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Summary

A scheme utilizing a temperature probe immersed in the working fluid to compensate for the dependence of hot-wire (or hot-film) velocity calibration on ambient temperature variations is analyzed. To achieve compensation, only properties of the anemometer bridge, the velocity and temperature probes need to be known. The scheme provides means for incorporating the temperature compensation a priori to conducting the experiments, without the need for temperature calibration. Experiments were conducted, using tungsten hot-wires in air, to check the performance of the analysis and to verify some of the assumptions that were made. For typical probes, with an ambient temperature increase of 20°C, maximum errors in the indicated velocity of 1% or less can be achieved.

Introduction

The temperature of the working fluid in many flow facilities does not remain constant for the entire duration of the experiment. These temperature changes may be deliberate, because of some aspect of the experiment, or incidental, e.g., due to the energy losses from the fan or pump providing the flow in the facility. In applications utilizing hot-wires or hot-films, such temperature variations are undesirable. Since the hot-wire sensor is not only sensitive to the flow velocity but also to the fluid temperature, density and viscosity, such variations lead in many occasions to substantial errors in the indicated velocity. A simple and inexpensive method of temperature compensation which would require no extensive additional calibration of the anemometer would be ideal. For example, an acceptable method would be one which can be accurately used once some basic properties of the probes are known. The accuracy of the method is of course measured by the errors introduced during temperature variations. Hence, an estimate of these errors is an essential part of the development and documentation of the method.

A number of schemes to measure and/or compensate for the ambient temperature variations are described in the literature and through commercially available compensators. Burchill and Jones [1] proposed a scheme in which the resistance in the anemometer bridge that controls the wire temperature is manually adjusted as the fluid temperature changes; the adjustment being based on calibration curves obtained earlier at different ambient temperatures. The amount of time wasted during an experiment is the major objection to this scheme. Other techniques developed by Chevray and Tutu [2] and similarly by Ali [3], involve a complex compensating scheme which is applied to the signal from the bridge, after it has been linearized, through the linearizer constants. Rose [4] determined a method by which values read from the anemometer were corrected through the use of a correction factor which is a function of temperature. His method was found to give 3% errors in indicated velocity due to only a 7°C change in temperature. Methods like those of Chevray and Tutu [2] and of Ali [3] are particularly suited when high frequency temperature changes are present in the flowfield.

A number of temperature compensating schemes proposed by the hot-film manufacturers depend on a temperature sensing probe which can be used in the anemometer bridge shown in Figure 1 in place of the bridge resistance R_3. The ideal probe for this purpose should have the following properties:

1. Its resistance R_T should be given by

$$R_T = r_o M R_C = M R_H$$

where M is the bridge ratio and r_o is the overheat ratio.

2. The dependence of R_T on temperature should be identical to that of the velocity probe operating resistance R_H.

3. The size of the probe should be sufficiently large so as not to be heated by the bridge current.

The latter requirement assures that the probe resistance is independent of the fluid velocity.

Such a probe is of course almost impossible to manufacture and even if it could be selected from a large statistical sample it could only be used with the one matched velocity probe. Fortunately most of the hot-film and hot-wire probes furnished by a manufacturer are quite similar in properties with their resistances falling within a limited range. A temperature compensating circuit similar to those outlined in Figure 1 can therefore be used, at least over some range of ambient temperature variation.

Nagib [5] and Tan-atichat *et al.* [6] used this scheme and were able to compensate to better than 1% change in indicated velocity over an 11°C change in water temperature. However, to determine the values of resistance in the network required a long and tedious iterative procedure. In spite of that, the scheme of Figure 1 was by far the simplest to apply in order to attain the desired accuracy, as long as the frequency response of the temperature probe was higher than the frequency of temperature changes. Therefore, it is one of our objectives to be able to analytically determine the values of these resistances knowing only a few basic properties of the velocity sensing probe and the temperature compensating probe, as well as the desired operating conditions. The maximum change of the fluid temperature, which can be tolerated without compensation, depends on the application and the accuracy required. However, a change in ambient temperature of 1°C for hot-film measurements in water and of 7°C for hot-wires in air flows is considered significant; i.e., errors larger or equal to 5% in the velocity measurements may be encountered.

Analysis

Based on the bridge balance condition of the anemometer circuit in Figure 1, we can write

$$R_1/R_2 = R_H/R_3 \equiv M \qquad (1)$$

where M is the bridge ratio and R_3 is the equivalent resistance of the temperature compensating circuit.

Considering only the first coefficient of the temperature dependence of R_C and R_T (i.e., ignoring second and higher order terms of temperature changes) we write

$$R_T = R_{To}[1+\alpha_T(T-T_o)] \quad (2)$$

and

$$R_C = R_{Co}[1+\alpha_C(T-T_o)] \quad (3)$$

where R_{To} and R_{Co} are the resistance of the temperature compensating probe and the cold resistance of the velocity probe, respectively, when the ambient temperature, T, is equal to the probe reference temperature T_o.

Assuming that α_r is independent of temperature, where

$$\alpha_r \equiv \frac{\alpha_T}{\alpha_C} \quad \text{and } \alpha_r > 0 \quad (4)$$

we define a probe resistance ratio as follows

$$\beta \equiv \frac{R_T}{R_C} \quad (5)$$

The resistance ratio β is dependent on temperature and its value at the original reference temperature T_o, (usually room temperature) is

$$\beta_o = \left[\frac{R_T}{R_C}\right]_{T=T_o} \quad (6)$$

We denote the operating resistance of the velocity sensor by R_H, i.e.,

$$R_H = R_C r_o(T) \quad \text{and } r_o(T) > 1 \quad (7)$$

where r_o is the overheat ratio. Since R_H is related to the temperature compensating circuit resistance through the bridge relation, it is also a function of the ambient temperature. First, we consider compensating circuit 1, where the equivalent resistance, R_3, is

$$R_3 = R_S + \frac{R_P R_T}{R_P + R_T} \quad (8)$$

From the bridge balance condition and by defining non-dimensional series and parallel resistances, S_r and P_r respectively, where

$$S_r \equiv \frac{R_S}{R_T} \quad \text{and } S_r \geq 0 \quad (9)$$

and

$$P_r \equiv \frac{R_P}{R_T} \qquad \text{and} \qquad P_r > 0 \tag{10}$$

we obtain the nondimensional bridge balance condition,

$$S_r + \left(1 + \frac{1}{P_r}\right)^{-1} = \frac{r_o}{\beta M} \tag{11}$$

The governing equation for the velocity sensor (King's law) can be written as

$$\frac{I_P^2 R_H}{R_H - R_C} = A + B\,U^n \tag{12}$$

where U is the fluid velocity and I_P is the current through the probe; A, B and n are nearly constant. Based on this relation and since for constant temperature operation the anemometer servo-amplifier maintains the probe temperature and hence its resistance R_H constant, one can conclude that two possible modes may be useful for keeping the output voltage constant, i.e., independent of ambient temperature. One can either maintain a constant overheat ratio or maintain a constant resistance difference.

Another possible mode exists if we rewrite King's law in terms of the bridge output voltage, i.e.,

$$E_b^2 = \left(1 - \frac{R_C}{R_H}\right)(R_1 + R_H)^2\;(A + BU^n) \tag{13}$$

and determine the condition necessary to keep the output (non-linearized or linearized) independent of ambient temperature. This mode assumes that the coefficients A, B and n are insensitive to temperature variations relative to the changes in R_C and R_H with ambient temperature (the validity of this assumption will be examined later).

Each of these modes will specify the dependence of the compensating circuit resistance with temperature necessary to maintain proper operation. This can be written as

$$\frac{dR_3}{dT} = \frac{R_{Co}\alpha_C}{M\eta} \tag{14}$$

where

$$\eta = \begin{cases} \frac{1}{r_o} & \text{for constant overheat ratio mode} \\ 1 & \text{for constant resistance difference mode} \\ \frac{1}{\tau(T)} & \text{for constant output voltage mode} \end{cases} \tag{15}$$

Assuming that A, B and n are independent of temperature, we find that

$$\tau(T) = \frac{1 + \frac{R_1}{R_H}}{2 + \frac{R_C}{R_H}\left[\frac{R_1}{R_H} - 1\right]} \tag{16}$$

The solution of Equation (14) along with the non-dimensional bridge relation for each circuit will yield values of the series and parallel resistances for each operating mode.

Compensating Circuit 1. Substitution of Equation (8) into Equation (14) while using Equations (5), (9) and (10) leads to the solution for the parallel resistance directly

$$P_r = (\sqrt{\alpha_r M\eta\, \beta_o} - 1)^{-1} \tag{17}$$

or

$$R_P = [R_T(\sqrt{\alpha_r M\eta\, \beta_o} - 1)^{-1}]_{T=T_o} \tag{18}$$

Then, substitution of Equation (17) into Equation (11) leads to

$$R_S = \left[R_T \left(\frac{r_o}{\beta M} - \frac{1}{\sqrt{\alpha_r M \eta \beta_o}} \right) \right]_{T=T_o} \qquad (19)$$

Here, the values of R_S and R_P are evaluated at the initial reference temperature T_o.

<u>Compensating Circuit 2</u>. Referring to Figure 3, we obtain

$$R_3 = \frac{R_P(R_S+R_T)}{R_P+R_S+R_T} \qquad (20)$$

Solving for the series and parallel resistences (for details see Drubka *et al*. [7]) one obtains

$$R_P = \left[\frac{R_T\ r_o\ \sqrt{\alpha_r M \eta\ \beta_o}}{\beta M\ (\sqrt{\alpha_r M \eta\ \beta_o} - 1)} \right]_{T=T_o} \qquad (21)$$

and

$$R_S = \left[R_T \left(\frac{r_o}{\beta M} \sqrt{\alpha_r M \eta\ \beta_o} - 1 \right) \right]_{T=T_o} \qquad (22)$$

Operating charts for the selection of the series and parallel resistances for circuits 1 and 2 are also plotted in Figures 2 and 3 respectively. These charts, or the equations given above, can be used by the reader to design or adjust either of the two compensating circuits. One needs only to know the properties of the anemometer bridge and the velocity and temperature probes, i.e., their resistances and their temperature dependence coefficients, in order to achieve the compensation, without any need for temperature calibration.

<u>Compensating Circuit 3</u>. For this case, the two equations governing the four unknown resistances lead to a family of solutions. Any of these solutions, which can be determined through the following procedure, satisfies the equations.

(1) Select R_{S1} such that

$$0 \leq R_{S1} \leq \left[R_T \left(\frac{r_o}{\beta M} \sqrt{\alpha_r M \eta \beta_o} - 1 \right) \right]_{T=T_o} \tag{23}$$

(2) Select R_{P1} such that

$$R_{P1} > \left[\frac{R_T + R_{S1}}{\sqrt{\alpha_r M \eta \beta_o} - 1} \right]_{T=T_o} \tag{24}$$

(3) Determine R_{S2} from

$$R_{S2} = \left[\frac{R_{P1}}{(R_{S1}+R_{P1}+R_T)} \left(\frac{R_T r_o \sqrt{\alpha_r M \eta \beta_o}}{\beta M} - R_{S1} - 1 \right) \right]_{T=T_o} \tag{25}$$

and (4) R_{P2} is given by

$$R_{P2} = \left[\frac{R_{P1} \sqrt{\alpha_r M \eta \beta_o}}{\beta M [R_{P1} \sqrt{\alpha_r M \eta \beta_o} - (R_{S1}+R_{P1}+R_T)]} \right]_{T=T_o} \tag{26}$$

Selection of a Compensating Probe. Ideally, it would be desirable to have one temperature compensating probe for all velocity probes. For typical probes this is not always possible. However, knowing the range of resistances of the velocity probes and their temperature coefficients, a temperature probe may be carefully selected for use in most of the cases. Based on the solution of compensating circuits 1, 2 and 3, and requiring that a real solution exists, the following inequality involving the parameters of both probes must hold:

$$\frac{R_{Co}}{M \alpha_r \eta(T_o)} < R_{To} \leq \frac{R_{Co} r_o^2 \alpha_r \eta(T_o)}{M} \tag{27}$$

With a knowledge of the properties of available velocity sensors, operating overheat ratios, anemometer bridge ratio and operation mode of the compensating circuit, a suitable temperature compensator may either be fabricated or purchased from a manufacturer based on this inequality.

Besides this restriction, the compensating probe must not be influenced by the electrical current passing through it. Otherwise, this results in temperature probes sensitive to velocity, which is not only an undesirable feature, but also affects the calibration curve of the velocity probe as discussed by Tan-atichat et al. [6]. The compensating circuit also has a frequency response to temperature variations which depends on the thermal time constant of the temperature probe (up to several cycles per second can be obtained using commercially available probes).

Error Estimates. When utilizing the constant output voltage mode, one can not expect ideal compensation from the proposed circuit configurations. Equation (14) prescribes the dependence of R_3 on ambient temperature for ideal compensation, while the introduction of a proposed circuit and a temperature probe forces a set function of temperature on R_3. Therefore, the "error" will be related to the difference between the two.

Once the velocity sensor is calibrated at the reference temperature, i.e.

$$E_b^2 = A' + B'U^n \ , \tag{28}$$

then the error in the indicated velocity is found to be [7]

$$\frac{U(T) - U(T_o)}{U(T_o)} = \left[\frac{E_b^2(T_o)e^I - A'}{E_b^2(T_o) - A'} \right]^{1/n} \tag{29}$$

where

$$I \equiv \int_{T_o}^{T} \frac{M\frac{\partial R_3}{\partial T}\left[2 - \frac{R_C}{MR_3} + \frac{R_1 R_C}{M^2 R_3^2}\right] - \alpha_C R_{Co}\left[1 + \frac{R_1}{MR_3}\right]}{\left(1 - \frac{R_C}{MR_3}\right)(R_1 + MR_3)} \, dT \tag{30}$$

Here, the integral I is indicative of the magnitude of the non-ideal behavior of the compensating circuit, i.e., the larger the value of I, the larger the percentage error in indicated velocity. If this integral has a value of zero, then the circuit behaves ideally, i.e., satisfying Equation (14) for all ambient temperatures. Since this integral is a function of $R_3(T)$, which depends on the compensating circuit configuration, its magnitude will be different for each of the three circuits of Figure 1. In general, the value of I decreases by going from circuit 1 to circuit 3.

The error estimates related to the integral I provide a means for determining the effectiveness of different compensators and hence for selecting the best one. In the case of the linearized output voltage, following a similar procedure, one finds that the corresponding error in the indicated velocity is given as

$$\frac{U(T) - U(T_o)}{U(T_o)} = e^{I/n} - 1 \qquad (31)$$

In circuit 1 and 2 there are two adjustable resistances which are uniquely determined. Therefore, the value of the integral I is predetermined. Circuit 3, however, has four undetermined resistances so that I can be minimized subject to two constraints, i.e., Equations (1) and (14). This minimization process [7] determines the values of the four resistances which would lead to the best possible performance by a particular compensator, i.e., lead to the smallest error in the indicated velocity as calculated by Equations (29) and (30).

Experimental Results and Discussion

Experiments were conducted in air to determine the validity of some of the assumptions made in the analysis and to evaluate the temperature dependence of certain parameters that were derived. The measurements were carried out utilizing the I.I.T. compressed-air driven hot-wire calibration tunnel described by Loehrke and Nagib [8]. A schematic of the entire setup is shown in Figure 4. Dry air was heated by a 4.5 kw "Chromalox" Model GCH3405 heater prior to entering the plenum chamber of the calibration tunnel. The air temperature was varied by regulating the current to the heater via a 3-phase voltage control unit. Air temperature was monitored in the test section of the calibration tunnel by a specially constructed circuit, utilizing a thermistor. A digital voltmeter indicated the temperature in °C directly with an accuracy and repeatability of the readings within 0.5°C over the range from 20 to 60°C. During the course of this experiment, the air temperature was measured approximately 2 cm. downstream of the hot-wire, to minimize errors due to heat loss to the surroundings. The flow rate was controlled by a Watts Model 110 pressure regulator in line with and ahead of the air heater. The velocity in the calibration jet was measured and monitored with the aid of a Valydine DP-45 pressure transducer. Before each reading, the regulator setting was adjusted to correct for the air temperature based on calibration information stored in a program of an HP-97 calculator.

The temperature coefficients of resistance of four compensating probes (three DISA Type 55FTC and one TSI Type 1310 AB) and several tungsten hot-wires ($d=0.00038$ cm, $\ell/d=580$) were obtained by exposing the probes to heated air streams of a known temperature. Probe resistance was measured using the bridge of a DISA 55D01 anemometer unit to within ±0.005 Ω. The temperature was varied and measurements taken throughout the range from 20 to 60°C. Graphs of probe resistance versus temperature were plotted and the temperature coefficients of resistance determined from the slope of the curve. In addition, a computer program utilizing the method of least squares was used to obtain accurate values of the temperature coefficients of the probes. The temperature dependence of the probe resistance for each case examined was found to be linear within 0.1% and the temperature coefficients were found to be accurate to within 1.5% of the mean value for all cases.

The hot-wire used in these measurements was calibrated for velocity over a range of overheat ratios and ambient temperatures. The parameters A, B and n in King's law were extracted from these results by minimizing the standard deviation of the data from Equation (13). The calculated values are plotted in Figure 5 and the outline of the calculation scheme is reported by Drubka et al. [7]. The curves indicate that the coefficient A has a slight temperature dependence while the coefficient B remains almost constant for all cases. The variations of the exponent n are within 4% of its mean value for all cases.

Guided by the analysis, three different operating modes were examined. In the first mode the operating overheat ratio was kept constant by manually adjusting the decade resistance of the anemometer at each temperature where data were taken. The second mode kept the operating resistance difference, $R_H - R_C$, constant by manually adjusting the decade resistance at each temperature. The third mode of operation involved adjusting the decade resistance of the anemometer to achieve constant output voltage at each fixed velocity as the air temperature was varied. The operating resistance as a function of ambient temperature, required to maintain the bridge output constant, was found from the latter approach.

Results from this portion of the experiment are shown in Figures 6 and 7. Figure 6 demonstrates that without compensation, the non-dimensional bridge voltage decreases linearly with increasing temperature, while maintaining the velocity constant. This result was found to be independent of flow velocity from 3.5 to 30 m/s. This drop in the bridge output voltage is dependent on the overheat ratio used and is largest for low overheats. Three overheats were used, namely, 1.4, 1.6 and 1.8; the latter being the most common value employed by various experimenters.

Holding the resistance difference constant, again while maintaining a constant velocity, the drop in output voltage was found to decrease. Ideally, the bridge output voltage for a velocity sensor should remain constant, for a constant velocity, as the ambient temperature changes. The constant overheat mode overcompensates but seems to approach the ideal compensation to a degree of accuracy greater than that for the constant difference mode. Both of these operating modes are found to be independent of fluid velocity and initial overheat ratio.

With the assumption that A, B and n are constants and by using Equation (13), the non-dimensional bridge voltages can be computed. These are compared in Figure 6 with experimental data for the uncompensated, constant resistance difference, and constant overheat cases. The figure demonstrates that there is excellent agreement between experiment and analysis for the uncompensated and constant resistance difference cases, while for the constant overheat case the analysis predicts non-dimensional bridge voltages slightly higher than the experimental results.

If the bridge output is forced to remain constant at a given velocity, i.e., the third mode of operation described above, the variation in the overheat ratio and resistance difference can be determined, as shown in Figure 7. The resistance difference has to increase substantially while the overheat ratio must drop slightly as the temperature increases in order to achieve ideal compensation. This tends to suggest that compensation using a constant overheat mode would give better results as compared to the constant difference mode, a result which is in agreement with Figure 6. Each of these cases, normalized in the proper way, is independent of fluid velocity and initial overheat ratio.

The response of the compensation resistance R_3 to changes in temperature is shown in Figure 8 for various modes of operation. In this and the following two figures, typical behavior of the variables is demonstrated for the typical conditions indicated on the figures. Similar behavior is found for other probes. The experimental results in Figure 8 demonstrate that the behavior of R_3 can be predicted very well by the analysis for the constant resistance difference and the constant overheat modes of operation. In the constant output mode, the analysis does not predict as much a rise in the resistance R_3 with increasing temperature as is actually observed in the experiment. The analysis of this mode was carried out on the assumption that the parameters in King's law, i.e., A, B and n, are independent of temperature. Based on Figures 6 and 7, and Equation (13), one can conclude that the variation in B and n, observed in Figure 5, may be neglected, since the normalized results appear to be independent of velocity. However, the temperature dependence of A is responsible for at least some of the observed discrepancy. The analysis can be modified to take into account the variation of the coefficient A with temperature. This variation, however, would have to be either assumed or experimentally determined. As a result of this, the compensating circuits could not be set prior to making the velocity measurements and the experimenter would have to calibrate his probe for both velocity and temperature.

The rate of change of R_3, i.e., dR_3/dT, obtained from the analysis for the constant output mode was lower than that required to hold the bridge output voltage constant. This is the main reason behind the discrepancy of Figure 8, in the case of constant output voltage operation. A systematic study of the parameters that could affect dR_3/dT revealed several interesting results which are displayed in Figure 9. If the temperature coefficient of resistance of the velocity probe is kept constant, take $\alpha_c = .0035/°C$ for example, the analysis indicates that dR_3/dT cannot be increased by changing the compensator resistance. A compensator with a high resistance results in a marked decrease in dR_3/dT as the temperature increases while a low resistance one will maintain a practically constant (decreasing very slightly) dR_3/dT. Compensators which have resistances anywhere in between the permissible high and low limits will exhibit a slight drop in dR_3/dT as the temperature increases. Use of circuit 1 instead of circuit 2 will also affect the slope of dR_3/dT with circuit 2 giving a smaller drop than circuit 1. The decrease in dR_3/dT as the temperature increases can be viewed as a decrease in sensitivity of the compensation circuits or in its ability to adjust the operating resistance of the velocity sensor. This behavior may or may not be desirable, depending on the resistance characteristics of the velocity sensor as a function of temperature. For our assumed, as well as measured, linear resistance change with temperature one would like to have the drop in dR_3/dT kept to a minimum for best compensation.

When the experimentally determined value of α_c for the hot-wire sensor is used in the analysis (α_c=.0035/°C), the resulting dR_3/dT is lower than that experimentally required to maintain a constant output. This explains the inability of the analysis in predicting the value of R_3 required to keep the output constant (at a fixed velocity) for the constant output mode of operation displayed in Figure 8. One way of correcting this deficiency is to use a fictitious or "pseudo" α_c which would yield the correct dR_3/dT i.e., sensitivity . The increase in α_c over the actual value required to affect this correction was found to be about 26%.

Careful examination of Figure 8 leads us to another method of achieving proper compensation from the analysis. The experimental values of the resistance R_3 for such compensation fall approximately halfway between the constant overheat analysis and the constant output analysis. The adjustable parallel and series resistances were then derived from the analysis to obtain the "modified" constant output mode. The procedure is to compute the values of the series resistance for constant overheat, $(R_S)_{COH}$, and for constant output, $(R_S)_{CO}$, modes of operation. An average value of the series resistance, $(R_S)_{MCO}$, is used for the modified constant output mode. The parallel resistance, $(R_P)_{MCO}$, is obtained by solving the equation pertaining to the circuit used (either 1 or 2) for $R_3(T_o) = r_o R_{Co}/M$. For circuits 1 and 2, the modified series resistance can be written as

$$(R_S)_{MCO} = \frac{(R_S)_{CO} + (R_S)_{COH}}{2} \tag{32}$$

where the constant output and constant overheat modes are described by Equation (15), and the series resistance for circuit 1 can be calculated from Equation (19) and that for circuit 2 is given by Equation (22). For circuit 1, the modified parallel resistance can then be written as

$$(R_P)_{MCO} = \left\{ \frac{R_T\left[\frac{r_o R_C}{M} - \frac{(R_S)_{CO}}{2} - \frac{(R_S)_{COH}}{2}\right]}{R_T + \frac{(R_S)_{CO}}{2} + \frac{(R_S)_{COH}}{2} - \frac{r_o R_C}{M}} \right\}_{T=T_o} \tag{33}$$

and for circuit 2

$$(R_P)_{MCO} = \left\{ \frac{\frac{r_o R_C}{M}\left[R_T + \frac{(R_S)_{CO}}{2} + \frac{(R_S)_{COH}}{2}\right]}{R_T + \frac{(R_S)_{CO}}{2} + \frac{(R_S)_{COH}}{2} - \frac{r_o R_C}{2}} \right\}_{T=T_o} \tag{34}$$

In the case of circuit 3, there are several ways one can go about obtaining a modified constant output mode to improve the level of compensation. For example, one can average R_{S1}, R_{S2} and R_{P1} as determined from the constant overheat or the constant output modes. Knowing the equivalent resistance R_3, one can then obtain the modified value of R_{p2}.

Using the modified values of the series and parallel resistances in the compensation circuit, the analysis yields values of R_3 which are very close to those measured from the experiment, as determined in Figure 8. The success of applying this mode of compensation to Wollaston (platinum) hot-wires or to hot-films has not been verified by the present experiments, although we anticipate similar results. Experiments using Wollaston hot-wires in air (instead of the present tungsten wires) and similar tests utilizing cylindrical fiber hot-films in water will be conducted to examine the performance of the analysis for different modes of operation.

Except for the conditions discussed earlier, which must be met in the selection of the compensator probe, Figure 10 demonstrates that it is possible to choose compensators from a range of permissible resistances. This range depends on α_r, the ratio between the temperature coefficient of resistance of the compensator and that of the velocity sensor. The mode of operation and the initial overheat will also affect this range of permissible values of R_{To} for a particular velocity probe. As the initial overheat ratio increases so does the range of permissible R_{To}'s, when α_r is held constant. Similarly, if we hold the overheat ratio constant, increasing α_r will increase this permissible R_{To} range. Figure 10 also demonstrates that even values of $\alpha_r \leq 1$ would permit compensation in this mode of operation. Holding all other conditions the same, the constant overheat mode produced the smallest range of permissible R_{To}'s, while the constant resistance difference operation produced the widest range with the constant output and modified constant output modes somewhere in between.

Error Evaluation

The experimental results presented in the previous section can be used to determine the exact level of compensation that each operating mode can deliver, using typical compensating circuits and probes. If $R_{HE}(T)$ is the experimentally determined operating resistance which maintains the bridge output voltage constant, then the actual change in output voltage with ambient temperature using one of the compensating circuits is given by

$$\frac{E_b(T) - E_b(T_o)}{E_b(T_o)} = \left[\frac{R_1 + MR_3(T)}{R_1 + R_{HE}(T)}\right]\left[\frac{1 - \dfrac{R_C(T)}{MR_3(T)}}{1 - \dfrac{R_C(T)}{R_{HE}(T)}}\right]^{\frac{1}{2}} - 1 \quad (35)$$

with the aid of Equation (28), the percent change in the indicated velocity for the various operating modes, employing compensating circuit 2 and a typical compensator, was calculated and is shown in Figure 11. Using the modified constant output mode a - 2.5% error exists in the velocity for a 40°C rise in ambient air temperature. This error is reduced to -1% for a 20°C change in ambient temperature. It can be further reduced by employing circuit 3. If no compensation is employed, an error of -50% in velocity is found from the measurements for a 40°C rise in air temperature at an operating overheat ratio of 1.8. This value increases to -75% for an initial overheat of 1.4.

As shown in Figure 10, operation in the constant output mode undercompensates by approximately 14%. The error estimated by Equation (29) for the same conditions is 2%. The difference between the error estimate and its actual value originates from the assumption that the coefficients A, B and n in King's law are independent of temperature. As indicated earlier, some typical temperature dependence of the coefficient A may be used to achieve more realistic error estimates by the analysis.

Conclusions

From the analysis of the proposed scheme for compensation of the anemometer output for ambient temperature variations, it was determined that:

1. One needs only to know the properties of the anemometer bridge and the velocity and temperature probes (i.e., their resistance and its temperature dependence coefficients) in order to achieve the compensation, without the need for temperature calibration. Hence, the scheme provides means for incorporating the temperature compensation a priori to conducting the experiments.

2. Solutions for the series and parallel resistances of the three proposed compensating circuits were obtained for four different operating modes; i.e., constant overheat ratio, constant overheat difference, constant output voltage and modified constant output voltage.

3. Constraints on the parameters of the temperature compensating probe, valid for all three circuit configurations, were derived. With this and the knowledge of the properties of available velocity sensors, operating overheat ratios, anemometer bridge ratio and operating mode, a suitable temperature compensating probe may either be fabricated or purchased.

4. Estimates of the errors introduced by the non-ideal behavior of the compensating circuits have been derived. The results suggest that when $R_3(T)$ is nearly linear, the error in the indicated velocity is proportional to the fourth power of the temperature variation. However, for typical temperature variations, i.e., less than 20°C, this error is approximately proportional to the temperature change. The estimated errors also depend on the circuit configuration utilized; circuit 3 being the optimum.

From the experiments on temperature compensation for tungsten hot-wires in air, the following conclusions were drawn:

1. The effective temperature compensation of all operating modes was found to be independent of fluid velocity and initial overheat ratio.

2. The coefficients A, B and n in King's law were found to vary slightly with air temperature. In view of the last conclusion, the variation in B and n can be ignored without introducing appreciable errors to the analysis.

3. Operating at an initial overheat of 1.8 and holding the velocity constant, the comparison between experiment and analysis reveals that for an air temperature rise of 40°C, which leads to a -50% error in the indicated velocity with no compensation, the following performance is achieved by the analysis, when utilizing circuit 2 and a typical compensator: the constant overheat mode overcompensates by 12%; the constant resistance difference mode undercompensates by 25%; the constant output mode undercompensates by 15%; and the modified constant output mode undercompensates by only 2.5%. The latter is considered very good in view of the simplicity of the technique and the large errors encountered with no compensation, i.e., 50%.

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Notation

Symbol	Definition
A, B	Coefficients in King's law
A',B'	Hot-Wire Velocity Calibration Constants
E_b	Anemometer Bridge Output Voltage
I	Integral Related to Non-ideal Behavior of Temperature Compensating Circuits
I_P	Current Passing Through Velocity Sensor
M	Bridge Ratio
n	Exponent in Probe Velocity Calibration and King's law
P_r	Parallel resistance ratio = R_P/R_T
R_C	Resistance of Velocity Sensor
R_{Co}	Resistance of Velocity Sensor at Reference Temperature T_o
R_H	Operating Resistance of Velocity Sensor
R_{HE}	Experimentally determined Operating Resistance of Velocity Sensor which Maintains Anemometer Output Voltage Constant as Ambient Temperature Changes
R_P, R_{P1}, R_{P2}	Parallel Resistances in Temperature Compensating Circuits
$(R_P)_{CO}$, $(R_S)_{CO}$	Parallel and Series Resistances Determined from Constant Output Mode of Operation
$(R_P)_{COH}$, $(R_S)_{COH}$	Parallel and Series Resistances Determined from Constant Overheat Mode of Operation
$(R_P)_{MCO}$, $(R_S)_{MCO}$	Parallel and Series Resistances Determined from Modified Constant Output Mode of Operation
R_S, R_{S1}, R_{S2}	Series Resistances in Temperature Compensating Circuits
R_T	Resistance of Temperature Compensator
R_{To}	Resistance of Temperature Compensator at Reference Temperature T_o

Symbol	Definition
R_1, R_2	Fixed Resistances in Anemometer Bridge
R_3	Equivalent Resistances of Temperature Compensating Circuits
r_o	Overheat Ratio = R_H/R_C
S_r	Series Resistance Ratio = R_S/R_T
T	Ambient Temperature
T_o	Ambient Reference Temperature; Usually Room Temperature
U	Time-Mean Velocity Normal to Sensor
α_C	Temperature Coefficient of Velocity Sensor
α_r	Ratio of Temperature Coefficients for Compensator and Velocity Sensor = α_T/α_C
α_T	Temperature Coefficients of Compensator Probe
β	Probes Resistance Ratio = R_T/R_C
β_o	Probes Resistance Ratio at Reference Temperature T_o
η	General Temperature Function defined by Equation (15)
τ	Resistive Function for Constant Bridge and Linearized Output Voltages defined by Equation (16)

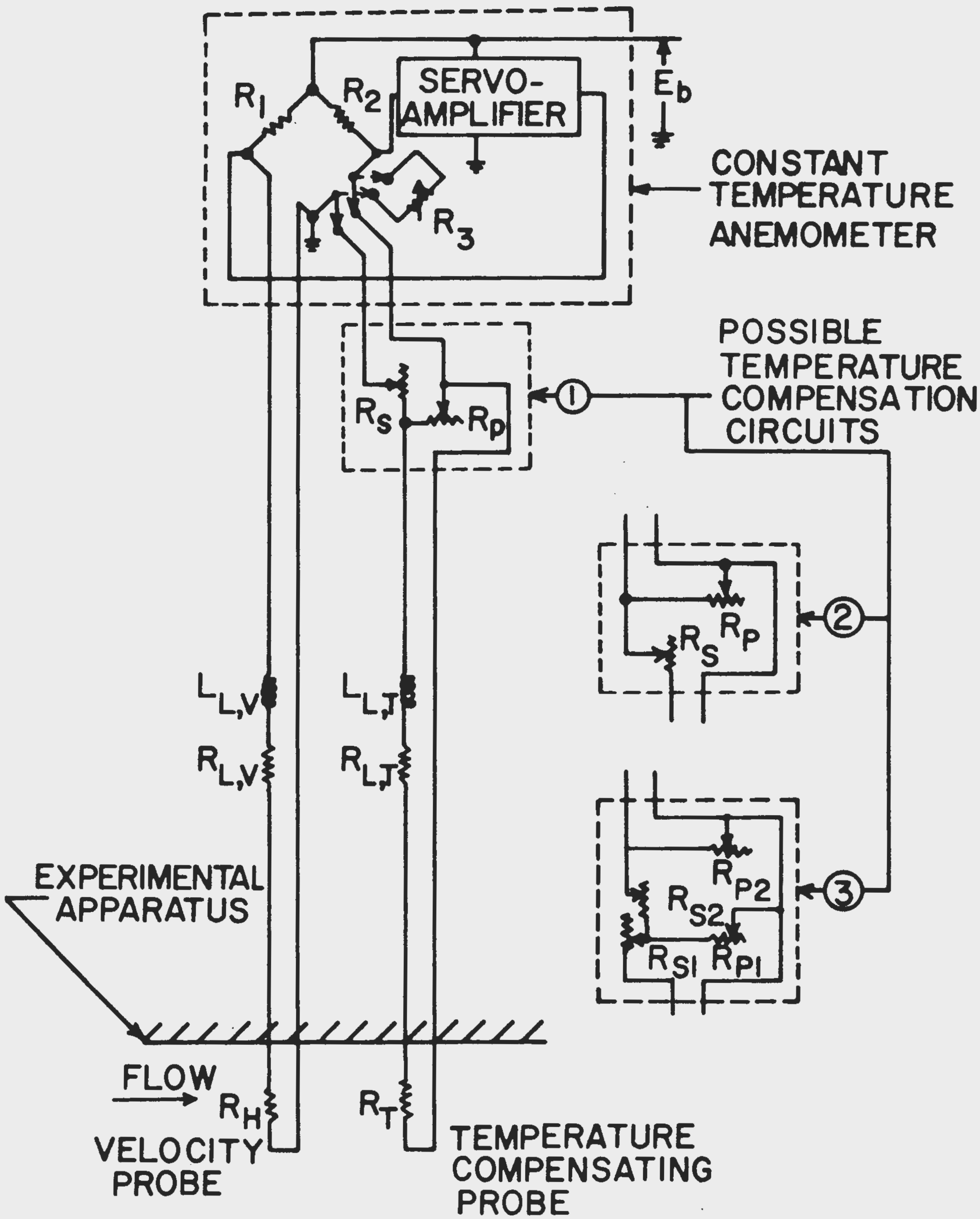

Figure 1. Schematic of Circuits Utilized in Schemes to Compensate for Ambient Temperature Variations

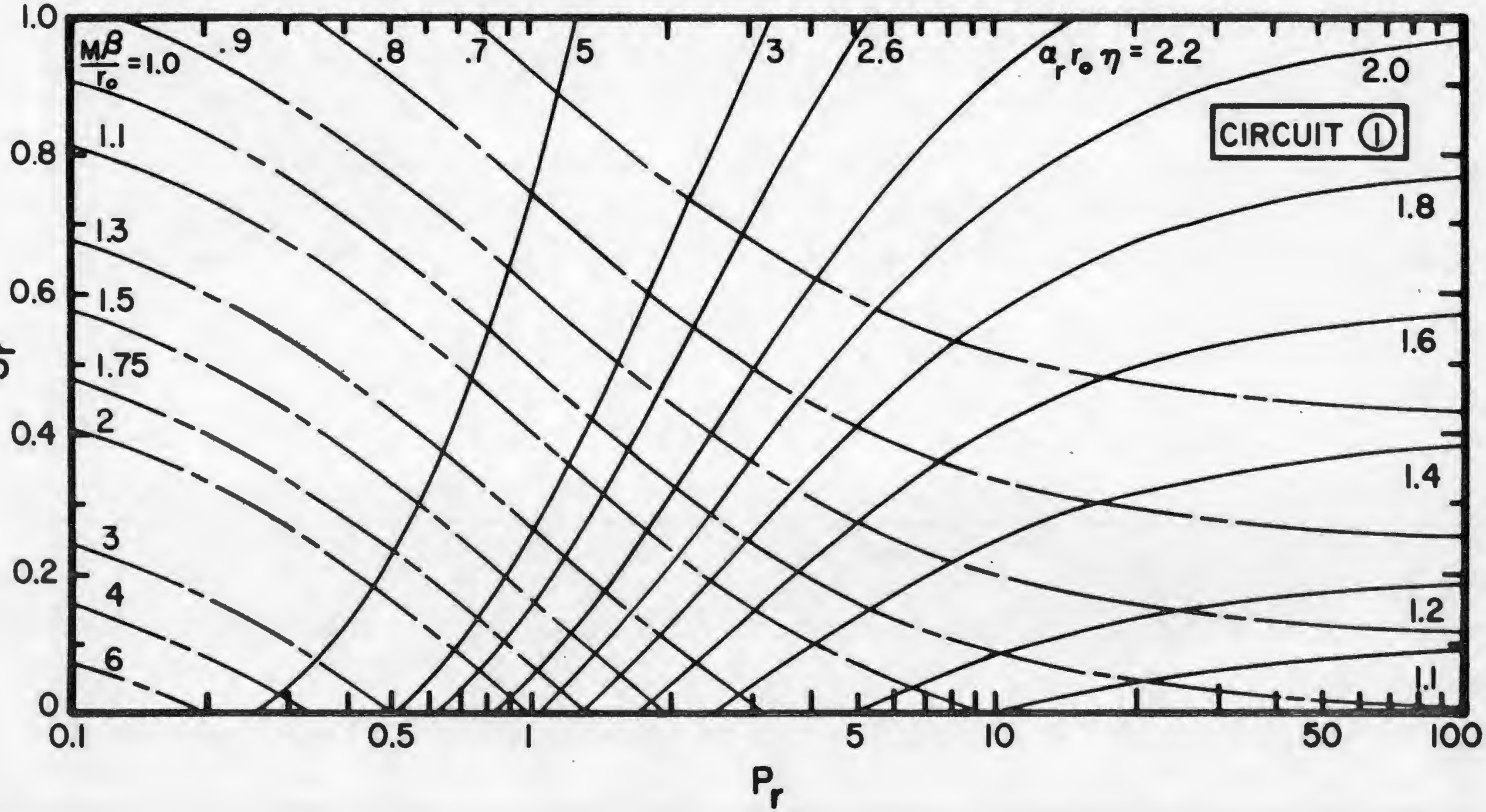

Figure 2. Non-dimensional Charts for Selecting Series and Parallel Resistance of Compensating Circuit "1"

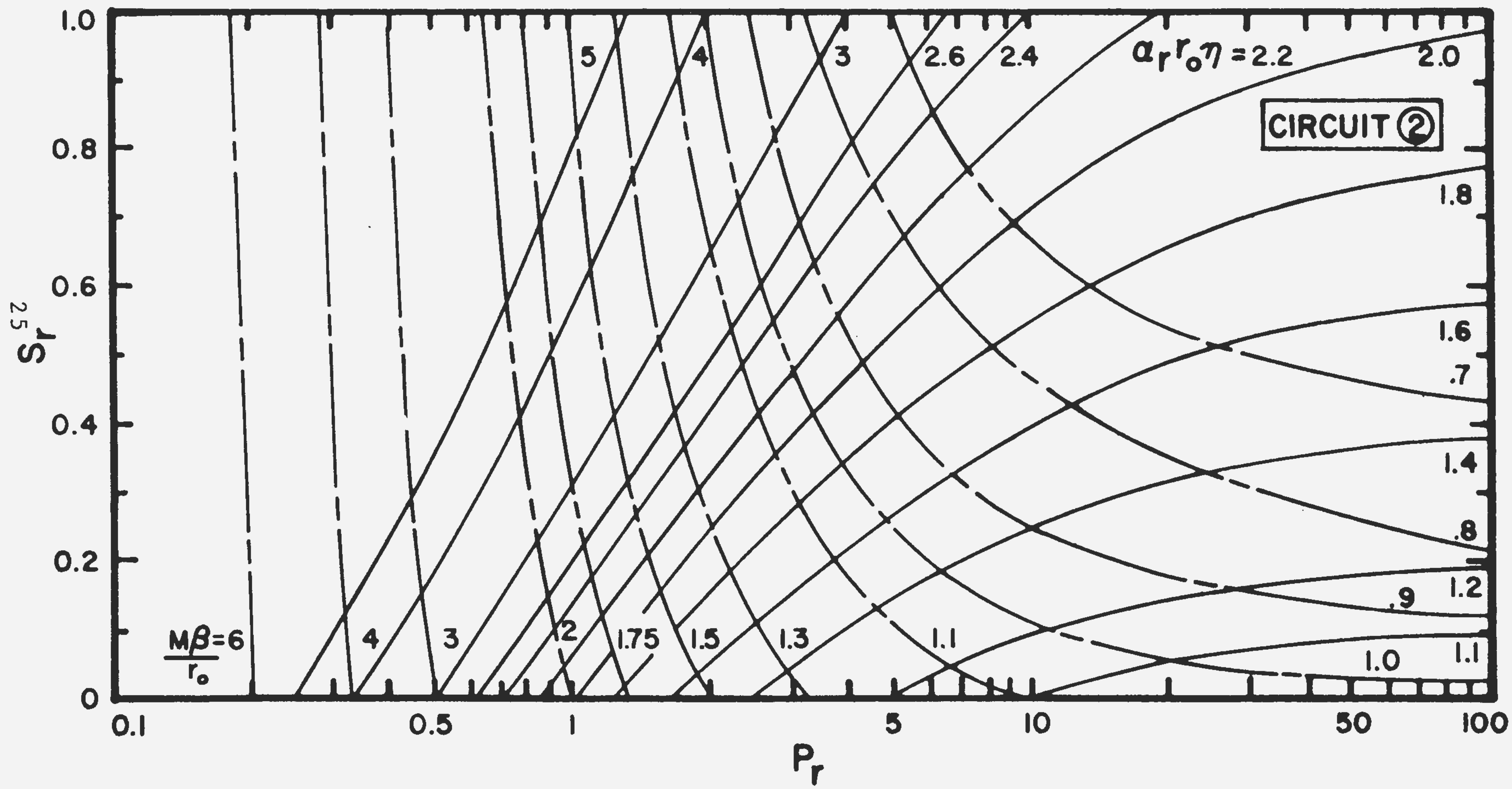

Figure 3. Non-dimensional charts for selecting parallel and series resistance of Compensating Circuit "2"

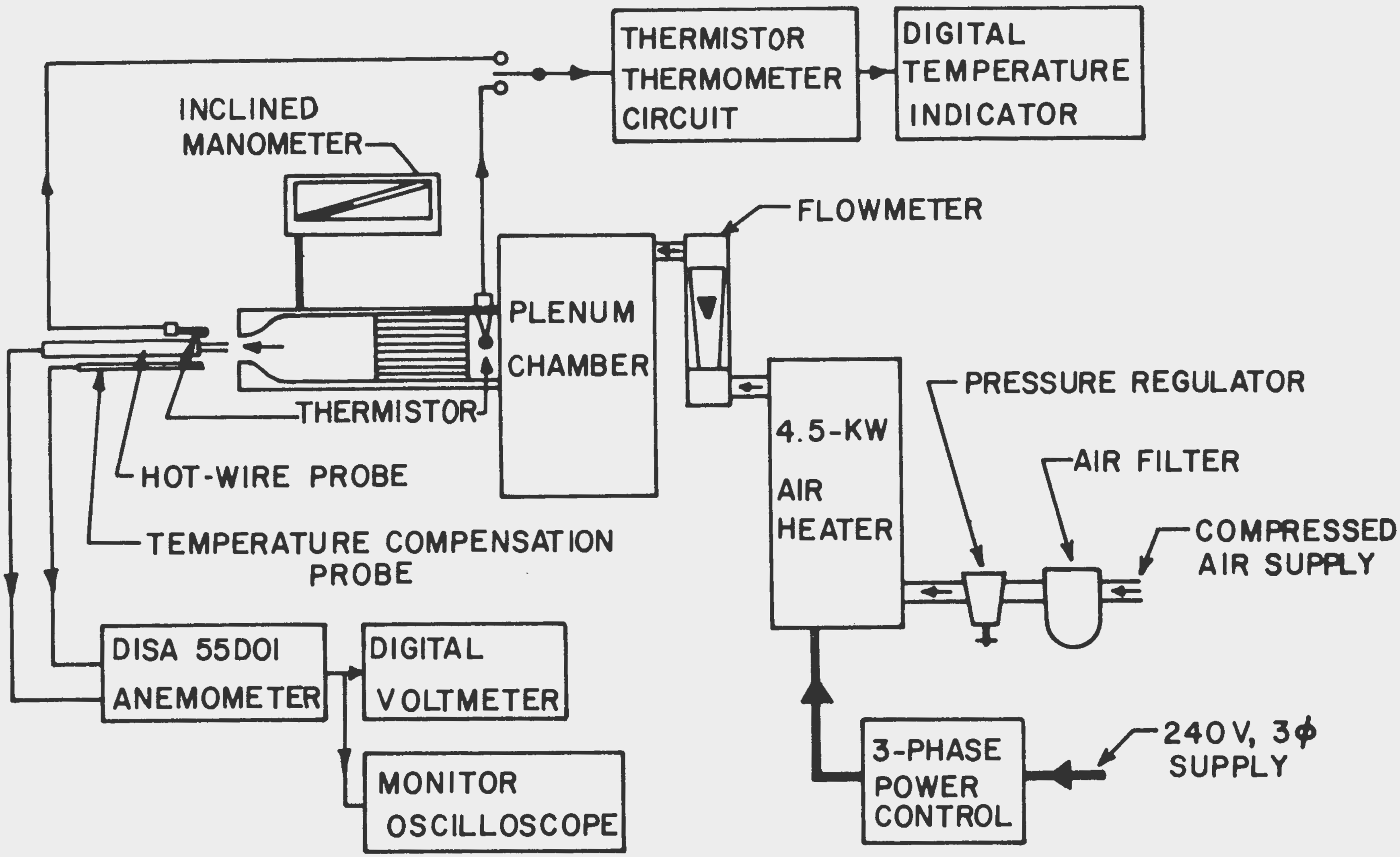

Figure 4. Schematic of Experimental Facility for Probe Calibration in Air at Different Ambient Temperatures

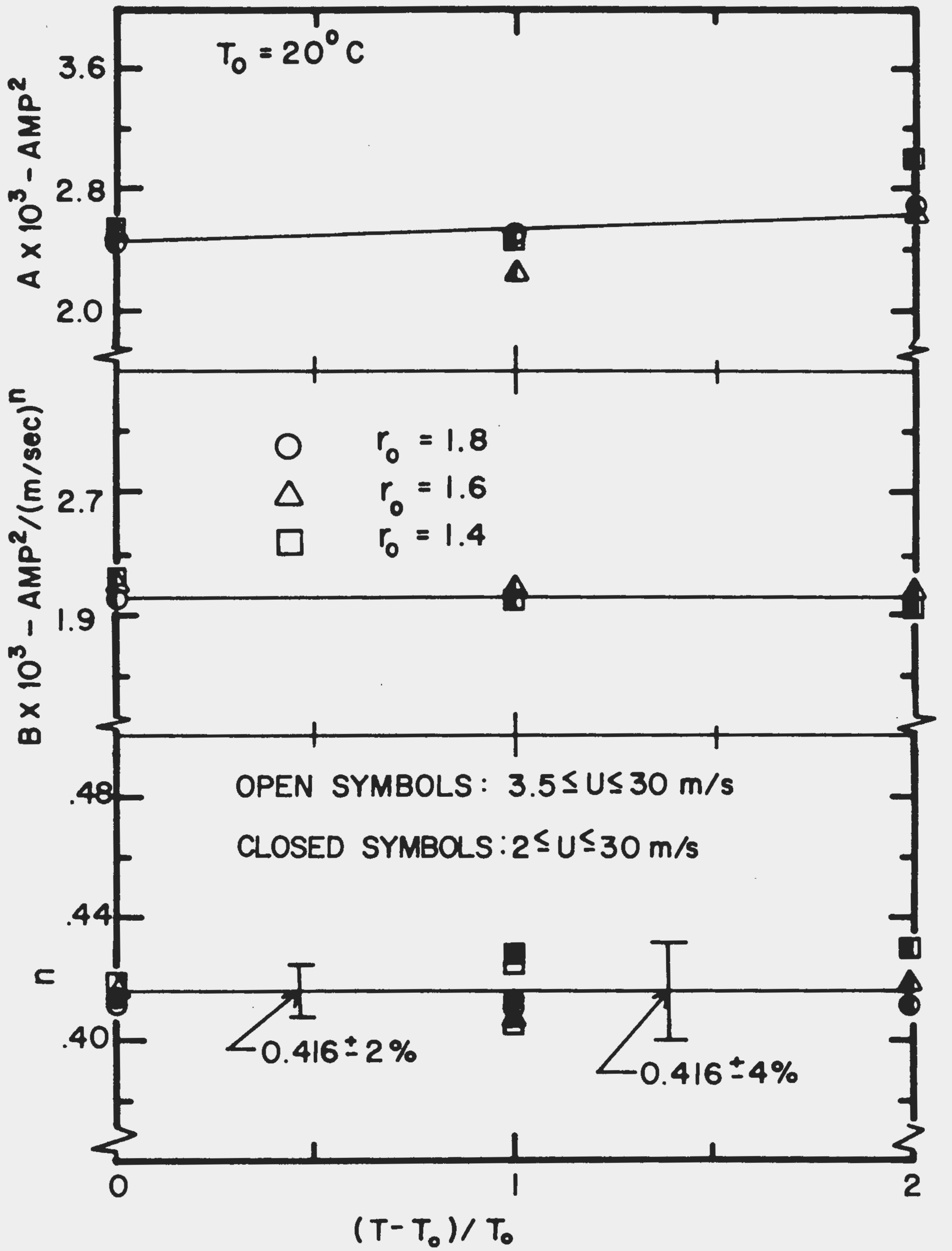

Figure 5. Temperature Dependence of King's Law Coefficients for Different Overheat Ratios

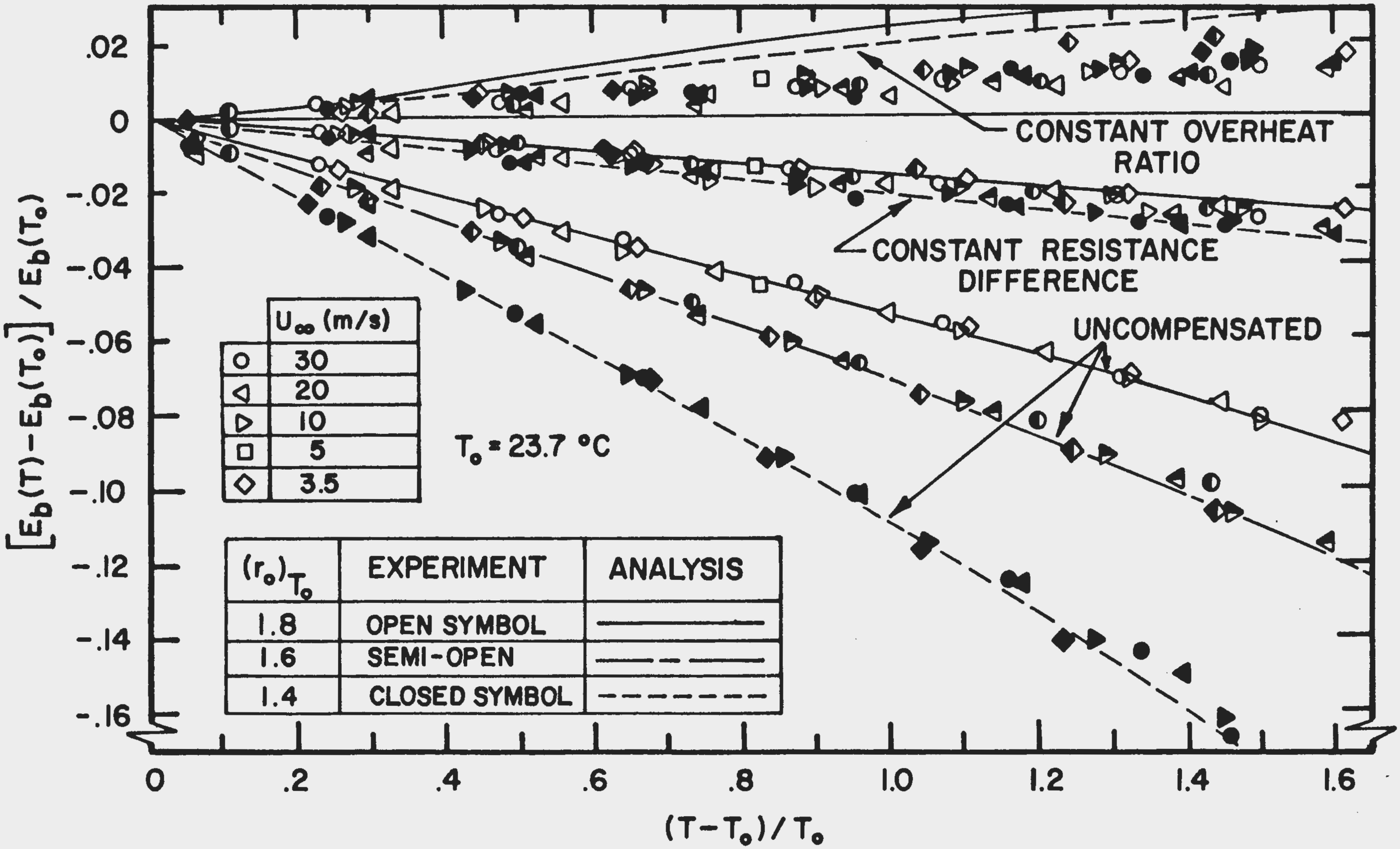

Figure 6. Non-dimensional Anemometer Output Operated in Constant Overheat Ratio, Constant Resistance Difference, and Uncompensated Modes at Various Overheat Ratios and Velocities

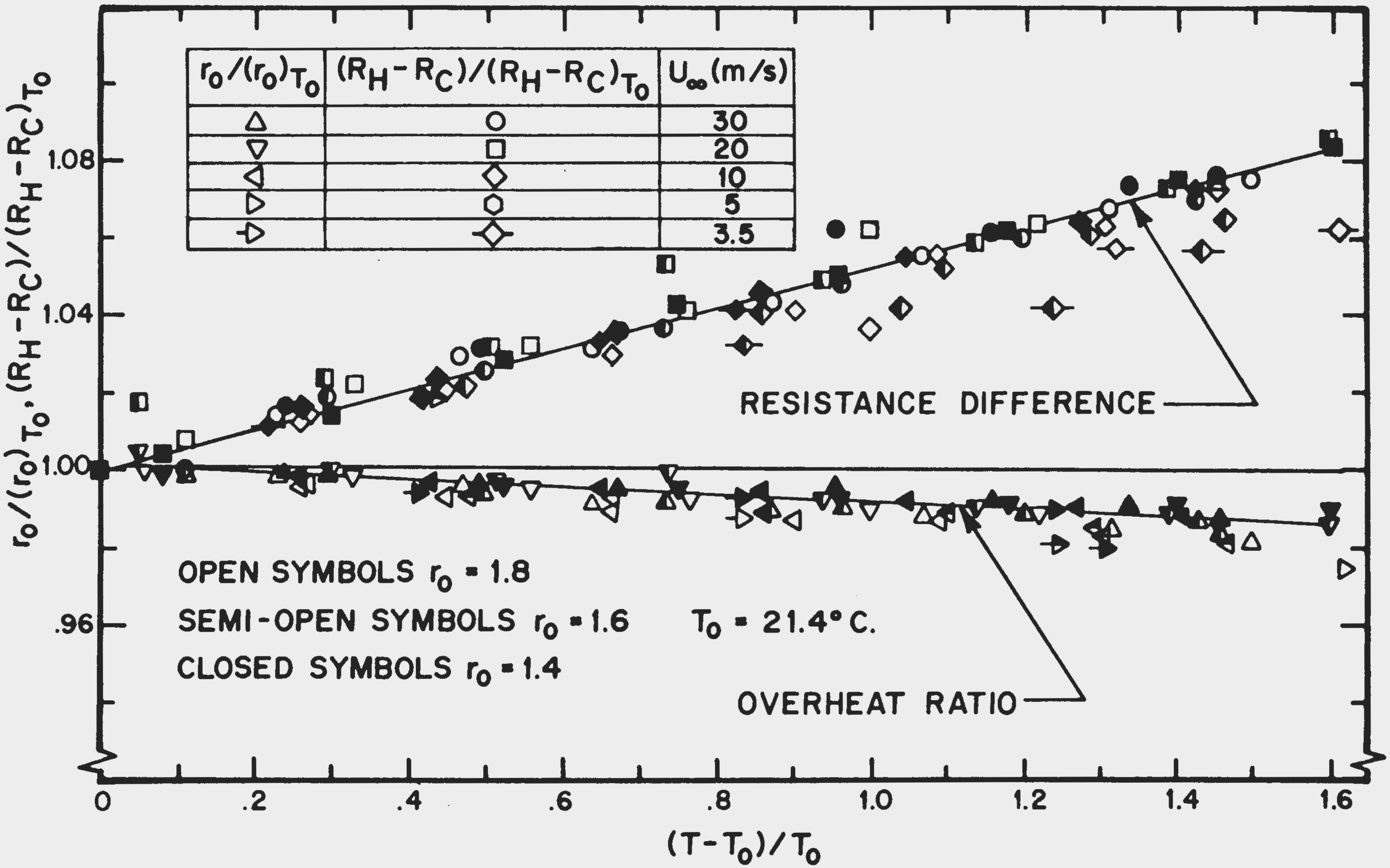

Figure 7. Variation of Overheat Ratio and Resistance Difference for Constant Output Operation with Air Temperature at Several Fixed Velocities

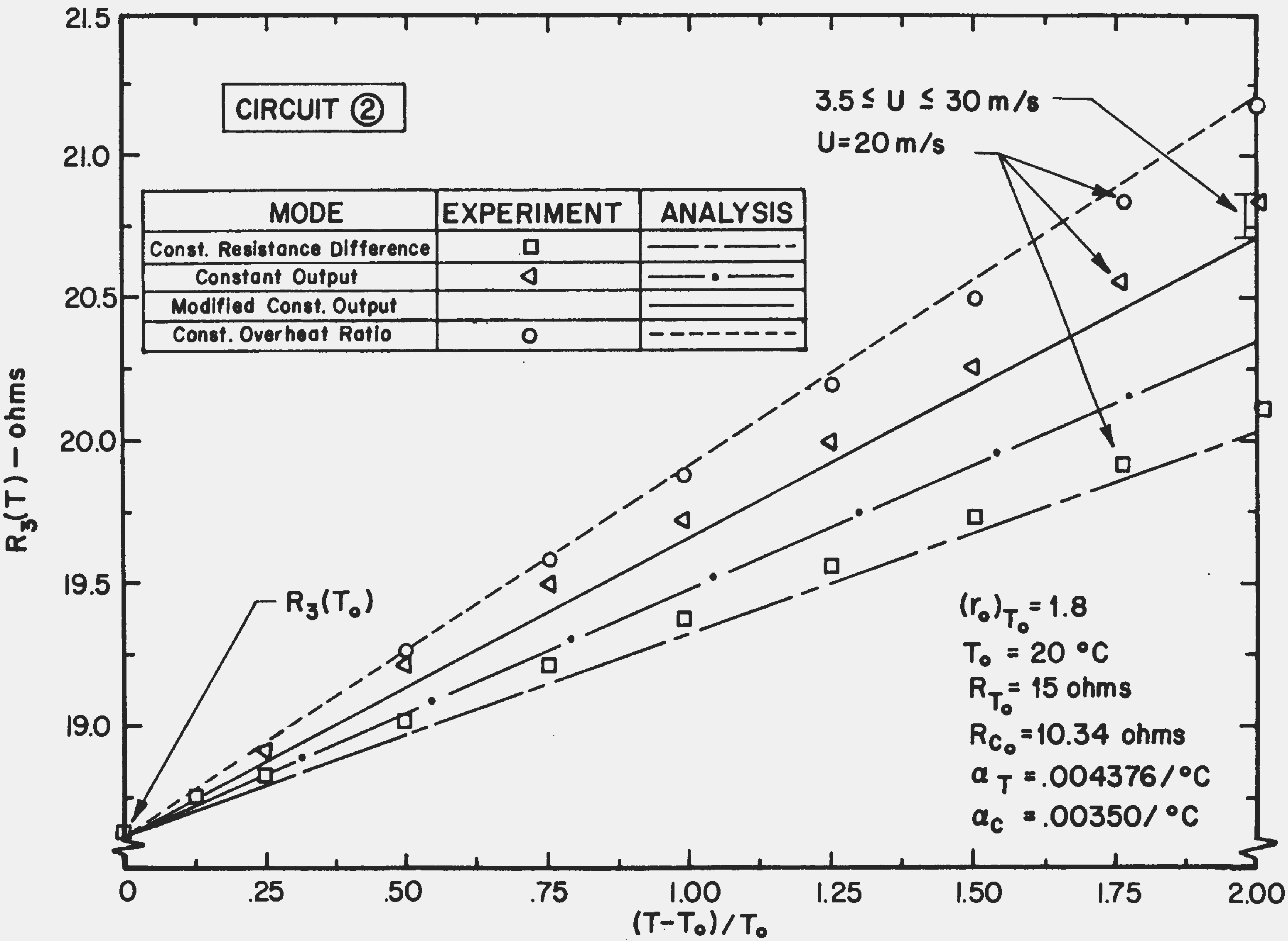

Figure 8. Variation of R_3 with Temperature for Constant Resistance Difference, Constant Output, Modified Constant Output, and Constant Overheat Ratio Modes of Operation Utilizing Compensating Circuit "2" and a Typical Compensator

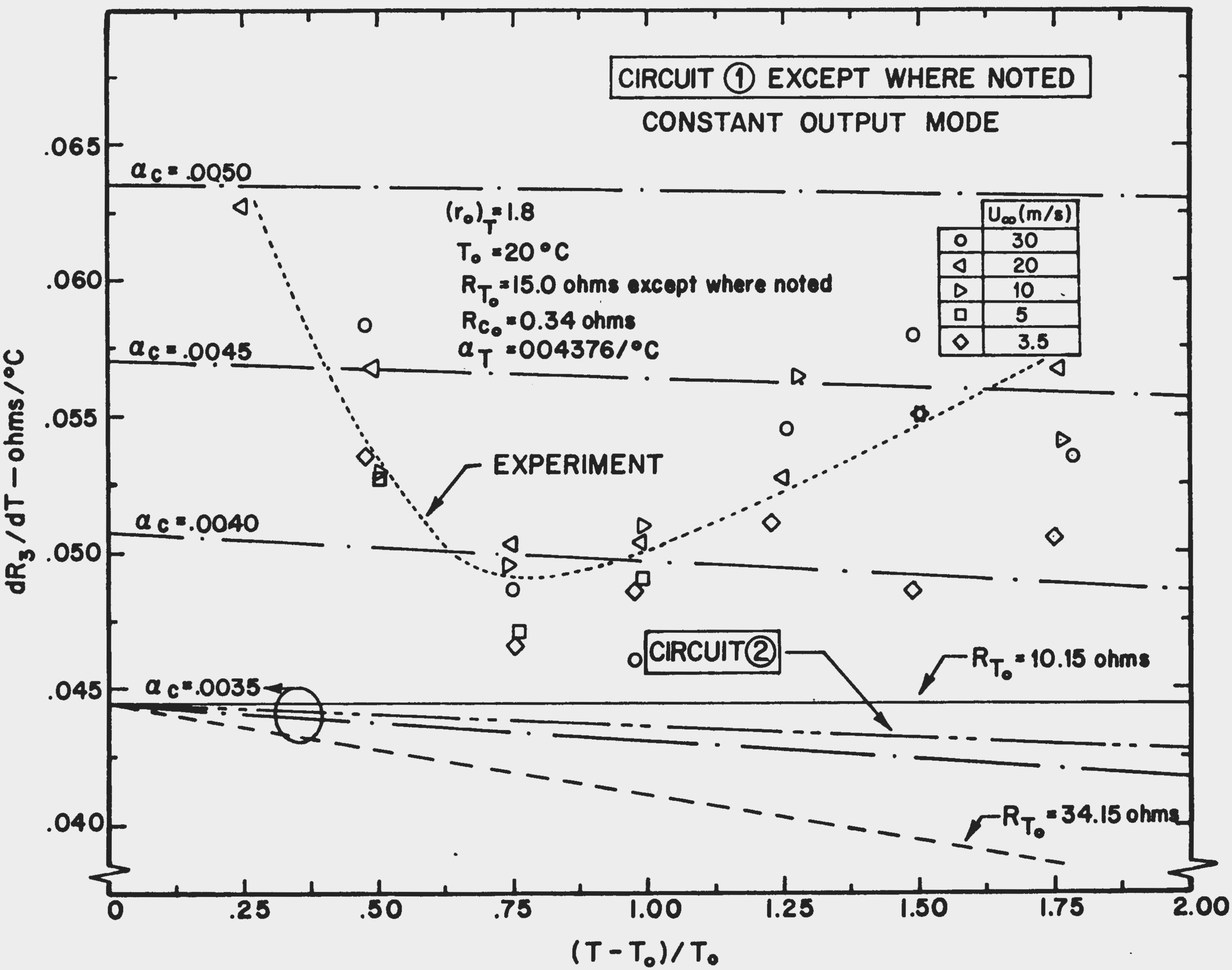

Figure 9. Sensitivity of dR3/dT to Ambient Temperature, Velocity Sensor Temperature Coefficient and Initial Compensator Resistance

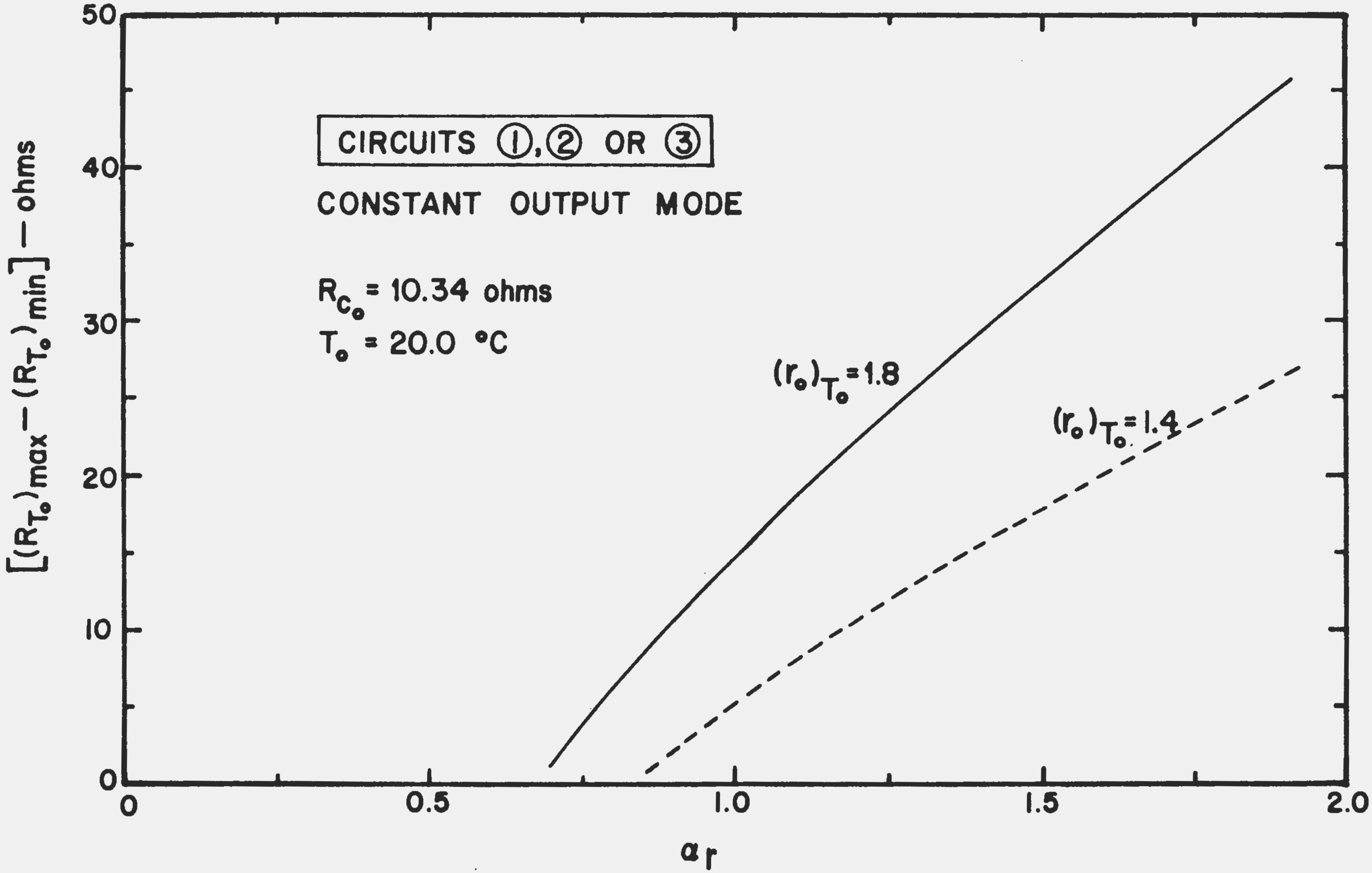

Figure 10. Effect of Initial Overheat Ratio on Allowable Resistance Range of Compensator Probe for a Typical Velocity Sensor as a Function of α_r

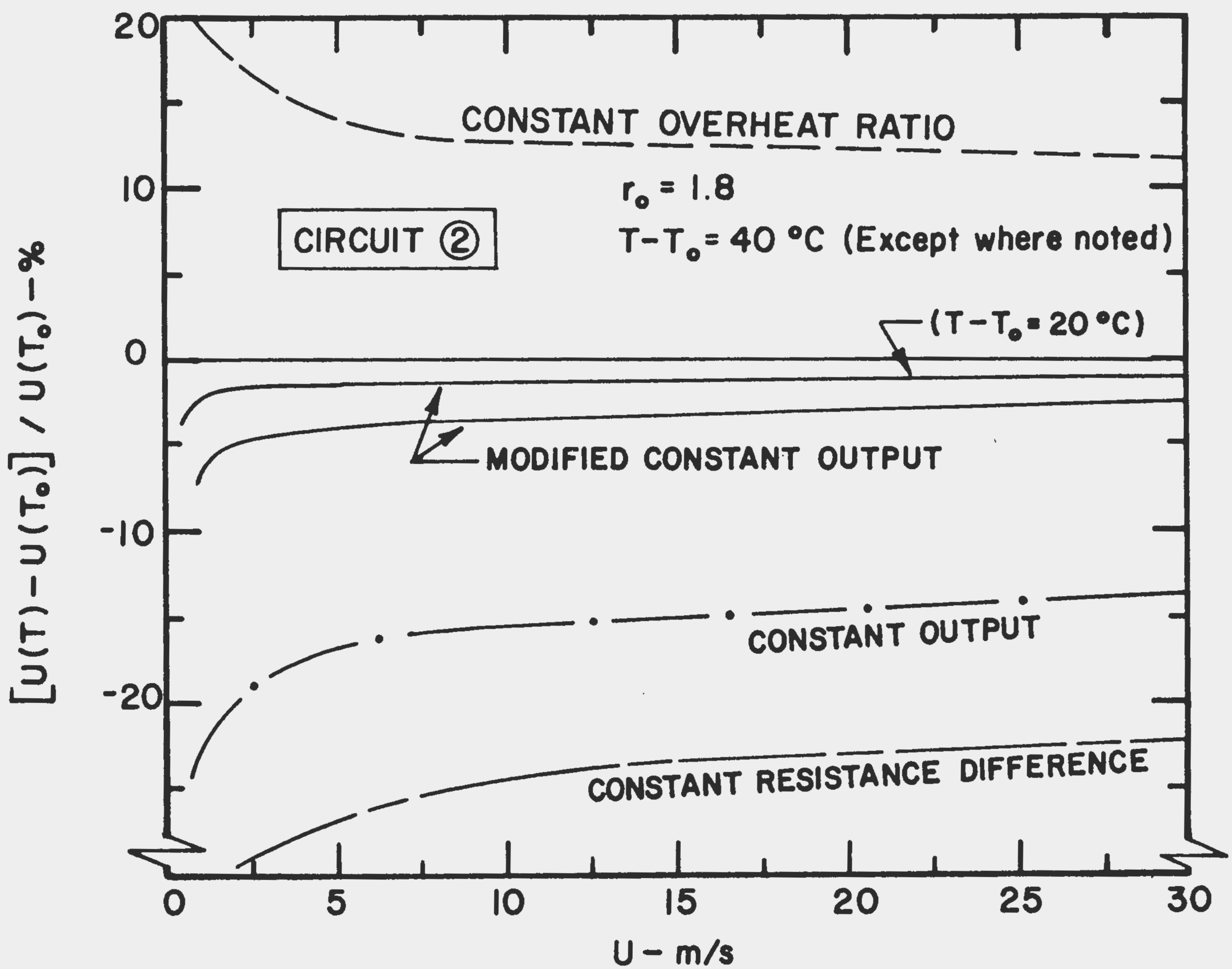

Figure 11. Non-dimensional Velocity Error for Various Modes of Operation as a Function of Free-Stream Velocity Using Compensating Circuit "2" and a Typical Compensator

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
A scheme utilizing a temperature probe immersed in the working fluid to compensate for the dependence of hot-wire (or hot-film) velocity calibration on ambient temperature variations is analyzed. To achieve compensation, only properties of the anemometer bridge, the velocity and temperature probes need to be known. The scheme provides means for incorporating the temperature compensation a priori to conducting the experiments, without the need for temperature calibration. Experiments were conducted, using tungsten hot-wires
(continued)

20. ABSTRACT (Continued)

in air, to check the performance of the analysis and to verify some of the assumptions that were made. For typical probes, with an ambient temperature increase of 20°C, maximum error in the indicated velocity of 1% or less can be achieved.