WADC TECHNICAL NOTE 56-488
ASTIA DOCUMENT No. AD 110562

A METHOD FOR CALCULATING THE ACOUSTICAL CHARACTERISTICS OF AIRCRAFT IN FLIGHT

J. N. COLE, 1/LT USAF
D. T. KYRAZIS, 1/LT USAF
HANS L. OESTREICHER

AERO MEDICAL LABORATORY

DECEMBER 1956

PROJECT No. 7210
TASK No. 71705

[redacted]

WRIGHT AIR DEVELOPMENT CENTER
AIR RESEARCH AND DEVELOPMENT COMMAND
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

Carpenter Litho & Prtg. Co., Springfield, O.
400 - March 1957

FOREWORD

This report was prepared by personnel of the Bio-Acoustics Branch under the authority of Project No. 7210, Task No. 71705, "Investigation and Control of the Acoustic Characteristics of Air Force Noise Sources," administered by the Aero Medical Laboratory, Directorate of Research, Wright Air Development Center. Lieutenants John N. Cole and Demos T. Kyrazis, USAF, and Dr. Hans L. Oestreicher were the prime contributors to the preparation of this report. Acknowledgement is made to Dr. Henning E. Von Gierke for his assistance in the project effort.

ABSTRACT

Information on the basic characteristics of aircraft noise sources during flight requires that a methodology be established which can adequately determine total acoustic power, directivity, and frequency characteristics from sound pressure measurements. A procedure is presented for transposing from sound pressure-time information received at a stationary ground point to information on the radiation characteristics of the moving source. Included are such physical mechanisms as propagation time lags, energy losses through a diverging sound field, atmospheric absorption losses, and Doppler shifts in frequency. A graphical approach is utilized wherever feasible to facilitate calculation. The method is equally applicable to cases where the microphone is in motion parallel to the axis of an axially symmetrical sound source at rest.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER:

Jack Bollerud
JACK BOLLERUD
Colonel, USAF (MC)
Chief, Aero Medical Laboratory
Directorate of Research

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

A METHOD FOR CALCULATING THE ACOUSTICAL CHARACTERISTICS OF AIRCRAFT IN FLIGHT

SECTION I

INTRODUCTION

Considerable speculation and uncertainty have existed concerning the effects of motion on a moving sound source. A theoretical treatment of the problem has indicated that the directivity and total acoustic power output of a sound source of constant velocity amplitude are functions of the source velocity relative to the medium through which it passes.[1]

Inherently, sound measurements on a moving source present obvious practical difficulties. The conventional manner of surveying a noise source is to measure its emitted radiation while it is stationary. Concise knowledge of over-all sound levels and levels in frequency bands are readily determined. However, when the source is moving, such information requires that a methodology be established which permits meaningful data to be obtained by feasible types of measurements. In short, how does one determine the radiation characteristics of a moving sound source?

This paper is part of a long range program aimed toward a more complete understanding of the noise generating characteristics of aircraft in flight. The purpose of this publication is to outline in detail a practical procedure for determining the directivity, spectrum, and acoustical power output of a moving noise source.

Although prime consideration will be given in the discussions to the study of sound radiation by in-flight aircraft, the methods outlined are equally applicable to any moving source. Conversely, measurements may be taken on a stationary source with a moving microphone to speed up the measurement procedure. Further consideration of these applications will be included in later sections of the paper.

[1]-Superscript numerals refer to references listed in the Bibliography.

Manuscript released by the author 28 December 1956 for publication as a WADC Technical Note.

SECTION II

AIRCRAFT IN FLIGHT - DEFINITIONS

When an airplane flies a straight path directly over a stationary recording point (G) with a constant speed and at a given altitude (Fig. 1), a certain sound pressure-time relationship is produced at the recording point (Fig. 2). The example to be used frequently throughout the discussions is a fly-over by a USAF F-100A jet aircraft, speed 630 mph, altitude 1600 feet, engine at full power.

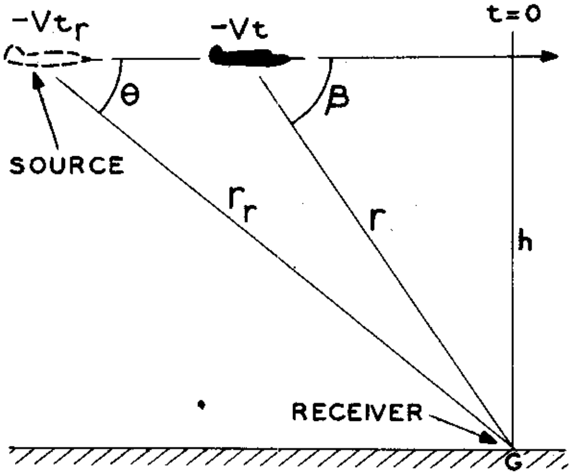

FIGURE I. TIME AND POSITION RELATIONSHIP OF THE AIRCRAFT TO THE STATIONARY RECORDING POINT.

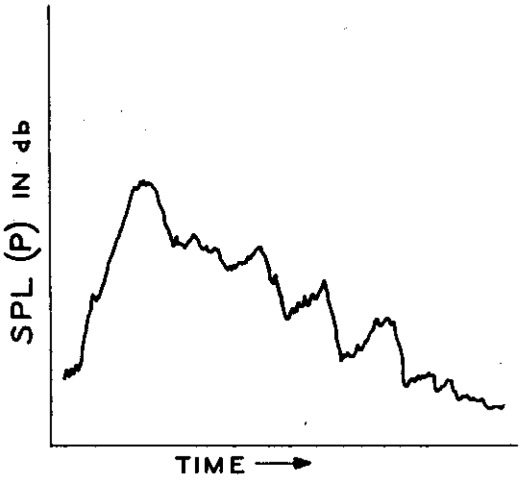

FIGURE 2. PRESSURE-TIME RELATIONSHIP PRODUCED AT THE STATIONARY RECORDING POINT.

Definitions:

t = time, defined as negative as the aircraft approaches point G, zero when the plane is directly above G, and positive after the plane passes G.

v = speed of aircraft

h = altitude of aircraft

r = slant range of aircraft at any given time t

β = angular position of aircraft relative to point G at any given time t

$p(t)$ = sound pressure observed at point G as a function of time t

c = velocity of sound propagation

f_S = frequency of sound emitted by aircraft source

f_R = apparent frequency received at point G corresponding to emitted frequency

M = Mach number of the aircraft = v/c

At any time t the pressure $p(t)$ being recorded at point G is that emitted from the source at an earlier associated time of t_r under an angle θ. The angle θ is measured between the nose direction of the aircraft and the direction toward the point of sound measurement. The distance through which this sound has propagated is r_r requiring a time of r_r/c. The subscript r is used to denote the associated or retarded position in space and time.

The aircraft is assumed to be radiating into free space; that is, the interaction of the ground with the sound field is considered negligible.

For aircraft or other noise sources moving along the ground in a straight line at constant speed past a stationary microphone, the situation and definitions involved are equivalent to those outlined above except for ground interference. The shortest distance from the microphone to the path of the source is equivalent to the altitude h, etc. The same descriptions specify the condition in which the source is at rest and the microphone is moving past in a straight line.

SECTION III

DIRECTIVITY CHARACTERISTICS OF AIRCRAFT IN FLIGHT

To adequately describe or evaluate a noise source and to compare its characteristics under varying speed conditions, its directivity pattern must be determined. Such a determination can be made for an aircraft in flight from the pressure-time relationship it produces at a stationary ground microphone, assuming certain knowledge of the aircraft's location and speed.

Employing the definitions in the preceding section and referring to Fig. 1, page 2,

$$t = t_r + \frac{r_r}{c}$$

This may be written

$$t = \frac{-h}{V \tan\theta} + \frac{h}{c \sin\theta} \qquad (1)$$

This expression relates desired angles of θ to time t. By applying this relation to the pressure-time curve recorded at position G, the points can be located which represent the sound pressure (or SPL) at an angle θ with the aircraft and at a distance r_r, Fig. 2, page 2.

1. OASL Directivity Patterns

To determine the directivity pattern at a constant radius R from the aircraft, then those SPL's measured at the stationary recording point must be corrected to yield the levels that would be measured at points at distance R from the aircraft and having zero relative motion with respect to the aircraft. Consideration must be given to frequency as well as level in transposing from information received at the recording point to information desired about the radiation pattern of the moving source. Assuming for the moment that only the over-all sound level is being considered, frequency shifts are unimportant. For this case, the following corrections must be added to the SPL's measured at the recording point G to obtain the SPL's on the circle with the radius R:

$$20 \log \frac{r_r}{R} + \alpha \left[\frac{r_r - R}{1000} \right] \text{db} \qquad (2)$$

The first term corrects for the spherical divergence of the sound field; and the second for atmospheric attenuation in propagating r_r - R distance where α is the attenuation constant in db/1000 ft. These corrected SPL's plotted versus the corresponding θ angles is the OASL directivity pattern at R distance from the aircraft. An example is given in Fig. 3, below,

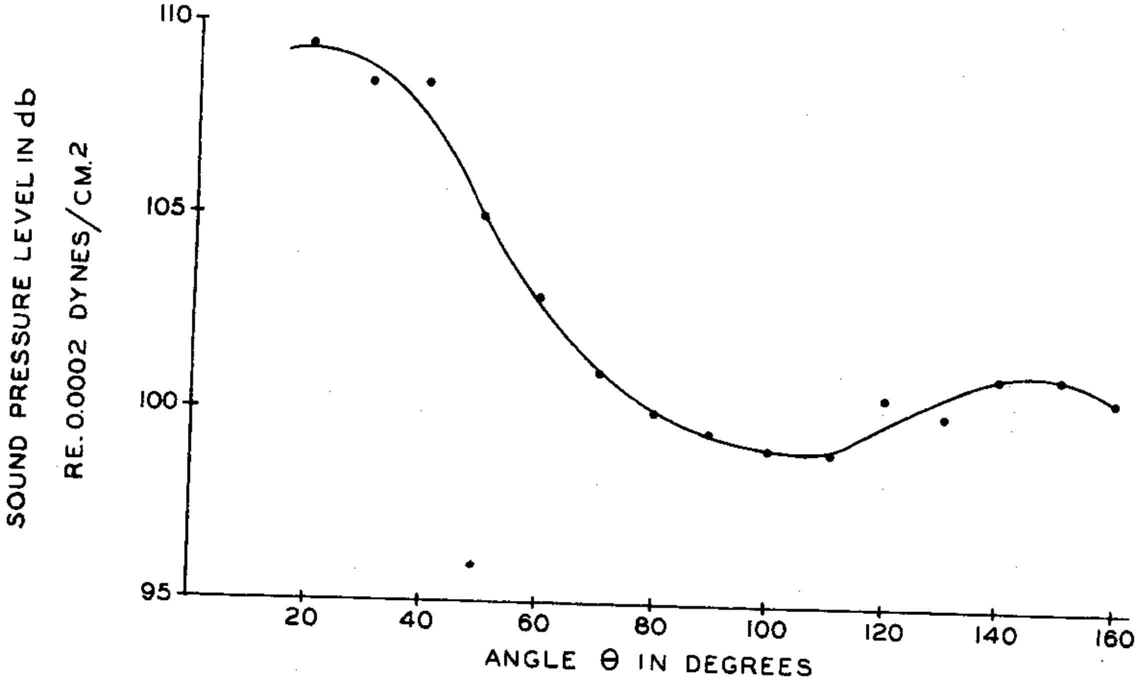

FIGURE 3. OASL DIRECTIVITY AT 500 FT. RADIUS FROM F 100A AIRCRAFT AT 630 M.P.H.

2. Directivity Patterns in Bands - Doppler Shifts in Frequency

If the radiation patterns of the source are desired in bands of frequency, Doppler frequency shifts must be taken into account.

At a particular value of θ (t) a given frequency observed at the receiver is not the actual frequency emitted by the moving source. The ratio of these frequencies is given by the familiar Doppler relation,

$$\frac{f_R}{f_S} = \frac{1}{1 - M\cos\theta} \tag{3}$$

This means, of course, that the SPL's in a given frequency band at the receiver are actually the levels in different bands of noise emitted by the source. For example, a SPL of 96 db in the 75/150 cps band, as analyzed at the receiver, is actually the level of the source noise emitted approximately in the 40/80 cps band when $\theta = 20^\circ$, $M = 0.5$ and $\frac{f_R}{f_S} = 0.53$ respectively. A graphical method is proposed as a practical way of determining the SPL's in given source frequency bands in order that the directivity pattern in those bands can be plotted. This method, presented in Step 8 (page 8) is approximate since it involves linear interpolation between sound pressure levels in received frequency bands.

It is also noted that Equation 3 is only a first approximation of the true phenomena since the information emitted by the moving source is distorted as observed at a stationary point. A pure sinusoidal source tone will be observed as a non-sinusoidal function at the stationary receiver. This function may be represented as a band of sinusoidal frequencies whose peak and width depend upon the degree of relative motion between source and receiver. For most practical applications, especially with respect to a source emitting random noise, Equation 3 can be assumed to be valid.

To facilitate calculation of the directivity pattern, Equation (1) is presented graphically in Figures 7 - 12, pages 15 - 20; Equation (2) in Figures 13 and 14, pages 21 and 22 ; and Equation (3) in Figure 15, page 23 .

3. Summary and Step Procedure

The following procedure illustrates and summarizes the method for determining the directivity pattern of an aircraft flying a straight path at a constant speed and altitude directly over the recording microphone:

(1) Calibrate the time scale of the pressure-time curve produced at the recording microphone. That is, locate the $t = 0$ point which is the time when the plane is directly above the recording point as determined by radar, theodolites, or such methods.

(2) Determine the θ, t relationship from Figures 7 - 12, pages 15 - 20 . Tabulate results. Typical θ values to employ are 10°, 20° — 170°. On Figures 7 - 12, which give the relationship for different θ and velocity ranges, select each value of θ ; extend a vertical line until it intersects that curve which corresponds to the aircraft speed in mph. From this point extend a horizontal line to the $\frac{t}{h} \times 10^3$ scale. Multiply this number by $\frac{h}{10^3}$ to obtain t.

(3) Employing these t values calibrate the t scale of the pressure-time curve in terms of θ .

(4) Read from the pressure-time curve and tabulate the SPL's corresponding to the θ values.

(5) Use Figure 13, page 21 , to determine the db correction factor to be added to the SPL's obtained in (4) because of the divergent sound field. On Figure 13 select each value of θ and extend a horizontal line until it intersects the curve which corresponds to the aircraft altitude h in feet. From this point extend a vertical line upwards until it intersects that curve corresponding to the distance R desired for the directivity pattern. From this point extend a horizontal line to the left until it intersects the db scale yielding the correction factor in db to be added to the SPL's tabulated in (3).

The line ABCD shown as a specific example is a typical path to be followed in using this chart; it is assumed that angle θ , altitude h, and distance R are 40°, 3500 feet, and 500 feet, respectively.

(6) Employ Figure 14, page 22 , to determine the db correction factor to be added, to the SPL's obtained in (4), because of atmospheric attenuation. On Figure 14 select each value of θ and extend a horizontal line until it intersects that curve corresponding to the aircraft altitude h. From this point extend a vertical line upward until it meets the first heavy horizontal line. Follow parallel to the dashed guide lines to the intersection with the horizontal line drawn out from the point on the distance scale which corresponds to the desired R distance. From this point extend a vertical line upwards until it intersects that line corresponding to the appropriate attenuation constant (db/1000 ft). From this point extend a horizontal line until it intersects the db scale yielding the correction to be added to the SPL's tabulated in (4).

The line ABCDEF shown as an example is a typical path to be followed in using this chart; it is assumed that angle θ , altitude h, distance R, and the absorption coefficient α are 40°, 4500 feet, 500 feet, and 2.5 db/1000 ft, respectively.

(7) Tabulate the SPL's as obtained in (4) and corrected by (5) and (6) versus θ . If the levels are OASL's, plot in polar or rectilinear form to display the desired directivity pattern. For levels in frequency bands proceed to steps (8), (9), (10), and (11).

(8) Plot for a given value of θ the tabulated SPL's of step (7) as a function of octave bands of frequency (assuming the analysis of the received sound to be in octave bands; if other bands are used the following procedure has to be modified accordingly.). Fig. 4, below, is an example.

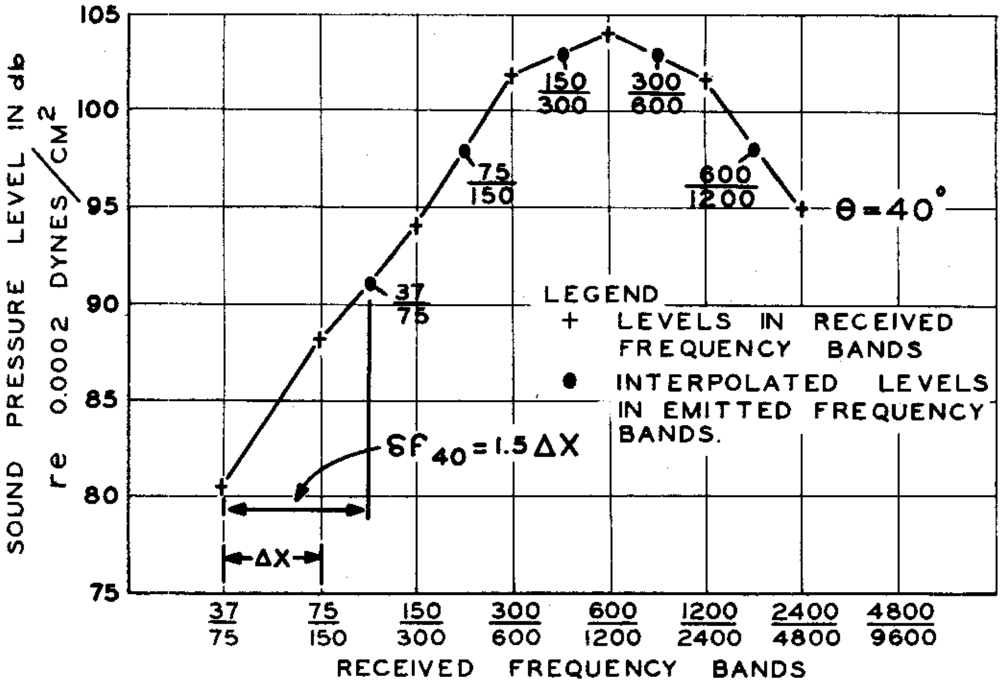

FIGURE 4. GRAPHICAL CORRECTION FOR DOPPLER SHIFT - F 100A AIRCRAFT

(9) Recalibrate the frequency scale in terms of sound emitted by the aircraft source. This is done for each θ curve as follows:

A. Determine from Figure 15 the ratio $\frac{F_R}{F_S}$ corresponding to θ .

B. Shift the entire frequency scale $\pm\delta f_\theta$ units distance on the abscissa where

$$\delta f_\theta = \frac{\Delta X}{0.3} \text{ LOG}\left[\frac{F_R}{F_S}\right] \text{ UNITS} \qquad (4)$$

ΔX is the unit distance separating consecutive octave bands. δ_θ is positive for $\theta < 90^\circ$.

(10) From each consecutive octave band point on the recalibrated scale extend a vertical line upwards to the θ curve. Read and tabulate the corresponding SPL's.

(11) Repeat steps (8), (9), and (10) for each θ value. Plot the directivities (i.e. the θ , SPL relations so obtained) which are for R distance.

SECTION IV

ACOUSTIC POWER OUTPUT OF AIRCRAFT IN FLIGHT

The acoustic power output of a noise source is obtained by integrating the intensity of the sound over a closed surface in the far field around the source. If a spherical surface is chosen, the power P is then,

$$P = \frac{R^2}{\rho c} \int_0^{\pi} \int_0^{2\pi} p^2 (\theta, \phi) \sin \theta \, d\theta \, d\phi \qquad (5)$$

where R = radius of sphere

ρc = characteristic impedence of air (at 760 mm Hg and 20°C = 41.5 dyne-sec/cm^3)

θ = angle of axial rotation in the (x, y) plane

ϕ = angle of axial rotation in the (x, z) plane

p = sound pressure measured at R distance at rotational angles of θ and ϕ.

If the sound field is axially symmetrical about the y axis, as is assumed as a first approximation for the jet engine sound field (Fig. 5, below), Equation (5) simplifies to:

$$P = \frac{2\pi R^2}{\rho c} \int_0^{\pi} p^2 (\theta) \sin \theta \, d\theta \qquad (6)$$

where θ is, as before, the angle between the aircraft nose direction and the point of sound measurement.

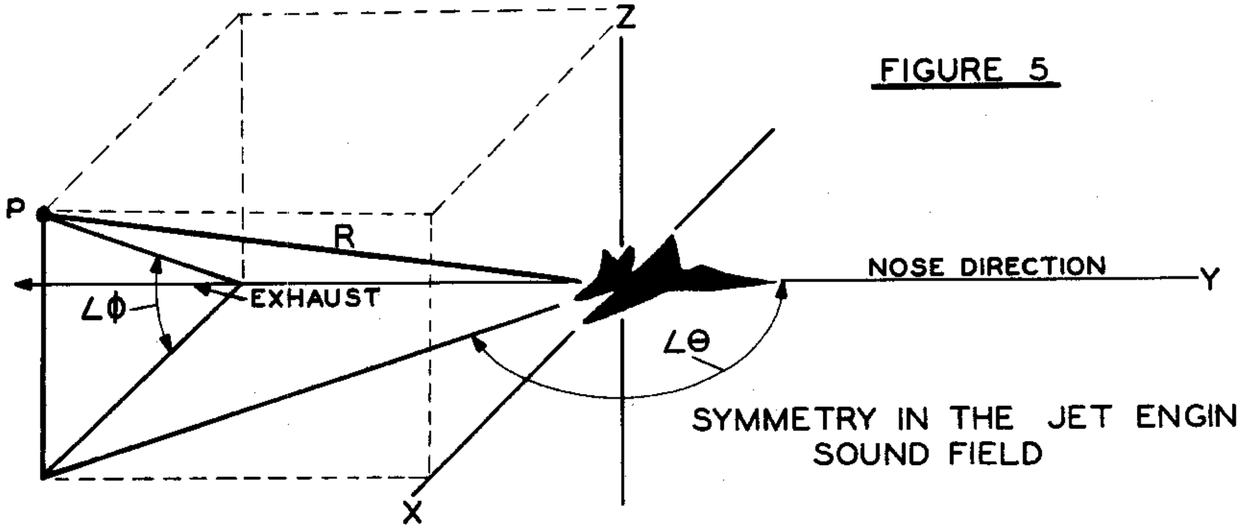

FIGURE 5

SYMMETRY IN THE JET ENGINE SOUND FIELD

By applying this equation to the directivity pattern of aircraft in flight, obtained as described earlier, the acoustic power output of these aircraft can be determined.

Equation (6) may be accurately evaluated in a graphical manner by plotting the SPL's on an ordinate (y) log scale (i.e. p on a linear scale) versus cos θ on the abscissa (x). See example in Fig. 6.

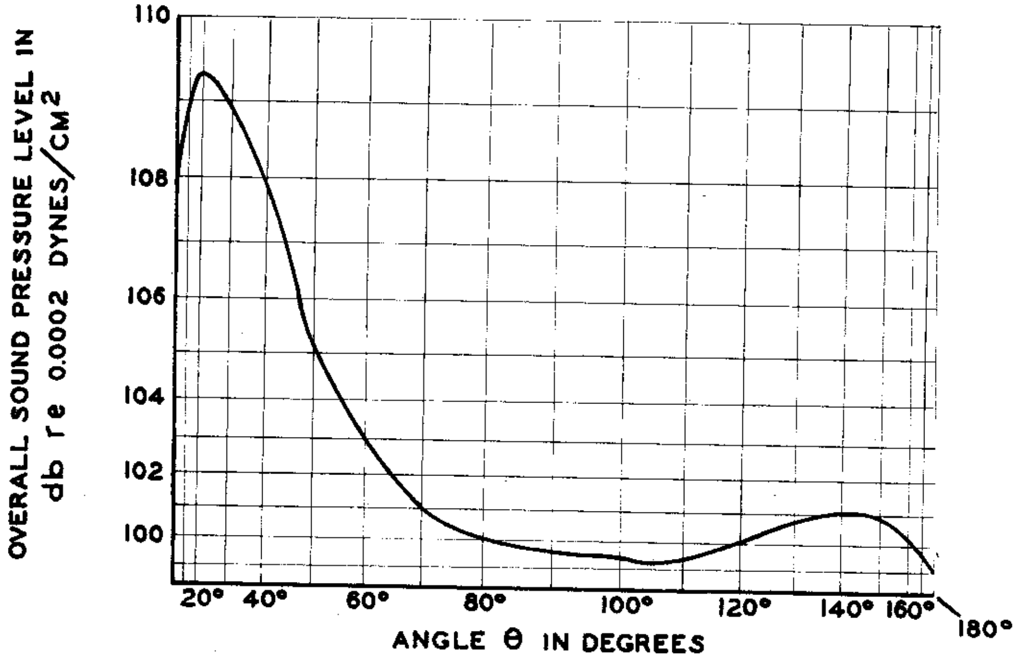

FIGURE 6. GRAPHICAL CALCULATION OF ACOUSTIC POWER

F 100A AIRCRAFT EXAMPLE

Planimetering the area under this curve with a $\int y^2 dx$ planimeter will yield a number A (in cm^3), which when multiplied by the appropriate scaling factors and constants, will yield the acoustic power P in watts. It can be readily verified that the power can be expressed as:

$$P = (2.82 \times 10^{-5}) \left(\frac{R^2}{XY^2} \right) \left[\mathrm{LOG}^{-1} \left(\frac{SPL_M - 74}{20} \right) \right]^2 \textcircled{A} \text{ WATTS} \qquad (7)$$

where R = radius of sphere in feet (equivalent to the R distance - in feet - of the directivity pattern)

SPL_M = SPL corresponding to the maximum length of the y scale

X, Y = physical length in cm of the entire x and y axes, respectively

Ⓐ = value of $\int y^2 dx$ in cm^3 from planimeter

This method of power calculation is equally applicable to the directivity pattern of jet aircraft measured statically on the ground where SPL measurements are taken in the ground plane. In this case the noise source is radiating only into half space; so the power P is half that indicated by Equations (6) and (7).

If an $\int y^2 dx$ planimeter is not available, the more common $\int y dx$ planimeter may be employed by plotting p^2 rather than p linearly on the ordinate. The scaling factors of Equation (7) must then be correspondingly changed.

An approximative, numerical method can be used to determine the power from the directivity pattern[2]. This approximation is based upon the fact that the total spherical surface surrounding the source can be divided into areas over each of which the pressure is essentially constant. Summation of the intensity-area products for each of these increments of area yields the approximate acoustic power output of the source.

It is noted that although the jet engine sound field is essentially symmetrical about the y-axis and Equation (6) is certainly applicable, the same is not necessarily true of the aerodynamic noise generated by the passage of the aircraft through the air. We must keep this fact in mind when applying the power equation (6) to aerodynamic noise.

If only the over-all power of the sound source is required, another simpler method can be employed. In this method the power is directly expressed in terms of the pressure-time relationship at the microphone without reference to the angle θ , as was done in Equation (6).

Considering p, r_r and θ as functions of t, Equation (6) can be written:

$$P = \frac{2\pi}{\rho c} \int_{-\infty}^{\infty} p^2(t)\, r_r^2(t) \sin\theta(t) \frac{d\theta}{dt} dt \qquad (8)$$

Differentiate Equation (1) and invert to yield:

$$\frac{d\theta}{dt} = \frac{V \sin^2 \theta}{h(1 - M \cos \theta)} \qquad \text{where} \qquad M = \frac{V}{C}$$

Substitute this $\frac{d\theta}{dt}$ expression into Equation (8).

Now from the trigonometric relations of Figure 1, page 2, determine sin θ , cos θ , and r_r as functions of v, h, M, and t. Using these expressions rewrite our power equation accordingly,

$$P = \frac{2\pi h^2 V}{\rho c} \int_{-\infty}^{\infty} \frac{p^2(t)}{\sqrt{(Vt)^2 + h^2(1 - M^2)}} \, dt \qquad (9)$$

This expression is suitable for ready computation by numerical or analogue techniques.

SECTION V

BIBLIOGRAPHY

1. Oestreicher, H. L., The Field of a Spatially Extended Moving Sound Source. WADC Technical Report 54-223. Wright Air Development Center, Wright-Patterson Air Force Base, Ohio, 1954.

2. Bolt, R. H. et al: Handbook of Acoustic Noise Control, Volume 1, Chapter 3. WADC Technical Report 52-204. Wright-Patterson Air Force Base, Ohio, 1952.

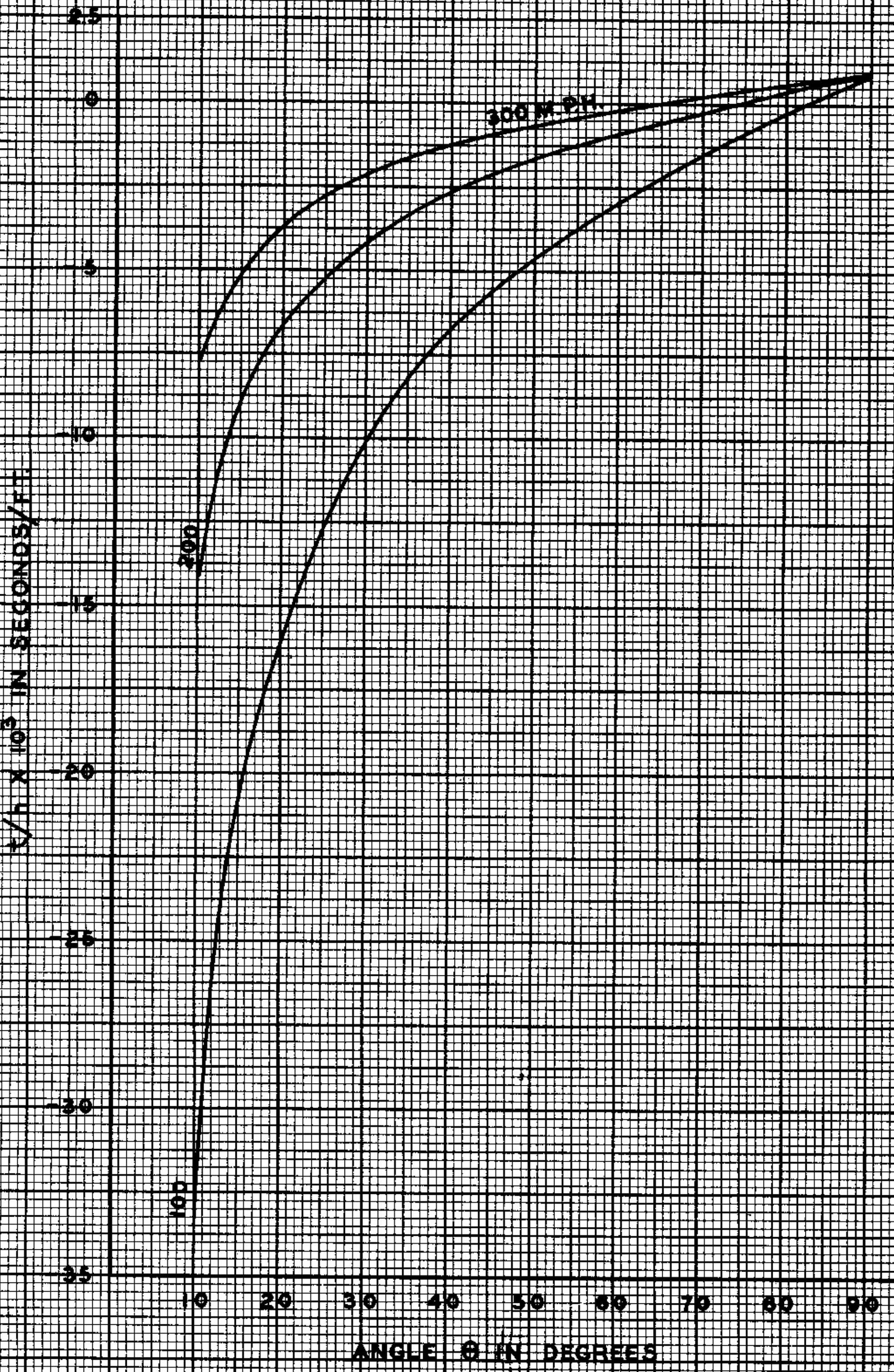

FIGURE 7. RELATIONSHIP OF ANGLE θ TO AIRCRAFT TIME POSITION

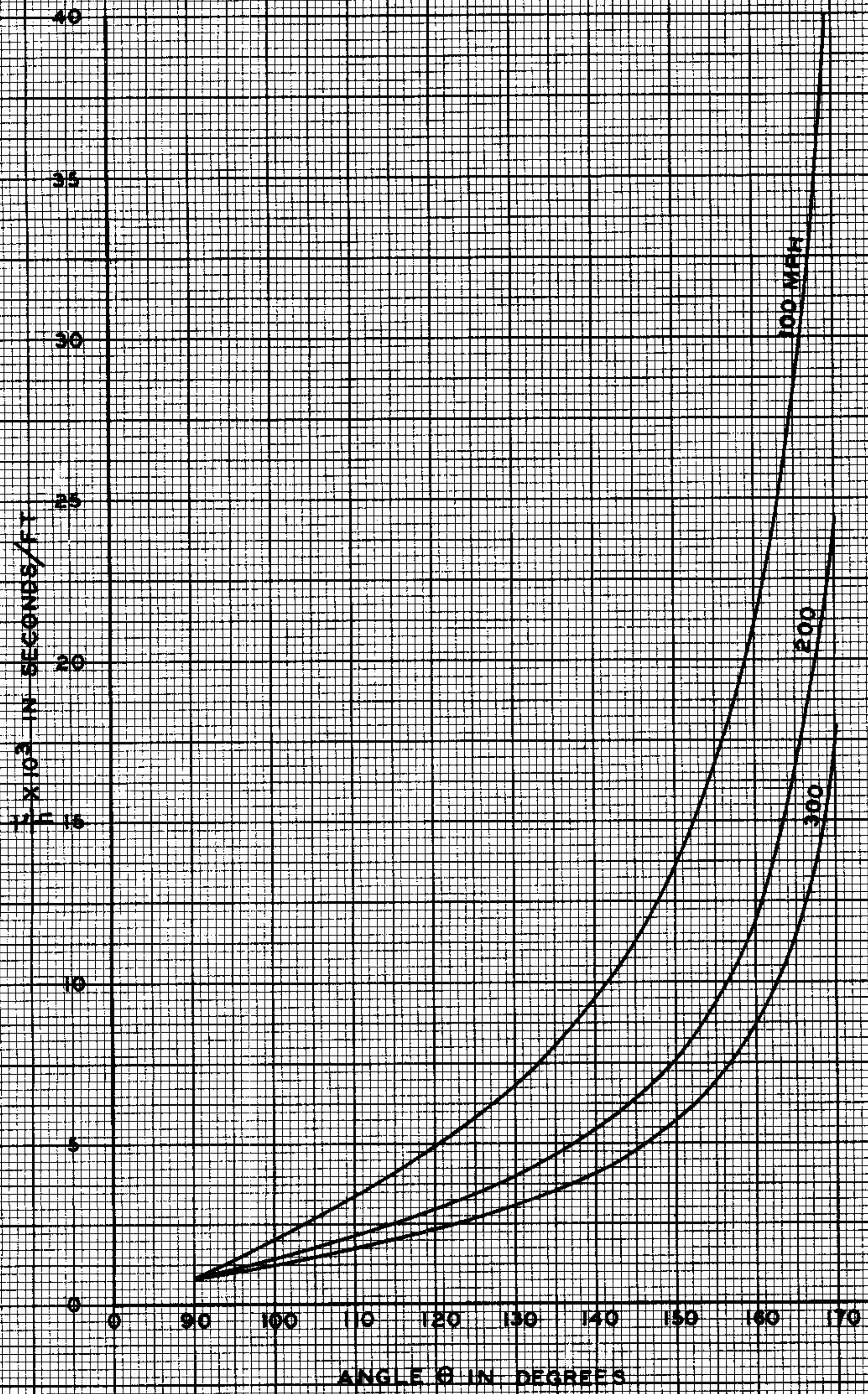

FIGURE 8. RELATIONSHIP OF ANGLE θ TO AIRCRAFT TIME POSITION

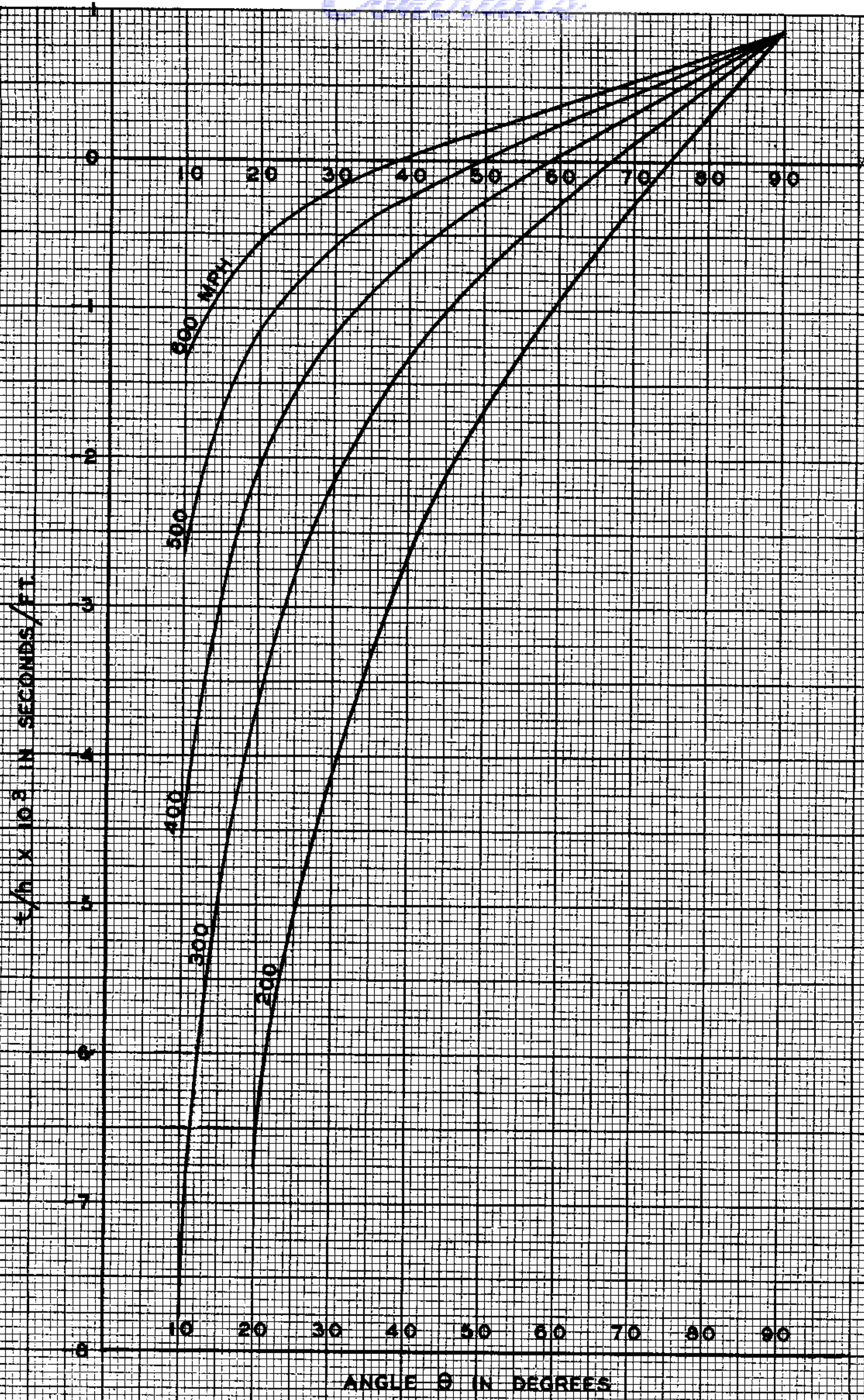

FIGURE 9. RELATIONSHIP OF ANGLE θ TO AIRCRAFT TIME POSITION

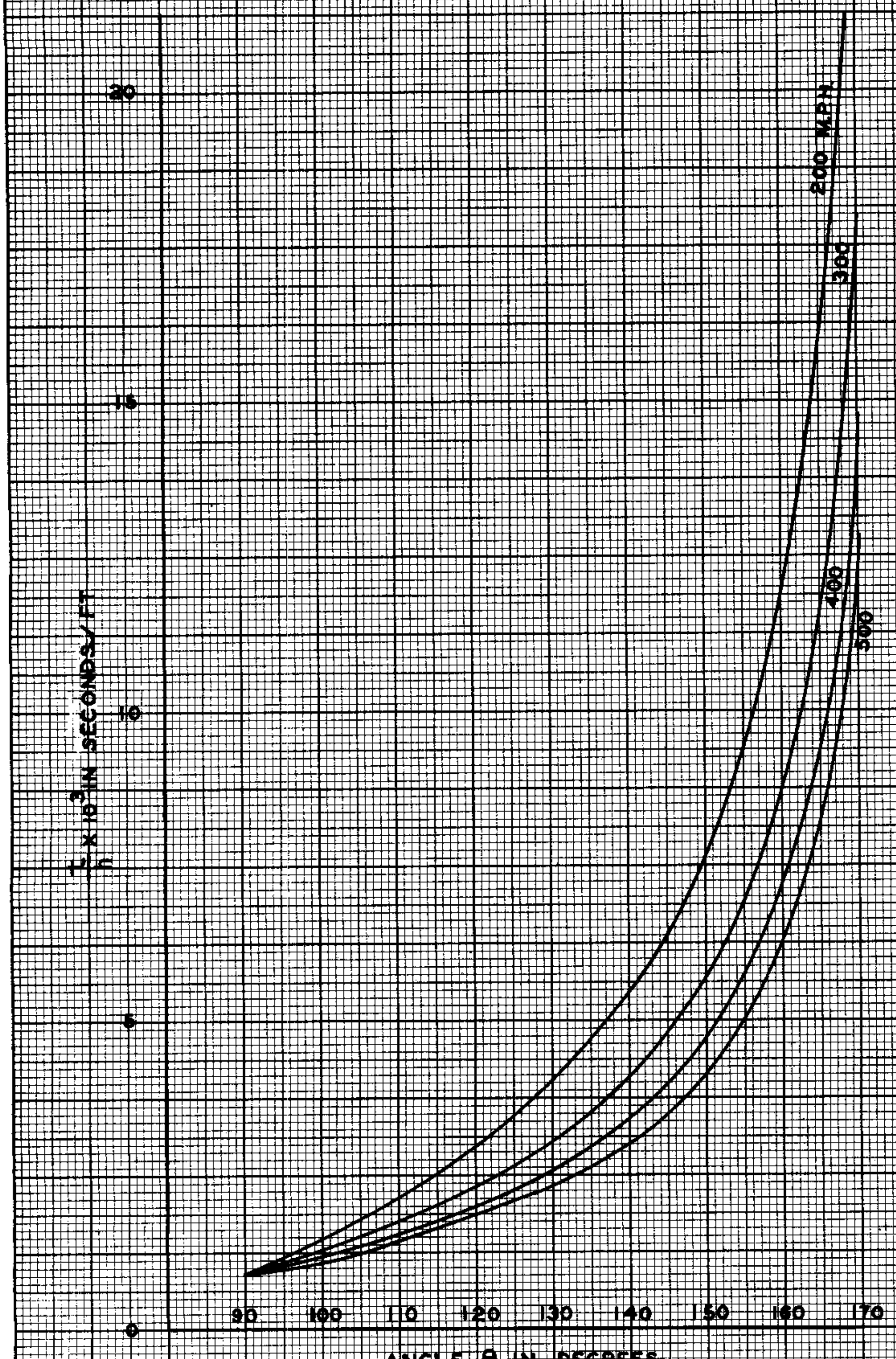

FIGURE 10. RELATIONSHIP OF ANGLE θ TO AIRCRAFT TIME POSITION

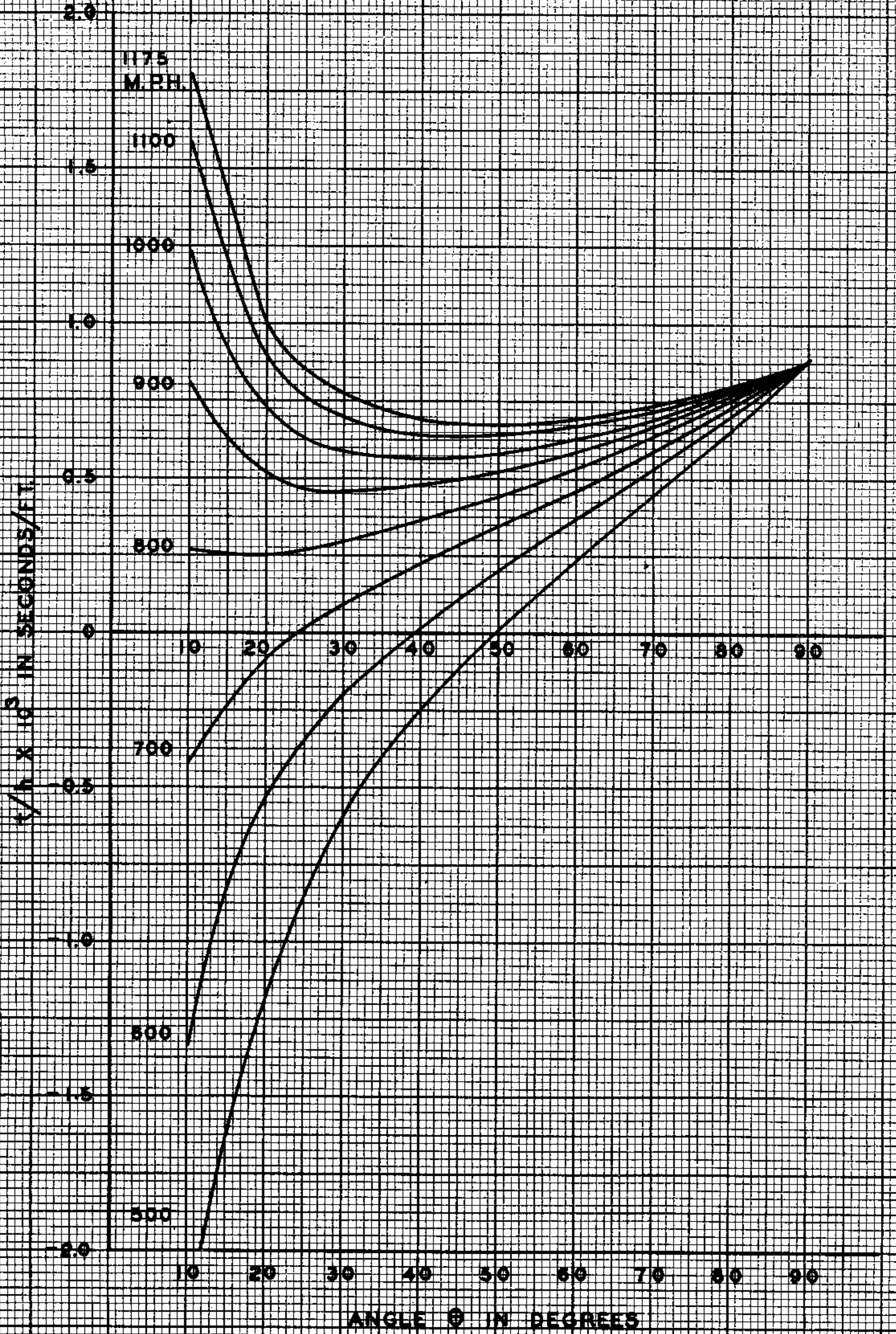

FIGURE 11. RELATIONSHIP OF ANGLE θ TO AIRCRAFT TIME POSITION

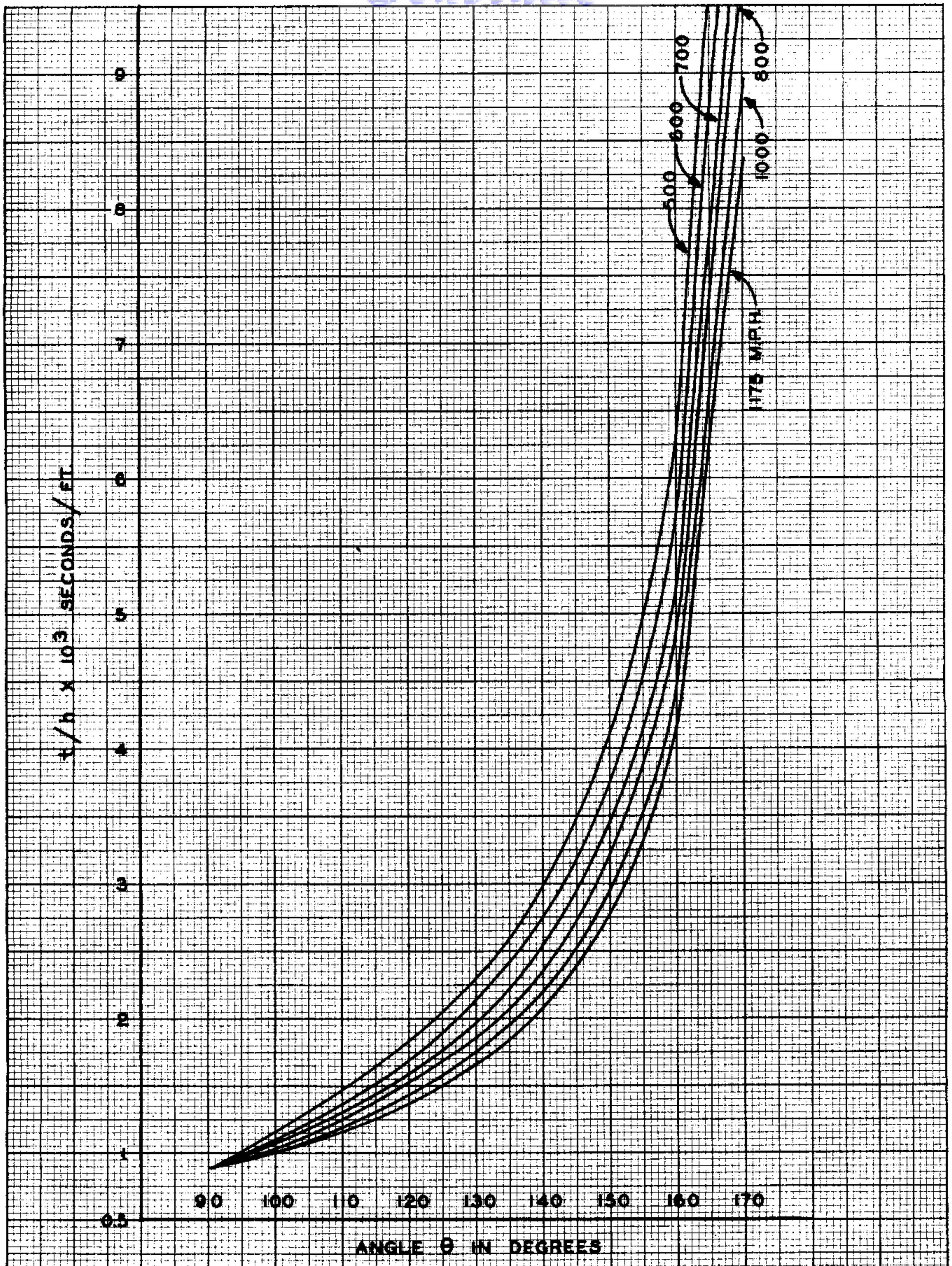

FIGURE 12. RELATIONSHIP OF ANGLE θ TO AIRCRAFT TIME POSITION

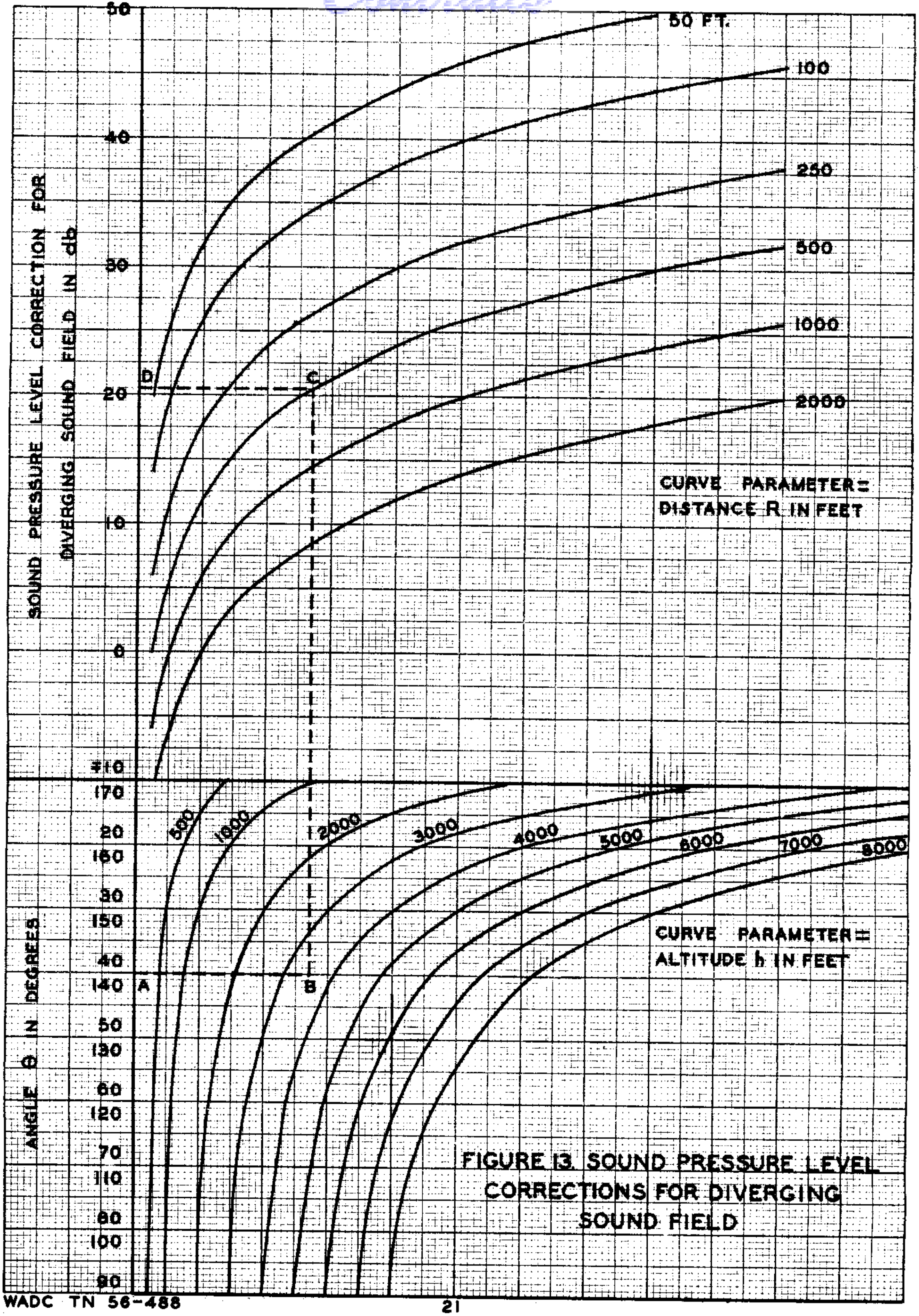

FIGURE 13. SOUND PRESSURE LEVEL CORRECTIONS FOR DIVERGING SOUND FIELD

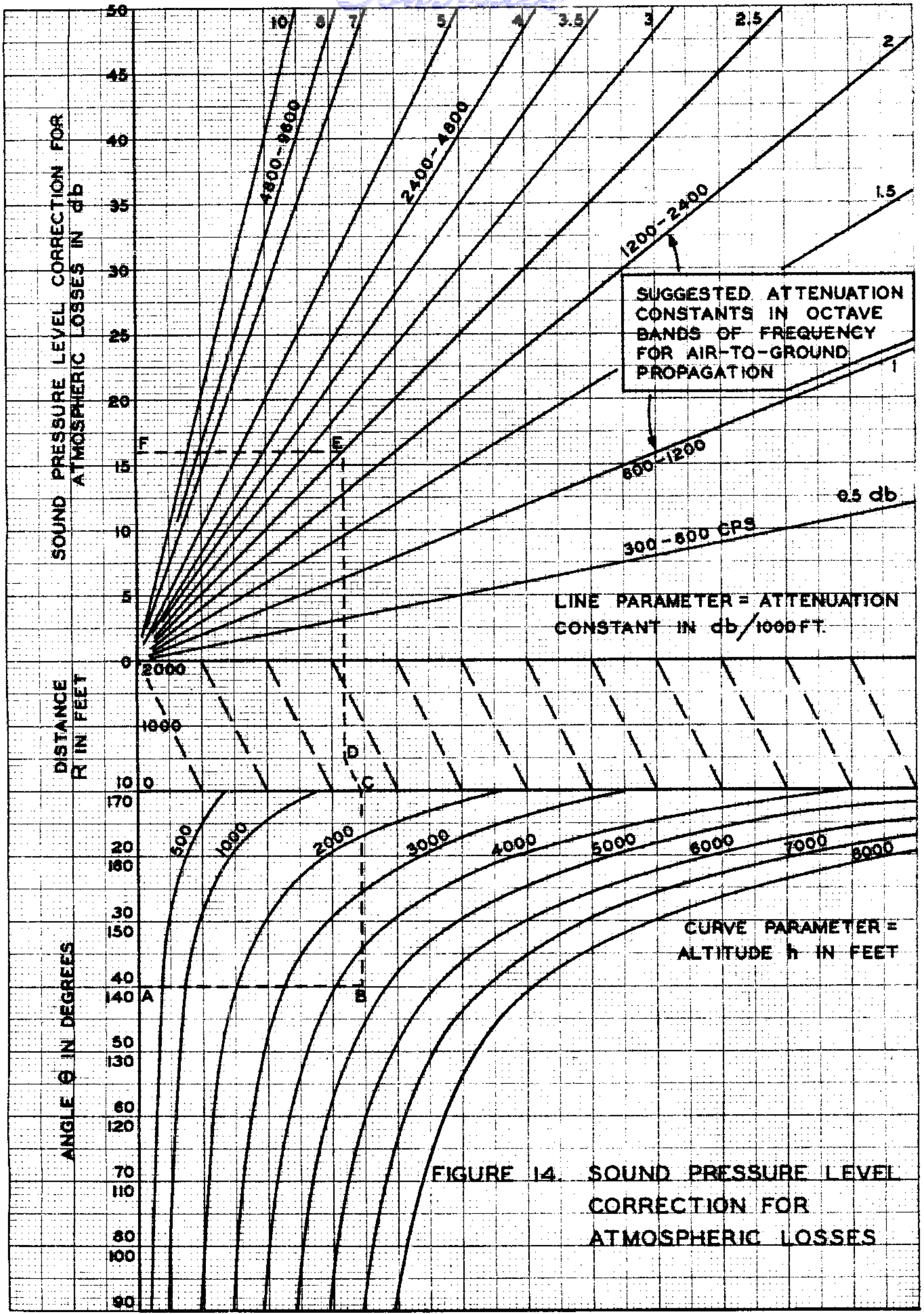

FIGURE 14. SOUND PRESSURE LEVEL CORRECTION FOR ATMOSPHERIC LOSSES

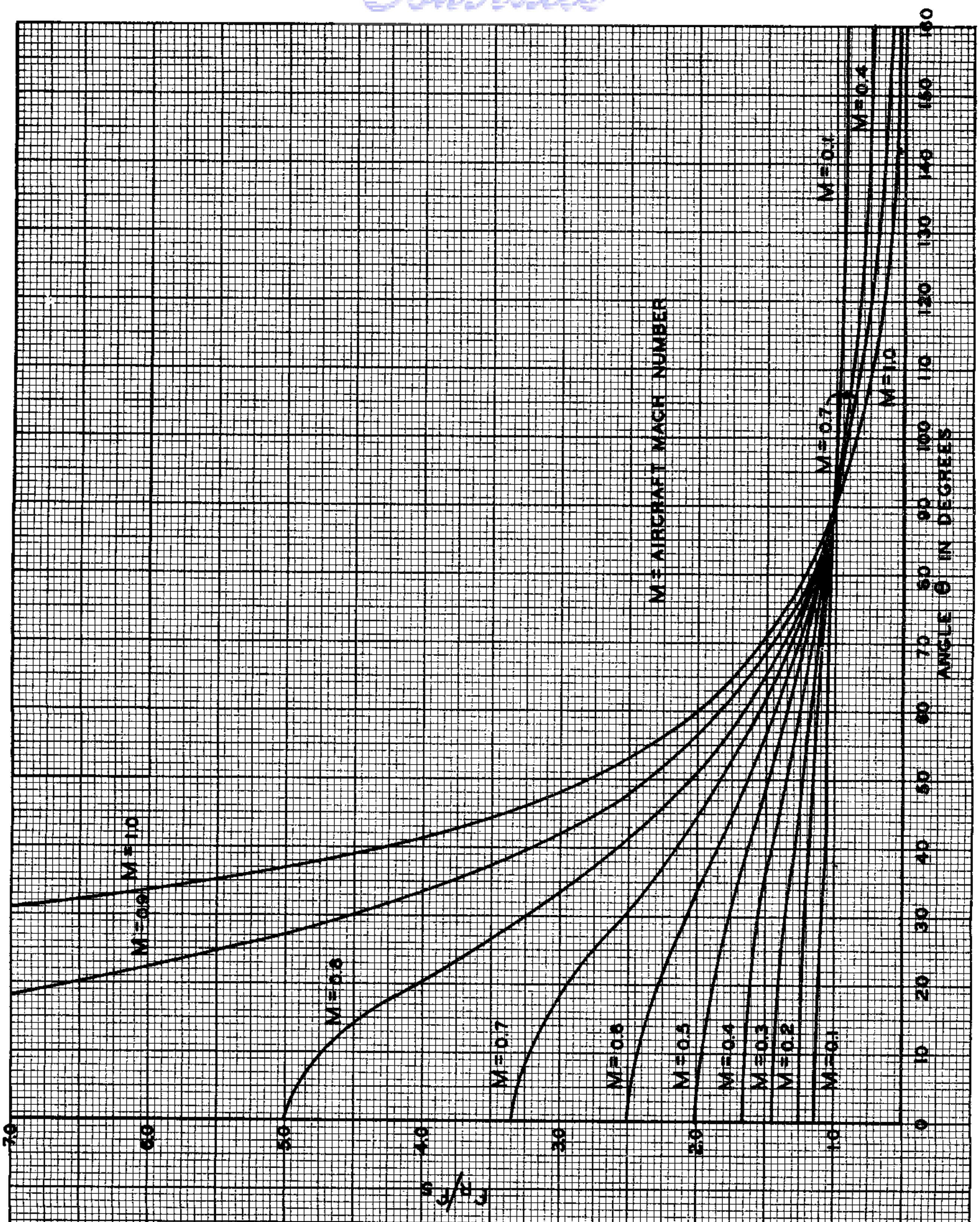

FIGURE 15. RATIO OF RECEIVED FREQUENCY f_R TO SOURCE FREQUENCY f_S