

DIRECTIONAL DAMPING OF THE GLOBAL VIBRATION MODES OF TUBULAR STRUCTURES BY CONSTRAINED-LAYER TREATMENTS

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ABSTRACT

Among the types of vibration modes that may need to be damped in thin-walled structures are those involving low-order, long-wavelength bending, torsion, or extension. These modes are characterized by vibratory stresses that are uniform or nearly uniform through the thickness of the wall. Although more commonly used for the control of panel bending, shell bending, and other local modes, segmented constrained-layer damping treatments can also provide effective damping for the control of such global vibrations. Methods described for the prediction of global-mode damping include new interpretations of closed-form solutions generated previously by Torvik. A finite-element implementation of the strain-energy principle of Ungar and Kerwin is also described. The axial length of the structure spanned by a single segment of damping treatment has been assumed much smaller than a vibration wavelength at the frequencies of interest, making static or quasi-static analyses useable. Damping loss factors calculated by each of these methods compare well with measurements on an assembly of damped, hollow, rectangular-cross-section beams.

INTRODUCTION

A need often arises to design passive vibration-damping treatments that can reduce responses of several classes of vibration modes of a given structure. Specifically, the low-frequency, long-wavelength, global bending modes, the torsional modes, and possibly even the axial modes of a thin-walled tube may all need to be damped. All of these global modes can play major roles in the transmission of low-frequency noise and vibration, and their responses are often difficult to control.

Effective applications of constrained-layer treatments to damp local plate-bending and shell-bending modes, (Figure 1b), are common. However, the use of these treatments to damp the global beam-bending modes of tubular structures, (Figure 1a), and their torsional and axial modes, is not frequently reported. A previous paper [1] showed that a given constrained-layer treatment, properly designed, can provide effective damping of the global bending modes, concurrent with high damping of the local plate- or shell-bending modes, of open- or closed-section, thin-walled beams. Segmentation of the constraining layers was shown to be vital in obtaining such combinations of global- and local-mode damping performance.

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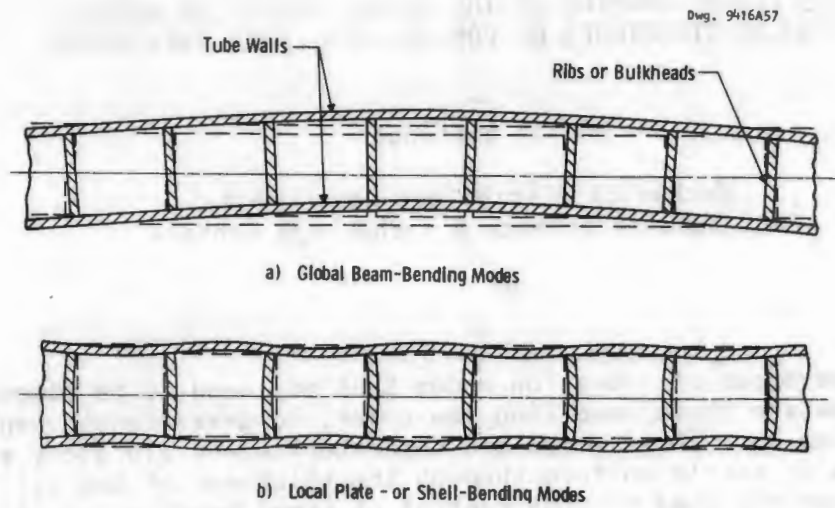


Figure 1 - Global vs. local bending modes of vibration in a thin-walled tubular structure

Constrained-layer treatments may also be required to damp the global modes of more complex structures in which each component simultaneously undergoes several different directions of vibration. One example would be the combined bending and torsion in members of a machinery support structure vibrating as illustrated in Figure 2. If the damping performance can be separately predicted for each direction of vibration in each component, the overall system damping values for combined modes of the entire structure can then be determined.

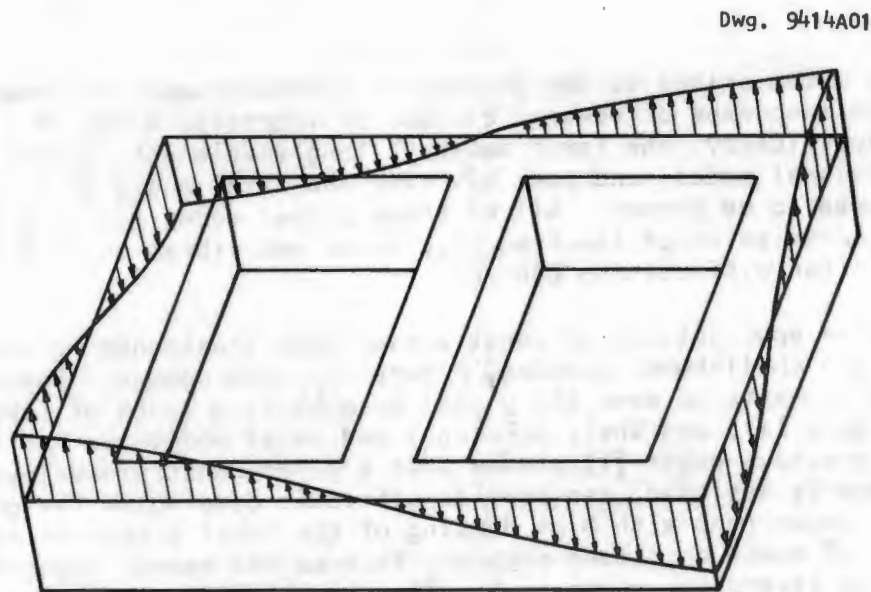


Figure 2 - Displacement shape for a combined torsional and bending vibration mode of a machinery support structure comprised of tubular members

This paper demonstrates that a given constrained-layer treatment can be effective in simultaneously damping the global bending, torsion, and axial vibrations of a tubular structural component. Methods are described for predicting or estimating the damping values for the individual directions of

vibration, including new interpretations of closed-form analyses performed previously by Torvik [2] and finite-element implementations of the strain energy principle of Ungar and Kerwin [3]. Experimental results on a damped box-beam test assembly, confirming the validity of these methods, are also given. Applications are primarily to closed tubular cross-sections, although the cross-section may be of almost any shape.

PREVIOUS ANALYSES OF DAMPING TREATMENT SEGMENTATION

The Ross-Ungar-Kerwin theory [4] is the basis of a method that is widely used for the design of constrained-layer treatments to damp flexural vibration waves in plates and beams. However, its application is valid only in cases where both the damping treatment and damped member are continuous or where any cuts between adjacent damping treatment segments happen to coincide with nodal points in a standing wave.

Parfitt [5] extended the Ross-Ungar-Kerwin theory to cases where the damping treatment is cut at uniform intervals along the length of a vibrating beam. Parfitt found that the low-frequency (long-wavelength) damping performance was dramatically improved by such axial segmentation, and he derived an expression for the optimum spacing of cuts in terms of the thicknesses and moduli of the constraining layer and the constrained VEM layer. Assuming constant VEM properties, the optimum segmentation would provide, at very low frequencies, damping performance almost as high as the peak damping of the continuous treatment. In this derivation the damping treatment was assumed sufficiently compliant to have no influence on the strain distribution in the base structure and to acquire negligibly small amounts of stored strain energy.

In a similar derivation of damping performance and optimum segment length by Plunkett and Lee [6], the same assumption of damping treatment compliance was made. Kress [7] derived an expression for optimum segment length which is sensitive to the properties of the base structure in addition to those of the damping layers.

Torvik [2] analyzed two different cases of quasi-static vibratory loading of constrained-layer-damped structural members, both of which can be applied to the global-mode damping of tubular structures having segmented damping treatments. Both of these analyses account for strain energy stored in all components of the system and are therefore applicable to stiff damping treatments such as are used in the experiments described later. These analyses are described and interpreted for use in global-mode damping applications in the next section.

DAMPING PERFORMANCE PREDICTION METHODS FOR GLOBAL MODES

Three alternate approaches for predicting or estimating the global-mode damping performance of segmented constrained-layer treatments on tubular structures are described below.

Closed-Form, Quasi-Static Solution for Vibratory Flexure

Torvik [2] analyzed the case of quasi-static moment loading of a finite-length cantilever beam covered by a single segment of constrained-layer treatment which is built-in at the root end and free at the opposite end as

shown in the uppermost view in Figure 3. Flexure in both the base member and the constraining layer is modeled using Bernoulli-Euler beam theory. The solution is also directly applicable to a free-free, moment-loaded beam of twice the length of the cantilever beam.

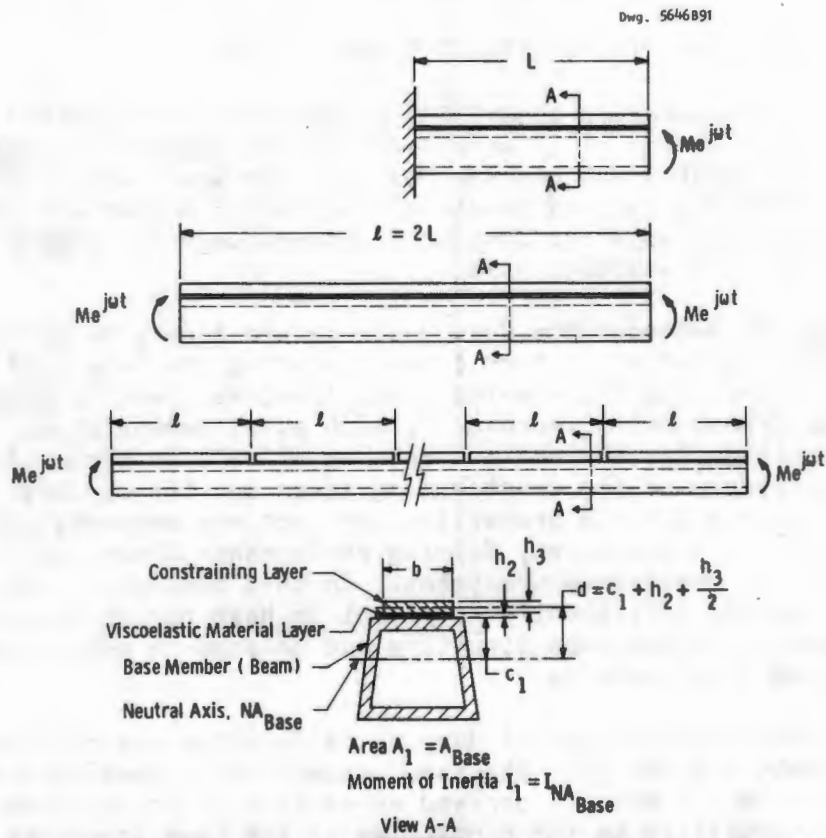


Figure 3 - One-side-damped cases represented by Torvik's closed-form, quasi-static solution for vibratory flexure

For sufficiently long wavelengths, the beam analyzed can also be viewed as a building block of a long, continuous vibrating beam of the same cross-section, damped by a segmented treatment as shown in a lower view in Figure 3. This analysis applies only to the installation of damping treatment on a planar surface that is oriented parallel to the neutral surface of the base member, but, as indicated by Figure 4, it can also be interpreted as applicable to structures that have symmetric cross-sections and are damped on two sides.* Torvik's analysis applies to the dynamic vibratory case, i.e., to the case in which there are transverse vibratory inertia forces, provided that the instantaneous standing-wave bending-moment distribution is nearly constant over the length of beam covered by each segment. That is, the bending wavelength must be long in comparison with the segment lengths.

*Setting area A_1 equal to infinity models the condition of zero extension at the base member centroid, brought about by symmetry.

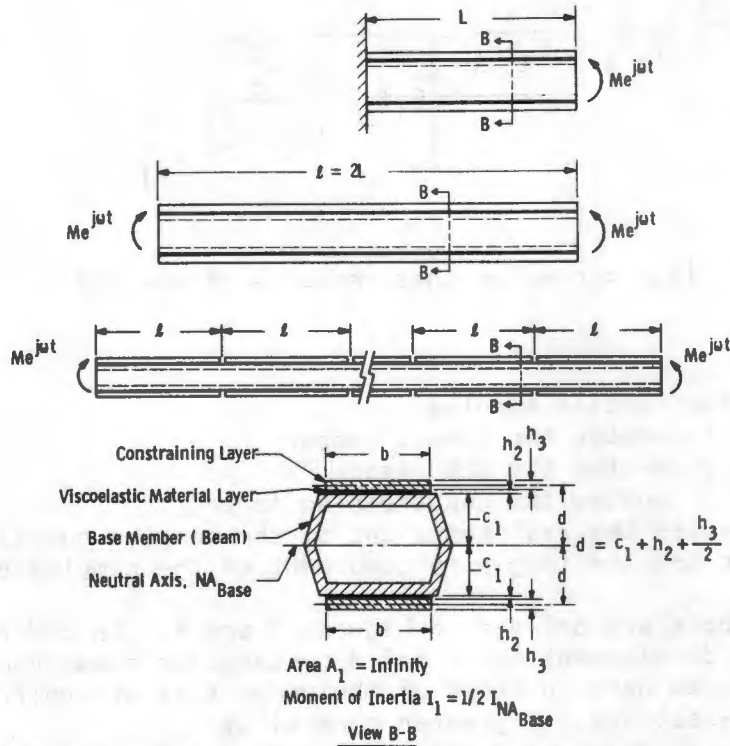


Figure 4 - Two-sides-damped cases represented by the closed-form, quasi-static solution for vibratory flexure

Torvik developed expressions for stored and dissipated energy leading to the following closed-form expression for system damping in flexure:

$$\eta_f = \frac{\eta_2 G_2 d^2}{h_2 \left[E_1 I_1 + \frac{E_3 b h_3^3}{12(1-\nu_3^2)} \right]} \cdot \frac{F(\theta)}{\text{Re} \left[\frac{L \tanh \theta}{\theta} - \frac{gL^3}{\theta^3} (\tanh \theta - \theta) \right]} \quad (1)$$

where η_f = flexural damping loss factor = 2 x percent critical damping /100, and

$$g = \frac{G^* b}{h_2} \left[\frac{1}{E_1 A_1} + \frac{1}{E_3 b h_3} \right]$$

$$F(\theta) = - \frac{1}{2} \frac{\text{Im} [\tan(i\theta)/(i\theta)]}{\text{Re} [\theta] \text{Im} [\theta]}$$

$$\theta = \left\{ \frac{G^* b L^2}{h_2} \left[\frac{1}{E_1 A_1} + \frac{1}{E_3 b h_3} + \frac{d^2}{E_1 I_1 + \frac{E_3 b h_3^3}{12(1-\nu_3^2)}} \right] \right\}^{1/2}$$

where $G^* = G_2(1 + j\eta_2) =$ complex shear modulus of the VEM

$$j = \sqrt{-1}$$

$E =$ Elastic tensile modulus

Subscript 1 denotes the damped member

Subscript 2 denotes the VEM layer

Subscript 3 denotes the constraining layer

$\text{Re}(z)$ denotes the real component of the complex quantity z

$\text{Im}(z)$ denotes the imaginary component of the complex quantity z

and all other symbols are defined in Figures 3 and 4. In contrast with Torvik's original development for a solid rectangular cross-section, these results are expressed here in terms of the properties of arbitrarily-shaped base member cross-sections for greater generality.

Closed-Form Quasi-Static Solution for Vibratory Extension

For thin-walled tubular structure cross-sections, the distribution of global-mode vibratory bending stresses is nearly uniform through the thickness of the wall. Under these conditions, which are illustrated in Figure 5, the bending-mode damping can be estimated by applying a solution of quasi-static extensional loading of a constrained-layer-damped structural member, derived by Torvik as an adaptation of an earlier analysis of lap joints by Avery [8]. This solution is also directly useable for predicting the damping of axial modes, and, as shown later, the torsional-mode damping values can also be reasonably close to the extensional damping value in some instances.

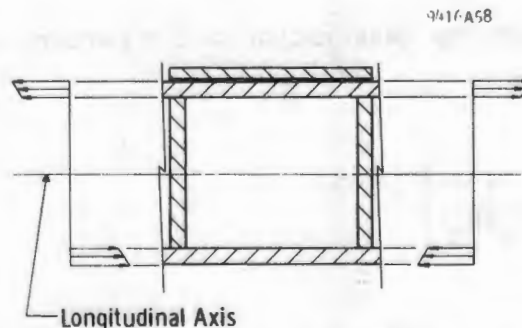


Figure 5 - Instantaneous stress distribution in a region of a damped tubular component undergoing global bending vibration, showing that the walls are in a state of extensional stress that is nearly uniform through the thickness

Torvik addressed the case of a member built-in at one end and covered by a single segment of damping treatment which is built-in at the root end and

free at the loaded end. This case is shown in the uppermost view of Figure 6. The solution also applies directly to a free-free, extensionally loaded bar of twice the length of the built-in bar, which is the case pictured in the middle view of Figure 6. Good agreement between calculated and measured extensional damping performance in constrained-layer-damped bars was obtained in an earlier study by the author [9].

Key assumptions in Torvik's derivation are that plane cross-sections in the damped member and in the constraining layer remain plane and translate without rotating. While this pure-translational deformation assumption does not apply in a strict sense, it is a good approximation to the actual conditions in a tubular structure in bending. Torvik derived the following formula for damping under vibratory extension:

$$\eta_e = \frac{\eta_2 G_2 b L^2}{E_1 A_1 h_2} \cdot \frac{F(\beta)}{\left[\frac{1}{\left[\frac{E_3 h_3 b}{E_1 A_1} + 1 \right]} + \frac{G_2 b L^2}{E_1 A_1 h_2} \operatorname{Re} \left[\frac{(1+j\eta_2) \tan(j\beta)}{j\beta^3} \right] \right]} \quad (2)$$

where η_e = extensional damping loss factor

$$F(\beta) = - \frac{1}{2} \frac{\operatorname{Im}[\tan(j\beta)] / (j\beta)}{\operatorname{Re}[\beta] \operatorname{Im}[\beta]}$$

$$\beta = \left\{ \frac{G^* b L^2}{h_2} \left[\frac{1}{E_1 A_1} + \frac{1}{E_3 b h_3} \right] \right\}^{1/2}$$

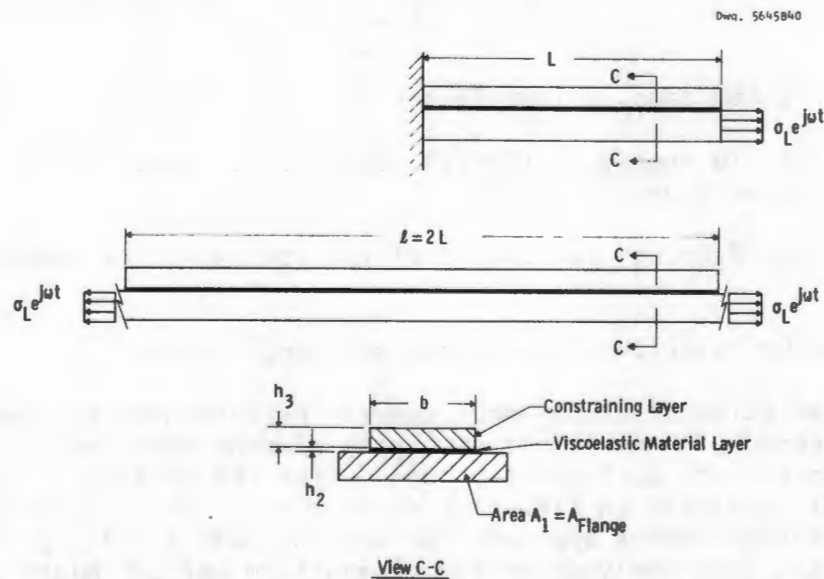


Figure 6 - The case of quasi-static, vibratory extension, analyzed earlier by Torvik, as applied to the portion of a structure wall beneath one segment of damping treatment

and all other quantities are defined in conjunction with Equation 1 above, or in Figure 6. Whereas Torvik assumed all three layers to be of equal width, which vanished in the formula he derived, his formula has been re-expressed here in terms of damping treatment width, b , and damped member area, A_1 , to apply to situations where the surface area is partially covered by the damping treatment.

In a separate treatment of the subject of segmentation, Kerwin and Smith [10] stated that achievable loss factors in extension are lower than those in flexure for a given system cross-section. This rule decidedly holds true for solid damped-member cross-sections.* However, as demonstrated in a later section, the difference diminishes for hollow cross-sections, and in the limit of beams whose section properties are dominated by flat, widely spaced parallel flanges, the optimum global bending loss factor tends toward equality with the optimum extensional loss factor.

To the extent that strain energy is stored in other regions of the tubular structure that are untreated, the extensional damping calculated in this manner will overestimate the true overall global-bending-mode loss factor. The experimental results in a later section demonstrate, however, that the extensional damping estimate can be reasonably close to the true bending-mode damping value for rectilinear-cross-section, thin-walled tubular beams having segmented damping treatment on all panels.

Finite-Element Modeling

In recent years there have been many applications of the finite-element method to predictions of constrained-layer damping performance. In many cases these predictions have made direct use of the strain-energy principle of Ungar and Kerwin [3], which is expressed symbolically as

$$\eta_s = \frac{\sum_j \eta_j U_j}{\sum_j U_j} \quad (3)$$

where η_s = system damping loss factor

U_j = strain energy in the j th region or component of the system in a given mode of vibration

η_j = the damping loss factor of the j th region or component.

The modeling to be described here is one such application.

The prediction of global-mode damping performance of segmented treatments introduces no new kinds of finite-element modeling requirements but may, if wavelengths are sufficiently long, offer the analyst the option to perform a static analysis in lieu of a dynamic one. Major incentives for choosing the finite-element approach for bending-mode damping predictions in preference to the previously-described closed-form methods might be curvature, slope, or irregularities such as cut-outs in the structure walls, which could

*A rule of thumb is that optimum flexural loss factors are roughly a factor of three times larger than the corresponding optimum extensional loss factors for solid cross-sections.

invalidate the use of the latter methods. In the case of torsional modes, no closed-form solutions appear to be available as alternates to finite-element modeling.

The finite-element modeling described here is an adaptation of an earlier approach of Killian and Lu [11] in which short, offset beam elements are used in lieu of brick elements to represent the VEM layer. Although originally used in conjunction with the direct frequency-response method using a complex-modulus representation for the VEM, this approach also lends itself to elastic modeling of the VEM layer using the modal strain energy principle. As in all other elastic-modeling implementations, which are quite common in finite-element predictions of constrained-layer damping, the results are approximations in that real-valued deformation shapes are generated in lieu of complex shapes. All modeling described here has been performed using the WECAN finite-element modeling program [12].

GLOBAL-MODE DAMPING EXPERIMENTS ON A TUBULAR STRUCTURE

Figure 7 is a photograph of an all-steel box-beam test assembly specially designed and constructed to verify global-mode damping

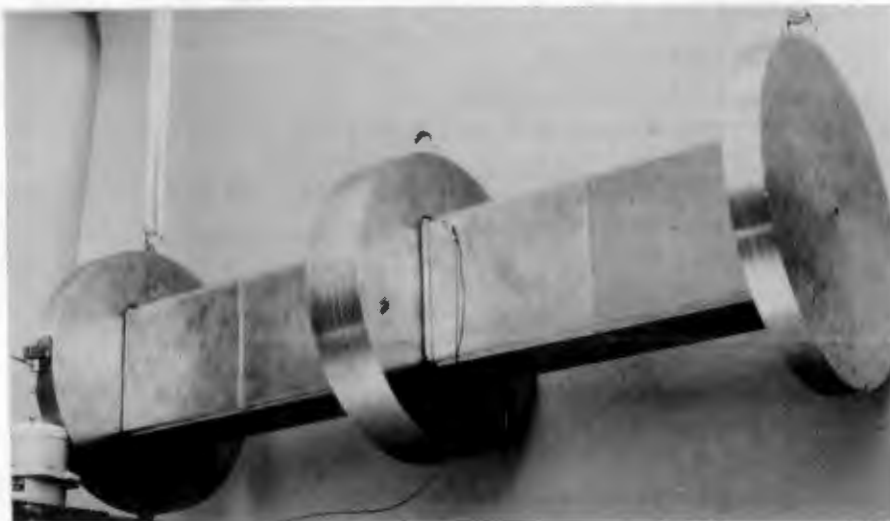


Figure 7 - Mass-loaded box-beam test assembly, with segmented damping treatment in place, undergoing torsional damping measurements

performance predictions, generated by each of the methods described above, for individual directions of vibration. The tubular portions represent the construction of the larger machinery support structure pictured in Figure 2, but with a one-dimensional configuration, it would be possible to measure the individual damping values without coupling among the various directions of vibration. The three solid circular discs made it possible to use a compact test configuration by providing mass and inertia loadings simulating reactions that would be present in a continuous beam of much greater length. Figure 8 shows the design details and illustrates that, in general, the cuts between damping treatment segments need not coincide with the placement of ribs or stiffeners. The next section addresses the fact that damping treatment was applied on all four sides of the rectangular-cross-section beams, whereas the closed-form solution for flexure applies to cases where only one or two sides are damped.

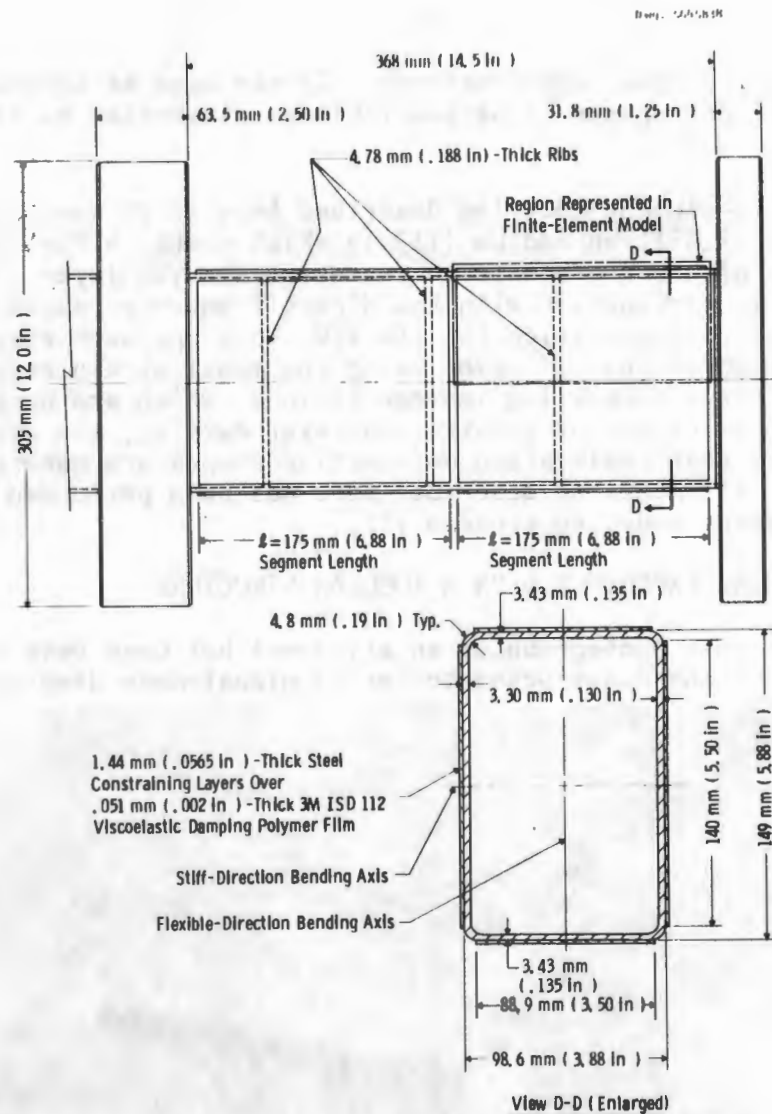


Figure 8 - Design details of the box-beam test assembly

The beams were fabricated by bevel-welding ribs of cold-rolled steel between short sections cut from 152 mm x 102 mm x 4.8-mm wall (6 in x 4 in x 3/16-in wall) structural-steel rectangular tubing. Flat surfaces were machined on the beam-wall exteriors before the weld-attachment of the discs and the installation of the damping treatment. Because of weld distortion there were sizeable variations in wall thickness, and the thickness values shown represent averages after machining. The total mass of the damped assembly was 91.6 kg (202lb), of which the undamped beams comprised 15.3 kg (33.7 lb) and the damping treatment comprised 3.6 kg (8.0 lb).

The damping treatment was installed in segments of 1.44 mm (0.0565 in)-thick constraining layer over 0.051mm (0.002 in)-thick 3M ISD 112 viscoelastic damping polymer, giving a fairly large stiffness ratio ($E_3h_3/E_1h_1 = 0.43$). Both sides of the damping polymer layers were bonded to the adjacent steel surfaces using 3M 1838 epoxy structural adhesive. Care was taken to avoid adhesive bridging between the constraining layer segments and the base structure. All damping performance calculations were performed using the frequency-dependent VEM property values plotted in Figure 9. These

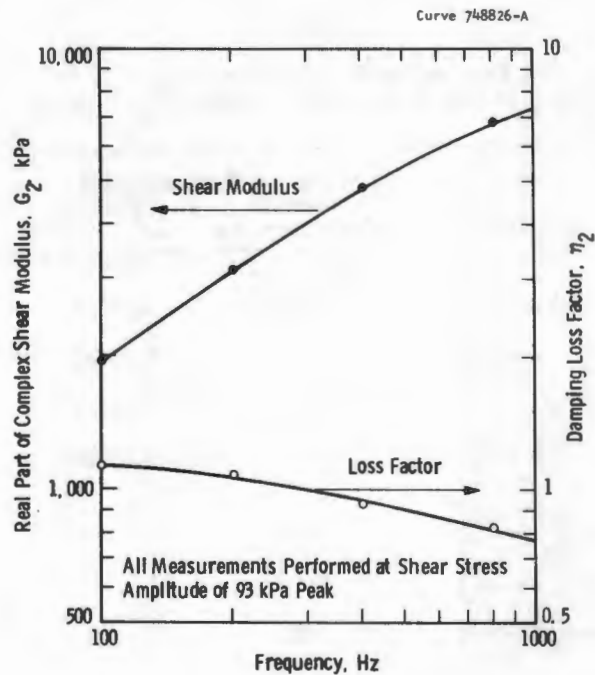


Figure 9 - Properties used in modeling the 3M ISD 112 constrained damping layers at the 22°C (72°F) test conditions

properties were measured on a different lot of the same damping polymer using a previously described measurement technique [13]. The extent of lot-to-lot variation in properties is not known.

The various bending and torsional modes of the assembly before and after installation of the damping treatment were individually excited in sinusoidal dwells using an eccentrically positioned shaker. Measurements of the higher-frequency axial modes were also performed by driving on the center axis. The damping values were measured by means of rates of decay from these dwells at shaker cutoff. All tests were performed at approximately 22°C (72°F) room temperature.












COMPARISON OF CALCULATIONS AND MEASUREMENTS

Table 1 lists measured natural frequency and damping values for the global modes of the test assembly before and after the installation of damping treatment. The results show that the damping due to the add-on treatment far exceeded the inherent damping of the base structure. The increases in natural frequencies show that the net effect of the damping treatment installation was to increase the dynamic stiffnesses in greater proportion than the increase in mass.

Although damping treatment was installed on all four sides of the beam portions, calculations using the closed-form solution for flexure with damping treatment on only two sides are nonetheless useful. Table 2 compares calculations for two different choices of area moment of inertia, one choice being half that of the full base-member cross-section as per Figure 4, and the other being that of only one of the two "flanges" (sides oriented parallel to the neutral axis). The former choice produces a lower bound to the true damping, because the damping contribution of the treatment on the sides perpendicular to the neutral axis is not taken into account. By the strain-

Table 1

Measured Natural Frequencies and Damping Values for Global Vibration
Modes of the Mass-Loaded Box-Beam Test Assembly

Mode Type	Mode Identity and Shape	Undamped Configuration		Damped Configuration	
		Natural Frequency, Hz	Damping Loss Factor	Natural Frequency, Hz	Damping Loss Factor
Flexible-Direction Beam Bending	1st Mode 	218	0.00088	230	0.043
	2nd Mode 	669	0.00085	712	0.041
	3rd Mode 	845	0.00088	896	0.036
	4th Mode 	987	0.00070	1052	0.043
Stiff-Direction Beam Bending	1st Mode 	305	0.00086	324	0.039
	3rd Mode* 	1145	P	1229	P
	4th Mode 	1360	P	--	--
Torsional	1st Mode 	357	0.0010	383	0.052
	2nd Mode 	511	0.0010	558	0.049
Axial	1st Mode 	942	0.0031	1005	0.035
	2nd Mode 	1323	0.0030	1387	0.028

* Indicates point of minimum motion amplitude.

P Indicates measurement rejected as invalid due to coupling with local bending modes of beam wall panels.

* A stiff-direction bending mode analogous to the second flexible-direction mode could not be identified but is believed to have existed at a frequency coincident with the first extensional mode.

energy principle, the latter choice should predict the true damping if the ratio of VEM strain energy to total strain energy in the sides perpendicular to the neutral axis were equal to the corresponding ratio in the flanges. However, the results of finite-element modeling, discussed below, indicate that in both the flexible and stiff bending directions, the fraction of VEM to total strain energy in the perpendicular sides is somewhat less than that in the flanges. Therefore the use of only the flange moment of inertia produces an upper bound to the true damping in instances where the thicknesses are nearly constant throughout the beam cross-section.

Figure 10 plots the calculated closed-form flexural damping values from Table 2 together with a curve of calculated extensional damping. Frequency affects all of these calculations only to the extent that it governs the VEM properties. The extensional damping curve has been generated using, for a base member, a flange having 3.35 mm (0.132 in) thickness, which represents a weighted average of the thicknesses of the four sides. This simplified approach can be taken because the stiffness ratio, E_3h_3/E_1h_1 , is nearly constant around the periphery. The base member width has been regarded as equal to that of the damping treatment, but to account for the incomplete coverage, an area ratio of 0.875 has been used to pro-rate the resultant loss

Table 2

Damping Values Calculated for Mass-Loaded Box-Beam Test Assembly
Using Closed-Form Solution for Vibratory Flexure

Mode Type	Mode Identity	Natural Frequency, Hz	Damping Loss Factor*	
			Using Moment of Inertia of Full Cross-Section	Using Moment of Inertia of Flanges Only
Flexible- Direction Bending	1st Mode	230	0.0530	0.0639
	2nd Mode	712	0.0411	0.0491
	3rd Mode	896	0.0381	0.0454
	4th Mode	1052	0.0359	0.0428
Stiff- Direction Bending	1st Mode	324	0.0417	0.0564

* Calculated in accordance with Equation (1) and Figure 4 using VEM properties at damped natural frequency for each mode. Resultant loss factors have been pro-rated by a 0.949 length ratio to account for incomplete coverage by the treatment.

factors. The fact that the extensional loss factor curve is bounded from above and below by the calculated flexural damping values illustrates its usefulness as an estimator of global bending-mode damping. The peak in the extensional damping curve is due to the occurrence of an optimum combination of G_2 , h_2 , and l for the specific E_1, h_1, E_3 , and h_3 values used.

Also included in Figure 10 are the results of quasi-static finite-element modeling of damping under both directions of pure bending-moment loading and under torsional and axial loading. Figure 11 depicts the 1/8-region finite-element mesh that modeled one of the tubular portions of the assembly (see also Figure 8) and shows how these loading conditions were simulated by imposing constant or linearly-varying displacement distributions on one of the ends while the opposite end was restrained. The VEM layer was modeled elastically. The storage modulus and loss factor values used were based on an arbitrarily-chosen frequency of 340 Hz for all loadings. The computed damping loss factors have been reduced by an empirically derived factor of 1/1.2. This factor corrects for the use of elastically calculated deflection shapes to compute the strain energies in a viscoelastically damped structure having a VEM loss factor of about 1.0.

The closeness among the finite-element results for the various directions of vibration adds further validity to the notion of using an extensional damping calculation to estimate the global bending-mode damping. Furthermore, these finite-element results are in very close proximity to the closed-form solution results. It seems at first surprising that the torsional damping is in such close proximity to that of the other directions. However, Figure 12 illustrates that the pure-shear deformation of the individual panels under torsion produces an extensional deformation of damped panel slices oriented at 45° to the tube axis, and that the effective lengths of these slices are comparable to the axial lengths of the segments.

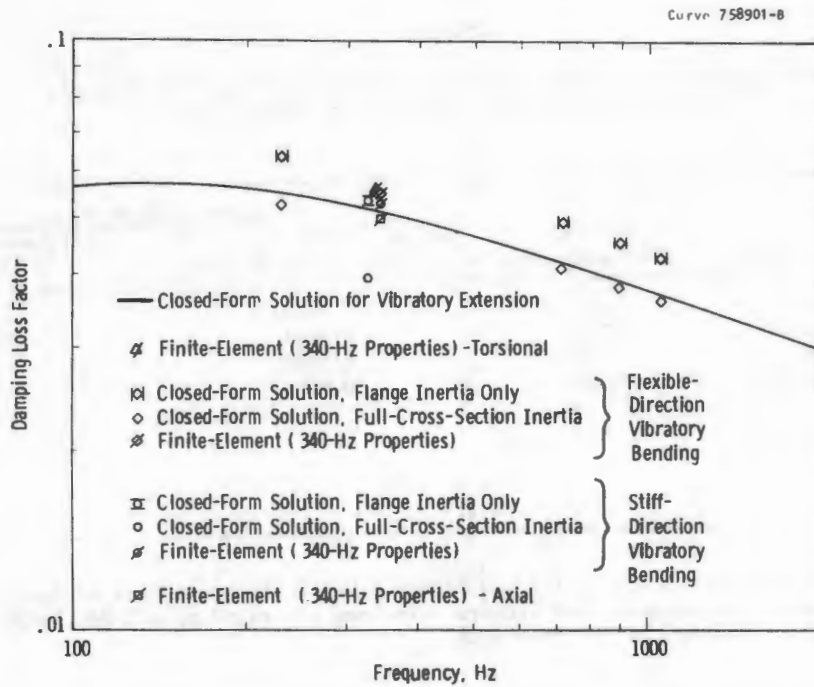


Figure 10 - Calculated damping performance for the box-beam test assembly

Finally, Figure 13 compares the closed-form extensional damping calculation with measured damping values for the global bending and extensional modes. There is considerable scatter in the measurements, but their trends appear reasonably well predicted by the extensional damping curve. The scatter may be partly attributed to the earlier-mentioned irregularities in wall thicknesses of the beam portions. In addition, transverse shear forces were undoubtedly present in some of the global bending modes, and these may have influenced the overall damping values differently for each of the modes.

DISCUSSION

The fact that the damping performance values plotted in Figures 10 and 13 are in the proximity of a peak is the result of the combination of damping treatment design parameters having been nearly optimized. This combination is particularly sensitive to the segment length parameter, l . The optimum global-mode loss factors are considerably smaller than the optimum loss factors for local-mode plate bending with this treatment (which would be in the vicinity of 0.15), but they do, nonetheless, represent useful and effective damping.

Circumferential segmentation of the damping treatment can play an important role in achieving good global-mode damping performance. Without segmentation circumferentially, the damping treatment would be forced to conform to the bending curvature of the base structure (compatible deformations) by way of transverse normal forces, and the VEM layer would sustain very little shear deformation as a result. This situation may be visualized by imagining the four circumferential damping treatment segments of the test assembly to be connected together rigidly at the corners as the beam sections undergo bending. This circumferential segmentation was the subject of an earlier analysis of damped, circular-cross-section tubes by the Russian authors Vinogradov and Chernoberevskii [14]. The conclusion of their study

was that the bending loss factors of tubes having axially continuous constrained-layer treatments can be substantially increased by slitting the constraining layer lengthwise into a number of arc-shaped segments. Especially for long axial segment lengths, the same result should apply to axially segmented treatments.

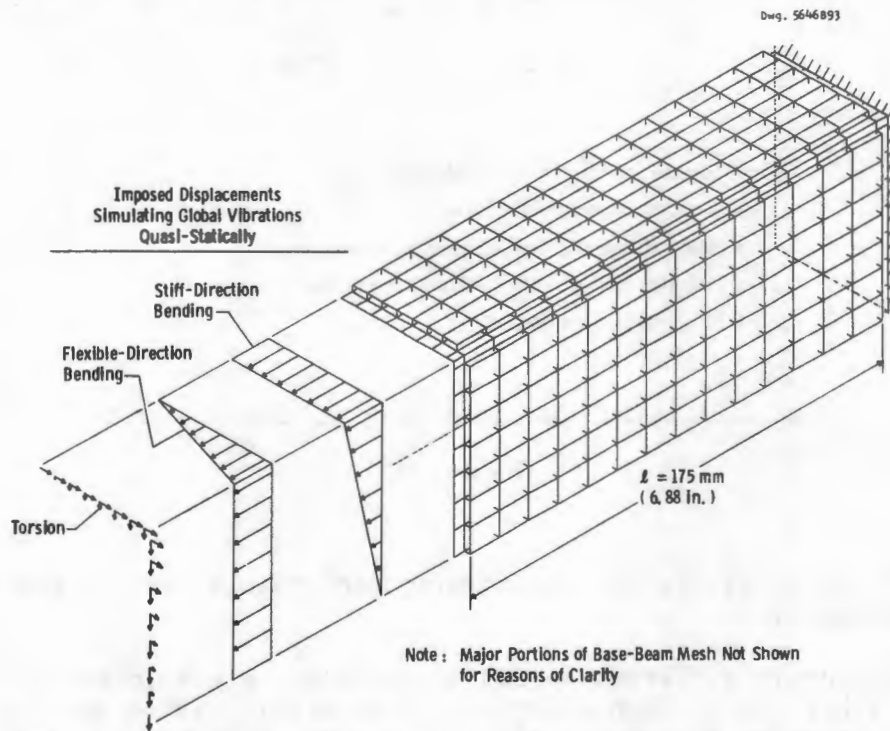


Figure 11 - Finite-element model of a one-eighth portion of one of the damped tubular portions of the box-beam test assembly

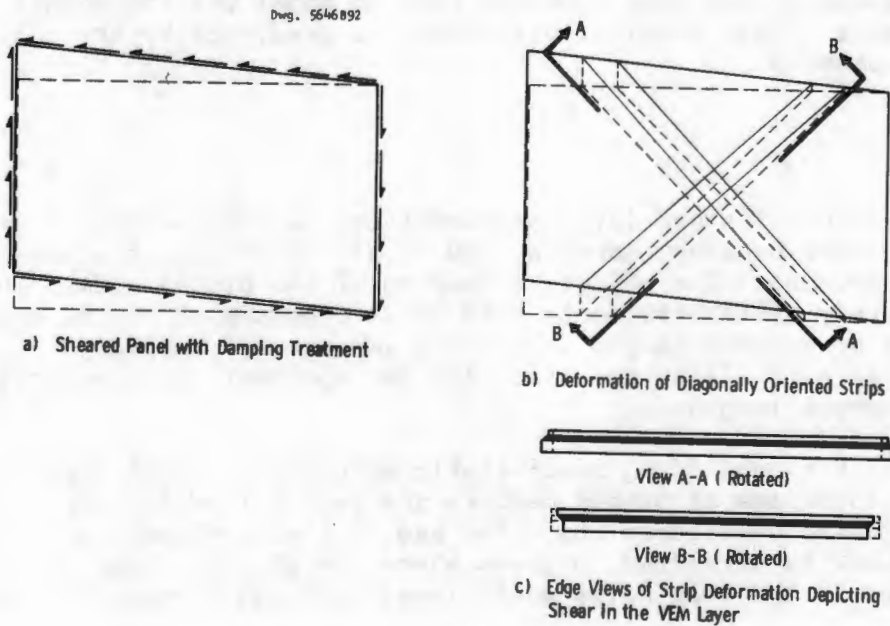


Figure 12 - Deformation of a damped box-beam panel under torsion-induced shear stress

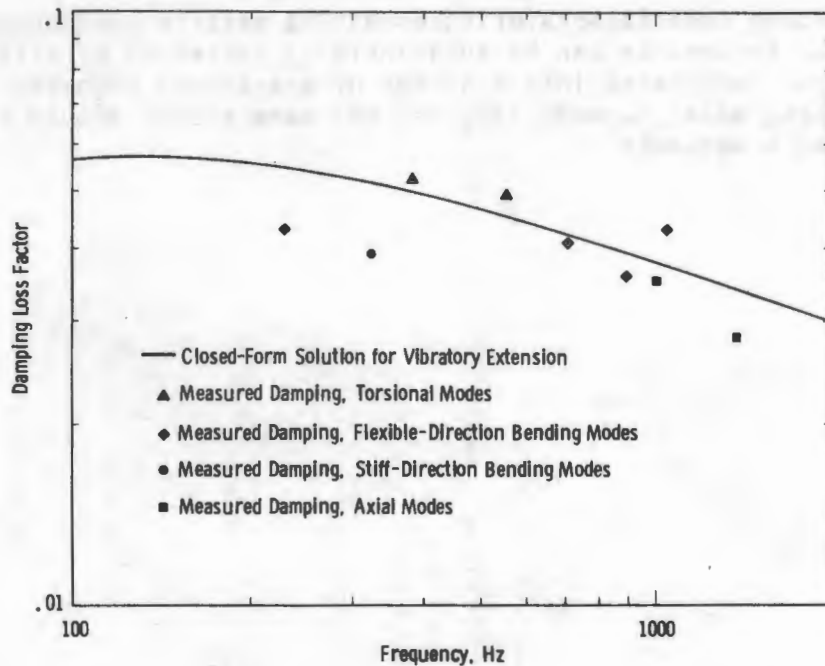


Figure 13 - Measured vs. calculated damping performance for the box-beam test assembly

If distinctly different values of damping are predicted for each direction of vibration in each component, the overall system damping value for each mode of a complex structural assembly can be predicted by using impedance or energy methods. Finite-element structural analysis codes such as MICA [15] enable the user to specify individual damping values corresponding to each degree of freedom of the beam elements used to model the components of a given structure, and a system damping loss factor is predicted by the code for each mode of the assembly.

CONCLUSIONS

A given constrained-layer treatment can be simultaneously effective in damping the global bending, torsion, and axial vibrations of a tubular structural component. The effective damping of the global modes requires the use of relatively stiff treatments, and axial subdivision of the treatment into segments of optimum length is vitally important. However, circumferential subdivision can also play an important role, especially for long axial segment lengths.

Torvik's closed-form, quasi-static solutions for the flexural and extensional vibrations of damped members are well-suited for performance estimates on these configurations. The use of finite-element modeling for this purpose may be warranted in cases where curvature, slope, or irregularities in the beam walls would invalidate the closed-form solutions.

The experiments on the damped, hollow, rectangular-cross-section beam assembly demonstrate the usefulness of each of these methods for calculating global-mode damping performance. Although derived for members damped on sides that are parallel to the neutral axis, the quasi-static flexural damping

prediction method is useable for performance estimates on rectangular-cross-section, tubular components having damping treatment on all sides. Measured and finite-element-calculated damping values for all global modes of the assembly agreed closely with the quasi-static extensional damping calculations, underscoring the usefulness of the latter method as a simple, but reasonably accurate, performance estimator.

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