $u_i = a_i U_F$ then the corresponding structure loads R_F are given by

$$R_F = a^T k a U_F + a^T S^*$$
$$= \sum a_i^T k_i a_i U_F + \sum a_i^T S_i^* \tag{2}$$

where a, k and S^* are compounded from the a_i's, k_i and S_i^*'s in the standard manner. (See notation).

For the sake of generality the full set of member displacements U_F must be related to a reduced set of true structure displacements U by the imposition of the external constraints. A wide variety of constraints (Reference 4), including the normal case of fixed boundary nodes, can be represented by a linear transformation matrix T such that

$$U_F = TU \tag{3}$$

The external forces R corresponding with the true displacements U are then given by

$$R = T^T a^T k a T U + T^T a^T S^*$$
$$= KU + R^* \tag{4}$$

. or

$$(R - R^*) = KU \tag{5}$$

where

$$K = T^T a^T k a T$$

and

$$R^* = T^T a^T S^*$$

The formal solution for the assembled structure including internal member loads (defined by the matrix U of displacements of the assembled structure) is thus obtained by the standard solution of the structure stiffness equations using a modified set of external loads $(R - R^*)$

Now suppose that the assembled structure forms the jth substructure of a higher level structural assembly. Certain of its nodes, referenced by suffix 1, will become terminal nodes in the higher structure while the remainder (suffix 2) will be internal nodes. For the purposes of analyzing the higher order structure it is again only necessary to define the terminal node stiffness matrix $\overline{K}_i$ (with internal freedom unconstrained) and the set of terminal node forces $\overline{R}_i$ which are required to equilibrate the internal forces (applied to the structure with terminal nodes constrained against deflection). By simple algebra we obtain

$$\overline{K}_1 = K_{11} - K_{12} K_{22}^{-1} K_{21} \tag{6}$$

and

$$\overline{R}_1 = K_{12} K_{22}^{-1} (R - R^*)_2 \tag{7}$$

For inclusion in the higher structure, the above terminal node matrices can be resolved into the member stiffness k_j and initial forces S^*_j by applying a standard direction cosine transformation where necessary

$$k_j = \lambda^T \overline{K}_1 \lambda \qquad\qquad S_j^* = \lambda^T \overline{R}_1. \tag{8}$$

We have established the basic algebra leading to the solution of structural assemblies in terms of substructures and to complete the formal theory it remains only to define the back-substitution phase of the calculation in which substructure internal behavior is related to the assembled structure solution and the internal loads and initial stresses.

If the displacements u_j of a substructure are determined from a higher structural analysis then the terminal node displacements U_1 in the substructure axes are given by

$$U_1 = \lambda u_j \tag{9}$$

The internal node displacements are

$$U_2 = K_{22}^{-1} \left[(R - R^*)_2 - K_{21} U_1 \right] \tag{10}$$

Equations 3, 9 and 10 yield the individual substructure member deflections given by:—

$$u_i = a_i U_F = a_i T \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \tag{11}$$

If member i is a substructure the same cycle of back-substitution is applied at the next structural level. If it is a simple element then u_i and the internal element forces are sufficient to define its complete behavior and if a compound element they define, by assumption, the behavior of interest to the analyst.

The above theoretical summary indicates the important fact that substructure analysis involves no mathematical operation in addition to those required in any general purpose analysis scheme. To include substructure analysis facilities in a general computational procedure is more a matter of organization and flexible programming than of analytical development. In the subsequent sections of this paper some of the requirements for a flexible analysis scheme are defined more fully.

2.0   SOME APPLICATIONS OF SUBSTRUCTURE CONCEPTS

There were originally two reasons for the introduction of substructure concepts into structural analysis: the natural breakdown of a problem, for convenience, into units representing separately designed and manufactured items and the need to overcome capacity limitations in early computer programmes. The second is not a generally valid reason and will not be considered further in the context of a modern computer analysis.

The convenience of subdividing a large and complex structure into its natural component structures is evident from the fact that the design of separate components may proceed at different times, may be in the hands of different groups and may be changed at any time without affecting other components. The individual components may be conveniently defined and analyzed in relation to different sets of axes. Finally, if the number of terminal nodes of each substructure is small in relation to the complete node set, then the analysis will proceed in manageable stages, each of which may be physically meaningful and provide convenient break points in computation. These are the obvious reasons for subdivision but it is arguable whether they would provide sufficient justification for the provision of automated facilities as proposed here. Several more applications of substructure analysis facilities can now be visualized which, taken together, offer substantial reductions in computing time and in the effort of problem formulation, and which improve the flexibility of analysis of real structures subject to continual design changes.

2.1   Partial Problem Solution

By a simple extension of the displacement analysis procedure or by careful formulation of a matrix force analysis we can obtain solutions to individual substructure analyses which give useful first approximations to the final solution. In the former case it is considered preferable to break out of the standard procedure outlined in Section 1 rather than modify that procedure to include special cases involving particular initial assumptions at substructure boundaries. Having derived the stiffness matrix K and the effective external loads $(R - R^*)$ for a substructure we can impose a particular set of displacement constraints or assumed reaction loads on the terminal nodes by the standard procedures of matrix analysis. For example, plane sections can be constrained to remain plane or rigid body motions eliminated by the application of a linear displacement transformation similar to Equations 3 and 4 or reactions of adjacent structure can be anticipated by applying suitably modified external loads at the appropriate points. The partial solution would be discarded on completion of the full analysis. In the case of matrix force analysis the partial solution could be introduced as the basic (b_0 in Argyris' notation) stress distribution prior to determination of the terminal redundant forces, and would thus form an integral part of the final solution.

2.2   Repetition of Identical and Handed Members

It frequently occurs that identical structural members or mirror image members occur in several places in the same structure. Treating these as substructures we can perform the basic stiffness computations once only and introduce local loadings as appropriate to each location. Particular examples are the frames and skin bays (including standard cutouts such as windows) in a cylindrical fuselage and the wings, tailplanes and engine mountings on many conventional aircraft. Useful savings, particularly in formulation effort, may be achieved.

2.3   Selection of Regions for Detailed Structural Study

When it is necessary to conduct a large scale analysis taking account of the influence of every local detail it is particularly appropriate to apply the multi-level substructure concepts so that the local problems may be conveniently separated from the overall problems.

Considerable economy of effort, both in computing and interpretation of results, can be achieved by selecting substructure regions for particular study by examination of the overall stress pattern in the higher order structures. Back-substitution to obtain local stresses need only be carried out in areas where general load levels warrant it. Other areas can always be investigated later with no loss of efficiency.

2.4   Automatic Subdivision of Continuous or Regular Regions of Structure

A more sophisticated set of facilities involving more than the simple substructure analysis techniques is envisaged under this heading. This could constitute the most revolutionary feature of the current development programme, extending the scope of general purpose numerical analysis procedures to embrace problems which would otherwise become unmanageable. The idea is very simple in principle but the possible ramifications are almost boundless. A region of continuous or regular structure may be defined by means of a very small amount of boundary geometry and a simple definition of the nature and location of regularly recurring features such as reinforcing stringers. We may then specify the characteristics of a uniform or graded grid to cover the region and programme the computer to identify nodes and elements, compute intermediate coordinates, calculate intermediate point loads from load distribution functions and perform the other necessary tasks to define completely a substructure. In this way fine mesh structural representation can be achieved with coarse mesh problem formulation. This principle provides the key to the problem of reducing elapsed time in the analysis of large structures on modern high speed computers.

It is currently a controversial question whether the analysis of continuous members (uniform plates or solid members) under rapidly varying stress is best conducted by refined element representation or by fine mesh application of standard simple elements. In real aircraft structures we usually encounter discrete reinforcing members (stringers, frames or corrugations) pitched at a spacing considerably smaller than the economical element size for complete structural analysis (Figure 3). In this case there can be little argument that a fine-mesh discrete element approach gives a means of analyzing elements, down to and below individual reinforcing member scale, which is more powerful than the use of special refined element representations.

Quite apart from the many instances where the stress analyst may be interested in the very detailed stress patterns in finely subdivided regions, it now seems probable that automatic substructure specification provides an economical means of idealizing the real structure for the purposes of a coarser mesh analysis. In the example of Figure 3 the integrally machined stiffeners must be idealized in some way so that their effective stiffness may be included at the nodes of the grid shown. The analyst may be spared this time consuming chore if he need only specify, for example, "6 equally spaced stringers of section S....." together with a grid size definition for a compound element stiffness matrix to be generated by automatic procedure.

This facility, together with the repetitive capability Paragraph 2.2 makes possible the inclusion of virtually all regular design features in a structure as a matter of routine, with no extra effort to the analyst and only a small increase in computing time due to the highly repetitive nature of these local features.

2.5   Modification of Local Structure

When suitably organized, the substructure approach is convenient for the introduction of local structural modifications. While techniques are available (References 2 and 4) for introducing modifications with the minimum amount of additional computation it is often simplest to organize a direct modification of the problem data and re-solution. When the structure is analyzed as a multi-level assembly of substructures the modifications need only be applied

to those "branches of the tree" containing the affected region. In assembling the modified structure stiffnesses and node loads this can save an appreciable amount of computation. During back-substitution to determine element displacements and stresses it is easy to isolate those portions of the structure which are significantly affected by the change and leave unmodified those which are not. If the stiffness and terminal load matrices are fully modified at all levels than an updated record of the effects of modification is obtained, to which cumulative modifications can be added indefinitely. Provided that solution and back-substitution proceed from the highest structure level an accurate solution is obtained even though previous solutions may not have been carried out fully.

2.6 Cyclic Redesign and Optimization

Automatic stress analysis can be used in the initial design of structures to improve the efficiency of the structure as well as confirm its safety. Comparison of detailed stress distributions with local strengths enables the analyst to specify a new set of structural dimensions. A new analysis is then required to determine accurately the effect of the modifications. This type of design improvement is currently carried out manually but will be undertaken automatically in the analysis system under development as a basic part of the scheduled optimization facilities.

If the majority of the individual elements of a structure are modified during any step in this procedure, the substructure approach has nothing to offer in improving the cycle. If modifications during each design step are local in nature then the advantages outlined in the previous Paragraphs 2.5 can be exploited again. The greatest advantage can be obtained if substructures are so chosen that design modification can be realistically effected by scaling the physical properties or dimensions of complete substructures at the highest possible structural level. If this technique can be adopted, modification of each substructure will then involve scaling the terminal node stiffnesses K, and will leave the node loads unaffected. This procedure can be efficiently organized with a considerable saving in computation time. It should be noted that when optimization routines are added to analysis facilities, many cycles of re-analysis can be involved and computational efficiency will then be at a premium as well as simplicity of problem formulation.

3.0 SUBSTRUCTURE FACILITIES IN A COMPREHENSIVE STRESS ANALYSIS SCHEME

In this section a brief account is given of some of the concepts embodied in a comprehensive stress analysis scheme currently under development at British Aircraft Corporation. While this scheme is envisaged as linking the functions of analysis, design and optimization we shall be concerned only with the first aspect.

The analytical procedures are based on a flexible matrix handling scheme, a central group of routines for the generation of structure behavior in matrix displacement or force terms and a specification and control language capable of linking together the basic routines with an 'open ended' sequence of ancillary programmes. The whole system is currently being programmed in FORTRAN IV for use on an IBM 7040 (though some of the most sophisticated facilities may not be incorporated until a more advanced machine of the System 360 type is available).

The organization of the scheme is conceived from the outset to incorporate automatic substructure facilities to any level of subdivision. This raises difficulties in specifying the problem in a concise manner, in extensive manipulation of the large, and often sparse, matrices involved and in making provision for some of the special selective facilities outlined in Section 2.

Development of the scheme can be visualized as taking place in 3 stages (though in reality there is no such clear cut division):—

Initially the central analysis procedures can be built up on the basis of comprehensive and explicit data being supplied at all stages and the presentation of a full solution (deflections and stresses for all loading cases in all structural elements).

To this central scheme we then add facilities for accepting the problem specification in its most concise form and data defining, as far as possible, the real structure rather than its idealized counterpart. Routine idealization of the structure and expansion of the data to the explicit form required for the basic procedures are carried out by linked-in subroutines.

Finally, facilities are introduced for interrupting the solution at various stages and following alternative paths either predetermined in the problem specification or deduced from inspection of the partial solution.

In order to describe in more detail the application of the scheme to a multi-level structure it is more convenient to subdivide the procedures in a manner following the progression of a calculation incorporating the use of all the above facilities. Figure 4 illustrates the 3 major steps in this progression — data expansion, structure behavior assembly and solution and back substitution — and indicates the direction in which they normally proceed. Each step is enlarged upon in the following sections.

3.1 Data Expansion

The data defining a structure, its loadings and constraints and the problem to be solved can be of two forms:—

Explicit data define point by point and member by member the numerical values and identification symbols of the constituents of the idealized problem to be solved.

Condensed data define rules and a reduced amount of numerical data from which explicit data can be derived as necessary.

Some typical examples of explicit and condensed data are given in Table I below. The examples of nodal data and element data are made specific as they represent a realistic situation in which considerable data economy can be achieved.

The normal sequence of data expansion will commence at the highest level of structural assembly and as the data at each level are expanded into explicit form it is logical to proceed down to the next lower level. There are exceptions to this rule (for which the programmes must make allowance) particularly when identical substructures or elements are specified. There will be many instances, for example a fully explicit specification, where data duplication occurs. This will mainly affect the geometrical definition of structures and their constituent substructures. Terminal node geometry may be defined in the substructure data, the assembly data or both, in the latter case probably related to different axes. For this reason the scheme will embody consistency checks and minor rectification procedures to ensure explicit data consistency to the high accuracy essential to numerical stress analysis. Where rectification is necessary it is most natural to adjust substructure data to be compatible with the higher level structure.

The normal sequence of data expansion operations at any one structural level and the lead-in to the next lower level are illustrated diagrammatically in Figure 5.

TABLE I

EXAMPLES OF EXPLICIT AND CONDENSED DATA

EXPLICIT DATA	CONDENSED DATA
NODAL DATA	
Identification symbols and coordinates for each of 300 node points on a non-circular cylinder with 20 similar sections.	Identification symbols and coordinates for 15 nodes constituting 1st section of cylinder; x-coordinates for remaining sections; Rules to generate symbols and coordinates for each node.
ELEMENT DATA	
Identification symbols, type designation, node identification, physical properties of each individual cylinder surface element.	Data to define physical property variation and commence identification sequence, rules to generate symbols, type designation, node identification (linked to nodal data rules) and physical properties for each surface element.
Terminal node identification and data storage location for each frame or bulkhead substructure	x-coordinate of each frame or bulkhead substructure and symbol to identify substructure; rules (if necessary) to generate terminal node identification.
CONSTRAINT DATA	
Identification symbols for degrees of freedom to be eliminated OR Identification symbols for variables to be constrained; identification symbols for new true variables; transformation matrix.	Rules to define degrees of freedom to be eliminated OR Identification symbols for local variables to be constrained and true variables; local transformation matrix; rules for repetitive application of constraints.
LOADING DATA	
Matrix of loads corresponding with all true variables at each structure node.	Numerical data defining load distribution and resultants; rules to define, check and correct individual node loads.

3.2 Structure Behavior Assembly

Under this heading we refer to the embodiment in the scheme of those facilities summarized symbolically in Equations 2 to 8 in Section 1. Facilities of this type have been available with varying degrees of generality and automation in many different organizations over the past ten years. The linking of the major operations is illustrated in Figure 6, a diagram which must have a familiar look to anyone involved in matrix displacement analysis applications. The main advances in the current scheme compared with most existing schemes known to the author are the elimination of all restrictions on size and interconnection of member stiffnesses (and corresponding load matrices), the removal of restrictions on the banded form of the assembled stiffness matrices (while retaining efficient handling and solution of sparse matrices), and the automatic facilities for storage and identification of substructure data and organization of the multi-level structure analysis without manual intervention.

The matrix handling facilities and efficient solution procedures are largely achieved through the use of a FORTRAN IV matrix handling scheme which is nearing completion. This enables matrix algebra operations to be called upon by statements corresponding to conventionally written algebra, each matrix symbol being able to represent a simple matrix array, a supermatrix (matrix composed of an unlimited number of sub-matrices) or one of many special forms, (sparse matrices, banded or diagonal matrices, Booleans, etc.). The scheme is idefinitely recursive, that is it can handle submatrices within submatrices to any level of nesting, as it is written with the current structural analysis scheme in mind.

The elimination of size and interconnection restrictions and the provision of a wide range of constraint and transformation procedures are achieved by devising an efficient and condensed language for problem specification and flexible routines for its interpretation. The ultimate objective of complete freedom from restrictions may not be achieved at the first pass, but the whole scheme is conceived on a modular basis with ease of extension and replacement of subroutines a paramount feature.

3.3 Solution and Back-Substitution

The final stage in the scheme is the solution phase, which for simple structures is entirely conventional, but which involves back-substitution loops when applied to multi-level structures. The major processes are illustrated diagrammtically in Figure 7. Once again the algebraic facilities for manipulation of the numerous matrices and submatrices are provided by the matrix handling scheme. Special features required at this stage are the control procedures and particularly the selective routines to determine the extent of the solution.

Selection may either be by predetermined instruction (which is relatively straightforward and does not involve the derivation and inspection of terminal loads as illustrated in Figure 7), or by inspection of the current level of solution. In simple terms the solution at any stage is extended to include the terminal loads of all substructures at the next level. These can be approximately converted into average measures of stress levels within the substructures by special procedures which must either be pre-programmed or supplied with the structure data. The loads or stresses thus estimated would be compared with permissible stress levels to determine whether each substructure is critical or not and a selection would be made on this basis. Quite unsophisticated selection procedures can suffice for this purpose since a broad tolerance can be placed on the selection criterion to ensure continued solution in marginal cases. For example selection could be based on end load or bending moment across substructure boundaries. By retention of a limited amount of intermediate data on tapes any further substructure solutions can be retrieved at a later stage should this be necessary.

Finally, mention will be made of a major problem, not specifically related to the principal subject matter of this paper, which will receive considerable attention once the main body of the current scheme is operational.

This concerns the vast amount of structural data which can be generated by a modern or near-future automated stress analysis procedure. Selective solution as outlined above will limit this quantity to some extent, but there will always be a tendency — more justified as the procedures become more rapid and automatic — to run large numbers of cases and include possible variants in design and loading. It is easy to reach the stage where this information cannot be assimilated by the analysts and designers themselves and output selection and processing is required. The eventual answer will be to link the analytical solution procedures with complementary procedures deriving structural strength and permissible loads and deformations from the basic design data. Routines to compare calculated and permissible values and printout selected critical and near-critical solutions can be envisaged.

As an intermediate goal some simple measure of severity of an element loading (e.g. the value of maximum principal direct or shear stress) can be specified as a criterion on which to base the selection of critical and near-critical solutions. This is a fairly simple extension of the procedure which is certain to be justified by saving time and better utilization of the analysis.

Where some level of automated idealization is employed, as discussed in Paragraph 2.4, then a supplementary routine can be introduced to convert back idealized structure stresses into stress levels in the real structure. Looking at Figure 4 it would be absurd to generate stresses in all the sub-elements but the maxima in a uniformly subdivided region would be of real significance in detail stressing.

REFERENCES

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2. Przemieniecki, J. S., "Matrix Structural Analysis of Substructures," AIAA Journal, Vol. 1, No. 1, January 1963.

3. Argyris, J. M., and Kelsey, S., Modern Fuselage Analysis and the Elastic Aircraft, Butterworths, London, 1963.

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NOTATION

a_i	Boolean-type matrix relating member i displacements to structure displacements
$a = \{a_i\}$	matrix obtained by compounding a_i's.
k_i	stiffness matrix for member i
$k = \lceil k_i \rfloor$	matrix obtained by diagonal compounding of k_i's
K, K_F	stiffness matrices in true (constrained) and all possible degrees of freedom respectively
$\bar{K}_i$	substructure terminal node stiffness matrix
R	externally applied loads (true degrees of freedom)
R^*	loads to constrain structure against initial deformation
$\bar{R}_i$	terminal node loads to equilibrate internal loads
S_i	element i boundary loads
S_i^*	loads to constrain element i against initial deformation
$S* = \{S_i*\}$	
T	constraint transformation matrix
u_i, u_j	node displacements of members i, j
U, U_F	structure displacements corresponding with K, K_F
λ	direction cosine transformation matrix

Subscripts 1 and 2 refer to substructure terminal nodes and internal nodes respectively.

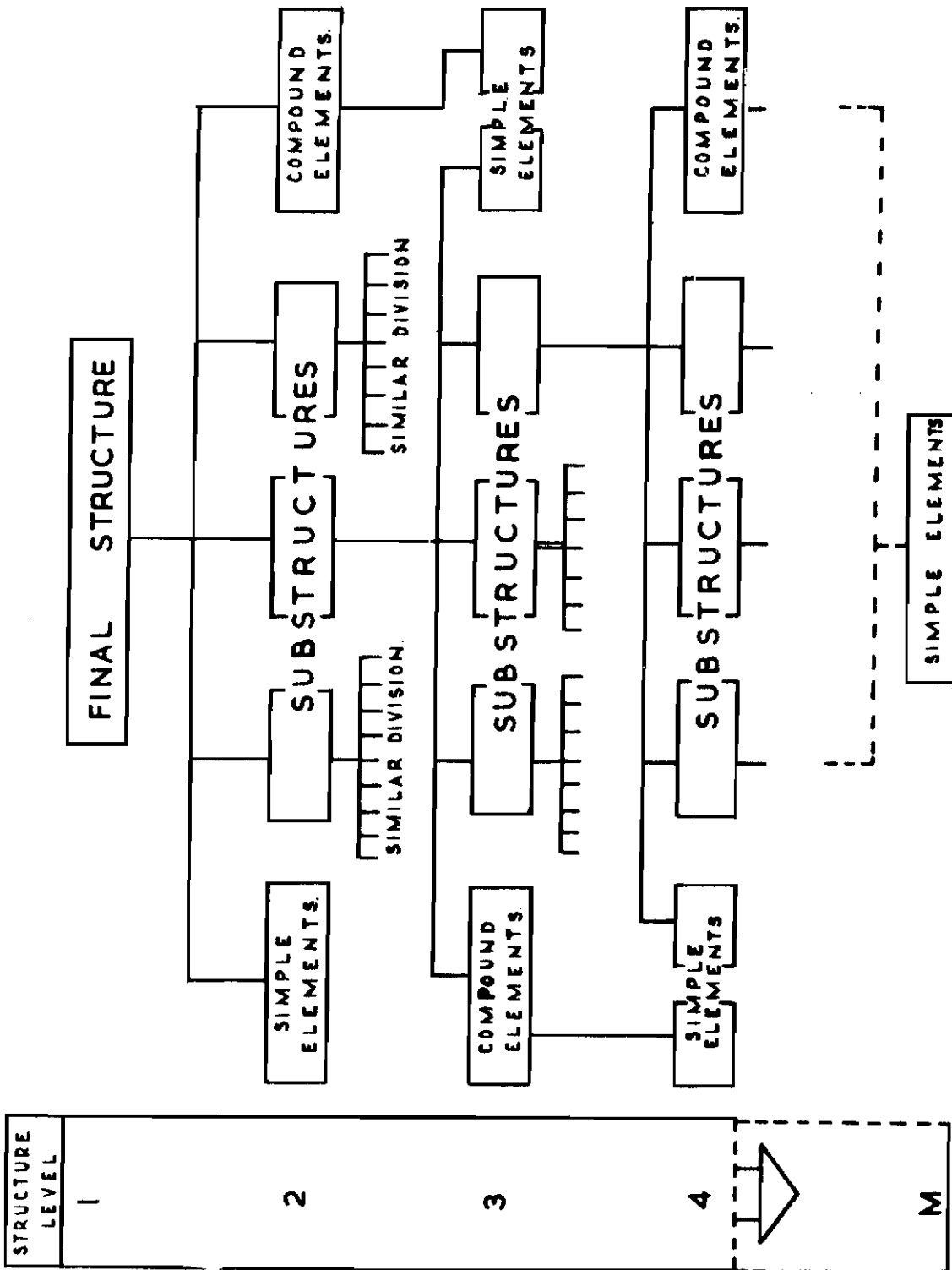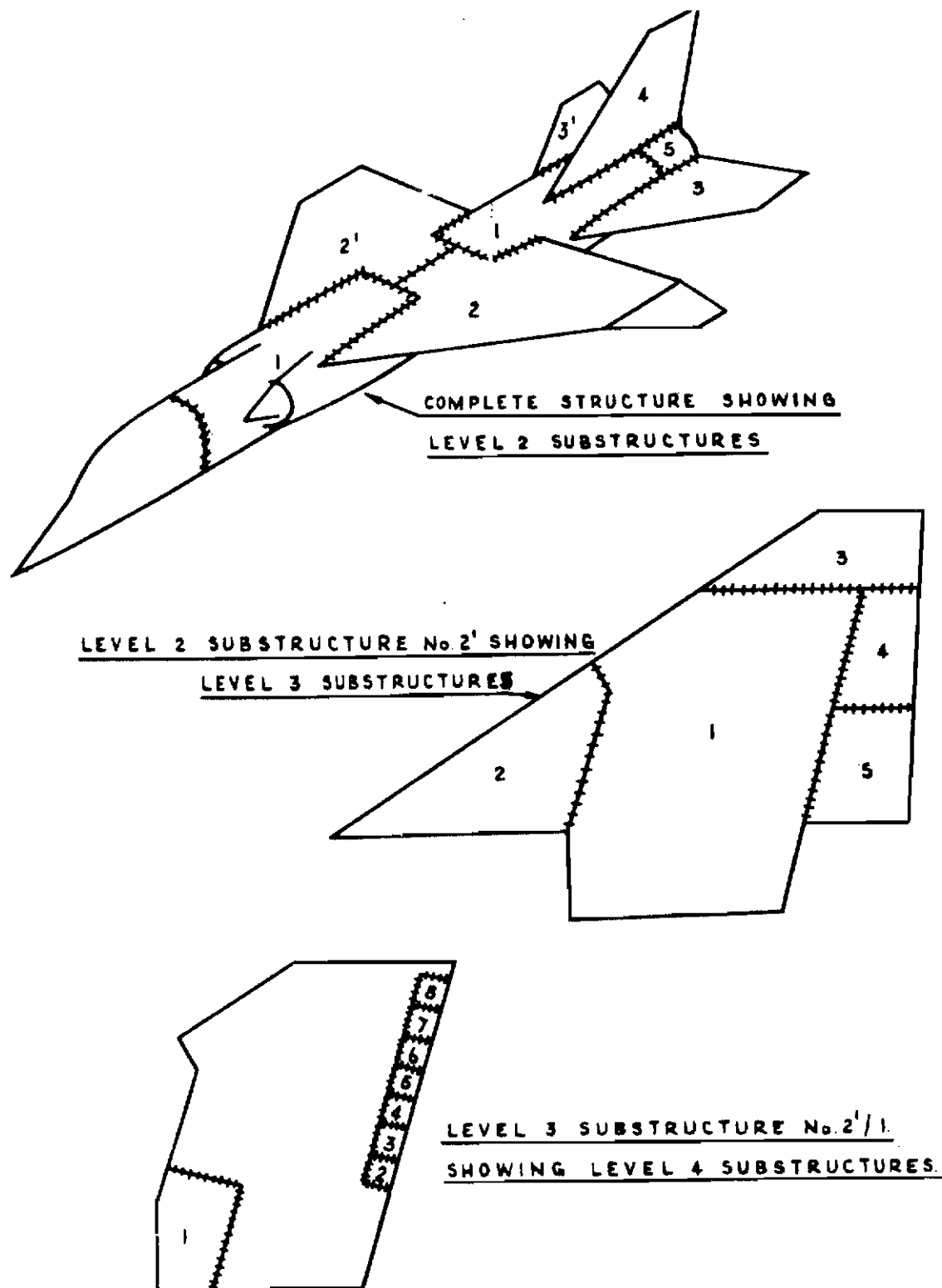Figure 1. Structure Hierarchy with 'M' Levels

COMPLETE STRUCTURE SHOWING
LEVEL 2 SUBSTRUCTURES

LEVEL 2 SUBSTRUCTURE No. 2' SHOWING
LEVEL 3 SUBSTRUCTURES

LEVEL 3 SUBSTRUCTURE No. 2'/1.
SHOWING LEVEL 4 SUBSTRUCTURES.

Figure 2. Practical Example of Multi Level Substructures

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ECONOMICAL GRID SIZE FOR
MAJOR STRUCTURAL ANALYSIS

POSSIBLE GRID FOR DETAILED
BOOM - MEMBRANE ANALYSIS.

POSSIBLE GRID FOR DETAILED
MEMBRANE + FLEXURE ANALYSIS.

IF BASIC PANEL IS A
RECTANGLE OR PARALLELOGRAM
NOTE THE REPETITIVE
NATURE OF THE SUBDIVISIONS.
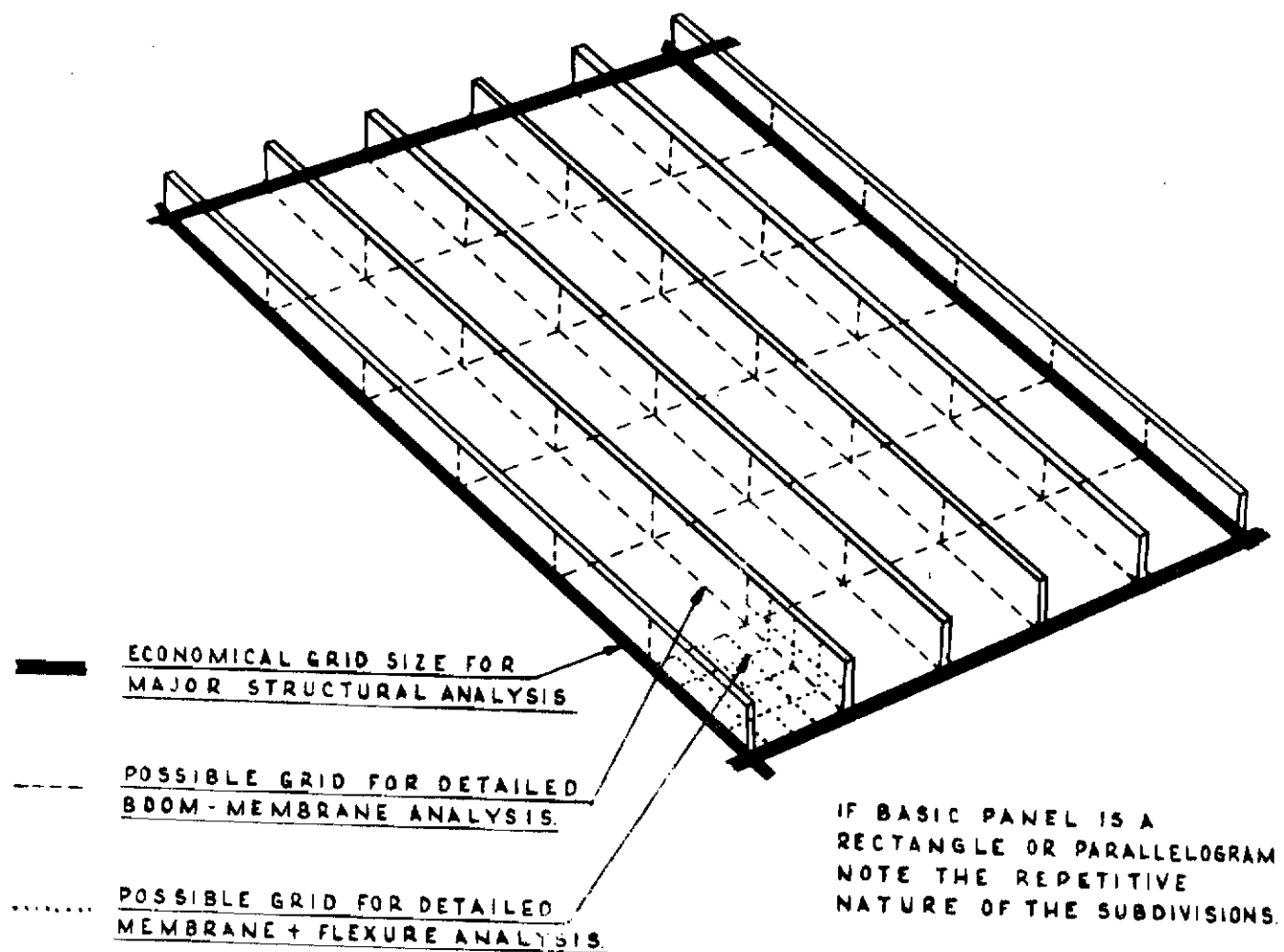
Figure 3. Subdivision of Reinforced Plate
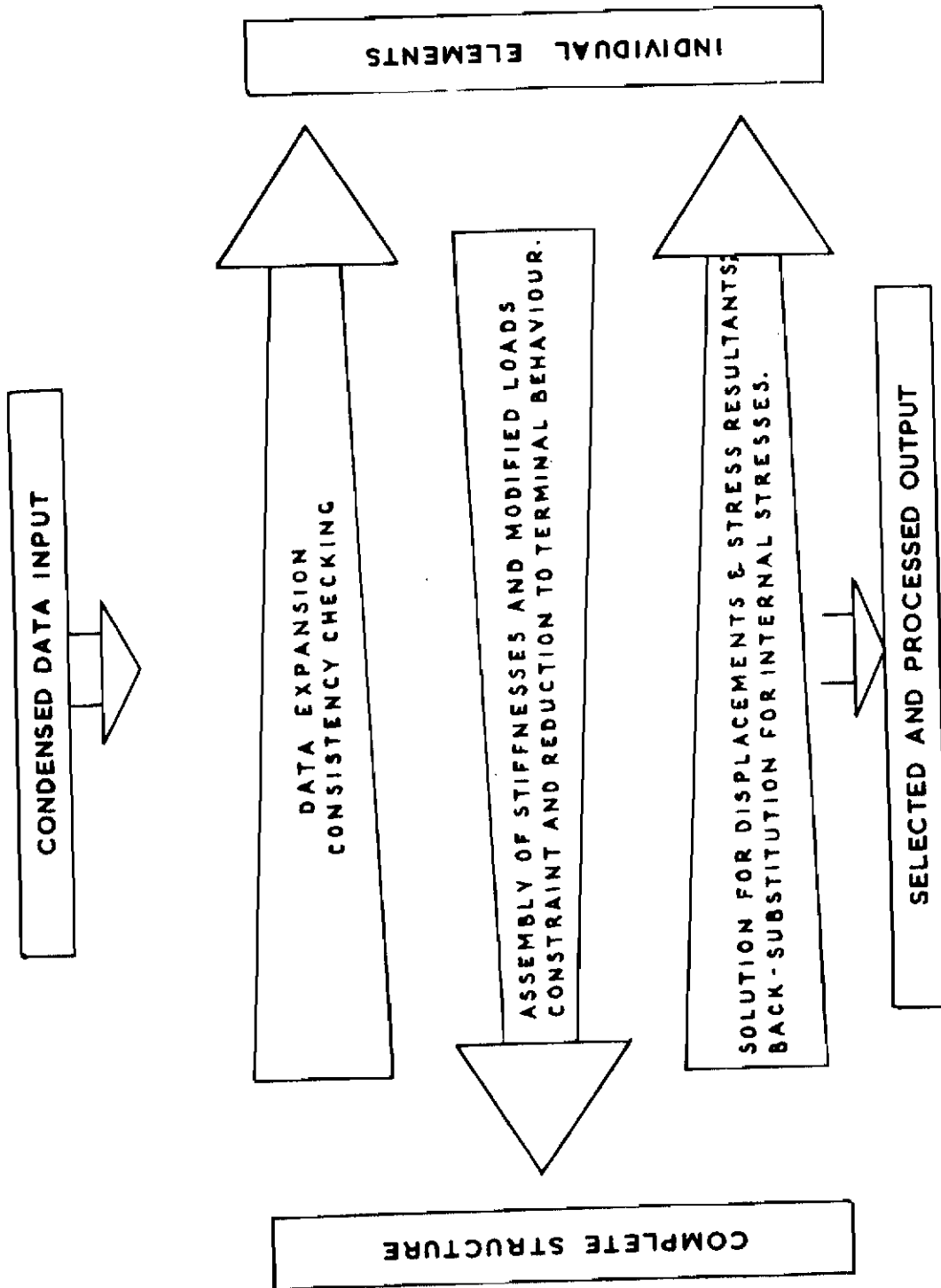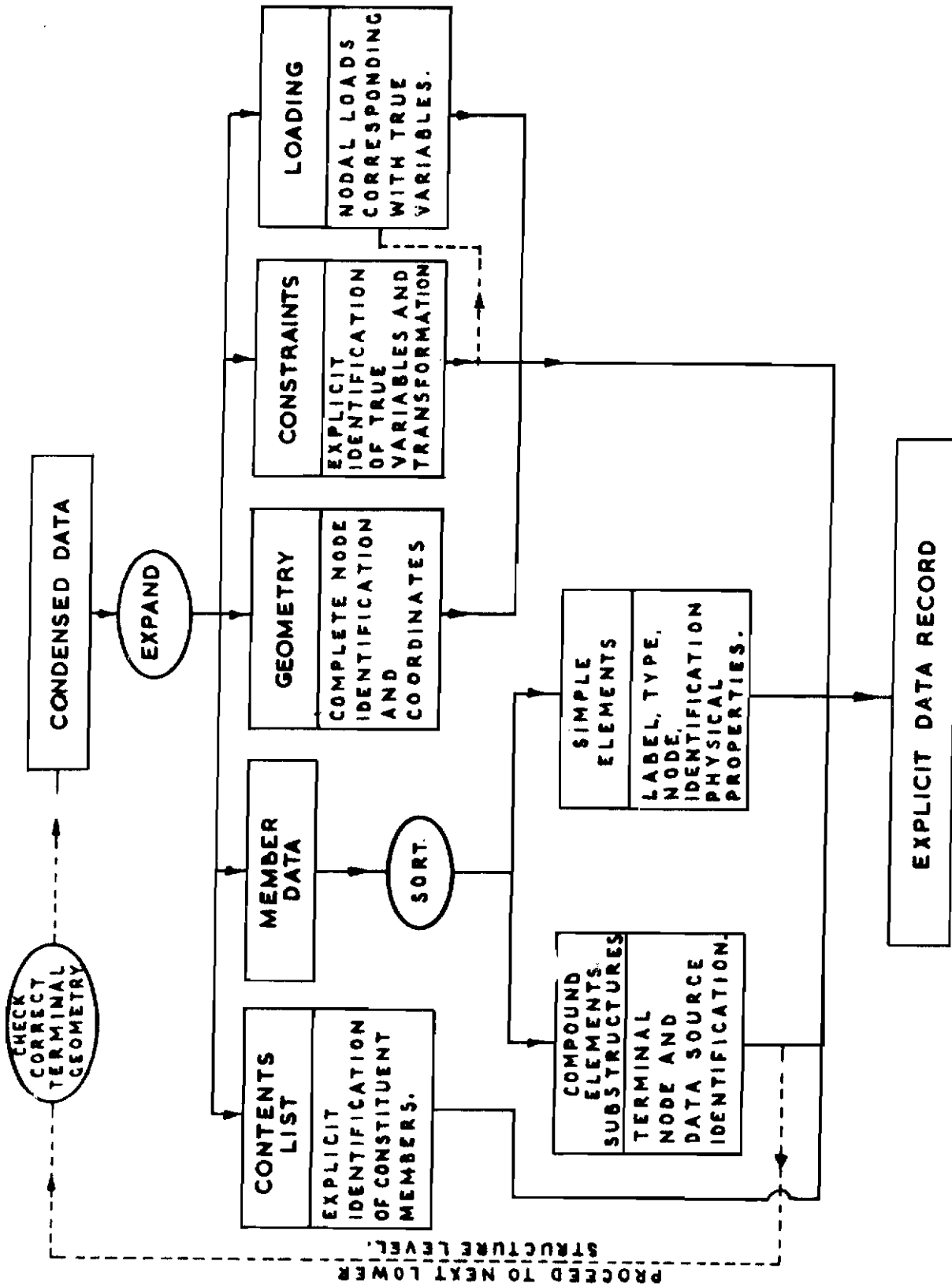
Figure 4. Idealized Progression of Substructure Analysis

Figure 5. Data Expansion Normal Sequence

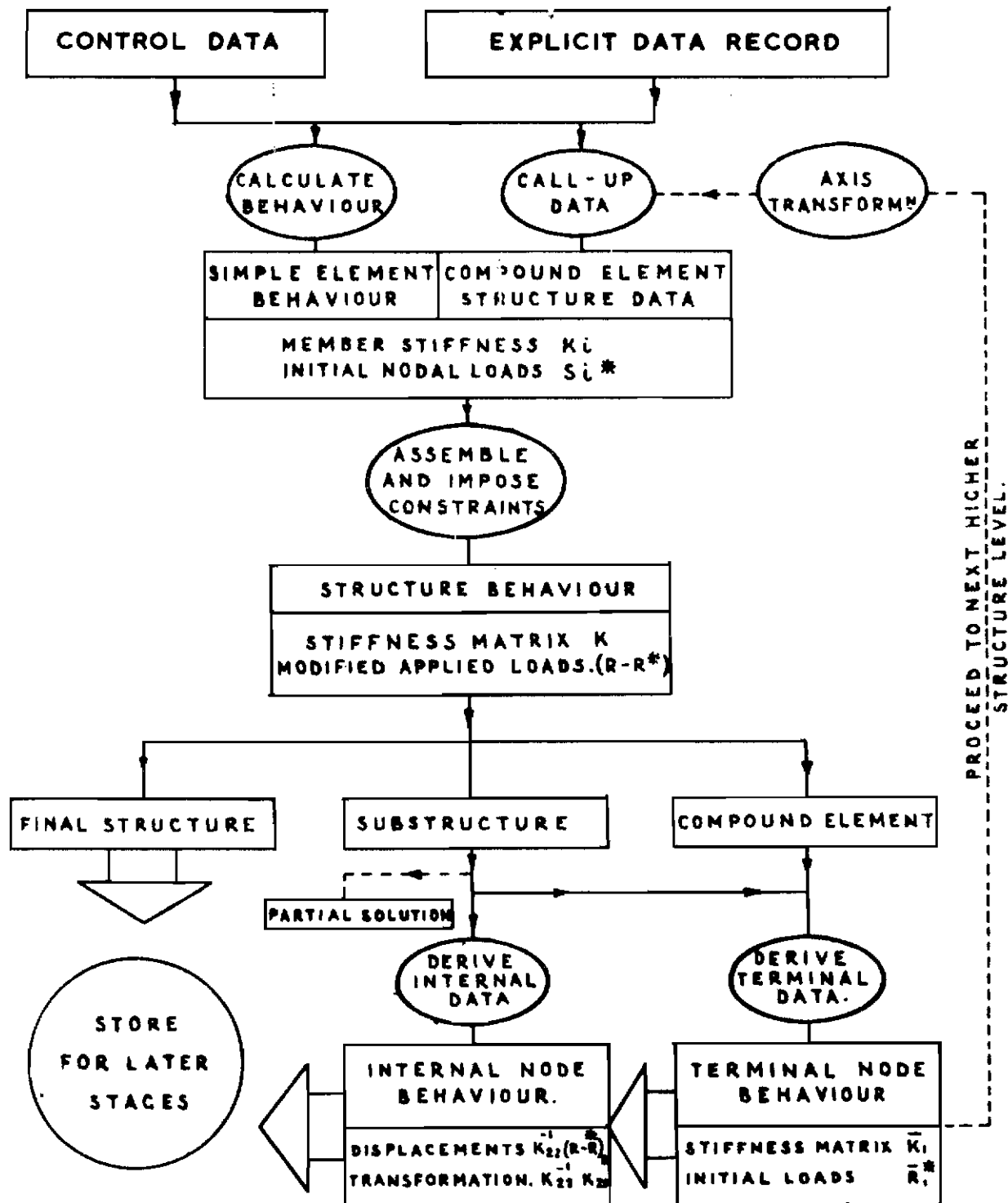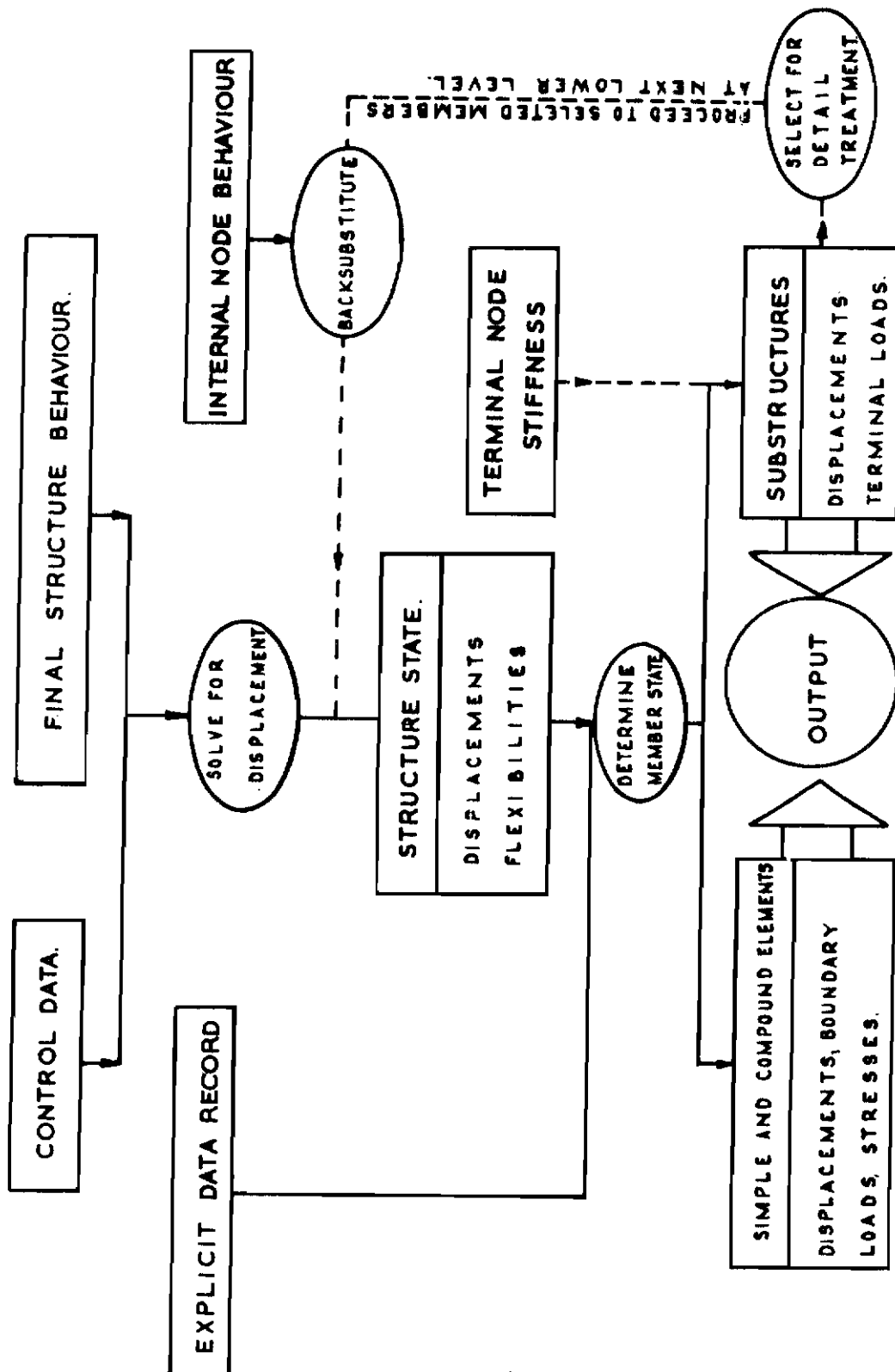Figure 6. Structure Behavior Assembly

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Figure 7. Solution and Back-substitution