VISCOELASTIC AND STRUCTURAL DAMPING ANALYSIS

Harry H. Hilton*
University of Illinois at Urbana-Champaign

ABSTRACT

The interrelationships between viscoelastic, Newtonian viscous and structural damping are analyzed in terms of Fourier transforms and complex moduli in the frequency domain and are also interpreted in terms of behavioral responses associated with real material compliances or moduli in the real time plane. It is shown that the correspondence between viscous and elastic structural damping is spurious, severely limited to only harmonic motion and that it does not extend to more complicated viscoelastic materials beyond Newtonian viscous flow dissipation. The dissipation energy generated by viscoelastic and structural damping is also examined. The effects of structural damping on elastic and viscoelastic bending-torsion flutter are evaluated with the help of numerical examples. The material considered is aluminum, but the analysis is general and can be applied to any viscoelastic material. It is shown that the presence of increased structural damping does not necessarily have a stabilizing effect by decreasing the viscoelastic or elastic flutter speed nor are the viscoelastic flutter speeds necessarily lower than the corresponding elastic ones.

INTRODUCTION

In flutter and vibration analysis, it is standard practice to augment elastic effects by the introduction of structural damping coefficients g [1-4], where the latter are essentially measures of losses due to material hysteresis and/or friction in structural joints. In both instances, the fundamental dissipation phenomenon is "dry" solid friction and as such, the associated force and displacement constitutive relations are explicitly independent of frequency and of displacement velocities, accelerations or their higher time derivatives. Analytically, the algebraic Hooke's law is maintained, but the actual, real elastic moduli are replaced by complex

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* Professor Emeritus of Aeronautical and Astronautical Engineering
104 S. Mathews
Urbana, IL 61801-2997
217-333-2653 FAX 217-244-7705
E-mail: HILTON AT UIUCVMC.BITNET

values, i.e., $E = E_o (1 + ig_1)$, where E_o is Young's modulus in the absence of structural damping. A similar expression is used for the elastic shear modulus, $G = G_o (1 + ig_2)$ and for the elastic bulk modulus, $K = K_o(1 + ig_3)$. The three g's displayed here may or may not be equal depending upon the particular damping encountered in a given structure.

Viscoelastic materials, on the other hand, obey differential and/or integral stresses-strain laws, which relate stresses, strains and their time derivatives of various orders [5]. The viscoelastic dissipation process is primarily an involved, highly frequency sensitive, material dependent viscous phenomenon with one or more coefficients of viscosity [5] and, as will be shown, totally unrelated to the structural damping mechanism. Historically, and in this paper as well, the term viscous damping refers to Newtonian flow, where the stresses are proportional to the strain velocities through at most only one coefficient of viscosity for shape changes and no more than one other for volume changes. While the structural damping phenomenon is well understood and experimental values for these damping coefficients are readily available [1-4], its interpretation vis-a-vis viscous damping appears confused [6, 7]. Fung [3], on the other hand, has correctly based his correspondence between viscous and structural damping on harmonic motion, but has restricted his analysis to only motion at the system's natural frequency. Under these conditions he shows that the structural damping coefficient is frequency independent. More recently, Dahl [8] has modeled solid friction damping in mechanical oscillators by using both linear and nonlinear formulations. His models are of interest, since they simulate decay behavioral patterns which approximate (a) Coulomb friction at high amplitudes and low frequencies, (b) viscous damping at mid amplitudes and mid frequencies and, finally, (c) structural damping at small amplitudes and high frequencies. However, these approximate similarities do not imply any relations between fundamental behavioral responses of solid and viscous damping phenomena. Saravanos and Chamis [9] present a hysteric damping analysis for composite laminates and include an extensive bibliography on damping.

Since viscoelasticity includes among other mechanisms both elasticity and viscous damping, i.e., velocity dependent Newtonian viscous dissipation, it can readily serve as a vehicle for the comparison of viscous and structural damping. In this paper, general linear viscoelastic stress-strain relations (including structural and viscous damping) are used to interpret the various damping processes by a critical examination of complex moduli in the frequency domain and of compliances in the real time plane. Such an approach makes it possible to treat generalized many degree of freedom systems and is not limited to the single mass, spring and damper combinations of References [6] and [7].

ANALYSIS

Flutter and Complex Moduli

The governing elastic equilibrium equations for flexible lifting surfaces, fuselages, etc. subjected to aerodynamic and inertial forces with generalized displacements $q_m(x,t)$, m=1, 2,--M, can be expressed in the generalized form

$$\sum_{m=o}^{M} [\sum_{n=o}^{N'} D^e_{mnk}\, \partial^n q_m\, (x,t)/\partial x^n = \sum_{n=1}^{N''} L_{mnk}\{V, q_m, \dot{q}_m, \ddot{q}_m\}] = F_k \quad (1)$$

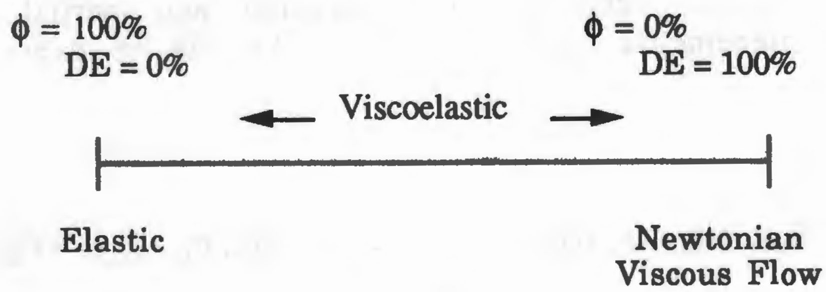
$$k = 1,2,...M; \qquad x = \{x_1, x_2, x_3\}$$

where L_{mnk} are differential operators describing inertia and unsteady aerodynamic contributions, V is the flight speed, F_k are generalized forces and the D^e_{mnk} are elastic stiffness terms depending primarily on material properties (i.e., Young's and shear moduli E_o and G_o), on structural geometry and on mass distributions. The elastic-viscoelastic analogy [5, 10, 11] consists of the application of Fourier transforms (F.T.) to Eqs. (1) and of the subsequent substitution of complex viscoelastic moduli $\overset{=}{E}$ and $\overset{=}{G}$ for the elastic moduli E_o and G_o, or essentially replacing 'the real and frequency independent elastic stiffnesses D^e_{mnk} by complex viscoelastic stiffness functions $\overset{=}{D}_{mnk}(\omega)$. This, then leads to governing viscoelastic relations in the F. T. plane

$$\sum_{m=0}^{M} [\sum_{n=0}^{N'} \overset{=}{D}_{mnk}(x,\omega)\partial^n \overset{=}{q}_m\, (x,\omega)/\partial x^n = \sum_{n=1}^{N''} \overset{=}{L}_{mnk}\{V, \overset{=}{q}_m, \omega\}] = \overset{=}{F}_k\, (x,\omega) \quad (2)$$

It can be readily shown [5] that for simple harmonic motion the F. T. variable ω is the oscillatory frequency and that in the case of flutter [10, 11] it becomes the flutter frequency, while V plays the role of the flutter speed. The latter two are, of course, pairs of eigenvalues at which a given flight structure can experience harmonic motion. The velocity V can readily be replaced by the flutter Mach number M_f.

Viscoelastic responses may also be characterized on an energy axis involving all potential energy at one end and all dissipation at the other which is shown schematically in Fig. 1. Elasticity and viscous damping represent the two degenerate viscoelastic extremes at opposite ends of the energy scale, i.e., elasticity is 100% potential energy and zero damping, while Newtonian viscous flow is all dissipation and no potential energy storage.



ϕ = potential energy
DE = dissipation energy

Fig. 1. Energy Representation of Material Properties

In general, linear viscoelastic material behavior (including elasticity and viscous damping), i.e. the stress-strain relations, can also be expressed by relations between generalized displacements q_m and forces F_m. For isotropic materials they are given by [5]

$$P_m \{F_m\} = Q_m \{q_m\} \tag{3}$$

and where P_m and Q_m are either differential or integral time operators. In particular, for each m these reduce to

$$F_m = E_{mo}(1 + ig_m)q_m \tag{4}$$

for elastic structural damping and to

$$F_m = E_{mo}\,(1+ ig_m)q_m + c\dot{q}_m \tag{5}$$

for combined viscous and elastic structural damping. (When g = 0 in Eq. (5), then only viscous damping takes place coupled with an elastic response.) Similar, but more involved, expressions may also be written for anisotropic materials [13], but will not be introduced here for the sake of simplicity. They are, however, treated briefly at the end of the next section.

The application of Fourier transforms to Eqs. (3), leads to [5, 10, 11]

$$\bar{\bar{F}}_m = \bar{\bar{E}}_m \bar{\bar{q}}_m \tag{6}$$

where the $\bar{\bar{E}}_m$ are frequency dependent viscoelastic complex moduli. Note that Eqs. (6) are symbolically equivalent to the F.T. of Eqs. (4) and that the F.T. of the elastic Eqs. (4) gives a complex modulus $\bar{\bar{E}}_m = E_{mo}(1 + ig_m)$ for structural damping, which is frequency insensitive. Since these complex moduli are expressible as $\bar{\bar{E}}_m = E_{mR}(x, \omega) + iE_{mI}(x, \omega)$, it follows that the viscoelastic stiffnesses are also complex, i.e., $\bar{\bar{D}}_{mnk}(x, \omega) = D_{mnkR}(x, \omega) + iD_{mnkI}(x, \omega)$, where the E_{mR}, E_{mI}, D_{mnkR}, and D_{mnkI} are all distinct real frequency functions. Such an omega dependence is due to the intrinsic nature of the time differential or integral viscoelastic stress-strain laws of Eqs. (3). It can be readily seen from Eqs. (5), that in the general viscoelastic case, the complex moduli with structural damping are

$$\bar{\bar{E}}_m(x, \omega) = (1 + ig_m)E_{mR}(x, \omega) + iE_{mI}(x, \omega) = E_{mR}(x, \omega) + iE'_{mI}(x, \omega) \tag{7}$$

where $E'_{mI} = g_m E_{mR} + E_{mI}$.

Furthermore, elastic structural damping is also included in Eqs. (2), by virtue of the complex moduli defined by Eqs. (4), except that then the D_{mnkR} and D_{mnkI} are frequency independent. In any event, the expressions on the right hand sides of Eqs. (2) (i.e., the generalized forces) are unaffected by the nature of the elastic or viscoelastic materials. Therefore, the fundamental difference is that in the elastic case with or without structural damping, D^e_{mnk} are frequency independent, while for viscoelastic materials the stiffness parameters $\bar{\bar{D}}_{mnk}$ are always frequency functions. For nonhomogeneous viscoelastic materials with structural damping, one needs only to replace the elastic stiffnesses in Eqs. (2) with $\bar{\bar{D}}_{mnk}(x, \omega) = D_{mnkR}(x, \omega) + iD'_{mnkI}(x, \omega)$ in the F. T. plane, where $D'_{mnkI} = g_m D_{mnkR} + D_{mnkI}$. Again note, that for elastic structural damping, the stiffness parameters in Eqs. (2) have a form identical to frequency independent viscoelastic ones. Table I illustrates the complex moduli representations in the four combinatorial cases considered.

TABLE I.

Complex Moduli $E = E_R + iE_I'$ and Compliances J_E (t)

Material	Real Part	Imaginary Part	Compliance J_E (t)
Elastic	E_o	0	
Elastic with structural damping	E_o	gE_o	$\delta(t)J_{Eo}/(1 + ig)$
Viscous damping with structural damping	E_o	$gE_o + c\omega$	$\exp[-(1 + g)t/\tau]/c$
Viscoelastic with structural damping	$E_R(\omega)$	$gE_R(\omega) + E_I(\omega)$	Eq. (14)

The structural damping terms igq may be thought of as out of phase components of the displacements q, and, as such, bear some resemblance to velocity effects, i.e., viscous damping. However, examination of the F. T. of the viscous damping term cq̇, in Eqs. (5), clearly shows that it is equal to $i\omega c\bar{\bar{q}}$ for a time independent viscosity coefficient c. Consequently, as long as structural damping coefficients g are frequency independent, they cannot phenomenologically relate to viscous damping, unless one postulates a c inversely proportional to ω - not the ordinary coefficient of viscosity, to be sure. Also note that the F. T. of Eq. (5) for g = 0 is $\bar{\bar{E}} = E_0(1+ic\omega^*/\sqrt{M^*E_0})$, where $\omega^* = \omega/\omega_N$ and the natural frequency $\omega_N^2 = E_0/M^*$, with M* the system mass. Therefore, the complex modulus for viscous damping is frequency dependent, and only at the natural frequency can a frequency independent correspondence be established between structural and viscous damping when $g = c\sqrt{M^*E_0}$. This relation between the complex moduli applies to any motion, and such a correspondence between g and c is not limited to harmonic motion as has been discussed earlier by Fung [3]. At all other frequencies, of course, the frequency dependent relationship $g = c\omega^*/\sqrt{M^*E_0}$ is valid for any elastic structural or viscous damping complex modulus, but is not physically realistic.

However, while such a proposition satisfies the consistency of expressions in the ω F. T. plane, Eqs. (4) and (5) demonstrate that even an inversely frequency dependent viscosity coefficient c or a constant one at the natural frequencies cannot restore correspondence in the time plane between the elastic and viscous damping cases for general displacement functions q(x,t) encountered in creep, relaxation and other non-oscillatory motions. As a matter of fact, even in relatively simple motion where q is proportional to a single exponential function exp(iωt), the correspondence between viscous and structural damping is lost in those non mechanical vibration problems, such as for instance flutter, which have highly nonlinear sensitivities to frequency eigenvalues. For convenience and completeness, one usually represents viscoelastic stress and strain behavior in terms of mechanical models, such as, for instance, the generalized Kelvin model (GKM) [5] shown in Fig. 2. Consequently, it follows from Eq. (5) and from an examination of the GKM that viscoelastic damping represents a much more complicated phenomenon than either elastic or viscous structural damping, since the complex compliances $\bar{\bar{J}}_E = 1/\bar{\bar{E}}$, $\bar{\bar{J}} = 1/\bar{\bar{G}}$, $\bar{\bar{J}}_v = 1/\bar{\bar{K}}$, etc. with $\bar{\bar{E}} = 3\bar{\bar{G}}/(1 + \bar{\bar{G}}/\bar{\bar{K}})$ are of the form

$$\bar{\bar{J}} = J_o/(1 + ig) + 1/i\omega\eta_{N+1} + \sum_{n=1}^{N} 1/\{G_n[1 + i(\omega\tau_n + g)]\} \qquad (8)$$

with similar relations for the other $\bar{\bar{J}}$'s and where the relaxation times $\tau_n = \eta_n/G_n$, η_n and G_n are all material property, temperature sensitive parameters [5] (Fig. 2), Viscoelastic compliances in the absence of structural damping are given by Eq. (8) with $g = 0$. Similarly, the expressions (8) also include viscous damping as a degenerate case of the form $N = 0$, $J_o = 0$ and with all $G_n = \infty$. The elastic case can be obtained from $\eta_{N+1} = G_n = \infty$.

These two distinct phenomena, i.e., structural and viscoelastic (including viscous) damping, may be interpreted in yet another fashion by examining their complex representations. For each generalized displacement q_m, the corresponding elastic modulus with structural damping can be represented by $E_o(1 + ig) = R_e\exp(i\Delta_e)$ and the expressions $R_e = E_o\sqrt{1 + g^2}$ and $\Delta_e = \tan^{-1}(g)$ are both frequency independent. (For the sake of simplicity of representation, the subscripts m are not included here.) Complex viscoelastic moduli may be written in a similar fashion as seen in Eq. (7) with

$$\bar{\bar{E}}(\omega) = R_{ve} \exp(i\Delta_{ve}) \tag{9}$$

where

$$R_{ve}(\omega) = E_R(\omega)\,\{1 + g^2 + 2g\,E''(\omega) + [E''(\omega)]^2\}^{1/2} \tag{10}$$

and

$$\Delta_{ve}(\omega) = \tan^{-1}\,[g + E''(\omega)] \tag{11}$$

are both frequency dependent with $E^*(\omega) = E_I(\omega)/E_R(\omega)$. These values are shown in Table II for 2024 aluminum [10, 11]. The R_{vemin} and R_{vemax} values correspond to $\omega = 0$ and ∞ (i.e. $t = \infty$ and 0) respectively and the E_I/E_R peak in the neighborhood of 15 Hz, which is of the order of magnitude of the flutter frequencies for the examples considered in Reference [11]. The R_e vectors for the elastic structural damping at the temperatures of Table II are equal to R_{vemax} and the angles Δ_e are equal to Δ_{vemin} at all temperatures.

TABLE II.

Viscoelastic Damping Properties of 2024 Aluminum [11]

Tempe-rature °F	Strucutral Damping Coefficient	(E_I'/E_R)	$R_{ve_{max}}$ psi x 10^{-7}	$R_{ve_{min}}$ psi x 10^{-7}	$\Delta_{ve_{max}}$ degrees	$\Delta_{ve_{min}}$ degrees
80	0	.00283	1.070	1.060	.162	0
80	.05	.0528	1.071	1.061	3.02	2.86
200	0	.00585	1.038	1.020	.335	0
200	.05	.0559	1.039	1.021	3.20	2.86
340	0	.0144	.990	.966	.826	0
340	.05	.0644	.991	.967	3.69	2.86
450	0	.0258	.954	.900	1.48	0
450	.05	.0758	.955	.901	4.33	2.86

Since the flutter Eqs. (2) are highly nonlinear functions of ω, an analytical comparison of viscoelastic and structural damping is not feasible. However, a reexamination of the bending-torsion supersonic flutter problem for a Timoshenko beam previously analyzed in Reference [11] based on the addition of structural damping effects as exemplified by Eqs. (7), leads to the results displayed in Table III.

TABLE III.

Some Flutter Results for
a 2024 AL Wing

	Elastic		Viscoelastic	
g	ω_f	M_f	ω_f	M_f
0	20.0001	1.3037	26.8586	1.5167
.005	19.9737	1.3056	26.8070	1.5081
.01	19.9465	1.3074	26.7550	1.4994
.05	19.7150	1.3239	29.0102	1.9887

These results are typical for metal wings in supersonic flow and fully account for the material property dependence on temperature as the flutter Mach number changes. It is to be noted that as the structural damping g increases the viscoelastic flutter Mach number M_f may increase (destabilizing) or decrease (stabilizing). For an elastic aluminum wing with the same mass distribution, geometry and aerodynamics, the corresponding flutter Mach numbers are smaller than the viscoelastic ones and an increase in structural damping for the elastic wing is destabilizing. Even though the viscoelastic action for 2024 aluminum at elevated temperatures is far from being as pronounced as it is in high polymers and composites, the viscoelastic flutter Mach numbers are significantly different from the corresponding elastic ones. (Table III) This is due to the highly nonlinear dependence on the flutter frequency ω_f and the attendant phase relations which shift in a complicated fashion. In References 10 and 11 it has been previously noted that viscoelastic flutter Mach numbers may be higher or lower than corresponding elastic ones for wings of identical geometry, mass distributions and aerodynamic properties. Dugunji [12] has noted similar behavior due to structural damping in elastic panel flutter.

Dissipation Energy

A comparison of the dissipation energies generated by viscoelastic, viscous and structural damping processes is next in order. They can be considered together by referring to the mechanical models of Fig. 2. For an

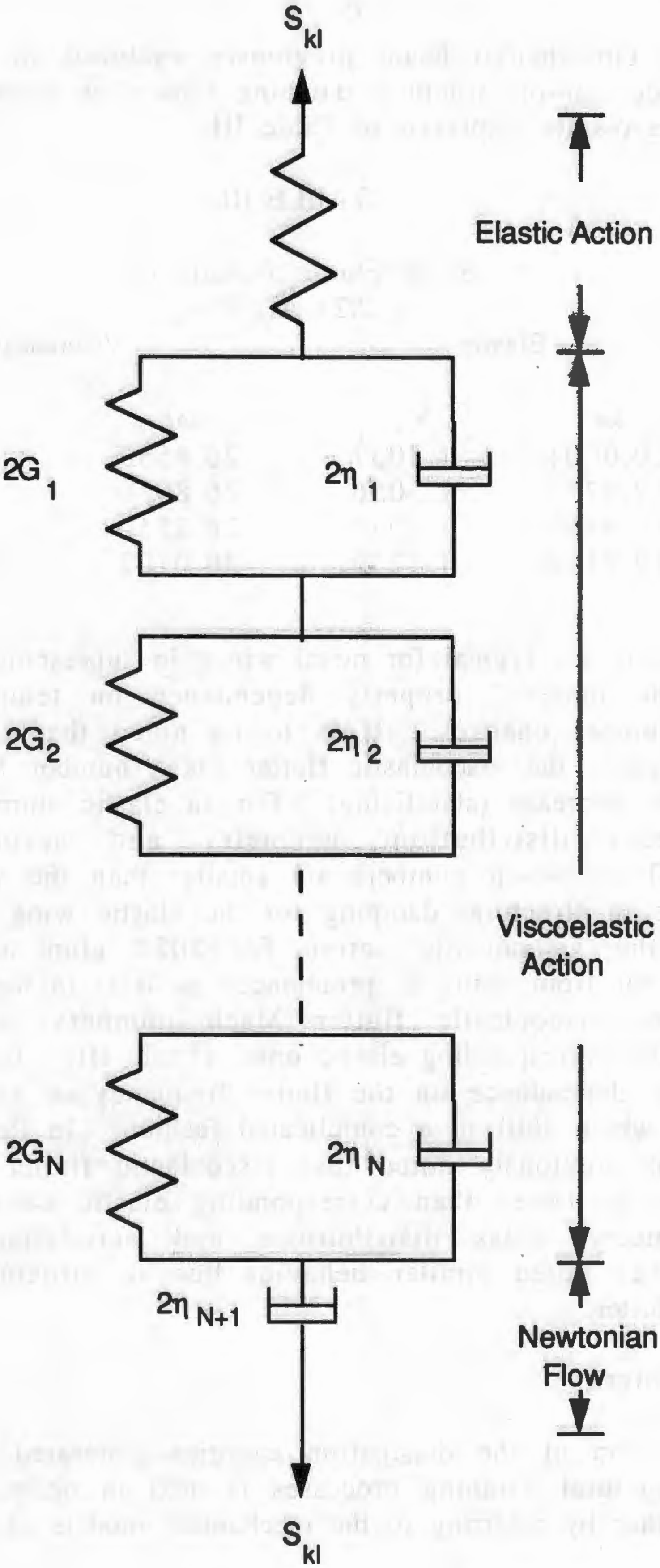


Fig. 2. The Generalized Kelvin Model

isotropic, linear viscoelastic material the stress-strain relations for change in shape and in volume are, respectively [5]

$$2\,E_{kl}(x, t) = \int_0^t J(x, t - t')\, S_{kl}\,(x, t')\, dt' \qquad (12)$$

$$\varepsilon(x, t) = \int_0^t J_v\,(x, t - t')\sigma(x, t')\, dt' \qquad (13)$$

where S_{kl} and E_{kl} are the stress and strain deviators, ε and σ the mean strains and stresses and where the compliances are

$$J(t) = \sum_{n=1}^{N+1} J_n(t) = \delta(t)\, J_o/(1 + ig) + 1/\eta_{N+1} + \sum_{n=1}^{N} \exp[-(1 + g)t/\tau_n]/\eta_n \qquad (14)$$

and with a similar expression for the volumetric compliance J_v, both of which are obtained by the F.T. inversion of Eq. (8). Note that Eq. (14) defines the general viscoelastic compliances in the presence of structural damping. These compliances may also be written in the manner of Eq. (7), i.e.

$$\bar{\bar{J}}\,(\omega) = 1/\bar{\bar{G}}\,(\omega) = J_R(\omega) - iJ_I'(\omega) = [G_R\,(\omega) - iG_I'(\omega)]/[G_R^2\,(\omega) + G_I'^2\,(\omega)] \qquad (15)$$

As can be seen from Eqs. (5) and (8), the introduction of structural damping effects into viscoelasticity results in changes of $G_n[1 + i(\omega\tau_n + g)]$ in the denominators of the sums of Eq. (5) and in first term multiplication by $1/(1+ig)$. Effectively, upon F.T. inversion, this serves to shift the time to $t+gt$ in the exponential terms of Eq. (14).

The dissipation energy per unit volume at any point $x = (x_1, x_2, x_3)$ and at any time $t > 0$ is

$$DE(x, t) = \int_0^t \sum_{n=1}^{N+1} S_{kl}^{(n)}\,(x, t')\, \dot{E}_{kl}^{(n)}\,(x, t')\, dt' + \int_0^t \sum_{m=1}^{M+1} \sigma^{(m)}(x, t')\, \dot{\varepsilon}^{(m)}(x, t')\, dt' \qquad (16)$$

where superscripts (n) and (m) denote quantities associated with each dashpot n or m in the Fig. 2 model. The stress-strain relations for each dashpot are given by [5]

$$S_{kl}^{(n)} = 2\eta_n \dot{E}_{kl}^{(n)} \tag{17}$$

and

$$2\, E_{kl}^{(m)}(x, t) = \int_o^t J_n(x, t-t')\, S_{kl}(x, t')\, dt' \tag{18}$$

where S_{kl} is the total stress deviator in the GKM model and is the sum of $S_{kl}^{(n)}$ of any dashpot and the stress deviator of its corresponding elastic paired spring. Equations similar to (17) and (18) can also be written for volumetric changes.

Differentiation of Eqs. (18) and substitution into (17) gives

$$S_{kl}^{(n)}(x, t) = \eta_n \{ \int_o^t \partial J_n(x, t - t')/\partial t'\, S_{kl}(x, t')\, dt' + J_n(x, 0)\, S_{kl}(x, t)\} \tag{19}$$

with a similar expression for $\sigma^{(m)}$. Finally, the introduction of Eqs. (17), (18) and (19) into (16) results in

$$DE(x, t) = \int_o^t \{\sum_{n=1}^{N+1} \frac{1}{2\eta_n} [\int_o^{t'} \frac{\partial J_n(x, t' - \xi)}{\partial t'} S_{kl}(x, \xi)\, d\xi$$

$$+ J_n(x, 0)\, S_{kl}(x, t')]\, [\int_o^{t'} \frac{\partial J_n(x, t' - \xi)}{\partial t'} S_{kl}(x, \xi)\, d\xi \tag{20}$$

$$+ J_n(x, 0)\, S_{kl}(x, t')]\}\, dt' + \int_o^t \{\sum_{m=1}^{M+1} \frac{1}{\eta_{vm}} [\int_o^{t'} \frac{\partial J_{vm}(x, t'-\xi)}{\partial t'} \sigma(x, \xi)\, d\xi$$

$$+ J_{vm}(x, 0)\, \sigma(x, t')]\}\, dt'$$

The total dissipative energy $DE_T(t)$ is the volume integral

$$DE_T(t) = \int_V DE(x, t)\, dx_1\, dx_2\, dx_3 \qquad (21)$$

where V is the total volume of the body. It is readily seen that Eqs. (20) and (21) depend both on material properties (J_n, J_{vm}) and the loading process (S_{kl}, σ, i. e. the stresses $\sigma_{kl} = S_{kl} + \delta_{kl}\sigma$). Consequently, Eq. (21) is an extremely useful expression for comparing the dissipation properties of various materials at one or more processes.

In the viscous no structural damping case, Eqs. (12) through (20) simplify to only one term in each of the J's, i.e.,

$$J = \exp[-(1 + g)t/\tau]/\eta \qquad (22)$$

and with a similar v subscripted expression for J_v and where $\tau = \eta/G_0$ and $\tau_v = \eta_v/K_0$ with, of course, the usual coefficient of viscosity c equal to the more general η. For elastic structural damping alone, only the first term of Eq. (14) remains and due to the nature of the Dirac delta functions the stress-strain relations (12) and (13) reduce to the usual algebraic elastic ones. The results are summarized in Table I.

For anisotropic materials, similar but more complicated flutter and dissipation energy expressions can readily be derived. However, they may require as many as 21 complex moduli or compliances (instead of the two isotropic ones used in the foregoing development) to fully describe anisotropic material behavior [13]. The anisotropic relations between stress σ_{kl} and strains ε_{kl} now become in the F. T. plane

$$\bar{\bar{\sigma}}_{kl}(x, \omega) = \sum_{m=1}^{3} \sum_{n=1}^{3} \bar{\bar{B}}_{klmn}(\omega)\, \bar{\bar{\varepsilon}}_{mn}(x, \omega) \qquad (23)$$

where $\bar{\bar{B}}_{klmn}$ are complex moduli. For the sake of economy of length, the anisotropic flutter and dissipation energy analysis and results are not included in this paper, however, the previous isotropic analysis can easily be extended to anisotropic materials by rewriting Eq. (3) as [11]

$$P_m \{F_m\} = \sum_{l=1}^{6} Q_{ml}\{q_{ml}\} \quad (24)$$

and subsequently redefining each $\bar{\bar{E}}_m, \bar{\bar{D}}_{mnk}$, etc, as $\bar{\bar{E}}_{ml}, \bar{\bar{D}}_{mnkl}$ in the relations following Eqs. (3) with l ranging from 1 to 6. This serves to expand the discussion from each isotropic $\bar{\bar{E}}_m$ to six anisotropic $\bar{\bar{E}}_{ml}$, but does not change any of the fundamental principles and interactions considered above.

Extensive damping properties of real materials may be found in References 14 and 15.

CONCLUSIONS

It is shown that for any general motion there is no relation between elastic structural damping and Newtonian viscous damping except at the natural frequencies of the system. The viscoelastic complex moduli are rederived to include structural damping.

The results indicate that in the presence of structural damping the real part is unaffected but the imaginary part includes effects due to the structural damping coefficient and the usual real and imaginary parts of the complex modulus. The illustrative examples for supersonic flutter of an aluminum wing indicate that an increase in the structural damping coefficient may increase or decrease the viscoelastic flutter speeds and the elastic flutter speeds are not necessarily higher than the corresponding viscoelastic ones. In other words neither structural nor viscoelastic damping necessarily produce stabilizing effects.

REFERENCES

1. Bisplinghoff, R. L., Ashley, H. and Halfman, R. L., *Aeroelasticity*, Addison-Wesley, N.Y., 1955.

2. Bisplinghof, R. L. and Ashley, H., *Principles of Aeroelasticity*, John Wiley and Sons, N.Y., 1962, p. 381.

3. Fung, Y. C., *An Introduction to the Theory of Aeroelasticity*, John Wiley and Sons, N.Y., 1955, pp. 477-479.

4. Scanlon, R. H., and Rosenbaum, R., *Introduction to the Study of Aircraft Vibration and Flutter*, Macmillan, N.Y., 1951, pp. 85-88, 196, 209-210, 341.

5. Hilton, H. H., "An Introduction to Viscoelastic Analysis," *Engineering Design for Plastics*, Reinhold Publ. Corp., N.Y., 1964, pp. 199-276.

6. Soroka, W. W., "Note on the Relation Between Viscous and Structural Damping Coefficients," *Journal of the Aeronautical Sciences*, Vol. 16, July 1949, pp. 409-410, 448.

7. Pinster, W., "Structural Damping," *Journal of the Aeronautical Sciences*, Vol. 16, Nov. 1949, p. 699.

8. Dahl, P. R., "Solid Friction Damping of Mechanical Vibrations," *AIAA Journal,* Vol. 14, Dec. 1976, pp. 1675-1682.

9. Saravanos, D. A. and Chamis, C. C., "Mechanics of Damping for Fiber Composite Laminates Including Hygrothermal Effects," *AIAA Journal*, Vol. 28, 1990, pp. 1813-1819.

10. Hilton, H. H., "Pitching Instability of Rigid Lifting Surfaces on Viscoelastic Supports in Subsonic and Supersonic Potential Flow," P*roc. of Third Midwestern Conf. on Solid Mechanics*, 1957, pp. 1-19.

11. Vail, C. F. and Hilton, H. H., "Bending-Torsion Flutter of Linear Viscoelastic Wings in Two Dimensional Flow," *University of Illinois Technical Report* AAE 76-1, 1976. Submitted for publication to *AIAA Journal.*

12. Dugunji, J., "Theoretical Considerations of Panel Flutter at High Supersonic Mach Numbers," *AIAA Journal*, Vol. 4, 1966, pp. 1257-1266.

13. Hilton, H. H., and Dong, S. B., "An Analysis for Anisotropic, Nonhomogeneous, Linear Viscoelasticity Including Thermal Stresses," *Development in Mechanics*, Pergamon Press, N.Y., 1964, pp. 58-73.

14 Lazan, B. J., *Damping of Materials and Members in Structural Mechanics,* Pergamon Press, N.Y., 1968.

15. Nashif, A. D., Jones, D. I. G. and Henderson, J. P., *Vibration Damping*, John Wiley & Sons, N.Y., 1985.