VEM CHARACTERIZATION PROGRAM

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ABSTRACT

The development and use of an interactive computer program for the characterization of complex modulus data is described. The program uses the collocation process which accurately fits the real part of the complex modulus data and then uses the lack of fit of the loss factor to adjust the temperature shift function. This iterative method, which has converged when both the real modulus and the loss factor are fit simultaneously (the real directly and the loss factor indirectly), yields the most accurate estimate of the temperature shift function possible.

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1. Introduction

Successful design of passive damping treatments using viscoelastic materials (VEM's) such as elastomers depends upon several factors. One important factor is accurate knowledge of the sensitivity of VEM properties to variations in temperature and frequency. Since it is impossible to test a viscoelastic material at every combination of temperature and frequency, the material is tested at discrete temperatures and frequencies and a mathematical relationship is developed that characterizes the material at all other combinations of temperature and frequency. This process is referred to as characterization.

The equations used in characterization are parametric in nature. They are easily represented on computers. The difficulty lies in correctly choosing the equation parameters so that they accurately represent the VEM's. Interactive computer graphics have greatly improved the process of choosing and adjusting the correct parametric values.

This paper describes a recently developed computer program which implements the Collocation process [1] to accurately characterize viscoelastic materials.

2. VEMINT MAC

VEMINT MAC is a computer program developed to run on Apple Macintosh II computers. It fully utilizes the Macintosh windowing environment to allow point-and-click manipulation of complex modulus data. VEMINT MAC incorporates new characterization models as well as many of the models used in the past.

Five analytical representations of the temperature shift function (TSF, or α_T) are available. They are

1. Spline fit of slope

2. WLF equation

3. $\log(\alpha_T)$ is an exponential

4. $d(\log(\alpha_T))/dT$ is quadratic in $1/T$

5. Arrhenius equation

The "Spline fit of slope" model is discussed later in more detail. VEMINT MAC also has the capability to use the historical, discretized (tabulated for each experimental temperature) representation of α_T.

GE SMRD 100B37-T2B

| 1 | GE SMRD 100B37-T2B | | | | | | SHEAR |

Alpha_T ✓ Unknown
      1   Spline fit of slope   #1
      2   WLF   ⌘2
G*:   3   Exponential   ⌘3
      4   Slope quadratic in 1/T   ⌘4
      5   Arrhenius   ⌘5
      6   Tabulated values   ⌘6

SI	Tempe			actor	Imag Mod	Alpha_T	Valid Pt
1	2.990			e-1	7.800e-2	2.645e-2	FALSE
2	2.990e+2	2.500e+0	2.340e-1	4.530e-1	1.060e-1	2.645e-2	FALSE
3	2.990e+2	3.750e+0	2.590e-1	5.160e-1	1.336e-1	2.645e-2	FALSE
4	2.990e+2	5.000e+0	2.790e-1	5.470e-1	1.526e-1	2.645e-2	FALSE
5	2.990e+2	6.250e+0	2.970e-1	5.940e-1	1.764e-1	2.645e-2	FALSE
6	2.990e+2	7.500e+0	3.120e-1	6.090e-1	1.900e-1	3.055e-2	TRUE
7	2.990e+2	8.750e+0	3.280e-1	6.410e-1	2.102e-1	3.055e-2	TRUE
8	2.990e+2	1.000e+1	3.400e-1	6.560e-1	2.230e-1	3.055e-2	TRUE
9	2.990e+2	1.125e+1	3.540e-1	6.720e-1	2.379e-1	3.055e-2	TRUE
10	2.990e+2	1.250e+1	3.660e-1	6.720e-1	2.460e-1	3.055e-2	TRUE
11	2.990e+2	1.375e+1	3.790e-1	6.880e-1	2.608e-1	3.055e-2	TRUE
12	2.990e+2	1.500e+1	3.880e-1	7.030e-1	2.728e-1	3.055e-2	TRUE

Figure 1. VEMINT MAC data window

Seven complex modulus equations are available in VEMINT MAC. They are

1. Ratio of factored polynomials

2. Rogers empirical

3. Bagley fractional

4. Huet fractional

5. Capps polynomial

6. Nashif 8 parameter

7. Nashif 15 parameter

Data and model parameters are displayed by VEMINT MAC in a spreadsheet-like window in which characterization models are chosen using popup menus (Figure 1). VEM data is read into VEMINT MAC in the form of ASCII text files with the format shown in Figure 2. Tab-delimeted and English data formats are also supported. The data in Figure 2 has already been characterized. New uncharacterized data must have the same general format, but only the first four data fields are necessary, as shown in Figure 3.

ALFA-T MODEL

NALF	NA	A(1)	A(2)	A(3)	A(4)	A(5)	A(6)
4	6	290.	220.	400.	5.016E-02	0.102	1.580E-03

COMPLEX MODULUS MODEL

NVEM	NB	B(1)	B(2)	B(3)	B(4)	B(5)	B(6)
4	6	4.190E-02	499.	4.640E+06	0.675	1.95	7.850E-02

COMPLEX MODULUS DATA AS A FUNCTION OF TEMPERATURE AND FREQUENCY

Temp (DEG K)	Freq (HZ)	MReal (MPA)	Eta	MImag (MPA)	Alpha-t	
227.6	1861.	261.1	7.5670E-02	19.76	2.7260E+04	T
227.6	3579.	289.1	7.3270E-02	21.18	2.7260E+04	T
227.6	5788.	307.7	6.6440E-02	20.44	2.7260E+04	T
241.5	1828.	170.1	0.2031	34.55	1576.	T
241.5	3486.	179.8	0.2133	38.35	1576.	T
241.5	5591.	188.0	0.2006	37.71	1576.	T
255.4	638.0	29.06	0.9018	26.21	129.9	T

Figure 2. VEM data previously characterized

The Macintosh windowing interface is fully utilized to allow the user to simultaneously display multiple plots and data side-by-side. Erroneous points, as might be observed in the Wicket or the reduced frequency plots, may be double-clicked on and their corresponding values are displayed in the data window.

The Collocation method is used to characterize all new data. The method uses the "Spline fit of slope" temperature shift model in conjunction with the "Ratio of factored polynomials" complex modulus model. This is described in more detail below.

Once characterized, hardcopy of plots, such as the International and Wicket plots, as well as numerical data may be laser printed. VEMINT MAC provides all the plots and data described in the proposed ISO standard [4]. These are

1. An updated tabulated data file

2. Plots of $\log(\eta)$ vs. $\log(G^*_M(f, T))$ in S.I. and English, (Wicket plot)

3. A plot of $\log(\alpha_T)$, $d(\log(\alpha_T))/dT$, and apparent activation energy vs. temperature

```
       1        3M ISD 113                                           SHEAR
----------+---------+---------+---------+---------+---------+---------+---------+
                            ALFA-T MODEL
NALF  NA     A
  0    0
----------+---------+---------+---------+---------+---------+---------+---------+
                         COMPLEX MODULUS MODEL
NVEM  NB     B
  0    0
----------+---------+---------+---------+---------+---------+---------+---------+
COMPLEX MODULUS DATA AS A FUNCTION OF TEMPERATURE AND FREQUENCY
  Temp       Freq      MReal       Eta
 (DEG K)     (HZ)      (MPA)
  227.6      1861.     261.1     7.5670E-02
  227.6      3579.     289.1     7.3270E-02
  227.6      5788.     307.7     6.6440E-02
  241.5      1828.     170.1     0.2031
  241.5      3486.     179.8     0.2133
  241.5      5591.     188.0     0.2006
  255.4      638.0     29.06     0.9018
    .
    .
```

Figure 3. Uncharacterized VEM data

4. A plot of $\log(\alpha_T)$ vs. 1/temperature

5. Plots of $\log$(frequency) vs. temperature, (S.I. and English)

6. Plots of $\log(G^*_R(f,T)), \log(G^*_I(f,T))$, and $\log(\eta)$ vs. temperature, (S.I. and English)

7. Plots of $\log(\eta)$ vs. $\log(G^*_R(f,T))$ with constant temperature lines and an experimental frequency axis (Reduced Wicket plot) in both S.I. and English

8. Plots of $\log(G^*_R(f,T)), \log(G^*_I(f,T))$, and $\log(\eta)$ vs. reduced frequency with constant temperature lines and an experimental frequency axis (International plot) in both S.I. and English

3. Temperature Shift Function

Historically, the WLF equation [2] has been used to define the TSF. This, however, has not been able to shift all viscoelastic material data correctly outside the transition. A new approach is to use a spline fit of the slope of $\log \alpha_T$ for a relatively small number of temperature points (e.g., 7 points) to define the TSF. The temperature points are calculated such that the corresponding α_T values are equally spaced on the vertical log scale.

The Wicket plot (Figure 4) is used interactively to obtain values for the maximum loss factor (η_{max}), the rubbery modulus asymptote (Ge), and the glassy modulus asymptote (Gg). The transition region is defined by η_{cutoff}, which is calculated as $0.7\eta_{max}$.

The reference temperature, T_{ref}, is obtained in two steps, first as the average of the temperatures defining the transition region, $T_{ref1} = (T_{cutoff\_min} + T_{cutoff\_max})/2$. In [3] it was shown that the time-dependent stress-strain relations for a viscoelastic material in the transition region is described by

$$G^*(f_R) = \frac{G_e + G_g\left(\frac{if_R}{f_{R0}}\right)^\beta}{1 + \left(\frac{if_R}{f_{R0}}\right)^\beta} \qquad (1)$$

where

$$\beta = \frac{2}{\pi}\arctan\left[\frac{\eta_{max}\left[(1-C^2) + 2(1-C)C^{1/2}(1+\eta_{max}^2)^{1/2}\right]}{\left[(1-C)^2 - 4C\eta_{max}^2\right]}\right]$$
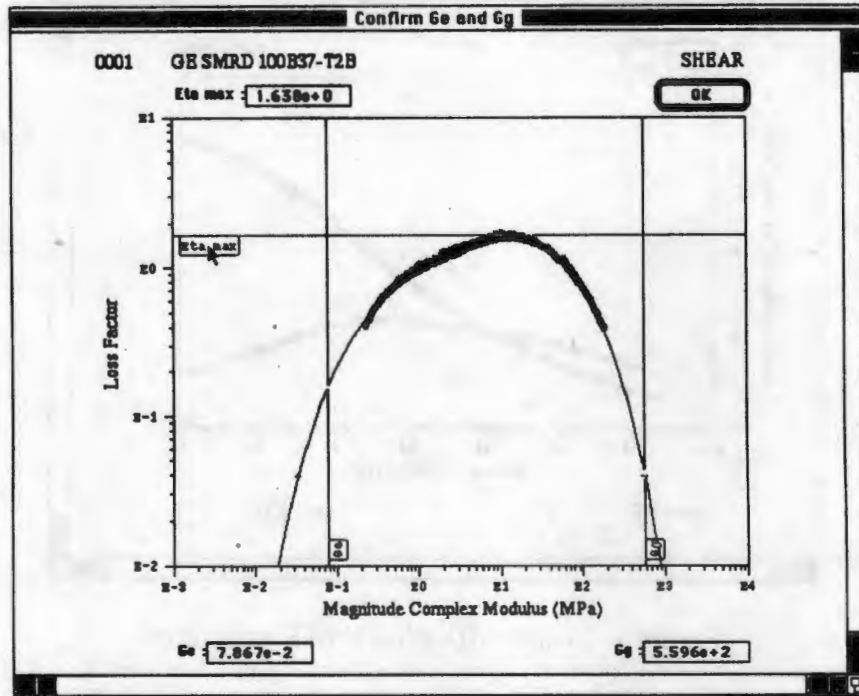
$$C = G_e/G_g$$

Figure 4. Wicket plot used to determine parameters

Solving for α_T / f_{R_0} gives

$$\frac{\alpha_T(T_i)}{f_{R_0}} = \frac{1}{j f_i} \left[\frac{G_i^* - G_e}{G_g - G_i^*} \right]^{\frac{1}{\beta}}$$

(2)

By fitting a quadratic through the data points defined within the transition by $\eta \geq \eta_{cutoff}$ an initial value for the slope of the log of α_T at T_{ref1} is calculated. Next, a quadratic fit of $\log \eta$ versus reduced temperature, where the reduced temperature is given by

$$T_{red_i} = \frac{T_{exp_i}}{1 + \frac{\log(f_{exp_i}/f_{ref}) T_{exp_i}}{S_{ref1} T_{ref1}^2}}$$

(3)

is used to calculate $T_{ref} = T_{red}$ evaluated at η_{max}. A value of 100 Hz is presently used for f_{ref}. The final reference slope, S_{ref}, is re-calculated from the quadratic through Equation 2.

A plot of $\log \alpha_T$ versus T is then displayed, where α_T is calculated from the "$d(\log(\alpha_T))/dT$ is quadratic in $1/T$" equation. As shown in Figure 5, the user may adjust the temperature shift function outside the transition by changing the values of $d(\log(\alpha_T))/dT$ (designated as SAL and SAZ) at the endpoints. Finally, α_T is calculated as the integral of the spline of the slope, where the constant of integration is given by $\alpha_T \equiv 1.0$ at T_{ref}.
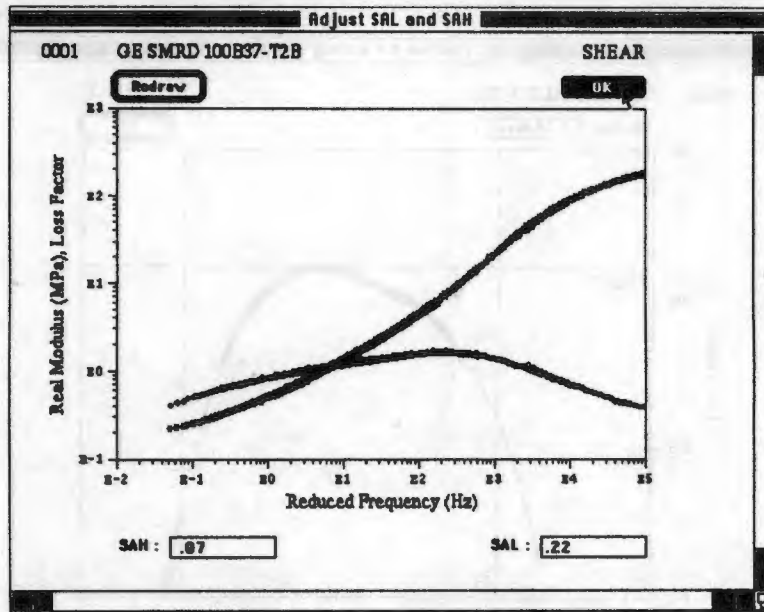
Figure 5. Adjust $d(\log(\alpha_T))/dT$ endpoints

The accuracy of the TSF parameters is checked by looking at a plot of the complex modulus data versus the reduced frequency. The spline knots may be interactively adjusted if any isotherm "shingles" are observed (Figures 6 and 7). Note, the reduced frequency at each temperature knot is calculated as

$$f_{R_i} = \alpha_T \left(T_{\text{knot}\,i}\right) f_{\text{ref}} \tag{4}$$

where f_{ref} here is calculated as the geometric mean of the experimental frequencies

$$\log\left(f_{\text{ref}}\right) = \left(\sum_{i=1}^{n} \log f_{\exp_i}\right)/n \tag{5}$$

4. Complex Modulus

The "Ratio of factored polynomials" model is given by

$$G\left(z\right) = G_e \prod_{i=1}^{N} \frac{1 + e\left(\frac{z}{r_i}\right)^{\beta_k}}{1 + \frac{1}{e}\left(\frac{z}{r_i}\right)^{\beta_k}} \tag{6}$$

where

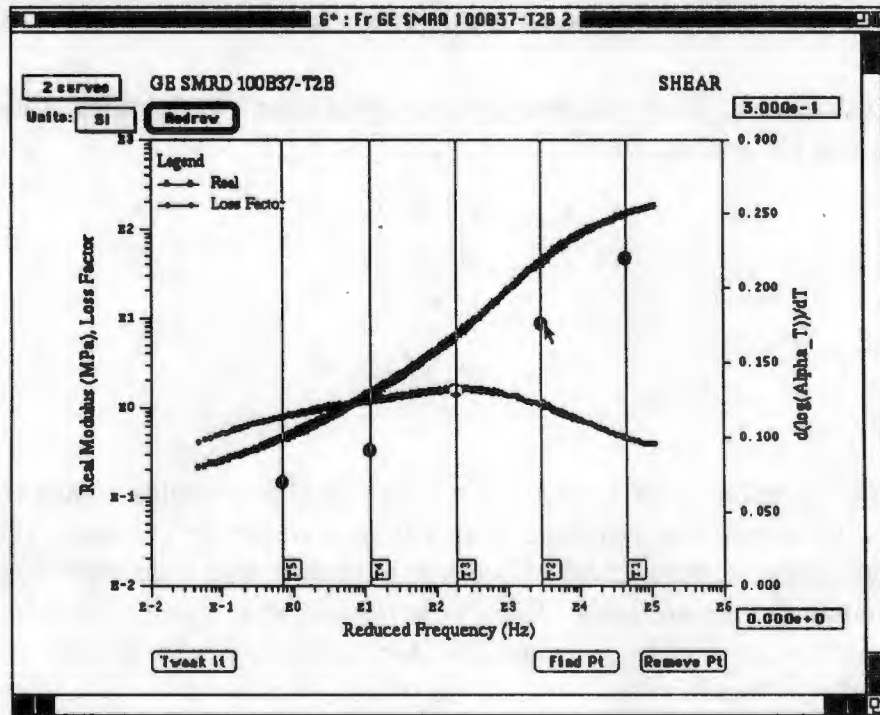$$z = j2\pi f_R$$
$$e = \left(G_g/G_e\right)^{1/2N}$$

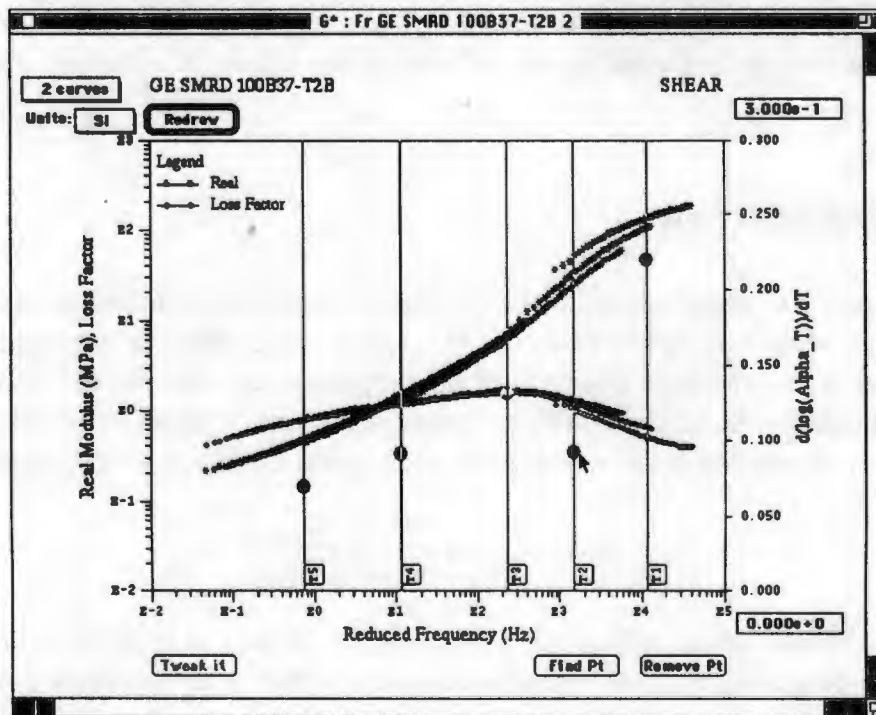Figure 6. Spline knots may be interactively adjusted

Figure 7. Reduced frequency after gross spline knot adjustment

In the present effort $\beta_k = 0.7$ although values from 0.5 to 1.0 have been used in the past.

Initial values of r_i are generated by first calculating logarithmically even spaced values between G_e and G_g with

$$
\begin{aligned}
G_{\text{fit}1} &= G_e e \\
G_{\text{fit}2} &= G_{\text{fit}1} e^2 \\
&\;\;\vdots \\
G_{\text{fit}n} &= G_{\text{fit}n-1} e^2
\end{aligned}
$$

A smoothing spline is fit to the magnitude complex modulus versus the reduced frequency. To ensure full coverage from rubbery to glassy plateaus, the spline is extended six decades below and above the minimum and maximum experimental reduced frequencies respectively. Each r_i is initialized as the reduced frequency at each $G_{\text{fit}i}$ on the spline. The r_i values are then iteratively refined with

$$
r_{i\text{new}} = r_{i\text{old}} \left(\frac{f_{R\text{fit}i}}{f_{R\text{curve}i}} \right)^{\zeta}
\tag{7}
$$

to fit Equation 6 to the magnitude modulus spline, where $f_{R\text{curve}i}$ is the reduced frequency for Equation 6 which returns $G_{\text{fit}i}$. The exponent ζ is set less than unity to keep individual iterative steps from overshooting the spline. A value of 0.5 is presently used.

5. Collocation

In addition to the smoothing spline fit through the magnitude modulus versus reduced frequency, a smoothing spline is also fit through η versus the reduced frequency. This is assumed to be the best estimate of the experimental loss factor. Starting with the TSF knot closest to the reference temperature and alternately working out, the corresponding reduced frequency is calculated using Equation 4. The equation

$$
S_{\text{new}i} = S_{\text{old}i} \frac{\arctan\left(\eta_{\text{curve}i}\right)}{\arctan\left(\eta_{\text{exp}i}\right)}
\tag{8}
$$

is then used to iteratively adjust the TSF based on the lack of fit of Equation 6. (Note, within the program, $S_{\text{new}i}$ is constrained to be within a user-defined percentage of $S_{\text{old}i}$.)

This is in contrast to [1] where the Wicket plot was used to make the comparison between $\eta_{i\text{curve}}$ and $\eta_{i\text{exp}}$. The change to using η versus reduced frequency alleviates
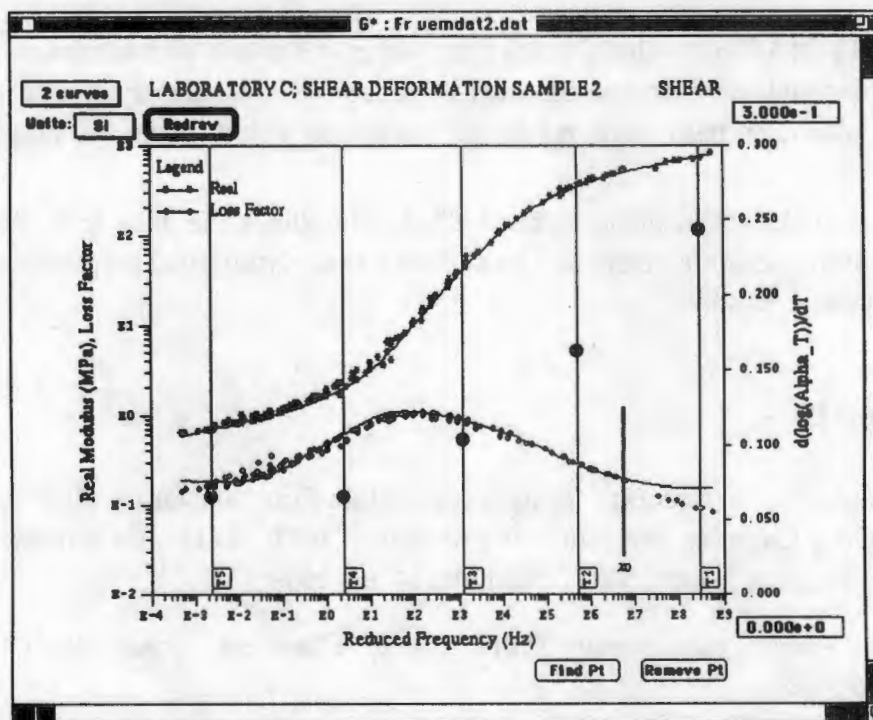
Figure 8. Loss factor divergence at x_0

possible problems encountered with numerical ill conditioning outside the transition where the slope of the Wicket typically gets steep. The plot of η versus reduced frequency is easily extended to allow the interactive adjustment of the TSF knots directly on the computer screen as seen in Figures 6 and 7.

After all the TSF knots have been adjusted, the smoothing splines are re-fit for both the magnitude modulus and the loss factor to the new reduced frequency and Equations 7 and 8 are repeated. The process stops when S_{new} is within 1% of S_{old} for every knot, or when a user specified number of iterations have taken place.

6. Observations

The need for good characterization has always been present. With advances in damping design tools the need has become even more critical.

The program VEMINT MAC has gone a long way toward more accurate characterization of VEMs; however, numerical difficulties still exist. It has been observed that outside the transition, η_{curve} and η_{exp} may begin to diverge at some reduced frequency. This may be especially true when coverage of experimental frequency is limited (Figure 10). A correction can be applied to the r_i's that does not change the modulus appreciably but does correctly adjust the loss factor and is the subject of further study.

VEMINT MAC, provides a great increase in the speed and accuracy with which damping material characterizations may be processed. The materials user may quickly find particular data points and set initial parameter values using the graphical interface.

The use of the Collocation method effectively allows the data to define the TSF, thus precluding possible errors due to operator bias. Numerical instabilities still need to be addressed, however.

References

[1] L. Rogers, "An Accurate Temperature Shift Function and a New Approach to Modeling Complex Modulus," Presented at 60th Shock and Vibration Symposium, Virginia Beach, VA, November 14-16, 1989.

[2] J. D. Ferry, *Viscoelastic Properties of Polymers*, John Wiley and Sons, 3rd ed., 1980.

[3] L. C. Rogers, "Operators and Fractional Derivatives for Viscoelastic Constitutive Equations," J. Rheology, Vol. 27, No. 4, 1983, pp. 351-372.

[4] L. Rogers, "Graphical Presentation of Damping Material Complex Modulus," *Proposed Standard*, ISO/TC108/WG13-N25, August 1987.