COMPUTER-DISPLAY FEASIBILITY STUDY FOR FLIGHT PERFORMANCE OPTIMIZATION

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FOREWORD

The research project outlined in this report was initiated 2 January 1968 by the Control Systems Research Branch, Flight Dynamics Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio. This report was prepared by Polhemus Associates Incorporated, Ann Arbor, Michigan, under USAF Contract F33615–68–C–1233. The contract was initiated under Project No. 6190, "Control Display," Task No. 619008, "Technology for Information Presentation," with Mr. John H. Kearns, Project Engineer. Mr. J. J. Moelker representing FDCR provided the technical supervision while Mr. Eldon Bobbett, also of FDCR, was the Task Engineer.

This report was prepared by Mr. D. F. Dunlap of Polhemus Associates, Inc. Included are conclusions resulting from analytical work performed by Mr. C. B. Jeffery and Mr. L. V. Ursel. Mr. W. L. Polhemus also contributed to the study.

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This report covers work conducted during the period January 1968 through July 1968. The report was submitted by the authors in July 1968.

This technical report has been reviewed and is approved.

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ABSTRACT

An airborne computer-display system is developed for the purpose of flying vertical plane optimum flight paths. System requirements are specified by first examining mission requirements, then operational requirements, and finally state-of-the-art constraints. The system is designed to provide command information for flying minimum time and fuel transition maneuvers, and maximum range and endurance cruise maneuvers. Optimal command data is derived, in the computer, from a series of stored profiles which reflect both the aircraft drag configuration and ambient weather conditions. System feasibility is demonstrated through a system description which consists of state-of-the-art computer and display equipments. A digital computer is shown to be more accurate, compact and flexible than a comparable analog system. Displays consisting of an attitude director indicator, a flight profile indicator, and Mach number and altitude indicators are suggested for presenting command information. Final system size and weight are less than one cubic foot and fifty pounds, respectively. As a convenience only, the study system is configured around F-104C performance criteria.

(Distribution of this Abstract is unlimited.)

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LIST OF SYMBOLS

Symbol	Definition
A	Moment of inertia about aircraft roll axis
a_t	Tail lift-curve slope
B	Moment of inertia about aircraft pitch axis
C	Moment of inertia about aircraft yaw axis
$\bar{c}$	Length of mean aerodynamic chord
C_D	Drag coefficient
C_{D_i}	Induced drag coefficient
C_{D_o}	Zero lift drag coefficient
C_{D_α}	$\frac{\partial C_D}{\partial \alpha}$
$\bar{c}_e$	Length of mean elevator chord
$C_{he_{\alpha_t}}$	$\frac{\partial C_{he}}{\partial \alpha_t}$
$C_{h\delta}$	$\frac{\partial C_{he}}{\partial \delta}$
C_L	Lift coefficient
C_{L_α}	Lift curve slope
C_{L_q}	$\frac{\partial C_L}{\partial q}$
C_m	Pitching moment coefficient

C_{m_q}	Pitch damping coefficient
C_{m_α}	Slope of pitching moment curve
$C_{m_{\cancel{q}\delta}}$	$\frac{\partial C_m}{\partial \delta}$
D	Pitch-yaw product of inertia. Also aerodynamic drag force.
d	Displacement from optimum flight path
E	Roll-yaw product of inertia
e	Oswald's efficiency factor. Also mass eccentricity of elevator surface
E_s	Specific energy $= h + \frac{V^2}{2g}$
F	Roll-pitch product of inertia
F_e	Elevator control force
F_x	Resultant of external forces acting in the aircraft longitudinal direction.
F_z	Resultant of external forces acting in plane of symmetry normal to F_x.
g	Gravitational constant
H_e	Aerodynamic elevator hinge moment
h	Altitude
h_x, h_y, h_z	Scalar components of angular momentum
I	Elevator moment of inertia
L	Aerodynamic lift force
l_t	Distance between aircraft center of gravity and aerodynamic center of tail

M	Mach number. Also resulting of moments about the aircraft pitch axis.
M_o	Mach number associated with optimum altitude-Mach number curve.
m	Aircraft mass
n	Aircraft load factor
N	Throttle setting
P	Aircraft rotation rate about roll axis
P_e	Elevator product of inertia
P_s	Aircraft optimization parameter ! Energy [illegible]
Q	Aircraft rotation rate about pitch axis
q	Dynamic pressure. Also pitch axis perturbation angular rate.
R	Aircraft rotation rate about yaw axis
r	Range
S	Aircraft wing area
s	Laplace operator
S_e	Area of elevator aft of hinge line.
U	Component of aircraft velocity in longitudinal direction
V	Component of aircraft velocity in plane of symmetry normal to U. Also total aircraft velocity.
V_H	Horizontal tail volume
W	Component of aircraft velocity normal to U, and V.
w	Aircraft weight. Also perturbation velocity in W direction.
$\dot{w}_F$	Fuel flow rate

w_v	Rotation rate of i coordinate system
AR	Aspect ratio
α	Angle of attack
γ	Flight path elevation angle
δ	Elevator deflection angle
ϵ	Downwash angle
θ	Aircraft pitch angle
θ_o	Pitch angle associated with optimum altitude - Mach number curve
ρ	Ambient air density
Σ_c	Computational error
Σ_I	Input error
Σ_o	Output error

Subscripts

o	Initial point. Also reference flight condition
1	Final point

Other

$(\dot{\ })$	Differentiation with respect to time

SECTION I

INTRODUCTION

Flight performance optimization has become an increasingly important factor in the margin between success and failure in airborne missions. With the coming generation of Mach 2.5 plus aircraft the three or four seconds one saves in an optimal maneuver can make the difference between a successful intercept and the loss of a defended target. Through the years, the benefits achieved through flight optimization have gained increasing recognition, and have led to a large volume of published material dealing with the subject. The objective of the present study is to apply this knowledge in the development of a flight path optimization system for the F-104C, a typical operational fighter.

As the performance levels of modern day fighter aircraft have increased, the margin of difference between competing aircraft has also become smaller. Such things as maintaining energy, turning rate, and fuel advantage during air-to-air combat are mainly a function of pilot knowledge and judgement. This is why it is important that careful consideration be given to the problem of preparation for combat. An initial advantage can make the vital difference. An understanding of the importance of optimizing a climb-out procedure, in terms of saving fuel, can be obtained from Figure 1. This is a plot showing the percentage of the total fuel used during the climb phase versus the specific energy level at the end of the climb. Since most fighter aircraft have combat times on the order of three to five minutes, it is clear that any fuel saved by means of applying an optimization technique can be of the utmost importance.

The purpose of the study was twofold. First, the status of flight path optimization theory was to be reviewed with the aim of determining where gaps in the theory exist as related to operational use. Second, recognizing the existence of gaps, specifications for an optimum flight path command system were to be developed. To this end a brief historical review of flight path optimization is given in Section II. Section III contains a survey of mission considerations which ultimately lead to the need for considering various kinds of optimized flight. The technique for arriving at operational requirements and those operational requirements which specifically arise for the F-104C is described in Section IV. Section V contains a discussion of system flight path and path tracking requirements analysis, while Sections VI and VII concern system display and computer requirements, respectively. Section VIII is a discussion of interface requirements. System specifications are given in Section IX and the conclusion follows in Section X.

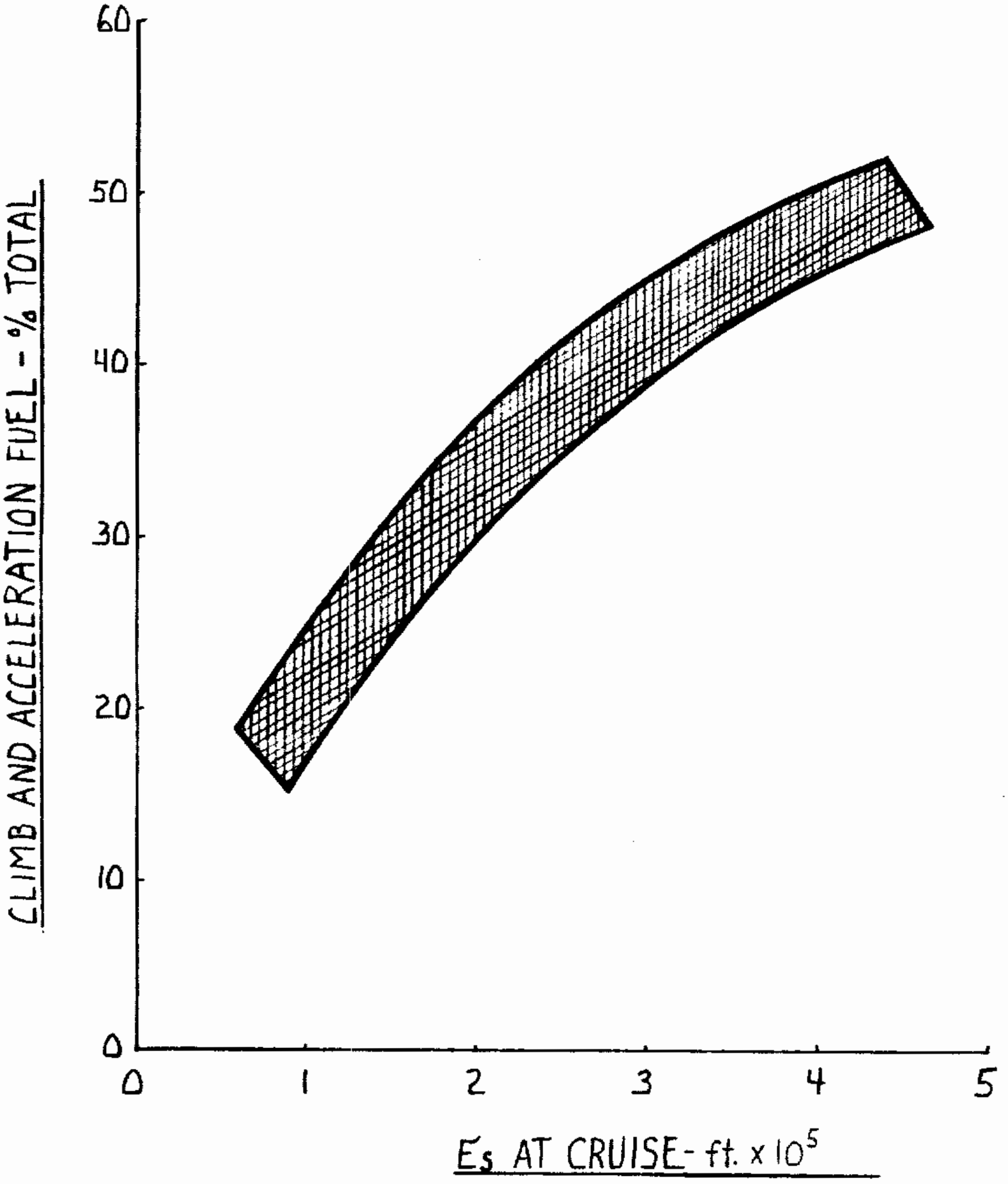


FIGURE 1 - FUEL FOR CLIMB AND ACCELERATION TO CRUISE
VS.
SPECIFIC ENERGY LEVEL

SECTION II

FLIGHT PATH OPTIMIZATION - HISTORICAL DEVELOPMENT

Aircraft performance optimization has undergone a period of revolution in the last ten to fifteen years, with the development of variational techniques as a means of analysis. This is most evident in the treatment of optimal climb strategies. In the classical treatment of minimum time to climb problems, it was generally assumed that maximum rate of climb occurred along the contour of maximum specific power [1]. This result was derived by assuming that accelerations along the flight path are negligible. As higher performance aircraft became available, this assumption led to results which were less and less accurate. Rate of climb corrections for higher performance aircraft were first suggested by Otten [2] and Phillips [3] in the early 1940's. Even with these corrections, however, the solution to minimum-time-to-climb problems, where a portion of the flight is supersonic, was essentially intractable. These problems were not effectively treated until the methods of the calculus of variations were brought to bear. Evidently the need for this approach was first realized by the Germans toward the end of World War II. A. Lippisch [4], designer of the ME-163, found that the high performance and limited endurance of this rocket-powered aircraft demanded a more accurate assessment of climb strategies than could be obtained with methods which were in common use at the time. Lush [5] and Rutowski [6] are generally given credit for extending this early work to a level where it could be applied to a good degree of accuracy by the practicing engineer. The generalization of optimum flight performance to cover such problems as optimum inclination of the thrust axis, maximum range at a given altitude, maximum endurance at a given altitude, maximum range for a given thrust control parameter, minimum fuel consumption climb to a given altitude, and many others, has probably reached its greatest level of differentiation at the hands of Miele [7]. Although presently inactive in the field, Miele has published more than twenty papers on the theory of flight path optimization since 1955.

Solution of flight path optimization problems, except in the more simplified cases, generally requires the use of digital computers. Two techniques have reached a degree of prominence over the years. These are the gradient methods and the method which involves the integration of the Euler-Lagrange equations. The first successful gradient programs were developed by Bryson [8] and Kelley [9]. With this technique, an optimum flight path is determined by comparison of an existing trajectory with its predecessors. The method can be made as exact as the model of the aircraft which is being simulated. In general, however, solutions are slow to converge, and oftentimes converge on a local minimum rather than the true minimum.

Integration of the Euler-Lagrange equations has been accomplished by Heerman [10] and Vincent [11]. The results are generally in the form of a flooded region of trajectories where some refinement is required to arrive at the desired solution. Programs of this type are characterized by instabilities and extreme sensitivity to particular parameters.

In general, existing optimization programs, while highly accurate, are far too slow and much too large for consideration as on-line, on-board, real time solution tools. Much research is being directed to improving these drawbacks, however. This will be commented upon further in Section V. For the present, then, on-board trajectory profiles must be determined from previously derived ground analyses.

SECTION III

MISSION CONSIDERATIONS

This section contains a review of various types of aircraft combat missions. This is done for the purpose of correlating missions with required optimum maneuvers.

Table I lists the missions and the associated optimal maneuvers which are required in each case. A number key to each of the optimal maneuvers is given as follows:

TABLE ~~II~~ I

MANEUVER NUMBER KEY

1. Minimum Time Climb
2. Minimum Time Altitude-Range Transition
3. Minimum Time Energy State Transition
4. Minimum Fuel Climb
5. Minimum Fuel Altitude-Range Transition
6. Minimum Fuel Energy State Transition
7. Maximum Range Climb
8. Maximum Range Cruise
9. Maximum Endurance Climb
10. Maximum Endurance Cruise
11. Range-Time-Altitude Rendezvous
12. Zoom Climb to Launch

In general most of the remarks in Table I are relatively straightforward. The only area which might be commented upon involves the need for the range-time-altitude rendezvous and the zoom-climb maneuvers. In order to understand the requirement for a range-time-altitude rendezvous, consider Figure 2. This is a plot of range versus time for both an incoming enemy aircraft and an outgoing interceptor. The plot of the enemy aircraft is shown from the point of initial detection until the crossing of the interceptor range-time envelope. It is assumed that:

a. The enemy aircraft is attempting to reach an objective which is close to the interceptor airfield.

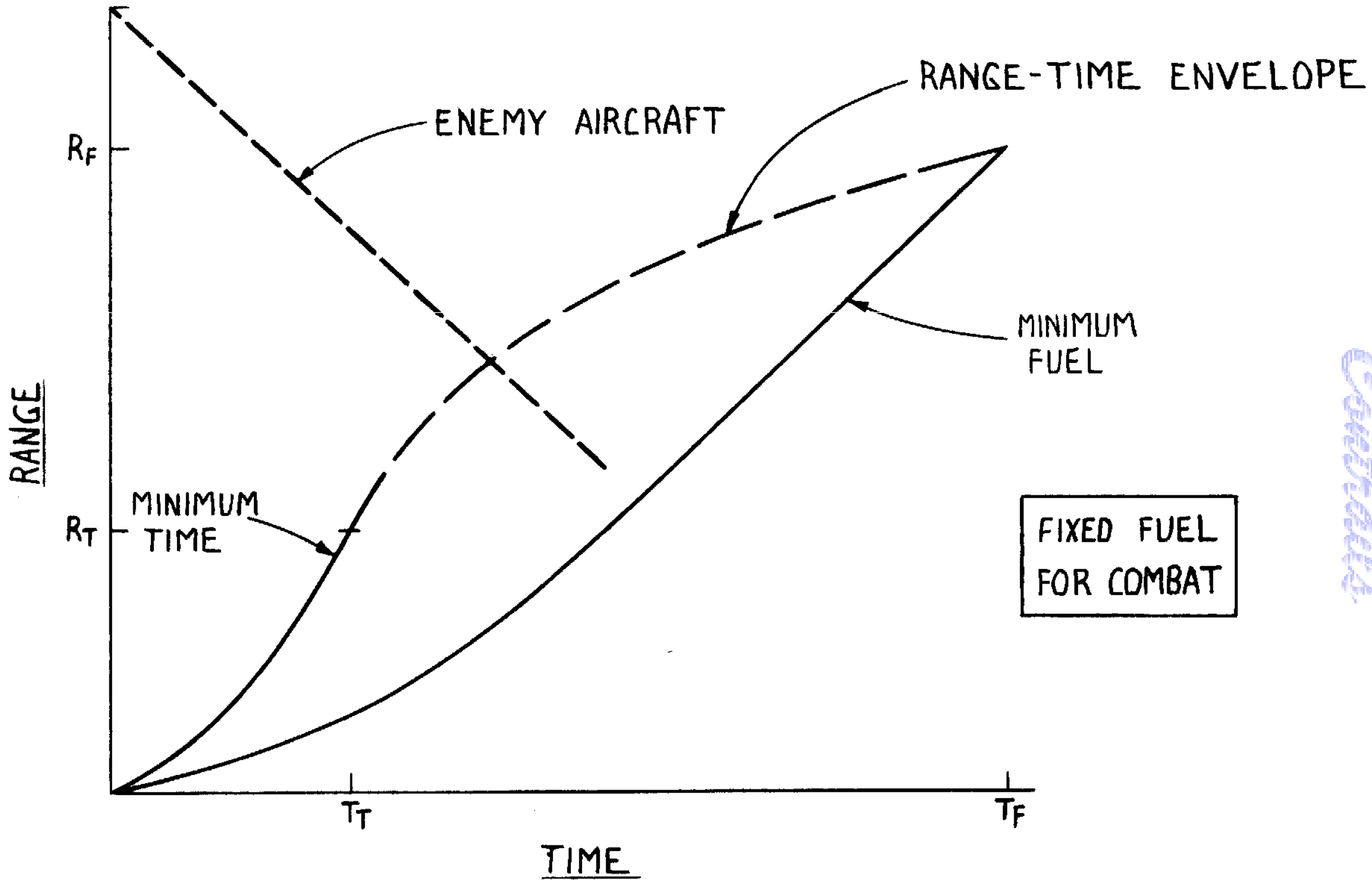


FIGURE 2 - INTERCEPT RELATED OPTIMIZED FLIGHT

TABLE I

OPTIMAL MANEUVER MISSION REQUIREMENTS

Mission Type	Mission Designation	Mission Task Description	Required Optimal Maneuvers
Fighter	Intercept	From initial ground alert status, defend friendly military objectives against approaching or penetrating enemy aircraft.	1., 4., 8., 11., 12.
	Combat Air Patrol	From airborne loiter position, defend friendly military objectives against approaching or penetrating enemy aircraft.	2., 3., 5., 6., 9., 10., 11., 12.
	Escort	Protect strike or transport missions from interference from enemy aircraft.	2., 3., 4., 5., 6., 7., 8., 11., 12.
	Fighter Sweep	Search out and attack enemy aircraft in contested airspace.	2., 3., 4., 5., 6., 7., 8., 11., 12.
Strike	Air Control	Strike air bases, air defense facilities supporting enemy forces against allied ground forces.	4., 5., 6., 7., 8., 11.
	Sea Control	Strike enemy forces afloat,naval bases and port facilities which support enemy naval forces.	4., 5., 6., 7., 8., 11.
	Close Air Support (CAS)	Strike troop concentrations,enemy armor, ground-supported artillery, missiles and command centers.	4., 5., 6., 7., 8., 11.
	Interdiction	Strike rail, road, and inland waterways serving enemy logistics systems which support enemy forces.	4., 5., 6,7., 8., 11.
	Deep Air Support (DAS)	Strike industrial facilities, supply depots, training camps, and billeting areas.	4., 5., 6., 7., 8., 11.
Other	Search and Rescue	Search out the location and assist in the rescue of downed aircraft pilots.	1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11.
	Reconnaissance	Gather photographic, radiation and visual information about enemy forces.	1., 2., 3., 4., 5., 6., 7., 8.
	Ferry	Fly aircraft from one site to another Under their own power.	4., 7., 8., 11.

b. The interceptor pilots wish to engage at a maximum range from the airfield.

c. The interceptor range is constrained by conserving an amount of reserve fuel both for combat and for the return flight.

The interceptor range-time envelope is bounded on the left by a minimum time flight profile and on the right by a minimum fuel path. Note, however, that the enemy crosses the outer range-time envelope before it crosses either of the minimum paths as it attempts to reach its objective. It is clear, then, that neither a minimum time nor a minimum fuel optimum flight path will be adequate for intercepting the enemy aircraft at maximum range. What is required is a path which is optimized for a range-time rendezvous. This will be a path which is jointly optimized in terms of range and fuel.

The zoom-climb maneuver stems from the requirement to intercept high flying bombers. Bombers that are flying at 70,000 feet and above cannot be attacked by an F-104C operating within its steady state flight envelope. The F-104C must go into a non-steady state zoom-climb before it executes a missile launch. The zoom, in general, must carry to within 10,000 feet of the altitude of the target to ensure good probability of kill.

No claim is made that this is a comprehensive review of combat missions. This is particularly true with respect to the F-104C. Information on F-104C missions has been gathered from several sources - none of which are classified. No comprehensive mission outline has been obtained, however. Therefore the most that can be said is that the missions given here are reasonably representative.

SECTION IV

OPERATIONAL REQUIREMENTS

This section contains a discussion of the operational requirements for flying optimal flight paths. A format for determining operational requirements is initially described and then applied to the F-104C.

1. OPERATIONAL REQUIREMENTS FORMAT

The following factors will be weighted in establishing optimal flight path operational requirements:

a. Frequency of mission - A

b. Frequency of optimal maneuver - B

c. Payoff in mission success - C

d. Payoff in increased performance - D

The term "frequency of mission" refers to the frequency percentage that an aircraft would fly a particular mission. This, of course, is directly related to the type of aircraft under consideration.

Item b. refers to the frequency percentage that a particular optimal maneuver would be flown in a given mission. For instance, even though a CAP mission may be flown rather infrequently, one would always loiter on such a mission. Therefore the weighting factor from this element would be 1.00.

"Payoff in mission success" pertains to an estimate of improved mission effectiveness which would be realized if the optimal maneuver were used, as opposed to its not being used. A good illustration here involves an intercept mission. If, as indicated in Section III, conditions warrant the use of a minimum time climb intercept, the improvement in mission success can be very large over what one might achieve with an arbitrary climb profile. A difference in climb time of three seconds can mean a difference in penetration of one nautical mile for a Mach 2.0 bomber. This difference alone could completely thwart the bomber and make the interceptor mission 100% effective.

The last term pertains to the increase in performance which results when a particular maneuver is optimized as opposed to when it is not. By referring to the intercept mission again, it may be that a 100% improvement in mission success results from only a 30% improvement in maneuver performance. Thus, the factor for mission success would be 1.00, while that for mission performance would be 0.30.

By properly weighting these factors, a measure of the operational requirements for a given optimal maneuver can be established. If A, B, C, and D are defined as noted, then a measure of effectiveness can be taken as

$$E = ABCD \tag{1}$$

2. F-104C FLIGHT PATH OPERATIONAL REQUIREMENTS

Since the F-104C is primarily an interceptor, the frequency of mission list is weighted accordingly. This is given in Table III. The strike missions are weighted toward the lower end, sea control and DAS being the least likely. Search and rescue and ferry are also rated rather low because of the expected infrequent use of the aircraft in this role.

A tabulation of all the factors, as a function, of mission is shown on Table IV for each maneuver. The following example is given to illustrate how the effectiveness of a given maneuver was established. Consider the minimum time climb maneuver. Table IV indicates that a minimum time maneuver might be flown in the intercept, search and rescue, and reconnaissance missions. According to Table III the frequency of the intercept mission (the factor A) is taken as .40. The frequency of a minimum time climb in an intercept mission, B, is taken to be 0.20. This reflects the reasoning, established in Section III, that most intercept missions will be flown along a range-time climb profile which is neither minimum time nor minimum fuel. The payoff in mission success, C, is taken to be 1.0, because of the fact that time is a very critical factor when a minimum time climb is selected. As noted in Section III, a difference as small as three seconds may mean the difference between complete success and failure. The payoff in increased performance, D, is taken as .30 in this case (note that this number is the same for each mission). It represents the percent improvement in climb time one might expect when using a minimum time climb, as opposed to using a suboptimal path. These four factors, when multiplied together, yield an effectiveness measure of .024 for the use of a minimum time climb in an intercept mission.

For the search and rescue mission the numbers A, B, C, and D are given, respectively, as .04, .80, .80, and .30. A, the mission frequency has been discussed previously. B, the maneuver frequency is taken to be .80 because of the general requirement of minimum response times in search efforts. The payoff in mission success, C, is also given a large value for the same reason. A few minutes lost in search could result in the death of the pilot through injuries, or in his capture by enemy forces. The factor 0.30 for the payoff in increased performance, D, is the same as for the intercept mission.

For the reconnaissance mission the factors of interest are the maneuver frequency, B, and the mission success payoff, C - 0.30 and 0.80, respectively. A low value of B was estimated because it was felt that a minimum time climb maneuver is not something that is done as a matter of course in reconnaissance missions. A minimum time climb-out, to avoid anti-aircraft fire, following a low pass photographic run might be a typical example. Even though a minimum time climb-out is not done on all missions, the payoff can be substantial when the maneuver is executed. Therefore, while the value of B is modest, the value of C is relatively high.

TABLE III

FREQUENCY OF MISSION FACTOR - A

Mission	Frequency Weight
Intercept	40
CAP	10
Escort	12
Fighter Sweep	7
Air Control	4
Close Air Support (CAS)	4
Sea Control	1
Interdiction	4
Deep Air Support (DAS)	1
Search and Rescue	4
Reconnaissance	12
Ferry	1

Table IV MANEUVER OPERATIONAL EFFECTIVENESS TABULATION

Mission	Mission Freq.	Minimum Time Climb				Minimum Time Altitude-Range Transition				Minimum Energy State		Time Transition		Minimum Fuel Climb				Minimum Fuel Altitude-Range Transition				Minimum Fuel Energy State Transition		
	A	B	C	D	E	B	C	D	E	B	C	D	E	B	C	D	E	B	C	D	E	B	C	D
Intercept	.40	.20	1.00	.30	.024	.00				.00				.20	.80	.30	.01920	.00				.00		
CAP	.10	.00				.05	1.00	.30	.0015	.15	1.00	.30	.0045	.00				.05	.80	.30	.00120	.15	.80	.30
Escort	.12	.00				.05	1.00	.30	.0018	.10	1.00	.30	.0036	.10	.40	.30	.00144	.05	.80	.30	.00144	.10	.80	.30
Fighter Sweep	.07	.00				.10	.20	.30	.00042	.10	1.00	.30	.0021	.50	.20	.30	.00210	.05	.80	.30	.00084	.05	.80	.30
Air Control	.04	.00				.00				.00				.10	.40	.30	.00048	.60	.40	.30	.00288	.40	.20	.30
Sea Control	.01	.00				.00				.00				.10	.40	.30	.00012	.60	.40	.30	.00072	.40	.20	.30
CAS	.04	.00				.00				.00				.10	.40	.30	.00048	.60	.40	.30	.00288	.40	.20	.30
Interdiction	.04	.00				.00				.00				.10	.40	.30	.00048	.60	.40	.30	.00288	.40	.20	.30
DAS	.01	.00				.00				.00				.10	.60	.30	.00018	.60	.40	.30	.00072	.40	.20	.30
S & R	.04	.80	.80	.30	.00768	.60	.20	.30	.00144	.20	.20	.30	.00048	.10	.60	.30	.00072	.05	.40	.30	.00024	.05	.10	.30
Recce.	.12	.30	.80	.30	.00864	.50	.40	.30	.0072	.50	.20	.30	.0036	.20	.40	.30	.00288	.50	.40	.30	.00720	.50	.20	.30
Ferry	.01	.00				.00				.00				.10	.50	.30	.00015	.00				.00		
Total E					.0403				.01236				.01428				.0282				.0210			

Table IV (Contd)

Mission		Maximum Range Climb				Maximum Range Cruise				Maximum Endurance Climb				Maximum Endurance Cruise				Range-Time-Altitude Rendezvous				Zoom Climb to Launch			
	E	B	C	D	E	B	C	D	E	B	C	D	E	B	C	D	E	B	C	D	E	B	C	D	E
Intercept		.00				.00				.00				.00				.60	.90	.20	.04320	.50	1.00	.60	.12000
CAP	.00360	.00				.00				1.00	.50	.10	.005	1.00	1.00	.20	.02	.20	.50	.20	.00200	.30	1.00	.60	.01800
Escort	.00288	.90	.80	.10	.00864	1.00	1.00	.20	.0240	.00				.00				.10	1.00	.20	.00240	.20	.80	.60	.01152
Fighter Sweep	.00084	.50	.50	.10	.00175	1.00	.20	.20	.0028	.00				.00				.30	.20	.20	.00084	.05	.20	.60	.00420
Air Control	.00096	.90	.80	.10	.00288	1.00	.80	.20	.0064	.00				.00				.20	.80	.20	.00128	.00			
Sea Control	.00024	.90	.80	.10	.00072	1.00	.80	.20	.0016	.00				.00				.20	.80	.20	.00032	.00			
CAS	.00096	.90	.80	.10	.00288	1.00	.80	.20	.0064	.00				.00				.20	.80	.20	.00128	.00			
Interdiction	.00096	.90	.80	.10	.00288	1.00	.80	.20	.0064	.00				.00				.20	.80	.20	.00128	.00			
DAS	.00024	1.00	.90	.10	.00090	1.00	1.00	.20	.0020	.00				.00				.50	.90	.20	.00090	.00			
S & R	.00006	1.00	.70	.10	.00280	.70	.80	.20	.0045	.05	.30	.10	.00006	.90	.80	.20	.00576	.70	.80	.20	.00448	.00			
Recce.	.00360	1.00	.60	.10	.00720	.80	.90	.20	.0173	.00				.00				.00				.00			
Ferry		.90	1.00	.10	.00090	1.00	1.00	.20	.0020	.00				.00				.40	.90	.20	.00072	.00			
Total E	.01434				.0316				.0734				.00506				.0258				.0587				.1537

By multiplying the A, B, C, and D values together for each of the missions, and then summing, the effectiveness for the minimum time climb maneuver is found to be .0403. The result, when this is done for all the maneuvers, is shown on Table V. The maneuvers are listed in decreasing order according to effectiveness number. The numbers given have been multiplied by a factor 10^3 over what is shown on Table IV.

TABLE V

MANEUVER EFFECTIVENESS

Maneuver	Effectiveness Number
Zoom Climb to Launch	153.70
Maximum Range Cruise	73.40
Range-Time Altitude Rendezvous	58.70
Minimum Time Climb	40.30
Maximum Range Climb	31.60
Minimum Fuel Climb	28.20
Maximum Endurance Cruise	25.80
Minimum Fuel Altitude-Range Transition	21.00
Minimum Fuel Energy State Transition	14.34
Minimum Time Energy State Transition	14.28
Minimum Time Altitude-Range Transition	12.36
Maximum Endurance Climb	5.06

SECTION V

SYSTEM REQUIREMENTS ANALYSIS

This section contains a discussion of the analysis leading to the development of system requirements for the component subsystems. The section begins by considering the system requirements for types of optimum flight paths. Next the research and analysis leading to the definition of optimum flight descriptors is described. This is followed by a discussion of the tracking problem involved in flying an optimum flight path.

1. SYSTEM FLIGHT PATH REQUIREMENTS ANALYSIS

a. Requirements Considerations

Flight path requirements will be determined by considering operational requirements, status constraints, and design considerations. Operational requirements refer to the ordering of optimal maneuvers, in terms of effectiveness, which was developed in Section IV. Status constraints refer to:

> The state of the art of flight path optimization theory in terms of yielding solutions to existing problems.
>
> The existence of required optimal flight path information, given that solution techniques exist.
>
> The existence of information from aircraft state sensors which is required to define on-board optimum flight paths.

Design constraints refer to considerations of cost, reliability, maintainability, and ease of implementation.

(1) Flight Path Optimization Status

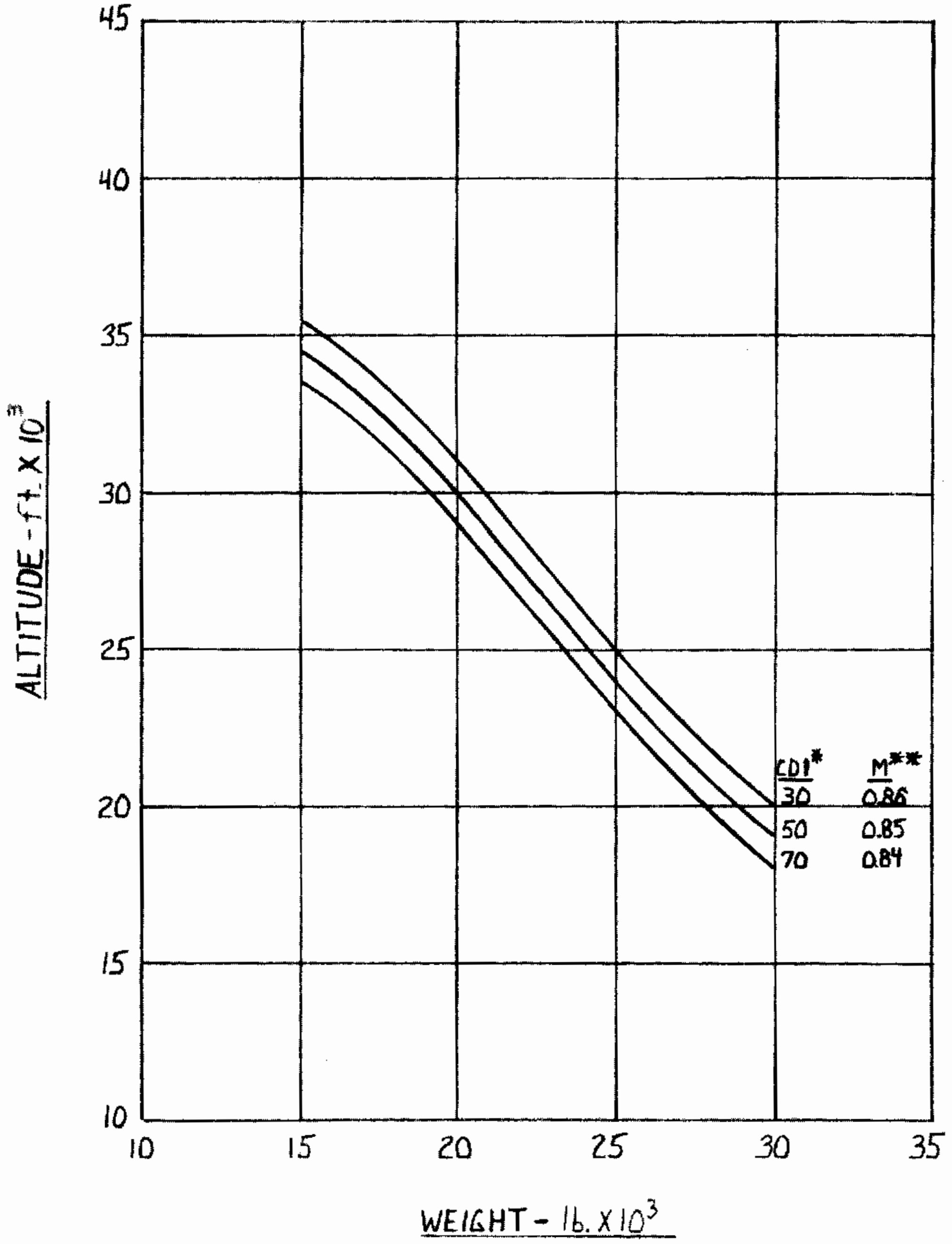
Research into the state of the art of flight path optimization analysis has shown that almost all of the optimal maneuvers listed in Table V have been examined by one or more investigators. The sole exception seems to be the "range-time-altitude rendezvous" maneuver. It may be, however, that this problem could be effectively handled with one of the gradient programs now in existence. This is not presently known. A summary of typical literature sources for each of the maneuvers is given as follows, where the numbers in brackets refer to references in the bibliography:

Zoom Climb to Launch [12]

Maximum Range Cruise [13]

Minimum Time Climb [14]

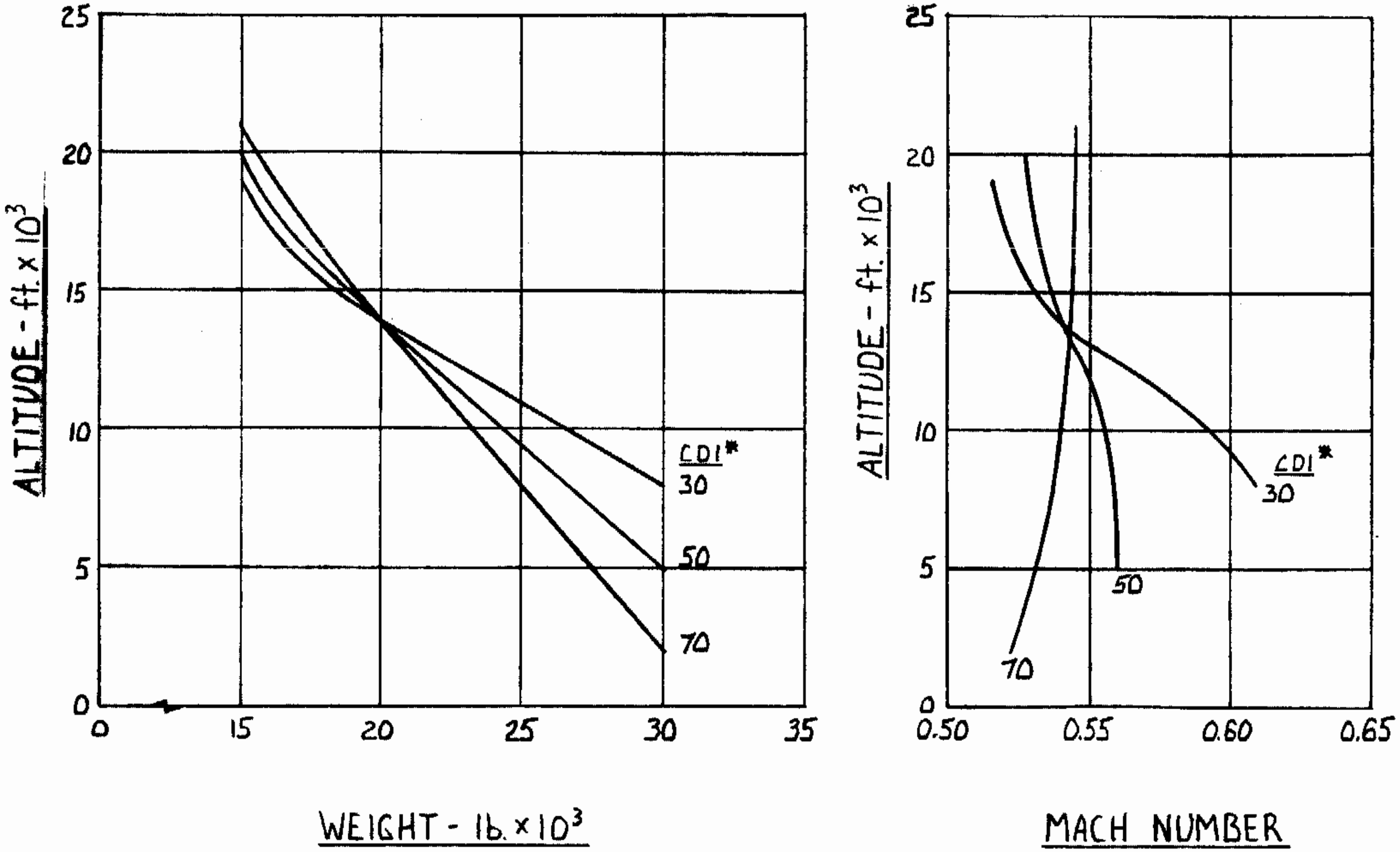
Maximum Range Climb [15]



* CONFIGURATION DRAG INDEX

** MACH NUMBER

FIGURE 3 - MAXIMUM RANGE CRUISE CONDITIONS FOR THE F-104 C



*CONFIGURATION DRAG INDEX

FIGURE 4 - MAXIMUM ENDURANCE CRUISE CONDITIONS FOR THE F-104 C

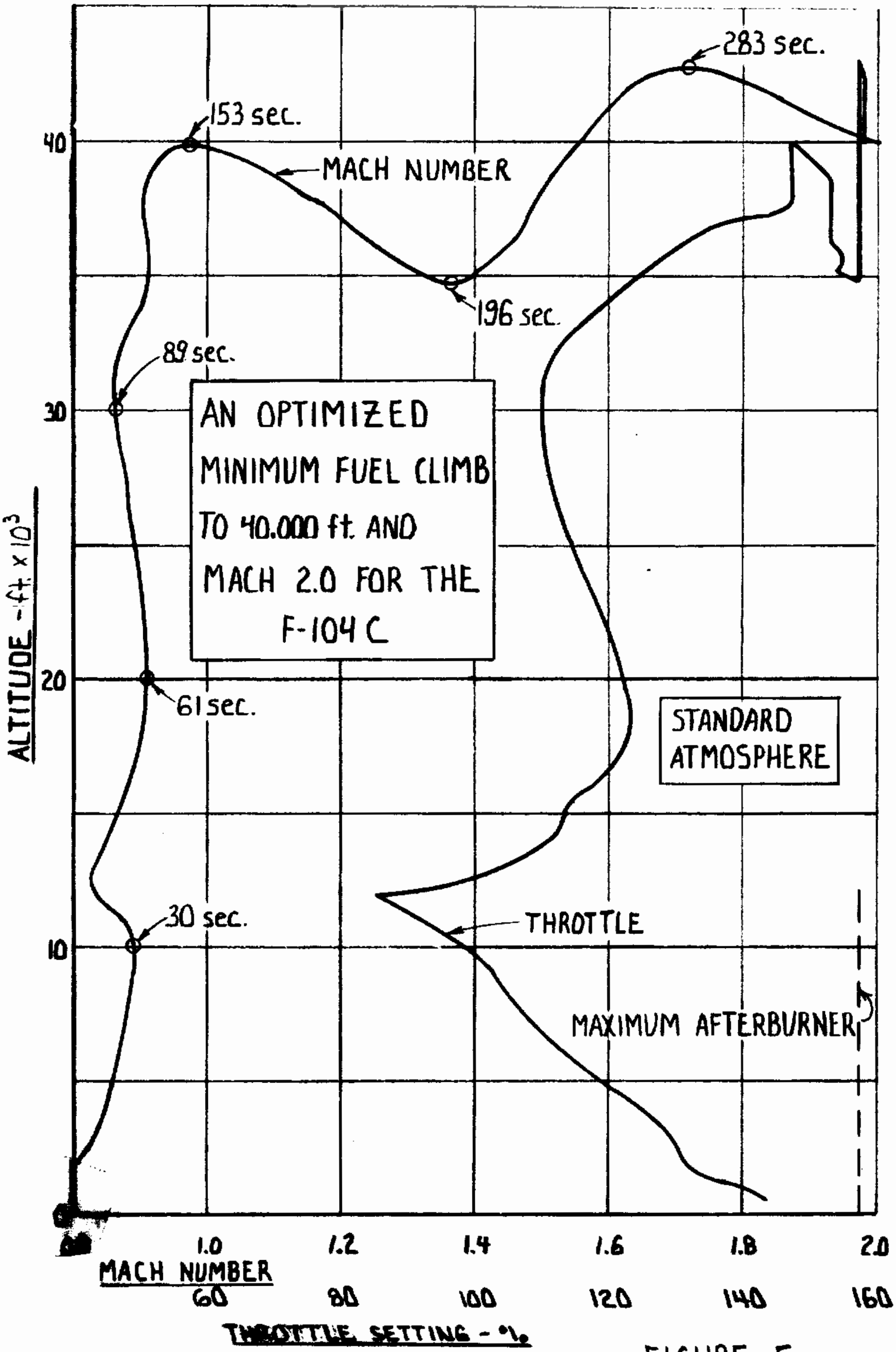

283 sec.
153 sec.
40
MACH NUMBER
196 sec.
89 sec.
30
AN OPTIMIZED MINIMUM FUEL CLIMB TO 40.000 ft. AND MACH 2.0 FOR THE F-104 C
ALTITUDE - ft. x 10^3
20
61 sec.
STANDARD ATMOSPHERE
30 sec.
THROTTLE
10
MAXIMUM AFTERBURNER
0
0.8
1.0
1.2
1.4
1.6
1.8
2.0
MACH NUMBER
60
80
100
120
140
160
THROTTLE SETTING - %

FIGURE 5

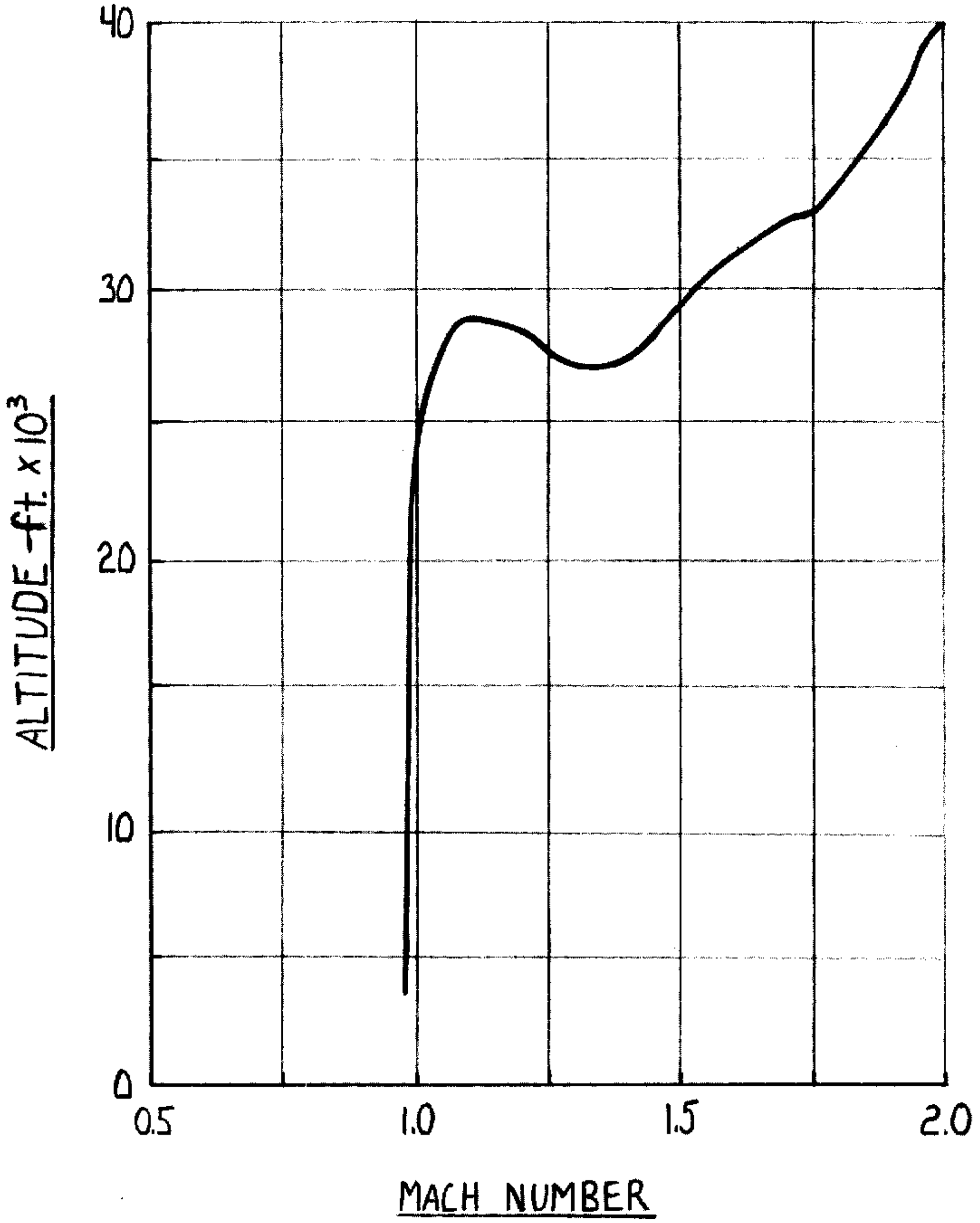


FIGURE 6 - AN OPTIMIZED MINIMUM TIME CLIMB TO 40.000 ft. AND MACH 2.0 FOR THE F-104 C

Minimum Fuel Climb [14]
Maximum Endurance Cruise [13]
Minimum Fuel Altitude-Range Transition [14]
Minimum Fuel Energy State Transition [14]
Minimum Time Energy State Transition [16]
Minimum Time Altitude-Range Transition [11]
Maximum Endurance Climb [15]

In general most of the references cited are directly applicable to the maneuver indicated. In some cases, however, the technique presented in the given reference would have to be extended. This is particularly true of Edelbaum's work [15] when applied to climb techniques. The work is primarily related to maximum range profiles. However, it seems reasonable that extensions could be made to the maximum endurance case. Failing this, one would probably resort to a gradient technique such as the STOP program [14]. From this discussion it seems clear that methods exist for handling almost any optimization program one would desire. This does not say, however, that data has been generated which can be applied to the F-104C. This, then, forces the consideration of the second status constraint - the existence of optimum flight path data.

(2) Optimum Flight Path Data

Adequate data exists to define maximum range cruise and maximum endurance cruise as a function of aircraft weight for standard atmospheric conditions (see Figures 3 and 4). This data is evidently representative of all drag configurations [13].

Adequate data exists for defining minimum time and minimum fuel climb paths for a standard atmosphere. These are shown on Figure 5 and 6, respectively, and were derived from the STOP program [14]. The minimum fuel path was optimized as a function of variable throttle position.

Data is marginal in relation to zoom climb and energy state transition maneuvers. If it is assumed that the Boyd rule of thumb [16], or something like it, can be made to represent transition maneuvers, then it seems likely that adequate paths can be developed which represent these maneuvers. Since the rule of thumb has not been adequately verified, however, this is only conjecture at the present time. This will be commented upon further later on in this section.

Information on the remaining optimal maneuvers is completely lacking, as it pertains to the F-104C, as far as has been ascertained. It would appear, however, that with existing computer models, all required information could be eventually defined. It seems more prudent at this time, however, to develop the present system by assuming this will not be done. Therefore, the system will be developed around the following optimal flight maneuvers:

1. Minimum time climb
2. Minimum time state transition
3. Minimum fuel climb
4. Minimum fuel state transition
5. Maximum range cruise
6. Maximum endurance cruise

(3) Sensor Information Limitations

Sensor information limitations do not restrict the group of optimal flight maneuvers listed above. The descriptors required to define optimal flight paths are in the form of altitude-Mach number profiles. In general, information from existing F-104C sensors is adequate (if perhaps not always as accurate as desired) to define the aircraft state relative to a given profile. This will be commented upon, in detail, later in the report.

b. Requirements Analysis and Research

At the beginning of the study there was a question as to whether optimum flight paths could be generated on board, on line, and in real time, or whether ground-derived approximations would be required. The results of the research carried on to answer this question are discussed in this section.

In order to begin the discussion of flight path optimization research, an outline of the solution objectives will be described. This will be followed by a development of the flight path optimization problem in terms which are suitable for mathematical solution. Next, several existing computer programs are described which are presently used for determining optimum flight paths. These are programs which could not be used for on-line computation, in the present application, but which may be suitable for limited on-line application in future generation airborne computers. It will be shown that one of these programs will be required for determining optimum flight path data which is to be stored on board, however.

(1) Solution Objective

The present flight path optimization problem is an optimal control synthesis problem in the vertical plane. Synthesis in general is defined as follows:

> Synthesis [17] - The problem of defining the optimal control in the form $u = v(x)$; i.e. the desired optimal control, at each time, depends only on the point of the space at which the phase point is located at the given time.

Note particularly that the objective is to define a control as a function of the state variables, rather than as a function of time.

A typical optimal control problem is shown in Figure 7. As shown, the problem is stochastic. Omission of the disturbing forces and errors of observation converts the problem into a deterministic one, if we assume, in addition, that the desired terminal state is deterministic.

If the observed state is correct only at the start of the solution, and observations are inadequate to define the state during the transformation process, the system is said to be unobservable, and the problem becomes an open loop problem.

If the loop is closed, by means of adequate observations, the system may be operated along a desired path. If the desired path causes some cost function to be minimized, it is an optimal path with respect to that cost function, as long as the system follows that path. If the system departs from the path for some reason such as disturbances or observation errors, the optimum policy does not in general consist in returning to the original optimal path, but in following a new path based on the new state of the system. A control algorithm that accomplishes this is said to be path adaptive.

An important class of problems arises when the desired state of the system is specified for some future time, t_f, with the events thereafter being immaterial. Normally the system state is known, in detail, at some prior time, t_o, therefore the problem is of the two point boundary value type. The problem may be thought of as a transformation, or mapping, of the system in state one into the system in state two. End point conditions may be fixed or variable. Instead of specifying t_f, the end point may be specified in terms of values or functions of some state variables. This is the type of problem under consideration here.

(2) System Equations

The equations of motion of the aircraft are of the form

$$\dot{\underline{x}} = F(\underline{x}, \underline{u}, t) \tag{2}$$

where

$\underline{x}$ is the state variable vector
$\underline{u}$ is the control variable vector
t is the independent variable, time

If it is assumed that the primary motion of interest in flight path optimization is the motion of the center of mass, and that rotational dynamics about the center of mass have a negligible effect, then, the vertical plane equations of motion can be derived as follows:

From Figure 8, it is seen that

$$\bar{V} = V\bar{i}_1 \tag{3}$$

Then

$$\dot{\bar{V}} = \bar{\dot{V}} + \bar{w}_v \times \bar{V} \tag{4}$$

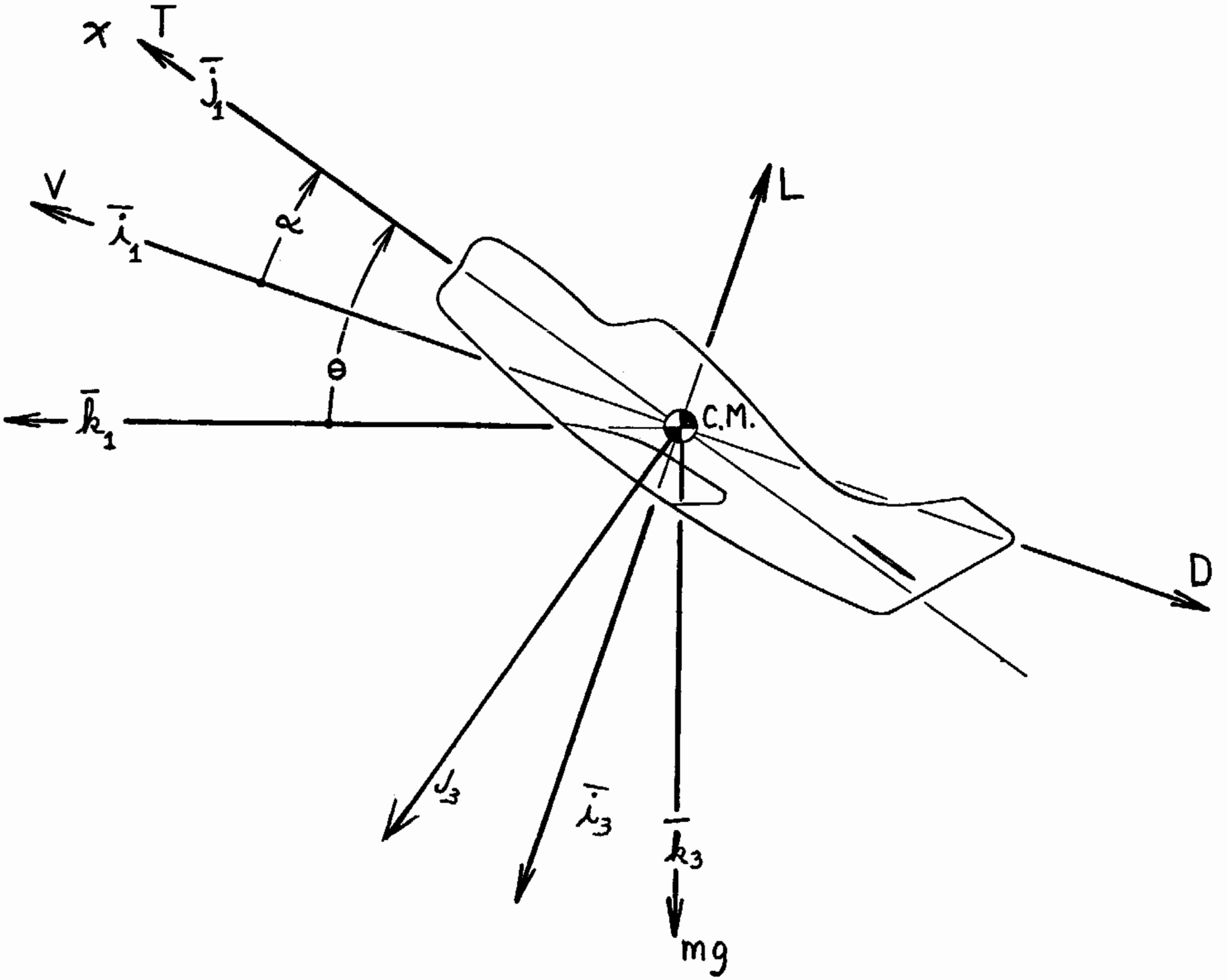


FIGURE 7- AIRCRAFT COORDINATE SYSTEM CONVENTIONS

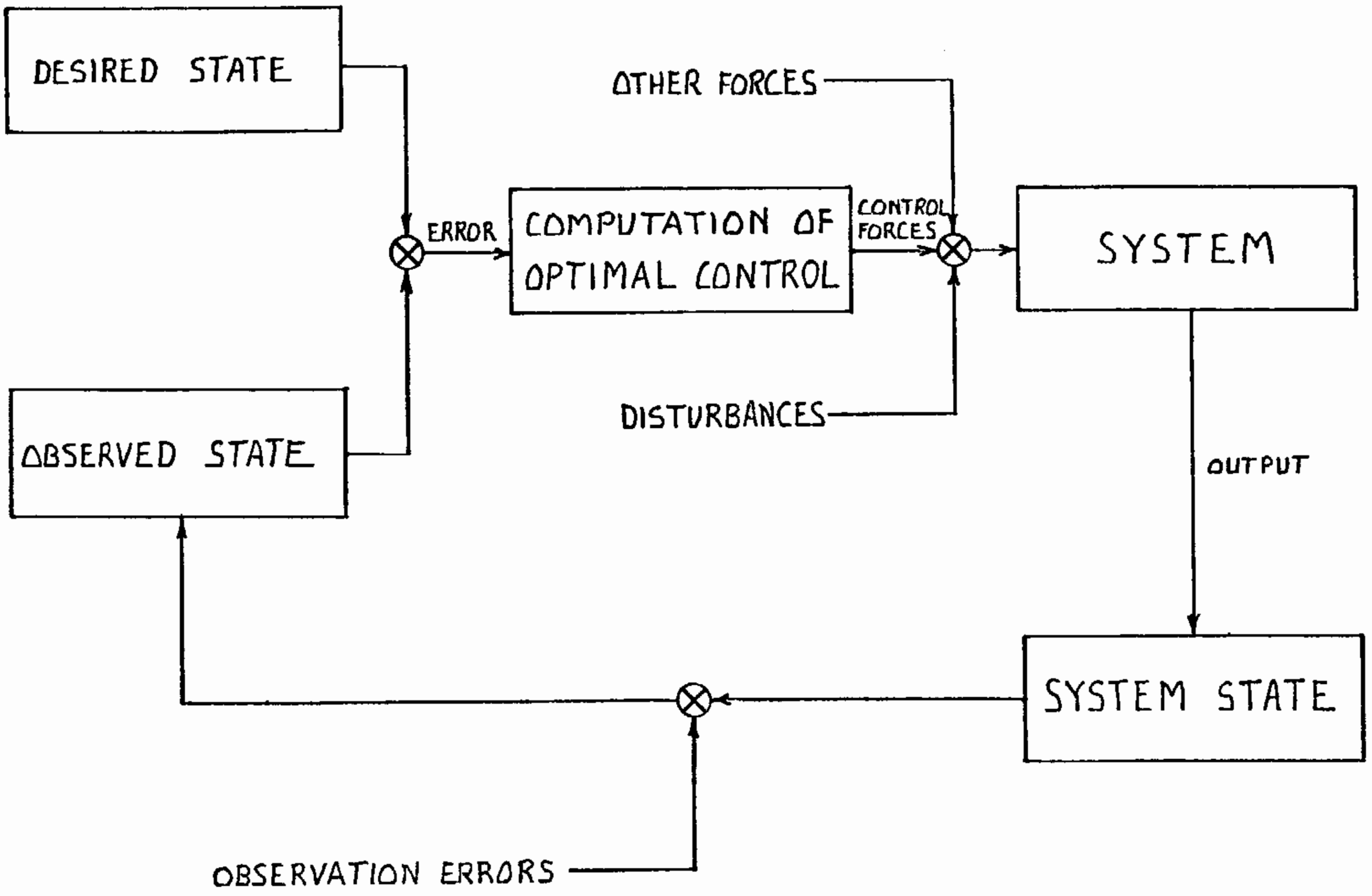


FIGURE 8 - TYPICAL OPTIMAL CONTROL SITUATION

$$= \dot{V}\,\bar{i}_1 - V\dot{\gamma}\,\bar{i}_3 \tag{5}$$

where

$$\bar{w}_V = \dot{\gamma}_1\,\bar{i}_2 \tag{6}$$

By making the summation of forces equal to the change in momentum,

$$\Sigma F = m\dot{V} \tag{7}$$

or

$$m(\dot{V}\bar{i}_1 - V\dot{\gamma}\,\bar{i}_3) = -L\bar{i}_3 - D\bar{i}_1 + mg\bar{k}_3 + T\bar{j}_1 \tag{8}$$

where it is assumed that changes in mass have a negligible effect, and the angle between the aircraft axis and thrust axis is zero.

Now the j and k systems are related to the i system through the following set of transformations:

$$\begin{bmatrix} \bar{k}_1 \\ \bar{k}_2 \\ \bar{k}_3 \end{bmatrix} = \begin{bmatrix} \cos\gamma & 0 & \sin\gamma \\ 0 & 1 & 0 \\ -\sin\gamma & 0 & \cos\gamma \end{bmatrix} \begin{bmatrix} \bar{i}_1 \\ \bar{i}_2 \\ \bar{i}_3 \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} \bar{i}_1 \\ \bar{i}_2 \\ \bar{i}_3 \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} \bar{i}_1 \\ \bar{i}_2 \\ \bar{i}_3 \end{bmatrix} \tag{10}$$

Therefore,

$$\bar{k}_3 = -\sin\gamma\,\bar{i}_1 + \cos\gamma\,\bar{i}_3 \tag{11}$$

$$\bar{i}_1 = \cos\alpha\,\bar{i}_1 - \sin\alpha\,\bar{i}_3 \tag{12}$$

and

$$\begin{aligned} m(\dot{V}\bar{i}_1 - V\dot{\gamma}\bar{i}_3) = {} & -L\bar{i}_3 - D\bar{i}_1 + mg(-\sin\gamma\,\bar{i}_1 + \cos\gamma\,\bar{i}_3) \\ & + T(\cos\alpha\,\bar{i}_1 - \sin\alpha\,\bar{i}_3) \end{aligned} \tag{13}$$

Equating like components on the left and right sides of equation (13) yields:

$$m\dot{V} = T\cos\alpha - D - mg\sin\gamma \tag{14}$$

$$mV\dot{\gamma} = L - mg\cos\gamma + T\sin\alpha \tag{15}$$

In addition, an examination of Figure 7 indicates an additional two equations may also be written. These are the range rate and the rate of climb, and are written as follows:

$$\frac{dr}{dt} = V\cos\gamma \tag{16}$$

$$\frac{dh}{dt} = V\sin\gamma \tag{17}$$

Finally, if it is assumed that changes in weight affect the static performance of the aircraft, but have a negligible dynamic effect in the sense of including $\dot{m}$ terms in equations (14) and (15), then the last equation of interest is given as follows:

$$\frac{dw}{dt} = \dot{w}_F \tag{18}$$

Equations (14) through (18) are the desired equations which define the aircraft state. Taking V, h, r, and w as the state variables and T and γ as the control variables we have

$$\underline{x} = \begin{pmatrix} V \\ h \\ r \\ w \end{pmatrix} \qquad W = mg \tag{19}$$

$$\underline{u} = \begin{pmatrix} T \\ \gamma \end{pmatrix} \tag{20}$$

and

$$\underline{\dot{x}} = \begin{pmatrix} \frac{dV}{dt} \\ \frac{dh}{dt} \\ \frac{dr}{dt} \\ \frac{dw}{dt} \end{pmatrix} = \begin{pmatrix} \frac{T\cos\alpha - D}{m} - g\sin\gamma \\ V\sin\gamma \\ V\cos\gamma \\ \dot{w}_F \end{pmatrix} \tag{21}$$

$\dot{\gamma}$, although it appears in the equation describing transverse motion (15), is now considered to be an independent control variable rather than a state variable.

Given the system equations and the solution objectives then, all that remains in specifying the flight path optimization problem is to specify the cost function which is to be minimized for a particular case. This is done next.

(3) Indices of Performance (Cost Functions)

There are several flight path optimization problems. These differ in criteria of performance, and to a lesser degree, in boundary conditions. In general, an optimization problem may be formulated in several ways. Consider the following problem wherein a function is sought which minimizes the functional

$$J = G(x,y_k)\Big|_i^f + \int_{x_i}^{x_f} H(x,y_k,\dot{y}_k)dx \qquad (22)$$

This is a formulation in the Calculus of Variations which is in the form of the problem of Bolza. If $G \equiv 0$ then the formulation is known as the problem of Lagrange, and if $H = 0$, as the problem of Mayer. In general any one of the three can be transformed into any other by a change of coordinates [18]. The Lagrange formulation will be used in this discussion.

In the case of a minimum time climb the function to be minimized is given as follows:

$$J = \min \int_{t_o}^{t_f} dt = \min (t_f - t_o) = \min T(h_f, V_f) \qquad (23)$$

$$= \min \int_{h_o}^{h_f} \frac{dh}{V_z} \qquad (24)$$

where $V_z = V \sin\gamma =$ rate of climb

Now, by multiplying equation (14) by $\frac{V}{w}$, it is seen that

$$\frac{V\dot{V}}{g} = (T\cos\alpha - D)\frac{V}{w} - V\sin\gamma \qquad (25)$$

or

$$V\sin\gamma = V_z = (T\cos\alpha - D)\frac{V}{w} - \frac{V}{g}\frac{dV}{dt} = \frac{dh}{dt} \qquad (26)$$

But,

$$\frac{dV}{dt} = \frac{dV}{dh}\,\frac{dh}{dt} \tag{27}$$

Therefore, substituting (27) into (26) and rearranging

$$V_z = \frac{(T\cos\alpha - D)\,\dfrac{V}{w}}{1 + \dfrac{V}{g}\,\dfrac{dV}{dh}} \tag{28}$$

and equation (24) becomes

$$J = \min \int_{h_o}^{h_f} \frac{\left(1 + \dfrac{V}{g}\,\dfrac{dV}{dh}\right) dh}{(T\cos\alpha - D)\,\dfrac{V}{w}} \tag{29}$$

$$= \min \int_{V_o}^{V_f} \frac{\left(\dfrac{dh}{dV} + \dfrac{V}{g}\right) dV}{(T\cos\alpha - D)\,\dfrac{V}{w}} \tag{30}$$

Now

$$T = T(h,\ V,\ \overset{\text{throttle setting}}{N}) \tag{31}$$

and

$$D = D(h,\ V, \alpha, w) \tag{32}$$

Therefore, the minimization cannot be handled in an analytical fashion unless simplifying assumptions are made. Many authors have done this [6], [19], [20]. The present discussion is concerned with more exact solutions, however. In general, these can only be done, in reasonable time, with the aid of high speed digital computers.

In addition to the minimum time problem, several other optimization problems may be postulated. In a minimum fuel transition problem one would be concerned with minimizing the integral

$$J = \min \int_{t_o}^{t_f} \dot{w}_F \, dt \tag{33}$$

Similarly for maximum range

$$J = \max \int_{w_{F_o}}^{w_{F_f}} \frac{r}{w_F} \, dw_F \tag{34}$$

In references 7 and 21, Miele has formulated the flight path optimization problem in terms of a 6 x 6 matrix. With the techniques provided there, one is able to mathematically formulate any optimization problem desired.

(4) State-of-the-Art Optimization Programs

Existing computer programs, used for determining solutions to optimal flight path problems, are described in this section. Several papers, describing digital computer solution techniques, have been examined. Some of these are discussed in the material that follows.

Methods for determining solutions to variational problems are generally separated into direct and indirect techniques. An indirect method is considered to be a method wherein a minimum is sought by means of a necessary condition for a minimum. A direct method is a method that depends upon direct comparison of the values of a function at two or more points. In general, analytic solutions are usually achieved by means of indirect methods. Direct methods are more applicable to numerical solution. Most of the discussion which follows will be concerned with direct methods.

(a) Gradient Methods

Gradient techniques such as those developed by Bryson [8], Kelley [22], Hague [23], Stein [15], et.al.,are the first of the direct methods which will be discussed. These methods provide precise control laws which are as accurate as the models being simulated. The advantages of the methods are in their conceptual simplicity and ease of programming. In addition, convergence is not contingent on a good initial estimate, improvement in each iteration is assured; control constraints are easily implemented; and the methods seek out relative minima rather than solutions which are merely stationary.

Disadvantages to the methods are in slow convergence and long computational time. In addition, large amounts of computer memory capacity are required. In the present application, methods with these characteristics cannot be made to run in real time and cannot be programmed within existing system memory constraints.

Various modified gradient techniques have been proposed, which converge more rapidly [24], [25], [26], [27]. Nevertheless these are still far too slow for real time operation and require orders of magnitude more of computer capacity than is available.

(b) Second-Variation Method

The second variation method is based on the theory of the second variation [28], [29] while gradient methods or methods of steepest descent (ascent) are based on the first-order theory. In the initial computation phases, the behavior of the method is similar to the gradient process. As the solution approaches the optimal function the solution progressively approaches a process which is conceptually similar to iteration on the Euler equations.

The advantages to the method are that near the solution, the convergence is quadratic; convergence is not dependent upon choosing a good starting function, improvement in each step is assured; and, in general, required computer time is less than is required for gradient techniques. The main disadvantage to the method is that computer programming is substantially more complicated than in the gradient method.

Even though the second-variation method is generally faster than present gradient techniques it is still not nearly fast enough for real time operation in an operational environment. In addition, computer capacity requirements are even larger. Therefore, methods of this type are also precluded as on board, on-line, real time, solution techniques.

(c) Newton-Raphson Methods

The Newton-Raphson method [30], [31], is usually considered to be an indirect method. It is similar, in many respects, however, to the second-variation method, which is a direct method. In the usual indirect approach, sets of initial conditions are successively mapped into new sets of initial conditions with the differential constraints satisfied. In the Newton-Raphson techniques a mapping is produced that transforms sets of functions into improved sets of functions. These do not satisfy the differential constraints until the method has converged, however.

The Newton-Raphson method is somewhat simpler to program than the second variation method and it shares the quadratic convergence property of the second method. A major problem, however, is that convergence is dependent upon the starting function. An additional disadvantage is that the method seeks out stationary solutions rather than relative minima.

Methods of this type also require more solution time and computer capacity than is practical to consider for the present application. It is of interest to note that an application of Newton's method has been made in the case of real time rocket guidance [32]. Iteration intervals for this particular program run anywhere from one to ten seconds, depending upon the length of trajectory remaining. The problem described was a ballistic, non-aerodynamic case, however, where the only forces involved were gravity and prescribed thrust. The thrust was programmed as a function of time and not as a function of the system state variables. Since the terrestrial flight optimization problem is much more complex than the ballistic case, a substantial increase in solution time can be expected. This would easily place the solution time out of the practical range.

(d) Dynamic Programming

Dynamic programming was developed by Richard Bellman in the 1950's [33], [34]. It has been accepted as a means of obtaining accurate solutions to a large variety of the optimization problems which occur in real life. It has the attractive property, not generally characteristic of variational problems, that the more one limits the options open to the system, the less calculation is required. Dynamic programming becomes unwieldy, however, in problems with more than two degrees of freedom.

Computer programs based on dynamic programming provide a "central field" of solutions, i.e., a field of trajectories, each starting at a distinct allowable point in space and proceeding in an optimal manner to a single target state without intersecting any other trajectory. Two specific flight path optimization programs have been examined which are based on the principles of dynamic programming [35], [36]. Even though each is restricted to a single terminal condition, (a highly restrictive situation!), computation times are excessive. This can be alleviated, to some extent, if the solution is restricted to a very small region near the terminal point. This, in effect, pre-supposes the characteristics of an optimal solution, however. The results can be unsatisfactory if a wrong guess is made. Indications are that dynamic programming solution times could be made fast enough for operational use, if the solution region is adequately constrained. In doing so, the utility of the program is restricted considerably beyond that required for the present application, however.

(e) Energy Maneuverability

The energy-maneuverability computer model [16] is based on the graphical optimization technique developed by Rutowski [6]. In the technique, contours of an aircraft performance parameter are first plotted within the flight envelope. The parameter is similar to an integrand in the classical Lagrange formulation in the calculus of variations. For minimum time the plotted parameter is:

$$P_s = \frac{(T-D)}{W} V \qquad (35)$$

For minimum fuel the parameter is:

$$P_s / \dot{w}_F \qquad (36)$$

Using the minimum time case as an example, it is shown that a minimum time path results when

$$\left(\frac{\partial P_s}{\partial V}\right)_{E_s = k} = 0, \qquad (37)$$

or when

$$\left(\frac{\partial P_s}{\partial h} \right)_{E_s = k} = 0. \qquad (38)$$

In effect, these equations indicate that a minimum time path results when contours of constant E_s are tangent to contours of constant P_s. The theoretical path, thus derived, is shown on Figure 9. The computer model described in reference [16] is a mechanization of this graphical process.

The path, as described by equations (37) and (38), is an extremal only for cases where the initial and final end points lie on the path. No mechanism is given for handling cases where the end points do not lie on this basic path. Boyd has, however, described a rule of thumb for optimum flight between arbitrary end points. This rule is given as follows and is presumably based upon inspection of results obtained by more detailed methods.

$$\Delta E_s = K \Delta h \qquad (39)$$

where

$$K = -1 \text{ when M must be increased}$$

$$= 2/3 \text{ when M must be decreased} \qquad (40)$$

In effect equations (39) and (40) describe transition paths from an arbitrary end point to the basic Rutowski path. Presently these equations are valid only in the subsonic flight regime, however. Therefore, the rule is more conceptual than actual and much development work would be required before an operational theory could be defined.

In the context of an on-board optimization system the energy-maneuverability technique has the following limitations:

> The relationship between the Rutowski path and the Boyd rule of thumb paths must be clearly defined.
>
> The transition portion of the Rutowski path between the subsonic and supersonic regions must be more adequately defined. In the Rutowski theory this path is shown as an arc of constant specific energy (see Figure 9). In actuality this path cannot be flown under the constant throttle assumption with which the path was derived. If a constant throttle is assumed, with thrust larger than drag, then energy is being added to the system and a constant specific energy path cannot be flown.
>
> A means of handling optimum paths with variable throttle position must be developed. All efforts, to date, are for a constant throttle.

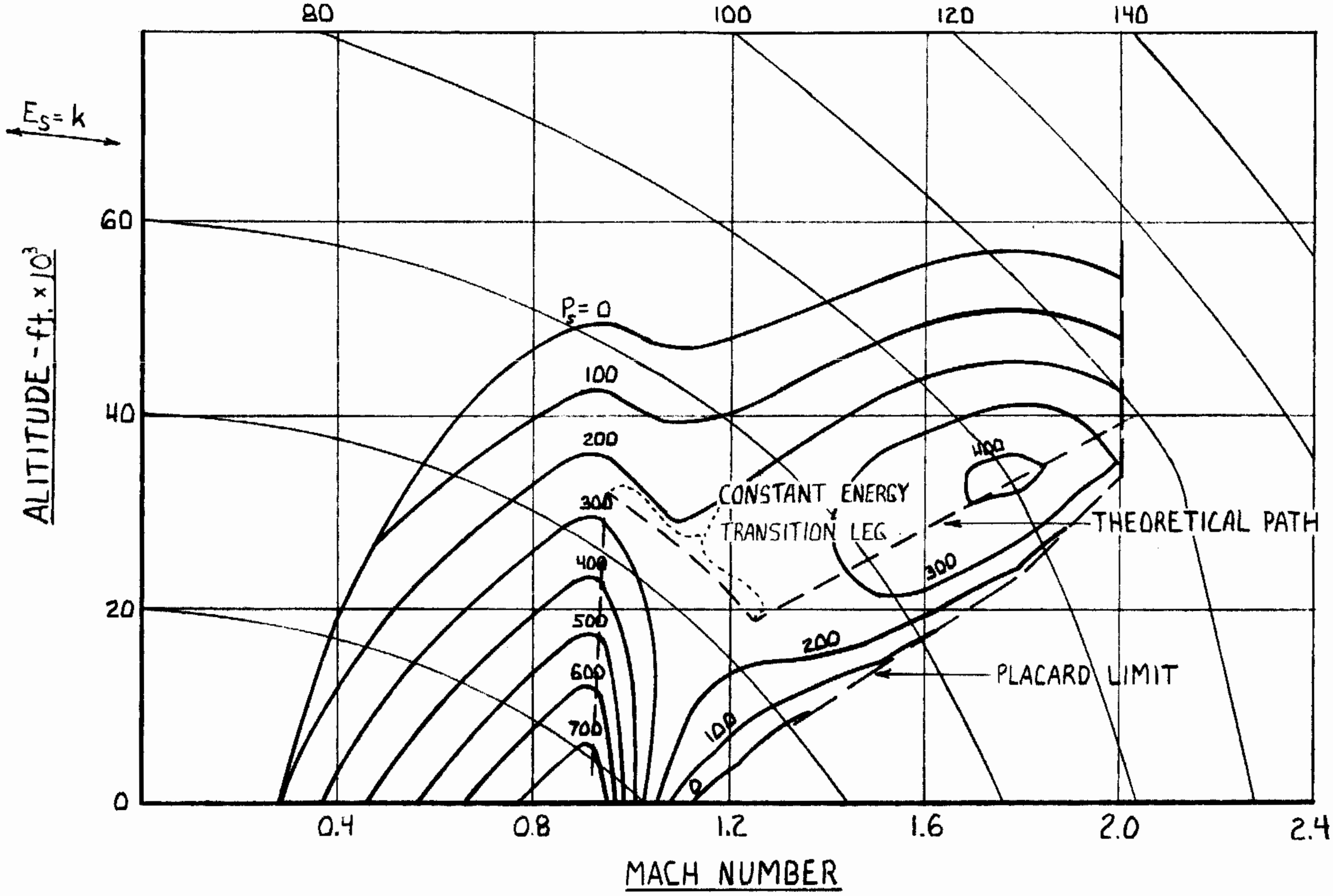


FIGURE 9 - THE THEORETICAL RUTOWSKI PATH SHOWN ON THE F-104C 1-9 ENERGY-RATE DIAGRAM

A means of handling optimum paths with variable load factor must be developed. The present methods are applicable to situations where a constant load factor is assumed for the entire flight envelope.

In general the energy maneuverability method is not highly flexible, but provides a means of obtaining approximate minimum time or fixed throttle minimum fuel solutions. The solutions are within ten percent of those obtained with a gradient technique. Required computer time using the energy maneuverability program was found to be 1.4 minutes for a comparison case given in reference 16 , as opposed to 92 minutes for the gradient method.

Programming would require about 5,000 x M words of storage, where M is the number of throttle settings required to describe the variable throttle adequately. If M is larger than two (and it may very well be in this application), then the storage problem will be acute for an airborne computer.

(f) Stored Solutions

The concept of stored solutions is distinct from the other methods discussed here in that one is now concerned with the selection of a particular solution rather than determining a solution. First of all, solutions must be calculated for each case for all significantly different sets of circumstances. Second, the solutions must be synthesized to provide flight profile command paths for any point in state space. Having done this, there are at least two approaches which may be followed.

In one case a family of solutions can be used to directly generate a flight profile [37]. The form of the profile is given as follows:

$$h = \sum_{i=1}^{N} C_i M^i (h) \qquad (41)$$

For a two dimensional state vector $\binom{h}{M}$, the profile for the aircraft case might have the form:

$$h = C_o + C_1 M + C_2 M^2 + C_3 M^3 + \ldots\ldots \qquad (42)$$

The storage of a finite number of constants - perhaps five, would characterize the required flight profile for the entire state space for each family of solutions.

A second approach [37] considers dividing up the state space into segments in each dimension. The profiles are approximated by separate parabolic arcs in each segment. The storage requirement is of the order 3nN, where n is the number of state variables, and N is the number of sub-divisions of each dimension.

Perhaps 3 x 2 x 3 = 18 constants for each segment would be required. This technique may require smoothing at segment intersections, but will give a more accurate solution than the first method if irregular solution patterns occur or if there are more than two state variables.

In general, stored solutions have the virtue of simplicity and speed of computation. Methods of this type can be readily mechanized for real time, on-board capability. Accuracy and performance should be investigated in subsequent studies, however.

c. Optimum Flight Path Mechanization

As indicated in the previous section there are two possible routes which may be followed in defining an onboard optimum flight profile. These are the on-line energy maneuverability computer program and the stored solution approach. Because of limitations in the energy program, a technique using stored solutions was adopted. The technique is based on an approach which was suggested by Boyd [16] in his energy maneuverability theory. This approach utilizes both a Rutowski path and the Boyd transition paths as a means of defining optimal flight paths. Rather than computing the Rutowski path on board, it is proposed that this path or paths, be stored in an on-board computer. More than one path is required for each type of optimal maneuver because of expected variations due to atmospheric, configuration drag, and weight effects.

In order to describe the character of the flight path mechanization scheme a more detailed discussion of the Rutowski-Boyd optimal path synthesis scheme will be given. This will be followed by a discussion of how non-standard effects tend to alter the paths which are developed for standard conditions. Finally the requirements for the overall number of distinct paths which must be stored will be determined.

(1) Rutowski-Boyd Optimal Path Synthesis

In this discussion a Rutowski path will be defined as the extremal path which is associated with an optimum climb profile such as those shown in Figures 5 and 6. This can be a path which is determined from any of the more exact optimization programs now in existence. It is not the graphical path which arises from the Rutowski theory, however. The name Rutowski is applied for convenience' sake and for purposes of recognizing the pioneering efforts in the flight path optimization field.

In the same vein a Boyd path will be referred to as an extremal arc with at least one end point that does not lie on a Rutowski path. Equations (39) and (40) describe the Boyd paths in terms of the required change in specific energy for a desired change in altitude. In his studies of flight path optimization Boyd has shown that combinations of Boyd and Rutowski paths, as well as intersecting Boyd paths result in near optimal maneuvers. Unfortunately, the concept has only been validated in the subsonic region, however. In this application it will be assumed that Boyd paths also exist in the supersonic region, even though these have not, as yet, been defined.

Examples of optimum paths between arbitrary end points using combined Boyd-Rutowski paths are shown on Figures 10 through 13. The examples shown are for minimum time maneuvers. As indicated, there are four types of paths which are of interest. Two of these are cases where the initial path is a Boyd path and the final path is a Rutowski path (Figure 10), and vice versa (Figure 11). The remaining two are cases where both the initial and final paths are Boyd paths. A Rutowski path may be intermediate between the two Boyd paths (Figure 12) or it may not (Figure 13).

It is clear from these figures that a Boyd path is either a dive to increase velocity or a zoom to increase altitude. Equations for the intersecting dive-climb and climb-dive arcs which represent the curves shown on Figure 12 are given as follows in terms of altitude and velocity:

Dive-Climb

Initial Dive:

$$h = h_o + \frac{V_o^2 - V_1^2}{4g} \tag{43}$$

Subsequent Climb:

$$h = h_1 + \frac{V_1^2 - V^2}{2/3\ g} \tag{44}$$

Climb-Dive

Initial Climb:

$$h = h_o + \frac{V_o^2 - V_1^2}{2/3\ g} \tag{45}$$

Subsequent Dive:

$$h = h_1 + \frac{V_1^2 - V^2}{4g} \tag{46}$$

These equations are derived in Appendix I.

The Boyd-Rutowski optimization technique, as proposed for use here, has

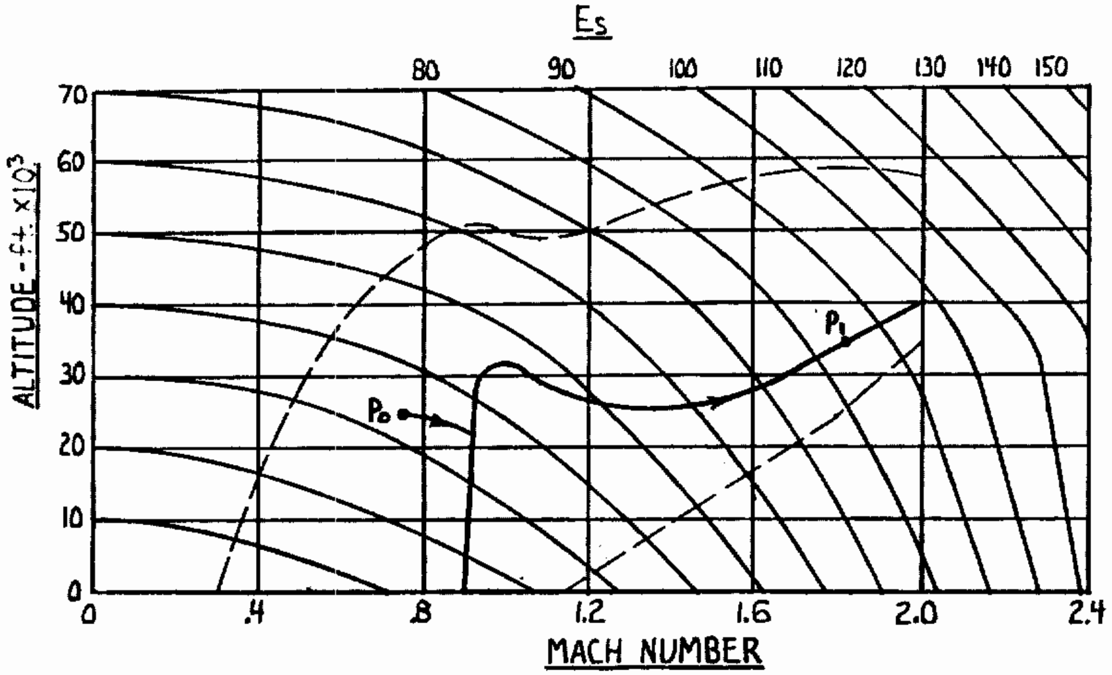


FIGURE 10 - BOYD START - RUTOWSKI FINISH

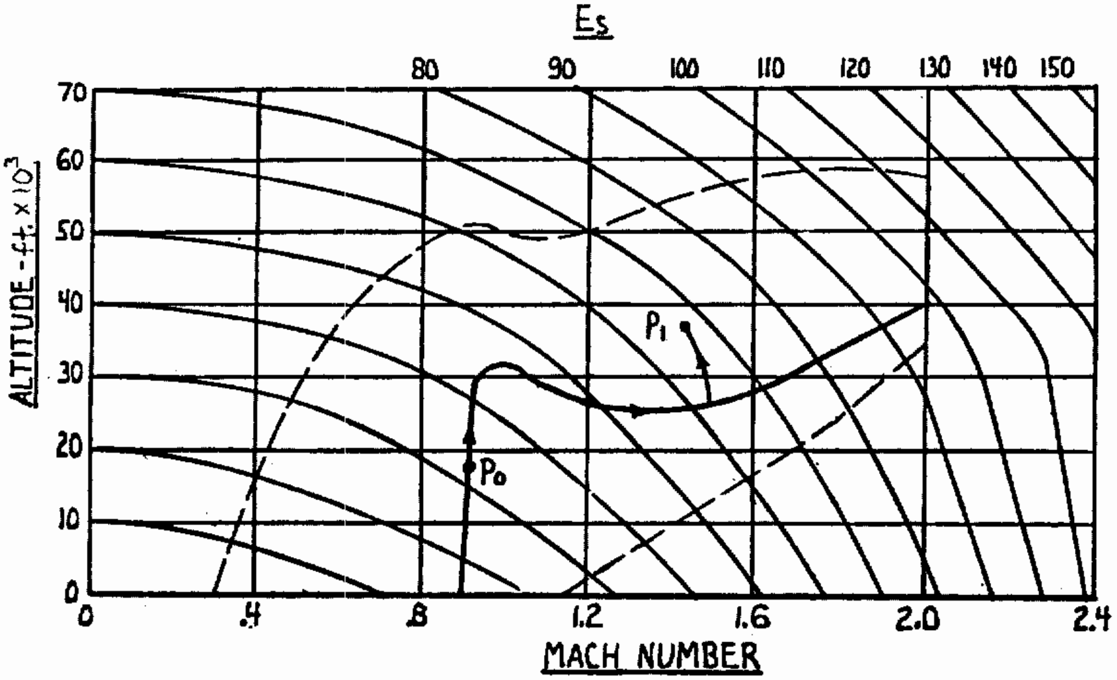


FIGURE 11 - RUTOWSKI START - BOYD FINISH

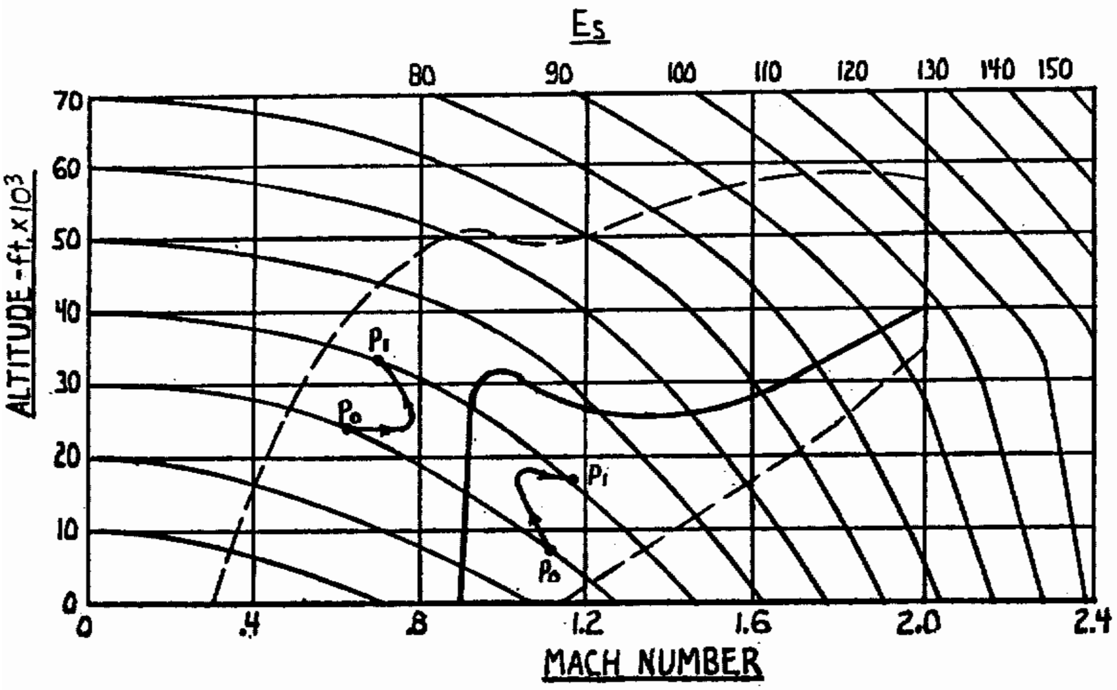


FIGURE 12 - BOYD START AND FINISH - NO RUTOWSKI CROSSING

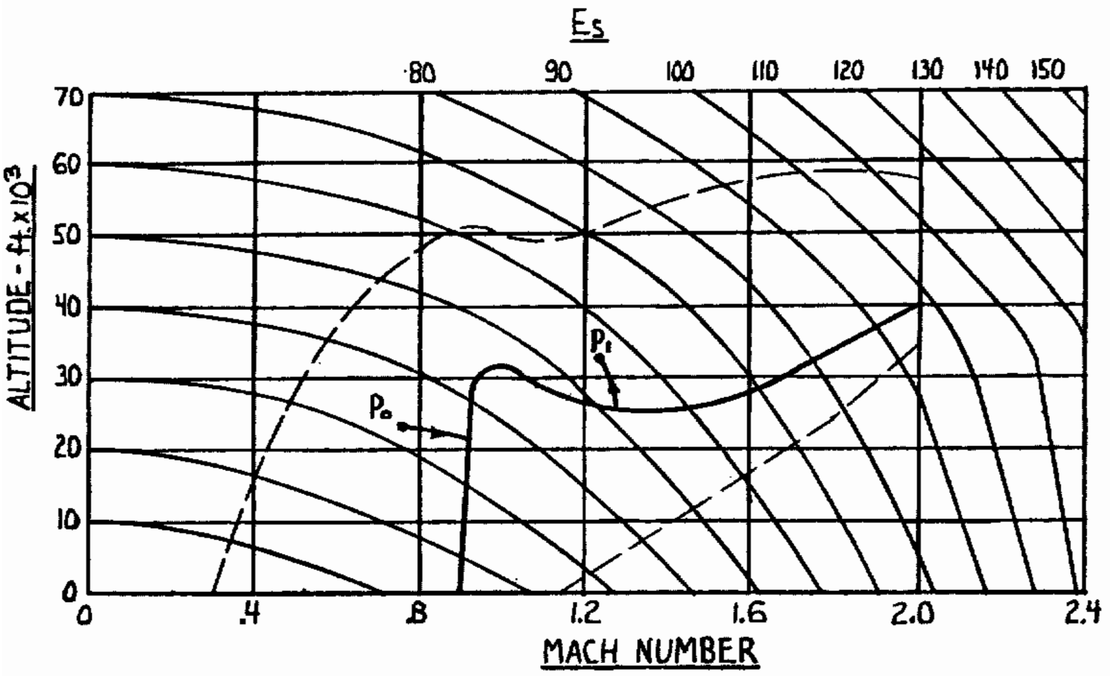


FIGURE 13 - BOYD START AND FINISH - RUTOWSKI CROSSING

not been completely validated as a feasible concept. In addition, it is not completely clear that this is the best method for storing flight profiles. It may be that the Longmuir and Bohn approach [37] is better, for instance. Available time and information was inadequate, in this regard, to make a valid comparison. It will be shown, however, that an operational system can be defined using the Boyd-Rutowski technique. Therefore, in terms of an intent to demonstrate system feasibility, the technique is more than adequate. Since it is clear that a Longmuir and Bohn approach is operationally feasible there is no question, then, as to system feasibility. The Boyd-Rutowski technique was chosen because the approach has the virtue of great simplicity and speed of computation. Its accuracy and sensitivity should be investigated in subsequent studies and compared with the Longmuir and Bohn approach. The latter method appears to be more accurate and have a greater development potential.

(2) Non-Standard Effects

An idea of the effects of non-standard conditions on flight path optimization can be obtained by examining the basic aircraft optimization parameter, P_s. It will be recalled that this parameter is given by the equation:

$$P_s = \frac{(T-D)\,V}{w} \qquad (35)$$

Given that V is dependent on the other three parameters, the quantities that determine the value of P_s are the thrust, drag, and weight. At a given flight condition the primary factor that influences thrust is ambient temperature. Exclusive of maneuvers, the primary factors that influence drag are aircraft frontal configuration and weight. Weapons loadings produce various frontal configurations and contribute to an increase in the zero lift drag coefficient, whereas weight contributes directly to induced drag. Weight, in addition to its influence on induced drag, is a direct factor in equation (35). The three main factors, then, that influence P_s contours, and therefore optimum flight path profiles, are ambient temperature, weight, and configuration drag. Each of these will be discussed in the following sections.

(a) Ambient Temperature

Variations in ambient temperature can have a significant effect on the flight profile of an air breathing aircraft. This can be illustrated by a consideration of the effect on SST climb profiles [38]. Five altitude-time profiles are shown on Figure 14 for as many different temperature profiles. The temperature profiles used are the International Standard Atmosphere, with ± 15°C and ± 30°C variations. All the flight profiles are based on the single climb schedule shown in the inset. It will be noted that there are variations of as much as 18,000 feet from one profile to another at a particular time point. In addition, it will be noted that the time to climb to 40,000 feet varies from ten to forty minutes. These are

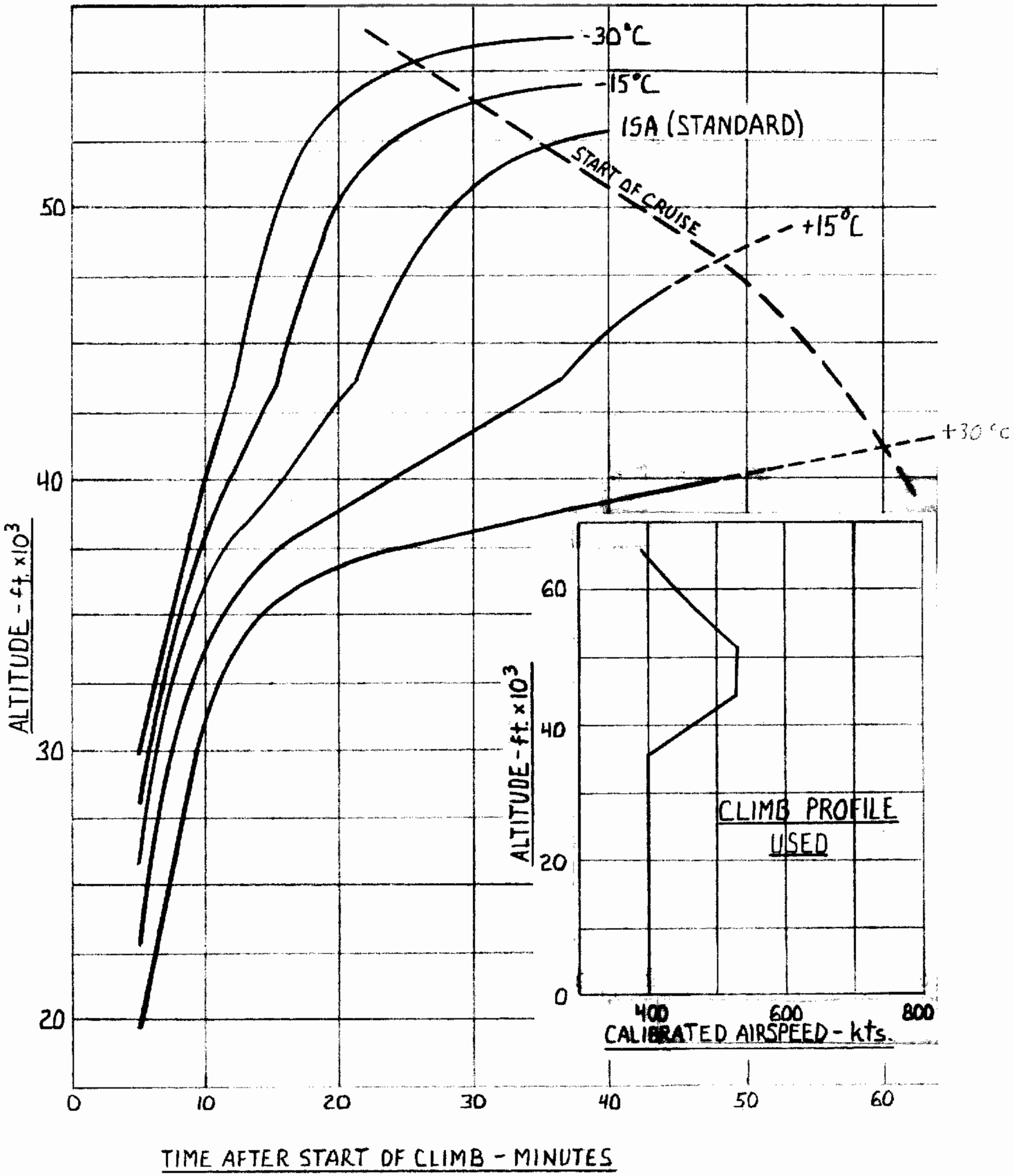


FIGURE 14 - SST HEIGHT/TIME VARIATIONS AS A FUNCTION OF TEMPERATURE

clearly significant variations. This is particularly true in minimum time applications.

To insure that these conclusions are of practical interest, mean temperature profiles for two widely separated localities are shown on Figure 15. These are mean profiles for North Atlantic Winter and for Tropical conditions. With mean variations from ISA in excess of 20°C it is obvious that individual day variations will be substantially larger.

With the wide variations in performance shown here, profile data for five temperature conditions will be stored in the computer. These will be ISA, ISA ± 15°C, and ISA ± 30°C. It will be presumed that interpolation between profiles will make it possible to simulate conditions such as those shown on Figure 15.

In actuality one would like to predict the temperature lapse rate, at altitude, as a function of ground measurements. As far as is known, this is not presently possible. In order to effectively optimize a flight profile, however, it is necessary to know, in advance, the entire environment through which the flight will pass. Point optimization will not lead to a totally optimized profile. Therefore, it seems worthwhile to consider this as a possible means of improving the operational capability of the system.

(b) Weight

The variation in an optimum profile with weight is a consequence of the change in induced drag which results from changes in weight. This is readily seen by noting the equation for drag coefficient which is given as follows:

$$C_D = C_{D_o} + C_{D_i} = C_{D_o} + \frac{C_L^2}{\pi \mathcal{A} e} \qquad (47)$$

Now, the lift generated during a maneuver is given by the equation

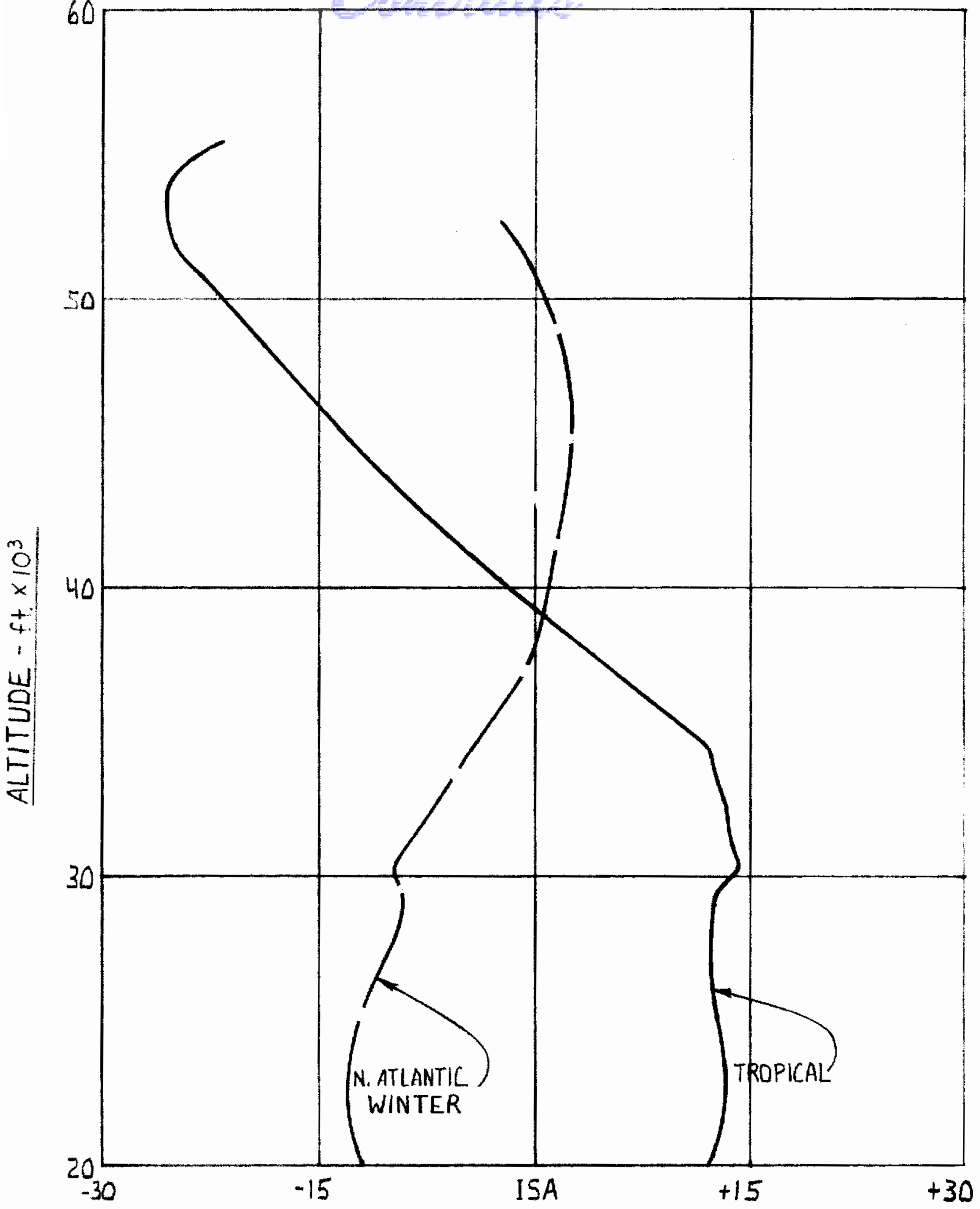
$$L = nw \qquad (48)$$

where n is the load factor. And since

$$L = C_L qS \qquad (49)$$

substitution into (47) yields

$$C_D = C_{D_o} + \frac{L^2}{q^2 S^2 \pi \mathcal{A} e}$$



TEMPERATURE DEPARTURE - °C

FIGURE 15 - COMPARISON OF MEAN TROPICAL AND NORTH ATLANTIC WINTER TEMPERATURE PROFILES WITH THE INTERNATIONAL STANDARD ATMOSPHERE

or

$$C_D = C_{D_o} + \frac{n^2 w^2}{q^2 S^2 \pi \mathcal{AR}\, e} \qquad (50)$$

It is obvious then, that the drag coefficient, and thus the drag, is proportional to the square of aircraft weight. It is not completely obvious how this affects an optimum flight profile, however.

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If a minimum time climb is used as an example, the energy rate diagram shown on Figure 9 can be used for discussion purposes. If the drag is increased through increased weight, it is obvious from equation (35) that the value of P_s will decrease. This can be seen by substituting the expression

$$D = C_D\, qS \qquad (51)$$

into equation (35) and then by substituting equation (50) for C_D. The result is:

$$P_s = \frac{(T - C_{D_o}\, qS)\, V}{w} - \frac{n^2 wV}{qS \pi \mathcal{AR}\, e} \qquad (52)$$

The term of interest is the last one on the right. This term shows that for a given set of flight conditions, P_s varies by an additive constant which is directly proportional to weight. Contours of constant P_s then, will move up or down, on Figure 9, directly as a function of weight.

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As stated earlier, the effect of this is not completely clear. The only evidence which is presently available is the effect of load factor on a climb profile. Figures 16 and 17 are energy rate diagrams for the F-104C for load factors of three and five g's,respectively. If the Rutowski minimum time paths are drawn on these figures as indicated (i.e. points where E_s contours are tangent to P_s contours are connected), comparison with Figure 9 shows that the initial part of the climb is not appreciably affected by changes in load factor. It would appear that transition to supersonic speeds would occur at a lower altitude, however.

Since both weight, w, and load factor, n, appear in the numerator of the rightmost term of equation (52) it seems likely that either would have about the same effect on P_s. It is obvious, of course, that the factor n is to the power two, while w is only to the power one. Regardless of this, Figures 16 and 17 can be interpreted as showing the effect on P_s due to alterations in the rightmost term. If this is accepted, it appears that the most obvious change in an optimum profile, due to changes in weight, will be a lowering of the transonic and supersonic portions of the path.

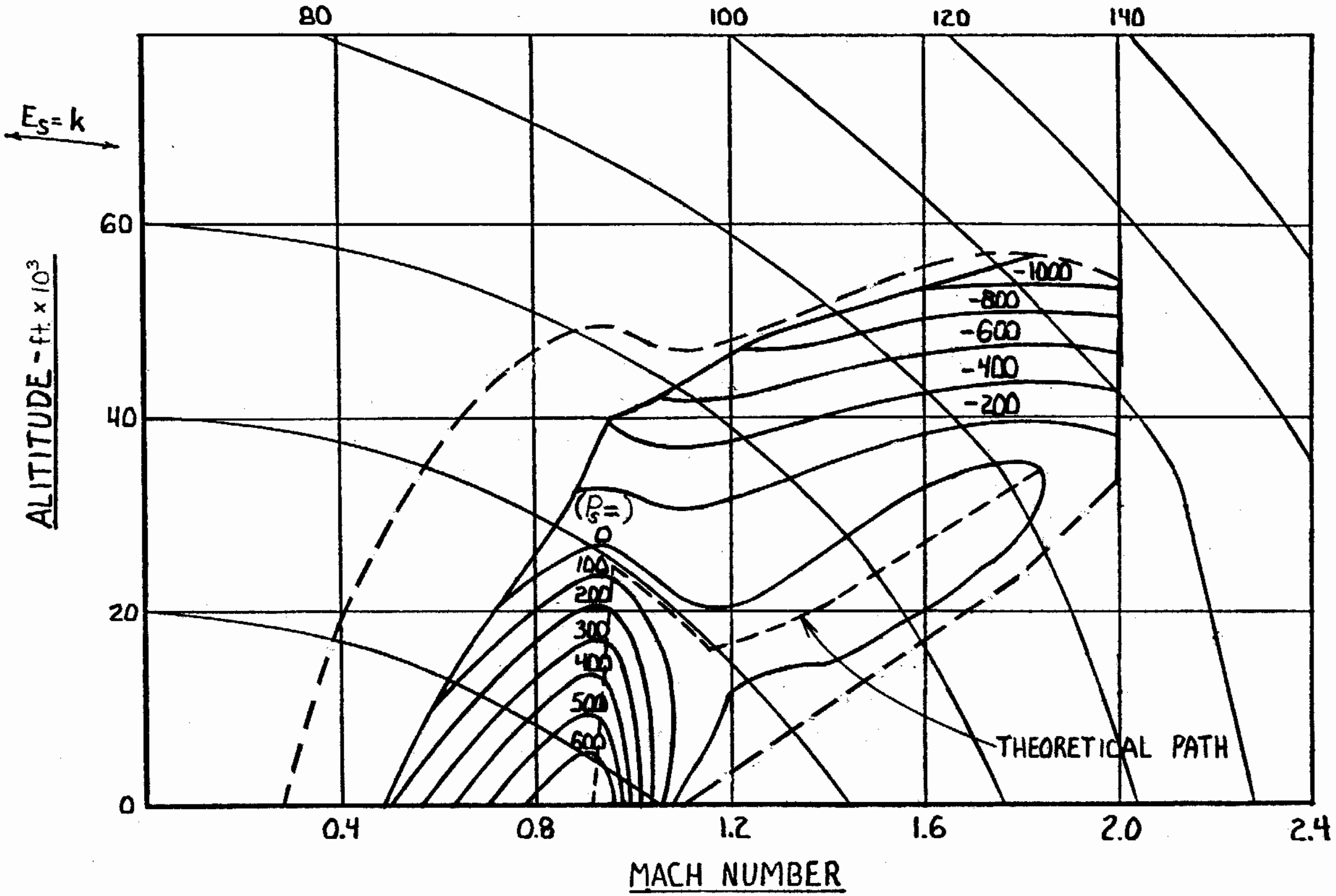


FIGURE 16 - F-104 C MAXIMUM POWER 3-g ENERGY RATE DIAGRAM

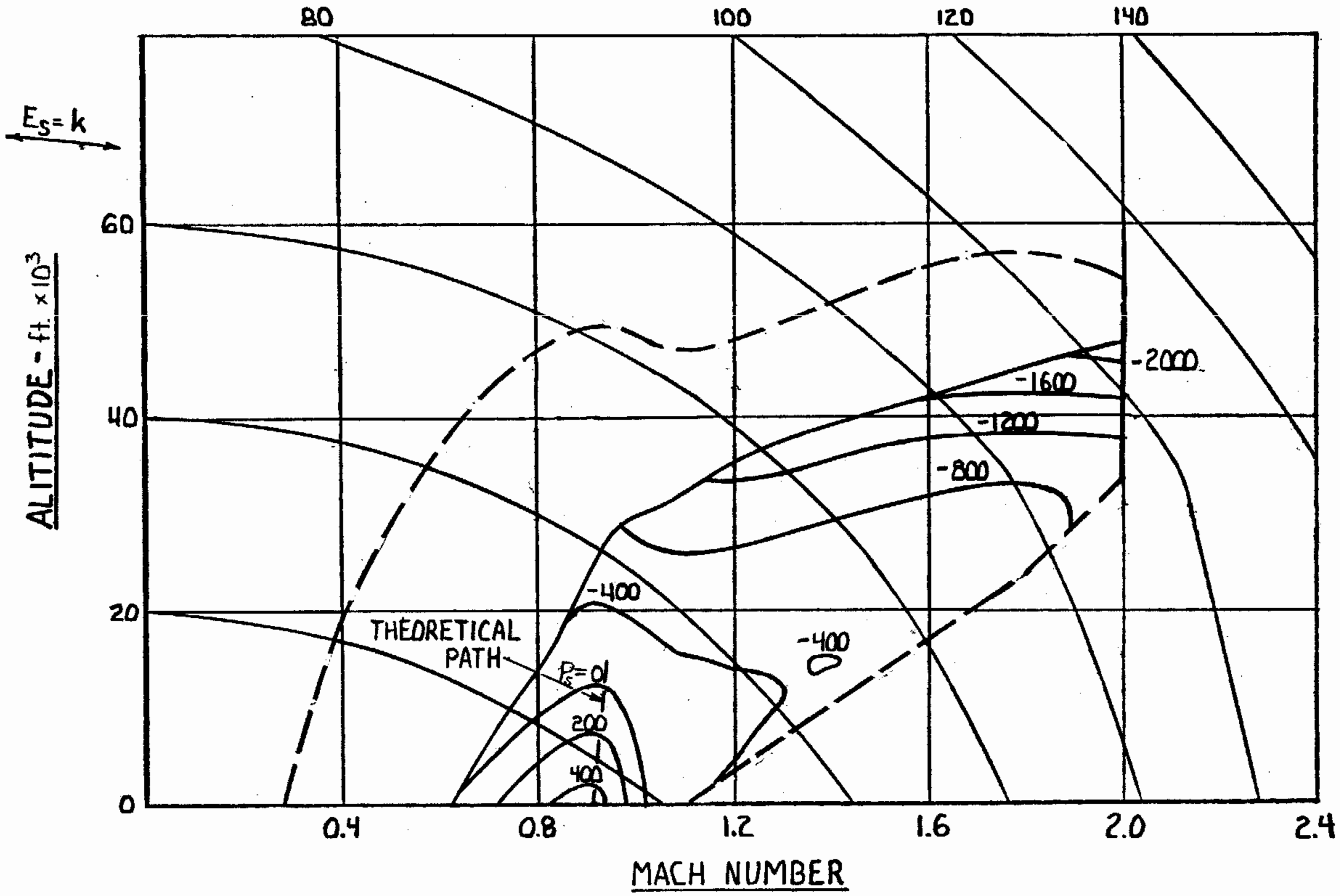


FIGURE 17 - F-104 C MAXIMUM POWER 5-g ENERGY RATE DIAGRAM

(c) Configuration Drag

Changes in configuration drag (i.e. changes due to weapons stores, tip tanks, etc.) have their largest effect on the term C_{D_o} in equation (47). If equation (52) is rewritten to isolate this factor, the result is:

$$P_s = \frac{(T - C_{D_i} qS)V}{W} - \frac{C_{D_o} qSV}{W} \qquad (53)$$

where C_{D_i} has been substituted for the right hand term in equation (50). Changes in P_s, then, are directly proportional to changes in C_{D_o}. It would appear, therefore, that the conclusions regarding changes in the flight profile due to C_{D_o} are the same as those pertaining to weight. In other words, the main effect will be a lowering of the transonic and supersonic portions of the path.

It is unfortunate, that direct data showing the influence of weight and configuration drag on optimum flight profiles is not available. Figures 16 and 17 which are for load factor variations, indicate much larger changes in the energy rate diagrams than one would expect from either weight or configuration drag.

For purposes of the present study, profile data for three different drag configurations will be stored in the computer. It will be assumed that interpolation between profiles is possible, and that the effects of either weight or configuration drag will be contained in these profiles. Previous discussions here, provide a convincing argument for this latter point in that changes in drag are directly proportional to separate changes in either weight or C_{D_o}.
Therefore, the effect which results from a change in weight can also be interpreted as an effect due to changes in C_{D_o}, and vice versa.

(3) Stored Profile Requirements

In order to determine profile storage requirements, consideration must be given to the types of descriptors which may be used to define a flight path, and to the types of descriptors which may be used for command purposes in following an optimum profile. Each of these considerations will be reflected in the following paragraphs. A more thorough justification for the choice of profiles will be given in part 2 of this section where flight path tracking research is described.

(a) Minimum Time Maneuvers

Flight profile descriptors which were considered for minimum time maneuvers are listed as follows:

- Pressure Altitude
- True Altitude
- Mach Number
- True Airspeed
- Ground Speed
- Calibrated Airspeed
- Normal Acceleration
- Flight Path Angle
- Pitch Angle
- Angle of Attack
- Angular Rates
- Time

From a control point of view the most useful of these are Mach Number, normal acceleration, angle of attack, and pitch angle. For this particular application it appears feasible to provide the pilot with a pitch angle command which is generated from an altitude - Mach number profile through the use of a control law. Other alternatives which were considered involved the use of either normal acceleration, flight path angle, or angle of attack as possible command quantities.

Pitch angle is a more meaningful control quantity to a pilot than is flight path angle. The former is under direct control of the pilot while the latter is controlled in a more indirect way. While the pilot can control both normal acceleration and angle of attack in a direct manner, time histories of these quantities are much more erratic than is pitch angle during a given maneuver. Therefore, pitch angle appears to be the best command variable. As stated previously, a control law can be devised which will allow direct computation of a commanded pitch angle from an altitude/Mach number curve.

(b) Minimum Fuel Maneuvers

Flight profile descriptors are the same for the minimum fuel case as they are for the minimum time problem, with the addition of throttle position and thrust magnitude. The minimum time problem is, by its very nature, a maximum throttle setting maneuver, whereas a minimum fuel maneuver will almost certainly involve continuously variable throttle settings.

Since the tracking task is the same as for the minimum time case, a pitch angle command, generated from an altitude/Mach number profile, will be used for this case as well. An additional altitude/throttle setting profile will also be provided.

Throttle setting will be assumed to be an independent variable relative to maintaining the correct speed along a minimum fuel path. It will be assumed that the primary means of controlling speed will be through pitch angle control. Because of the expected wide variation in throttle setting during a given maneuver (see Figure 5), it may be that automatic throttle setting is preferable. This will be commented upon further, later on.

(c) Maximum Range Maneuvers

1,15

Maximum range maneuvers involve much slower changes in state variables than do either minimum time or minimum fuel maneuvers. As indicated on Figure 3, maximum range cruise altitude is almost a linear function of gross weight. Cruise Mach number is almost a constant and is only slightly dependent upon drag configuration. Pitch command data is no longer required, and the task is one of flying at an altitude which varies as a function of gross weight. Throttle adjustments would be made to maintain a constant Mach number. Two types of profiles are therefore required: altitude/gross weight, and throttle setting/Mach number. [illegible]

(d) Maximum Endurance Maneuvers

Maximum endurance flight is very similar to maximum range maneuvers. The major difference, as indicated on Figure 4, is that the optimum Mach number varies with altitude. Therefore, three types of profiles are required. These are altitude/gross weight, altitude/Mach number, and throttle setting/Mach number.

(4) Requirements Summary

In summary then the following types of profiles are required:

Minimum Time Maneuvers

- Altitude/Mach Number

Minimum Fuel Maneuvers

- Altitude/Mach Number
- Altitude/Throttle Setting

Maximum Range Maneuvers

- Altitude/Gross Weight
- Throttle Setting/Gross Weight

Maximum Endurance Maneuvers

- Altitude/Gross Weight
- Altitude/Mach Number
- Throttle Setting/Mach Number

As indicated in the section named Non-standard Effects, these profiles must be stored for three drag configurations, and five temperature profiles. Since there are eight separate profiles listed here, a total of 8 x 3 x 5 = 120 profiles must be stored. Computer requirements for this will be commented upon in Section VII.

2. SYSTEM PATH TRACKING REQUIREMENTS

Path tracking requirements will be determined by considering path characteristics, aircraft response characteristic, human path tracking performance, tracking control laws, and improvements which can be realized through quickening and smoothing. Each of these topics, with the exception of path characteristics, will be discussed in the following sections. Path characteristics have already been thoroughly discussed.

A system block diagram is given in Figure 18. The relationship between the various factors listed above is clearly shown here. The diagram is drawn primarily to show perturbation effects. Entire functions (i.e. functions which include both the standard and perturbation values of a variable) are shown as inputs to the computer. A discussion of how these factors interact will follow. Since the dynamic manner in which these interactions occur is not well understood, the discussions will be more qualitative than quantitative.

a. Aircraft Response Characteristics

In order to determine an aircraft's longitudinal response characteristics, changes in the state of the aircraft due to throttle and elevator control deflections must be considered. Almost every treatment of the problem, in the past, has been concerned with attitude response characteristics, only. Since the path to be tracked, in this case, involves controlling velocity, both attitude response as well as velocity response must be considered.

In order to do this, one must start with the basic equations of motion and proceed to the development of system transfer functions. This has been done, and it involves a very lengthy procedure. The details and results are included as Appendix II.

The transfer functions for pitch angle and angle of attack are given as equations (221) and (222) in this appendix. As shown, the denominator, or attitude response characteristic equation, of each of these transfer functions is fourth order. The roots of this equation establish the response of the aircraft to attitude perturbations. In general there are two sets of complex conjugate roots, one describes a long period lightly damped oscillation, while the other is of much higher frequency and is relatively well damped. These are called the phugoid and short period modes, respectively.

In general, angle of attack disturbance is negligibly small in the phugoid mode, and speed disturbance is negligible in the short period mode. Since the present task involves controlling speed through pitch changes, controlling phugoid mode dynamics is certainly desirable. In addition, it is quite likely that short period mode dynamics will also be of importance when pilot transfer characteristics are included. Therefore, the frequency range of interest from the standpoint of control, covers both the phugoid and short period modes.

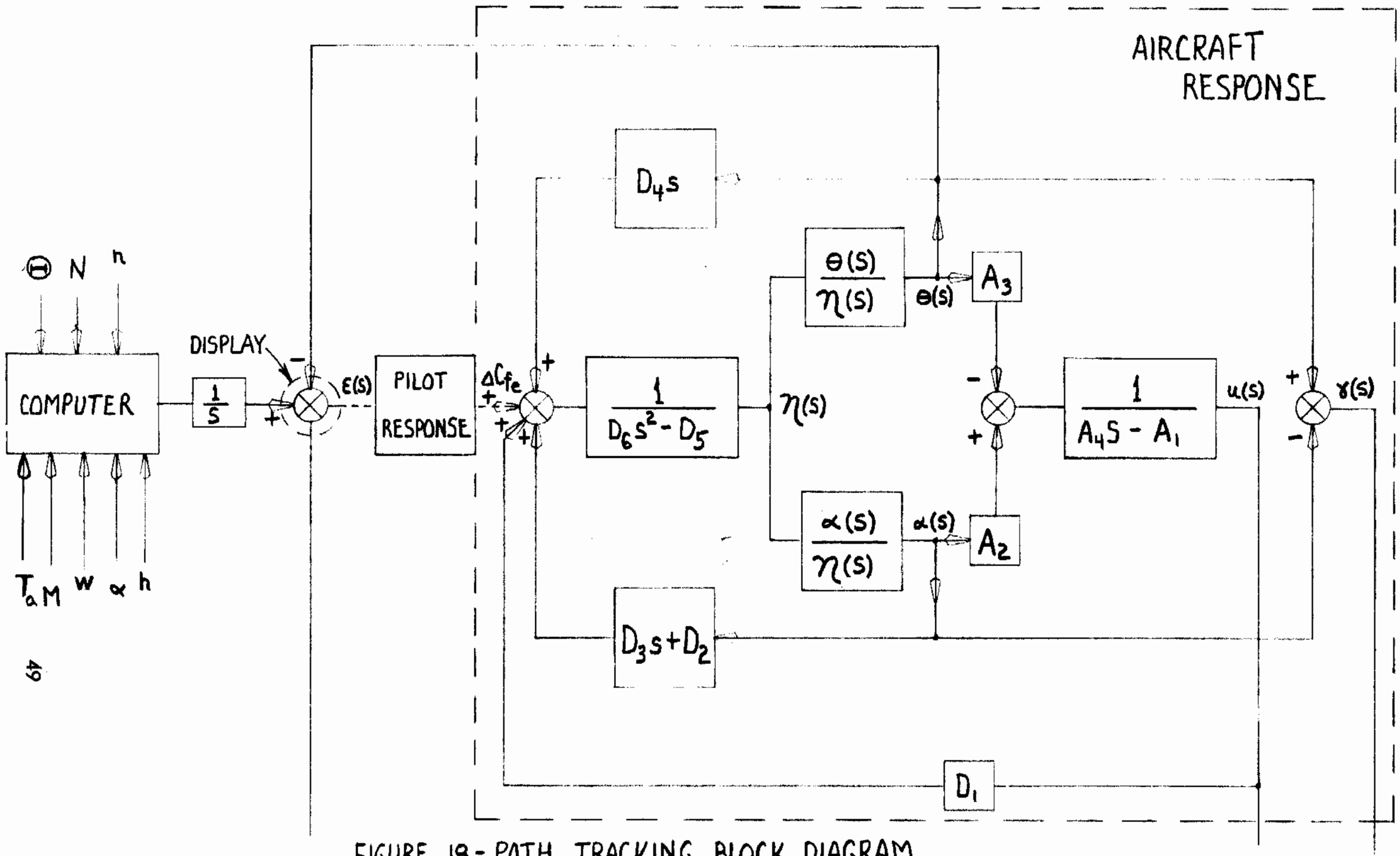


FIGURE 18 - PATH TRACKING BLOCK DIAGRAM

The system block diagram on Figure 18 shows that changes in velocity result from changes in both pitch angle and angle of attack. The equation which shows this is equation (217) of Appendix II and is rewritten here as follows:

$$u(s) = \frac{A_2 \alpha(s) - A_3 \theta(s)}{A_4 s - A_1} \tag{54}$$

b. Human Path Tracking Performance

Human path tracking response characteristics are a large function of the type of task being performed. Factors which are known to influence performance include display presentation, nature of the input signal, external stresses, number of practice trials, type of control, motivation, fatigue, and certain personal variables. Attempts to develop a human transfer function have met with increasing success over the last several years, but research is still far from complete.

One of the linear models which has been suggested takes the form [39]

$$Y_p = \frac{K_p e^{-\tau s}(\tau_L s + 1)}{(\tau_N s + 1)(\tau_I s + 1)} \tag{55}$$

where the time constants τ_L, τ_I, and τ_N, represent anticipation, smoothing, and neuro-muscular lag. Representative values of the various quantities are

τ = 0.2 sec. ± 20%

τ_L = 0 to 2.5 sec

τ_I = 0 to 20 sec

τ_N = 0.1 sec ± 20%

K_p = 1 to 100

K_p, τ_L, and τ_I are varied by the performer to suit the task. For example, evidence [40] indicates that for a single control, single display task, the transfer function varies as follows:

$$Y_p = \frac{K_p e^{-\tau s}}{(\tau_I s + 1)(\tau_N s + 1)} \qquad \text{for } Y_C = K_C \tag{56}$$

$$Y_p = \frac{K_p e^{-\tau s}}{(\tau_N s + 1)} \qquad \text{for } Y_C = \frac{K_C}{s} \tag{57}$$

$$Y_p = \frac{K_p e^{-\tau s}(\tau_L s + 1)}{(\tau_N s + 1)} \qquad \text{for } Y_C = \frac{K_C}{s^2} \tag{58}$$

where Y_C represents the transfer function of the controlled process.

Other types of human transfer function models, which in many ways better represent actual performance, include:

Nonlinear [41]

Time Variable [42]

Discrete [43] [44] [45]

Adaptive [46]

In general all techniques are representative of the human controller in at least one task. None represents the controller in all tasks, however. Further, in multi-mode, multi-display systems, the model must be altered significantly.

c. Tracking Control Laws

In following an altitude-velocity profile, a pilot has the option of using aircraft pitch and/or throttle setting as control variables. In this application it will be assumed that the primary means of controlling velocity will be by means of pitch angle. Throttle setting will be assumed an independent influence. In other words, it will be assumed that

Velocity will be specified as a function of altitude

Throttle will be specified as a function of altitude, and altered accordingly, and

Velocity will be controlled by controlling pitch angle.

A control law is necessary, then, that relates required pitch angle to desired velocity.

In the course of the study two types of control laws have been suggested.

Each has its advantages and disadvantages. The first is given as follows:

$$\theta_c = \theta_o - K_1 (M_o - M) \frac{(t_1 s + 1)}{(t_2 s + 1)} \tag{59}$$

This law dictates that the commanded value of pitch angle, θ_c, is equal to flight path related angle, θ_o, minus a term related to Mach number error. The sign of the Mach error term is chosen such that θ_c is decreased if M is less than M_o, and vice versa. The lead-lag compensation reflects smoothing and quickening considerations. These will be commented upon further in the next section.

If this law is used, both θ_o and M_o must be specified as functions of altitude, for a desired flight path. If the aircraft is on the desired path, then

$$M = M_o$$

and

$$\theta_c = \theta_o$$

If θ_o is not exactly the value which keeps the aircraft at M_o, however, the aircraft will seek a different velocity. Therefore, it seems likely that the aircraft will oscillate in velocity.

θ_o, in fact, is related to aircraft weight, ambient conditions, and configuration drag as well as Mach Number. Therefore, at this writing it appears that there would be considerable difficulty in implementing a control law such as that given by equation (59). It may be, however, that θ_o is relatively insensitive to the various factors listed here. If so, this type of law could be quite useful. Therefore, it will remain in consideration until more definite data is available. The law has a singular advantage in that the command is exact, if θ_o is specified correctly.

The second control law to be considered is given as follows:

$$\dot{\theta}_c = K_2 (M_o - M) f(s) \tag{60}$$

With this law, an attitude rate is dictated solely by the error in desired Mach number. A value of θ_o need not be specified. Therefore variations in θ_o as a function of weight, atmospheric conditions, and drag configuration need not be considered. This would result in an immediate savings in computer storage requirements.

The disadvantage lies in the fact that a command rate is specified, rather than an angle. This, in effect, introduces 90° of lag into the control loop. (Note that $\theta_c = \frac{\dot{\theta}_c}{s}$, and that 90° of phase lag is associated with a $\frac{1}{s}$ factor). Intuitively, this means that even though the commanded angular rate is zero, the command angle may not be the value which keeps the rate at zero. If the system oscillates about the zero rate point for any length of time, however, the command soon becomes the proper value which keeps the rate zero. The lag which is introduced may be undesirable, however.

Much of the problem can be substantially alleviated by specifying proper loop compensation. This means specifying the form of the factor $K_2 f(s)$. K_2 is a gain and f(s) is a general expression for some sort of a lead-lag term. By properly simulating the tracking task it seems reasonable that a form of f(s) can be specified which will yield satisfactory performance. As stated previously considerations of smoothing and quickening should also be included here.

Of the two control laws given here the form given by equation (60) seems to offer the most promise. Therefore it will be considered as a part of the system design in future discussions. A control law cannot be specified with any certainty, of course, until a simulation of the tracking task has been completed.

d. Smoothing and Quickening

In order to ensure that the path tracking task, being considered here, is performed as efficiently as possible it is necessary to consider tracking aids such as smoothing and quickening. In terms of equation (60) this refers, in part, to specifying the form of f(s). It also may pertain to reshaping a commanded flight path so as to eliminate sharp corners and reduce the required path following bandwidth. (This latter type of compensation is sometimes called predictive aiding.)

Quickening is the situation where the command quantity and its first, and perhaps second, and higher, order derivatives are displayed to the pilot in some type of combined signal. Smoothing, on the other hand, relates to including integrals of the command quantity to the pilot. In control system language quickening introduces lead to the system, whereas smoothing introduces lag. Therefore, the numerator of f(s) would reflect system quickening and the denominator, system smoothing.

The objectives of quickening and smoothing are to reduce the pilot transfer function to a simple gain. If the system is perfectly compensated, this will allow the pilot to use his control as a simple positioning device. In the problem at hand, it will not be practical to design a perfectly compensated system since the quickening and smoothing terms will vary with the aircraft transfer function.

e. Tracking Loop Considerations

In order to establish the proper quickening and smoothing compensation, it is necessary to consider the entire path tracking task. On Figure 18 this means that consideration must be given to the dynamic character of the pilot, elevator mechanism, and airframe. The form of the compensation can be determined by ensuring that the overall bandwidth of the system is greater than that contained in the path to be tracked. This, in turn, requires a control loop analysis.

The methods for carrying out this analysis are well established and there appear to be no problems associated with doing so. The analysis has not been done, because of lack of required information, but should be considered as an important part of future efforts. This should not be considered as an end in itself however, since analysis will, at best, provide only an estimate of required compensation. The analysis should be accompanied by a simulation of the tracking task which includes an examination of different control laws and flight conditions.

SECTION VI

SYSTEM DISPLAY REQUIREMENTS

Display requirements will be evaluated by considering display variables, tracking task characteristics, and the state of the art of display technology. Each of these topics will be discussed in order in a section entitled System Display Considerations. This will be followed by a section wherein typical display formats are suggested. Next, display accuracy and update rates are evaluated and this is followed by a short discussion of pilot control panels.

The discussion of display requirements given in this section is in no way meant to be firm because of the many variables (e.g. type of control law, quickening and smoothing compensation, etc.) which must be evaluated before a final decision can be made. The suggested displays are good enough, however, to yield data which is necessary for determining computer requirements. The discussions which follow reflect these objectives.

1. DISPLAY CONSIDERATIONS

The operational objective of this study has been to utilize the concepts of flight path optimization in the vertical plane. This has resulted in the definition of four basic operating modes - minimum time, minimum fuel, maximum range, and maximum endurance. These modes have, in turn, given rise to a requirement for the display of command and situation data, which will allow the pilot to achieve an optimized flight objective.

Because of the single plane restriction, it is evident that misleading display design recommendations may result from the study. This should be recognized, and further study should be carried out to define three axis control display requirements, if necessary. Bearing in mind the possible existence of these types of deficiencies, an analysis of system display requirements has been carried out. This is begun as follows:

a. Display Variables

Stored profile requirements were called out in Section V. In addition, comments were given regarding the types of command variables which would be best for flying a particular profile. The display system can be initially defined by determining those command variables which are required for control purposes, and those which are needed for status monitoring. This is done as follows:

Minimum Time Maneuvers

Control	Monitor
Pitch Angle	Altitude

Mach Number

Minimum Fuel Maneuvers

Control	Monitor
Pitch Angle	Altitude
Throttle Position	Mach Number

Maximum Range and Endurance Maneuvers

Control	Monitor
Altitude	Fuel Flow Rate
Mach Number	
Throttle Position	

As indicated in Section V, pitch angle has been selected as the primary control parameter for minimum time maneuvers. Thrust will be constant (maximum afterburner), and Mach number (or airspeed) and altitude will be treated as dependent variables.

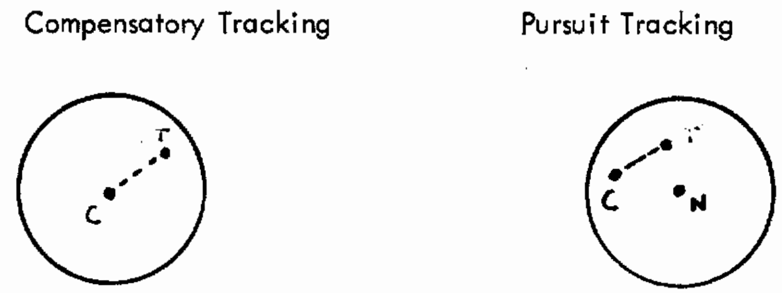
Similar comments apply to the minimum fuel case except that the throttle will be variable. As suggested in Section V, there seems to be a strong case for an automatic throttle due to the wide variation in throttle setting in a typical minimum fuel maneuver.

Maximum range and endurance maneuvers place different requirements on the display system. These requirements are generally far less stringent, however, because the rates of variation are much lower. Both altitude and Mach number must be controlled, but it is felt that this can be accomplished by modifying existing dynamic display equipment already on the F-104C. What will be required is a display which shows the proper altitude and Mach number which the particular gross weight situation demands. Modifications of existing displays should then be adequate for achieving and maintaining the proper cruise flight conditions. After the required altitude has been attained, Mach number can be controlled with throttle adjustments.

b. Tracking Display Requirements

Prior to deciding on a particular display, it is necessary to consider whether a pursuit or compensatory type is to be used. Pursuit displays are those in which the locations of both the target and the controlled element are shown. A compensatory display is one where only the difference between the target and controlled element is shown. The two displays are shown conceptually on Figure 19 [47].

The factors which affect the selection of one or the other are task complexity, nature of the input, machine dynamics, and the order of the system being controlled. When there is only one desired task objective, either kind of display is satisfactory. Pursuit displays become relatively better, however, as the operator begins to time-share his actions with other tasks. As the bandwidth of the tracking input increases the advantage also begins to shift to pursuit displays. For low bandwidth signals, however, there seems to be little



Compensatory Tracking

C(Control): fixed
T(Target) : movable
C---T : error

Pursuit Tracking

C(Control): movable
T(Target) : movable
N(Null) : fixed
C---T : shows difference in relative locations

Figure 19 TRACKING DISPLAY CLASSIFICATIONS

significant difference in performance. Similar comments apply to the order of the control device. Comparisons of the performance experienced with a zero order position controller with rate (first order), acceleration (second order), and higher order controllers indicate that performance degrades as the order of the controller goes up. As the task becomes more difficult, however, the degradation with pursuit displays is less than that for compensatory displays. For the simpler tasks there is little difference, however.

The basic advantage in pursuit tracking is that the operator has the opportunity to know what course the path has followed, and can frequently make predictions about the future course. There is no consistent superiority of either compensatory or pursuit tracking under all circumstances, however. If the task is relatively simple, either display is satisfactory and considerations involving items like cost and mechanization will probably determine the final choice. As the task gets more difficult, pursuit displays are generally superior.

These factors are considered next in selecting the type of display which is best for the various kinds of tracking tasks under consideration.

(1) Minimum Time and Minimum Fuel Maneuvers

The two command quantities associated with these maneuvers are pitch angle and throttle setting (recognizing, of course, that throttle setting is only important in the minimum fuel case). From Figures 20 and 21, which are histories of flight path angle, it is clear that command pitch angle will have a relatively complex history. Amplitude and angular rate variations can be expected to be relatively large — 50° and 4°/sec., respectively. Since pitch control amounts to a zero order position tracking task, these factors suggest the use of a pursuit display.

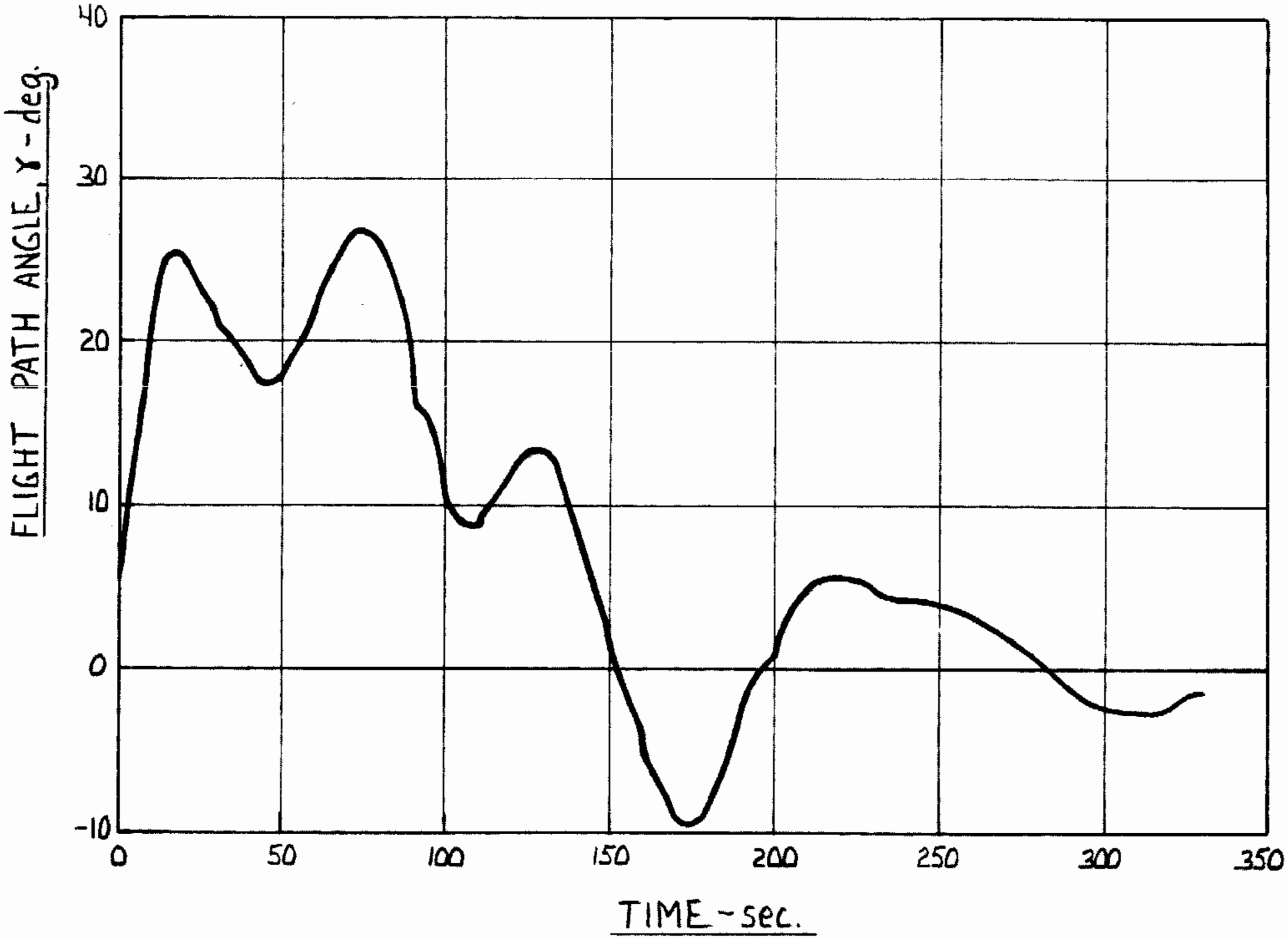


FIGURE 20 - FLIGHT PATH ANGLE HISTORY FOR A MINIMUM FUEL CLIMB (SEE FIGURE 5)

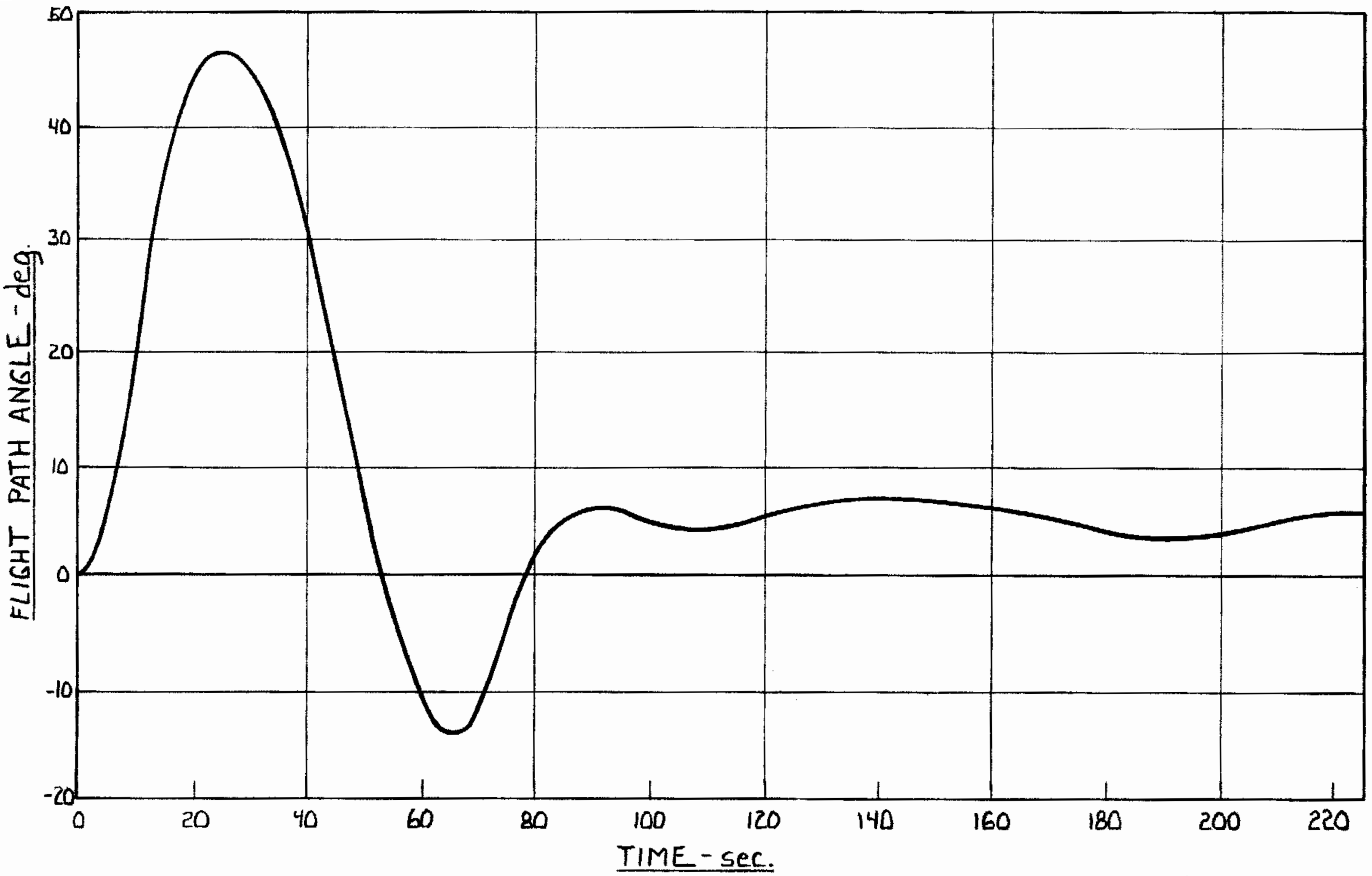


FIGURE 21 - FLIGHT PATH ANGLE HISTORY FOR A MINIMUM TIME CLIMB (SEE FIGURE 6)

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A throttle setting profile for minimum fuel climb is shown on Figure 5. A continuous throttle variation is indicated. If this is required in operational situations, serious consideration must be given to using an automatic throttle. If, on the other hand, it is shown that acceptable performance can be achieved with a set of discrete throttle changes, a warning display along with a discrete readout of throttle setting command is recommended.

The two monitored quantities associated with minimum time and minimum fuel maneuvers are altitude and Mach number. Since these quantities will not be tracked (at least in this application), a display indicating necessary status information is all that is required. Actual values of these quantities rather than simple errors are most meaningful to a pilot, therefore it is suggested that the information be presented in the form of a pursuit display. This means that both desired and actual values of Mach number and altitude should be shown on the display.

(2) Maximum Range and Endurance Maneuvers

The command parameters here are altitude, Mach number, and throttle setting. These quantities are slowly varying and will experience modest changes during a typical cruise maneuver. Therefore either compensatory or pursuit tracking displays should be satisfactory. Again it follows from operational considerations, however, that actual values of command quantities are more useful than error quantities. Therefore pursuit displays are recommended.

Fuel flow rate variations need only be monitored as a secondary task in these types of maneuvers. The requirement here seems to be one of monitoring gross weight to ensure that the computer generated altitude and Mach number commands are consistent with intuition.

(3) Summary

From the previous remarks it was concluded that, (1) pitch, Mach number, and altitude displays will be of the pursuit type, (2) throttle setting commands will be displayed in the form of a discrete readout and warning annunciator, or the throttle may be automatically controlled with a discrete readout for status monitoring, (3) fuel flow rate will be displayed for status monitoring purposes.

For minimum time and minimum fuel maneuvers the pitch display will be used for path following and the Mach number and altitude displays will be used for status monitoring. For maximum range and maximum endurance maneuvers the Mach number and altitude displays will be used for control purposes.

The throttle will be designed with both automatic and manual mode capability. The automatic throttle will be used for minimum fuel maneuvers whereas a discrete readout display with manual settings will be used in the other maneuvers. A discrete display will also be available during the automatic mode operation for status monitoring.

Fuel flow rate will be used for status monitoring during maximum range and endurance maneuvers.

c. Display State of the Art

The display requirements listed in this section are recognized to be somewhat unique in that functions are needed which have hitherto been unnecessary in interceptor aircraft. Accordingly, a display survey was initiated, and data was solicited on currently existing displays and planned developments. The discussions which follow reflect some of the background information that was obtained as well as ideas which have been collected as a consequence of association with the industry.

(1) Display Classifications

Basically there are two categories which displays can be classified under. These are head-up and head-down. Head-Up Display (HUD) development has received considerable attention in recent years as a consequence of the need for a device which simultaneously displays command and situation data and which allows the pilot to view the external visual environment. They are generally most useful in weapon delivery, landing, and terrain avoidance applications. Since the method allows the integration of situation, command and discrete data, it is apparent that this type of display may be employed in the present system. (It is doubtful, however, if an HUD could be installed in the F-104C.) There doesn't seem to be much advantage in doing so, however. Nevertheless, an alternative HUD format will be suggested in the next section. If the case for an HUD can be supported for other reasons, then a display like that suggested might be used.

Head-down displays are advantageous when the normal functions of the pilot require monitoring instrument panel readings. If the pilot is already concerned with reading these head-down instruments, it makes no sense to require that he read an HUD, as well, unless he is particularly concerned with the outside environment. Otherwise the pilot would be continuously nodding his head in order to monitor both classes of instruments.

In following an optimum flight path, it would seem that a head-down type display is better. One would normally be flying at a reasonable altitude during these maneuvers and would not normally be concerned with visually searching for enemy aircraft until the maneuver has been completed. A pilot's interest would center more on a radar display for purposes of monitoring external surroundings. There may be situations where this is not true, however. Therefore, a head-down type of display format will be recommended with an HUD as an alternate.

(2) Display Types (Head Down)

Head down displays which suggest themselves are dial/pointer and pictorial types. Dial/pointer displays are, in general, used for the display of director data, while pictorial displays are more useful as situation monitors. Dial/pointer displays are in common use in contemporary aircraft and include such instruments as attitude director indicators (ADI), Mach meters, altimeters, etc. Various types of ADI instruments have been developed for purposes of aiding in instrument landings. It seems quite reasonable that a contemporary design, of this type, could be modified to function both as a landing aid and as an aid in flying an optimum flight path.

Pictorial display of vertical situation data is a relatively new concept which is being suggested for SST and VTOL applications. The technique appears to be advantageous for this application, as well, in that it allows the simultaneous display of a number of parameters as well as trend data. This could result in the elimination of several numerical or dial pointer displays. Since the pictorial displays being developed make use of solid state devices, the devices will have an inherent flexibility which will allow the display of data pertinent to the operation of other systems. Descriptions of some typical display formats which appear to be adequate for the flight path tracking task are given in the following material.

2. DISPLAY FORMAT SUGGESTIONS

The recommendation of a display system for purposes of energy management is dependent upon operational objectives, parameters to be displayed, and display philosophy. It is also heavily dependent upon aircraft type. If the F-104C is taken as the aircraft, then a display configuration of the type shown on Figures 22, 23 and 24 is recommended. The configuration shown on Figure 22 contains an ADI. Figure 23 shows a Mach meter and an altimeter and Figure 24 is an illustration of a pictorial display.

a. Attitude Director Indicator

The ADI format which is recommended is patterned after the Lear Siegler Model 4058AC. This indicator can be used to present pitch command, speed error, and an indication of status relative to the required flight profile. It can also be used for directional steering.

The split axis profile status indicator is primarily designed for following a landing glide slope. It is recommended here for use in a dual role which includes both landing and optimum path following. For vertical plane optimization, interest is primarily in the vertical path director indicator. This indicator is programmed for optimum path tracking efficiency for the particular path and dynamic system of interest. Generally in instruments like this, the displacement of the indicator is programmed according to a law like the following:

$$\theta_R = C_1\theta_c + C_2\dot{\theta} + C_3 n + C_4 d \qquad (61)$$

Thus, the required pitch angle is a weighted function of commanded pitch angle and actual values of pitch angular rate, normal acceleration, and displacement from the

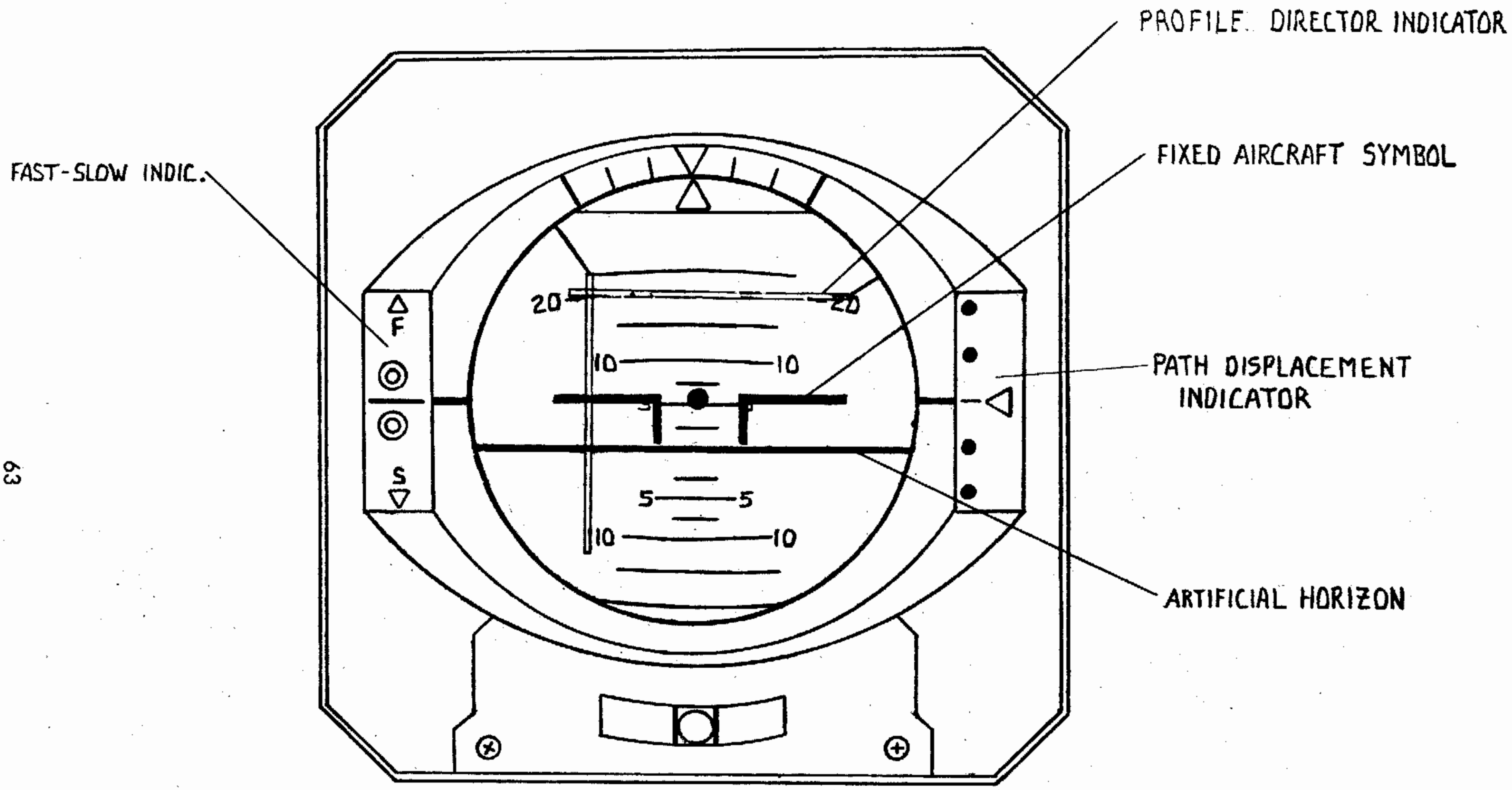

FIGURE 22 - ATTITUDE DIRECTOR INDICATOR

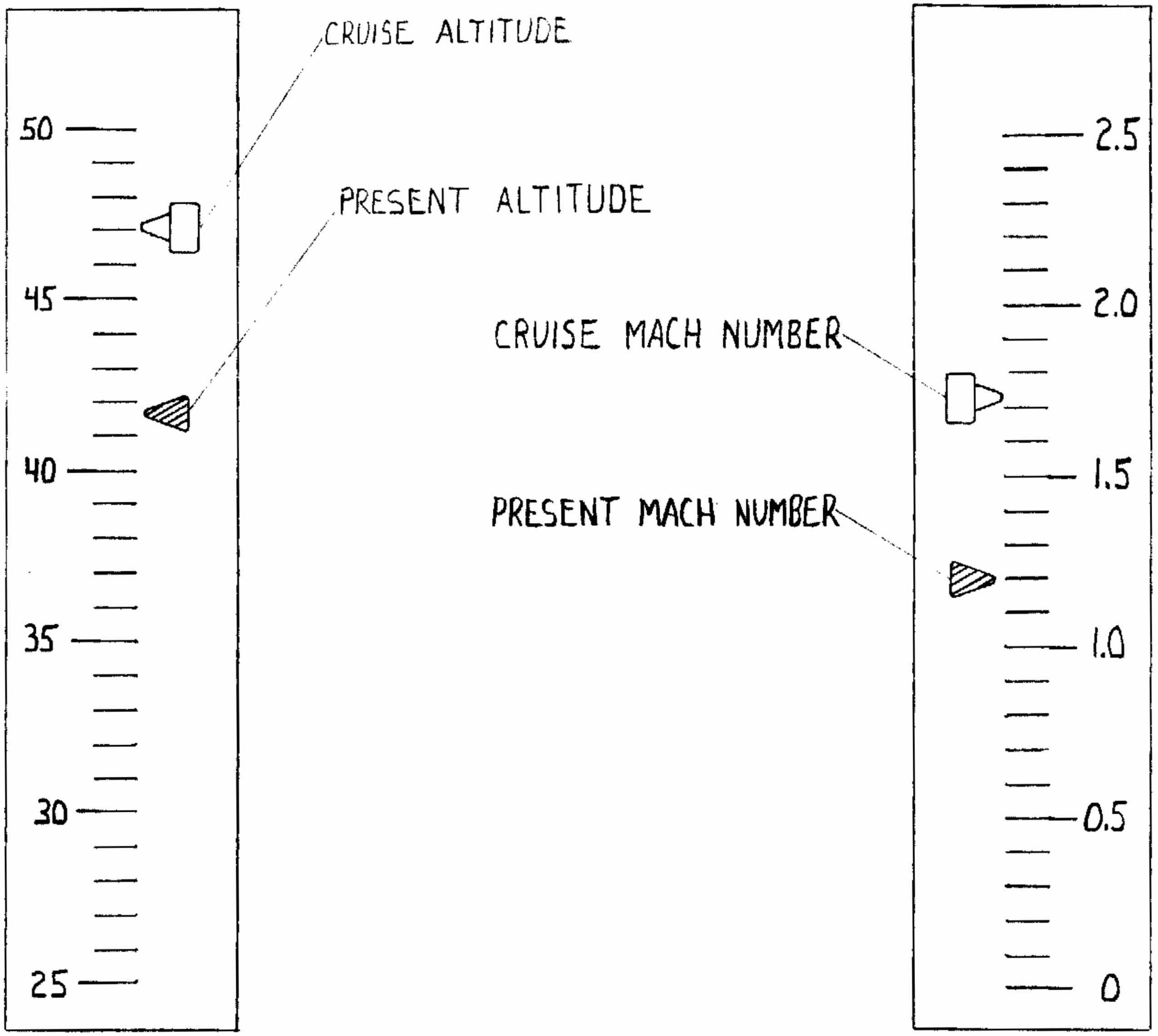

FIGURE 23 - ALTITUDE AND MACH NUMBER PURSUIT DISPLAYS

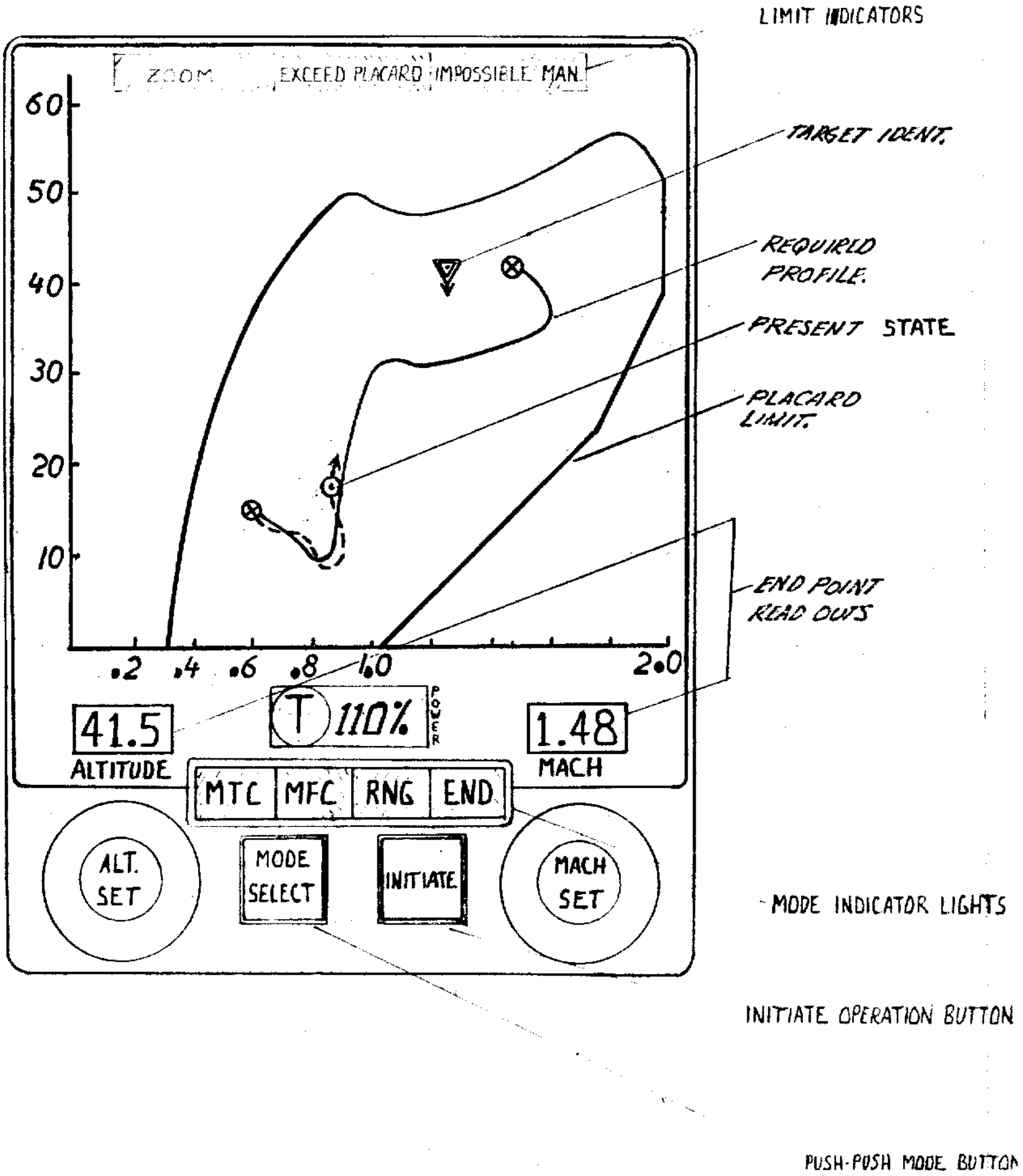

FIGURE 24 - FLIGHT PROFILE INDICATOR AND CONTROL PANEL

desired path. Therefore, it is quite possible that a zero command could be recorded while one is still somewhat removed from the flight path. A zero displacement would mean, however, that one is progressing satisfactorily toward the path. The gains associated with each of the terms in equation (61) are determined by analysis, simulation, and flight test of the particular dynamic system under consideration.

The aircraft symbol is fixed to the case, as is commonly done, and is used as a reference for on course tracking.

The fast-slow indicator is included to give an indication of speed status relative to the optimum path. The central lights go on if velocity is within a fixed tolerance. The slow light remains on if speed is slow and the fast light remains on if the speed is fast.

The pointer on the right side of the display is for the purpose of indicating high or low with respect to the flight path. This information which is additional to that already available from the ADI.

b. Mach Number and Altitude Displays

Mach number and altitude displays can be engendered by modifying or replacing existing instruments. The requirement is for an instrument display of the pursuit type showing the present value of the parameter and the current command value. It may also be desirable to display the end point value, which is characteristic of the particular maneuver, on this instrument. In the simplest case, an existing instrument would be modified by adding one computer-driven pointer and a manually-settable pointer.

c. Pictorial Display

The pictorial display shown on Figure 24 is configured in the form of a vertical situation display or flight profile indicator. The following information is shown:

- Selected altitude/Mach number profile
- Present aircraft state in altitude/Mach number space
- Current altitude/Mach number vector
- Flight path history
- Placard and steady state flight envelope boundaries
- Initial and final end point state values
- Target aircraft state

The display might also be used for showing altitude/range profiles as well as aircraft and weapon status data.

d. HUD Display

An alternative display using the head-up concept is shown on Figure 25. Information shown on this display includes everything contained in the ADI, altitude and Mach number displays.

e. Summary

The recommended display format is composed of the ADI, the altitude and Mach number displays, and the pictorial display. For minimum time and minimum fuel maneuvers, the ADI and pictorial displays would be used. The ADI would be used for the pitch profile following task, and the pictorial display would be used for altitude and Mach number status monitoring. The pictorial display would also be used to provide a 'precognitive' [48] status situation to the pilot in the sense that it will give him the ability to predict the future behavior of the pitch command indicator. Past performance, present state, and present direction along the flight path are also indicated here.

For maximum range and endurance maneuvers, the altitude and Mach number displays would be used. The pilot's task here is relatively easy and would require nothing more than manipulating the aircraft controls until the cruise altitude and Mach number have been attained.

3. CONTROL PANEL CONSIDERATIONS

In order to fly an optimum flight path it is necessary for the pilot to select a particular optimum mode, and dial in the end point conditions. In addition, warning indicators must be included which indicate whether the flight path satisfies specific aircraft performance limits. A control panel sufficient for this purpose is shown on Figure 24.

A push-push switch is shown for selecting the type of maneuver; dials are included for allowing the pilot to set in the maneuver end points; and warning lights are shown so as to indicate whether part of the particular path chosen is impossible, in the zoom region, or requires operation beyond the placard limits. These lights are required because of the variability of the flight envelope with ambient conditions and aircraft drag configuration.

4. DISPLAY ACCURACY AND UPDATE RATES

Display accuracy and update rate requirements will be determined by assuming accuracy requirements of the following order for pitch, velocity and altitude.

Command Pitch Angle	0.25°
Velocity	1 kt
Altitude	50 ft.

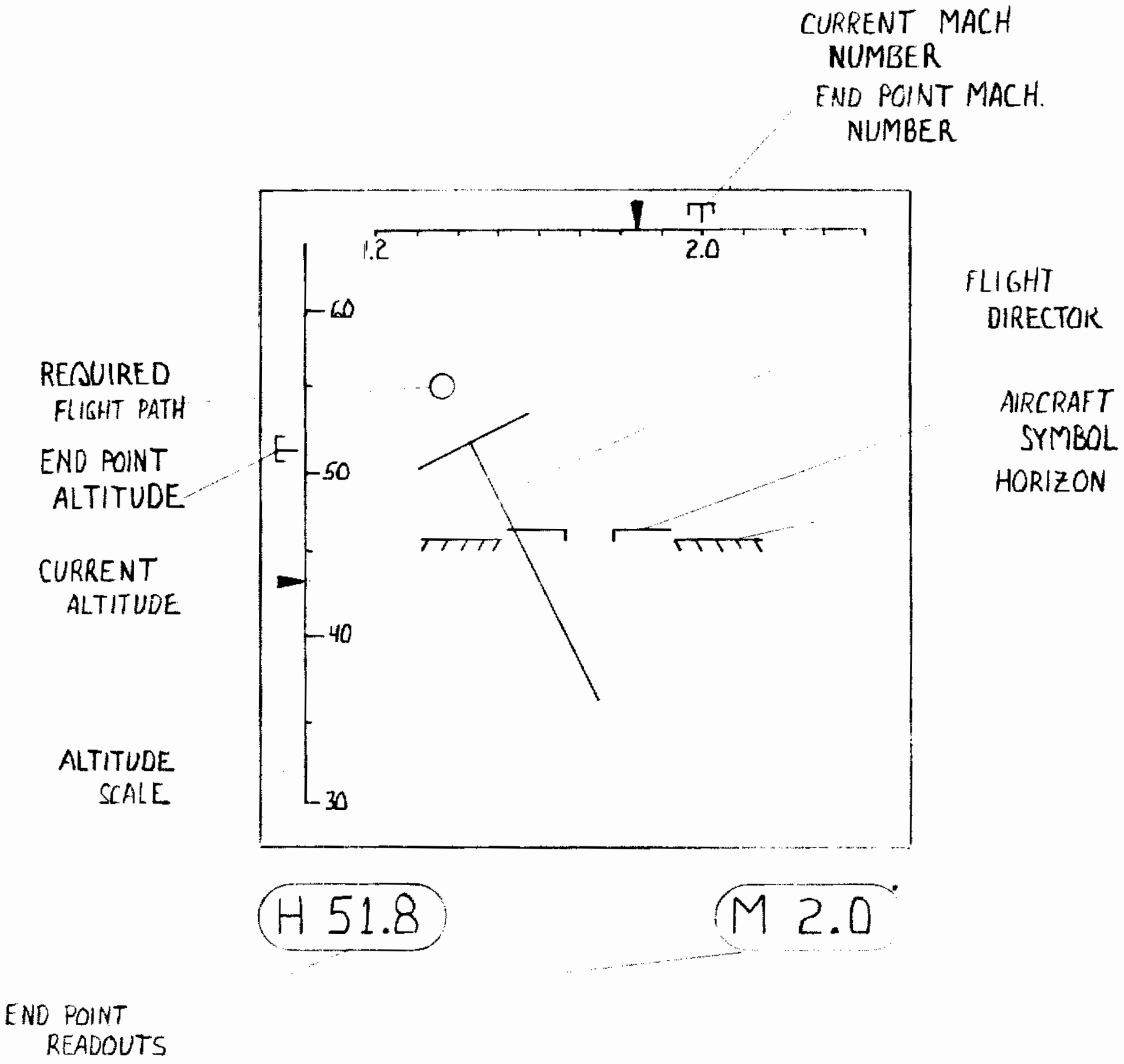

FIGURE 25-HEAD-UP DISPLAY FORMAT

a. Pitch Angle

A pitch command error will result in an unwanted aircraft longitudinal acceleration. This can be seen by noting Figure 18. If the aircraft is initially in steady state flight (i.e. thrust equal to drag), then the angular error $\Delta\theta$ will result in an acceleration, a, as given by the equation

P

$$a = g\Delta\theta \tag{62}$$

Therefore, a 0.25° value of $\Delta\theta$ results in an acceleration error of about 0.14 ft/sec^2.

Next the input/output accuracy associated with angular data will be determined. If it is assumed that inaccuracies in computation, input and output are equal and have the same type of distribution, then the equation defining the total RSS error is:

$$\Sigma_S = \sqrt{\Sigma_I^2 + \Sigma_c^2 + \Sigma_o^2} \tag{63}$$

(If digital computation is used, it is unlikely that the computational error will significantly contribute to the total error. Pending the selection of a computational technique, it will be carried here, however. The result obtained will be conservative, regardless of the computational technique which is used.) If it is assumed then, that the total error is 0.25°,

$$\Sigma_S = 0.25^\circ = \sqrt{3\Sigma_I^2} \tag{64}$$

and $\Sigma_I = 0.144^\circ$. If it is further assumed that the range of variation is ± 90°, then an accuracy of 0.08% is required.

The update rate for pitch angle can be determined by considering maneuvers which result in the most rapid rate of change of this quantity. Data presently available indicates that the maximum pitch angular rate occurs during minimum time maneuvers. Figure 21 indicates that the maximum angular rate for the minimum time climb maneuver is in excess of 4°/sec. In order to ensure that the pitch angle is specified to within 0.25° it must be updated at $\frac{4}{.25} = 16$ times per second. In practice it has been found that a factor of five should be applied to this is ensure a continuously appearing presentation to the pilot. Because of this, the required update rate will be 80 times per second.

b. Airspeed and Mach Number

Controlling airspeed to within one knot is equivalent to specifying Mach number to within .001. For a maximum Mach number of 2.0 an accuracy of .05% is needed. Equivalent accuracy is required for airspeed, whether indicated or calibrated.

Computational update rates for airspeed and Mach number are determined by considering the maximum acceleration along the flight path. For the F-104C this is at most .78g. Therefore the maximum velocity rate of change is 15 knots/sec. To maintain an accuracy of one knot in computing commanded pitch angle Mach number must be updated at a rate of 15 per second.

Actual Mach number is displayed in two places. For cruise operations it will appear on the Mach indicator, and for minimum time and minimum fuel maneuvers it will appear on the flight profile indicator. Under cruise conditions transients in velocity are generally due to turbulence. In these circumstances, the normal procedure is to let the transients die out of their own accord, rather than to actively null out the error. Therefore, the display of Mach number should probably represent a heavily filtered version of the actual Mach number. An update rate of about one per second, then, seems entirely adequate.

For transition maneuvers the actual Mach number update rate should correspond more with the maximum acceleration requirement mentioned earlier. Therefore, if a factor of five is applied to the 15 per second rate mentioned there, the update rate becomes 75 per second. This is the update rate requirement for actual Mach number on the flight profile indicator.

Commanded Mach number information is displayed only during cruise operations. Here commanded Mach number changes only as a function of gross weight. An update rate of one per minute seems entirely adequate.

c. Altitude

An altitude accuracy of 50 feet is typical of modern digital air data systems. It will be assumed that this accuracy is necessary in this system, as well. The range over which this accuracy is to be maintained will be 60,000 ft. Thus, an accuracy of 0.08% is required.

The required update rate for altitude is determined by considering maximum rate of climb. For the F-104C the maximum rate of climb is just under 800 ft/sec. Thus, the required computational update rate is 16 per second.

As with Mach number, actual altitude is also displayed in two places - for cruise operations on the altitude indicator, and for transition maneuvers on the flight profile indicator. During cruise, altitude transients are allowed to stabilize as with Mach number, and an update rate of about one per second seems adequate. The displayed quantity should again represent a filtered version of the actual altitude.

For transition maneuvers the update rate will reflect the maximum rate of climb requirement. With the factor of five this becomes 80 per second. This is the update rate requirement for actual altitude on the flight profile indicator.

Command altitude also varies as a function of gross weight as per the comments related to command Mach number during cruise. An update rate of one per minute seems adequate here, as well.

d. Throttle Setting

Throttle setting accuracy can be determined by assuming that the acceleration caused by an error in throttle setting (i.e. a resulting thrust error) can be no greater than the acceleration caused by an error in pitch angle. From part a_2 of this subsection the acceleration resulting from an error in pitch angle was .14 ft/sec^2. If it is assumed that an error in thrust causes an error in acceleration, the magnitude of the thrust error is given by the expression

$$\Delta T = ma \qquad (65)$$

$$= \frac{(.14 \text{ ft/sec}^2)(15{,}000 \text{ lbs.})}{32.2 \text{ ft/sec}^2.} = 65 \text{ lb.}$$

For the F-104C maximum thrust is 18,900 lbs. Therefore, the thrust must be controlled to within 0.34%.

[illegible]

The required update rate for throttle setting can be approximately obtained from Figure 5. During the first 35 seconds of flight throttle setting varies from 144% to 85% - a total change of 59%. Thus the maximum rate of change is about 1.7% per second. If 18,900 lb. represents maximum thrust of 158%, then 65 lb. represents about 0.55% change in throttle setting. Therefore the update rate for throttle setting is about $\frac{1.7}{.55}$ = 3 per second. With the factor of 5 this becomes 15 per second.

e. Summary

The various display accuracy and update rates are summarized as follows:

TABLE VI

DISPLAY ACCURACY AND UPDATE RATE REQUIREMENTS

Quantity	Display Accuracy	% Full Scale	Update Rate
Command Pitch Angle	0.25°	0.08	80 per second
Command Mach Number	0.001	0.05	1 per minute
Mach Number	0.001	0.05	75 per second
Command Altitude	50 ft.	0.08	1 per minute
Altitude	50 ft.	0.08	80 per second
Throttle Setting	0.5%	0.34	15 per second

f. Size, Weight and Power Specifications

Size, weight, and power values for the display system are difficult to specify at present because of the possibility of integrating flight path management displays with existing displays. Each of the displays recommended here can be integrated with a display already on the F-104C. The attitude director would simply be a modification of the one already present; altitude and Mach number display would replace the existing altimeter and Mach meter; and the flight profile indicator could be integrated with the radar display. In addition, power requirements for these displays are easily compatible with that already available.

It seems reasonable to conclude, therefore, that display size and weight will account for no more than 20% of the totals available for the entire computer-display system. This includes the possible requirements for expansion to the three dimensional case. Thus size and weight requirements for displays should not exceed 0.2 cu. ft. and 10 lb., respectively.

g. Conclusion

Before leaving the subject of displays it seems worthwhile to again point out that the recommendations given here are for a vertical plane command system. Conclusions could be substantially different when the entire three-dimensional problem is considered. In addition, much effort is required in the way of analysis and simulation to determine the interaction of displays, control laws, and tracking dynamics. Therefore, the recommendations given here must be considered tentative and subject to revision as more information is collected.

SECTION VII

SYSTEM COMPUTER REQUIREMENTS

Computer requirements will be determined by initially considering the requirements for a digital system. Following this an alternate analog system will be developed. It will be clear at that point that a digital system is superior.

The requirements for the digital system will be determined by making a detailed analysis of minimum time maneuvers and extrapolating the results to cover the other cases. A computer program is developed for this case and computer memory, speed, and input/output requirements are obtained. Based on this, overall system computer requirements are then developed.

1. MINIMUM TIME COMPUTER PROGRAM

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The flow chart shown on Figure 26 illustrates the sequence of computations required to generate pitch commands for an optimized minimum time maneuver. The program indicated is designed to accommodate initial and final energy states where there is a net increase in energy. The various cases are listed as follows:

The initial and final energy states lie on a Rutowski Path (see Figure 27.)

The initial energy state lies on a Boyd path and the final energy state lies on a Rutowski path (see Figure 10).

The initial energy state lies on a Rutowski path and the final energy state lies on a Boyd path (Figure 11).

Both end states lie on Boyd paths. (Figures 12 and 13)

In mechanizing these types of maneuvers it was assumed that:

Boyd's rule of thumb equations are applicable over the entire flight envelope. At most modification of the constant K (see equation (39)) will be required to describe optimum supersonic paths.

Paths constructed by combining segments of Boyd and Rutowski paths are very nearly optimal.

Rutowski paths can be defined by curve segments which are fourth order or less.

With these assumptions, it is possible to develop the computer program for minimum time maneuvers. The descriptions which follow outline the methods which are employed at each of the steps indicated on the flow chart.

Steps 1, 2, & 3 - (1) The computer reads in the aircraft configuration, temperature, present Mach number, and altitude, and the final state dial settings. (These latter quantities being set by the pilot.) The minimum time mode light is activated. The final energy state demanded is checked to determine if it is, (2) within the steady-state envelope and (3) does not exceed placard limits. The zoom region and placard limits are defined by a series of simple curves. The check is carried out via a number of limit tests, equation solutions, and comparisons — the result being obtained from a decision table. The Zoom Region or Exceed Placards indicator is activated if applicable. (If the terminal point exceeds placard limits a new terminal point may be set, or the pilot may elect to continue despite exceeding limits.)

Steps 4 & 5 - (4) Pressure Altitude, H_p, is converted to true altitude, H_T, using a polynominal equation. (5) The received Mach number, M, may also require conversion from an indicated to a true value, M_T.

Steps 6 to 12 - The next step in the computation is to determine the applicable Rutowski path which corresponds to the particular aircraft configuration and atmospheric conditions of interest. (6) This is done by interpolating both between adjacent temperature and adjacent configuration profiles. The path which is established will be defined by a group of stored polynomial equations. Next it is necessary to establish whether the end points are on Rutowski or Boyd paths. This is achieved by determining whether the end points lie within a tolerance region surrounding the Rutowski paths. Present data indicates that this region may be adequately defined by a number of straight lines (see Figure 28.) Hence, the result may be derived by a decision table using the end points of the straight lines as the logic test points. (7) In carrying out the process, the segment of the Rutowski path which lies nearest to the initial energy state is identified. (8) The tolerance test for this end point is then carried out. (9) If this point lies outside the tolerance region, a flag is set. (10), (11) Next, the segment corresponding to the terminal point is idenfified, and the proximity tests are carried out for it. (12) At the conclusion of these tests, the type of path is defined and the program branches to the routine associated with a particular one.

a. Rutowski Path (Figure 27)

Step 13 - On entering this phase of the task the computer has completed most of the decision operations and assumes, in effect, the role of the control unit in a feed-back control system. Most computations in this loop are involved in the calculation of pitch control commands. Either equation (59) or (60) may be used for this purpose. As indicated in Part 2 of this section, the former implies a more detailed knowledge of the relationship between the variables (h/M) and the control parameter (pitch angle). The steps outlined in the flow chart are those required for this technique. The following two paragraphs outline the two approaches and thereby indicate the differences. In both techniques the Rutowski path is defined by a set of equations relating altitude and Mach number. Also the deviation of the present position in the h/M plane from the Rutowski path is obtained. This is achieved by substituting one of the energy parameters (M or h) into the equation

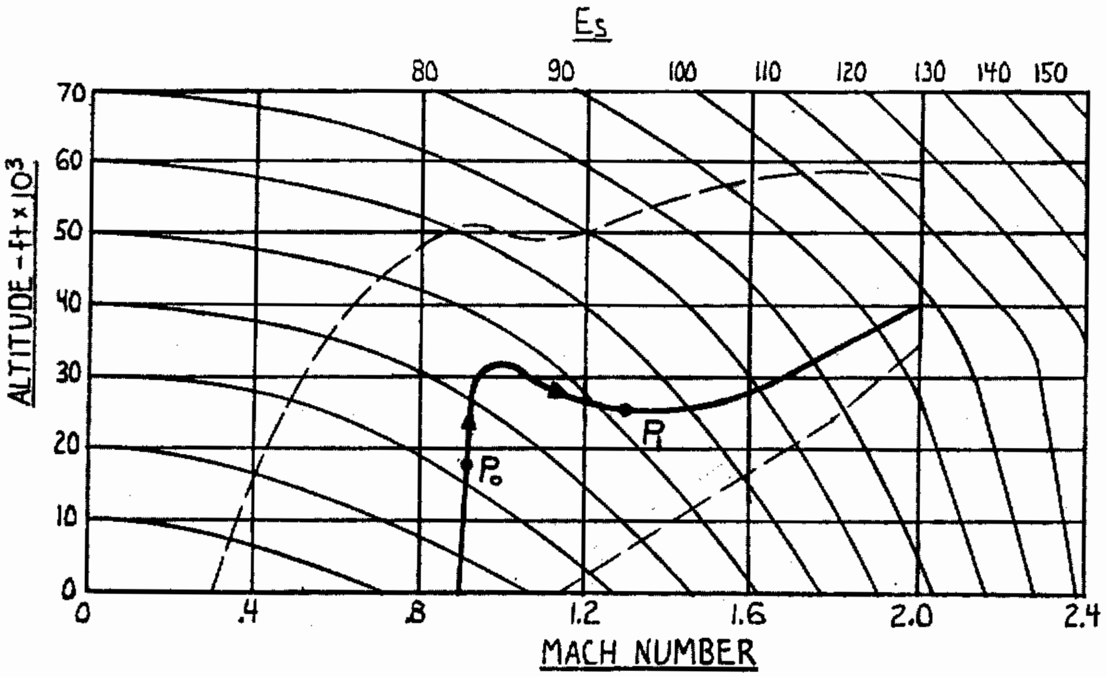

FIGURE 27- RUTOWSKI START AND FINISH

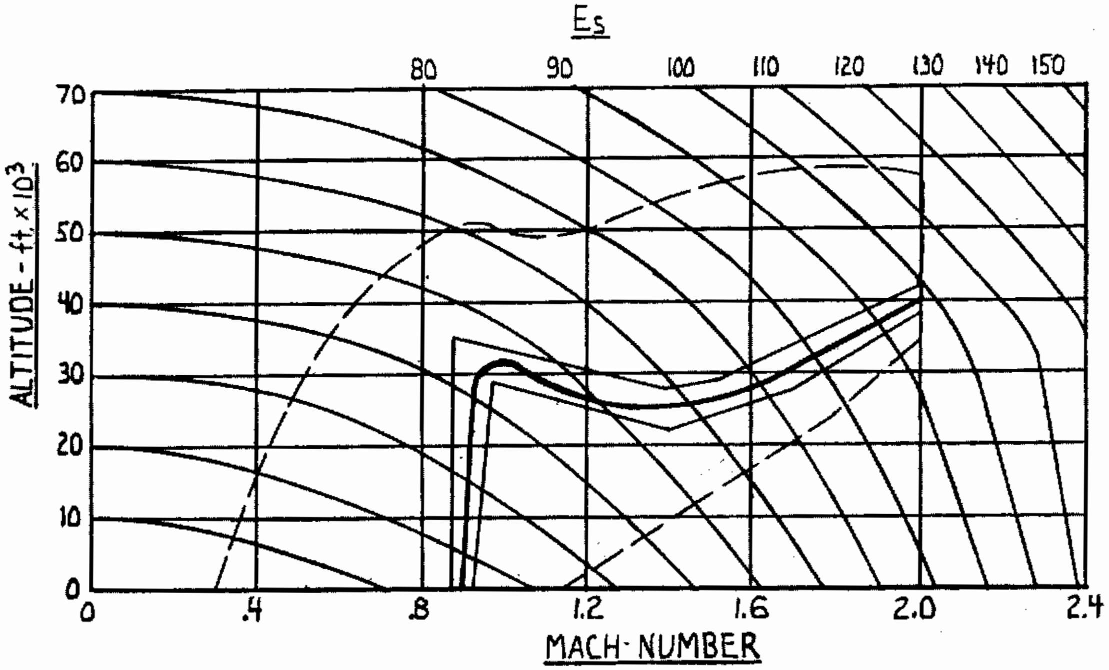

FIGURE 28- RUTOWSKI TOLERANCE REGION

which defines the Rutowski path. The value obtained for the second parameter is compared with the value at the present position. The difference between a Rutowski point (the calculated value) and the present position is then derived.

Step 14 - When equation (59) is used (as in the flow chart), the programming and storage required for these steps increase as compared to the alternate approach outlined in the next paragraph. The basic difference is in the requirement for storing the pitch angle profile which is associated with the desired optimum flight. The relationship between this desired pitch angle, and the ability to fly the optimum path is a function of aircraft flight configuration (i.e. weight and drag characteristics), and atmospheric conditions. As pointed out in part 2 of this section, an inexact knowledge of the desired pitch angle can result in the aircraft oscillating in velocity.

The alternative technique, involving the use of equation (60), is much simpler in that Step (14) is no longer required. This technique aims at minimizing the deviation from the Rutowski path with only a basic knowledge of aircraft-handling characteristics. The sign and magnitude of the difference is translated, via a Mach number or Altitude high/low decision function, into a required pitch angle. This is achieved in Step (15) which is similar for both techniques.

While the second of the techniques is obviously simpler and less demanding in terms of memory and programming, it does not present an automatic choice. Prior to the selection of a method, a simulation must be carried out to define the sensitivity between the control law, aircraft, display and pilot dynamics.

Step 15 - The pitch angle command is checked by comparing it with absolute limiting values, the present pitch angle, and the previous pitch angle command in order to assure that a smoothed output occurs. It is then converted to the required output format and transmitted to the applicable displays.

The first control sequence is complete at the conclusion of Step (15).

The control iteration rate depends on aircraft performance requirements and computer speed. As a first approximation the iteration rate is equal to the display update rate compensating for computer time lag.

Subsequent pitch control sequences begin with the recomputation of true altitude and Mach number [Steps (4) and (5)]. Present position on the h/M plane is compared with the final position defined by the pilot. If they are within tolerance limits, the program branches to check the validity of the stored final position. This is necessary since a 'pseudo' end point is inserted as part of other path control sequences which are explained later. If it is a true end point, then the minimum time indicator is set. Assuming the final position comparison is negative, then the Rutowski check [Steps (7) and (8)] is repeated. The program then branches to Step (13), and the control sequence [Steps (14) to (16)] is iterated.

b. Boyd Start - Rutowski Finish (Figure 10)

If the initial aircraft position on the altitude/Mach number plane is not within the specified tolerance of the Rutowski path, the aircraft must fly a path defined by one of the Boyd equations derived in Appendix I. These equations describe either a climb or dive path, depending on the initial position relative to the Rutowski curve.

Steps 22 to 26 - The relative position on the altitude/Mach number plane is computed in a manner similar to Step (13). The climb or dive equation is selected using a decision table, and the equations are completely specified by inserting initial h/M values. Step (23) does not apply to the initial control cycle. It is implemented on subsequent control cycles when deviations will exist. The only difference from Step (13) is the substitution of a Boyd equation rather than a Rutowski equation.

The pitch angle control computations, as indicated, are exactly the same as Steps (14) and (15). Subsequent iterations commence with Steps (4), (5) and (8) and continue until a positive result to Step (8) causes a program branch into the Rutowski path control routines. Certain modifications must be incorporated if a dive path is required in order to ensure that the altitude remains above a safe minimum. This may be achieved by automatically changing the Boyd constant for discrete low level/low energy start positions. Alternatively, a look-ahead computation, which would iterate to an optimum modifier for minimum altitude conditions, could be carried out prior to applying the Boyd equation in control computations.

The Step (7) test gives a positive result when the present position enters the Rutowski path region. This tolerance region will allow a smooth transition between the Boyd and Rutowski paths without incurring an obvious overshoot condition.

c. Rutowski Start - Boyd Finish (Figure 11)

If the terminal end point on the altitude/Mach number plane is not within the tolerance limits of the Rutowski path, then the final leg of the path will be a Boyd path. The Boyd equation which is selected is one which corresponds to the second leg of the applicable dive/climb, or climb/dive maneuvers listed in Appendix I. That is, if the terminal point were below the Rutowski path, equation (84) would be selected. If it were above, equation (82) would apply.

Steps 27 to 30 - Initially, a marker which is used to validate the end-point condition in the Rutowski path routines is unset. The climb/dive sequence is then defined in a similar manner to Step (22) and the Boyd equation is specified by inserting the terminal conditions. The intersection point of the Boyd curve and the Rutowski region is obtained by tracking the Boyd curve until the Rutowski region test [Step (7)] yields a positive result. The direction of tracking is obtained from a decision table using the positional information available. Tracking is carried out by solving the Boyd equation for incremental changes in one parameter (M or h). The position on the h/M plane which yields the positive result, (i.e. it is in the tolerance region of the Rutowski curve) is inserted as a 'pseudo-end-point' into the Rutowski control routines. The program then branches to the Rutowski control computations which apply to the initial portion of the optimum path.

On reaching and detecting the pseudo end-point, the program branches to the terminal path control computations.

Steps 31 to 35 - This control sequence is very similar in computational requirements to Steps (23) to (25). Step (31) is similar to Step (23) with the curve selected in Step (28) being used. The pitch control sequence is the same as Steps (14) and (15). Subsequent control iterations begin with Steps (4) and (5). An end-point test is conducted in Step (34). When the end-point test is positive the indicator is serviced via Step (35).

d. Boyd Start and Finish (Figures 12 and 13)

Both the start and finish conditions are off the Rutowski curve for this case. The initial decision logic is greater with this path than it is for Cases a. and b.

Step 36 - It is necessary to determine the relative positions of both the start and finish conditions with respect to the Rutowski path. This is equivalent, in computation, to two cycles of Step (28). Should the test reveal that the terminal conditions are on the same side of the Rutowski path the program branches to Step (37).

Step 37 - The climb/dive sequence is obtained as a by-product of Step (38). The terminal conditions are inserted into the Boyd equations which are then completely specified.

Step 38 - The transition point or intercept of the two Boyd paths specified above is calculated by simple substitution and quadratic equation solution.

Step 39 - The existence of intercepts between the climb/dive curves and the Rutowski curve the transition point lies. If the transition point is on the opposite side of the Rutowski curve to the terminal conditions, then the intersections must occur and the program branches to Step (40).

Steps 40 and 41 - Step (40) is composed of Steps (27), (29), and (30). Step (41) is merely relocating data and setting branch points for the finishing region of the flight path. The program then jumps into Step (23) as for Case b. and computations continue as outlined in Case b. and Case c.

If there is no intercept [Step (39)] then the program branches to Step (42).

Steps 42 to 45 - This control loop is very similar to Steps (23) and (26). The only difference is that the final test, Step (45), is used to determine if the present state is in the region of the transition point rather than the Rutowski path. On obtaining a positive result, the program branches to Step (46).

Step 46 – Step (46) is merely an initializing procedure prior to a branch into the control computations for the non-Rutowski path of Case c. To achieve this, the program branches into Step (31), after setting the finish point and appropriate other markers in required memory locations. Steps (31) to (35) are implemented as outlined earlier for Case c.

Steps 47 to 48 – Should the test in Step (36) yield a negative result, (i.e. the start and finish points are on opposite sides of the Rutowski path) then the program branches to Step (47) where the climb/dive sequence is specified as in Step (37). The intercepts with the Rutowski path are computed in a manner similar to that outlined in Step (29).

Steps 49 and 52 – Prior to any control computations it is necessary to establish whether the Rutowski portion of the flight involves an increasing or decreasing energy state path. A decreasing energy state Rutowski path is possible if the initial Boyd path intercepts the Rutowski curve at a higher level than the final Boyd path. The situation is shown on Figure 29 for the case of a climb-climb sequence. Even though the final energy state is higher than the initial one, the plane must fly "down" the Rutowski path to accomplish the maneuver. With a constant maximum throttle setting this is not possible.

In fact, if the throttle is held constant, there is always a net gain in energy per unit time whenever the aircraft is operating within the flight envelope. Since

$$\frac{dE_s}{dt} = P_s = \left(\frac{T-D}{w}\right)V, \qquad (66)$$

the rate at which the system is gaining energy is given by the value of P_s which corresponds to the aircraft state. Thus, the limiting minimum energy change, for a given minimum time maneuver, is not zero, but some finite value above zero. Mathematically this is stated as follows:

$$\Delta E_s = \int_{t_0}^{t_f} P_s \; (h, V)\, dt > 0 \qquad (67)$$

It is for this reason that one cannot fly the constant energy transition maneuver which is a part of the theoretical Rutowski path (see Figure 9).

Getting back to the program, if a "down" leg on the Rutowski path is indicated, a loop is entered in which the Boyd constants [i.e. the values of K in equation (39)] are modified and Step (49) is repeated. This modification decreases the slope of both Boyd paths and reduces the length of the down leg of the Rutowski path. This modification

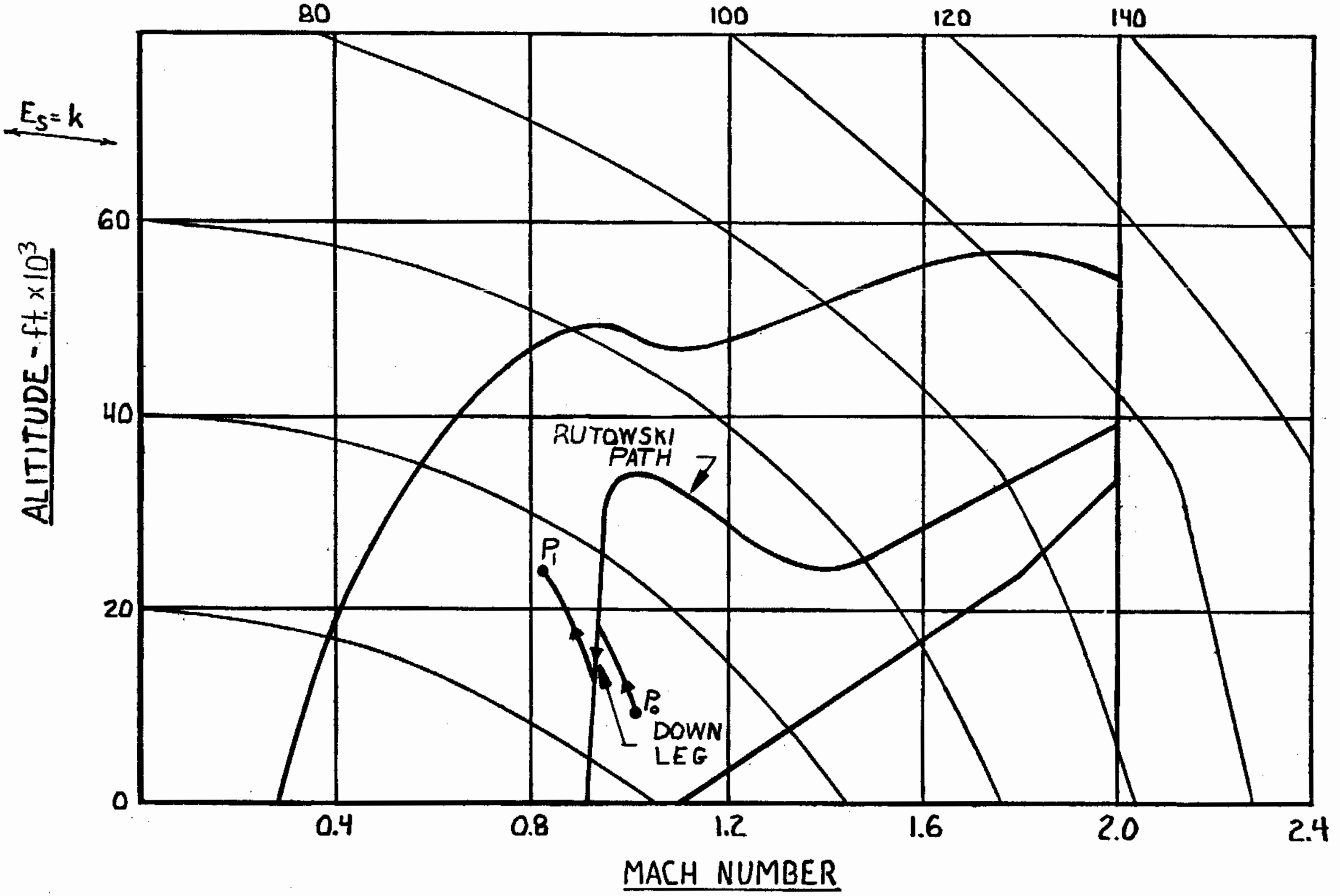

FIGURE 29-A CLIMB-CLIMB SEQUENCE WITH AN IMPOSSIBLE "DOWN" RUTOWSKI LEG

process is repeated in successive iterations until the down leg is eliminated.

The final slope for the Boyd curves which result must still be compatible with the condition of equation (67). Therefore, a test will be required to ensure that the net change in energy along the path is at least as large as the minimum given by this equation. When the down leg has been removed, the program branches to Step (51) where the end points are tested for compatibility with the energy constraints. If the maneuver end points are not within limits, the impossible maneuver indicator is set and the pilot is WIERD. required to select a new end point.

Steps 53 and 54 – The computations contained in Steps (53) and (54) are similar to Steps (40) and (41). The only difference is in the specific constants of the modified Boyd Paths.

2. COMPUTER LOADING ESTIMATES

The computer loading, in terms of the amount of memory required and number of instructions that must be executed, may be estimated by examining the flight paths which were considered on the flow chart. The computations required to implement the routines indicated may be classified under four main headings. These are start-up, testing, control sequence, and input/output and displays.

a. Start-Up

The start-up sequence, which includes Steps (1) to (12) on the flow chart, involves checking the demanded final Mach/Altitude state against flight envelope constraints and determining the type of path required. The aircraft placard/zoom region limits may be defined by approximately 5 equations. Each of these equations (possibly straight lines) may be solved in approximately 10 instructions giving, with the simple decision tables involved, a total of approximately 70 instructions. The computation of true altitude and Mach number involves the solution of polynomials, each requiring approximately 20 instructions. The Rutowski tests for the terminal conditions involve computations similar to Step (1). Setting indicators and testing flags require approximately 5 instructions each. The total instruction count for this sequence is listed below:

Steps (1), (2), (3)	80
Steps (4), (5)	40
Steps (6), (7), (8), (10), (11)	60
Steps (9), (12)	10
Approximate Total	190

b. Testing

It is necessary, via test routines, to establish the present flight parameters prior to entering the control routine. These tests are contained in Steps (16), (17), (18), and

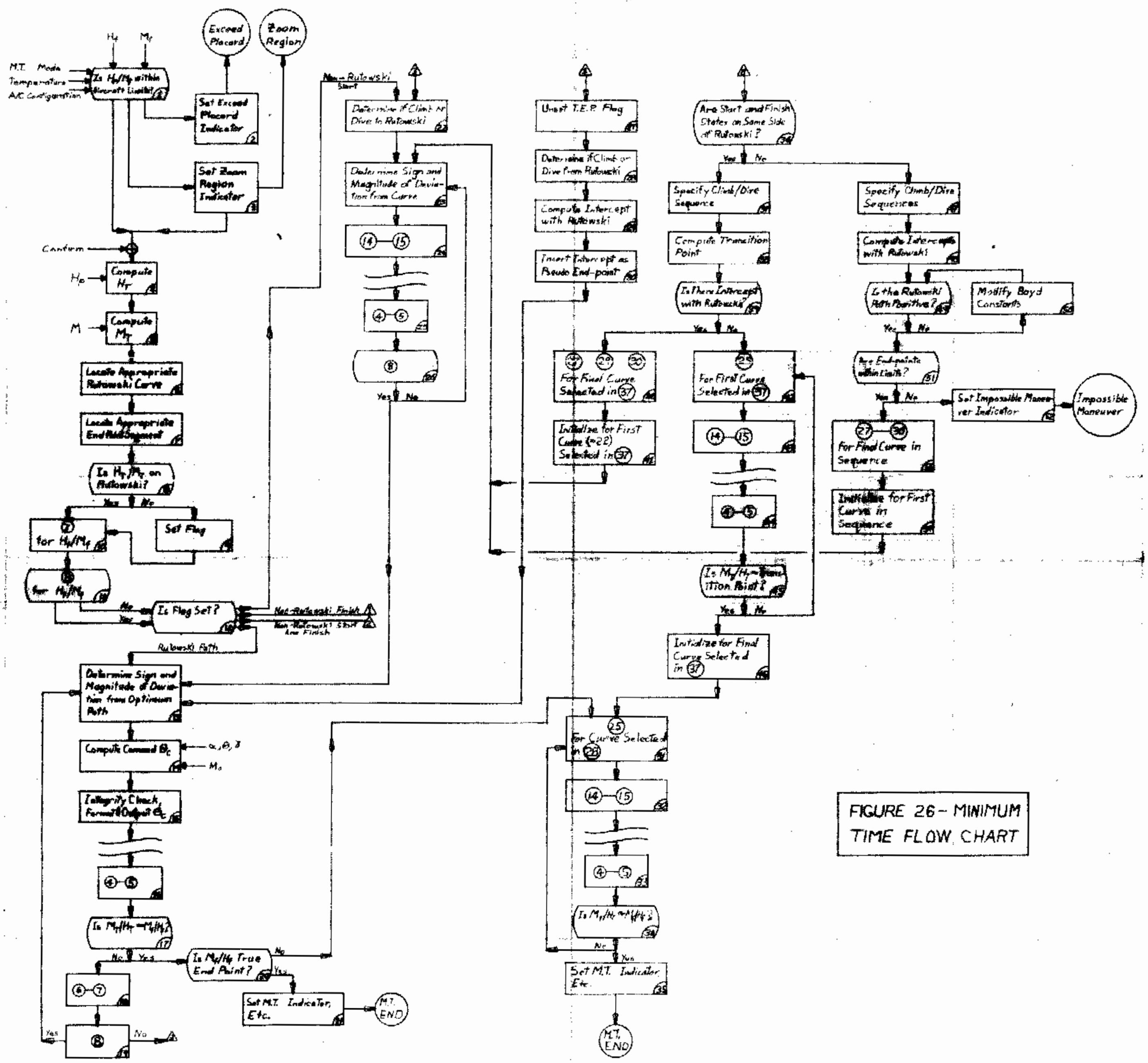

FIGURE 26 - MINIMUM TIME FLOW CHART

(19), which are carried out for all except the first control iteration. As indicated on the flow chart they contain computations common to earlier steps.

Step (16) [Steps (4), (5)]	40
Steps (18), (19) [Steps (6), (7), (8)]	30
Step (17)	20
	90

The test routines are similar for all flight paths. The only major difference occurs in Case a. where the test sequence may be more complicated. The high estimate (190 + 90 = 280 instructions) refers to the case where the program has to sequence through Steps (36) to (39). This only occurs for one control iteration.

c. Control Sequence

The control sequence is the same for each flight path - Steps (14) and (15) being common to all loops. The only differences occur due to the form of the equations defining the particular flight path. The Rutowski path has the longest sequence due to the possibility of higher order equations. The steps considered are (13) to (15). Step (13) requires polynomial solution as well as a small decision table. Step (14) involves a polynomial solution after a decision table to locate the correct equation and also, possibly, interpolation. Step (16) is an aircraft control equation which must be applied to the results of both Steps (13) and (14). The final step is merely a number of checks against stored data and possibly some formating (half-word byte-transmission).

Step (13)	50
Step (14)	70
Step (15)	50
	170

d. Input/Output and Displays

The input/output instructions are implemented at the start and finish points of the subroutines where required. Mach number, altitude, temperature, aircraft configuration, pitch angle, and mode signals must be received during each computation cycle while Mach number, altitude, and pitch signals must be transmitted to displays. These amount to approximately 10 input-output addresses, some of which will require more than one data transfer.

e. Minimum Time Requirements Summary

The instruction count is summarized in Table VII. The slightly higher figures in the table are intended to account for program linkages and test sequences.

The memory requirements, as compiled in the table, do not consider the extensive

Table VII

LOADING ESTIMATES FOR MINIMUM TIME MANEUVERS

Instructions	Rutowski	Boyd Start-Rutowski Finish	Rutowski Start-Boyd Finish	Boyd Start and Finish Step (37) Sequence	Boyd Start and Finish Step (47) Sequence	Total
Start-Up	220	220	220	220	220	
Testing	110	110	110	280	220	
Control Sequence	220	210	200	200	220	
Input/Output and Display	20	20	20	20	20	
Subtotal Words	570	560	550	720	680	
Memory						
Programs	570	120	110	300	240	1,340
Data and Stored Constants	150	25				175
Total Words	720	145	110	300	240	1,515

use of common subroutines. The programming estimate is, therefore, approximately equal to the total instruction count for each flight path minus the common start-up and control routines. The stored constants are required for the coefficients in the polynomial equations which are employed. It is presumed that each constant will require one-half of a word of storage. For the Rutowski path, separate curves will be stored for five temperature levels and three aircraft configurations. It will be assumed that it requires four, fourth order polynomial segments to define each of these fifteen curves. Therefore, the number of constants required to define the Rutowski path is as follows:

$$(4 \text{ segments/curve}) \ (15 \text{ curves}) \ (5 \text{ constants/segment}) = 300 \text{ constants.}$$

It will further be assumed that the Boyd constants will require modification for each of the temperature and configuration situations. This will result in thirty stored Boyd constants with two constants required for each situation. In addition, the flight envelope boundary limits must be stored. This is included, with the Boyd constants, in the estimated storage requirement of 50 constants for the entire group of non-Rutowski paths. Using the one-half word per constant rule, a total of 175 words $\left(= \frac{300 + 50}{2} \right)$ will be required for data and stored constants.

The total memory requirement for minimum time program and data storage requirements is between 1,500 and 1,800 words. After the start-up sequence has been completed, and an optimum path has been selected, the control loop, consisting of testing, control sequencing, and input/output sequencing requires about 400 instructions.

3. Overall System Computer Requirements

Analysis of the Minimum Time Climb path has resulted in the computer memory and speed requirements shown in Table VII. These results will be used to estimate the total computer requirement.

a. Additional Optimum Maneuver Requirements

The additional tasks to be accommodated in the computer comprise optimization for minimum fuel maneuvers, and for maximum range and endurance cruise operations. Each of these is discussed in the following paragraphs.

(1) Minimum Fuel Maneuvers

Minimum fuel optimization requires the storage of additional flight path descriptor data and the generation of throttle commands. The additional profiles to be stored include altitude/Mach number and altitude/throttle setting profiles. Each of these profiles is required for five temperature and three aircraft configurations. If each curve requires four, fourth degree polynomials each requiring 5 constants, a total of 660 constants must be stored; with sixty of these being required for defining Boyd paths. By using one-half word for each constant, the number of additional words required is 330.

The programming required is increased over that required for the minimum time case. It is assumed that the profile following routine shown on Figure 26 can also be employed for the minimum fuel case. Throttle setting command generation would not need to be nearly so complex, however. Specifying an altitude/Mach number profile also results in specifying an altitude/throttle command profile. Therefore, a short routine connected with throttle setting need only be added to the already existing program. Common subroutines (polynomial curve fit, for example) would be used and start-up and input-output routines would not increase substantially. It is reasonable to estimate an additional 150 words of program memory will be required for this mode.

Computer speed requirements would be increased by a proportionate amount and an estimate of 500 instructions per cycle will be used.

The total additional requirement for Minimum Fuel Climb case is therefore:

Data Storage	330
Program Storage	150
Total	480
Instructions	500 per cycle

(2) Maximum Range and Endurance Maneuvers

The increment in computer requirement to accommodate maximum range and endurance maneuvers is largely associated with storage of appropriate flight condition data. The data required is given as follows:

Maximum Range

Altitude vs Gross Weight
Throttle Setting vs Gross Weight

Maximum Endurance

Altitude vs Gross Weight
Altitude vs Mach Number
Throttle Setting vs Gross Weight

It should be noted that the data to be stored here is not of the same type as that required for minimum time and minimum fuel maneuvers. The maneuvers called for here are not specifically concerned with transitions in altitude/Mach number space. Rather, the requirement is to fly at a specific altitude and Mach number which maximizes range and endurance. One might transition to this flight condition along a minimum fuel path, however. Data to be stored then, is in the form of a set of steady state flight conditions which are largely a function of gross weight.

A total of five curves for each of five temperature conditions and three aircraft configurations are to be stored. An examination of typical profiles (see Figures 3 and 4) shows that they are quite well behaved, and in some cases nearly linear. It will be assumed that a single fourth degree polynomial will adequately describe any of the curves. Therefore, it will be necessary to store a total of

(75 curves) (1 segment/curve) (5 constants/segment) = 375 constants

Using one-half word of storage per constant, the total number of words required is 188. Programming and speed requirements are not significantly changed.

(3) Executive and Self Test

Executive and self test functions for a program of this size typically require between three and five hundred words. It will be assumed, in this case that the requirement is 400 words. In addition, it will be assumed that about 300 instructions per cycle will be required here.

(4) Summary

The overall memory and speed requirements for the computer system are given in Table VIII. The speed requirement was derived by using a requirement of 500 operations per cycle for the control loop and 300 operations per cycle for self test. The total is about 300 operations per cycle. If an average update rate of seven per second is used, the result is a speed requirement of 5,600 per second.

It is normal practice when performing an analysis of this type to allow safety factors in the computer characteristics derived. The magnitude of the safety factor is dependent on the detail with which the problem is assessed. In this case it is felt that the storage requirement has been estimated with reasonable allowances for unknowns. However, it is clear that operating speeds have been estimated as a consequence of many assumptions and therefore the result must be used with caution. Accordingly, the speed should be increased by at least a factor of three. The speed requirement, determined in this way, is termed characteristic speed. Therefore tentative requirements for memory and characteristic speed are approximately 2,600 words and 17,000 operations per second, respectively.

Reserve capacity for inclusion of the three dimensional problem will be discussed in part c. of this subsection.

b. Alternate Analog System

An alternate analog computational system was also developed so as to be certain that the choice of a digital system was not premature. In actuality the system falls more in the hybrid class rather than in the class of completely analog systems. The system was

TABLE VIII

MEMORY REQUIREMENTS

Function	Data Memory	Program Memory	Total Memory
Minimum Time Climb	175	1,340	1,515
Minimum Fuel Climb	330	150	480
Maximum Range/ Endurance	188		188
Executive and Self Test		400	400
TOTAL			2,583

Speed Requirement - 5,600 operations per second set up with the same type of format that was used for the digital computer. A detailed design was developed and the documentation that would be required is quite extensive. Therefore, only the results of the design, in terms of equipment requirements, weight, and volume will be reported upon here. If further interest in an analog system is noted, more of the detail can be supplied.

(1) Equipment List

The equipment required for a representative analog computational system is listed in Table IX. The totals for weight and volume are considered to be optimistic and are probably smaller than is practical with state of the art equipment. Even so, the values are far in excess of the weight and volume constraints (i.e. 50 lb. and one cu. ft., respectively) which the final design must meet. If these numbers are representative, and there is every reason to believe that they are, then it is clear that an analog system is not practical within the present system design constraints.

(2) Other Considerations

In addition to limitations in meeting weight and volume constraints, an analog system will almost certainly be less accurate, less reliable, and less flexible than a comparable digital device. Accuracy in analog computers is related to amplifier stability, nonlinear computation error, and limited variable range. Amplifiers have reached a high degree of development in recent years, but in an airborne environment drift stabilization problems will undoubtedly exist. Nonlinear computation error results from inherent inaccuracies in solid state function generation equipment, as well as in the aging characteristics of these devices. Again, equipment of this type is highly sophisticated in today's electronic world, but the more accurate equipment is heavier and more bulky. An amplifier is a voltage sensitive device and there is a limited range over which it can accurately operate. Thus, the requirement for measuring small differences in large quantities is a difficult one. Measuring to 0.25° accuracy in a 30° pitch angle command, for example, would appear to be rather difficult with an airborne analog system.

Reliability in digital equipment is generally better than for comparable analog equipment. With the advent of solid state devices the difference is not large, however, unless mechanical equipment is required as a part of the analog design.

Finally, changing an analog program is generally more difficult than changing a digital one. In general, analog programs are usually quite inflexible because of their hard wire nature. Special purpose components such as multipliers, for instance, are required for specific operations, whereas in digital machinery the same component can be used for both multiplication and addition.

Regardless of weight and volume considerations the choice of a digital system could be made strictly on the basis of merit.

c. Digital Computer Recommendations

TABLE IX

ANALOG SYSTEM WEIGHT AND VOLUME BREAKDOWN

Computer Components

Number	Component	Unit Weight oz.	Total Weight oz.	Subsystem Weights lbs.
225	Differential Operational Amplifiers	2.0	450	
241	Operational Amplifiers	2.0	482	
2004	Precision Resistors (0.05%)	0.03	60	
11,832	Precision Resistors (0.5%)	0.03	355	
4,680	Trimmer Potentiometers	0.1	468	
307	Subminiature Relays	0.5	154	
20	Reed Relays	0.3	6	
2,077	Diodes	0.05	104	
17	Zener Diodes	0.1	2	
26	Capacitors	0.5	13	
22	Flip-flops (Integrated Circuit)	0.05	1	
33	AND's, OR's, NAND's, etc.	0.05	2	
24	Solid State Analog Switches	3.0	72	
8	High Impedance Analog Switches	3.0	24	
10	Square Law Function Generators	4.0	40	
6	Square Root Function Generators	4.0	24	
52	Multipliers (Quarter-square)	8.0	416	
	Subtotal		2,673 oz.	167
Power Supply				25
Circuit Board Mounts				20
Cabinet				30
Circuit Boards, Connectors, etc.				10
	TOTAL WEIGHT			252 lb.
	MINIMUM VOLUME			2.2 cu. ft.

Prior to the selection of a computer for the system a survey of currently available machines was conducted. The first phase of the survey consisted of obtaining information from all possible manufacturers. These included firms in the military mobile computer market as well as the known manufacturers of airborne computers. The requirement was specified as aerospace so that all possible machines could be considered. By comparing machines with the requirements of the Work Statement and the system study, the list of machines to be considered was reduced. Detailed information was requested, where necessary, on the remaining computers. The data are not listed because of restrictions relative to the publication of comparative manufacturers' specifications.

(1) Characteristics

Data was compiled under the following major characteristic headings.

Machine Organization
Memory
Commands
Data Transfer
Technology
Peripherals
Software Facilities
Physical Features
Special Characteristics

(2) Governing Requirements

In terms of the Work Statement and the system study, the specifications are most stringent with respect to physical features. By adding a tolerance to these requirements and using this as a 'filter', the number of computers to be considered was greatly reduced.

(a) Weight and Volume

The specified system weight and volume limits are 50 lbs and 1.0 cu. ft. The computer element of this total was assumed to be 30 lbs and 0.50 cu. ft.

(b) Technology

The incorporation of integrated circuits as the major technology is considered a mandatory requirement. While a fixed memory is desirable for reasons of system integrity, it is undesirable during the development phase and for logistics purposes. This indicates a requirement for an Electrically Alterable Non-Destructive Read Out (EANDRO) device rather than a fixed Read Only Memory (ROM). The device most commonly employed in this field is microbiax which offers high speed, packing density, and reliability. Alternative techniques of implementing a Non-Destructive Read Out (NDRO) memory are discussed in Appendix III.

In addition to the NDRO feature, the programs require DRO memory due to the large number of intermediate results which are obtained.

The best memory configuration is therefore a mixture of DRO and EANDRO. The absence of NDRO in a computer does not exclude the machine. When transient protection, automatic shut-down and parity checking are incorporated in a DRO memory computer, then the machine achieves the requisite performance and may be considered compliant with the specification. It must be added that most of the computers have a DRO coincident-current core memory. The 'missing core' technique described in Appendix III may be used to achieve an NDRO memory without major redesign of the machine.

(3) Conclusions

On the basis of the survey it was concluded that there are several commercial computers which are capable of meeting system requirements. Selections were based on an initial examination of weight, size, logic content, memory capacity, and ability to meet the requirements stated in part 3. of this section. The tendency was to select a machine with considerable reserve capacity. A factor of three was employed in the area of characteristic speed, while a 50% margin was added to the memory requirement with the additional proviso that the memory be expandable. The new speed and memory requirements are therefore increased to approximately 3,900 words and 50,000 operations/second. This was done to ensure that the machines selected had the requisite computing capability. Accordingly the tentative specifications of part 3. were modified to read as follows:

Memory	4096 words (expandable)
Word Length	24 bits
Memory Type	EANDRO/DRO
Software	Higher Language and Diagnostics
Weight	30 lbs.
Volume	0.5 cu. ft.
Operating Speed	50,000 operations/second

The following four computers are typical of those that meet these physical and functional characteristics:

Raytheon	RAC-230
Hughes	HCM 205
IBM	4PI/TC
Univac	1824

SECTION VIII

INTERFACE REQUIREMENTS

Interface requirements are primarily embodied in the input and output equipment surrounding the computer. In order to understand the interface relationship between these equipments, a simplified block diagram is shown on Figure 30. As indicated, the system elements are composed of a number of data sources, a computer, and a number of data destinations. The data sources are an air data computer, engine sensors, an angle of attack sensor, a stable platform, the control panel, and certain discrete inputs. The type of data and signal form emanating from each of these sources is listed on Table X.

Data destinations include aircraft instruments, warning devices and displays. The type and form of signal typically required for these devices are given on Table XI.

In computer system terminology the input/output system consists of three function areas. These are:

Digital Control Logic

Data Source Input Converters

Display Output Converters

In the discussion which follows each of these functions will be described to a level which will allow the definition of input/output requirements.

The digital control logic can only be specified, at this stage, to a logical function and signal flow level. The computer selected for the system is assumed to be one of the small airborne computers called out in Section VIII, which have a single interrupt line and a programmed input/output system. The inputs from the Air Data system are assumed to be compatible with standards common to modern aircraft. The data and control signals entered by the pilot are digital inputs to assure computer interface simplicity. A block diagram of the input/output system is shown in Figure 31.

1. DIGITAL CONTROL LOGIC

All data transfers and most control signals are initiated by the programmed input/output instructions implemented in the computer. The remaining control signals, initiated by pilot action, enter the computer via the interrupt line. During an input instruction the computer transmits an address and function code together with control strobes to the input/output section. The address code is decoded to determine the selected input converter and channel number. The function code is interpreted as a mode command, i. e., initiate convert, transfer register, etc. Two instructions are required to transfer an analog signal into the computer. The first instruction specifies the converter and channel, and initiates an analog to digital conversion. The second

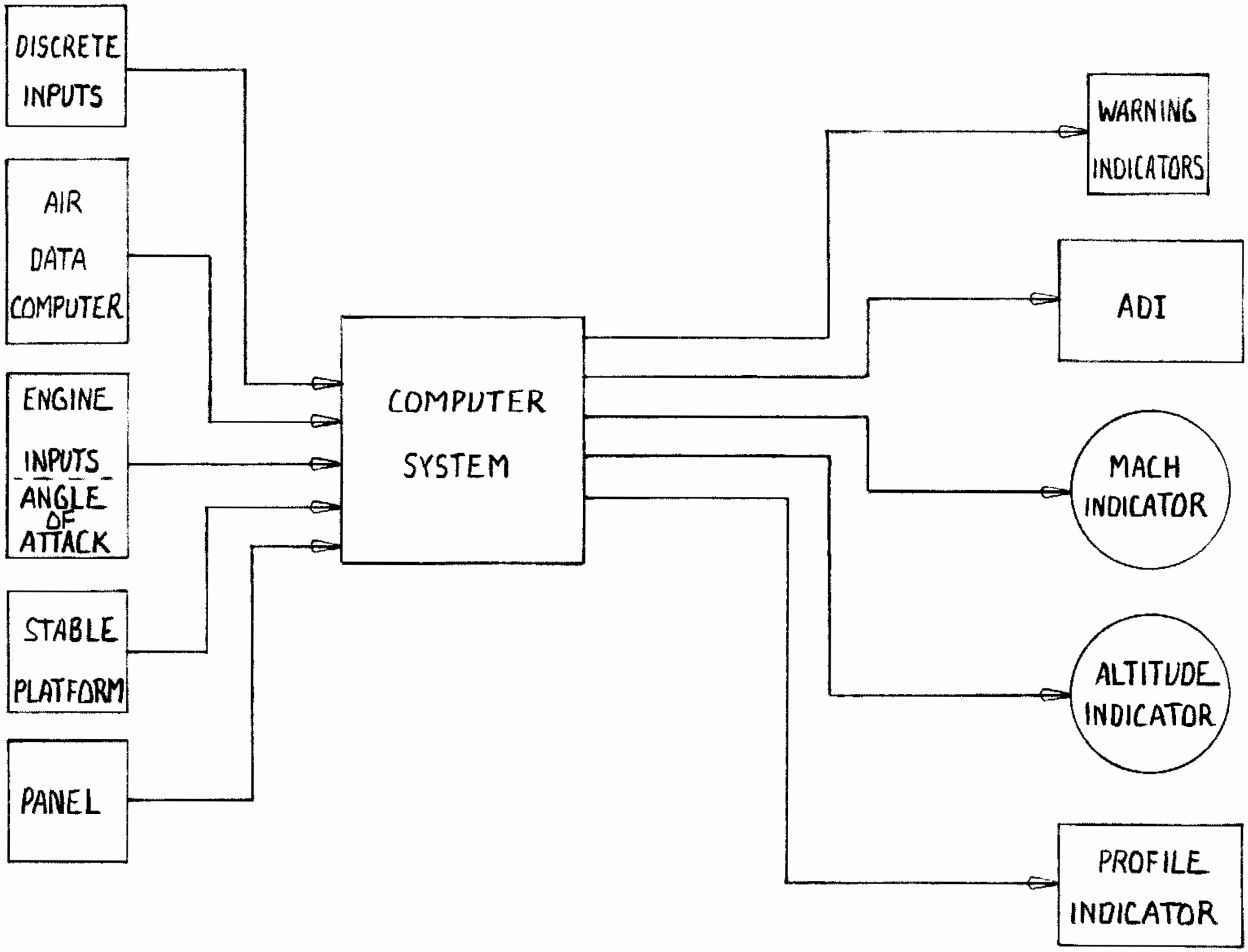

FIGURE 30 - INTERFACE RELATIONSHIPS

TABLE X

DATA SOURCES AND SIGNAL FORMS

Data Source		Parameter	Electrical Characteristic
Air Data Computer	-	Mach Number	Synchro
		Temperature	Voltage
		Altitude	Synchro
		True Airspeed	Synchro
Stable Platform	-	Pitch Angle	Synchro
		Pitch Angular Rate	Potentiometer
Angle of Attack Sensor	-	α, Angle of Attack	Synchro or Potentiometer
Engine Sensor	-	Fuel Flow Rate	Frequency
Aircraft Sensors	-	Weapon Load	Discretes of Voltage Levels
		Fuel Load	Digital
Panel	-	Mode Selection Numerical Data Command Signals Test Signals	As Required

TABLE XI

DATA DESTINATIONS AND REQUIRED SIGNAL FORM

Data Destination		Parameter	Characteristic
Attitude Director Indicator	-	Pitch Command	Synchro
		Profile Elevation	Voltage
		Fast/Slow Indication	Voltage
Mach Indicator	-	Required Mach Number	Voltage
		True Mach Number	Voltage
Altitude Indicator	-	Required Altitude	Voltage
		True Altitude	Voltage
Profile Display	-	Present Mach Number	Digital
		Present Altitude	Digital
		Vector ΔM	Digital
		Vector Δh	Digital
		End Point Height	Digital
		End Point Mach Number	Digital
		Target Mach Number	Digital
		Target Altitude	Digital

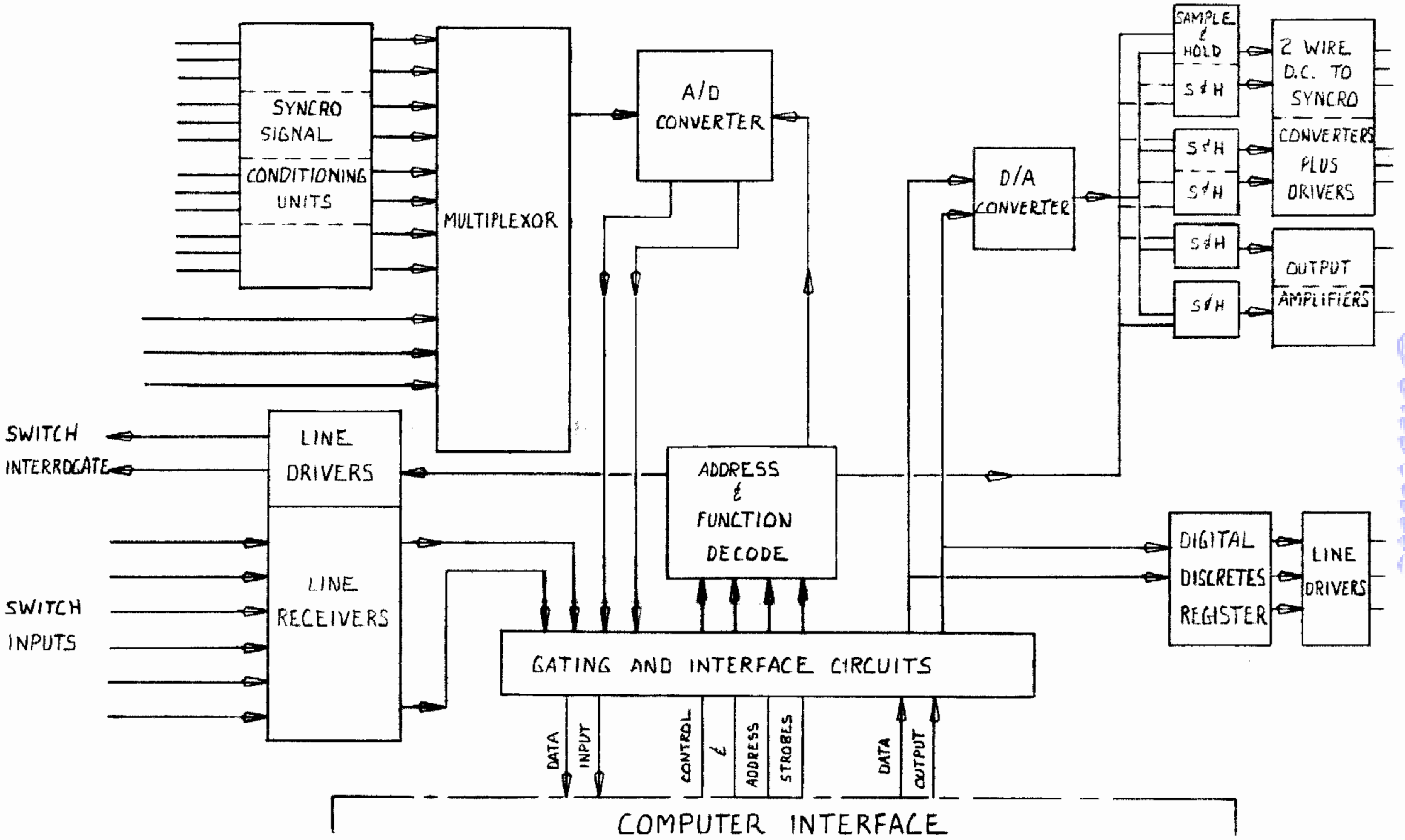

FIGURE 31 - INPUT OUTPUT SYSTEM

instruction specifies the same address and orders the digital result of the conversion to be loaded into the computer. Digital inputs (from the pilot's control panel) require only one input instruction to be strobed into the computer. The assumed input signals and their form are listed in Table X. Each output display function is serviced via a single output instruction. The computer specifies the converter command, channel address, and transmits the data to be converted.

2. INPUT CONVERTERS

The input signals are received in both analog and digital form. The analog signals are a.c. synchros and d.c. voltage potentiometer type inputs. The digital signals consist of coded and discrete inputs. A block diagram of a typical input system is shown in Figure 32.

a. Analog Converters

The analog inputs are most economically converted using a single voltage to digital converter, multiplexed between all input channels. Since all of the inputs require a 10 or 11 bit conversion accuracy, there is negligible redundancy in a common 11 bit converter. The synchro signals require processing prior to conversion. Hence, the analog conversion system consists of signal processors, a multiplexor, and a voltage to digital (A/D) converter.

(1) Signal Processing

It is necessary to condition the synchro waveform prior to directing it to the voltage converter. The technique involves the transformation of the synchro waveform into two d.c. voltages which are proportional to the sine and cosine of the synchro angle. This is achieved using an electronic equivalent of the Scott-T transformer, sample and hold circuits, and control and timing logic. Electronic Scott-T devices employing integrated circuit operational amplifiers, with adequate accuracy, have been produced by airborne electronics manufacturers. They are small, light-weight and inexpensive. Sample and hold devices are required on the sine and cosine channels from the electronic Scott-T devices. They can be implemented using integrated circuit operational amplifiers, and FET switches. The sample and hold control pulses are synchronized to the synchro excitation signal. In this way, the hold period starts near the peak of the carrier and ensures better resolution. This is achieved using a zero-crossing detector. In order to reduce conversion delays to a minimum, the sample and hold circuits normally run asynchronously to the computer. Hence, the sample and hold circuits function on each successive cycle of the carrier. During a single carrier period the hold amplifier output suffers negligible decay and the change in angle is insignificant. Hence the output of the hold amplifier may be directed by the computer to the voltage converter at almost any instant. The maximum possible delay occurs when a sample is being taken at the same instant the output is being called for. The delay incurred is of the order of 20 μsec. Normally, however, the computer command locks out the sample mode while the conversion is being implemented.

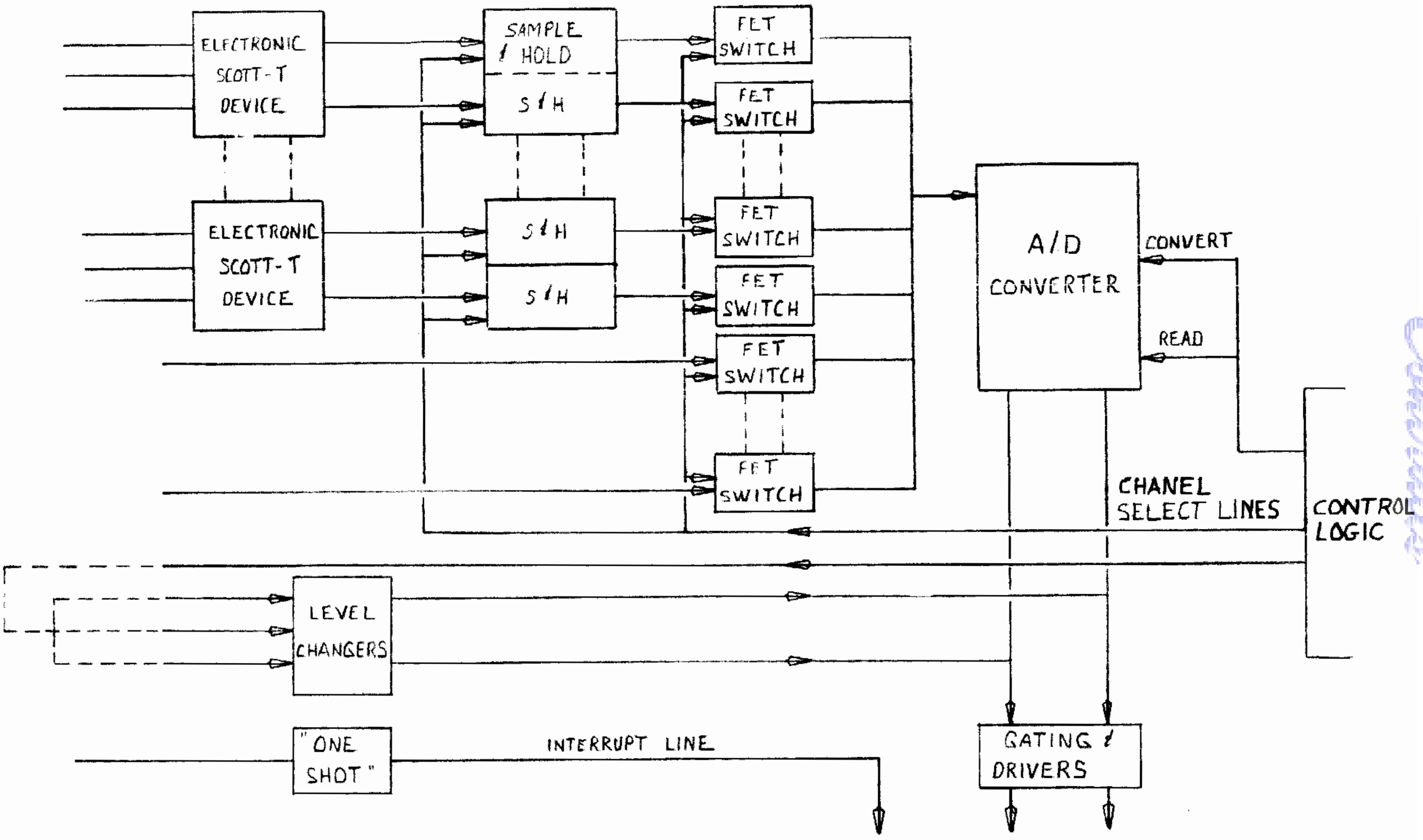

FIGURE 32- INPUT SYSTEM BLOCK DIAGRAM

(2) Multiplexor

A twelve channel multiplexor is required to accommodate the eleven system inputs plus a test signal. This is achieved by switching the inputs into the zero potential point of the A/D converter comparator using FET switches. These exhibit zero offset voltage and low 'ON' resistance. The worst case switching time is of the order of 2 μ sec.

(3) A/D Converter

The A/D converter is of the conventional successive approximation type and presents no peculiar difficulties — especially as there is no requirement for high speed. Converters are commonly available with bit conversion times of 2 - 20 μ sec per bit and 12 bit accuracy.

b. Digital Signal Conversion

The pilot's control panel is the source of the digital signals. The flight mode switch and numerical data (demanded Mach/Altitude) may be entered here. The computer treats these devices as simulated encoders and interrogates them, via a single instruction, as required. This is achieved using a line driver (switch interrogate), a diode isolating and encoding matrix, and line receivers. Instantaneous signals, i.e., initiate demanded flight path, must be stored. The simplest method of transmission to the computer is via an external interrupt line. The computer is thereby forced to change its program sequence and to determine the source of the interrupt. In the absence of an interrupt facility, the computer must scan the digital signal stores at an adequate rate. This can be achieved with tightly controlled programming sequences.

3. OUTPUT CONVERTER SYSTEM

The computer must transmit data to service displays which require both analog and digital signals. The analog displays are required for flight parameters while the digital displays are also used for warning and status indicators.

The parameter displays are driven by d.c. voltages and synchro signals. The conversion technique employed is similar, in reverse, to the analog input system. A single digital to d.c. voltage (D/A) converter is coupled to a multiplexor which is followed by hold circuits and signal continuing devices. Signal continuing is required to transform the two d.c. voltages (proportional to the sine and cosine of the commanded synchro angle), present at the sample and hold outputs, into a three-wire synchro waveform. This is achieved using a 400 Hz modulator, a power amplifier and a Scott-T transformer. The d.c. voltages are modulated and power amplification is subsequently required to drive the Scott-T transformer. The d.c. voltage outputs are derived from sample and hold circuits via a buffer amplifier which supplies load current.

The digital outputs are in the form of discrete signals which service status indicators. As such, a storage register (one bit flip-flop) is required which is individually addressable by the computer with set or reset commands.

4. WEIGHT AND VOLUME

A packaging estimate must be referenced to a standardized packaging configuration. The following estimates are for an ARINC 1/4 ATR box (5" x 8" x 5") in which the Printed Circuit Boards (PCB) are mounted transversely in the box (approximately 4-1/2" x 7") or the estimates are based on the use of currently available devices and commonly employed PCB packaging techniques.

All of the digital control, gating, and computer interface logic could be mounted on one board. The A/D converter would require another board. Two synchro signal conditioning units could be mounted on a single additional board. Hence, two boards will be required for the four units. The multiplexor could also be mounted on a single board or it could be contained on the digital logic and A/D converter board. Therefore, a total of approximately six printed circuit boards would be required for interfacing. In addition to this, a precision power supply will be needed.

SECTION IX

SYSTEM SPECIFICATION

The elements of the system have been described in the previous sections. The data presented in this section is collected and summarized in the form of an outline specification. The specification is, of necessity, somewhat general in character. This is a consequence of the many assumptions made in the course of defining the system requirements. Therefore, a conservative approach has been taken in the design in order to assure a realistic estimate of hardware and software requirements. The following paragraphs outline the specifications. Over design assured.

?

1. SCOPE

The flight path optimization system shall be designed to optimize aircraft performance in the longitudinal plane. The system shall be designed to incorporate lateral plane performance optimization, at a later time, if desired. It shall operate as a stored program function generator under the control of the pilot.

2. SYSTEM FUNCTIONS

The system shall accept the inputs shown in Table XII, process them and provide the outputs in Table XIII*. Capability will be provided for defining optimized paths between any two arbitrary points on the altitude/Mach number flight envelope, and for establishing optimum cruise conditions. Specifically, the system shall assure that aircraft performance is optimized for:

Minimum Time Maneuvers
Minimum Fuel Maneuvers
Maximum Range Cruise
Maximum Endurance Cruise

These modes shall be individually selected by the pilot.

The system shall contain in its memory the data required to define these profiles for three different aircraft configurations. In addition, the system shall also

---

* Accuracy requirements, in bits, were determined by redefining the percentage error requirements into bit accuracy requirements. From Table VI, for instance, an 0.08% error limitation on command pitch angle translates to an error of one bit in eleven. Thus, the output accuracy requirement is 11 bits. The input requirement is slightly more accurate at 12 bits. To correlate further, an error of one bit in eight covers a percentage error range from 0.39% to 0.78%; one bit in nine - 0.196% to 0.39%; one bit in ten - 0.098 to 0.195%etc.

TABLE XII

SYSTEM INPUTS

Input Quantity	Source	Form	Accuracy	Input Rate
Pitch Angle	Stable Platform	Synchro	12 bits	80 per second
Normal Acceleration	Stable Platform	Synchro	11 bits	80 per second
Mach Number	Air Data Computer	Synchro	12 bits	75 per second
Altitude	Air Data Computer	Synchro	12 bits	80 per second
Ambient Temperature	Air Data Computer	Voltage	8 bits	2 per second
Angle of Attack	Angle of Attack Sensor	Potentiometer	8 bits	10 per second
Throttle Setting	Engine Sensor	Potentiometer	10 bits	15 per second
Aircraft Weight	Fuel Weight Sensor	Voltage	9 bits	1 per second
Aircraft Configuration	Stores Panel	Digital	8 bits	1 per second
Mode Selection	Pilot Panel	Digital	11 bits	1 per second
End Point Altitude	Pilot Panel	Digital	11 bits	1 per second
End Point Mach Number	Pilot Panel	Digital	11 bits	1 per second
Target Altitude	Data Link	Digital	9 bits	1 per second
Target Mach Number	Data Link	Digital	9 bits	1 per second

TABLE XIII

SYSTEM OUTPUTS

Output Quantity	Destination	Form	Accuracy	Output Rate
Pitch Command	ADI	Voltage	11 bits	80 per second
Profile Displacement	ADI	Voltage	7 bits	10 per second
Fast/Slow Indication	ADI	Voltage	9 bits	75 per second
Throttle Command	Profile Indicator	Digital	9 bits	15 per second
Mach Number	Profile Indicator	Digital	11 bits	75 per second
Altitude	Profile Indicator	Digital	11 bits	80 per second
Vector M	Profile Indicator	Digital	11 bits	75 per second
Vector h	Profile Indicator	Digital	11 bits	80 per second
Target Mach Number	Profile Indicator	Digital	8 bits	1 per second
Target Altitude	Profile Indicator	Digital	8 bits	1 per second
Command Mach Number	Mach Indicator	Digital	11 bits	1 per minute
Command Altitude	Altitude Indicator	Digital	11 bits	1 per minute

contain the data required to define optimum maneuvers for other than standard atmospheric conditions. For optimum flight path maneuvers, the system shall generate an output pitch command which is computed from a law containing smoothing, quickening, and other functions which aid in path tracking. For cruise operations, the system will generate optimum altitude and Mach number.

The system shall display command and status data to the pilot which will allow him to carry out the optimum commands in an efficient and timely manner. For optimum path maneuvers, command information will be displayed on an attitude director indicator and on a flight profile indicator. Pursuit displays of altitude and Mach number will be provided for cruise operations.

The system shall be capable of being reprogrammed to provide optimized command functions for aircraft drag configurations other than those considered here, without redesigning and rewiring the existing system.

3. SYSTEM ELEMENTS

The system shall contain the elements shown in Figure 33. These are composed of:

- Central Computer Unit
- Input/Output Unit
- Pilot's Control Panel
- Aircraft External Configuration Unit
- Profile Display Unit
- Direct Memory Access Channel
- Attitude Director Indicator
- Mach Number and Altitude Command Displays

These elements are described in the following sections.

a. Central Computer Unit

The system computer shall be an airborne general purpose digital type. As a minimum, the computer performance characteristics shall be the following:

Memory - 4,096 words of 24 bit length for program and data storage.

Memory Type - Memory shall be EANDRO for program storage. Working registers shall be contained either in a DRO or hardware stores. Display data storage shall be DRO.

Memory Protection - Provision shall be made to protect working storage from the effects of temporary power interruptions. Power shall maintain operating voltage for a time equivalent to the longest operation time.

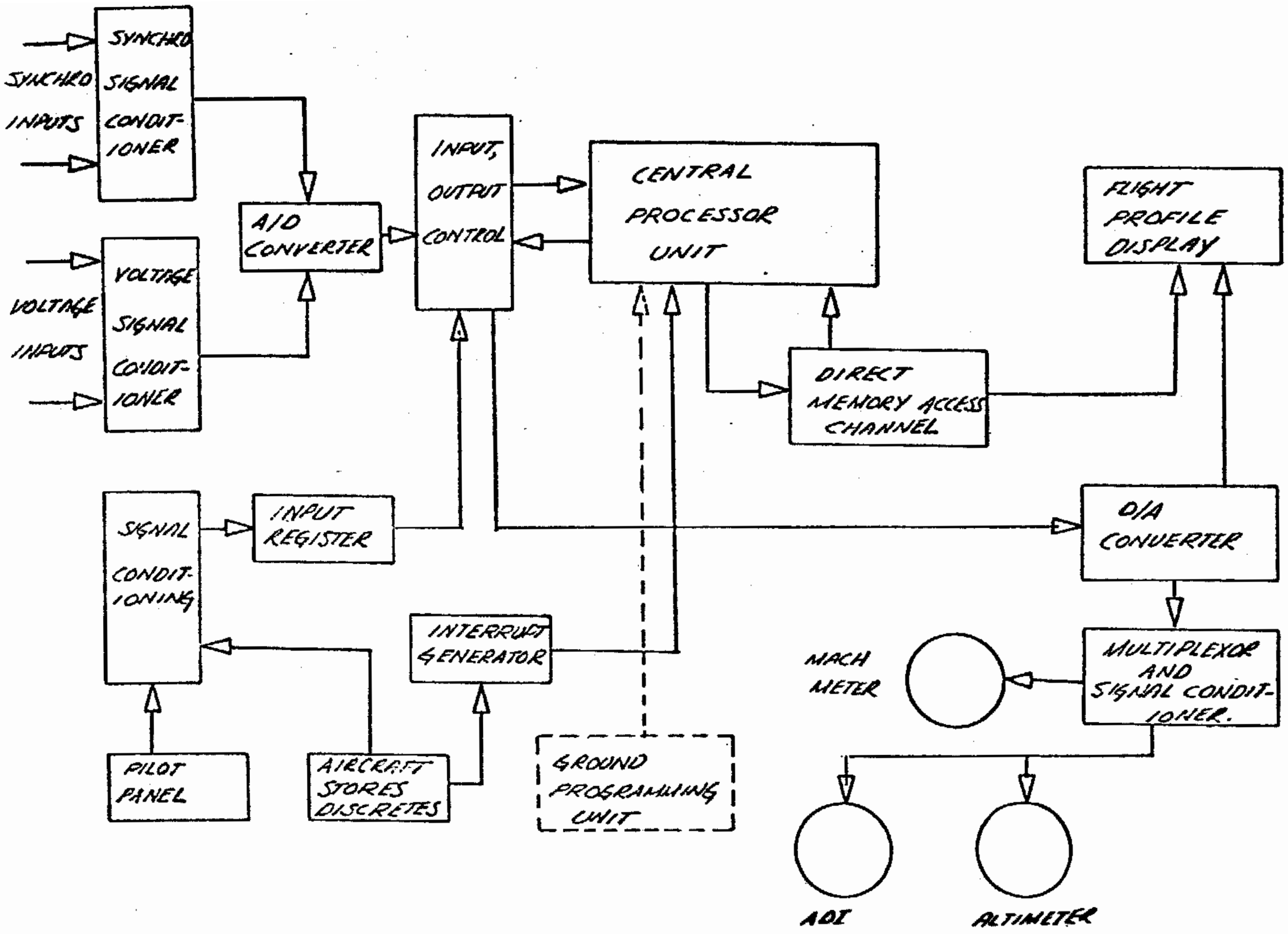

FIGURE 33 - FLIGHT PATH OPTIMIZATION SYSTEM BLOCK DIAGRAM

Direct Memory Access - A direct memory access channel shall be provided to allow data readout without interfering with the Central Processor Unit functions.

Memory Growth - Memory expansion to at least 8,192 words shall be possible.

Processing Speed - The computer shall be capable of executing at least 50,000 operations per second. At least 20% of these will be long commands of the multiply or divide type.

Interrupt - Although not a requirement for the basic system, it is preferable that the computer contain an external interrupt system. This system shall be capable of causing the program to jump to the highest priority interrupt occuring at any time. Priority decision shall be taken using the external hardware and shall include the level of the program currently being executed.

Input/Output - The computer input/output shall be capable of processing at least 1,000 inputs and 1,000 outputs per second. Transfer to the computer shall be bit serial, word serial.

Size & Weight - The computer shall occupy no more than 0.50 cu. ft. and weigh no more than 30 lbs.

Military - The computer shall conform to the following MIL Specs:
MIL E 5400 Class II Environment
MIL I 6181 Electromagnetic Interference
MIL STD 704 Power Supply Characteristics Cat. B

Technology - Microelectronic integrated circuits shall be used to the greatest extent possible.

Software - The following software shall be available:
Compiler - Fortran IV
Symbolic Assembler
Diagnostics
Self Test Program

b. Input/Output Unit

The input/output unit for the system shall be capable of processing input voltage, synchro, discrete, and digital data into a format which can be transferred directly to the computer. It shall also accept outputs from the computer for transfer to specified output and control devices.

(1) Inputs

The unit shall accept inputs from an air data computer, a stable platform and other aircraft performance sensors. The inputs to the unit are shown in Table XII together with accuracies, characteristics and transfer rates. The input unit shall provide growth potential to accommodate at least twice as many inputs as are shown.

The input unit shall contain circuitry for conditioning input data. It shall be an objective of the design to use a single multiplexed analog to digital converter. The circuitry shall contain provision for selection, conversion and transfer of input quantities. The unit shall use integrated circuits to the greatest extent possible.

(2) Outputs

The input/output unit shall provide the outputs shown in Table XIII at the rates and accuracies tabulated there.

(3) MIL Specifications

The system shall conform with the specifications listed for the computer.

(4) Size & Weight

The input/output unit (including direct memory access channel) shall occupy a maximum of 0.20 cu. ft. and weigh not more than 10 lbs.

c. Pilot's Control Panel

The pilot's control panel shall contain provision for control of the system. This shall include flight mode selection, test mode selection, and power control.

There shall be a minimum of four selectable flight modes with provision for at least twice that number. Actuation of the flight mode selector shall cause the system computer to branch to the appropriate software program and execute the functions associated with that mode. The operating modes shall be:

Flight Standby

Minimum Time Transition

Minimum Fuel Transition

Maximum Range Cruise

Maximum Endurance Cruise

Provision shall also be made for test modes as follows:

Ground Test

Flight Test

Actuation of the mode selector to either of these positions shall cause a system self-test program to be executed and, if successful, the display of fixed data on the ADI and Flight Profile Displays.

System power turn on shall be controlled from the panel. Three positions shall be provided - Off, Standby, and Operate.

d. Aircraft External Configuration Unit

Provision shall be included for automatically specifying the aircraft external stores configuration to the central computer unit. The unit shall specify the pylon position and the store carried thereon. Store identity will be established on the ground and the unit will define a drag configuration based on the continuing presence of each store.

e. Profile Display Unit

A profile display unit shall be provided which displays aircraft state and command data. The display unit shall have the capacity to display:

- The required altitude/Mach number profile.
- The limiting flight envelope of the aircraft for the particular external store configuration, and ambient conditions which prevail.
- The current aircraft position on the altitude/Mach number plane.
- The current aircraft vector direction on the altitude/Mach number plane.
- Numerical read out of end point altitude and Mach number.
- The current target position on the altitude/Mach number plane.
- The current target vector direction on the altitude/Mach number plane.
- A throttle command magnitude and annunciator - the annunciator to be extinguished when the command is satisfied.
- Display mode selection (four modes to be defined).
- Provision for similar pictorial data display on the altitude/distance plane.
- Provision for numerical end point selection.

The display shall be provided using a Cathode Ray Tube (CRT) with an effective working diameter of six inches. The display capacity shall be adequate for displaying the following:

- 8 Symbols each made up of an equivalent line length no longer than the display diameter
- 36 Alphanumeric symbols
- 16 Tactical symbols
- 3 axis symbols

In addition, the display shall meet the following specifications:

Accuracy	0.50%
Resolution	0.25%
Frame Rate	30 per second with P-1 phosphor or equivalent
Brightness	Adequate for viewing in the presence of an ambient brightness of 1,000 ft. L.

f. Direct Memory Access

The data to be displayed on the profile indicator shall be stored in the destructive read out portion of computer memory. The direct memory access channel shall be designed to read this data out of memory, re-write it and send it to the display unit in digital format. Data to be read out shall include symbol coordinate and bright up control. The data read out cycle shall cause a minimum of interference with the central processor unit.

g. Attitude Director Indicator

An attitude director indicator shall be provided to display the following:

- Pitch Command
- Fast/Slow Indication
- Position relative to required profile
- Aircraft pitch angle

The display of these data shall be under the control of the central processor unit. Display accuracies shall be:

	Accuracy	Range
Attitude Command	0.25 deg.	± 90 deg.
Fast/Slow Indication Bracket	0.005M	0.6 - 2.5
Vertical Position Relative to Required Profile	1.0%	500 ft.
Aircraft Attitude	0.5 deg.	± 90 deg.

h. Mach Number and Altitude Command Displays

It shall be a requirement during the cruise modes of operation to provide Mach number and height command outputs on the Mach meter and altimeter. These outputs shall be provided to 0.001M and 50 feet respectively in the form of a digital output to the respective instruments.

4. OTHER FEATURES

a. Self-Test

The system shall contain provision for the execution of self test programs and routines to allow the isolation of faults at the printed circuit level. A minimum amount of ground equipment shall be necessary to achieve this objective.

b. System Software

The basic techniques used for execution of system tasks shall be polynomial curve fitting for Rutowski profile generation, solution of the Boyd equations for definition of profiles associated with arbitrary end points, and generation of the control commands according to an equation determined by analysis and simulation. In addition, there shall be provision for automatic start up and shut down programs, an on-line system self test, and a maintenance self test.

SECTION X

CONCLUSION

It has been shown that aircraft vertical plane flight performance optimization can be simplified to a form which is manageable in an airborne computer-display system. Further, it has been demonstrated with reasonable certainty that a system can be developed from state-of-the-art equipment. While system specifications are somewhat general due to the many assumptions made in developing system requirements, derived system weight and volume requirements seem well within achieveable limits. These remarks remain true even if one reserves a sizeable portion of computer capacity for growth into the three dimensional optimization area.

The system is not one which is easily implemented in an F-104C, however, because this particular aircraft has few of the elements which are common to modern avionics systems. For example, the F-104C has no air data system, nor does it have a stable platform or advanced display devices. The F-104C was chosen as a representative aircraft because of the optimum flight path data that was already available through previous studies. In general, applications of the system to developmental aircraft seem more practical in that integration with other systems can be considered in the initial design stages. Application of the system requires the development of optimum path data, however, regardless of what aircraft is involved.

Finally, the system is capable of providing optimum flight performance information, but only in terms of a limited set of performance criteria. Even though the system recommended here covers a good portion of the optimal maneuvers, there are several other cases which should be eventually incorporated in an operational system. As pointed out in Section IV the most important of these are zoom climb to launch, and range-time-altitude rendezvous. Before embarking on a system development program, then, more thought should be given to defining a system operational requirement.

APPENDIX I

TRANSITION FROM ARBITRARY POINTS IN STATE SPACE TO RUTOWSKI PATHS

Colonel J.R. Boyd [16] has developed a rule of thumb for determining "optimum" flight paths from arbitrary points in state space to Rutowski paths. The basic rule of thumb is of the following form:

$$\Delta E_s = K \Delta h \tag{68}$$

where $K = 2/3$ if Mach number is to be decreased

or $K = -1$ if Mach number is to be increased

and where

$$E_s = h + V^2/2g \tag{69}$$

The rule of thumb is used to transition between energy states on a height/Mach number diagram. For example, if one wishes to transition from the state P_o to the state P_1 on Figure 34, the rule of thumb indicates there are two possible ways. One way involves a dive/climb where Mach number is first increased and then decreased. The other involves a climb/dive where the opposite occurs. In general, only one path approaches a true optimum, however.

In the course of Boyd's studies it has become evident that the criterion for deciding on which path is optimum depends on the relationship of the end points to the basic Rutowski path. If the basic path is to the right or above the end points, one always dives before climbing. If the basic path is to the left or below, one always climbs before diving. The criterion is illustrated on Figure 35.

If, during the course of a transition maneuver, the transition curves intersect the basic path, one follows the basic path from the point of initial intersection until the second transition path is encountered. One then follows the second transition curve until the terminal point is reached. The technique is illustrated on Figure 13.

Despite the elaborate set of rules which have been suggested, the rule of thumb is actually rather conceptual. The rule has only been developed for subsonic flight. In addition, it has not been tested to the extent that there is a great deal of confidence in the values of K which have been suggested. Nevertheless, the development of a practical on-board computer system requires that rules of this type be employed. Therefore, it will be assumed that rules exist for all parts of the flight regime, and that with sufficient exercise of existing computer programs their form can be ascertained.

In order to determine the form of equations which define actual transition paths, consider equations (68) and (69). If, in equation (69), an incremental change in each of the variables is made, the result is:

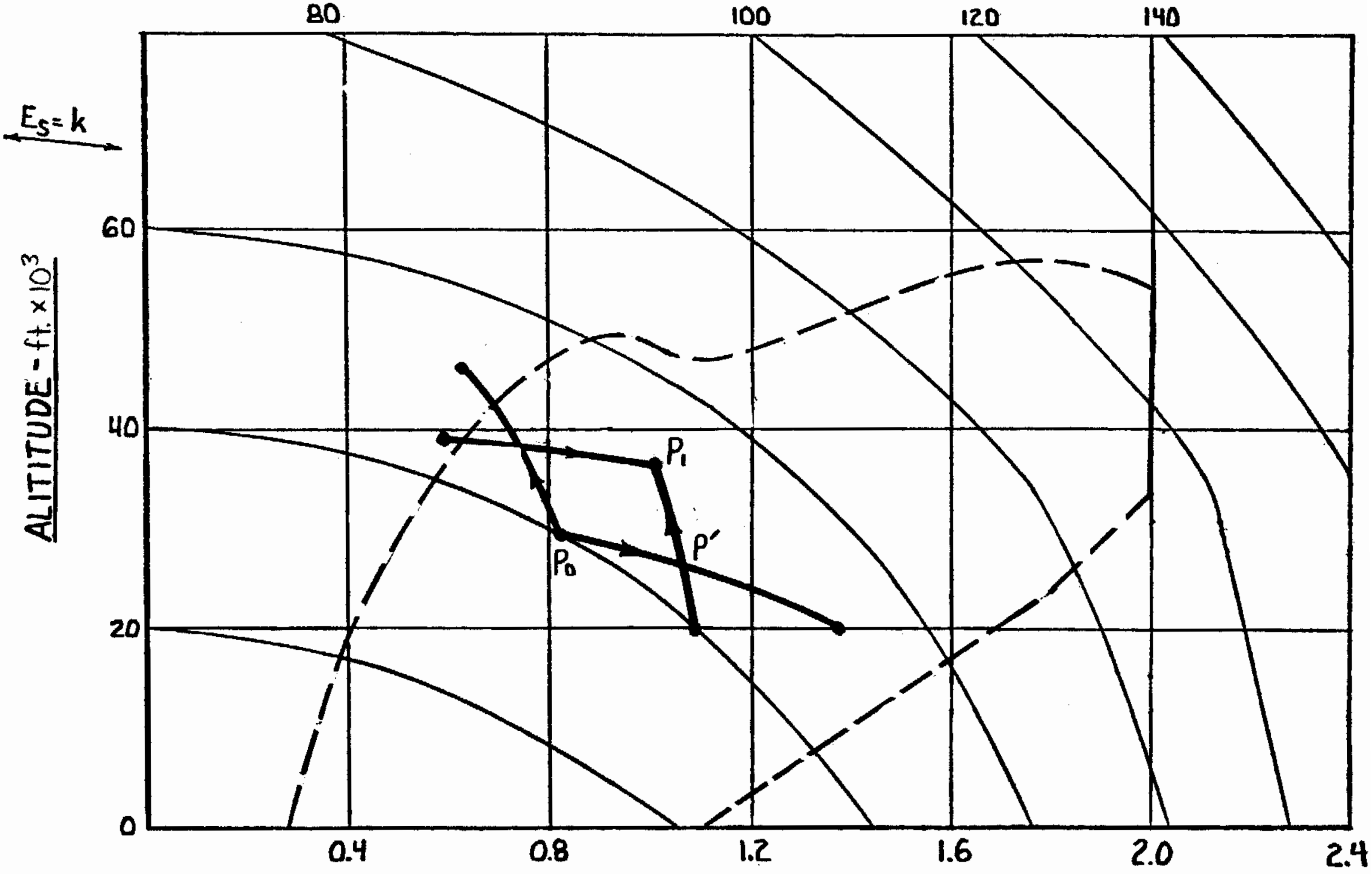

FIGURE 34-ALTERNATIVE BOYD FLIGHT PATHS

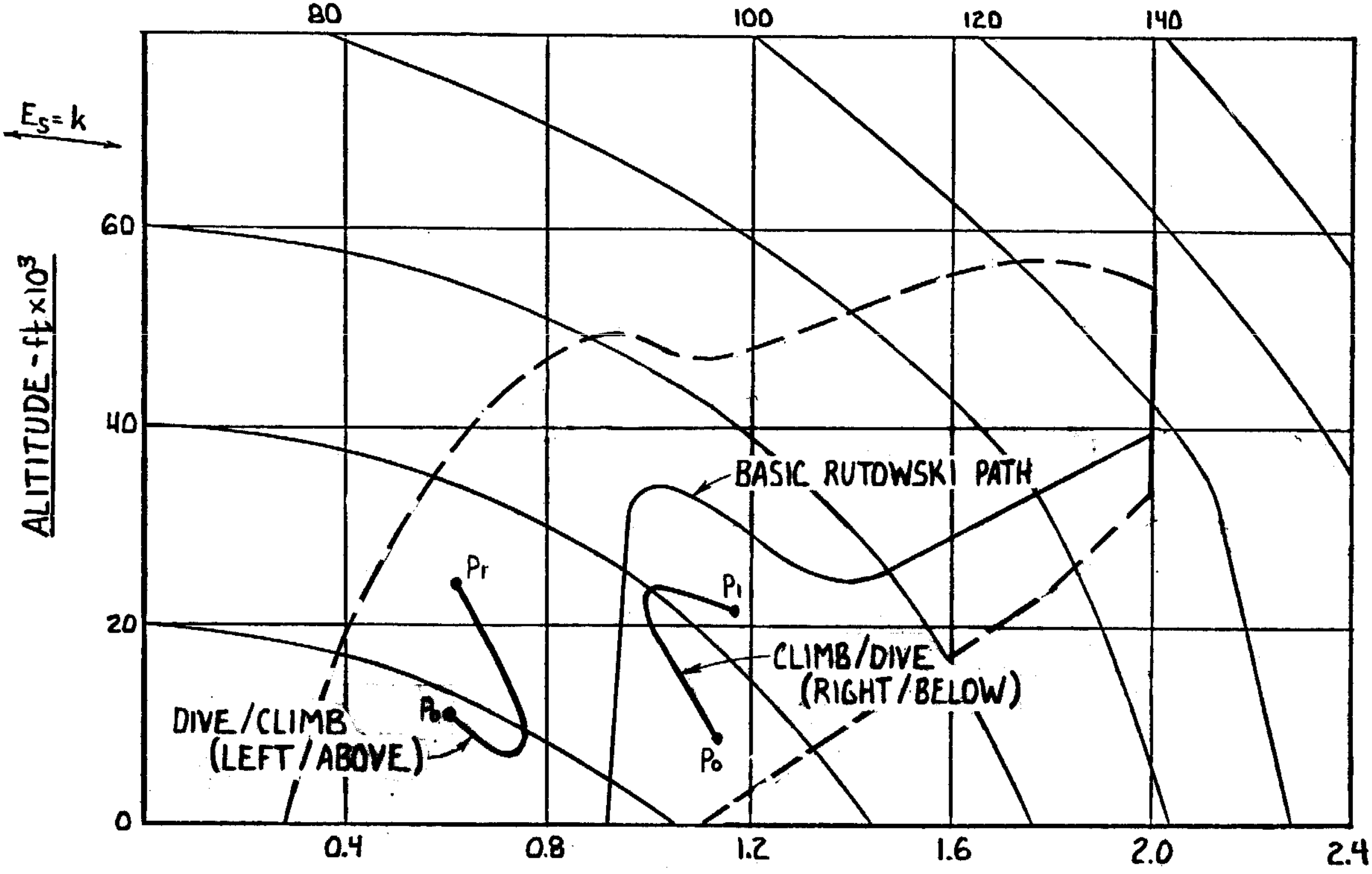

FIGURE 35 - BOYD PATH DIRECTION CONVENTION

$$E_s + \Delta E_s = h + \Delta h + \frac{(V + \Delta V)^2}{2g} \tag{70}$$

and since $E_s = h + \frac{V^2}{2g}$

$$\Delta E_s = \Delta h + \frac{2V \Delta V + \Delta V^2}{2g} \tag{71}$$

But from equation (68),

$$\Delta E_s = K \Delta h \tag{72}$$

Therefore,

$$K \Delta h = \Delta h + \frac{2V \Delta V + \Delta V^2}{2g} \tag{73}$$

Taking the limit, now, as Δh and ΔV approach zero yields

$$\lim_{\Delta h \to 0} K \Delta h = \lim_{\Delta h \to 0} \Delta h + \lim_{\Delta V \to 0} \frac{2V \Delta V + \Delta V^2}{2g}$$

$$\text{or} \quad K dh = dh + \frac{2V dV + (dV)^2}{2g} \tag{74}$$

where the term $(dV)^2$ is second order as compared to VdV

Therefore,

$$dh = \frac{-VdV}{g(1-K)} \tag{75}$$

This equation can now be integrated. Bearing in mind that equation (70) can represent either a climb or a dive, integration must include two segments. On Figure 34 , for instance, if a dive/climb profile is required we must first integrate from P_o to P' with $K = K' = -1$.

Following this, integration must be carried out over the climb portion of the transition. $K = K'' = 2/3$ P' to P_1 . Using this reasoning, the dive portion of the profile is

$$\int_{h_o}^{h} dh = -\int_{V_o}^{V} \frac{VdV}{g(1-K')} \tag{76}$$

or
$$h-h_o = \frac{V_o^2 - V_1^2}{4g} \qquad (77)$$

and the climb portion is

$$h_1 - h = \frac{V^2 - V_1^2}{2/3\ g} \qquad (78)$$

If the maneuver was climb/dive rather than dive/climb, the equations would take the form:

$$h - h_o = \frac{V_o^2 - V^2}{2/3\ g} \qquad (79)$$

$$h_1 - h = \frac{V^2 - V_1^2}{4g} \qquad (80)$$

Summarizing, the equations for the paths for the two maneuvers are listed as follows:

Dive-Climb

Initial Dive:

$$h = h_o + \frac{V_o^2 - V^2}{4g} \qquad (81)$$

Subsequent Climb:

$$h = h_1 + \frac{V_1^2 - V^2}{2/3\ g} \qquad (82)$$

Climb-Dive

Initial Climb:

$$h = h_o + \frac{V_o^2 - V^2}{2/3\ g} \qquad (83)$$

Subsequent Dive:

$$h = h_1 + \frac{V_1^2 - V^2}{4g} \qquad (84)$$

All that remains is to define the transition point between the dive and climb portions of the maneuver. These are given as follows:

Equating (81) and (82) yields:

$$h_o + \frac{V_o^2 - V^2}{4g} = h_1 + \frac{V_1^2 - V^2}{2/3\ g} \tag{85}$$

$$V^2 = 4/5\ g(h_1 - h_o) + 6/5\ V_1^2 - 1/5\ V_o^2 \tag{86}$$

Substituting this into equation (81)

$$h = 1/5\ (6\ h_o - h_1) + \frac{3}{10g}\ (V_o^2 - V_1^2) \tag{87}$$

Equating (83) and (84) yields:

$$V^2 = \frac{-V_1^2}{5} + 6/5\ V_o^2 - 4/5\ g(h_1 - h_o) \tag{88}$$

Substituting this into equation (84)

$$h = 1/5\ (6h_1 - h_o) + \frac{3}{10g}\ (V_1^2 - V_o^2) \tag{89}$$

The transition altitudes and velocities are:

Dive-Climb

$$V = \sqrt{4/5\ g(h_1 - h_o) - 6/5\ V_1^2 - 1/5\ V_o^2} \tag{90}$$

$$h = 1/5\ (6\ h_o - h_1) + \frac{3}{10g} \tag{91}$$

Climb-Dive

$$V = \sqrt{6/5\ V_o^2 - 1/5\ V_1^2 - 4/5\, g\ (h_1 - h_o)} \tag{92}$$

$$h = 1/5\ (6h_1 - h_o) + \frac{3}{10g}\ (V_1^2 - V_o^2) \tag{93}$$

Using similar techniques equations for dive-dive and climb-climb maneuvers can also be determined.

APPENDIX II

DERIVATION OF AIRCRAFT VELOCITY AND PITCH RESPONSE CHARACTERISTICS

The character of the derivation and the notation used here will be similar to that found in reference 49.

Using a stability axis approach, as noted on Figure 7 with moments of inertia representing the i coordinate system, the three longitudinal equations of motion are given as follows:

$$F_x = m(\dot{U} + QW - RV) \tag{94}$$

$$F_z = m(\dot{W} + PV - QU) \tag{95}$$

$$M = \dot{h}_y + Rh_x - Ph_z \tag{96}$$

where, by means of Euler transformations shown on Figure 36.

$$h_x = AP - FQ - ER \tag{97}$$

$$h_y = -FP + BQ - DR \tag{98}$$

$$h_z = -EP - DQ + CR \tag{99}$$

Since the present study is concerned with longitudinal aircraft motions

$$P = R = V = \dot{P} = \dot{R} = 0$$

and equations (94), (95), and (96) become

$$F_x = m(\dot{U} + QW) \tag{100}$$

$$F_z = m(\dot{W} - QU) \tag{101}$$

$$M = B\dot{Q} \tag{102}$$

1. ELEVATOR EQUATION

In addition to these three equations, equations representing control functions must also be considered. An equation for the elevator, as shown in reference 49, takes the following form:

$$I\ddot{\delta} + mea + P_e(PR - \dot{Q}) = H_e + F_e \tag{103}$$

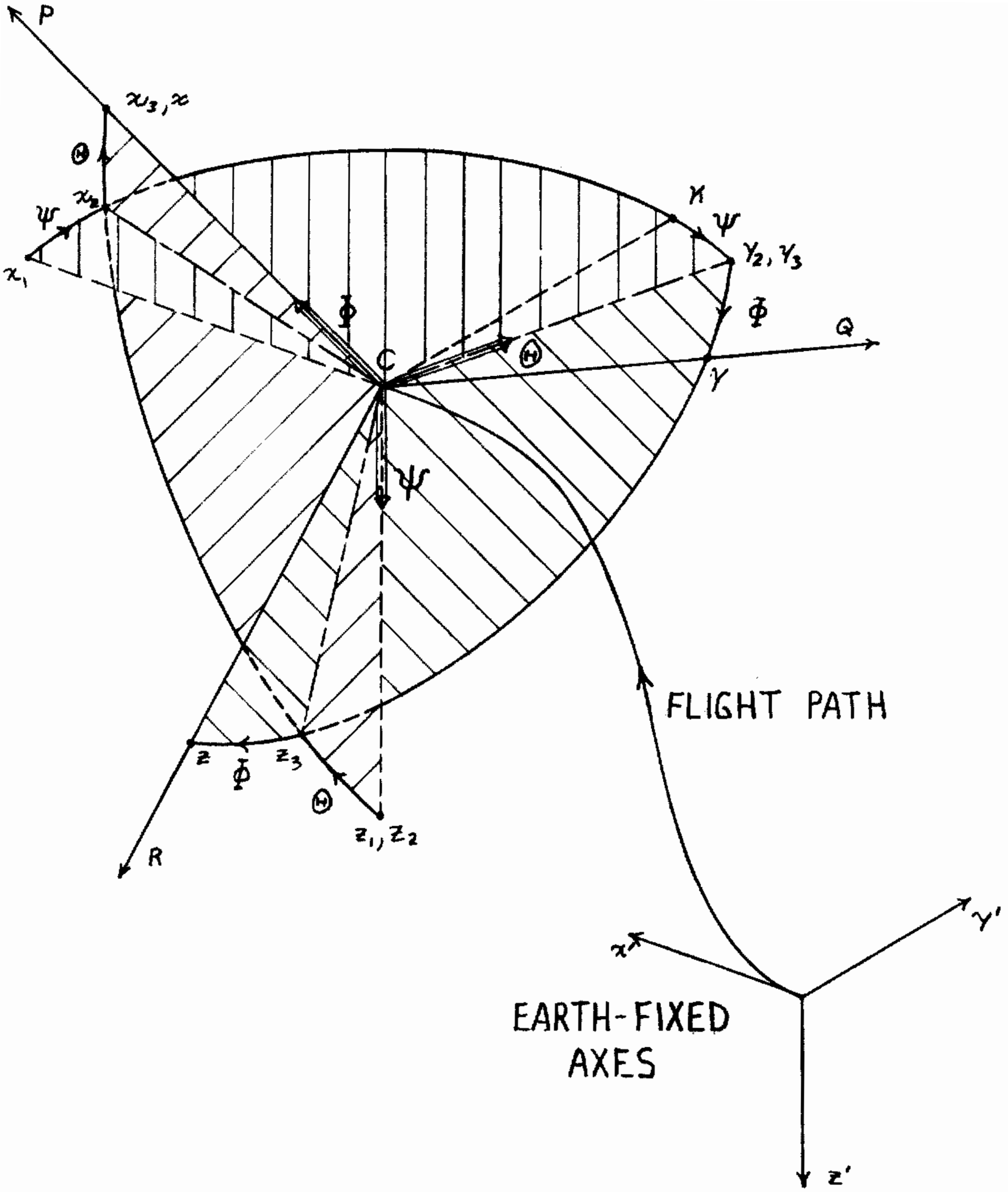

FIGURE 36- AIRCRAFT ORIENTATION

If it is assumed that the elevator is dynamically balanced, then the terms e and P_e are zero. (This is a good assumption in that the elevator must be carefully balanced to avoid flutter difficulties.) Equation (103) then becomes:

$$I\ddot{\delta} = H_e + F_e \tag{104}$$

An Additional control equation could be written for the throttle. It is assumed, however, that the dynamics of the throttle mechanism are fast enough so as to be unimportant in the present application.

2. APPLIED FORCES

Applied forces include aerodynamic, propulsive, and gravitational forces. The terms F_x, and F_z in equations (94) and (95) are, therefore, defined accordingly:

$$F_x = X - mg \sin \Theta \tag{105}$$

$$F_z = Z + mg \cos \Theta \tag{106}$$

where X and Z are the aerodynamic and propulsive forces. Equations (100) and (101) then become:

$$X - mg \sin \Theta = m(\dot{U} + QW) \tag{107}$$

$$Z + mg \cos \Theta = m(\dot{W} - QU) \tag{108}$$

3. SMALL DISTURBANCE THEORY

In determining the aircraft's response to pitch and throttle changes the perturbations of interest are generally small in nature. Therefore, the variables of interest can be defined in terms of a reference value and a small perturbation which is continuously variable. Using this reasoning the following quantities are defined:

$$U = U_o + u = V + u \tag{109}$$

$$W = W_o + w \tag{110}$$

$$Q = Q_o + q \tag{111}$$

$$\delta = \delta_o + \eta \tag{112}$$

$$\Theta = \Theta_o + \theta \tag{113}$$

$$M = M_o + \Delta M \tag{114}$$

$$X = X_o + \Delta X \tag{115}$$

$$Z = Z_o + \Delta Z \tag{116}$$

$$H_e = H_{eo} + \Delta H_e \tag{117}$$

$$F_e = F_{eo} + \Delta F_e \tag{118}$$

Further,

$$\sin\Theta = \sin(\Theta_o + \theta) = \sin\Theta_o \cos\theta + \cos\Theta_o \sin\theta$$

$$= \sin\Theta_o + \theta \cos\Theta_o \tag{119}$$

where $\cos\theta \cong 1$ (120)

$$\sin\theta \cong \theta \tag{121}$$

Similarly,

$$\cos\Theta = \cos\Theta_o - \theta \sin\Theta_o \tag{122}$$

For a particular standard optimal maneuver it will be assumed that,

$$X_o - mg \sin\Theta_o = 0 \tag{123}$$

$$Z_o + mg \cos\Theta_o = 0 \tag{124}$$

$$M_o = 0 \tag{125}$$

$$H_{eo} + F_{eo} = 0 \tag{126}$$

$$W_o = 0 \tag{127}$$

$$q_o = 0 \tag{128}$$

Also it is clear that,

$$\dot{U} = \dot{u} \tag{129}$$

$$QW = qw \tag{130}$$

$$QU = qV + qu \cong qV \tag{131}$$

where in the last equation the term qu is considerably smaller than qV. (Note that the V here is not the same as the V used earlier in equations (94) and (95).)

Using the equations (109) - (118), (119), (122), and (129) - (131), equations (102) and (104) - (106) become:

$$X_o + \Delta X - mg(\sin\Theta_o + \theta\cos\Theta_o) = m(\dot{u} + qw) \qquad (132)$$

$$Z_o + \Delta Z + mg(\cos\Theta_o - \theta\sin\Theta_o) = m(\dot{w} - qV) \qquad (133)$$

$$M_o + \Delta M = B\dot{q} \qquad (134)$$

$$I\ddot{\eta} = H_{eo} + \Delta H_e + F_{eo} + \Delta F_e \qquad (135)$$

If now, motions along the standard path are removed, by subtracting out equations (123) to (128), only the perturbation effects remain. Equations (132) - (135) then become:

$$\Delta X - mg\theta\cos\Theta_o = m\dot{u} \qquad (136)$$

$$\Delta Z - mg\theta\sin\Theta_o = m(\dot{w} - qV) \qquad (137)$$

$$\Delta M = B\dot{q} \qquad (138)$$

$$I\ddot{\eta} = \Delta H_e + \Delta F_e \qquad (139)$$

where in equation (136) the term mqw on the right side has been dropped as being second order in relation to mu. These last four equations are the equations of interest.

In order to define the incremental aerodynamic forces and moments, an approach similar to the following will be used.

$$\Delta A = \frac{\partial A}{\partial u}u + \frac{\partial A}{\partial \dot{u}}\dot{u} + \cdots + \frac{\partial A}{\partial \dot{\eta}}\dot{\eta} + \frac{\partial A}{\partial \ddot{\eta}}\ddot{\eta} + \cdots \qquad (140)$$

$$= A_u u + A_{\dot{u}}\dot{u} + \cdots + A_{\dot{\eta}}\dot{\eta} + A_{\ddot{\eta}}\ddot{\eta} + \cdots \qquad (141)$$

Thus,

$$\Delta X = \Delta X(u,w) = X_u u + X_w w \qquad (142)$$

$$\Delta Z = \Delta Z(u,w,q,\eta) = Z_u u + Z_w w + Z_q q + Z_\eta \eta \qquad (143)$$

$$\Delta M = \Delta M(u,w,q,\eta) = M_u u + M_w w + M_q q + M_\eta \eta \qquad (144)$$

$$\Delta H_e = \Delta H_e(u,w,w,q,\eta) = H_{eu}u + H_{ew}w + H_{e\dot{w}}\dot{w} + H_{eq}q + H_{e\eta}\eta \qquad (145)$$

Each of the partial derivatives in equations (142) - (145) must be further evaluated to completely define the incremental functions. This is a lenghty process and would require a large amount of space. Therefore, in order to illustrate the procedure and still keep the narrative brief, only one term will be developed in detail. The remaining terms will be listed in order following this.

The term which will be developed in detail is X_u. If the coefficient C_x is introduced then the X term can be defined as follows:

$$X = \frac{1}{2} C_x \rho V^2 S \tag{146}$$

and

$$X_u = \left(\frac{\partial X}{\partial u}\right)_o = C_{x_o} \rho V S \frac{\partial V}{\partial u} + \frac{1}{2} \rho V^2 S \left(\frac{\partial C_x}{\partial u}\right)_o \tag{147}$$

where the subscript zero indicates that the quantity is evaluated at the reference flight condition.

From Figure 7 the aerodynamic and thrust forces on the aircraft are given as follows in the x direction:

$$X = T + L\sin\alpha - D\cos\alpha \tag{148}$$

$$\cong T + L\alpha - D \tag{149}$$

for all α. Also,

$$C_x = C_T + C_L\alpha - C_D \tag{150}$$

Therefore,

$$\left(\frac{\partial C_x}{\partial u}\right)_o = \left(\frac{\partial C_T}{\partial u}\right)_o + \left(\alpha \frac{\partial C_L}{\partial U}\right)_o - \left(\frac{\partial C_D}{\partial u}\right)_o \tag{151}$$

$$= \left(\frac{\partial C_T}{\partial u}\right)_o - \left(\frac{\partial C_D}{\partial u}\right)_o \tag{152}$$

where at the reference condition $\alpha = 0$.

Now

$$T = T(h, V, N) \tag{153}$$

$$= \frac{1}{2} C_T \rho V^2 S \tag{154}$$

Therefore,

$$\left(\frac{\partial C_T}{\partial u}\right)_o = \left(\frac{\partial T}{\partial u}\right)_o \left(\frac{1}{1/2\ \rho V^2 S}\right) - \frac{2T_o}{1/2\ \rho VS} + \left(\frac{\partial T}{\partial h}\frac{\partial h}{\partial u}\right)_o \times \left(\frac{1}{1/2\ \rho V^2 S}\right) + \left(\frac{\partial T}{\partial N}\frac{\partial N}{\partial u}\right)_o \left(\frac{1}{1/2\ \rho V^2 S}\right) \tag{155}$$

Also

$$C_D = C_D(\alpha, M) \tag{156}$$

$$\frac{\partial C_D}{\partial u} = \frac{\partial C_D}{\partial M}\frac{\partial M}{\partial u} \tag{157}$$

and since

$$M = \frac{V+u}{a} \tag{158}$$

$$\frac{\partial M}{\partial u} = \frac{1}{a} \tag{159}$$

Therefore,

$$\frac{\partial C_D}{\partial u} = \frac{1}{a}\frac{\partial C_D}{\partial M} \tag{160}$$

and equation (152) becomes

$$\left(\frac{\partial C_x}{\partial u}\right)_o = \left(\frac{\partial T}{\partial u}\right)_o \left(\frac{1}{1/2\ \rho V^2 S}\right) - \frac{2C_{T_o}}{V} + \left(\frac{\partial T}{\partial h}\frac{\partial h}{\partial u}\right)_o \times \left(\frac{1}{1/2\ \rho V^2 S}\right) + \left(\frac{\partial T}{\partial N}\frac{\partial N}{\partial u}\right)_o \left(\frac{1}{1/2\ \rho V^2 S}\right) + \frac{1}{a}\frac{\partial C_D}{\partial M} \tag{161}$$

This can then be substituted into equation (147) to yield

$$X_u = \left(\frac{\partial T}{\partial u}\right)_o + \left(\frac{\partial T}{\partial h}\frac{\partial h}{\partial u}\right)_o + \left(\frac{\partial T}{\partial N}\frac{\partial N}{\partial u}\right)_o - C_{D_o}\rho VS - M\frac{\partial C_D}{\partial M}\frac{1}{2}\rho VS \tag{162}$$

Using similar techniques

$$X_w = \frac{1}{2}\rho VSC_{L_o}\left(1 - \frac{2C_{L\alpha}}{\pi \mathit{AR}\, e}\right) \tag{163}$$

$$Z_u = -\frac{1}{2}\rho VS\left(M\frac{\partial C_L}{\partial M} + 2C_{L_o}\right) \tag{164}$$

$$Z_w = -\frac{1}{2}\rho VS\left(C_{L_\alpha} + C_{D_o}\right) \tag{165}$$

$$Z_q = -\bar{c}a_t V_H \frac{1}{2}\rho VS \tag{166}$$

$$Z_\eta = -C_{L_\delta}\frac{1}{2}\rho V^2 S \tag{167}$$

where

$$Z = C_Z \frac{1}{2}\rho V^2 S = -\left(C_L + C_{D_\alpha}\right)\frac{1}{2}\rho V^2 S \tag{168}$$

$$M_u = \left(M\frac{\partial C_m}{\partial M} + 2C_{m_o}\right)\frac{1}{2}\rho VS\frac{\bar{c}}{2} \tag{169}$$

$$M_w = \frac{1}{2}\rho VS\frac{\bar{c}}{2}C_{m_\alpha} \tag{170}$$

$$M_q = \frac{1}{2}\rho VS\left(\frac{\bar{c}}{2}\right)^2 C_{m_q} \tag{171}$$

$$M_\eta = \frac{1}{2}\rho V^2 S\frac{\bar{c}}{2}C_{m_\delta} \tag{172}$$

where

$$M = C_m\frac{1}{2}\rho V^2 S\frac{\bar{c}}{2} = \left(C_{m_o} + C_{m_\alpha}\alpha + C_{m_\delta}\delta\right)\frac{1}{2}\rho V^2 S\frac{\bar{c}}{2} \tag{173}$$

Also,

$$H_{eu} = \frac{1}{2}\rho VMS_e\bar{c}_e\frac{\partial C_{he}}{\partial M} + C_{heo}\rho VS_e\bar{c}_e \tag{174}$$

$$H_{ew} = \frac{1}{2}\rho VS_e\bar{c}_e\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)C_{hea_t} \tag{175}$$

$$H_{e\acute{w}} = \rho S_e\bar{c}_e l_t\frac{\partial \epsilon}{\partial \alpha}C_{hea_t} \tag{176}$$

$$H_{eq} = \frac{1}{2}\rho VS_e\bar{c}_e l_t C_{hea_t} \tag{177}$$

$$H_{e\eta} = \frac{1}{2}\rho V^2 S_e\bar{c}_e C_{h_\delta} \tag{178}$$

where

$$H_e = C_{he}\frac{1}{2}\rho V^2 S_e \bar{c}_e \tag{179}$$

Also,

$$\Delta F_e = \Delta C_{fe}\frac{1}{2}\rho V^2 S_e \bar{c}_e \tag{180}$$

Substituting equations (162) & (163) into equation (142) and then equation (142) into equation (136) yields

$$\begin{aligned} m\dot{u} = {} & \left[\left(\frac{\partial T}{\partial u}\right)_o + \left(\frac{\partial T}{\partial h}\frac{\partial h}{\partial u}\right)_o + \left(\frac{\partial T}{\partial N}\frac{\partial N}{\partial u}\right)_o - C_{D_o}\rho VS \right. \\ & \left. -M\frac{\partial C_D}{\partial M}\frac{1}{2}\rho VS\right]u + \frac{1}{2}\rho V^2 SC_{L_o}\left(1 - \frac{2C_{L\alpha}}{\pi A\!\!\!R\, e}\right)\alpha \\ & -\frac{1}{2}\rho V^2 SC_{L_o}\theta \end{aligned} \tag{181}$$

where

$$\alpha \cong \frac{w}{V} \tag{182}$$

and

$$L_o = mg\cos\Theta_o = C_{L_o}\frac{1}{2}\rho V^2 S \tag{183}$$

Similarly equations (137) - (139) can be revised by using equations (143) - (145), (164) - (167), (169) - (172), (174) - (178) and (180). The resulting equations are listed as follows:

$$\begin{aligned} -mV\dot{\gamma} = {} & \frac{1}{2}\rho VS\left(M\frac{\partial C_L}{\partial M} + 2C_{L_o}\right)u + \frac{1}{2}\rho V^2 S\left(C_{L_\alpha} + C_{D_o}\right)\alpha \\ & + \frac{1}{2}\rho VS\bar{c}a_t V_H\dot{\theta} + \frac{1}{2}\rho V^2 SC_{L_\delta}\eta + \frac{1}{2}\rho V^2 SC_{L_o}\theta\tan\Theta_o \end{aligned} \tag{184}$$

where

$$\dot{\theta} \cong q \tag{185}$$

$$\dot{w} = V\dot{\alpha} \tag{186}$$

$$\dot{\gamma} = \dot{\theta} - \dot{\alpha} \tag{187}$$

and

$$mg\theta\sin\Theta_o = L_o\theta\tan\Theta_o = \frac{1}{2}\rho V^2 SC_{L_o}\theta\tan\Theta_o \tag{188}$$

$$B\ddot{\theta} = \frac{1}{2}\rho VS\frac{\bar{c}}{2}\left(M\frac{\partial C_m}{\partial M} + 2C_{m_o}\right)u + \frac{1}{2}\rho V^2S\frac{\bar{c}}{2}C_{m_\alpha}\alpha$$

$$+ \frac{1}{2}\rho VS\left(\frac{\bar{c}}{2}\right)^2 C_{m_q}\dot{\theta} + \frac{1}{2}\rho V^2S\frac{\bar{c}}{2}C_{m_\delta}\eta \tag{189}$$

$$I\ddot{\eta} = \frac{1}{2}\rho VS_e\bar{c}_e\left(M\frac{\partial C_{he}}{\partial M} - 2C_{heo}\right)u + \frac{1}{2}\rho V^2S_e\bar{c}_e\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right)C_{he\alpha_t}\alpha$$

$$+ \rho VS_e\bar{c}_e l_t\frac{\partial\epsilon}{\partial\alpha}C_{he\alpha_t}\dot{\alpha} + \frac{1}{2}\rho VS_e\bar{c}_e l_t C_{he\alpha_t}\dot{\theta}$$

$$+ \frac{1}{2}\rho V^2S_e\bar{c}_e C_{h_\delta}\eta + \frac{1}{2}\rho V^2S_e\bar{c}_e\Delta C_{fe} \tag{190}$$

where

$$\Delta F_e = \frac{1}{2}\rho V^2S_e\bar{c}_e\Delta C_{fe} \tag{191}$$

If, now, equations (181) and (184) are divided by $\frac{1}{2}\rho V^2S$, equation (189) is divided by $\frac{1}{2}\rho V^2S\frac{\bar{c}}{2}$, and equation (190) by $\frac{1}{2}\rho V^2S_e\bar{c}_e$, the resulting four equations are:

$$\frac{m\dot{u}}{1/2\ \rho V^2S} = \left[\left(\frac{\partial C_T}{\partial u}\right)_o + \left(\frac{\partial C_T}{\partial h}\frac{\partial h}{\partial u}\right)_o + \left(\frac{\partial C_T}{\partial N}\frac{\partial N}{\partial u}\right)_o - \frac{2C_{D_o}}{V}\right.$$

$$\left. - \frac{1}{a}\frac{\partial C_D}{\partial M}\right]u + C_{L_o}\left(1 - \frac{2C_{L_\alpha}}{\pi \mathit{R}\ e}\right)\alpha - C_{L_o}\theta \tag{192}$$

$$\frac{mV(\dot{\theta} - \dot{\alpha})}{1/2\ \rho V^2S} = \left(\frac{1}{a}\frac{\partial C_L}{\partial M} + \frac{2C_{L_o}}{V}\right)u + \left(C_{L_\alpha} + C_{D_o}\right)\alpha + \frac{\bar{c}a_tV_H}{V}\dot{\theta}$$

$$+ C_{L_\delta}\eta + C_{L_o}\theta\tan\Theta_o \tag{193}$$

$$\frac{B\ddot{\theta}}{1/2\ \rho V^2S\frac{\bar{c}}{2}} = \left(\frac{1}{a}\frac{\partial C_m}{\partial M} + \frac{2C_{m_o}}{V}\right)u + C_{m_\alpha}\alpha + \frac{\bar{c}}{2V}C_{m_q}\dot{\theta}$$

$$+ C_{m_\delta}\eta \tag{194}$$

$$\frac{I\ddot{\eta}}{1/2\ \rho V^2S_e\bar{c}_e} = \left(\frac{1}{a}\frac{\partial C_{he}}{\partial M} + \frac{2C_{heo}}{V}\right)u + \left(1 - \frac{\partial\epsilon}{\partial\alpha}\right)C_{he\alpha_t}\alpha$$

$$+ \frac{2}{V}l_t\frac{\partial\epsilon}{\partial\alpha}C_{he\alpha_t}\dot{\alpha} + \frac{l_t}{V}C_{he\alpha_t}\dot{\theta} + C_{he\delta}\eta$$

$$+ \Delta C_{fe} \tag{195}$$

These four equations are representative of the longitudinal dynamics of the aircraft for small perturbations from the standard path.

4. TRANSFER FUNCTIONS

In order to further refine the analysis for purposes of determining aircraft response, it is necessary to rework equations (192) - (195) into the form of transfer functions. To do this, each of the coefficients in these equations will be abbreviated. This is done as follows:

For equation (192), let

$$A_1 = \left(\frac{\partial C_T}{\partial u}\right)_o + \left(\frac{\partial C_T}{\partial h}\frac{\partial h}{\partial u}\right)_o + \left(\frac{\partial C_T}{\partial N}\frac{\partial N}{\partial u}\right)_o - \frac{2C_{D_o}}{V} - \frac{1}{a}\frac{\partial C_D}{\partial M} \tag{196}$$

$$A_2 = C_{L_o}\left(1 - \frac{2C_{L_\alpha}}{\pi \mathit{AR}\, e}\right) \tag{197}$$

$$A_3 = C_{L_o} \tag{198}$$

$$A_4 = \frac{m}{1/2\ \rho V^2 S} \tag{199}$$

For equation (193), let

$$B_1 = \frac{1}{a}\frac{\partial C_L}{\partial M} + \frac{2C_{L_o}}{V} \tag{200}$$

$$B_2 = C_{L_\alpha} + C_{D_o} \tag{201}$$

$$B_3 = \frac{\bar{c} a_t V_H}{V} \tag{202}$$

$$B_4 = C_{L_\delta} \tag{203}$$

$$B_5 = C_{L_o} \tan \Theta_o \tag{204}$$

$$B_6 = \frac{m}{1/2\ \rho V S} \tag{205}$$

For equation (194), let

$$C_1 = \frac{1}{a}\frac{\partial C_m}{\partial M} + \frac{2C_{m_o}}{V} \tag{206}$$

$$C_2 = C_{m_\alpha} \tag{207}$$

$$C_3 = \frac{\bar{c}}{2V} C_{m_q} \tag{208}$$

$$C_4 = C_{m_\delta} \tag{209}$$

$$C_5 = \frac{B}{1/2\ \rho V^2 S \frac{\bar{c}}{2}} \tag{210}$$

For equation (195), let

$$D_1 = \frac{1}{a}\frac{\partial C_{he}}{\partial M} + \frac{2C_{heo}}{V} \tag{211}$$

$$D_2 = \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right) C_{he\alpha_t} \tag{212}$$

$$D_3 = \frac{2}{V} l_t \frac{\partial \epsilon}{\partial \alpha} C_{he\alpha_t} \tag{213}$$

$$D_4 = \frac{l_t}{V} C_{he\alpha_t} \tag{214}$$

$$D_5 = C_{he\delta} \tag{215}$$

$$D_6 = \frac{I}{1/2\ \rho V^2 S_e \bar{c}_e} \tag{216}$$

With these definitions, equations (192) – (195) become:

$$(A_4 s - A_1)u(s) - A_2\alpha(s) + A_3\theta(s) = 0 \tag{217}$$

$$B_1 u(s) + (B_6 s + B_2)\alpha(s) + \left[(B_3 - B_6)s + B_5\right]\theta(s) + B_4\eta(s) = 0 \tag{218}$$

$$C_1 u(s) + C_2 \alpha(s) - (C_5 s^2 - C_3 s)\theta(s) + C_4 \eta(s) = 0 \tag{219}$$

$$- D_1 u(s) - (D_3 s + D_2)\alpha(s) - D_4 s\theta(s) + (D_6 s^2 - D_5)\eta(s) = \Delta C_{fe} \tag{220}$$

where the designated variables now represent the Laplace transformed counterparts of those used previously.

Equations (217) - (220) can be reworked to yield the transfer functions $\frac{\theta(s)}{\eta(s)}$ and $\frac{\alpha(s)}{\eta(s)}$. This can be done by eliminating u(s) from each of these equations, and algebraically manipulating the results. The resulting transfer functions are:

$$\frac{\theta(s)}{\eta(s)} = \frac{B_4[A_2C_1 + C_2(A_4s - A_1)] - C_4[A_2B_1 + (B_6s + B_2)(A_4s - A_1)]}{[(B_6 - B_3)s - B_5][A_2C_1 + (A_4s - A_1)C_2] + A_3[B_1C_2 - (B_6s + B_2)C_1] - (C_5s^2 - C_3s)[A_2B_1 - (A_4s - A_1)(B_6s + B_2)]} \tag{221}$$

$$\frac{\alpha(s)}{\eta(s)} = \frac{B_4[A_3C_1 + (C_5s^2 - C_3s)(A_4s - A_1)] - C_4\{A_3B_1 - (A_4s - A_1)[(B_3 - B_6)s + B_5]\}}{[(B_6 - B_3)s - B_5][A_2C_1 + (A_4s - A_1)C_2] + A_3[B_1C_2 - (B_6s + B_2)C_1] - (C_5s^2 - C_3s)[A_2B_1 - (A_4s - A_1)(B_6s + B_2)]} \tag{222}$$

These two equations along with equations (217) and (220) can be used to determine the complete response of the aircraft to elevator and throttle commands.

APPENDIX III

AIRBORNE COMPUTER STATE OF THE ART

1. INTRODUCTION

The airborne computer may be considered to consist of three main functional units. These are the memory, input/output section, and Central Processing Unit (CPU). These are discussed below.

2. FUNCTIONAL UNITS

a. Memory

The memory in spaceborne computers can be thought of as being divided into one section for the instructions and constraints ("Program memory"), another section for the variable data and temporary data ("data memory"), and a third section used within the central processing unit. Sometimes this division is physical as well as conceptual, in which case the program memory is usually read-only, and the data memory read-write; the CPU may have a read-only memory, a read-write memory, or both.

(1) Data Memory

Data memories are built with devices which are capable of being read from and written into in real time. Most contemporary computers use core memories for data storage. A few use destructive readout (DRO) thin film memories and biax devices, however, because of the widespread use of core memories in spaceborne as well as ground-based computers, ample information is available, and these will not be discussed.

(2) Program Memory

In machines without a separate program memory, the program and the data are both stored in a read-write memory. The program can then be easily modified, either by itself, (e.g., to set up the return address when beginning a subroutine) or by external sources (as when entering a new program). It is possible, however, to write a program in a manner that requires no direct modification of instructions. The program can then be stored in a read-only memory, which reduces the probability of inadvertent damage to the program. The read-only memory could be built using conventional DRO cores with an automatic restore cycle after every read cycle but, more often, it is built with special devices which allow a non-destructive readout (NDRO) technique to be used. Most contemporary computers, which have separate program memories, employ NDRO techniques that are implemented with multi-aperture devices, thin-film elements, or wired arrays.

(a) Multi-aperture Devices

Transfluxors and BIAX elements are multi-aperture devices which are used in computers. Readout from a BIAX element is completely nondestructive, so that no time is needed to restore information read from the element. A transfluxor has NDRO characteristics, but some degradation of the information occurs and the information is destroyed by repeated read cycles. This is usually compensated for by applying, after each read operation, a "reinforce" pulse to all cores without regard to the address which was read. In some cases, the reinforce pulse is applied only after four read operations have been completed.

These elements are electrically alterable, although usually the write operation is longer and requires more power than the write operation for a DRO core. Nonetheless, applications are usually in situations where information is read much more often than it is written. This is particularly true in program storage where there is only an occasional need to alter the program in real time.

The primary application of these elements, however, is in implementing read-only memories. Their read-time (1 μsec or less) is typically 1/10 as long as their write-time. This read-time compares favorably with the access time of DRO cores which are used in spaceborne memories.

(b) Thin Films

Thin film sandwich elements (not to be confused with DRO thin film devices) are another example of NDRO devices used in spaceborne computers. They can be electrically altered, but the write time is typically 1000 times the read time. Therefore, information is usually electrically entered by external equipment and no provision is made for real-time write operation.

(c) Wired Arrays

With these memories, information is wired directly into an array. Information can be changed only by disassembly and rewiring. There are two types. The "missing core" memory is implemented by omitting or removing specific cores or by shorting cores with a single-turn winding. The "core rope" memory, which is the type used for program storage in the Apollo Guidance Computer, is implemented by having the sense or drive winding bypass specific cores. Both types of wired arrays are useful for storing a well defined program. The probability of error, due to a power transient or to an inadvertent attempt to write over an instruction, is greatly reduced with these types of devices.

(3) Memories used in CPU

In addition to the implicitly addressed central register, a central processor sometimes has explicitly addressed storage locations for temporary storage of intermediate results. In addition to being explicitly addressed, this "scratch pad" memory is characterized by being faster than main memory and by providing more storage than is normally available in central registers.

(4) Memory Capacity

Memory sizes of spaceborne computers vary widely. Most have a memory with at least 4096 words as a standard size. Maximums may range all the way to 132,000 words, however. In general, most computers have about 1,000 words set aside for data (read/write) storage if a read-only memory is used for program storage.

Memory capacity plays an important role in comparing the physical characteristics of computers, since the memory accounts for a significant portion (often as much as 50%) of the computer's overall volume and weight.

b. Input/Output (I/O) Section

This section of a computer is usually designed for a specific application and often includes interface equipment. Therefore, only the more general facilities and requirements are discussed. The requirement for real-time operation implies a need for one or a combination of the following types of design philosophies:

(1) A computer which sequentially performs its assigned tasks fast enough so that all tasks are accomplished during a given period.

(2) A computer which periodically scans its tasks (even though it may be in the process of doing a particular task) and does each in an order based on a preassigned priority.

(3) A computer which has external interrupts so that, when a task needs to be performed, the program in process can be interrupted and resumed after the task called for by the interrupt has been accomplished.

In terms of speed requirements or, alternatively, the number of tasks that can be handles with a given speed, (2) above requires less speed than (1), and (3) requires less speed than (2) for a given set of tasks. Both (2) and (3) require a method of retaining the status of the program so that it can be resumed at a later time, however.

With few exceptions, contemporary spaceborne computers have at least one interrupt level. Many provide hardware implementation for program status retention rather than requiring software to handle this function.

Most of the computers in this survey have provisions for the input and output of both serial and parallel digital data as well as for discrete signals. A separate I/O unit may be needed, though, to handle inputs from shaft-position encoders, real-time clocks, and analog devices or to provide outputs of analog voltages or pulse trains of controlled frequency. A separate I/O unit is particularly desirable if it provides for program continuation during completion of I/O transfers, assignment of priorities to I/O channels, and simultaneous operation of two or more I/O channels. Most computers either incorporate these I/O features, or are flexible enough that these features could be added.

a. Central Processing Unit

Memories are often taken "off-the-shelf", and I/O configurations are usually designed for specific application. The central processor is the unique, distinguishing unit, of a given computer, however. The features in the CPU which distinguish it from other computers include the type of arithmetic used, the method of addressing, the word length, the instruction repertoire, and the components used to build the CPU.

(1) Arithmetic

With few exceptions, spaceborne computers are parallel, binary, fixed point, general purpose machines with negative numbers represented in complementary notation. Generally, there is good reason for not choosing the alternatives. The choice is not always obvious, however. In most cases, a serial machine would be slower but its potential savings in hardware and reduced complexity must be considered. If the computational task requires a set of serial operations, then a serial machine is just as fast as a parallel type.

Fixed point data representation can provide a wide enough number range for aerospace problems so that the increased hardware necessary for floating point representation can be avoided. On the other hand, fixed-point number representation makes the choice of word length more important, and also adds to the programmer's burden. It is likely that as hardware becomes smaller, lighter, and more reliable, floating-point representation will become more common.

(2) Addressing

Early ground-based digital computers used a four-address instruction word where the addresses of two operands, the address of the result, and the address of the next instruction were specified. Changes have been made since then, however, in an attempt to better utilize available memory and to increase the number of addressable words for a given instruction word length. Hardware, in the form of an instruction counter or current address register, and software, in the form of a jump instruction, have replaced the address of the next instruction. Hardware, in the form of an addressable register, and software in the form of a'load register' instruction and a 'store register' instruction, have replaced the address of one of the operands and the address of the result. These changes resulted in single-address machines, both for ground-based and aerospace applications.

Further changes in instruction word formats have resulted in what can be termed modified single address formats. Such formats have become commonplace in current ground-based computers. A few spaceborne computers also use a modified single address format. These machines provide for one operand to be addressed in memory and another operand or result address to be chosen from registers in the central processing unit. In addition, at least one machine has several different formats, one of which specifies two operand addresses in main memory. In general, advances in addressing techniques have been accomplished by an historical trend to a single address, followed by multiple formats that allow the earlier techniques to be used when desirable.

(3) Word Length

Choice of computer word length is influenced by both instruction and data word length requirements. These requirements are not always compatible. Instruction word length is strongly influenced by the number of commands to be addressed. Data word length depends heavily on the precision required in the data word. Most spaceborne computers are used for guidance and navigation, and have 24 to 32-bit instruction and data words. There are, however, two other choices if the required data word length is longer than is necessary for the instruction word. First, both the instruction word and the data word can be made shorter. Second, double precision operations can be used where necessary.

(4) Instruction Repertoire

A basic set of instructions for a computer includes provisions to perform arithmetic operations, logic operations, data transfers to and from memory, data transfers to and from external equipment, shift operations, and decision operations. Many machines have more than 50 (and as many as 135) instructions in their repertoire and these are composed primarily of machines whose date of introduction is mid-1966 or later. This trend toward a greater number of programmable instructions requires more bits in the operation code, but can provide savings in the program execution time and in ease of programming. For instance, one manufacturer has developed machines which have a MOVE instruction to allow any number of bits in a directly addressed register to be selected by a mask and moved to any desired position in a central register. This instruction can be used in place of a shift instruction followed by a transfer instruction. These machines also have a GATED COMPARISON instruction, which allows a value in memory to be compared with a value in a central register plus or minus a designated "gated" value. This allows approximate comparisons to be made with a single instruction, instead of comparing with both ends of a range when some tolerance is permissable.

3. HARDWARE COMPONENTS

Integrated Circuits (IC's) are incorporated in virtually all computer CPU's which have been developed since 1963. These IC's are generally monolithic bipolar silicon circuits rather than metal oxide silicon (MOS) circuits. Prior to 1963 the CPU design relied on discrete components or hybrid thin film circuits.

4. OPERATIONAL AND PHYSICAL CHARACTERISTICS OF AEROSPACE COMPUTERS

a. Speed

Probably one of the most important characteristics of a computer is its "speed". Speed here is defined as the time required to accomplish the task for which the computer is programmed. Since computers are programmed for a wide variety of tasks, it is difficult to measure speed when defined in this way. Therefore other measures of speed have arisen. In particular, memory access or cycle times, instruction execution times, and input-output times are used separately or collectively to describe the speed of a machine. These times are useful indicators of machine speed in that an upper limit on the time required to perform a task can be fixed.

(1) Instruction Execution Time

A good indication of instruction execution times can be obtained by determining the execution time of the add, multiply and divide instructions. Typically, add times will be much smaller than multiply times and multiply times will be slightly smaller than divide times. Other operations such as subtract, transfer, jump and logic operations require execution times roughly equivalent to the add time. Most of the recently developed machines have parallel arithmetic units, and have add times less than 10 μ sec. Some notable exceptions, however, have add times which go as high as 20 or 30 μ sec.

It is more difficult to generalize when discussing multiply and divide times. The variety of ways in which these operations can be done results in a variety of times. For instance, one machine with 5 μ sec add time has multiply and divide times of 9 and 18 μ sec, respectively. Another with a 2 μ sec add time requires multiply and divide times of 12 and 30 μ sec, respectively. The best generalization that can be made is that multiply operations take approximately 2 to 60 times as long as add operation, with an average of about 5 times as long. Divide operations, on the other hand, take approximately 3 to 70 times as long as an add operation, with an average of about 8 times as long.

(2) Memory Cycle Time

Memory cycle time plays an important role in determining instruction execution times since the instruction execution time includes the time to fetch the instruction and, for some instructions, the time to fetch an operand. Of course, as previously mentioned, instruction look-ahead and memory overlap have been used at times to reduce the contribution that memory cycle time makes to the average instruction execution time. For example, in one particular machine four instructions are read from main memory at one time, and then stored in an instruction look-ahead memory in the CPU. In another machine a 4 μsec add time is decreased to 2 μ sec if advantage is taken of memory overlap.

b. Software

Almost all spaceborne computers are provided with specially prepared assembly programs which can be run on commercial ground-based machines. There are a few which are also equipped with compilers. Most also have diagnostic programs for self-testing. All are provided with software to allow simulation on a commercial machine so that programs can be checked out and debugged.

Some computers can be grouped into families of aerospace machines - all with the same instruction set but with different size and operational capabilities. This concept provides flexibility in sizing a machine to a job and allows software to be shared.

Another concept is that of a family of ground-based and aerospace machines, which alleviates the need for a simulator and special compilers and assemblers. In these cases, the instruction set of the aerospace machine is a subset of the ground-based machine. This is significant for two reasons. First, a simulator program is not required and program debugging becomes more of a routine problem. Second, much of the software experience gained from work on the ground machine is applicable to the spaceborne machine.

c. Physical Characteristics

The physical characteristics of aerospace computers of recent design fall into one of two broad categories, depending on the application for which they were designed. Those designed for limited, specific applications in space are extremely small (on the order of 0.2 - 0.4 cu. ft.) and light (on the order of 25 - 40 lbs.), and have a power requirement of about 100 watts. Those designed either for general use in space applications or for specific, highly demanding space application weigh in the 50 - 75 lbs. range, have a volume less than but close to 1 cu. ft., and a power requirement in the 100 to 300 watt range. Most machines are designed for use in very stringent environments without the need for specially produced conditions.

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13. ABSTRACT

An airborne computer-display system is developed for the purpose of flying vertical plane optimum flight paths. System requirements are specified by first examining mission requirements, then operational requirements, and finally state-of-the-art constraints. The system is designed to provide command information for flying minimum time and fuel transition maneuvers, and maximum range and endurance cruise maneuvers. Optimal command data is derived, in the computer, from a series of stored profiles which reflect both the aircraft drag configuration and ambient weather conditions. System feasibility is demonstrated through a system description which consists of state-of-the-art computer and display equipments. A digital computer is shown to be more accurate, compact and flexible than a comparable analog system. Displays consisting of an attitude director indicator, a flight profile indicator, and Mach number and altitude indicators are suggested for presenting command information. Final system size and weight are less than one cubic foot and fifty pounds, respectively. The system is tentatively developed for the F-104C, a contemporary aircraft for which a reasonable amount of optimum performance data is available. The system seems more applicable to developmental aircraft of the present era, however.

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