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# PROPULSION SYSTEM FLOW STABILITY PROGRAM (DYNAMIC),

## PHASE I FINAL TECHNICAL REPORT,

## PART XX — COMPUTER PROGRAMS FOR THE ROOT LOCUS ANALYSIS OF LINEARIZED TURBOJET AND TURBOFAN CONTROL SYSTEMS,

E. L. Lum
AUTONETICS DIVISION OF NORTH AMERICAN ROCKWELL CORPORATION

TECHNICAL REPORT AFAPL-TR-68-142, PART XX

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Air Force Aero Propulsion Laboratory
Air Force Systems Command
Wright-Patterson Air Force Base, Ohio

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PART XX

# PROPULSION SYSTEM FLOW STABILITY PROGRAM (DYNAMIC)

## PHASE I FINAL TECHNICAL REPORT

## PART XX. COMPUTER PROGRAMS FOR THE ROOT LOCUS ANALYSIS OF LINEARIZED TURBOJET AND TURBOFAN CONTROL SYSTEMS

E. L. Lum

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# FOREWORD

This report describes work accomplished in Phase I of the two-phase program, "Propulsion System Flow Stability Program (Dynamic)" conducted under USAF Contract F33615-67-C-1848. The work was accomplished in the period from 20 June 1967 to 30 September 1968 by the Los Angeles Division of North American Rockwell Corporation, the prime Contractor, and the Subcontractors, the Allison Division of General Motors Corporation (supported by Northern Research and Engineering Corporation), the Autonetics Division of North American Rockwell Corporation (supported by the Aeronautical Division of Honeywell, Incorporated), and the Pratt & Whitney Aircraft Division of United Aircraft Corporation.

The program was sponsored by the Air Force Aero Propulsion Laboratory, Wright-Patterson Air Force Base, Ohio. Mr. H. J. Gratz, APTA, Turbine Engine Division, was the Project Engineer.

This volume is Part XX of twenty parts and was prepared by the Autonetics Division of North American Rockwell Corporation.

Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

Ernest C. Simpson
Chief, Turbine Engine Division

# ABSTRACT

This part describes computer programs developed to handle the computational tasks for the linear stability analysis of propulsion controls by root locus techniques. The descriptions, flowcharts, and Fortran program listings of two main programs TJET and TFAN developed for the analysis of turbojet and turbofan engine control systems are presented. Also presented are the descriptions of three subroutines specially developed to facilitate data handling for this study. Examples are given to illustrate the application of computer programs TJET and TFAN.

TJET and TFAN have been valuable in the linear analysis of the propulsion system. However, it is recommended that these computer programs be expanded to include the capability of analyzing coupled control loops and to include a limited transient analysis.

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# I. INTRODUCTION

Computer programs TJET and TFAN were developed to aid in the linear analysis of the turbojet and turbofan engine control systems, respectively, under the Propulsion System Flow Stability Program. These computer programs compute the closed loop characteristic of the engine control systems for stability analysis by root locus techniques. A brief introduction and discussion of root locus techniques follows.

The Laplace transform representation of a simple control loop is shown in Figure 1, where $K_c$ is the control gain which may be varied and $G_p$ is the s-plane transfer function of the open loop system, or plant. The numerator, $N_p$, and denominator, $D_p$, of the s-plane transfer function, $G_p$, are both polynomials in s. The roots of $N_p$ are called the open loop zeros and the roots of $D_p$ are called the open loop poles.

The transfer function of the closed loop system of Figure 1 is

$$\frac{C}{R}(s) = \frac{K_c G_p}{1 + K_c G_p} = \frac{K_c N_p}{D_p + K_c N_p} \tag{1}$$

The denominator of Equation (1) characterizes the closed loop behavior of the system and is thus called the closed loop characteristic, $D_{CL}$, or

$$D_{CL} = D_p + K_c N_p \tag{2}$$

By root locus techniques, the roots of $D_{CL}$, or closed loop poles, are plotted in the complex s-plane as a function of varying $K_c$, from which the stability and frequency response characteristics of the closed loop system may be observed. The operations required to obtain the root locus of $D_{CL}$ is (a) perform the polynomial addition indicated by the right hand side of Equation (2), and (b) factor the resulting polynomial to obtain the roots.

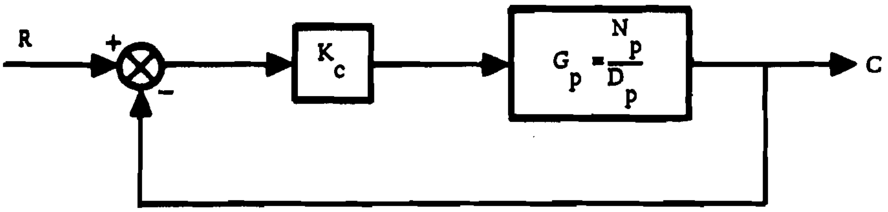

Figure 1. Simple Control Loop Block Diagram

For the general case, a complex control system may have many control loops with many control gains to be determined. However, any linear control system can be reduced to the form of the simple case of Figure 1, with $G_p$ representing a system transfer function with control loops already closed. Actually, computing $G_p$ for coupled control systems is a rather complicated process, especially when all actuators and sensor dynamics are incorporated. But once $G_p$ is determined for a particular control loop, all that is required to compute the closed loop characteristic for that loop is the simple procedure of Equation (2). In this fashion, one may closed control loops around other control loops, or may close loops simultaneously, which is what TJET and TFAN were designed for.

The present versions of TJET and TFAN as presented in this section are useful only for the analysis of a single loop closure because the necessary cross-coupling terms discussed in Part XIX, were not included in the equations used in programming TJET and TFAN. This omission was made because the available transfer function data did not include the necessary cross-coupling terms anyway. Hence, although TJET and TFAN are set up for sequential loop closures, the results for varying more than one control gain will not be valid unless the control loops involved are not strongly coupled, which is not the case here. The present versions of TJET and TFAN do, however, provide the framework for coupled multiloop control analysis of the respective engines, and with a few modifications, the required cross-coupling terms can be included in TJET and TFAN, if and when such an analysis is desirable.

Main programs TJET and TFAN and all non-library subprograms required are programmed in Fortran IV, level H for the IBM 360. Compiler option (OPT = 2) was used to optimize the execution speed and reduce the size of the object deck. Descriptions, flowcharts and listings of the Fortran programs developed for the Propulsion System Flow Stability Program linear control analysis follow.

## II. DESCRIPTION OF MAIN PROGRAM TJET

### BLOCK DIAGRAM AND NOMENCLATURE

The two control systems for the turbojet engine under study are (1) the turbojet rotor speed control loop, or Speed Control, in short, and (2) the turbojet turbine discharge temperature control, or the Temperature Control, in short. The block diagrams for the subject control systems are shown in Figure 2. The nomenclature used for this figure and for the control equations are defined in Table I. $K_{WNA}$, $K_{AT}$, $K_{NT}$, $K_{TN}$, $K_{N2A}$ and $K_{T6A}$ are the control gains to be determined.

Program TJET allows for the variation of the control gains $K_{WNA}$, $K_{AT}$, $K_{NT}$ and $K_{TN}$, individually, or in sequence. The integrator gains $K_{N2A}$ and $K_{T6A}$ are preselected. Individual control loops are investigated by setting the control gains in the other control loops to zero. For example, to investigate the Speed Control loop, one sets $K_{AT} = K_{TN} = 0$.

To illustrate how TJET closed control loops sequentially, the following notation is used,

$$D_{CL} = F(K_c, G_p) \tag{3}$$

where F denotes the polynomial addition process indicated by the right hand side of Equation (2). Then, if it is desired to vary $K_{WNA}$, $K_{AT}$, $K_{NT}$ and $K_{TN}$ sequentially, TJET will perform the following operation

$$D_{CL} = F_4(K_{WNA}, F_3(K_{AT}, F_2(K_{NT}, F_1(K_{TN}, G_p)))) \tag{4}$$

where $F_i$ (i = 1, 2, 3, 4) indicates the operation defined by Equation (3) with $F_1$ evaluated first, $F_2$ next, etc. In the programming of TJET, this is implemented by nested DO loops (see flowchart and listing of TJET). If one or more of the gains $K_{WNA}$, $K_{AT}$, $K_{NT}$ or $K_{TN}$ is set to zero, then the operation indicated by Equation (3) is bypassed for that control gain by logic statements in the program, enabling the investigation of individual control loops separately.

### EQUATIONS FOR LINEAR ANALYSIS OF TURBOJET ENGINE

The closed loop characteristic (DF) of the turbojet engine control, without coupling between control loops, is (using the same notation as defined for the arrays of TJET)

$$\begin{aligned}(DF) = (CA) + \left[K_{WNA}(CB) + (CC)\right] + K_{AT}(CD) \\ + K_{NT}(CE) + K_{TN}(CF)\end{aligned}$$

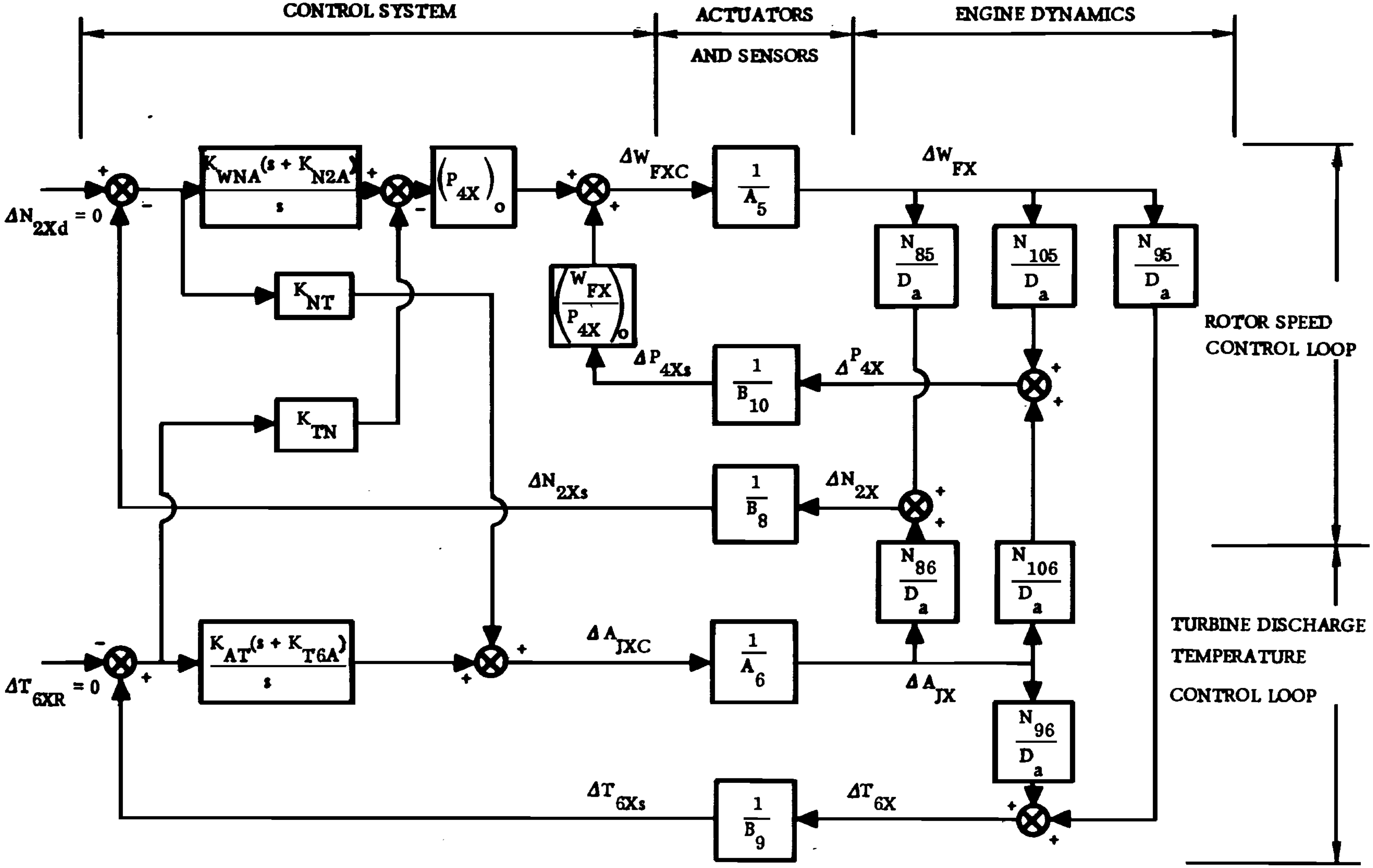

Figure 2. Turbojet Engine Linear Control System Block Diagram

Table I. Nomenclature for Analysis of Turbojet Engine Control

a) Sensed and Actuated Variables

Actuated Variables, $x_j$

$x_5 = W_{FX}$ Turbojet Main Combustor Fuel Flow (lb/hr)

$x_6 = A_{JX}$ Turbojet Exhaust Nozzle Area ($ft^2$)

Sensed Variables, $y_i$

$y_8 = N_{2X}$ Turbojet Rotor Mechanical Speed (RPM)

$y_9 = T_{6X}$ Turbojet Turbine Discharge Temperature (°R)

$y_{10} = P_{4X}$ Turbojet Compressor Discharge Pressure (psia)

b) Actuator and Sensor Definitions

$W_{FX}$ Actuator: $1/A_5$

$A_{JX}$ Actuator: $1/A_6$

$N_{2X}$ Sensor: $1/B_8$

$T_{6X}$ Sensor: $1/B_9$

$P_{4X}$ Sensor: $1/B_{10}$

where $A_j$ is the denominator of the actuator lag for $x_j$, and $B_i$ is the denominator of the sensor lag for $y_i$.

c) Transfer Function Numerators

$N_{85}$ = Numerator of $\partial N_{2X}/\partial W_{FX}$ transfer function

$N_{86}$ = Numerator of $\partial N_{2X}/\partial A_{JX}$ transfer function

$N_{95}$ = Numerator of $\partial T_{6X}/\partial W_{FX}$ transfer function

Table I. Concluded

$N_{96}$ = Numerator of $\partial T_{6X}/\partial A_{JX}$ transfer function

$N_{105}$ = Numerator of $\partial P_{4X}/\partial W_{FX}$ transfer function

In general, $N_{ij}/D_a$ is the transfer function for the i-th dependent variable $y_i$, due to the j-th independent variable $x_j$. $D_a$ is the open loop denominator of the turbojet engine dynamics.

where

1. Open-Loop Denominator ($D_p$ of Equation 2)

   $$(CA) = sA_5A_6B_8B_9B_{10}D_a$$

2. Numerators for speed loop closure

   $$(CB) = (P_{4X})_o\, A_6B_9B_{10}(s + K_{N2A})N_{85}$$

   $$(CC) = -s(W_{FX}/P_{4X})_o\, A_6B_8B_9N_{105}$$

3. Numerator for Temperature Loop Closure

   $$(CD) = -A_5B_8B_{10}(s + K_{T6A})N_{96}$$

4. Numerator for Variation in $K_{NT}$

   $$(CE) = sA_5B_9B_{10}N_{86}$$

5. Numerator for Variation in $K_{TN}$

   $$(CF) = s(P_{4X})_o\, A_6B_8B_{10}N_{95}$$

Actually, TJET has logic statements which omit certain actuator poles, $A_i$, and sensor poles, $B_j$, from these equations if they are not affected by one of the loop closures. For example, if only the speed loop is closed, then $A_6$ and $B_9$ are not required in the computations.

## FLOWCHART AND PROGRAM LISTING OF TJET

Table II defines the nomenclature used in the programming of computer program TJET. The flowchart for main program TJET is shown on Figure 3. The print-out statements are left out of the flowchart for clarity. The circled numbers in Figure 3 correspond to the statement numbers in the Fortran H program listing of TJET shown on Table III.

Table II. List of Subprograms and Variables Used for TJET

a) List of Subprograms Required for TJET

| | |
|---|---|
| Non-library: | in Fortran H for IBM 360 |
| (1) XPNP5 | Expand a polynomial with 5 sets of first order lags |
| (2) GENP | Generate a polynomial from data of form given by P&WA |
| (3) XPNTAU | Expand a polynomial with a given set of first order lags |
| (4) PRTX[1] | Polynomial root extraction using Bairstow and Newton-Raphson iterations |
| (5) FLIP[1] | Reverse order of elements in an array |
| (6) POLYX[2] | Multiply two polynomials |
| (7) GPOL[2] | Multiply a polynomial by a constant |
| (8) POLADD[2] | Add two polynomials |
| (9) WRT[2] | Print roots of a polynomial |
| (a) WRTPOL[2] | Print coefficients of a polynomial |
| (10) PRIN[2] | Print non-zero elements of an array |
| Library: | for Fortran IV on IBM 360 |
| (11) DECRD | Read variable length data in standard format |

b) Arrays of Variables Used in TJET

| | | |
|---|---|---|
| DA | = $D_a$ | Open-loop denominator |
| N85 | = $N_{85}$ | Numerator[3] of $\partial N_{2X}/\partial W_{FX}$ transfer function |

---

[1] Developed by J. C. Long of NAR/Space Division

[2] Originally developed for ALODE's I and II, Automatic Linear Optimal Design and Evaluation Computer Programs

[3] Expressed either in terms of (1) coefficients of polynomial, or (2) first-order lags of polynomial, in which case FLG $\neq 0$.

Table II. (Cont)

| | | |
|---|---|---|
| N86 | = $N_{86}$ | Numerator[3] of $\partial N_{2X}/\partial A_{JX}$ transfer function |
| N95 | = $N_{95}$ | Numerator[3] of $\partial T_{6X}/\partial W_{FX}$ transfer function |
| N96 | = $N_{96}$ | Numerator[3] of $\partial T_{6X}/\partial A_{JX}$ transfer function |
| N105 | = $N_{105}$ | Numerator[3] of $\partial P_{4X}/\partial W_{FX}$ transfer function |
| KWNA | = $K_{WNA}$ | Control gains for rotor speed loop |
| KAT | = $K_{AT}$ | Control gains for temperature loop |
| KNT | = $K_{NT}$ | "Decoupling" gain |
| KTN | = $K_{TN}$ | "Decoupling" gain |
| TA5 | = $A_5$ | Time constants of $W_{FX}$ actuator lag |
| TA6 | = $A_6$ | Time constants of $A_{JX}$ actuator lag |
| TB8 | = $B_8$ | Time constants of $N_{2X}$ sensor lag |
| TB9 | = $B_9$ | Time constants of $T_{6X}$ sensor lag |
| TB10 | = $B_{10}$ | Time constants of $P_{4X}$ sensor lag |
| F | | Array for input of data using DECRD |
| TITLE | | Title of run in alpha-numeric characters (up to 72 characters) |
| CA, CB, CC, CD, CE, CF | | Arrays used for temporary storage of intermediate polynomials used in computations |
| DCL, DD, DE, DF | | Arrays used for temporary storage of polynomials used to compute closed-loop characteristic polynomial |
| CAR, CAC (Complex) | | Arrays used for temporary storage of the real and complex roots of a polynomial |

---

[3] Expressed either in terms of (1) coefficients of polynomial, or (2) first-order lags of polynomial, in which case FLG $\neq 0$.

Table II. (Cont)

c) **Real *4 Variables**

| | |
|---|---|
| CP0 | 1. if checkout print option desired |
| FLG | $\neq 0.$ if data entered in form supplied by P&WA |
| G85 | Steady-state gain for $\partial N_{2X}/\partial W_{FX}$ transfer function, only if FLG$\neq$0 |
| G86 | Steady-state gain for $\partial N_{2X}/\partial A_{JX}$ transfer function, only if FLG$\neq$0 |
| G95 | Steady-state gain for $\partial T_{6X}/\partial W_{FX}$ transfer function, only if FLG$\neq$0 |
| G96 | Steady-state gain for $\partial T_{6X}/\partial A_{JX}$ transfer function, only if FLG$\neq$0 |
| G105 | Steady-state gain for $\partial P_{4X}/\partial W_{FX}$ transfer function, only if FLG$\neq$0 |
| KN2A = $K_{N2A}$ | Integrator gain for speed loop |
| KT6A = $K_{T6A}$ | Integrator gain for temperature loop |
| P4A = $(P_{4X})_o$ | Initial steady-state value of $P_{4X}$ |
| WFA = $(W_{FX})_o$ | Initial steady-state value of $W_{FX}$ |
| SF | Scale factor for roots (used as a dummy argument when calling subroutine PRTX) |

d) **Integer *4 Variables**

| | |
|---|---|
| NDA | Degree of open loop denominator polynomial (array DA) |
| NN85 | Degree of numerator for $\partial N_{2X}/\partial W_{FX}$ transfer function |
| NN86 | Degree of numerator for $\partial N_{2X}/\partial A_{JX}$ transfer function |
| NN95 | Degree of numerator for $\partial T_{6X}/\partial W_{FX}$ transfer function |
| NN96 | Degree of numerator for $\partial T_{6X}/\partial A_{JX}$ transfer function |
| NN105 | Degree of numerator for $\partial P_{4X}/\partial W_{FX}$ transfer function |

Table II. (Cont)

| | |
|---|---|
| NTA5 | Number of first-order lags for $W_{FX}$ actuator |
| NTA6 | Number of first-order lags for $A_{JX}$ actuator |
| NTB8 | Number of first-order lags for $N_{2X}$ sensor |
| NTB9 | Number of first-order lags for $T_{6X}$ sensor |
| NTB10 | Number of first-order lags for $P_{4X}$ sensor |
| NWNA | Number of $K_{WNA}$ gains for which root locus is computed |
| NAT | Number of $K_{AT}$ gains for which root locus is computed |
| NNT | Number of $K_{NT}$ gains for which root locus is computed |
| NTN | Number of $K_{TN}$ gains for which root locus is computed |

The following integers are used for indexing only: I, NA, NB, NC, ND. The following integers are used to indicate size of arrays in computations: (Example, NCAR = No. of real roots stored in array CAR.) NCA, NCB, NCC, NCD, NCE, NCF, NDD, NDE, NDF, NCAC, NCAR, NDCL

e) Logical constants used for TJET

A logical constant is TRUE if the expression to the right of the colon (:) is true; FALSE, otherwise.

Symbols: a|b means a or b; a&b means a and b.

| | | |
|---|---|---|
| LA: | NWNA > 0 | Speed Control loop closed |
| LAN: | NWNA ≤ 0 | Speed Control loop open |
| LB: | NAT > 0 | Temperature Control loop closed |
| LBN: | NAT ≤ 0 | Temperature Control loop open |
| LC: | NNT > 0 | $K_{NT} \neq 0$ |
| LCN: | NNT ≤ 0 | No decoupling gain $K_{NT}$ |
| LD: | NTN > 0 | $K_{TN} \neq 0$ |

Table II. Concluded

| | | |
|---|---|---|
| LDN: | NTN $\leq 0$ | No decoupling gain $K_{TN}$ |
| LEN: | $K_{N2A} = 0$ | No integrator in speed loop |
| LFN: | $K_{T6A} = 0$ | No integrator in temperature loop |
| LH: | (LEN&LFN)\|(LAN&LFN)\|(LBN&LEN)\|(LAN&LBN)<br>If LH is true, then a free s term exists. | |
| LAC: | LA\|LC | Speed loop closed or $K_{NT} \neq 0$ |
| LAD: | LA\|LD | Speed loop closed or $K_{TN} \neq 0$ |
| LBC: | LB\|LC | Temperature loop closed or $K_{NT} \neq 0$ |
| LBD: | LB\|LD | Temperature loop closed or $K_{TN} \neq 0$ |
| LIN: | $(W_{FX} = 0) \mid (P_{4X} = 0)$ | No $P_{4X}$ term, see equations. |
| LP: | CPO = 1. | If true, then print-out provided for checkout of program |

## FORMAT OF INPUT DATA FOR TJET

The following pages (Table IV) show the format of input data for TJET. The data cards should follow the [//G . SYSIN DD *] control card and should be arranged as follows:

1. Title card, description of run in alpha-numeric characters which will be printed on top of first page of run. This card must be first card of each run.

2. Data cards for each run. All data must be Real *4 and are read into array F of main program TJET. Relative locations of the variables assigned to array F are described on the following data sheets. Blanks will leave data from previous run unchanged. A minus sign must be in Column 1 on last data card of each run.

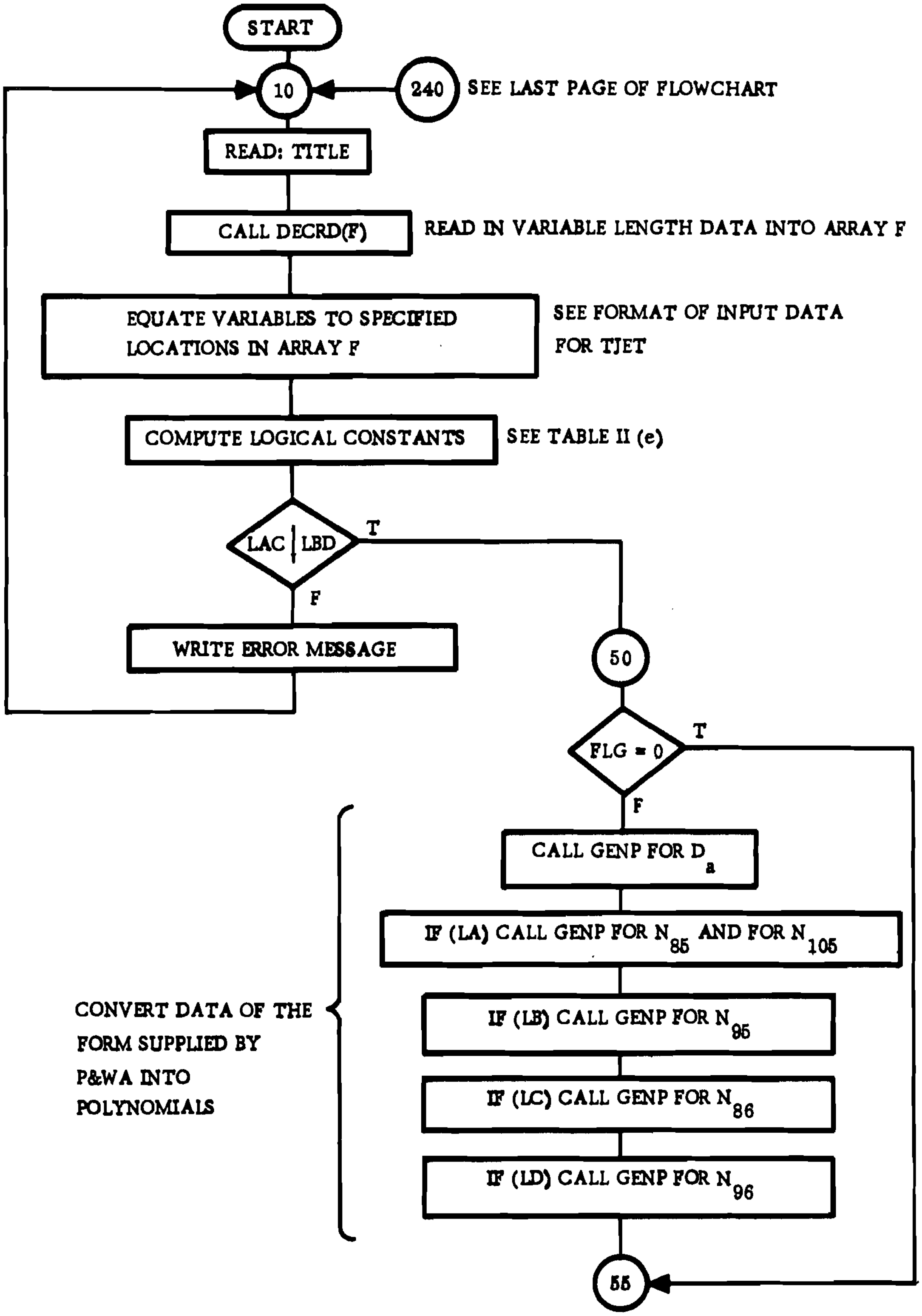

Figure 3. Flowchart of Main Program TJET

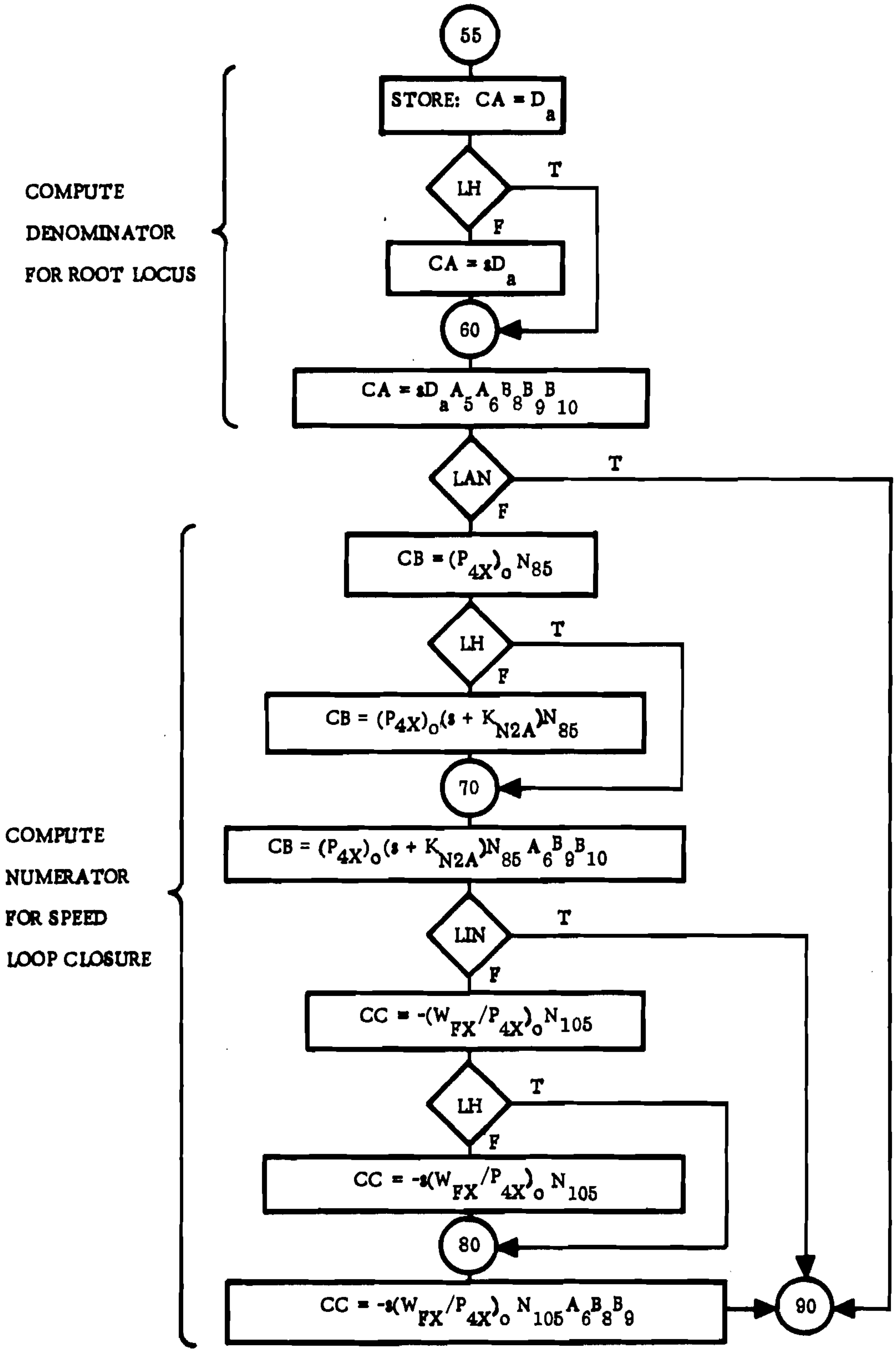

Figure 3. (Continued)

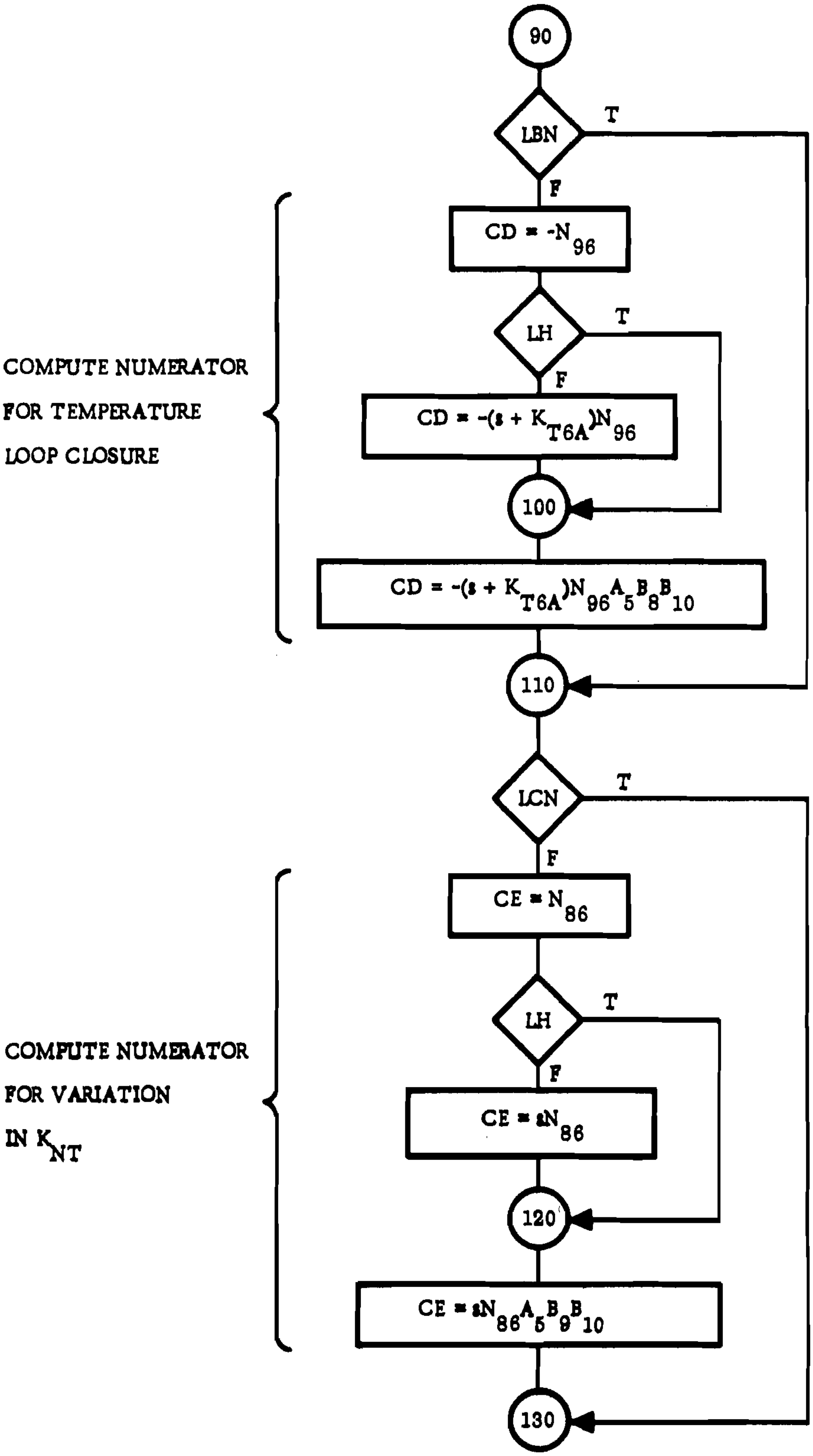

**Figure 3.** (Continued)

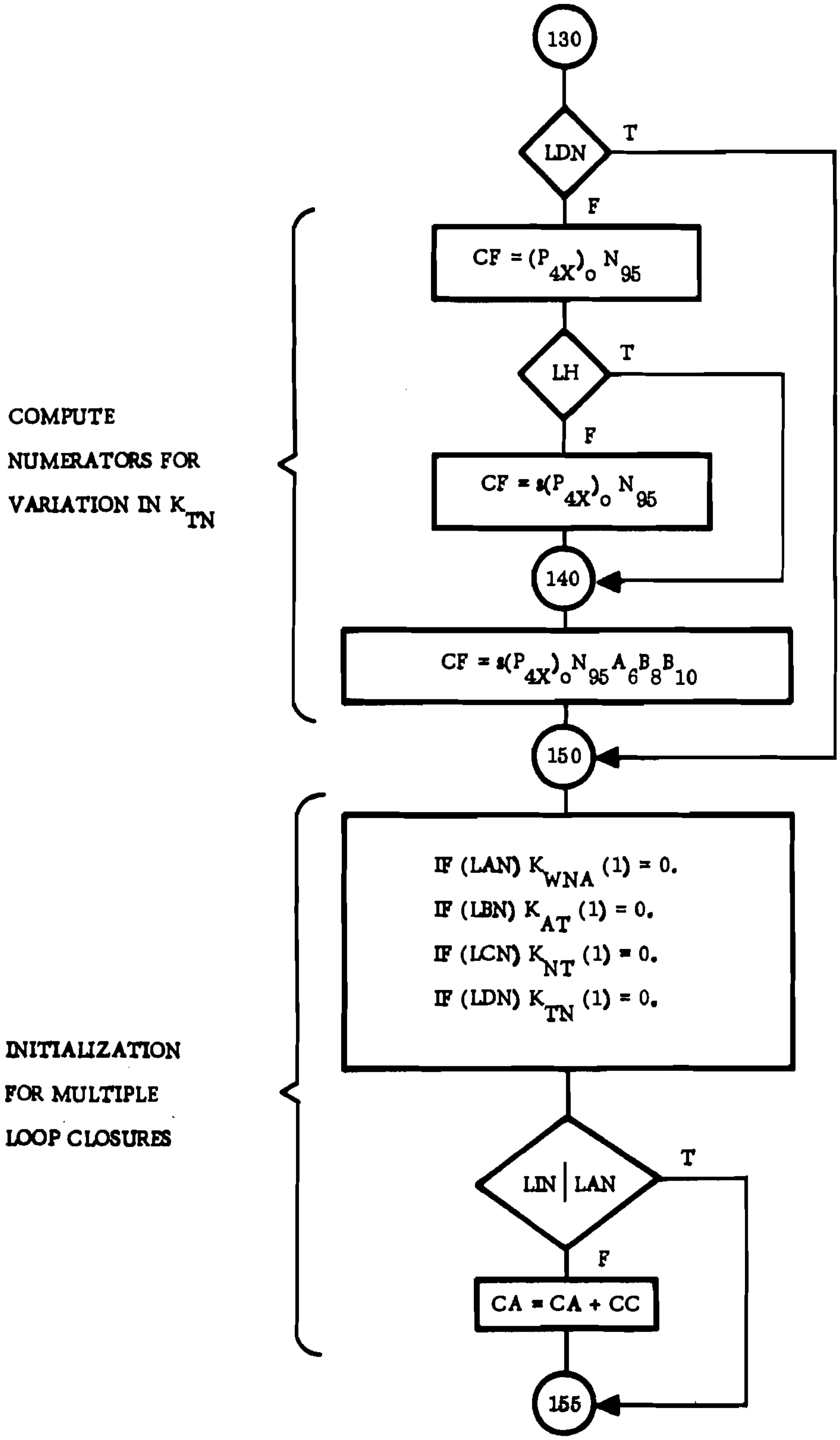

Figure 3. (Continued)

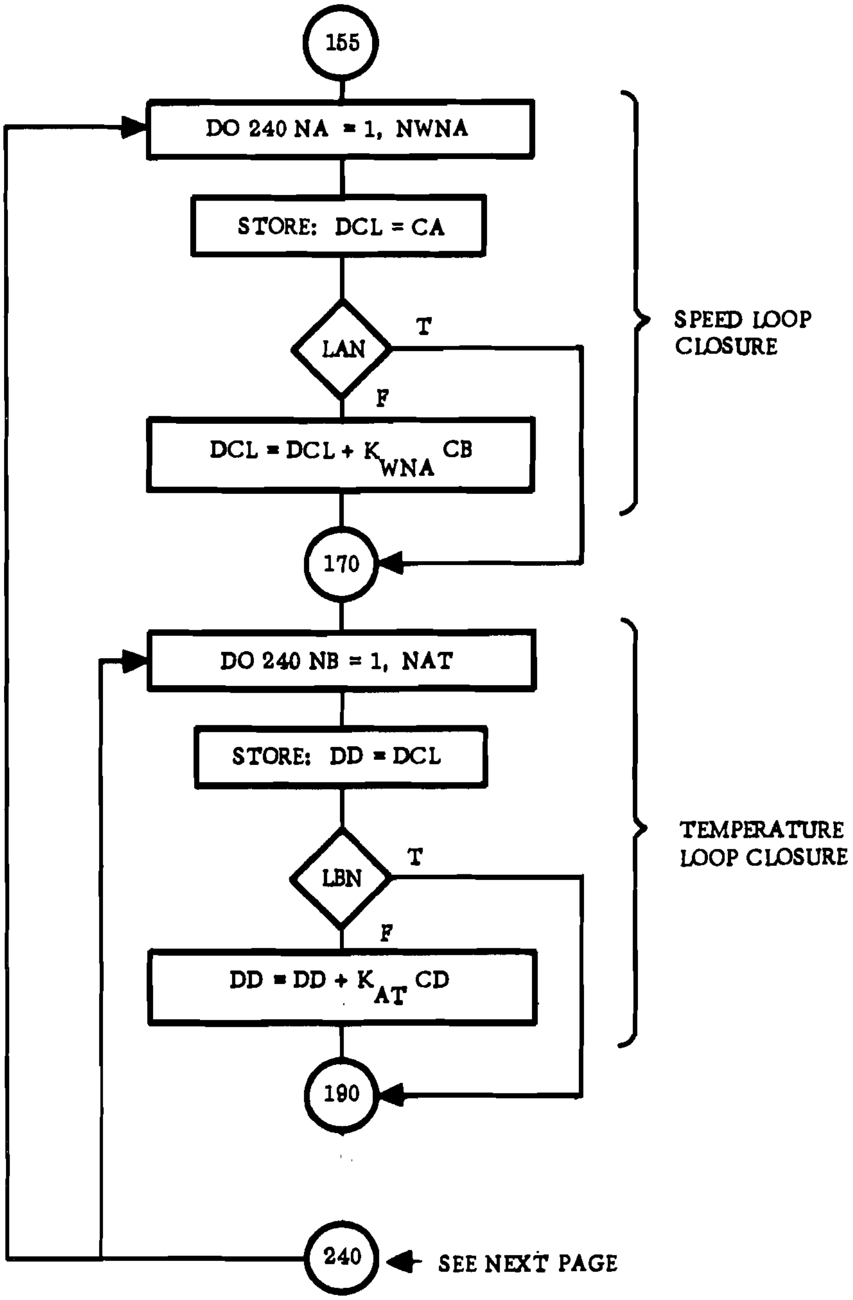

Figure 3. (Continued)

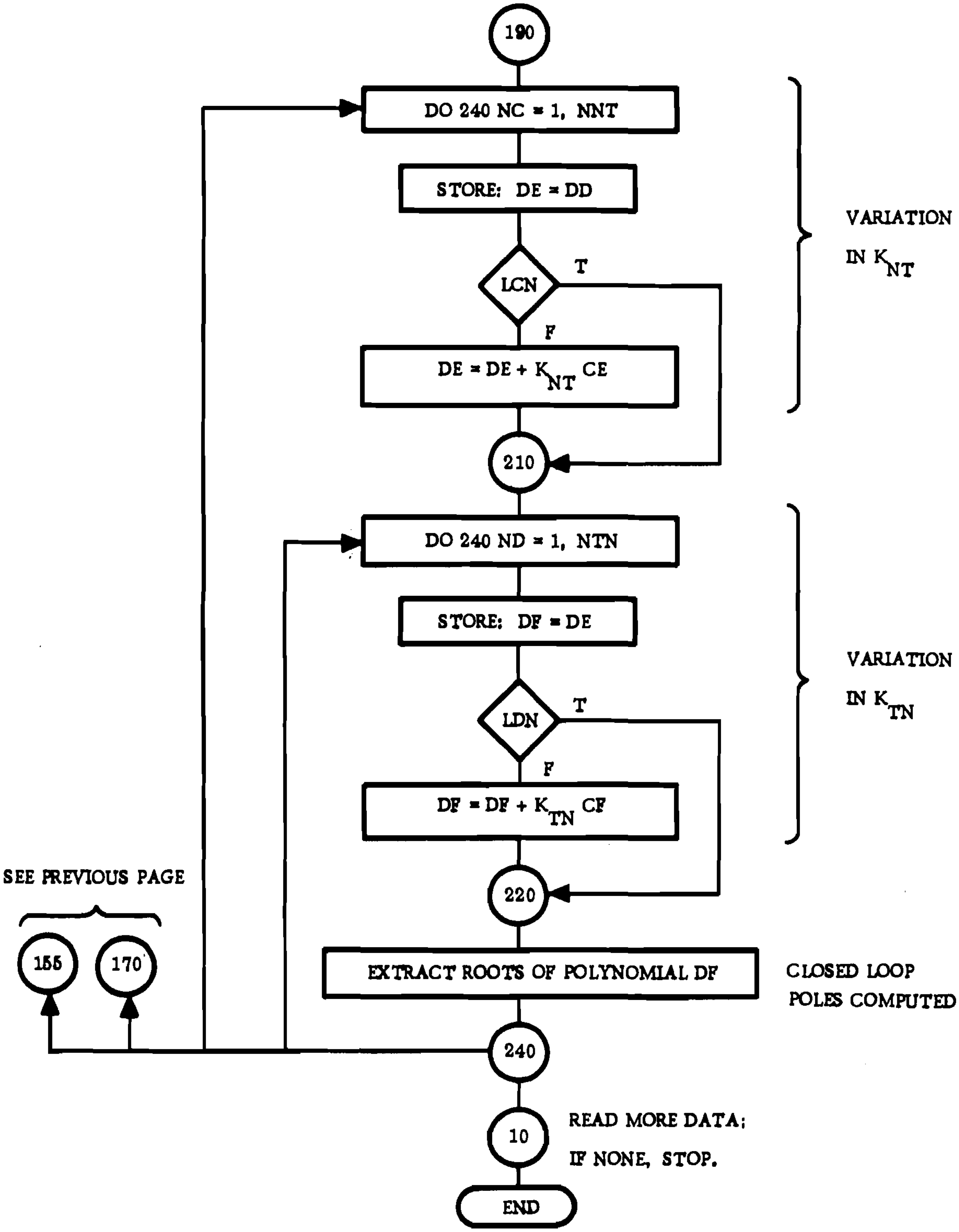

Figure 3. (Concluded)

# Table III. Fortran H Program Listing of TJET

```
C                    MAIN PROGRAM                                       00000030
C     ROOT LOCUS FOR MULTIPLE LOOP CLOSURES FOR TURBOJET    ENGINE      00000040
C     DEVELOPED BY E.L.LUM FEBRUARY 1968                                00000050
C                                                                       00000060
      IMPLICIT LOGICAL*4(L)                                             00000070
      COMPLEX*8 CAC(10)                                                 00000080
      REAL*4 DA(8),N85(8),N86(8),N95(8),N96(8),N105(8)                  00000090
      REAL*4 KWNA(19),KAT(19),KNT(19),KTN(19),KN2A,KT6A                 00000100
      REAL*4 TA5(4),TA6(4),TB9(4),TB10(4),TITLE(18),F(170),TB8(4)       00000110
      REAL*4 CA(20),CB(20),CC(20),CD(20),CE(20),CF(20),CAR(20)          00000120
      REAL*4 DCL(20),DD(20),DE(20),DF(20)                               00000130
      COMMON /CMB/TA5,TA6,TB8,TB9,TB10,NTA5,NTA6,NTB8,NTB9,NTB10        00000145
      EQUIVALENCE (F(2),DA),(F(12),N85),(F(22),N86),(F(32),N95),        00000150
     1(F(42),N96),(F(52),N105),(F(61),P4A),(F(62),WFA),(F(63),KN2A),    00000160
     2(F(64),KT6A),(F(65),CPO),(F(67),KWNA),(F(87),KAT),(F(107),KNT),   00000170
     3(F(127),KTN),(F(10),FLG),(F(20),G85),(F(30),G86),(F(40),G95),     00000180
     4(F(50),G96),(F(60),G105)                                          00000185
      SF=1.                                                             00000190
      DO 5 I=1,170                                                      00000194
    5 F(I)=0.                                                           00000196
   10 READ(5,20)TITLE                                                   00000200
   20 FORMAT(18A4)                                                      00000210
      WRITE(6,25)TITLE                                                  00000220
   25 FORMAT(1H1,4X,'ROOT LOCUS FOR MULTIPLE LOOP CLOSURES FOR  TURBOJET 00000230
     1 ENGINE'//5X,18A4)                                                00000240
      CALL DECRD(F)                                                     00000250
      WRITE(6,30)                                                       00000260
   30 FORMAT(1H0,4X,'NON-ZERO INPUT DATA'//4(10X,'I',7X,'F(I)',5X))     00000270
      CALL PRIN(F,170)                                                  00000280
      DO 35 I=1,4                                                       00000282
      TA5(I)=F(I+146)                                                   00000283
      TA6(I)=F(I+151)                                                   00000284
      TB8(I)=F(I+166)                                                   00000285
      TB9(I)=F(I+156)                                                   00000286
   35 TB10(I)=F(I+161)                                                  00000287
      NDA=F(1)                                                          00000290
      NN85=F(11)                                                        00000300
      NN86=F(21)                                                        00000310
      NN95=F(31)                                                        00000320
```

## Table III. (Continued)

```
      NN96=F(41)                                                        00000330
      NN105=F(51)                                                       00000340
      NWNA=F(66)                                                        00000350
      NAT=F(86)                                                         00000360
      NNT=F(106)                                                        00000370
      NTN=F(126)                                                        00000380
      NTA5=F(146)                                                       00000390
      NTA6=F(151)                                                       00000400
      NTB9=F(156)                                                       00000410
      NTB10=F(161)                                                      00000420
      NTB8=F(166)                                                       00000425
      CAR(1)=1.                                                         00000427
      LA=NWNA.GT.0                                                      00000430
      LAN=.NOT.LA                                                       00000440
      LB=NAT.GT.0                                                       00000450
      LBN=.NOT.LB                                                       00000460
      LC=NNT.GT.0                                                       00000470
      LCN=.NOT.LC                                                       00000480
      LD=NTN.GT.0                                                       00000490
      LDN=.NOT.LD                                                       00000500
      LEN=KN2A.EQ.0.                                                    00000510
      LFN=KT6A.EQ.0.                                                    00000520
      LH=LEN.AND.LFN.OR.LAN.AND.LFN.OR.LBN.AND.LEN.OR.LAN.AND.LBN       00000530
      LAC=LA.OR.LC                                                      00000540
      LAD=LA.OR.LD                                                      00000550
      LBC=LB.OR.LC                                                      00000560
      LBD=LB.OR.LD                                                      00000570
      LIN=WFA.EQ.0..OR.P4A.EQ.0.                                        00000590
      LP=.TRUE.                                                         00000600
      IF(LAC.OR.LBD)GO TO 50                                            00000640
      WRITE(6,40)                                                       00000650
40    FORMAT(1H0,4X,'OPEN LOOP, NO CONTROL GAINS SPECIFIED')            00000660
      GO TO 10                                                          00000670
50    IF(FLG.EQ.0.)GO TO 55                                             00000672
      WRITE(6,52)                                                       00000674
52    FORMAT(1H0,2X,'OPEN LOOP TRANSFER FUNCTION POLYNOMIALS GENERATED')00000676
      CALL GENP(DA,NDA,1.,'DA(S)   ',2)                                 00000678
      IF(LA)CALL GENP(N85,NN85,G85,'N85(S)  ',2)                        00000680
      IF(LC)CALL GENP(N86,NN86,G86,'N86(S)  ',2)                        00000682
      IF(LD)CALL GENP(N95,NN95,G95,'N95(S)  ',2)                        00000684
```

## Table III. (Continued)

```
      IF(LB)CALL GENP(N96,NN96,G96,'N96(S)  ',2)                        00000686
      IF(LA)CALL GENP(N105,NN105,G105,'N105(S) ',2)                     00000688
C        COMPUTE DENOMINATOR CA(S)                                      00000694
   55 CALL GPOL(DA,NDA,1.,CA,NCA)                                       00000696
      IF(LH)GO TO 60                                                    00000700
      NCA=NCA+1                                                         00000710
      CA(NCA+1)=0.                                                      00000720
   60 CALL XPNP5(CA,NCA,LAD,LBC,LAC,LBD,LA,'OPEN LOOP DENOMINATOR, CA(S) 00000740
     1',7,'OPEN LOOP POLES ',4,&10)                                     00000750
      IF(LAN)GO TO 90                                                   00000790
      WRITE(6,310)                                                      00000795
  310 FORMAT(1H0,1X,'SPEED LOOP CLOSED')                                00000800
      CALL GPOL(N85,NN85,P4A,CB,NCB)                                    00000810
      IF(LEN)WRITE(6,350)                                               00000814
  350 FORMAT(1H0,4X,'NO INTEGRATOR IN SPEED LOOP')                      00000816
      IF(LH)GO TO 70                                                    00000820
      WRITE(6,370)KN2A                                                  00000830
  370 FORMAT(1H0,4X,'INTEGRATOR GAIN, KN2A=',G12.5)                     00000835
      CAR(2)=KN2A                                                       00000840
      CALL POLYX(CB,NCB,CAR,1,CB,NCB)                                   00000850
   70 CALL XPNP5(CB,NCB,.FALSE.,LBC,.FALSE.,LBD,LA,'NUMERATOR ASSOCIATED 00000860
     1 WITH KWNA, CB(S)    ',10,'ZEROS ASSOCIATED WITH KWNA  ',7,&10)   00000870
      IF(LIN)GO TO 90                                                   00000915
      CALL GPOL(N105,NN105,-WFA/P4A,CC,NCC)                             00000920
      IF(LH)GO TO 80                                                    00000930
      NCC=NCC+1                                                         00000940
      CC(NCC+1)=0.                                                      00000950
   80 CALL XPNP5(CC,NCC,.FALSE.,LBC,LAC,LBD,LAN,'NUMERATOR ASSOCIATED WI 00000960
     1TH P4A, CC(S)',9,'ZEROS ASSOCIATED WITH P4A    ',7,&10)           00000970
   90 IF(LBN)GO TO 110                                                  00001010
      WRITE(6,320)                                                      00001015
  320 FORMAT(1H0,1X,'TEMPERATURE LOOP CLOSED')                          00001020
      CALL GPOL(N96,NN96,-1.,CD,NCD)                                    00001030
      IF(LFN)WRITE(6,360)                                               00001034
  360 FORMAT(1H0,4X,'NO INTEGRATOR IN TEMPERATURE LOOP')                00001036
      IF(LH)GO TO 100                                                   00001040
      WRITE(6,380)KT6A                                                  00001050
  380 FORMAT(1H0,4X,'INTEGRATOR GAIN, KT6A=',G12.5)                     00001055
      CAR(2)=KT6A                                                       00001060
      CALL POLYX(CD,NCD,CAR,1,CD,NCD)                                   00001070
```

## Table III. (Continued)

```
  100 CALL XPNP5(CD,NCD,LAD,.FALSE.,LAC,.FALSE.,LA,'NUMERATOR ASSOCIATED 00001080
     1 WITH KAT, CD(S)',9,'ZEROS ASSOCIATED WITH KAT    ',7,&10)          00001090
  110 IF(LCN)GO TO 130                                                   00001130
      WRITE(6,330)                                                       00001135
  330 FORMAT(1H0,1X,'VARIATION IN KNT')                                  00001140
      CALL GPOL(N86,NN86,1.,CE,NCE)                                      00001150
      IF(LH)GO TO 120                                                    00001160
      NCE=NCE+1                                                          00001170
      CE(NCE+1)=0.                                                       00001180
  120 CALL XPNP5(CE,NCE,LAD,.FALSE.,.FALSE.,LBD,LA,'NUMERATOR ASSOCIATED 00001190
     1 WITH KNT, CE(S)',9,'ZEROS ASSOCIATED WITH KNT    ',7,&10)          00001200
  130 IF(LDN)GO TO 150                                                   00001250
      WRITE(6,340)                                                       00001255
  340 FORMAT(1H0,1X,'VARIATION IN KTN')                                  00001260
      CALL GPOL(N95,NN95,P4A,CF,NCF)                                     00001270
      IF(LH)GO TO 140                                                    00001280
      NCF=NCF+1                                                          00001290
      CF(NCF+1)=0.                                                       00001300
  140 CALL XPNP5(CF,NCF,.FALSE.,LBC,LAC,.FALSE.,LA,'NUMERATOR ASSOCIATED 00001310
     1 WITH KTN, CF(S)',9,'ZEROS ASSOCIATED WITH KTN    ',7,&10)          00001320
C          MULTIPLE LOOP CLOSURES                                        00001360
  150 IF(LAN)KWNA(1)=0.                                                  00001370
      IF(LBN)KAT(1)=0.                                                   00001380
      IF(LCN)KNT(1)=0.                                                   00001390
      IF(LDN)KTN(1)=0.                                                   00001400
      LP=CPO.EQ.1.                                                       00001402
      IF(LIN.OR.LAN)GO TO 155                                            00001410
      CALL POLADD(CC,NCC,1.,CA,NCA,1.,CA,NCA)                            00001415
      CALL WRTPOL(CA,NCA,'CA=CA+CC',2)                                   00001420
  155 WRITE(6,160)TITLE                                                  00001425
  160 FORMAT(1H1,4X,18A4//5X,'BEGIN LOOP CLOSURES')                      00001430
      DO 240 NA=1,NWNA                                                   00001435
      CALL GPOL(CA,NCA,1.,DCL,NDCL)                                      00001437
      IF(LAN)GO TO 170                                                   00001440
      CALL POLADD(CB,NCB,KWNA(NA),DCL,NDCL,1.,DCL,NDCL)                  00001450
  170 DO 240 NB=1,NAT                                                    00001470
      CALL GPOL(DCL,NDCL,1.,DD,NDD)                                      00001500
      IF(LBN)GO TO 190                                                   00001505
      CALL POLADD(CD,NCD,KAT(NB),DD,NDD,1.,DD,NDD)                       00001510
  190 DO 240 NC=1,NNT                                                    00001530
```

Table III. (Concluded)

```
      CALL GPOL(DD,NDD,1.,DE,NDE)                                       00001550
      IF(LCN)GO TO 210                                                  00001560
      CALL POLADD(CE,NCE,KNT(NC),DE,NDE,1.,DE,NDE)                      00001580
  210 DO 240 ND=1,NTN                                                   00001600
      CALL GPOL(DE,NDE,1.,DF,NDF)                                       00001620
      IF(LDN)GO TO 220                                                  00001640
      CALL POLADD(CF,NCF,KTN(ND),DF,NDF,1.,DF,NDF)                      00001650
  220 WRITE(6,230)KWNA(NA),KAT(NB),KNT(NC),KTN(ND)                      00001660
  230 FORMAT(1H0//'  CONTROL GAINS: KWNA=',G12.5,', KAT=',G12.5,', KNT=' 00001670
     1,G12.5,', KTN=',G12.5)                                            00001680
      IF(LP)CALL WRTPOL(DF,NDF,'CLOSED LOOP DENOMINATOR ',6)            00001690
      CALL PRTX (DF,NDF,CAR,NCAR,CAC,NCAC,50,1.E6,SF,&10)               00001700
  240 CALL WRT(CAR,NCAR,CAC,NCAC,'CLOSED LOOP POLES    ',5)             00001710
      WRITE(6,260)                                                      00001810
  260 FORMAT(1H0,4X,'END OF LOOP CLOSURES')                             00001820
      GO TO 10                                                          00001830
      END                                                               00001840
```

# Table IV. Input Data Format for TJET

DECK NO. TJET PROGRAMMER E. L. LUM DATE 5-29-68 PAGE 1 of 5 JOB NO. ________

|  | NUMBER | IDENTIFICATION | DESCRIPTION DO NOT KEY PUNCH |
|---|---|---|---|
| 1 | Title card |  | Must be first card of run. |
| 13 | description of |  |  |
| 25 | run in alpha-numeric |  |  |
| 37 | characters |  |  |
| 49 |  | 73 80 |  |
| 61 |  | Identification |  |
| 1 | 1 |  | Relative data field location in array F of main program TJET. |
| 13 | NDA |  | 1 Degree of open-loop denominator polynomial |
| 25 | DA(1) Variable names |  | 2 Open-loop denominator polynomial, Da(s) |
| 37 | ↓ assigned to data |  | ↓ See special instructions |
| 49 | DA(8) fields shown | 73 80 | 9 |
| 61 | FLG |  | 10 See special instructions on previous page. |
| 1 | 1 1 |  |  |
| 13 | NN85 |  | 11 Degree of numerator $N_{85}(s)$ |
| 25 | N85(1) |  | 12 Numerator polynomial $N_{85}(s) = \partial N_{2X}/\partial W_{FX}$, for Speed Control |
| 37 | ↓ |  | ↓ See special instructions |
| 49 | N85(8) | 73 80 | 19 |
| 61 | G85 |  | 20 Gain of $N_{85}(s)$, used only if FLG $\neq$ 0. |
| 1 | 2 1 |  |  |
| 13 | NN86 |  | 21 Degree of numerator $N_{86}(s)$ |
| 25 | N86(1) |  | 22 Numerator polynomial $N_{85}(s) = \partial N_{2K}/\partial A_{JX}$, |
| 37 | ↓ |  | ↓ See special instructions |
| 49 | N86(8) | 73 80 | 29 |
| 61 | G86 |  | 30 Gain of $N_{86}(s)$, used only if FLG $\neq$ 0. |

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## Table IV. (Cont)

DECK NO.______ PROGRAMMER ______ DATE ______ PAGE 2 of 5 JOB NO. ______

| | NUMBER | IDENTIFICATION | | DESCRIPTION DO NOT KEY PUNCH |
|---|---|---|---|---|
| 1 | 3 1 | | | |
| 13 | NN95 | | 31 | Degree of numerator $N_{96}(s)$ |
| 25 | N95(1) | | 32 | Numerator polynomial $N_{95}(s) = \partial T_{6X}/\partial W_{FX}$, |
| 37 | ↓ | | ↓ | See special instructions |
| 49 | N95(8) | 73 80 | 39 | |
| 61 | G95 | | 40 | gain of $N_{95}(s)$, used only if $FLG \neq 0$. |
| 1 | 4 1 | | | |
| 13 | NN96 | | 41 | Degree of numerator $N_{96}(s)$ |
| 25 | NN96(1) | | 42 | Numerator polynomial $N_{96}(s) = \partial T_{6X}/\partial A_{JX}$, for Temperature Control |
| 37 | ↓ | | ↓ | See special instructions |
| 49 | N96(8) | 73 80 | 49 | |
| 61 | G96 | | 50 | gain of $N_{96}(s)$, used only if $FLG \neq 0$ |
| 1 | 5 1 | | | |
| 13 | NN105 | | 51 | Degree of numerator $N_{105}(s)$ |
| 25 | N105(1) | | 52 | Numerator polynomial $N_{105}(s) = \partial P_{4X}/\partial W_{FX}$, |
| 37 | ↓ | | ↓ | See special instructions |
| 49 | N105(8) | 73 80 | 59 | |
| 61 | G105 | | 60 | gain of $N_{105}(s)$, used only if $FLG \neq 0$. |
| 1 | 6 1 | | | |
| 13 | P4A | | 61 | $(P_{4X})_o$, initial steady state value of $P_{4X}$ |
| 25 | WFA | | 62 | $(W_{FX})_o$, initial steady state value of $W_{FX}$ |
| 37 | KN2A | | 63 | $K_{N2A}$, integrator gain in Speed Control loop |
| 49 | KT6A | 73 80 | 64 | $K_{T6A}$, integrator gain in Temperature Control loop |
| 61 | CPO | | 65 | Check-out print option, 1. if print-out desired |

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## Table IV. (Cont)

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE 3 of 5 JOB NO. ______

| | NUMBER | IDENTIFICATION | DESCRIPTION DO NOT KEY PUNCH |
|---|---|---|---|
| 1 | 6 6 | | |
| 13 | NWNA | | 66 No. of Control gains, $K_{WNA}$; 0. if Speed Control loop open |
| 25 | KWNA(1) | | 67 |
| 37 | ↓ | | Set of control gains $K_{WNA}$ for Speed Control loop |
| 49 | | 73 80 | closure for which closed-loop characteristic |
| 61 | KWNA(19) | | 85 is computed. |
| 1 | 8 6 | | |
| 13 | NAT | | 86 No. of Control gains, $K_{AT}$; 0. if Temperature Control loop open |
| 25 | KAT(1) | | 87 |
| 37 | ↓ | | Set of control gains $K_{AT}$ for Temperature Control |
| 49 | | 73 80 | loop closure for which closed loop |
| 61 | KAT(19) | | 105 characteristic is computed. |
| 1 | 1 0 6 | | |
| 13 | NNT | | 106 No. of decoupling gains $K_{NT}$ |
| 25 | KNT(1) | | 107 |
| 37 | ↓ | | Set of decoupling gains $K_{NT}$ for |
| 49 | | 73 80 | which closed loop characteristic is |
| 61 | KNT(19) | | 125 computed. |
| 1 | 1 2 6 | | |
| 13 | NTN | | 126 No. of decoupling gains $K_{TN}$. |
| 25 | KTN(1) | | 127 |
| 37 | ↓ | | Set of decoupling gains $K_{TN}$ for |
| 49 | | 73 80 | which closed loop characteristic is |
| 61 | KTN(19) | | 145 computed. |

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## Table IV. (Cont)

DECK NO. ________ PROGRAMMER ________ DATE ________ PAGE 4 of 5 JOB NO. ________

| | NUMBER | IDENTIFICATION | DESCRIPTION DO NOT KEY PUNCH |
|---|---|---|---|
| 1 | 1 4 6 | | |
| 13 | NTA5 | | 146 No. of time constants for $A_5$. |
| 25 | TA5(1) | | 147 |
| 37 | ↓ | | 148 Set of time constants for |
| 49 | | 73 80 | 149 $W_{FX}$ actuator lags, $1/A_5$ |
| 61 | TA5(4) | | 150 |
| 1 | 1 5 1 | | |
| 13 | NTA6 | | 151 No. of time constants for $A_6$ |
| 25 | TA6(1) | | 152 |
| 37 | ↓ | | 153 Set of time constants for |
| 49 | | 73 80 | 154 $A_{JX}$ actuator lags, $1/A_6$ |
| 61 | TA6(4) | | 155 |
| 1 | 1 5 6 | | |
| 13 | NTB9 | | 156 No. of time constants for $B_9$ |
| 25 | TB9(1) | | 157 |
| 37 | ↓ | | 158 Set of time constants for |
| 49 | | 73 80 | 159 $T_{6X}$ sensor lags, $1/B_9$ |
| 61 | TB9(4) | | 160 |
| 1 | 1 6 1 | | |
| 13 | NTB10 | | 161 No. of time constants for $B_{10}$ |
| 25 | TB10(1) | | 162 |
| 37 | ↓ | | 163 Set of time constants for |
| 49 | | 73 80 | 164 $P_{4X}$ sensor lags, $1/B_{10}$ |
| 61 | TB10(4) | | 165 |

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## Table IV. (Concluded)

DECK NO. TJET PROGRAMMER E. L. LUM DATE 5-29-68 PAGE 5 of 5 JOB NO.

| | | NUMBER | IDENTIFICATION | DESCRIPTION DO NOT KEY PUNCH |
|---|---|---|---|---|
| 1 | - | 1 6 6 | | |
| 13 | | NTB8 | | 166 No. of time constants for $B_8$ |
| 25 | | TB8(1) | | 167 |
| 37 | | ↓ | | 168 Set of time constants for |
| 49 | | | 73 80 | 169 $N_{2X}$ sensor lags, $1/B_8$ |
| 61 | | TB8(4) | | 170 |
| 1 | | | | |
| 13 | | Title card for next run | | |
| 25 | | | | |
| 37 | | | | |
| 49 | | | 73 80 | |
| 61 | | | | |
| 1 | - | x x x | | |
| 13 | | Last card of next run | | |
| 25 | | | | |
| 37 | | | | |
| 49 | | | 73 80 | |
| 61 | | | | |
| 1 | / | * | | This card must be last card of entire deck |
| 13 | | | | |
| 25 | | | | |
| 37 | | | | |
| 49 | | | 73 80 | |
| 61 | | | | |

FORM [illegible]-C47 REV. 7-[illegible]

a. Special instructions for data entered into locations 2 through 9, 12 through 19, 22 through 29, 32 through 39, 42 through 49, and 52 through 59 (transfer function data):

   (1) If FLG (location 10) = 0, then these locations contain the coefficients of the indicated polynomials.
   (2) If FLG $\neq$ 0, then these locations contain the first-order lags, $\omega_i$, of the form supplied by P&WA, that is,

$$p(s) = g(1 + s/\omega_1)(1 + s/\omega_2) \ldots (1 + s/\omega_n)$$

   where g is the steady state gains of p(s). The $\omega_i$ 's are destroyed by the program, and for the next run, these locations will contain the coefficients of the computed polynomial p(s).

3. Repeat 1 and 2 above for as many runs as necessary.

4. Last card of entire deck must contain/* in Columns 1 and 2.

## EXAMPLE CASE USING TJET

The example case used is for a turbojet engine operating at sea level, Mach 0.8, military power setting. The necessary transfer functions polynomials required for the Speed Control are:

$$D_a = -0.554335s^2 - 95.0994s - 337.734$$

$$= -0.554335\ (s + 3.62811)\ (s + 167.928)$$

$$N_{85} = -5.55574s - 1883.08 = -5.55574\ (s + 338.943)$$

$$N_{105} = -15.4052s^2 - 2674.94s - 20258.1$$

$$= -15.4052\ (s + 7.9360)\ (s + 165.702)$$

The actuator and sensor lags are:

$$A_5 = 0.02\ (s + 50)$$

$$B_8 = 0.05\ (s + 20)$$

$$B_{10} = 0.02\ (s + 50)$$

The initial conditions are:

$(P_{4X})_o$ = 380.58 psia

$(W_{FX})_o$ = 3.92222 lb/sec

The integrator gain is set to $K_{N2A} = 1.0$ sec$^{-1}$.

For the speed loop closure only, we set $K_{AT} = K_{NT} = K_{TN} = 0$.

The following computer print-out (Table V) shows the computation of the closed loop characteristic and poles for the speed loop closure, for control gain $K_{WNA}$ varying between 0.0 and 0.05. A plot of the root locus of the closed loop poles are shown on Figure 4.

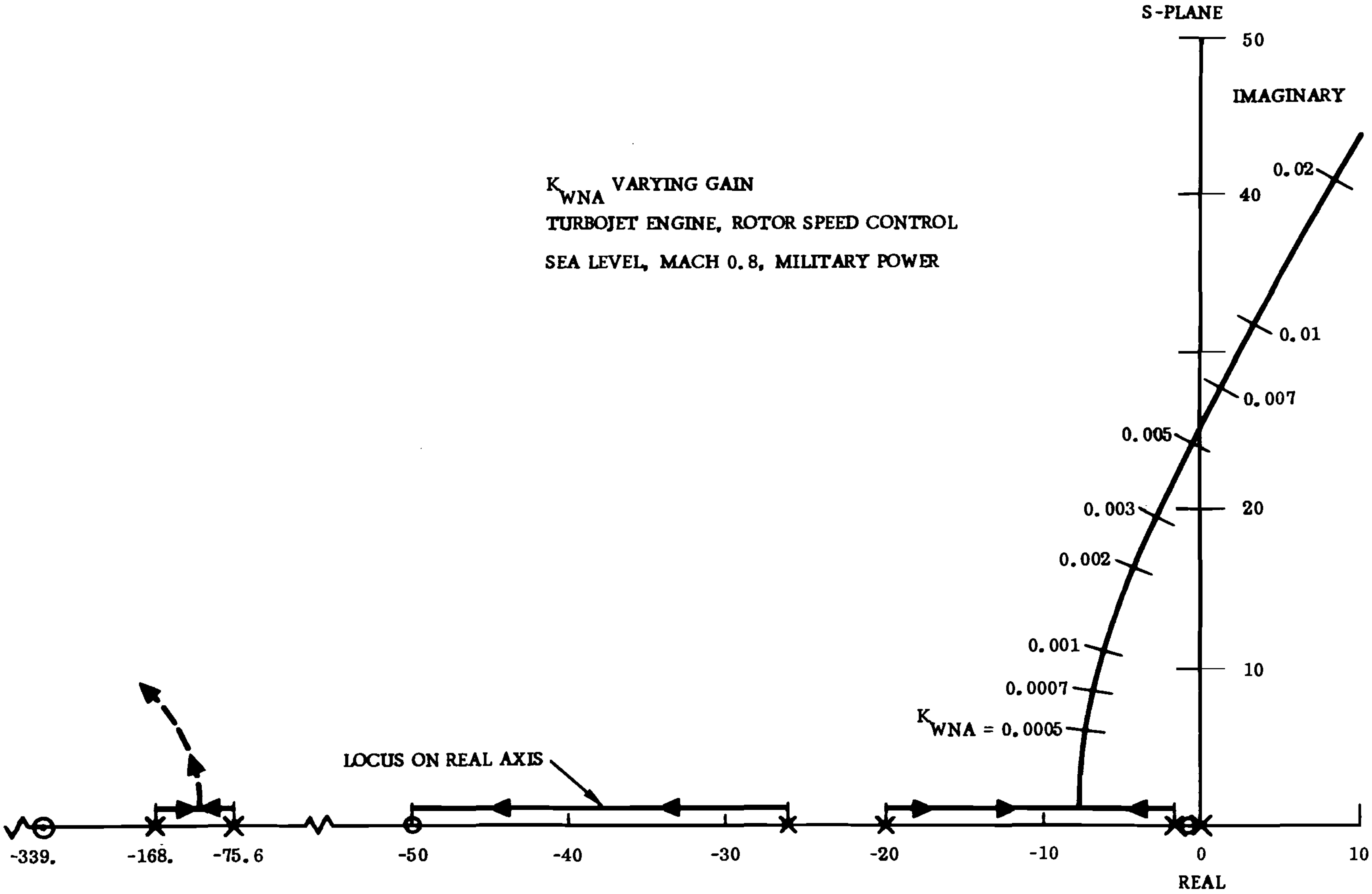


Figure 4. Root Locus for Example Case Using TJET

## Table V. Computer Printout for Example Case Using TJET

```
ROOT LOCUS FOR MULTIPLE LOOP CLOSURES FOR SATELLITE ENGINE

  TURBOJET  , MIL POWER, H=0, M=0.8, SPEED LOOP CLOSURE  ONLY

NON-ZERO INPUT DATA SEE FORMAT OF INPUT DATA

     I         F(I)              I        F(I)              I        F(I)              I        F(I)
     1      2.00000              2   -0.554335              3    -95.0994              4    -337.734
    11      1.00000             12    -5.55574             13    -1883.08             22    -61.5205
    31      2.00000             32    -168.621             33    -25573.8             34    -65030.0
    41      1.00000             42     212.246             43     1613.31             51     2.00000
    52     -15.4052             53    -2674.94             54    -20258.1             61     380.580
    62      3.92222             63     1.00000             64     5.00000             65     1.00000
    66      19.0000             68     0.200000E-04        69    0.300000E-04         70    0.500000E-04
    71     0.700000E-04         72     0.100000E-03        73    0.200000E-03         74    0.300000E-03
    75     0.5C0000E-03         76     0.700000E-03        77    0.100000E-02         78    0.200000E-02
    79     0.300000E-02         80     0.500000E-02        81    0.700000E-02         82    0.100000E-01
    83     0.200000E-01         84     0.300000E-01        85    0.500000E-01         91    0.300000E-03
    92     0.500000E-03         93     0.700000E-03        94    0.100000E-02         95    0.200000E-02
    96     0.300000E-02         97     0.500000E-02        98    0.700000E-02         99    0.100000E-01
   100     0.2C0000E-01        101     0.300000E-01       102    0.500000E-01        103    0.700000E-01
   104     0.100000E 00        105     0.200000           146     1.00000            147    0.200000E-01
   151      1.00000            152     0.200000E-01       156     1.00000            157     1.00000
   161      1.C0000            162     0.200000E-01       166     1.00000            167    0.500000E-01

 OPEN LOOP DENOMINATOR, CA(S)
  PCLYNOMIAL OF ORDER  6, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
  -0.110867E-04    -0.323239E-02    -0.284883          -9.92384          -125.495
   -337.734          0.0

 OPEN LOOP POLES
  NUMBER OF REAL ROOTS=  6
   -3.62811          -19.9999          -50.0001          -50.0002          -167.928
   0.0
```

Compute open-loop
denominator for
speed loop closure

```
SPEED LOOP CLOSED

  INTEGRATOR GAIN, KN2A=  1.0000

 NUMERATOR ASSOCIATED WITH KWNA, CB(S)
  POLYNOMIAL OF ORDER  3, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
   -42.2880          -16489.9          -733110.          -716662.

 ZEROS ASSOCIATEC WITH KWNA
```

Compute numerators
for speed loop closure

## Table V. (Continued)

```
 NUMBER OF REAL ROOTS=  3
  -50.0000          -1.00000          -338.943

NUMERATOR ASSOCIATED WITH P4A, CC(S)
 PCLYNOMIAL OF ORDER  4, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
  0.793825E-02       1.53715           38.0065           208.778          0.0

ZEROS ASSOCIATED WITH P4A
 NUMBER OF REAL ROOTS=  4
  -20.0000          -7.93600          -165.702          0.0

CA=CA+CC
 POLYNOMIAL OF ORDER  6, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
 -0.110867E-04     -0.323239E-C2     -0.276945          -8.38669         -87.4889
  -128.957          0.0
```

Compute numerators for speed loop closure

# Table V. (Continued)

```
  SATELLITE, MIL POWER, H=0, M=0.8, SPEED LOOP CLOSURE ONLY

 BEGIN LOOP CLOSURES
                Variable Gain            All of these gains are set to zero.

CONTROL GAINS: KWNA= 0.0          , KAT= 0.0          , KNT= 0.0          , KTN= 0.0

  CLOSED LOOP DENOMINATOR
   POLYNOMIAL OF ORDER  6, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
   (-0.110867E-04)s^6 -(0.323239E-02)s^5 -(0.276945) s^4   -(8.38669)s^3      -(87.4889) s^2     Check open-
    -(128.957)s    +  0.0                                                                          loop poles

  CLOSED LOOP POLES
   NUMBER OF REAL ROOTS=  6
    -1.75134         -26.1284          -20.0001          -168.045          -75.6312
    0.0


CONTROL GAINS: KWNA= 0.20000E-04, KAT= 0.0          , KNT= 0.0          , KTN= 0.0

  CLOSED LOOP DENOMINATOR
   POLYNOMIAL OF ORDER  6, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
   -0.110867E-04    -0.323239E-02    -0.276945         -8.38754          -87.8187        First gain change
    -143.619         -14.3332                                                            K_WNA = 0.00002

  CLOSED LOOP POLES
   NUMBER OF REAL ROOTS=  6
    -1.85717         -0.106690         -19.2933          -75.6579          -26.6006
    -168.040


CONTROL GAINS: KWNA= 0.30000E-04, KAT= 0.0          , KNT= 0.0          , KTN= 0.0

  CLOSED LOOP DENOMINATOR
   POLYNOMIAL OF ORDER  6, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
   -0.110867E-04    -0.323239E-02    -0.276945         -8.38796          -87.9836
    -150.950         -21.4999

  CLOSED LOOP POLES
   NUMBER OF REAL ROOTS=  6
   -0.156493         -1.91675          -26.8064          -18.9667          -168.037
```

### Table V. (Continued)

```
     -75.6720


CONTROL GAINS: KWNA= 0.50000E-04, KAT= 0.0          , KNT= 0.0          , KTN= 0.0

   CLOSED LOOP DENOMINATOR
    POLYNOMIAL OF ORDER  6, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
    -0.110867E-04    -0.323239E-02    -0.276945          -8.38881          -88.3134
     -165.612         -35.8331

   CLOSED LOOP POLES
    NUMBER OF REAL ROOTS=  6
     -2.05030         -0.248536         -27.1777          -18.3476          -168.033
     -75.6986


CONTROL GAINS: KWNA= 0.70000E-04, KAT= 0.0          , KNT= 0.0          , KTN= 0.0

   CLOSED LOOP DENOMINATOR
    POLYNOMIAL OF ORDER  6, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
    -0.110867E-04    -0.323239E-02    -0.276945          -8.38965          -88.6432
     -180.274         -50.1664

   CLOSED LOOP POLES
    NUMBER OF REAL ROOTS=  6
     -2.20417         -0.330248         -27.5069          -17.7607          -168.028
     -75.7259


CONTROL GAINS: KWNA= 0.10000E-03, KAT= 0.0          , KNT= 0.0          , KTN= 0.0

   CLOSED LOOP DENOMINATOR
    POLYNOMIAL OF ORDER  6, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
    -0.110867E-04    -0.323239E-02    -0.276945          -8.39092          -89.1379
     -202.268         -71.6662

   CLOSED LOOP POLES
    NUMBER OF REAL ROOTS=  6
    -0.433966         -2.47540          -27.9434          -16.9158          -168.021
     -75.7663
```

Table V. (Continued)

```
CONTROL GAINS: KWNA= 0.10000E-01, KAT= 0.0        , KNT= 0.0         , KTN= 0.0

  CLOSED LOOP DENOMINATOR
   POLYNOMIAL OF ORDER  6, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
   -0.110867E-04    -0.323239E-02    -0.276945         -8.80957         -252.388
    -7460.05         -7166.62

  CLOSED LOOP POLES
   NUMBER OF REAL ROOTS=  4
   -0.992900         -88.0103         -43.6979         -165.499
   NUMBER OF COMPLEX PAIR ROOTS=  1
     3.32245      31.8092


CONTROL GAINS: KWNA= 0.20000E-01, KAT= 0.0        , KNT= 0.0         , KTN= 0.0

  CLOSED LOOP DENOMINATOR
   POLYNOMIAL OF ORDER  6, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
   -0.110867E-04    -0.323239E-02    -0.276945         -9.23245         -417.287
    -14791.2         -14333.2

  CLOSED LOOP POLES
   NUMBER OF REAL ROOTS=  4
   -0.996455         -46.1716         -162.499         -98.6472
   NUMBER OF COMPLEX PAIR ROOTS=  1
     8.37923      41.0215


CONTROL GAINS: KWNA= 0.30000E-01, KAT= 0.0        , KNT= 0.0         , KTN= 0.0

  CLOSED LOOP DENOMINATOR
   POLYNOMIAL OF ORDER  6, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
   -0.110867E-04    -0.323239E-02    -0.276945         -9.65533         -582.187
    -22122.2         -21499.9

  CLOSED LOOP POLES
   NUMBER OF REAL ROOTS=  4
   -0.997639         -47.2520         -158.772         -108.603
   NUMBER OF COMPLEX PAIR ROOTS=  1
     12.0348      47.3382
```

Table V. (Concluded)

```
CONTROL GAINS: KWNA= C.50000E-01, KAT= 0.0          , KNT= 0.0          , KTN= 0.0

  CLOSED LOOP DENCMINATOR
   PCLYNOMIAL OF ORCER  6, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
   -0.110867E-04    -0.323239E-02    -0.276945          -10.5011          -911.985
    -36784.4         -35833.1

  CLOSED LOOP POLES
   NUMBER OF REAL ROOTS=  4
   -0.998584         -48.2449         -141.754          -135.568
   NUMBER OF COMPLEX PAIR ROOTS=  1
     17.5046       56.4327

   END OF LOOP CLOSURES
```

# III. DESCRIPTION OF MAIN PROGRAM TFAN

## BLOCK DIAGRAM AND NOMENCLATURE

The control systems for the turbofan under study are:

1. the turbofan low pressure rotor speed control, or Fan Speed Control, in short,
2. the fan discharge corrected air flow control, or Corrected Air Flow Control, in short,
3. the turbofan high pressure rotor speed control, or H. P. Rotor Speed Control, in short.

The block diagrams for the individual control loops are shown on Figure 5. The nomenclature used in the figure and in the control equations are defined by Table VI. Control gains $K_{AN1}$, $K_{AW}$, $K_{WN2}$, $K_{N1}$, $K_W$, and $K_{N2}$ are to be determined by the linear analysis.

Program TFAN allows for the variation of control gains $K_{AN1}$, $K_{AW}$ or $K_{WN2}$ to obtain the root locus of the corresponding control loop. Integrator gains $K_{N1}$, $K_W$ and $K_{N2}$ are preselected. Individual control loops are investigated by setting the control gains for the other loops to zero. For example, to obtain a root locus for the Fan Speed Control only, one must let $K_{AW} = K_{WN2} = 0$.

The present version of TFAN is useful only for the analysis of single loop closures because the cross-coupling transfer functions necessary for the analysis of two or more coupled control loops were not included in the equations used in programming TFAN. This omission was made because the cross-coupling transfer functions for the turbofan engine were not available anyway. Hence, although TFAN is set up for sequential loop closures, the results for varying more than one control gain simultaneously will not be valid.

## EQUATIONS FOR LINEAR ANALYSIS OF TURBOFAN ENGINE

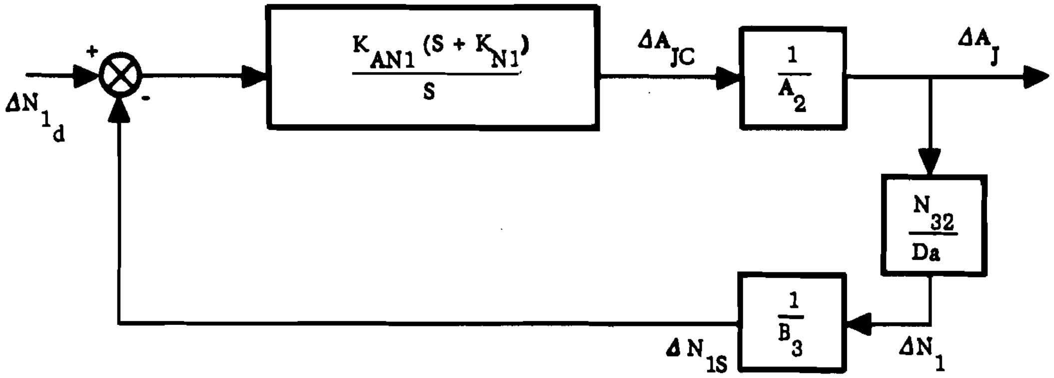
The closed loop characteristic, (RC), of the turbofan engine control, without coupling between control loops, is (using the same notation as defined for the arrays in TFAN).

$$(RC) = (PA) + K_{AN1}(PB) + K_{AW}(PC) + [(PE) + K_{WN2}(PD)]$$

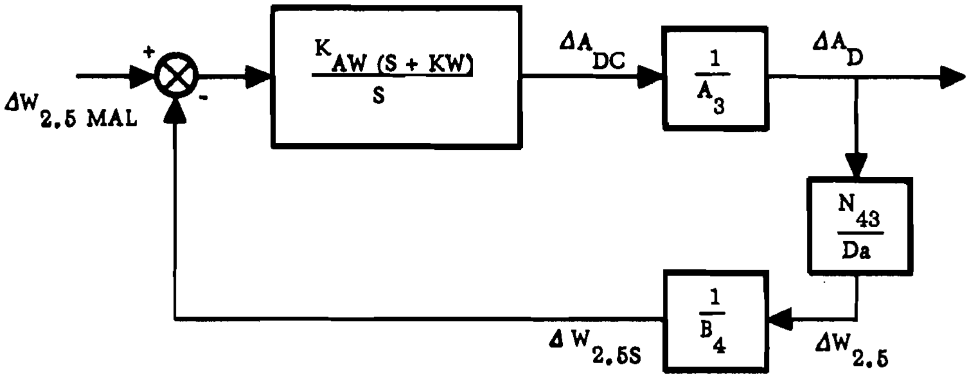
where

1. Denominator for root locus

$$(PA) = s(A_2B_3)(A_3B_4)(A_4B_5B_6B_7)D_a$$



(a) Fan Speed Control



(b) Fan Corrected Air Flow Control

**Figure 5. Turbofan Engine Linear Control System Block Diagram**

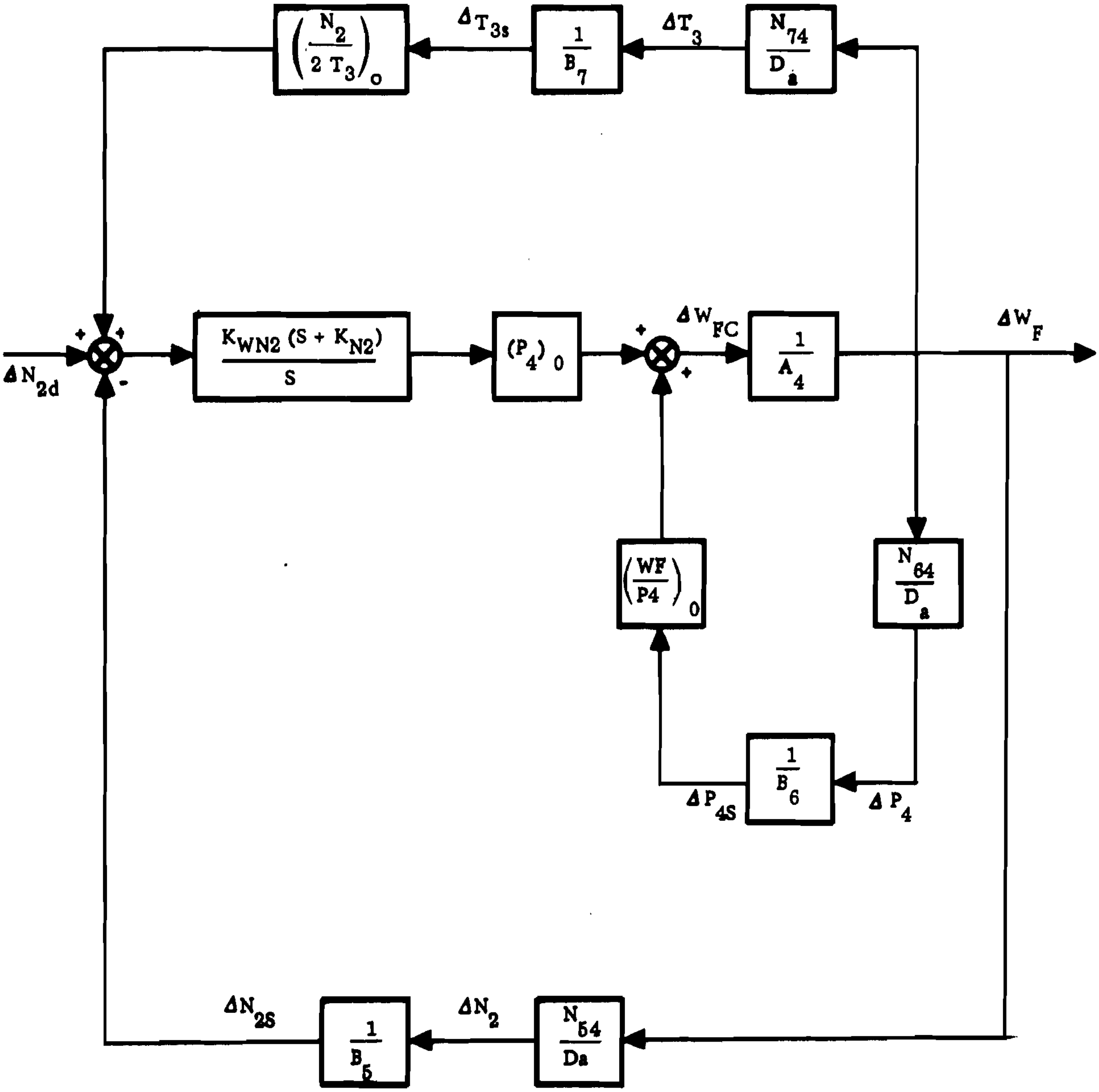


Figure 5. (Concluded)

Table VI. Nomenclature for Linear Analysis of Turbofan Engine Control

a) Actuated and Sensed Variables

| Actuated Variables | Description |
|---|---|
| $x_2 = A_D$ | Turbofan Main Exhaust Nozzle Area ($ft^2$) |
| $x_3 = A_J$ | Fan Duct Exhaust Nozzle Area ($ft^2$) |
| $x_4 = W_F$ | Turbofan Fuel Flow (lb/hr) |
| **Sensed Variables** | |
| $y_3 = N_1$ | Fan Rotor Mechanical Speed (RPM) |
| $y_4 = W_{2.5}$ | Fan Discharge Corrected Air Flow |
| $y_5 = N_2$ | Turbofan High Pressure Rotor Mechanical Speed (RPM) |
| $y_6 = P_4$ | Turbofan High Pressure Rotor Compressor Discharge Pressure (psia) |
| $y_7 = T_3$ | Turbofan High Pressure Rotor Compressor Inlet Temperature (°R) |

b) Actuator and Sensor Definitions

$A_J$ Actuator $= 1/A_2$

$A_D$ Actuator $= 1/A_3$

$W_F$ Actuator $= 1/A_4$

$N_1$ Sensor $= 1/B_3$

$W_{2.5}$ Sensor $= 1/B_4$

$N_2$ Sensor $= 1/B_5$

$P_4$ Sensor $= 1/B_6$

$T_3$ Sensor $= 1/B_7$

Table VI. Concluded

where $A_j$ is the denominator of the actuator lag for $x_j$, and $B_i$ is the denominator lag for $y_i$.

c) Transfer Function Numerators

$N_{32}$ = Numerator of $\partial N_1/\partial A_J$ Transfer Function

$N_{43}$ = Numerator of $\partial W_{2.5}/\partial A_D$ Transfer Function

$N_{54}$ = Numerator of $\partial N_2/\partial W_F$ Transfer Function

$N_{64}$ = Numerator of $\partial P_4/\partial W_F$ Transfer Function

$N_{74}$ = Numerator of $\partial T_3/\partial W_F$ Transfer Function

In general, $N_{ij}/D_a$ is the transfer function for the i-th dependent variable, $y_i$, due to the j-th independent variable $x_j$. $D_a$ is the open loop denominator of the turbofan engine dynamics.

2. Numerator for Fan Speed Control

$$(PB) = N_{32}(s + K_{N1})(A_3B_4)(A_4B_5B_6B_7)$$

3. Numerator for Corrected Air Flow Control

$$(PC) = N_{43}(s + K_W)(A_2B_3)(A_4B_5B_6B_7)$$

4. Numerators for H.P. Rotor Speed Control

$$(PD) = (s + K_{N2})(P_4)_o B_6[N_{54}B_7 - (N_2/2T_3)N_{74}B_5](A_2B_3)(A_3B_4)$$

$$(PE) = -s(W_F/P_4)_o N_{64}(B_5B_7)(A_2B_3)(A_3B_4)$$

Actually, TFAN has logic statements which omit certain actuator poles, $A_i$, and sensor poles, $B_j$, from these equations if they are not affected by one of the loop closures. For example, if only the Fan Speed Loop is closed, then $(A_3B_4)$ and $(A_4B_5B_6B_7)$ are set to unity.

## FLOWCHART AND PROGRAM LISTING OF TFAN

Table VII defines the nomenclature used in the programming of computer program TFAN. The flowchart for mainprogram TFAN is shown on Figure 6. The printout statements are left out of the flowchart for clarity. The circled numbers in Figure 6 correspond to the statement numbers in the Fortran H program listing of TFAN shown on Table VIII.

## FORMAT OF INPUT DATA FOR TFAN

The following pages (Table IX) show the format of input data for TFAN. The data cards should follow the [//G.SYSIN DD*] control card and should be arranged as follows:

1. Title card, description of run in alpha-numeric characters which will be printed at top of first page of print-out of the run. This card must be the first card of each run.
2. Data cards for each run. All data must be real *4 variables and are read into array F of main program TFAN. The relative locations of the variables assigned to array F are described on the following pages. Blanks will leave data from the previous run unchanged. A minus sign must be in Column 1 on the last data card for each run.

    a. Special instructions for data entered into locations 2 through 9, 12 through 19, 22 through 29, 32 through 39, 42 through 49, and 52 through 59 (transfer function data):

        (1) If FLG (location 10) = 0, then these locations contain the coefficients of the indicated polynomials.
        (2) If FLG $\neq 0$, then these locations contain the first-order lags, $\omega_i$, of the form supplied by P&WA, that is,

$$p(s) = g(1 + s/\omega_1)(1 + s/\omega_2)\ldots(1 + s/\omega_n)$$

where g is the steady state gain of p(s). The $\omega_i$ 's are destroyed by the program, and for the next run these locations will contain the coefficients of the computed polynomial p(s).

3. Repeat 1 and 2 above for as many runs as necessary.
4. The last card of the entire deck must contain/* in Columns 1 and 2.

Table VII. List of Subprograms and Variables Used for TFAN

a) List of Subprograms Required for TFAN

Non-library: in Fortran H for IBM 360

| | |
|---|---|
| 1. XPNP3 | Expand a polynomial with three sets of first-order lags |
| 2. GENP | Generate a polynomial from data of the form given by P&WA |
| 3. XPNTAU | Expand a polynomial with a given set of first-order lags |
| 4. PRTX[4] | Polynomial root extraction using Bairstow and Newton-Raphson iteration algorithms |
| 5. FLIP[4] | Reverse order of elements in an array |
| 6. POLYX[5] | Multiply two polynomials |
| 7. GPOL[5] | Multiply a polynomial by a constant |
| 8. POLADD[5] | Add two polynomials |
| 9. WRT[5] | Print roots of a polynomial |
| a. WRTPOL[5] | Print coefficients of a polynomial |
| 10. PRIN[5] | Print non-zero elements of an array |
| 11. STREAL[5] | Store two arrays in one array |
| a. STR3[5] | Store three arrays in one array |

Library: for Fortran IV on IBM 360

| | |
|---|---|
| 12. DECRD | Read variable length data in standard format |

---

[4] Developed by J. C. Long of NAR/Space Division.

[5] Subprograms originally developed for ALODES's I and II, Automatic Linear Optimal Design and Evaluation, computer programs.

Table VII. (Cont)

b) Arrays of Variables used in TFAN

| | | |
|---|---|---|
| DA | $= D_a$ | Open loop denominator polynomial[6] |
| N32 | $= N_{32}$ | Numerator polynomial[6] of $\partial N_1/\partial A_J$ transfer function |
| N43 | $= N_{43}$ | Numerator polynomial[6] of $\partial W_{2.5}/\partial A_D$ transfer function |
| N54 | $= N_{54}$ | Numerator polynomial[6] of $\partial N_2/\partial W_F$ transfer function |
| N64 | $= N_{64}$ | Numerator polynomial[6] of $\partial P_4/\partial W_F$ transfer function |
| N74 | $= N_{74}$ | Numerator polynomial[6] of $\partial T_3/\partial W_F$ transfer function |
| KAN1 | $= K_{AN1}$ | Control gains for Fan Speed Control |
| KAW | $= K_{AW}$ | Control gains for Corrected Air Flow Control |
| KWN2 | $= K_{WN2}$ | Control gains for H. P. Rotor Speed Control |
| TA2 | $= A_2$ | Time constants of $A_J$ actuator |
| TA3 | $= A_3$ | Time constants of $A_D$ actuator |
| TA4 | $= A_4$ | Time constants of $W_F$ actuator |
| TB3 | $= B_3$ | Time constants of $N_1$ sensor |
| TB4 | $= B_4$ | Time constants of $W_{2.5}$ sensor |
| TB5 | $= B_5$ | Time constants of $N_2$ sensor |
| TB6 | $= B_6$ | Time constants of $P_4$ sensor |
| TB7 | $= B_7$ | Time constants of $T_3$ sensor |

---

[6] Polynomials expressed either in terms of (1) the coefficients of the polynomial, or (2) the time constants and gain of the polynomial. See input data format for TFAN.

Table VII. (Cont)

| | |
|---|---|
| F | Array for the input of data using DECRD |
| TITLE | Title of run in alpha-numeric characters (up to 72 characters) |
| PA, PB, PC,<br>PD, PE | Arrays used for the temporary storage of the coefficients of polynomials in intermediate computations |
| RA, RB, RC | Arrays used for the temporary storage of coefficients of polynomials used to compute closed-loop characteristic polynomial |
| RR, CC (Complex) | Arrays used for the temporary storage of the real and complex roots of a polynomial |
| $HA = (A_2, B_3)$ | Array used to store time constants of actuator and sensor of Fan Speed Control Loop |
| $HB = (A_3, B_4)$ | Array used to store time constants of actuator and sensor of Corrected Air Flow Control Loop |
| $HC = (A_4, B_5, B_6, B_7)$ | Array used to store time constants of actuator and sensor of H. P. Rotor Speed Control Loop |

c) <u>Real *4 Variables</u>

| | | |
|---|---|---|
| FLG | $\neq 0$ if data entered in form supplied by P&WA | |
| G32 | Steady-state gain for $\partial N_1/\partial A_J$ transfer function | |
| G43 | Steady-state gain for $\partial W_{2.5}/\partial A_D$ transfer function | |
| G54 | Steady-state gain for $\partial N_2/\partial W_F$ transfer function | Used only if FLG$\neq$ 0. |
| G64 | Steady-state gain for $\partial P_4/\partial W_F$ transfer function | |
| G74 | Steady-state gain for $\partial T_3/\partial W_F$ transfer function | |
| $KN1 = K_{N1}$ | Integrator gain for Fan Speed Control Loop | |

Table VII. (Cont)

| | | | |
|---|---|---|---|
| KW | $= K_{W1}$ | Integrator gain for Corrected Air Flow Control Loop | |
| KN2 | $= K_{N2}$ | Integrator gain for H. P. Rotor Control Loop | |
| P4 | $= (P_4)_o$ | Initial Steady State Value of $P_4$ | |
| WF | $= (W_F)_o$ | Initial Steady State Value of $W_F$ | Used only for H. P. Rotor Speed Control |
| N2 | $= (N_2)_o$ | Initial Steady State Value of $N_2$ | |
| T3 | $= (T_3)_o$ | Initial Steady State Value of $T_3$ | |
| CPO | | 1.0 if coefficients of closed loop characteristic polynomial printed out. | |
| SF | | Scale factor for roots (used as a dummy argument when calling subroutine PRTX) | |

d) Integer *4 Variables

| | |
|---|---|
| NDA | Degree of open loop denominator polynomial, $D_a$ |
| NN32 | Degree of numerator polynomial of $\partial N_1/\partial A_J$ transfer function |
| NN43 | Degree of numerator polynomial of $\partial W_{2.5}/\partial A_D$ transfer function |
| NN54 | Degree of numerator polynomial of $\partial N_2/\partial W_F$ transfer function |
| NN64 | Degree of numerator polynomial of $\partial P_4/\partial W_F$ transfer function |
| NN74 | Degree of numerator polynomial of $\partial T_3/\partial W_F$ transfer function |
| NTA2 | Number of time constants for $A_J$ actuator |
| NTA3 | Number of time constants for $A_D$ actuator |
| NTA4 | Number of time constants for $W_F$ actuator |
| NTB3 | Number of time constants for $N_1$ sensor |
| NTB4 | Number of time constants for $W_{2.5}$ sensor |

Table VII. (Cont)

| | |
|---|---|
| NTB5 | Number of time constants for $N_2$ sensor |
| NTB6 | Number of time constants for $P_4$ sensor |
| NTB7 | Number of time constants for $T_3$ sensor |
| NAN1 | Number of control gains $K_{AN1}$ for which root locus is computed |
| NAW | Number of control gains $K_{AW}$ for which root locus is computed |
| NWN2 | Number of control gains $K_{WN2}$ for which root locus is computed |

The following integers are used for indexing only: I, NA, NB, NC.

The following integers are used to indicate degree of polynomials for intermediate computations:

NPA, NPB, NPC, NPD, NPE, NRA, NRB, NRC

Example: NPA indicates that the first (NPA + 1) locations of array PA are used to store coefficients of a polynomial.

The following integers are used to indicate the number of locations used in an array for intermediate computations:

NHA, NHB, NHC, NCC, NRR

Example: NHA indicates that the first (NHA) locations of array HA are being used for computation.

### e) List of Logical Constants for TFAN

Logical constant is .TRUE. if the expression to the right of the colon (:) is true; .FALSE. otherwise.

Symbols: a|b means a or b; a & b means a and b.

| | | |
|---|---|---|
| LA: | NAN1 > 0 | Fan Speed Control Loop closed |
| LAN: | NAN1 ≤ 0 | Fan Speed Control Loop open |
| LB: | NAW > 0 | Corrected Air Flow Control Loop closed |
| LBN: | NAW ≤ 0 | Corrected Air Flow Control Loop open |

Table VII. Concluded

| | | |
|---|---|---|
| LC: | NWN2 > 0 | H. P. Rotor Speed Control Loop closed |
| LCN: | NWN2 ≤ 0 | H. P. Rotor Speed Control Loop open |
| LDN: | $K_{N1} = 0$ | No integrator gain in Fan Speed Control Loop |
| LEN: | $K_W = 0$ | No integrator gain in Corrected Air Flow Control Loop |
| LFN: | $K_{N2} = 0$ | No integrator gain in H. P. Rotor Control Loop |
| LH: | (LDN & LEN & LFN) \| (LAN & LBN & LFN) \|<br>(LBN & LCN & LDN) \| (LAN & LCN & LEN)<br>If LH is true, then a free s term can be factored out. | |
| LJN: | $[(W_F)_o = 0] \mid [(P_4)_o = 0]$ | No $P_4$ term in H. P. Rotor Speed Control<br>See Equations. |
| LP: | CPO = 1 | If true, then coefficients of closed loop characteristic are also printed out. |

## EXAMPLE CASE USING TFAN

The example case used is for a typical turbofan engine operating at sea level, zero Mach, and military power setting. The necessary transfer function polynomials required for the H. P. Rotor Speed Control (Figure 5c) are:

$D_a = (1 + s/1.8)(1 + s/14)$

$N_{54} = 0.54$

$N_{64} = 0.03(1 + s/4)$

$N_{74} = 0.016$

The initial conditions are:

$(P_4)_o = 354.7$ psia

$(W_F)_o = 5822.2$ lb/hr

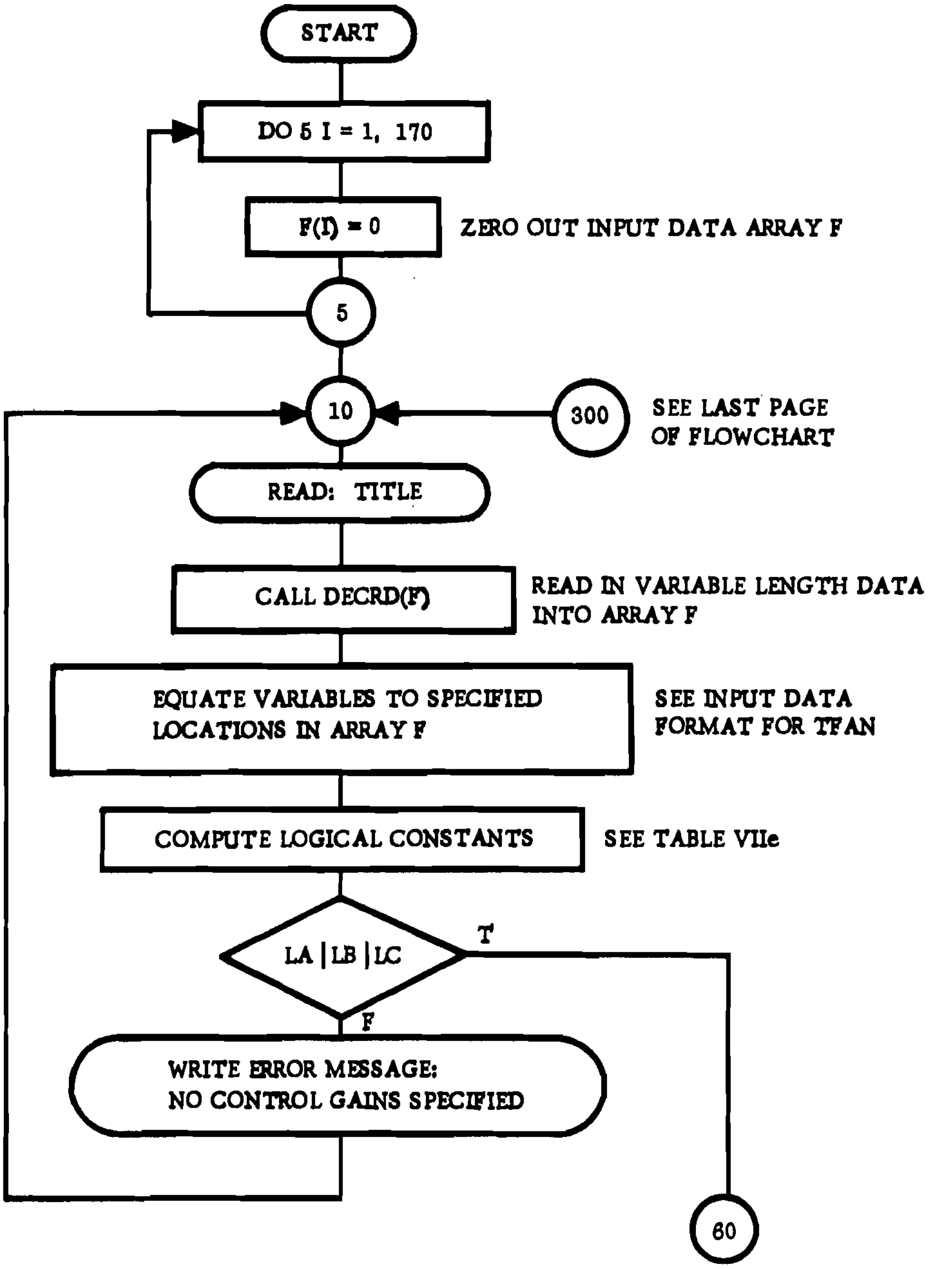


Figure 6. Flowchart of Main Program TFAN

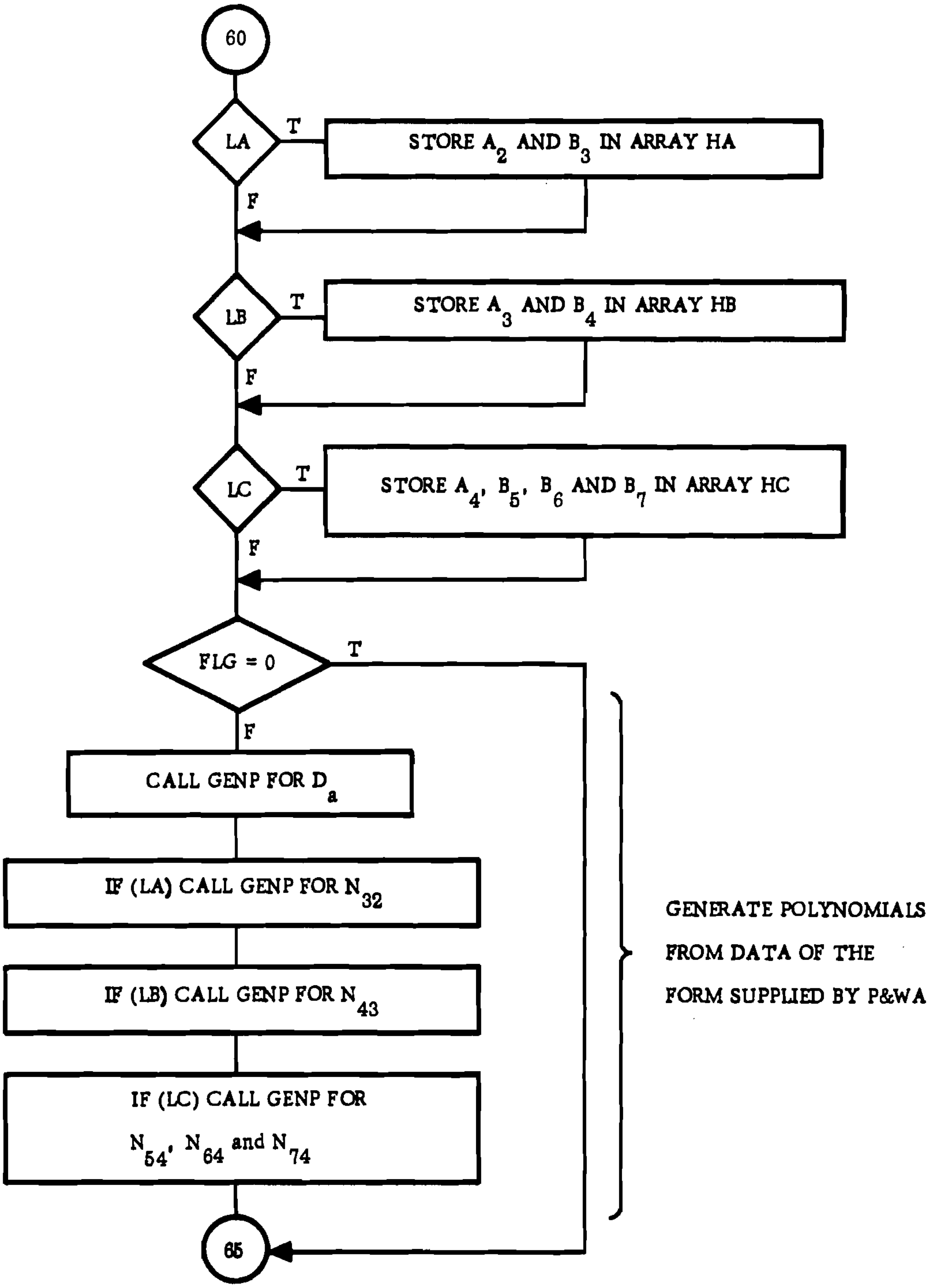


Figure 6. (Continued)

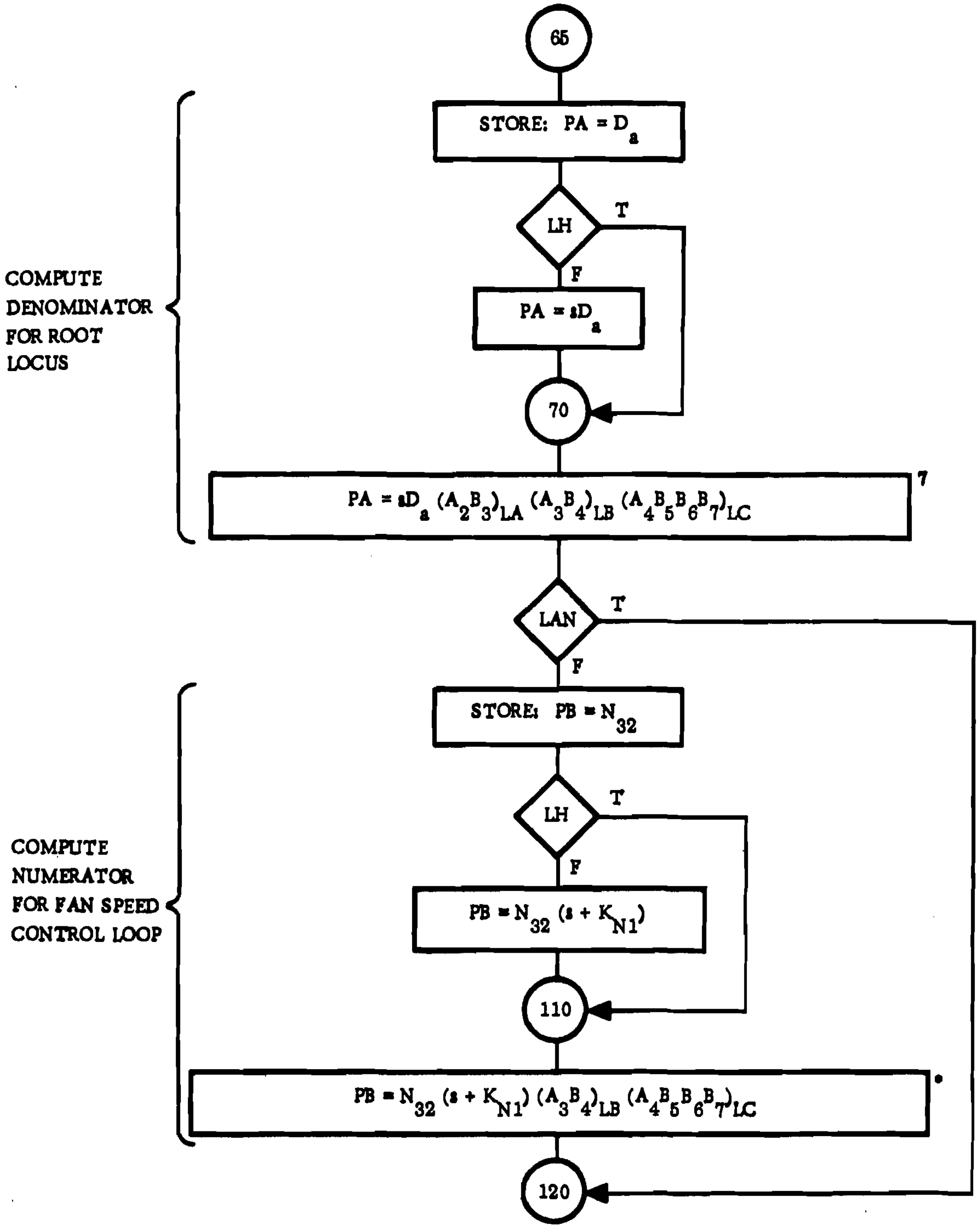


Figure 6. (Continued)

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[7] $(A_2B_3)_{LA}$ means $(A_2B_3)$ is included in the polynomial if LA is true, etc.

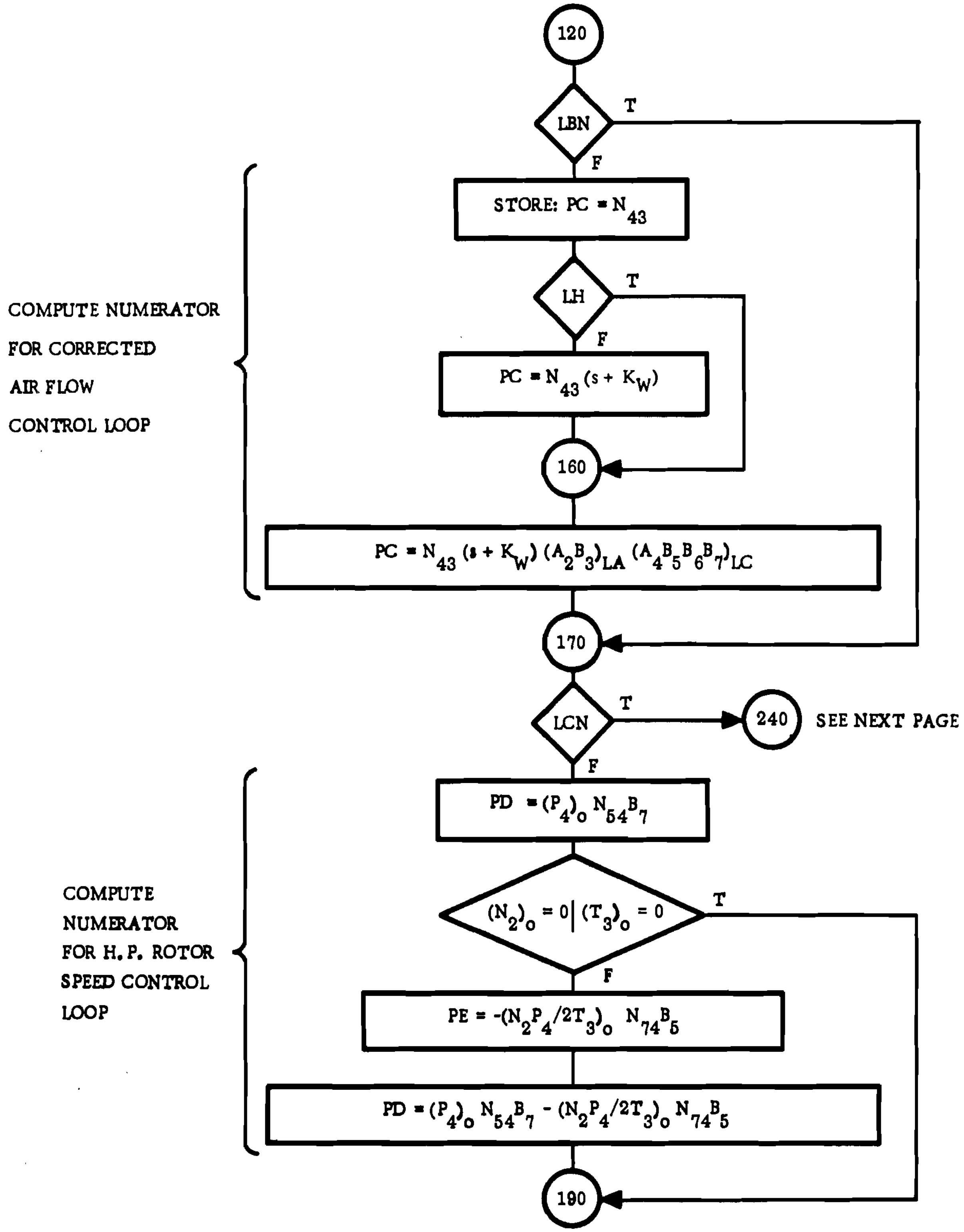
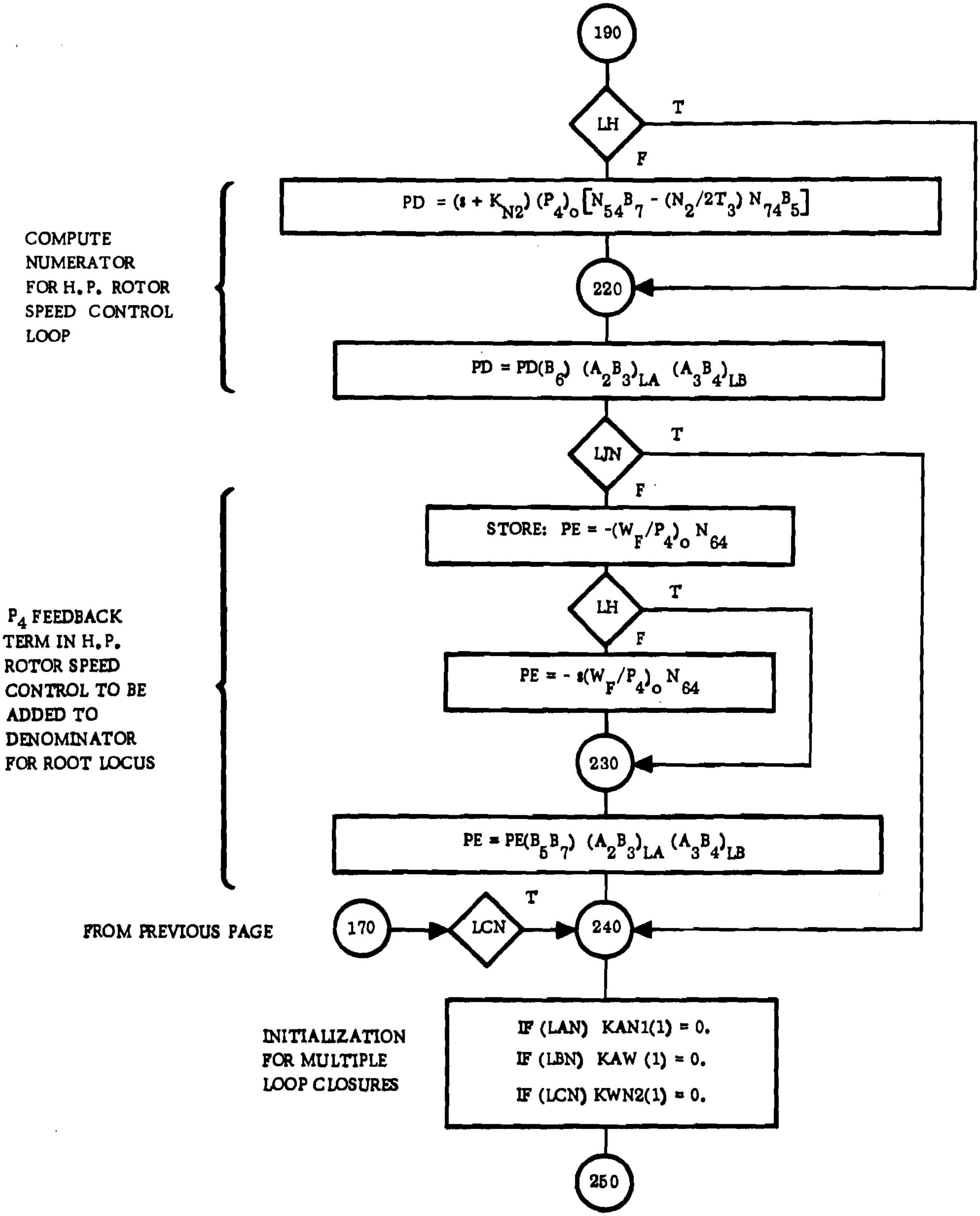


Figure 6. (Continued)



COMPUTE NUMERATOR FOR H.P. ROTOR SPEED CONTROL LOOP

$P_4$ FEEDBACK TERM IN H.P. ROTOR SPEED CONTROL TO BE ADDED TO DENOMINATOR FOR ROOT LOCUS

FROM PREVIOUS PAGE

INITIALIZATION FOR MULTIPLE LOOP CLOSURES

Figure 6. (Continued)

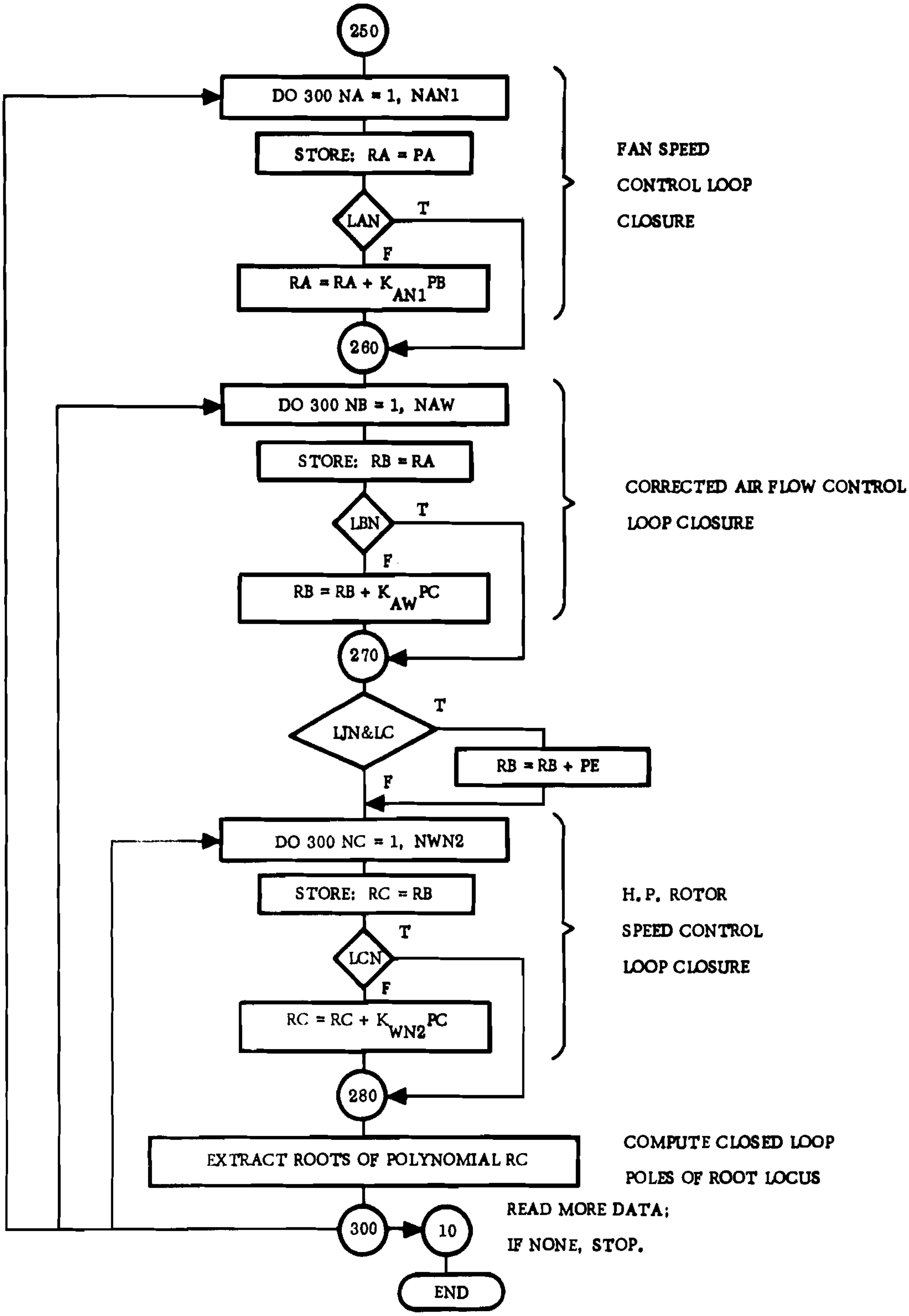


Figure 6. (Concluded)

$(N_2)_o$ = 12,800 RPM

$(T_3)_o$ = 619.42 °R

The integrator gain is set at $K_{N2} = 1.0 \text{ sec}^{-1}$.

Control gains for the other control loops are set to zero, i.e.,

$K_{AN1} = K_{AW} = 0.$

The actuator and sensor lags are:

$A_4 = 0.02 (s + 50)$

$B_5 = 0.05 (s + 20)$

$B_6 = 0.02 (s + 50)$

$B_7 = (s + 1)$

The following computer print-out (Table X) shows the computation of the root locus for the H.P. Rotor Speed Control, for control gain $K_{WN2}$ varying from 0. to 0.5. A plot of the root locus for this case is shown on Figure 7.

Table VIII. Fortran H Program Listing of TFAN

```
C                     MAIN PROGRAM
C     ROOT LOCUS FOR MULTIPLE LOOP CLOSURES FOR TURBOFAN ENGINE
C     E.L.LUM, FEBRUARY 1968
C
      IMPLICIT LOGICAL*4(L)
      COMPLEX*8  CC(10)
      REAL*4 DA(8),N32(8),N43(8),N54(8),N64(8),N74(8)
      REAL*4 KAN1(19),KAW(19),KWN2(19),KN1,KW,KN2,N2,TITLE(18),F(170)
      REAL*4 TA2(4),TA3(4),TA4(4),TB3(4),TB4(4),TB5(4),TB6(4),TB7(4)
      REAL*4 PA(20),PB(20),PC(20),PD(20),PE(20)
      REAL*4 RA(20),RB(20),RC(20),HA(8),HB(8),HC(16),RR(20)
      COMMON /CMB/HA,HB,HC,NHA,NHB,NHC
      EQUIVALENCE (F(2),DA),(F(12),N32),(F(22),N43),(F(32),N54),
     1(F(42),N64),(F(52),N74),(F(61),P4),(F(62),WF),(F(63),N2),
     2(F(64),T3),(F(66),KN1),(F(67),KW),(F(68),KN2),(F(72),KAN1)
     3,(F(92),KAW),(F(112),KWN2),(F(132),TA2),(F(137),TA3),(F(142),TA4)
     4,(F(147),TB3),(F(152),TB4),(F(157),TB5),(F(162),TB6),(F(167),TB7)
     5,(F(10),FLG),(F(20),G32),(F(30),G43),(F(40),G54),(F(50),G64)
     6,(F(60),G74),(F(65),CPO)
      SF=1.
      LP=.TRUE.
      DO 5 I=1,170
    5 F(I)=0.
   10 READ(5,20)TITLE
   20 FORMAT(18A4)
      WRITE(6,30)TITLE
   30 FORMAT(1H1,4X,'ROOT LOCUS FOR MULTIPLE LOOP CLOSURES FOR TURBOFAN
     1ENGINE'//5X,18A4)
      CALL DECRD(F)
      WRITE(6,40)
   40 FORMAT(1H0,4X,'NON-ZERO INPUT DATA'//4(10X,'I',7X,'F(I)',5X))
      CALL PRIN(F,170)
      NDA=F(1)
      NN32=F(11)
      NN43=F(21)
      NN54=F(31)
      NN64=F(41)
      NN74=F(51)
```

Table VIII. (Continued)

```
      NAN1=F(71)                                                        00000380
      NAW=F(91)                                                         00000390
      NWN2=F(111)                                                       00000400
      NTA2=F(131)                                                       00000410
      NTA3=F(136)                                                       00000420
      NTA4=F(141)                                                       00000430
      NTB3=F(146)                                                       00000440
      NTB4=F(151)                                                       00000450
      NTB5=F(156)                                                       00000460
      NTB6=F(161)                                                       00000470
      NTB7=F(166)                                                       00000480
      RA(1)=1.                                                          00000490
      LA=NAN1.GT.0                                                      00000500
      LAN=.NOT.LA                                                       00000510
      LB=NAW.GT.0                                                       00000520
      LBN=.NOT.LB                                                       00000530
      LC=NWN2.GT.0                                                      00000540
      LCN=.NOT.LC                                                       00000550
      LDN=KN1.EQ.0.                                                     00000560
      LEN=KW.EQ.0.                                                      00000570
      LFN=KN2.EQ.0.                                                     00000580
      LH=LDN.AND.LEN.AND.LFN.OR.LAN.AND.LBN.AND.LFN.OR.LBN.AND.LCN.AND. 00000590
     1LDN.OR.LAN.AND.LCN.AND.LEN                                        00000600
      LJN=WF.EQ.0..OR.P4.EQ.0.                                          00000610
      IF(LA.OR.LB.OR.LC)GO TO 60                                        00000620
      WRITE(6,50)                                                       00000630
   50 FORMAT(1H0,4X,'OPEN LOOP, NO CONTROL GAINS SPECIFIED')            00000640
      GO TO 10                                                          00000650
   60 IF(LA)CALL STREAL(TA2,NTA2,TB3,NTB3,HA,NHA)                       00000660
      IF(LB)CALL STREAL(TA3,NTA3,TB4,NTB4,HB,NHB)                       00000670
      IF(LC)CALL STREAL(TA4,NTA4,TB5,NTB5,HC,NHC)                       00000680
      IF(LC)CALL STR3(HC,NHC,TB6,NTB6,TB7,NTB7,HC,NHC)                  00000690
      IF(FLG.EQ.0.)GO TO 65                                             00000692
      WRITE(6,62)                                                       00000694
   62 FORMAT(1H0,2X,'OPEN LOOP TRANSFER FUNCTION POLYNOMIALS GENERATED')00000696
      CALL GENP(DA,NDA,1.,'DA(S)   ',2)                                 00000698
      IF(LA)CALL GENP(N32,NN32,G32,'N32(S)  ',2)                        00000700
      IF(LB)CALL GENP(N43,NN43,G43,'N43(S)  ',2)                        00000702
```

Table VIII. (Continued)

```
      IF(LC)CALL GENP(N54,NN54,G54,'N54(S)   ',2)
      IF(LC)CALL GENP(N64,NN64,G64,'N64(S)   ',2)
      IF(LC)CALL GENP(N74,NN74,G74,'N74(S)   ',2)
C        COMPUTE DENOMINATOR, PA(S)
   65 CALL GPOL(DA,NDA,1.,PA,NPA)
      IF(LH)GO TO 70
      NPA=NPA+1
      PA(NPA+1)=0.
   70 CALL XPNP3(PA,NPA,LA,LB,LC,'OPEN LOOP DENOMINATOR, PA(S)',7,
     1'OPEN LOOP POLES ',4,&10)
      IF(LAN)GO TO 120
      WRITE(6,80)
   80 FORMAT(1H0,2X,'FAN SPEED CONTROL LOOP CLOSED')
      CALL GPOL(N32,NN32,1.,PB,NPB)
      IF(LDN)WRITE(6,90)
   90 FORMAT(1H0,4X,'NO INTEGRATOR IN FAN SPEED LOOP')
      IF(LH)GO TO 110
      WRITE(6,100)KN1
  100 FORMAT(1H0,4X,'INTEGRATOR GAIN, KN1=',G12.5)
      RA(2)=KN1
      CALL POLYX(PB,NPB,RA,1,PB,NPB)
  110 CALL XPNP3(PB,NPB,LAN,LB,LC,'NUMERATOR ASSOCIATED WITH KAN1, PB(S)
     1    ',10,'ZEROS ASSOCIATED WITH KAN1   ',7,&10)
  120 IF(LBN)GO TO 170
      WRITE(6,130)
  130 FORMAT(1H0,2X,'CORRECTED AIR FLOW CONTROL LOOP CLOSED')
      CALL GPOL(N43,NN43,1.,PC,NPC)
      IF(LEN)WRITE(6,140)
  140 FORMAT(1H0,4X,'NO INTEGRATOR IN CORRECTED AIR FLOW LOOP')
      IF(LH)GO TO 160
      WRITE(6,150)KW
  150 FORMAT(1H0,4X,'INTEGRATOR GAIN, KW=',G12.5)
      RA(2)=KW
      CALL POLYX(PC,NPC,RA,1 ,PC,NPC)
  160 CALL XPNP3(PC,NPC,LA,LBN,LC,'NUMERATOR ASSOCIATED WITH KAW, PC(S)'
     1,9,'ZEROS ASSOCIATED WITH KAW    ',7,&10)
  170 IF(LCN)GO TO 240
      WRITE(6,180)
```

Table VIII. (Continued)

```
180 FORMAT(1H0,2X,'H.P. ROTOR SPEED CONTROL LOOP CLOSED')
    CALL GPOL(N54,NN54,P4,PD,NPD)
    CALL XPNTAU(TB7,NTB7,PD,NPD,NPD)
    IF(N2.EQ.0..OR.T3.EQ.0.)GO TO 190
    CALL GPOL(N74,NN74,-N2/2.*P4/T3,PE,NPE)
    CALL XPNTAU(TB5,NTB5,PE,NPE,NPE)
    CALL POLADD(PD,NPD,1.,PE,NPE,1.,PD,NPD)
190 IF(LFN)WRITE(6,200)
200 FORMAT(1H0,4X,'NO INTEGRATOR IN H.P. ROTOR SPEED LOOP')
    IF(LH)GO TO 220
    WRITE(6,210)KN2
210 FORMAT(1H0,4X,'INTEGRATOR GAIN, KN2=',G12.5)
    RA(2)=KN2
    CALL POLYX(PD,NPD,RA,1,PD,NPD)
220 CALL XPNTAU(TB6,NTB6,PD,NPD,NPD)
    CALL XPNP3(PD,NPD,LA,LB,LCN,'NUMERATOR ASSOCIATED WITH KWN2, PD(S)
   1   ',10,'ZEROS ASSOCIATED WITH KWN2  ',7,&10)
    IF(LJN)GO TO 240
    CALL GPOL(N64,NN64,-WF/P4,PE,NPE)
    IF(LH)GO TO 230
    NPE=NPE+1
    PE(NPE+1)=0.
230 CALL STREAL(TB5,NTB5,TB7,NTB7,RA,NRA)
    CALL XPNTAU(RA,NRA,PE,NPE,NPE)
    CALL XPNP3(PE,NPE,LA,LB,LCN,'NUMERATOR ASSOCIATED WITH P4, PE(S) '
   1,9,'ZEROS ASSOCIATED WITH P4',6,&10)
C       MULTIPLE LOOP CLOSURES
240 IF(LAN)KAN1(1)=0.
    IF(LBN)KAW(1)=0.
    IF(LCN)KWN2(1)=0.
    LP=CPO.EQ.1.
    WRITE(6,250)TITLE
250 FORMAT(1H1,4X,18A4//5X,'BEGIN LOOP CLOSURES')
    DO 300 NA=1,NAN1
    CALL GPOL(PA,NPA,1.,RA,NRA)
    IF(LAN)GO TO 260
    CALL POLADD(PB,NPB,KAN1(NA),RA,NRA,1.,RA,NRA)
260 DO 300 NB=1,NAW
```

Table VIII. (Concluded)

```
      CALL GPOL(RA,NRA,1.,RB,NRB)
      IF(LBN)GO TO 270
      CALL POLADD(PC,NPC,KAW(NB),RB,NRB,1.,RB,NRB)
  270 IF(.NOT.LJN.AND.LC)CALL POLADD(PE,NPE,1.,RB,NRB,1.,RB,NRB)
      DO 300 NC=1,NWN2
      CALL GPOL(RB,NRB,1.,RC,NRC)
      IF(LCN)GO TO 280
      CALL POLADD(PD,NPD,KWN2(NC),RC,NRC,1.,RC,NRC)
  280 WRITE(6,290)KAN1(NA),KAW(NB),KWN2(NC)
  290 FORMAT(1H0//2X,'CCNTROL GAINS: KAN1=',G12.5,', KAW=',G12.5,', KWN2
     1=',G12.5)
      IF(LP)CALL WRTPOL(RC,NRC,'CLOSED LOOP DENOMINATOR ',6)
      CALL PRTX (RC,NRC,RR,NRR,CC,NCC,50,1.E6,SF,&10)
  300 CALL WRT(RR,NRR,CC,NCC,'CLOSED LOOP POLES  ',5)
      WRITE(6,310)
  310 FORMAT(1H0,4X,'END OF LOOP CLOSURES')
      GO TO 10
      END
```

# Table IX. Input Data Format for TFAN

DECK NO. DATA PROGRAMMER E. L. LUM DATE 6-30-68 PAGE 1 of 5 JOB NO.

|  | NUMBER |  | IDENTIFICATION | DESCRIPTION DO NOT KEY PUNCH |
|---|---|---|---|---|
| 1 | Title card - - |  | ▨ | Must be first card of run |
| 13 | description of run |  | ▨ |  |
| 25 | in alpha-numeric |  | ▨ |  |
| 37 | characters |  | ▨ |  |
| 49 |  |  | 73 ▨ 80 |  |
| 61 |  |  | identification | Relative data field location in array F of main program TFAM |
| 1 | 1 |  | ▨ |  |
| 13 | NDA |  | ▨ | 1 Deggee of open-loop denominator, $D_a(s)$ |
| 25 | DA(1) | Variable names | ▨ | 2 Open-loop denominator polynomial, $D_a(s)$ |
| 37 | ↓ | assigned to data | ▨ | ↓ See special instructions on previous page. |
| 49 | DA(8) | fields shown | 73 ▨ 80 | 9 |
| 61 | FLG |  |  | 10 See special instructions on previous page. |
| 1 | 1 1 |  | ▨ |  |
| 13 | NN32 |  | ▨ | 11 Degree of numerator, $N_{32}(s)$ |
| 25 | N32(1) |  | ▨ | 12 Numerator polynomial $N_{32}(s) = \partial N_1/A_J$ |
| 37 | ↓ |  | ▨ | ↓ for Fan Speed Control |
| 49 | N32(8) |  | 73 ▨ 80 | 19 See special instructions on previous page. |
| 61 | G32 |  |  | 20 Gain of $N_{32}(s)$, used only if FLG≠0 |
| 1 | 2 1 |  | ▨ |  |
| 13 | NN43 |  | ▨ | 21 Degree of numerator, $N_{43}(s)$ |
| 25 | N43(1) |  | ▨ | 22 Numerator polynomial $N_{43}(s) = \partial W_{2,5}/\partial A_D$ |
| 37 | ↓ |  | ▨ | ↓ for Corrected Air Flow Control. |
| 49 | N43(8) |  | 73 ▨ 80 | 29 See special instructions on previous page. |
| 61 | G43 |  |  | 30 Gain of $N_{43}(s)$, used only if FLG≠0. |

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## Table IX. (Cont)

DECK NO. DATA PROGRAMMER E. L. LUM DATE 6-30-68 PAGE 3 of 3 JOB NO. ______

| | NUMBER | IDENTIFICATION | DESCRIPTION DO NOT KEY PUNCH | |
|---|---|---|---|---|
| 1 | 3 1 | | | |
| 13 | NN54 | | 31 Degree of numerator, $N_{54}(s)$ | |
| 25 | N54(1) | | 32 Numerator polynomial, $N_{54}(s) = \partial N_2/W_F$ | |
| 37 | | | for H. P. Rotor Speed Control | |
| 49 | N54(8) | 73 80 | 39 See special instructions | |
| 61 | G54 | | 40 Gain of $N_{54}(s)$, used only if $FLG \neq 0$. | |
| 1 | 4 1 | | | |
| 13 | NN64 | | 41 Degree of numerator, $N_{64}(s)$ | |
| 25 | N64(1) | | 42 Numerator polynomial, $N_{64}(s) = \partial P/\partial W_F$ | |
| 37 | | | | |
| 49 | N64(8) | 73 80 | 49 See special instructions | |
| 61 | G64 | | 50 Gain of $N_{64}(s)$, used only if $FLG \neq 0$ | |
| 1 | 5 1 | | | |
| 13 | NN74 | | 51 Degree of numerator, $N_{74}(s)$ | |
| 25 | N74(1) | | 52 Numerator polynomial, $N_{74}(s) = \partial T_3/\partial W_F$ | |
| 37 | | | for H. P. Rotor Speed Control | |
| 49 | N74(8) | 73 80 | 59 See special instructions | |
| 61 | G74 | | 60 Gain of $N_{74}(s)$, used only if $FLG \neq 0$ | |
| 1 | 6 1 | | | |
| 13 | P4 | | 61 $(P_4)_o$ initial steady-state value of $P_4$ | |
| 25 | WF | | 62 $(W_F)_o$ " " " " " $W_F$ | For H. P. |
| 37 | N2 | | 63 $(N_2)_o$ " " " " " $N_2$ | Rotor Speed |
| 49 | T3 | 73 80 | 64 $(T_3)_o$ " " " " " $T_3$ | Control Only |
| 61 | CPO | | 65 Check-out print option, 1. if print-out desired | |

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## Table IX. (Cont)

DECK NO. DATA PROGRAMMER E. L. LUM DATE 6-30-68 PAGE 3 of 5 JOB NO.

| | NUMBER | IDENTIFICATION | DESCRIPTION DO NOT KEY PUNCH |
|---|---|---|---|
| 1 | 6 6 | | |
| 13 | KN1 | | 66 Integrator gain $K_{N1}$ for Fan Speed Control |
| 25 | KW | | 67 Integrator gain $K_W$ for Corrected Air Flow Control |
| 37 | KN2 | | 68 Integrator gain $K_{N2}$ for H. P. Rotor Speed Control |
| 49 | Not Used | 73 80 | 69 |
| 61 | Not Used | | 70 |
| 1 | 7 1 | | |
| 13 | NAN1 | | 71 No. of control gain, $K_{AN1}$; 0. if Fan Speed Loop open |
| 25 | KAN1 (1) | | 72 Set of control gains $K_{AN1}$ for which closed-loop |
| 37 | | | characteristic is computed for Fan Speed |
| 49 | | 73 80 | Control Loop. |
| 61 | KAN1 (19) | | 90 |
| 1 | 9 1 | | |
| 13 | NAW | | 91 No. of control gains, $K_{AW}$; 0. if Corrected Air Flow Loop open |
| 25 | KAW(1) | | 92 Set of control gains $K_{AW}$ for which closed loop |
| 37 | | | characteristic is computed for Corrected |
| 49 | | 73 80 | Air Flow Control Loop. |
| 61 | KAW(19) | | 110 |
| 1 | 1 1 1 | | |
| 13 | NWN2 | | 111 No. of control gains, $K_{WN2}$; 0. if H. P. Rotor Speed Loop open |
| 25 | KWN2(1) | | 112 Set of control gains $K_{WN2}$ for which closed-loop |
| 37 | | | characteristic is computed for H. P. Rotor |
| 49 | | 73 80 | Speed Control Loop. |
| 61 | KWN2(19) | | 130 |

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## Table IX. (Cont)

DECK NO. DATA PROGRAMMER E. L. LUM DATE 6-30-68 PAGE 4 of 5 JOB NO.

|  | NUMBER | IDENTIFICATION | DESCRIPTION DO NOT KEY PUNCH |
|---|---|---|---|
| 1 | 1 3 1 |  |  |
| 13 | NTA2 |  | 131 No. of time constants for $A_2$ |
| 25 | TA2(1) |  | 132 |
| 37 |  |  | 133 Set of time constants for |
| 49 |  | 73 80 | 134 $A_J$ actuator 1/A2 for Fan Speed Control. |
| 61 | TA2(4) |  | 135 |
| 1 | 1 3 6 |  |  |
| 13 | NTA3 |  | 136 No. of time constants for $A_3$ |
| 25 | TA3(1) |  | 137 |
| 37 |  |  | 138 Set of time constants for |
| 49 |  | 73 80 | 139 $A_D$ actuator $1/A_3$ for Corrected Air Flow Control |
| 61 | TA3(4) |  | 140 |
| 1 | β 1 4 1 |  |  |
| 13 | NTA4 |  | 141 No. of time constants for $A_4$ |
| 25 | TA4(1) |  | 142 |
| 37 |  |  | 143 Set of time constants for |
| 49 |  | 73 80 | 144 $W_F$ actuator $1/A_4$ for H. P. Rotor Speed Control |
| 61 | TA4(4) |  | 145 |
| 1 | 1 4 6 |  |  |
| 13 | NTB3 |  | 146 No. of time constants for $B_3$ |
| 25 | TB3(1) |  | 147 |
| 37 |  |  | 148 Set of time constants for $N_1$ Sensor $1/B_3$ |
| 49 |  | 73 80 | 149 for Fan Speed Control |
| 61 | TB3(4) |  | 150 |

## Table IX. (Concluded)

DECK NO. DATA PROGRAMMER E. L. LUM DATE 6-30-68 PAGE 5 of 5 JOB NO.

|  | NUMBER | IDENTIFICATION | DESCRIPTION DO NOT KEY PUNCH |
|---|---|---|---|
| 1 | 1 5 1 |  |  |
| 13 | NTB4 |  | 151 No. of time constants for $B_4$ |
| 25 | TB4(1) |  | 152 |
| 37 |  |  | 153 Set of time constants for $W_{2.5}$ sensor $1/B_4$ |
| 49 |  | 73 80 | 154 for Corrected Air Flow Control |
| 61 | TB4(4) |  | 155 |
| 1 | 1 5 6 |  |  |
| 13 | NTB5 |  | 156 No. of time constants for $B_5$ |
| 25 | TB5(1) |  | 157 |
| 37 |  |  | 158 Set of time constants for $N_2$ sensor $1/B_5$ |
| 49 |  | 73 80 | 159 for H. P. Rotor Speed Control |
| 61 | TB5(4) |  | 160 |
| 1 | 1 6 1 |  |  |
| 13 | NTB6 |  | 161 No. of time constants for $B_6$ |
| 25 | TB6(1) |  | 162 |
| 37 |  |  | 163 Set of time constants for $P_4$ sensor $1/B_6$ |
| 49 |  | 73 80 | 164 for H. P. Rotor Speed Control |
| 61 | TB6(4) |  | 165 |
| 1 | 1 6 6 |  | Minus sign must be on last card of run. |
| 13 | NTB7 |  | 166 No. of time constants for $B_7$ |
| 25 | TB7(1) |  | 167 |
| 37 |  |  | 168 Set of time constants for $T_3$ sensor $1/B_7$ |
| 49 |  | 73 80 | 169 for H. P. Rotor Speed Control |
| 61 | TB7(4) |  | 170 |

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## Table X. Computer Printout for Example Case Using TFAN

```
  ROOT LOCUS FOR MULTIPLE LOOP CLOSURES FOR TURBOFAN ENGINE

    EXAMPLE CASE USING TFAN, H.P. ROTOR SPEED CONTROL

  NON-ZERO INPUT DATA (SEE FORMAT OF INPUT DATA)

        I        F(I)               I        F(I)               I        F(I)               I        F(I)
        1     2.00000               2     1.80000               3     14.0000              10     1.00000
       40     0.540000             41     1.00000              42     4.00000              50     0.300000E-01
       60     0.160000E-01         61     352.700              62     5822.20              63     12800.0
       64     619.420              65     1.00000              68     1.00000             111     14.0000
      113     0.100000E-02        114     0.200000E-02        115     0.300000E-02        116     0.500000E-02
      117     0.100000E-01        118     0.200000E-01        119     0.300000E-01        120     0.500000E-01
      121     0.700000E-01        122     0.100000E 00        123     0.200000            124     0.300000
      125     0.500000            141     1.00000             142     0.200000E-01        156     1.00000
      157     0.500000E-01        161     1.00000             162     0.200000E-01        166     1.00000
      167     1.00000

 OPEN LOOP TRANSFER FUNCTION POLYNOMIALS GENERATED

  DA(S)
   POLYNOMIAL OF ORDER  2, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
    0.3968253E-01     0.6269843          1.000000

  N54(S)
   POLYNOMIAL OF ORDER  0, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
    0.5400000

  N64(S)
   POLYNOMIAL OF ORDER  1, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
    0.7499997E-02     0.3000000E-01

  N74(S)
   POLYNOMIAL OF ORDER  0, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
    0.1600000E-01

  OPEN LOOP DENOMINATOR, PA(S)
   POLYNOMIAL OF ORDER  7, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
    0.7936504E-06     0.1085714E-03     0.5203962E-02     0.1036072          0.8154953
     1.716984          1.000000         0.0

  OPEN LOOP POLES
   NUMBER OF REAL ROOTS=  7
    -1.799993         -1.000002         -19.99992         -14.00016          -49.99997
```

**Polynomials generated by subroutine GENP**

**Compute open-loop denominator**

## Table X. (Continued)

```
   -49.99986          0.0

 H.P. ROTOR SPEED CONTROL LOOP CLOSED

   INTEGRATOR GAIN, KN2=  1.0000

  NUMERATOR ASSOCIATED WITH KWN2, PD(S)
   POLYNOMIAL OF ORDER  3, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
     3.750852            193.9365            322.3364            132.1510

  ZEROS ASSOCIATED WITH KWN2
   NUMBER OF REAL ROOTS=  3
   -0.9999949          -0.7046493          -50.00000

  NUMERATOR ASSOCIATED WITH P4, PE(S)
   POLYNOMIAL OF ORDER  4, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
   -0.61903311E-02     -0.1547577          -0.6437926          -0.4952251           0.0

  ZEROS ASSOCIATED WITH P4
   NUMBER OF REAL ROOTS=  4
    -4.000005           -1.000000           -19.99997           0.0
```

**Compute numerator for root locus**

**$P_4$ term to be added to open-loop denominator**

## Table X. (Continued)

```
EXAMPLE CASE USING TFAN, H.P. ROTOR SPEED CONTROL

BEGIN LOOP CLOSURES
```

**These gains are set to zero.** **Variable gain**

```
CONTROL GAINS: KAN1= 0.0          , KAW= 0.0          , KWN2= 0.0

 CLOSED LOOP DENOMINATOR
  POLYNOMIAL OF ORDER  7, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
   0.7936504E-06     0.1085714E-03     0.5203962E-02     0.9741688E-01     0.6607376
   1.073191          0.5047749         0.0

 CLOSED LOOP POLES
  NUMBER OF REAL ROOTS=  5
   -1.060050         -1.000072         -19.99988         -10.32411         0.0
  NUMBER OF COMPLEX PAIR ROOTS=  1
   -52.20795         13.41269
```

**Denominator for root locus, $P_4$ term added to open-loop denominator**

```
CONTROL GAINS: KAN1= 0.0          , KAW= 0.0          , KWN2= 0.10000E-02

 CLOSED LOOP DENOMINATOR
  POLYNOMIAL OF ORDER  7, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
   0.7936504E-06     0.1085714E-03     0.5203962E-02     0.9741688E-01     0.6644884
   1.267127          0.8271112         0.1321510

 CLOSED LOOP POLES
  NUMBER OF REAL ROOTS=  5
   -0.2333040        -1.280692         -1.000019         -9.326349         -20.56915
  NUMBER OF COMPLEX PAIR ROOTS=  1
   -52.19524         13.43887
```

**First control gain change, $K_{WN2}$ = 0.001**

```
CONTROL GAINS: KAN1= 0.0          , KAW= 0.0          , KWN2= 0.20000E-02

 CLOSED LOOP DENOMINATOR
  POLYNOMIAL OF ORDER  7, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
   0.7936504E-06     0.1085714E-03     0.5203962E-02     0.9741688E-01     0.6682392
   1.461063          1.149447          0.2643021

 CLOSED LOOP POLES
```

## Table X. (Continued)

```
 NUMBER OF REAL ROOTS=  5
 -1.000001      -0.3928037         -8.304079          -1.668417          -21.06929
 NUMBER OF COMPLEX PAIR ROOTS=  1
 -52.18265      13.46566


CONTROL GAINS: KAN1= 0.0         , KAW= 0.0          , KWN2= 0.30000E-02

 CLOSED LOOP DENOMINATOR
  POLYNOMIAL OF ORDER  7, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
  0.7936504E-06    0.1085714E-03    0.5203962E-02    0.9741688E-01    0.6719901
   1.655000         1.471784        0.3964531

 CLOSED LOOP POLES
  NUMBER OF REAL ROOTS=  5
  -0.4867685       -2.293941        -1.000003         -7.159027          -21.52005
  NUMBER OF COMPLEX PAIR ROOTS=  1
  -52.17001      13.49223


CONTROL GAINS: KAN1= 0.0         , KAW= 0.0          , KWN2= 0.50000E-02

 CLOSED LOOP DENOMINATOR
  POLYNOMIAL OF ORDER  7, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
  0.7936504E-06    0.1085714E-03    0.5203962E-02    0.9741688E-01    0.6794918
   2.042872         2.116455        0.6607550

 CLOSED LOOP POLES
  NUMBER OF REAL ROOTS=  3
  -0.9999929      -0.5159929         -22.31532
  NUMBER OF COMPLEX PAIR ROOTS=  2
  -4.309188      1.935585         -52.14510        13.54541


CONTROL GAINS: KAN1= 0.0         , KAW= 0.0          , KWN2= 0.10000E-01

 CLOSED LOOP DENOMINATOR
  POLYNOMIAL OF ORDER  7, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
  0.7936504E-06    0.1085714E-03    0.5203962E-02    0.9741688E-01    0.6982461
   3.012554         3.728137        1.321509
```

## Table X. (Continued)

```
   CLOSED LOOP POLES
    NUMBER OF REAL ROOTS=  3
    -0.9999968         -0.6435101          -23.92505
    NUMBER OF COMPLEX PAIR ROOTS=  2
     -3.532336      4.981854          -52.08340      13.67929


 CONTROL GAINS: KAN1= 0.0          , KAW= 0.0          , KWN2= 0.20000E-01

   CLOSED LOOP DENOMINATOR
    POLYNOMIAL OF ORDER  7, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
     0.7936504E-06     0.1085714E-03     0.5203962E-02     0.9741688E-01     0.7357545
      4.951920          6.951502          2.643021

   CLOSED LOOP POLES
    NUMBER OF REAL ROOTS=  3
    -0.6751729         -0.5999958          -26.33441
    NUMBER OF COMPLEX PAIR ROOTS=  2
     -2.431242      7.667241          -51.96394      13.95483


 CONTROL GAINS: KAN1= 0.0          , KAW= 0.0          , KWN2= 0.30000E-01

   CLOSED LOOP DENOMINATOR
    POLYNOMIAL OF ORDER  7, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
     0.7936504E-06     0.1085714E-03     0.5203962E-02     0.9741688E-01     0.7732631
      6.891284          10.17487          3.964531

   CLOSED LOOP POLES
    NUMBER OF REAL ROOTS=  3
    -0.6852608         -0.9999967          -28.20828
    NUMBER OF COMPLEX PAIR ROOTS=  2
     -1.602321      9.317381          -51.85086      14.23878


 CONTROL GAINS: KAN1= 0.0          , KAW= 0.0          , KWN2= 0.50000E-01

   CLOSED LOOP DENOMINATOR
    POLYNOMIAL OF ORDER  7, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
     0.7936504E-06     0.1085714E-03     0.5203962E-02     0.9741688E-01     0.8482801
      10.77001          16.62157          6.607552
```

## Table X. (Continued)

```
 CLOSED LOOP POLES
  NUMBER OF REAL ROOTS=  3
  -0.9999901         -0.6931489          -31.14755
  NUMBER OF COMPLEX PAIR ROOTS=  2
  -0.3313532       11.55170          -51.64830       14.82947


CONTROL GAINS: KAN1= 0.0          , KAW= 0.0          , KWN2= 0.70000E-01

 CLOSED LOOP DENOMINATOR
  POLYNOMIAL OF ORDER  7, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
   0.7936504E-06     0.1085714E-03     0.5203962E-02     0.9741688E-01      0.9232971
    14.64874          23.06830          9.250572

 CLOSED LOOP POLES
  NUMBER OF REAL ROOTS=  3
  -0.9999917         -0.6964747          -33.45343
  NUMBER OF COMPLEX PAIR ROOTS=  2
   0.6600275       13.14172          -51.48506       15.44758


CONTROL GAINS: KAN1= 0.0          , KAW= 0.0          , KWN2= 0.10000E 00

 CLOSED LOOP DENOMINATOR
  POLYNOMIAL OF ORDER  7, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
   0.7936504E-06     0.1085714E-03     0.5203962E-02     0.9741688E-01      1.035822
    20.46681          32.73839          13.21510

 CLOSED LOOP POLES
  NUMBER OF REAL ROOTS=  3
  -0.6989459         -0.9999989          -36.16368
  NUMBER OF COMPLEX PAIR ROOTS=  2
   1.858982        14.94720          -51.32767       16.40396


CONTROL GAINS: KAN1= 0.0          , KAW= 0.0          , KWN2= 0.20000

 CLOSED LOOP DENOMINATOR
  POLYNOMIAL OF ORDER  7, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
   0.7936504E-06     0.1085714E-03     0.5203962E-02     0.9741688E-01      1.410908
    39.86046          64.97203          26.43021
```

Table X. (Concluded)

```
   CLOSED LOOP POLES
    NUMBER OF REAL ROOTS=  3
    -0.9999959         -0.7018111          -41.42575
    NUMBER OF COMPLEX PAIR ROOTS=  2
      4.677513        18.86580           -51.51369         19.44998


 CONTROL GAINS: KAN1= 0.0          , KAW= 0.0           , KWN2= 0.30000

   CLOSED LOOP DENOMINATOR
    POLYNOMIAL OF ORDER  7, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
     0.7936504E-06     0.1085714E-03     0.5203962E-02     0.9741688E-01       1.785992
      59.25410          97.20566          39.64529

   CLOSED LOOP POLES
    NUMBER OF REAL ROOTS=  3
    -0.9999958         -0.7027591          -43.87103
    NUMBER OF COMPLEX PAIR ROOTS=  2
      6.661975        21.44598           -52.27505         21.91109


 CONTROL GAINS: KAN1= 0.0          , KAW= 0.0           , KWN2= 0.50000

   CLOSED LOOP DENOMINATOR
    POLYNOMIAL OF ORDER  7, COEFFICIENTS IN DESCENDING ORDER OF VARIABLE
     0.7936504E-06     0.1085714E-03     0.5203962E-02     0.9741688E-01       2.536162
      98.04141          161.6730          66.07552

   CLOSED LOOP POLES
    NUMBER OF REAL ROOTS=  3
    -0.9999953         -0.7035168          -46.07709
    NUMBER OF COMPLEX PAIR ROOTS=  2
     -54.07115        25.49562            9.561440         25.04507

    END OF LOOP CLOSURES

 IHC217I

 TRACEBACK FOLLOWS-    ROUTINE     ISN    REG. 14

                       IBCOM              82014108
```

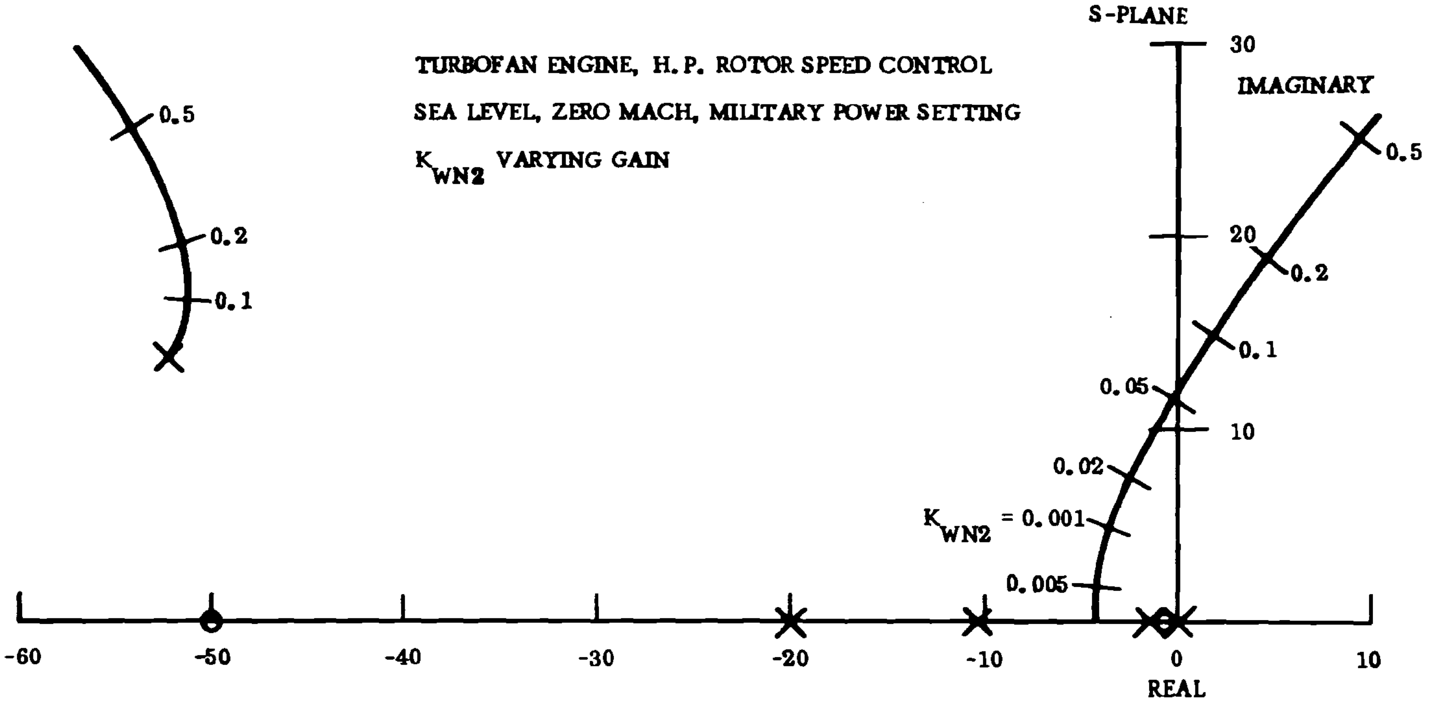

Figure 7. Root Locus for Example Case Using TFAN

## IV. NEW COMPUTER SUBPROGRAMS DEVELOPED FOR TJET AND TFAN

The following subroutines were developed specifically for use with main programs TJET and TFAN to (1) expand polynomials with the appropriate sets of actuator or sensor lags given in terms of time constants, and (2) to facilitate handling of data of the form received from P&WA (See description of GENP). The following pages contain

1. Subroutine XPNTAU - Flowchart and program listing
2. Subroutine XPNP5 - Flowchart and program listing
3. Subroutine GENP - Flowchart and program listing

Subroutine XPNP5 is required for main program TJET only. Subroutine XPNP3 for main program TFAN only is similar to XPNP5 except that XPNP3 expands a polynomial with up to three sets of first-order lag time constants rather than five sets of time constants as in XPNP5.

### DESCRIPTION OF SUBROUTINE XPNTAU

This subroutine is used to combine the appropriate actuator or sensor poles with the polynomials used in computing the root locus. The actuator or sensor poles are expressed as a set of first-order time constants. Subroutine XPNTAU expands a polynomial $p_1(s)$ to the form

$$p_n(s) = p_1(s)\,(\tau_1 s + 1)\,(\tau_2 s + 1)\,.\,.\,.\,(\tau_{ntau} s + 1)$$

The flowchart for subroutine XPNTAU is shown on Figure 8, and the Fortran H program listing is shown on Table XI. The arguments for XPNTAU are:

| | |
|---|---|
| TAU | Array of time constants, $\tau_i$, $i = 1, 2, \ldots,$ ntau |
| NTAU | Number of time constants in array TAU |
| P | Coefficients of polynomial to be expanded, $p_k$, $k = 1, \ldots$ np |
| NPI | Initial degree of polynomial $p_1(s)$ |
| NP | Final degree of expanded polynomial $p_n(s)$ |

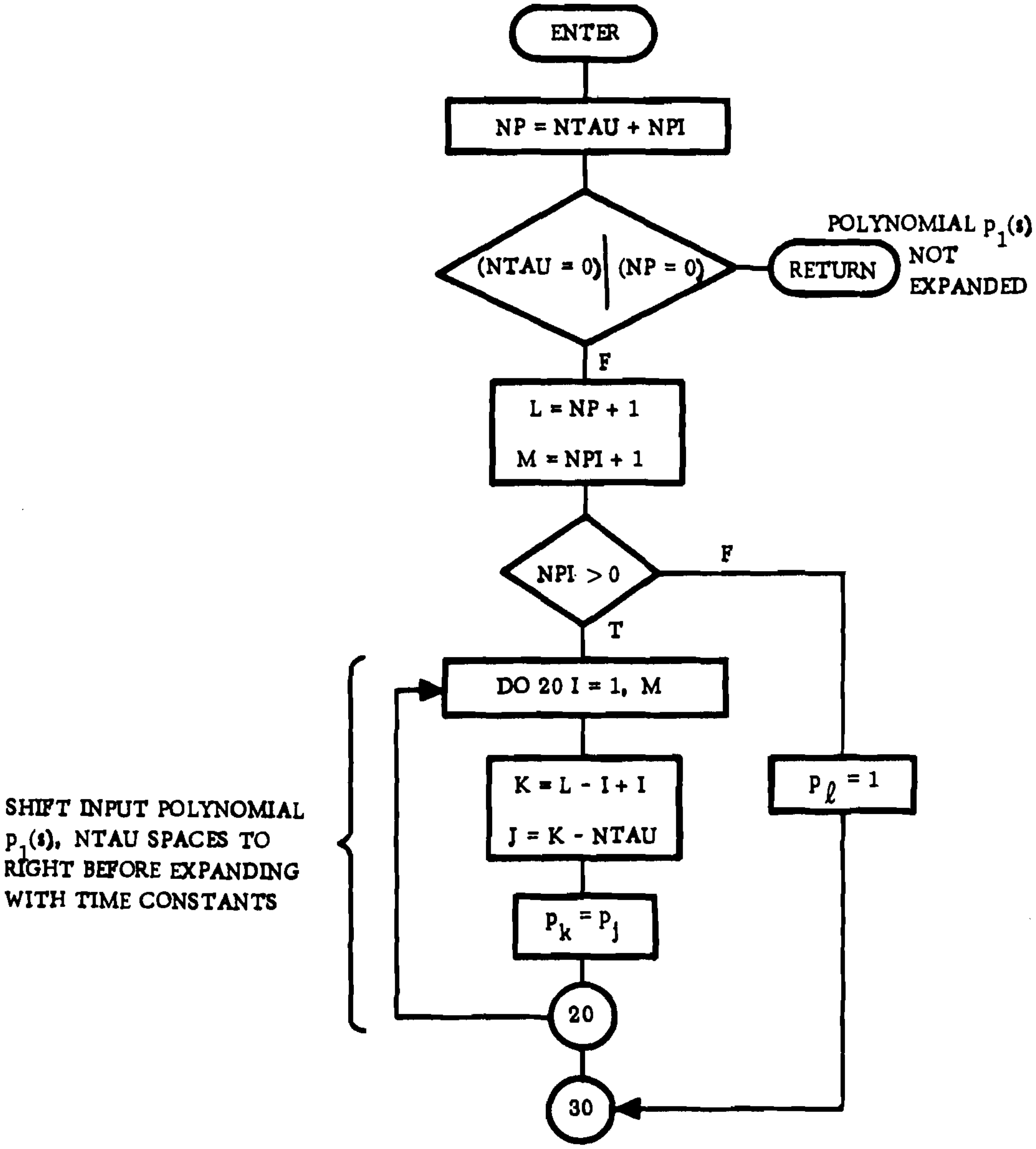

Figure 8. Flowchart for Subroutine XPNTAU

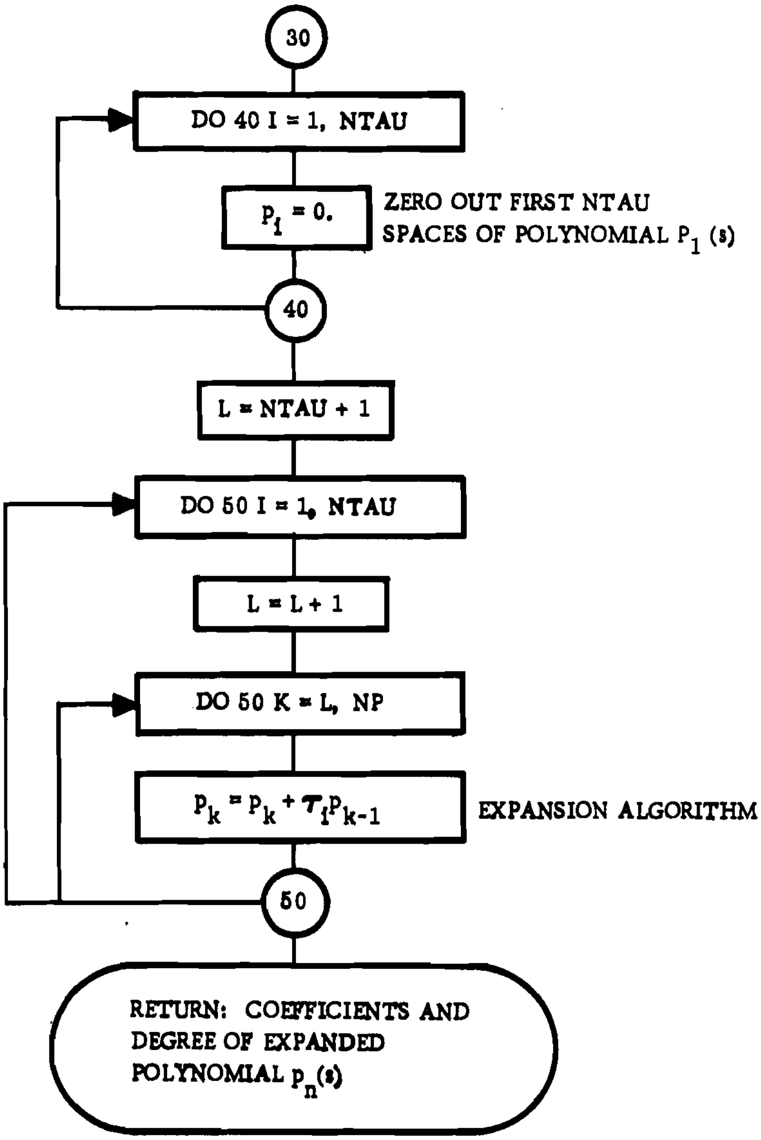

Figure 8. (Concluded)

Table XI. Fortran H Program Listing for Subroutine XPNTAU

```
      SUBROUTINE XPNTAU(TAU,NTAU,P,NPI,NP)                              D1000010
C        THIS SUBROUTINE EXPANDS A POLYNOMIAL P(S) TO THE FORM          D1000020
C     P(S)*(TAU(1)*S+1)*...*(TAU(NTAU)*S+1)                             D1000030
C     DEVELOPED BY E.L.LUM FEBRUARY 1968                                D1000040
C                                                                       D1000050
      REAL*4 TAU(1),P(1)                                                D1000060
      NP=NTAU+NPI                                                       D1000070
      IF(NTAU.EQ.0.OR.NP.EQ.0)RETURN                                    D1000080
      L=NP+1                                                            D1000090
      M=NPI+1                                                           D1000100
      DO 20 I=1,M                                                       D1000140
      K=L-I+1                                                           D1000150
      J=K-NTAU                                                          D1000160
   20 P(K)=P(J)                                                         D1000170
      DO 40 I=1,NTAU                                                    D1000180
   40 P(I)=0.                                                           D1000190
      L=NTAU+1                                                          D1000200
      DO 50 I=1,NTAU                                                    D1000210
      L=L-I                                                             D1000220
      DO 50 K=L,NP                                                      D1000230
   50 P(K)=P(K)+P(K+1)*TAU(I)                                           D1000240
      RETURN                                                            D1000250
      END                                                               D1000260
```

## DESCRIPTION OF SUBROUTINE XPNP5

This subroutine is used to combine five or less sets of actuator and sensor poles with a polynomial. The actuator and sensor poles are expressed as first-order time constants. Logical statements are used to determine which poles are to be combined with the given polynomial $p_1(s)$, i.e.,

$$p_n(s) = p_1(s)\,(A)_{LA}\,(B)_{LB}\,(C)_{LC}\,(D)_{LD}\,(E)_{LE}$$

where $(A)_{LA}$ means that the poles from set A are combined with $p_1(s)$ if LA is true, etc.

The flowchart for subroutine XPNP5 is shown on Figure 9, and the Fortran H program listing is shown on Table XII. The arguments for XPNP5 are:

| | |
|---|---|
| P | Polynomial to be expanded, p(s) |
| NP | Initial degree of polynomial p(s); returned as final degree of p(s) |
| LA, LB, LC, LD, LE | Logical constants, if true expands polynomial p(s) by first-order lags from set A, B, C, D, or E, respectively |
| TA | Title to be printed with expanded polynomial p |
| NTA | Length of title TA (Number of characters in multiples of 4) |
| TB | Title to be printed with extracted roots of expanded polynomial p |
| NTB | Length of title TB (Number of characters in multiples of 4) |
| * | Non-standard return in case of error in extracting roots |

The variables that are passed by COMMON to simplify the programming and to save time and storage are:

| | |
|---|---|
| A, B, C, D, E | Sets of first-order lags expressed in terms of time constants |
| NA, NB, NC, ND, NE | Number of first-order lags in set A, B, C, D, or E, respectively |

## DESCRIPTION OF SUBROUTINE GENP

This subroutine is used to facilitate data handling by converting transfer function data of the form supplied by P&WA into coefficients of s-plane polynomials. The transfer function data are expressed in terms of the first-order lag frequencies ($a_i$) and the final steady state gain (g) of the transfer function. Subroutine GENP generates the coefficients of a polynomial of the form

$$a(s) = g(1 + s/a_1)(1 + s/a_2) \ldots (1 + s/a_{na}).$$

The flowchart for subroutine GENP is shown on Figure 10, and the Fortran program listing is shown on Table XIII. The arguments used for GENP are:

| | |
|---|---|
| A | Array of first-order lag frequencies, $a_i$, i=1, 2, ..., na; also array in which generated polynomial is returned, a(s) |
| NA | Number of first-order lags in A |
| G | Steady state of expanded polynomial a(s) |
| T | Title to be printed with expanded polynomial a(s) |
| NT | Length of title T (Number of characters in multiples of 4) |

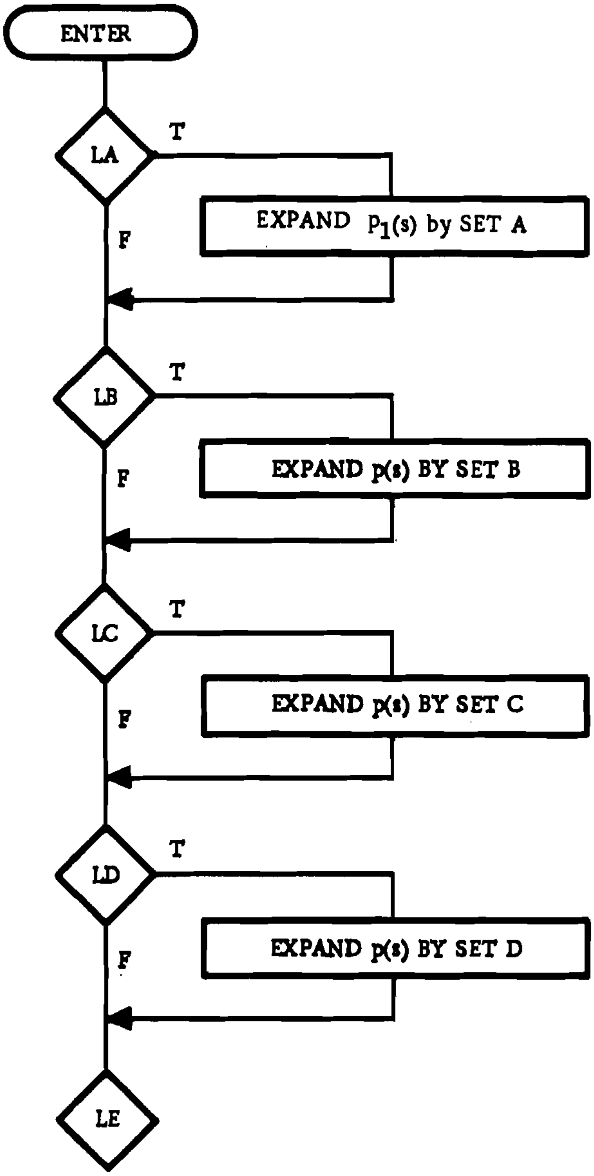

Figure 9. Flowchart for Subroutine XPNP5

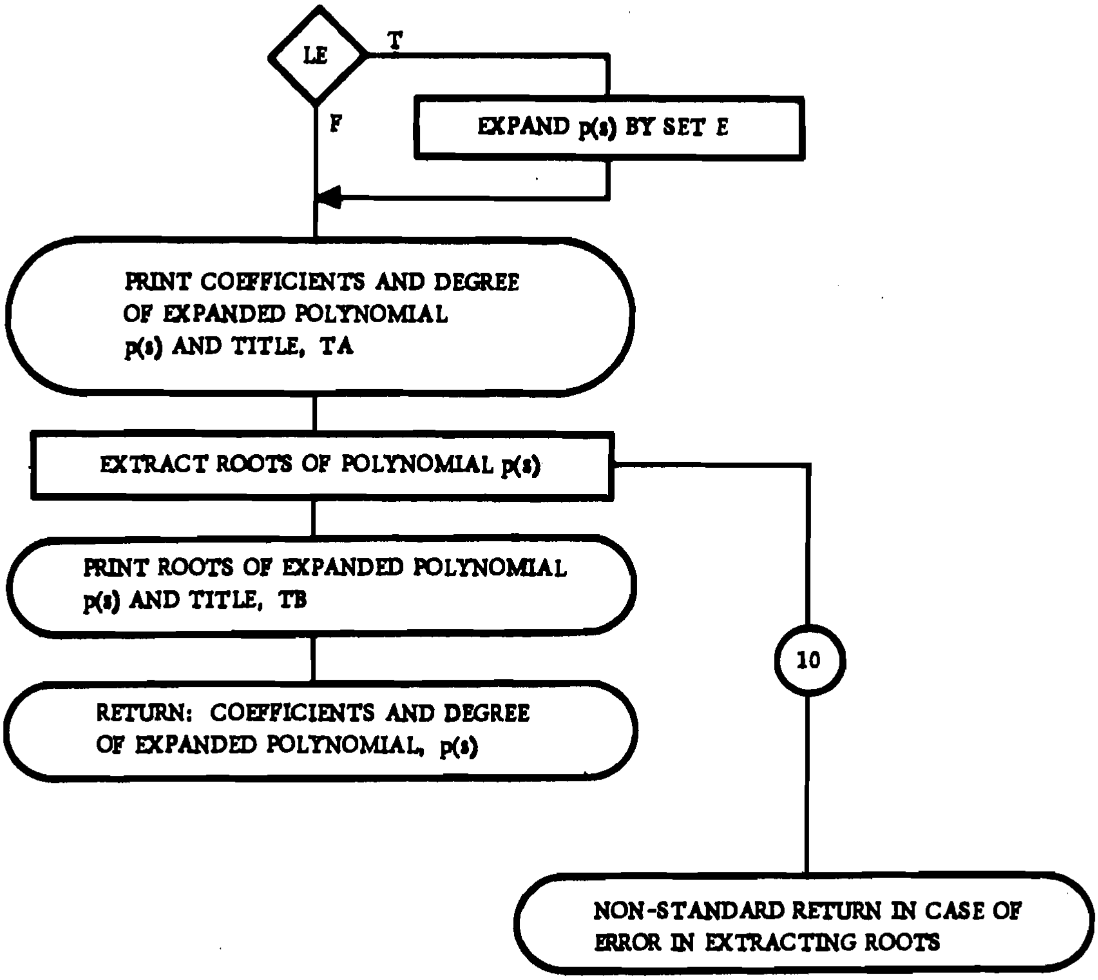

Figure 9. (Concluded)

## Table XII. Fortran H Program Listing for Subroutine XPNP5

```
      SUBROUTINE XPNP5(P,NP,LA,LB,LC,LD,LE,TA,NTA,TB,NTB,*)             X0000010
C        THIS SUBROUTINE EXPANDS A POLYNOMIAL WITH UP TO FIVE SETS OF   X0000020
C     DYNAMIC LAGS SPECIFIED BY TIME CONSTANTS.                         X0000030
C                                                                       X0000040
      LOGICAL*4 LA,LB,LC,LD,LE                                          X0000050
      COMPLEX*8 CC(10)                                                  X0000060
      REAL*4 P(1),TA(1),TB(1),A(4),B(4),C(4),D(4),E(4),RR(20)           X0000070
      COMMON /CMB/A,B,C,D,E,NA,NB,NC,ND,NE                              X0000090
      IF(LA)CALL XPNTAU(A,NA,P,NP,NP)                                   X0000100
      IF(LB)CALL XPNTAU(B,NB,P,NP,NP)                                   X0000110
      IF(LC)CALL XPNTAU(C,NC,P,NP,NP)                                   X0000120
      IF(LD)CALL XPNTAU(D,ND,P,NP,NP)                                   X0000130
      IF(LE)CALL XPNTAU(E,NE,P,NP,NP)                                   X0000140
      CALL WRTPOL(P,NP,TA,NTA)                                          X0000150
      CALL PRTX (P,NP,RR,NRR,CC,NCC,50,1.E6,1.,&10)                     X0000160
      CALL WRT(RR,NRR,CC,NCC,TB,NTB)                                    X0000165
      RETURN                                                            X0000170
   10 RETURN 1                                                          X0000180
      END                                                               X0000190
```

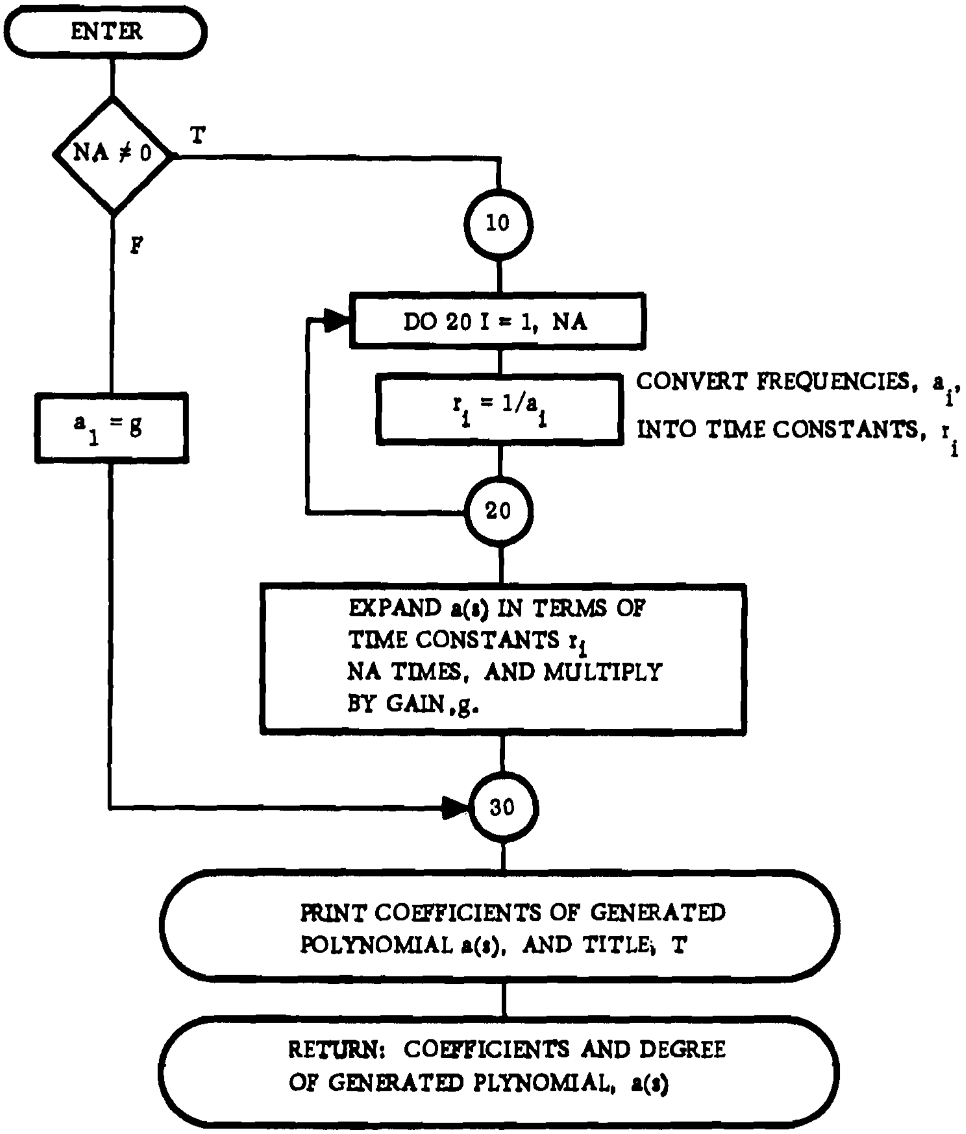

Figure 10. Flowchart for Subroutine GENP

## Table XIII. Fortran H Program Listing for Subroutine GENP

```
      SUBROUTINE GENP(A,NA,G,T,NT)                                      X1000010
C        THIS SUBROUTINE GENERATES A POLYNOMIAL OF THE FORM             X1000020
C     G*(1+S/A(1))(1+S/A(2))...(1+S/A(NA))                              X1000030
C                                                                       X1000040
      REAL*4 A(1),T(1),R(8)                                             X1000050
      IF(NA.LT.0)RETURN                                                 X1000060
      IF(NA.NE.0)GO TO 10                                               X1000070
      A(1)=G                                                            X1000080
      GO TO 30                                                          X1000090
   10 DO 20 I=1,NA                                                      X1000100
   20 R(I)=1./A(I)                                                      X1000110
      A(1)=1.                                                           X1000115
      CALL XPNTAU(R,NA,A,0,NA)                                          X1000120
      CALL GPOL(A,NA,G,A,NA)                                            X1000130
   30 CALL WRTPOL(A,NA,T,NT)                                            X1000140
      RETURN                                                            X1000150
      END                                                               X1000160
```

## V. CONCLUSION AND RECOMMENDATIONS

Computer programs TJET and TFAN have been valuable in the linear control analysis for the Propulsion System Flow Stability Program. The efficiency of these programs are such that many flight cases can be evaluated in a relatively short time to achieve the primary goal of designing rapid responding closed loop control systems.

The reasons that TJET and TFAN were developed rather than using an existing root locus computer program for the analysis are:

1. Some control loops (the turbojet rotor speed control, and turbofan high pressure rotor speed control loops) have more than one feedback, which requires more than one computer run on a general root locus computer program. TJET and TFAN combine all the necessary polynomial terms for the root locus analysis of these control loops automatically in one computer run.

2. Data handling is facilitated by (a) the conversion of transfer function data of the form supplied by P&WA in terms of the natural frequencies and steady-state gains into s-plane polynomial coefficients, (b) the actuator and sensor time constants for each control loop is combined automatically with the appropriate polynomial terms, and (c) all gains due to transfer function dynamics, actuator and sensor lags and initial operating point data are computed correctly, eliminating the time consuming process and reducing the probability of error in computing the correct gains by hand calculations.

3. Each control loop and control gain in the print-outs of TJET and TFAN are identified as such for easy reference.

It is recommended that the following modifications and additions to TJET and TFAN be made to increase the efficiency in designing and evaluating linear control systems for the turbojet and turbofan engines.

1. Include the capability of analyzing coupled engine control systems by adding the necessary cross-coupling terms in the polynomial equations to account for the interaction between control loops. The structures of TJET and TFAN are set up such that the cross-coupling numerators necessary for the analysis of coupled control loops can be easily included with minor modifications to TJET and TFAN.

2. Include the capability of analyzing the inlet duct control loops. This capability, when coupled to (1) above, should enable analysis of the complete system with cross-coupling (interaction).

3. To facilitate the transient analysis and the design of rapid responding control systems, an exponential least squares fit subprogram described in Part XIX, Appendix B should be included so that the effective closed loop step response time constants can be printed out along with the root locus.

4. If the cross-coupling transfer functions are included at a later date, then it should be possible to compute and minimize by decoupling gains the integral square error in one control loop due to a reference input into another loop.

5. To facilitate the root locus analysis, the outputs from TJET and TFAN can be modified to include

    a. CRT (Cathode Ray Tube) plots of the root locus as an option, and
    b. the print-outs of the damping ratios and natural frequencies of the computed complex pair closed loop poles of the root locus.

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13. ABSTRACT

This part describes computer programs developed to handle the computational tasks for the linear stability analysis of propulsion controls by root locus techniques. The descriptions, flowcharts, and Fortran program listings of two main programs TJET and TFAN developed for the analysis of turbojet and turbofan engine control systems are presented. Also presented are the descriptions of three subroutines specially developed to facilitate data handling for this study. Examples are given to illustrate the application of computer programs TJET and TFAN.

TJET and TFAN have been valuable in the linear analysis of the propulsion system. However, it is recommended that these computer programs be expanded to include the capability of analyzing coupled control loops and to include a limited transient analysis.

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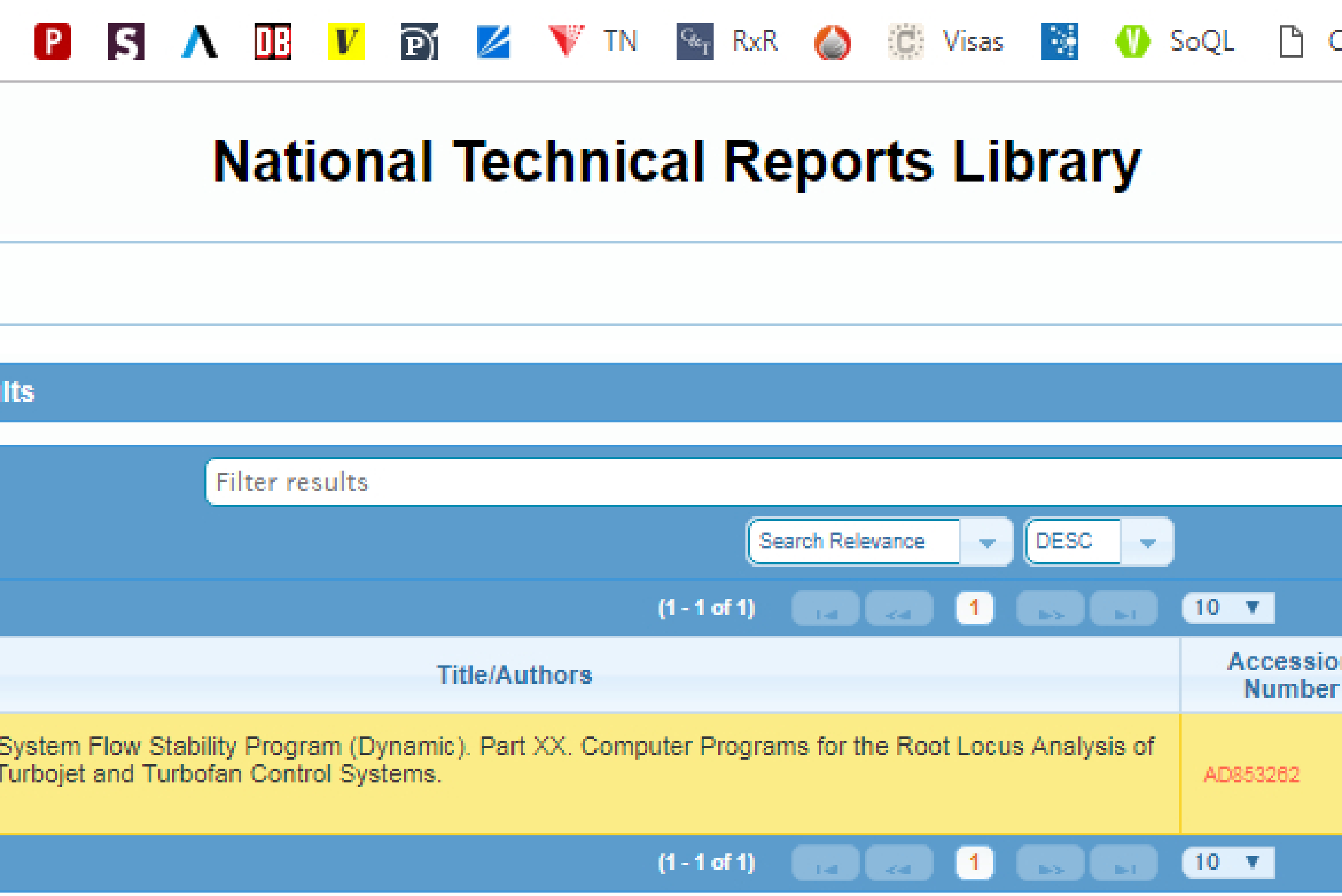
| 14. KEY WORDS | LINK A | | LINK B | | LINK C | |
|---|---|---|---|---|---|---|
| | ROLE | WT | ROLE | WT | ROLE | WT |
| Root Locus | | | | | | |
| Turbojet Control Loop Analysis | | | | | | |
| Turbofan Control Loop Analysis | | | | | | |
| Computer Program for Gas Turbine Engine Control Analysis | | | | | | |



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