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PART 1

SECURITY INFORMATION

EVALUATION OF AIRCRAFT ACCESSORY POWER TRANSMISSION SYSTEMS BY SELECTED ANALYTICAL METHODS

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FOREWORD

This report was prepared by the Armour Research Foundation of the Illinois Institute of Technology under USAF Contract No. AF33(038)-21751. The work performed in this program was under the supervision of the Power Plant Laboratory, Directorate of Laboratories, Wright Air Development Center, with Mr. J. D. Delano, Jr. as project engineer, and is covered by RDO No. R-536-232, "Power Plant Power Transmission Systems".

This report is presented in two parts. This part (Part 1) is concerned with the basic assumptions for the various types of aircraft accessory power transmission systems concerned, while Part 2 consists of the derivations of the equations which were utilized in Part 1.

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ABSTRACT

A method is presented for evaluating four possible types of aircraft accessory power transmission systems as listed below.

1. Pneumatic
2. Hydraulic
3. Electric
4. Mechanical

This evaluation method is intended for use in deciding which type of accessory transmission system should be used in a given type aircraft.

These systems were analyzed on a minimum weight basis. Weights as well as the increase in fuel weight to compensate for power extracted from the engines were considered on the basis of mission profile and power characteristics of the accessory systems. The minimum weight was mainly determined by the transmission line sizes in the respective systems, while other components were essentially constants in the analysis.

This report is divided into two parts. This part (Part 1) is devoted to listing, for each system, the basic assumptions and required data, and to presenting a step by step procedure for calculating the minimum total weight of each basic system. Part 2 contains derivations of the equations which were utilized in Part 1 without derivation or detailed explanation.

The security classification of the title of this report is "UNCLASSIFIED". While each individual section of this report is not considered classified, the compendium of information contained in this report is considered to be of sufficient importance to require the protection afforded by a RESTRICTED classification.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER:

NORMAN C. APPOLD, Colonel, USAF
Chief, Power Plant Laboratory
Directorate of Laboratories

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SECTION I

INTRODUCTION

This study was undertaken to develop a method of evaluating accessory power transmission systems on aircraft. It is based on the design and mission requirements of the airplane under study.

For a better understanding of this evaluation a short explanation of the reason for making this study, a general description of the basic accessory transmission systems, an explanation of the analysis method, and a brief summary of the initial assumptions and required data are included here.

A. The Problem of Accessory Power in Modern Aircraft

Modern military aircraft such as fighters, bombers or cargo planes require, in addition to the power plant needed for their propulsion, a variety of accessories or auxiliaries for safe operation and improved performance. These accessories can be grouped into two categories. The first group consists of engine accessories needed for proper operation of the engine such as the oil pump, fuel pump, engine controls and ignition parts. The second group consists of accessories required for the operation of the airplane such as the generator, alternator, hydraulic pump, and air compressor. In the following discussion, the first group will be referred to as "engine components", while the term "accessory" will be used for all auxiliary equipment not required for the operation of the power plant.

Recent trends in aircraft configurations, aircraft engine development, and accessory power requirements have progressively caused serious difficulties in regard to mounting and driving the accessories. Aircraft are becoming faster, thereby increasing the importance of drag reduction. Aircraft engines are becoming more compact, offering less frontal area for mounting accessories. The power required to drive the accessories is increasing due to additional electronic equipment and other accessories needed on larger and faster airplanes.

In the past, the accessories have been mounted on the engine. The trend in engine development indicates that the space available for mounting accessories on the engine is steadily decreasing. The trend in accessory power requirement indicates that the space required for mounting the accessories on the engine is steadily increasing. The difficulties arising from this space problem are manifest in current aircraft.

The difficulties arising from the space problem on the engine can be circumvented by mounting the accessories inside the airplane, remotely located from the engine and driving them by means of remote power transmission.

Several methods or systems exist for driving remotely located accessories. In designing a new aircraft it is necessary to determine at an early stage which of these methods should be used. This decision has to be based, in part, on the optimum performance to be expected, considering the design and mission requirements of the particular aircraft.

B. Remotely Driven Accessory Power Systems

A remotely driven accessory power system shall be defined as a system that consists of a transmission system and a distribution system, as shown diagrammatically in Fig. I-1. The transmission system consists of a power extraction unit, a transmission line, a power conversion unit and an accessory gear box. The distribution system consists of accessory power units, distribution lines and the accessories.

Since the accessory power system must supply the particular requirements of the specific airplane for which it is designed, the basic system may have many variations.

During the design of an accessory power system, three problems must be analyzed. First, the airplane must be analyzed to determine the accessory functions that must be performed. For example, the bomb bay doors must be opened and closed, the landing gear must be raised and lowered, and the radar equipment must be operated. The number and type of accessory functions and the power required for each function varies with each airplane type.

Second, each function must be analyzed to determine the type of actuating device to be used to perform the function. The selection of hydraulic, pneumatic, or electric actuating devices is influenced by the accessory function, the availability, reliability, and development stage of a particular device. The actuating devices utilizing the same kind of energy are generally supplied from a common distribution system and a single accessory power unit. This is shown schematically in Fig. I-1 where pneumatic, hydraulic and electric power units supply pneumatic, hydraulic and electric actuating devices, respectively.

Third, the system must be analyzed to determine the method of extracting the power from the power plant and transmitting it to the accessory gear box. The power plant may be the main engine or an auxiliary engine. The power can be extracted mechanically from a power take-off drive, or pneumatically by bleeding air from the main engine compressor. If the power is extracted mechanically, it can be transmitted in one of three ways, mechanically, hydraulically, or electrically, as shown in Figs. I-2, I-3, and I-4. If the power is extracted pneumatically, it is transmitted through an air duct in the form of relatively high pressure, high temperature air to the accessory gear box, as shown in Fig. I-5. Here again the transmission system selected depends largely on the particular airplane under consideration.

The distribution lines and the power transmission lines are not always clearly defined as indicated in Fig. I-1. The power extracted from the engine may in part be directly distributed to some of the accessories without being converted into some other form of energy. This is done in the case of bleed-air system (Fig. I-5) where some of the air is used for hot-air anti-icing, and cabin air conditioning. Thus, some overlap may exist between the power transmission and the distribution or actuating systems.

In larger aircraft the accessory power may be extracted from more than one engine, and the transmission line may distribute the extracted power to several power conversion units. This indicates the wide variety of possible configurations encountered in aircraft accessory power transmission systems.

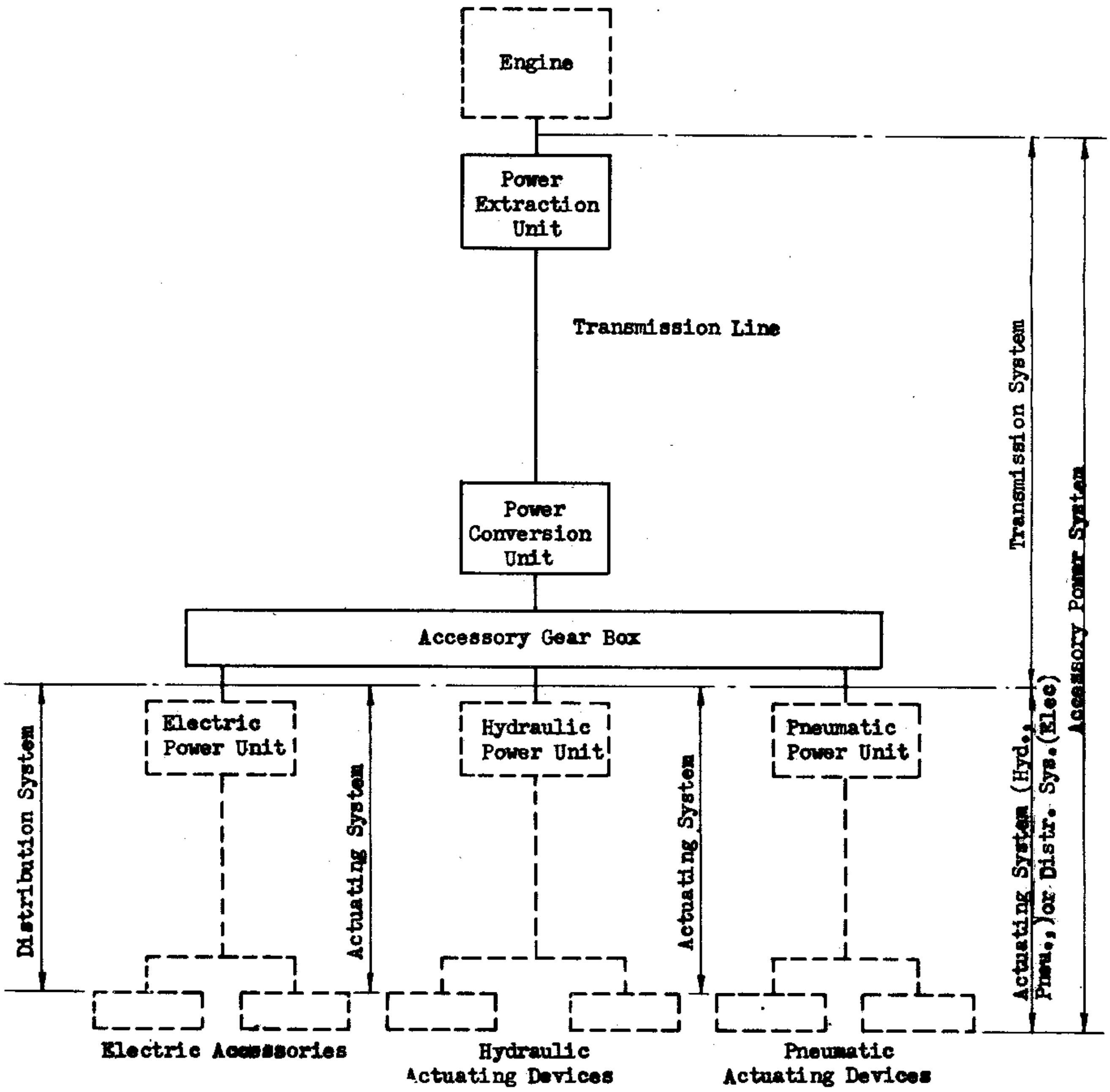


Fig. I-1 BASIC ACCESSORY POWER SYSTEM

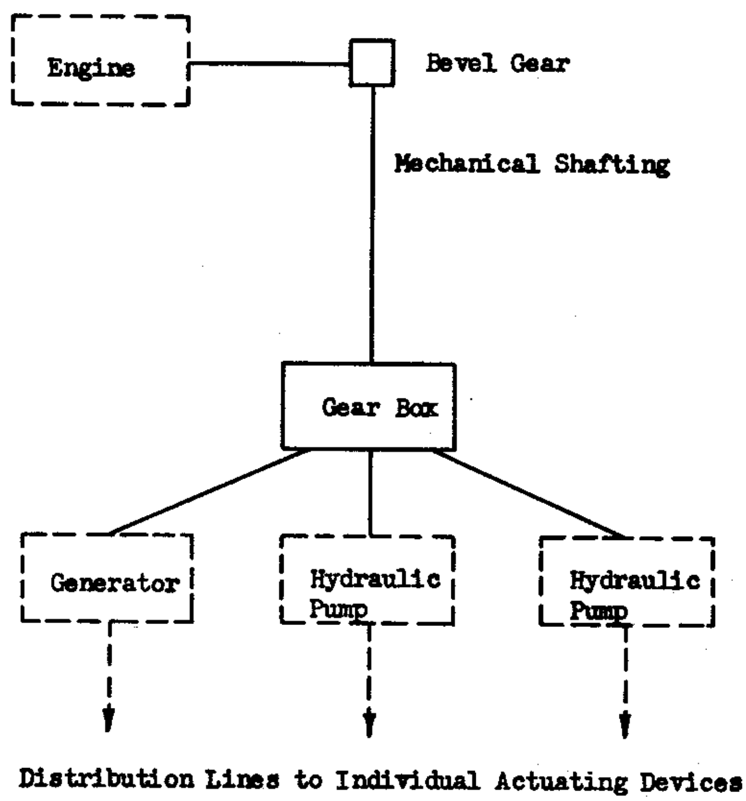


Fig. I-2 MECHANICAL POWER TRANSMISSION SYSTEM

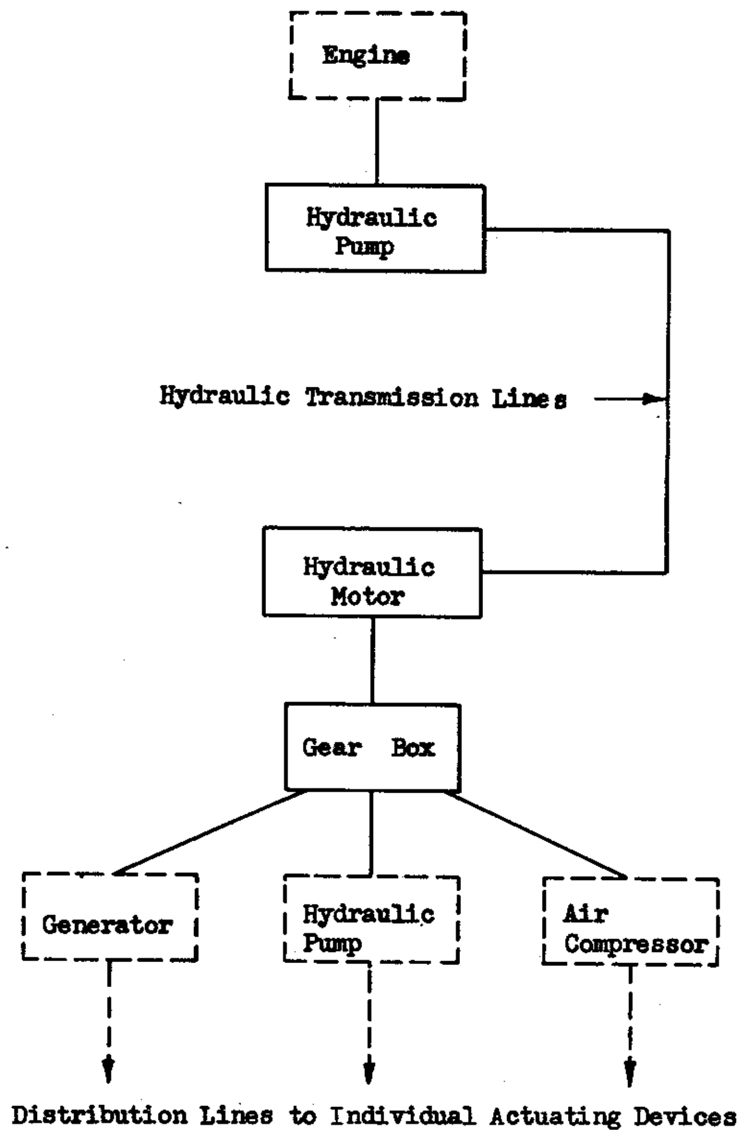


Fig. I-3 HYDRAULIC POWER TRANSMISSION SYSTEM

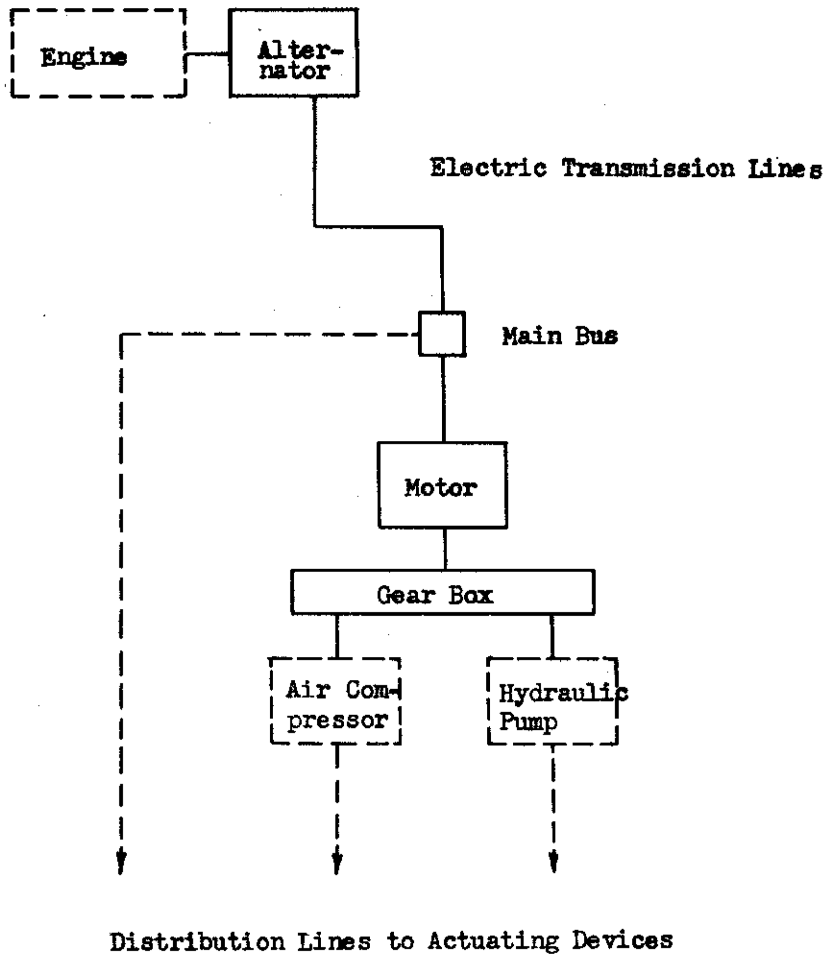


Fig. I-4 ELECTRIC POWER TRANSMISSION SYSTEM

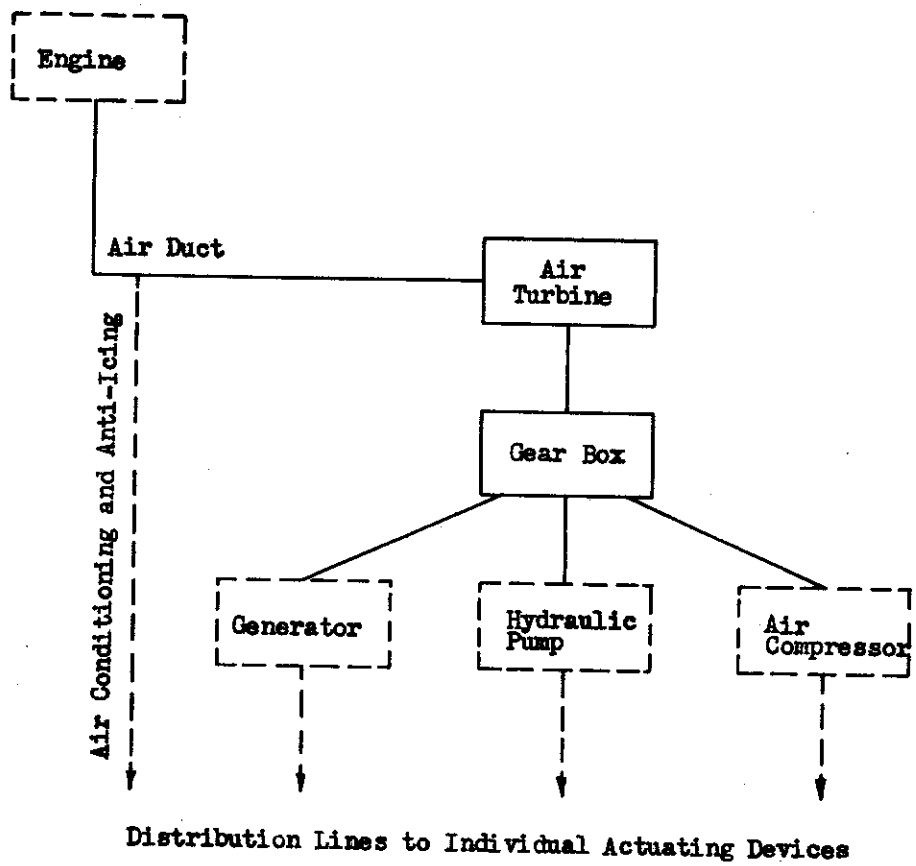


Fig. I-5 PNEUMATIC POWER TRANSMISSION SYSTEM

C. Method of Evaluating Power Transmission Systems

This analysis is concerned with the basic power transmission systems which consist of one extraction unit, one conversion unit, a gear box, and the transmission line, as shown in the upper portion of Fig. I-1.

Four basic types of transmission systems are considered:

1. Pneumatic

2. Hydraulic

3. Electrical

4. Mechanical

The relative preference of the four basic types of systems is evaluated by determining their estimated minimum weight. The evaluation of the weight is based on experienced values of component weights, the weight of the minimum transmission lines, and the additional weight of the fuel required for operating the transmission system, which is based on the flight profile of the aircraft under consideration.

It is realized that this method of evaluation gives only a partial comparison, since other important parameters, such as reliability, vulnerability, maintenance problems, and life expectancy are not included. The reason for this omission is that these parameters do not lend themselves to a mathematical analysis. However, by first determining the weights of the four basic types, and later modifying the obtained weight rating on the basis of experience factors, taking into account the reliability, vulnerability, etc., a more complete evaluation can be made. With sufficient experience data available a relative number system can be worked out for subsequent regrouping of the preference given on the weight basis.

D. Initial Data and Assumptions

The initial data and assumptions required to evaluate an accessory power transmission system for a given aircraft are briefly summarized here.

It is assumed that this analysis will be applied at the stage of airplane design when the following data are available: the flight profile or mission, the engine performance characteristics, and the approximate location and magnitude of the required accessory power.

The flight profile indicates the average cruise speed and altitude, the maximum altitude, and the range of the airplane. These factors are important in evaluating engine operating conditions and the fuel consumption chargeable to the accessories. In addition, the maximum speed and altitude and the minimum speed are required to determine the transmission system components' weight and efficiency.

The engine performance characteristics are necessary in determining the effect of power extraction on fuel consumption. If an extraction unit, such as an alternator or hydraulic pump, is driven from the engine shaft, the fuel consumption of the engine will be increased. If power is extracted from the engine in the form of bleed air, it is necessary to know the pressure, temperature, and amount of air available for bleed at various engine and airplane operating conditions. Bleeding air from the engine will also increase the engine fuel consumption. This information is obtainable from the engine manufacturer's model specifications.

The amount of power required and the approximate location in the airplane where this power is needed determines the size and length of the transmission lines and the size of the extraction and conversion units.

At this stage of development the dimensional outline of the airplane and the location of the main engines should be well established. The power requirement and the load schedule for each accessory must be estimated. This in turn is used to determine the grouping of the accessories and location of the distribution points (power conversion units).

The dimensional outline of the airplane, showing the location and the amount of power required of the power conversion units, constitutes a framework into which each type of transmission system has to be fitted.

It may happen that the power required and location of the conversion units change during the period of development of the airplane. If these changes are not radical, maintaining the original pattern and essentially the same power and layout dimensions the result of the transmission system evaluation will not be affected appreciably. If drastic changes are made, the transmission system should be re-evaluated.

E. Presentation of Method for Evaluating Transmission Systems

Part 1 of this report is devoted to listing the basic assumptions for each system, the required data, and presenting a step by step procedure for calculating the minimum total weight of each basic system. The list of data indicates the information to be obtained and the most probable source. The procedure indicates the quantity to be computed and the equation or curve to be used in calculating it. Where necessary explanatory notes are included. The equations and curves used in Part 1 are derived and discussed in Part 2.

ANALYSIS OF THE WEIGHT OF PNEUMATIC POWER TRANSMISSION SYSTEMS

Section II

A. Introduction

This section shows the procedures for determining the approximate weignts of the two types of pneumatic transmission systems which are presently being used or contemplated for use in modern airplanes. These systems are:

1. Straight bleed system
2. Bleed and burn system

The straight bleed system consists essentially of a bleed port in the jet engine which provides for extraction of relatively high pressure, high temperature air from the engine compressor; ducting which transmits the bleed air from the jet engine to an air turbine; and the air turbine which converts the energy of the bleed air to shaft power. In addition, controls are needed to regulate the speed of the air turbine. A schematic diagram of a typical straight bleed system is shown in Fig. II-1. The thermodynamic cycle from the inlet of the compressor at am to the outlet of the turbine at bo is shown on a temperature-entropy diagram in Fig. II-2.

The horsepower demand on the air turbine is determined by the requirements of the individual accessories. This includes any emergency overloads which may be expected. The air turbine must satisfy these power requirements by drawing from energy available in the bleed air. The energy in the bleed air available to the turbine is determined by the operating conditions of the airplane engine and the pressure drop in the duct. For a given airplane and engine operating condition, a decrease in the accessory power requirement tends to increase the turbine speed. This is sensed by the control which acts to decrease the amount of bleed air delivered to the turbine. For a given accessory load any change in airplane or engine operating conditions which decreases the bleed air temperature and pressure will tend to decrease the turbine speed. In this instance the control acts to increase the amount of bleed air delivered to the turbine.

The pneumatic system must be designed so that the maximum or overload requirements will be satisfied under the most adverse operating conditions of the airplane and engine. Selection of the proper duct diameter has to be made accordingly. The pressure drop, and consequently the energy loss in a duct, increases with the velocity of the air passing through it. For given accessory load and inlet conditions a smaller duct causes a larger pressure drop and greater energy loss. The smallest duct that is capable of transmitting the necessary amount of air without undue energy loss must be determined for the most adverse operating conditions.

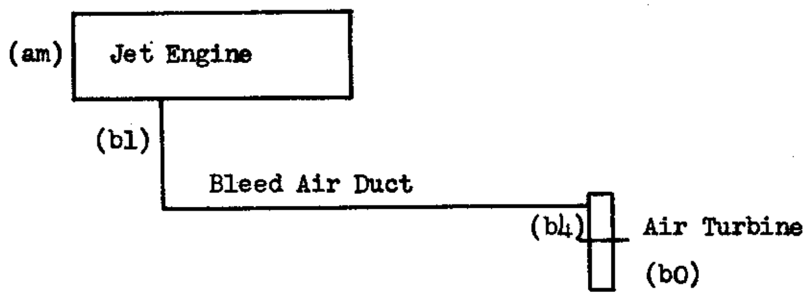


Fig. II-1 SCHEMATIC DIAGRAM FOR A STRAIGHT BLEED AIR SYSTEM

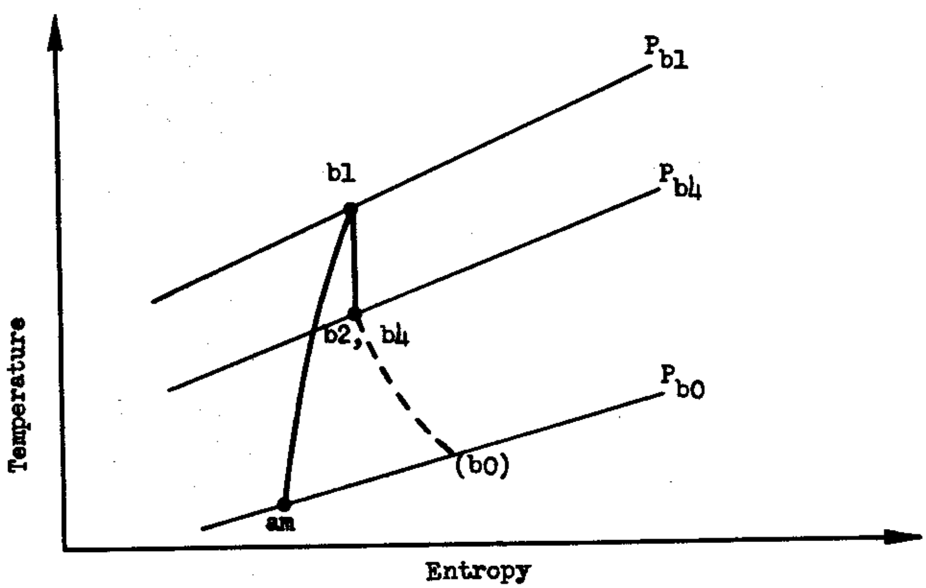


Fig. II-2 TEMPERATURE ENTROPY DIAGRAM FOR STRAIGHT BLEED AIR SYSTEM

The most adverse or critical operating conditions occur when the airplane is making a landing approach or descends from a high altitude with the engines idling. Then the energy available per pound of bleed air is low due to low temperature and pressure of the air discharged from the engine compressor, making a large duct diameter necessary. It is possible that other circumstances may represent the critical operating conditions for different airplanes with different airplane engines, and different overload specifications. Therefore, the minimum permissible duct diameter should be calculated for every condition requiring a large amount of bleed air due to a low pressure ratio. The largest among the minimum duct diameters which is needed indicates the most adverse operating conditions.

When using a pneumatic system, it is generally advantageous from a weight viewpoint to combine the cabin conditioning system with the accessory power transmission system. The cabin conditioning duct diameter is increased to accommodate the air requirements of the accessory power unit. Thus, for the duct the transmission system is charged only for the increment in duct weight due to the increase in duct diameter.

When the accessory power system is combined with the cabin conditioning system, the maximum allowable pressure drop through the duct is given. For proper functioning the cabin conditioning and pressurization system requires that the inlet pressure to the cabin conditioning turbine be kept above a specified minimum at high altitude. This minimum pressure is determined by the requirements of the conditioning unit and is used as a design criterion in calculating the duct diameter.

The bleed and burn system is identical to the straight bleed system, except that a burner is inserted just ahead of the turbine. The burner raises the air temperature thereby increasing the energy available to the turbine per pound of bleed air. Figs. II-3 and II-4 show a schematic diagram and a temperature-entropy diagram for this system. In many cases the bleed and burn system is superior to the straight bleed system from a weight and performance viewpoint. However, there are two rather serious objections to the bleed and burn system; the control problem is complex, and the additional burner represents a safety hazard. The choice between a straight bleed and a bleed and burn system has to be based on considerations of safety. Therefore, the choice cannot be made solely on the basis of minimum weight requirements, but is largely a matter of judgment and policy.

For a fair comparison with other systems it is necessary to determine the optimum pneumatic system which is defined as the lightest system capable of fulfilling all power requirements of the airplane design mission. The weight of the system is composed of the following items:

1. Weight of system components (duct, air turbine, governor or controls).

2. Weight of fuel required by engine to compress the air which was bled from main engine compressor to operate the accessories.

3. Weight of fuel required to overcome any additional aerodynamic drag caused by transmission system.

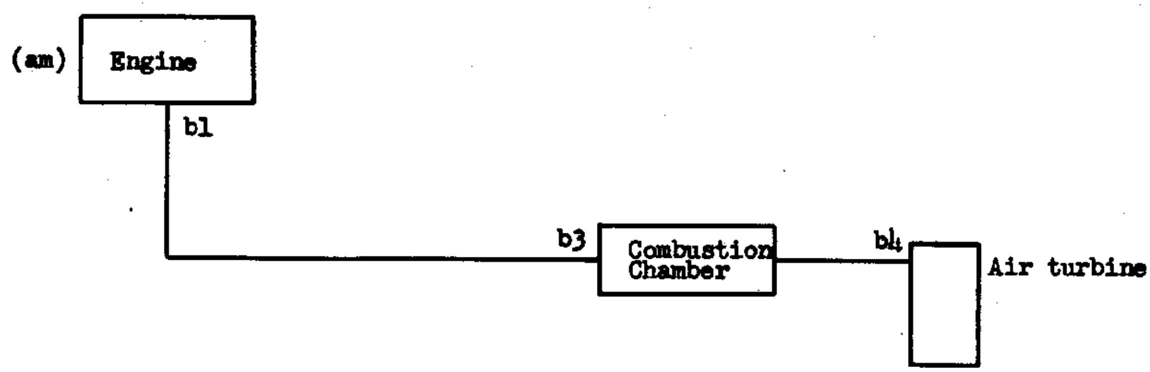


Fig. II-3 SCHEMATIC DIAGRAM OF A BLEED AND BURN SYSTEM

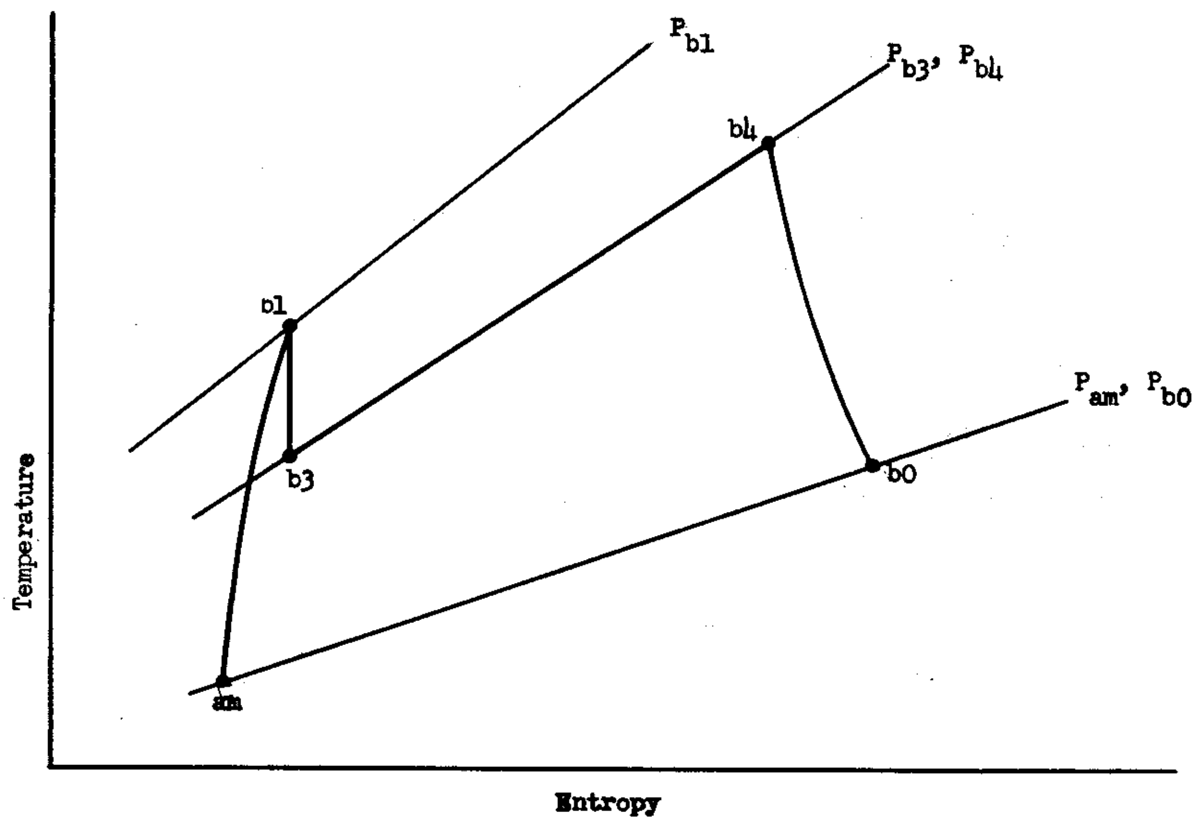


Fig. II-4 TEMPERATURE - ENTROPY DIAGRAM FOR BLEED AND BURN SYSTEM

4. Any airplane structural weight increase caused by the installation of the system in the airplane.

This report includes an analysis of the first two items listed above for the systems under consideration. Since the other two items are determined primarily by the airplane design concept, they are treated as constant for each particular type of system and airplane. An analysis of a specific airplane should include a study of these factors.

The optimum pneumatic system must be capable of transmitting the maximum required accessory power at all operating conditions of the aircraft. Since the pressure at the engine bleed port is determined by the operating conditions, the design quantity remaining variable is the duct diameter. The smallest duct diameter that can satisfy the power requirements ordinarily results in the lightest installed weight, while the fuel weight required to furnish the bleed air is relatively large. The major contribution to the fuel weight occurs during cruise conditions, while the critical operating conditions which determine the minimum duct diameter usually occur at other operating conditions.

In order to determine the optimum duct diameter it is usually more convenient to start with the minimum duct diameter, determine the corresponding weight of ducting and turbines, and calculate the required fuel weight. Since no smaller duct diameter can satisfy the system requirements it suffices to check whether a larger diameter results in a greater or smaller total system weight. Should the weight be smaller, when a larger duct is assumed, the process is repeated until the minimum total weight is determined. The maximum power condition is used to determine the duct diameter which in turn determines the capacity or size of the system components. The weight of the system is calculated from the size or capacity of the system components. The flight profile of the airplane and the load schedule of the accessories are used to determine the fuel required by the accessories.

The optimum pneumatic system is determined by a trial and error procedure. Beginning with the minimum permissible duct diameter capable of transmitting the maximum required accessory power, successively larger diameters are assumed and the system weight is calculated for each duct diameter. Increasing the duct diameter effectively decreases the losses in the system and, hence, increases the system efficiency. This saves fuel at the expense of increased installed weight. It may occur for certain design flight missions that the savings in fuel weight will more than offset the increase in installed weight due to the larger duct diameter. Hence, the optimum duct diameter may be larger than the minimum permissible diameter. A plot of total weight versus duct diameter will indicate the optimum diameter which will give the minimum total weight.

Under some circumstances it is possible to calculate an approximate optimum duct diameter directly for a straight bleed system, i.e., the duct diameter at which minimum total system weight occurs. The optimum duct analysis, (presented in part D-3 of this section), assumes that the airplane is operating at cruise conditions, that the weight of the required turbine is not affected by changes in duct diameter, that no overload is specified, and that the turbine is designed for the power required from the accessory system at cruise conditions. Therefore, when the overload horsepower requirements are only slightly larger than the cruise horsepower requirements, and when the turbine weight is a small percentage of the total system weight, the optimum duct diameter can be approximated by direct calculation.

The method for determining the minimum permissible duct diameter and the total system weight is based on several assumptions. It is assumed that the turbine is controlled by a variable area nozzle. Due to the uncertainty as to the exact duct length and number and location of fittings, the pressure drop in the duct can only be approximated. For this analysis, it is more desirable to calculate a pressure drop that is too large rather than too small. Therefore, a constant temperature, equal to the duct inlet temperature, is assumed for purposes of estimating the pressure losses in the duct. It is assumed that the heat lost from the bleed air due to heat transfer through the duct walls is equal to the heat gained due to friction and turbulence. The flow from the bleed point at the engine to the end of the duct therefore can be treated like an isentropic process. It is assumed that the combustion in the bleed and burn system takes place at constant pressure.

The procedure for evaluating a pneumatic power transmission system consists of three steps:

1. Establishment of design conditions required for evaluation. These include the mission profile of the airplane, the accessory load schedule (including overloads), approximate location of the accessories within the airframe, type of system permitted (straight bleed or bleed and burn) and pressure drop limitations.

2. Determination of minimum duct diameter.

3. Determination of minimum total system weight.

Throughout the minimum weight analysis several groups or arrangements of quantities keep reappearing. To facilitate computation, these quantities are assigned names and symbols, such as head loss parameter, β, dimensionless horsepower parameter, HP^*, and dimensionless pressure drop parameter, Y^*. These parameters are generally determined separately, in some cases by mathematical computation and in others from curves. They are then used in later computations.

B. <u>Nomenclature</u>

A — annular area of the rotor, $in.^2$

a — taper

C_{bl} — thrust correction factor

C'_{bl} — fuel flow correction factor

C''_{bl}	specific fuel consumption chargeable to the accessories, lb of fuel per hr/lb of bleed air per sec
C_u	nozzle coefficient
c	per cent increase in rotor blade height over nozzle blade height
c_p	specific heat at constant pressure, Btu/lb°F
D	duct diameter or diameter, ft
D^*	dimensionless duct parameter
d	turbine tip diameter, in.
F_n	thrust of engine, lb
f	friction factor, factor, or function
g	acceleration due to gravity, ft/sec^2, or function of X^* (see page 40)
h	blade height, in.
HP	horsepower
HP^*	dimensionless horsepower parameter
K	head loss coefficient or proportionality constant
ΣK	total head loss coefficient
k	ratio of specific heats
L	length of duct, ft
N	revolutions per minute, 1/min
P	absolute pressure, lb/ft^2
R	gas constant, ft-lb/lb°R
Re	Reynolds number
r	ratio or radius, in.
S	ratio of fitting weight to weight of duct alone, or pitch, in.
s	allowable stress, psi
T	taper factor or temperature, °R
t	thickness, in.

V	velocity, ft/sec
W	weight, lb
W_a	weight rate of air flow, lb/sec
W_{BL}	bleed air flow, lb/sec
W^*_{BL}	dimensionless bleed air flow parameter
W_f	fuel flow for engine, lb/hr
W_F	weight of fuel required to operate accessory system for time, τ, lb
w	width, in.
X^*	dimensionless pressure drop parameter
α	inverse of pressure ratio, P_{b0}/P_{bl}
β	head loss parameter
γ	weight density, lb/ft^3
δ	aspect ratio
η	efficiency
θ	admission angle
σ	blade solidity
μ	dynamic (absolute) viscosity, lb-sec/ft^2
τ	duration of power extraction, hr
Φ	horsepower parameter
ψ	specific thrust fuel consumption, lb/hr-lb
ω	duct weight coefficient, lb/ft^2

Subscripts

A	area
a	air
am	ambient condition

BL	bleed air
b	blade
b1	conditions at compressor outlet
b3	conditions at burner inlet
b4	conditions at turbine inlet
b0	conditions at turbine discharge
C	casing
c	cruise
cr	critical
D	duct
d	disk
E	engine
f	fuel
G	governor or gears
GB	gear box
i	insulation
j	jet
m	mean
N	nozzle
p	pitch line
T	turbine, or total
t	tangential, throat
V	velocity
W	wheel

C. Initial Data

The following is a list of the initial data required to perform the pneumatic system analysis. The data are divided into groups according to the probable source of information.

1. From Design Flight Mission

Establish typical flight conditions for the airplane. The altitude, speed, and thrust required from the engine should be obtained for each flight condition. The following is a list of typical flight conditions:

Cruise

Landing (minimum power at sea level)

Take-off

Military power

Descent at altitude (minimum power)

2. From Engine Specifications

Bleed pressure at each flight condition, psia

Bleed temperature at each flight condition, °R

Thrust correction factor due to extraction of bleed air, C_{bl}, (at cruise conditions)

Fuel flow correction factor due to extraction of bleed air, C'_{bl}, (at cruise conditions)

Engine thrust at cruise conditions, F_n, lb

Fuel flow at cruise conditions, W_f, lb/hr

Air flow at cruise conditions, W_a, lb/sec

Specific thrust fuel consumption, ψ, (see Fig. III-2)

3. From Accessory System Specifications

Horsepower requirements of the accessory system at various flight conditions, including overload

Duct length, L, ft

Duration of power extraction, τ, hours (depends also on design mission)

Approximate number of bends, turns, valves, etc.

Determine if there are maximum pressure drop requirements

Determine if a bleed and burn system is permissible

4. From Initial Assumptions

Several decisions must be made in regard to the type of component equipment and materials to be used, such as type of turbine and controls, kind of duct insulation, kind of fuel, etc. These decisions must be based on the stage of development, availability, ease of maintenance, etc. of the particular component or material. The following design data are then based on the best available information concerning these components and materials:

t_D duct wall thickness, ft

γ_D weight density duct material, lb/ft^3

t_i insulation thickness, ft

γ_i weight density of insulation, lb/ft^3

S ratio of fitting weight to weight of duct alone

HV heating value of the fuel, Btu/lb

T_{b4} maximum turbine inlet pressure, °R

η_T turbine efficiency, per cent

Both full and part load turbine efficiencies should be obtained for the type of turbine to be used. A reasonable estimate of these efficiencies can be taken from Ref. II-4 for turbines designed for this type of application.

To determine the weight of an axial flow turbine, the following information is necessary:

a taper, ratio of blade cross-sectional area at the tip to the cross-sectional area at the root

θ turbine admission angle, degree (see Fig. II-8)

s allowable stress, psi

δ blade aspect ratio, ratio of blade height to blade width

t average casing thickness, in.

w_C casing width, in.

γ_C weight density of casing material, lb/ft^3

f_V velocity factor, ratio of wheel pitch line velocity to tangential velocity of the gas

f_A area factor or gaging, ratio of nozzle throat area to annular area

c per cent increase in rotor blade height over nozzle blade height

D. Analysis Procedure for Straight Bleed System

A step by step procedure is presented for determining the minimum duct diameter and the minimum total system weight for a straight bleed system as shown in Fig. II-1.

The minimum duct diameter is calculated in one of two possible ways depending on the maximum pressure drop stipulation. If the accessory transmission system does not incorporate the cabin conditioning system, there is no maximum allowable pressure drop specified. It is only required that the maximum load be satisfied at the most adverse flight condition. The procedure for calculating the minimum duct diameter under these conditions is presented in step 1 below.

If the power transmission system does incorporate the cabin conditioning system, a maximum allowable pressure drop is specified. It is required that the maximum accessory load be satisfied and that the maximum pressure drop not be exceeded at the most adverse flight condition. The procedure for determining the minimum duct diameter under these conditions is outlined in step 2 below.

Under certain special conditions the optimum duct diameter, i.e.,the duct diameter which will give the minimum total system weight, can be computed directly. This makes it possible to avoid the trial and error procedure in determining the minimum total system weight. The procedure for calculating the optimum duct diameter is presented in step 3 below.

Once a duct diameter is established the approximate total system weight can be calculated. Where the airplane flight conditions and the transmission system specifications are such that the optimum duct diameter cannot be determined directly, the system weight is computed for the minimum duct diameter and several larger diameters. A plot of the total system weight versus duct diameter will indicate the optimum duct diameter and the minimum system weight. The procedure for calculating the system weight is presented in step 4.

1. Determine Minimum Duct Diameter at Critical Flight Condition - Case One

Case one considers maximum load at adverse flight conditions with no pressure drop stipulation from the cabin conditioning system.

a. Determine the Head Loss Parameter, β

The head loss parameter is found from (derivation of this equation is in Part 2 of this report)

$$\beta = \frac{16\ R\ T_{bl}\ \Sigma K}{g \pi^2\ P_{bl}^{\ 2}} \qquad \text{(II-1)}$$

where:

R = gas constant, ft-lb/lb °R

P_{bl} = bleed air pressure at compressor, lb/ft^2 abs.

ΣK = total head loss coefficient expressed in number of velocity heads

g = acceleration due to gravity, ft/sec^2

T_{bl} = bleed air temperature at compressor, °R

The total head loss coefficient, expressed in number of velocity heads, ΣK, must be estimated from the airplane configuration and empirical data, as follows:

From the dimensional outline of the airplane, the location of the engine and power conversion unit, estimate the approximate number of elbows, valves and tees, and the length of straight ducting. Determine the individual head loss coefficient of each fitting and length of duct. The total velocity head loss coefficient is equal to the sum of the individual head loss coefficients.

The head loss coefficient for each fitting or length of duct is defined as:

$$K = \frac{\Delta P}{\frac{\gamma}{g} V^2} \qquad \text{(II-2)}$$

where:

K = head loss coefficient or the number of velocity heads lost

ΔP = pressure drop, lb/ft^2

γ = weight density of the air, lb/ft^3

V = velocity of the air in the duct, ft/sec

For straight lengths of circular ducts,

$$K = f \frac{L}{D} \qquad \text{(II-3)}$$

where:

f = friction factor

L = length of straight duct, ft

D = duct diameter, ft

The head loss coefficient varies according to the type and design of each fitting and, to a lesser degree, with the Reynolds Number of the flow through the fittings and ducts. These coefficients have been determined experimentally and tabulated. A large number of such coefficients for various duct components correlated with Reynolds Number are given in Refs. II-1, II-2, and II-3. By assuming a value for Reynolds Number, which will be checked later, an estimate of the individual head loss coefficients may be obtained from these references. A reasonable initial assumption for Reynolds Number can be computed from the following equation by assuming a flow rate of bleed air and a duct diameter:

$$Re = \frac{4W_{BL}}{\pi \, g \, D \, \mu} \qquad \text{(II-4)}$$

where:

W_{BL} = flow of bleed air, lb/sec

D = duct diameter, ft

μ = dynamic (absolute) viscosity of the air, lb-sec/ft^2, (based on duct inlet temperature)

b. <u>Determine the Maximum Dimensionless Horsepower Parameter, HP^*</u>

The maximum dimensionless horsepower parameter can be taken from Fig. II-5. For an inverse pressure ratio, α, existing at the given flight and load conditions the maximum value of HP^* can be read on the ordinate. This also determines a value for the dimensionless pressure drop parameter, X^*, read on the abscissa. The inverse pressure ratio is defined as

$$\alpha = \frac{P_{b0}}{P_{b1}} \qquad \text{(II-5)}$$

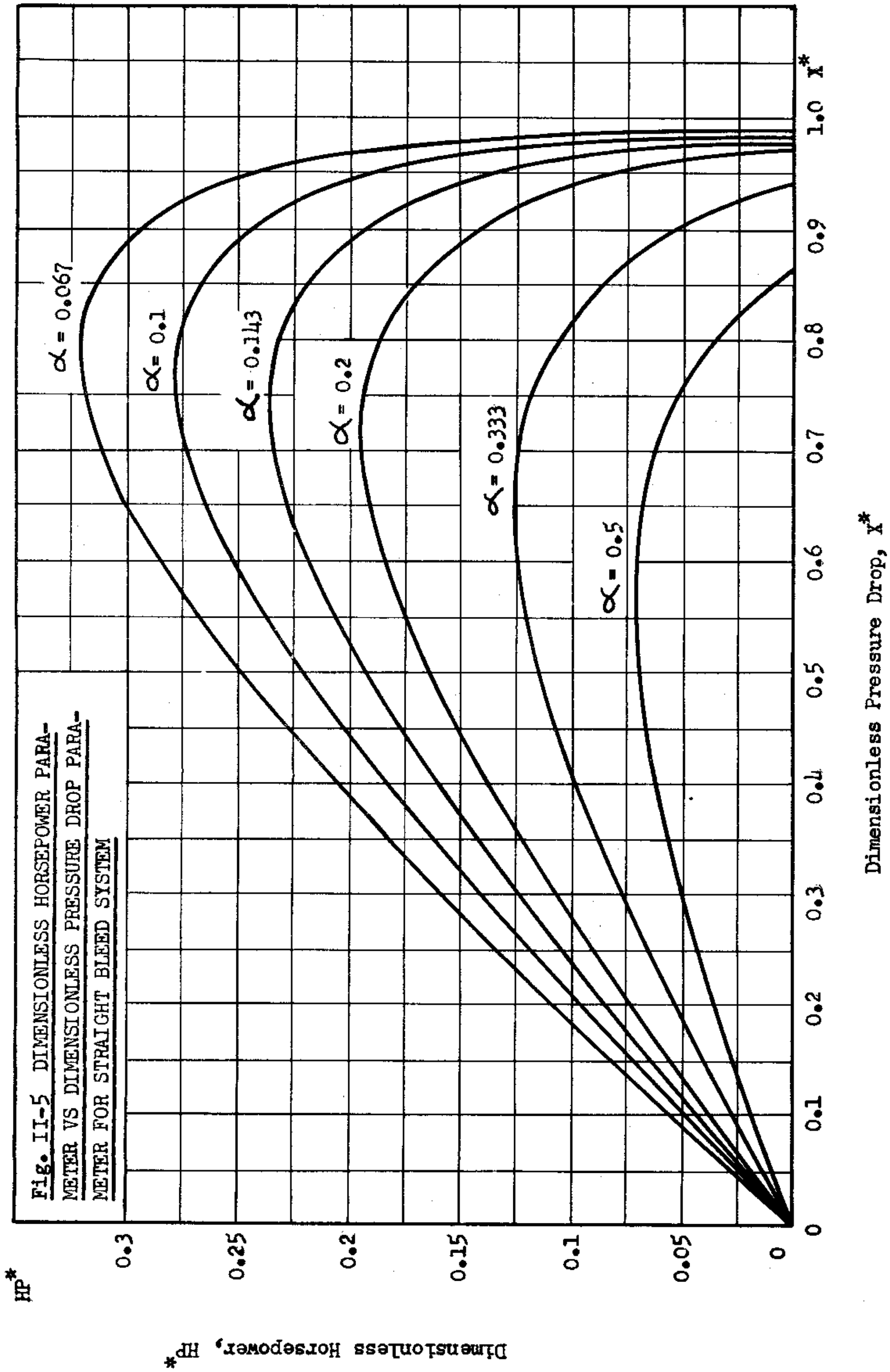


Fig. II-5 DIMENSIONLESS HORSEPOWER PARAMETER VS DIMENSIONLESS PRESSURE DROP PARAMETER FOR STRAIGHT BLEED SYSTEM

c. Calculate the Minimum Duct Diameter

Now the minimum duct diameter can be calculated from

$$D_{min} = \sqrt{\frac{2.95 \sqrt{\beta}\ HP_{max}}{\eta_T \quad T_{bl} \quad HP^*}} \tag{II-6}$$

d. Calculate the Bleed Air Flow, W_{BL}

The bleed air flow can be found from

$$W_{BL} = 0.61 \frac{P_{bl} \ D^2_{min} \ X^*}{\sqrt{T_{bl} \ \Sigma K}} \tag{II-7}$$

The dimensionless pressure drop parameter, X^*, was determined for these conditions in step 1-b.

e. Check the Value of ΣK Used in Step 1-a

Using the calculated values of W_{BL} and D_{min}, recalculate the Reynolds Number from Eq. (II-4). The total head loss coefficient, ΣK, must be estimated again and compared with the ΣK used to calculate β in step 1-a.

Repeat steps 1-a through 1-e until the two values of ΣK agree within 15 per cent. Since the flow in the duct is generally turbulent, ΣK varies only slightly with appreciable changes in Reynolds Number. Hence, satisfactory agreement should be obtained in one or two calculations.

2. Determine Minimum Duct Diameter at Critical Flight Conditions - Case Two

Case two considers maximum load at adverse flight conditions with a maximum allowable pressure drop specified by the cabin conditioning system.

a. Calculate the Dimensionless Pressure Drop Parameter, X^*

The dimensionless pressure drop parameter can be calculated from

$$X^* = \sqrt{1 - \left[\frac{P_{b4}}{P_{bl}}\right]^2} \tag{II-8}$$

where:

$\frac{P_{b4}}{P_{b1}}$ = ratio of absolute turbine inlet pressure to absolute compressor outlet pressure. The turbine inlet pressure, P_{b4}, is established by the specified pressure drop.

b. Determine the Dimensionless Horsepower, HP^*

Using the value of X^* calculated above and the value of α existing at the specific flight condition, HP^* can be taken from Fig. II-5.

c. Calculate the Head Loss Parameter, β

The value of β is found by the same method described in Section 1-a.

d. Calculate the Minimum Duct Diameter, D_{min}

The minimum duct diameter is calculated from Eq. (II-6).

e. Calculate the Bleed Air Flow, W_{BL}

The bleed air flow is calculated from Eq. (II-7). Use the value of X^* calculated in step 2-a.

f. Check the Value of ΣK Used to Calculate β

The method for checking ΣK is described in step 1-e.

3. Optimum Duct Diameter

The procedure for calculating the optimum duct diameter is based on the following assumptions:

The airplane is operated at cruise conditions throughout the flight

Accessory power turbine weight is not affected by changes in duct diameter

No specified overload requirements

The turbine is designed for the power required of the transmission system at cruise conditions

The method for computing the optimum duct diameter is presented below.

a. Determine Duct Weight Coefficient, ω

The duct weight coefficient is the total weight of the duct, including fittings and insulation per unit length per unit diameter.

$$\omega = \frac{W_D}{LD} \qquad \text{(II-9)}$$

where:

W_D = total duct weight including fittings and insulation, lb

L = duct length, ft

D = inside duct diameter, ft

If possible the weight coefficient should be based on information from previous installations. If no data are available, the weight coefficient can be calculated from the following:

$$\omega = \pi \left[(1 + S)\, t_D\, \gamma_D + t_i\, \gamma_i \right] \qquad \text{(II-10)}$$

t_D = duct wall thickness, ft

γ_D = weight density of duct material, lb/ft^3

t_i = insulation thickness, ft

γ_i = weight density of insulating material, lb/ft^3

S = ratio of fitting weight to the weight of the duct alone

b. Determine the Specific Fuel Consumption of the Accessories, C''_{bl}

The specific fuel consumption is found from

$$C''_{bl} = \frac{\psi C_{bl} \; F_n + C'_{bl} \; W_f}{W_a} \qquad \text{(II-11)}$$

where:

ψ = specific thrust fuel consumption, lb/lb-hr (see Fig. III-2 in hydraulic section)

C_{bl} = thrust correction factor

C'_{bl} = fuel flow correction factor

F_n = engine thrust, lb

W_f = fuel flow, lb/hr

W_a = air flow, lb/sec

All of the factors in Eq. (II-11) can be obtained from the engine specifications. For this particular duct calculation these factors should be taken at airplane cruise conditions.

c. Determine the Horsepower Parameter, Φ

The horsepower parameter is found from

$$\Phi = \frac{2.95\ HP_c}{\eta_T\ T_{bl}} \tag{II-12}$$

where:

HP_c = horsepower required by accessories at cruise conditions

T_{bl} = temperature of bleed air at compressor outlet, °R

η_T = turbine efficiency, per cent (assumed)

The turbine efficiency must be estimated from the best information available on the type of turbine being considered at the time of analysis. A good indication of the variation of turbine efficiency with load and inlet conditions is obtained from Ref. II-4.

d. Determine the Head Loss Parameter, β

The head loss parameter is found by the same method described in Step 1-a except the various temperatures, pressures, etc. are taken at cruise conditions.

e. Calculate the Dimensionless Time Parameter, τ^*

The time parameter is found from

$$\tau^* = \frac{C''_{bl}\sqrt{\Phi}}{\omega L\ \sqrt[4]{\beta}}\ \tau \tag{II-13}$$

where:

τ = total time that power is extracted from the engine, hr

All of the factors in this equation have been determined.

f. Determine the Optimum Value of the Pressure Drop Parameter, X^*

The optimum value of the pressure drop parameter is taken from Fig. II-6 for the calculated value of the dimensionless time parameter, τ^*, and the value of the inverse pressure ratio, α, existing in the engine at cruise conditions.

g. Calculate the Optimum Duct Diameter

Determine the dimensionless horsepower parameter, HP^*, from Fig. II-5 at the optimum X^* and the existing α. Calculate the optimum duct diameter from

$$D_{opt} = \sqrt{\frac{2.95 \sqrt{\beta}\ HP_c}{\eta_T\ T_{bl}\ HP^*}} \tag{II-14}$$

h. Check the Value of ΣK Used and Calculate β

Determine the bleed air flow, W_{BL} from

$$W_{BL} = \frac{D^2_{opt}}{\sqrt{\beta}}\ X^* \tag{II-15}$$

Using the values of D_{opt} and W_{BL} check the value of ΣK as described in Section 1-e.

4. Determine the Minimum Total Weight of the System

The total system weight is equal to the sum of the component weights. The weight of each component is determined as a function of duct diameter. The total system weight is calculated first for the minimum permissible duct diameter and then for a few assumed duct diameters at nominal increments. A plot of total system weight versus duct diameter will indicate the minimum total system weight. The total system weight, ΣW, is calculated from

$$\Sigma W = W_T + W_D + W_F + W_k \tag{II-16}$$

where:

W_T = turbine weight, lb

W_D = total duct weight, including fittings and insulation, lb

W_F = fuel weight, lb

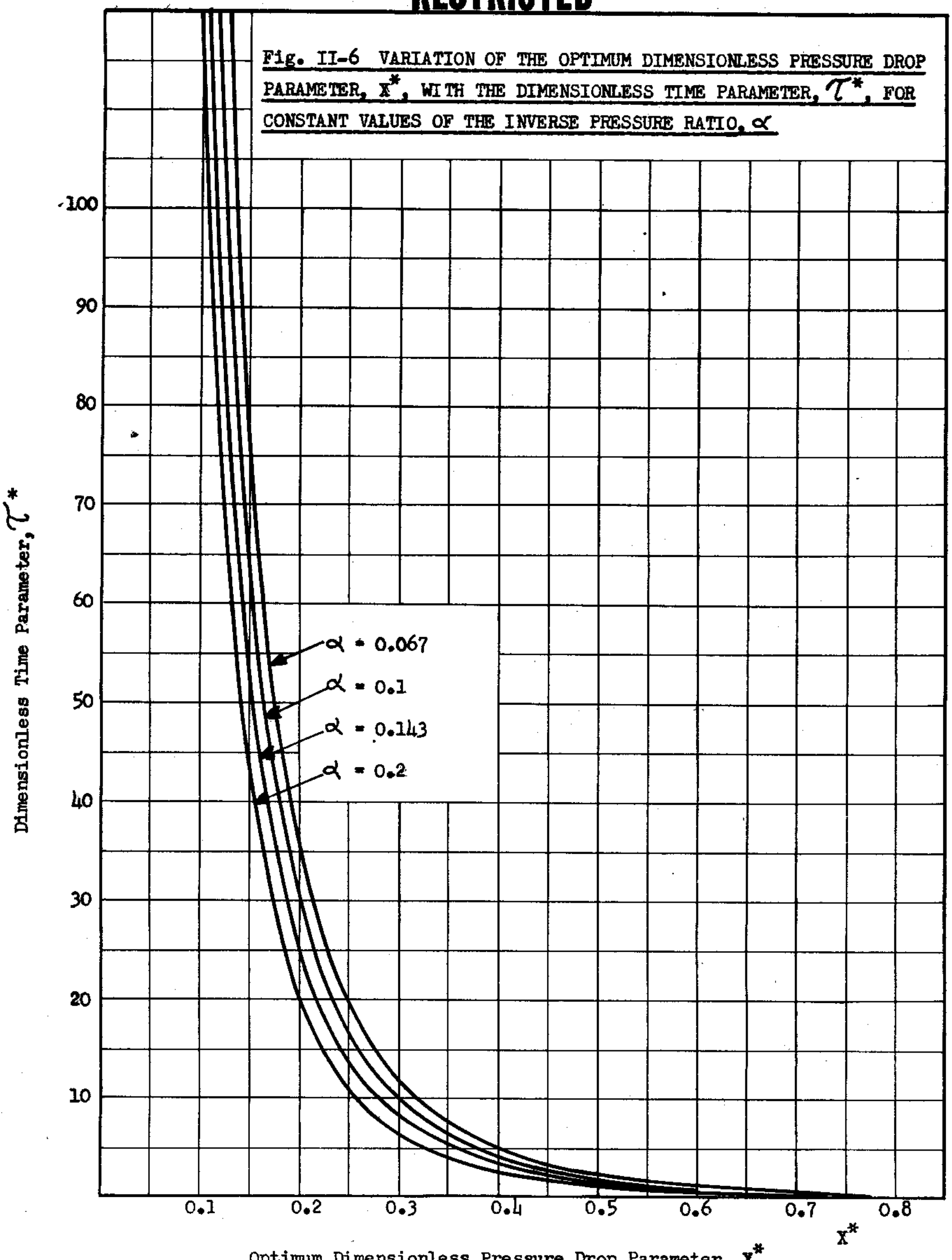


Fig. II-6 VARIATION OF THE OPTIMUM DIMENSIONLESS PRESSURE DROP PARAMETER, X*, WITH THE DIMENSIONLESS TIME PARAMETER, τ*, FOR CONSTANT VALUES OF THE INVERSE PRESSURE RATIO, α
α = 0.067
α = 0.1
α = 0.143
α = 0.2
100
90
80
70
60
50
40
30
20
10
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
X*
Dimensionless Time Parameter, τ*
Optimum Dimensionless Pressure Drop Parameter, X*

W_k = increment of total aircraft weight due to additional structural requirements and to the fuel required to overcome any increased aerodynamic drag chargeable to the transmission system, lb

The method of determining the weight of each component shown in Eq. (II-16) is outlined below.

a. Determine the Turbine Weight, W_T

Part F of this section presents a method for calculating the turbine weight.

b. Determine the Duct Weight, W_D

The duct weight is calculated from

$$W_D = \omega DL \qquad \text{(II-17)}$$

where:

ω = duct weight coefficient (determined in step 3-a)

c. Determine the Fuel Weight, W_F

If the accessory load schedule and the flight profile are such that the specific fuel consumption and the bleed air flow can be assumed constant throughout the mission of the airplane, the total fuel weight can be calculated from the following:

$$W_F = C''_{bl} \, W_{BL} \, \tau \qquad \text{(II-18)}$$

where:

C''_{bl} = specific fuel consumption chargeable to accessories, lb of fuel per hr/lb of bleed air per sec

W_{BL} = bleed air flow at cruise conditions, lb/sec

τ = duration of power extraction, hr (from initial data)

If the accessory load schedule and the flight profile are such that C''_{bl} and W_{BL} cannot be assumed constant, then the flight is divided into short time intervals over which this assumption is valid. The sum of the fuel weights for these intervals is the total weight of fuel required for the mission.

The bleed air flow, W_{BL}, is determined as follows:

Determine the dimensionless horsepower, HP^*, from

$$HP^* = \frac{4.84\sqrt{\Sigma K'}\,HP}{\eta_T P_{bl}\sqrt{T_{bl}}\,D^2} \tag{II-19}$$

Determine the value of X^* corresponding to HP^* and the inverse pressure ratio, α, from Fig. II-5.

Calculate W_{BL} from

$$W_{BL} = \frac{D^2}{\sqrt{\beta}}\,X^* \tag{II-20}$$

The head loss parameter, β, is determined from Eq. (II-1).

d. Determine the Incremental Airplane Weight, W_k

The value of W_k is primarily determined from the design concept of the particular airplane and type of accessory system under consideration. Therefore, W_k is treated as a constant for each accessory system and airplane.

The total system weight is computed from Eq. (II-16) for the minimum duct diameter, and several larger duct diameters. A plot of total weight versus duct diameter will indicate the minimum total weight of the straight bleed system. This figure may be used for comparing the straight bleed system with other types of accessory transmission systems.

E. Analysis Procedure For a Bleed and Burn System

A step by step procedure is presented for determining the minimum duct diameter and the minimum total system weight for a bleed and burn system as shown in Fig. II-3.

The procedure is similar to that for a straight bleed system. The minimum duct diameter is determined for two cases. Case one considers maximum accessory load at adverse flight conditions with no pressure drop stipulation. Case two considers maximum accessory load at adverse flight conditions with a specified maximum pressure drop. The procedure for computing the total system weight is presented in step 3 below. Here again it is necessary to calculate the total weight for several duct diameters to determine the minimum total weight.

1. Determine Minimum Duct Diameter at Critical Flight Conditions - Case One

Case one considers maximum load at adverse flight conditions with no pressure drop stipulation.

a. Obtain the Maximum Value of the Dimensionless Horsepower, HP^*

Determine the maximum value of the dimensionless horsepower, HP^*, from Fig. II-7 at the prevailing inverse pressure ratio, α, for the critical flight condition. This also determines a corresponding value of X^*.

b. Determine the Value of the Head Loss Parameter, β

The head loss parameter, β, is determined as shown in Section D, step 1-a.

c. Calculate the Minimum Duct Diameter

The minimum duct diameter is calculated from

$$D_{min} = \sqrt{\frac{2.95\sqrt{\beta}\ HP_{max}}{\eta_T\ T_{b4}\ HP^*}} \qquad \text{(II-21)}$$

where:

HP_{max} = maximum horsepower required of accessories

T_{b4} = turbine inlet temperature, °R (This is the temperature after combustion.)

β = head loss parameter determined from Eq. (II-1)

d. Check the Value of ΣK Used to Calculate the Head Loss Parameter, β

Using the value of X^* corresponding to HP_{max} in Fig. II-7, calculate the bleed air flow, W_{BL}, from

$$W_{BL} = \frac{D^2_{min}}{\sqrt{\beta}} X^* \qquad \text{(II-22)}$$

With the calculated values of D_{min} and W_{BL}, compute the Reynolds Number from Eq. (II-4) and estimate the value of ΣK as outlined in step 1-a of Section D. Steps 1-a through 1-d of this section should be repeated until the calculated value of ΣK agrees with the original estimate within 15 per cent.

2. Determine Minimum Duct Diameter at Critical Flight Conditions - Case Two

Case two considers maximum load at adverse flight conditions with a specified maximum pressure drop.

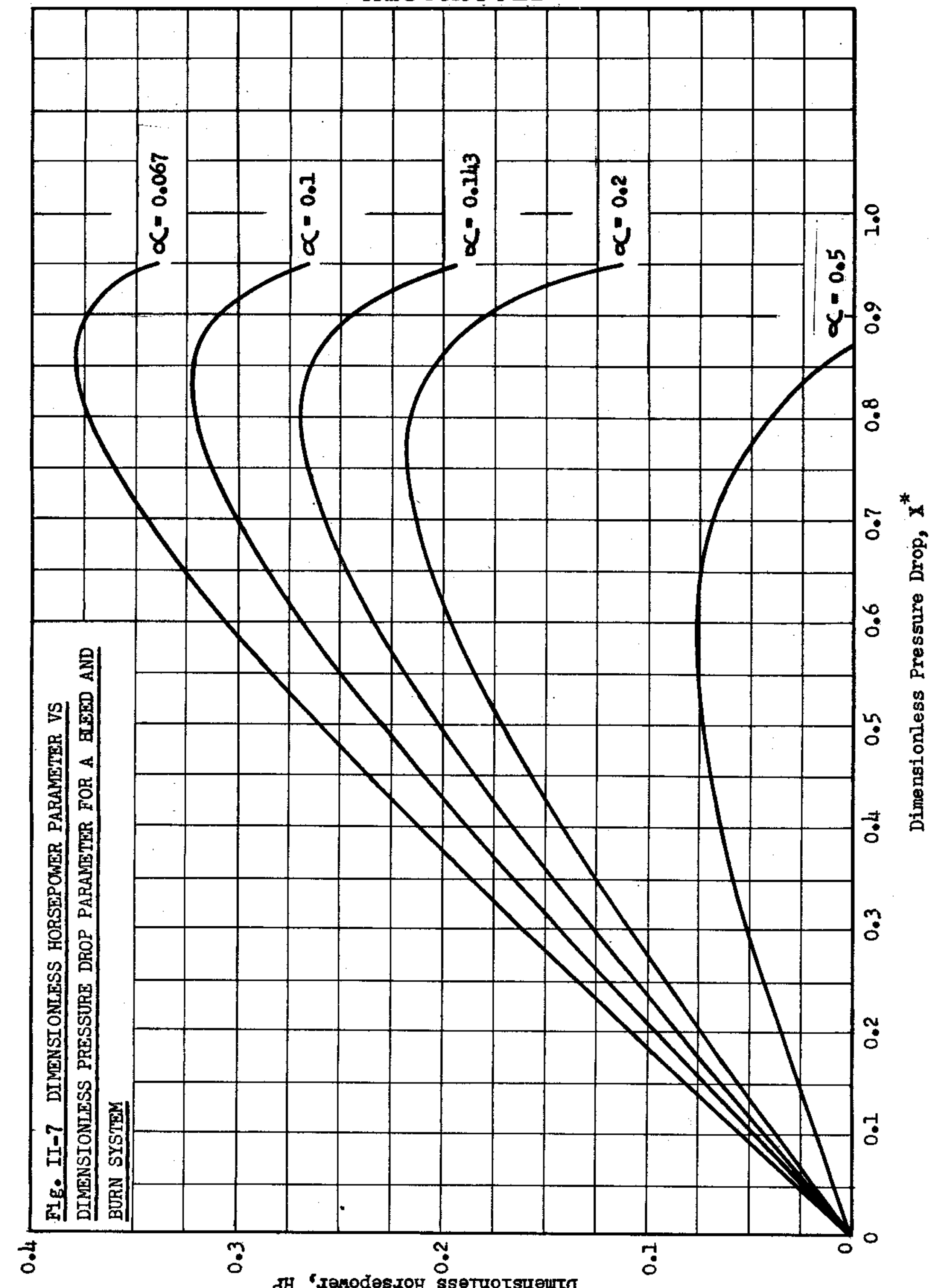


Fig. II-7 DIMENSIONLESS HORSEPOWER PARAMETER VS DIMENSIONLESS PRESSURE DROP PARAMETER FOR A BLEED AND BURN SYSTEM

a. Calculate the pressure drop parameter, X^*, from Eq. (II-8)

b. Determine the dimensionless horsepower from Fig. II-7 for the value of X^* calculated above and the inverse pressure ratio, existing at given flight condition.

c. Calculate the minimum duct diameter from Eq. (II-21)

d. Check the value of ΣK as shown in step 1-d of this section.

3. Determine the Minimum Total System Weight

The total system weight is calculated from Eq. (II-16). The method of calculating the turbine weight, W_T, is presented in Part F of this section. The duct weight, W_D, is calculated from Eq. (II-17). The weight of the fuel, W_F, is determined from

$$W_F = W_{BL} \tau \left[C''_{b1} + \frac{3600 \, (T_{b4} - T_{b3}) \, c_p}{HV \, \eta_{comb}} \right] \qquad \text{(II-23)}$$

where:

W_{BL} = weight flow of bleed air, lb/sec

τ = duration of power extraction, hr

C''_{b1} = specific fuel consumption chargeable to accessories as determined from Eq. (II-11), lb of fuel per hour/lb of air per sec

T_{b4} = turbine inlet temperature, °R

T_{b3} = air temperature at the burner inlet, °R

c_p = specific heat of air at constant pressure, Btu/lb °F

HV = heating value of fuel, Btu/lb of fuel

η_{comb} = combustion efficiency, per cent

Since the pressure drop through the duct is assumed to be isentropic, the temperature at the end of the duct, T_{b3}, is found from

$$T_{b3} = T_{b1} \left[\frac{P_{b3}}{P_{b1}} \right]^{\frac{k-1}{k}} \qquad \text{(II-24)}$$

The bleed air flow, W_{BL}, is determined from cruise conditions as follows:

Calculate HP from

$$HP^* = \frac{2.95\sqrt{\beta}\ HP}{D^2\ \eta_T\ T_{b4}} \qquad \text{(II-25)}$$

Determine X^* corresponding to HP^* and the inverse pressure ratio, α, at cruise conditions from Fig. II-7.

Calculate the bleed air flow, W_{BL}, from Eq. (II-20)

$$W_{BL} = \frac{D^2}{\sqrt{\beta}}\ X^*$$

The total system weight is calculated for the minimum duct diameter and several larger diameters. The minimum total weight is obtained from the curve of total weight versus diameter. This minimum weight may then be used to compare the bleed and burn system with other types of accessory systems.

F. Weight of the Air Turbine

In calculating the total weight of a pneumatic system from Eq. (II-16), it is necessary to know the air turbine weight. This section presents a method for estimating the turbine weight.

The total weight of an air turbine can be expressed as the sum of the weights of the turbine wheel, the turbine casing, the governing device and the gear box. Thus:

$$W_T = W_W + W_C + W_G + W_{GB} \qquad \text{(II-26)}$$

where:

W_T = total weight of the turbine, lb

W_W = weight of the turbine wheel, lb

W_C = weight of the turbine casing, lb

W_G = weight of the governor, lb

W_{GB} = weight of the gear box, lb

The weight of the gear box is determined from torque and speed reduction requirements. The procedure for calculating this weight is presented in Section VI.

The weight of the governing system is largely dependent on the type of governing used; that is, whether a variable nozzle area control or a throttle control is used and whether a mechanical or electronic governing system is used. Once the method and type of governing are chosen, the weight of the governing system becomes a function of the power output of the turbine. The weight of the governing system must be obtained from the manufacturer's data. This weight may vary from 5 to 10 pounds for a simple throttle control and may be many times this figure for a partial admission turbine. For the bleed and burn system the burner weight is included with the governor weight.

The weight of the turbine wheel and casing depend on the type of turbine used. Two basic types of air turbine are considered here, the axial flow type and the radial flow type. Schematic diagrams of an axial flow and a radial flow turbine are shown in Figs. II-8 and II-9, respectively.

To determine accurately the weight of these turbine wheels by analytical methods would be highly impractical. In an aircraft accessory power turbine installation, the turbine wheel weight represents only a small percentage of the total turbine weight, and the weight of the turbine casing and governor can only be estimated. Therefore, it is sufficient to establish approximate methods for estimating the wheel weight.

The weight of these turbines is based on the geometric dimensions of the wheel. The geometric dimensions of the axial flow turbine are determined by the flow requirements and the stress limitations of the disk and blades. For a given set of conditions the flow is indicated by the maximum required nozzle throat area. The stresses are indicated by the pitch line velocity of the wheel, which in turn is determined from the jet velocity of the gas leaving the nozzle, (see Fig. II-8). Thus, the geometry of the axial turbine is determined once the maximum throat area and jet velocity are known. The weight can then be estimated from the wheel dimensions.

The geometric dimensions of the radial flow turbine depend almost entirely on the flow requirements. Here again, the flow is indicated by the maximum nozzle throat area. Therefore, the weight of a radial flow turbine can be determined once the maximum nozzle throat area is known.

The method for determining the maximum nozzle throat area and the jet velocity is different for the straight bleed and the bleed and burn systems. Steps 1 and 2 below describe the procedure for calculating the maximum nozzle throat area and the jet velocity for a straight bleed system, while steps 3 and 4 describe the procedure for calculating these quantities for a bleed and burn system. This supplies the necessary data for calculating the weight of either an axial flow or a radial flow turbine. Steps 5, 6 and 7 describe the procedure for calculating the approximate weight of an axial flow turbine. Step 8 describes the procedure for calculating the approximate weight of a radial flow turbine.

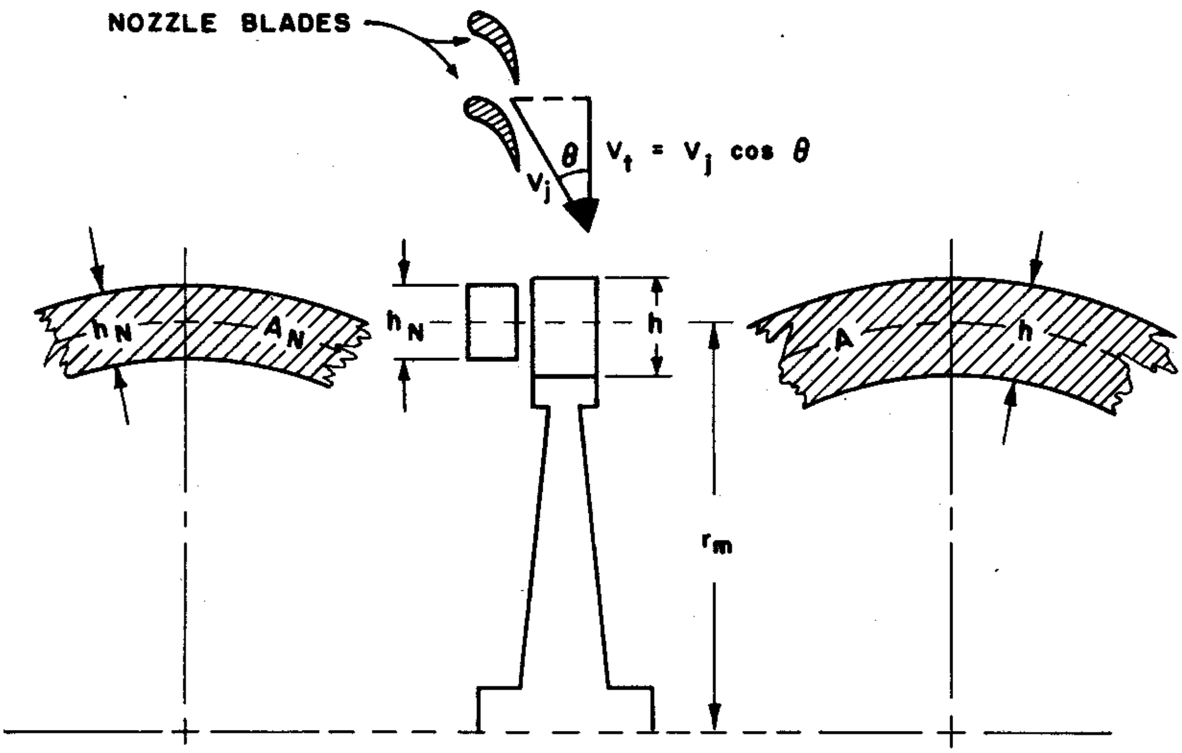


Fig. II-8 SCHEMATIC DIAGRAM OF TURBINE WHEEL AND VELOCITY DIAGRAM AT NOZZLE EXIT

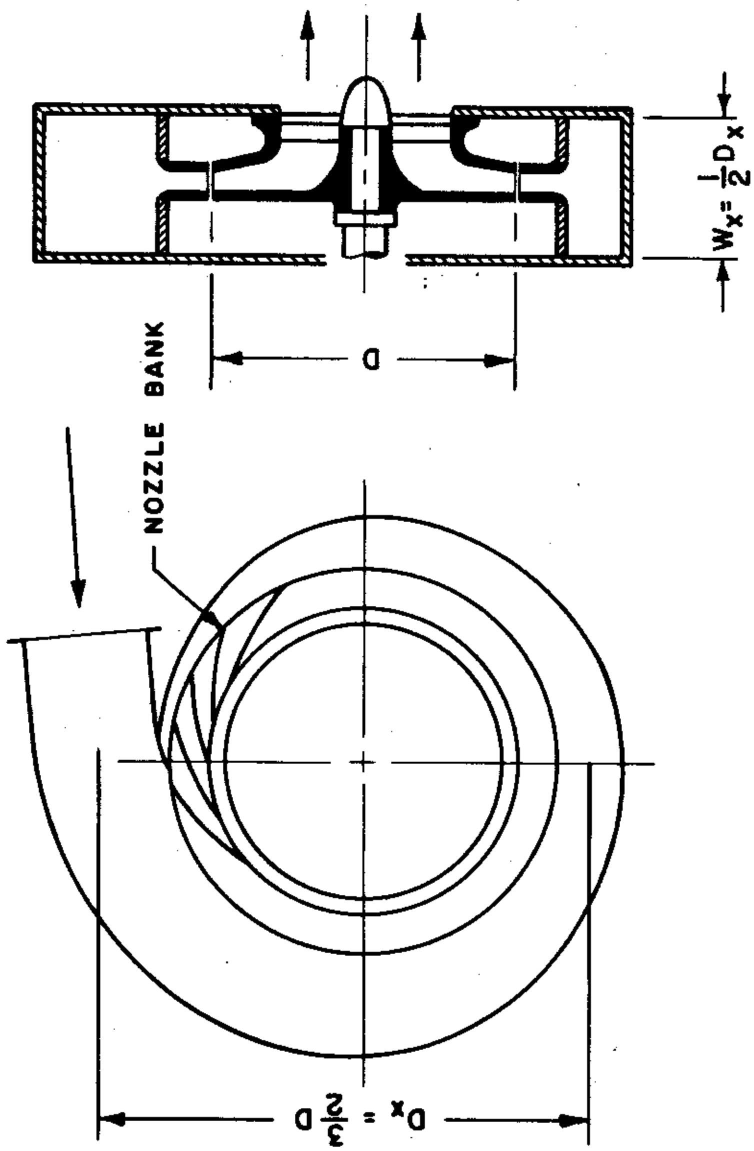


Fig. II-9 SCHEMATIC DIAGRAM OF RADIAL TURBINE

1. Maximum Required Nozzle Throat Area, A_t, - Straight Bleed System

The following procedure is used to determine the required throat area A_t, for a straight bleed system.

a. Determine the Dimensionless Horsepower Parameter, HP^*

For the most adverse flight conditions and maximum required horsepower, calculate HP^* from

$$HP^* = \frac{2.95\sqrt{\beta}\ HP_{max}}{\eta_T\ T_{bl}\ D^2} \qquad \text{(II-27)}$$

The head loss parameter, β, is determined from Eq. (II-1) for the critical flight condition.

b. Determine the Dimensionless Pressure Drop Parameter, X^*

Using the value of HP^* calculated above and the value of α prevailing at the most adverse flight condition, X^* can be taken from Fig. II-5.

c. Determine the Value of the Dimensionless Pressure Drop Function For the Straight Bleed System, $g(X^*)$

Knowing X^*, the value of $g(X^*)$ can be taken from Fig. II-10.

d. Calculate the Maximum Required Throat Area, A_t

The maximum required throat area is found from

$$A_t = 1.15 \frac{D^2}{\sqrt{\Sigma K}} g(X^*) \qquad \text{(II-28)}$$

2. Jet Velocity - Straight Bleed System

For the straight bleed system, the jet velocity of the gas issuing from the nozzle can be closely approximated from

$$V_j = \sqrt{\frac{k}{k+1} 2g\ RT_{bl}} \qquad \text{(II-29)}$$

where:

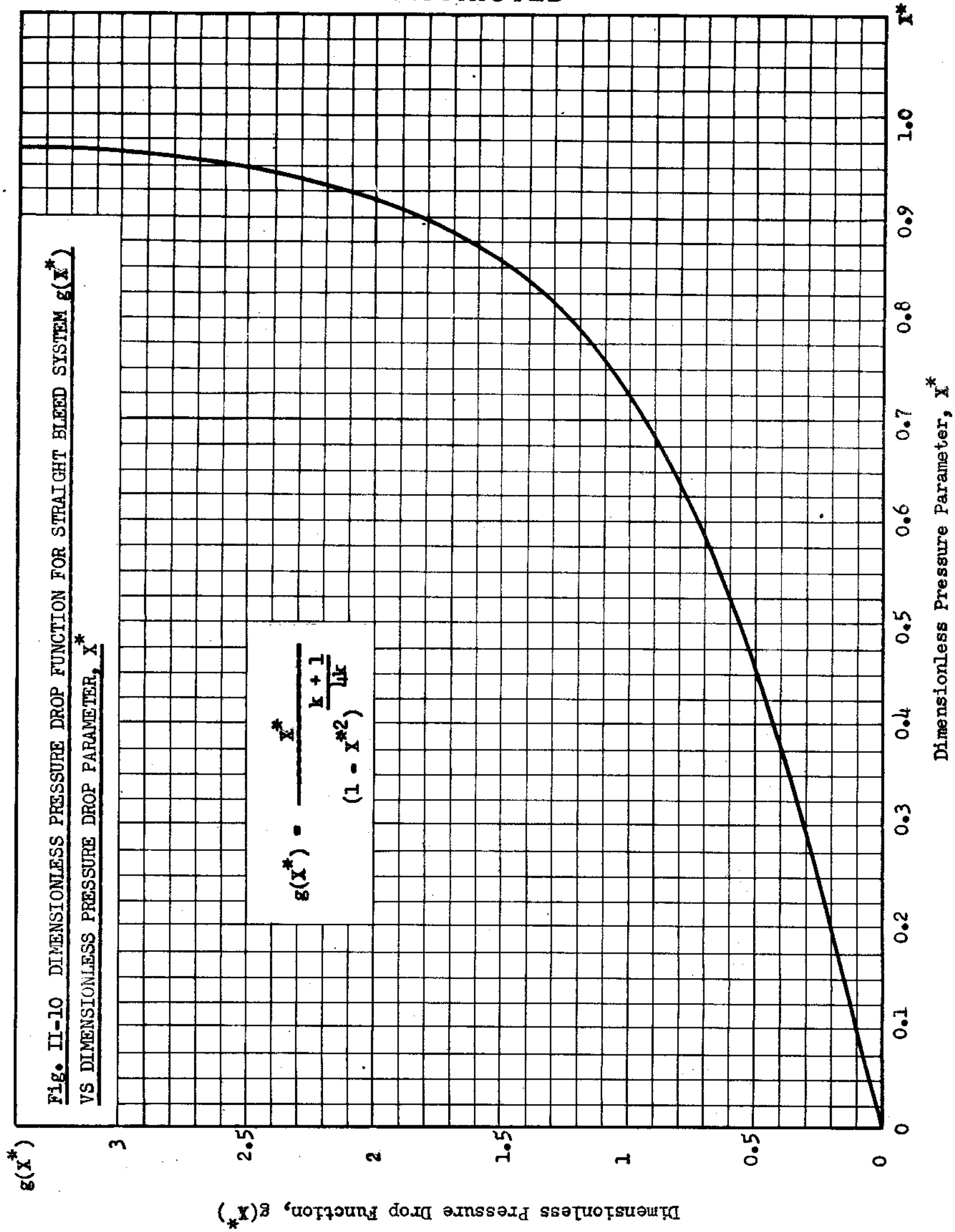


Fig. II-10 DIMENSIONLESS PRESSURE DROP FUNCTION FOR STRAIGHT BLEED SYSTEM $g(X^*)$ VS DIMENSIONLESS PRESSURE DROP PARAMETER, X^*

k = ratio of specific heats

g = acceleration due to gravity, ft/sec^2

R = gas constant, ft-lb/lb °R

T_{b1} = discharge temperature of compressor, °R

3. Maximum Required Nozzle Throat Area - Bleed and Burn System

The following procedure is used to determine the required throat area, A_t, for a bleed and burn system.

a. Determine the Dimensionless Horsepower Parameter, HP^*

For the most adverse flight conditions and the maximum required horsepower, calculate HP^* from

$$HP^* = \frac{2.95\sqrt{\beta}}{D^2 \ \eta_T \ T_{b4}} HP_{max} \qquad \text{(II-30)}$$

The head loss parameter, β, is determined from Eq. (II-1) for the critical flight condition.

b. Determine the Dimensionless Pressure Drop Parameter, X^*

Using the value of HP^* calculated above and the value of α prevailing at the most adverse flight condition, X^* can be taken from Fig. II-7.

c. Determine the Value of the Dimensionless Pressure Drop Function For the Bleed and Burn System, $h(X^*)$

Knowing X^*, the value of $h(X^*)$ can be taken from Fig. II-11.

d. Calculate Maximum Required Throat Area, A_t, for Bleed and Burn System

Maximum required throat area is found from

$$A_t = 1.15 \frac{D^2}{\sqrt{\Sigma K}} \sqrt{\frac{T_{b4}}{T_{b1}}} \ h(X^*) \qquad \text{(II-31)}$$

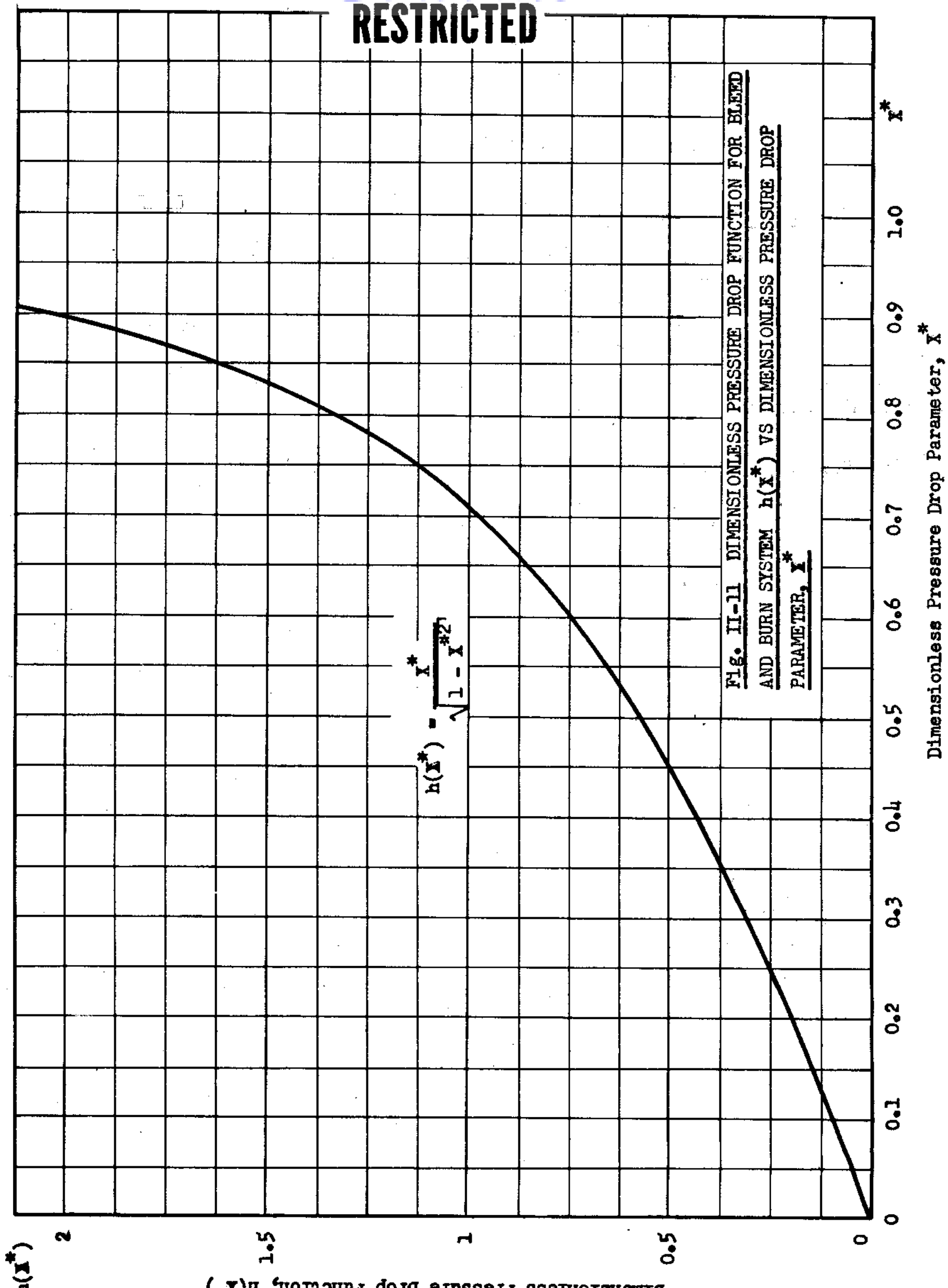


Fig. II-11 DIMENSIONLESS PRESSURE DROP FUNCTION FOR BLEED AND BURN SYSTEM $h(X^*)$ VS DIMENSIONLESS PRESSURE DROP PARAMETER, X^*

where:

T_{b4} = turbine inlet temperature, °R

4. Jet Velocity - Bleed and Burn System

For the bleed and burn system the velocity of the gas issuing from the nozzle is given by

$$V_j = C_u \sqrt{\frac{k}{k+1} \; 2gRT_{b4}} \qquad \text{(II-32)}$$

Since the pressure ratio is normally greater than critical, C_u should be taken equal to unity.

5. Weight of the Turbine Wheel for an Axial Flow Turbine

Having determined the jet velocity and the maximum required nozzle throat area, the following procedure may be used to determine the turbine wheel weight:

a. Determine the Annular Area of the Rotor, A

The annular area of the rotor (see Fig. II-8) is found from

$$A = \frac{1 + c}{f_A} A_t \qquad \text{(II-33)}$$

where:

c = per cent increase in rotor blade height over the nozzle blade height

f_A = area factor or gaging

The area factor is defined as follows:

$$f_A = \frac{A_t}{A_N}$$

where:

A_N = annular area of the nozzle ring, in.^{2} (see Fig. II-8).

b. Determine Pitch Line Velocity, V_p

The pitch line velocity is found from

$$V_p = f_V \, V_j \cos \theta \qquad \text{(II-34)}$$

where:

f_V = velocity factor, ratio of pitch line velocity to tangential velocity (V_p / V_t)

θ = admission angle, deg.

c. Determine the Ratio of Blade Height to Mean Radius, h/r_m

The blade-radius ratio, h/r_m, is calculated from (Ref. II-6):

$$\frac{h}{r_m} = 4630 \frac{s}{T} \frac{1}{\gamma_b V_p^2} \qquad \text{(II-35)}$$

where:

s = allowable stress, lb/in.^{2}

T = taper factor

γ_b = weight density of blade material, lb/ft^3

The taper factor, T, is introduced to correct the stress at the blade root for the decrease in stress due to tapered blading. The value of this factor is shown in Fig. II-12 for several values of taper, a. The taper is defined as the ratio of blade cross-sectional area at the tip to the cross-sectional area at the root.

The ratio, h/r_m, may be taken directly from Fig. II-13, which shows Eq. (II-35) drawn for several values of s/T and for an assumed material weight density of 540 lbs/ft^3.

d. Determine the Blade Height, h

The mean radius, r_m, can be calculated from

$$r_m = \sqrt{\frac{A}{2\pi} \frac{1}{h/r_m}} \qquad \text{(II-36)}$$

The mean radius may be taken from Fig. II-14 where h/r_m is plotted against r_m for several values of annular area. Knowing the mean radius and the ratio, h/r_m, the blade height, h, can now be calculated from

$$h = \left(\frac{h}{r_m}\right) r_m \qquad \text{(II-37)}$$

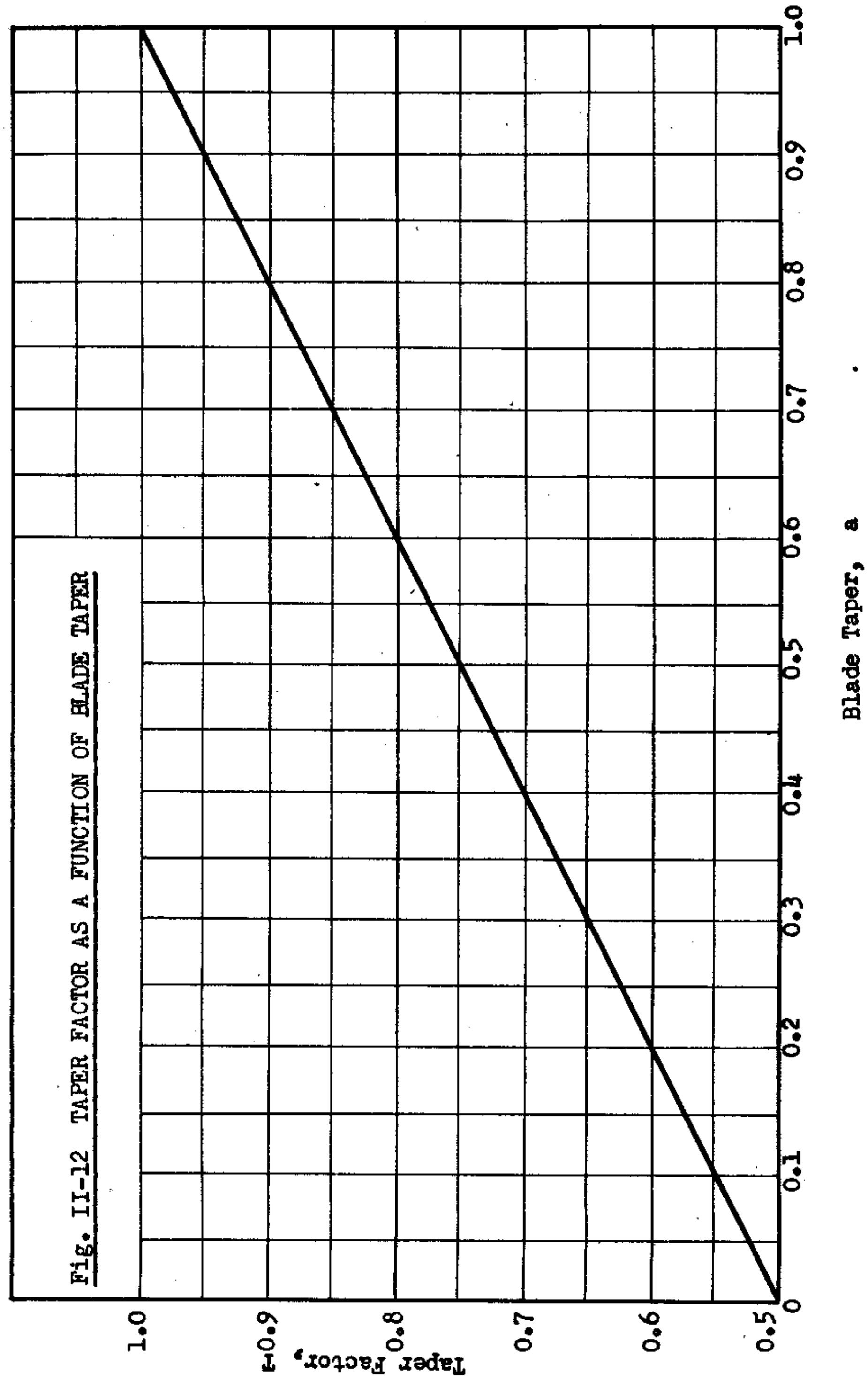


Fig. II-12 TAPER FACTOR AS A FUNCTION OF BLADE TAPER

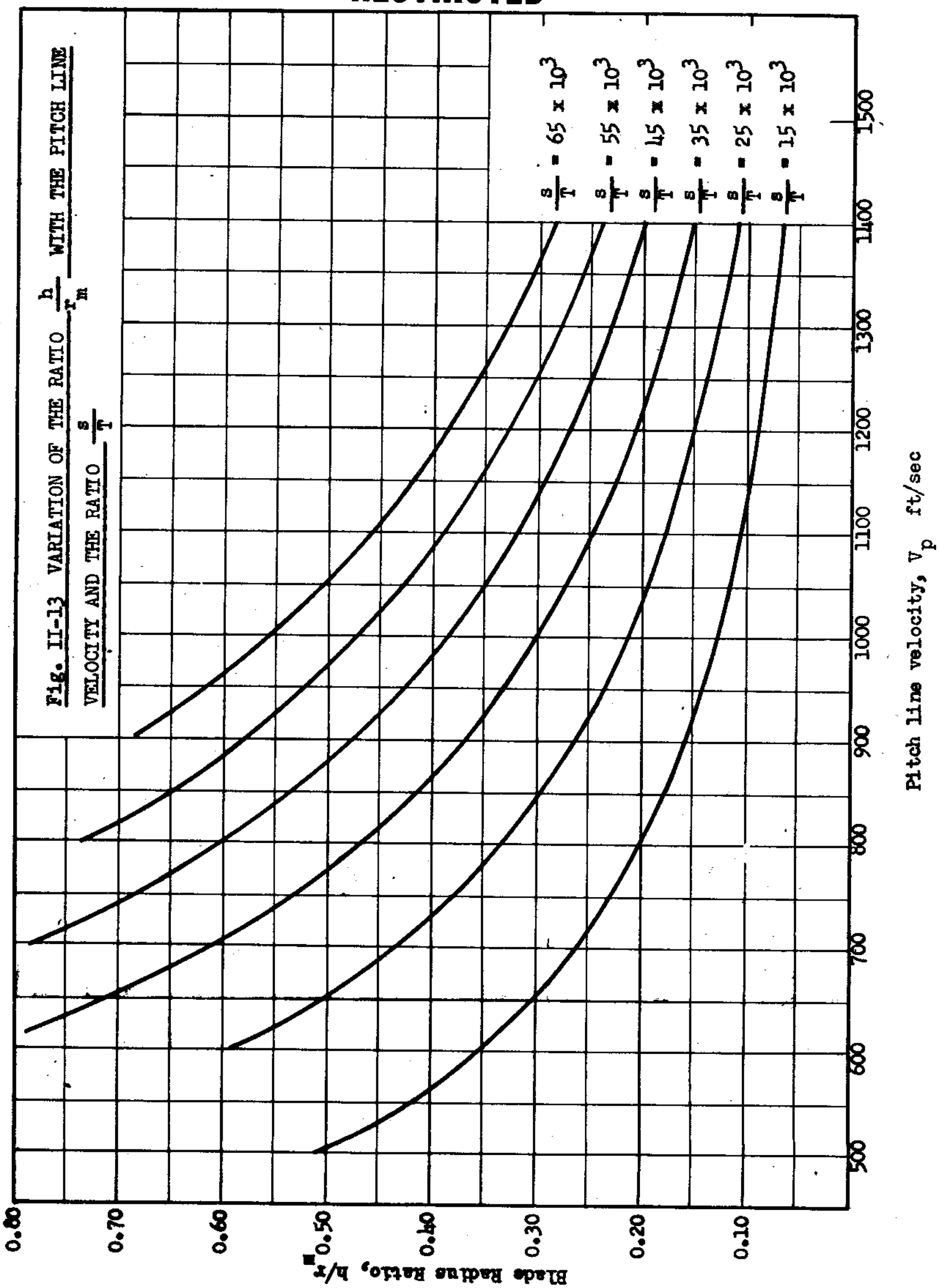


Fig. II-13 VARIATION OF THE RATIO $\frac{h}{r_m}$ WITH THE PITCH LINE VELOCITY AND THE RATIO $\frac{s}{T}$

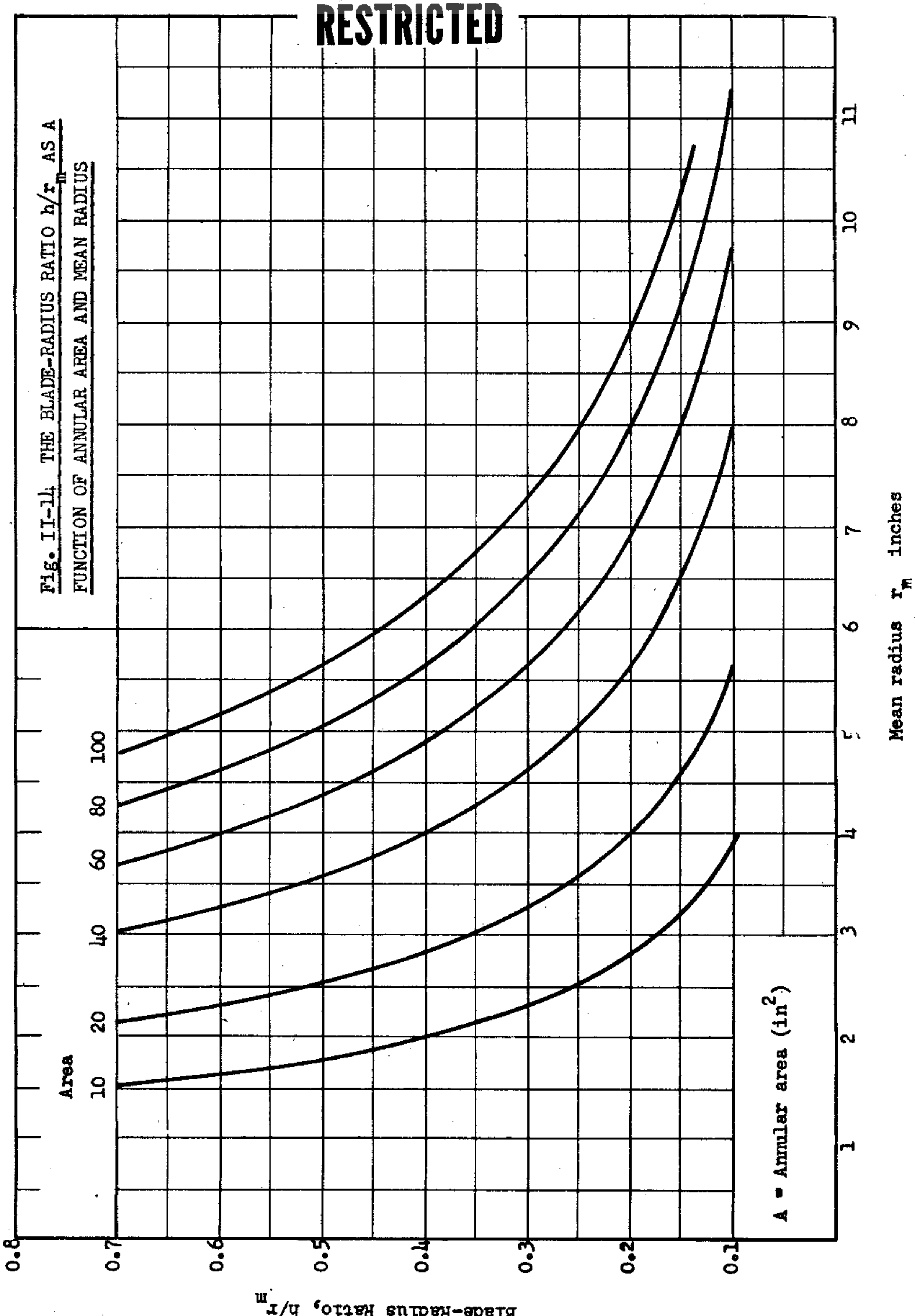


Fig. II-14 THE BLADE-RADIUS RATIO h/r_m AS A FUNCTION OF ANNULAR AREA AND MEAN RADIUS

e. Determine the Wheel Speed, N

The rpm of the turbine wheel is calculated from the known values of the mean radius and the pitch line velocity.

$$N = \frac{10^3}{8.74} \frac{V_p}{r_m} \qquad \text{(II-38)}$$

The rpm can be taken from Fig. II-15, where the mean radius is plotted against pitch line velocity for several wheel speeds.

f. Calculate the Turbine Wheel Weight, W_W

The effect of several design variables on turbine wheel weights has been investigated by the NACA (Ref. II-5). The resulting relationship, as shown by this reference, can be approximated by:

$$\frac{W_W \, \delta}{A} = F\left(\frac{h}{r_m}\right) h \qquad \text{(II-39)}$$

or the wheel weight can be calculated from

$$W_W = \frac{A}{\delta} F\left(\frac{h}{r_m}\right) h$$

where:

W_W = weight of the turbine wheel

δ = aspect ratio, blade height/blade width

A = annular area behind rotor, $in.^2$

h = blade height, in.

$F\left(\frac{h}{r_m}\right)$ = a function of the blade height to mean radius ratie

The values of $F(h/r_m)$ are shown in Fig. II-16 for the following representative design conditions:

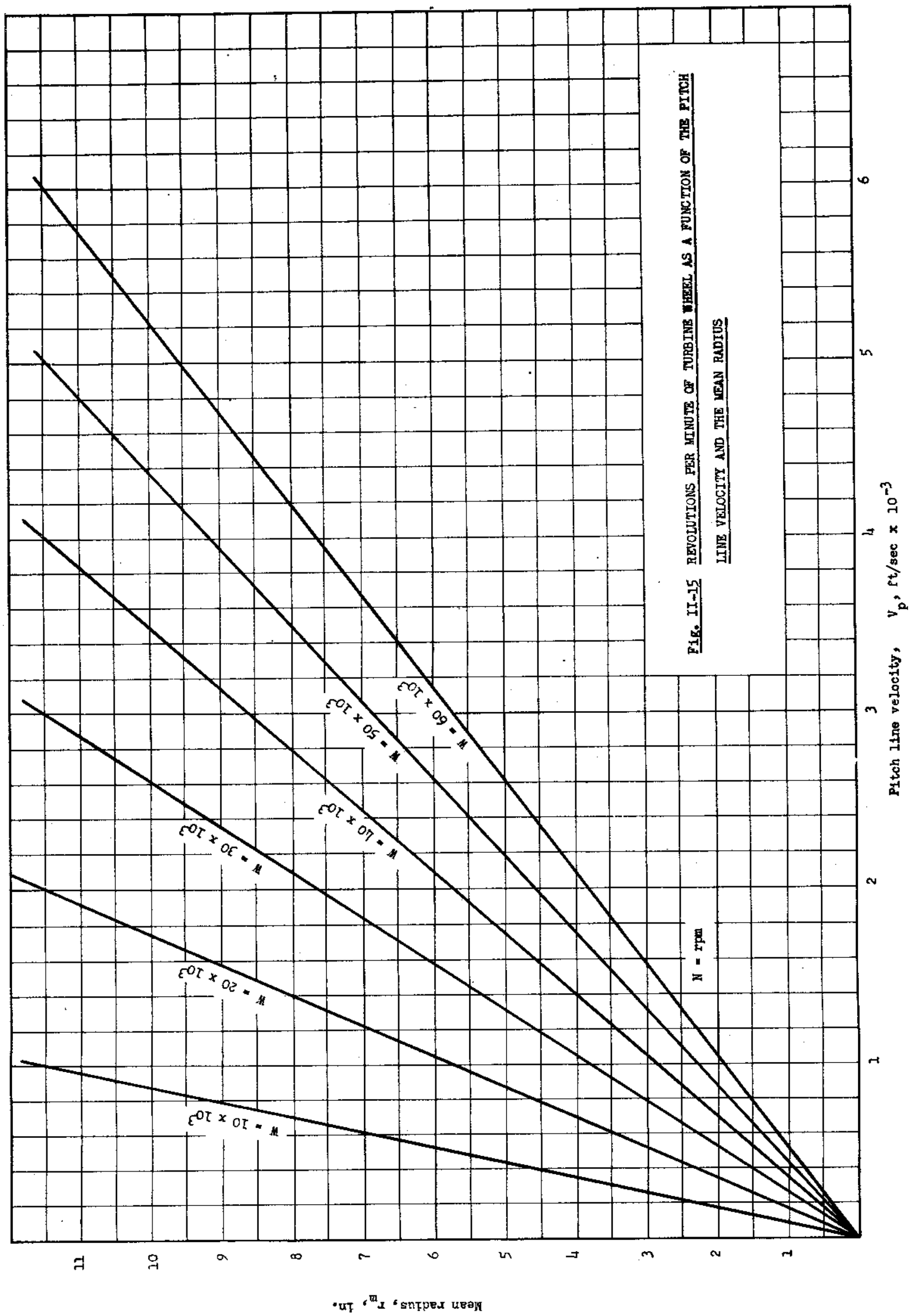


Fig. II-15 REVOLUTIONS PER MINUTE OF TURBINE WHEEL AS A FUNCTION OF THE PITCH LINE VELOCITY AND THE MEAN RADIUS

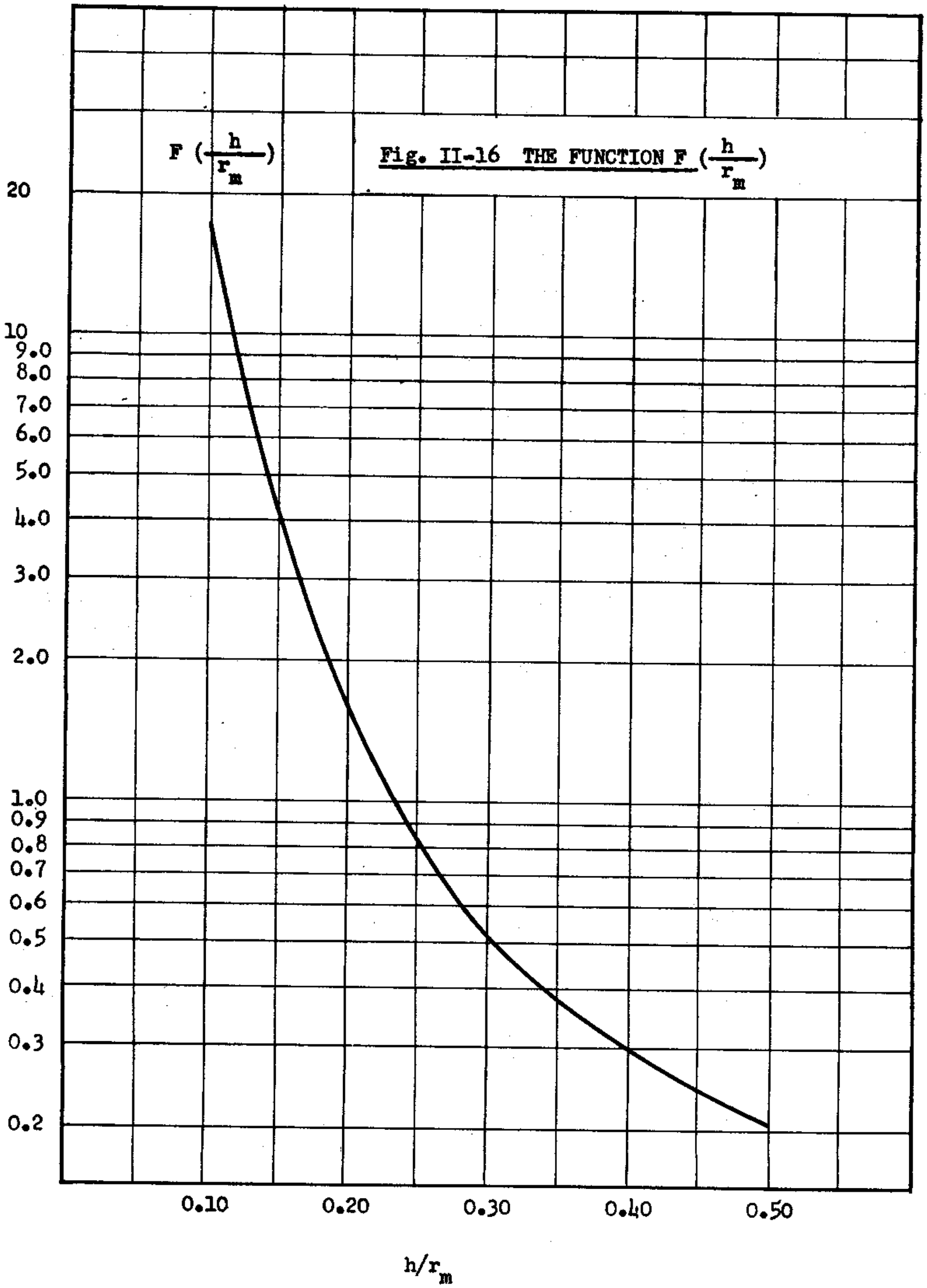


Fig. II-16 THE FUNCTION $F\left(\frac{h}{r_m}\right)$

r_s = 0.9

λ = 1.5

a = 0.3

K = 0.15

γ_d = 510 lb/ft^3

γ_b = 540 lb/ft^3

where:

r_s = stress ratio of stress in the blade to the stress in the disk

λ = blade solidity (ratio of blade width to the blade pitch)

a = taper (ratio of blade cross-sectional area at the tip to the cross-sectional area at the root)

K = proportionality constant (ratio of cross-sectional area at the blade root to the square of the blade width)

γ_d = weight density of the disk material, lb/ft^3

γ_b = weight density of the blade material, lb/ft^3

From Eq. (II-40), lines of constant h/r_m ratio can be drawn as shown in Fig. II-17. On the same graph, lines of constant annular area, A, are shown. The equation of these lines is given by:

$$A = 2\pi r_m h = 2\pi h^2 \left(\frac{h}{r_m}\right)^{-1} \qquad \text{(II-41)}$$

or

$$h = \sqrt{\frac{A}{2\pi}}\sqrt{\frac{h}{r_m}} \qquad \text{(II-42)}$$

Fig. II-17 is drawn on two sheets to facilitate reading the low values of the parameter, $\frac{W_W \delta}{A}$

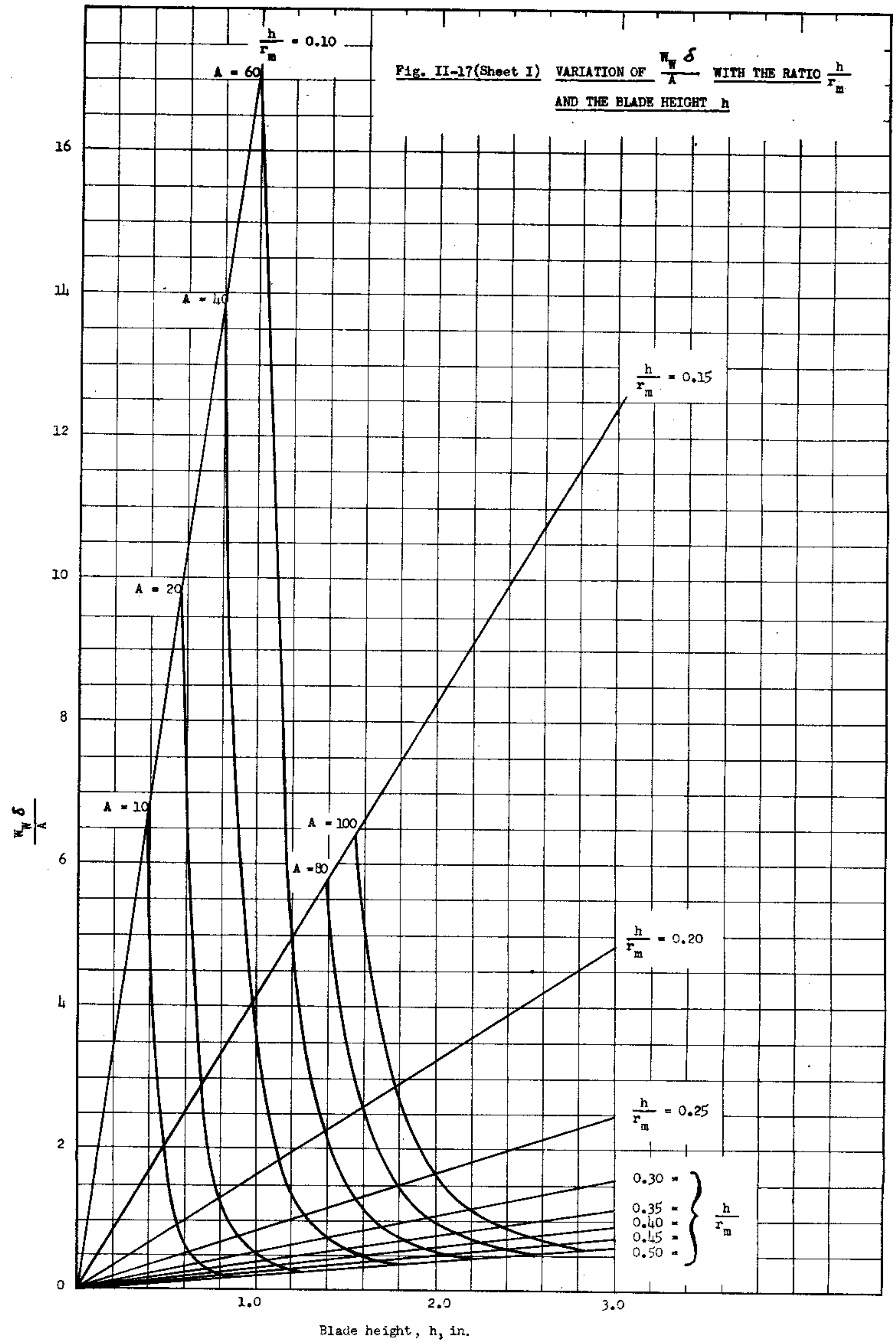
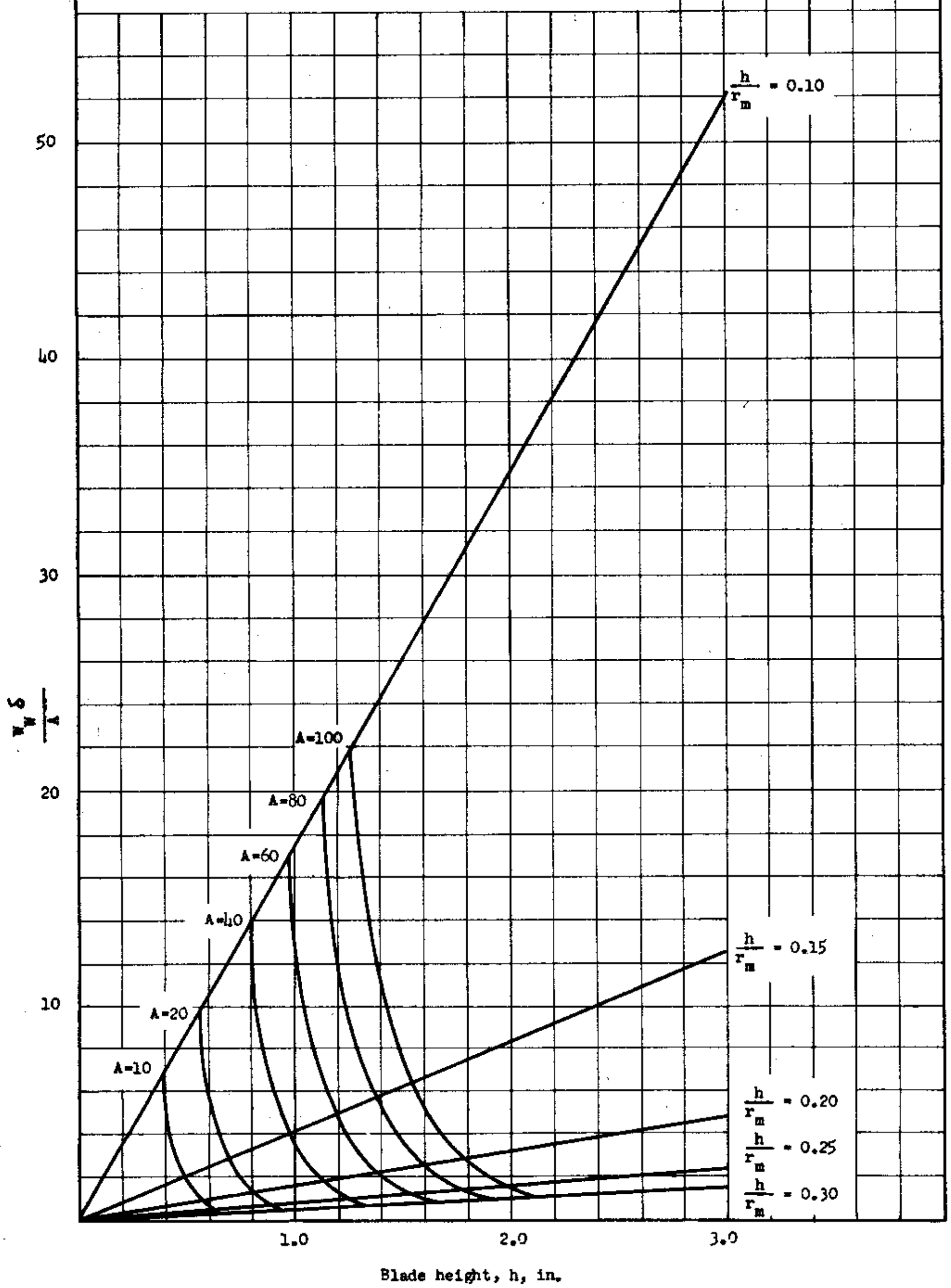


Fig. II-17 (Sheet I) VARIATION OF $\frac{W_W \delta}{A}$ WITH THE RATIO $\frac{h}{r_m}$ AND THE BLADE HEIGHT h

Fig. II-17(Sheet II) VARIATION OF $\frac{W_w \delta}{A}$ WITH THE RATIO $\frac{h}{r_m}$ AND THE BLADE HEIGHT h



6. Weight of the Turbine Casing, W_C, for an Axial Flow Turbine

The weight of the turbine casing can be approximated by replacing the casing with a cylindrical enclosure having the same diameter as the tip diameter of the turbine wheel. The weight of this enclosure can be expressed in terms of the blade-radius ratio, annular area, and design constant as follows:

$$W_C = \frac{At\gamma_C}{1728}\left[\frac{w_C}{d} + \frac{1}{2}\right]\frac{\left[1 + \frac{h}{2r_m}\right]^2}{\frac{h}{2r_m}} \qquad \text{(II-43)}$$

or

$$\frac{1728\, W_C}{At\gamma_C\left[\frac{w_C}{d} + \frac{1}{2}\right]} = \frac{\left[1 + \frac{h}{2r_m}\right]^2}{\frac{h}{2r_m}} \qquad \text{(II-44)}$$

where:

w_C = width of casing (assumed), in.

d = tip diameter at turbine wheel = $2\, r_m + h$, in.

t = thickness of turbine casing (assumed), in.

γ_C = weight density of casing material (assumed), lb/ft^3

Eq. (II-44) is shown graphically in Fig. II-18. Knowing the blade-radius ratio, h/r_m, a value for the left member of Eq. (II-44) can be taken from Fig. II-16. Since all of the factors except W_C are known, W_C can be calculated.

7. Combined Weight of Axial Flow Turbine Wheel and Casing, W_{WC}

The validity of the system evaluation will not be materially impaired

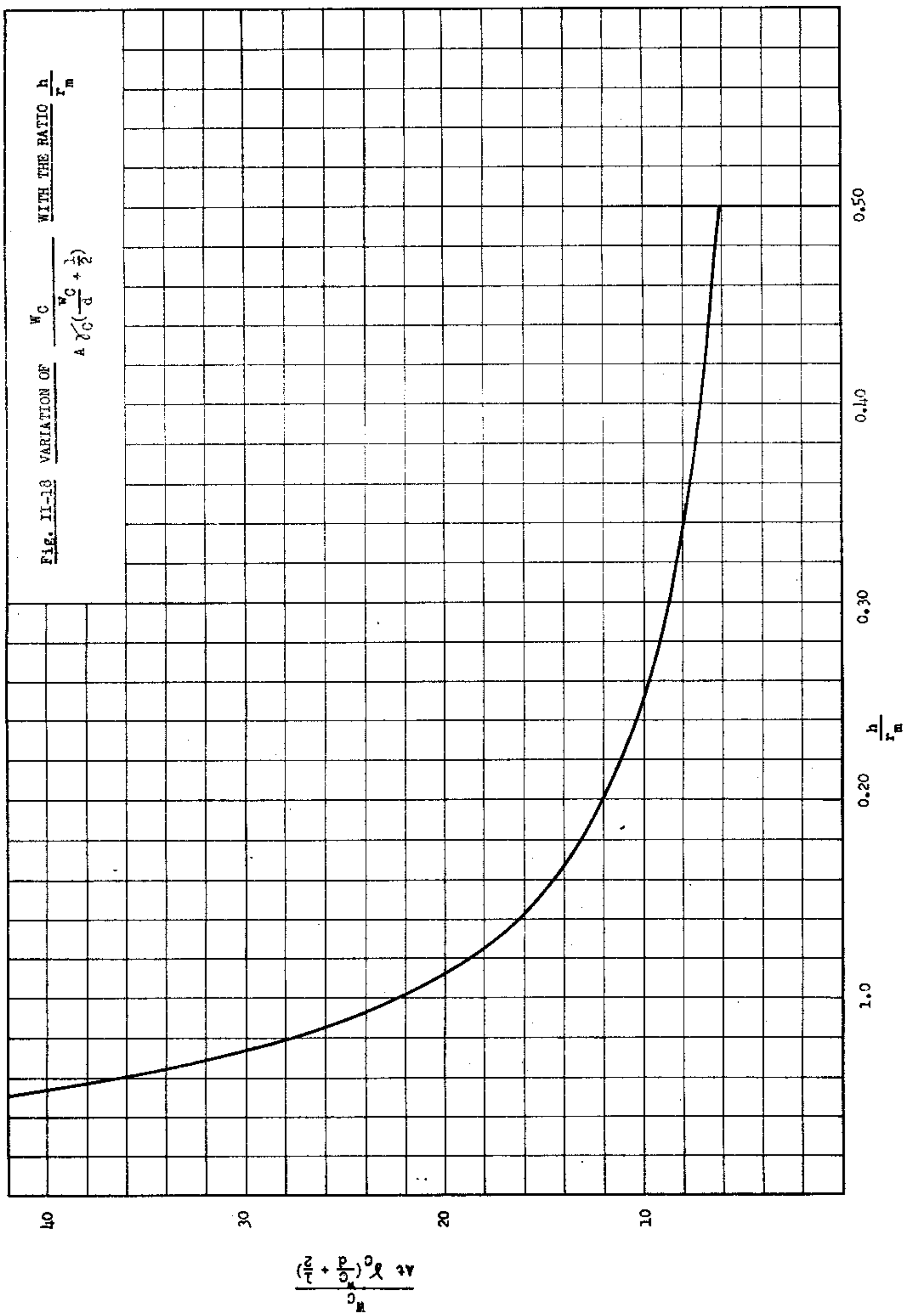


Fig. II-18 VARIATION OF $\frac{W_C}{A\gamma_C(\frac{W_C}{d}+\frac{1}{2})}$ WITH THE RATIO $\frac{h}{r_m}$

if the total weight of the turbine and casing is determined as follows. The weight of the turbine wheel is given by Eq. (II-40) and the weight of the casing is given by Eq. (II-43). The combined weight, W_{WC}, the sum of the two, can be written as:

$$W_{WC} = \frac{F\left[\frac{h}{r_m}\right]\sqrt{\frac{h}{r_m}}}{\sqrt{2\pi}\,\delta} A^{3/2} + \frac{t\,\gamma_C\left[\frac{w_C}{d} + \frac{1}{2}\right]\left[1 + \frac{h}{2r_m}\right]^2}{1728\,\frac{h}{2r_m}} A \qquad \text{(II-45)}$$

In order to simplify the determination of this combined weight, the following representative values are assigned to the design parameters:

$$\delta = 3$$

$$t = 0.10 \text{ in}$$

$$\frac{w_C}{d} = 0.5$$

$$\gamma_C = 490 \text{ lb/ft}^3$$

By fixing these values, the weight becomes a function of the area alone. The ratio h/r_m can be used as a parameter, the value of which can be determined for a given pitch line velocity from Fig. II-14. The variation of the weight, W_{WC}, with annular area, A, (determined in step 5-a), for several values of the parameter h/r_m is shown in Fig. II-19 which can be used for estimating the combined weight of the turbine wheel and the casing.

8. Weight of a Radial Flow Turbine

Approximate formulas have been developed to determine the weight of a radial flow turbine as a function of wheel diameter.

a. Determine the Wheel Diameter

For aircraft accessory power turbines the ratio of nozzle area A_t, to the square of the wheel diameter, d, may vary between 0.08 and 0.10, (Ref. II-4). Using the value of 0.10 for the weight analysis gives

$$d = 3.16\sqrt{A_t} \qquad \text{(II-46)}$$

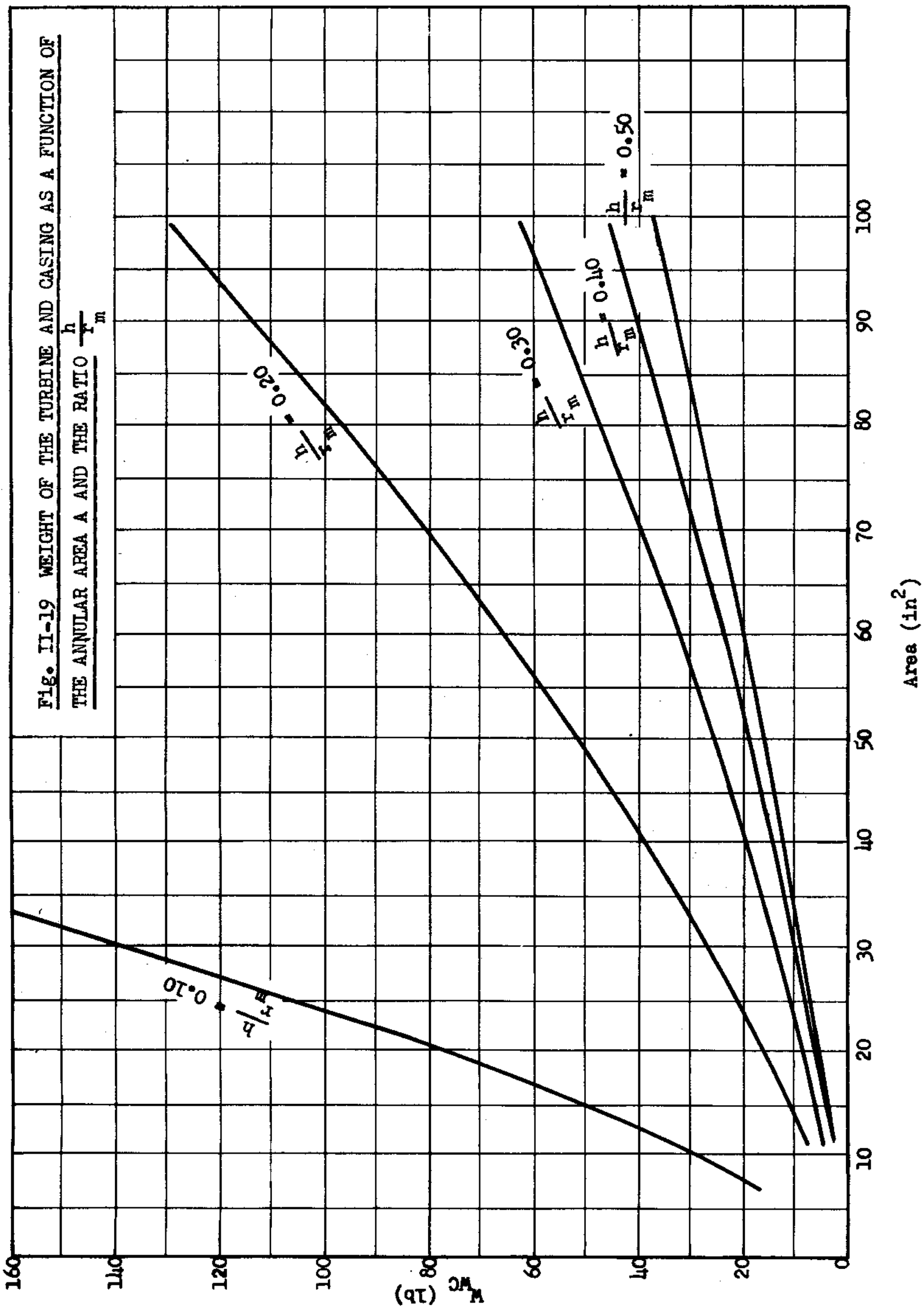


Fig. II-19 WEIGHT OF THE TURBINE AND CASING AS A FUNCTION OF THE ANNULAR AREA A AND THE RATIO $\frac{h}{r_m}$

b. Determine the Wheel Weight

Assuming that the weight of the wheel is equal to weight of a solid cylinder with the same outside diameter but a smaller or average equivalent width, it was found that the wheel weight is proportional to the cube of the diameter. The proportionality constant was calculated for steel wheels from available data on actual turbines. The weight of a turbine wheel was found to be

$$W_W = 0.0182\ d^3 \qquad \text{(II-47)}$$

c. Determine the Turbine Casing Weight

The weight of the turbine casing can be approximated by replacing the casing with a cylindrical enclosure. The diameter of this enclosure, d_x, can be taken as 1.5 times the wheel diameter (see Fig. II-9) while the width, w_x, can be taken as 0.5 d_x. This results in the following for the casing weight:

$$W_C = \frac{\pi\, d_x^2\ t\, \gamma_C}{1728}\left(\frac{1}{2} + \frac{w_x}{d_x}\right) \qquad \text{(II-48)}$$

where:

d_x = diameter of equivalent cylindrical enclosure, in.

w_x = width of equivalent cylindrical enclosure, in.

t = thickness of equivalent cylindrical enclosure, in.

γ_C = weight density of material, lb/ft^3

Using the assumptions

$d_x = 1.5\ d$

$w_x = 0.5\ d_x$

and assuming

$t = 0.10$ in.

$\gamma_C = 490$ lb/ft^3

then

$$W_C = 0.2\ d^2 \tag{II-49}$$

d. Combined Weight of Turbine Wheel and Casing

Adding Eqs. (II-47) and (II-49) gives the combined weight of the turbine wheel and casing.

$$W_{WC} = 0.0182\ d^3 + 0.2\ d^2 \tag{II-50}$$

Eq. (II-49) is shown graphically in Fig. II-20. This figure can be used to obtain the approximate weight of a radial flow turbine having a wheel diameter, d.

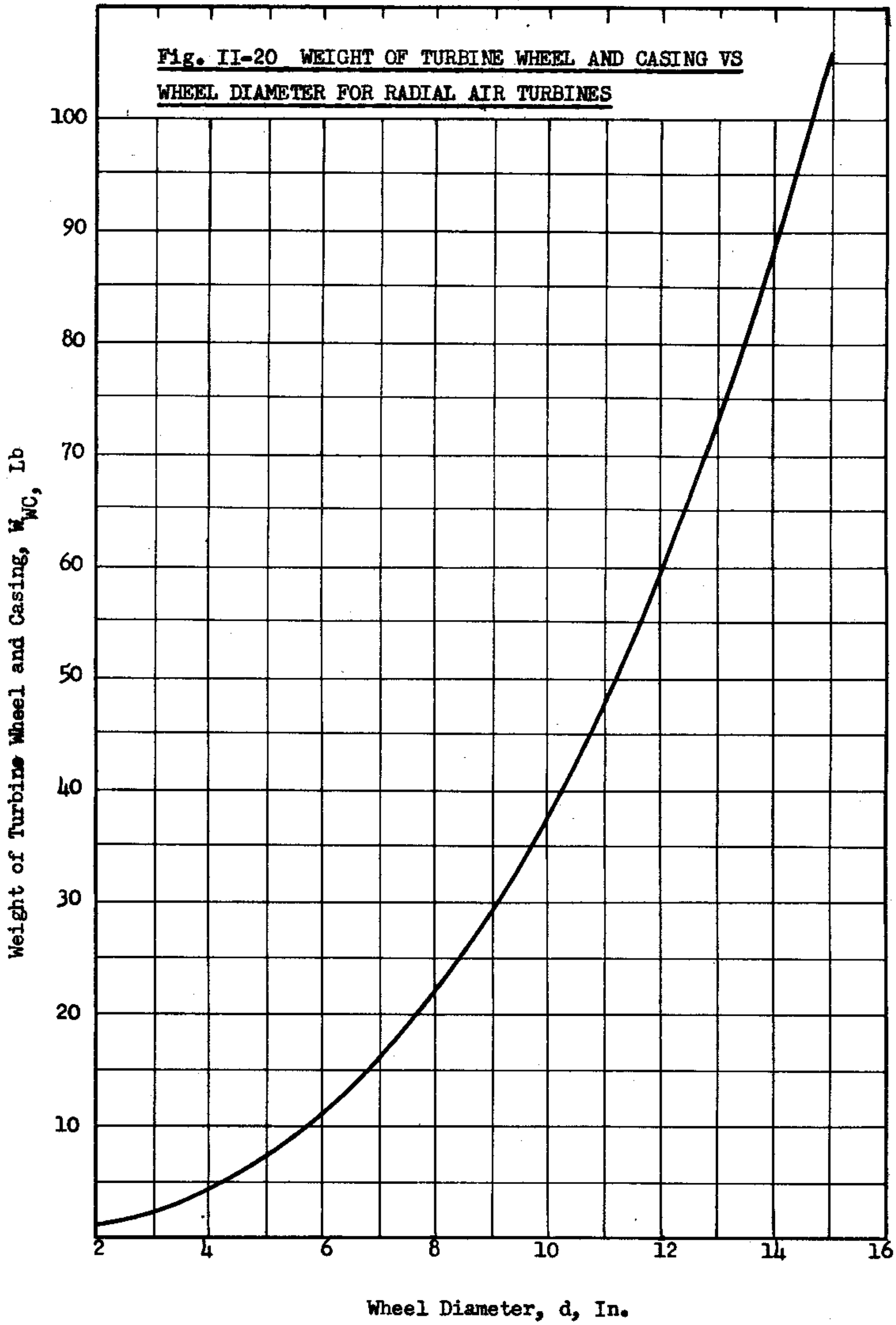


Fig. II-20 WEIGHT OF TURBINE WHEEL AND CASING VS WHEEL DIAMETER FOR RADIAL AIR TURBINES
100
90
80
70
60
50
40
30
20
10
Weight of Turbine Wheel and Casing, W_{WC}, Lb
2
4
6
8
10
12
14
16
Wheel Diameter, d, In.

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II-5 LaValle, V. L., and Huppert, M. C., Effects of Several Design Variables on Turbine Wheel Weight, NACA TN No. 1814, February, 1949

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ANALYSIS OF THE WEIGHT OF HYDRAULIC POWER TRANSMISSION SYSTEMS

Section III

A. Introduction

A hydraulic transmission system consists essentially of a pump for extracting power from the engine, fluid transmission lines for transmitting the hydraulic power to the motor, and a hydraulic motor for reconversion of the hydraulic power to mechanical power. In addition, a reservoir is required for fluid storage and de-aeration; an oil cooler must be provided to dissipate the heat generated within the system, and controls are needed for regulating the flow and pressure of the system. Such a system is shown in Fig. III-1.

The speed of the motor is controlled by its displacement and rate of flow. The power output is given by the motor speed and the torque as determined by the pressure difference acting on the motor.

In this section, the procedures are shown for determining the approximate weights of two types of systems which may be used on aircraft. These systems are:

1. Constant flow, variable pressure system

2. Constant pressure, variable flow system

The constant flow, variable pressure system incorporates a variable displacement pump and a constant displacement motor. As the engine speed changes, the pump displacement is varied to maintain a constant flow. Consequently, the power output of the motor is determined by the system pressure.

The constant pressure, variable flow system utilizes variable displacement units for both the pump and the motor. Under constant load conditions, the pump displacement is altered to compensate for any changes in engine speed, so that a constant output load is maintained. A change in load conditions tends to result in a change in output speed. A speed or torque sensing servo-mechanism alters the motor displacement to provide the torque required for restoring the speed. This change in motor displacement is transmitted to the pump servo-mechanism to cause a similar change in pump displacement, thereby providing the flow necessary to maintain the new motor load at the rated speed.

The constant flow, variable pressure system is best suited for single load application and can be operated at constant motor speed. It is capable of supplying a limited overload for short time intervals by operating above the rated pressure of the pump and motor.

The constant pressure, variable flow system is best suited to multiload

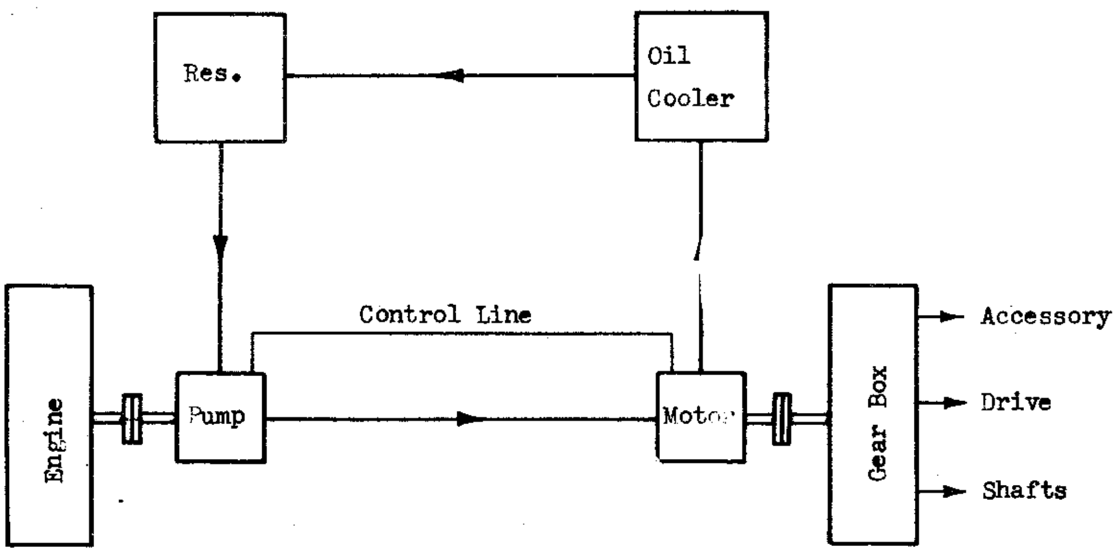


Fig. III-1 SCHEMATIC DIAGRAM OF SIMPLE HYDRAULIC POWER TRANSMISSION SYSTEM applications which do not require a constant motor speed, but which require continuous operation. Applications of this system include flight control power boosts, fuel pumps, etc. The system must be designed for maximum load requirements, because it cannot be operated above the rated pressure of the pump and motor. Since the units must be larger, the system weight is greater than that of a constant flow, variable pressure system.

For the purposes of comparison, an optimum hydraulic system is defined as the lightest system capable of fulfilling all power requirements of the design mission. The weight of this system includes the following items:

1. Weight of the system components (pump, motor, transmission lines and fluid, oil cooler, reservoir, etc.)

2. Additional fuel required by engine to provide the power extracted by the hydraulic pump.

3. Additional fuel required by engine to provide the power required to overcome any additional airplane drag caused by the transmission system.

4. Any structural weight increase caused by the incorporation of the system in the airplane.

In the system analysis presented in this report the first two items listed above are considered. Since the other two items are determined primarily by the airplane design concept, they are introduced as constants which have to be evaluated before the analysis can be carried out.

With design concepts introduced as constants, the weight of the optimum hydraulic transmission system is dependent upon factors influencing the losses in the system. A large part of the losses are represented by the transmission line losses as given by the pressure drop in the lines. If the lines are designed too small, the pressure drop will be high. The pump, motor and oil cooler must therefore be larger in order to compensate for the increased losses. If the lines are designed too large, the losses will be decreased; but the line weight will be disproportionately large. The optimum system, therefore, is one which has a pressure drop that results in the minimum system weight.

A method is presented here by which the optimum pressure drop for a given application can be determined. It is based on the assumption that the flow in the lines is turbulent, and the pressure drop proportional to the square of the velocity. The pressure drop in the straight portions of the transmission lines is assumed to be small compared to that in the bends and fittings.

The evaluation of hydraulic transmission systems can be divided into four steps:

1. Establishment of the design conditions required for the evaluation.

These include the mission profile of the airplane, the power schedule of the accessories, and the approximate locations of the accessories within the airframe.

2. Establishment of the parameters used to describe the characteristics of the system and of the units comprising the system. The characteristic parameters include those describing the engine specific fuel consumption for the power supplied to the system, and those describing the system flow and energy requirements for the power delivered by the motor. Other parameters are those expressing the transmission line and fluid weight density, and pump and motor parameters which reflect design features of these components.

3. Determination of the optimum pressure drop. The parameters established in Step 2 are utilized to determine graphically the optimum pressure drop for the system. The curves expressing the optimum pressure drop in terms of the characteristic parameters were obtained by minimizing the total weight equation with respect to the pressure drop. Separate curves are shown for constant flow-variable pressure systems and for constant pressure-variable flow systems.

4. Calculation of the total system weight and the weights of its components. From the optimum pressure drop, the required power output, and the defined parameters, the total system weight is obtained. Equations are shown which will give the weights of the individual components if desired.

B. Nomenclature

A	constant in Eq. (III-15), representing the fixed weight of constant flow variable pressure hydraulic power transmission system, lb, defined by Eq. (III-14)
B	constant in Eq. (III-15) defined by Eq. (III-8)
C	constant defined by Eq. (III-9)
C_{PX}	thrust correction factor of engine due to power extraction
C'_{PX}	fuel flow correction factor due to power extraction
C''_{PX}	specific fuel consumption of engine for the increment of total engine power which is extracted by the power transmission system, lb/HP-hr
D	hydraulic power transmission line diameter, ft
F_n	engine thrust at design cruise speed and altitude, lb
G	energy parameter, sec^2/ft^2, defined by Eq. (III-2)
H	constant in Eq. (III-30) representing the fixed weight of constant pressure variable flow hydraulic power transmission system, lb, defined by Eq. (III-29)

HP_{cr}	power requirements from the system at cruise conditions, hp
HP_m	maximum power required from system, including overloads, hp
HP_n	power required by accessory at its normal rating, hp
HP_r	rated power of pump-motor combination at rated pressure, hp
HP_{ref}	a reference power of the jet engine, arbitrarily taken as 10 per cent of the jet power at sea level static conditions, hp
J	average weight of control and auxiliary lines and fluid, lb
K	velocity head loss
L	length of hydraulic transmission lines, ft
L_3	combined length of control and auxiliary lines, ft
M	parameter defined by Eq. III-27
N_r	rated speed of hydraulic motor, 1/min
Q	flow rate, ft^3/sec
P	pressure, lb/ft^2
P_m	maximum permissible operating pressure of pump, lb/ft^2
P_r	rated working pressure of pump, lb/ft^2
P_s	maximum gage pressure of pump, lb/ft^2
R	parameter defined by Eq. (III-26)
S	parameter defined by Eq. (III-28)
τ	duration of power extraction, hr
V	fluid velocity, ft/sec
W	weight, lb
W_c	weight of oil cooler, lb
W_{CL}	weight of control and auxiliary lines, lb
W_F	weight of fuel consumed by engine to deliver power extracted by the accessory drive system, lb
W_f	engine fuel consumption rate at thrust, F_n, lb/hr

W_L	weight of hydraulic power transmission lines, lb
W_r	weight of reservoir, lb
W_s	weight of pump-motor combination which, when closely coupled, has same power output as that required from system, lb
W_u	weight of required pump-motor combination, lb
W_k	increment of total aircraft weight due to additional structural requirements and to the fuel required to overcome any increased aerodynamic drag chargeable to the transmission system, lb
g	acceleration of gravity, 32.2 ft/sec^2
h	ratio of weight of filled reservoir to weight of fluid in it
m	ratio of weight of hydraulic lines, including fittings and clamps, to weight of tubing only
s	maximum permissible working stress in tube walls, lb/ft^2
x^*	pressure loss coefficient
x^*_{cr}	pressure loss coefficient evaluated at cruise power output conditions
x^*_m	pressure loss coefficient evaluated at maximum power output conditions
α	fraction of hydraulic line fluid volume carried in the reservoir for fluid de-aeration, expansion and contraction purposes
β	fraction of hydraulic line fluid volume carried in the reservoir per hour of power extraction to compensate for small seepage leaks
ΔP	pressure loss through the transmission lines, lb/ft^2
ΔW	change in weight, lb
η_a	efficiency of the hydraulic pump, expressed as a decimal
η_b	efficiency of the hydraulic motor, expressed as a decimal
η_L	efficiency of the transmission lines, expressed as a decimal
γ	weight density, lb/ft^3
γ_f	weight density of the fluid, lb/ft^3
γ_m	weight density of the tube wall metal, lb/ft^3
ΣK	total velocity head loss

ΣW total weight of the system, lb

ϕ line density parameter, lb/ft^3

ψ specific thrust fuel consumption of engine, lb/lb-hr

Ω flow coefficient, ft^3/sec

C. Required Design Information for the Evaluation of a Hydraulic Transmission System

To determine an optimum hydraulic power transmission system for a given aircraft, the following design information is required:

1. From preliminary design data and airplane specification:

Locations of pumps and motors in the airplane

Design mission cruising speed

Design mission cruising altitude

τ Duration of power extraction, hrs

HP_{cr} Average power extracted from transmission system during design mission cruise, hp

HP_m Maximum power, including overloads, extracted from drive system. This includes overloads due to electrical system faults, hp

2. From the engine manufacturers' performance specification:

F_n Engine thrust at design mission cruising speed and cruising altitude, lbs

W_f Fuel consumption rate at thrust, F_n, lb/hr

C_{PX} Thrust correction factor of engine due to power extraction, (see Ref. III-1)

C'_{PX} Fuel flow correction factor due to power extraction, (see Ref. III-1)

HP_{ref} A reference horsepower, arbitrarily taken as 10 per cent of the jet horsepower at sea level static conditions (see Ref. III-1) This rating is used as a base for comparing engine performance at any operational condition, hp

3. From the pump and motor manufacturers' data:

HP_r Rated power of units at rated pressure, hp

P_m Maximum allowable working pressure of units, lb/ft^2

P_s Maximum gage pressure of units, lb/ft^2
This pressure is the maximum working pressure plus any supercharge pressure used in a pressurized system

P_r Rated pressure of units, lb/ft^2

η_a Pump efficiency

η_b Motor efficiency

N_r Rated speed of hydraulic motor, rpm

4. From miscellaneous sources:

γ_f Weight density of hydraulic fluid, lb/ft^3

γ_m Weight density of hydraulic line material, lbs/ft^3

s Working stress of line material, lb/ft^2
This is the rated tensile stress divided by the required safety factor

5. Estimated Data:

The following data must be estimated from the design information:

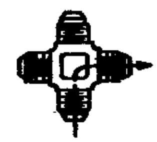
L Length of the transmission lines, ft
This should be estimated as closely as possible from the airplane configuration and the location of the system components within the airframe, keeping in mind the need of clearing the major structural and functional components of the airplane.

L_3 Total length of auxiliary and control lines, ft
Number of bends and fittings in each line - The general line configuration will be an indication of the number and types of fittings required.

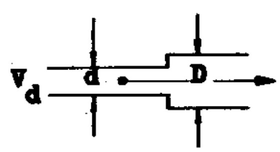
ΣK The total number of velocity heads lost in the transmission line. This is a function of the total line length, the number of bends, elbows and other fittings, and is influenced by the Reynolds Number of the flow in the transmission line. Since ΣK will vary for each airplane model, it must be evaluated individually for each particular model and no estimate of ΣK can be given here. Table III-1 shows representative K values for some hydraulic fitting configurations at various Reynolds Numbers, and may be used as a guide in estimating the value of the total velocity head loss, ΣK. Further data will be found in Refs. III-2, and III-3. As a check on the validity of the assumed values

TABLE III-1

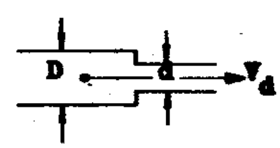
REPRESENTATIVE K VALUES FOR VARIOUS HYDRAULIC TUBE FITTINGS

Reynolds Number	AN-821 Elbow 	AN-824 Tee 90° Flow 	AN-827 Cross 90° Flow 
200	4.5	4.0	5.0
500	3.0	2.9	3.4
1,000	2.4	2.8	2.8
2,000	2.1	2.4	2.4
5,000	1.4	1.7	1.7
10,000 and up	1.0	1.0	1.2

Sudden Expansion*



Sudden Contraction*



Ratio d/D	Sudden Expansion*	Sudden Contraction*
0.10	0.98	0.64
0.18	0.88	0.61
0.40	0.66	0.48
0.61	0.32	0.22
0.82	0.12	0.12

*Based on Turbulent Flow and Velocity V_d

WADC TR 53-36

of ΣK, the line pressure drop should be recalculated after the line diameter and the fluid velocity have been determined from Eqs. (III-11) and (III-13), and the range of Reynolds Numbers checked. If the discrepancy is large, a new value of ΣK should be determined.

m Fraction, added to the tube weight, to account for the additional weight of tube fittings and clips. It is the ratio of the tube fitting and clip weight to the bare tube weight. It can be estimated or obtained by a survey of hydraulic systems in existing aircraft.

α Fraction of the line fluid volume which corresponds to the reservoir volume that is provided for fluid de-aeration, expansion, and contraction.

β Fraction of line volume to be carried in the reservoir for every hour of power extraction to compensate for small seepage leaks.

α and β should be so proportioned as to give a reservoir of reasonable size when considered in the light of current aircraft practice.

h Ratio of total reservoir weight to the weight of the fluid stored in it.

D. Evaluation of Design Parameters

In the course of the minimum weight analysis a number of parameters were introduced for the convenience of later applications and graphical presentation. In this paragraph, these parameters are enumerated and explained, where possible, in physical terms. It should be kept in mind that the primary purpose of introducing these parameters was to write the analytical relations in a simpler form, particularly suited for graphical presentation.

In applying these parameters, they must be evaluated for the data of the design point (cruising and/or maximum conditions). If they are meant to refer to overload, or maximum power conditions, the subscript 'm' is used, eg:

$$\Omega_m = \frac{550 \ HP_m}{\eta_b P_m}$$

1. The Flow Parameter, Ω, ft^3/sec, represents a rate of flow, and is expressed by

$$\Omega_m = \frac{550 \ HP_m}{\eta_b P_m} \qquad \text{(III-1)}$$

2. The Pressure Loss Parameter, G, sec^2/ft^2, represents the pressure

losses per unit velocity. It is given by,

$$G_m = \frac{8\ \gamma_f \Sigma K}{\pi^2 g\ P_m} \qquad \text{(III-2)}$$

3. Line Density Parameter, ϕ, lb/ft^3, represents the specific weight of the line and the fluid enclosed by it. It is represented by the equation

$$\phi = \frac{\pi P_s \gamma_m (1 + m)(1 + \frac{P_s}{2s})}{2s} + \frac{\pi \gamma_f}{4} \qquad \text{(III-3)}$$

4. Specific Thrust Fuel Consumption, ψ, lb/hr/lb, is the rate of change of engine fuel consumption per unit thrust at the design mission cruising speed and altitude. It is defined by

$$\psi = \frac{dW_f}{dF_n} \qquad \text{(III-4)}$$

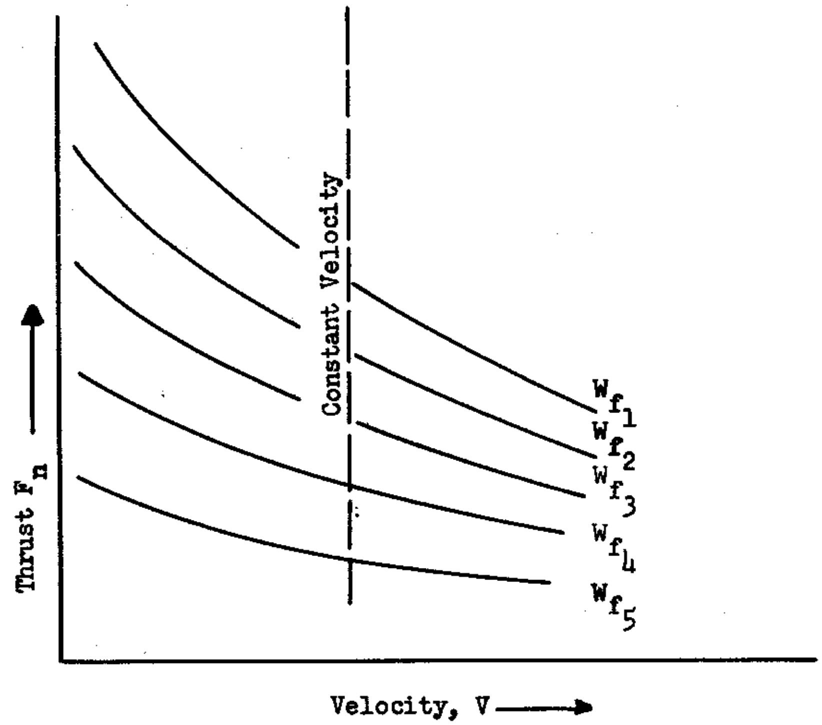
This parameter is evaluated as the tangent to the curve of the specific thrust fuel consumption at the specified thrust output. Fig. III-2 shows graphically the procedure involved.

5. Specific Transmission Fuel Consumption, C''_{PX}, lb/HP-hr, is the increment of fuel consumed per hour by the engine, due to a unit of power extracted by the power transmission system. It is obtained by correcting the fuel consumption to maintain the normal cruise thrust output of the engine at the required level.

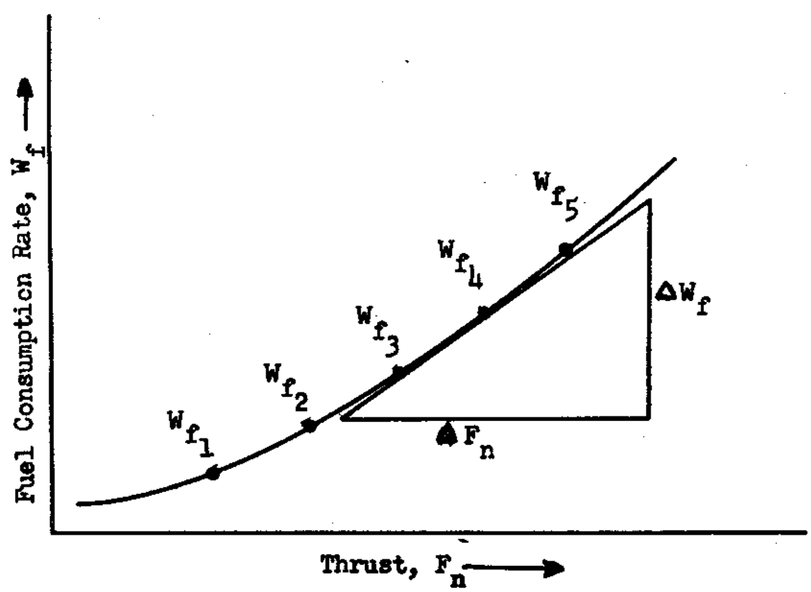
$$C''_{PX} = \frac{\psi F_n C_{PX} - W_f C'_{PX}}{HP_{ref}} \qquad \text{(III-5)}$$

6. Power Units Weight Factor, dW_s/dHP_m, lb/HP, represents the weight per horsepower for a hydraulic pump and motor combination without transmission lines. This is obtained as the slope of the plot of the pump-motor weight as a function of maximum power output. During this investigation a correlation of weight and power parameters was made for units meeting the requirements of Ref. III-5. The equation obtained from this correlation was then modified to reflect the weight of pump and motors in combination, allowing for additional weight increments for variable volume and supercharging features, as well as other auxiliaries. The resulting equation, for pump-motor combinations operating at a 3000 to 6000 rpm input speed range, and constant 6000 rpm output speed, is

$$\frac{dW_s}{dHP_m} = \frac{68{,}140}{N_r^{2/3} \left(\frac{P_m^2}{P_r}\right)^{1/3} HP_m^{1/3}} \qquad \text{(III-6)}$$



(a) Engine Fuel Consumption Data As Presented in Engine Manufacturer's Specification



(b) Fuel Consumption Curve Replotted from (a) showing Method of Obtaining $\psi = dW_f/dF_n$

Figure III-2 Method of Evaluating ψ

If no other data on weights of pumps and motors are available, Eq. (III-6) may be used for an approximation. The change in weight of these pump-motor combinations with change in power output is shown graphically in Fig. III-3. Care should be used to evaluate dW_s/dHP_m at the design point.

E. Evaluation of a Constant Flow Variable Pressure System

The constant flow, variable pressure hydraulic transmission system utilizes a variable displacement pump and a constant displacement motor. If the engine speed changes, the variable pump displacement adjusts itself so that the flow remains constant. Since the flow is constant, the resulting pressure loss in the line is constant.

The power output of the motor is determined by the acting pressure difference. As the load increases, the system pressure increases. The maximum power output of the system depends upon the maximum permissible system pressure.

The transmission line pressure drop represents only a small portion of the total system pressure at maximum power output. However, as the power output decreases, the system pressure decreases, until at zero power output the system pressure is just sufficient to overcome the line resistance. Fig. III-4 is a diagram of the pressure-power output relationship for such a system, where the maximum horsepower, HP_m, is twice the rated horsepower, HP. It may be seen that for a given design point, the slope of the pressure vs power curve is determined by the pressure drop, ΔP. If ΔP is too large, an inefficient system results. If ΔP is too small, the installed weight is unnecessarily high. The lightest system has the optimum pressure drop.

In the following, the procedure will be outlined for obtaining the optimum weight of the pump-transmission line-motor combination including the required auxiliaries, in terms of the horsepower rating, the diameter and wall thickness of the transmission line, all based on the given aircraft configuration, mission profile, and accessory power schedule.

The weight of the system to be optimized consists of the weight of the pump-motor combination, including governor equipment, W_u, the weights of the oil cooler, W_c, the transmission line, W_L, the reservoir, W_r, and the increment of total aircraft weight due to the additional structural requirements and to the fuel required to overcome any increased aerodynamic drag chargeable to the system, W_k. With the exception of the last item, which is considered to be a constant for any type system, the weights of the above components in terms of the system parameters are shown in Eqs. (III-16) to (III-21).

1. Determination of the Optimum Transmission Line Losses

The system weight is required to be a minimum, and the optimum pressure loss in the transmission lines connected with this minimum is to be determined. This was done mathematically by optimizing the total weight equation (shown in Part 2) with regard to a quantity, x^*, which is the square root of the pressure loss written as a fraction of the working pump

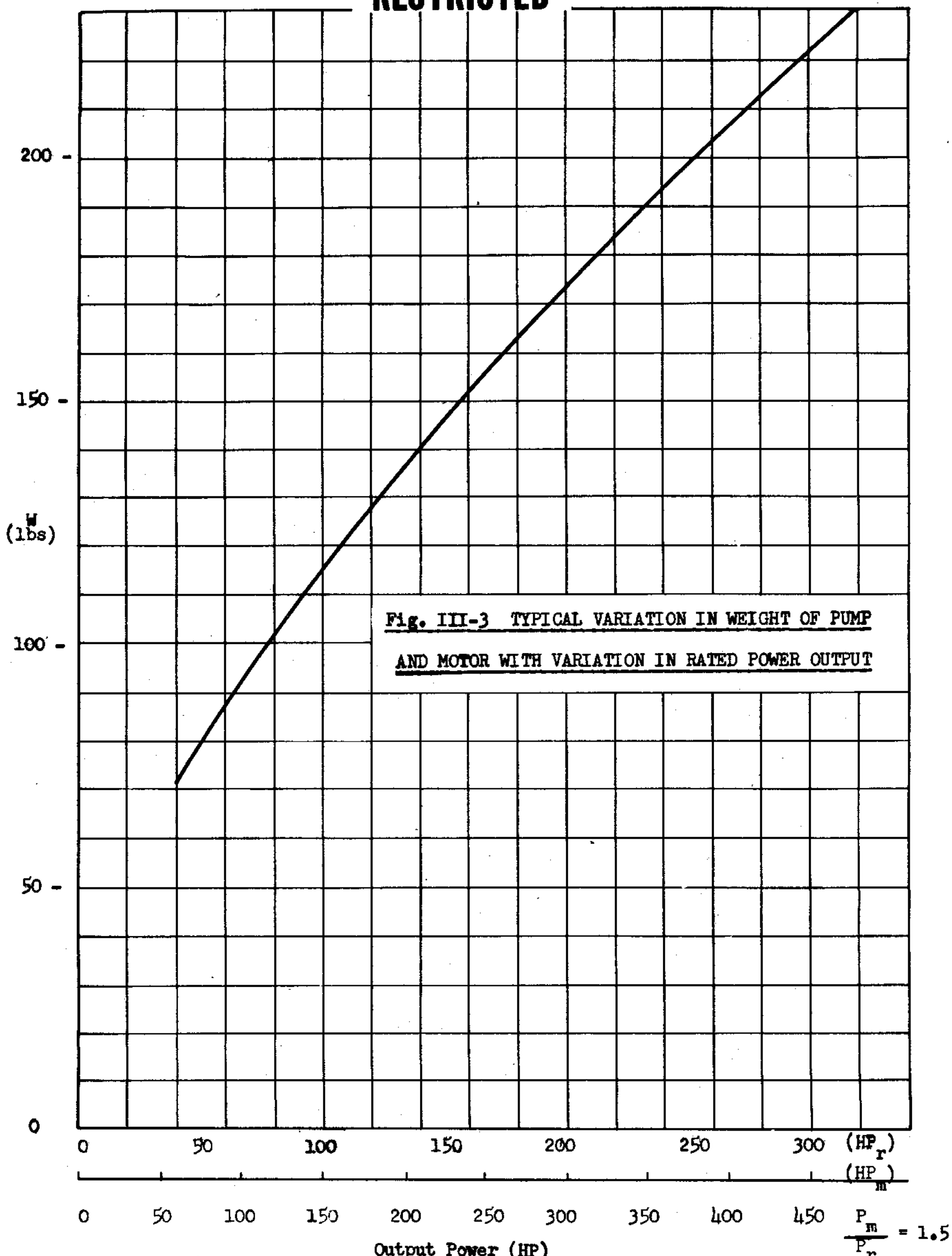


Fig. III-3 TYPICAL VARIATION IN WEIGHT OF PUMP AND MOTOR WITH VARIATION IN RATED POWER OUTPUT

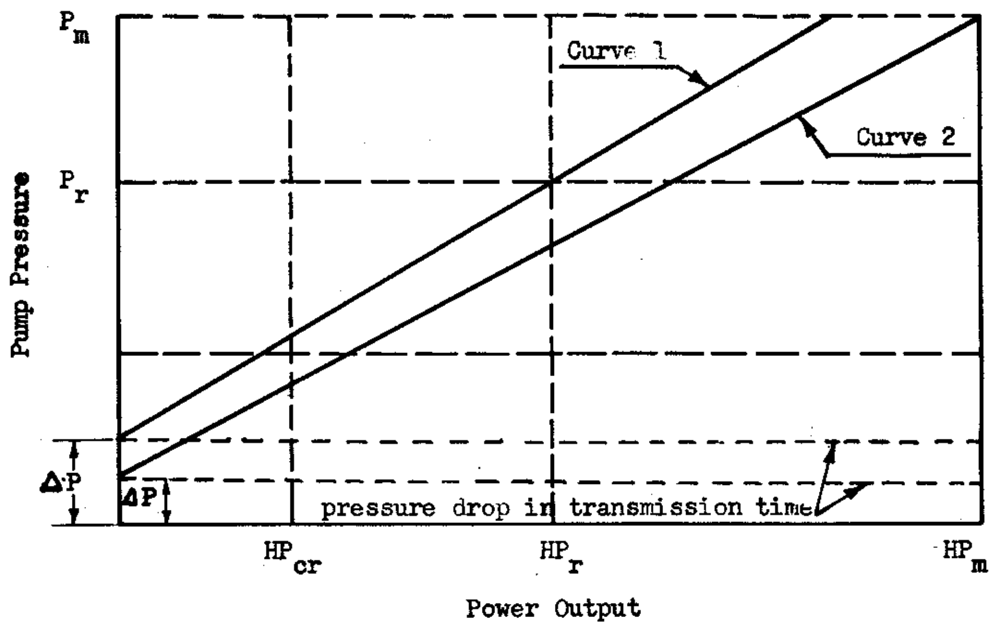


Fig. III-4 GENERAL RELATIONSHIP BETWEEN PRESSURE AND POWER OUTPUT OF A CONSTANT FLOW, VARIABLE PRESSURE HYDRAULIC POWER TRANSMISSION SYSTEM

For two typical cases. Curve 1 represents the relationships for a system designed for maximum power at maximum pressure. Curve 2 represents those for a design power at rated pressure. ΔP and $\Delta P'$ represent the respective line pressure drops for the two cases.

Legend

HP_m - Maximum power

HP_r - Design, or rated power

HP_{cr} - Cruise power requirements

P_m - Maximum permissible operating pressure of pump and motor

P_r - Rated Operating pressure of pump and motor pressure, $\sqrt{\frac{\Delta P}{P}}$. The quantity x^* was introduced for the convenience of calculation, and is called the pressure loss coefficient.

$$x^* = \sqrt{\frac{\Delta P}{P}} \qquad \text{(III-7)}$$

The condition for minimum weight as obtained from optimizing the total weight equation with respect to the pressure loss coefficient x^* is

$$\frac{2\,B}{C} = \frac{1 - 3\,x^{*2}}{x^{*3}} \qquad \text{(III-8)}$$

where
B is representative of specific pump and motor unit weight, oil cooler weight and fuel weight due to transmission line losses

$$B = \frac{dW_s}{dHP_m} + \frac{0.385}{\eta_a\,\eta_b} + \frac{C''_{PX}\,\tau}{\eta_a\,\eta_b} \qquad \text{(III-9)}$$

and
C is representative of the weights of transmission lines and reservoir

$$C = \left[\frac{\pi}{4}\ (\alpha + \beta\tau)\ \gamma_f h + \phi\right] \frac{L\,G^{1/2}\,\Omega}{HP} \qquad \text{(III-10)}$$

The relationship expressed by Eq. (III-8) is shown graphically in Fig. III-5. The values of B and C are determined first from the available data on aircraft configuration, mission, engine characteristics, etc. Then x^* is found for the corresponding value of 2B/C. From x^*, the pressure drop is found, see Eq. (III-10).

Having determined the optimum transmission line pressure loss, the line diameter, fluid flow rate, and fluid velocity can be calculated.

The line diameter is a function of x^*_m, and is given by the equation

$$D = \frac{G_m^{1/4}\ \Omega_m^{1/2}}{\left[x^*_m\ (1 - x^{*2}_m)\right]^{1/2}} \qquad \text{(III-11)}$$

The fluid flow rate within the system is found from the equation

$$Q = \frac{\Omega_m}{1 - x^{*2}_m} \qquad \text{(III-12)}$$

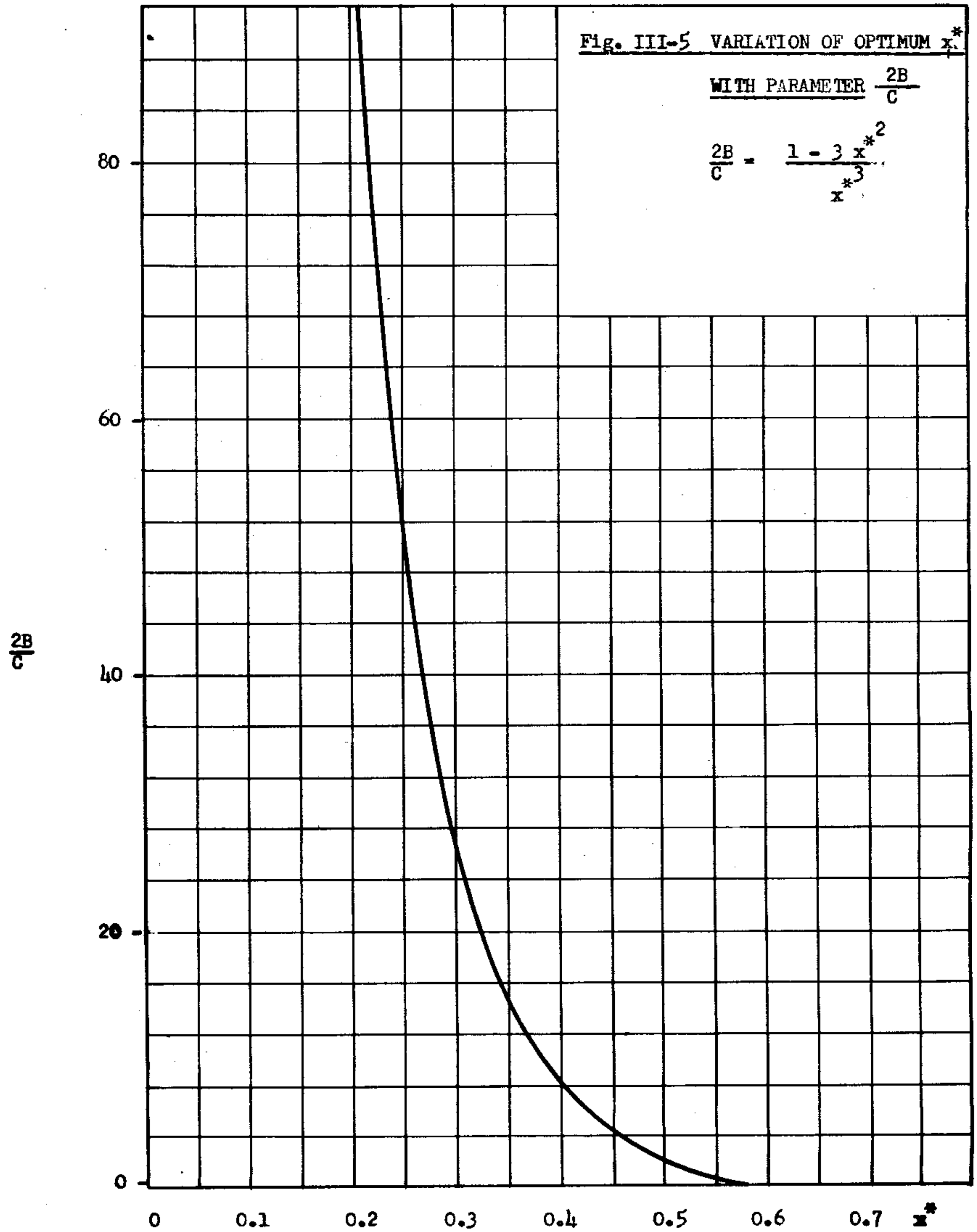


Fig. III-5 VARIATION OF OPTIMUM x^*
WITH PARAMETER $\frac{2B}{C}$
$\frac{2B}{C} = \frac{1 - 3x^{*2}}{x^{*3}}$
80
60
40
20
0
$\frac{2B}{C}$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
x^*

The fluid velocity in the transmission lines is

$$V = \frac{4\, x_m^*}{\pi\, G_m^{1/2}} \qquad \text{(III-13)}$$

From the line diameter, the fluid velocity, and the viscosity of the hydraulic fluid used, the Reynolds Number can be obtained and the value of ΣK can be checked for conformance with the value obtained on the basis of an assumed Reynolds Number. If the discrepancy is large, a new value of ΣK should be assumed and the foregoing calculations repeated.

2. Total Weight of the System

The total weight of the system may be considered to be comprised of three components: a fixed weight, A, which represents the weight of the system without the transmission lines and reservoir and the variable weights represented by B and C as shown by Eqs. (III-8) and (III-9), respectively. The fixed weight of A is defined by the equation

$$A = W_s + \frac{0.385\, HP_n}{\eta_a\, \eta_b}(1 - \eta_a \eta_b) + 2 + \frac{C_{PX}\, \tau\, HP_{cr}}{\eta_a\, \eta_b} + W_{CL} + W_k \qquad \text{(III-14)}$$

where:

W_s = weight of the pump-motor combination which, when closely coupled (transmission line of zero length) has the same power output as that required from the system. W_s is to be taken at the point at which dW_s/dHP_m was determined.

HP_n = power required by the driven accessories at their rated output

W_{CL} = weight of control and auxiliary lines. This may be estimated from the line lengths and the unit weight of each line size involved.

The total weight of the system can then be expressed as

$$W = A + B\, HP_m \frac{x_m^{*2}}{1 - x_m^{*2}} + C \frac{HP_m}{x_m^* (1 - x_m^{*2})} \qquad \text{(III-15)}$$

3. Weights of System Components

The weights of the system components may be calculated individually from the equations given below. The derivation of these equations will be given in Part 2 of this report.

a. Pump-Motor Weight

The weight of the pump-motor combination is assumed to include the servo units required to vary the pump displacement. The weight is given by

$$W_u = W_s + \frac{dW_s}{dHP_m} HP_m \frac{x_m^{*2}}{1 - x_m^{*2}} \qquad \text{(III-16)}$$

b. Oil Cooler Weight

The oil cooler weight is assumed to meet the requirements of Ref.III-4. It is further assumed that overloads will be of very short duration, and, therefore, no additional cooling capacity will be required for them. However, the system should be capable of running continually at the full rated power of the driven accessories without undue heating. The oil cooler weight is approximated by the following equation, obtained from a curve fitted to the weights and face areas of round tubular air cooled oil coolers.

$$W_c = \frac{0.385}{\eta_a \eta_b} HP_n (1 - \eta_a \eta_b) + HP_m \frac{x_m^{*2}}{1 - x_m^{*2}} + 2 \qquad \text{(III-17)}$$

where:

HP_n = rated power required by the driven accessories

c. Transmission Line Weight

The weight of the hydraulic power transmission lines is given by

$$W_L = \frac{\phi L G_m^{1/2} \Omega_m}{x_m^* (1 - x_m^{*2})} \qquad \text{(III-18)}$$

d. Reservoir Weight

The reservoir weight is assumed roughly proportional to its capacity. This in turn is proportional to the line capacity, the design cruise power, and the duration of power extraction. On this basis the weight is

$$W_r = \frac{\pi L (\alpha + \beta \tau) \gamma_f h G_m^{1/2} \Omega_m}{4 x_m^* (1 - x_m^{*2})} \qquad \text{(III-19)}$$

e. Fuel Weight

The weight of the fuel consumed by the engine in generating the power extracted by the transmission system is based on the design mission cruise power output only. The fuel weight is

$$W_F = \frac{C^*_{PX} \tau HP_{cr}}{\eta_a \ \eta_b} \ \frac{1}{1 - x^{*2}} \qquad \text{(III-20)}$$

f. Weight of Control and Auxiliary Lines

The weight of control and auxiliary lines is considered to be unaffected by the system efficiency, and is shown as

$$W_{CL} = JL_3 \qquad \text{(III-21)}$$

where:

J = the average weight of control lines per unit length

L_3 = total length of control lines

F. Evaluation of Constant Pressure Variable Flow System

A constant pressure, variable flow hydraulic system is one in which the transmission line pressure remains constant and the flow is varied in accordance with the accessory demand. The output speed is not constant if fixed displacement motors are used, but it can be made nearly constant with variable displacement motors, particularly if units with linear torque-displacement characteristics are used.

Since the pressure is constant in this type system, the power output is a function of the system flow. Fig. III-6 is a graphical representation of the rate of flow vs power. It may be noted that the power output increases as the flow increases, up to a maximum value. At this maximum power, the pressure loss equals one third of the pump working pressure ($\Delta P = \frac{P}{3}$), which can be readily shown by optimizing $HP = (P - \Delta P)\ (\Delta P/k)^{1/2}$ with regard to ΔP. Any increase in flow beyond this value causes a decrease in power output, due to the rapid increase in the pressure drop. Ultimately the flow becomes so great that the pressure energy of the fluid is completely dissipated in overcoming the fluid friction, and no power is available for driving the accessories.

The flow at the maximum power output must be limited to such a value that the pressure drop does not exceed one third of the pump working pressure. The flow at design cruise power output is, therefore, a function of the maximum power as well as of the cruise power.

The constant pressure variable flow system must be designed to provide maximum power output at rated pump pressure. The flow and energy parameters

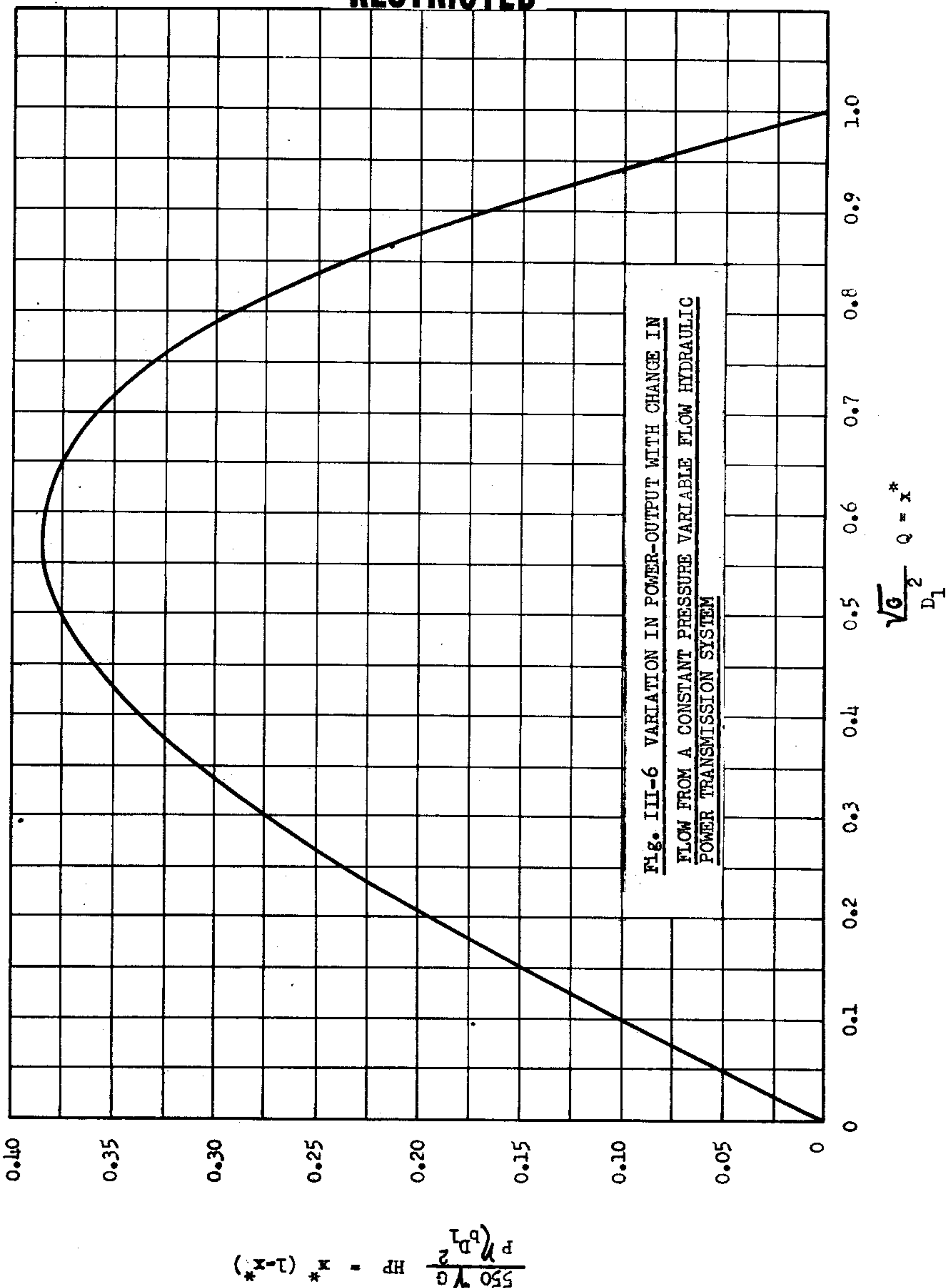


Fig. III-6 VARIATION IN POWER-OUTPUT WITH CHANGE IN FLOW FROM A CONSTANT PRESSURE VARIABLE FLOW HYDRAULIC POWER TRANSMISSION SYSTEM given by Eqs. (III-1) and (III-2) must also be evaluated at rated pressure. The pump and motor must be capable of handling the maximum flow requirement. Thus, the weight parameter given by Eq. (III-3) is calculated for rated pump and motor pressures.

1. Optimum Transmission Line Losses

The procedure for minimizing the weight of a constant pressure variable flow system is similar to that explained for a constant flow variable pressure system. However, varying the flow instead of the pressure results in a somewhat different expression for the optimum transmission line losses. The result and its derivation is given in detail in Part 2.

The optimum losses are most readily found graphically after the determination of the following parameters:

R is representative of the pump, motor and oil cooler weight changes due to transmission line losses.

$$R = \frac{dW_u}{dHP_m} + \frac{0.1925}{\eta_a \ \eta_b} \qquad \text{(III-22)}$$

M is representative of the fuel weight increment required due to transmission line losses.

$$M = \frac{C''_{PX} \tau}{\eta_a \ \eta_b} \qquad \text{(III-23)}$$

S is representative of line and reservoir weights.

$$S = \left[\phi + \frac{\pi}{4} \gamma_f h \ (\alpha + \beta \tau) \right] \quad L \frac{G^{1/2} \Omega}{HP} \qquad \text{(III-24)}$$

$\frac{HP_m}{HP_{cr}}$ = ratio of maximum power requirement to cruise power requirement (III-25)

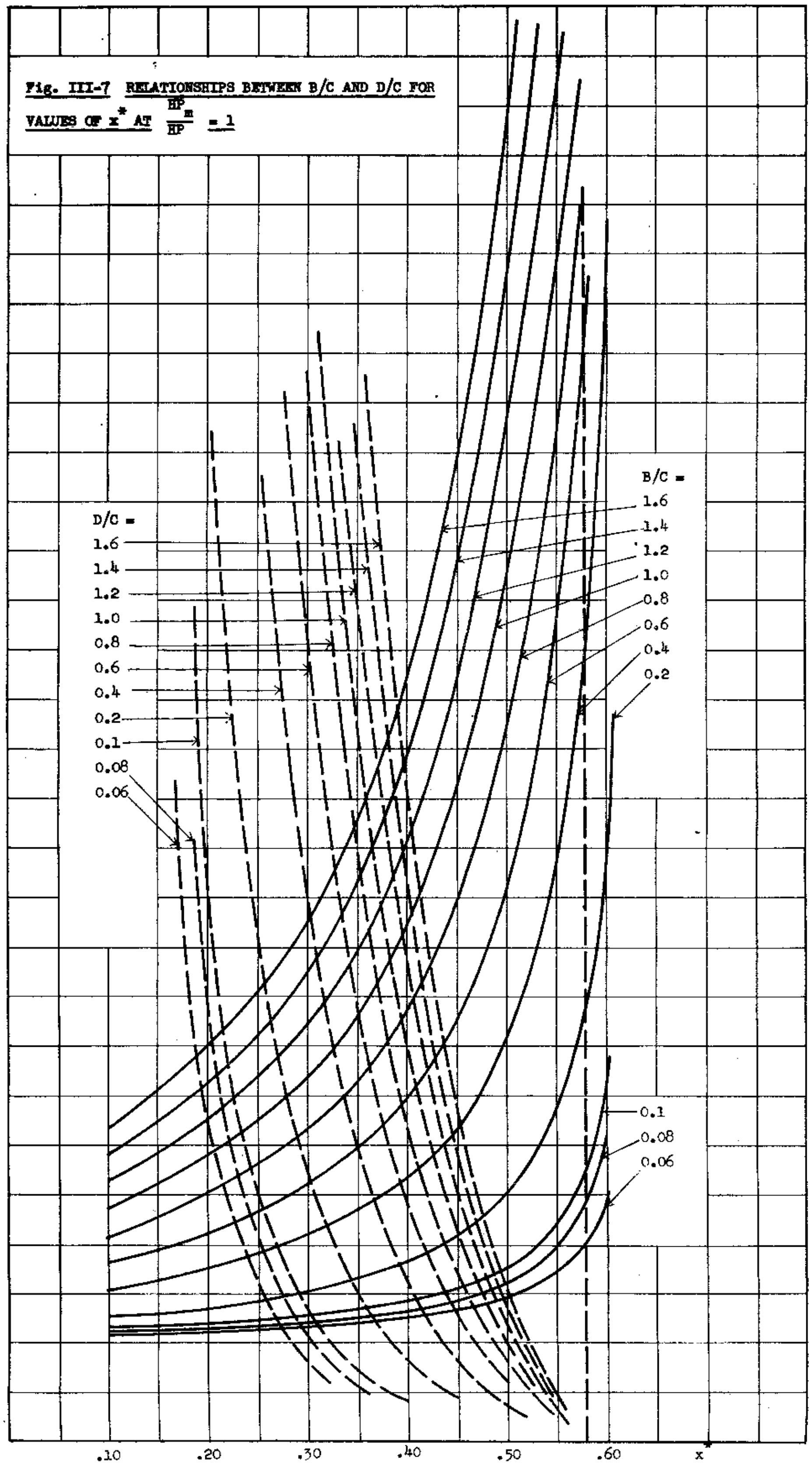
$\frac{R}{M}$ = ratio of parameter R to parameter M (III-26)

$\frac{S}{M}$ = ratio of parameter S to parameter M (III-27)

Having determined the parameter ratios listed in Eqs. (III-25) to (III-27) the value of x^* is obtained from the curves shown on Figs. III-7 to III-10, inclusive. The proper curve is selected in accordance with the $\frac{HP_m}{HP_{cr}}$ ratio.

If the ratio lies between two integral ratios of $\frac{HP_m}{HP_{cr}}$, the value of x^* is determined by interpolation.

Fig. III-7 RELATIONSHIPS BETWEEN B/C AND D/C FOR VALUES OF x^* AT $\frac{HP_m}{HP} = 1$



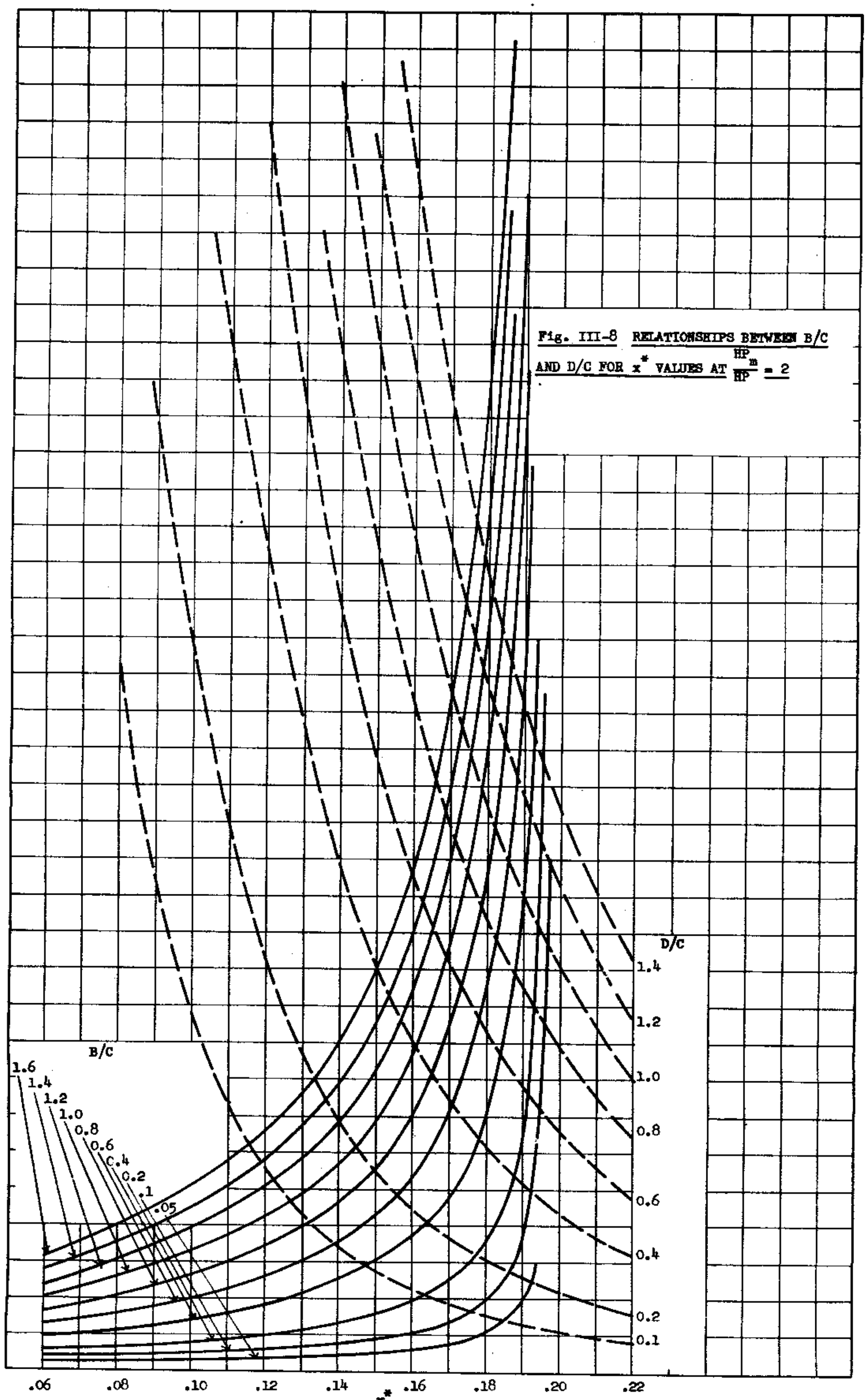


Fig. III-8 RELATIONSHIPS BETWEEN B/C AND D/C FOR x* VALUES AT HPm/HP = 2
D/C
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.1
B/C
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
.1
.05
.06
.08
.10
.12
.14
.16
.18
.20
.22
x*

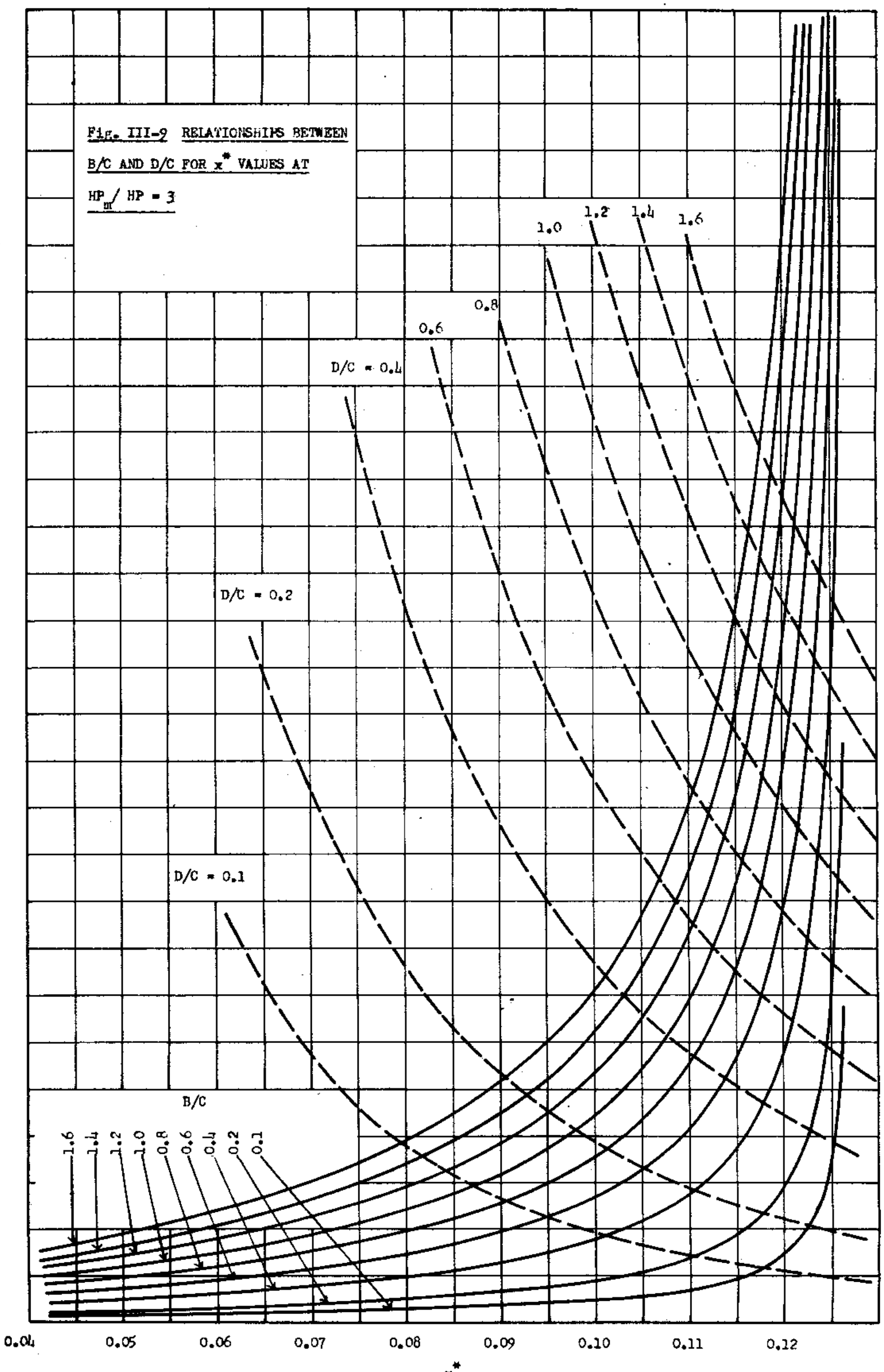


Fig. III-9 RELATIONSHIPS BETWEEN B/C AND D/C FOR x* VALUES AT HP_m/ HP = 3
1.0
1.2
1.4
1.6
0.8
0.6
D/C = 0.4
D/C = 0.2
D/C = 0.1
B/C
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.1
0.04
0.05
0.06
0.07
0.08
0.09
0.10
0.11
0.12
x*

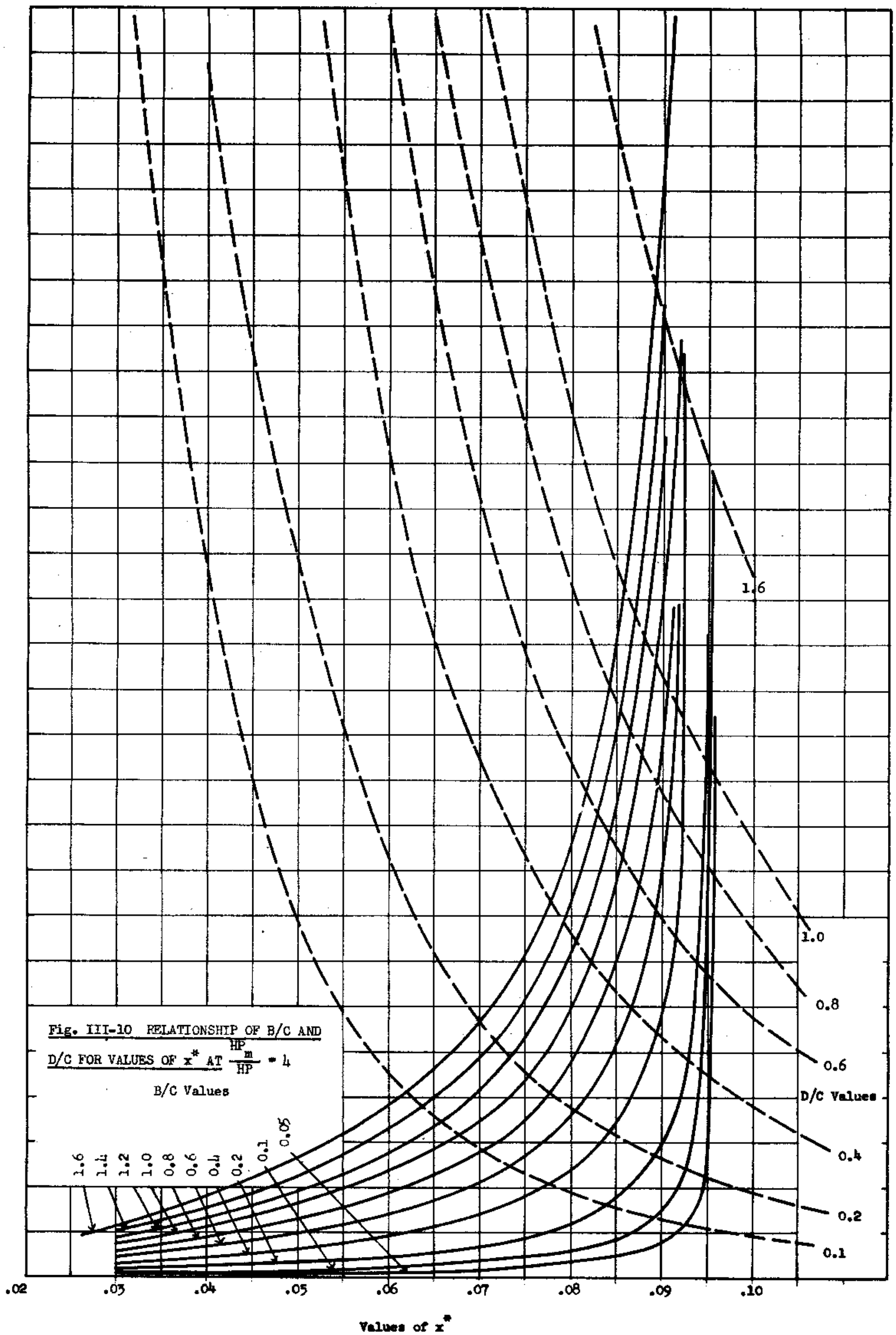


Fig. III-10 RELATIONSHIP OF B/C AND D/C FOR VALUES OF x^* AT $\frac{HP_m}{HP} = 4$
B/C Values
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.1
0.05
D/C Values
1.6
1.0
0.8
0.6
0.4
0.2
0.1
.02
.03
.04
.05
.06
.07
.08
.09
.10
Values of x^*

The value of x^* is found on the abscissa scale directly below the intersection of the $\frac{R}{M}$ and $\frac{S}{M}$ parameter curves.

Since $x^{*2} = \frac{\Delta P}{P}$, and the system operates constantly at the rated pump pressure, the optimum pressure drop for cruise power conditions can now be determined.

The pressure drop coefficient, x_m^*, at maximum power conditions is determined from Fig. III-11, reading up from the x_{cr}^* - value to the proper $\frac{HP_m}{HP_{cr}}$ curve, and then directly to x_m^* on the ordinate. The pressure drop at maximum power conditions is then calculated as before.

The equations whose solutions are represented graphically in Figs. III-6 to III-10 are derived in Part 2 of this report.

The line diameter, flow rate, and fluid velocity may be determined from Eqs. (III-13) and (III-14), evaluating Ω, G, and x^* at the proper design values. The validity of the assumed value of ΣK can then be checked, as outlined previously.

2. Total Weight of a Constant Pressure Variable Flow Hydraulic Power Transmission System

The weight of the system may be expressed similarly to that of the constant flow variable pressure system. In this instance, however, there is a fixed weight, H, plus three incremental weights. The fixed weight represents the weight of a system without a reservoir or transmission line. The incremental weight represents that of the lines and reservoir, as well as the added weight due to the line losses.

Let the fixed weight be represented by the symbol H.

Then

$$H = W_s - \frac{dW_s}{dHP_m} HP_m - 0.1925 HP_m + 2 + JL_3 + W_k \qquad \text{(III-28)}$$

where

W_s is evaluated at maximum power and rated pump pressure.

The total weight of the system is expressed by

$$W = H + R \frac{HP_m}{1 - x_m^{*2}} + M \frac{HP_{cr}}{1 - x_{cr}^{*2}} + S \frac{HP_{cr}}{x_{cr}^*(1 - x_{cr}^{*2})} \qquad \text{(III-29)}$$

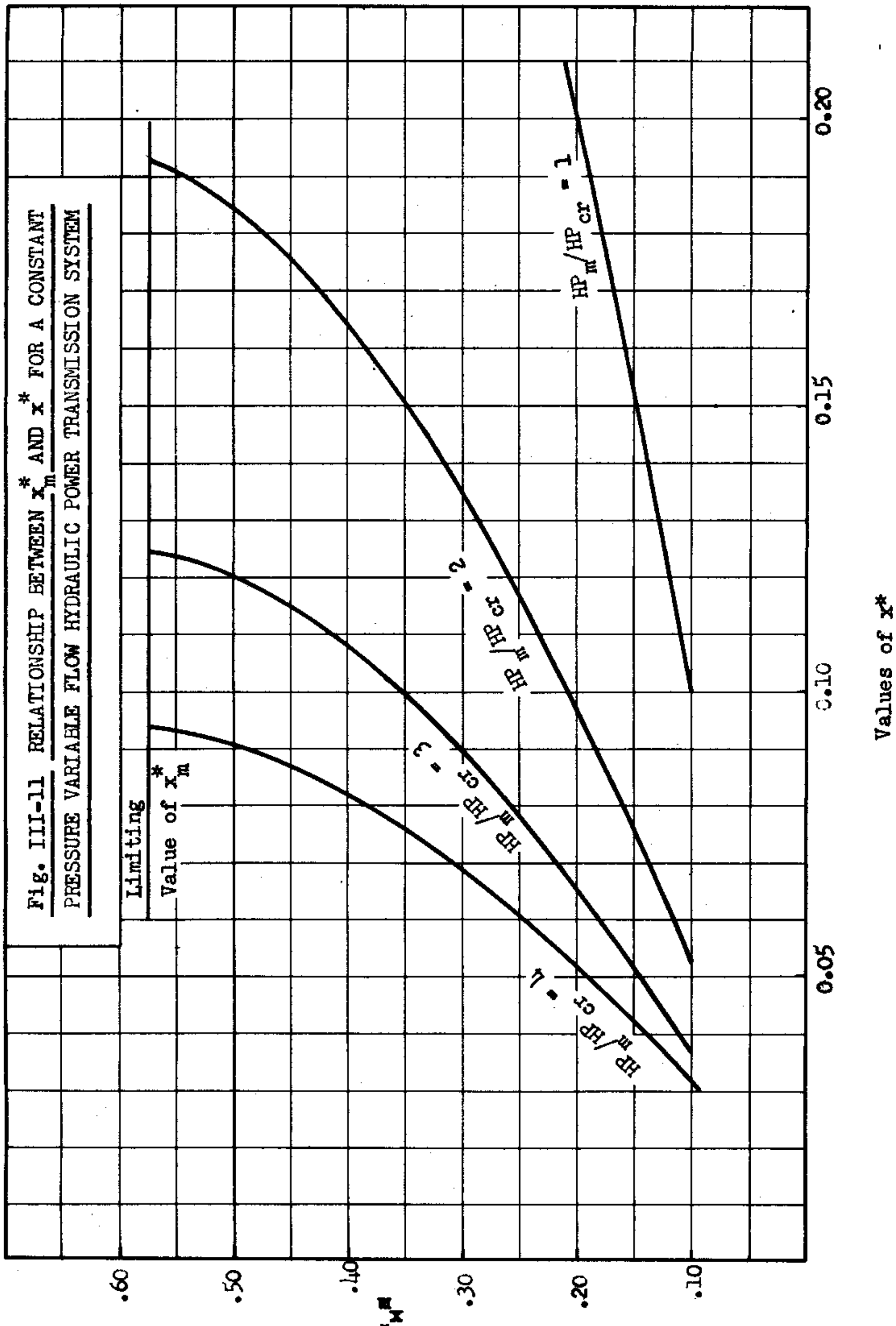


Fig. III-11 RELATIONSHIP BETWEEN x_m^* AND x^* FOR A CONSTANT PRESSURE VARIABLE FLOW HYDRAULIC POWER TRANSMISSION SYSTEM

3. Weights of the System Components

The equations for the weights of the individual components of a constant pressure, variable flow system are the same as those for a constant flow variable pressure system, with the exception of the equation for the oil cooler.

The equations for the weights of the system components are summarized below.

a. Pump-Motor Combination

$$W_u = W_s + \frac{dW_s}{dHP_m} HP_m \left[\frac{x_m^{*2}}{1 - x_m^{*2}} \right] \qquad \text{(III-30)}$$

The oil cooler weight is based on a heat rejection of one half the losses at maximum power. This assumption is valid for most practical purposes. However, if the rated input of the accessories is greater than one-half the maximum design power, the oil cooler size should be increased to handle the additional load.

b. Oil Cooler Weight

$$W_c = 0.1925\ HP_m \left[\frac{1}{\eta_a\ \eta_b} \ \frac{1}{1 - x_m^{*2}} - 1 \right] + 2 \qquad \text{(III-31)}$$

c. Transmission Line Weight

$$W_L = \frac{\phi L\, G^{1/2} \Omega_m}{x_m^* (1 - x_m^{*2})} \qquad \text{(III-32)}$$

d. Reservoir Weight

$$W_r = \frac{\pi (\alpha + \beta \tau)\ L \gamma_f\, h\, G^{1/2} \Omega_m}{4 x_m^* (1 - x_m^{*2})} \qquad \text{(III-33)}$$

e. Fuel Weight

$$W_F = \frac{M\ HP_{cr}}{1 - x_{cr}^{*2}} \qquad \text{(III-34)}$$

f. Control and Auxiliary Line Weight

The weights of the control lines and auxiliary lines are considered to be constant for relatively small changes in power output of the system. Their weight may be represented as:

$$W_{CL} = J\, L_3$$

where

W_{CL} = weight of control and auxiliary lines

J = average unit weight of the lines

L_3 = total length of the lines

REFERENCES

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III-2 Nels M. Sverdrup, Calculating the Energy Losses in Hydraulic Systems, "Product Engineering", pp 146 - 152, April 1951.

III-3 J. E. Campbell, Investigation of the Fundamental Characteristics of High Performance Hydraulic Systems, USAF T. R. 5997, June 1950.

III-4 Military Specification MIL-C-5637, Coolers; Oil, Tubular, Aircraft, 15 February, 1950.

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ANALYSIS OF THE WEIGHT OF AN ELECTRICAL POWER TRANSMISSION SYSTEM

Section IV

A. Introduction

An electric transmission system consists essentially of a generator or alternator for extracting power from the engine, electric transmission cables for transmitting the electric power to the motor, and an electric motor for reconversion of the electric power into mechanical power. In addition, a constant speed drive is required if constant frequency output is desired; an oil cooler must be provided to dissipate the heat generated by mechanical losses (oil used as cooling medium) and controls are needed for current control. Such a system is shown in Fig. IV-1. For most aircraft applications, the electric transmission system is interconnected with the electric distribution system. This is shown in Fig. IV-2.

For purposes of comparison, an optimum electric system is defined. This is the lightest system capable of fulfilling all power requirements of the design mission. The weight of the system includes the following items:

1. Weight of the system components, (generator, motor, transmission lines, control equipment, fuses, etc.).

2. Additional fuel required by engine to provide the power extracted by the generator or alternator.

3. Additional fuel required by engine to provide power required to overcome any additional drag caused by the transmission system.

4. Any structural weight increase caused by the incorporation of the system into the airplane.

In the system analysis presented in this report the first two items listed above are considered in detail. The other two items are determined primarily by the airplane design concept. They are introduced here as constants which have to be evaluated before the analysis can be carried out.

With the design concepts introduced as constants, the weight of the electric transmission system is dependent upon factors influencing the losses in the system. A large part of these losses are represented by the transmission line losses as given by the voltage drop along the lines. If the cables chosen are too small, the voltage drop will be high and the generator must have a larger capacity since it must supply more power to overcome these losses. If the cables are chosen too large, the losses will decrease, but the cable weight will be disproportionately large. The optimum system, therefore, is one which has a voltage drop that results in the minimum system weight.

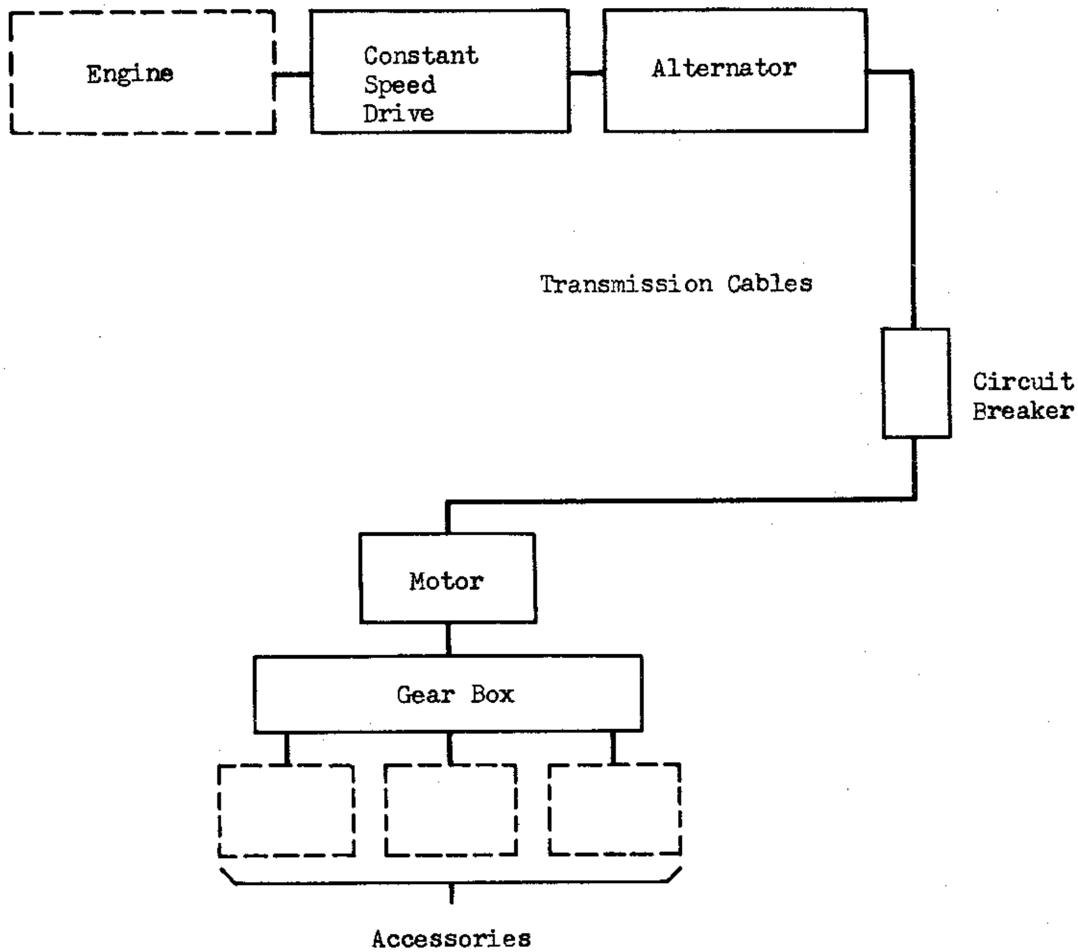


Fig. IV-1 SIMPLE ELECTRIC POWER TRANSMISSION SYSTEM

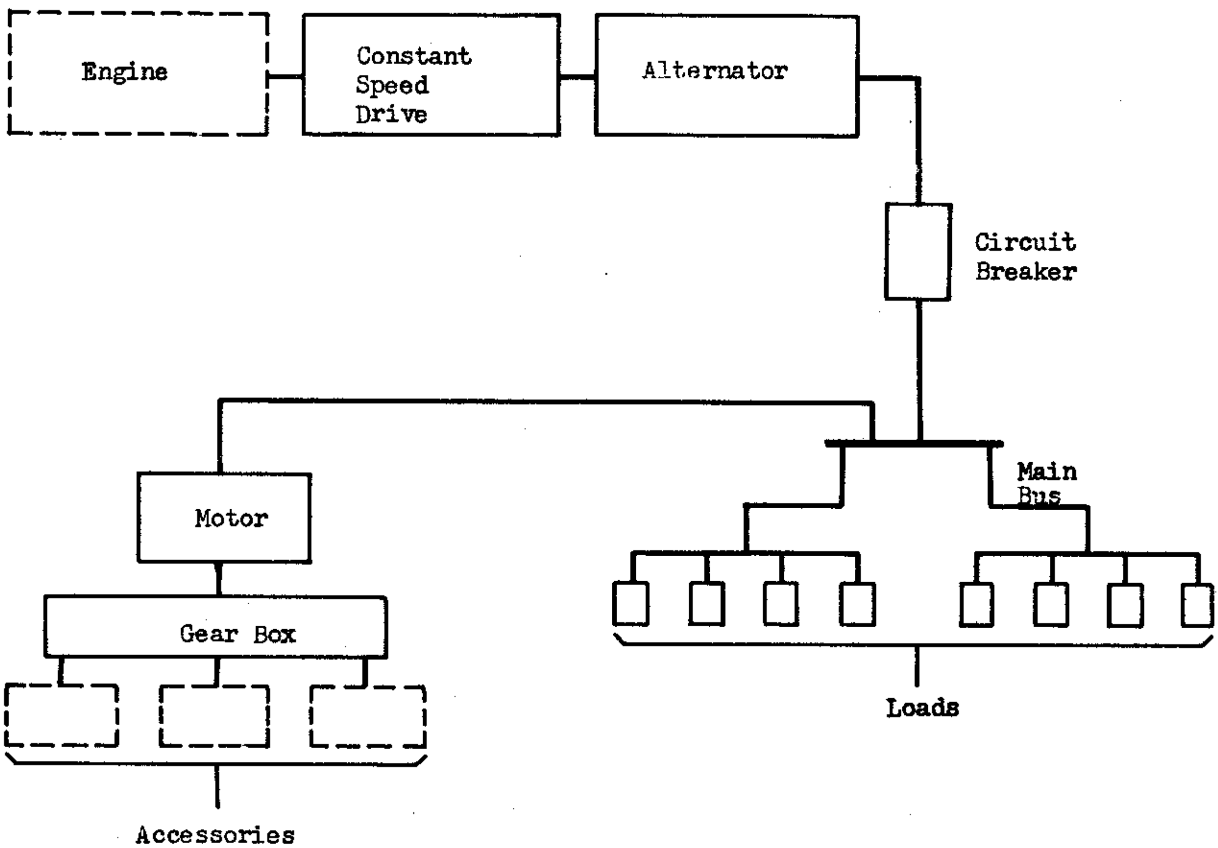


Fig. IV-2 ELECTRIC POWER SYSTEM

If the electric transmission system is interconnected with the distribution system, the maximum allowable voltage drop is dictated by the allowable regulation at the load. In addition to this, the total voltage drop along the cables is limited by the heat dissipating capacity of the cable. That is, the maximum allowable voltage drop per unit length is prescribed for the several cables used for aircraft application. The voltage drop used for the system design cannot exceed the limits imposed by these restrictions. In some cases, a voltage drop smaller than the maximum allowable may be desirable from a system weight viewpoint.

In this section the procedures are outlined for determining the weights of electric systems which may be used in aircraft. These systems are illustrated in Fig. IV-3. They are:

1. Two wire d-c system
2. Grounded d-c system
3. One phase, two wire a-c system
4. One phase, grounded a-c system
5. Three phase, grounded a-c system

The most commonly used systems are the 28 volt direct current system and the 200/115 volt 3 phase alternating current system.

B. Nomenclature

A	cross-sectional area of conductor - ft^2
C_{PX}	thrust correction factor
C'_{PX}	fuel flow correction factor
E	voltage, volt
F_n	thrust of engine, lb
HP	horsepower
I	current, amp
L	length of cable, ft
N	rated speed, rpm
P	power, watt
T	torque, ft-lb
W	weight, lb

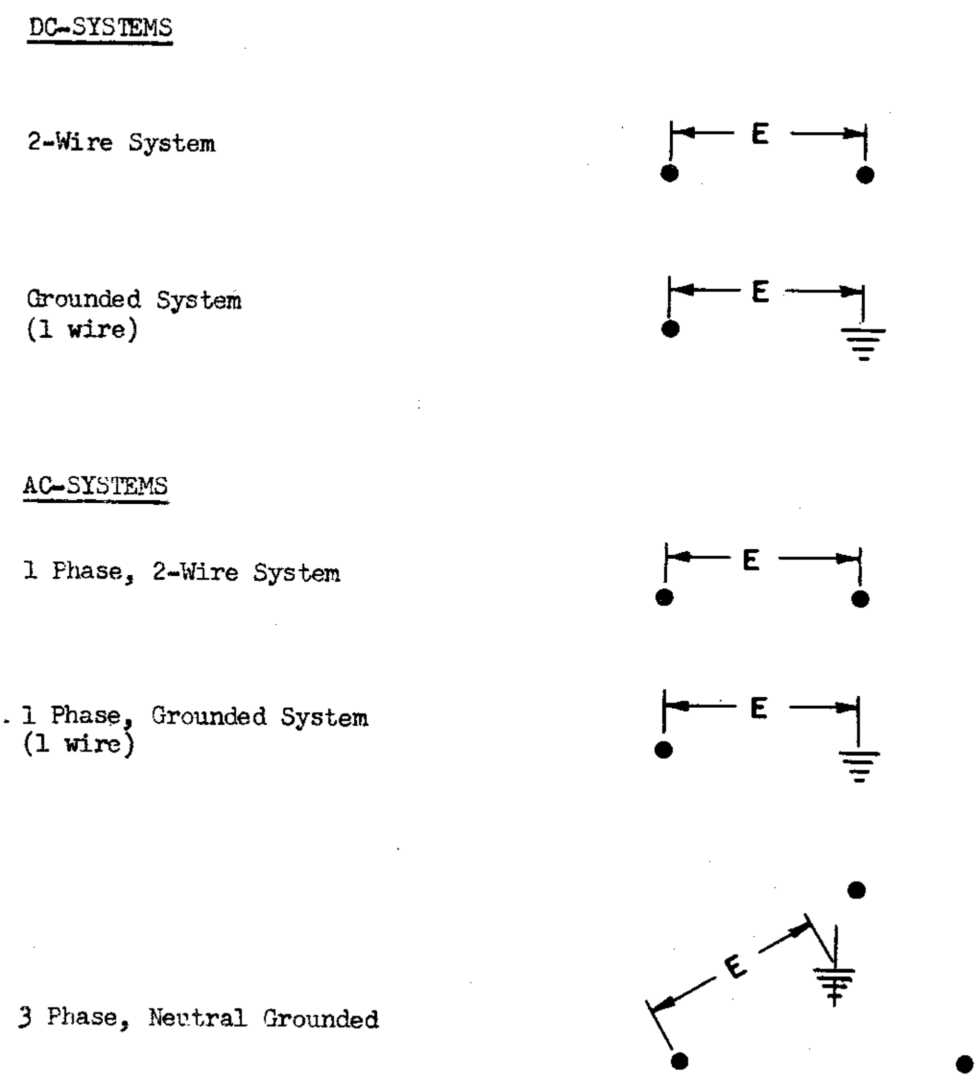


Fig. IV-3 SCHEMATIC DIAGRAM OF ELECTRIC POWER SYSTEMS

n_c number of cables

pf power factor

γ combined specific weight density of cable, lb/ft^3

η efficiency, per cent

ρ resistivity, ohm-ft

τ time, hr

ΣW total weight, lb

ΔE voltage drop, volt

Subscripts

c power control

cs constant speed

F fuel

g generator or alternator

GB gear box

m motor

o output

r rated

C. Required Design Information for the Evaluation of an Electric Transmission System

To determine the optimum electric power transmission system for a given aircraft, the following design information is required:

1. From preliminary design data and airplane specification

Locations of generator or alternator and motors in the airplane

Design mission cruising speed

Design mission cruising altitude

τ Duration of power extraction, hr

P_o Average power extracted from transmission system during design mission

P_{or} Rated power, extracted from drive system

2. From the engine manufacturers' performance specification

F_n Engine thrust at design mission cruising speed and cruising altitude, lb

W_f Fuel consumption rate at thrust, F_n, lb/hr

C_{PX} Thrust correction factor of engine due to power extraction (See Ref. IV-1)

C'_{PX} Fuel flow correction factor (See Ref. IV-1)

HP_{ref} A reference horsepower, arbitrarily taken as 10 per cent of the jet horsepower at sea level static conditions (see Ref. IV-1). This rating is used as a base for comparing engine performance at any operation condition, HP.

3. From the electric equipment manufacturers' data

P_r Rated power of units

η_g Efficiency of generator or alternator

η_m Efficiency of the motor

N Rated or base speed of the unit, rpm

4. Estimated data

The following data must be estimated from the design information:

L Length of the transmission lines, ft. This should be estimated as closely as possible from the airplane configuration and the location of the system components within the airframe, keeping in mind the need of cleaning structural and functional components on the airplane.

5. Calculated Data

The specific fuel consumption of the transmission system, C''_{PX}, must be calculated from the basic engine data. The method used is shown in detail in the hydraulic section (D-4 and -5).

D. Evaluation of an Electric Transmission System

The total weight of an electric power transmission system is the sum of the weight of the elements of the system, that is,

$$\Sigma W = W_t + W_g + W_m + W_F + W_{cs} + W_{GB} + W_c + W_k \tag{IV-1}$$

where:

ΣW = total weight of the system, lb

W_t = weight of the transmission cables, lb

W_g = alternator or generator weight, lb

W_m = weight of motor, lb

W_F = weight of fuel required to provide the power required by the system, lb

W_{cs} = weight of constant speed drive, lb

W_{GB} = weight of gear box, lb

W_c = weight of power controls, lb

W_k = other weight costs chargeable to the system such as fuel required to overcome additional drag, lb

The first four items are functions of the losses in the system, and can therefore be expressed as functions of the voltage drop.

The information required to obtain the weights of the system components is presented here. The derivations of the equations are shown in Part 2 of this report. For the a-c systems it was assumed that the power factor at the alternator is equal to the power factor at the load and that the voltage drop along the transmission line is only due to the resistivity of the lines. In aircraft applications these conditions are very nearly met.

1. Generator or Alternator Weight

The power output of the generator or alternator must be sufficient to supply the required load and the transmission losses. The total output power required by a generator is given by

$$P_{gr} = \frac{P_{or} / \eta_m}{1 - \frac{\Delta E}{E_g}} \tag{IV-2}$$

and from an alternator by

$$P_{gr} = \frac{P_{or} / \eta_m}{1 - \frac{\Delta E}{E_g (pf)}} \tag{IV-3}$$

where:

P_{gr} = rated power output of the generator, watts

P_{or} = rated power delivered to gear box by the motor, watts

η_m = efficiency of motor

E_g = generator voltage (see Fig. IV-3), volt

ΔE = voltage drop between generator and motor, volt

pf = power factor

The weight of the machine can be calculated from the following equation:

$$W_g = 34.5 \left[\frac{P_{gr}}{N} \right]^{0.56} \tag{IV-4}$$

where:

N = the base speed of the generator, rpm

This equation represents an empirical relationship between the generator or alternator weight and the output.

The weight versus power output is plotted from Eq. (IV-4) for various rated speeds. The resulting family of curves are shown in Fig. IV-4. In terms of the voltage drop, the generator weight is given by

$$W_g = 34.5 \left[\frac{P_{or} / \eta_m}{N \left(1 - \frac{\Delta E}{E_g}\right)} \right]^{0.56} \tag{IV-5}$$

and the alternator weight by

$$W_g = 34.5 \left[\frac{P_{or} / \eta_m}{N \left(1 - \frac{\Delta E}{E_g (pf)}\right)} \right]^{0.56} \tag{IV-6}$$

2. <u>Fuel Weight Expressed as a Function of Voltage Drop</u>

The fuel weight is determined by the power extracted from the engine. This can be expressed in terms of the generator output power, and

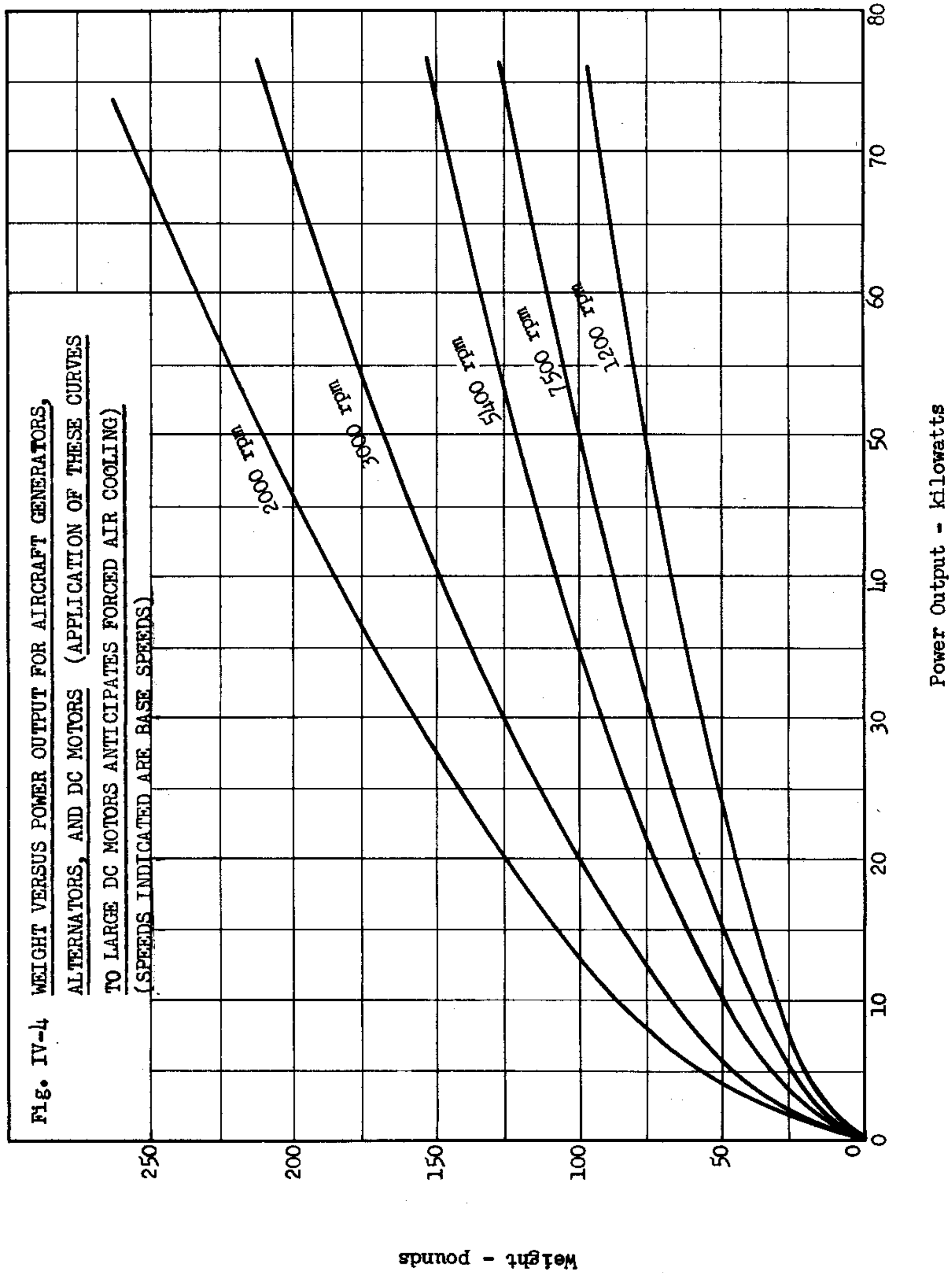


Fig. IV-4 WEIGHT VERSUS POWER OUTPUT FOR AIRCRAFT GENERATORS, ALTERNATORS, AND DC MOTORS (APPLICATION OF THESE CURVES TO LARGE DC MOTORS ANTICIPATES FORCED AIR COOLING) (SPEEDS INDICATED ARE BASE SPEEDS) the generator efficiency as

$$W_F = \frac{1340 \; C''_{PX} \; P_g \; \tau}{\eta_g} \qquad \text{(IV-7)}$$

where:

P_g = power output of the generator, watts

τ = duration of power extraction, hr

1340 = conversion from watts to horsepower

η_g = generator efficiency

C''_{PX} = specific fuel consumption of transmission system, lb/HP hr

In terms of the power at the load and the voltage drop, the fuel weight is given for a generator by

$$W_F = \frac{1340 \; C''_{PX} \; P_o \; \tau}{\eta_m \; \eta_g (1 - \frac{\Delta E}{E_g})} \qquad \text{(IV-8)}$$

and for an alternator by

$$W_F = \frac{1340 \; C''_{PX} \; P_o \; \tau}{\eta_m \; \eta_g \left[1 - \frac{\Delta E}{E_g (pf)}\right]} \qquad \text{(IV-9)}$$

3. Cable Weight Expressed as a Function of Voltage Drop

The weight of the cable or cables is given by

$$W_c = \gamma A L \; n_c = \frac{\gamma L \; I}{\frac{I}{A}} n_c \qquad \text{(IV-10)}$$

where:

γ = combined weight density of cable and insulation (619 lb/ft^3 for copper aircraft cable and 260 lb/ft^3 for aluminum cable)

A = cross-sectional area of conductor - ft^2

L = length of cable, ft

n_c = number of cables in system (one or two wire system)

In terms of the power output and voltage drop this can be expressed as shown below.

For d-c systems,

$$W_c = \frac{\gamma \rho L^2 (P_{or} / \eta_m)\, n_c^2}{E_g \left(1 - \frac{\Delta E}{E_g}\right) \Delta E} \qquad \text{(IV-11)}$$

For 1 phase a-c systems,

$$W_c = \frac{\gamma \rho L^2 (P_{or} / \eta_m)\, n_c^2}{E_g\, (pf) \left[1 - \frac{\Delta E}{E_g (pf)}\right] \Delta E} \qquad \text{(IV-12)}$$

For 3 phase a-c systems,

$$W_c = \frac{\gamma \rho L^2 (P_{or} / \eta_m)}{E_g\, (pf) \left[1 - \frac{\Delta E}{E_g (pf)}\right] \Delta E} \qquad \text{(IV-13)}$$

where

ρ = resistivity, ohm-ft (5.66×10^{-8} for copper wire and 9.0×10^{-8} for aluminum wire)

In case multiple channels are used to supply a load, the channels should be able to supply 25 per cent overload without exceeding the design voltage drop. The load to be supplied by each channel, therefore, becomes $(1.25\, P_{or})$/(no. of channels). This value should be used in the cable weight calculation for multiple channels.

For a single channel system the load to be supplied is P_{or}.

4. Motor Weight

The weight of the motor is a function of the torque requirement. The output torque of the motor is given by

$$T = 5250\, HP_{or} / N \qquad \text{(IV-14)}$$

where

HP_{or} = rated horsepower output of unit, HP

N = rated speed, rpm

T = torque, ft-lb

The weight of the motor required to furnish this torque can be obtained from Fig. IV-5.

5. Weight of the Constant Speed Drive

In the case of an a-c system, it is usually more economical to provide constant frequency current. This arrangement makes it possible to parallel alternators and no auxiliary devices are required to produce constant frequency current for special equipment. The weight of the drive can be obtained from Fig. III-3 (hydraulic section) for a given power rating.

6. Weight of the Gear Box

The weight of the accessory gear box, in the case under consideration, will depend only on the speed of the motor driving the gear box, since the required output speeds and torques are fixed. The method to be used in this weight analysis of a gear box is presented in Section VI of this report.

7. Weight of Generator or Alternator Controls

In order to maintain adequate control of the electric power system certain controls and protective devices are required for each generator used.

Because of standardization of this equipment the combined weight of the control equipment is practically a fixed value for all presently used systems of the same voltage.

The devices used in conjunction with a d-c generator are:

a) Voltage Regulator

b) Field Relay

c) Differential Current Relay

d) Over Voltage Relay

e) Differential Voltage - Reverse Current Relay and Contactor

f) Circuit Breaker

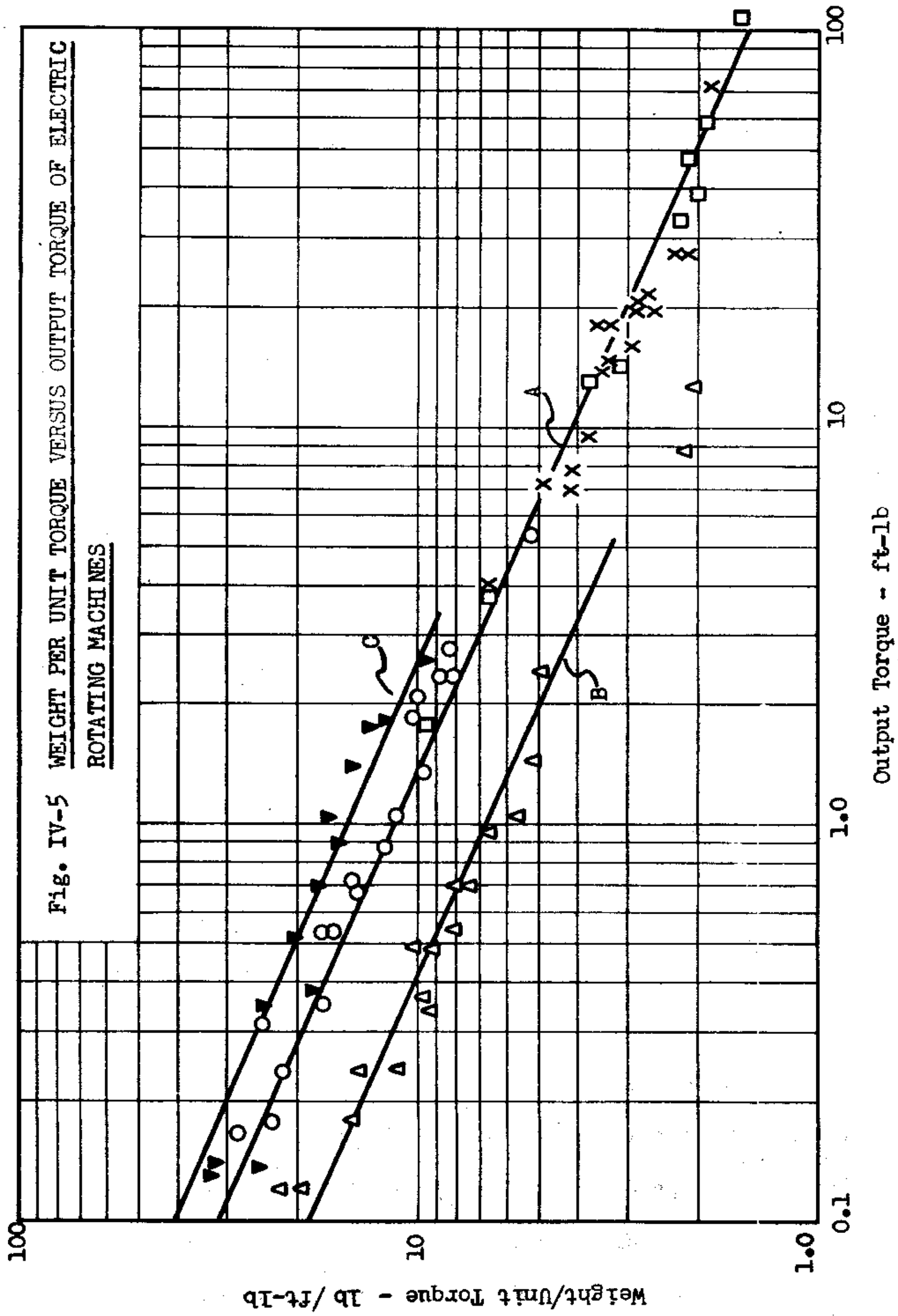


Fig. IV-5 WEIGHT PER UNIT TORQUE VERSUS OUTPUT TORQUE OF ELECTRIC ROTATING MACHINES

CURVE A
- O - DC motor - open
- □ - 400 cycle alternator - self-excited
- x - DC generator - self-excited

CURVE B ▵ - 400 cycle induction motor - open

CURVE C ▼ - V-DC motor - total enclosed

The devices used in conjunction with an alternator are:

a) Voltage Regulator

b) Differential Current Protection Relay

c) Current Transformers

d) Excitor Protection Relay

e) Excitor Control Relay

f) Reactive Power Equalizer Circuit

g) Real Power Equalizer Circuit

h) Circuit Breaker

i) Frequency Control Circuit

The combined weight of this equipment for the systems used is shown in Table IV-1.

TABLE IV-1

Weight of Generator or Alternator Controls

System	Weight - lb
28 - Volt d-c	15.5
115 - Volt d-c	25.8
200/115 - Volt a-c	38.5

E. Determination of Design Voltage Drops

In order to be able to calculate the total system weight from the expressions shown above, the voltage drop must be known. The design voltage drop to be used is the smallest of the following voltage drops:

1) Maximum allowable voltage drop for voltage regulator

2) Maximum allowable voltage drop per foot of cable

3) Voltage drop which minimizes the total weight of the system, or the optimum voltage drop

In most applications the design voltage drop is determined by the regulation requirements. The theoretical optimum voltage drop is usually large and cannot be used in actual applications due to high current densities, which result in large voltage drops per foot of cable.

The voltage regulations required for aircraft applications are tabulated in Table IV-2.

TABLE IV-2

Allowable Voltage Drops

Nominal System Voltage E	Maximum Allowable Voltage Drop Continuous, E	Regulation $\frac{E}{E}$, %
28	1	3.57
115	4	3.48
200	7	3.5

The maximum allowable voltage drop for the cable is determined by the current carried by the cable. The allowable current capacity for copper and aluminum cables is shown in Table IV-3.

The current carried by the cable can be obtained in terms of power output for the several systems considered from Table IV-4. For multiple channels, the current carried by each channel is obtained by using 1.25 P_{or} for the rated load and dividing by the number of channels used.

TABLE IV-3

Current Carrying Capacity for Cables

	Cable Size	Continuous Current Amps: Single Cable in Free Air	Continuous Current Amps: Cables in Conduit or Bundles
Copper	AN - 20	11	7.5
	AN - 18	16	10
	AN - 16	22	13
	AN - 14	32	17
	AN - 12	41	23
	AN - 10	55	33
	AN - 8	73	46
	AN - 6	101	60
	AN - 4	135	80
	AN - 2	181	100
Aluminum	AL - 8	60	36
	AL - 6	83	50
	AL - 4	108	66
	AL - 2	152	90
	AL - 1	174	105
	AL - 0	202	123

TABLE IV-4

Current Required by Electric Systems

DC - Systems	Current Carried in Cables, amp.
2-wire system and grounded system (1 wire)	$I = \dfrac{P_{or} / \eta_m}{E_g \left(1 - \dfrac{\Delta E}{E_g}\right)}$
AC - Systems	
1 Phase, 2-wire system and 1 Phase, grounded system, (1 wire)	$I = \dfrac{P_{or} / \eta_m}{E_g (pf) \left[1 - \dfrac{\Delta E}{E_g (pf)}\right]}$
3 Phase, neutral grounded	$I = \dfrac{P_{or} / \eta_m}{3 E_g (pf) \left[1 - \dfrac{\Delta E}{E_g (pf)}\right]}$

REFERENCES

IV-1. Military Specification MIL - E 5008, Model Specifications for Aircraft, Turbo-Jet Engines, 19 July, 1949.

IV-2. Military Specification MIL - E 7563, Electrical Equipment, Aircraft, Installation of, General Specification for, 1 March 1949.

IV-3. Military Specification, MIL - W 5088, Wiring, Aircraft, Installation of.

ANALYSIS OF THE WEIGHT OF A MECHANICAL POWER TRANSMISSION SYSTEM

Section V

A. Introduction

A schematic diagram of the mechanical power transmission system used as the basis of this analysis is shown in Fig. V-1. It consists of a series of hollow torque tubes and coupling units. The length of the torque tube is dictated by critical speed requirements and is a function of the tube diameter and wall thickness. Each coupling unit (Fig. V-2) consists of two adapters, a flexible coupling, intermediate solid shafts, and two bearings. The adapters serve to reduce the hollow torque tube to a solid shaft at the coupling unit. This avoids excessive size and weight for the bearings, pillow blocks, and the flexible coupling which would be needed for the larger tube diameters. A shaft housing is included, since specifications require such a housing in some cases. If a constant output speed is required, as, for example, when driving an alternator, a constant speed drive must be included since the engine speed varies.

It is assumed that there are "n" coupling units and shaft units in the system, each end of the shaft having the equivalent of one half of a coupling unit.

B. Nomenclature

A cross-sectional area which must be added to standard pillow block to provide clearance for torque tube, in.^{2}

c radial clearance between outside diameter of the shaft and the inside diameter of the shaft housing, in.

C_1 parameter defined by Eq. (V-6)

C_2 parameter defined by Eq. (V-7)

C_{PX} thrust correction factor due to power extraction

C'_{PX} fuel flow correction factor due to power extraction

C''_{PX} specific fuel consumption of transmission system, lb/HP-hr

D_b outside diameter of bearing, in.

D_h inside diameter of housing, in.

D_i inside diameter of torque tube, in.

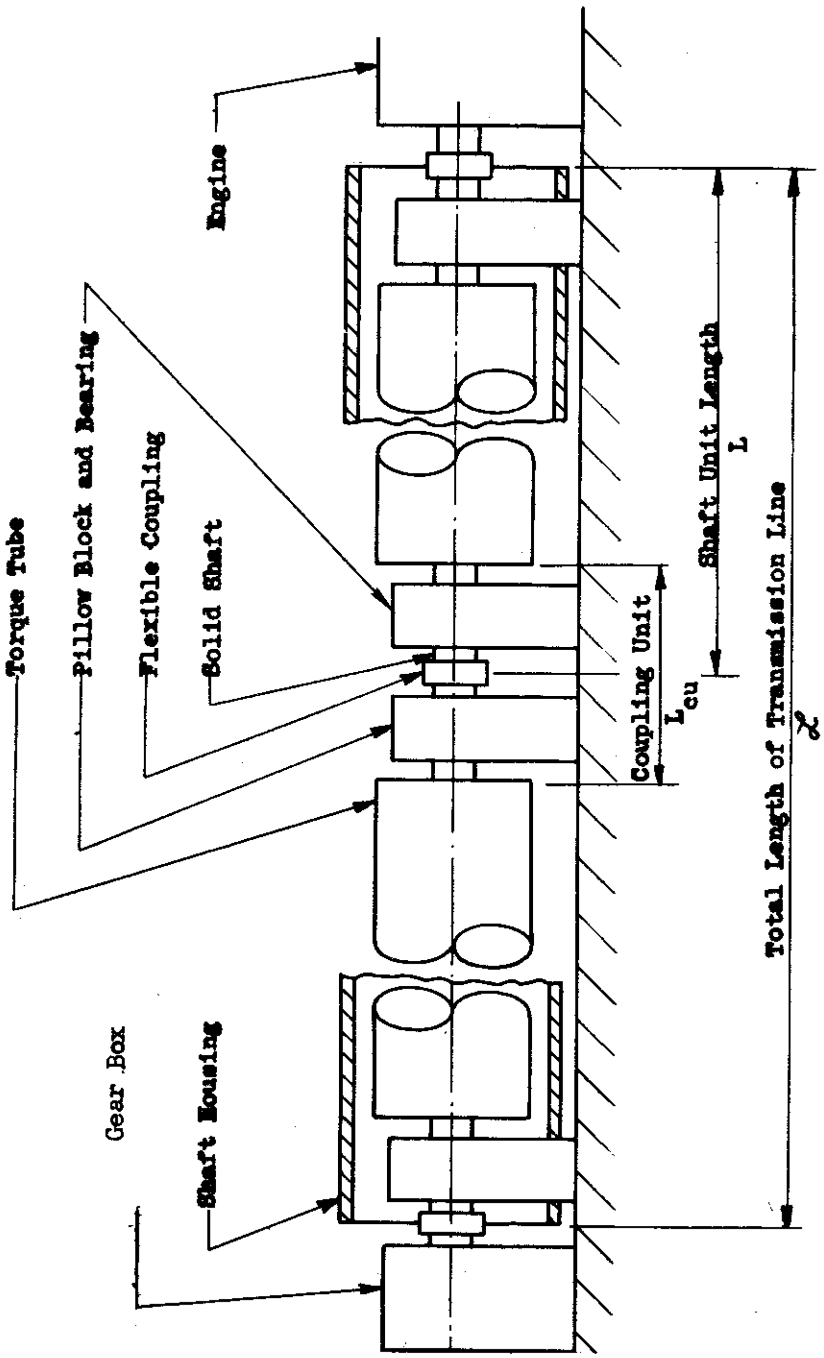


Fig. V-1 SCHEMATIC DIAGRAM OF MECHANICAL POWER TRANSMISSION SYSTEM

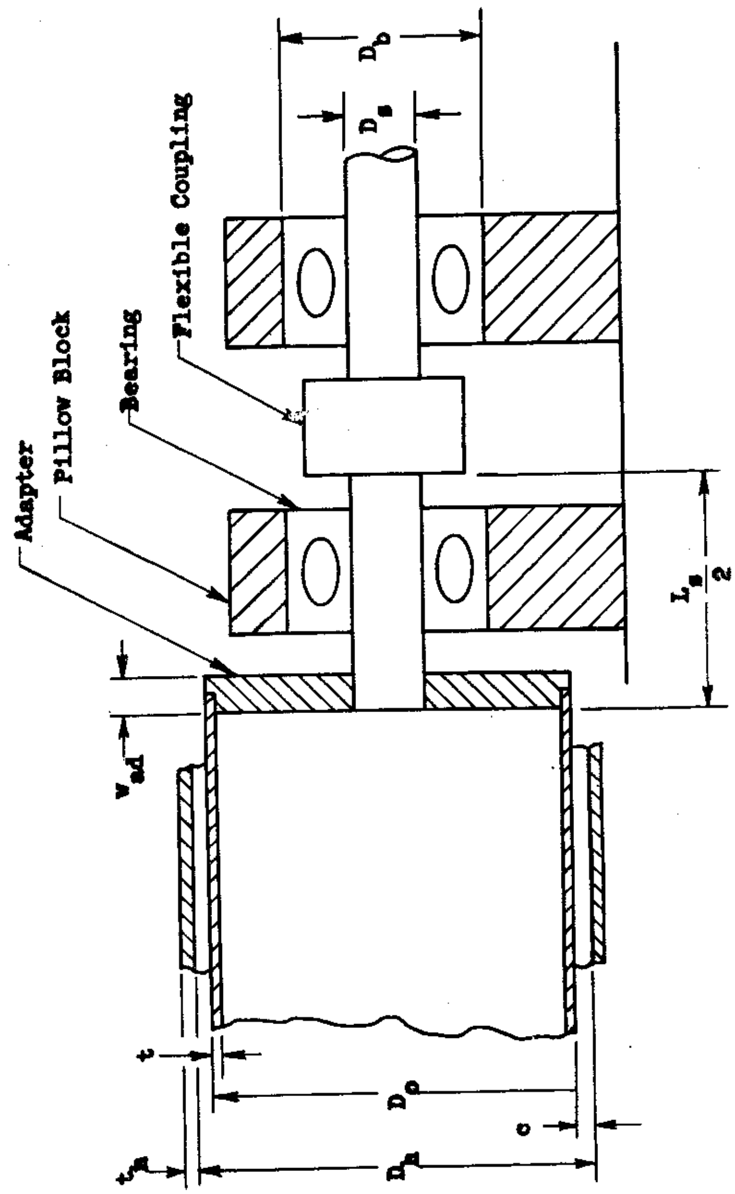


Fig. V-2 SCHEMATIC DIAGRAM COUPLING UNIT COMPONENTS

D_o outside diameter of torque tube, in.

D_s solid shaft diameter, in.

E modulus of elasticity, lb/in.^{2}

F_n engine thrust at design cruise speed and altitude, lb.

g acceleration due to gravity, in./sec.^{2}

HP_r normal rated horsepower output of transmission system, hp

HP_c power requirement from the system at cruise conditions, hp

HP_{REF} reference power of the jet engine arbitrarily chosen as 10 per cent of jet power at sea level static conditions, hp

HP_x horsepower extracted from engine, hp

I rectangular moment of inertia, in.^{4}

K diameter ratio, D_i/D_o

K_t shock factor

$\mathcal{L}$ total distance accessory power to be transmitted, ft

L shaft unit length, in.

L_{cu} length of coupling unit, in.

L_s length of solid shaft, in.

n number of shaft units

N normal rated shaft speed, rpm

N_c shaft speed at cruise conditions, rpm

N_{cr} critical speed of shaft, rpm

R area ratio of pillow block

S_s design shear stress, psi

t practical minimum wall thickness of torque tube, in.

t_h wall thickness of shaft housing, in.

T torque, lb-in.

T_s static torque, lb-in.

w_{ad} width of adapter, in.

w_p width of the pillow block, in.

W torque tube weight, lb

W_b bearing weight, lb

ΣW total weight of transmission system, lb

W_{cs} weight of the constant speed drive, lb

W_{cu} coupling unit weight, lb

W_F weight of fuel required to operate the accessory system for a given time, τ, lb

W_f fuel flow of unburdened engine, lb/hr

W_{fc} flexible coupling weight, lb

W_h weight of shaft housing, lb

W_k Increment of total aircraft weight due to additional structural requirements and to the required fuel to overcome any increased aerodynamic drag chargeable to transmission system, lb

W_p weight of pillow block, lb

W_s intermediate solid shaft weight, lb

α adapter proportionality constant for weight, lb/in^2

ω primary proportionality constant for pillow block weight, $lb/in.^2$

β secondary proportionality constant for pillow block weight, $lb/in.^2$

ε ratio of housing clearance to outside diameter of the shaft $= c/D_o$

η coupling unit efficiency

η_{cs} efficiency of constant speed device

γ weight density of torque tube material, $lb/in.^3$

γ_{ad} weight density of adapter material, $lb/in.^3$

γ_h weight density of shaft housing material, $lb/in.^3$

γ_p weight density of pillow block material, $lb/in.^3$

γ_s weight density of solid shaft material, $lb/in.^3$

τ duration of power extraction, hr

Φ horsepower parameter defined by Eq. (V-2)

ψ specific thrust fuel consumption, lb/lb-hr. [see Eq. (III-4)]

C. Initial Data to be Obtained

The initial data and information required to evaluate a mechanical power transmission system, are listed below.

1. From Flight Profile of Airplane

The flight profile of the airplane will furnish the following data:

cruise speed of airplane

cruise altitude

maximum altitude

approximate duration of power extraction

thrust required from each engine at various flight conditions

2. From Engine Specifications

The engine manufacturer's specifications for the engine to be used on the airplane will provide the following data:

Maximum speed at accessory drive pad on engine (corresponds to maximum engine speed), rpm

Normal rated speed at accessory drive pad on engine (corresponds to normal rated engine speed), rpm

Speed at accessory drive pad on engine under cruise conditions, rpm

C_{PX} thrust correction factor due to power extraction at cruise conditions

C'_{PX} fuel flow correction factor due to power extraction at cruise conditions

HP_{REF} reference horsepower of engine

W_f fuel flow of unburdened engine at cruise conditions, lb/hr

3. From Tentative Specifications for Transmission System

These specifications will supply the following data:

HP_r normal rated horsepower required of transmission system

HP_c horsepower output required of the transmission system at cruise conditions

τ duration of power extraction, hr.

$\mathcal{L}$ distance between main engine and constant speed drive, ft

4. From Design Constants for System Components

Several decisions or assumptions must be made in regard to the type of components and materials to be used in the system. Based on current aircraft practice and the stage of component development, the following must be assumed:

kind of materials for the shaft, pillow blocks, and housing

approximate configuration of coupling unit

type of flexible couplings

type of bearings

type of conversion unit

The design constants should be based on the best information available for these components. If no suitable data are obtainable, reasonable assumptions should be made. The assumptions chosen in the sample calculations may be used as a guide.

The following component design constants are necessary:

L_{cu} length of coupling unit, in.

L_s length of solid shaft, in.

K_t shock factor for shaft

c radial clearance between outside diameter of the shaft and the inside diameter of the shaft housing, in.

N_{cr} minimum critical speed of shaft, rpm (based on maximum shaft speed)

η coupling unit efficiency

η_{cs} efficiency of constant speed device

γ weight density of torque tube material, $lb/in.^3$

γ_{ad} weight density of adapter material, $lb/in.^3$

γ_h	weight density of shaft housing material, lb/in.^{3}
γ_p	weight density of pillow block material, lb/in.^{3}
W_{fc}	flexible coupling weight for various torque ratings, lb
D_b	outside diameter of bearings for the range of shaft sizes under consideration, in.
S_s	design shear stress, psi
t	practical minimum wall thickness of torque tube, in.
t_h	practical wall thickness of shaft housing, in.
w_{ad}	width of adapter, in.
w_p	width of pillow block, in.
W_b	bearing weight for various shaft sizes, lb
W_{cs}	weight of the constant speed drive, lb
γ_s	weight density of solid shaft material, lb/in.^{3}
ω	primary proportionality constant for pillow block weight lb/in.^{2}

D. System Analysis Procedure

For the purposes of this analysis, the transmission system is divided into a number of identical units. Each of these units consists of a hollow shaft having half a coupling unit on each end of the shaft.

The shaft must satisfy two basic requirements. First, the critical speed, N_{cr}, must be equal to or greater than the specified value, and second, the shaft must be strong enough to transmit the required horsepower indicated by the torque parameter, Φ. The critical speed of a hollow shaft is determined by its wall thickness, diameter, and length. The torque capacity of the shaft is determined by the wall thickness and the diameter. It is necessary to establish a combination of the practical minimum wall thickness, t, of the torque tube, the shaft diameter, D, and the shaft unit length, L, such that these two conditions are satisfied.

For a minimum practical wall thickness and a given critical speed, there exists a length which will exactly satisfy the torque requirements. The number of shaft units corresponding to this length is referred to as the critical number, n_{cr}. If more shaft units are used and they are designed for the required critical speed, they are not able to transmit the required torque. If the number of units is less than the critical number, the shafts can transmit more than the required torque.

The optimum number of shafts is defined as the number of shaft units which results in a minimum total weight of the transmission system, including the fuel required to supply the power extracted from the engine. The optimum number of shaft units must be equal or less than the critical number if the shaft satisfies the critical speed and the torque requirements.

The procedure for analyzing a mechanical power transmission system consists of three phases, calculation of the number of shaft units which will satisfy critical speed and horsepower requirements, determination of the optimum shaft diameter, and computation of the minimum total system weight. A step by step procedure is presented for calculating each of these quantities.

To simplify the computation, recurrent parameters and constants such as the torque parameter, Φ, and the auxiliary constants, C_1 and C_2, are evaluated separately or taken from curves. They are then used in subsequent calculations.

1. Calculation of the Number of Shaft Units Which Just Satisfy Critical Speed and Horsepower Requirements, n_{cr}

   The maximum number of shaft units is determined by a trial and error procedure.

   a. Assume a value for n, the number of shaft units.

   b. Compute the shaft unit length

$$L = \frac{\mathcal{L} \times 12}{n} \qquad \text{(V-1)}$$

where

$\mathcal{L}$ = total length of transmission line, ft

   c. Compute the critical speed parameter, $L\sqrt{N_{cr}}$

   d. Calculate the torque parameter, Φ,

$$\Phi = \frac{K_t \; HP_r}{N \; S_s \; \eta_{cs} \; \eta^n} \qquad \text{(V-2)}$$

where

K_t = shock factor

HP_r = normal rated horsepower of system, hp

N = normal rated shaft speed, rpm

S_s = design shear stress of shaft material, psi

η_{cs} = efficiency of the constant speed drive

η = coupling unit efficiency

n = number of shaft units

e. Plot the value of t and $L\sqrt{N_{cr}}$ on Fig. V-3.

At this point the shaft diameter, D_o, is determined such that the critical speed requirement is just satisfied. If this point lies above the Φ value, calculated in step 1-d, the shaft is capable of transmitting the horsepower and the horsepower requirement is satisfied. If this point lies below the Φ value, the shaft is not capable of transmitting the horsepower and the horsepower requirement is not satisfied.

f. Repeat steps a through e until the critical number of shafts (n_{cr}) which will just satisfy the horsepower requirement is determined.

2. Calculation of the Approximate Optimum Torque Tube Diameter

The approximate optimum torque diameter is also determined by a trial and error procedure.

a. Determine solid shaft diameter, D_s, from Fig. V-4 for the torque parameter, Φ , corresponding to n_{cr}.

b. Determine the weight for a unit length of torque tube, $\frac{W}{L}$. The outside diameter of the torque tube, D_o, is taken from Fig. V-3 at the value of $L\sqrt{N_{cr}}$ and "t" used to obtain n_{cr}. The weight per unit length of the torque tube is then determined from Fig. V-5.

c. Determine the adapter proportionality constant, α.

$$\alpha = \frac{\pi \, \gamma_{ad} \, w_{ad}}{4}, \qquad \text{(V-3)}$$

where

γ_{ad} weight density of adapter material, lb/in.^{3}

w_{ad} width of the adapter, in. (see Fig. V-2)

d. Determine secondary proportionality constant, β, for pillow block weight.

When $D_o \leq 1.25\, D_b$,
$\beta = 0$

When $D_o > 1.25\, D_b$,
$\beta = R \gamma_p w_p$ (V-4)

where

R area ratio of pillow block

γ_p weight density of pillow block material, lb/in.^{3}

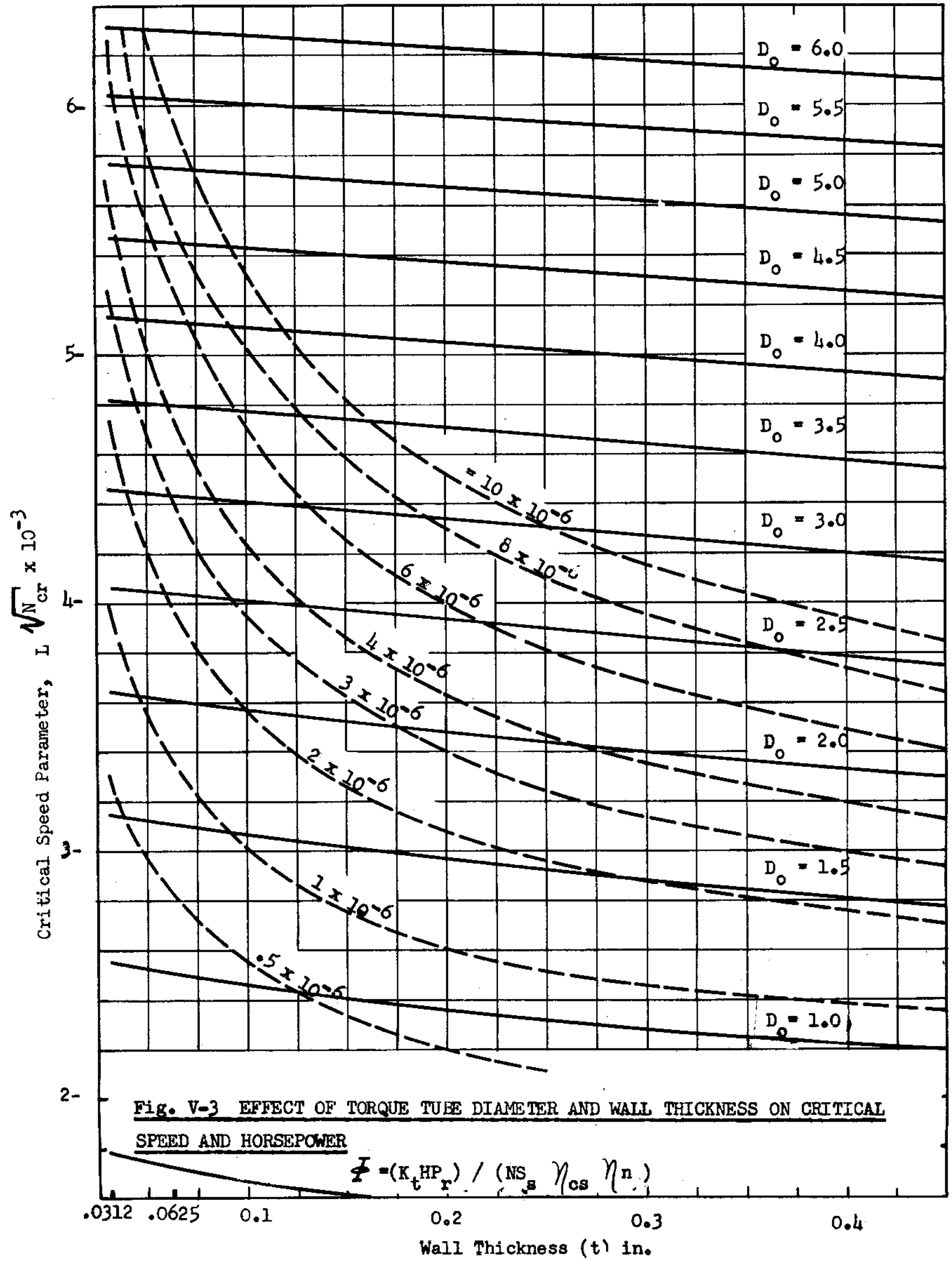
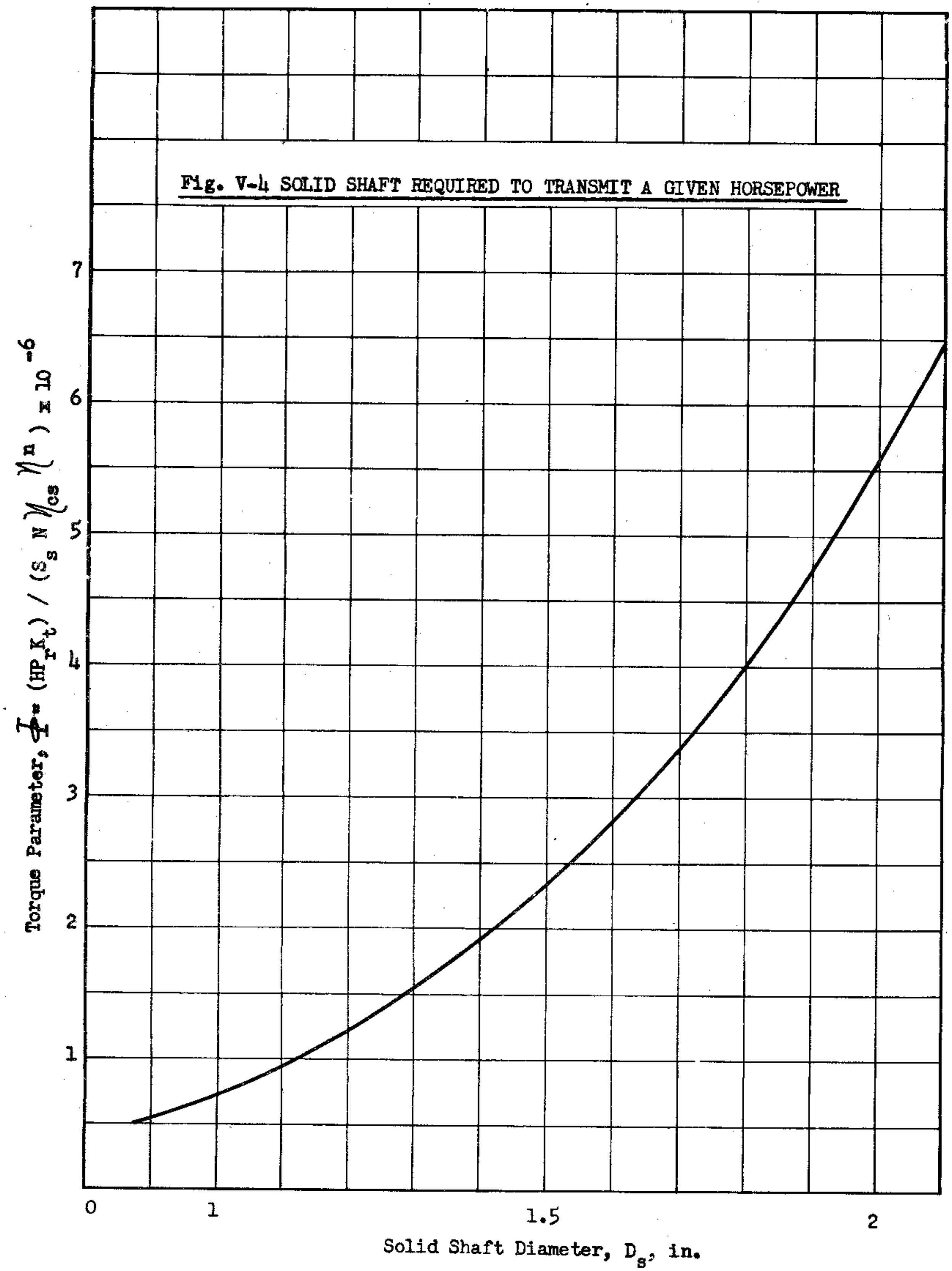


Fig. V-3 EFFECT OF TORQUE TUBE DIAMETER AND WALL THICKNESS ON CRITICAL SPEED AND HORSEPOWER

$\Phi = (K_t HP_r) / (NS_s \eta_{cs} \eta_n)$

Fig. V-4 SOLID SHAFT REQUIRED TO TRANSMIT A GIVEN HORSEPOWER



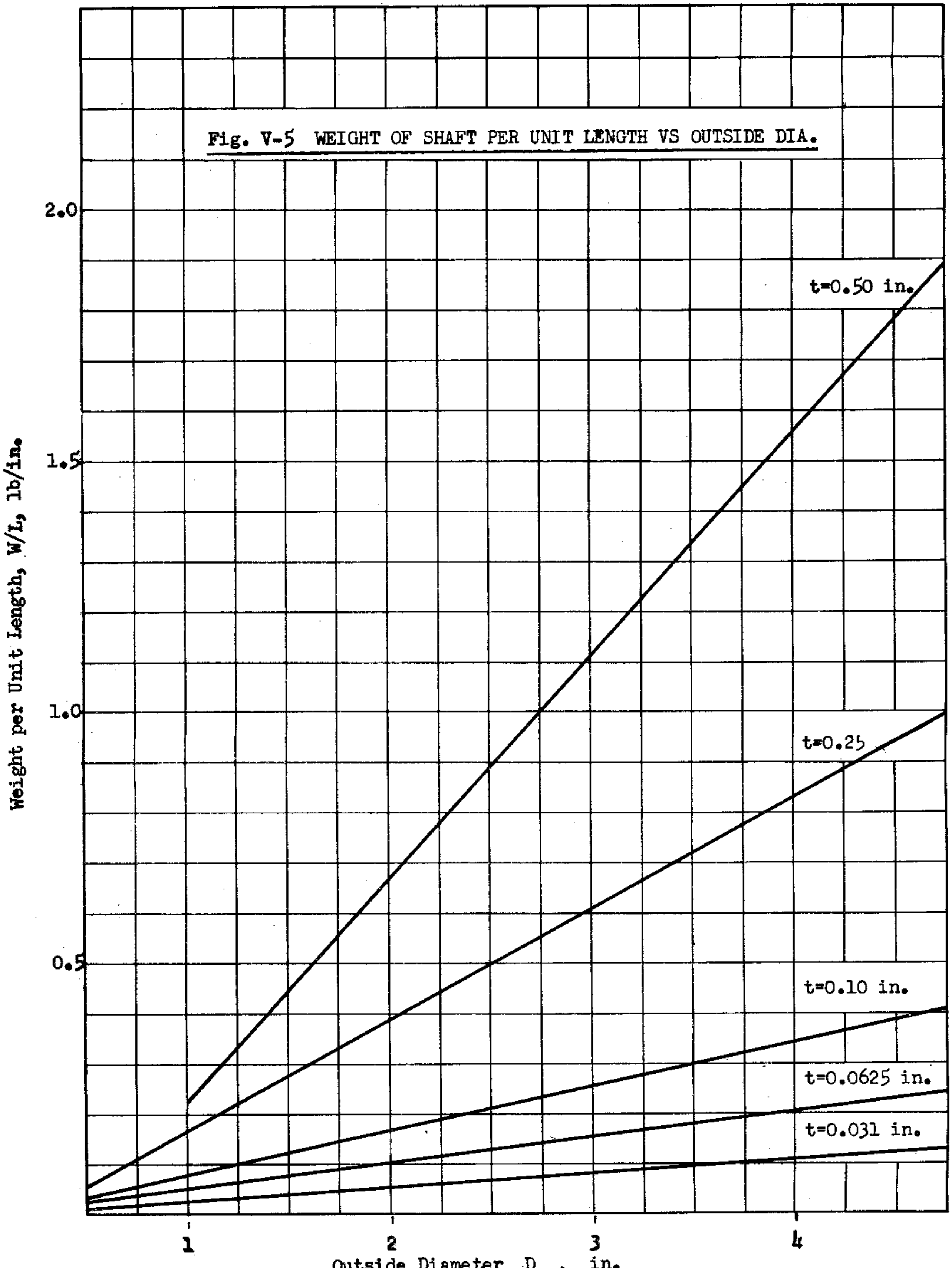


Fig. V-5 WEIGHT OF SHAFT PER UNIT LENGTH VS OUTSIDE DIA.
t=0.50 in.
t=0.25
t=0.10 in.
t=0.0625 in.
t=0.031 in.
Weight per Unit Length, W/L, lb/in.
2.0
1.5
1.0
0.5
1
2
3
4
Outside Diameter D_o , in.

w_p = width of the pillow block, in.

D_o = outside diameter of torque tube, in.

D_b = outside diameter of bearing, in.

The area ratio of the pillow block is found from

$$R = \frac{4A}{\pi (D_o^2 - D_b^2)}$$

where

A is the area that must be added to standard pillow block to provide clearance for large diameter torque tubes (see Fig. V-6).

e. Determine the specific fuel consumption of the accessories, C''_{PX}.

$$C''_{PX} = \frac{\psi \; F_n \; C_{PX} - W_f \; C'_{PX}}{HP_{REF}} \qquad (V\text{-}5)$$

where

C''_{PX} = specific fuel consumption of the transmission system, lb/HP-hr

F_n = engine thrust at design cruise speed and altitude, lb

ψ = specific thrust fuel consumption, lb/lb-hr (see Eq. (III-4))

C'_{PX} = fuel flow correction factor due to power extraction

C_{PX} = thrust correction factor due to power extraction

W_f = fuel flow of unburdened engine, lb/hr

HP_{REF} = reference HP, arbitrarily taken as 10 per cent of jet horsepower at sea level and stationary conditions

All of the factors in the above equation can be obtained from engine specifications.

f. Determine the auxiliary constants C_1 and C_2.

To determine the minimum total weight of the system, the equation for the total system weight is differentiated with respect to the torque tube diameter. The solution of the equation for the torque tube diameter which will give the minimum total system weight is shown in the supplement to this report. In solving this optimum diameter it is convenient to introduce the auxiliary constants, C_1 and C_2. These constants are defined as:

<u>Case I</u>: $D_o \leq 1.25\ D_b$

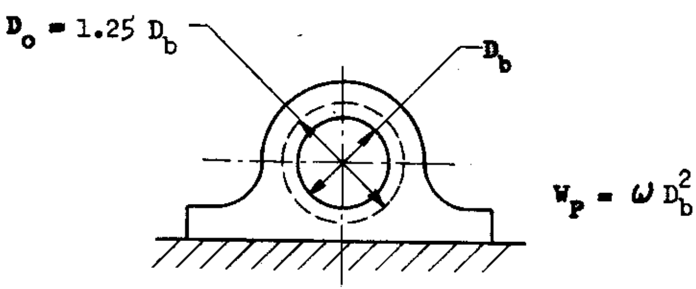


Fig. 10a

<u>Case II</u>: $D_o > 1.25\ D_b$

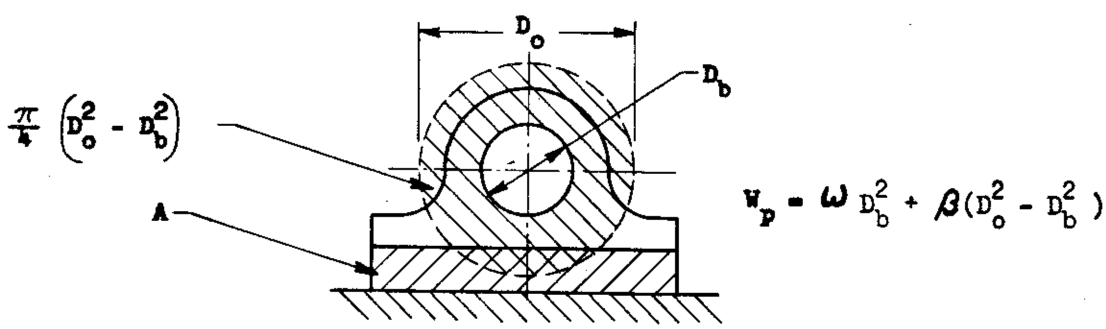


Fig. 10b

Fig. V-6 **SCHEMATIC DIAGRAM OF PILLOW BLOCK CONFIGURATIONS**

$$C_1 = \frac{6(\alpha + \beta)\eta^n}{\left\{2 W_b + W_{fc} + \left[\frac{\pi \gamma_s L_s D_s^2}{4} - \frac{W}{L} L_{cu}\right] - 2\left[\alpha D_s^2 + (\beta - \omega) D_b^2\right]\right\}\eta^n - \frac{HP_r C''_{PX} \tau \ln \eta}{\eta_{cs}}} \qquad \text{(V-6)}$$

$$C_2 = \frac{136.9\eta^n \left[\gamma t + \gamma_h t_h (1 + 2\varepsilon)\right]\sqrt{1 - \frac{t}{D_o}}}{\left\{2 W_b + W_{fc} + \left(\frac{\pi \gamma_s L_s D_s^2}{4} - \frac{W}{L} L_{cu}\right) - \left[2\alpha D_s^2 + (\beta - \omega) D_b^2\right]\right\}\eta^n - \frac{HP_r C''_{PX} \tau \ln \eta}{\eta_{cs}}} \qquad \text{(V-7)}$$

All of the factors in these two equations have been determined previously.

g. Determine the approximate optimum torque tube diameter, $(D_o)_{opt}$. On Fig. V-7 locate the lines of constant auxiliary constants C_1 and C_2 calculated above. The intersection of these two integral curves determines the value for the approximate optimum torque tube diameter, $(D_o)_{opt}$.

h. Determine the optimum number of shaft units, n_{opt}. From Fig. V-3 at the optimum diameter, $(D_o)_{opt}$ and the practical minimum wall thickness, t, obtain $(L\sqrt{N_{cr}})_{opt}$. Then,

$$n_{opt} = \frac{\mathcal{L} \times 12}{L_{opt}} \qquad \text{(V-8)}$$

i. Compare the optimum number of shaft units, n_{opt}, to the critical number of shaft units, n_{cr}. The value of n_{opt} will be equal to, greater than, or less than n_{cr}.

If $n_{opt} = n_{cr}$, the value of D_o corresponding to n_{cr} is the optimum torque tube diameter.

If $n_{opt} > n_{max}$, the optimum torque tube diameter lies below the

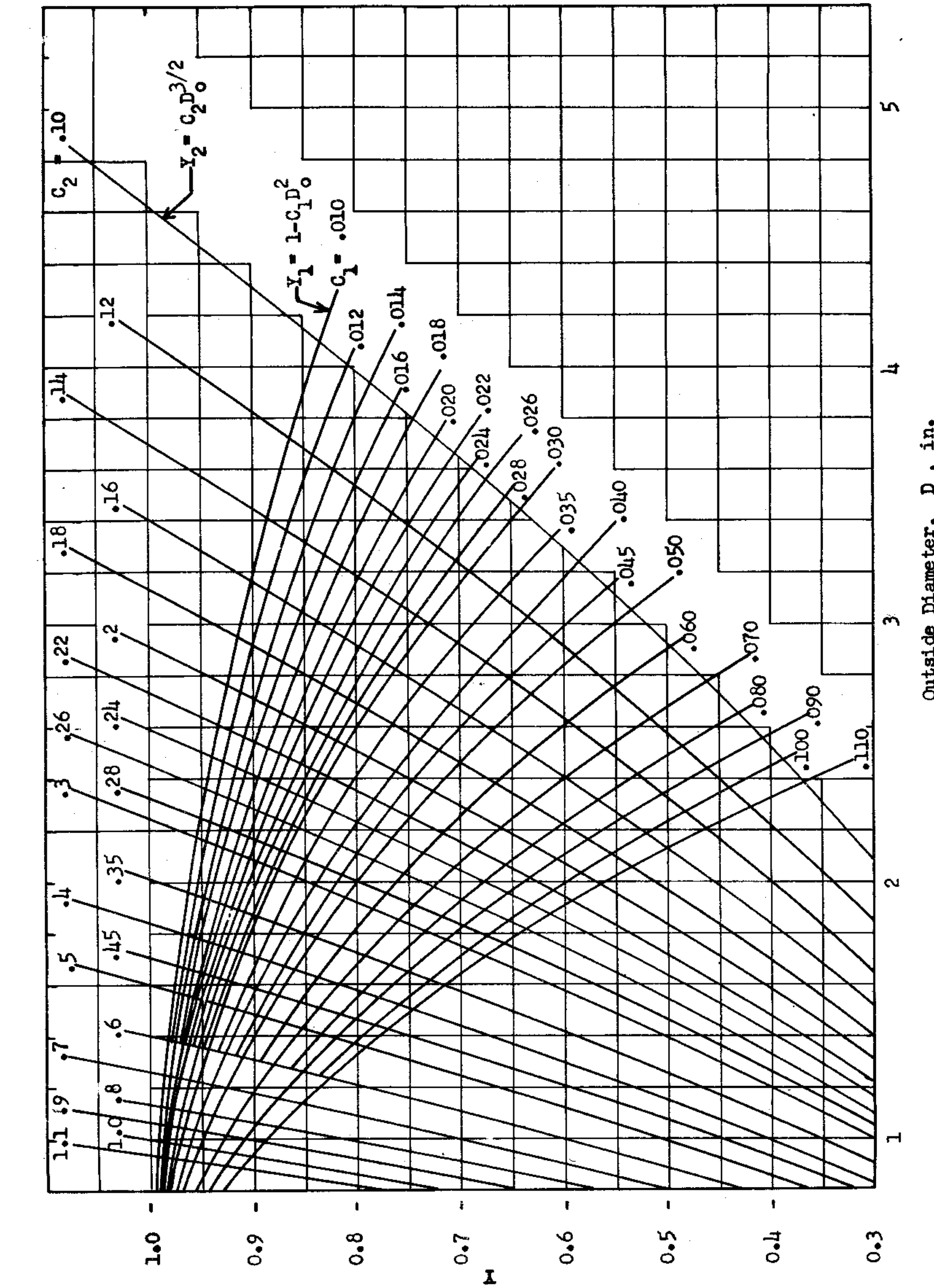


Fig. V-7 GRAPHICAL SOLUTION OF $C_2 D_o^{3/2} = 1 - C_1 D_o^2$

corresponding line of constant torque parameter, Φ , and, therefore, the torque tube is unable to transmit the required horsepower. In this situation, the torque tube diameter corresponding to n_{cr} should be used.

If $n_{opt} < n_{max}$, the optimum torque tube diameter lies above the corresponding line of constant torque parameter, Φ. In this case it is necessary to assume a new value of $n < n_{cr}$, and calculate a new n_{opt}. If the new n_{opt} agrees with the assumed n within one shaft unit, the D_o corresponding to the new n_{opt} is the approximate optimum torque tube diameter. If the new n_{opt} does not agree with the assumed n, another n must be assumed, and the process repeated until the calculated value agrees with the assumed value within one shaft unit.

3. Calculation of Minimum Total Weight of the System

The calculated value for the optimum number of shaft units, n_{opt}, is usually a fraction. Therefore, the total weight of the system is calculated twice, for the torque tube diameter corresponding to the next whole number of shaft units above the calculated n_{opt}, and for the torque tube diameter corresponding to the next whole number of shaft units below the calculated n_{opt}. The lowest of the two total weight values is the approximate minimum total weight of the system.

The total weight of the system is given by:

$$W = n\left[\frac{W}{L} L + W_{cu}\right] + W_F + W_h + W_{cs} + W_k + W_{GB} \qquad (V\text{-}9)$$

where

n = number of shaft units

W = weight of one shaft unit, lb

L = length of shaft unit, in.

W_{cu} = weight of coupling unit, lb

W_F = weight of fuel required to operate the accessory system for a given time, τ, lb

W_h = weight of shaft housing, lb

W_{cs} = weight of the constant speed drive, lb (see Fig. III-3)

W_k = increment of total aircraft weight due to additional structural requirements and to the fuel required to overcome any increased aerodynamic drag chargeable to transmission system, lb

W_{GB} = weight of the gear box, lb (see Section VI)

Each of the terms in the above equation is evaluated as follows:

a. The torque tube weight per unit length, $\frac{W}{L}$.

The torque tube weight is found from Fig. V-5 for the practical minimum wall thickness, t, and the optimum torque tube diameter, $(D_o)_{opt}$.

b. The coupling unit weight, W_{cu}

The coupling unit weight, W_{cu}, is the sum of the component weights as follows: (See Fig. V-2)

$$W_{cu} = 2W_{ad} + 2W_p + 2W_b + W_{fc} + W_s \qquad \text{(V-10)}$$

where

W_{ad} = weight of the adapter, lb

W_p = weight of the pillow block, lb

W_b = weight of the bearing, lb (from manufacturer's data)

W_{fc} = weight of the flexible coupling, lb (from manufacturer's data)

W_s = weight of solid, intermediate shaft, lb

These coupling unit component weights are determined as shown below. The weight of the adapter is found from:

$$W_{ad} = \alpha(D_e^2 - D_s^2) \qquad \text{(V-11)}$$

where

$$\alpha = \frac{\pi \gamma_{ad} w_{ad}}{4}$$

D_s diameter of intermediate solid shaft, in.(from Fig V-4)

γ_{ad} weight density of adapter material, lb/in.^{3} (from manufacturer's data)

w_{ad} width of the adapter, in. (see Fig. V-2) (from manufacturer's data)

The equation for the pillow block weight is:

$$W_P = \omega D_b^2 + \beta (D_o^2 - D_b^2) \frac{\pi}{4} \tag{V-12}$$

where

D_b outside diameter of bearing, in. (from manufacturer's data)

ω primary proportionality constant for pillow block weight determined from the design of the particular pillow blocks, lb/in.^{2}

β secondary proportionality constant for pillow block weight determined as shown in section 2-d, lb/in.^{2}

The weight of the solid intermediate shaft is determined from the following equation:

$$W_s = \frac{\pi \gamma_s D_s^2 L_s}{4} - \frac{W}{L} L_{cu} \tag{V-13}$$

where

γ_s weight density of solid shaft material, lb/in.^{3}

L_s length of solid shaft determined by the coupling unit configuration, in.

L_{cu} length of coupling unit determined by the coupling unit configuration, in.

The last term in the above equation is a correction factor, taking into account the fact that the torque tube does not extend over the full length, L. (see Fig. V-1)

c. The fuel weight, W_F.

The fuel weight is obtained from the following equation:

$$W_F = \frac{C''_{PX} HP'_c}{\eta_{cs} \eta^n} \tag{V-14}$$

where

τ duration of power extraction, hr

C''_{PX} specific fuel consumption of accessories, lb/HP-hr (determined as shown in section 2-e)

HP_c horsepower required by accessories under cruise conditions

d. The shaft housing weight, W_h.

The housing weight is found from:

$$W_h = 37.7\, \gamma_h\, t_h\, (1 + 2\,\mathcal{E})\mathcal{L} D_o \qquad \text{(V-15)}$$

where

γ_h weight density of housing material, lb/in.^{3}

t_h wall thickness of housing, in. (assumed)

$\mathcal{E}$ ratio of housing clearance to outside diameter of torque tube, $\frac{c}{D_o}$. The clearance, c, is assumed.

E. Sample Calculation

The procedure for determining the minimum total weight of the shaft system is illustrated by the following sample calculation.

1. Initial Data

The following conditions are assumed for an aircraft operating at cruise conditions:

Speed of aircraft	400 knots
Altitude	35000 ft
Engine thrust	3000 lbs
Engine performance	as shown in Ref. 7

The following specifications for the transmission system are assumed:

normal rated power required by accessories, HP_r	50 HP
normal rated speed of shaft, N	6000 rpm
maximum shaft speed, N_{max}	10,000 rpm

minimum critical speed, N_{cr}	14,000 rpm
duration of power extraction, τ	
case (a)	2 hr
case (b)	20 hr
distance between main engine and gear box, $\mathcal{L}$	60 ft
shock factor, K_t	1.5
design stress for steel shaft, S_s	25,000 psi
density of torque tube material, γ	0.284 lb/in.^{3}
practical minimum thickness, t, for torque tube	0.062 in.
shaft housing thickness, t_h	0.062 in.
density of shaft housing material, γ_h	0.284 lb/in.^{3}
density of intermediate, solid shaft material, γ_s	0.284 lb/in.^{3}
efficiency of constant speed drive, η_{cs}	0.85
coupling unit efficiency, η	0.97
width of adapter, w_{ad}	0.5 in.
area ratio, R	0.125
length of solid shaft, L_s	4.0 in.
coupling unit length, L_{cu}	6 in.
diameter of the bearing, D_b	2 x D_s
proportionality constant for the adapter weight, α	0.111 lb/in.^{2}
proportionality constants for pillow block weight, β	0.223 lb/in.^{2}
β (when $D_o \leq 1.25\ D_b$)	0
ω	0.565 lb/in.^{2}
bearing weight	1.0 lb/in of D_s
flexible coupling weight	4.5 lb
specific fuel consumption of accessories, C''_{PX}	0.5 lbs/HP-hr
ratio of housing clearance to outside diameter of shaft, ε.	0.06

2. Determination of Critical Number of Shaft Units

a. Assume a value for n

$n = 25$

b. Compute the shaft unit length

$$L = \frac{60 \times 12}{25} = 28.8 \text{ in.}$$

c. Compute $L\sqrt{N_{cr}}$

$$L\sqrt{14{,}000} = 3.41 \times 10^3$$

d. Calculate the parameter, Φ, from Eq.(V-2)

$$\Phi = \frac{1.5 \times 50}{6000 \times 25{,}000\ (0.97)^{25}\ (0.85)}$$

$$= 1.26 \times 10^{-6}$$

e. Plot items c and d on Fig. V-3 at t= 0.062

It is seen that the line of $\Phi = 1.26 \times 10^{-6}$ crosses the t = 0.062 line at $L\sqrt{N_{cr}} = 3.54 \times 10^3$. Step c indicates that a 25-unit system requires $L\sqrt{N_{cr}} = 3.41 \times 10^3$, and, thus, will not transmit the required horsepower.

f. Assume a new value for n

$n = 24$

g. Compute new shaft length

$$L = \frac{60 \times 12}{24}$$

$$= 30 \text{ in.}$$

h. Compute new $L\sqrt{N_{cr}}$

$$L\sqrt{14{,}000} = 3.54 \times 10^3$$

i. Calculate new parameter, Φ

$$\Phi = \frac{1.5 \times 50}{6000 \times 25{,}000\ (0.97)^{24}\ (0.85)}$$

$$= 1.22 \times 10^{-6}$$

j. Plot items h and i on Fig. V-3 at t = 0.062 in.

It is seen that the new $L\sqrt{N_{cr}}$ lies above the intersection of

$\Phi = 1.22 \times 10^{-6}$, and $t = 0.062$. Thus, a 24-unit system will satisfy the horsepower requirements, whereas a 25-unit system will not. Therefore,

$$n_{cr} = 24 \text{ units.}$$

This value of n is used as a first approximation for n in calculating the optimum torque tube diameter.

3. Determination of Approximate Optimum Torque Tube Diameter

a. Determine solid shaft diameter

From Fig. V-4 at $\Phi = 1.22 \times 10^{-6}$

$$D_s = 1.20 \text{ in.}$$

b. Determine weight for unit length of torque tube

The D_o of the torque tube is taken from Fig.V-3 at $L\sqrt{N_{cr}} = 3.54 \times 10^3$ and $t = 0.062$ in.

$$D_o = 1.94 \text{ in.}$$

The weight per unit length of the torque tube, W/L, is determined from Fig. V-5 at $D_o = 1.94$ in. and $t = 0.062$ in.

$$\frac{W}{L} = 0.1 \text{ lbs/in.}$$

c. Calculate Parameter, C_1

For the assumed conditions and $\tau = 2$ hours, Eq.(V-6) gives:

$$C_1 = 0.0368$$

d. Calculate Parameter, C_2

For the assumed conditions and $\tau = 2$ hours, Eq.(V-7) gives:

$$C_2 = 0.280$$

e. Determine approximate optimum torque tube diameter

From Fig. V-7 for:

$C_1 = 0.0368$

$C_2 = 0.280$

$(D_o)_{opt} = 2.08$ in.

f. Calculate optimum number of shaft units, n_{opt}.

From Fig. V-3 determine $L\sqrt{N_{cr}}$ at t = 0.062 in. and D_o = 2.08 in.

$$L\sqrt{N_{cr}} = 3.67 \times 10^3$$

Then,

$$L = 31 \text{ in.}$$

and $n_{opt} = 23.2$

This value of n_{opt} is within one shaft unit of the n_{cr} found in step 2 - j, above.

Repeating steps a to f for τ = 20 hours gives:

$(D_o)_{opt} = 3.00$ in.

$n_{opt} = 19.15$

F. Determination of Approximate Minimum Total Weight

Calculate the total weight for the next larger and the next smaller whole number of shaft units from Eq. (V-9).

For τ = 2 hr:

$n = 23$	$n = 24$
$L = 31.3$ in.	$L = 30$ in.
$D_o = 2.11$ in.	$D_o = 1.95$ in.
$D_s = 1.19$ in.	$D_s = 1.20$ in.
$\Sigma W = 690$ lbs	$\Sigma W = 696$ lbs

For τ = 20 hr:

$n = 20$	$n = 19$

$L = 36$ in.	$L = 37.9$ in.
$D_o = 2.78$ in.	$D_o = 3.07$ in.
$D_s = 1.15$ in.	$D_s = 1.14$ in.
$\Sigma W = 1670$ lbs	$\Sigma W = 1680$ lbs

Thus, for the 2-hour system the approximate minimum total weight is 690 lbs.

For the 20-hour system the approximate minimum total weight is 1670 lbs.

The results and pertinent design data for this sample calculation are tabulated in Table V-1.

Table V-1

TABULATION OF PERFORMANCE DATA FOR A MECHANICAL ACCESSORY POWER TRANSMISSION SYSTEM

Design Data		2-hour system	20-hour system
τ	Duration of power extraction (hr)	2	20
HP_o	Output horsepower	50	50
$(D_o)_{opt}$	Torque tube diameter, in.	2.11	2.78
n	Number of shaft units	23	20
$\Sigma W-W_F$	Installed weight, lb.	571	583
W_F	Weight of fuel, lb.	119	1087
W_F/τ	Fuel rate for accessories, lb/hr	59.5	54.35
ΣW	Min. total weight of system, lb.	690	1670

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V-3 Vallance and Doughtie, Design of Machine Members, McGraw Hill Book Company, 1943.

V-4 Army Air Force Specification No. X-28474-A, May 8, 1946.

V-5 Army Air Force Specification No. MIL-S-7470, January 13, 1953.

V-6 Army Air Force Specification No. MIL-S-7471, January 13, 1953.

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ANALYSIS OF THE WEIGHT OF AIRCRAFT

ACCESSORY GEAR BOXES

Section VI

A. Introduction

Aircraft accessory gear boxes are generally designed to satisfy the particular requirements of a specific installation. These designs result in a wide variety of gear boxes, since the number of drives, the configuration of the gear box housing, center distances, and flange and drive sizes depend on the requirements of the individual application. Also, many accessory gear boxes are incorporated in the accessory unit, as in the case of most pneumatic turbines.

In the evaluation of aircraft accessory power transmission systems, it is desired to estimate the weight of the required gear boxes. The basic type of gear box is a single-step speed reducer with a single drive. A method for determining the approximate weight of this type of gear box is developed in this section.

When necessary, the weight of a multi-drive gear box can be obtained by the same method of analysis; however, a considerable amount of specific design information in regard to the particular installation is required.

1. Description of Single-Step Speed Reducer

A schematic diagram of a single-step speed reducer is shown in Fig. VI-1. The gears are mounted on hollow shafts and connected to driving and driven units by means of splines and standard AND drive and flange. The casing is just large enough to contain the gears and bearings. The size of the drive and flange is dictated by the torque and overhung moments of the driving unit and the accessory upon which the gear box is mounted.

2. Method of Analysis

For the weight analysis, the gear box in Fig. VI-1 is modified to look like the gear box shown in Fig. VI-2. All of the parts are lumped into three major equivalent components: the gears, the housing, and the drive and flange. These components are analyzed individually to determine their weight. The total weight of the gear box is the sum of the equivalent component weights.

For the type gear box under consideration the gear box total weight decreases with a decrease in pinion diameter. Thus, the smallest pinion which will transmit the required torque should be used to obtain minimum weight. Therefore, the gear box should be designed with the smallest possible pinion diameter which will transmit the required torque.

The smallest permissible size of the gears is first determined from strength considerations. The size of the gears determines the size of the casing. The type of AND Standard drives and flanges selected is determined by the torque transmitted and the overhung moment imposed on the gearbox. The size of non-standard drives and flanges is determined in a similar manner.

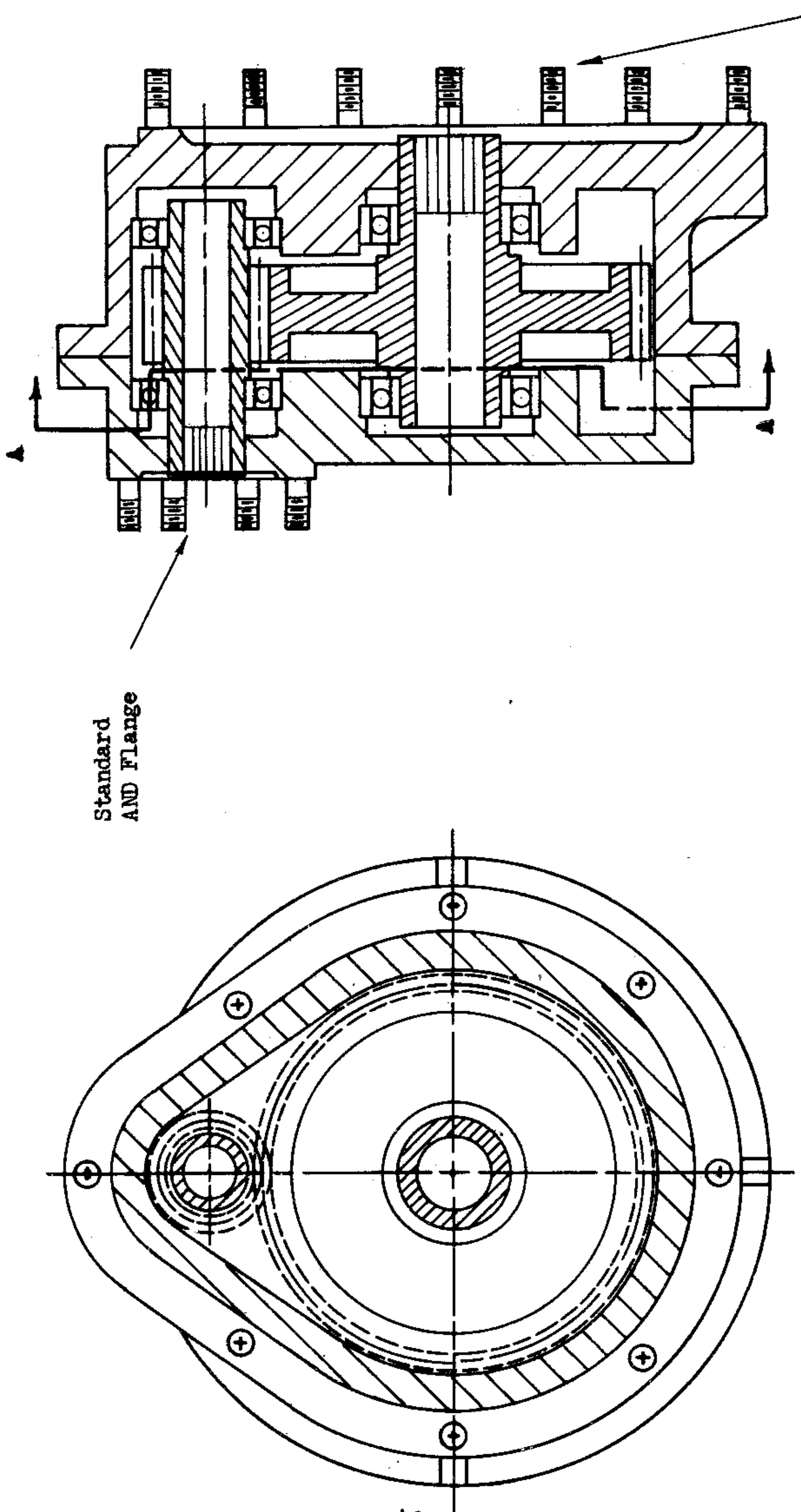


Fig. VI-1 SCHEMATIC DIAGRAM OF SINGLE-STEP SPEED REDUCER

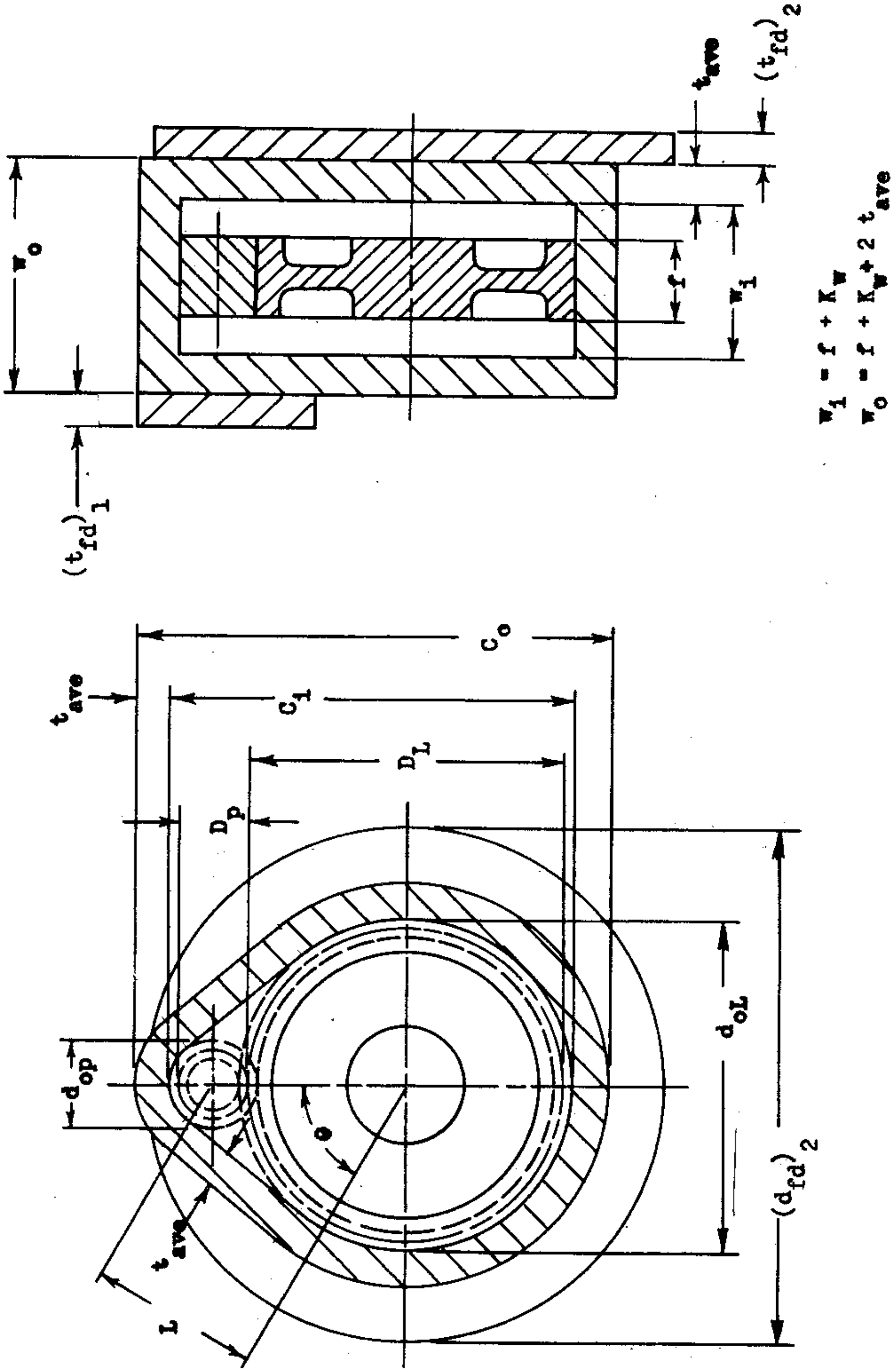


Fig. VI-2 SINGLE-STEP SPEED REDUCER MODIFIED FOR WEIGHT ANALYSIS

Once the size of these equivalent components is determined, their volume and weight can be computed from geometrical considerations.

B. Nomenclature

A	projected area perpendicular to the centerline of the gears, $in.^2$
a	weight factor
C	height of gear box, in.
c	hole factor
D	pitch diameter, in.
d	diameter, in.
F	tangential force at pitch line, lb
F(R)	function determined from the geometry of the projected areas of the gears
f	face width of the gears, in.
h	spline addendum, in.
HP	horsepower, hp
j	scale factor
K_t	shock factor for shaft
K_w	housing width factor, in.
L	length of line tangent to outside diameter of both gears, in.
m	minimum thickness of material above keyway, in.
N	speed, rpm
n	number of gear teeth
P	diametral pitch, 1/in.
R	gear reduction ratio
S	number of spline teeth
(SF)	service factor for gears
s_w	working stress for gear teeth, psi
s_s	design stress for shaft, psi
T	torque, lb-in.

t	thickness, in.
V	pitch line velocity, ft/min.
W	weight, lb/
w	width of gear housing, in.
Y	form factor for Lewis Equation
z	diametral pitch x addendum
Φ	pressure angle of gears, degrees
γ	weight density, lb/in.^{3}
θ	angle between line through gear centers and a line perpendicular to common tangent of the outside diameters of the gears, degrees

SUBSCRIPTS

1	input
2	output
ave	average
e	equivalent
G	gears (both pinion and large gear)
GB	gear box
H	hub
h	housing
i	inside
L	large gear
o	outside
p	pinion
fd	flange or drive
r	rim
sp	spline
w	web

C. Initial Data

The following basic data are required to calculate the total weight of a single-step speed reducer:

1. From Accessory System Specification

HP	rated horsepower, hp
N_1	rated input speed, rpm
HP_{max}	maximum or overload horsepower
N_{min}	minimum input speed, at maximum power, rpm
R	gear reduction ratio

2. From Assumptions Based on Experience and Judgment

K_t	shock factor for shaft
(SF)	service factor for gears
s_s	design shear stress for shaft, psi
s_w	design tensile stress for gears, psi
γ_G	weight density of gear material, lb/in^3
γ_h	weight density of housing material, lb/in^3
t_{ave}	average wall thickness of casing, in.

D. Analysis Procedure

The total weight of the gear box is the sum of the individual component weights.

$$W_{GB} = W_G + W_h + (W_{fd})_1 + (W_{fd})_2 \qquad \text{(VI-1)}$$

where:

W_{GB} = total weight of the gear box, lb

W_G = weight of the gears, lb

W_h = weight of the housing, lb

$(W_{fd})_1$ = weight of flange, lb

$(W_{fd})_2$ = weight of drive, lb

The weight of these components is determined from their size. Their size and weight are determined by the following procedure:

1. Size of Gears

The size of gears is effectively determined when the following quantities are known:

outside diameter of the pinion shaft
pitch diameter of the pinion
face width of the gears

The value of these factors stem from the solid shaft diameter which will transmit the required horsepower. The method for calculating these factors is shown below.

a. Determine Solid Shaft Diameter

The diameter of the solid shaft which will transmit the required horsepower at a given speed is found from

$$\frac{HP \ K_t}{N \ s_s} = 3.12 \times 10^{-6} \ d_s^{\ 3} \qquad \text{(VI-2)}$$

where:

HP = horsepower transmitted by shaft, hp

K_t = shock factor

N = shaft speed, rpm

s_s = design shear stress for shaft, psi

d_s = diameter of solid shaft, in.

This equation is shown graphically in Fig. VI-3.

. Determine the Outside Diameter of Pinion Shaft

The solid shaft drives the pinion through an internally splined hollow shaft. Since the hollow shaft must be as strong as the solid shaft, it is possible to obtain the following expression relating the ratio d_o/d_s and the number of spline teeth.

$$\left[\frac{d_o}{d_s}\right]^4 - \frac{d_o}{d_s} = \left[\frac{(S + 1.8)}{(S - 1.8)}\right]^4 \qquad \text{(VI-3)}$$

where:

d_o = outside diameter of hollow shaft, in.

S = number of spline teeth.

A curve, showing the relationship between d_o/d_s and S is shown in Fig. VI-4.

Assume the number of spline teeth and determine the diameter ratio, d_o/d_s, from this curve. Knowing the diameter ratio, the outside diameter of the hollow shaft can be calculated.

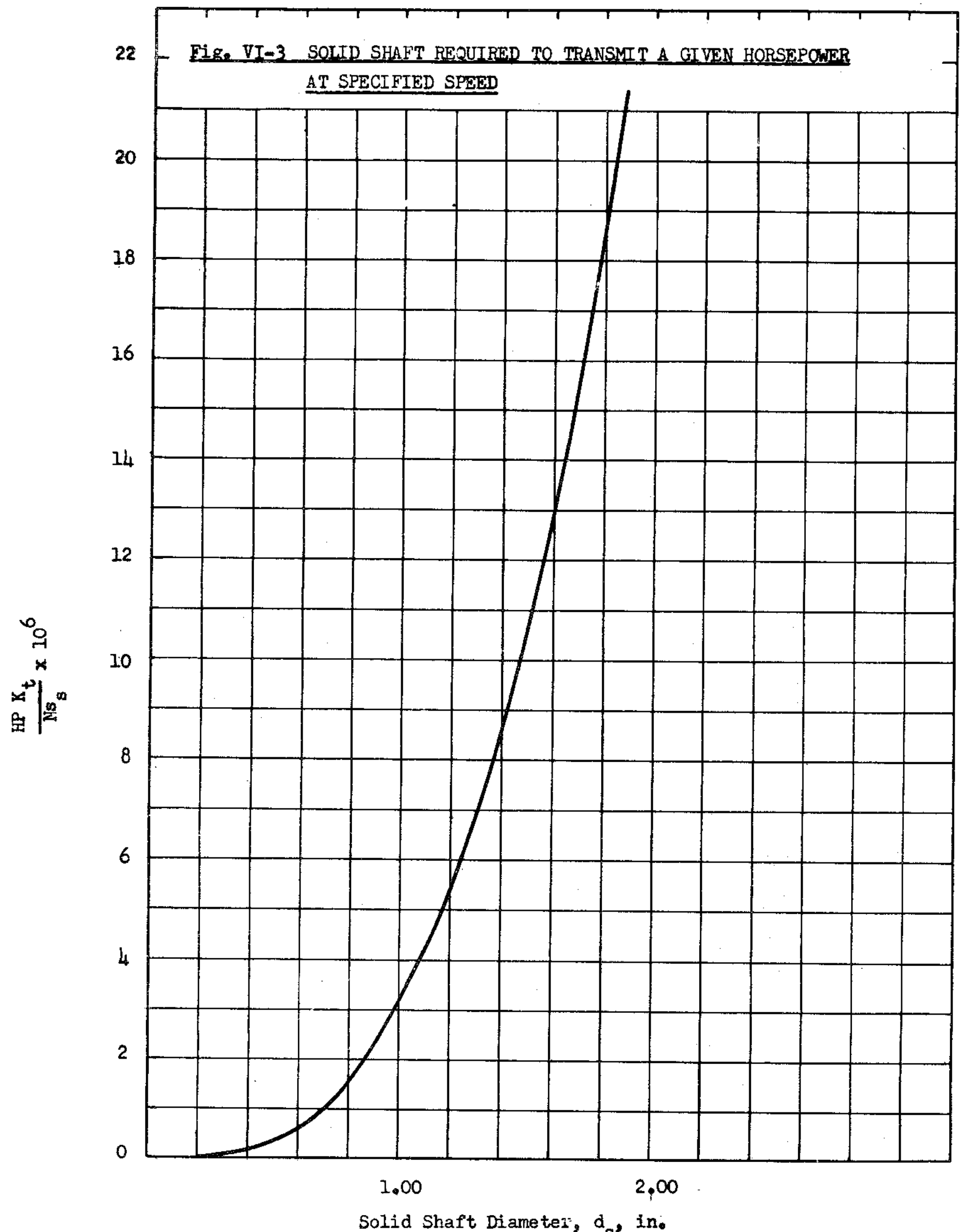


Fig. VI-3 SOLID SHAFT REQUIRED TO TRANSMIT A GIVEN HORSEPOWER AT SPECIFIED SPEED
22
20
18
16
14
12
10
8
6
4
2
0
$\frac{HP\ K_t}{Ns_s} \times 10^6$
1.00
2.00
Solid Shaft Diameter, d_s, in.

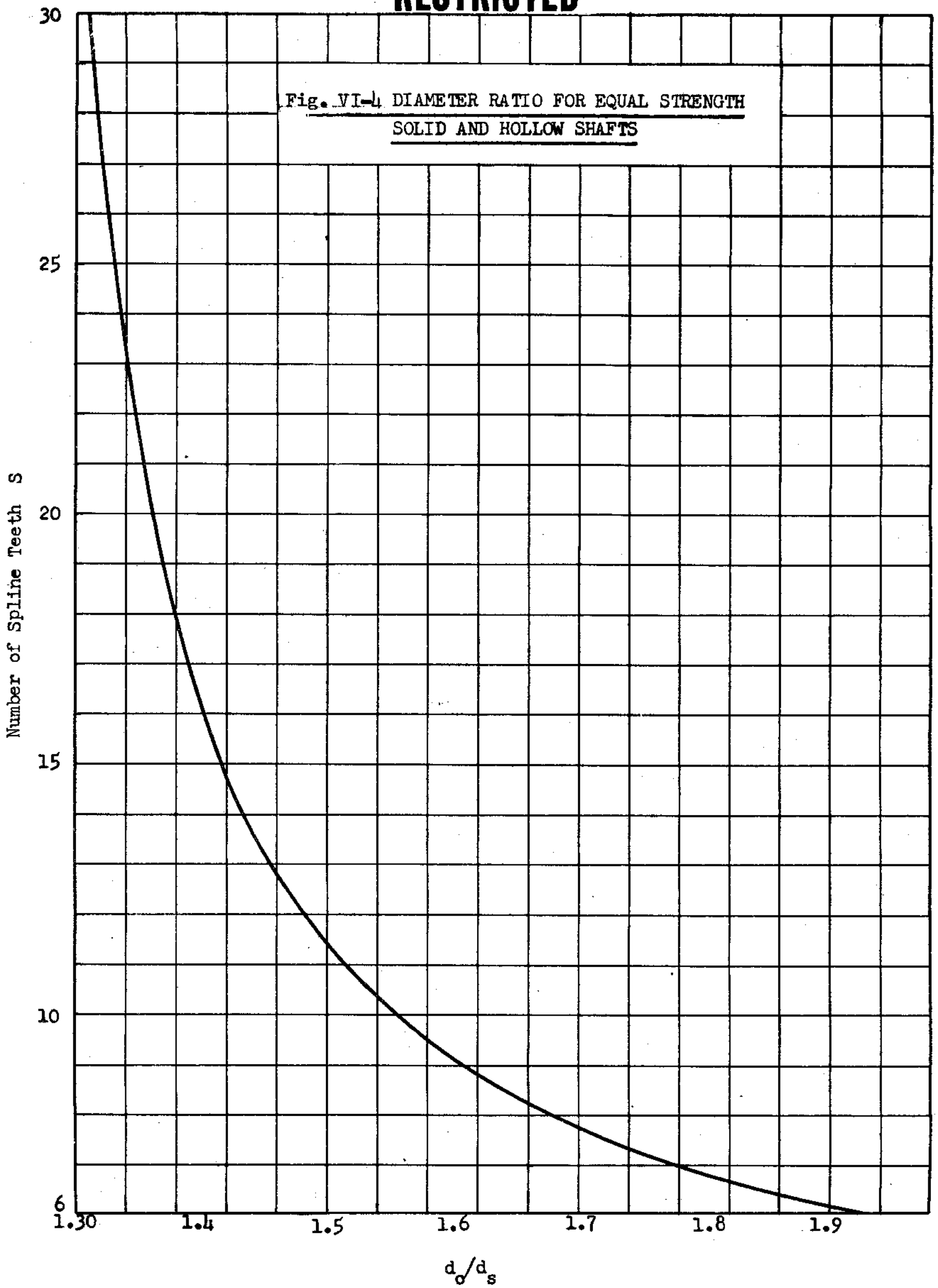


Fig. VI-4 DIAMETER RATIO FOR EQUAL STRENGTH
SOLID AND HOLLOW SHAFTS
Number of Spline Teeth S
30
25
20
15
10
6
1.30
1.4
1.5
1.6
1.7
1.8
1.9
d_o/d_s

c. Determine the Minimum Pitch Diameter of Pinion

The root diameter of the pinion is assumed to be equal to the outside diameter of the hollow shaft. Then, for standard tooth proportions, calculate the pitch diameter of the pinion, D_p, from

$$D_p = \frac{d_o}{1 - \frac{2.314}{n_p}} \qquad \text{(VI-4)}$$

where:

n_p = minimum number of teeth on pinion.

The value of n_p is found from

$$n_p = \frac{4\,R + \sqrt{16\,R^2 + 16\,\sin^2\,\Phi(1 + 2\,R)}}{2\,\sin^2\,\Phi(1 + 2\,R)} \qquad \text{(VI-5)}$$

where:

R = gear ratio
Φ = pressure angle, degrees

This equation is shown graphically in Fig. VI-5 for 20 degree full depth, involute teeth.

d. Determine the Face Width of the Gears

The face width of the gears is determined from the Lewis equation for precision cut gears (Ref. VI-1)

$$f = \frac{F\,P\,(78 + \sqrt{V})}{78\,s_w\,Y} \qquad \text{(VI-6)}$$

where:

f = face width, in.
F = tangential force at pitch line, lb.
P = diametral pitch, 1/in.
V = pitch line velocity, ft/min.
s_w = working stress of gear material, psi
Y = form factor for Lewis equation

The tangential force, F, is determined from the design horsepower, speed, and the pinion pitch diameter as follows:

$$F = \frac{126{,}000\ HP}{(SF)\ D_p\ N_p} \qquad \text{(VI-7)}$$

where:

(SF) = service factor which depends on the type of loading and service expected from the installation.

The pitch line velocity, V, is determined from the following:

$$V = \frac{\pi\,D_p\,N_p}{12} \qquad \text{(VI-8)}$$

To facilitate computation, Eqs. (VI-7) and (VI-8) are shown graphically in Figs. VI-6 and VI-7, respectively.

The form factor, Y, for the Lewis Equation has well established values which may be found in tabulated form in almost any mechanical handbook. For convenience values of Y for 20-degree, full-depth, involute teeth are plotted against the number of teeth in Fig. VI-8.

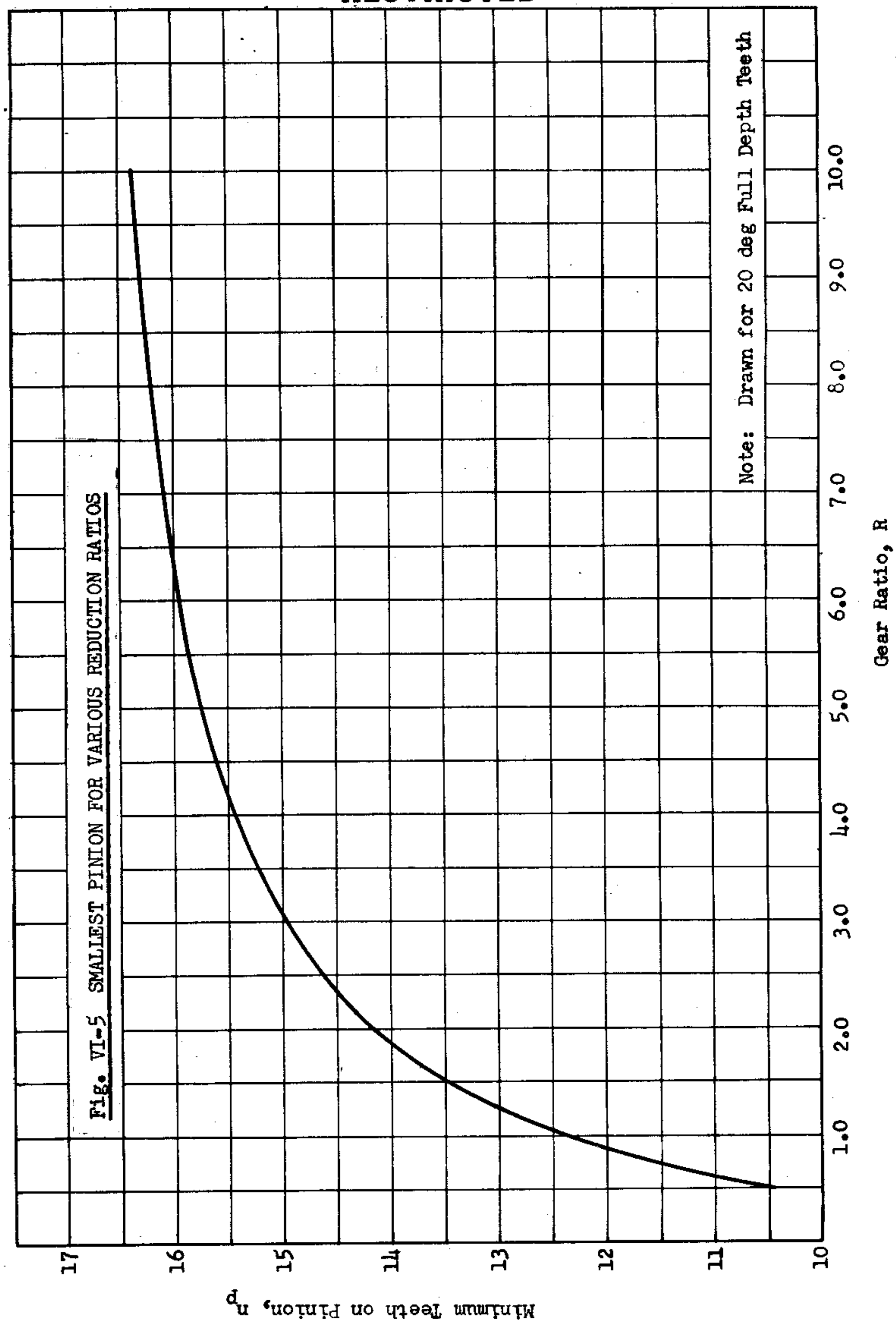


Fig. VI-5 SMALLEST PINION FOR VARIOUS REDUCTION RATIOS

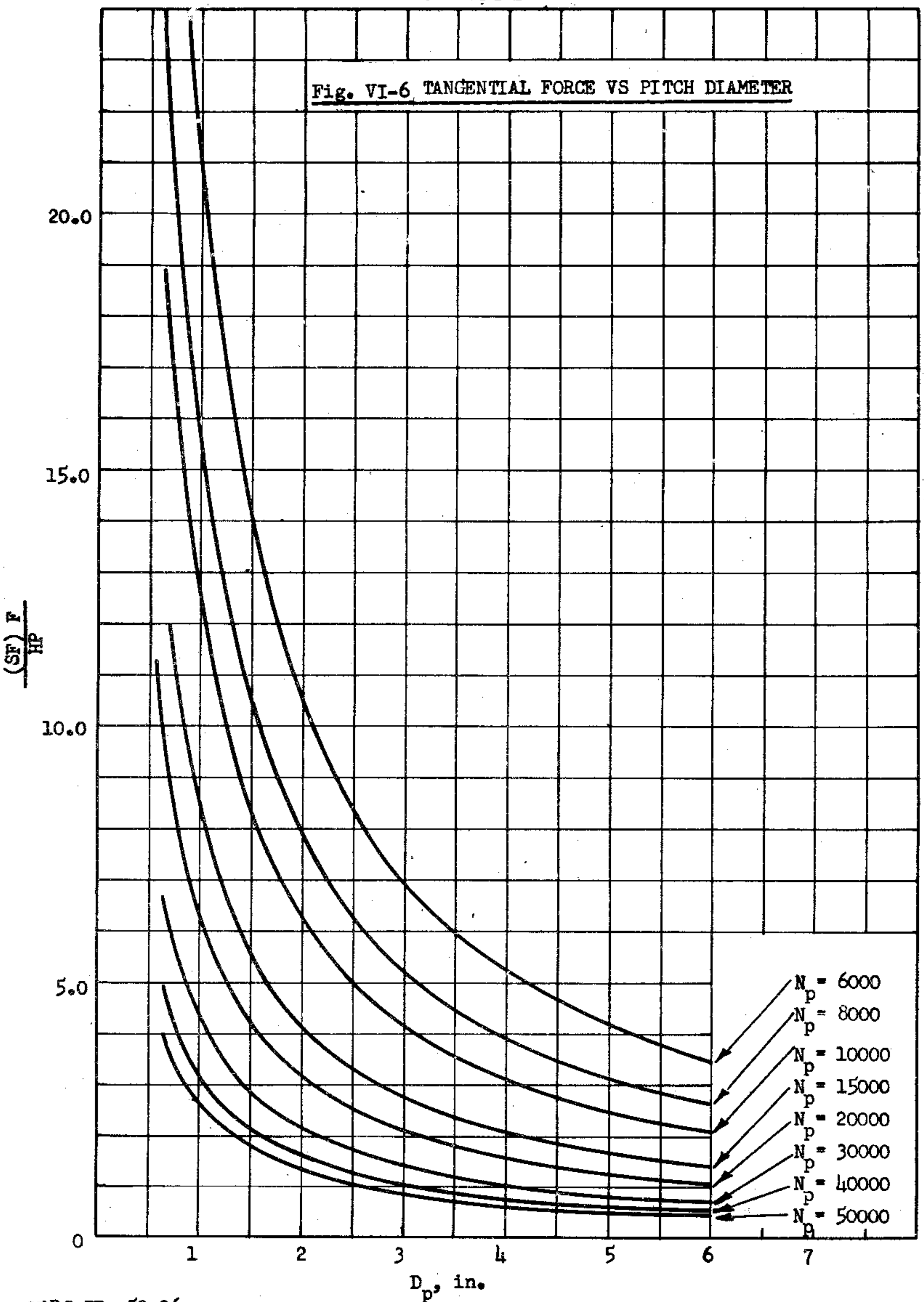


Fig. VI-6 TANGENTIAL FORCE VS PITCH DIAMETER
20.0
15.0
10.0
5.0
0
$\frac{(SF)\ F}{HP}$
1
2
3
4
5
6
7
D_p, in.
N_p = 6000
N_p = 8000
N_p = 10000
N_p = 15000
N_p = 20000
N_p = 30000
N_p = 40000
N_p = 50000

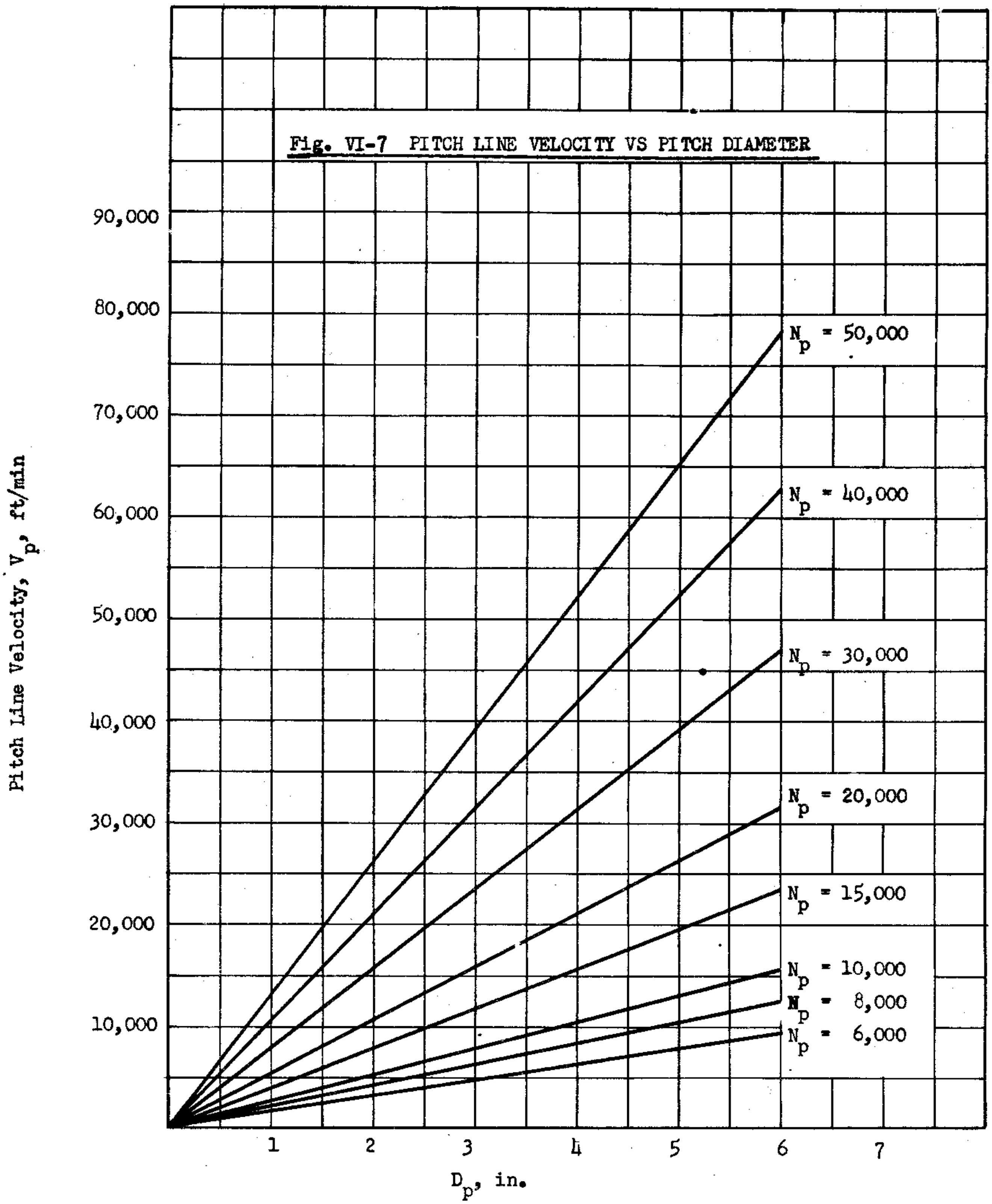


Fig. VI-7 PITCH LINE VELOCITY VS PITCH DIAMETER
90,000
80,000
70,000
60,000
50,000
40,000
30,000
20,000
10,000
Pitch Line Velocity, V_p, ft/min
N_p = 50,000
N_p = 40,000
N_p = 30,000
N_p = 20,000
N_p = 15,000
N_p = 10,000
N_p = 8,000
N_p = 6,000
1
2
3
4
5
6
7
D_p, in.

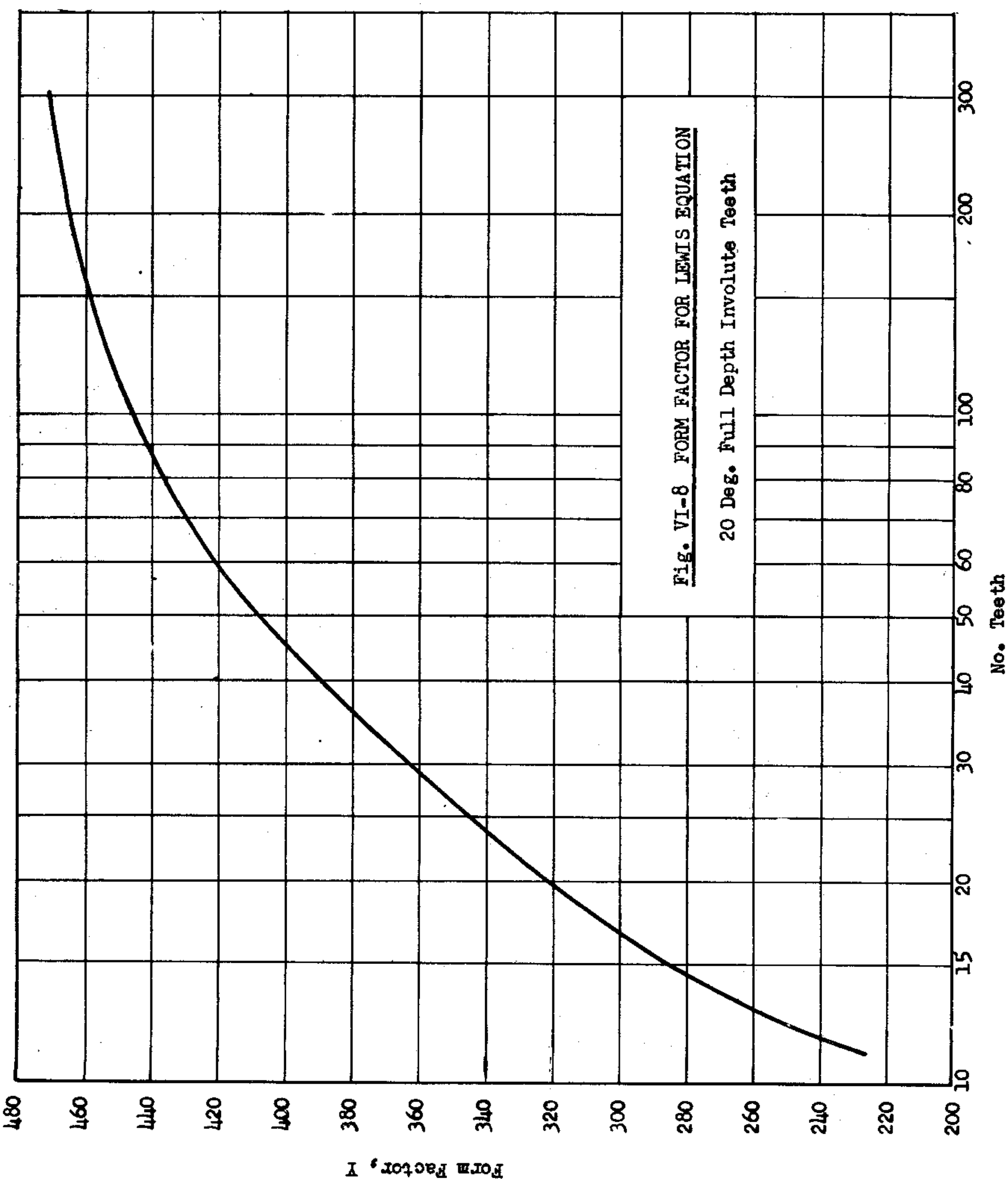


Fig. VI-8 FORM FACTOR FOR LEWIS EQUATION

20 Deg. Full Depth Involute Teeth

2. Weight of Gears

Calculate the total weight of the two gears, including shafts from

$$W_G = \frac{\pi}{4} \gamma_G f (a_p D_p^2 + a_L D_L^2) \tag{VI-9}$$

where:

W_G = weight of pinion and large gears including shaft, lb.

γ_G = weight density of gear material, lb/in^3

f = face width of gears, in.

a_p = weight factor for pinion (weight of actual pinion/weight of solid pinion)

a_L = weight factor for gear (weight of actual gear/weight of solid gear)

D_L = pitch diameter of large gear, in.

It is standard practive in aircraft accessory gear boxes to mount the gears on hollow shafts in order to reduce weight. It is assumed for this analysis that the shaft material extending on either side of the gear will just fill the hole at the center of the gear. (See Figs. VI-1 and VI-2). Thus Eq. (VI-9) gives the weight of two right circular cylinders of height f and diameters D_p and D_L.

The factors a_p and a_L take into consideration the weight reduction due to webs and lightening holes in the pinion and gear.

When the pitch diameter of the gear is large compared to the outside diameter of the shaft, reduction in weight is obtained through the use of webs and lightening holes. Although there is a wide latitude of possible designs and applications of the gears, it is possible to make reasonable estimates of the weight factor from empirical data and theory.

The weight factor for a gear is obtained from the following equation:

$$a = 1 - (1 - B)(X^2 - Z^2) \tag{VI-10}$$

where:

a = weight of actual gear/weight of solid gear

$B = c\,(0.157 + 0.125\,\frac{1}{f})$

$X = 1 - \frac{(2 + \pi)f}{10D}$

$Z = \frac{d_o}{D} + \frac{1}{3.54}\sqrt{\frac{f}{D}}$

where:

c = hole factor, (weight ratio of gear web with holes to solid gear web)

As can be seen from Eq. (VI-10), the weight factor is a function of the pitch diameter, face width, and the outside diameter of the shaft. Either a_p or a_L is determined by substituting the appropriate values for the pinion or the gear, whichever the case may be. For the range of gears of interest in this investigation, manufacturers' data indicate that the hole factor, c, remains approximately constant at a value of 0.45. Figure VI-9 shows the weight factor vs. pitch diameter for several different values of d_o and f which are likely

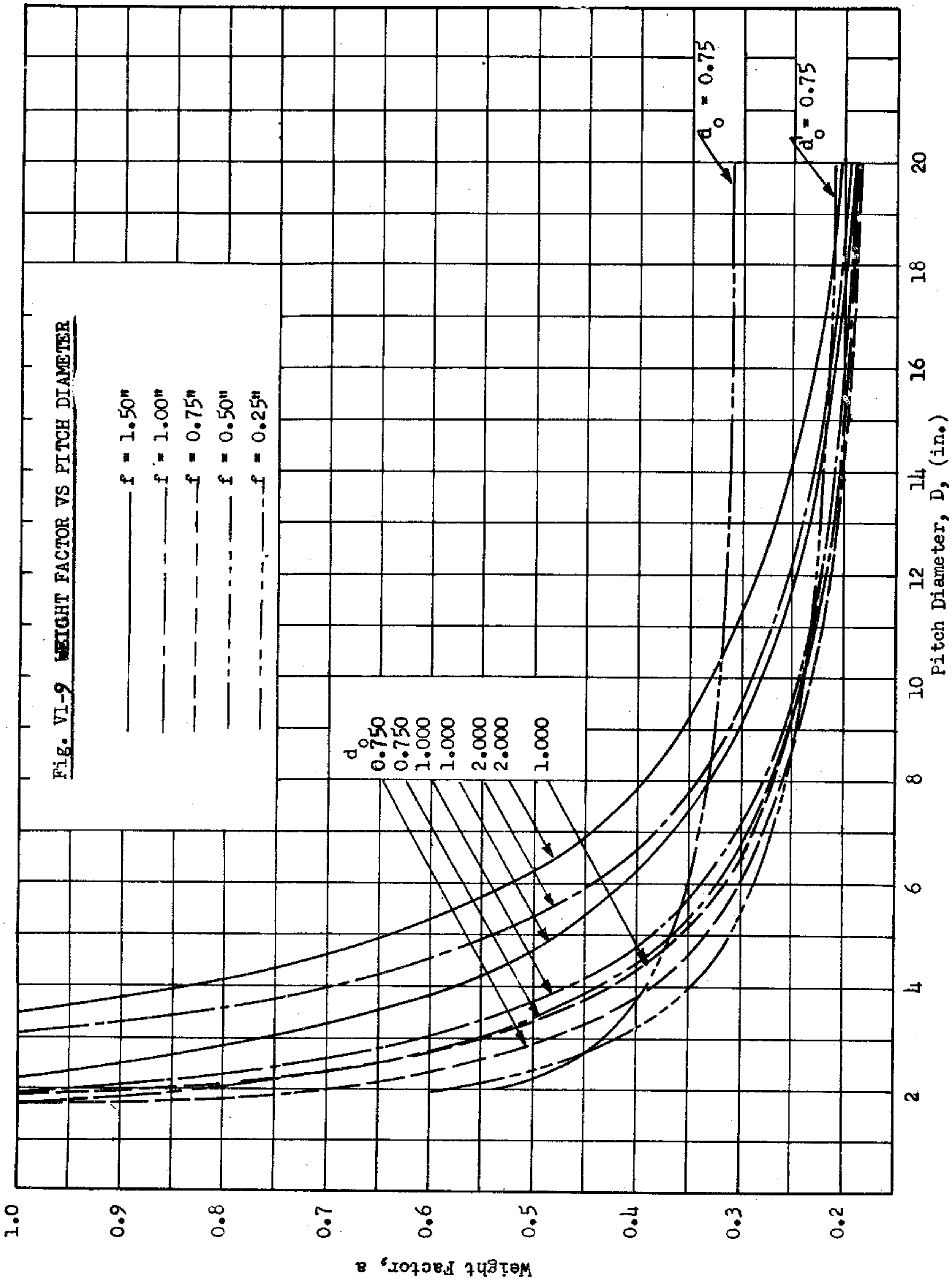


Fig. VI-9 WEIGHT FACTOR VS PITCH DIAMETER to be encountered in aircraft accessory drives. These curves can be interpolated within the accuracy of the curves themselves.

3. Weight of Housing

The functional purpose of the gear housing is to provide support for the bearings, maintain required center distance for the gears, retain the lubricant, and keep dirt and foreign material away from the gears and bearings. The gear case must fulfill these requirements and at the same time be as light as possible. It then follows that simplicity of design is sacrificed in order to reduce the weight to a minimum. As can be seen from Fig. VI-1, the gear housing has a rather complicated shape even in this simplified sketch. The basic casing is built up with bosses and webs to support the bearings and increase the rigidity.

Although the wall thickness of a gear casing is very irregular, it is reasonable to assume that the average wall thickness (including bearings, bearing bosses, stiffening webs, and joining flanges) is constant for gear boxes designed for approximately the same horsepower over similar speed ranges and ratios. Therefore, for this analysis, a modified gear case is assumed in which all of the discontinuities are evenly distributed over the entire surface, giving an average wall thickness as indicated in Fig. VI-2.

The weight of the modified gear case is obtained by considering two volumes, the inner volume ($w_i\ A_i$) bounded by the inside surface of the modified gear housing (see Fig. VI-10) and outer volume ($w_o\ A_o$) bounded by the outside surface of the modified gear housing (see Fig. VI-11). The volume of material in the gear case, is equal to the difference between the outer and inner volumes. Multiplying by density gives the gear case weight. Thus, the housing weight can be found from

$$W_h = \gamma_h (w_o\ A_o - w_i\ A_i) \qquad \text{(VI-11)}$$

Using the geometric relationships of the gears and housings, Eq. (VI-11) can be written as

$$W_h = \gamma_h D_p^{\ 2} F(R) \left[(f + K_w + 2t_{ave}) \left(1 + \frac{2}{n_p} + \frac{2t_{ave}}{D_p\ (1 + R)} \right)^2 - (f + K_w) \left(1 + \frac{2}{n_p} \right)^2 \right] \qquad \text{(VI-12)}$$

where:

K_w = housing width factor

$F(R)$ = function determined from the geometry of the projected area of the gears.

The housing width factor must be assumed. It is defined by

$$K_w = w_i - f \qquad \text{(VI-13)}$$

See Fig. VI-2.

Values for $F(R)$ can be obtained from Fig. VI-12.

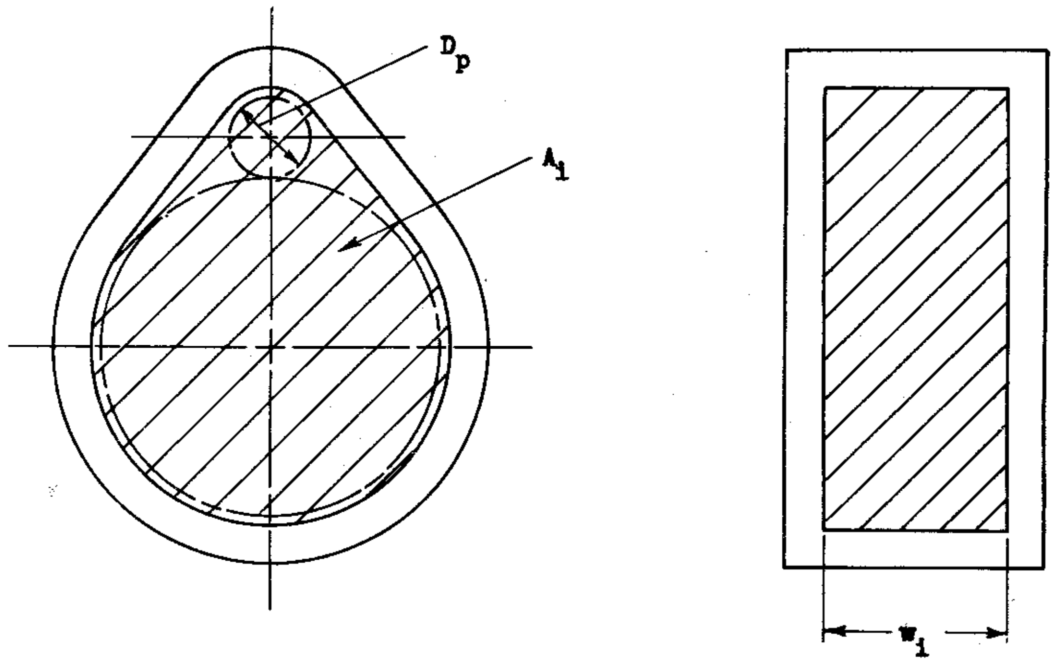


Fig. VI-10 MODIFIED GEAR CASE (INNER VOLUME)

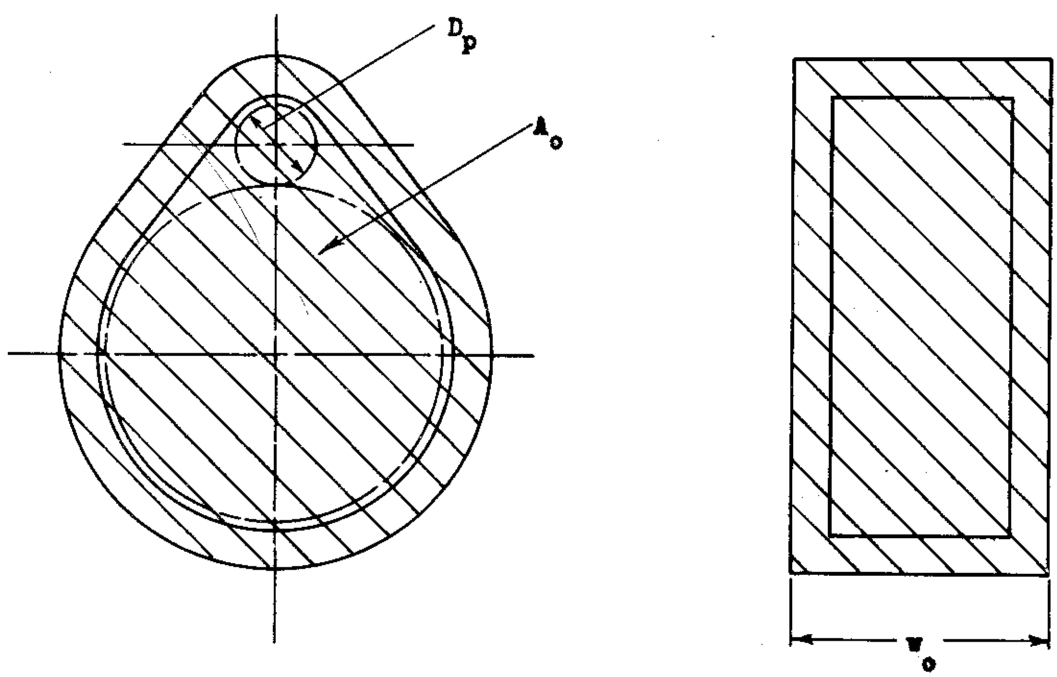
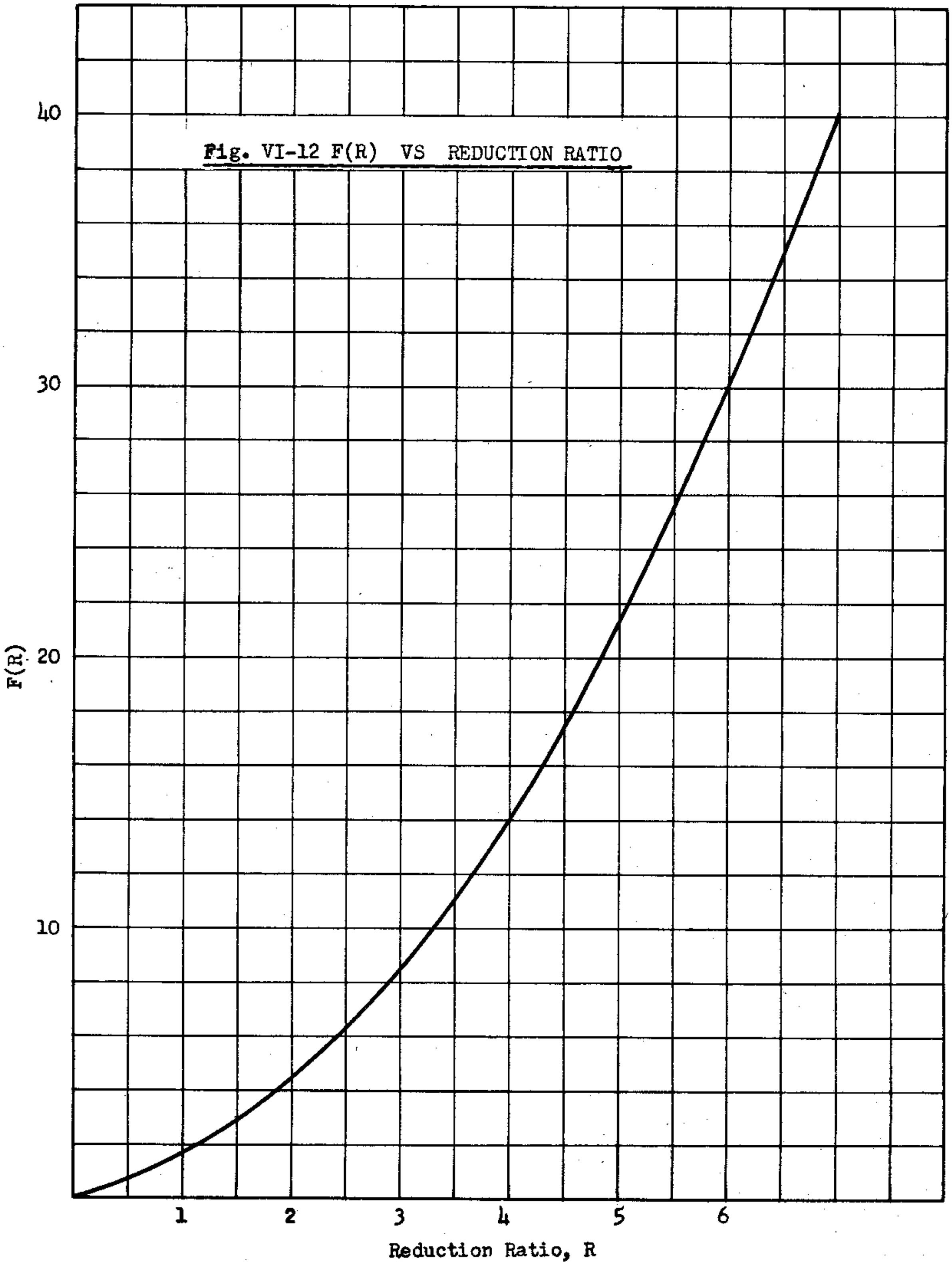


Fig. VI-II MODIFIED GEAR CASE (OUTER VOLUME)

Fig. VI-12 F(R) VS REDUCTION RATIO



4. Weight of Drives and Flanges

The size of the drives and flanges used on a gear box is largely determined by the overhung moment of the accessory being driven by the gear box and the overhung moment of the machine driving the gear box. AND Standards which cover a wide range of sizes should be used. It is left to the judgement of the analyst, performing the calculation of gear box weight, to select the proper AND Standard.

The flange and drives are assumed to be solid discs with a thickness and outside diameter given by the AND Standards. In most cases the drive is made integral with the gear casing. If the drive happens to be larger than the gear housing, the housing is flared out to the proper diameter and the drive in general is faired in with the over-all design of the installation. Bosses and webs are added where necessary for the flange bolts and for added stiffness. Thus, although the solid disc assumption gives a heavy drive, the extra weight compensates for the added weight of the bosses and webs.

The weight of the flange or drive is found from the following for round and square configurations respectively:

$$W_{fd} = \frac{\pi}{4} \gamma_h t_d d_{fd}^2 \qquad \text{(VI-14)}$$

$$W_{fd} = \gamma_h t_{fd} d_{fd}^2 \qquad \text{(VI-15)}$$

where:

γ_h = density of the housing material, lb/in^3

d_{fd} = outside dimension of flange or drive, in.

t_{fd} = thickness of the flange or drive, in.

E. Multi-Drive Gear Boxes

The weight analysis of a multi-pad gear box is approached in the same manner as the single-step speed reducer. The unit is reduced to three components, the gears, the casing, and the pads.

To determine the weights of the components, the following design information is necessary:

1. Maximum horsepower, speed and direction of rotation for each drive or flange.
2. Approximate overhung moment and torque for each drive or flange.
3. Center distances between drives.
4. Input speed.

The weight of the gears, drives and flanges can be found using the same equations and curves developed in this report. It is necessary to make a scale layout of the gear box to determine the configuration of the gear case. From this the outer and inner projected areas can be determined graphically.

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VI-2. V. L. Maleev, Machine Design, International Textbook Company, 1946.

VI-3. Specification MIL-G-6641, General Specification for Aircraft Accessory Gearboxes, 3 August 1951.