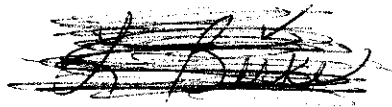


**PROCEEDINGS OF THE SECOND CONFERENCE  
ON MATRIX METHODS IN STRUCTURAL  
MECHANICS**

Proceedings of the conference held at  
Wright-Patterson Air Force Base, Ohio  
15-17 October 1968



DECEMBER 1969

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1100 - February 1970 - CO455- 108-2398

**PROCEEDINGS OF THE SECOND CONFERENCE  
ON MATRIX METHODS IN STRUCTURAL  
MECHANICS**

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**PREFACE**

The Second Conference on Matrix Methods in Structural Mechanics, sponsored by the Air Force Institute of Technology (AFIT), Air University, and the Air Force Flight Dynamics Laboratory (AFFDL), Air Force Systems Command, was held on 15-17 October 1968 in the AFIT auditorium at Wright-Patterson Air Force Base, Ohio. The purpose of the conference was to discuss the recent developments in matrix structural analysis and design of structural systems. This volume contains all the papers presented at the conference.

Of the 255 invited scientists and engineers, thirty-one came from the following foreign countries: Belgium, Canada, France, Germany, Holland, India, Ireland, Japan, Norway, Portugal, and the United Kingdom. The United States' participation was as follows: 46 from universities, 77 from government agencies, and 101 from industrial organizations.

Forty papers were presented at the conference in seven sessions entitled: Structural Weight Optimization, Dynamics, General Elements, Curved Elements, Applications, General Methods, and Nonlinear Analysis. The papers covered practically all major aspects of recent research and development work on matrix methods in structural mechanics.

The members of the Technical Committee for the conference, who also served as the Technical Editors, express their appreciation to the authors and session chairmen for their contributions toward the success of the conference, and to all those who attended and participated in the discussions. Special appreciation is extended to Brig Gen Raymond A. Gilbert, Director of Laboratories, Air Force Systems Command, for his excellent keynote address. Wholehearted thanks are also offered to The Honorable Alexander H. Flax, Assistant Secretary of the Air Force (Research and Development),\* for the outstanding address he presented at the conference banquet.

Grateful acknowledgment is made to Mr. John W. Thomas, Publications Editor, Editorial Branch, Technical Services Division, Directorate of Engineering Standards for his efforts in preparing these proceedings for publication.

October 1968  
Wright-Patterson Air Force Base  
Dayton, Ohio 45433

Technical Editors

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\*Now, President of the Institute for Defense Analyses, Arlington, Virginia

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