ELASTIC-VISCOPLASTIC RESPONSE OF CLAMPED BEAMS UNDER UNIFORMLY DISTRIBUTED IMPULSE

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FOREWORD

This report was prepared by the Department of Mechanical Sciences, Southwest Research Institute, San Antonio, Texas under USAF Contract No. AF 33(615)-5338. The contract was initiated under Project No. 7351, "Metallic Materials," Task No. 735106, "Behavior of Metals." The work was monitored by the Metals and Ceramics Division, Air Force Materials Laboratory, Air Force Systems Command, with Dr. T. Nicholas, MAMD, as project scientist.

This report covers work conducted from April 1967 to July 1968. The manuscript of this report was released by the authors November 1968 for publication.

This technical report has been reviewed and is approved.

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ABSTRACT

Experiments are performed with rigidly clamped beams loaded impulsively by an explosive technique. The transient and residual deflection and strain are measured. Experimental results are compared with predictions from approximate rigid-plastic theory and a more exact elastic-viscoplastic numerical solution.

Results obtained indicate that elastic strains are not negligible for the system studied. For this reason, the approximate rigid-plastic analysis is less accurate than the more exact numerical solution. The dynamic stress-strain properties of the beam material were determined from separate experiments conducted over a wide range of strain rates up to 1000/sec. These strain-rate dependent properties, when incorporated into the numerical solution, allow an accurate prediction of the response of the beam. The strain-rate dependence of the material is shown to influence the response amplitude of the beam.

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SECTION I

INTRODUCTION

The work to be described was undertaken in order to investigate the influence of the strength properties of metals, especially strain-rate sensitivity, upon the dynamic deformation of impulsively loaded structures. In previous work(1)*, the strength properties of several metals had been obtained from uniaxial tension and compression tests over a wide range in strain rate. In the present work, beam specimens, fabricated from two of these metals from which accurate strength properties were known, are loaded impulsively with measurements made of pertinent loading and response parameters. Comparisons of these measurements with available analyses are made. Items of concern include the suitability of the analyses to predict transient inelastic response and the relative importance of including the effect of rate of strain in describing the strength properties of the beam material.

A complete survey of dynamic plastic deformation of structures has been made recently by Symonds.(2) Therefore, only a few of the references more pertinent to the present work will be cited. Since the complexity of the problem is such as to rule out exact analytical solutions, a great deal of emphasis has been placed upon developing approximate methods for computing permanent deformations. A starting assumption which allows analytical treatment of the problem is to idealize the material as rigid-plastic. One consequence of this idealization is the formation of regions of localized plastic bending referred to as a plastic hinge. Deformation (rotation) is localized at these hinge points, which may be either moving or stationary, with the hinges separating rigid sections of the structure.

The elementary rigid-plastic theory neglects the material characteristics of elasticity, plastic work-hardening, and possible influence of rate of strain upon strength properties. Elastic energy is usually neglected by restricting the solutions to cases where the initial kinetic energy imparted to the structure is large in comparison with the maximum elastic energy that can be stored. Analytical corrections have been made to the elementary theory to include work-hardening, strain-rate effects, and the effects of finite deformation of the structure.(3, 4, 5, 6) The geometry of the structure and its constraint conditions may require additional inclusion of the influence of axial constraint(6) and shear(7) in combination with bending effects.

Bodner(8) has emphasized that with respect to material properties, the strain-rate dependence of the yield stress is the most significant

*Superscript numerals in parentheses refer to List of References, Section VI.

correction to the elementary rigid-plastic theory. He points out, however, that since strain rate varies both spatially and with time in the structure, an overall correction factor on the rigid-plastic theory is not generally applicable. Such an overall correction factor may be more applicable with the mode approximation techniques of Martin and Symonds.[9] Inclusion of a rate sensitive yield stress also serves to eliminate the concentrated traveling hinge that results from the rigid-plastic assumption. The effect of rate-hardening tends to stiffen the hinge and to spread plastic deformation to adjacent areas.

Experiments which have been performed, primarily with beams, show that the rigid-plastic analysis is a reasonable first approximation. Including corrections, such as for strain rate, generally improves the correlation between theory and experiment. However, in these cases, it is not always easy to determine the relative importance or magnitude between the effects included in the analysis and those omitted. These effects are not always uncoupled. Similarly, the experiments themselves are not free from uncertainty. The dynamic boundary conditions are hard to maintain, and accurate measurement of the time history and distribution of the loading and of the response is difficult. Usually, the loading is considered impulsive and the permanent deformation is the primary correlative variable. Seldom have the mechanical properties for the material from which the structure is fabricated been measured for the range in deformation, rate, and temperature, at which the experiment is performed.

In order to avoid many of the uncertainties in the approximate analyses, it is possible to go directly to computer-oriented numerical solutions. Among the first to do this were Witmer et al.[10] at MIT. They reformulated the governing differential equations in finite difference form suitable for numerical computation. The continuous structure is effectively broken up into a network of concentrated mass points connected by extensible but straight links. The material description includes elastic, strain-hardening, and strain-rate effects. Large deflections may be handled, but the effects of rotary inertia and transverse shear deformation are neglected. Computer programs have been formulated for beams, plates, rings and shells subject to impulsive loading. The response of each mass point is computed at each time increment, so that complete information on the deformation history is available.

In the present work we describe experiments performed with aluminum and with titanium beams, clamped at each end and loaded by means of explosive over one entire surface. Similar experiments with clamped beams have been previously reported by Humphreys[11] and by Krieg and Duffey.[12] The experiments were instrumented to measure accurately the total impulse imparted to the beam, the transient maximum midspan deflection, the transient midspan strain, and the permanent deflection. The strength properties of both the aluminum and the titanium had previously been measured[1] in uniaxial tension and compression at strain rates to 1000 in/in/sec. The experimental results are compared with predictions based upon the approximate rigid-plastic theory[13] and with the MIT DEPROSS computer code.[14]

The results show that, for the system studied, the elastic effects are very important. For this reason, the rigid-plastic analysis is less accurate while the more exact numerical solution gives very good agreement. The experiments in conjunction with the numerical results show that the concentrated plastic hinge concept is subject to some interpretation physically, and is strongly coupled with elastic effects in the early stages of deformation; strain-rate effects can be important.

SECTION II

EXPERIMENTAL PROCEDURE

The experiments were designed to minimize, as far as possible, the number of uncertainties. The beams were machined from 3/4-in.-diameter bar stock. The mechanical property data reported in Reference 1 was obtained from the same bar material, so that the strength properties in tension and compression were well known. Two metals were used: titanium 6AL-4V and aluminum 7075-T6. These were chosen because the titanium was shown to be moderately rate sensitive at room temperature, whereas the aluminum alloy was insensitive to strain rate over the range of interest.

The beam was clamped in a test fixture as shown in Figure 1. The round end sections, which were integral with the machined flat section of

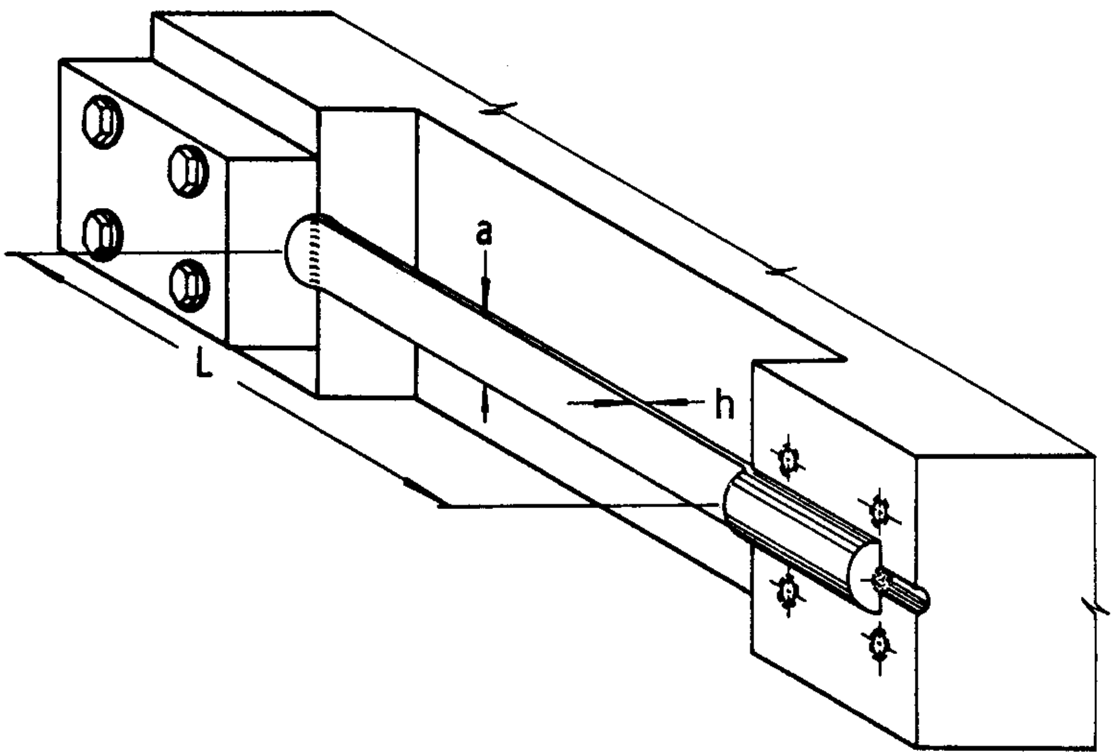

FIGURE 1

SCHEMATIC OF BEAM SHOWING END RESTRAINTS the beam, were clamped rigidly between two blocks to prevent rotation. Axial motion of the ends was prevented by both the clamping surface and a restraining bolt threaded axially into each end section. The reduced beam section was faired into each end with a small fillet. The dimensions of each beam are given in Table I along with the material constants.

The beams, with mounting fixture, were suspended as a ballistic pendulum as shown in Figure 2. The loading was applied by spray depositing a thin layer of light-sensitive explosive (silver acetylide - silver nitrate) over one entire surface of the beam and producing simultaneous detonation by means of an intense flash from a xenon flash tube. The details of this technique are described in Reference 15. The explosive imparts an impluse to the surface which is simultaneous to within 1 μsec and has a duration of less than 10 μsec as determined from independent calibrations. The weight of explosive deposited on each beam is measured and determines the impulse level produced upon detonation. In all cases, peak surface pressures produced were less than the yield strength of the beam material, so that there was no chance of spallation or shock hardening of the beam due to the explosive loading.

While the weight of explosive deposited was used to predict the impulse level in advance, for each shot, the total impulse was measured directly by recording the motion of the ballistic pendulum. The pendulum motion was tracked by means of a long stroke, linear differential transformer. The period of the pendulum was approximately 1/3 sec and was very lightly damped.

Initially, attempts were made to measure the detailed pressure-time history of loading. This was done by sandwiching the explosive between the beam and a clear plexiglass sheet of the same cross sectional area. The explosive was detonated by flashing light through the plexiglass. The explosive pressure imparted simultaneously to the beam and the plexiglass was monitored as a pressure pulse in the plexiglass by means of surface-mounted, resistance strain gages. While the pressure-time histories could be recorded by this technique, the containment of the detonation extended the loading time to such an extent that it could no longer be considered as impulsive for the present tests. Therefore, the uncontained detonation was used with the measurement of total impulse only. Available analyses all require an initial velocity distribution or impulsive loading.

The strain response of the beam was monitored by two strain gages mounted on the opposite side of the beam from the explosive. One was located at the center of the beam and the other near the root. Each strain gage was one leg of a bridge circuit whose output was recorded on an oscilloscope with a camera. The scopes were triggered from the circuit initiating the flash tubes.

TABLE I

DIMENSIONAL AND MATERIAL CONSTANTS

	Titanium 6Al-4V	Aluminum 7075-T6
Length, L	5.10 in.	5.10 in.
Width, a	0.75 in.	0.75 in.
Thickness, h	0.075 in.	0.125 in.
Elastic Modulus, E	16.1×10^6 lb/in^2	10.2×10^6 lb/in^2
Plastic Modulus, E_p	2.81×10^5 lb/in^2	12.4×10^4 lb/in^2
Yield Parameter, σ_0	130,000 lb/in^2	82,500 lb/in^2
Rate Constant, D	3.4×10^6/sec	--
Exponent, p	7.65	--
Density, ρ	4.12×10^{-4} lb-sec^2/in^4	2.62×10^{-4} lb-sec^2/in^4

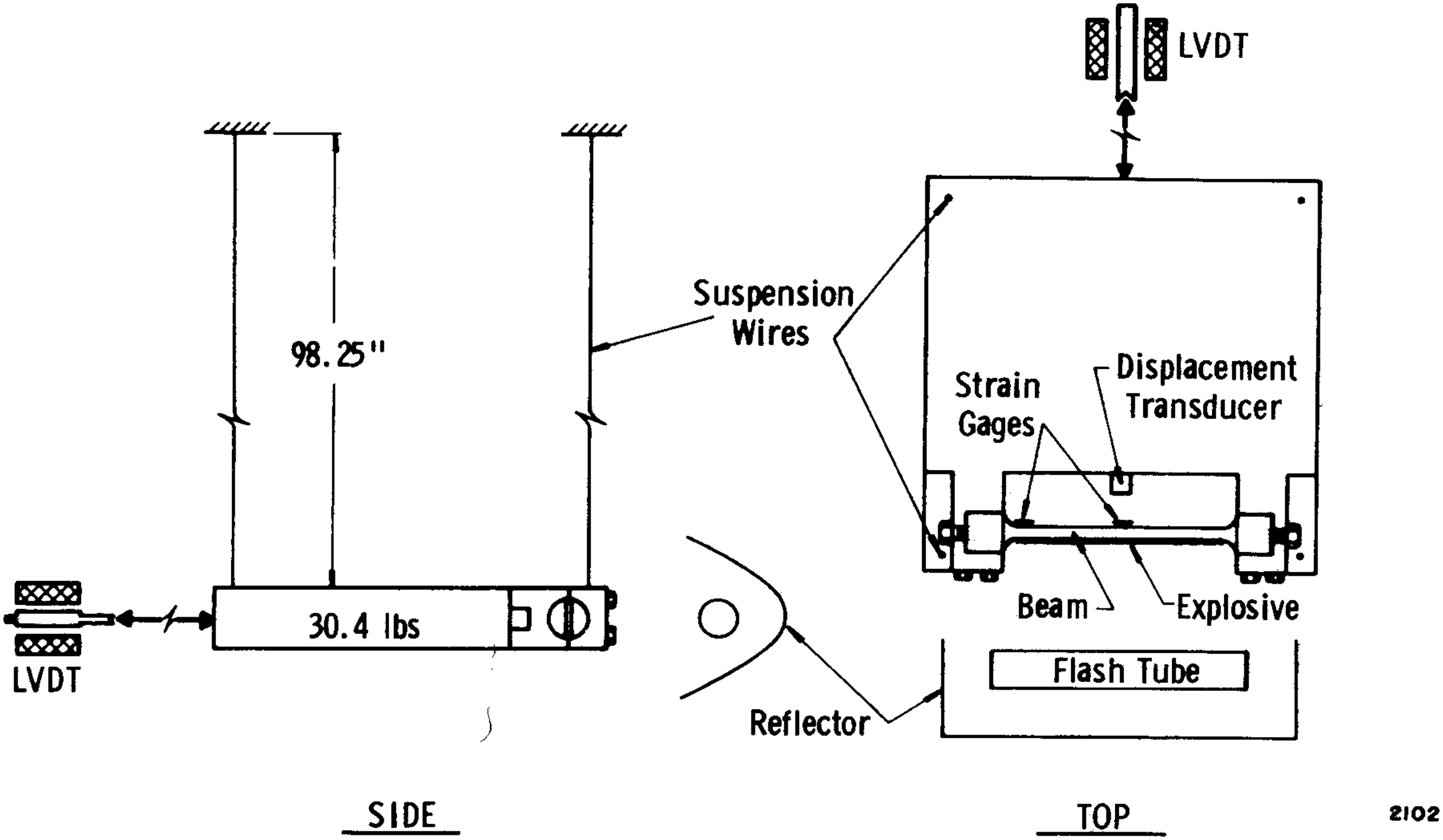

FIGURE 2. ARRANGEMENT OF BALLISTIC PENDULUM, DETONATION AND RECORDING DEVICES

The transient midspan deflection was recorded with a Bently-Nevada displacement transducer attached to the pendulum and referenced to the back surface of the beam. The range of this transducer was not great enough to follow the total motion, thus only the maximum transient deflection was recorded. The permanent or residual deformation of each beam was measured after each test while the beam was still in its fixture. High speed motion pictures (10,000 frames/sec) were taken of several shots; however, this maximum framing rate available was not sufficient to resolve the initial beam motion.

SECTION III

ANALYSIS

The experimental results will be compared, where possible, with both the approximate rigid-plastic analysis and with the more exact numerical techniques of Witmer et al.[10, 14] Symonds and Mentel[13] have treated the specific case of a rectangular beam, clamped at each end and subject to a uniformly distributed impulse. This analysis explicitly takes into account the effects of the axial constraints. The material is idealized as rigid-plastic and strain-rate effects are not considered in this case. The deformation is assumed to take place in two stages. The first stage consists of two stationary hinges at the fixed boundaries and two moving hinges originating at the boundaries and propagating toward the center. When the two moving hinges meet at the midpoint of the beam, the second stage commences with fixed hinges at the center and each boundary. Deformation then continues in this mode until the initial kinetic energy is absorbed. For finite deflections, large inplane tensile forces are developed and the beam behaves as a "plastic string". Symonds and Mentel estimate an upper and lower bound for the final plastic deformation when this string mode dominates. These bounds are given by (Eq. 60a, Ref. 13):

$$\frac{IL}{2h\sqrt{\rho\sigma_0}} - h < w_m < \frac{IL}{2h\sqrt{\rho\sigma_0}} - \frac{h}{2} \qquad (1)$$

where w_m is the maximum transverse deflection of the center of the beam, I is the impulse, and the remaining dimensions and material parameters are those indicated in Table I.

The first term in the above inequality can be shown to be related to pure extensional plastic strain. This is seen by equating the initial kinetic energy inparted to the beam to the plastic energy the beam may absorb as a uniformly stretched "string" in the mode depicted in Figure 3. The initial kinetic energy is related to the impulse by

$$KE = \frac{1}{2}\frac{aL}{\rho h} I^2 \qquad (2)$$

Equating (2) to the strain energy (assuming rigid-plastic behavior)

$$\frac{1}{2}\frac{aL}{\rho h} I^2 = ahL\int_0^{\epsilon_m} \sigma d\epsilon$$

$$= ahL\sigma_0\epsilon_m \qquad (3)$$

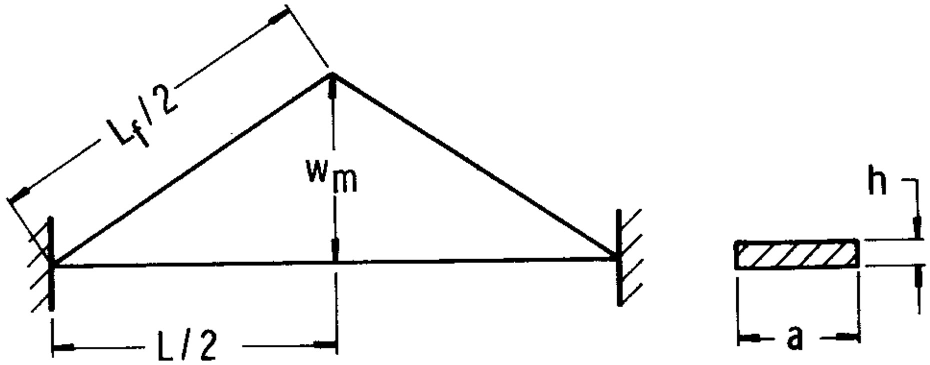

FIGURE 3

STRETCHED STRING MODE

where ϵ_m is the maximum uniform strain at maximum deflection, w_m. The transverse displacement is related to ϵ_m by

$$w_m^2 = \frac{1}{4}(L_f^2 - L^2)$$

$$= \frac{L^2}{4}[(1+\epsilon_m)^2 - 1]$$

$$\simeq \frac{L^2}{2}\epsilon_m \tag{4}$$

Substituting (4) in (3) and rearranging yields

$$w_m \simeq \frac{IL}{2h\sqrt{\rho\sigma_0}}$$

This is the displacement which would occur if all the energy was absorbed in the pure stretching mode of Figure 3. The additional term in the inequality (1) represents an added contribution from bending. Since $(L_f - L)$ is an upper bound on the amount of extension that can occur, any bending or curvature of the beam will reduce the midspan deflection.

In the computations made with the MIT DEPROSS program [14], the beam was broken up into 30 segments along the beam semispan (symmetry assumed) and six segments through the thickness. In our application of the program, the incremental time step at which the finite difference equations were solved was approximately 0.2 μsec. The computation was carried out to 200 μsec with printout at 10-μsec intervals. Typical computer time for a single run was on the order of 1 minute on a CDC 3600 computer.

The program requires a uniform input velocity for the beam (or some segment of the beam symmetric about the center). This velocity is given by

$$V_0 = \frac{I}{\rho h}$$

where I is the specific impulse. The uniaxial stress-strain curve for the beam material is assumed piecewise linear. For both the aluminum and titanium, a bilinear approximation was found adequate. Maximum strains were always less than 5 percent. The yield point (or, in general, each transition point between linear segments of the stress-strain curve) can be prescribed rate dependent according to the relation

$$\sigma_y = \sigma_0\left[1 + \left(\frac{\dot{\epsilon}}{D}\right)^{1/p}\right] \tag{5}$$

The constants, σ_0, D and p given in Table I were found to give a good fit to the data presented in Reference 1. For the aluminum, $\sigma_y = \sigma_0$, a constant. For the titanium, the yield stress at 10^{-2} in/in/sec is 140,000 lb/in^2 and at 10^3 in/in/sec it increases to 175,000 lb/in^2, a 25 percent increase.

An additional loading on the beam is the drag force as the beam moves through the air. This drag force is given by

$$D = \frac{1}{2} C_D \rho_a S V^2$$

where C_D is a dimensionless drag coefficient, ρ_a is the air density, S is the surface area of the beam (= aL), and V is the transverse velocity of the beam (= $\dot{w}$). The drag force would act to reduce the amplitude of the deflections. However, simple calculations can show that the overall effect is negligible in the present situation.

SECTION IV

RESULTS AND DISCUSSION

The transient peak midspan deflection and permanent midspan deflection for the aluminum and the titanium beams are presented in Figures 4 and 5. The solid curve in each case is based upon the numerical computations[14] using the parameters listed in Table I. Also shown are the bounds as given by (1) for the rigid-plastic approximation. The permanent deflection values are experimental only, since the numerical computations were not carried out for sufficient time to establish permanent deflections. Because of the large amount of elastic energy stored, the beams oscillated for a long period before coming to rest.

For both the aluminum and the titanium beams, the measured peak deflections are consistently somewhat lower for a prescribed impulse than predicted by computation. This difference is of the order of 10 percent, which is certainly not bad for this type of problem. The measured permanent deflections are much less than the transient deflections, indicating a great deal of elastic recovery. For this reason, it is not surprising that the rigid-plastic approximate analysis does not yield good agreement with permanent displacements. It is somewhat surprising, however, that with increasing impulse the bounds from rigid-plastic theory and the experimental permanent deflections diverge. With increasing impulse a greater proportion of the input kinetic energy is absorbed in plastic deformation as versus elastic deformation, leading to the usual assumption that the elastic energy may be neglected. The same divergent trend is noted in the uncorrected data of Humphreys[11]. (The present experiments encompass the same range in impulse parameter as Humphreys' experiments.) Humphreys observed from transient photographs only about 5 percent elastic recovery based upon peak displacement. This is certainly much less than in the present experiments and results, in part, from the higher yield stress of the present materials (although the 7075-T6 aluminum should be comparable with the hardened steel used by Humphreys).

Calculated energy distribution in the titanium beams as a function of the impulse or initial kinetic energy imparted is given in Figure 6. The energy distribution given for each impulse level is that existing at the time the midpoint of the beam attains its peak displacement. At this time the kinetic energy goes through a minimum but does not vanish because the velocity of all points of the beam do not go through zero simultaneously. For the low impulse or elastic case, there are obviously several normal modes of oscillation excited simultaneously. The large percentage of elastic energy stored results from both the inplane boundary constraints on the beam and the high yield strengths of the structual alloys considered. Most previous comparisons with rigid-plastic theory have been made for beams without axial constraint when bending is dominant over inplane extension.

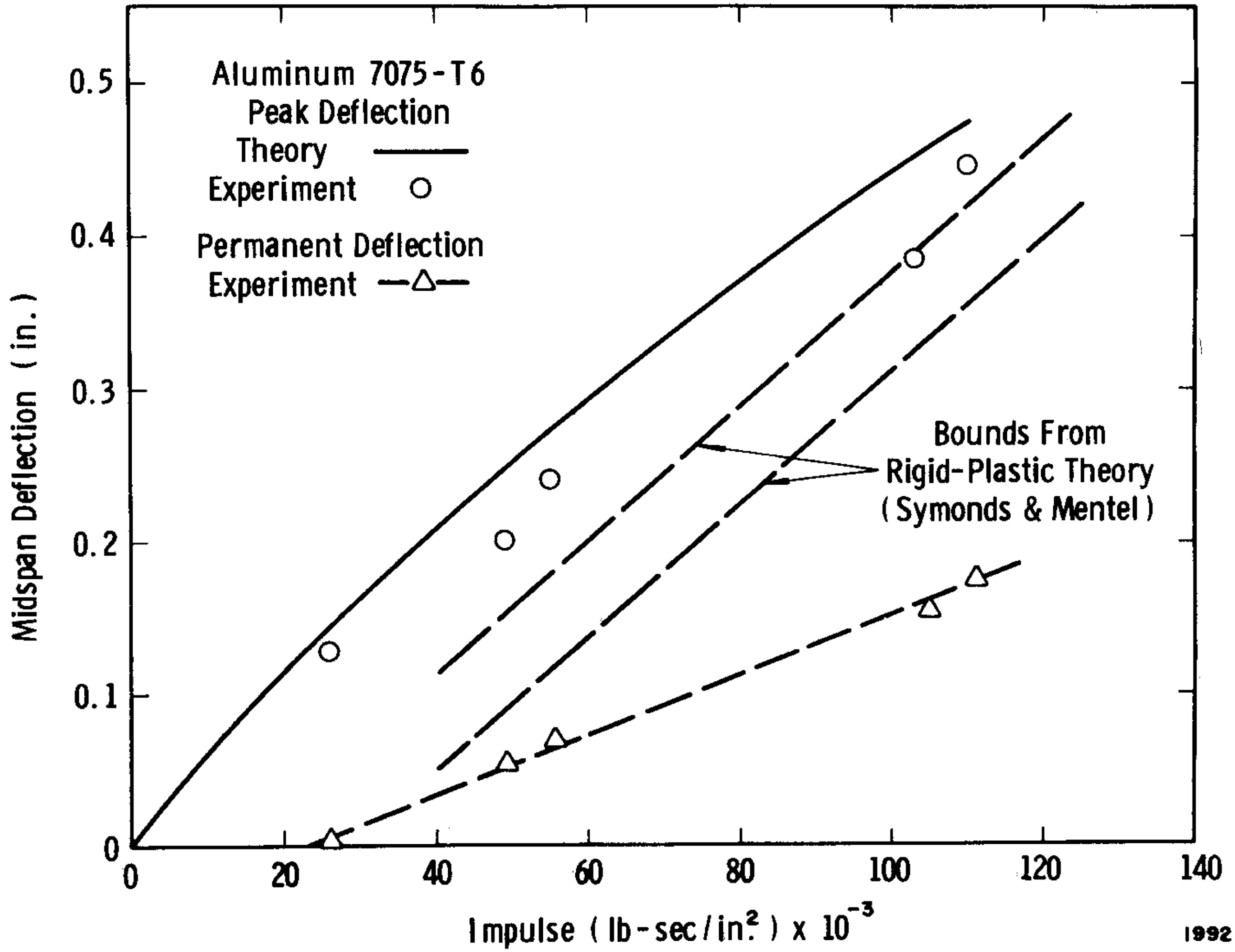

FIGURE 4. MAXIMUM MIDSPAN DEFLECTION AS A FUNCTION OF IMPULSE FOR ALUMINUM BEAMS

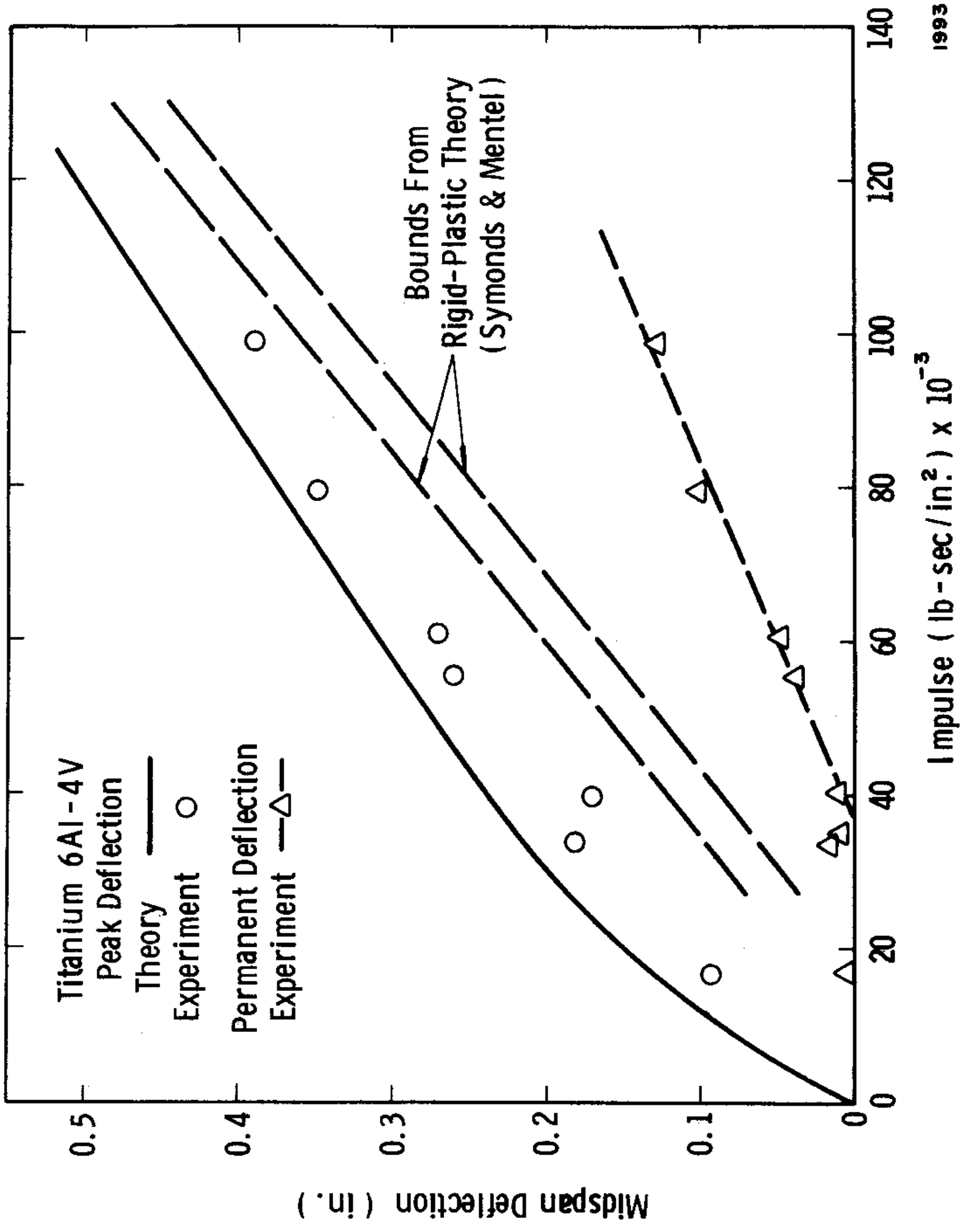

FIGURE 5. MAXIMUM MIDSPAN DEFLECTION AS A FUNCTION OF IMPULSE FOR TITANIUM BEAMS

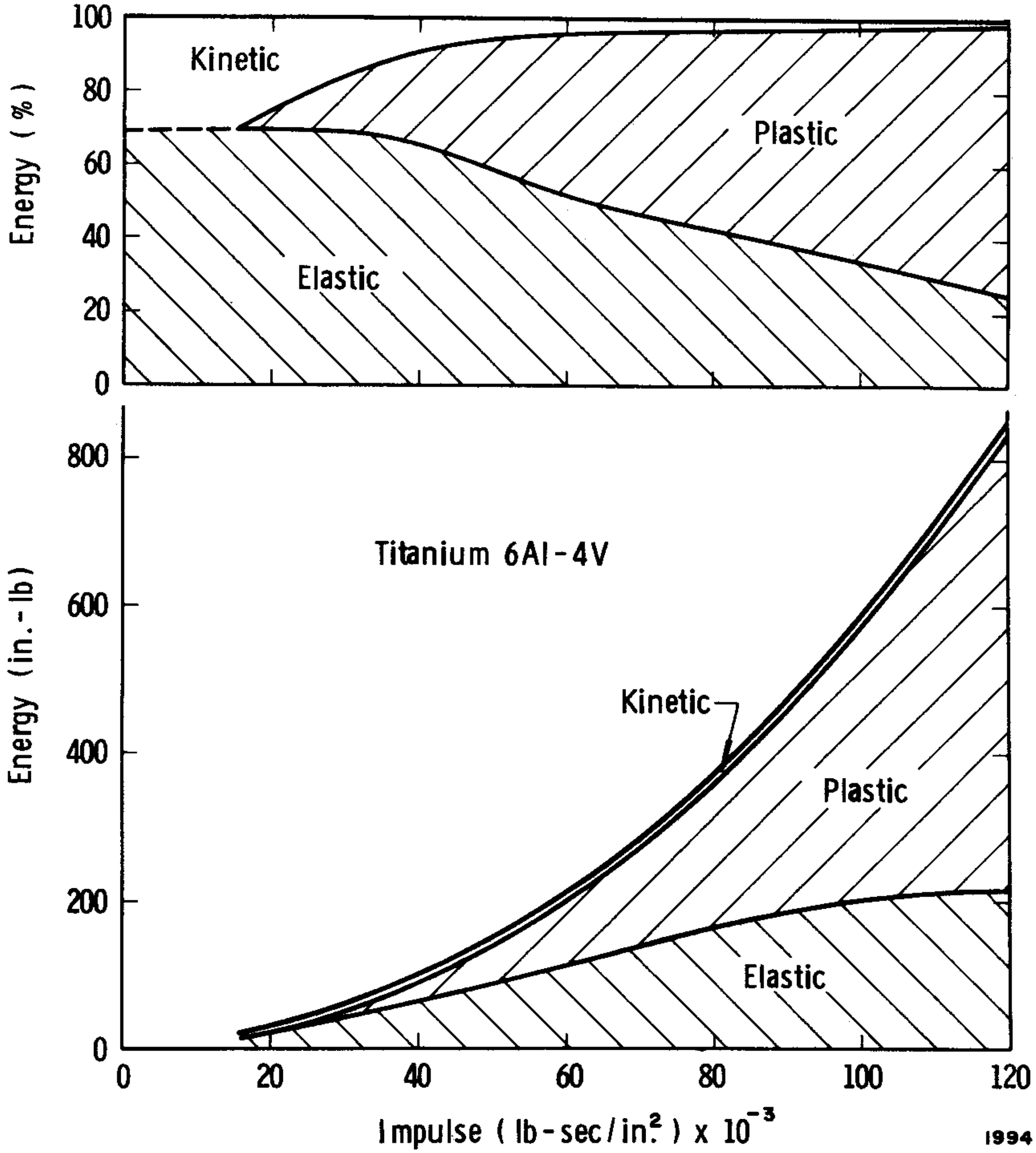

FIGURE 6. ENERGY DISTRIBUTION AT TIME OF MAXIMUM DEFLECTION FOR TITANIUM BEAMS

In Figure 7, the maximum transient strain measured at the midpoint of the titanium beams is plotted and compared with the computer results. The agreement here is very good. The strain measurements are felt to have a higher degree of confidence than the displacement measurements. Strain measurements are lacking at the highest impulse levels because the limits of the strain gages were exceeded. The computations resulted in a linear correlation between maximum strain and impulse, at least for this one point on the beam.

Typical transient strain histories for an aluminum and a titanium beam are given in Figures 8 and 9. These records and calculations are again for the center of the beam on the side opposite to the explosive. The main features of the experimental records are reproduced by the DEPROSS program. The detailed "wiggles" in the response are influenced strongly by the input parameters, so that exact agreement is not to be expected. Large, dominantly elastic vibrations precede the main inelastic response. The duration of the loading is less than 10 μsec so that the loading is accomplished before significant elastic response occurs at the center of the beam, a requirement of the impulse assumption.

For the aluminum beam, the solid curve is calculated with a constant yield stress appropriate for a rate-insensitive material. For the titanium beam, the solid curve includes rate sensitivity in the calculation according to (5). Two additional calculations were made for the titanium beam to compare the effect of using an equivalent "static" and "dynamic" constant yield stress. These results are given by the curves labeled σ_0 = 140 ksi and 175 ksi in Figure 9. The 175 ksi corresponds to the yield stress associated with a uniform strain rate of 1000/in/in/sec. This is roughly the maximum strain rate occurring during the major plastic loading portion of the response (between 80 and 120 μsec in Figure 9). For this example, the calculated response based upon the single "dynamic" stress-strain curve and the strain-rate dependent curve differ only slightly. This would indicate that the major plastic strain occurred at a rate close to the average dynamic rate chosen. In general, however, the strain rate varies both in time and with position in the beam, so that use of an average stress-strain curve or an overall strain-rate correction may be subject to error. Use of the static stress-strain curve results in a "softer" beam and the magnitude of the error will be obviously in proportion to the magnitude of the strain-rate effect.

Computed transient deflection mode shapes are given in Figure 10 for titanium beams at three different impulse levels. Measured permanent deflections are also given for one case. It is observed that a region of sharp curvature in the bending mode progresses from the fixed boundary towards the center of the beam. This region has generally been interpreted as the location of the "plastic hinge."[5,11] However, the lowest impulse case in Figure 10 corresponds to a fully elastic response, and the same progressing wave is observed. Further, the rate of progression of the wave, as determined

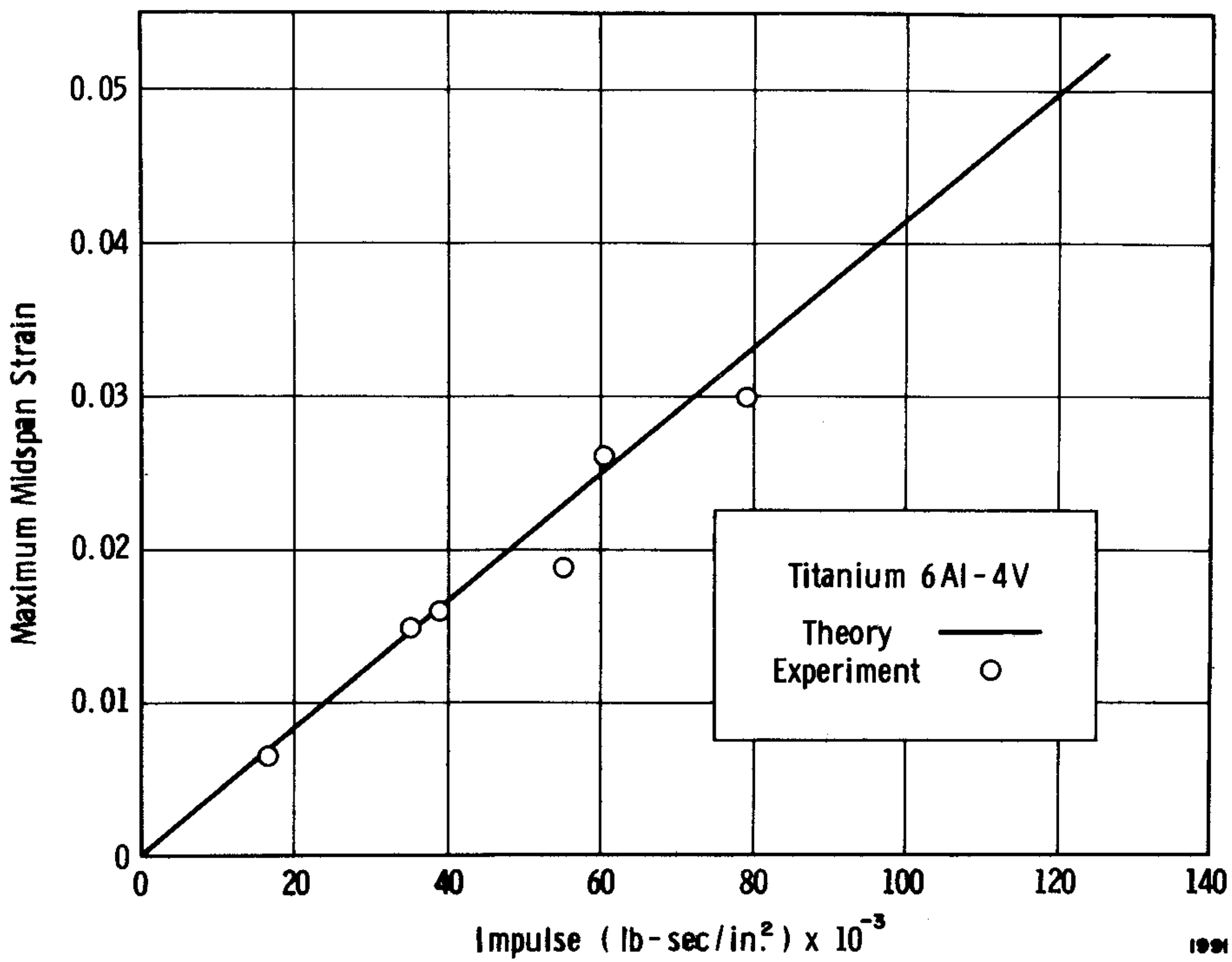

FIGURE 7. MAXIMUM MIDSPAN STRAIN AS A FUNCTION OF IMPULSE FOR TITANIUM BEAMS

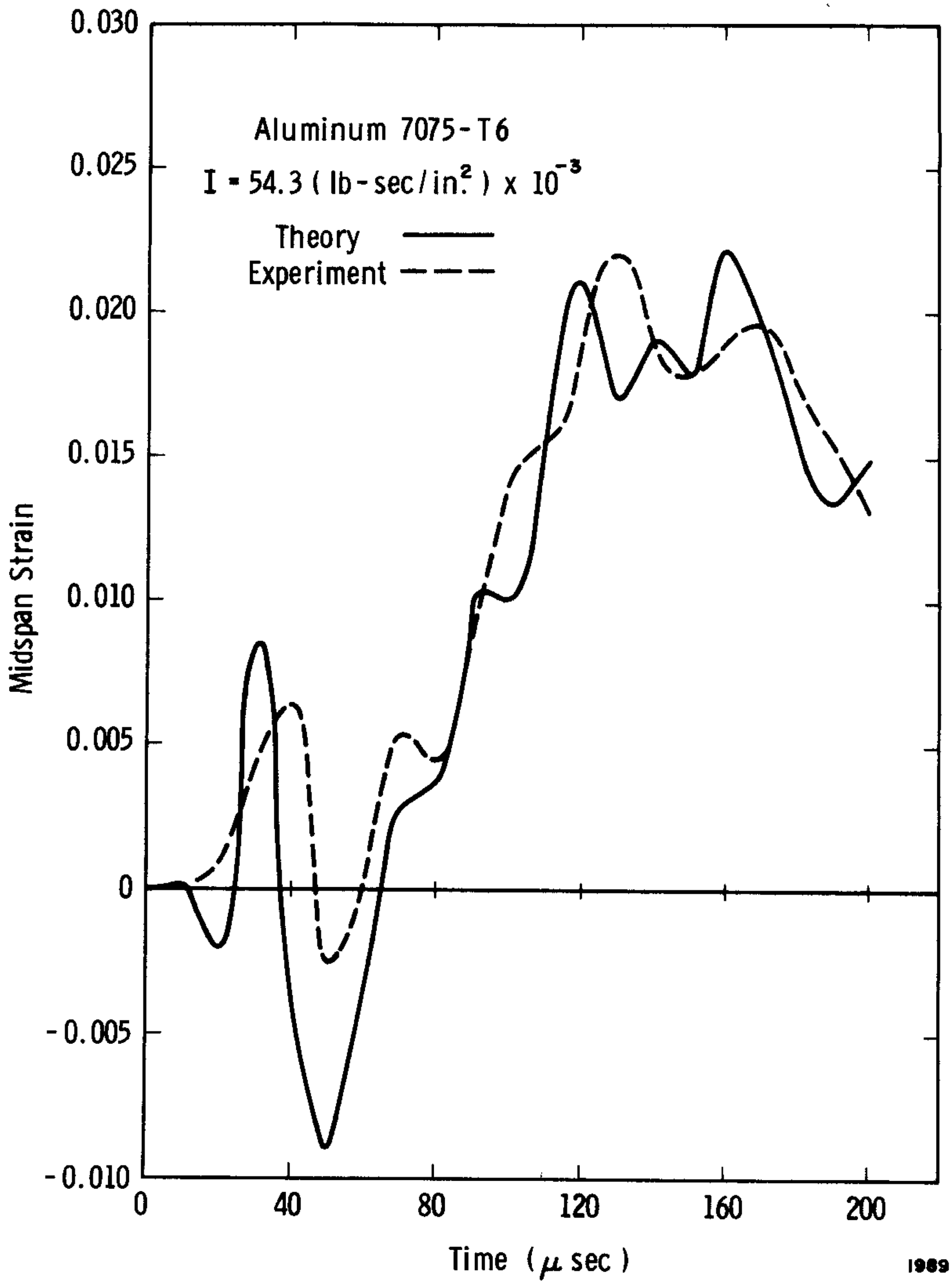

FIGURE 8. TRANSIENT STRAIN HISTORY FOR AN ALUMINUM BEAM

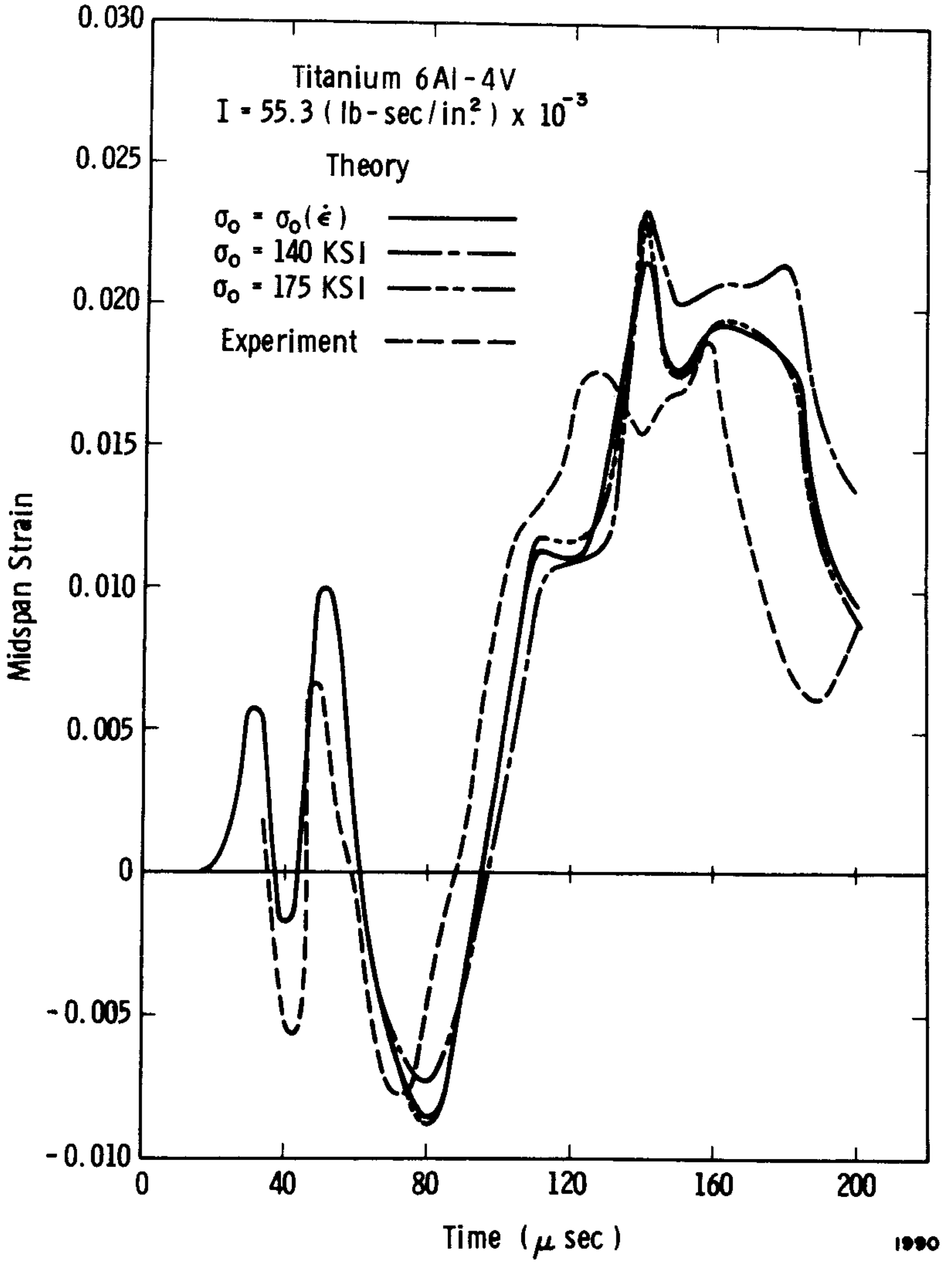

FIGURE 9. TRANSIENT STRAIN HISTORY FOR A TITANIUM BEAM

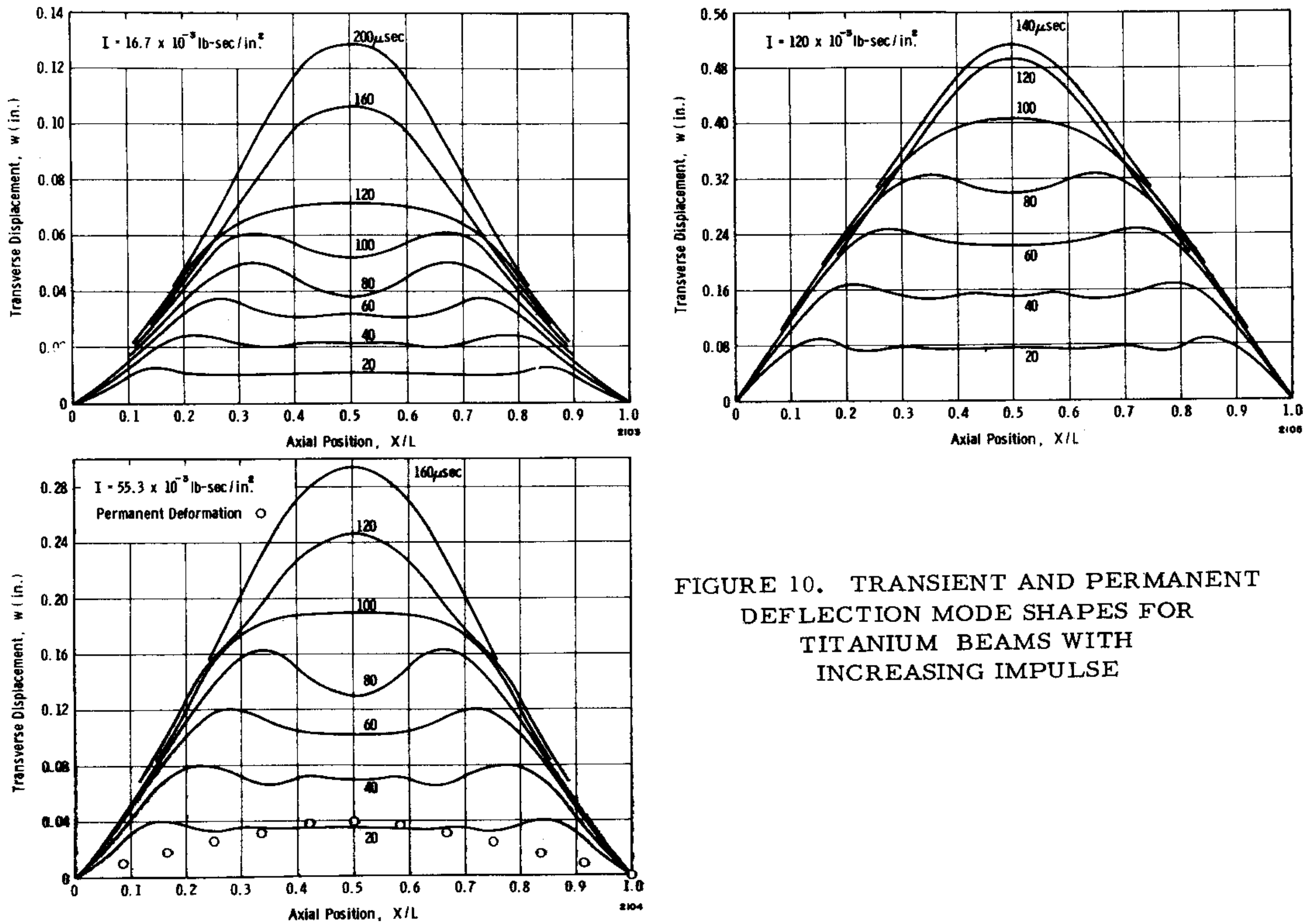

FIGURE 10. TRANSIENT AND PERMANENT DEFLECTION MODE SHAPES FOR TITANIUM BEAMS WITH INCREASING IMPULSE

by plotting the local maximum in w versus time, does not change noticeably with increasing impulse amplitude, i.e., with change from elastic to elastic-plastic response. For the 0.120 lb-sec/in^2 impulse, plastic yielding is initiated in this region after about 30 μsec. Thus, it would appear that the traveling wave or "hinge" is dominantly an elastic phenomena and not a consequence of rigid-plastic behavior. The predicted mode shapes in Figure 10 are in agreement with the high speed photographs of Humphreys(11) for a fully clamped beam and also those of Florence and Firth(5) for a beam fixed from rotation but axially unrestrained.

Yielding is initiated at regions of sharp curvature in the elastic mode. First yielding occurs at the fixed support. Subsequently, one or more plastic regions are initiated at local regions of sharp curvature in the bending mode. The growth of the plastic zones is illustrated in Figure 11 for the case of an intermediate impulse of a titanium beam. The corresponding deflection mode shapes are given in the center plot of Figure 10. The shaded regions in Figure 11 correspond to points on the beam where the surface strain (obtained from the computations) is greater than the yield strain. The plastic zones occur on the side of the beam which is in tension in bending. The zones which form toward the center of the beam occur on the face opposite to the loading, except for the central region occurring at 80 μsec which results from the reverse curvature indicated on the mode shape, Figure 10. The plastic zones do not progress in a uniform manner and are influenced by the elastic vibrations.

For the present problem, the elastic effects obviously cannot be neglected. This results from the relatively high yield strength of the structural alloys used and the axial constraint imposed which results in a large mean tensile strain and thereby maximum storage of elastic energy. Therefore, the rigid-plastic material assumption is inadequate for this case. The general role of elastic effects should be considered, especially for structural materials which have a high yield stress. In all cases, the initial deformation is elastic and the elastic modes influence the development of the plastic zones. This effect was noted by Florence and Firth(5) who obtained experimental mode shapes similar to those of Figure 10. Nevertheless, they obtained correlation between the measured "elastic-plastic hinge" position and rigid-plastic theory. For cantilever beams impulsively loaded at the tip, Bodner has indicated, both from photographic(16) and strain gage(8) records, the absence of a distinct traveling plastic hinge. Actually, the elastic, strain-rate, and strain-hardening effects all tend to eliminate the distinct plastic hinge in dynamic problems.

The importance of including strain-rate dependence of the stress-strain relation in the dynamic analysis depends on several factors. The first is, of course, the relative strain-rate dependence of the strength properties. Most studies of strain-rate corrections to the approximate rigid-plastic theories have been checked against experiments with mild steel beams. Mild steel exhibits an unusually high rate sensitivity, especially in the region of the yield

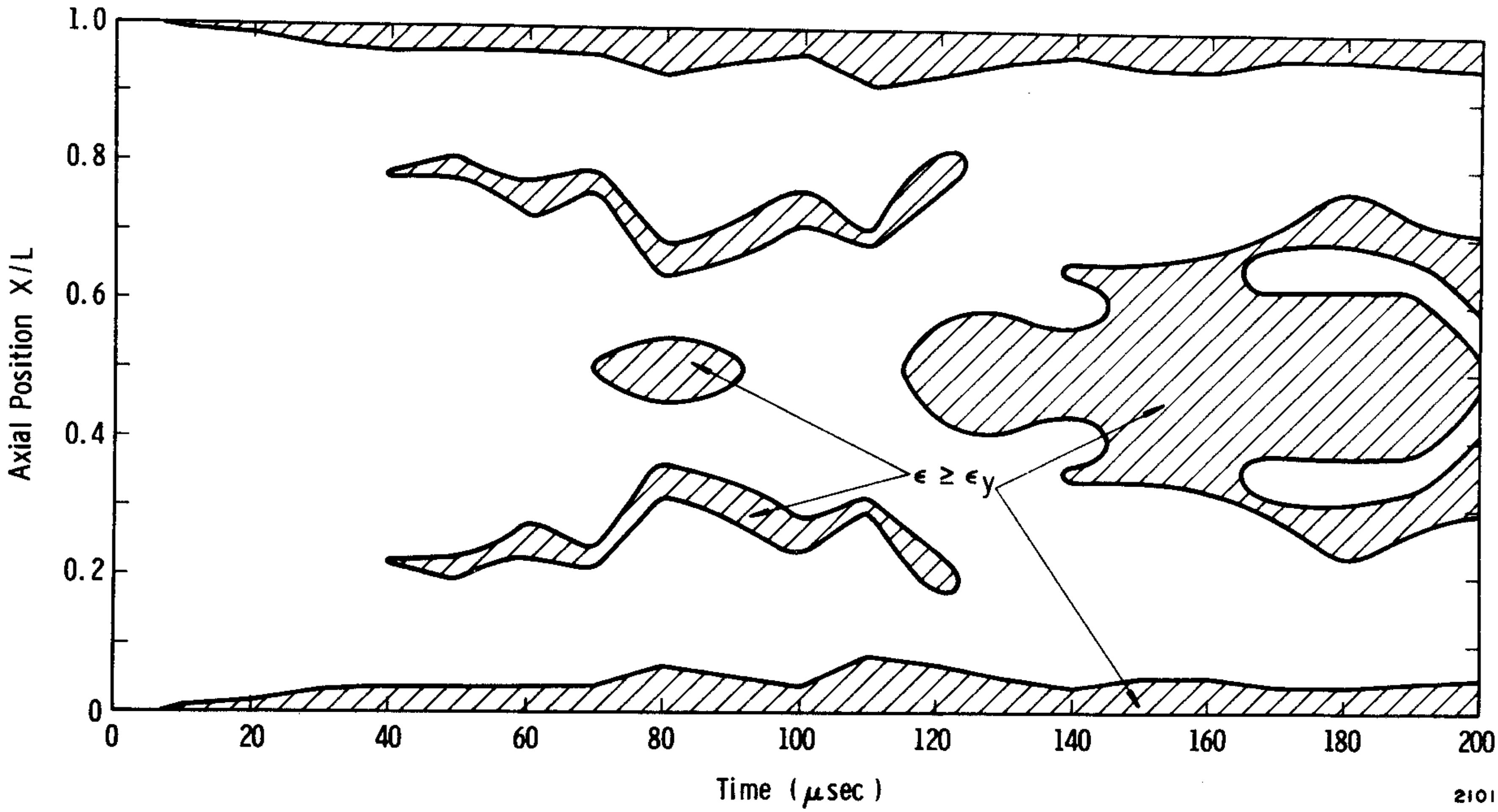

FIGURE 11. GROWTH OF PLASTIC ZONES IN TITANIUM BEAMS, $I = 55.3 \times 10^{-3}$ lb-sec/in^2

point.* Other structural alloys are generally less rate sensitive. In these cases, it may be uncertain as to whether the correction due to strain-rate effects is indeed significantly greater than other effects which have been neglected. Thus, the decision upon inclusion of rate effect is dependent on the overall accuracy of the analysis being performed. For the computer oriented numerical techniques, quite complex material description may be programmed in without appreciable additional difficulty. It appears that, at least in some cases, an overall strain-rate correction factor or an average dynamic stress-strain curve may be employed with an otherwise rate-independent analysis. Bodner(8) has applied an overall strain-rate correction factor successfully with the mode approximation solution for cantilever beams. Perrone(17) has suggested approximate solutions for rate-sensitive structures using a constant flow stress associated with the initial or maximum strain rate. The present calculations (Figure 9) indicate that an appropriately chosen constant, dynamic stress-strain relation yields results in close agreement with the complete strain-rate dependent solution.

Another effect of strain rate that has not been considered here is its effect upon ductility. While high strain rates may allow the use of increased yield strengths, ductility may be adversely affected, particularly for high strength metals with initially low ductility. Structures made of such metals may be more susceptible to fracture under dynamic loads. On the other hand, initially ductile metals may exhibit increased ductility at higher strain rates. Dynamic failure, as well as yield, criteria are needed.

*In most experiments, the maximum strains are restricted to 3 to 5 percent.

SECTION V

CONCLUSIONS

The transient elastic-viscoplastic response of impulsively loaded beams can be predicted accurately with available computer programs based upon numerical solution of the governing differential equations. For the clamped and axially restrained beam, the approximate rigid-plastic anaylsis was found inadequate because of the strong influence of elastic effects.

SECTION VI

ACKNOWLEDGEMENTS

The authors wish to express their gratitude to Professor E. A. Witmer for his assistance in the application of the DEPROSS computer program and to Professor S. R. Bodner for his several helpful discussions. Also our gratitude is expressed to Mr. L. M. Yeakley for his assistance in the experiments and to Dr. P. H. Francis for running the computer programs.

SECTION VII

REFERENCES

1. U. S. Lindholm, L. M. Yeakley, and R. L. Bessey, "An Investigation of the Behavior of Materials Under High Rates of Deformation," AFML-TR-68-194, July 1968.

2. P. S. Symonds, "Survey of Methods of Analysis for Plastic Deformation of Structures Under Dynamic Loading," Brown University, Final Report, Contract Nonr 3248(01)(X), June 1967.

3. P. S. Symonds, "Viscoplastic Behavior in Response of Structures to Dynamic Loading," Proc. of Colloq. on Behavior of Materials Under Dynamic Loading, ASME, New York (1965).

4. S. R. Bodner, "Strain Rate Effects in Dynamic Loading of Structures," Proc. of Colloq. on Behavior of Materials Under Dynamic Loading, ASME, New York (1965).

5. A. L. Florence and R. D. Firth, "Rigid-Plastic Beams Under Uniformly Distributed Impulses," J. Appl. Mech., 32, pp. 481-488 (1965).

6. N. Jones, "Influence of Strain-Hardening and Strain-Rate Sensitivity on the Permanent Deformation of Impulsively Loaded Rigid-Plastic Beams," Int. J. Mech. Sci., 9, pp. 777-796 (1967).

7. T. Nonaka, "Some Interaction Effects in a Problem of Plastic Beam Dynamics," Part I, II, III, J. Appl. Mech., 34, pp. 623-643 (1967).

8. S. R. Bodner, "Deformation of Rate-Sensitive Structures Under Impulsive Loading," in Engineering Plasticity, Heyman and Leckie, Eds., Cambridge Univ. Press (1968).

9. J. B. Martin and P. S. Symonds, "Mode Approximations for Impulsively-Loaded Rigid-Plastic Structures," J. Eng. Mech. Div., Proc. ASCE, EM5, pp. 43-66 (1966).

10. E. A. Witmer, H. A. Balmer, J. W. Leech, and T. H. H. Pian, "Large Dynamic Deformations of Beams, Rings, Plates and Shells," AIAA Journal, 1, 8, pp. 1848-1857 (1963).

11. J. S. Humphreys, "Plastic Deformation of Impulsively Loaded Straight Clamped Beams," J. Appl. Mech., 32, pp. 7-10 (1965).

12. R. D. Krieg and T. A. Duffey, "The Large Deflection Dynamic Elastic-Plastic Response of Clamped Beams Subjected to Explosive Loading," SC-RR-67-848, Sandia Laboratory, Albuquerque (1968).

13. P. S. Symonds and T. J. Mentel, "Impulsive Loading of Plastic Beams with Axial Constraints," J. Mech. Phys. Solids, 6, pp. 186-202 (1958).

14. H. A. Balmer, "Improved Computer Programs - DEPROSS 1, 2 and 3 to Calculate the Dynamic Elastic-Plastic Two-Dimensional Responses of Impulsively-Loaded Beams, Rings, Plates, and Shells of Revolution," Report ASRL TR 128-3, MIT Aeroelastic and Structures Research Laboratory, August 1965.

15. F. O. Hoese, C. G. Langner, and W. E. Baker, "Simultaneous Initiation Over Large Areas of a Spray-Deposited Explosive," Experimental Mechanics, 8, pp. 392-397 (1968).

16. S. R. Bodner and P. S. Symonds, "Experimental and Theoretical Investigation of the Plastic Deformation of Cantilever Beams Subjected to Impulsive Loading," J. Appl. Mech., 29, pp. 719-728 (1962).

17. N. Perrone, "On a Simplified Method for Solving Impulsively Loaded Structures of Rate Sensitive Materials," J. Appl. Mech., 32, pp. 489-492 (1965).

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Department of Mechanical Sciences Southwest Research Institute San Antonio, Texas	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

ELASTIC-VISCOPLASTIC RESPONSE OF CLAMPED BEAMS UNDER UNIFORMLY DISTRIBUTED IMPULSE

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Summary Report -- April 1967 to July 1968

5. AUTHOR(S) (First name, middle initial, last name)

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Roger L. Bessey

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
January 1969	27	17

8a. CONTRACT OR GRANT NO. AF 33(615)-5338	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO. 7351	Technical Report No. 2
c. TASK NO. 735106	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	AFML-TR-68-396

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Metals and Ceramics Division Air Force Materials Laboratory Wright-Patterson Air Force Base, Ohio

13. ABSTRACT

Experiments are performed with rigidly clamped beams loaded impulsively by an explosive technique. The transient and residual deflection and strain are measured. Experimental results are compared with predictions from approximate rigid-plastic theory and a more exact elastic-viscoplastic numerical solution.

Results obtained indicate that elastic strains are not negligible for the system studied. For this reason, the approximate rigid-plastic analysis is less accurate than the more exact numerical solution. The dynamic stress-strain properties of the beam material were determined from separate experiments conducted over a wide range of strain rates up to 1000/sec. These strain-rate dependent properties, when incorporated into the numerical solution, allow an accurate prediction of the response of the beam. The strain-rate dependence of the material is shown to influence the response amplitude of the beam.

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DD FORM 1 NOV 65 1473

UNCLASSIFIED

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Dynamic beam response Strength of metals Strain-rate effects						