AN INTERNAL DAMPING CONFIGURATION
FOR TUBES AND HOLLOW PANELS

by

Edward M. Kerwin, Jr.
BBN Systems and Technologies Corporation

Abstract

The provision of an effective internal damping treatment for the bending vibrations of a hollow structure is a difficult task. This report on Work in Progress explores the possible use of a stiff-skin laminate with a thick viscoelastic core as a damping insert for a box beam. The work of Kurtze and Watters [J. Acoust. Soc. Am., 31, 739-748 (1959)] has shown that in its mid-frequency, core-shear range such a laminate has its dominant elastic energy of transverse-wave deformation in shearing of the core, with negligible extension of the stiff skins. As a result the transverse-wave loss factor of the laminate alone is the loss factor of the core, making the laminate an attractive candidate as an internal damper. Coupling to the interior of the box beam would be through normal forces driving the transverse displacement, with relative tangential displacements allowed by a "slip" layer of low shear stiffness. The analysis to date explores the wave propagation and damping properties of the composite structure (box beam plus insert). The results show that high damping could be realized if the core-shear mode of the insert laminate could be made to dominate the composite properties. Unfortunately this clear dominance would require that the coupled-skin bending stiffness of the insert greatly exceed the bending stiffness of the box beam, a result unlikely to be achieved in practice. Continuing work will explore the utility of the laminate insert under more realistic requirements. Other insert-damper configurations also await evaluation.

1. INTRODUCTION

The damping of bending waves of hollow structures by internal treatments has long represented a vexing problem. Standard approaches (such as simple free or constrained viscoelastic layers) are not very effective, principally because of geometric and kinematic limitations imposed by trying to work inside a structure, as well as the ever present bounds on the properties of dissipative materials. This paper is a first report on work-in-progress, and proposes a treatment concept that can in principle provide effective damping (although the required properties of the structural parts of the treatment appear to be elusive).

The particular damping treatment configuration chosen for first study is a core-shear composite adapted from the the work of Kurtze and Watters on the control of the speed of transverse waves in acoustical panels. The proposed damping treatment is unusual in that it is to be driven by the transverse displacement and normal forces (vs. interface shear forces) of the structure to be damped.

In this paper we describe the concept, illustrate its potential to provide significant elastic energy storage in the viscoelastic core, and describe the dynamic functional behavior of the components of the combined system (hollow structure plus internal damper), including the expected system loss factor. In our conclusions we note the material-properties limitations that appear to block the realization of an ideal damping treatment as first proposed. However, alternatives and further anlayses are both indicated.

2. THE DAMPING PROBLEM CHOSEN

2.1 The Box Beam

The structure to be damped is the box beam of Figure 1. The walls are of equal thickness on all sides, and are thin relative to the cross dimension of the beam. We are to assume that it is important to the problem at hand that an effective level of damping be provided for the (free) bending vibrations of this beam. The objective is to be pursued agressively, including the consideration of heroic (heavy etc.) measures.

A common first suggestion as a damping treatment for such a hollow beam is that it be filled with a dissipative material. Such an approach effectively places a simple viscoelastic beam within the structural beam. The result as is seen in Figure 2 is that the neutral planes of the two beams coincide; they undergo the same transverse displacements and rotations of cross sections. No shear is generated between the two, and the elastic energies of deformation for maximum displacement y_o at wave number k are those of individual bending as follows:

$$V_{Box\ Beam} \equiv V_b = \frac{1}{4} B_b k^4 y_o{}^2$$

$$V_{VE} \equiv V_{ins} = \frac{1}{4} B_{ins} k^4 y_o^2 \tag{1}$$

where, in each case, B = EI, the bending stiffness

E = Young's modulus

I = area moment of the cross section

and

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} ,$$

the wavenumber* of the transverse displacement, and

λ = wave length

f = frequency

c = wave speed.

If we assign a loss factor η_i to the insert and assume the box beam is (relatively) without losses, we have for the loss factor of the composite (beam plus core damper)

$$\eta = \frac{\Sigma \eta_j V_j}{\Sigma V_j} = \frac{V_{ins}}{V_b + V_{ins}} \eta_{ins} . \qquad (2)$$

For a homogeneous insert within a thin walled beam we will find

$$I_{ins} \doteq I_b \qquad (3)$$

(The two are equal where the wall thickness is 0.08 times the outside dimension of the beam), and we expect that $E_{ins} \ll E_b$.

It then follows that $V_{ins} \ll V_b$; and Eq. (2) become

$$\eta \doteq \frac{E_{ins}}{E_b} \eta_{ins} . \qquad (4)$$

Thus, in this case the system loss factor is only a small fraction of that of the insert material, and such a treatment is ineffective.

An alternative suggestion that is frequently made is that one employ a structural beam as an insert, with a thin layer of viscoelastic material between it and the inside walls of the box beam. For such a treatment one argues as before that there is no relative motion generated in bending of the composite, hence no shear deformation of the viscoelastic layer, and essentially no damping.

---

*The wavenumber $k = 2\pi/\lambda$ is very useful in describing phenomena that are harmonic in space with _spatial_ period λ. By analogy with the circular frequency $\omega = 2\pi f = 2\pi/T$, which describes a phenomenon that is harmonic in time with _temporal_ period T=1/f, the wavenumber is sometimes called the spatial frequency.

2.2 The Shear-Core Configuration

Clearly, one needs to find a way to generate more deformation in the damping insert. The work of Kurtze and Watters[1] suggests such a mechanism. They were concerned about controlling the speed of bending waves in acoustical panels, because good acoustic transmission loss could be preserved if a way could be found to keep the bending wave speed below sonic speed through enough of the audible frequency range.[2-4] This bending-wave-speed problem had become more important with the advent of lighter, stiffer panels.

Kurtze and Watters evolved a stressed-skin panel construction (see Figure 3) in which the shear properties of the thick core placed the broad transition region between coupled-skin bending at low frequencies, and uncoupled-skin bending at high frequencies so as to keep the "bending" (i.e., transverse) wave speed subsonic, as the Figure shows. In the transition region, the panel skins appear essentially inextensible so that core shear controls the stiffness of the panel in transverse deformation. As a result, the wave speed is essentially constant at a level determined by the panel mass and the core shear stiffness. A consequence of this behavior, as Kurtze and Watters recognized, is that in the transition region the panel loss factor is essentially that of the core material. If such a "panel" could be adapted as an insert damper that could control the transverse motion of the composite (box beam plus damping insert), then high system loss factors might be realized. It is this possibility that lead us to investigate the shear-core insert.

Figure 4 sketches the desired transition-region, core-shear behavior in comparison to the simple bending insert discussed earlier and shown in Figure 2. In core-shear deformation, cross sections do <u>not</u> rotate but rather stay vertical and parallel. This motion is forced by the stiff skins, which must "slip" relative to the inside surface of the box beam. As a result, the core shears throughout (if it is homogenous), and the core stores an average elastic energy V_{cs} per unit length of

$$V_{CS} = \frac{1}{4} G A k^2 y_o^2 \tag{5}$$

where G = shear modulus of the core
A = cross section of the core
y_o = maximum transverse displacement.

Comparing the elastic energies of the shear core and the viscoelastic beam inserts (See Equation 1), taking them to be made of the same material, we find

$$\frac{V_{CS}}{V_{VE}} = \frac{GA}{Bk^2} = \frac{12G}{E(kh)^2} \doteq \frac{4}{(kh)^2} \text{ (Large).} \tag{6}$$

Note that the ratio is large because for simple bending of the system we would find $(kh)^2 = (2\pi h/\lambda)^2$ to be small relative to unity. Thus the shear core can store much more elastic energy than a viscoelastic beam of the same material.

As we noted earlier, the shear core laminate has three distinct free transverse wave types in different frequency ranges:

low frequency: coupled-skin bending with dominant elastic energy in stretching of the skins

$$V_{BC} = \frac{1}{4} B_c k^4 y_o^2 \tag{7}$$

transition region: core-shear transverse deformation with dominant elastic energy in core shear; skins essentially inextensible

$$V_{CS} = \frac{1}{4} GA k^2 y_o^2 \tag{8}$$

high frequency: uncoupled skin bending with dominant elastic energy in the bending of the two skins

$$V_{BU} = \frac{1}{4} \cdot 2B_{skin} k^4 y_o^2 \tag{9}$$

These regions are indicated asymptotically in Figure 5 as wavenumber vs. frequency. In each of the bending regions, coupled bending (BC) at low frequencies and uncoupled bending (BU) at high frequencies, the bending wave speed increases as $f^{1/2}$. In the transition region (if it is broad enough), the wavespeed is essentially constant so that wavenumber increases as f.

The "break" frequencies between these regions occur at wavenumbers k_I and k_{II}, at which the elastic energies for the two adjacent transverse wave types would be equal. That is, where

$$V_{BC} = V_{CS},$$
$$k^2 = k^2_I = \frac{GA}{B_c},$$

and where

$$V_{CS} = V_{BU},$$
$$k^2 = k^2_{II} = \frac{GA}{B_u} = \frac{GA}{2B_{skin}}. \tag{10}$$

It follows that the transition range of core-shear behavior in wavenumber and in frequency is

$$\frac{f_{II}}{f_I} = \frac{k_{II}}{k_I} = \frac{B_c}{2B_{skin}}^{1/2}. \tag{11}$$

Kurtze and Watters, see Figure 3, showed a panel design with a transition range of two decades. A little thought shows that a broad range should also, in principle, be achievable in a damping insert.

FBD-5

3.   THE COMPOSITE BOX BEAM WITH SHEAR CORE INSERT

Figure 6 shows the composite beam that we wish to consider; a stressed-skin insert with a viscoelectric shear core placed as a damper within the box beam. The "slip" interface that is required between insert and box beam is assumed to transmit normal forces between the two, but to allow relative shear displacement with only minimal shear stresses. For the moment, let us assume that this is accomplished with a thin layer having a low shear modulus.

3.1 Component Impedances for Transverse Motion

In determining the characteristics of the composite beam in bending, i.e., its wavenumber (or wavespeed) and loss factor, we require the impedances of both box beam and insert for transverse motion. In either case, the impedance is the ratio of a transverse force at wavenumber k and frequency f to the resulting transverse velocity at k and f. For a beam in simple bending one has

$$Z_p = i \frac{Bk^4}{\omega} \left[1-\left(k_p/k\right)^4\right] = -i\omega m\left[1-\left(k/k_p\right)^4\right] \tag{12}$$

where B = Bending stiffness (ratio of bending moment to resulting curvature)

   m = Mass per unit length of the beam, and

$k_p^4 = \omega^2 m/B$, the wavenumber of free bending waves.

With the appropriate parameters B and m, this expression also describes the impedance of the insert in its bending regimes at low and high frequencies. Since our convention here is a time dependence $e^{-i\omega t}$, positive reactance represents stiffness, and negative reactance represents mass.

The condition for the propagation of free waves is that the impedance go to zero (in the case of losses, one sets the imaginary part equal to zero). Thus the wavenumber for free bending waves is k_p, as defined above.

The impedance expression of Equation (12) shows that at a given frequency one finds the following:

$k < k_b$, $Z_b \rightarrow$ mass reactance

$k > k_b$, $Z_b \rightarrow$ stiffness reactance

The physical interpretation of these results is that if we try to bend a beam dynamically at a wave length <u>longer</u> than its freewave length ($k<k_p$), its behavior is as a mass. Correspondingly, if we try to bend the beam at a wavelength <u>shorter</u> than its free wavelength ($k > k_p$), it appears as a stiffness that increases rapidly with increasing k.

For the insert in its core-shear region, we can show that the transverse impedance is the following:

$$Z_{cs} = i \frac{GAk^2}{\omega} \left[1 - (k_s/k)^2 \right]$$

(13)

$$= -i\omega m_s \left[1 - (k/k_s)^2 \right]$$

where $k_s = \omega^2 m/GA$, the free wavenumber.

For the insert in core-shear the wave-number dependence of the impedance at a frequency is qualitatively like that of the bending beam, that is, a massive reactance for $k < k_s$ and stiffness reactance for $k > k_s$. However, the variations with k are less rapid than for the beam (see the following section).

3.2 Characteristics of the Composite Beam

In Figure 7 we show the wavenumber dependence of the beam and shear core impedances. Each impedance is normalized by ωm, the mass reactance at the frequency considered, i.e., the transverse reactance found as $k \to o$, simple transverse translation. Here m represents m_b or m_s. Also, each reactance is plotted as a function of the wavenumber relative to its respective free wavenumber. In each case, the reactance is massive approaching ωm for small wavenumber, and dropping to zero at the free wave-number.

Above the free wavenumber, the reactance is stiff, and increases rapidly with k. This is especially true for the bending beam (note the right-hand scale which is compressed 100 times relative to the left-hand scale. Also note that the stiffness reactance for the shear core is plotted at 10 times its value, i.e., the reactance itself has a value of 10 where the right hand scale shows 10 x Insert = 100).

To illustrate the desired behavior of the shear-core insert as a damper in the box beam, we show the free wavenumber characteristics of both as functions of frequency in Figure 8. The three characteristics appear for the shear core, the core-shear branch being the one of principal interest for damping. In each case, the nature of the transverse impedance is indicated qualitatively as "S" for stiffness above the free wavenumber line, and "M" for massive below. The relative frequency and wavenumber scales are each unity at the crossover "c" of the beam and core-shear characteristics.

The free-wavenumber characteristic for the composite beam will lie between the box-beam characteristic and the dominant characteristic of the insert, i.e., the one closest to the beam characteristic at the frequency of interest. (The true insert characteristic will fair smoothly from one of its branches to another. The asymptotic lines are being used here to illustrate the behavior.)

Figure 9 shows the combined wavenumber-frequency characteristic for the composite beam. In the region well below f_c, the crossover of box-beam bending and insert core shear, the composite approaches a line lying above the core-shear characteristic, by the following factor

$$k/k_s \rightarrow \left(1 + m_b/m_s\right)^{1/2}$$

$$\eta \rightarrow \eta_s \qquad\qquad f \ll f_c \qquad\qquad\qquad (14)$$

and the expected composite loss factor approaches η_s, that of the shear core. This happy result depends, however, on the assumption that the insert behavior is controlled by core shear, and that the transition to coupled-skin bending lies far enough below the frequency and wavenumber range shown. (See Conclusions below.)

At frequency f_c the free wavenumber of the composite coincides with those of the box beam and the insert individually. Because only the insert has significant losses, the composite loss factor is

$$\eta = \eta_s / \left(1 + m_b/m_s\right) \ , \ f = f_c \ . \qquad\qquad (15)$$

That is, the composite loss factor is reduced by the ratio of insert mass to total mass at this crossover frequency. This result is indicative of the relative vibrational energies in the two components of the composite.

Again, as shown in Figure 9, at frequencies well above f_c, the stiffness of the box beam controls, and

$$k/k_b \rightarrow \left(1 + m_s/m_b\right)^{1/4}$$

$$\eta/\eta_s \rightarrow \frac{f_c/f}{\left[\left(m_b/m_s\right)\left(1 + m_b/m_s\right)\right]^{1/2}} \qquad f \gg f_c \qquad\qquad (16)$$

In this high-frequency region where the box-beam bending dominates progressively, the composite loss factor decreases inversely with frequency; and again the system mass parameters conveniently describe the results.

The loss factor variation over the frequency range $0.1 \, f_c$ to $10 \, f_c$ is shown in Figure 10 as the ratio of composite loss factor to η_s. The result presented is for the example $m_s = m_b$, i.e., the insert and box beam have equal mass. The performance shown represents a significant fraction of the core loss factor and can be realized if, as we have assumed, the core-shear properties of the insert are effective over the frequency range, especially at the lower frequencies.

4. EVALUATION AND CONCLUSIONS

In this preliminary assessment of the damping potential of the shear core insert we have used asymptotic expressions to characterize the insert. Our focus has been on the interaction between the box beam and the insert in the frequency region in which core shear dominates the behavior of the insert.

Implicit in this approach are several assumptions about the insert's characteristics -- namely,

a) that core shear dominates over a broad frequency range so that the expected high levels of damping will be realized over a sufficient bandwidth, and

b) that the lower break frequency f_I between core shear and coupled-skin bending (at lower frequencies) lies well below f_c where the core-shear freewave characteristic intersects the box-beam free bending wave characteristic (see Figure 8).

The first of these does not of itself appear troublesome. As we saw in Figure 3, Kurtze and Watters demonstrated a core-shear region (actually a spread between break frequencies) of two decades in frequency.

On the other hand, the second assumption imposes the requirement that

$$\left(\frac{B}{m}\right)_{\substack{\text{insert} \\ \text{coupled-skin}}} \gg \left(\frac{B}{m}\right)_{\text{box beam}} .$$

This is indeed a condition that may prove difficult, if not impossible to meet. Further, the damping results of Equations 14-16 as indicated in Figure 10 show that the mass of the insert must not be too small, and probably should be comparable to the mass of the box beam. Thus, a requirement for effective damping may be

$$B_{\substack{\text{insert} \\ \text{coupled-skin}}} \gg B_{\text{box beam}} .$$

Energy arguments would support such a result.

It would be difficult to achieve a coupled-skin bending stiffness of the insert that is much larger than the bending stiffness of the box beam particularly since the insert must fit within the box beam. (We note that the coupled-skin bending stiffness varies approximately as

$$B_{\text{coupled-skin}} \sim E_{\text{skin}} h_1 h_2^2$$

so that the core thickness is a strong determinant of the bending stiffness. The box beam is assumed to be made of a material with high elastic modulus, so that E_{skin} cannot be very large in comparison.)

It appears unlikely that the idealized conditions first assumed can be realized. (i.e., $B_{\text{insert}} \gg B_{\text{box beam}}$). However, the damping achievable with less extreme requirements should be determined in the continuing work. In addition, the simple shear-core configuration is only one of several damping insert designs presently under consideration.

Incidentally, we note that since the shear-core configuration is driven by the lateral motion of the beam-to-be damped, its performance is not dependent on its being inside the beam. Thus there may be cases in which an external core-shear damping treatment (with its dimensions not restricted by the inner dimensions of a hollow beam or panel) may prove effective.

ACKNOWLEDGMENT

Note: The damping treatment considered here is one of several that are the subjects of a pending patent.

REFERENCES

1. Kurtze, G., and B.G. Watters, "New Wall Design for High Transmission Loss or High Damping", Journal of the Acoustical Society of America 31, 739-748 (1959).

2. Cremer, L., Akustische Zeitschrift, 7 p. 81 (1942).

3. See discussion in L. Beranek, Ed., Noise Reduction, McGraw-Hill, New York 1960 (p. 290,ff).

4. Cremer, L., M. Heckl, E.E. Ungar, Structure-Borne Sound, Second Edition, Springer-Verlag, New York (1988).

FBD-10

Fig. 1. Box beam.

BOX BEAM

$2y_o$

VISCOELASTIC BEAM INSERT

$<V> = \frac{1}{4} Bk^4 y_o^2$
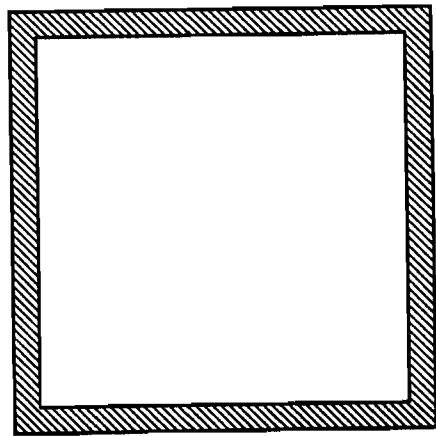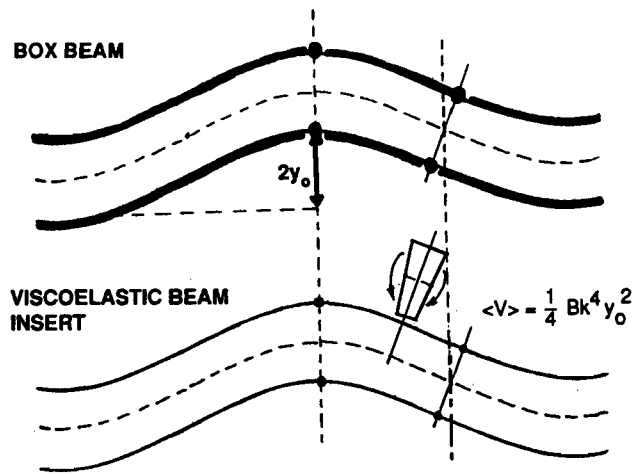
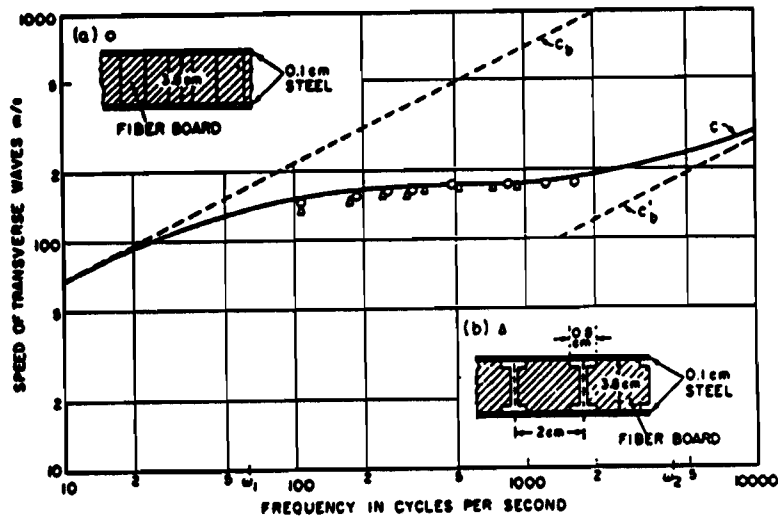**Fig. 2. Bending deformations of box beam and of viscoelastic beam insert.**

Fig. 3. Experimentally determined transverse wave speed versus frequency, for sandwich bars of wood fiber-board cores and steel skins. (a) For core material arranged with the grain perpendicular to bar (for lowest shear stiffness). (b) For grain of core layer parallel to axis of bar, shear stiffness reduced by cutouts (Kurtze and Watters, Ref. 1).

BOX BEAM

$2y_o$

VISCOELASTIC BEAM INSERT

$$\langle V \rangle = \tfrac{1}{4} B k^4 y_o^2$$

CORE SHEAR INSERT

$$\langle V \rangle = \tfrac{1}{4} G A k^2 y_o^2$$
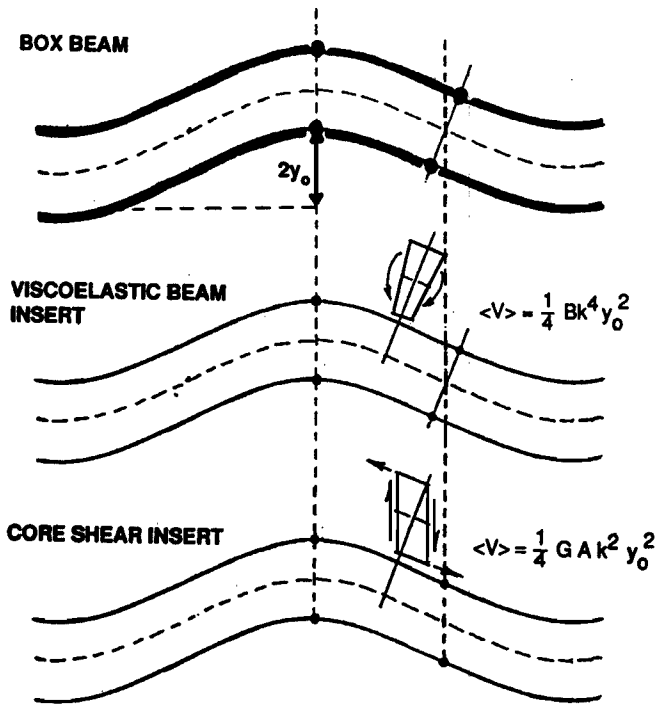
Fig. 4. Transverse deformation of box beam, viscoelastic beam, and core shear insert.
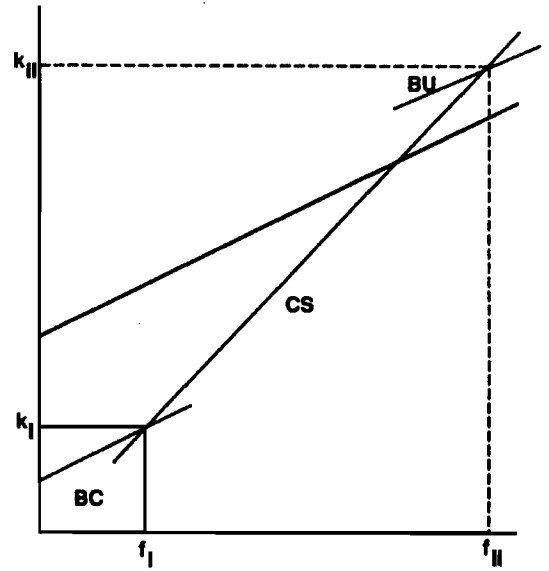

k_{II}

BU

CS

k_I

BC

f_I f_{II}

Fig. 5. Transverse wavenumber regimes for shear-core insert; BC; coupled-skin bending, CS; core shear, BU; uncoupled-skin bending.
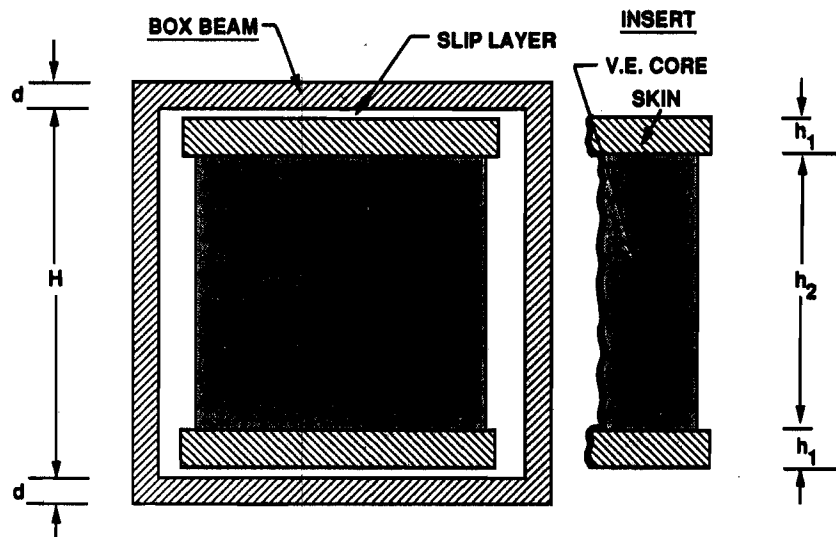

BOX BEAM —   — SLIP LAYER   **INSERT**

— V.E. CORE

SKIN

d

H

d

h_1

h_2

h_1

Fig. 6. Core-shear insert damper for box beam.

FBD-12
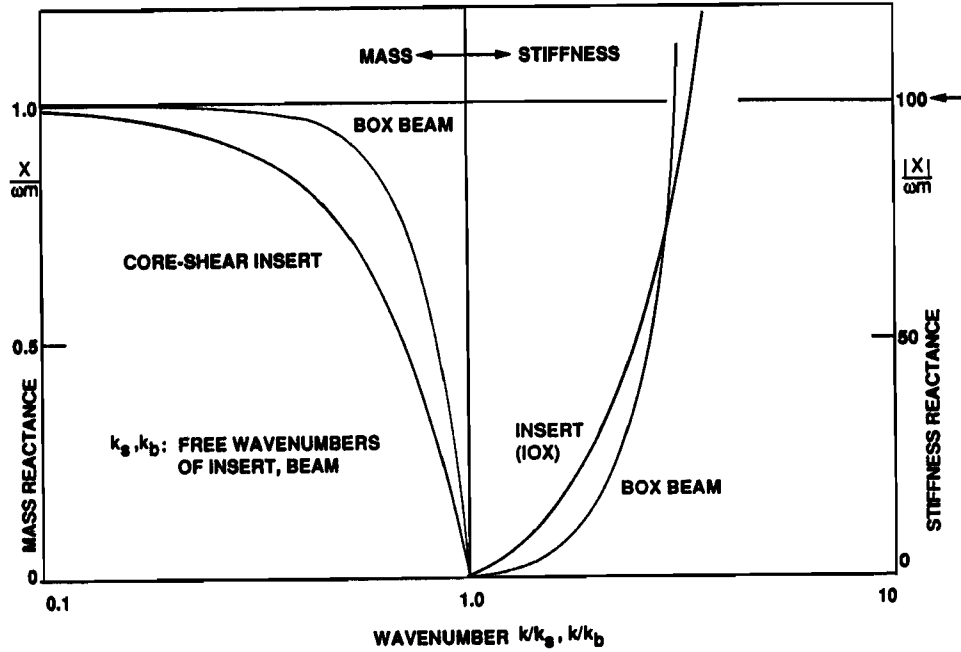
Fig. 7. Transverse reactance vs wavenumber for box beam
and core-shear insert at frequency ω.

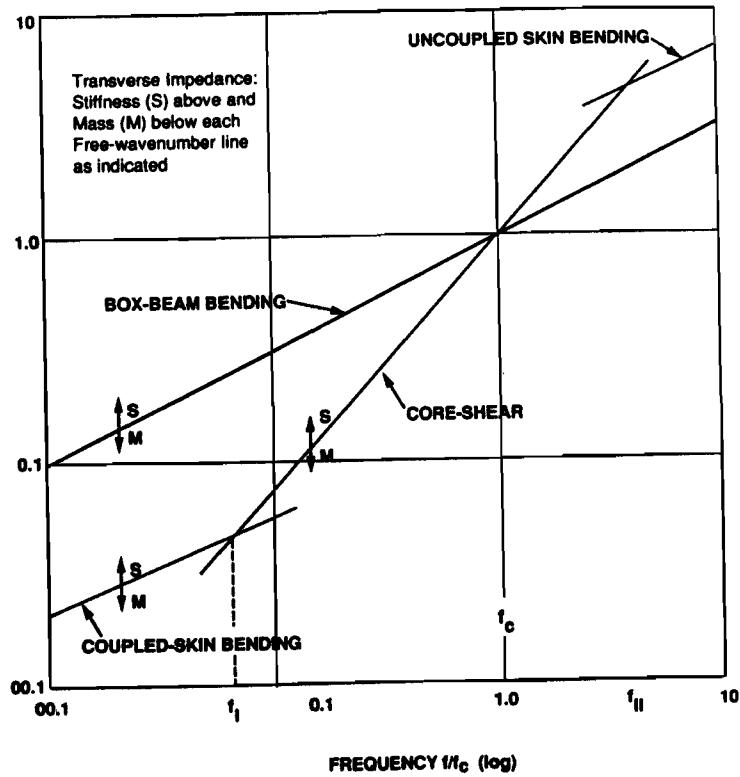
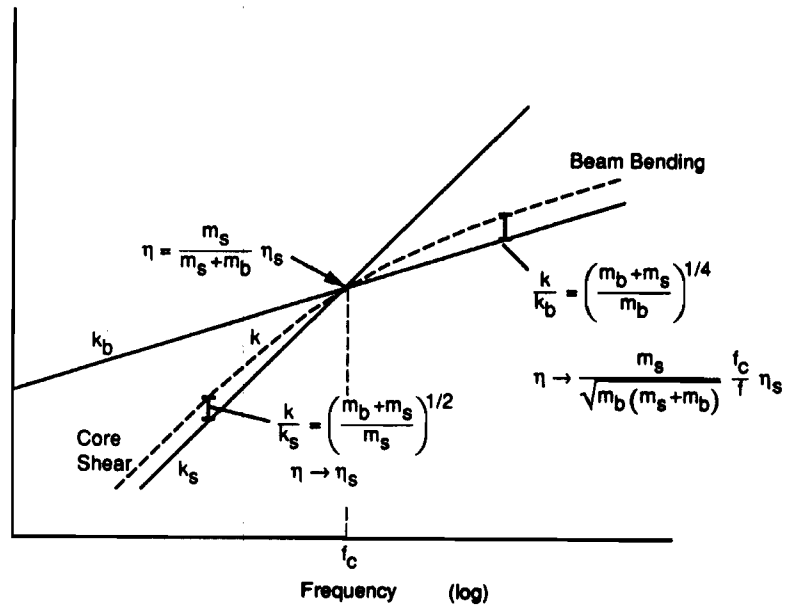Fig. 8. Wavenumber vs frequency characteristics of box beam
and of core-shear insert.

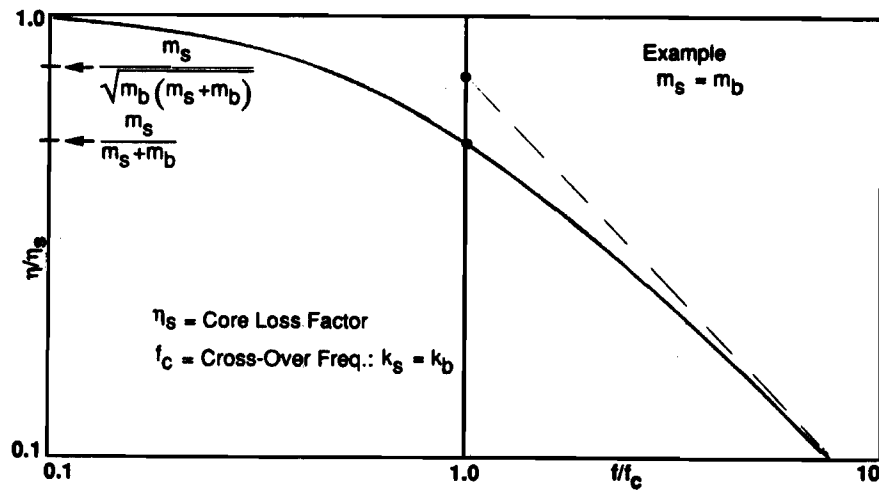**Fig. 9. Wavenumber k of the box beam with core shear insert.**

Fig. 10. Box beam with core-shear insert. Relative loss factor η/η_s vs frequency.