NON-LINEAR DYNAMIC ANALYSIS WITH FREQUENCY-DEPENDENT DAMPING

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ABSTRACT

Methods of dynamic structural analysis in the frequency domain are strongly indicated when the structural and damping properties are frequency-dependent. Typical problems in which this dependency occurs are soil-structure interaction problems. Moreover physical non-linearlities can be present due to high strain levels in the soil.

In the present work a method of dynamic analysis in the frequency domain is presented that performs a rigorous analysis of non-linear structural systems with physical non-linearities and frequency-dependent properties. Stiffness and damping properties can be considered as frequency-dependent and the stiffness property can also be strain-dependent. The method is a step-by-step incremental one with linearized steps. In each step the integration is performed in the frequency domain through a FFT algorithm.

Examples of the analysis of SDOF soil-structure interaction system are presented in order to assess the applicability and stress the features of the method and to show the influence of frequency-dependent damping in the structural response.

INTRODUCTION

Methods of dynamic structural analysis in the frequency domain are strongly indicated when the structural and damping properties depend on the excitation frequency. Typical problems in which this dependency occurs are soil-structure interaction problems. On the other hand physical non-linearities can be present due to high strain levels in the soil.

Linear Structural dynamic analysis in the frequency domain is well known and had a great development with the use of the FFT algorithm. Nevertheless only recently methods of non-linear dynamic structural analysis in the frequency domain have been developed. Kawamoto [1983] presented the so called Hybrid Frequency-Time Domain(HFTD) method in which the non-linearities are treated as pseudo-forces. Darbre and Wolf [1987] demonstrated the convergence for the partial version of the HFTD method.

In the present paper a method for non-linear dynamic structural analysis in the frequency domain is presented that works for Single-Degree-Of-Freedom(SDOF) systems with frequency-dependent properties and physical non-linearities. Stiffness and damping properties can be considered as frequency-dependent and the stiffness property can also depend on the displacement. The method uses a step-by-step incremental technique with linearized steps and a secant stiffness. In each step the integration of the dynamic equilibrium equation is performed exactly in the frequency domain through a FFT algorithm taking into account the frequency-dependent properties.

Examples of the analysis of non-linear SDOF systems with frequency-dependent properties submitted to transient excitation are presented. The influence of the frequency-dependent damping upon the structural response is highlighted in these examples.

FORMULATION OF THE PROBLEM

Consider the SDOF system of Fig. 1 submitted to an arbitrary excitation p(t). The spring stiffness k depends on the displacement v due to the system non-linearity and the damping coefficient depends on the frequency of the excitation, ω. The problem is then to integrate the dynamic equilibrium equation

$$m\ddot{v} + c(\omega)\dot{v} + k(v)v = p(t). \qquad (1)$$

As the damping coefficient is ω dependent a frequency-domain analysis has to be performed and, as the stiffness depends on the displacement, a linearization technique must be employed. Consequently the present method is a Step-by-Step Incremental Linearization in the Frequency Domain (SILFD) method. In each linearized step a secant stiffness is considered.

THE SILFD METHOD

In order to calculate the response of the system governed by Eq.1 two approximations are made. The first one is the approximation of the given load by piecewise linear segments. The total time interval in which the response is to be calculated is divided in intervals $\Delta t_t = t_j - t_{j-1}$; p_j and p_{j-1} are the values of p(t) in times t_j and t_{j-1}, respectively, and $\Delta p_j = p_j - p_{j-1}$, Fig.2a. The load variation in time interval Δt_j is given by, Fig. 2a,

$$p(\tau) = p_{j-1} + \frac{\Delta p_j}{\Delta t_j}\tau . \tag{2}$$

where τ is the current time in Δt_j $(0 \leq \tau \leq \Delta t_j)$. The second approximation refers to the spring force versus displacement curve. This curve is also approximated by piecewise linear segments as indicated in Fig. 2c. The levels of these two approximations depend on the accuracy with which the load and the stiffness variation can have a good representation.

The response of the system is calculated through the linearized steps along the time intervals Δt_j in which the spring is considered linear with stiffness k_j, Fig. 2b. The linearized dynamic equilibrium equation in time interval Δt_j is

$$m\ddot{v} + c(\omega)\dot{v} + k_j v = p(\tau) \tag{3}$$

with the initial conditions v_{j-1} and $\dot{v}_{j-1}$, Fig. 2a. Taking the Fourier Transform (FT), (F), of both sides of Eq. 3 and considering that F is a linear operator one obtains

$$F[m\ddot{v}] + F[c(\omega)\dot{v}] + F[k_j v] = F[p(\tau)]. \tag{4}$$

The following equations are now considered by definition:

$$F[m\ddot{v}] = \int_0^\infty m\,\ddot{v}\,e^{-i\omega\tau}\,d\tau \;; \tag{5a}$$

$$F[c(\omega)v] = \int_0^\infty c(\omega)\;v\;e^{-\omega\tau}\,d\tau \;; \tag{5b}$$

$$F[k_j v] = \int_0^\infty k_j v\,e^{-i\omega\tau} d\tau = k_j\;V(\omega) \;; \tag{5c}$$

$$F[p(\tau)] = \int_0^\infty p(\tau)\, e^{-i\omega\tau}\, d\tau = P(\omega) \; , \tag{5d}$$

noting that in Eq. 5c k_j is constant. Integration by parts of the right-hand sides of Eqs. 5a and b gives respectively

$$F[m\ddot{v}] = - m\dot{v}_{j-1} - i\omega\, mv_{j-1} - \omega^2 mV\,(\omega) \; ; \tag{6a}$$

$$F[c(\omega)\dot{v}] = - c(\omega)v_{j-1} + i\omega c(\omega)\; V(\omega) \; . \tag{6b}$$

Introducing now Eqs. 6a, 6b, 5c, and 5d into Eq.4, the following equation is obtained

$$[H(\omega)]^{-1}\, V(\omega) = \bar{P}(\omega) \; . \tag{7}$$

In this equation

$$\bar{P}(\omega) = P(\omega) + m\dot{v}_{j-1} + i\omega mv_{j-1} + c(\omega)v_{j-1} \; . \tag{8}$$

is the FT of the load with due regard of the initial conditions, v_{j-1} and $\dot{v}_{j-1}$, and

$$H(\omega) = [-\omega^2 m + i\omega c(\omega) + k_j]^{-1} \tag{9}$$

is the system complex frequency response function in time interval Δt_j . The Fourier Transform of $v(\tau)$ is obtained, by inversion in Eq. 7, as

$$V(\omega) = H(\omega)\; \bar{P}(\omega) \; . \tag{10}$$

The response $v(\tau)$ in time interval Δt_j is then the inverse FT of $V(\omega)$, Eq. 10. In this way

$$v(\tau) = \frac{1}{2\pi} \int_0^\infty V(\omega)\; e^{i\omega\tau}\; d\omega \; . \tag{11}$$

The FT of the velocity is

$$\bar{V}(\omega) = i\omega\bar{P}(\omega)\; H(\omega) \; . \tag{12}$$

The velocity response is the inverse FT of $\bar{V}(\omega)$ and is given by

$$v(\tau) = \frac{1}{2\pi} \int_0^\infty \bar{V}\;(\omega)\; e^{i\omega\tau}\; d\omega \; . \tag{13}$$

COMPUTATIONAL PROCEDURE

The computational procedure of the SILFD method consists of the following steps:

i. for each time interval Δt_j Fig. 2a, obtain k_j from the spring force versus displacement curve, Fig. 2c.

ii. extend the load function in time Δt_j with a trail of zeroes in order to total 2^m points (m integer); calculate the discrete FT's $P(\omega_k)$, Eq. 5d, and $\bar{P}(\omega_k)$, Eq. 8, $(k = 1,2, \ldots, 2^m)$, Fig.3

iii. calculate $H(\omega_k)$, considering $c(\omega_k)$, for $k = 1,2, \ldots, 2^m$.

iv. calculate the discrete FT's $V(\omega_k)$, Eq.10, and $\bar{V}(\omega_k)$, Eq.12, for $k = 1,2, \ldots, 2^m$.

v. calculate the inverse discrete FT's of $V(\omega_k)$ and $\bar{V}(\omega_k)$ to obtain $v(\tau)$ and $v(\tau)$.

vi. if $v_j < \bar{v}_k$, Fig. 2c, go to time interval Δt_{j+1} ; if $v_j > \bar{v}_k$, Fig. 2c, reduce Δt_j and go to ii.

EXAMPLES

The first example is the SDOF system of Fig.1 with mass $m = 1.0 \times 10^6$kg. The spring force versus displacement variation is bi-linear and is given in Fig. 4 where the initial stiffness is $k_o = 15 \times 10^9$ N/m. Several damping coefficient versus frequency variations are considered which are shown in Fig. 5. The maximum values of these curves correspond to damping ratios of 5%, 10%, 20%, 30%, and 40% of the critical damping of the linear system. The system is submitted to the load given in Fig. 6. The curves ACC and A10 in Fig 7 are the system responses with a constant damping coefficient corresponding to 10% of the critical damping and with the damping coefficient given by the 10% curve in Fig. 5, respectively. The consideration of variable damping, in this case, leads to greater values for the displacements. The curves in Fig.8 display the response with damping coefficients given by the 5%, 20%, and 40% curves in Fig.5. The great influence of the damping variation on the pseudo-period of the response and the great difference in the obtained displacements, after the load develops, is to be noted.

The second example is a SDOF linear system with $m = 1.0 \times 10^6$kg , $k = 1.5 \times 10^{10}$ N/m and an hysteretic damping coeffient $D = 0.10$ submitted to a transient excitation. The analysis is performed considering a viscous damping ratio $\xi = 0.10$ and and equivalent viscous damping coefficient defined by $c_{eq} = \frac{2Dk}{\omega}$. The variation of c_{eq} with ω is shown in Fig.9. As this variation has a singularity for $\omega = o$ the following approximations are considered: horizontal (1); tangent (2); and parabolic (3). Fig. 10 shows the responses obtained with a viscous damping coefficient corresponding to a damping ratio of 10% and with equivalent viscous damping coeffients with the horizontal (CHR), tangent (CTG), and parabolic (CPA) approximations. From Fig. 10 it can be observed that, for a transient excitation, the consideration of the frequency content of the excitation leads to greater values of the response. On the other hand, for a resonant harmonic excitation, the response can be calculated with a constant equivalent damping coefficient equal to the hysteretic damping coefficient.

CONCLUSIONS

The method presented herein is applicable to the analysis of the dynamic response of non-linear systems in the frequency domain. The stiffness and damping properties can depend on the excitation frequency, as in soil-structure interaction systems, and the stiffness can depend on the displacement in face of non-linearities.

The use of a FFT algorithm turns out the method to be competitive with methods in time domain. On the other hand the method is mandatory for systems in which the properties depend on the frequency content of the excitations. The system non-linearities ar treated by incremental linearized steps with a secant stiffness.

Great differences in the responses considering constant and frequency-dependent damping are observed in the analysed cases mainly when the transient load dies out.

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2. Kawamoto, J.D. [1983], "Solution of Nonlinear Dynamic Structural Systems by a Hybrid Frequency-Time Domain Approach", MIT Research Report R83-5, Department of Civil Engineering, Cambridge, 1983.

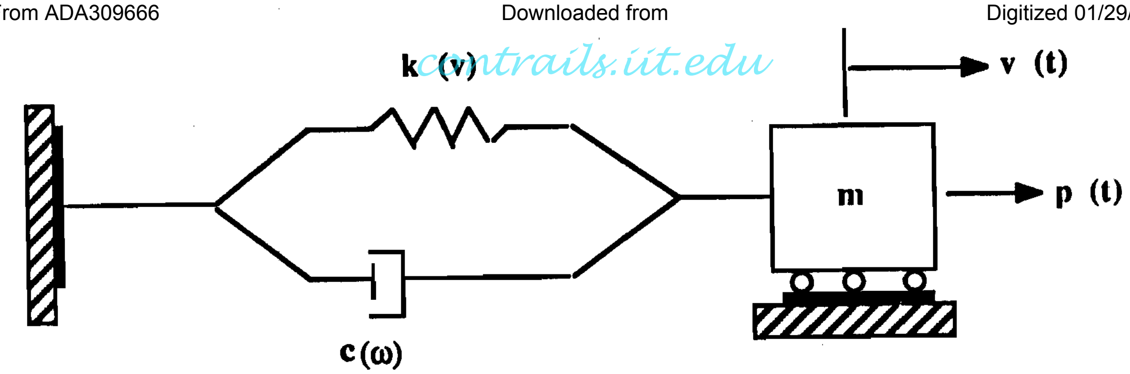

Figure 1.- SDOF System

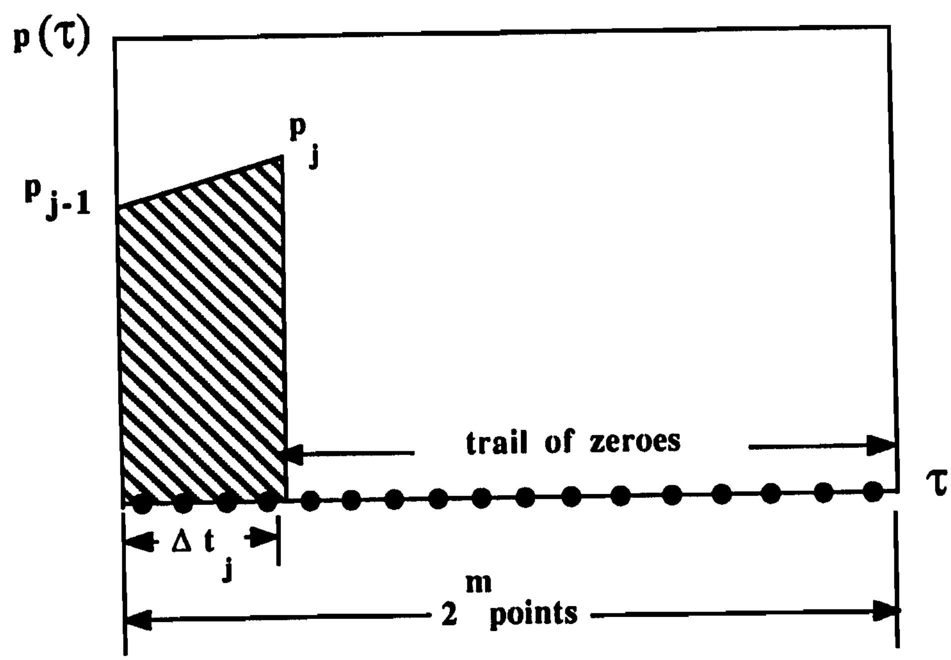

Figure 3.- Extended Load Function

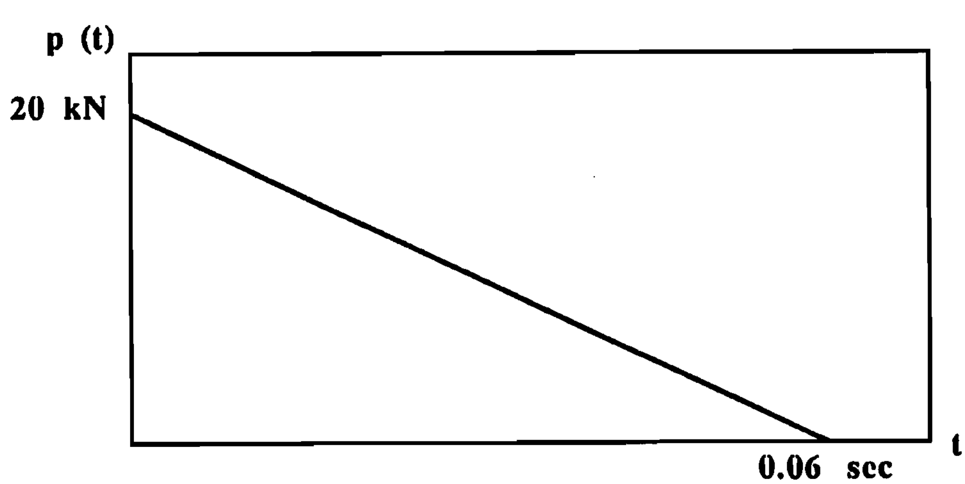

Figure 6.- Load

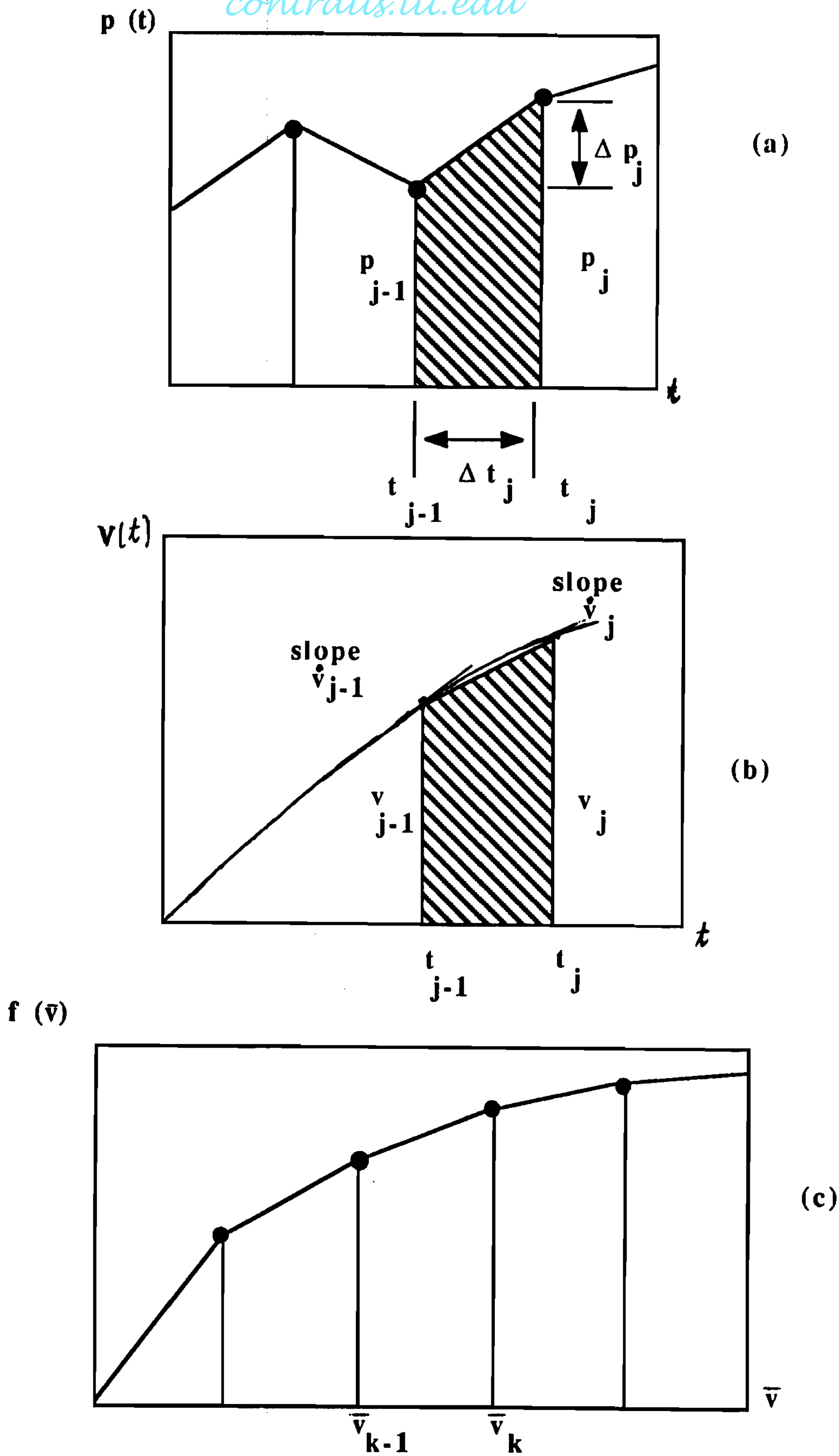

Figure 2.- a) Load; b) Displacement responce; c) Spring force

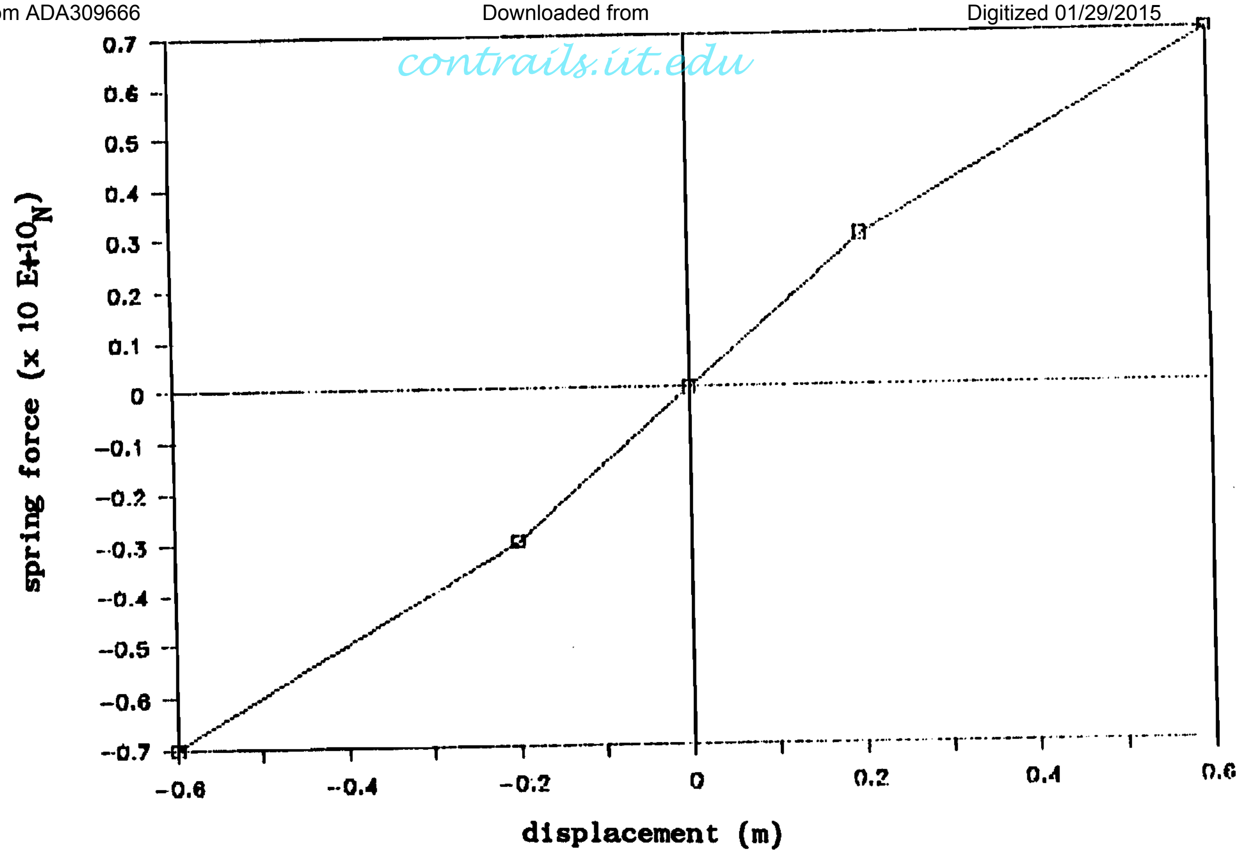

Fig. 4 - Spring force versus displacement

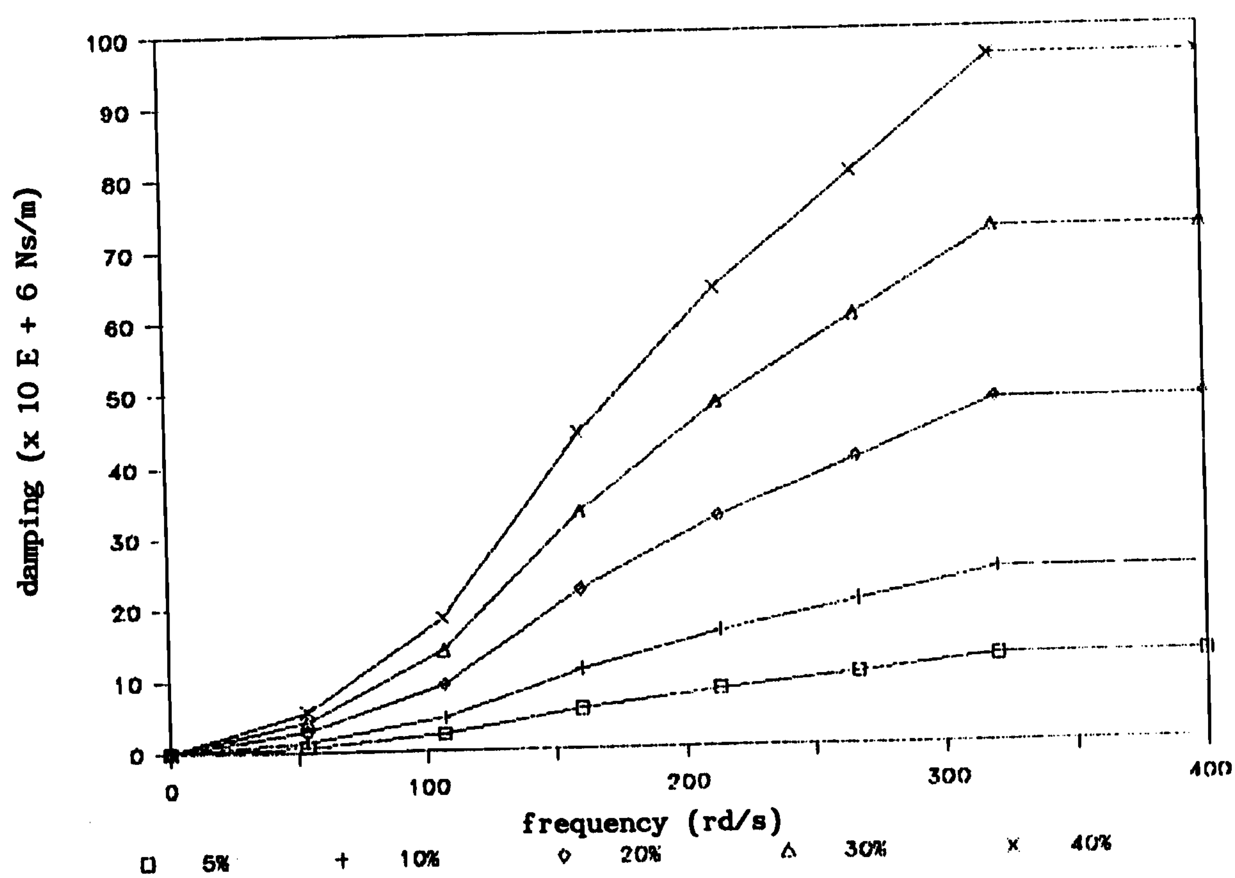

Fig. 5 - Damping coefficients versus frequency

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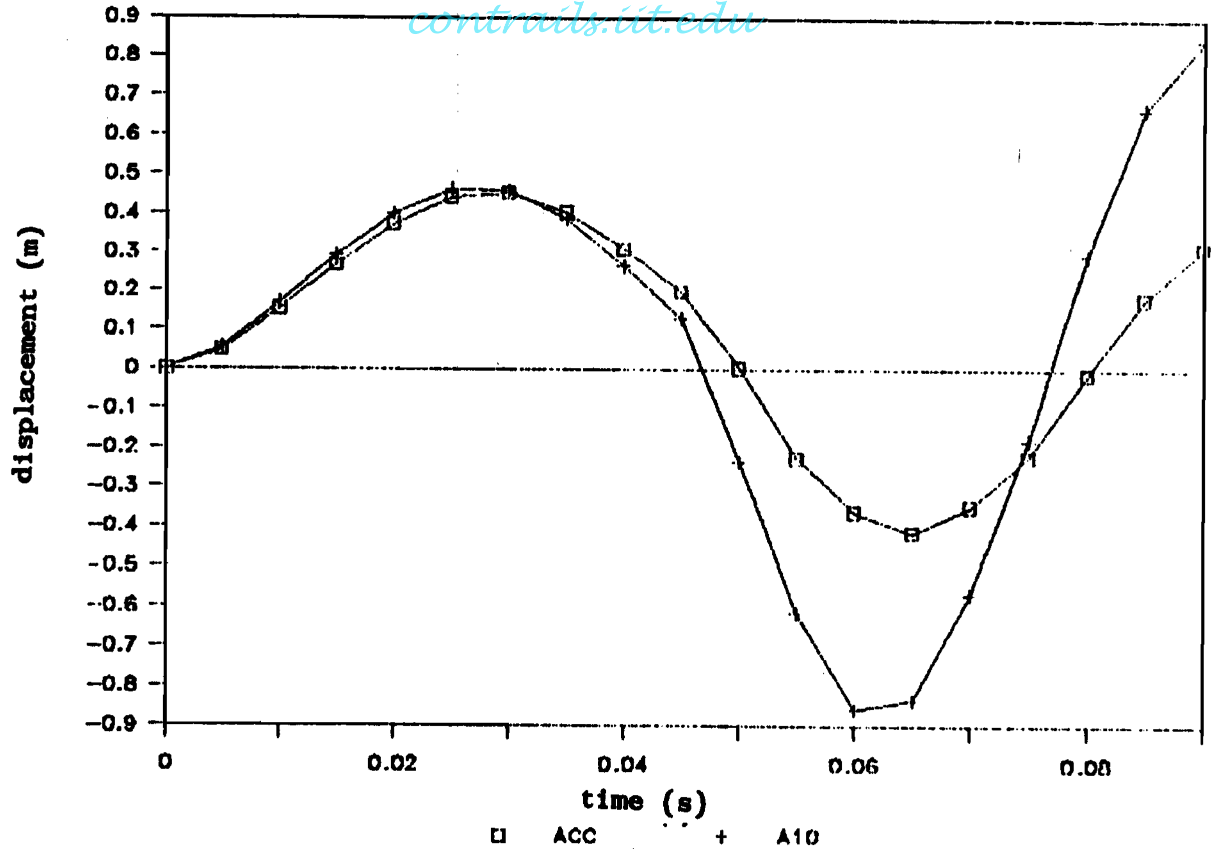

Fig. 7 - System responses

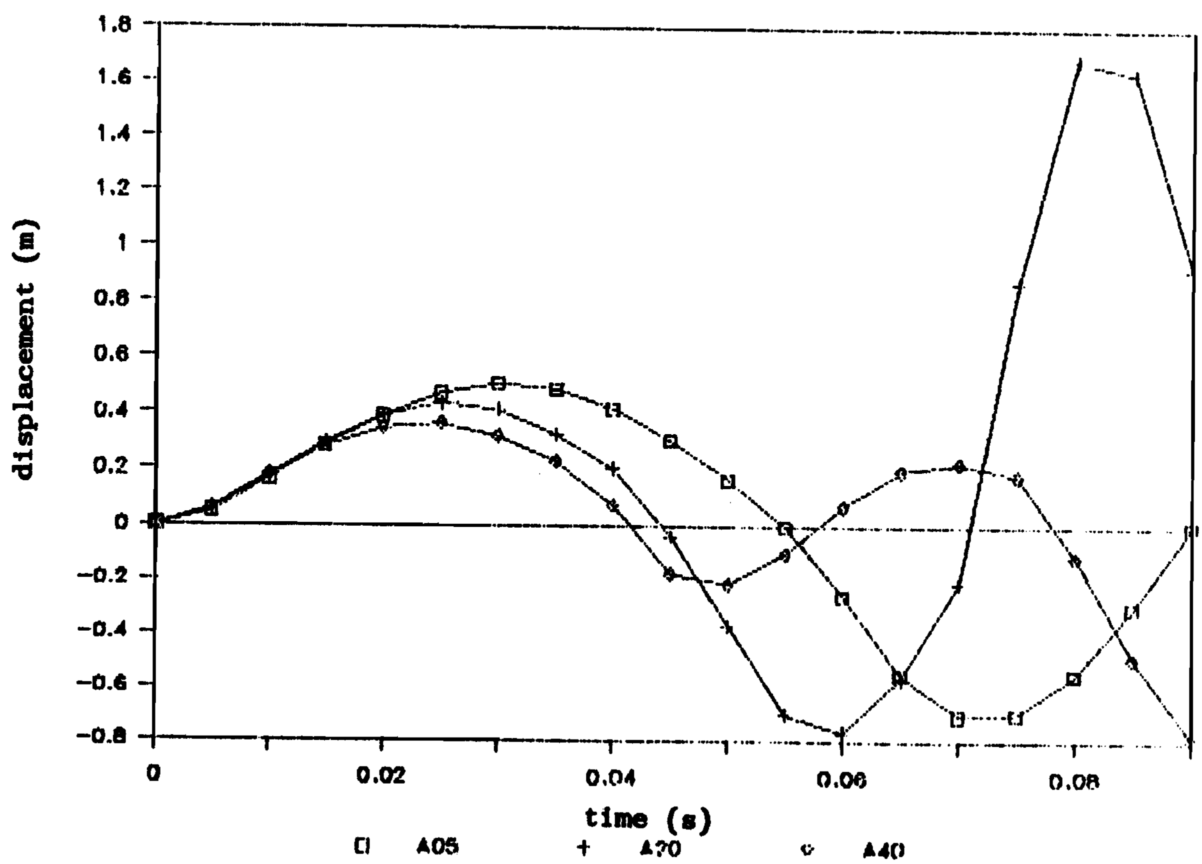

Fig. 8 - System responses

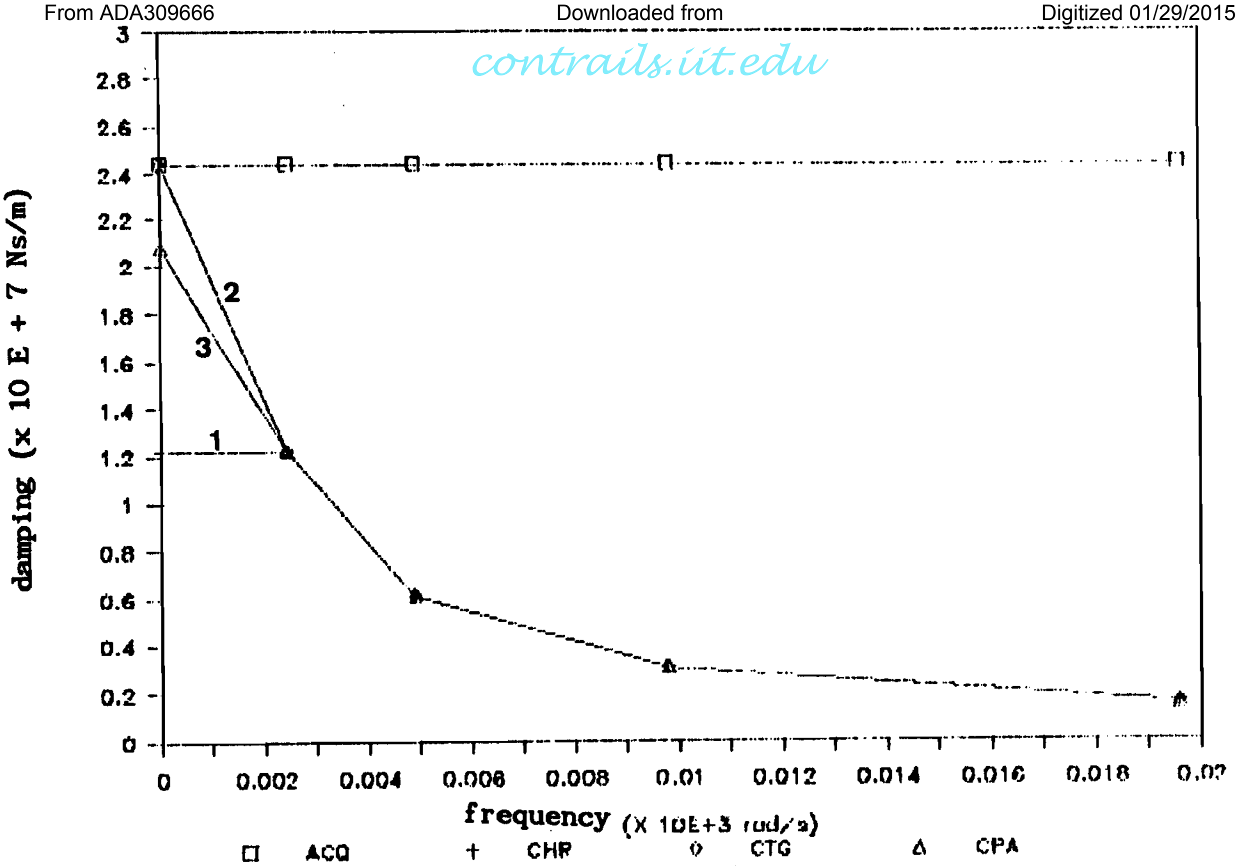

Fig. 9 - Equivalent viscous damping versus frequency

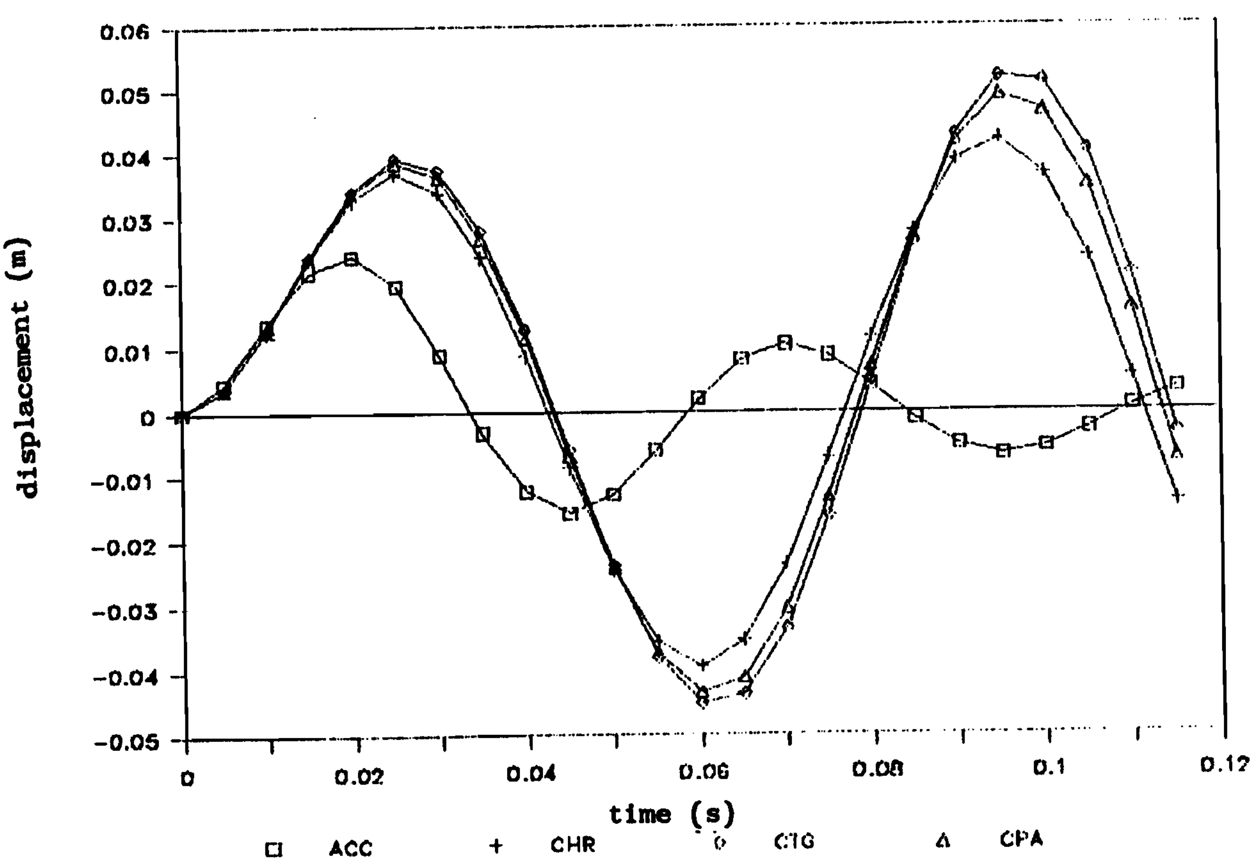

Fig. 10 - System responses