Identification of System Parameters in a Slewing Control Experiment

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Abstract

Presented is a method for state space model improvement using measured free decay time histories of a dynamic system. This methodology is called the matrix exponential method (MEM). MEM assumes that some state space model exist for a dynamic system, and that the model's system matrix, $A(\boldsymbol{P})$, is a function of various parameters, P_i's. MEM uses a first order Taylor series expansion of the measurement time histories to determine what changes in the system parameters, ΔP_i's, are necessary such that the dynamic response of our model more closely resembles the measured response of the physical system. This paper will present the results of this parameter correction method to a control structure interaction problem, namely, a slewing control experiment. MEM will be used to correct some unknown, and often unpredictable, parameters, e.g., the angular viscous damping. Comparisons of the model's time histories and frequencies will be compared with measurements from the experiment.

I. Introduction

Arriving at a mathematical model that accurately reflects events that occur in physical systems can often be extremely difficult. In many instances certain parameters of a dynamic system can be measured and/or calculated precisely. For example, knowing the mass distribution and geometry of a rigid body allows dynamicists to obtain accurate values for the inertia of such bodies. Similarly, in elastic structures, the measurement of force to displacement yields precise information about stiffness, in the sense that the predicted inertia and stiffness for a system can be calculated and/or measured. Damping can be considered to be a dynamic parameter in that it can only be measured using the response of the system to some known force, or a free-decay from some initial condition.

Typically, models are developed based on our knowledge of a system's parameters. However, simulation results quite often do not correspond to the dynamic behavior of the physical system. In the finite element method, users often adjust models by using "equivalent lengths" or "equivalent mass" elements so that the eigenvalues of the FEM model and those of the physical system appear similar. The goal of MEM is simply to determine what changes in the system parameters, ΔP_i's, are necessary such that the model's response simulation results more closely resemble the measured response of the physical system.

The matrix exponential method(MEM) was developed by Tsen and Mook in Ref.1 and 2. Another time domain identification scheme[3] identifies entire system matrices, i.e., the mass, stiffness, and damping. However, this method assumes that all acceleration, velocity and displacement time histories are measurable. The MEM approach is different than other approaches because it simply determines which parameters in an existing model should be adjusted in order to improve a model's prediction of the physical system's dynamic response, rather than assuming the need to develop a new or realized model.

II. Matrix Exponential Method

Let us consider the following linear, time-invariant state space model.

$$\dot{x} = Ax \tag{1a}$$
$$y = Cx \tag{1b}$$

where

x - $2n$x1	state vector
A - $2n$x$2n$	system matrix
B - $2n$xm	input matrix
C - px$2n$	output matrix

where, $x(0)$ is an $2n$x1 vector of initial states. Let the system matrix A, be a function of system parameters, P_i's, as follows,

$$A = A(P) \tag{2}$$

where, $P^T = [P_1, P_2, \ldots]$.

Now let us assume the measurements, Y_{meas}, are equal to the value of those predicted by the model Y, if we could adjust the parameters of the model by ΔP , e.g.,

$$Y_{meas} = Y(P + \Delta P) \tag{3}$$

Taking a Taylor series expansion of Eq. (3) we can say that,

$$Y_{meas} = Y(P) + \frac{\partial Y}{\partial P}\Delta P + \ldots \text{ higher order terms} \tag{4}$$

Note that $Y(P)$ is the output vector of the model. We now drop the higher order terms and solve for ΔP,

$$\Delta P = \left[\frac{\partial Y}{\partial P}\right]^{-1} (Y_{meas} - Y) \quad (5)$$

The model's output sensitivity with respect to a parameter, $\partial Y/\partial P_i$, is determined as follows. Take the partial derivative of Eq.(1b) with respect to the parameter, P_i,

$$\frac{\partial Y}{\partial P_i} = \frac{\partial C}{\partial P_i} x + C \frac{\partial x}{\partial P_i} \quad (6)$$

Since $\frac{\partial C}{\partial P_i} = 0$,

$$\frac{\partial Y}{\partial P_i} = C \frac{\partial x}{\partial P_i} \quad (7)$$

To find $\frac{\partial x}{\partial P_i}$, we take the derivative of Eq. (1a) with respect to the parameter in question, P_i ,i.e.,

$$\frac{\partial \dot{x}}{\partial P_i} = \frac{\partial A}{\partial P_i} x + A \frac{\partial x}{\partial P_i} \quad (8)$$

We now define a new state space, $Z^T = [x^T : \partial x/\partial P_i{}^T]$, so that, and solve Eqs. (8) and (1a) simultaneously, i.e.,

$$\dot{Z} = A' Z \quad (9)$$

where

$$A' = \begin{bmatrix} A & . & 0 \\ & \cdots & . \\ \frac{dA}{dP_i} & . & A \end{bmatrix}$$

and

$$Z(0) = \begin{bmatrix} x(0) \\ \cdots \\ 0 \end{bmatrix}$$

Equation (9) is solved by finding a matrix exponential of the matrix A'. Here it has been assumed that the initial conditions of the physical system, hence, those of the model $x(0)$, do not depend upon the parameter, P_i, i.e.,

$$\frac{\partial x(0)}{\partial P_i} = 0 \quad (10)$$

The state vector $Z(t)$ contains the response of our model, $x(t)$, and the sensitivity of the state vector $\frac{\partial x}{\partial P_i}(t)$.

Computational Implementation

A more general form of Eq. (5) can be written as follows. If the number of parameters that are being adjusted is j, and k measurements of the time histories are measured, Eq. (6) becomes,

$$\begin{bmatrix} Y_{meas}(t_0) & - & Y(t_0) \\ Y_{meas}(t_1) & - & Y(t_1) \\ & \vdots & \\ Y_{meas}(t_{k-1}) & - & Y(t_{k-1}) \end{bmatrix} = \begin{bmatrix} \frac{\partial Y(t_0)}{\partial P_1} & \cdots & \frac{\partial Y(t_0)}{\partial P_j} \\ \vdots & \vdots & \vdots \\ \frac{\partial Y(t_0)}{\partial P_1}(t_{k-1}) & \cdots & \frac{\partial Y}{\partial P_j}(t_{k-1}) \end{bmatrix} \begin{bmatrix} \Delta P_1 \\ \vdots \\ \Delta P_j \end{bmatrix} \tag{11}$$

or, in matrix form,

$$E = S\,\Delta P \tag{12}$$

where E - $k{\cdot}m$ x 1, vector of model error

S - $k{\cdot}m$ x j, matrix of output sensitivities

ΔP - j x 1, vector of parameter adjustments

Equation (12) is an over-determined problem which can be solved using the least squares approach[4], i.e.,

$$\Delta P = (S^T S)^{-1}\, S^T E \tag{13}$$

In large order problems where the inverse of $S^T S$ may not exist, and it may be necessary to apply a rank decomposition technique, such as, singular value decomposition.

Algorithm

The only rational reason for dropping the higher order terms from Eq. (4) was convenience. Therefore, there is no reason to believe a first order expansion of the measurements is accurate. Hence, in order to find the total change in parameters, ΔP_i's we must apply Eq. (5) in a recursive manner. First an initial value is chosen for all P_i's (this may be $P = 0$), and then following procedure is applied.

(1) Find the model's output time histories $Y(t_0)...Y(t_{k-1})$ and subtract this form the measurements, $[Y_{meas}(t_{l-1}) - Y(t_{l-1})]$ where $l = 1,...,k$.

(2) Find the output sensitivities, $\frac{\partial Y}{\partial P_i}$ $i = 1,...,j$. (Recall that sensitivities and predicted outputs are solved simultaneously.)

(3) Find ΔP

(4) Add ΔP to P, update the model and repeat (1)-(3) until $|\Delta P_i| << |P_i|$ for $i = 1,...,j$.

Note that only one set of measurements are needed.

III. Modeling

The following formulation is similar to the modeling approach formulation found in Ref.6. The apparatus used in the slewing control experiment can be described as a flexible beam clamped to a rigid hub(see figure 1), where, Y - X is in the horizontal plane, τ is a torque applied to the hub, and $y(x,t)$ is the displacement of the beam with respect to x. The hub parameters are the radius, a,and the hub inertia, I_H. The beam has the following properties, EI , the flexural rigidity , ρ, the mass density per unit length, and L , the beam length.

The Lagrangian, L_g, is the total kinetic energy of the system minus the total potential, so that,

$$L_g = \frac{1}{2}\int_0^L (\rho(a+x)\,\dot{\theta} + \dot{y})^2 dx + I_H\,\dot{\theta}^2 - \frac{1}{2}\int_0^L EI\,(\frac{\partial^2 y}{\partial x^2})^2\, dx \tag{14}$$

We assume a separation of variables solution for the motion of the beam, i.e.,

$$y(x,t) = \sum_{i=1}^{n} q_i(t)\ \phi_i(x) \tag{15}$$

where, n is the finite number of of the beam modes included in the model. The functions, $\phi_i(x)$, are the beam's eigenfunctions, and the equations of motion are derived from Hamilton's principle.

$$\delta \int_{t_f}^{t_0} [\, L_g + W_{nc}\,]dt = 0 \tag{16}$$

where, $W_{nc} = \tau\theta$, is the work performed by non-conservative forces, i.e., the applied torque. The eigenfunctions of the beam are orthogonal and normalized with respect to mass density.[5] Using a three mode expansion for the motion of the beam, we arrive at the open loop equations of motion.

$$\tilde{M}\ddot{q} + \tilde{K}q = B\tau \tag{17}$$

where, $q^T = [\theta, q_1, q_2, q_3]$, B^T=[1, 0, 0, 0],

$$\tilde{M} = \begin{bmatrix} \int_0^L \rho\,(a+x)^2 dx + I_H & I_1 & I_2 & I_3 \\ I_1 & 1 & 0 & 0 \\ I_2 & 0 & 1 & 0 \\ I_3 & 0 & 0 & 1 \end{bmatrix}$$

$$\tilde{K} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \omega_1^2 & 0 & 0 \\ 0 & 0 & \omega_2^2 & 0 \\ 0 & 0 & 0 & \omega_3^2 \end{bmatrix}$$

and

$$I_i = \int_0^L \rho(a+x)\phi_i(x)dx.$$

The terms q_i are the modal coordinates of the beam.

Actuator Dynamics

The torque applied to the hub is generated by an armature controlled electric motor whose dynamics are characterized by the circuit in figure 2. By applying Kirchoff's law to this circuit, and summing the moments about the motor's armature, the following expression for the torque is derived as a function of armature voltage.

$$\tau = \frac{K_t}{R_a}\, e_a - I_M\,\ddot{\theta} - (C_v + \frac{K_b K_t}{R_a})\,\dot{\theta} \tag{18}$$

Since the motor inductance is small in our experiment, its contribution to the dynamics is neglected in Eq. (18).

Closed Loop Response
A simple closed loop response is given by setting the armature voltage proportional to the position,

$$e_a = K_p(\theta_{ref.} - \theta) \tag{19}$$

where, K_p is the position feedback gain.

Combining Eqs. (18) and (19) with the beam dynamics of Eq. (17), we arrive at Eq. (20) which describes the response of the closed loop system with the simple position control law.

$$M\ddot{q} + D\dot{q} + Kq = B\theta_{ref} \tag{20}$$

where

$$M = \tilde{M} + \begin{bmatrix} I_m & 0_{1\times 3} \\ 0_{3\times 1} & 0_{3\times 3} \end{bmatrix} \tag{21}$$

$$D = \begin{bmatrix} (C_v + K_b K_t/R_a) & 0_{1\times 3} \\ 0_{3\times 1} & \mathrm{DIAG}(2\zeta_1\omega_1, 2\zeta_2\omega_2, 2\zeta_3\omega_3) \end{bmatrix}$$

$$K = \begin{bmatrix} K_p K_t/R_a & 0_{1\times 3} \\ K_p K_t/R_a \begin{Bmatrix} \Gamma_1 \\ \vdots \\ \Gamma_n \end{Bmatrix} & \mathrm{DIAG}(\omega_1^2, \omega_2^2, \omega_3^2) \end{bmatrix}$$

and

$$B^T = [\,1.0,\ K_p K_t/R_a\,\Gamma_1, \ldots, K_p K_t/R_a\,\Gamma_n]$$

Note that the modal damping terms, $2\zeta_i\omega_i$, were added to the damping matrix, D. The assumed modes of vibration for the beam were found using Euler-Bernoulli beam analysis where the boundary conditions were assumed to be spring-hinged at the slewing axis and free at the other end.[9] The value of the spring constant is determined by the servo-stiffness term, $K_p K_t/R_a$. The modal participation factors were added to the stiffness and control matrices, K and B. The modal participation factors account for the direct transmission of the control's restoring torque to the modal deflections of the flexible structure.

IV. Set Up and Experimental Response

The experimental apparatus, depicted in figure 1, consists of a steel beam clamped to a rotating axis. The hub is the rigid mechanical interface between the beam and the slewing axis. The beam is 3" wide and 1/32" thick. An Electrocraft permanent magnet DC electric motor (Model #586) is attached concentrically to the axis of rotation. Angular sensing is performed using a potentiometer, and a tachometer provides an angular velocity sensor. A strain gauge, located at the root of the beam, is used for additional sensing. The physical characteristics of the motor/beam system are as follows.

$EI/\rho = .857$ Nm^3/kg	$K_b = 5.539\times10^{-2}$ volts/rad/sec
$I_m = 6.02\times10^{-2}$ kg m^2	$K_t = 5.508\times10^{-2}$ Nm/A
$L = 1.19$ m	$L_a = 2.3\times10^{-3}$ H
$a = .06$ m	$R_a = 2.0$ ohms

$I_H = 3.49\text{x}10^{-3}\ \text{kgm}^2$

The position control was implemented using an EAI 2000 analog computer, where the feedback constant was set to 8 volts/rad. The viscous damping parameter, C_v, is unknown, hence, is one of the parameters in the identification procedure.

The beam was set to an initial position in the slewing plane. The feedback circuit was then closed and the beam returned to the zero voltage reference point of the potentiometer. The potentiometer, tachometer, and strain gauge signals were recorded during the response (figure 3). The responses were sampled at a frequency of 128 Hz, although only a 12.8 Hz sampling frequency was used in the identification procedure to reduce the number of data points, hence, the computational effort. The flatness in the peaks of the position response were due to the discrete number of windings in the potentiometer.

V. Results of the Identification Procedure

The position response of the system was dominated by the rigid body motion of the structure, θ. The effects of the structural modes on the measured position response of the beam were secondary. The only contribution to the position measurement came from the 1st flexible mode of the structure. The measured position response was used to apply MEM to the model.

The initial unknown parameters of the model were arbitrarily set to C_v = .0528 and the modal damping term was set $2\zeta_1\omega_1 = 0.5$. The higher mode damping terms were set to $2\zeta_2\omega_2 = 2\zeta_3\omega_3 = 1$. Since the modes of the beam did not participate strongly in the position response, the modal damping was not included in the identification procedure. The focus here was to find the correct viscous damping constant which would yield a more accurate model. Figures 4a and 4b contain plots of the measurements, the initial model and the improved model for the single parameter tuning of C_v, and the multiple parameter tuning of C_v and K_p, respectively. Figures 5a and 5b show the convergence of the algorithm. The error is defined as,

$$\|Y_{error}\| = (Y_{\text{meas}} - Y_{\text{model}})^T\,(Y_{\text{meas}} - Y_{\text{model}}) \tag{22}$$

where, Y_{error}, Y_{meas} and Y_{model} are vectors in time. Table 1 contains the changes in parameters for our model, and table 2 contains the natural frequencies and damping ratios for the measurements and the model responses. Measurement frequencies and damping ratios were determined by a version of ERA[7] found in Ref. 8.

Run 1: Single Parameter Adjustment
The simple case was considered where only one parameter, C_v, is adjusted. Figure 5a shows the model error, $\|Y_{\text{error}}\|$, was reduced from 40.21 to 8.78 and shows the algorithm converged smoothly in a few iterations. In this case the response of the final model simply contains too much damping (see Fig. 4a), i.e., the 1st modal damping of the closed loop response is $\zeta_1 = 24.4\%$.

Run 2: Multiple Parameter Adjustment
Due to current limits in the amplifier circuit, simulations indicated that some amplifier saturation occurred. To correct for this, the model's position feedback constant was also adjusted. The final model of Run 1 was used as the initial model for this run. The model error was further reduced from 8.78 to 3.65. The first natural frequency error was reduced from 6.7% in our initial model to 3.1% in our final model. However, the model error, $\|Y_{\text{error}}\|$, did not show a smooth convergence.

VI. Discussion and Conclusions

An important point of this analysis is that the damping term C_v, and the final positions feedback, K_p, adjusted our linear model to account for what is often nonlinear effects in the actual system. For instance, analysis in the effects of air damping on slewing experiments show that air drag effects are nonlinear functions.[10] Our resulting improved model is a linear approximation to the linear and nonlinear parameters of the physical system, while not perfect, such linear models are of much interest for design purposes for the control engineer.

Uniqueness is another important topic in modeling improvement. Although, the least squares solution of Eq.(12) is a unique solution[4], the recursive procedure does not guarantee the final solution is a global minimum of the modeling error. The algorithm proposed is similar to the classic Newton-Raphson method, and as in the Newton-Raphson method, very ignorant initial guesses can lead to numerical instabilities. Moreover, numerical minimums may not be physically significant, i.e., in *Run 1* the numerical minimum simply contained too much damping. This is because the degree to which the modeling error is minimized depends on a judicious choice of parameters which are tuned.

MEM fine tunes a current model such that there is better agreement between the predicted response of the response of the physical system. The algorithm requires only one set of measurements, and is a time domain technique which requires only modest computational effort. The MEM method has been applied to a slewing control experiment. This method reduced the disagreement between the model and the measurements.

VII. Acknowledgment

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VIII. References

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Table 1 Parameter Values

		Initial	Final	ΔP_i's
Run 1:				
	C_v	.0528	.2048	.1520
	$\|\|Y_{error}\|\|$	40.2109	8.7819	
Run 2:				
	C_v	.2048	.1299	-.0741
	K_p	8.0	6.5753	-1.4247
	$\|\|Y_{error}\|\|$	8.7819	3.6512	

Table 2 Modal Parameters Closed Loop Response

	Mode 1		Mode 2	
	ω_1	ζ_1	ω_2	ζ_2
Measurements.	.1421	9.92%	1.5648	3.10%
Run 1:				
Initial	.1520	6.32%	1.72%	2.31%
Final	.1519	24.48%	1.724%	2.46%
Run 2:				
Final	.1377	17.73%	1.7249	2.42%

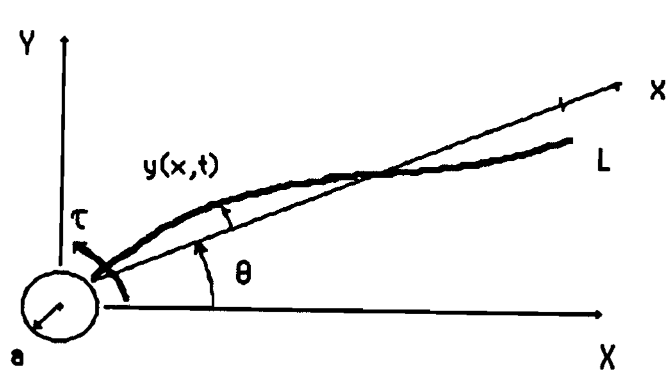

figure 1. Flexible Beam Attached to Rigid a Hub.

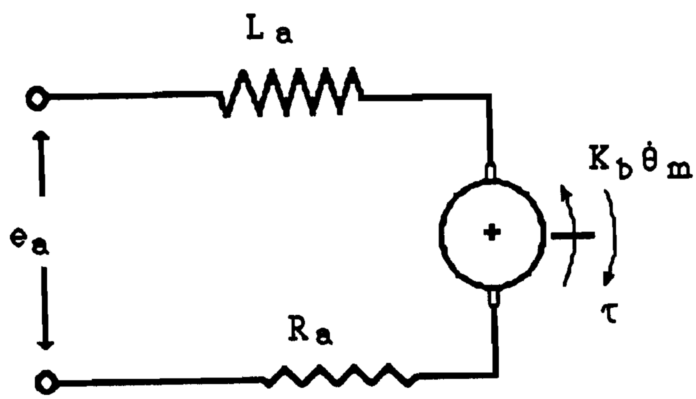

figure 2. Motor Armature Circuit.

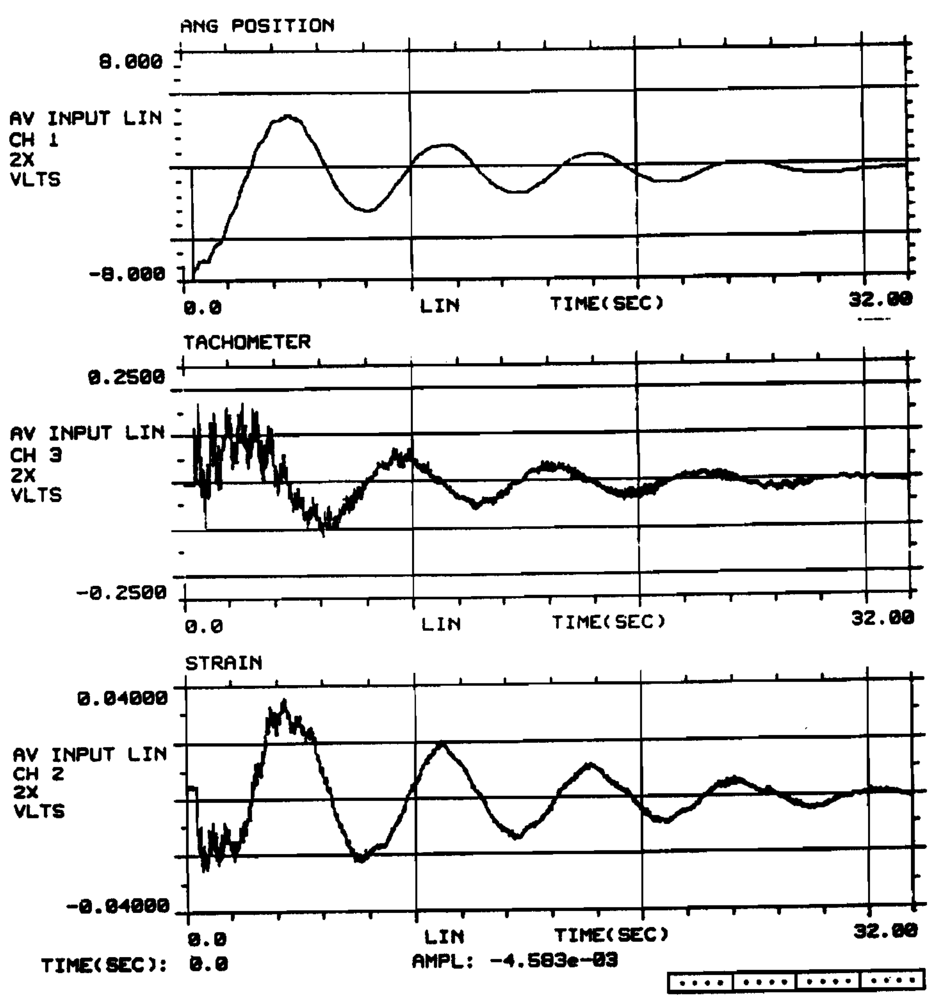

Figure 3. Measured Time Histories- Free Decay.

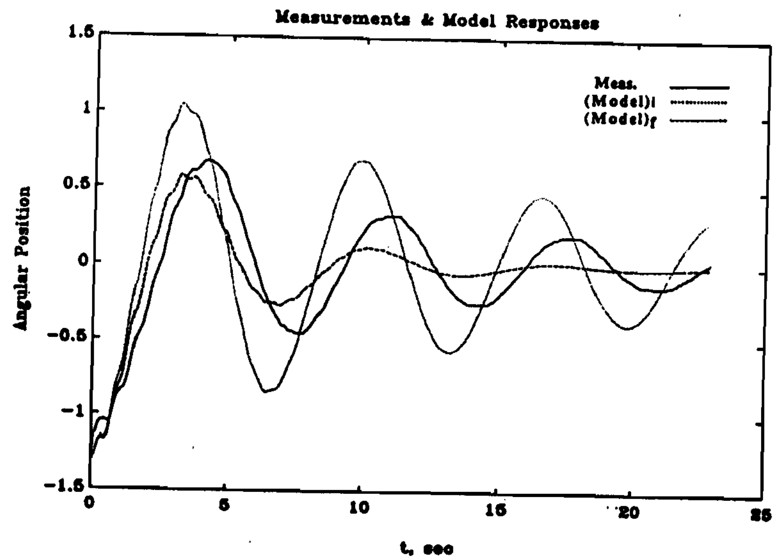

Figure 4a. Measurements and Initial and Final Models-*Run 1*.

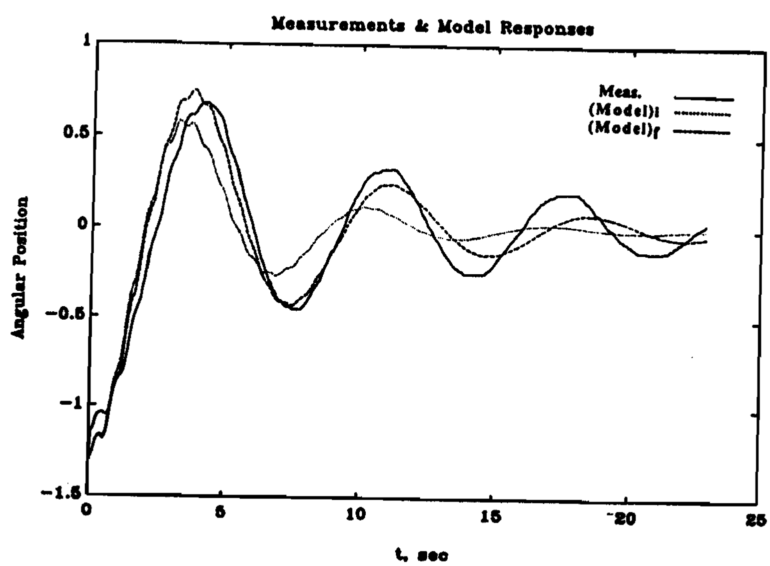

Figure 4b. Measurements and Initial and Final Models-*Run 2*.

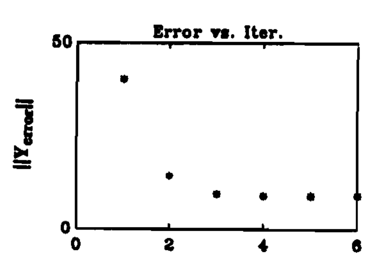

Figure 5a. Algorithm Convergence-*Run 1*.

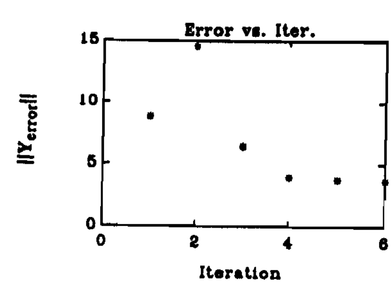

Figure 5b. Algorithm Convergence-*Run 2*.