THE EFFECT OF COLLIMATION ERROR ON PROPORTIONAL NAVIGATIONAL SYSTEMS

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Proportional navigation is a guidance system which corrects to a collision course by attempting to make the vehicles rate of change of heading proportional to the rotation rate of the line of sight to the target. In mathematical notation, then

(1) $\dot{\gamma} = \lambda\dot{\sigma}$

γ and σ are, respectively, the angles which the vehicle's velocity vector and the line of sight from that vehicle to the target make with an arbitrary reference line in inertial space. $\dot{\gamma}$ and $\dot{\sigma}$ are the time rates of change of these angles. λ, often called the navigation constant, is simply a constant of proportionality.

When line of sight angle information is provided by a radar seeker, filtering of this information is used to smooth target noise. Typically, the system may resemble that provided by two simple lo-pass filters in series. In practice, then, the navigation equation looks like this

(2) $$\tau_1 \tau_2 \frac{d^2 \dot{\gamma}}{dt^2} + (\tau_1 + \tau_2) \frac{d\dot{\gamma}}{dt} + \dot{\gamma} = \lambda\dot{\sigma}$$

τ_1 and τ_2 are the time constants of the two lo-pass filters. This equation can be recognized as a description of damped simple harmonic motion.

Radome, or more properly, collimation errors cause a displacement of the apparent line of sight from the true line of sight. If the component of this displacement in the direction of motion of the true line of sight is called η, then the effect of this error must be taken into account by adding $\lambda\dot{\eta}$ to the right hand side of Equation (2). Now the system is operating on an apparent line of sight motion, $\dot{\sigma}_A$.

(2a) $$\tau_1 \tau_2 \frac{d^2 \dot{\gamma}}{dt^2} + (\tau_1 + \tau_2) \frac{d\dot{\gamma}}{dt} + \dot{\gamma} = \lambda(\dot{\sigma} + \dot{\eta}) = \lambda\dot{\sigma}_A$$

It is assumed that in the limited portion of the radome with which we are concerned the change of the collimation error, η, is linear with changes in the radar antenna offset angle from the vehicle's fore and aft axis. If this offset angle is called β, this statement of linearity may be written

(3) $\dot{\eta} = k\dot{\beta}$

The constant, K, is the collimation or radome boresight slope. It should be noted that K is positive for a change in apparent line of sight in the same direction as the change in offset angle which caused it.

To show how the collimation boresight error interacts with the vehicle's turning dynamics, some further substitutions must be made. First, the offset angle, β, may be written in terms of the angle, θ, which the vehicle axis makes with the original inertial reference line and the true line of sight σ. This can be done strictly only when the antenna axis is identical with the true line of sight. In practice, tracking errors are very small compared to offset angles and this simplification gives accurate results where K is of the order of 0.1 or less. With this understanding, then,

(4) $$\dot{\beta} = \dot{\theta} - \dot{\sigma}$$

Here, again, $\dot{\beta}$ is positive if the offset angle is increasing. Substituting once more into Equation (2a)

(5) $$\tau_1 \tau_2 \frac{d^2 \dot{\gamma}}{dt^2} + (\tau_1 + \tau_2) \frac{d\dot{\gamma}}{dt} + \dot{\gamma} = \lambda\dot{\sigma} + K\lambda\dot{\theta} - K\lambda\dot{\sigma}$$

$$= \lambda(1 - K)\,\dot{\sigma} + K\lambda\dot{\theta}$$

The axis of a turning vehicle is not in general coincident instantaneously with its velocity vector. The angular difference is called the angle of attack, α, and in general for smooth turning is in the same plane as the turning trajectory. One may describe this mathematically as follows:

(6) $$\alpha = \theta - \gamma$$

or rearranging and differentiating with respect to time

(6a) $$\dot{\theta} = \dot{\alpha} + \dot{\gamma}$$

The configuration of the vehicle determines the angle of attack necessary to cause a given turn rate. Thus it may be written

(7) $$\alpha = \tau_S \dot{\gamma}$$

The constant of proportionality $_S$ is called the vehicle's turning time constant. Substituting (7) and (6a) back into (5),

(8) $$\tau_1 \tau_2 \frac{d^2 \dot{\gamma}}{dt^2} + (\tau_1 + \tau_2) \frac{d\dot{\gamma}}{dt} + \dot{\gamma} = \lambda(1 - K)\,\dot{\sigma} + K\lambda\left(\dot{\gamma} + \tau_S \frac{d\dot{\gamma}}{dt}\right)$$

or

(8a) $\tau_1 \tau_2 \frac{d^2 \dot{\gamma}}{dt^2} + (\tau_1 + \tau_2 - k\lambda\tau_S) \frac{d\dot{\gamma}}{dt} + (1 - k\lambda) \dot{\gamma} (1 - k) \dot{\sigma}$

Equation (8a) is more complicated than Equation (2) but also describes damped simple harmonic motion. It can be seen that when a steady state condition is reached, i.e. the higher derivatives have vanished, the perturbed proportional navigation equation becomes

(9) $\dot{\gamma} = \frac{\lambda (1 - k)}{1 - k\lambda} \dot{\sigma}$.

It can be seen that if k becomes as large as $1/\lambda$, the system gain becomes infinite and further increases in k will actually cause acceleration in the wrong direction. This, then, describes one stability limit on collimation boresight slope i.e.

(10) $k\lambda \leq 1$

In order that damped simple harmonic motion be in fact damped and therefore stable, the first order or damping term must be positive. Therefore, another condition for guidance stability in the presence of a boresight slope is derived by setting the first order term in (8a) equal to, or larger than zero, hence,

(11) $\tau_1 + \tau_2 - k\lambda\tau_S \geq 0$

or

(11a) $k\lambda \leq \frac{\tau_1 + \tau_2}{\tau_S}$

These equations describe then the maximum positive collimation boresight slope consistant with guidance stability.

Negative stability limits can be derived in many cases of interest by including in the guidance equation the vehicle's transient turn rate response to rudder signals. This response is in general of the form of another damped harmonic oscillator. If the resonant frequency of the vehicle's transient response is considerably larger than the tracking system resonant frequency, the following approximate negative stability criterion may be derived.

(12) $k\lambda > \frac{-2 \zeta_0 \omega_0 \tau_1 \tau_2}{\tau_2}$

Here ω_0 and ζ_0 describe the resonant frequency and damping constant of the simple harmonic turn rate response, and τ_1, τ_2 and τ_S are as described before.

If the mathematical model of the guidance system is extended to include both the pitch and yaw planes of guidance it is found that similar stability criteria exist with regard to the component of collimation error which is at right angles to η. A collimation cross-talk slope k_x is described as follows:

(13) $$\dot{\eta}_x - k_x \dot{\beta}$$

Here $\dot{\eta}_x$ is the apparent motion of the line of sight at right angles to the change in offset angle. In this two dimensional case the stability limits for crosstalk and boresight slopes are interdependent and a typical stability boundary diagram is shown in Figure II.

It should be noted, however, that the effect of collimation errors on guidance accuracy become pronounced long before the system becomes unstable. A typical curve of guidance miss due to scintillation noise as a function of positive boresight collimation slope alone is shown in Figure III.

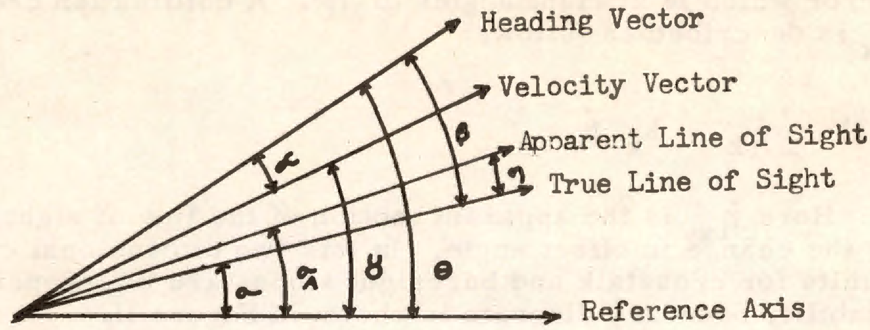


Figure 1

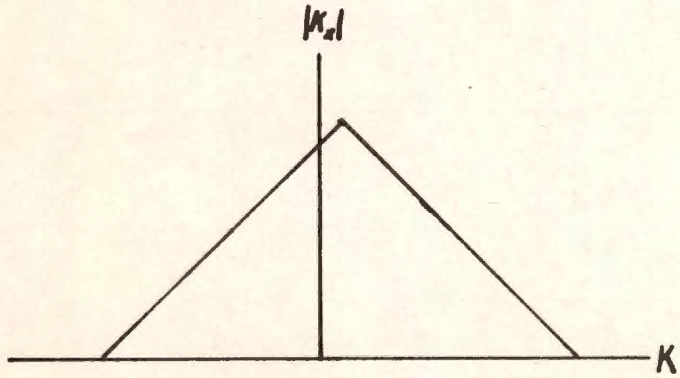
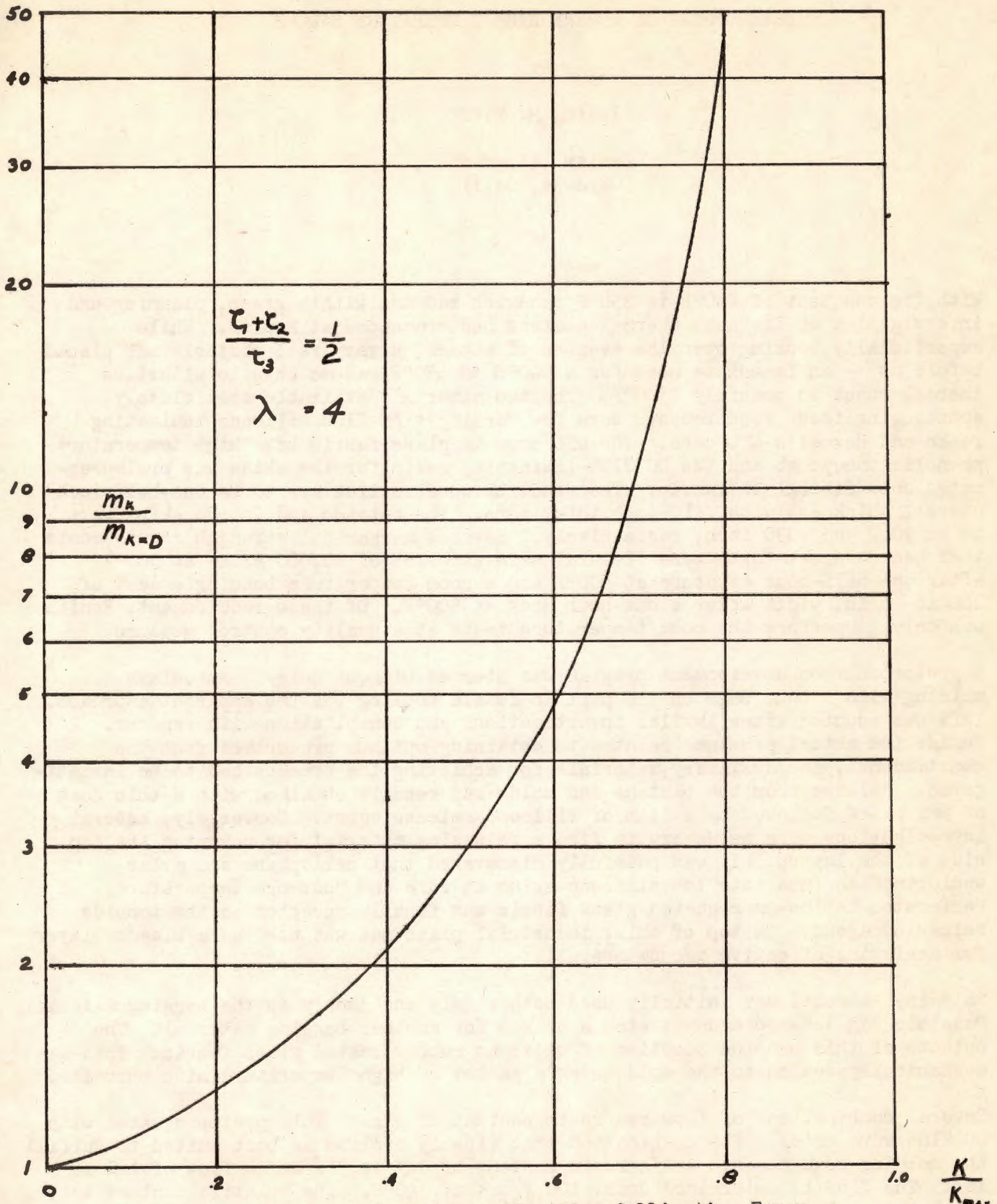


Figure 2



Increase in Miss Due to Positive Collimation Error Slope when Guidance System is Excited by White Noise

Figure 3