DISCUSSION OF THE PAPERS BY PROF. FREUDENTHAL AND PROF. WEIBULL

Alan Powell.- I would like to ask Professor Freudenthal whether instead of Miner's rules he has tried correlating the results with Shanley's modification, namely one over n to some power. Do you think there is any future in this at all?

A. M. Freudenthal.- We have in fatigue and in crack propagation two effects, the effect of non-linearity of damage-accumulation at constant stress and the effect of interaction between stress-levels. Shanley's modification applies entirely to the question of non-linearity; instead of using a linear cycle ratio he uses a cycle ratio to a power. In the test which we performed and which we attempted to interpret we found that the effect of non-linearity is of much less importance than the effect of stress-interaction. Thus, while Shanley's concepts may be valid with respect to the non-linear crack-propagation under a constant amplitude, it does not consider the main effects of interaction between various stress levels.

P. W. Smith.- This question is for Professor Freudenthal. In relation to the fictitious S-N diagram, I believe you said it depends on the spectrum and it is derived for some measure of the amplitude of the constant spectrum. This measure might be the rms stress. I do not see where the introduction of the fictitious S-N diagram is valuable over a plot of life against rms stress for that particular spectrum.

A. M. Freudenthal.- I think we are talking about two different things. The spectrum I have in mind is a load spectrum and it is derived from gust records. For a given load spectrum applied in a randomly run test, the rms stress is not clearly related to fatigue damage.

P. W. Smith.- Why couldn't the load information itself be used directly without any necessity of introducing the combination with the S-N diagram. For each spectrum you have derived a life that could be associated with some measure of amplitude. This could be perhaps an rms load.

A. M. Freudenthal.- What we are trying to do is not to derive information for the material specimen itself but to deduce from the information which we obtain from the material specimen the behavior of complex structures. In other words, we are faced with the problem of having to design large scale structures without actually testing them. We know that fatigue is a highly statistical phenomenon but we can never test a sufficient number of large scale structures to get into a probability range which would be acceptable by the designer. Thus, we have to test small specimens of which we can have sufficient replication to study not only the trend of fatigue life but also its statistical distribution. Direct plots of rms stress versus life do not provide information that is useful, unless you test the actual structure. On the other hand, the use of our investigation in the future should be to provide a means by which, from the knowledge of the constant amplitude S-N diagram of the structure, the trend of which can be obtained or at least estimated from the large number of existing constant-amplitude tests of structural parts and structures, to deduce the expected service fatigue life of the structure under a random gust sequence belonging to a given spectrum. For this purpose, the relation between the S-N diagram and the fatigue life under variable stress amplitudes has to be known rather than the life related to the rms stress of the spectrum. Only in this way does it become possible to develop a relatively simple design procedure based on a quasi-linear relation of damage accumulation in terms of the "fictitious" S-N diagram.

J. R. Fuller.- With random phenomena very often and particularly for broad bands you cannot pick out a particular stress cycle. If the phenomenon is truly random it is random and you cannot make stress out of it. I would like criticism of this idea. Since we know that the power spectrum is one way of describing a random phenomenon, this together with a distribution of the random function itself gives a fair description of this phenomenon.

Would it therefore not be possible to find a direct relationship between spectrum and life ? For a single sinusoidal constant amplitude, you have a spike as a power spectrum; for an essentially constant amplitude or frequency with random amplitude, you have two spikes which are close together, so there again the problem reduces to the one which we are familiar with. In other words, we have a narrow band problem. Now suppose we had other types of power spectra like trapezoidal or two triangles or something like this, it would seem that they should be uniquely related to life, without going through any intervening approximate theory like Miner's rule or a more elegant theory.

Waloddi Weibull.- A power spectrum does not provide necessary and sufficient information in that you could have different power spectra which would do exactly the same fatigue damage and you could have the same power spectra producing different fatigue damage. This means that you have to give additional information to the power spectrum which is not enough to specify the damaging effect. If you assume a random noise for which the statistical representation corresponds to the normal distribution, then, I think, it may perhaps be possible that you have a unique representation of the damage but you would certainly have provided superfluous information since the distribution of the power over the frequency band is not essential to know.

A. M. Freudenthal.- I think that if the power spectrum itself is applied, you are leaving out a very important part, which is the response of the structure; the power spectrum refers to the applied load, not to the structural response. In the simplest case, you can excite one mode, but under complicated conditions you probably excite not only the five or six modes which Professor Richards showed but, particularly if you have a damaged structure, additional shear modes which have not been in Professor Richards' diagram and which, in the damaged structure, may be in a low enough range to become structurally significant.

H. T. Corten.- This question is for Professor Freudenthal. In looking over some of the data and literature on notched sections of welded sheets and joints, it appears as though the range of stress as well as the shape and the material may affect the value of the slope of the hypothetical S-N diagram that you are speaking of. Has this been your experience and how does it vary as you look the data over including full scale wings.

A. M. Freudenthal.- It is obvious that the mean stress will have quite an effect and stress concentrations will have an effect. We have been running tests on random rotating beam machines with zero mean stress; our results which are intended as showing a trend rather than quantitatively reliable values are thus results based on zero mean stress. We are now running some tests on stress concentration and we find quite significant differences; we also expect to find considerable effect of mean stress. However, the point which I am trying to make is that we have been concerned with establishing a workable procedure that can show the effects of stress interaction. Incidentally, I want to mention that the stress interaction must not always be negative. We can very well imagine conditions under which the fictitious S-N diagram may go up beyond the conventional diagram; for instance, if we apply an initial high intensity prestress to the structure we may get a longer life at any stress amplitude below this prestress. Thus, the specific results will obviously be affected by mean stress, by stress concentrations, by residual stresses due to prestraining, and by a number of other effects.

P. M. Belcher.- One of the things we like to come out of this conference is some notion of what all of these concepts mean when you get down to hardware. Two concepts have come up today about which this sort of thinking could take place: one is the question of stress interaction, the other the question of correlation. My question is this. Is stress interaction an important phenomenon for acoustical fatigue and if it is important, is it important for both long life and short life, that is for the transport aircraft and missile, or is it important for just one? I think it is possible to represent the number of cycles to failure under random load by an equation according to which the fatigue life is one over the sum of the spectral ratios $(\frac{p_s}{N_s})$ where p_s represents the frequencies in the stress spectrum and N_s represents the number of cycles from a conventional S-N diagram. We are worrying about a large number of cycles, perhaps as many as 10^9, which is still not an infinite number, and I think we can roughly put in intervals and say we will treat these functions as continuous functions. I think that there should be some sort of a factor in the sum of spectral ratios. Can we just put an interaction coefficient alpha in there and say that this takes care of the stress interaction if we make that alpha a function of our stress spectrum?

A. M. Freudenthal.- We are looking for a walking stick on which we can base our design. What we have found is that if you have a spectrum with a small number of high stress levels its effect on low stress levels is to reduce the life. An expedient way in which this information can be represented is by reducing the life which we get from the constant amplitude test by a multiplication factor representing this interaction. This factor is obviously a function of all stress levels of the spectrum above the considered level.

P. M. Belcher.- I wish to consider a case which is of interest to us. Using a conventional S-N diagram for 2024 aluminum I impose a real distribution of stresses that produces a life of 10^9 cycles with an rms stress of 4250 psi. On the basis of the conventional diagram with $N = 10^9$ at a stress amplitude of 15,000 psi, most of the damage seems to be taking place at something like $3\frac{1}{2}$ to $4\frac{1}{2}$ times the rms value. Suppose we change the S-N curve to one of the fictitious S-N curves, this changes the answer a little bit; the higher stresses are doing more damage than I said they were before, therefore the life is getting shorter. As a result of having a more realistic representation of damage, we are therefore getting a lower life. We are talking about working at a rms value which is 1/13 to 1/15 of the ultimate; therefore, all damage is taking place in the Miner case at something like less than 25% of the ultimate of the material, with few stresses up in the higher region. I suspect that the difference will not be very large for long life. If you talk about short lives and missiles, we are working at much higher stresses; here the difference may be larger.

A. M. Freudenthal.- I would just like to make one more remark here concerning the stress interaction; as we consider a different (fictitious) curve, this is a curve with a steeper slope for which the maximum of the specific damage goes down. Thus, the effect of the stress interaction is to push the Miner point of maximum specific damage down, not to push it up. The concept of stress interaction has no meaning in terms of short life, as we are in the region of high amplitude fatigue; even if there is an interaction it can hardly be noticed. The main effect of stress interaction is in the low range and its main result is to push the amplitude of maximum interaction down. I do not know how important that is in acoustical fatigue; the importance in structural fatigue is that it may change the significant stress level at which the constant stress amplitude test should be run in order to represent the variable stress service performance of the material, at least as far as this is possible in a constant amplitude test.