WADC TECHNICAL REPORT 57-96
ASTIA DOCUMENT No. 151114

A SUMMARY OF COMPRESSIVE-CREEP CHARACTERISTICS OF METAL COLUMNS AT ELEVATED TEMPERATURES

R. L. CARLSON
G. K. MANNING

BATTELLE MEMORIAL INSTITUTE

APRIL 1958

MATERIALS LABORATORY
CONTRACT NO. AF33(616)-3317
PROJECT NO. 7360

WRIGHT AIR DEVELOPMENT CENTER
AIR RESEARCH AND DEVELOPMENT COMMAND
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

Carpenter Litho & Prtg. Co., Springfield, O.
600 — May 1958

FOREWORD

This report was prepared by Battelle Memorial Institute, Columbus, Ohio, under Contract No. AF 33(616)-3317. The investigation summarized was initiated under Contract No. AF 33(038)-9542, Project No. 7360, "Materials Analysis and Evaluation Techniques", Task No. 73605, "Design Data for Metals". The work was administered under the direction of the Materials Laboratory, Directorate of Laboratories, Wright Air Development Center, with Mr. E. L. Horne acting as project engineer.

This report summarizes the work performed during the period February 1, 1950, to December 1, 1955.

The authors wish to acknowledge the guidance of Mr. L. R. Jackson and Mr. C. B. Voldrich, who served in the capacity of Technical Directors.

ABSTRACT

A summary of an extensive study of the creep buckling of metal columns is presented. The column behavior prior to collapse is described and the column action at the time of buckling is interpreted in terms of stability.

Solutions to creep buckling are discussed. It is concluded that in spite of certain limitations, Shanley's time-dependent tangent-modulus method has several practical advantages over available analytical solutions.

An application of the time-dependent tangent-modulus method to four structural metals indicates that estimates are consistently conservative for small values of column imperfection. Imperfection variations were generally observed to have a very marked effect on the column lifetime.

The possible existence of a lower column-load limit below which time-dependent collapse will not occur is discussed. From a rational consideration of known creep behavior it is concluded that there may be a temperature below which finite lower limits exist and above which the lower limit is zero.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER:

Richard R. Kennedy

RICHARD R. KENNEDY
Chief, Metals Branch
Materials Laboratory

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS
(Continued)

A SUMMARY OF COMPRESSIVE-CREEP PROPERTIES OF METAL COLUMNS AT ELEVATED TEMPERATURES

INTRODUCTION

The creep of engineering materials has been recognized as an important problem in design for many years. In instances where such considerations have been necessary, methods have been evolved to cope with the problems associated with such behavior. For the most part, the methods are specialized, and they are formulated to accommodate the particular requirements of the design situation at hand. A notable example of an area of design in which an essentially "tailored" philosophy of design has been evolved is that of power-plant equipment. Here, though there certainly are problems that remain to be solved, there is an enormous backlog of knowledge and experience from which to draw. Many "guides" to design are available.

The advent of high-speed aircraft and missiles in recent years has introduced a new area in which elevated-temperature phenomena such as creep* must be accounted for in design. As might be anticipated, many of the requirements in this relatively "new area" differ from those previously encountered in elevated-temperature problems. It is also true that in some instances special problems that are being found are almost unique to the operating conditions being met. It is only natural, then, that the perspective required, or the philosophy of design should differ in this area from that which has evolved in, say, the power-plant equipment area.

One of the special problems that has become the subject of increasing interest in the aircraft industry during the past eight years is the creep buckling of structural elements. The study of this problem, as it manifests itself in columns, has been the subject of Battelle's investigation (Reference 1) under Contract No. AF 33(038)-9542 for approximately five years. During that time, effort has been concentrated on the acquisition of data on several representative structural metals, and on the evolution of methods for predicting creep-buckling behavior by the use of data from standard static (short-time) tests and creep tests.

*Creep, may, of course, occur in some materials at room temperature. Creep in aircraft structural materials, however, is rarely a design consideration at room temperature.

Manuscript released by the authors 13 March 1957 as a WADC Technical Report.

GENERAL DISCUSSION

Statement of the Problem

As the designation would imply, column creep buckling is a stability phenomenon. After a certain period of time, during which the column deflection is increasing steadily due to creep, collapse will occur; i.e., the column will no longer be able to support its load. To illustrate how creep buckling is in essence similar to static or short time column buckling, reference will be made to Figure 1 which is a plot of column average stress versus column deflection (2). The solid curve, starting at the origin, describes the short time (no creep effects) variation of average stress with deflection. For a column load producing an average stress such as σ_1, stability can be examined by an infinitesimal increase in stress, $d\sigma$. If $\frac{d\sigma}{d\delta}$ is greater than zero, the column is stable under σ_1. When the column average stress is increased, the level designated as σ_m will ultimately be reached. Here,

$$\frac{d\sigma}{d\delta} = 0$$

and the column is unstable; i.e., collapse will occur and deflections greater than that corresponding to the maximum average stress can be realized only for average stresses less than σ_m (see the dashed curve).

The stability of a column whose deflection is steadily increasing due to creep (under a constant load) can be examined in essentially the same manner. Initially, the column is loaded to an average stress of σ_1. At time zero, the column is stable in the same sense as previously. If no creep occurred, the stress-deflection point would remain on the solid curve. If creep can occur, however, and the load remains constant, the stress-deflection point will move horizontally and to the right with increasing time. A sequence of times t_1, t_2, ... can be depicted, then, as in Figure 1. To examine the stability of the loaded column after a time t_1 has elapsed, the stability test described earlier can be utilized. An infinitesimal increase in stress, $d\sigma$, can be applied instantaneously*. If, as is indicated at t_1, the value $\frac{d\sigma}{d\delta}$ is greater than zero, the column is stable at the time t_1. Subsequent examinations at t_2, t_3, ... would indicate that as the column deflection has increased with time, the value of $\frac{d\sigma}{d\delta}$ would approach zero. On the verge of collapse, the value of $\frac{d\sigma}{d\delta}$ would be zero as illustrated at t_f.

The physical significance of the collapse phenomenon is that the internal column "fibers" are no longer able to provide an internal moment

*Since creep can occur, the stress increase should not take any time.

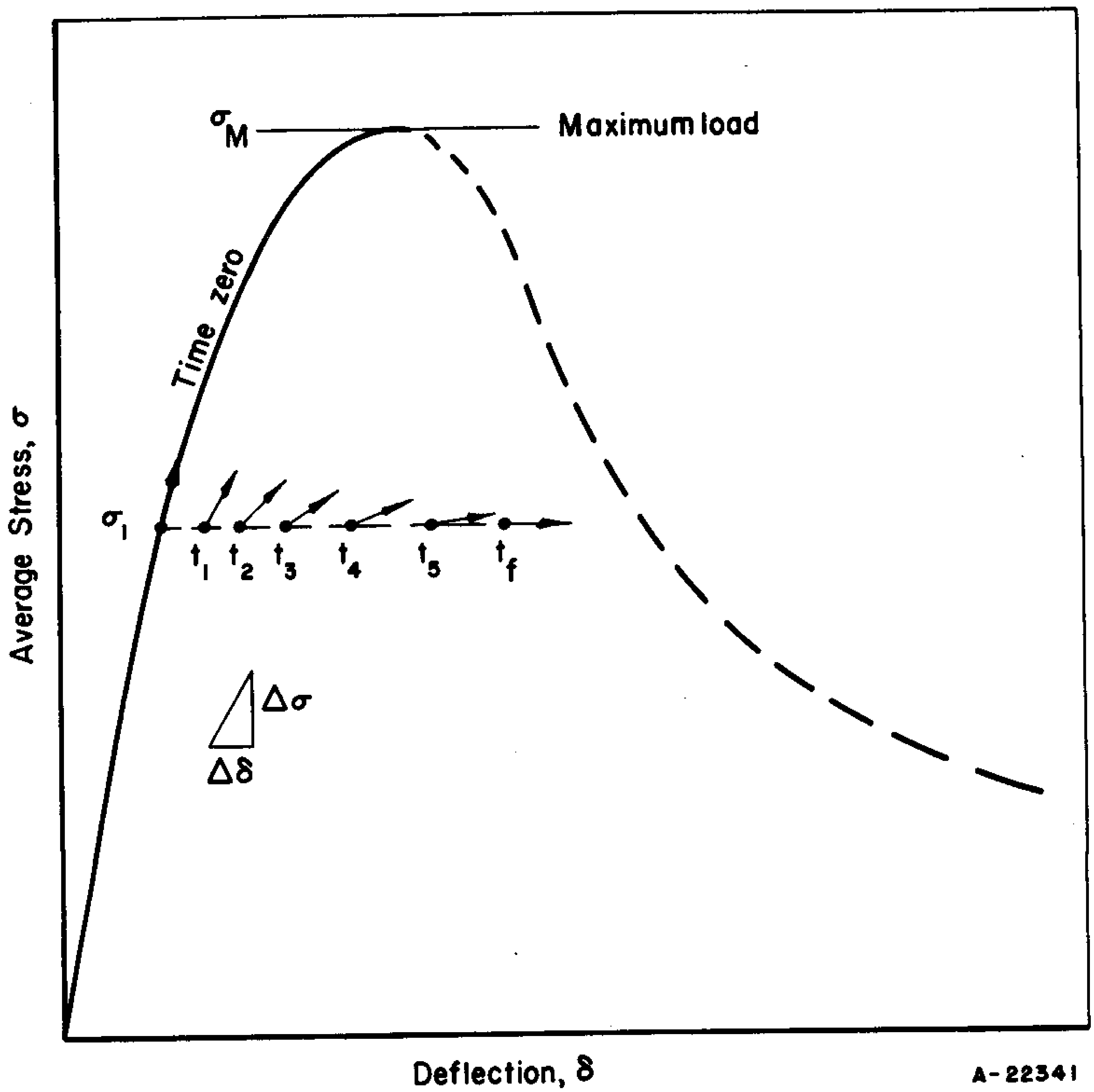


FIGURE 1. COLUMN AVERAGE STRESS VERSUS DEFLECTION

that can resist the external moment, i.e., its capacity to resist the moment is exceeded. In the static or short time case, this limiting moment is reached by an increasing column load and accompanying deflection increases. For the creep buckling case, the moment is increased to a limiting value by deflection increases alone as the external load is constant. In both instances, however, the internal moment capacity is ultimately exhausted.

Although the plot in Figure 1 does illustrate the stability aspect of the creep buckling problem, it does not describe the deflection history in detail. To illustrate the form of the deflection history curves obtained in this study, typical curves shown in Figure 2 have been prepared. As shown in Figure 2(a), the column deflects immediately upon application of the load to the time-zero equilibrium position. During the initial part of the test, the deflection rate decreases with time. After a period during which the rate is essentially constant, the rate begins to increase with time, and failure or collapse ensues.

The exact shape of the curves in terms of deflection and length of the time periods associated with deflection rate (decreasing, constant, increasing) vary with slenderness ratio, imperfection, material, and temperature. Each of the features described, however, has been present to some degree for all of the tests conducted in this investigation.

The fact that the deflection rate decreases with time during the early part of the deflection history may seem difficult to accept when it is remembered that bending moment increases continuously with time. It must be remembered, however, that metals have the capacity to harden as they creep. This, in fact, is the reason that a "primary stage" of creep, during which the creep rate decreases, is observed.

During the early portion of the column creep test, this "hardening" effect evidently is more than just able to compensate for the increasing bending moment, and the decreasing deflection rate is observed. As the deflection continues, however, the hardening effect becomes less dominant and ultimately, it is completely overshadowed by the increasing bending moment. Another factor which contributes to the ultimate acceleration in deflection rate is the fact that the creep "hardening" effect does not continue, but eventually gives rise to a state described in uniform stress tests as the "secondary stage"*.

The differences in behavior that exist for different average stresses and column imperfections (in these examples, the slenderness ratio is constant) are also illustrated by the curves in Figure 2(a). For the same imperfection, $C = C_1$, the time to a given deflection increases with decreasing stress.

*During the "primary stage" hardness increases, i.e., the time necessary to produce a given amount of creep increases with time. During the "secondary stage" hardness is constant, i.e., the time necessary to produce a given amount of creep is constant with time.

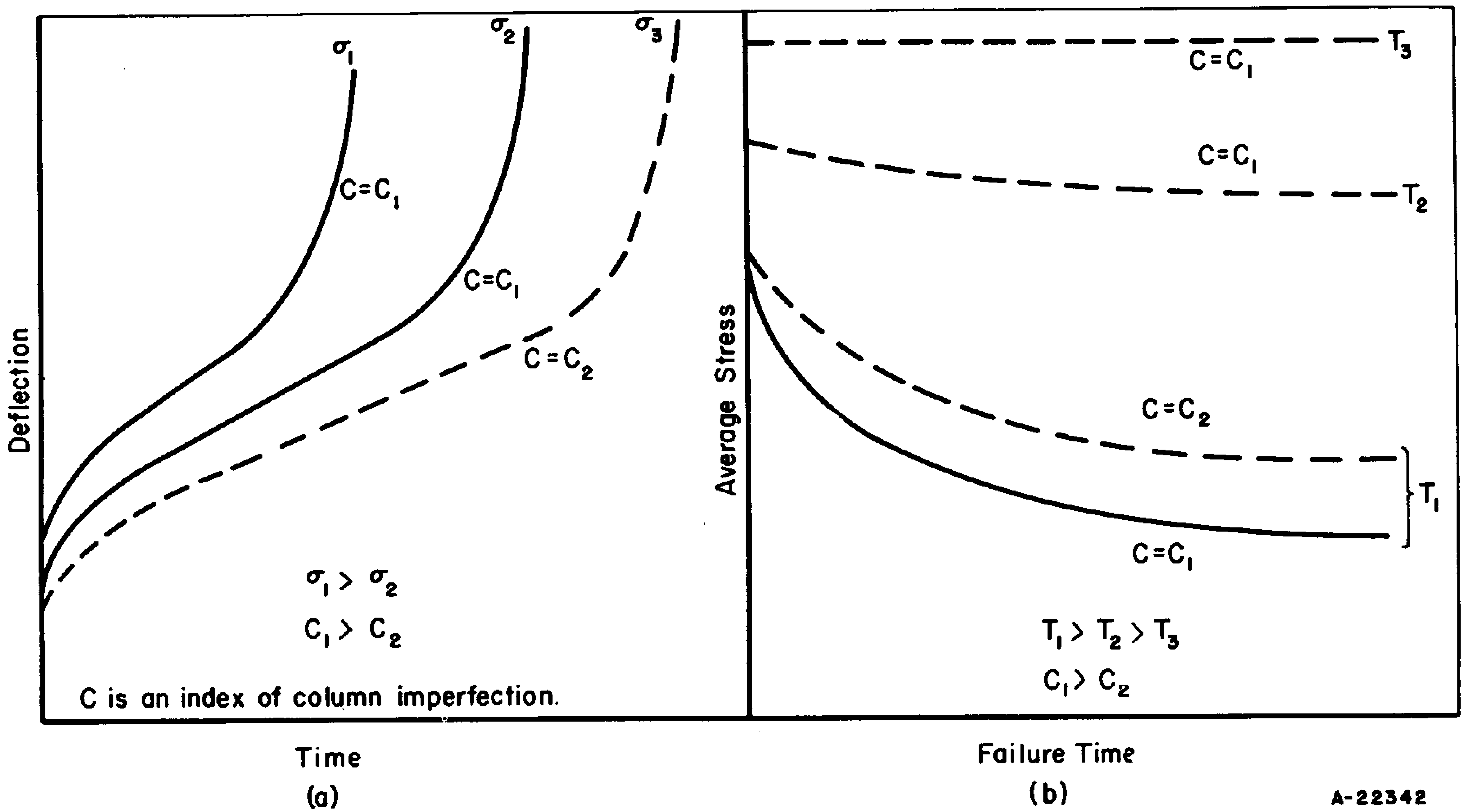


FIGURE 2. TYPICAL PLOTS OF COLUMN CREEP BUCKLING DATA

For a given average stress, the time to a given deflection increases with decreasing imperfection.

If a plot of average stress-versus-failure or collapse time is prepared from test data, the results appear as shown in Figure 2 (b). Figure 2 (b) also illustrated the effect of temperature; as the temperature decreases, the curves become flatter. The curves as shown suggest that for each condition, there may be a limiting stress below which time-dependent collapse will not occur. In terms of design philosophy, the possible existence of such a lower limit could have important implications. For this reason, this point will be discussed in greater detail in another section of the report.

Figure 2 (b) also illustrates, for a given temperature, the effect of imperfection. Both Figures 2 (a) and 2 (b) suggest that the effect of imperfection can be quite significant. This effect will be discussed in more detail in the report and actual data will be presented to support this representation. Referring once again to the evolution of design methods, it should be noted that the effect — in terms of variation of behavior — of imperfection is quite important. If the deflection at a given time or the collapse load for a given time is only slightly effected by changes in imperfection, then including imperfection as an ingredient in a design procedure is not too important. If imperfection does have a pronounced effect on behavior, then methods not accounting for it must be used with caution until the errors involved have been thoroughly evaluated.

Solutions of the Problem

A number of "solutions" to the problem of column creep buckling have been advanced. This is disturbing if one is accustomed to thinking in terms of "exact" solutions such as exist for certain problems in the mathematical theory of elasticity. It is true, of course, that approximate solutions are obtained in elasticity. In such instances, however, they are of little value unless their relationship to the "exact" solution is known; i.e., is the approximate solution conservative or nonconservative?

The success of the theory of elasticity in providing solutions to engineering problems is based on the fact that the idealized model used in the mathematical analysis can be made to agree with the actual structure or machine. The major model requirements that must be satisfied are:

(1) It must have the same form or shape.

(2) The boundary conditions (surface loads) must be the same.

(3) The response relationship between load and deformation must be known*.

The requirements for the solution of problems involving creep are, of course, essentially the same. The difficulty that is encountered in the solution of these problems can be traced to the difficulty in satisfying the third requirement with a relationship that is simple enough to permit solutions.

The normal course followed in attempting to solve problems that do not, upon a first evaluation, lend themselves to solution is to simplify the model used to represent them. In analyzing the plastic buckling of short columns, for example, Shanley(3) made use of an idealized, two flange column model that permitted an elementary analysis. In this instance, the structural element was simplified to permit an examination of the behavior of interest (this corresponds to Item 1 above).

Another example in which a simplification has been introduced to permit solutions is in the theory of plasticity. Here, the "perfectly plastic" solid has been introduced; i.e., a material element is elastic in accordance with Hooke's Law until a certain critical flow condition is satisfied; thereafter it can flow at a constant intensity of loading**. In these instances, the response relationship has been simplified (this corresponds to Item 3 above).

Further simplifications can be wrought by manipulation of the condition under Item 2 (replacement of complex surface loadings by "equivalent", simpler loading that permits solution at regions sufficiently distant from the applied loading). In this discussion, emphasis will be centered on the types of simplifications possible under Items 1 and 3, however, since these are the ones that have been utilized in the column creep buckling problem.

It is obvious, of course, that when a solution is obtained for an idealized or simplified model, the solution is exact only for the model as prescribed. How well the solution may "fit" a real problem is another matter. If the material response relationship (Item 3) is a poor approximation, the solution will very likely be a crude approximation. If the model (Item 1) utilized is an oversimplification, essential features of true behavior probably will be distorted; it is also conceivable that features unique to the particular model may be emphasized.

A number of solutions, based on the types of simplifications described above, have been made. One approach, used by Freudenthal(5), Hilton(6), Rosenthal and Baer(7), and Kempner(8) makes use of a material response

* The cases described here as being readily amendable to solution are those for which the relationship is linear; i.e., they obey Hooke's Law.

** Still further simplification has been introduced in some solutions by the use of a "plastic-rigid" material (see Reference 4).

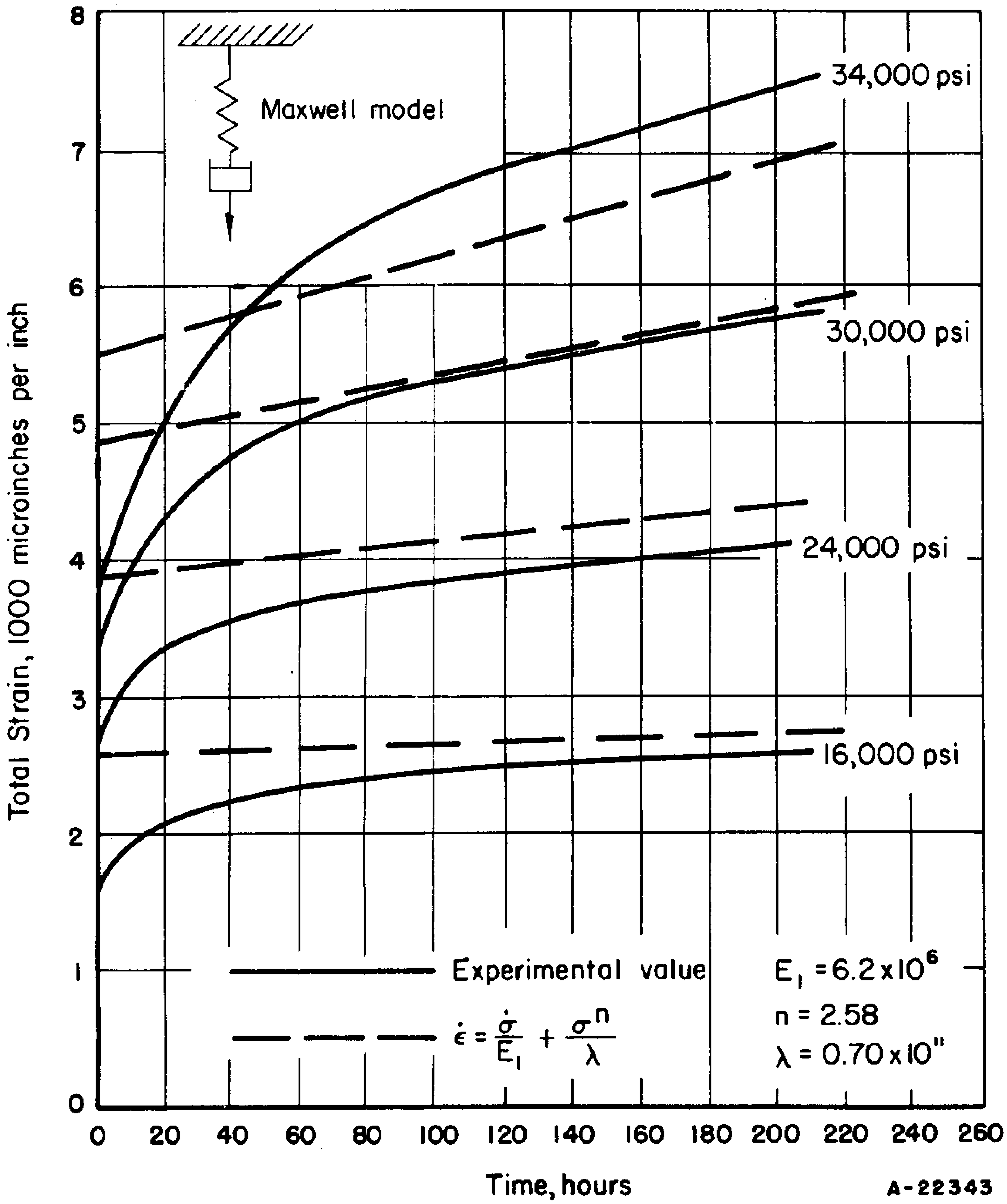


FIGURE 3. CORRELATION OF TENSILE CREEP DATA FOR 2024-T4 ALUMINUM ALLOY AT 350 F BY THE USE OF A NON-LINEARLY VISCO-ELASTIC RESPONSE MODEL

described by viscoelastic elements. The main advantage of this approach is that it results in mathematically solvable equations. The principal disadvantage is that many real materials do not obey such laws at temperatures for which materials are used.

As an illustration of the shortcomings or limitations of such response laws, an attempt has been made to "fit" a law of the type used by Kempner(8) to creep data for the aluminum alloy 2024-T4 tested at 350 F(1). The solid curves of Figure 3 represent actual creep data, and the dashed lines represent one of the best "fits" that could be obtained.

Although the creep rates predicted by the law beyond about 100 hours are fairly accurate, the "fit" for times less than 100 hours is poor. Predictions based on such a law would have the advantage of being conservative, but they may well be unacceptably drastic. One may argue that beyond 100 hours, predictions would be good. This argument, however, is not good, because:

(1) One may be interested in capacities short of 100 hours.

(2) In creep buckling, the bending moment increases with time (in contrast to pure bending in which the moment would be constant with time), hence, it is not possible to hope for a "steady-state" condition in which the predicted behavior becomes correct.

Although the law described in Figure 3 has been made more flexible by making the visco-elastic element "nonlinear", it is still not flexible enough to describe actual behavior. It should also be noted that a simple solution using this law was possible only for an idealized column containing a two-flange cross section. This must certainly be considered as another restriction to its general usefulness.

Another important limitation restricting the practical use of such laws is associated with the method of evaluation of the relationships. The constants in the equations would, if one were to actually use them, be derived from constant load or constant stress tests. Strictly speaking, then, the laws can be expected to be valid only for constant load or constant stress behavior. The laws are not generally valid for variable stress behavior. Creep buckling, of course, occurs under continuously changing stresses.

It has been demonstrated a number of times (9, 10, and 11) that not all laws derived from constant stress tests can be valid for variable stress. Roberts(10) and Cottrell(11) indicate, for example, that the functional form of an equation that is possibly valid is

$$\dot{\epsilon}_c = F(\sigma, \epsilon_c) \tag{1}$$

where

$\dot{\epsilon}_c$ is the strain or creep rate,
σ is the stress,

and

$\dot{\epsilon}_c$ is the creep strain.

The corresponding functional form of the relationship of Figure 3 is

$$\dot{\epsilon}_c = G(\sigma).$$

A second type of approach has been suggested that emphasizes the use of real, rather than idealized material-response behavior. Because this can usually be done only by the use of complex empirical relations, simple mathematical solutions are possible only for a column with an idealized, two-flange cross section. For solid cross sections, these methods require lengthy step-by-step-type computations.

Several methods of analyzing creep buckling according to this second type of approach have been suggested. Two of these methods, by Higgins[12], and by Libove[13], utilize a deformation law suggested by Shanley[14], but the procedures of analyses are different. As noted above, they require lengthy computations for columns with solid cross sections.

It is seen from this summary that the problem of column creep buckling is so complex that in formulating solutions, one is forced to simplify and idealize to obtain straightforward solutions.

If one wants a solution that can be used as a quantitative guide to a real design problem, a question immediately arises with regard to how much simplification can be afforded.

If one only wants to describe the mechanics of the process or the "column action", there is a tendency to accept a greater degree of simplification. We should not however, fail to realize that if we adopt this attitude, we risk the possibility of either obscuring "real" features of the process of interest or introducing artificial properties that are unique to the simplified model*.

*These conclusions are generally valid, but they are particularly timely with respect to problems involving creep.

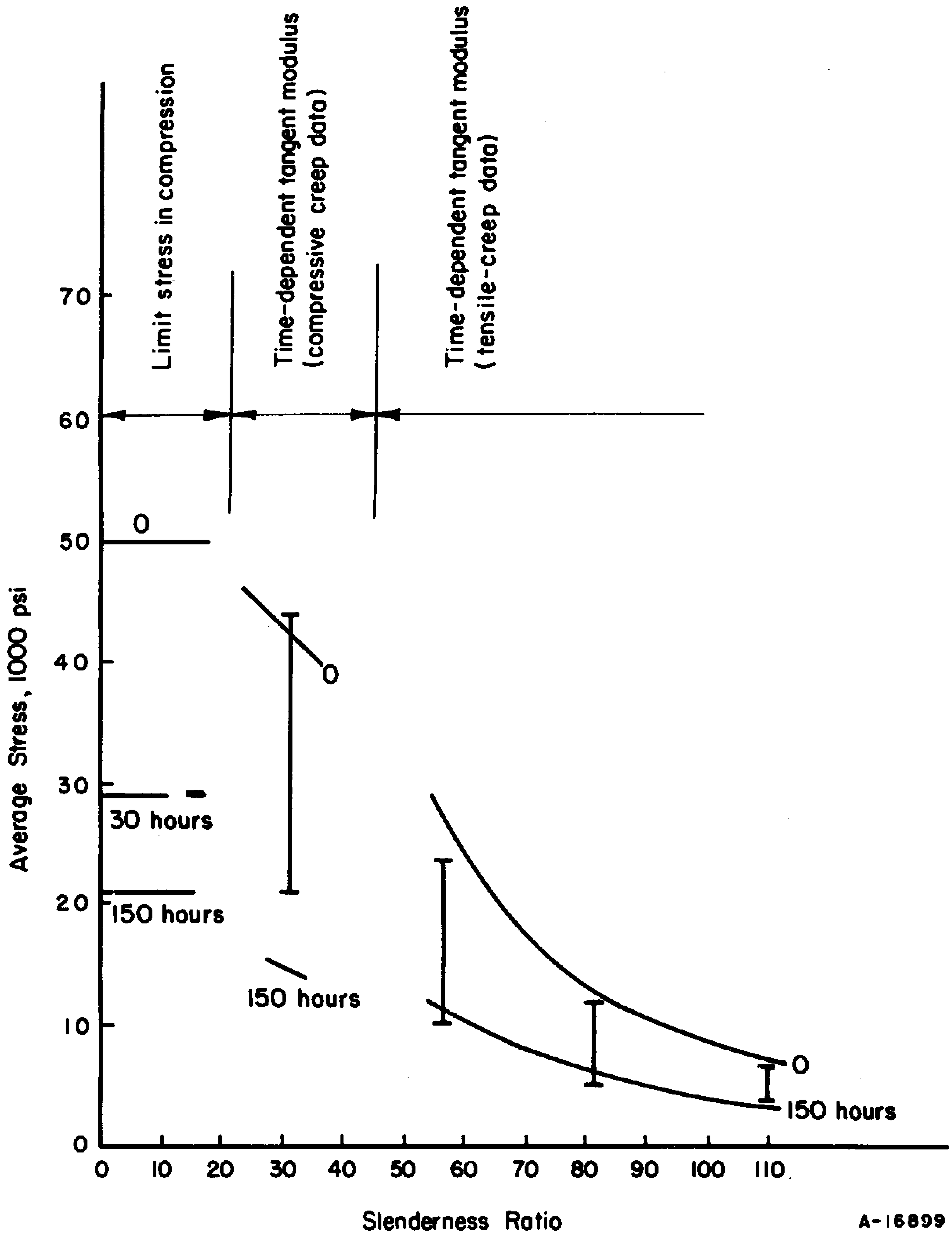


FIGURE 20. AVERAGE STRESS VERSUS SLENDERNESS-RATIO DATA FOR 2024-T4 ALUMINUM ALLOY COLUMNS AT 450 F

The tests conducted for each test material and at each test temperature were as follows:

(1) Static compression tests were performed to obtain stress-strain curves. In some instances tensile tests were also performed.

(2) Tensile and compressive creep tests were conducted to obtain strain-time curves. Usually the greater effort was devoted to tensile creep testing. Limited compressive creep testing was conducted to obtain comparisons of the tensile and compressive creep resistance.

(3) Creep buckling tests were conducted on columns of various slenderness ratios and imperfections.

The tests performed under Items 1 and 2 above are standard tests, and since the details of the test conditions have been presented, they will not be discussed further at this point. It should not, however, be inferred from this method of presentation that these data are of secondary importance. Actually, they are essential to an analysis of the column creep buckling data.

Since the tests performed under Item 3 are not standard tests, they will be described at this time to clarify the discussions to follow.

All column tests were conducted on lever-type loading stands of the kind shown in Figure 4. Load applied at the left end of the overhead lever was transmitted through a plunger to the column specimen. Hardened steel knife edges were fixed to the ends of the column specimen to provide "hinged-end" support.

The test procedure used was as follows:

(1) Trial loading at room temperature was conducted to obtain column load-deflection data that could be used to obtain an estimate of the column imperfection. A Southwell Plot[2] was used to obtain this estimate.

(2) The column specimen was enclosed in the hinged, split-furnace and heated to the test temperature.

(3) After obtaining a zero-load reading, load was applied in increments in from 2 to 3 minutes. Loading was stopped short of the zero time buckling load, and the zero-time reading was noted. The subsequent deflection history was then recorded continuously. Curves of the type shown in Figure 2a were obtained from these records.

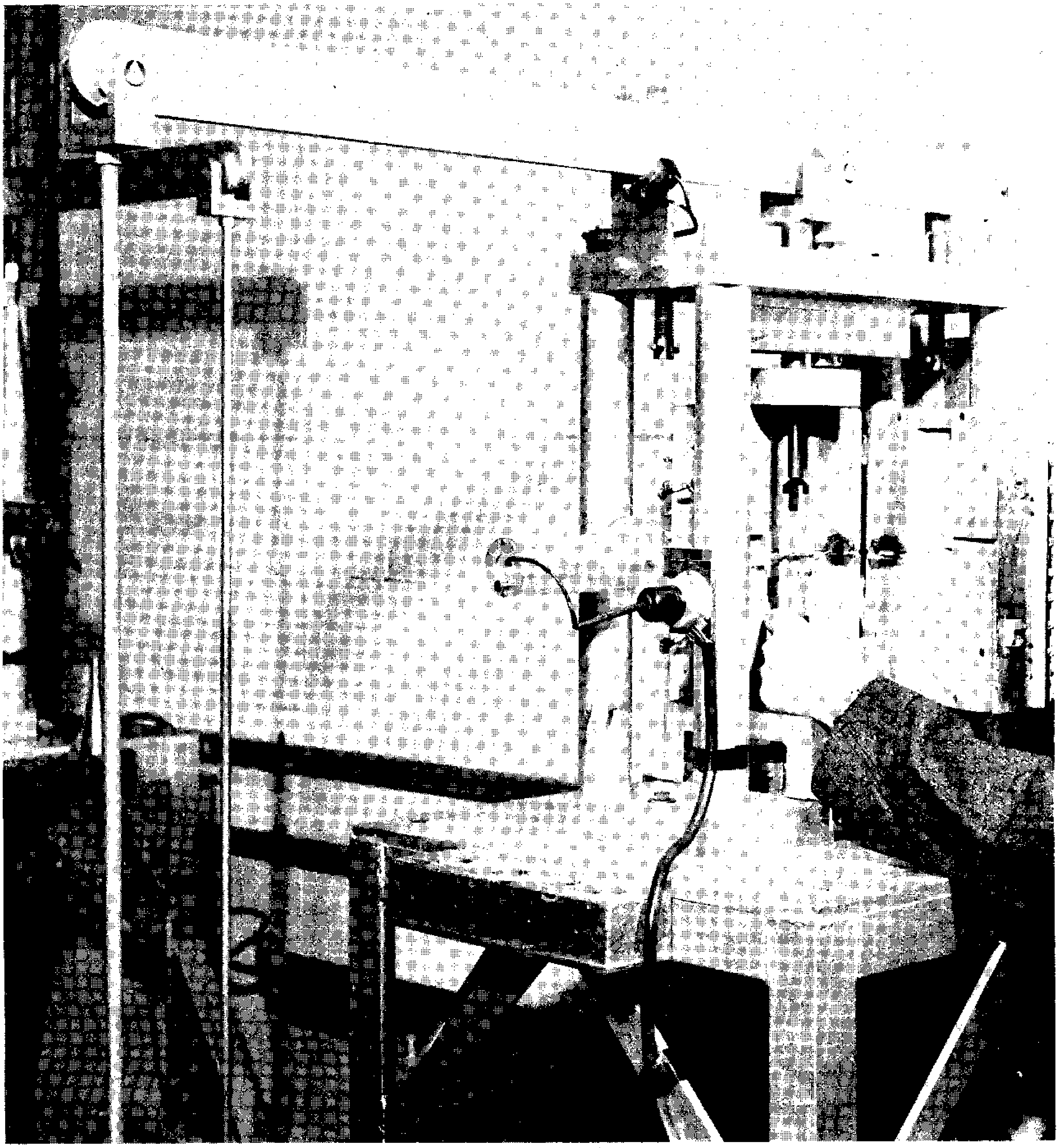


FIGURE 4. COLUMN LOADING STAND

The above testing procedure was followed in all of the experimental work conducted. In some instances, additional steps or special precautions were necessary.

Some data were obtained, for example, for which the imperfection was adjusted, prior to testing, to a prescribed value. This was accomplished by the use of shim adjustments that produced the imperfection (eccentricity of loading) desired.

Special precautions had to be observed during testing whenever the test material was metallurgically unstable at the test temperature. In these instances, the same heating period prior to testing was always repeated. With this procedure, some change of properties occurred prior to testing, but the "state" at the initiation of each test was the same.

During the experimental studies conducted as a part of the investigation summarized, the following materials were used:

(1) Aluminum alloy 2024* stabilized at 600 F for 100 hours prior to testing.

(2) Aluminum alloy 2024-T4.

(3) Titanium alloy C-110M**.

(4) Stainless steel Type 302.

(5) Stainless steel Type 17-7PH (TH1050).

(6) Aluminum alloy 2024-T3 in the form of square tubing.

With the exception of the sixth material, all column specimens had solid, rectangular cross sections.

THE SIGNIFICANCE OF THE TIME-DEPENDENT TANGENT-MODULUS AS APPLIED TO COLUMN CREEP BUCKLING

During this investigation, the relation between approximate stress distributions derived from isochronous stress-strain curves and the actual stress distributions for columns subject to creep was studied. Since some of the ramifications of this approximate approach are not apparent, it is felt that a detailed discussion is appropriate. Before beginning, however, a brief review of the basis for the method will be given. In this review,

*Formerly designated as 24S.

**Formerly designated as RC-130A.

reference to Figure 5 will be helpful. The t = 0 line is the elastic stress-strain curve, and the t_i curve is the iso-stress-strain curve for the time t_i. That is, for example, the amount of creep occurring during the time t_i at the stress σ_D is given by D-G.

In discussing column stress distributions, it will be convenient to consider one cross section of a column of given dimensions and imperfection. The cases considered are only those for which the load has produced an initial stressing which is elastic throughout.

If a load is applied to the given column, the initial stress distribution can be illustrated by the use of the compression stress-strain diagram of column material. In Figure 5, the ordinate B-A represents the stress on the convex side of the column, and D-C represents the stress on the concave side. The ordinates along B-D represent intermediate, initial-stress values.

As can be seen, the stresses are sufficiently large to cause creep. The nonuniform stress distribution will, moreover, cause nonuniform creep, which in turn, will result in an increase in the column deflection. To maintain an internal and external-moment balance, the stresses must readjust. To satisfy the requirement for a larger internal moment, the stress σ_D will increase, and, since the external load remains constant, the stress σ_B will decrease. If, after a finite time t_i, σ_D increases to σ_D', and σ_B decreases to σ_B', the stress-strain points will move to D' and B', respectively. This can be shown to be true by using extreme paths of change. For σ_D, for example, these are D-G-E and D-H-F.

Since D' and B' represent the end points of the stress distribution for the time t_i, a curve connecting them, such as shown dashed, describes the actual stress distribution at t_i.

As can be seen, the stress-distribution curve is more steeply inclined than the t_i curve in the region of the average stress σ_A. If, now, a segment of the t_i curve was to be used as an approximation* to the real stress distribution B'-D', it would appear as B''-D'' in the inset of Figure 5. As can be seen, the use of the approximate stress distribution would indicate that the column was less stiff than it actually is. This property of the iso-stress-strain curves then forms the basis for a means of estimating allowable-column-load capacities.

Before proceeding, it should be emphasized that, whereas the use of the t_i segment will yield conservative estimates, the use of the tangent to the t_i curve will not necessarily be conservative, as anticipated by Shanley(14).

*Force and moment balance requirements must, of course, be satisfied, and these would determine the end points of the approximate stress distributions.

To show this, an example advanced by Rosenthal(17) will be used. In Figure 6 the diagram of Figure 5 is repeated, except that the average column stress σ_A is now below the lowest stress for which creep can occur*. As can be seen, the tangent at σ_A remains unchanged, in spite of the fact that the column can creep under the stress σ_D on the concave face. As also can be seen, the t_i segment is, however, still less steep than the real stress distribution B'-D', and it can, hence, be used as a basis for a conservative estimate.

It should be noted that, in this example, it is probably necessary that the initial imperfection be sufficiently large for collapse to occur eventually. It may be that, for column imperfections normally encountered, the average stress of the example may be below values for which eventual collapse can occur**. Nevertheless, the example serves to indicate that the role of the time-dependent tangent-modulus load derived from iso-stress-strain curves should be more clearly interpreted. An attempt will be made in this section to clarify this point.

Before discussing the relation of the time-dependent tangent-modulus load to actual creep-buckling loads, however, a review of the method for estimating these load capacities by the use of iso-stress-strain-curve segments will be given, because it is pertinent to the discussion.

Earlier in this discussion, it was indicated that such segments could be used to obtain a conservative description of the creep behavior of columns. To illustrate how this actually may be accomplished, an answer to the following question will be given.

For a column of a given slenderness ratio $L/r = R_1$ and initial imperfection $c = c_1$, what is the maximum-load-capacity estimate that can be obtained for the time $t = t_1$ by using t_1 iso-stress-strain curve segments as approximate stress distributions*** ?

A procedure which can be followed to obtain the foregoing value can be illustrated by references to Figure 7 in the following discussion. Only the essential features will be considered here, as details of actual computations of this type can be found in references 2 and 18.

The basis for the computation is derived from a consideration of the condition for equlibrium; that is, force and moment balance requirements must be satisfied. These can be summarized for a given cross section by

*It may be argued that no such lower limit exists, and that creep will always occur for a finite stress. Even if this were true, however, cases in which the creep was not sufficient to modify the tangent significantly would certainly be found.

**Experimental evidence indicates that there may be, for each column slenderness ratio and imperfection, a lower load limit below which collapse will not occur. This possibility needs to be examined more thoroughly, however, and it is discussed in a later section of this report.

***For reasons noted earlier, the actual column will be more "stiff" than indicated by the approximation for a given average stress.

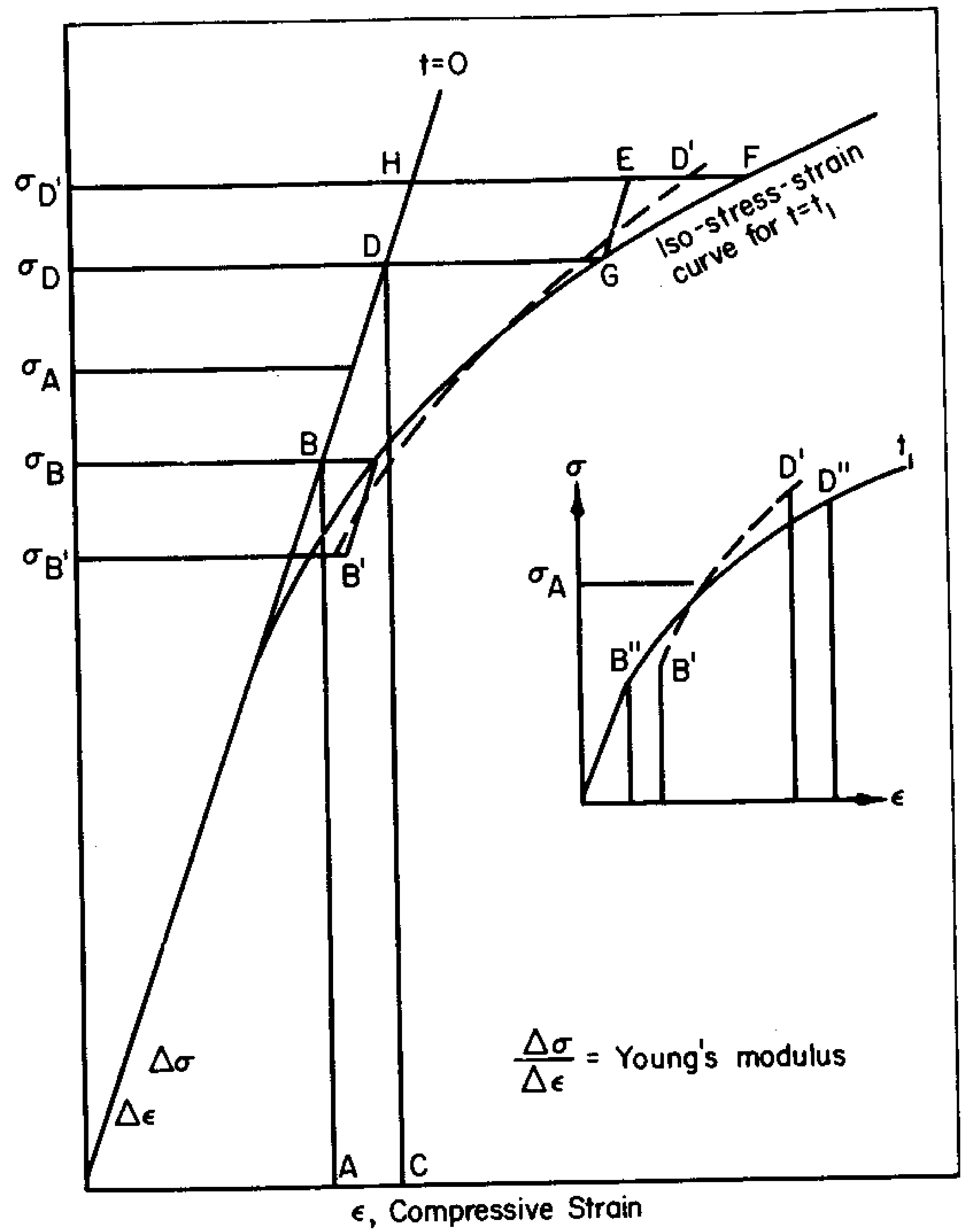


FIGURE 5. GRAPHIC REPRESENTATION OF STRESS DISTRIBUTION ACTING ON A COLUMN CROSS SECTION

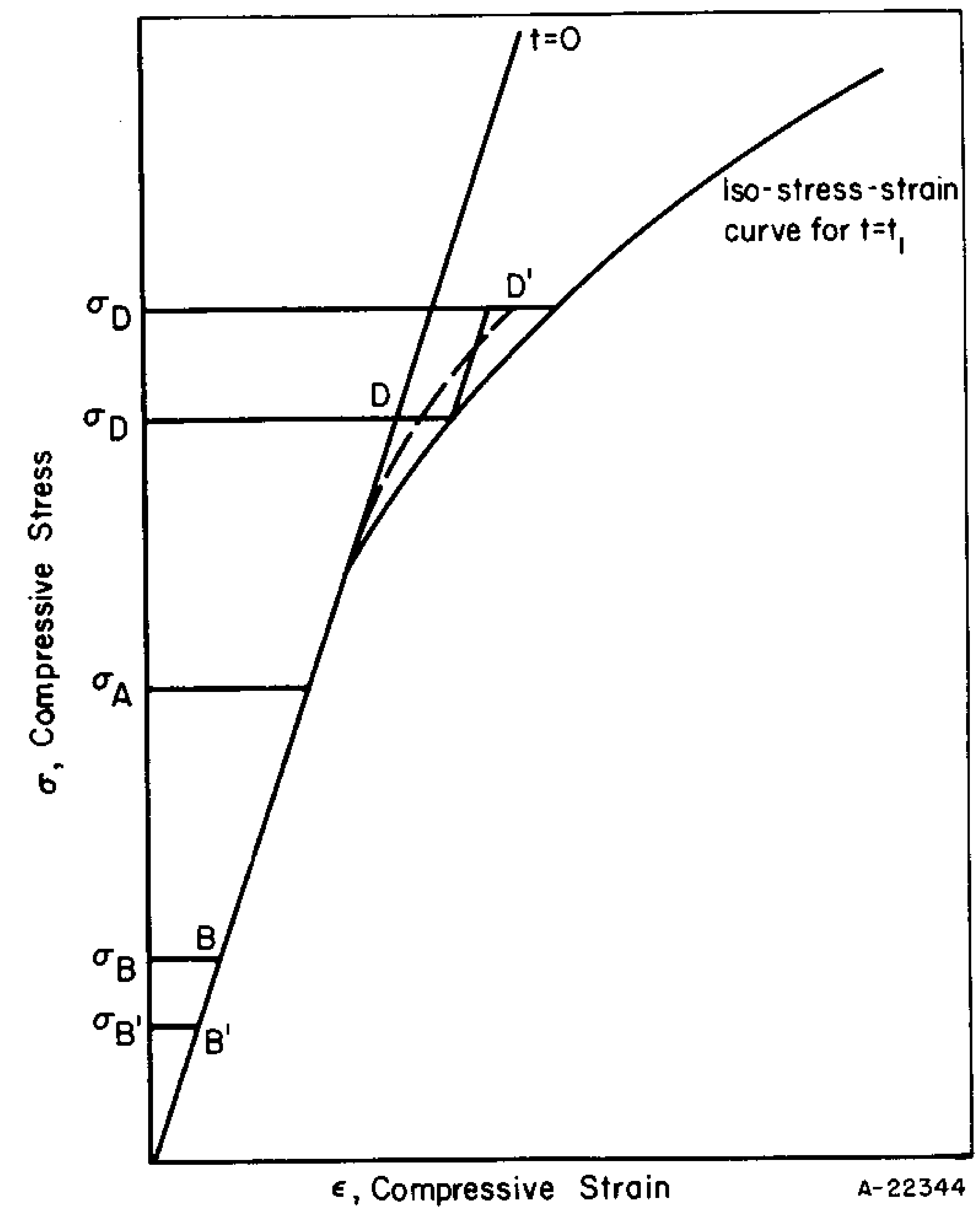


FIGURE 6. GRAPHIC REPRESENTATION OF STRESS DISTRIBUTION

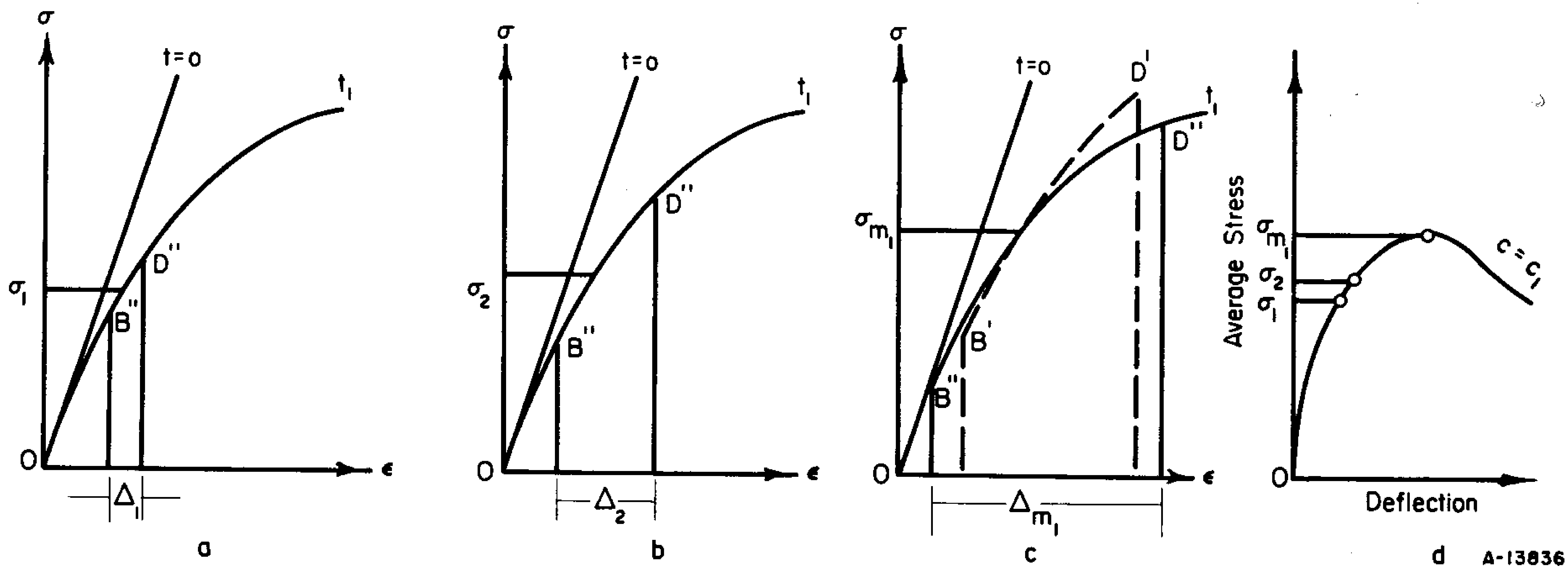


FIGURE 7. GRAPHIC REPRESENTATION OF PROCEDURE FOR COMPUTING APPROXIMATE STRESS DISTRIBUTIONS FROM THE t_1 ISO-STRESS-STRAIN CURVE

the equation

$$y = \frac{M}{P} \qquad (2)$$

where y = total deflection resulting from initial imperfection and bending due to the column load, M = bending moment, P = column load.

The values of M and P can be shown(2, 18) to be given by integrals associated with the stress distribution, as illustrated in Figure 5 by the curves B-D, B'-D', and B"-D"; M is the internal moment and P is the internal force associated with each distribution.

To find the maximum average stress for the given column from the t_i curve of Figure 7, one can take a sequence of trial values $\sigma_1, \sigma_2, \ldots \sigma_n$ and satisfy Equation (2) for each value. In Figure 7a, for example, B"-D" illustrates a segment which satisfied Equation (2). The location of B" and D" can be determined by evaluating Equation (2) for trial positions. Only correct locations will balance the equation.

Insets a, b, and c illustrate the process for successive values of average stress. In Figure 7d the sequence of average stresses has been plotted as a function of the deflection due to the load*. As can be seen, the trial values of average stress rise to a maximum and then fall**. For the time t_i, then σ_{m1} represents, for the given column, the highest average stress that can be obtained using a segment of the t_i iso-stress-strain curve as an approximate stress distribution. For purposes of comparison, the actual stress distribution, B'-D', at the time t_1 for the average stress σ_{m1} is shown dashed in Figure 7c.

The procedure just outlined can now be seen to have been merely a device by means of which it was possible to obtain the maximum allowable estimate of the load capacity of the given column for the time t_1.

To determine the relationship between these conservative or allowable-load-capacity estimates and the so-called time-dependent tangent-modulus load, it is useful to refer to a diagram similar to that of Figure 7d. For this purpose, Figure 8 has been constructed. In addition, now, to the estimate σ_{m1} for an imperfection of c_1, and estimate σ_{m2} for an imperfection of c_2 has been included. As indicated, c_2 is less than c_1. Also, an estimate σ_{m3} for an imperfection of c = 0 is included. The value of σ_{m3} corresponds to the time-dependent tangent-modulus load, as derived from the iso-stress-strain curve for the time t_1.

It is thus seen that the time-dependent tangent-modulus method can be interpreted as a limiting case of the allowable-load-capacity method. As the initial imperfection approaches zero, the load-capacity estimate

* The deflection is a function of the strain difference(2, 18).

** This characteristic behavior is to be expected, because the computation procedure followed is simply that used to compute maximum column loads(2, 18).

approaches the time-dependent tangent-modulus load. This follows, because, as B and D (see Figure 5) approach one another, B'' and D'' become closer, and a tangent to the t_1 curve at σ_A approaches the curve segment at that point.

To illustrate more clearly the relationship between the load-capacity approximations and the actual load capacity, the graph of Figure 9 has been prepared. The graph shown is a plot of column average stress as a function of failure time for a given column slenderness ratio. To serve as reference values, the relative positions of the reduced or double-modulus load and the tangent-modulus load have been indicated. As von Karman has indicated(19), these can be considered as the upper and lower limits, respectively, of the critical load for an initially straight, axially load column.

The result of using tangents to iso-stress-strain curves to derive approximate load-capacity estimates is illustrated by the indicated curve. Since the time-zero iso-stress-strain curve coincides with the actual stress-strain curve, the curve intersects the average-stress axis at the value of the tangent-modulus load.

Consider, now, an imperfect column whose immediate-failure time load is, as indicated, below the tangent-modulus load*. Since the time-failure curve corresponding to such a column starts below the tangent modulus load, it seems reasonable to expect that, for a sufficiently large initial imperfection it will remain below the time-dependent tangent-modulus time-failure curve throughout.

The lowest curve in the graph represents the time-failure curve obtained by using curve segments of the iso-stress-strain curve as approximate stress distributions. These estimates are, as indicated earlier, conservative for the given column. It should be noted that this curve intersects the actual time-failure curve on the average-stress axis. This again follows from the fact that zero-time iso-stress-strain curve coincides with the actual stress-strain curve.

The relationship between the two approximate methods of obtaining load-capacity estimates (the one using tangents and the other using curve segments) is made clear by Figure 9. It also can be seen that the curves may lie fairly close to one another. The spacing will be established, of course, by the amount of imperfection present. In some instances, it is probable that the top two curves will intersect at some value of time. If this occurred, the time-dependent tangent-modulus load would be nonconservative up to that time, and conservative afterward.

*For imperfect "inelastic" columns, the immediate-failure load or maximum load can be greater than the tangent-modulus load. As the column slenderness ratio increases, however, the maximum loads tend to become less than the tangent modulus loads. The columns studied in this investigation were all within the "elastic" column range (large slenderness ratios), so the immediate-failure or maximum load can justifiably be picked to be less than the tangent modulus load.

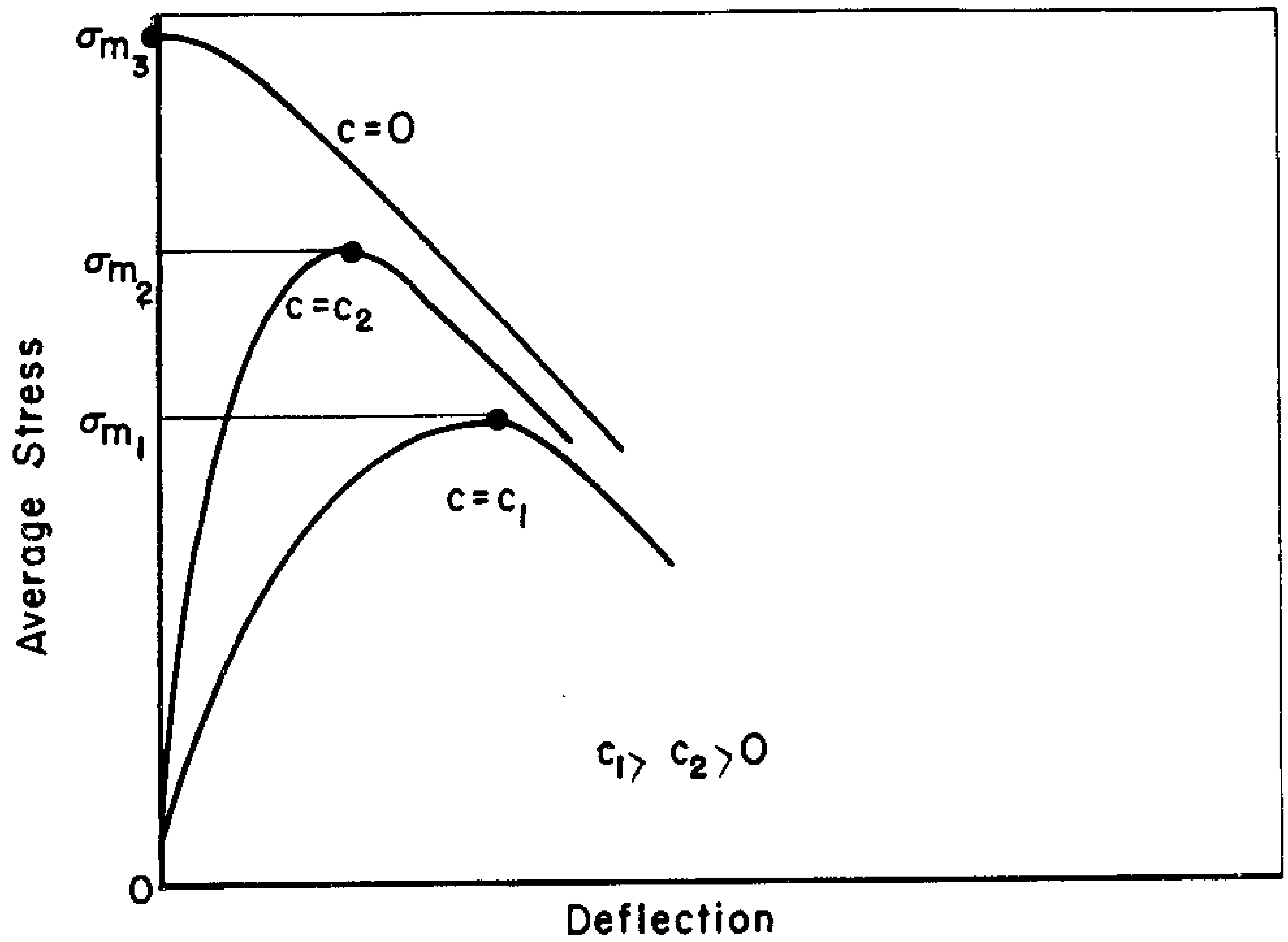


FIGURE 8. VARIATION OF MAXIMUM ALLOWABLE AVERAGE STRESS WITH COLUMN IMPERFECTION FOR TIME t_1

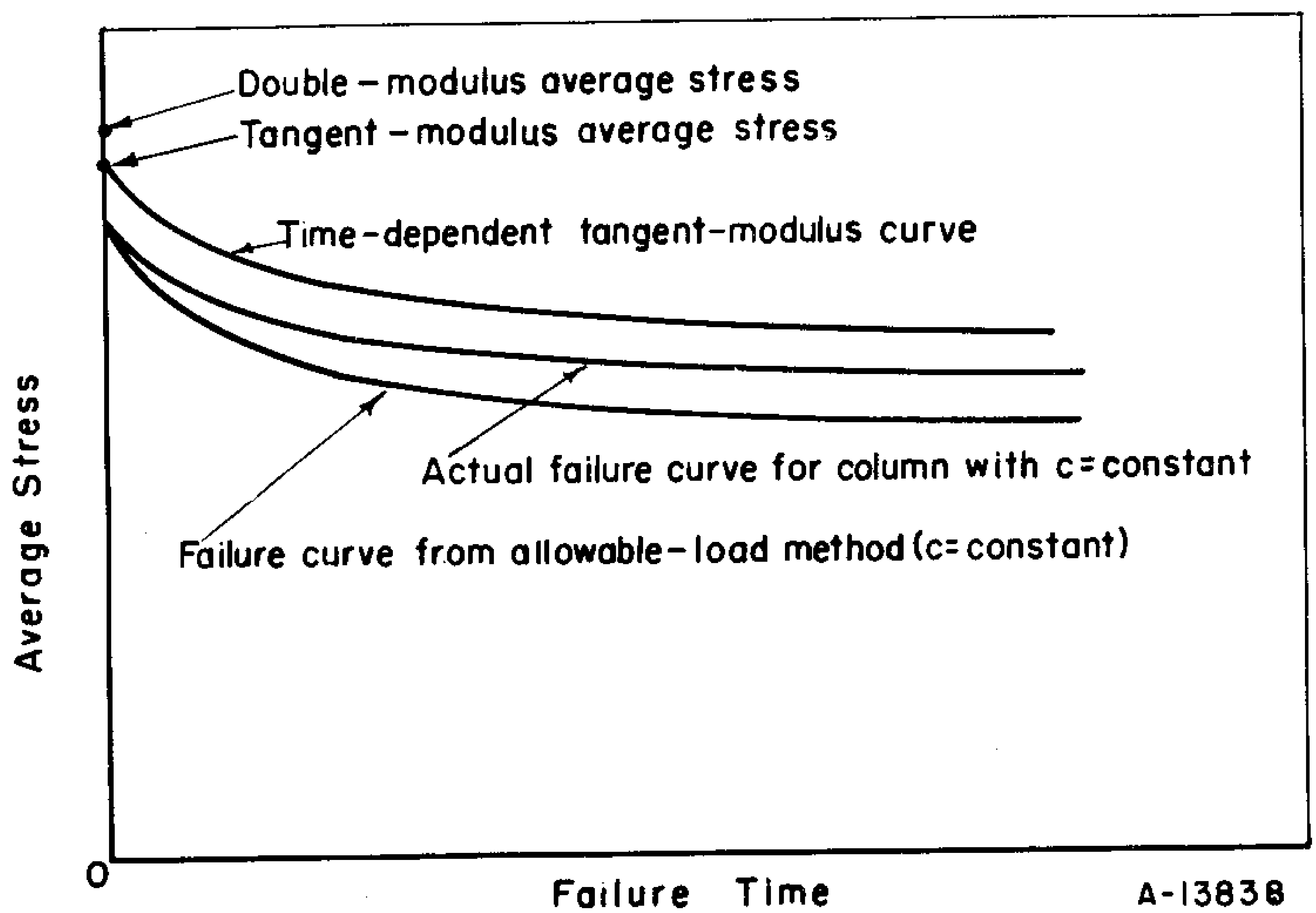


FIGURE 9. RELATIVE POSITIONS OF ACTUAL AND APPROXIMATE COLUMN-FAILURE CURVES

In this discussion, the relation of the time-dependent tangent-modulus load — as conceived by Shanley — to actual column capacity has been clarified. It may be interpreted as a limiting case of the allowable-load capacity which has been shown to be a conservative estimate. The time-dependent tangent-modulus load is therefore, an approximation to a conservative estimate. The approximation, however, may be either conservative or nonconservative.

Because the time-dependent tangent-modulus load is so easily computed, it offers promise as a means for obtaining engineering estimates of column-load capacity in spite of the fact that it may sometimes be nonconservative. It should be noted, moreover, that only a method that can account for column imperfection will be free of the possibility of nonconservatism. A method free from this possibility, however, will undoubtedly be relatively complex, so that its advantage will probably be at least partially offset.

APPLICATION OF THE TIME-DEPENDENT TANGENT-MODULUS

In the previous section, the use of the time-dependent tangent-modulus was discussed. It was concluded that its use for obtaining capacity estimates for columns subject to creep buckling can be either conservative or nonconservative. It was also suggested, however, that the method may prove useful in spite of the possibility of nonconservatism.

To clarify the manner in which the estimate varies from actual column creep buckling capacities, all of the column creep data obtained in this investigation have been plotted in Figure 10. In this graph the error of the estimate* has been plotted as the ordinate, and the imperfection of the test column as the abscissa. Values above the horizontal axis indicate that the estimated capacity was conservative or less than the actual capacity for the given failure time. Values below the horizontal axis were nonconservative. The imperfection, C, has been plotted as a fraction of the column length, L. The value of $C = 10 \times 10^{-4}L$, for example, is often written as:

$$C = \frac{L}{1000} \; .$$

The data indicated by the first six symbols in the identification list represent individual test values. The last two symbols, for 2024-T4, indicate the variation of data for a number of tests. As indicated, the

*The error was based on the relation $\dfrac{\text{Estimated Capacity} - \text{Actual Capacity}}{\text{Actual Capacity}} \times 100.$

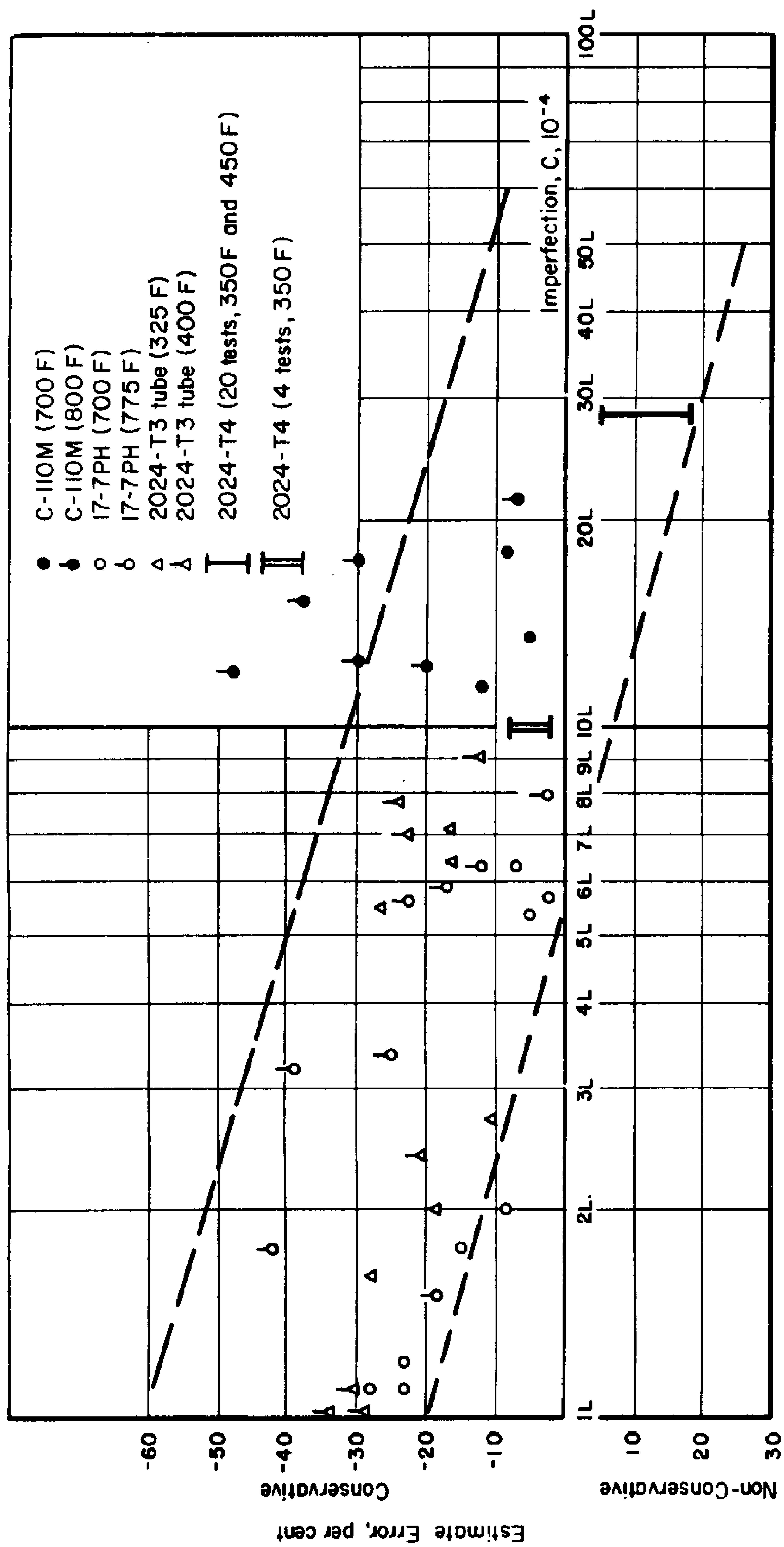


FIGURE 10. ERROR OF ESTIMATE VERSUS IMPERFECTION FOR ACCUMULATED DATA

imperfection was adjusted to be constant within the last two groups of tests.

The inclined dashed lines have been included to suggest the probable trend of the data. Most of the data, which included results for different materials, test temperatures, and slenderness ratios, fall within the dashed lines. As would be expected from the discussion of the previous section, the estimates are predominantly conservative for small imperfections, and tend toward nonconservatism as the imperfection increases. The data for 2024-T4 are of particular interest as they illustrate the transition from conservative to nonconservative.

Although the scatter is significant, it should be expected. In static column behavior, for example, it is known that mechanical properties influence the column capacity*. The trend, then, does not purport to reveal a universal relationship. It rather indicates the trend of the error of the estimate for several structural materials.

One further point of interest can be detected from these data. For each group of tests the data for the higher of two test temperatures are indicated by the symbol with the vertical dash mark. An internal comparison of data for each material indicates that the estimate is quite consistently more conservative for the higher temperatures. This is apparently associated with the fact that the isochronous stress-strain curves become more "spread out" as the temperature increases, and it suggests that the loss in capacity indicated by the estimate becomes exaggerated.

It is also of interest to note that data for the tubular columns appears to follow the general trend set by the data for the columns with solid, rectangular cross sections. There does not, therefore, appear to be any special difficulty associated with the application of the time-dependent tangent-modulus to such special shapes.

For all of the data obtained, it appears that the estimates computed are safe or conservative for the test columns whose value of C is less than about 5×10^{-4} L ($C = \frac{L}{2000}$). Actually much of the data indicate conservatism for imperfection well above this value. In any event, these data suggest that ultimately it may be possible to use the time-dependent tangent-modulus method for estimating column creep buckling loads safely if the column imperfections are not excessive.

*Consider columns in the "slender" or "elastic" range. For two such columns with the same slenderness ratio and imperfection, and the same elastic modulus, for example, the maximum column load will be greater for the column with the greater resistance to plastic flow. The difference between the maximum loads and the Euler load (which is the same for both columns) will, hence, be different.

SCATTER IN LIFETIME

It has been apparent from the data obtained during this investigation that the scatter in lifetime can be quite significant. The implications that derive from a consideration of these data can, of course, be expected to exert an influence on practical methods of analyzing creep buckling. It is felt, therefore, that some time should be spent discussing the factors responsible for scatter. Although the observations contained in this discussion are based primarily on the results obtained in this study, they will be generalized to include some of the factors that may be expected to be operative for columns that are part of actual structural assemblies.

Probably the primary factor controlling scatter in lifetime for creep buckling is imperfection. An example* of how imperfection can influence column capacity is presented in Figure 11. Each of the plotted points represents an actual column creep buckling experiment. As indicated, the imperfections were adjusted to the indicated values. The variation in lifetime for the selected values of imperfection is both distinct and significant.

Imperfection is usually represented in column behavior as either an eccentricity of load application or an initial bowing or curvature of the column. Both of these types of imperfections can be introduced during the fabrication processes that are a part of its manufacture. Any forming process such as rolling, drawing, or extruding can conceivably produce variations that can manifest themselves as imperfections. Various machining operations can also introduce imperfections.

Still another fabrication variable that may in some instances introduce significant degrees of imperfection is heat treatment. This is particularly true when the heat treatment utilized produces warping in the structural element. In instances such as this, the order in which heat treatment and finish machining are accomplished can be expected to produce different results.

With regard to the application of load, which is in reality a form of imperfection, this depends largely on the method of load transmission. In tests of the type conducted in this program, the end caps used to transmit the column load are the primary source of eccentricity. In actual structural members, the method of connecting elements in installation will determine the degree of eccentricity.

From the above discussion, it appears that the primary sources of imperfection can be grouped under fabrication and installation. The amount of scatter, then, for a given system of production can be expected to produce a certain distribution of lifetimes for a given load. If the fabrication process is not too well controlled, the imperfections will vary considerably

* See Part II of Reference 1.

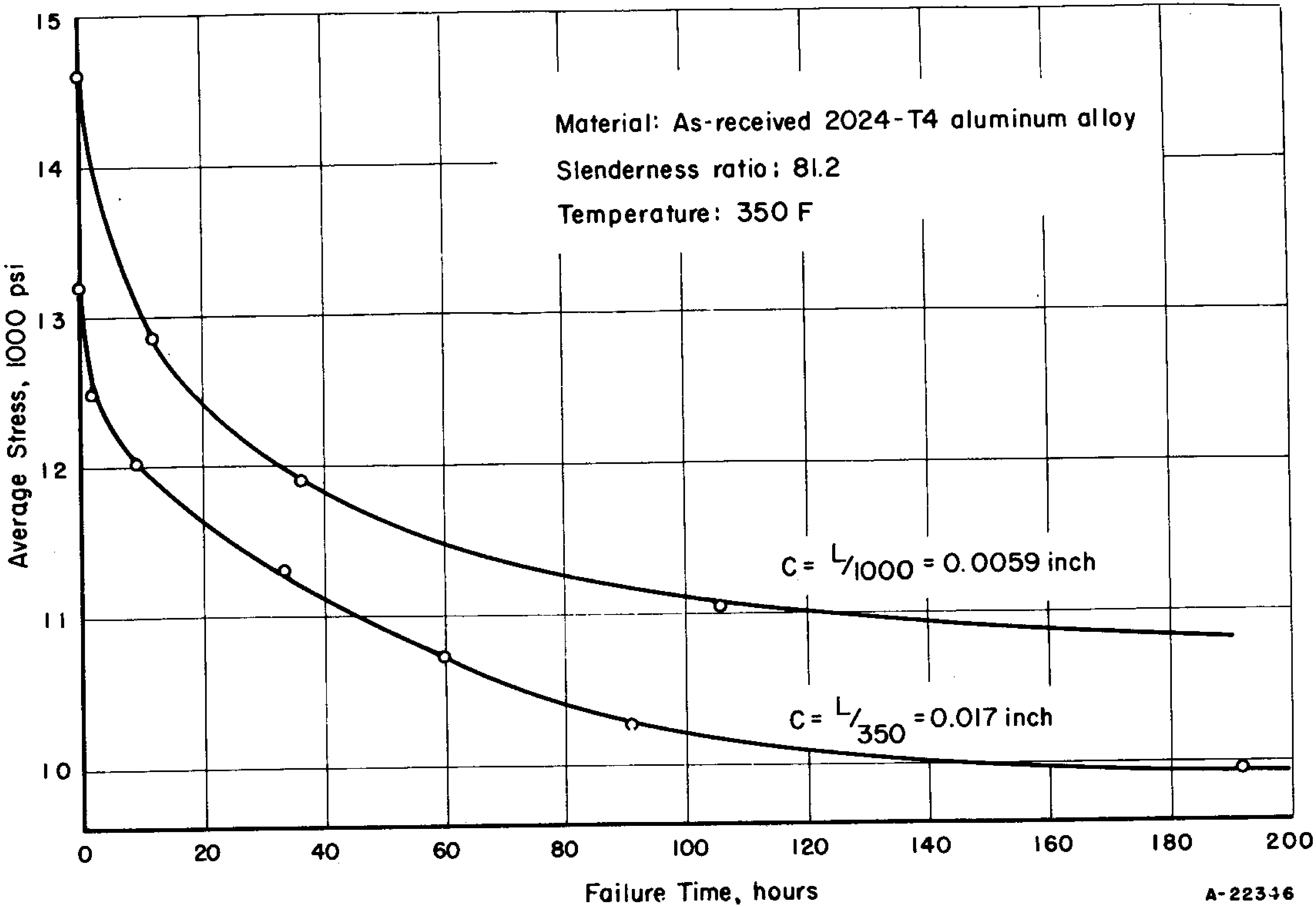


FIGURE 11. THE EFFECT OF IMPERFECTION ON COLUMN CAPACITY

from member to member, and the scatter in lifetime can be expected to be significant. If the process is closely controlled, scatter in lifetime will be small. With these observations in mind, it can be realized that a statistical study of a phenomenon such as creep buckling would produce results that reflected primarily the control existing for the fabrication method used. The results could not be used to anticipate the behavior of even the same material, if the fabrication process were different.

The above discussion emphasizes the fact that imperfection variations can cause scatter. The amount of scatter that can result from a given variation in imperfections depends on the properties or response of the given material. To illustrate its effect, it will be helpful to consider first short-time or static column behavior.

Consider two columns of the same slenderness ratio and the same amount of imperfection. Both have the same elastic modulus, but the proportional limits and subsequent plastic properties are different. In comparing two such columns, it is well known that the capacity or maximum load of the column whose material has the higher stress-strain curve (greater resistance to plastic flow) will be the greater. Its capacity to provide an internal moment to resist bending will be greater.

It can be seen from this example that the sensitivity to imperfection depends on the response properties of the material. That is, the variation in maximum loads for a given set of imperfections will be greater for the material with the lesser resistance to plastic flow.

The extension of this behavior to column creep buckling can be expected to follow naturally. The spread in lifetime for a given set of imperfections can be expected to be sensitive to the creep response of the given material. The poorer the resistance to creep, the greater the spread in lifetime for a given set of imperfections.

In laboratory tests, scatter due to variations in properties from specimen to specimen can be minimized by the use of material from a single heat or in some extreme cases by the use of material from a single rod or bar. In service application, of course, a given part will be fabricated from material from many different heats. This will introduce some property variation. Also, if heat treatments are necessary, it is probable that variations due to slight differences in the treatment will arise. Particularly in actual structural parts, then, there is always a good possibility of some scatter in performance due to property variations. This contribution to scatter is, of course, independent of variations in imperfection of the type discussed above.

The column creep buckling data obtained during this investigation indicate that the scatter in lifetime that occurs is sufficiently significant to warrant special consideration. The two sources of scatter singled-out are column imperfections associated with the column shape and manner of loading, and variations in mechanical properties.

A COMPARISON OF THE ALLOYS STUDIED

To compare the relative efficiency of some of the alloys tested, a special plot has been prepared, and it is presented in Figure 12. The graph contains curves of a column capacity index versus the temperature.

The index, I, for time zero is defined* as

$$I = \frac{\text{Young's Modulus in Compression}}{\text{Density of the Alloy}}.$$

The index for a time of 25 hours is defined as

$$I = \frac{\text{Modulus in Compression}}{\text{Density}} \times \frac{\text{Capacity for time of 25 hours}}{\text{Capacity for time of zero (at temperature)}}.$$

Values for the second ratio in the index for 25 hours were based on experimentally observed decreases in capacity for the given alloys.

Although plots of the type shown in Figure 12 can be useful guides for alloy selection, they must be interpreted with caution. A number of factors can influence alloy selection. Some of the more obvious ones are contained in the following list:

(1) Application: long- or short-time life.

(2) Design efficiency: strength-weight or stiffness-weight considerations.

(3) Economics: cost.

(4) Design procedure: simple or elaborate.

The first two factors can be incorporated as a part of graphs like Figure 12.

The third factor — economics — entails in itself a multitude of factors. Among them are:

(1) Basic material cost.

*The critical average stresses for two columns with the same slenderness ratio are proportional to the respective values of modulus.

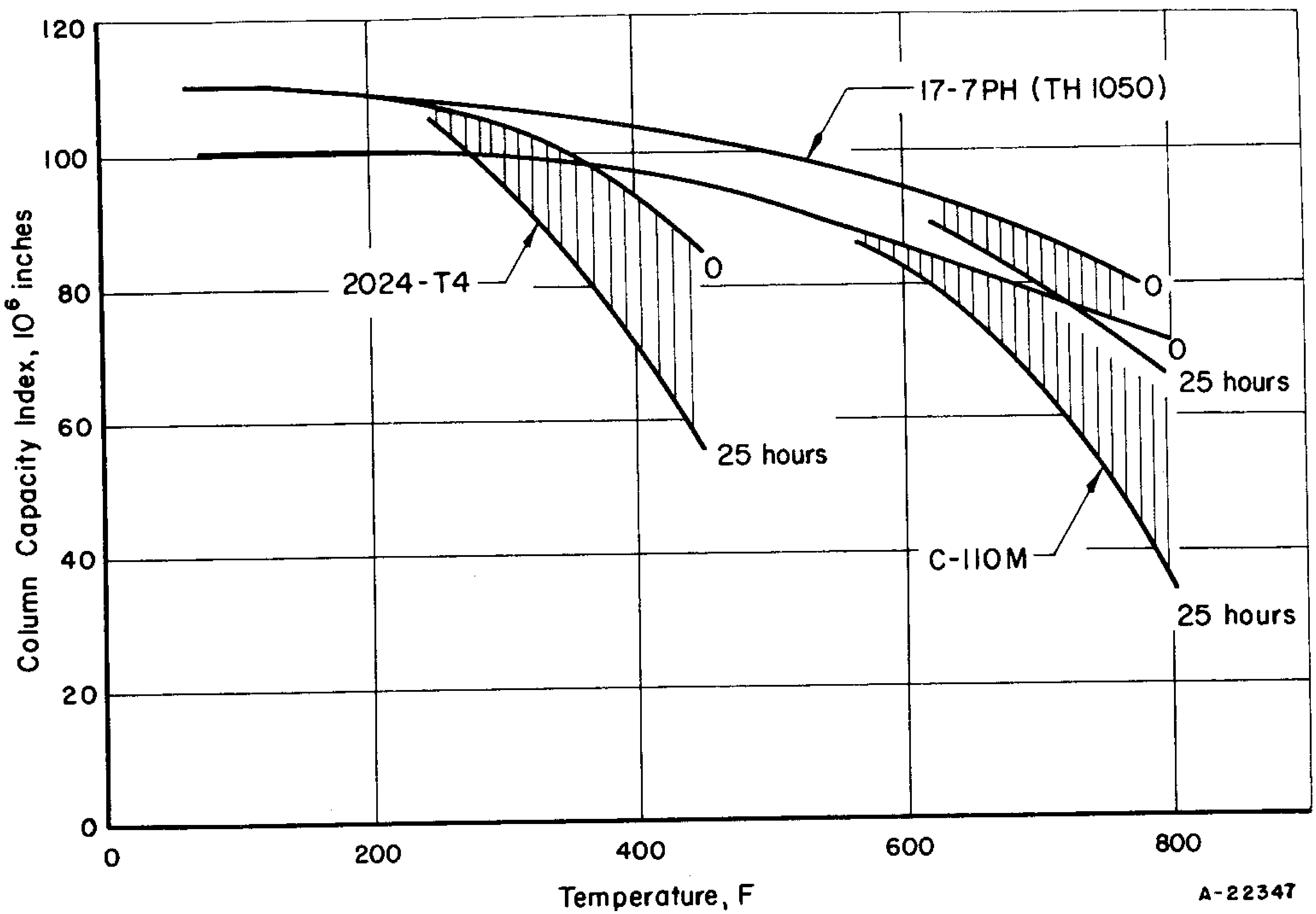


FIGURE 12. COLUMN-CAPACITY INDEX VERSUS TEMPERATURE

(2) Ease of fabrication.

(3) Availability of design and fabrication experience.

(4) Adaptability to standard methods of design and fabrication.

For most of these economic factors, the aircraft industry no doubt finds the replacement of aluminum alloys difficult to accept. The loss in stiffness and strength as temperatures rise is, of course, the primary reason for considering possible substitutes.

The fourth factor — design procedure — can be illustrated by an example. For a column at 275°F with a lifetime of 25 hours C-110M and 2024-T4 are both acceptable*. Designwise, C-110M would be "simple" to use, however, since no time-effects need be considered. Here, the use of 2024-T4 would be "marginal".

The temperature at which a transition from, say 2024-T4, to 17-7 PH (TH 1050) or C-110M, becomes necessary, depends on the application. For structures with "short" life expectancies, 2024-T4 may compete satisfactorily up to 400 F on the basis of the results of Figure 12. For life times greater than 25 hours, an upper limit of the order of 300 F is indicated. It is thus seen that the application can influence the limiting temperature for a given alloy. In general, a final decision as to the proper selection of an alloy must be governed by a combined consideration of the relative importance of all of the factors discussed above as they pertain to the given case.

SHORT-COLUMN BEHAVIOR

Introduction

To provide data in the short-column range, creep-buckling tests were conducted on columns with slenderness ratios of 16 and 31 during this investigation. The material used for these studies was the aluminum alloy 2024-T4.

In preparing for this study, it was recognized that certain necessary data were already available from the previous study. This included static data for test temperatures of 350 F and 450 F, and tensile-creep and limited compressive-creep data. The stress range of the creep data available, however, did not extend to high enough stresses for use in this

*Disregard 17-7 PH in this comparison.

study. It was, therefore, necessary to obtain additional creep data. Since the creep at the higher stresses produces significant cross-sectional area changes in short times, compressive-creep tests were conducted.

Compressive-Creep Tests

As in previous studies on this material, the material was tested in the "as-received" condition. Since the material is unstable at the test temperatures used, care had to be exercised in reproducing the same heating time prior to testing. This time was one hour.

Test specimens were right circular cylinders with a diameter of 0.312 inch and lengths of 1.25 inches and 0.87 inch. The shorter length was used at the highest stress levels tested. This became necessary when it was observed that, for the higher values of stress, creep buckling occurred too soon to obtain satisfactory creep data.

The test results for the compressive-creep tests are plotted in Figures 13 and 14. In Figure 13, the curve for 48,000 psi is partially dashed because the alignment was bad during the latter part of the test.

One particularly interesting feature of the curves of Figures 13 and 14 is the appearance of an inflection point after which the creep rate steadily increases. In view of the steadily increasing cross-sectional area that would be present in a compression-creep test, the accelerated creep rate may at first be difficult to understand. It was thought at first that alignment may have had some influence on this behavior, so this was checked for all of the tests conducted at high stress levels. It was found, however, that in nearly all instances, alignment was good within the strain ranges studied.

A check of the literature revealed that Finlay and Hibbard[20] observed what they called an "intermittent creep rate" for a four per cent aluminum-copper alloy. In tests that they conducted, it was observed that the presence of the "intermittent creep rate" could be associated with precipitation phenomena.

If precipitation occurring during a test is responsible for the form of the curves of Figures 13 and 14, one would expect that precipitation resulted in hardening of the test material up to times of approximately 10 to 20 hours. Figure 15, which is a plot of yield strength versus aging time for several test temperatures, tends to support this thesis. The curves of Figure 15 were obtained by Dix[21] for 2024-T4 flat sheet. Although these curves were obtained for the material in a different form, the aging characteristics should be expected to be similar to those of the test material used in this investigation. As can be seen, hardness increases abruptly to

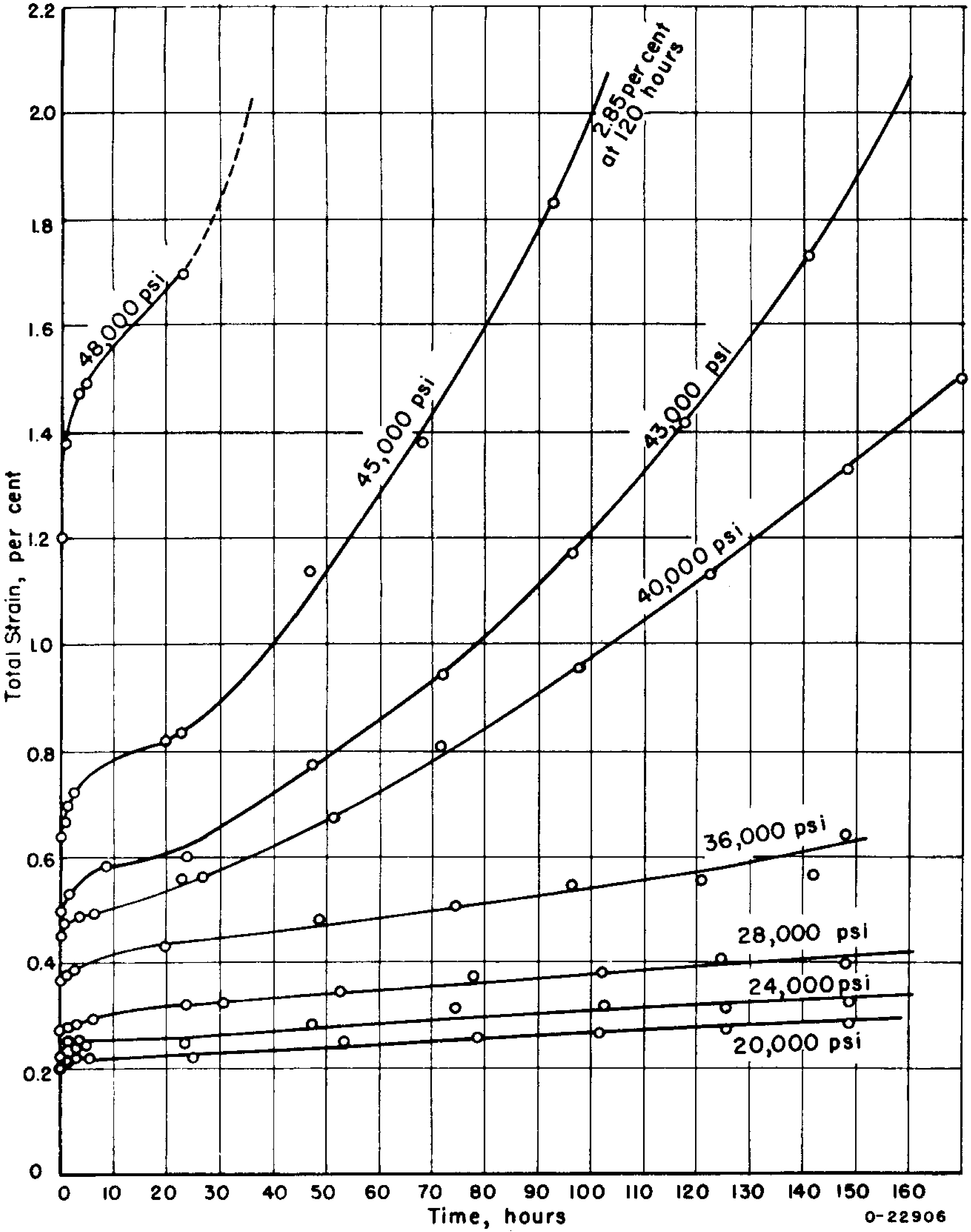


FIGURE 13. TOTAL STRAIN VERSUS TIME CURVES FOR AS-RECEIVED 2024-T4 ALUMINUM ALLOY TESTED IN COMPRESSION AT 350 F

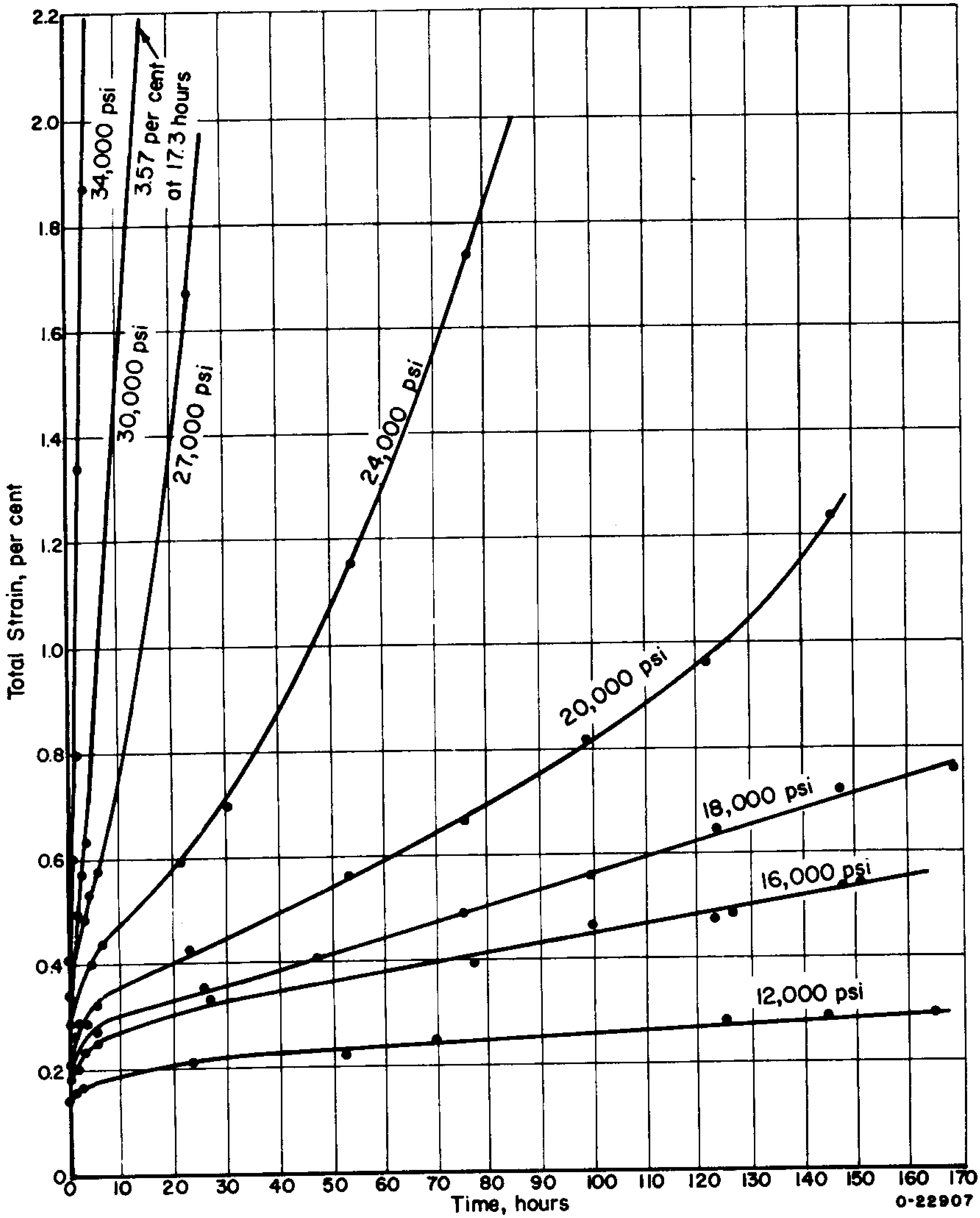


FIGURE 14. TOTAL STRAIN VERSUS TIME CURVES FOR AS-RECEIVED 2024-T4 ALUMINUM ALLOY TESTED IN COMPRESSION AT 450 F

approximately 16 hours for a temperature of 350 F. Thereafter, the hardness increases only slightly to 30 hours. Unfortunately, the curves were not extended beyond 32 hours. The curves in Figure 13, however, would tend to indicate that no further increase in hardness would be experienced. There might, in fact, be a slight decrease after 32 hours.

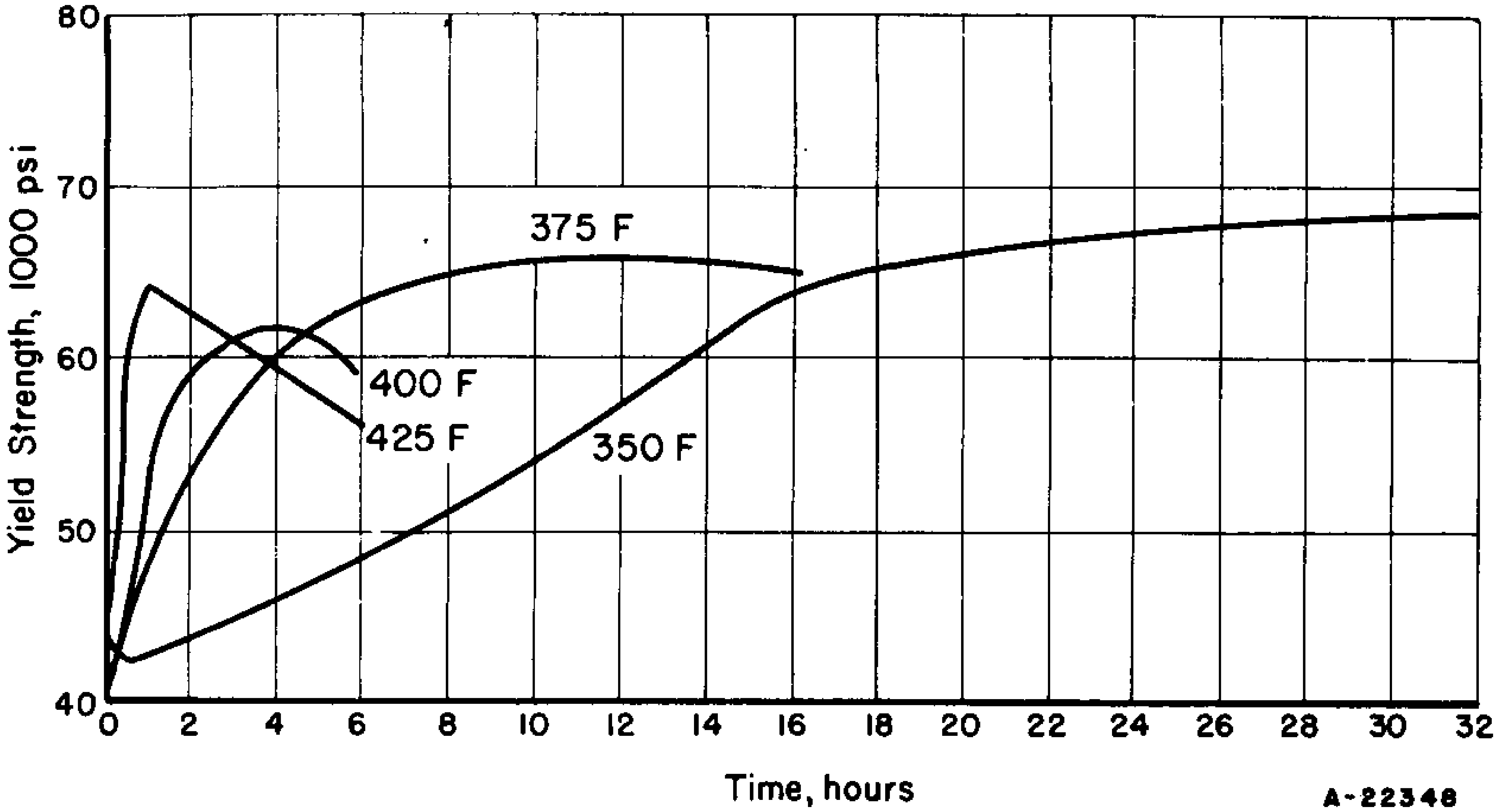


FIGURE 15. ARTIFICIAL AGING CURVES FOR 2024-T4 FLAT SHEET (AFTER DIX)

In passing, it should also be noted that the precipitation that occurs during creep may be expected to be influenced by the strain that has occurred. Since the aging curves of Figure 15 were obtained for specimens in an unstressed state, their form with respect to obtaining times for response changes must be tempered with caution.

Unfortunately, the plot of Figure 15 does not include an aging curve for the temperature of 450 F. The forms of those presented, however, do suggest what may be expected. The curves given would tend to indicate that an increase in hardness would occur, but that hardness would not remain at a high level much beyond 4 hours. Although it is somewhat difficult to fix the time of the inflection points from the curves of Figure 14, they appear to occur in the vicinity of approximately 6 hours.

Column Tests

Two short-column specimens were tested. Both specimens had a square 7/16-inch by 7/16 inch cross section. A slot 5/16 inch wide and

3/16 inch deep was cut into the end of the specimens, and hardened steel inserts were made to transmit the column load. V-notches were cut in the inserts and the distance between the root of the notches at each end of the column was the column length. By this measurement, one column length was 3.9 inches and the other was 2.0 inches. The computed slenderness ratios for the above columns were 31 and 16.

The results of column creep-buckling tests for the columns with a slenderness ratio of 31 are summarized in Figure 16. As indicated on the graph, the data from the two batches of the specimens are identified by solid and open points. As can be seen, the trends for these two batches, as indicated by the dashed and solid upper curves, suggest that the slots for the second batch were centered somewhat more accurately than those of the first batch. It is apparent that the resulting differences in imperfection for the two batches can result in marked differences in lifetime for a given load. Unfortunately, the deflections associated with the load application at the beginning of each of these tests was so small that no accurate evaluation of imperfections was possible. It is apparent from the results that differences do exist, however.

Using the compressive-creep data, isochronous stress-strain curves were constructed and the column stress, σ, was computed by the use of the Euler formula, $\sigma = \frac{\pi^2 E_T}{\left(\frac{L}{r}\right)^2}$, where E_T is the time-dependent tangent modulus.

The results of these computations are shown in Figure 16 together with the experimentally obtained data. As can be seen, the column-capacity estimates obtained by the use of the time-dependent tangent-modulus method are quite markedly conservative for both test temperatures.

Although the correlation data presented previously for this material suggest that the tendency toward conservatism observed here does not extend to columns with larger slenderness ratio values (see Figure 10), it is difficult to make any final conclusions without considering the possible effect of differences in imperfection between the two sets of data. To obtain a final or more satisfactory resolution of this question, it is necessary to study the effect of imperfection more thoroughly.

Although imperfection can certainly alter the effects discussed above, it should be noted that an inherent difference in the behavior of long columns which are initially elastically loaded throughout, and short columns which are initially inelastically loaded should be anticipated. This stems from the fact that the columns are creeping and bending under a constant load, and, therefore, loading can be expected to occur on the concave side of the cross section, and unloading can be expected to occur on the convex side of the cross section. For the column which is initially elastically loaded throughout, the instantaneous time-independent adjustments that occur while the column is creeping will be governed both in

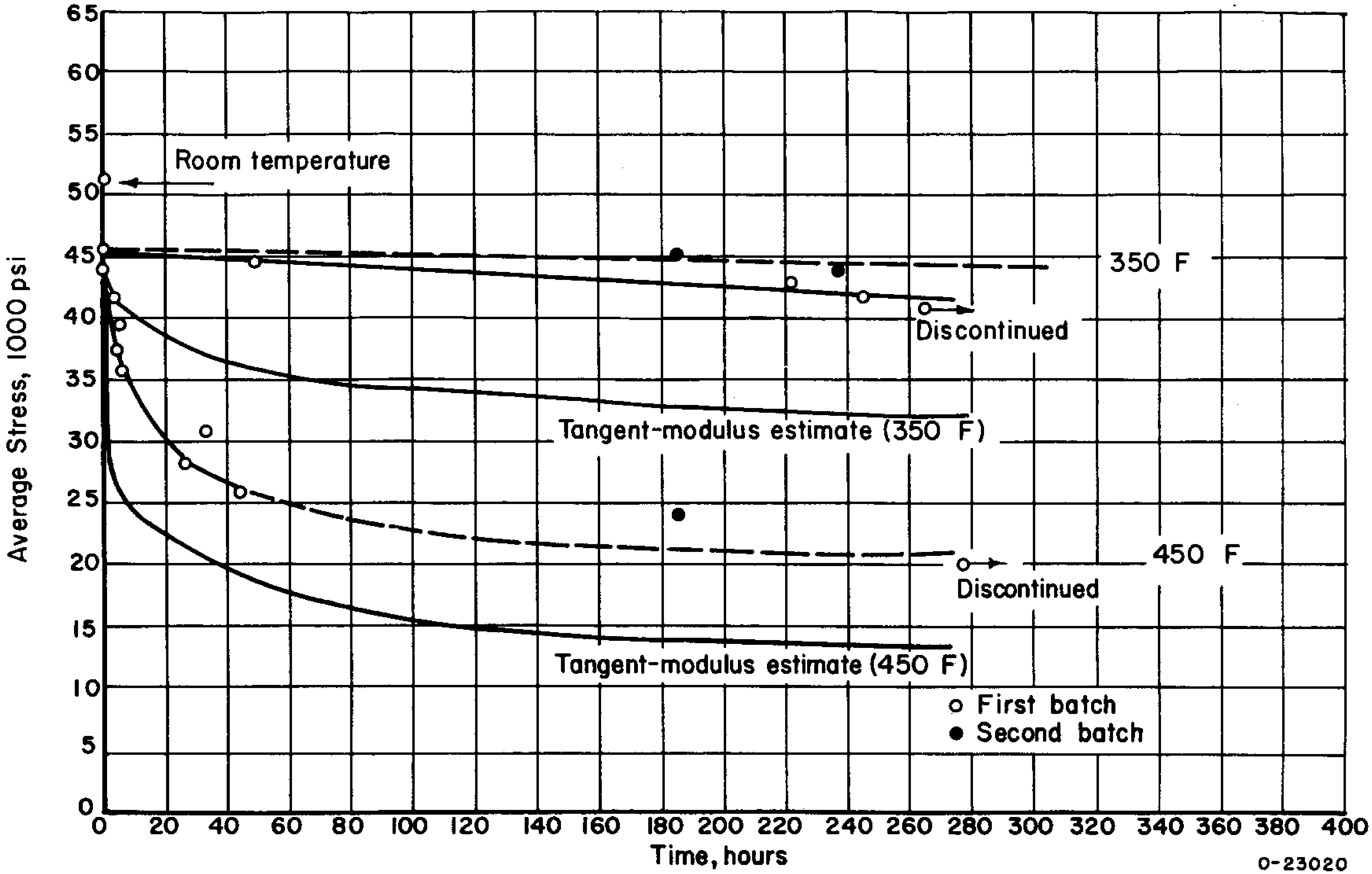


FIGURE 16. AVERAGE STRESS VERSUS FAILURE TIME FOR 2024-T4 ALUMINUM ALLOY COLUMNS WITH $L/r = 31$

loading and unloading by the elastic modulus. For the inelastically loaded column, however, such adjustments will be governed by a tangent modulus for loading, and by the elastic modulus for unloading.

The use of the time-dependent tangent-modulus results in essentially an approximate load-capacity estimate, and it does not, as applied here, account for differences in column action that exist between an initially elastic load and an initially inelastic load. Since, moreover, the action associated with an inelastic case is related to a double modulus type of behavior, it might be anticipated that the use of a method based on the tangent-modulus concept would result in an indication that the trend of increase in column capacity with decreasing slenderness ratio was slower than would actually be the case. A comparison of load estimates and experimental data for both long and short columns would tend to support this explanation.

Preliminary tests on the 2.0-inch specimen indicated that the possible column loads for this column were high enough to cause damage to the equipment used. To decrease the level of loads studied, tests on this column were limited to the test temperature of 450 F, and loads short of failure were studied.

Due to the limitations of testing necessary for this specimen, it was decided to alter the usual procedure. Instead of varying loads and getting column-capacity versus time curves, the load was held constant and various imperfections were tested. Differences in imperfection were achieved by grinding a layer of material from one of the column surfaces parallel to the knife-edge slots of the specimen. Although the amounts removed were small, this created in essence a shift in the slot and produced an eccentricity of loading. The amount of eccentricity introduced in each case was equal to one-half the layer of material removed. The results of these tests are summarized in the tabulation given below and in Figure 17. The value of average stress was 28,900 ± 100 psi for all tests.

Failure Time, hours	Eccentricity, 10^{-3} inch
25	11.0
38	9.8
41	9.0
42	8.2
161	6.2

It is apparent from these tests that imperfection — eccentricity in this instance — can have a marked effect on the lifetime of "short" columns. Figure 17 illustrates the variation for only one average stress. As indicated in the graph, additional curves for other stress values exist, and they would produce a family of curves of time versus eccentricity.

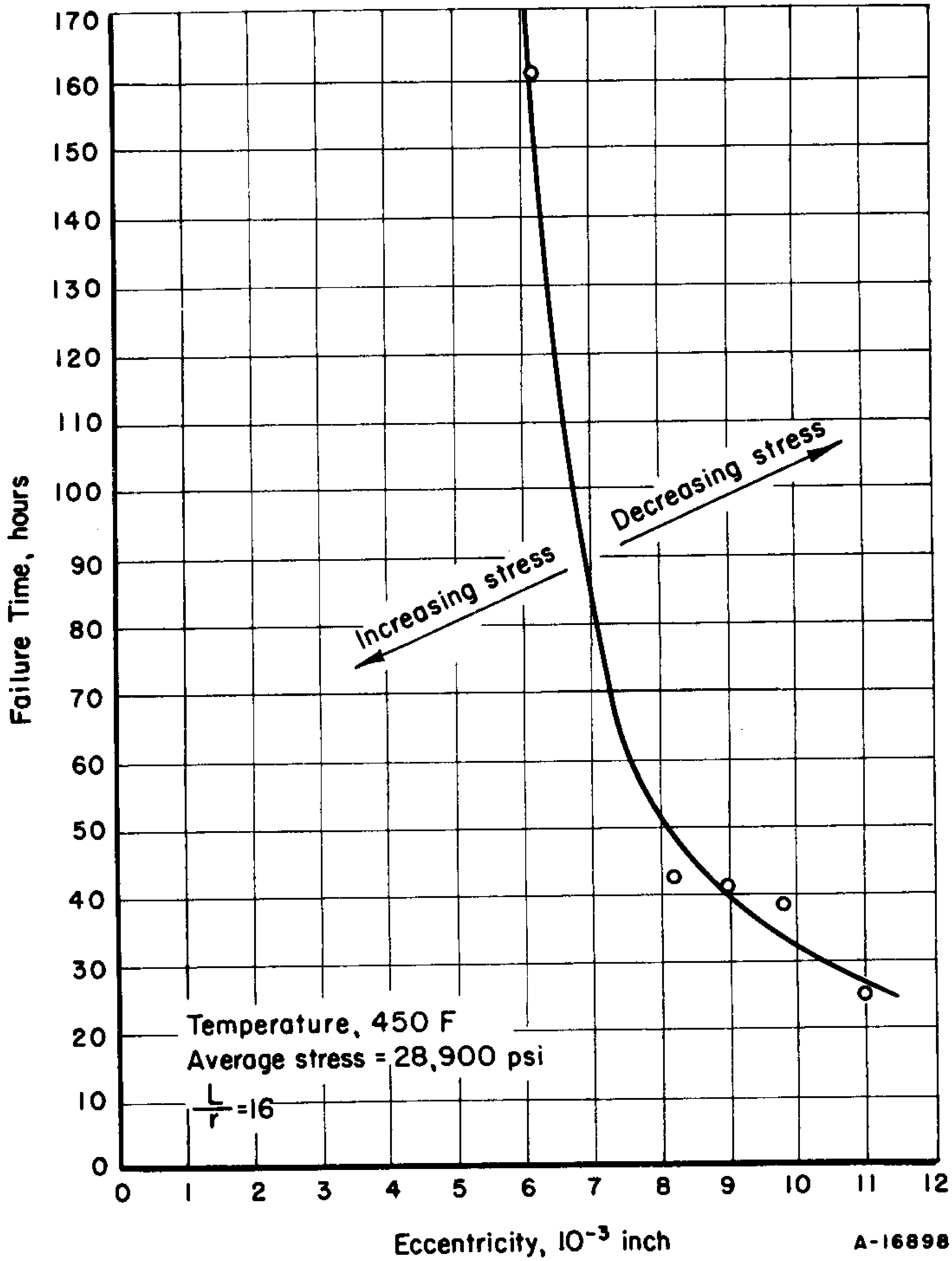


FIGURE 17. FAILURE TIME VERSUS ECCENTRICITY FOR A "SHORT" 2024-T4 ALUMINUM ALLOY COLUMN

A Limit Stress in Compression

The analysis of column members with slenderness ratios of the order of 16 and less presents many difficulties. For this reason, an attempt was made to evolve a relatively simple, rational approach to the problem that would provide a method of establishing a limiting load.

In tensile applications, the limiting load is frequently associated with the ultimate load. This does not mean that the elements of the structure would be required to sustain loads producing stresses of the order of the ultimate stress. The ultimate stress is used merely as an upper limit. The elements are designed in such a fashion that the stresses sustained will be something less than the ultimate load. In elevated-temperature applications where creep is possible, the selection of an ultimate stress requires, in essence, a new definition. One possible definition is based on the expected life of the structure. For example, if the structure is to last 10 hours, stress versus failure time for stress-rupture data can be used to select the proper stress for 10 hours. As in the previous case of the ultimate load criterion, however, the stress so defined serves merely as an upper limit.

In designing for compression, stability considerations must be made. This, of course, again introduces the criterion of a limiting load. It is conceivable, however, that if the compressive member has a small enough slenderness ratio, the criterion for design may be an allowable amount of deformation rather than an allowable load. The possibility of rupture in compression in ductile materials need not be considered, hence, the use of the limiting load, analagous to the ultimate load, does not at first seem possible.

The purpose of this discussion is to suggest a derivation or definition of a limiting load which may in some senses be considered analogous to limiting loads derived from stress-rupture data for tension. The considerations here are confined to cases in which the slenderness ratio of the compressive member is very small.

To illustrate how the concept of a limiting load may be evolved for compressive loading, compressive-creep data for the aluminum alloy 2024-T4 for a temperature of 450 F will be utilized. For the present application, the data have been replotted in Figure 18 in the form of isochronous stress-strain curves. In addition to the curves for the various times, dashed horizontal lines have been included. As can be seen, there is a dashed line for each time curve. The dashed lines were positioned in such a manner as to show the leveling-off trend for each curve. That is, each curve appears to become asymptotic to its corresponding dashed line. Consider now the isochronous stress-strain curve for 5 hours. As can be seen from this curve, very large strains will occur under a stress of 35,000 psi in 5 hours' time. Thus, if one is concerned with times under

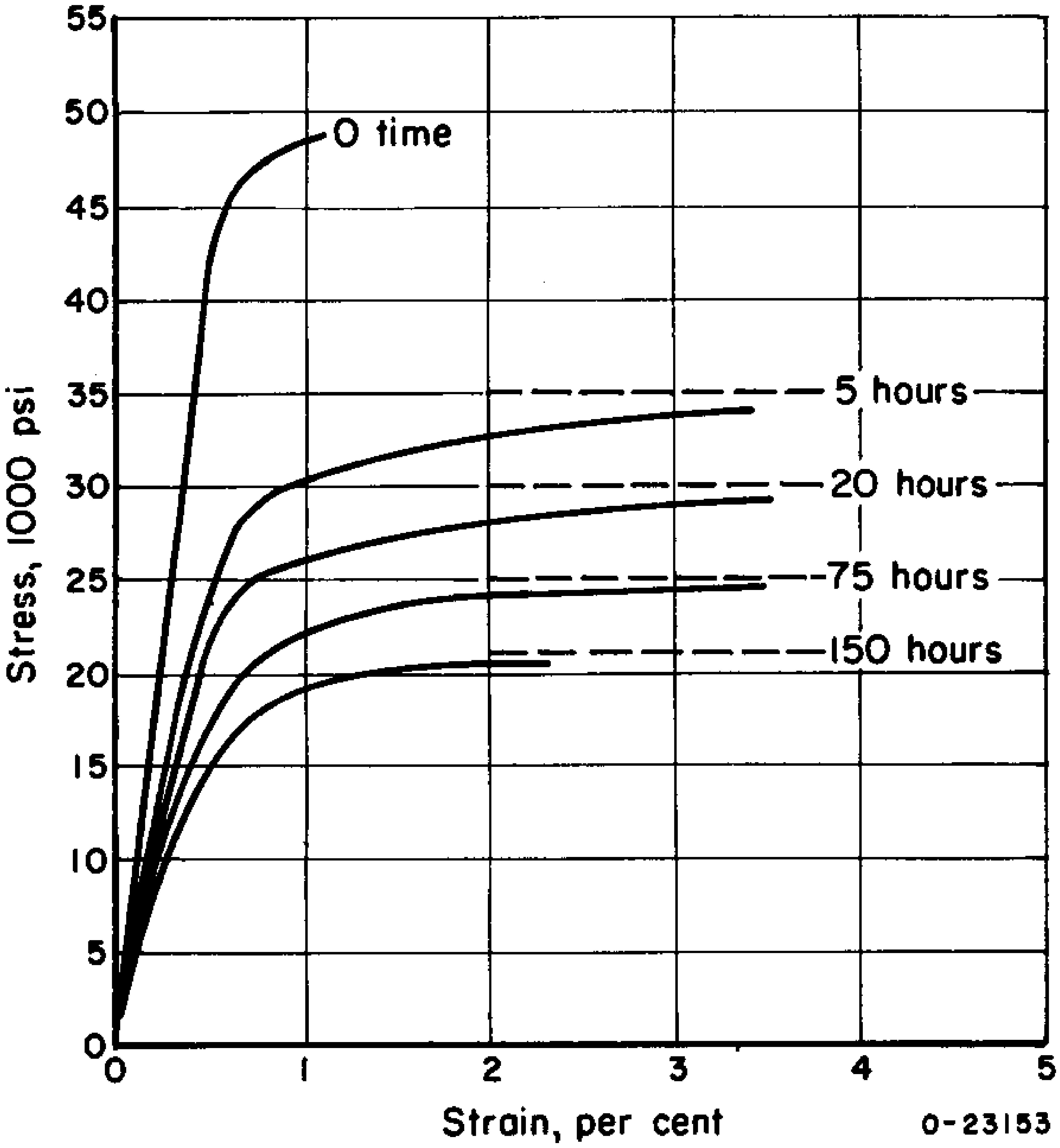


FIGURE 18. ISOCHRONOUS STRESS-STRAIN CURVES FOR ALUMINUM ALLOY 2024-T4 TESTED IN COMPRESSION AT 450 F

stress of about 5 hours, 35,000 psi may be thought of as a limiting stress. In a similar manner, 30,000 psi may be thought of as the limiting stress for 20 hours, 25,000 psi as the limiting stress for 75 hours, and 21,000 psi as the limiting stress for 150 hours.

Using the horizontal dashed curves to select limiting stress values, it is possible to construct a stress versus time curve whose form will be similar to that of stress-rupture curves. For each value of time, it will be possible to find the limiting value of stress for which the capacity to resist creep deformation is greatly reduced. The curve of Figure 19 has been plotted by selecting the stress values corresponding to the dashed lines of Figure 18.

The form of the isochronous stress-strain curves in reference to the manner in which they tend to level off is dependent upon temperature. For the example cited, it can be seen that the capacity to resist deformation for long times will be relatively small when compared to the zero time or static strength. The individual isochronous stress-strain curves tend, in fact, to originate at the origin. For lower temperatures, the general level of the time curves is closer to the zero time curve, and they appear to originate at a finite value of stress. This, in turn, would produce a "flatter" limit curve.

CAPACITY ESTIMATES FOR "LONG" AND "SHORT" COLUMNS

With a tentative method for estimating load capacities for "very short" columns available now, it should be possible, in conjunction with methods available for "long" columns, to obtain load capacity estimates for all slenderness ratio values. To examine this possibility, data for a range of slenderness ratio values were used to prepare Figure 20.

The solid curves within each section of the plot were obtained by the use of the methods indicated. The times of 0 and 150 hours were selected because this was the range of times over which data were available for most of the conditions.

The vertical lines indicate the experimentally observed variation in column capacity in going from 0 to 150 hours lifetime. The solid bar at $\frac{L}{r} = 16$ represents the data of Figure 17. Since four of the failures for $\frac{L}{r} = 16$ were in the vicinity of 30 hours, the "limit stress" for that time has also been indicated.

In general, the methods used to estimate the loss in capacity with increasing time appear to reflect the actual loss quite well. These data,

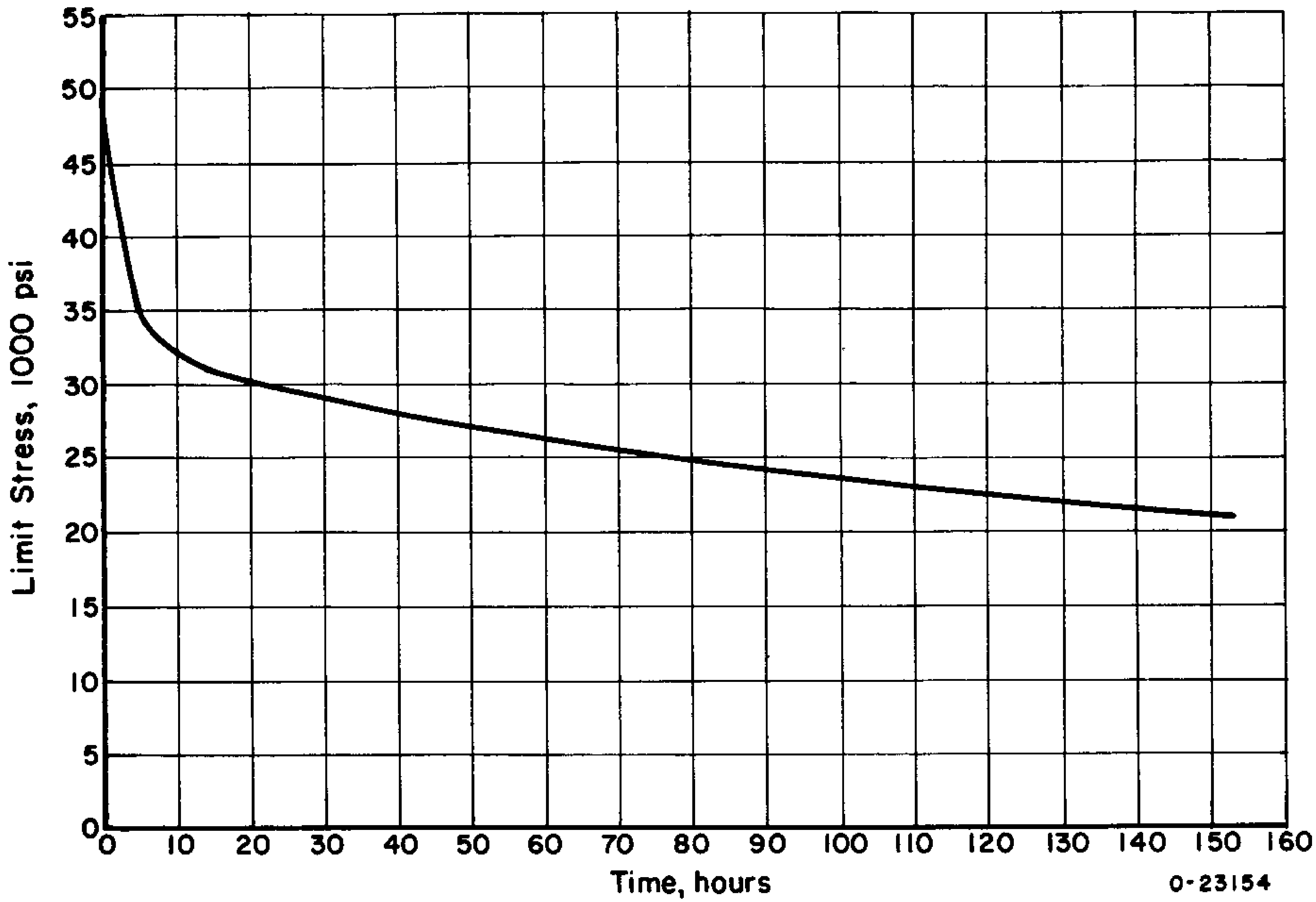


FIGURE 19. LIMIT STRESS IN COMPRESSION VERSUS TIME FOR ALUMINUM ALLOY 2024-T4 AT 450 F

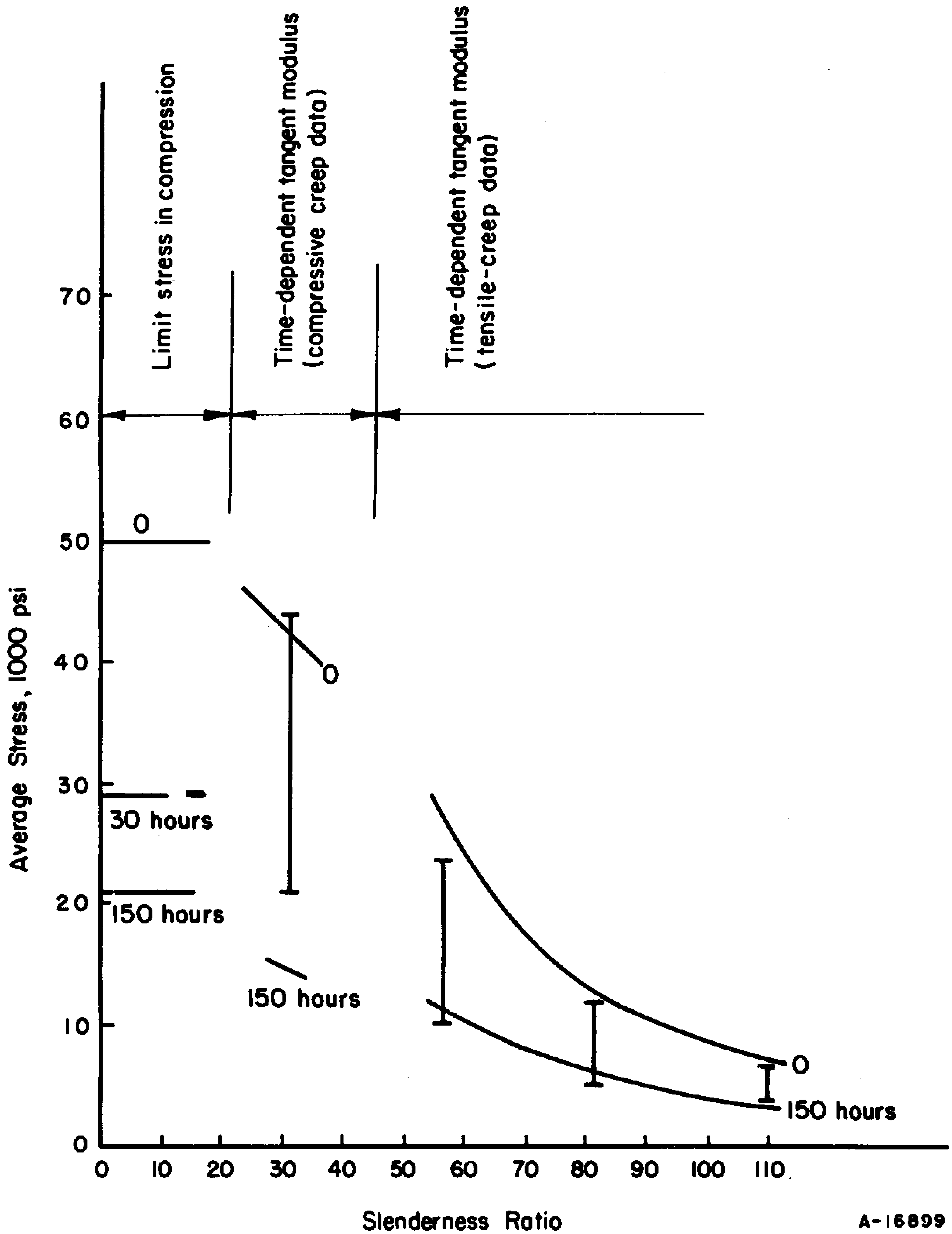


FIGURE 20. AVERAGE STRESS VERSUS SLENDERNESS-RATIO DATA FOR 2024-T4 ALUMINUM ALLOY COLUMNS AT 450 F

therefore, suggest that relatively simple methods may be used to obtain estimates of column-load capacity for all slenderness ratio values.

ON THE EXISTENCE OF LOWER LIMIT TO CREEP BUCKLING

In most of the experimental work conducted as a part of this program, it has been observed that for a column of a given slenderness ratio and imperfection, there appears to be a lower load limit below which buckling or collapse due to creep will not occur. The average stress versus failure or collapse time curve usually falls quickly up to about 50 hours. After about 150 hours, however, the curve falls very slowly and appears to be leveling off to a lower limit. Average stresses below the apparent limit were not observed to result in collapse even though the tests were allowed to continue well beyond 200 hours.

It may, of course, be argued that the test times indicated above are not long enough to form any definite conclusion. An argument of this type is difficult to resolve. If the time of testing without failure is doubled or even tripled, it can still be said that the time was insufficiently long.

Hoff[22] has stated that: "Creep deformations increasing the initial deflections take place continuously under any compressive load however small. Thus, the column buckles eventually even if the loading acting on it is much smaller than the Euler buckling load calculated from the elastic properties of the material at the particular temperature".

If one assumes that all materials creep, even at low temperatures, it can be inferred from Hoff's statement that there is never a finite lower limit below which creep buckling will not occur. If one assumes the existence of a "transition temperature" below which creep will not occur, creep buckling will not occur below it, but no finite "lower limit" exists above it. Actually, neither of these possibilities is very appealing.

The question of whether or not a finite lower limit exists below which creep buckling will not occur can be important because it may influence the formulation of design methods. If such a limit exists, the selection of design stresses for long time service can soundly be based on the limiting value. If no such limit exists, acceptable design values are time-dependent. For the former possibility, a single estimate suffices for all long times. For the latter possibility, a single estimate is insufficient.

It is natural to hope that a satisfactory resolution of this question can be based on theoretical rather than on experimental information. Unfortunately, however, creep-buckling theories that are available do not provide a general answer. The theories of Hoff(22) and Kempner(8), for example, are rigorously valid only for materials obeying a viscoelastic response, and for columns with idealized two-flange cross sections*. The theory of Libove(13) attempts to describe real material behavior, but it contains limitations that tend to weaken its answer to the question under discussion. These will be discussed in more detail below.

Libove's theory indicates that the life of a column under a given load approaches infinity for an initial imperfection approaching zero, and zero for an initial imperfection approaching infinity. This suggests that, for a finite initial imperfection, there is a "critical time", and life is finite. This refutes the idea of a finite "lower limit" below which buckling due to creep will not occur.

It should be noted, however, that the conclusions derived from Libove's theory can be expected to be rigorously applicable only for columns whose material obeys Shanley's creep law as applied to the empirical strain-time relationship for constant uniaxial stress,

$$\epsilon_c = A(\exp B\,\sigma)\, t^K \tag{3}$$

where

ϵ_c is the creep strain,
σ is the constant stress,
t is the time,

and A, B, and K are material constants.

By the use of Shanley's creep law, the differential change in total strain (creep strain and elastic strain), $d\epsilon$, can be written as

$$d\epsilon = \frac{K\left[A \exp B\sigma\right]^{1/K} dt}{\left[\epsilon - \frac{\sigma}{E}\right]^{\frac{(1-K)}{K}}} + \frac{d\sigma}{E} \ . \tag{4}$$

With regard to the validity of this law, Libove's comments(13) are quite pertinent to this discussion: "The most questionable aspect of the theory is probably the assumption that the creep rate under constant stress is a function only of the instantaneous values of the stress and strain and not of their histories. The assumption is questionable in view of some scattered experimental evidence which indicates that history does affect the constant-stress creep rate of alloys. The magnitude of this effect has not been systematically investigated. Where the effect is large, the present analysis would not apply."

* See discussion in the section on Solutions of the Problem.

The lack of definite knowledge indicated in the above statement creates an uncertainty that tends to preclude the possibility of general conclusions.

Another, somewhat more straightforward, limitation arises from the nature of Equation 3. It will be noted that even for a stress, σ, equal to zero, the creep strain ϵ_c, is finite and increases with time. This suggests that Equation 4 cannot be valid for small values of stress.

At high temperatures*, the error introduced by the use of Equation 4 may not be significant due to low creep resistance. At intermediate temperatures (which are incidentally more practical in regard to material usage) the "fit" of a relationship such as Equation 3 may become poor as the stresses necessary for measurable creep become large.

From the above discussion, it is clear that predictions for <u>general</u> creep-buckling behavior cannot be derived from available theories. The theories available utilize creep laws which in each instance restrict their validity to materials that obey the given laws. It also follows that the possible existence of a finite "lower limit" to creep buckling cannot be generally proven or disproven analytically by available theories.

Since an argument based on theoretical grounds cannot be used to either prove or disprove the existence of a "lower limit", an argument based on real creep behavior has been developed. Suppose we consider a material whose creep-time behavior is as illustrated in Figure 21(a). At the lowest stresses, σ_1, and σ_2, the creep behavior is "transient" in nature. Cottrell(11) describes such behavior as "transient flow which soon dies away almost completely, so that the dimensions of the specimen become stable and the applied load can be supported safely for an indefinitely long time". For larger values of stress, σ_3 and σ_4, strain in Figure 21(a) increases with time in the manner indicated.

Cottrell's statement does not say that in transient creep, the flow ultimately stops completely. The suggestion is strong, however, that for engineering purposes, it may be considered to stop**. If we assume, however, that it does stop, and then study the ramifications of the assumption, we may be able to present reasons for the observed apparent "lower limit". Before proceeding, however, two points should be noted:

---

*This is relative, of course, for a given material. For aluminum alloys, temperatures in excess of 400°F are "high".

**Actually, this question has not been completely resolved, and it is, therefore, advisable not to be dogmatic.

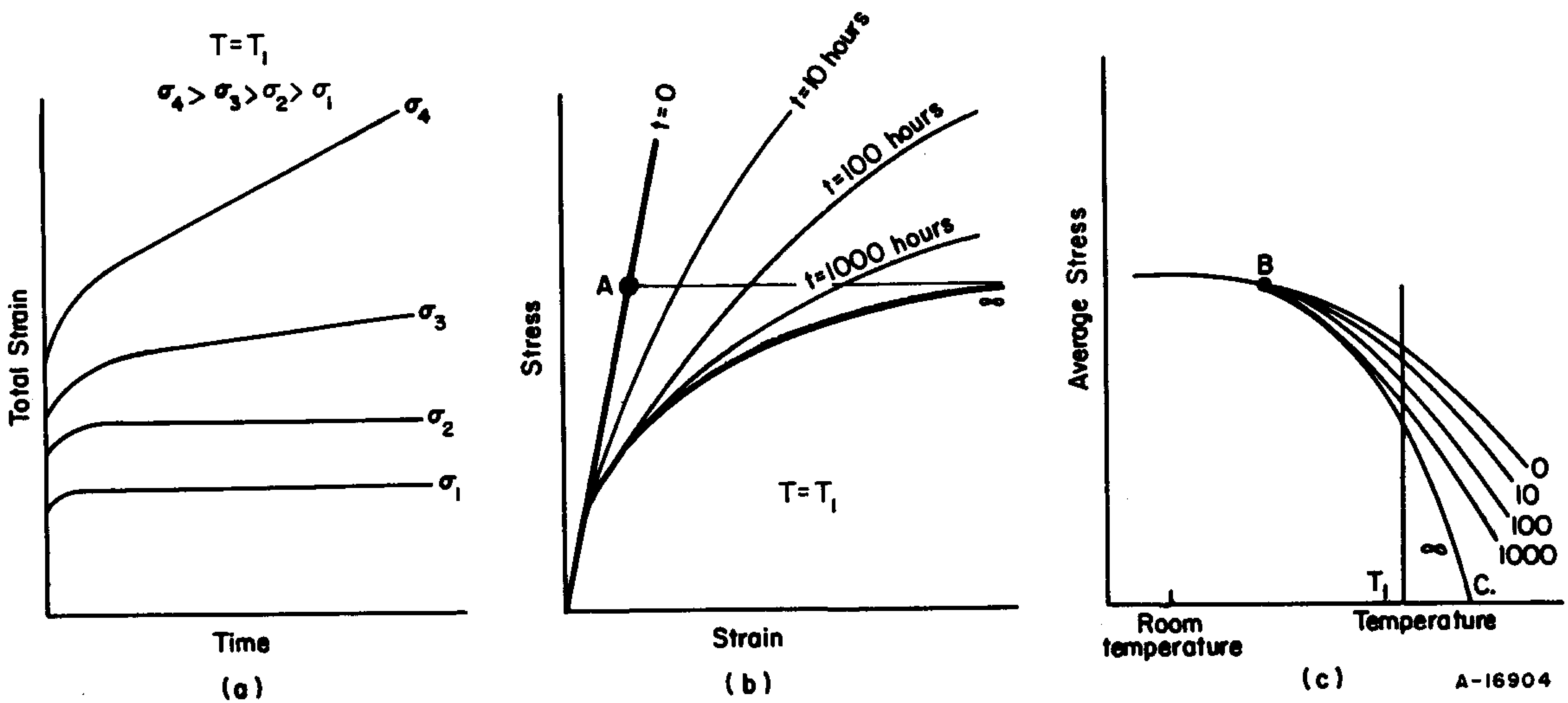


FIGURE 21. GRAPHIC RELATION OF CREEP BEHAVIOR TO COLUMN CAPACITY

(1) Creep curves for structural materials usually have the appearance of the curves of Figure 21(a) for temperatures that are within what would normally be considered their useful range.

(2) Creep laws of the type discussed earlier in this section are not flexible enough to "match" the behavior indicated for the range from σ_1 to σ_4.

To illustrate the creep behavior in another manner, isochronous stress-strain curves for the material of Figure 21(a) are shown in Figure 21(b). In this figure, the heavy curve marked "∞" indicates, for each stress level, the strain at which the creep rate tends to zero.

Returning now to the creep-buckling problem, it would appear that, as the average stress for a column of a given imperfection decreases, the possibility of obtaining stresses that produce only transient creep should be good provided the imperfection is not too large. In Figure 21(b), average stresses below the Point A should be eligible for this transient-type behavior; that is, a stable configuration should ultimately be possible.

In Figure 21(c), the consequences of the above reasoning are shown for the variation in average stress with temperature for a given column. Up to the Point B, creep buckling is not observed. Any column load less than the maximum or collapse load results in a stable behavior; that is, collapse will not occur. It might be said, then, that up to B, all times, 0, 10, 100, 1000, and ∞ are contained in the single curve. For temperatures above B, however, time-dependent collapse is possible, and the "fanned" curves illustrate this behavior. In accordance with the argument for a "lower limit", the curve labeled "∞" is also included. Since it seems reasonable that the lower limit should be zero when the temperature becomes sufficiently high, the "∞" curve has been shown to drop to zero.

For temperatures such as T_1 (within the range from B to C) the type of column behavior possible when the material response is as illustrated in Figure 21(b) is shown. For temperatures above the Point C, no finite lower limit exists, and the "∞" curve of Figure 21(b) would coincide with the strain axis.

The test temperatures of this investigation have apparently been confined to temperatures that are slightly below those indicated by the Point C. Figure 22 is a plot of actual column creep buckling data presented in the manner illustrated in Figure 21(c). These results tend to support the explanation described above. That is, the "lower limit" appears to tend toward a finite value below about 450 F and zero above about 450 F.

The behavior indicated in Figure 21(c) requires that the nature of creep behavior vary with temperature. It is a well-known fact that this is

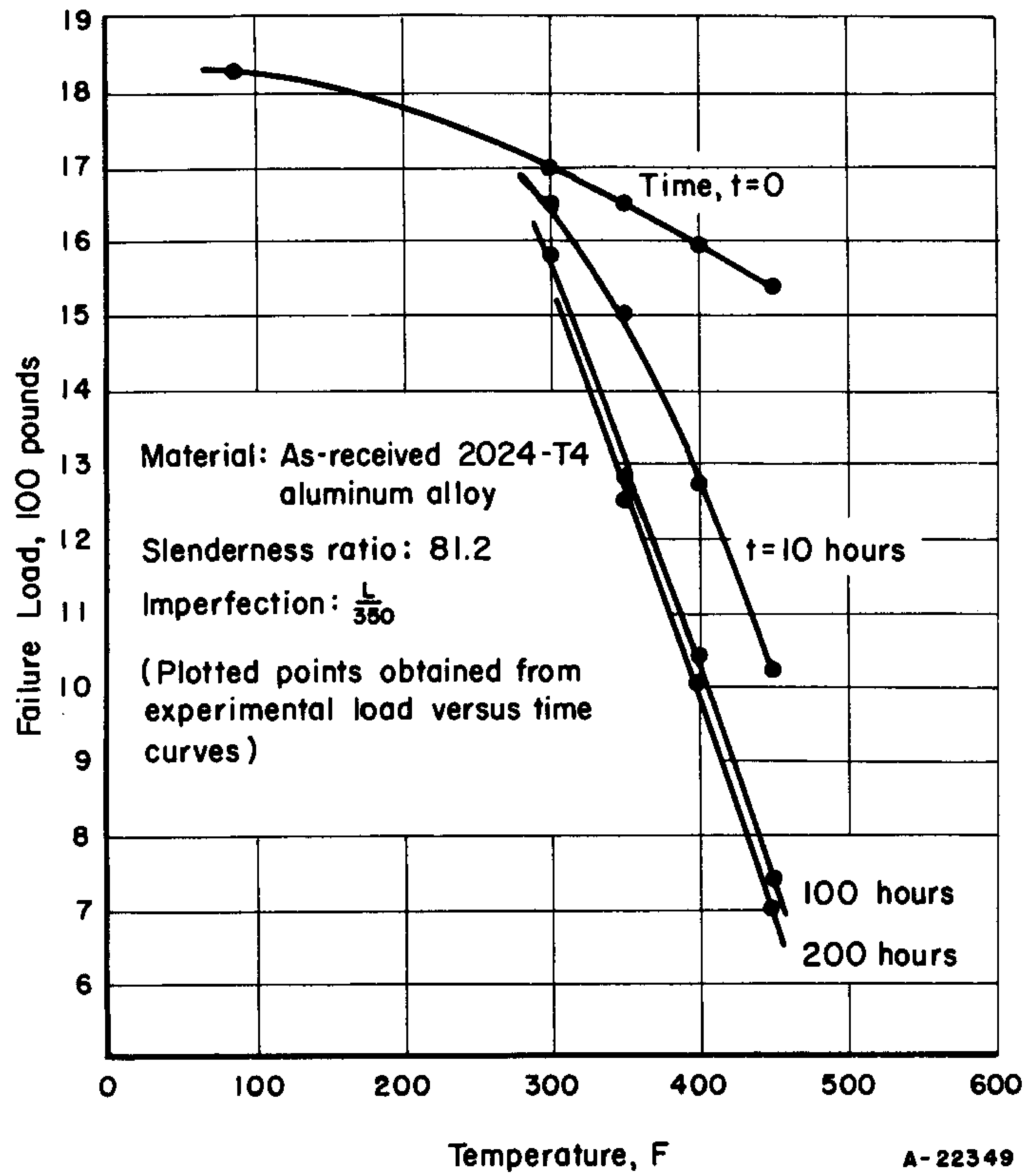


FIGURE 22. COLUMN CAPACITY VERSUS TEMPERATURE FOR VARIOUS FAILURE TIMES

true. The possible modes of deformation increase with increasing temperature. This suggests that a creep law that is good over a wide range of temperatures can be expected to be far more complex than the engineering type laws suggested to date. This does not mean that the use of laws that have an approximate validity over a limited temperature range should not be used. It should be remembered, however, that conclusions derived from analyses based on these laws can be expected to yield a correct estimate of behavior only in the given, limited range. They should not be used to predict behavior outside their range of validity.

With reference to the real behavior illustrated in Figure 22, the following conclusions, based on the preceding discussion, can be made:

(1) The column behavior can be well described by the use of the static or short-time properties (stress-strain curve) of the given material up to temperatures of about 250 F. The "lower limit" and the t = 0 curve coincide.

(2) A theoretical account of the behavior observed between about 250 F and about 450 F must make use of a creep law that can properly describe transient creep effects. In this range, the "lower limit" is below and very close to the t = 200 hour curve.

(3) The use of creep laws that predict that no finite "lower limit" exists should be confined to temperatures above 450 F. Above 450 F, the "lower limit" is zero.

SUMMARY AND CONCLUSIONS

The preceding sections have contained a summary and an analysis of results from an extensive study of the creep buckling of metal columns. The conclusions listed below summarize the findings.

(1) Although a number of solutions to the problem of column creep buckling have been advanced, few have practical applicability. The limitations associated with these solutions usually can be traced to use of idealized column models (two flange column cross-section) and unrealistic material response laws.

(2) The relation of the time-dependent tangent-modulus load — as conceived by Shanley — to the actual column capacity is clarified. It is interpreted as a limiting case of the allowable-load capacity which is shown to be a conservative estimate. The time-dependent tangent-modulus load is, therefore, an approximation to a conservative estimate. The approximation, however, may be either conservative or nonconservative.

(3) An application of the time-dependent tangent-modulus method to four representative structural materials indicates that estimates are consistently conservative for small values of column imperfection.

(4) Data from all phases of the investigation indicate that variations in imperfection can result in marked differences in column lifetime.

(5) Alloy selection for a structural element is dependent on the expected lifetime, strength-weight or stiffness-weight considerations, material and fabrication costs, and on the design methods necessary (must time-effects be considered or are short-time methods satisfactory?)

(6) Metallurgical changes such as aging or precipitation can influence the resistance of a metal to creep. For the aluminum alloy 2024-T4 creep rates in compression were observed to increase with time when softening due to aging could be expected to be occurring.

(7) For the aluminum alloy 2024-T4 loaded in uniaxial compression, there is, for each stress, a time beyond which the resistance to creep is very low. A limit stress-versus-time curve (analogous to the stress-versus-rupture time for tension) can be based on this behavior. The resulting correlation appears to have promise for estimating load capacities for "very short" columns.

(8) The possible existence of a lower column-load limit below which time-dependent collapse will not occur is discussed. It is concluded that available analytical methods cannot be used to predict such behavior because of their limited validity. A rational consideration of known creep behavior, however, indicates that there may be a temperature below which finite lower limits exist. Above this temperature, the lower limit value is zero.

BIBLIOGRAPHY

(1) Carlson, R. L., and Manning, G. K., "Investigation of Compressive-Creep Properties of Aluminum Columns at Elevated Temperatures", WADC Technical Report 52-251, Parts 2 (1954), 3 (1955), 4 (1955).

(2) Timoshenko, S., Theory of Elastic Stability, McGraw-Hill Book Company, Inc., New York, page 177 (1936).

(3) Shanley, F. R., "Inelastic Column Theory", Jour. of Aeronautical Science, page 261 (1947).

(4) Hill, R., The Mathematical Theory of Plasticity, Oxford University Press, London, page 38 (1950).

(5) Freudenthal, A. M., The Inelastic Behavior of Engineering Materials and Structures, John Wiley and Sons, Inc., New York, N. Y. (1950).

(6) Hilton, H. H., "Creep Collapse of Viscoelastic Columns With Initial Curvature", Jour. of the Aeronautical Sciences, Vol. 19, page 844 (1952).

(7) Rosenthal, D., and Baer, H. W., "An Elementary Theory of Creep Buckling of Columns", Proceedings of the First U. S. National Congress of Applied Mechanics, ASME, page 603 (1952).

(8) Kempner, J., "Creep Bending and Buckling of Nonlinearly Viscoelastic Columns", NACA TN 3137 (1954).

(9) Davenport, C. C., "Correlation of Creep and Relaxation Properties of Copper", Trans. Am. Soc. Mechanical Engineers, Jour. of Applied Mechanics, Vol. 60 (1938).

(10) Roberts, I., "Prediction of Relaxation of Metals from Creep Data", Proceedings, Am. Soc. Testing Materials (1951).

(11) Cottrell, A. H., Dislocations and Plastic Flow in Crystals, Oxford University Press, London, page 195 (1953).

(12) Shanley, F. R., Weight-Strength Analysis of Aircraft Structures, McGraw-Hill Book Company, Inc., New York, N. Y., Chapter 20 (1952), "Effect of Creep on Column Deflection", by T. P. Higgins, page 359.

(13) Libove, C., "Creep-Buckling Analysis of Rectangular-Section Columns", NACA TN 2596 (1953).

(14) Shanley, F. R., loc. cit., Chapter 16.

(15) Shanley, F. R., loc. cit., Chapter 19.

(16) Carlson, R. L., and Schwope, A. D., "A Method for Estimating Allowable Load Capacities of Columns Subject to Creep", Proceedings of the Second U. S. National Congress of Applied Mechanics, ASME, page 563 (1955). See also Part 2 of reference 1 above.

(17) Rosenthal, D., and Hasonovitch, D., WADC Technical Report 54-402, (1954).

(18) Bleich, F., Buckling Strength of Metal Structures, McGraw-Hill Book Company, Inc., New York, N. Y. (1952).

(19) Von Karman, T., Discussion to "Inelastic-Column Theory", by F. R. Shanley, Jour. of the Aeronautical Sciences, Vol. 14, page 267 (1947).

(20) Finlay, W. L., and Hibbard, W. R., "Some Effects of Applied Stresses on Precipitation Phenomena", Trans. AIME, page 255 (1949).

(21) Dix, E. H., "New Developments in High-Strength Aluminum Alloy Products", Trans. ASM, Vol. 35, page 130 (1945).

(22) Hoff, N. J., "Rapid Creep in Structures", Jour. of the Aeronautical Sciences, page 661, October (1955).

APPENDIX

COMPUTATION OF THE TIME-DEPENDENT TANGENT MODULUS FROM ISOCHRONOUS STRESS-STRAIN CURVES

The procedure involved in computing time-dependent tangent-modulus values for estimating column creep buckling loads can be illustrated by the use of the graphs in Figure 23.

Starting as in inset (a), with total strain (initial, instantaneous strain plus creep or time dependent strain) versus time curves, values of strain for various values of stress at a time, t_1, can be obtained. Using the corresponding values of stress and strain, the isochronous stress-strain curve for time, t_1, can be plotted as illustrated in inset (b). The static or time $t = o$ curve is also shown. By obtaining tangents to the curve of inset (b), a curve of time-dependent tangent-modulus versus stress can be constructed as shown in inset (c).

The computation of creep buckling average stresses is based on the use of the Euler formula,

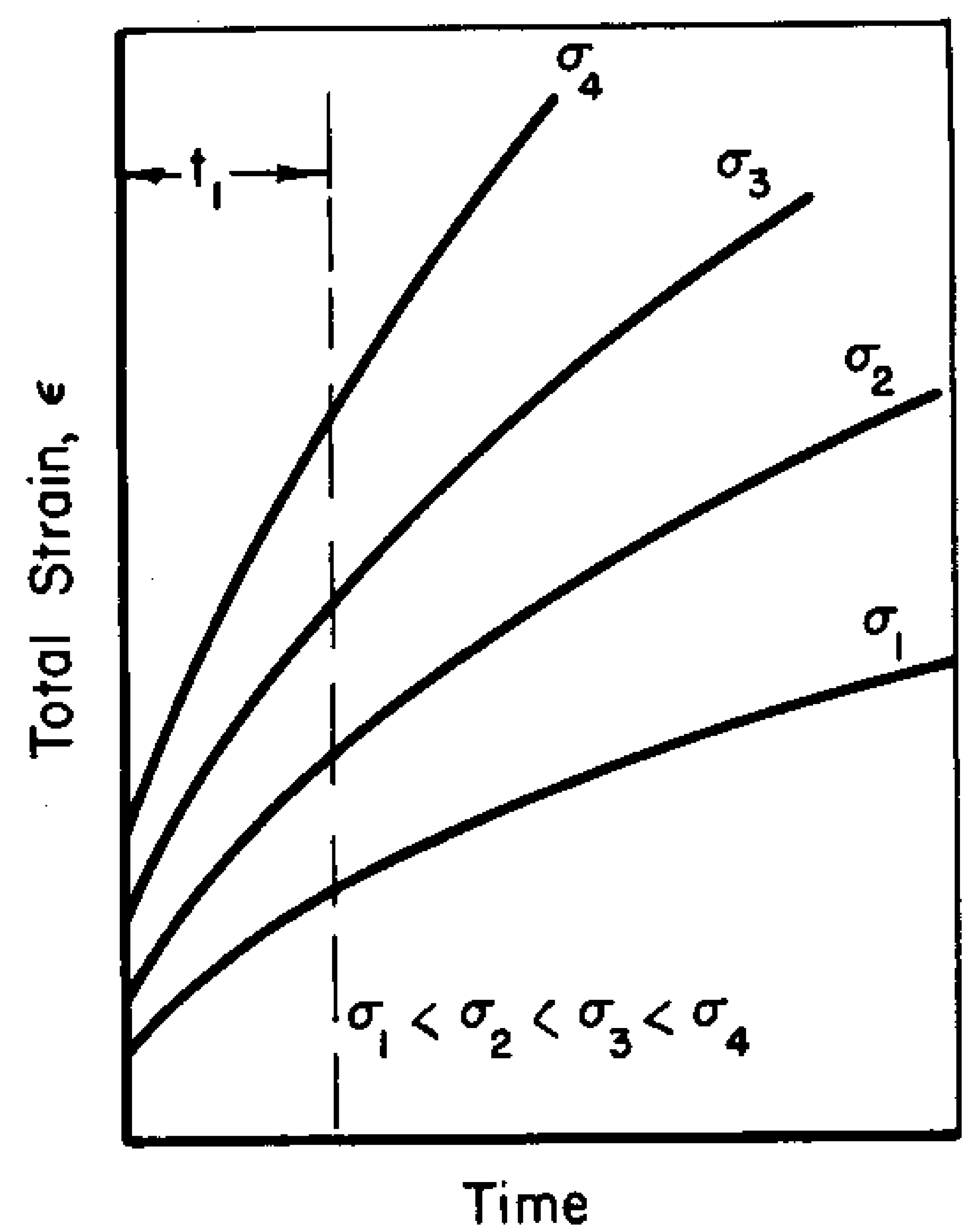
$$\sigma = \frac{\pi^2 E_T(t)}{\left(\frac{L}{r}\right)^2} .$$

where the elastic modulus is replaced by the time-dependent tangent-modulus, $E_T(t)$.

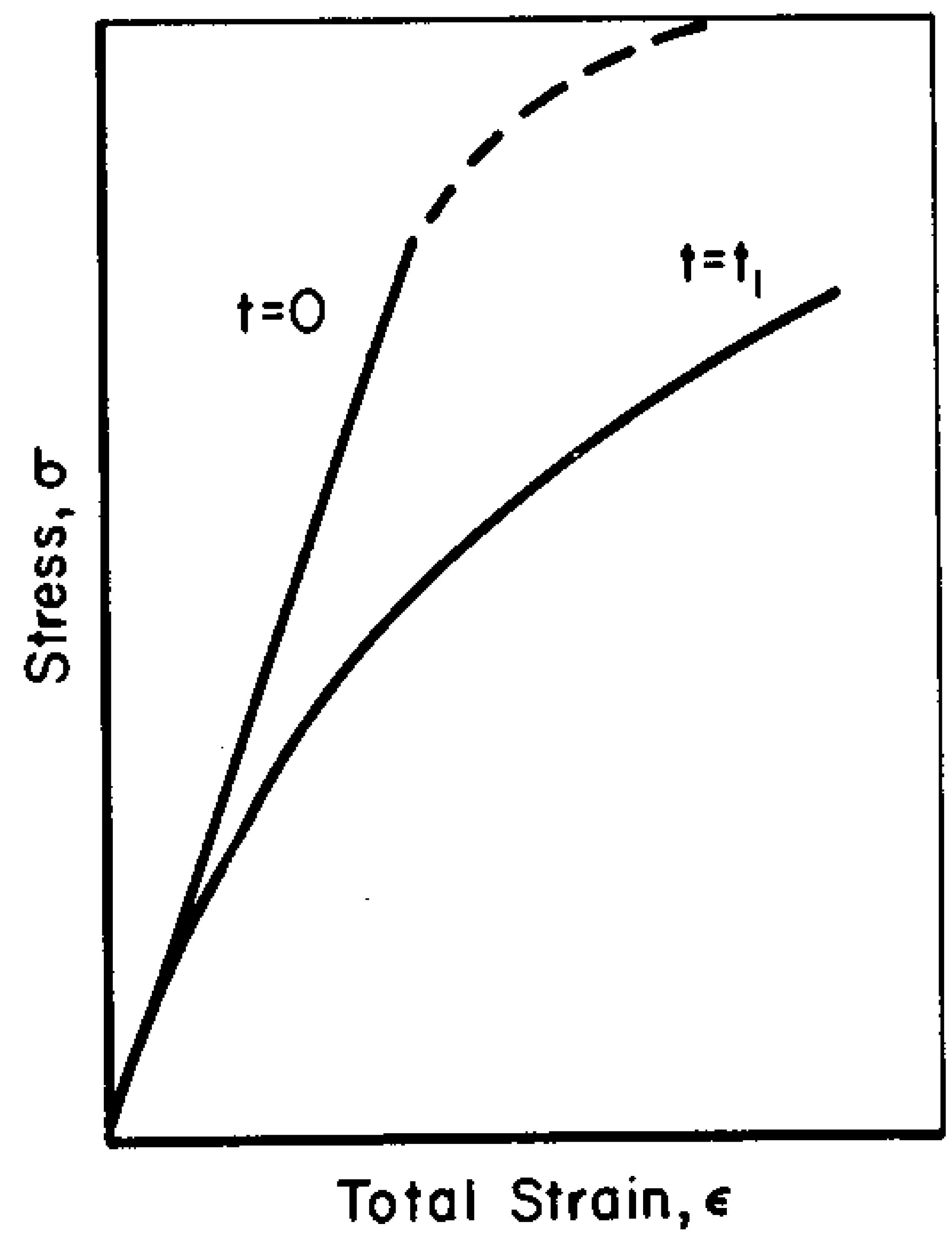
For convenience of computation, this can be rearranged to read

$$\frac{L}{r} = \pi \left(\frac{E_T}{\sigma}\right)^{1/2} .$$

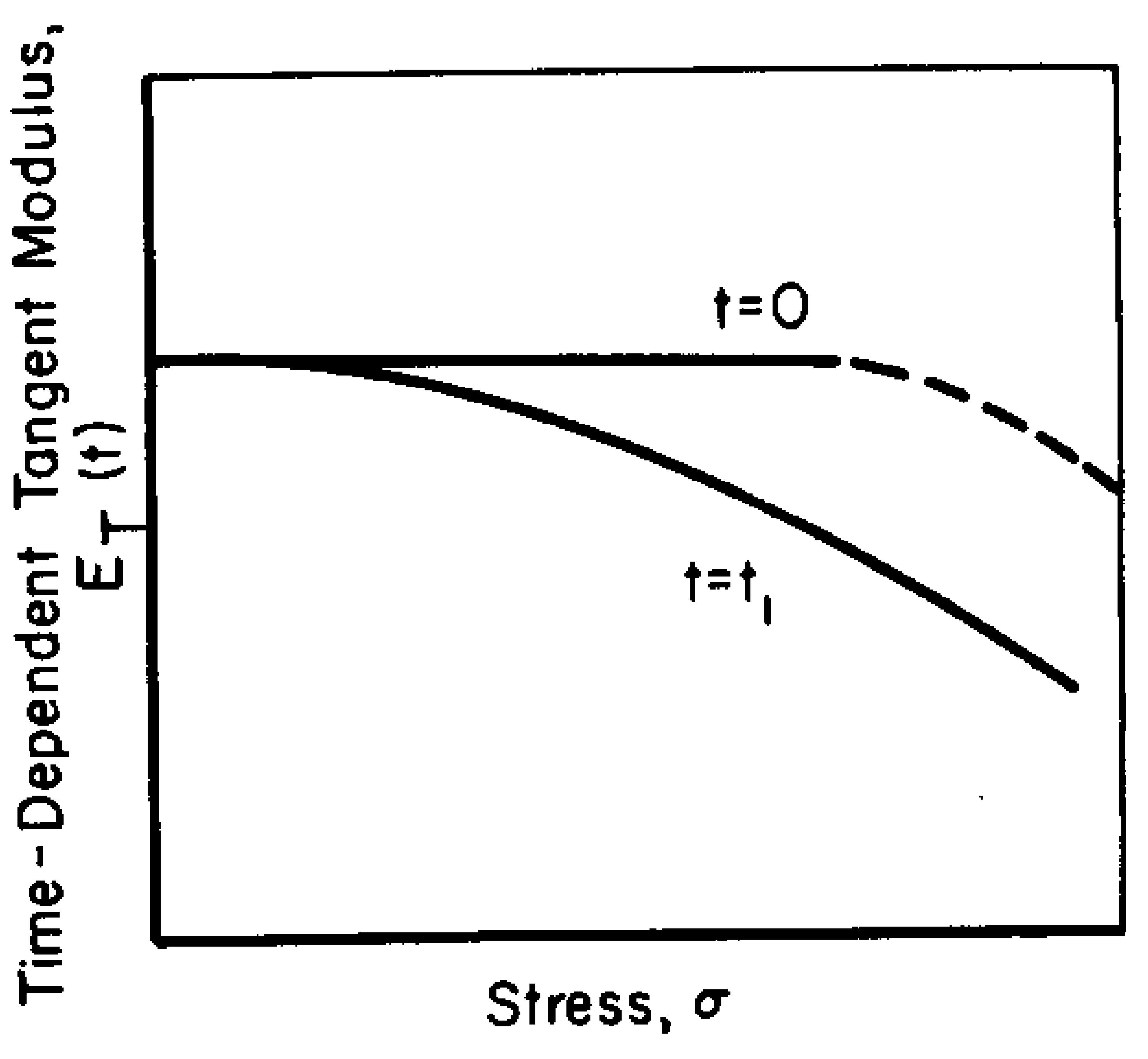
A table of the form shown below can then be prepared to yield curves of the type shown in inset (d).



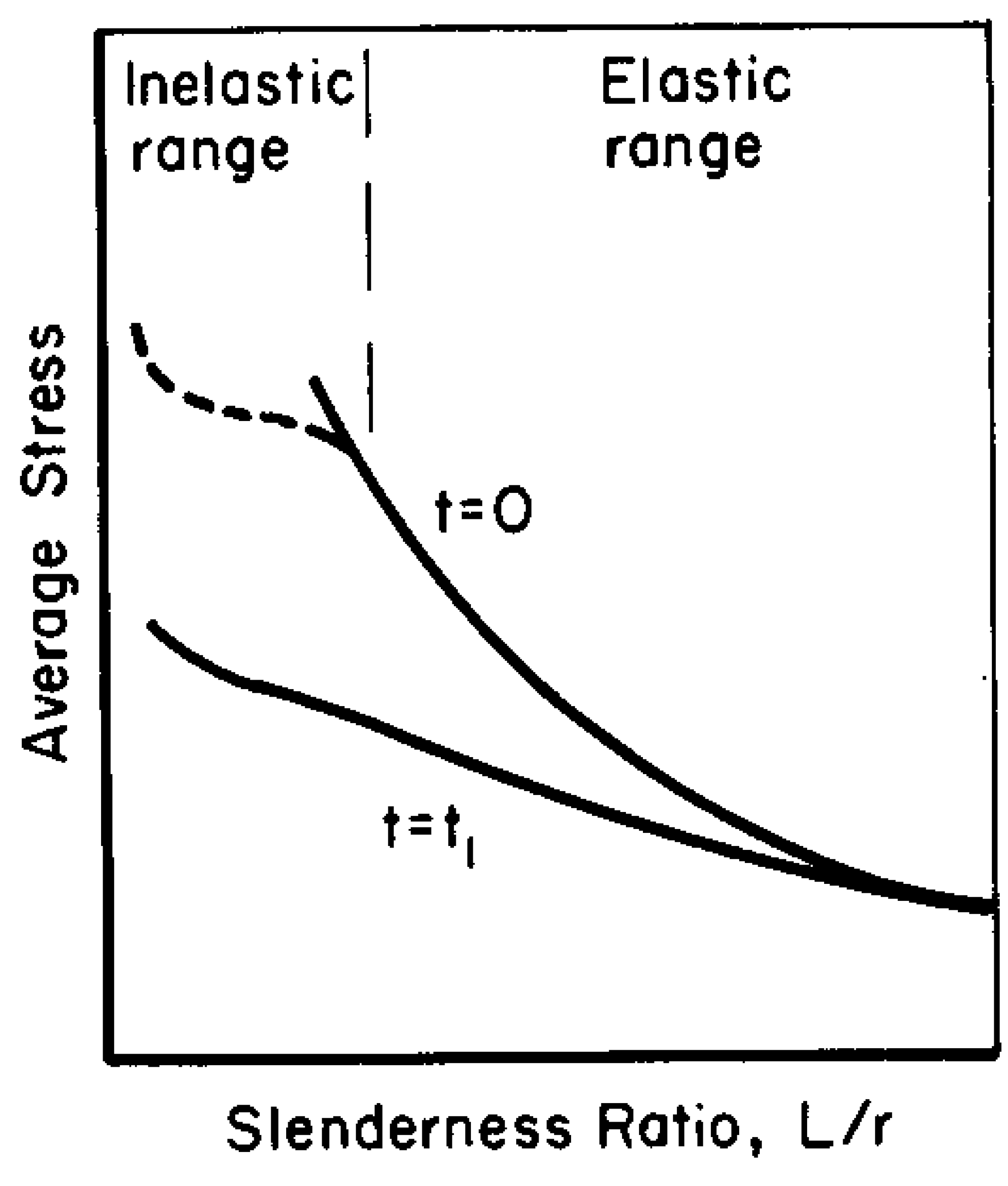
a. Strain-Versus-Time Data



b. Isochronous Stress-Versus-Strain Curve



c. Time-Dependent Tangent Modulus Versus Stress



d. Critical Average Stress-Versus-Slenderness Ratio

A-22350

FIGURE 23. USE OF THE TIME-DEPENDENT TANGENT MODULUS

$E_T(t_1)$	σ	$\pi\left(\frac{E_T}{\sigma}\right)^{1/2}$ or $\frac{L}{r}$
(psi)	(psi)	--
E_{T1}	σ_1	$\left(\frac{L}{r}\right)_1$
E_{T2}	σ_2	$\left(\frac{L}{r}\right)_2$
--	--	--

The preparation of average stress versus slenderness ratio curves for additional times can be accomplished simply by repeating the above procedure for the desired times.