WADC TECHNICAL REPORT 55-393

BINOCULAR DISPARITY AS A CODING DIMENSION FOR PICTORIAL INSTRUMENT AND RADAR DISPLAYS

JEROME COHEN

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FOREWORD

This report was prepared by Antioch College under USAF Contract No. AF 18(600)-50. The contract is a task under Project 7186, "Visual Presentation of Information." The task is 71545, "Psychological Factors in the Design of Instrument Displays." The contract is administered by the Psychology Branch of the Aero Medical Laboratory, Directorate of Research, Wright Air Development Center, with Mr. William J. White acting as Task Scientist. Mrs. Constance G. Norris and Mr. Paul C. Friedman were the research assistants in the collection and analysis of the data.

ABSTRACT

Two experiments were done in order to determine the increments of binocular disparity angle which would result in equal discriminability of absolute depth judgments. On the basis of the results, we can specify the discrete amounts of binocular disparity to present to the operators of three dimensional cathode ray tube displays; or conversely, we can state the relative accuracy with which operators would be able to make depth discriminations over the range of binocular disparities between the upper and the lower limits of fusion.

The first experiment employed nineteen subjects who recorded depth judgments on a continuous scale, and the second employed twenty subjects who recorded their depth judgments in eleven discrete categories. Twenty-two pairs of test slides with four disparate targets on each slide were presented to the subjects in the same stereoscope in both studies. The responses were scaled by the method of the R scale transformation; this involves normalizing the data, and separating the response categories by equal standard deviation distances. The computed R scale increments were found to be proportional to the logarithm of the response category increments.

There is an almost linear relationship between discriminability and the logarithm of binocular disparity angles. The discriminability is most accurate at zero disparity, and decreases proportionately to the logarithm of disparity angles. Suggested angles of binocular disparity for equipment use are presented in the paper, as well as a discussion on the effects of disparity on target location on the horizontal dimension.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER:

JACK BOLLERUD
Colonel, USAF (MC)
Chief, Aero Medical Laboratory
Directorate of Research

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INTRODUCTION

Binocular disparity resulting in the perception of apparent depth has been considered as a method for adding a third dimension to a two dimensional cathode ray tube display (5). Whether binocular disparity would be a practical addition to the other coding dimensions already available depends on how well people can discriminate between different amounts of disparity and respond appropriately. The lack of information about that point is readily apparent in the radar chapter of the Joint Services Human Engineering Guide (2), where the problem of binocular disparity for the coding of information could not be considered because of the paucity of appropriate data. Three dimensional displays would be more naturalistic in some of the integrated instrument displays which Grether (10) suggests might be made more pictorial, such instruments as used for blind landing, navigation and air traffic control.

The binocular disparity angle is defined as the difference in the angles subtended at the two eyes by two points in space.1/ Because of the horizontal separation of the two eyes, two points at different distances subtend different angles at the two eyes. Visual angles on the horizontal axis will be changed by different amounts in the two eyes by changing the relative distance of the two points, but the angular separation in the vertical axis will be changed by the same amount in the two eyes. If the nearer of two points is fixated, then the relative location of the image of the farther point in the right eye will be to the right of the relative position of the same image in the left eye: this condition is known as uncrossed disparity, and the angle is assigned a positive value throughout this paper. If the farther of the two points is fixated, then the image of the nearer point in the right eye will be to the left of the position of the corresponding image in the left eye: this will be referred to as crossed disparity, and assigned a negative value throughout. The basis for the creation of three dimensional perception from flat pictures in a stereoscope is that the angular separation between similar points in the two pictures will differ according to the values which they would have in each eye when looking at the normal scene. The combined single view which we see when looking in a stereoscope is geometrically similar to the naturalistic view which would exist in order to present the disparities on the slide; and the analogous depth relationships are recreated in our visual experience.

1/ The binocular disparity angle is often referred to as the binocular parallax angle. It should not be confused with the term visual angle which is the angle subtended at the entrance pupil by two points in the visual field. The minimum angular separation which permits a person to distinguish as separate the two points defines his visual acuity.

If binocular disparity were used for three dimensional representation, cues which otherwise would have been required for this function could serve as additional sources of information to help in the identification of the targets. If all the information in the display could be presented on one three dimensional scope, the interpretation of the information would be more immediate and easier, than if one were required to combine the readings from two two dimensional scopes in order to read the range, bearing and altitude of a target. Two general methods of scope display should be considered. Either one scope face with two directions of light polarization could be used with polaroid glasses, or two scopes could be perceptually combined by the use of prisms, as in a stereoscope. The present paper is a report of research which has more relevance to the second method of scope presentation, and a subsequent paper will deal with the first method which is similar to the method used in commercial three dimensional movies.

PAST APPLICATIONS AND RESEARCH

Binocular disparity has been successfully applied to military problems for two kinds of range finding. The wander-mark method of range finding is a true stereoscopic range finder, where the operator combines the disparate images of the two eyes and adjusts a mark to the same apparent depth as the target. The error, in judged distance, is related to the amount of disparity which can be displayed without resulting in an apparent depth difference between the mark and the target in the stereoscopic range finders. The more common range finder in use in the American Armed Services is a split field range finder in which the two disparate images are the top and bottom half of the visual field, and the operator aligns them to form a single, unified picture, but with no depth effect. The discrimination is based on vernier acuity rather than stereoscopic acuity[2/]. In both kinds of range finders the binocular disparity is effectively increased by increasing the length of the arms which contain the primary lenses of the optical system, which in effect increases the interocular distance.

Almost all of the experiments in the literature on stereoscopic depth are directed toward measuring the absolute thresholds for binocular disparity under various conditions (6, 18, 19). The brightness of the field has about the same effect on stereoscopic acuity as on visual acuity, both showing a rod-cone discontinuity (3, 17). Two experimenters have found that larger targets result in lower stereo-acuity thresholds (14, 15), but no relationship has been found between the thickness of the stimulus bars and stereo-acuity (4). This interesting discrepancy between the effects of increased width and size indicate that the variables which are related to changes in apparent depth thresholds are related to

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[2/] Vernier acuity is measured by the least lateral gap between the ends of two adjacent lines which is discriminable. Stereoscopic acuity is measured by the least binocular disparity angle which allows the subject to see two images at different distances.

size; but the exact relationships have yet to be worked out. The optimum angular separation of the targets from each other or from a comparison line has been determined (9, 12). Also the type of target has been investigated in terms of its effects on stereo-ranging which, as mentioned before, involves the stereo-acuity threshold (11). Some attention has been paid to the problem of incorrect fusion of reticle lines when they are close enough together to result in multiple fusion possibilities (8). Only one study has been found on the measurement of the threshold of stereopsis with moving targets (13). Many experimenters have studied stereoscopic acuity and ranging abilities as functions of personal and physiological variables as well as the distribution of the ability throughout the population, but our concern at this stage should be with the stimulus variables, and our references have been limited to a few representative studies. A good general discussion of binocular disparity and apparent depth is presented in Woodworth's Experimental Psychology (24).

THE PROBLEM

The utilization of apparent depth as a coding dimension depends on obtaining knowledge about how accurately binocular disparity angles may be discriminated over the whole range of disparities from no disparity to the disparity at which the apparent depth is maximum. Since the only relevant experiments in the literature are concerned with absolute thresholds, new experiments must be done in order to obtain the basic data which are necessary before binocular disparity can be used successfully as a coding dimension for cathode ray tube presentations. Firstly, is stereo-acuity good enough to furnish a sufficient number of discriminable steps? If so, the second concern is that the stimulus parameters of the display should be manipulated in order to determine the maximum number of apparent depth steps under the most favorable conditions; and the effects of the stereoscopic display on the other discriminations which must be made, such as the horizontal and vertical location of the targets, must be determined.

The experimental method was chosen to yield data which would be used to derive an equal discriminability scale of binocular disparities, in order to equalize the response confusion between adjacent stimulus categories. We are interested in determining the optimal spacing of stimulus categories to minimize the errors made by the subjects using a stereoscopic instrument. The technique of analyzing the data in this way will be discussed later in the paper when the results are presented.

THE EXPERIMENTS

Two experiments were done in order to determine the relationships between perceived distance and binocular disparity, with the purpose of establishing a scale of equal discriminability. The experiments were alike in all respects except for the following differences: 1) Nineteen subjects were used in the first, and twenty different subjects were used in the second. 2) The subjects in the first experiment were asked to report on the depth, and the horizontal and vertical location of the targets, while the subjects in the second experiment reported on the depth and horizontal location only. 3) The depth judgements in the first experiment were made on a continuous scale, while in the second experiment they were made on a discrete scale.

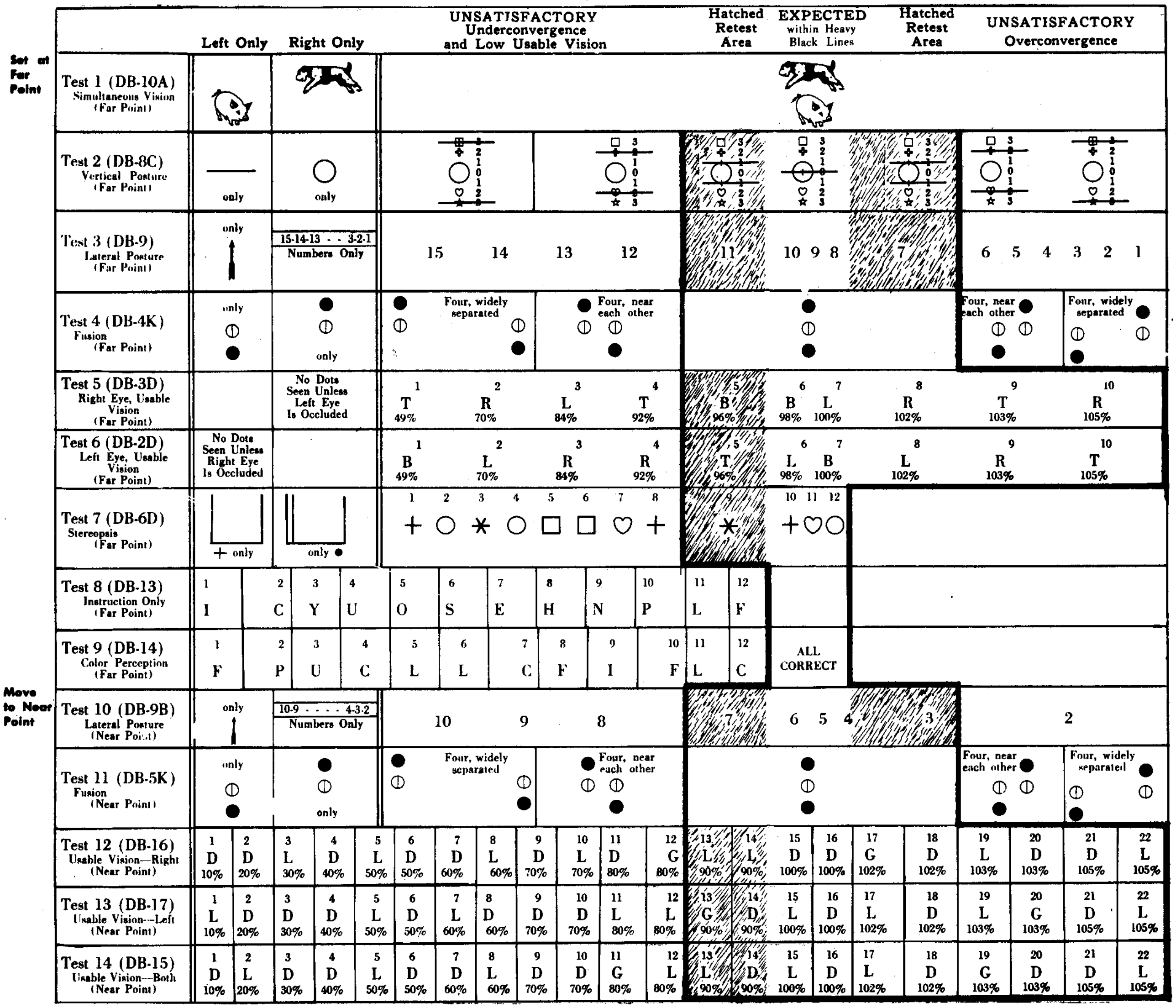
Since the experiments were so similar and yielded similar results, they will be discussed together and unless specified in the text, the statements made apply to both experiments.

Subjects

Undergraduate college students of both sexes, ranging in age from 16 to 26 years, were used as subjects. Only one subject had training in binocular range finding, and the others had either no experience or experience limited to having occasionally viewed depth pictures through viewers or at commerical three dimensional movies. They were naive about the principle of binocular disparity and apparent depth, and some showed surprise at first observing the practice slide, since it was their first experience at seeing apparent depth from the combination of two flat pictures. All the subjects fell within the limits on the Keystone Telebinocular Test as shown in the profile of Figure 1. Failure of any test except the color vision test in the series disqualified students as subjects in this experiment. They were also given **the Keystone** Aviators Series DC31-53 tests for stereopsis, and subjects with all levels of stereopsis were accepted; they ranged from 30% to 85%.

Apparatus

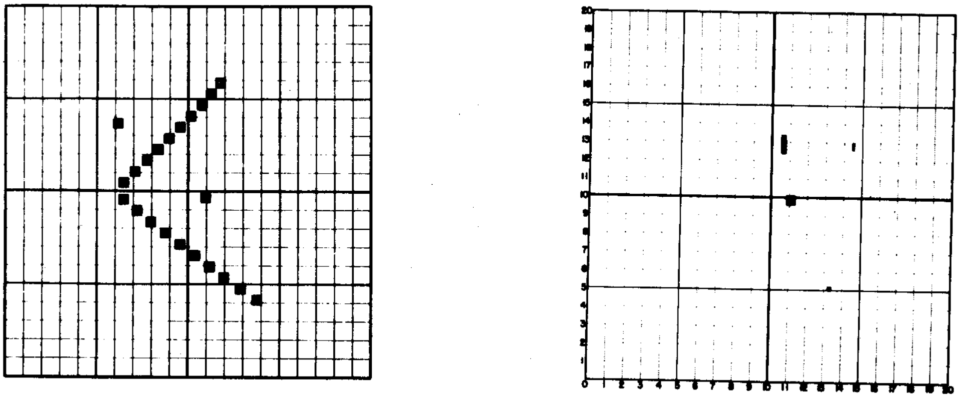
The same telebinocular stereoscope used for visual screening was also used in the experiment, but it was modified in order to provide transillumination of the test slides. The source of illumination was a ten and a half inch eight watt standard cool white fluorescent tube mounted on the back of the slide holder. The light holder was backed by an aluminum reflector, and sand blasted plastic was placed in front of the tube so that the light shining through the slides was diffuse and even over the whole surface of the slide.



	Left Only	Right Only	UNSATISFACTORY Underconvergence and Low Usable Vision	Hatched Retest Area	EXPECTED within Heavy Black Lines	Hatched Retest Area	UNSATISFACTORY Overconvergence
Set at Far Point Test 1 (DB-10A) Simultaneous Vision (Far Point)							
Test 2 (DB-8C) Vertical Posture (Far Point)	only	only					
Test 3 (DB-9) Lateral Posture (Far Point)	only	15-14-13 - - 3-2-1 Numbers Only	15 14 13 12	11	10 9 8	7	6 5 4 3 2 1
Test 4 (DB-4K) Fusion (Far Point)	only	only	Four, widely separated Four, near each other				Four, near each other Four, widely separated
Test 5 (DB-3D) Right Eye, Usable Vision (Far Point)		No Dots Seen Unless Left Eye Is Occluded	1 T 49% 2 R 70% 3 L 84% 4 T 92%	5 B 96%	6 B 98% 7 L 100%	8 R 102%	9 T 103% 10 R 105%
Test 6 (DB-2D) Left Eye, Usable Vision (Far Point)	No Dots Seen Unless Right Eye Is Occluded		1 B 49% 2 L 70% 3 R 84% 4 R 92%	5 T 96%	6 L 98% 7 B 100%	8 L 102%	9 R 103% 10 T 105%
Test 7 (DB-6D) Stereopsis (Far Point)	+ only	only ●	1 + 2 ○ 3 ✱ 4 ○ 5 □ 6 □ 7 ♡ 8 +	9 ✱	10 + 11 ♡ 12 ○		
Test 8 (DB-13) Instruction Only (Far Point)	1 I	2 C 3 Y 4 U	5 O 6 S 7 E 8 H 9 N 10 P	11 L 12 F			
Test 9 (DB-14) Color Perception (Far Point)	1 F	2 P 3 U 4 C	5 L 6 L 7 C 8 F 9 I 10 F	11 L 12 C	ALL CORRECT		
Move to Near Point Test 10 (DB-9B) Lateral Posture (Near Point)	only	10-9 - - - - 4-3-2 Numbers Only	10 9 8	7	6 5 4	3	2
Test 11 (DB-5K) Fusion (Near Point)	only	only	Four, widely separated Four, near each other				Four, near each other Four, widely separated
Test 12 (DB-16) Usable Vision—Right (Near Point)	1 D 10% 2 D 20%	3 L 30% 4 D 40%	5 L 50% 6 D 50% 7 D 60% 8 L 60% 9 D 70% 10 L 70% 11 D 80% 12 G 80%	13 L 90% 14 L 90%	15 D 100% 16 D 100% 17 G 102%	18 D 102%	19 L 103% 20 D 103% 21 D 105% 22 L 105%
Test 13 (DB-17) Usable Vision—Left (Near Point)	1 L 10% 2 D 20%	3 D 30% 4 D 40%	5 L 50% 6 D 50% 7 L 60% 8 D 60% 9 D 70% 10 D 70% 11 L 80% 12 L 80%	13 G 90% 14 L 90%	15 L 100% 16 D 100% 17 L 102%	18 D 102%	19 L 103% 20 G 103% 21 D 105% 22 L 105%
Test 14 (DB-15) Usable Vision—Both (Near Point)	1 D 10% 2 L 20%	3 D 30% 4 D 40%	5 L 50% 6 D 50% 7 D 60% 8 L 60% 9 D 70% 10 D 70% 11 G 80% 12 L 80%	13 L 90% 14 D 90%	15 L 100% 16 D 100% 17 L 102%	18 D 102%	19 G 103% 20 D 103% 21 D 105% 22 L 105%

Fig. 1. Keystone Visual Survey Test record, reproduced with the permission of the Keystone View Co., Meadville, Pennsylvania. The subjects selected for the experiments scored between the heavy black lines on all of the test cards.

Twenty-two pairs of test slides were made by photographing twenty inch square preparations on graph paper, lined every tenth of an inch. Black paper targets were cemented on the graph paper in the proper positions to create the desired disparities when the slides were viewed in the stereoscope. The area of the negatives was 1/100 that of the original drawings, or two inch squares with 1/100 inch separation between the fine grid lines. The practice slides used in the first experiment and one of the test slides are shown in Figures 2. The practice slides for the two experiments shown in Figure 2a, differed somewhat in the apparent distance pattern of the targets presented on each. The subjects in the first experiment saw equally spaced disparities in front of and behind the plane of the grid; the top target was the farthest and the bottom target the nearest. The extreme targets were at the same disparities as the extremes presented in the experimental slides, but there were more disparity steps each way in the practice slide. The practice slide for the second experiment presented the two middle targets in the bend of the angle, at the same distance as the grid, but the same disparity steps were used as in the experimental slides. The scales for the horizontal and vertical directions of the target were always presented on the left eye slide. All four targets shown in Figure 2b were presented on all the test slides, and were randomly placed on the graphs between the horizontal and vertical limits of 5 and 10. The coordinates were selected from a table of random numbers.



a. practice slide

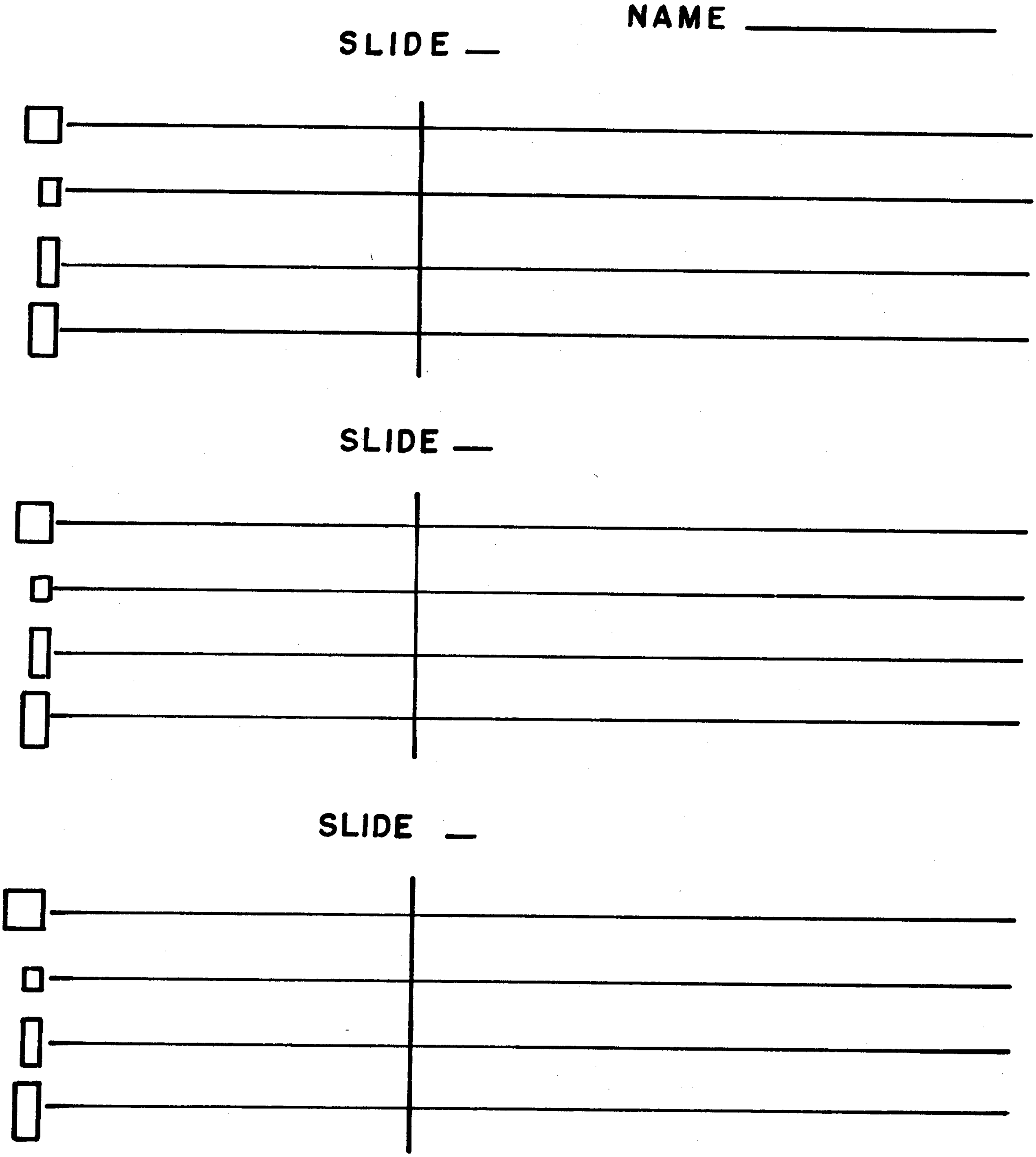
b. experimental slide

Fig. 2. Slides used in the experiment, full size reproductions. The slides used in the experiment were negatives, so that the lines and targets were transparent, and the spaces were opaque.

Procedure

Each subject was tested for three sessions spaced about one week apart. His visual screening and stereopsis tests were given in the first session. In the second session he was shown the practice slide, and the position of the slide holder was adjusted as close to the focal plane of the lens or the zero point (160 mm.) as possible, without interfering with clear vision. He was asked to describe what he saw, and usually there was no difficulty in seeing the stepwise spatial and depth arrangement of the targets. The practice slides were perceived correctly if the top nine targets were seen beyond the grid distance, and the bottom targets were in front of the grid. If all the targets looked either in front of or behind the grid, the subject was fusing the grid incorrectly. He was told to examine the left and right borders of the grid as well as the vertical centerline. If the vertical lines were seen as single, and the incorrect spatial orientation persisted, he was told how the pattern ought to look and the distance between slides adjusted until correctly fused. The horizontal and vertical scales were pointed out to the subject, and the coding of the graduation marks was explained. The subject was asked at the first session to tell the horizontal and vertical coordinates of the lower left corner of each target, accurately to the nearest tenth of a scale unit. He was told to look with both eyes at all times, and to fixate anywhere on the slide. The location to be reported was the perpendicular projection from the apparent position of the target to the plane of the grid. He read the two coordinates of all the targets on the test slide, and the experimenter called upon the subject to make corrections on any targets that were read with appreciable errors. The subject was also told to report any time that the target was seen double, or any time that he felt that he was not fusing the targets or grid lines correctly. After satisfactory completion of the practice slide, the subject was presented with the experimental series of slides, and he read the horizontal and vertical coordinates in turn for each of the four targets. As mentioned previously, in the second experiment in which the subject made discrete depth judgments, the horizontal location only was reported, because the analysis of results in the first experiment showed that the vertical location of the targets did not vary with their apparent distance. The first session, as well as producing information on the apparent direction of disparate targets, also was a training session for the second part of the experiment. The subject saw the target slides in their depth relationships, although he was giving other information about the targets.

The subject was told that he was going to see the same series of slides at the beginning of the second experimental session, but this time he would be concerned with reporting the third dimension or the depth of targets rather than their location in the frontal-parallel plane. He was first shown the practice slide, and he described the spatial relations of the targets and the grid. The subject was then asked to judge the distance between his eyes and each target in relation to the apparent distance of the grid, and mark a record sheet. Figures 3 and 4 show the record sheets used in the two experiments. In the first experiment, the depth



NAME ____________

SLIDE __

SLIDE __

SLIDE __

Fig. 3. Subject record sheet for the first experiment.

NAME ______________ DATE ____________ ADDRESS ______________

		NEAR			GRID			FAR		
5	6	7	8	9	10	11	12	13	14	15

Fig. 4. Subject record sheet for the second experiment.

of the target was indicated by marking a distance on a horizontal line; in the second experiment, by checking the appropriate box on a line of eleven boxes, each box representing one of the eleven depth catagories. The practice slide was also shown to the subject after the eleventh experimental slide, in order for the subjects to see the limits again. The subject looked at the slide for as long as he wanted, but did not record his responses during the second presentation.

The order of presentation of the slides was the same for all subjects in both experiments, and the subject could record the distance of the four targets on a slide in whatever order he wished. The subject removed his head from the stereoscope during the changing of slides and looked in again when the experimenter said, "Ready." It took an average of about forty-five minutes to go through the eighty-eight targets on the twenty-two pairs of slides used in the session. All of the targets on the first slide should be seen behind the grid. If the subject saw any target in front of the grid, he was coached to achieve proper fusion until the grid was seen nearer than any of the targets. The subject was instructed to examine each target individually in relation to the grid before reporting its distance. The subject tried to make absolute distance judgments and report them as relative linear extents on a piece of paper. Responses to the rest of the slides were not questioned by the experimenter, unless it was felt that there was a probability of misrecording or misidentifying a target. The subject was permitted and encouraged to take all the time he wanted in order to respond accurately and consistently, and his times were not recorded. The subjects were offered a rest period of a few minutes after the eleventh slide, but mostly they wanted to continue until completion of the experiment.

RESULTS

We shall first consider the analysis of the data concerning the scaling of depth judgments. To summarize the experimental design: There were nineteen subjects in the first experiment in which the depth judgment was made by placing a check mark on a continuous line. Twenty subjects in the second experiment reported their depth judgments by marking the target in one of eleven discrete spaces on a line **representing depth** . There were four randomly placed targets on each slide,with eleven equal increments of disparity from 45 to -45 minutes of arc, so a total of eighty-eight target depths were judged by each subject.

The subject's absolute depth judgments in the first experiment were assigned numerical values by measuring the position of the subject's mark on the line. Forty-one response categories from -20 through 0, at the same distance as the reference grid, to +20 were used in scoring the response sheets, that is four times the number of stimulus categories on either side of the grid distance. In the second experiment, the subjects responded by marking one of the eleven categories for each target, so there were as many response categories as stimulus categories. The statistical treatment of the results was the same for both experiments. The purpose of the analysis was to derive an equal discriminability scale, and the procedure used by Garner and Hake (7), the R scale transformation was used, with the difference that the data for all subjects in each experiment were added together to derive the transformed response scale. Since the response limits were set by the record sheets, we assumed that all the subjects could be treated together in order to increase the reliability. Actually two subjects were discarded from the first experiment because their individual curves were quite different from the rest of the subjects. One had a narrow range of responses, and the other responded to many of the farther targets as if they were seen in front of the grid. One subject in the second experiment was discarded for not being able to maintain the proper fusion of the grid pattern while making the depth judgments. The plots of means and sigmas of the responses to each stimulus category were not markedly different for all the other subjects. A typical subject's mean response curve is illustrated in Figure 5a.

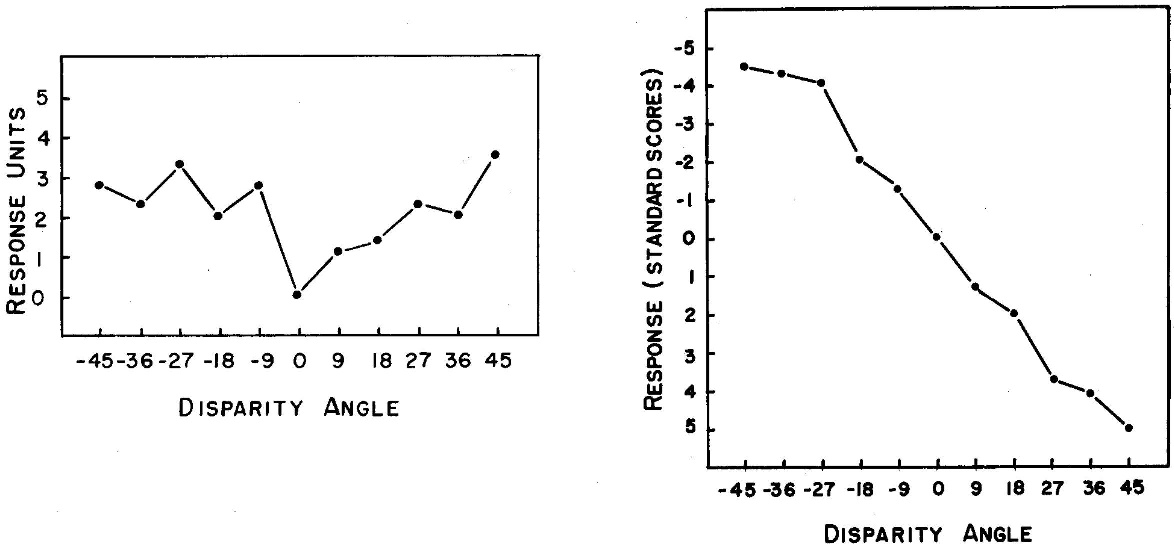


Fig. 5. a. Mean response for different disparity angles of one subject in experiment II. b. Standard deviation of responses for different disparity angles for the same subject.

The responses from all the subjects were combined in a separate matrix for each of the four target configurations. The R scale transformation was computed for each target type. Since the same relationship to the response scale was found, and the numerical values of the scales were not essentially different for the targets taken separately, they were combined into one matrix for each experiment. Target differences were small and inconsistent, so the results for all the targets analyzed together will be discussed.

The computational steps for the R scale transformation are the following:

1. The data are arranged in a stimulus-response matrix. Each column represents a stimulus category and each row a response category.

2. A percentile cumulative frequency distribution is then computed for each column. The extremes of the distribution, below about 5% and above 95%, can be ignored.

3. The distributions are normalized by entering a cumulative normal curve table with the percentile scores for each cell, and recording the equivalent x/σ scores. We now have a distribution of standard scores for each response category in a given stimulus category. It is helpful to graph these data on linear graph paper with a separate plot for each stimulus. If the data in a given stimulus category are normal, the plots will be straight lines, and the slope will indicate the consistency of response to a given stimulus.

4. The next step is to adjust the response scale, so that the intervals between response categories are such as to make the stimulus lines homoscedastic, or equally sloped. The average difference in sigma units between each two adjacent response categories, for all the stimulus categories represented in both is computed.

5. The lowest response category "equal to presented distance" is assigned a value of zero, and the average sigma for each successive response category is added to the last, to obtain the standard score value for each successive response category. The "R scale" value of each response category is the assigned value in standard score units, which is a linear scale. The response categories will now be non linear, with more space between adjacent categories which are confused less often in response to the stimuli. The R scale values may be smoothed to conform to the obtained linear relationship between the log of the response categories and the log of the computed R scale values, as shown in Figure 7. If each stimulus category is now graphed on the new R scale, the lines will be homoscedastic, as in Figure 6.

6. The R scale is a scale of equal response discriminability. In order to select stimuli which will result in equal response discriminability, the median ($x/\sigma = 0$) R scale value for each stimulus category should be

plotted against the stimulus category value; as shown in Figure 8, the discriminability is proportional to the log disparity. Stimulus values may then be selected by moving along the R scale in equal increments and reading the stimulus values from the best fit line. We therefore derive an equal discriminability scale of stimulus values.

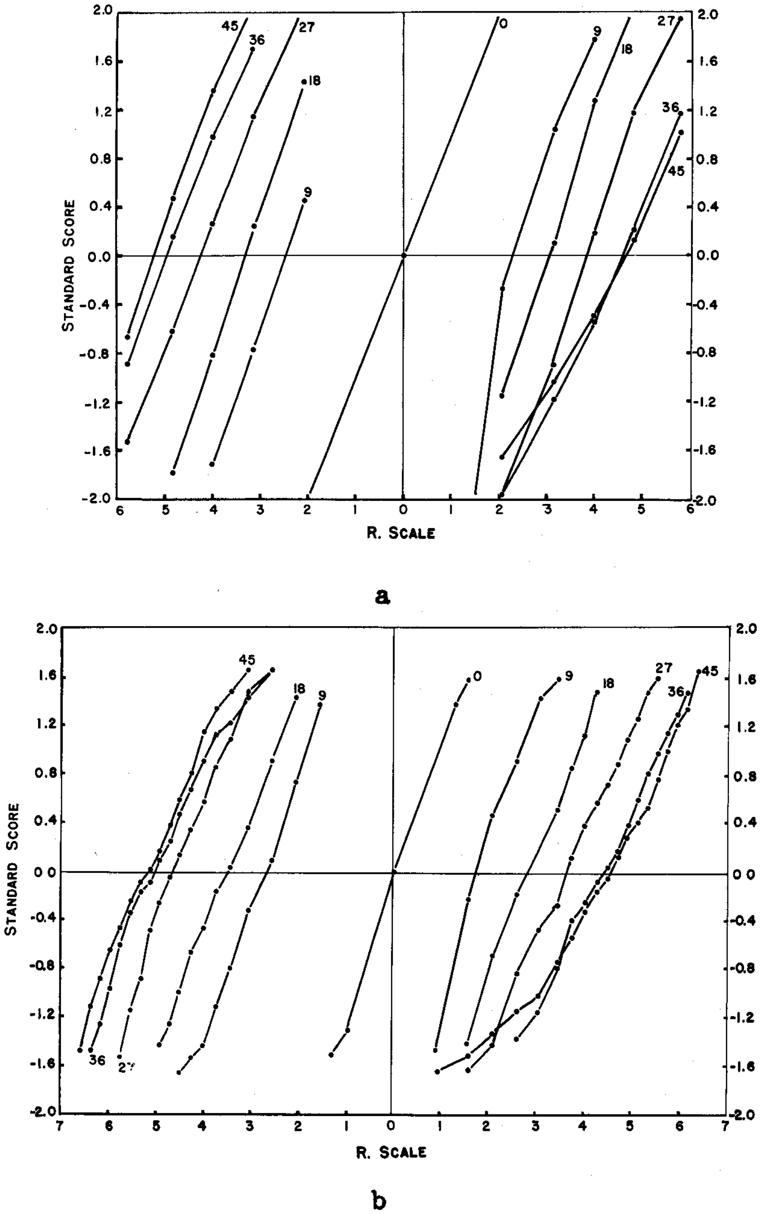


Fig. 6. Normalized response curves plotted on the R scale axis. The left half of each figure represents crossed disparities and judgements nearer the grid, and the right half represents uncrossed disparities and judgements farther than the grid. The top graph, a, is for the first experiment, and b is for the data of the second.

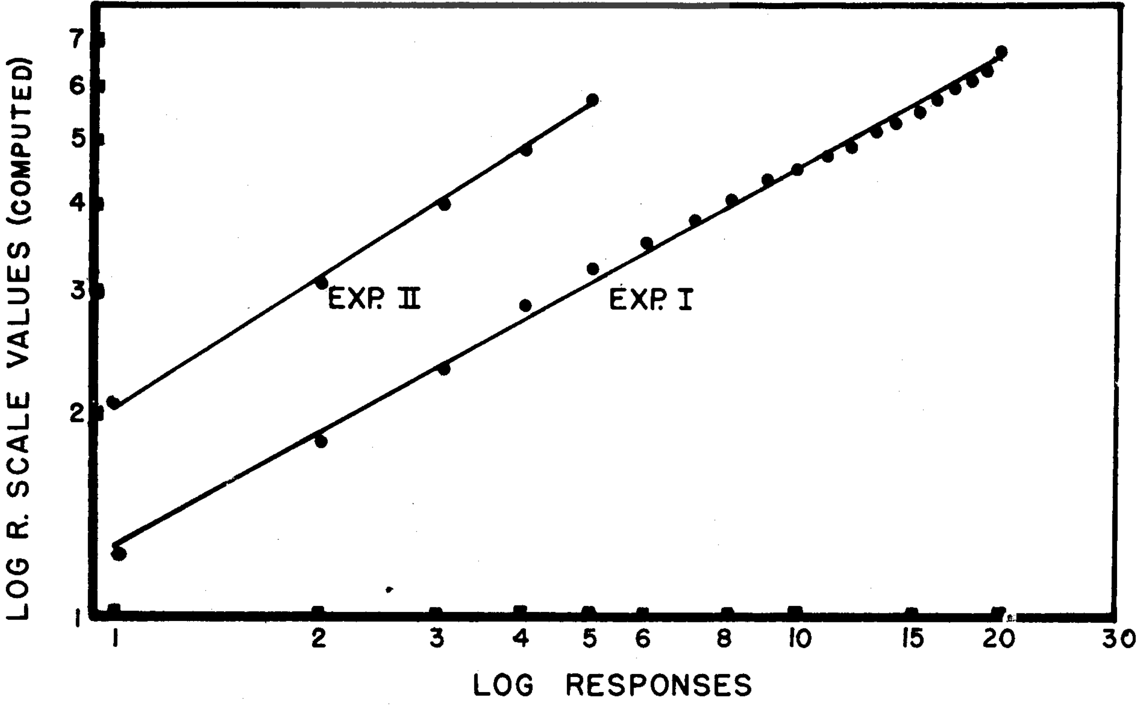


Fig. 7. Log log relationship between the obtained R scale values and the response categories for both experiments.

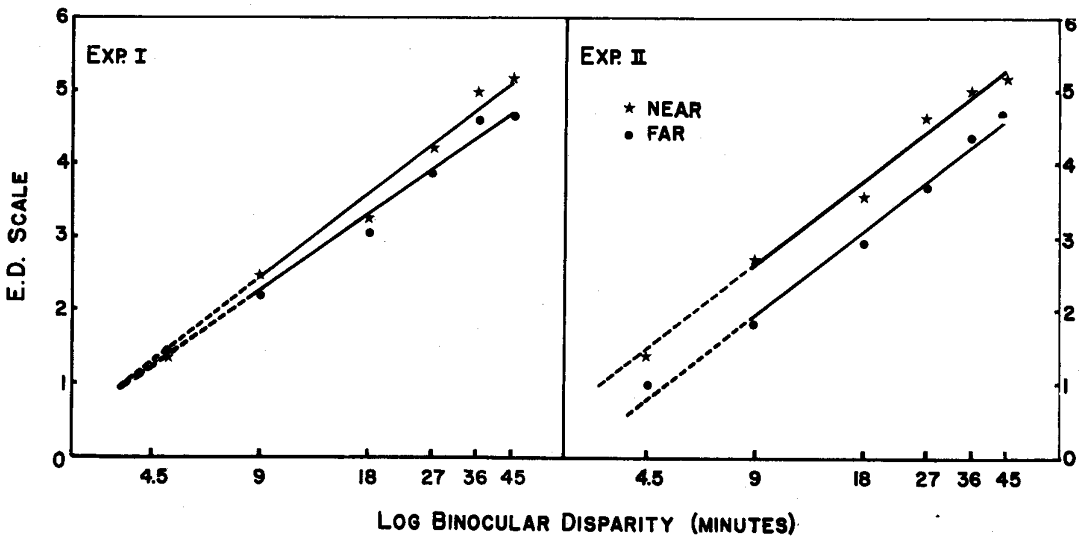


Fig. 8. Equal discriminability as a function of log disparity. The stars represent crossed disparities and the dots represent uncrossed disparities.

DISCUSSION

The accuracy with which various amounts of binocular disparity can be identified as absolute depth increments has been determined as a function of the amount of disparity. Figure 8 shows that there is a linear relationship between discriminability and log binocular disparity. It means that the difference in the amount of binocular disparity necessary to produce a perception of a difference of depth is a constant ratio of the amount of disparity presented. Thus the minimal average error in the reporting of depth over the whole scale, and the sensitivity of the instrument employing binocular disparity would be maximized if the presented amounts of binocular disparity increase logarithmically. The values of the stimulus categories of binocular disparity which yield equal increments of discriminability can be chosen by reference to Figure 8. For example, if we can allow 32% of the responses to a given stimulus to be in error, either by confusion with the next highest or lowest stimulus category, then the stimuli should be chosen so that their E.D. response values on the graph are two units apart. If we expect half the responses to be correct and half in error, then we should select the stimuli to be 1.35 units apart. If the categories are selected three E.D. units apart, then 87% of the responses should be correct and 6.5% should be confused with the adjacent stimulus either way. One E.D. scale unit equals one standard deviation of the normalized response curves, for any portion of the curve, so that a normal curve table can be used to specify the amount of response overlap between adjacent stimulus categories, depending on the separation of the categories. We are dealing with a two tailed overlap, so in a one tailed normal curve table, we would enter the Area column with one half the percentage of correct responses desired, and double the entry in the adjacent z or x/σ column in order to tell how many E.D. units apart the stimulus categories should be, in order to yield the desired percentage of correct responses.

The two experiments yielded the same logarithmic functions; so for the viewing conditions and the apparatus, it seems that the obtained functions will hold for different ways of responding. Each subject's response matrix had less consistency for large disparities than for small disparities. Figure 5b shows the relationship between the stimulus disparity and the standard deviation of the responses for a typical subject. Ogle (20) found a similar exponential relationship between the amount of stimulus disparity and the standard deviation of responses, when subjects tried to equate the apparent distance of a target in front of the fixation point with the apparent distance of a target behind the fixation point with an adjustable mirror stereoscope. He also found no significant difference in the accuracy of distance judgements between crossed and uncrossed disparities. Ogle's subjects were asked to maintain steady fixation, and the exposure duration of the targets was 0.2 second in order to obviate eye movements. It is significant that, with such different experimental conditions, the results of the two studies are similar in so far as the experiments are comparable. Ogle's data are from another subject and himself, while our data are based on thirty-nine naive subjects with a wide range of stereoptic acuity. Superior subjects show a steeper slope of the regression line in Figure 8, and more E.D. scale units would be included between their limits of fusion. Poor subjects in this ability would include fewer E.D. units within their limits of fusion, and a regression line of less slope. Another experiment is planned in which many more measurements will be made on each subject, so that stable individual curves may be generated in order to determine whether the linear relationship between E.D. units and log binocular disparity holds for each subject. The experiment will employ the stereoptometer (1) in which the experimenter will set the disparities, so that slides will not be used.

Before deciding on the use of the third dimension for coding, we must consider the effect of binocular disparity on the accuracy of reading the other two dimensions. Our subjects were required to give the two dimensional location of the targets while looking at the three dimensional slides. We found that disparity had no effect on the vertical location of the targets, since there was no vertical disparity. The accuracy of reading the horizontal location of the targets was markedly reduced with large disparities, since the depth effect depends on horizontal disparity alone. Figure 9 shows the relationship between the error in the horizontal location of the targets and binocular disparity. There is an appreciable increase beyond 36' disparity, so the criteria for accuracy in the horizontal dimension must be known [3]. The error tolerance may set the practical limits for the disparities used in operational equipment, in order to be sure that the information gain in the third dimension is not cancelled by the information loss in the first or second dimension. If one of the two dimensional extents is to be read more accurately than the other, then it should be coded as the vertical dimension on the display.

In order to improve the accuracy of target location, could the equipment be modified to correct for constant errors, and therefore overcome the horizontal errors introduced by disparity? Only further research can fully answer that question, but tentatively it seems unlikely, since there is no typical constant error pattern among subjects. Many subjects seem to have a fairly consistent error pattern, but there is no basis for guessing how response consistency could be improved except by continued practice with the apparatus.

The midpoint between the lower left hand corners of the two disparate targets is the correct horizontal location in our calculation of the horizontal

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[3] Actually there is a linear relationship of log error to disparity angles between 9' and 45' of arc.

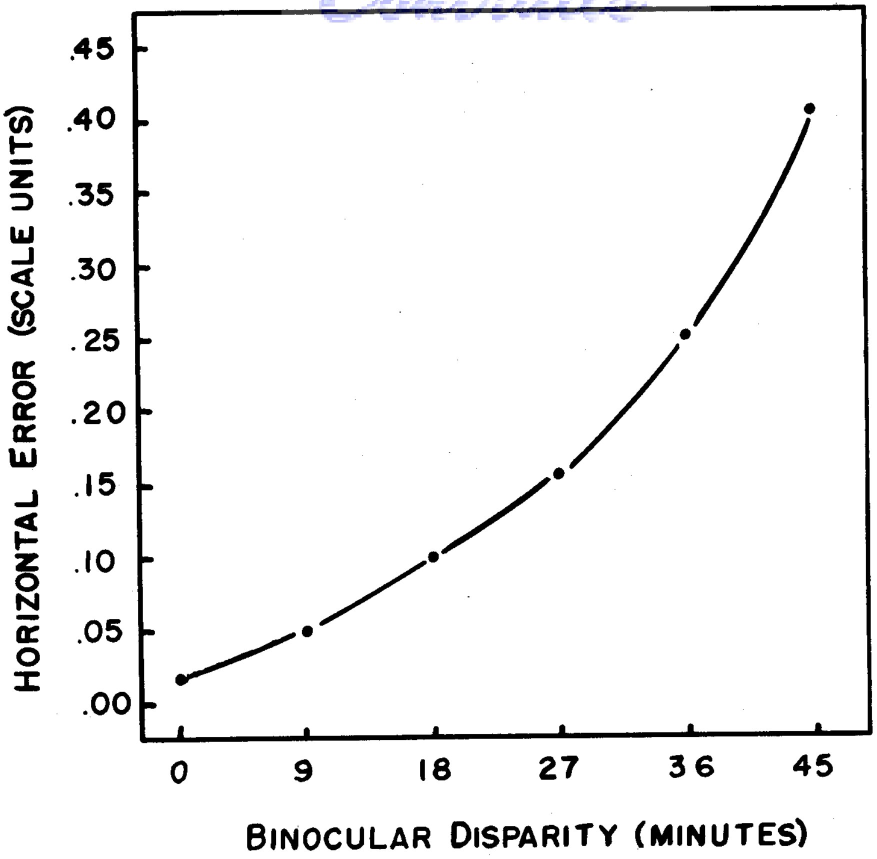

Fig. 9. Horizontal error in scale units as a function of binocular disparity. The curve is based on the mean values from all the subjects in both experiments.

errors. It would be the actual geometric location of the two lines of sight projected to each of the lower left corners of the target images if they were displaced toward each other by an equal amount during the process of displacement which results in fusion. A paper by Werner (23) discusses image displacement due to horizontal binocular disparity, and attributes the perception of depth to the displacement. Our results indicate that the amount of relative displacement of the two disparate images is an individual matter, and subjects differed markedly in the consistency of their fusion patterns.

The target could logically be located anywhere between the positions of the left and right eye images. According to Walls' theory of visual direction (22) and eye dominance (21), the dominant eye determines the visual direction of the perceived object. Each subject was given a sighting preference test, similar to the Miles A B C test (16), except that we made our own sighting pyramid out of stiff paper. According to Walls' theory, we should have expected the subject in our experiment to locate the object by its relative position on the right eye slide if he was right eye dominant, or at the left eye position if left eye dominant. The nondominant eye would function in the depth location by presenting a disparate image, but all of the displacement would accrue in the nondominant eye's image. According to our results, there is very little relation between the visual direction of the targets and sighting eye dominance. Only 11 subjects consistently located the targets toward the dominant eye's image. If enough subjects reported the location toward one or the other eye consistently, then we would think that it was a manifestation of yet another kind of eye dominance, which is not related to "sighting eye dominance." But a result which we cannot explain is that some subjects displaced the target location toward one eye for targets in front of the grid, and toward the other eye for targets behind the grid. These results are certainly an argument against an eye dominance determiner of visual location, because they require a double mechanism which favors one eye at one apparent distance, and the other for another distance. Until the problem of visual direction is solved, perhaps the best suggestion for requiring accurate location is for the observer to close one eye when he needs to read the visual direction of the target; or perhaps, to switch off one of the pictures to either eye. If that were the case, the correct horizontal location could be put into the picture that remained, and the other picture would contribute the disparate images.

These research findings assure the feasibility of stereoscopic binocular disparity as a desirable coding dimension for radar or pictorial information displays. Sufficient information was obtained to specify the limits and the discrete increments of disparity which may be presented in order to get the optimum from the instrument in terms of response discriminability. If the stimulus input is a continuous rather than a discrete function, the accuracy is best at zero disparity, and decreases with increasing disparity to the limits somewhat beyond the limits of fusion. If the accuracy requirements are the same throughout the scale, then the most frequent target depths should be presented at the depth location of the grid.

If discrete steps of binocular disparity are presented, then one of the steps should be 0° disparity, since "equal distance" is a very accurate and fast judgment. The author recommends the use of three steps of crossed and three steps of uncrossed disparity in addition, which are based on equal increments on the E.D. scale. Binocular disparities about 1 2/3 units apart are recommended; 0, 6, 18, and 42 minutes of crossed and uncrossed disparities are suitable, if the horizontal error tolerance permits the inclusion of such a range. With that distribution of disparities, about 60% of the responses would be correct for all the stimuli, and about 20% would be confused with the adjacent stimulus category on either side of the presented value. That seems like a practical error tolerance, since very few errors would be expected to be in error by more than one category; and it is likely that with experience with the instrument, the subjects would be able to increase their percentage of correct responses.

Some questions were raised by the experiments, but remain unanswered, and should be attacked in future research. There are individual differences in the range of fusion due to unknown factors. If a person has a wider range of fusion, will he include more E.D. units in his range, or will the slope of the function be less, so that no greater range is included? In selecting operators for stereo-equipment, how does the stereo-threshold relate to the slope of his E.D. curve, and the limits of fusion? This must be known in order to select the best operators. The results were obtained with an optical device which presented separate images of the grid and targets to the two eyes, and they were fused and seen at an indefinite location in front of the subject. A new experiment is planned which will present reference lines at a definite distance in front of the subject, and a target circle will be introduced stereoptically between the reference distance and the subject, covering the same range of crossed disparities as in the present experiment. The question to be answered is whether the same relationship between equal discriminability and log disparity holds when the reference plane is at a definite absolute distance from the subject, and the targets seem to have an absolute location in real space. Probably, in that case, the discriminability will be related to the actual projected target distance as well as to the binocular disparity.

SUMMARY

The suggestion has been made that binocular disparity could be a coding dimension for use in apparent three dimensional radar displays, in which the apparent depth of an object in front or in back of the plane of the scope face may represent the height, distance, or some other dimension of the target location. Two experiments were done in order to determine whether subjects could discriminate differences in binocular disparity with sufficient accuracy to make its inclusion in a cathode ray tube display worth-while. We conclude on the basis of our observations that three dimensional radar is practical, and we state the accuracy with which absolute differences in binocualr disparity can be discriminated without presenting the subject a scale by which he can compare relative amounts of apparent depth.

Subjects can most easily discriminate differences in binocular disparity near zero disparity, and become progressively less accurate for increasing crossed or uncrossed disparities. There was found to be a linear relationship between discriminability of differences in disparity and the logarithm of the disparity from zero disparity to plus or minus 45 minutes of disparity, which was the approximate limit of fusion for most subjects in the experiments.

The recommendation is made that if discrete disparities are presented by an operational instrument, the disparities should be selected so that the disparity between two adjacent categories is a constant percentage of the absolute amount of disparity. This will enable the operator to make judgments of equal accuracy over the range of binocular disparities presented, which would minimize the average error if the judgments had to be made over the whole range with equal frequency. If the demands for accuracy differ over the range of presented disparities, then the apparent distance of the targets which must be discriminated most accurately should appear at the same depth as the plane of the scope face.

Subjects may respond either with continuous or categorized judgments since the difference in the method of report was the main difference between the two experiments, and the scaling results were similar in both experiments. Nineteen subjects in the first experiment and twenty subjects in the second reported on the three dimensional location of a total of eighty-eight targets on twenty-two slides presented in a stereoscope. Binocular disparity increased horizontal errors in target location, in a way that the mean error increased proportionately by larger amounts for each successive linear increase in binocular disparity. The vertical location of the targets was as accurate for large as for small amounts of binocular disparity. The results are based on naive subjects; and with additional practice, the discriminations probably would become more accurate, but the pattern of accuracy of response for any given subjects would be expected to remain the same.

BIBLIOGRAPHY

1. Alluisi, E.A. and Harken G.S. The Stereoptometer - A Simple Haploscopic Instrument for the Study of Binocular Space Perception. Science, Volume 117, 1953. pp. 682-683.

2. Baker, C.A. and Grether, W.F. Visual Presentation of Information. United States Air Force Technical Report No. 54-160, Aero-Medical Laboratory, Wright Air Development Center, Ohio, 1954.

3. Berry, R.N., Riggs, L.A. and Duncan, C.P. The Relations of Vernier and Depth Discrimination to Field Brightness. Journal of Experimental Psychology, Volume 40, 1950. pp. 349-354.

4. Berry, R.N., Riggs, L.A. and Richards, W. The Relation of Vernier and Depth Discrimination to Thickness of the Bars. Journal of Experimental Psychology, Volume 40, 1950. pp. 520-522.

5. Fitts, P.M. Engineering Psychology and Equipment Design. Handbook of Experimental Psychology, Edited by S.S. Stevens. John Wiley, New York, 1951. Chapter 35, pp. 1287-1340.

6. Fry, G.A. Measurement of the Threshold of Stereopsis. Optometric Weekly, Volume 33, 1942. pp. 1029-1032.

7. Garner, W.R. and Hake, H.W. The Amount of Information in Absolute Judgments. Psychological Review, Volume 58, 1951. pp. 446-459.

8. Graham, C.H., Hammer, Elaine R., Mueller, Roberta D. and Mote, F.A. Stereoscopic Settings with Reticles Providing Multiple Reference Ranges: The Perception of Spatially Repeating Patterns. Journal of Psychology, Volume 27, 1949. pp. 209-216.

9. Graham, C.H., Riggs, L.A., Mueller, C.G. and Soloman, R.L. Precision of Stereoscopic Settings as Influenced by Distance of Target from a Fiducial Line. Journal of Psychology, Volume 27, 1949. pp. 203-207.

10. Grether, W.F. Discussion of Pictorial versus Symbolic Aircraft Instrument Displays. United States Air Force, Engineering Division Memorandum Report No. TSEAA-694-8B, Aero-Medical Laboratory, Wright Air Development Center, Ohio, 1947.

11. Harvard University, Howe Laboratory of Opthalmology. A Comparison of the Accuracy of Stereo-ranging on Targets of Different Types. Publication Board No. L62425, U.S. Department of Commerce, Washington, D.C., 1942.

12. Hirsh, M.J. and Weymouth, F.W. Distance Discrimination: II. Effect on Threshold of Lateral Separation of Test Objects. A.M.A. Archives of Opthalmology, Volume 39, 1948. pp. 224-231.

13. Ludvigh, E.J. Accuracy of Stereopsis as a Function of the First and Second Derivatives of Disparateness with Respect to Time. Publication Board No. 55792,U.S. Department of Commerce, Washington, D.C., 1947.

14. McCulloch, C. and Crush, M. Clinical Aspects of Stereopsis. A.M.A. Archives of Opthalmology, Volume 36, 1946. pp. 171-188.

15. Mead, L.C. The Influence of Size of Test Stimuli, Interpupillary Distance and Age on Stereoscopic Depth Perception. Journal of Experimental Psychology, Volume 33, 1943. pp. 148-158.

16. Miles, W.R. Ocular Dominance Demonstrated by Unconscious Sighting. Journal of Experimental Psychology, Volume 12, 1929. pp. 113-126.

17. Mueller, C.G. and Lloyd, V.V. Stereoscopic Acuity for Various Levels of Illumination. Proceedings of National Academy of Science, Volume 34, Washington, D.C., 1948. pp. 223-227.

18. Nordenson, J.W. On the Measure of Stereoscopic Acuity of Vision. British Journal of Opthalmology, Volume 32, 1948. pp. 913-914.

19. Ogle, K.M. On the Limits of Stereoscopic Vision. Journal of Experimental Psychology, Volume 44, 1952. pp. 253-259.

20. Ogle, K.M. Precision and Validity of Stereoscopic Depth Perception from Double Images. Journal of the Optical Society of America, Volume 43, 1953. pp. 906-913.

21. Walls, G.L. A Theory of Ocular Dominance. A.M.A. Archives of Opthalmology, Volume 45, 1951. pp. 387-412.

22. Walls, G.L. The Problem of Visual Direction. American Journal of Opthalmology and Archives of the American Academy of Optometry, Monograph 117, 1951. pp. 1-101.

23. Werner, H. Dynamics in Binocular Depth Perception. Psychological Monographs, Volume 49, No. 2, 1937. pp. 1-127.

24. Woodworth, R.S. Experimental Psychology. Henry Holt, New York, 1938.