FOREWORD

This is Volume II of a two-volume report on an investigation of methods for allocating system reliability and testing for compliance to reliability requirements. The development of procedural allocation and testing methods is explained in Volume I.

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ABSTRACT

A step-by-step procedure for implementing the allocation model described in Volume I of this report is presented together with a discussion of methodology and required inputs. The basic data inputs, derived from an analysis of failure data representing over two-million equipment operating hours, and procedures for using them,are described in Appendix A. Appendix B describes a procedure for determining the feasibility of the over-all system reliability requirement. Detailed examples of the complete allocation procedure are presented in Appendix C. Appendix D contains a list of Figures and Tables.

PUBLICATION REVIEW

The publication of this report does not constitute approval by the Air Force of the findings or conclusions contained herein. It is published only for the exchange and stimulation of ideas.

FOR THE COMMANDER:

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STEP-BY-STEP RELIABILITY ALLOCATION PROCEDURE

1. INTRODUCTION

1.1 Scope

This document establishes uniform procedures for specifying, at the design stage, the reliability values that must be met by components, equipments, and subsystems to satisfy the reliability requirement established for a given Air Force system.

1.2 Need for Allocation Procedure

Quantitative system reliability requirements are now considered to be a system performance parameter. A reliability requirement at the system level, however, tends to lose its significance because responsibility for development is necessarily shared at system sublevels within the military department and among its contractors. It is therefore necessary to allocate the system reliability requirement among the system sublevels at which manufacturing and procuring responsibility is clearly defined.

1.3 Criteria for Effective Allocation

The initial allocation model should satisfy the following criteria:

(1) The model must be generally applicable.

(2) The model should be based on the ultimate use of standard input data.

(3) The methods provided in the model must be economically feasible.

(4) The model must yield realistic and attainable requirements.

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1.4 General Approach

This document presents mathematical formulas which reflect the influence of certain factors on allocated requirements and at the same time meet the criteria in Section 1.3, above. In the development and application of the allocation procedures, some approximations are necessary, and some factors -- such as weight and cost -- are not explicitly considered in the mode.† The procedural routine for allocating reliability entails the following general steps:

(1) Identification of the basic levels (units) of the system over which the system requirement will be apportioned.

(2) Determination of the feasibility of the system requirement by estimating a measure of system complexity and referring to a reliability-feasibility chart.

(3) Assignment to each unit of numerical factors which, through allocation formulas, determine the relative unit state-of-the-art.

(4) Use of a basic equation or an appropriate chart for determination of each unit's allocated reliability.

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† In Section 4 of this document, the role these factors play in adjusting the allocated requirement is discussed.

2. BASIS OF ALLOCATION

2.1 Level of Allocation

The system level to which reliability requirements are allocated is usually determined by the division of responsibilities within the military department and among its contractors. This division of responsibilities may occur at various system sublevels, and within such segments it may be desirable to allocate requirements to even lower levels of subdivisions. In this document, the term "system" shall apply to the level at which an over-all reliability requirement exists. Any component, equipment, or other subdivision of a system to which a reliability requirement is being allocated will be called a "unit." The division of a system into units must be accomplished prior to the allocation of the over-all reliability requirement.

2.2 Elements Considered in the Allocation

2.2.1 System and Failure Definitions

The system under consideration must be clearly defined in terms of its functions and boundaries. From a determination of the operational demands and the functional requirements of the system, the conditions that constitute failure or unsatisfactory performance can be determined. These conditions can then be translated into measurable unit characteristics. The boundaries surrounding the system and each unit must be clearly defined to insure that important items are neither neglected nor considered more than once.

2.2.2 System Reliability Requirement

The primary element in a reliability allocation model is the system reliability requirement. This requirement is usually determined on the basis of ultimate user requirements and feasibility, but it may derive from an allocation performed at a higher echelon. The requirement may be stated in any appropriate measure such as mean life, system failure rate, or, preferably, a reliability over a fixed period of time.

If the possibility exists that a system which is performing all its designed functions satisfactorily can still fail to accomplish the mission, the system requirement may be subject to misinterpretation. The following factors must be considered:

System Effectiveness $S^*(T)$ = probability that the system can successfully meet a stated operational demand for T hours of operation under stated conditions.

System Reliability $R^*(T)$ = probability that the system will satisfactorily perform its designed functions for T hours of operation under stated conditions.

System Design Adequacy D_s = probability that satisfactory performance of designed functions will lead to accomplishment of the mission.

Probability-of-success requirements on systems which have design adequacies less than one (1.0) shall be considered to be system effectiveness requirements unless otherwise stated. The system reliability requirement is related to the system effectiveness requirement by the formula[†]

$$R^*(T) = \frac{S^*(T)}{D_s} \quad .$$

Design adequacy must be determined prior to allocation in order to obtain $R^*(T)$. Theoretical investigations, Monte Carlo simulations, or experimentations may be necessary to estimate D_s. Since design adequacy is usually a function of many variables such as system accuracy, environmental conditions, and system inputs, an average value for D_s may be used by considering the relative frequency distribution of these parameters. It is probable that, at the design stage, system design adequacy will have to be assigned on an intuitive basis after careful consideration of the operational demands on the system and the abilities of various units to meet these demands.

[†] For the general case, system effectiveness is related to reliability by the formula, $S^*(T) = R^*(T) \cdot D_s \cdot P_{or}$, where P_{or} is the operational readiness defined as the probability that at any point in time a system is either operating satisfactorily, or ready to be placed in operation on demand, when used under stated conditions. For the purpose of allocation, P_{or} is assumed to be one. A thorough discussion of these concepts will be found in Reference 1.

2.2.3 Relationships Between Unit and System Failure

The relationships between unit failure and system failure must be determined before the allocation is made. Four types of basic relationships for which allocation methods are presented are as follows:[†]

(1) Serial system: no functional duplicates exist and each unit must operate successfully for system success.

(2) Modified serial system: no functional duplicates exist but units can fail without necessarily causing system failure.

(3) Redundant system: components of the system are duplicated for increased reliability but each redundant path or mode of operation is equally effective in performing its function.

(4) Multimodal system:[†] redundant paths or modes of operation are not equally effective in performing their function.

From these unit/system failure relationships, it is possible to develop the two concepts of unit essentiality and modal design adequacy, which are discussed below and will be employed in the allocation procedures set forth in Section 3 of this document.

2.2.3.1 Unit Essentiality

The concept of essentiality, used to describe the effect of unit failure on mission success, is considered only if a failed unit does not have a functional duplicate. It is defined as follows:

> The essentiality of a unit is the probability that the system will fail to accomplish its mission if the unit fails while all other units perform satisfactorily.

Unit essentiality must be considered in the allocation of reliability of modified serial systems; it may also be involved in redundant and multimodal systems. At the design stage of system development, the likelihood is that the essentiality of various units within the system will have to be assigned intuitively, on the basis of experience gained with

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[†] In this instance, the allocation method presented is applicable only to bimodal systems.

similar systems. If appropriate system failure data is available, essentiality can be estimated by the ratio,

$$E_j = \frac{\text{Number of mission failures due only to } j^{th} \text{ unit failure}}{\text{Number of } j^{th} \text{ unit failures}}$$

2.2.3.2 Modal Design Adequacy

A mode of operation is defined to be a unique combination of components which are required to perform the system function. Modal design adequacy is defined as follows:

> Modal design adequacy is the probability that, given satisfactory operation in the mode, the system will accomplish its mission.

A redundant system is defined to be one in which there is more than one mode of operation (because of functional duplicates) but where the design adequacies of each mode are equal (e.g., the components in each mode are identical). For allocation purposes, components which have functional duplicates are termed redundant units.

A multimodal system is defined as one incorporating more than one mode of operation (because of functional duplicates), each mode having a different design adequacy (e.g., secondary modes result in some degradation in performance). For allocation purposes, components which have functional duplicates are termed modal units. In the determination of values for modal design adequacy, the discussion of system design adequacy in Section 2.2.2 is applicable.

2.2.4 Basic Failure Data

2.2.4.1 State-of-the-Art Measures

The allocation procedure is based on the relative reliabilities to be expected of various units of a system, as determined from past experience. These are computed as part of the procedure, from the relative failure rates given in Appendix A, which have been derived from field reliability surveillance on various systems and are considered to be representative of the state of the art. Effective application of the recommended procedures requires understanding of these inputs and the reasoning behind their development.

2.2.4.2 Level of Analysis

In the phase of design for which this allocation procedure is intended, it is possible to determine functional performance levels on a part group basis. The input data is derived from a reliability comparison of the various functions; hence the allocation will be based on estimates of functional population within the system and the relative difficulty of achieving high reliability for each function. For this purpose, the relative failure rate, k_i, compared to a standard function has been chosen to represent the past reliability experience of the various functions. (The subscript, i, is an index of the function being considered.) Available relative failure rates are given in Appendix A.

2.2.4.3 Electronic Functions

The electronic functional levels to which this procedure is applicable correspond to the functions performed by individual element groups (AEG's). An active element group is defined as consisting of an active element (one part, such as a tube, capable of performing valving or controlling action) plus the associated passive parts; examples of active element groups include amplifiers, oscillators, mixers, and rectifiers. It should be noted that in the preparation of the input data for the allocation procedure, dual-function tubes have generally been counted as representative of two separate active element groups if they perform two distinct functions.

AEG's have been classified by functional categories which cover all standard electronic functions. A functional category represents a group of functions of similar nature, subject to similar stresses, and with relatively common part populations. For example, audio amplifiers, audio detectors, and audio oscillators are all grouped into the audio category.

Failure data from field studies of a number of equipments has been analyzed, and functional AEG failure rates (k_i) -- expressed as ratios to the audio failure rate -- provide the common basis for allocating reliability.† Relative failure rates and definitions of the functional categories will be found in Appendix A.

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† Functions for which the classification is inadequate may be evaluated by one of several estimating methods given in Appendix A.

2.2.4.4 Non-Electronic Functions

The definition of an active element group does not always lend itself to non-electronic functions, and available failure data often does not allow analysis based on part groups as opposed to individual parts. The principle which has been followed in these instances is that the significant functions which can be identified at the design stage are performed by individual parts for which relative failure rates are given (e.g., electric motors are considered without any associated parts such as capacitors, leads, or switches). For ease in presentation, the term AEG is also used to represent the non-electronic parts which are utilized in the allocation.†

2.2.4.5 Present Data Inadequacies

It is not possible to list all conceivable functions and functional categories that might appear in future systems, and suitable data is not always available for many of the standard functions, particularly in non-electronic areas. If a relative failure rate is not listed in Appendix A for a given functional requirement, one of the estimating procedures given in the Appendix must be used. Later revisions to Appendix A will reflect additional failure data as it becomes available.

2.2.5 Unit Duty Cycles

Duty cycles are taken into account in the allocation procedures, to reflect any variance in unit operational time requirements. Units which have a limited operational period because of a low duty cycle should have a relatively high reliability allocation over the system operating period.

2.2.6 Gross Unit Environment

It is assumed that the relative failure rates given in Appendix A are independent of the uniform gross environment of the system, i.e., the relative failure rates are the same for ground-based, shipboard, airborne, and other environments. Some systems, however, have large differences in gross environmental characteristics from unit to unit. In such cases, provisions are made for adjusting the rates; these are also given in Appendix A.

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† It should be noted that in many cases the failure rates given in Appendix A are relative to the failure rate of the average electronic AEG, rather than the audio AEG. Such rates must be adjusted according to the procedure given in the Appendix before they are used in the allocation formulas.

2.2.7 Part Selection

The relative functional failure rates are assumed to be the same for any system in which the respective part classes are homogeneous -- e.g., tubes are used consistently, without resort to transistors in some applications. A problem does arise if it is decided to employ a mixed population, such as in a partially transistorized system. In such instances, adjustments similar to those discussed for gross environmental conditions in Section 2.2.6 will be made according to the procedures and data given in Appendix A.

3. PROCEDURAL STEPS FOR ALLOCATION

This section contains step-by-step procedures for determining allocated unit reliabilities for: serial and modified serial systems, redundant systems, and bimodal systems. The basic data inputs to the procedures are the relative functional failure rates and the modification factors reflecting the influences of special or nontypical conditions.

Two assumptions are required for proper utilization of the allocation procedures. They are:

(1) That allocation levels can be so chosen that failure probabilities are independent, i.e., dependent components can be grouped into one unit, making failure probability of this unit independent of the state of the other units.

(2) That the unit state can be described in discrete terms of success and failure through analysis of the system reliability requirement and the functional relationship between unit and system operation.

The procedural steps have been designed to accommodate each of the specific system types. Attention is called to the fact, however, that five steps are applicable to all systems. These are described in Section 3.1, and are preliminary to any steps that are peculiar to the specific system types. Steps applying to serial and modified serial systems will be found in Section 3.2. Steps for redundant systems and bimodal systems are described in Sections 3.3 and 3.4, respectively.

Appendices are provided for several of the relatively complicated steps in the following sections. Appendix A contains the basic inputs to the allocation procedure and provides step-by-step instruction for using and modifying these inputs as dictated by the allocation method. Appendix B describes fully the steps determining feasibility for all system types. Detailed examples for using the complete allocation procedure for each of the four system types are given in Appendix C.

3.1 Steps Applicable to All System Types

The following steps apply to all of the system types considered in this procedure and must be accomplished first. Then the steps peculiar to each system type are to be performed in the order listed. (See Sections 3.2., 3.3 or 3.4.) The sample allocation work sheet on the facing page can be used for all system types.

▶ (a) Define the units for which the system reliability requirement is to be allocated by constructing a reliability block diagram showing the units (blocks) of the system in logical sequence.

▶ (b) Determine the system type (serial or modified serial, redundant or bimodal) by referring to the definitions given in Section 2.2.3. The term redundant configuration shall apply to the group of redundant units (see Section 2.2.3.1) and the term bimodal configuration shall apply to the group of modal units (see Section 2.2.3.2).

▶ (c) Refer to Appendix A and follow the procedures outlined to obtain unit failure indices K_j. These K_j's should be listed on an allocation worksheet similar to that attached to this section.

▶ (d) From the definition of system success and the operational demands imposed on each unit, estimate the essentiality, E_j, of each series unit and the redundant or bimodal configuration if applicable. (For unmodified serial systems, E is equal to one for all units.) List the unit essentialities in the appropriate columns of the work sheet.

▶ (e) If the system requirement is in terms of a probability of successful operation for T hours, estimate the average operating time, t_j, of each unit and the redundant or bimodal configuration during T system hours of operation. If the requirement is in terms of mean life or failure rate, a value of T should be chosen to represent a significant period of system operation such as average mission-time or average maintenance-period. Provision for listing the t_j's is also made on the work sheet.

After the completion of the above steps, refer to the section in this report pertaining to the specific system type under consideration.

DESIGN STAGE RELIABILITY ALLOCATION WORKSHEET

System ____________________ Date ____________

Primary Mission ______________ System Operating Time - T ____________

System Reliability Requirement† ____________

	Series Units				Redund. Config.	Redundant Units	
Unit or Configuration Identification U_j	U_1	U_2	. . .	U_m	$U_r^{\dagger\dagger}$	$U_{r_1}^{\dagger\dagger}$	$U_{r_2}^{\dagger\dagger}$
Essentiality E_j	E_1	E_2	. . .	E_m	E_r		
Operating Time t_j	t_1	t_2	. . .	t_m	t_r		
Failure Index K_j	K_1	K_2	. . .	K_m	$K_r^{\dagger}$	K_{r1}	K_{r2}
Failure Index Ratio $w_j^{\dagger}$	w_1	w_2	. . .	w_m	w_r	w_{r1}	w_{r2}
Allocated Reliability $\hat{R}(t_j)^{\dagger}$	$\hat{R}(t_1)$	$\hat{R}(t_2)$	. . .	$\hat{R}(t_m)$	$\hat{R}(t_r)$	$\hat{R}_{r_1}(t_r)$	$\hat{R}_{r_2}(t_r)$

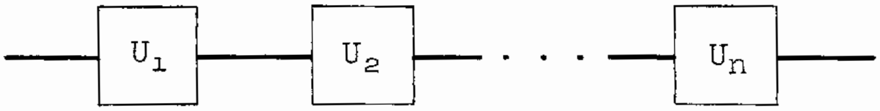
† Obtained according to appropriate steps of procedures

†† Subscript b for bimodal systems

3.2 Final Steps -- Serial or Modified Serial Systems

Assuming that the preliminary steps described in Section 3.1 have been completed, then the following steps for serial or modified serial systems should be performed in the order listed.

The following unit identification notation will be used in this section:



U_1 to U_n are series or modified series units

▶ 1. Determine the system reliability requirement by first estimating system design adequacy. If the design adequacy is less than one (1.0), the original system requirement shall be considered to be a system effectiveness requirement unless otherwise specified. The equivalent system reliability requirement is given by

$$R^*(T) = \frac{S^*(T)}{D_s} \tag{1}$$

where $R^*(T)$ is the system reliability requirement for T system or mission hours of operation.

$S^*(T)$ is the system effectiveness requirement for T system or mission hours of operation.

D_S is the system design adequacy.

NOTE: If the original requirement is given in terms of mean life or failure rate, a value of T should be chosen to represent a significant period of system operation such as average mission time or average maintenance period. The equivalent system effectiveness requirement is obtained from the equation

$$S^*(T) = e^{-T/\overline{\theta}^*} \qquad (2)$$

or

$$S^*(T) = e^{-\overline{\lambda}^* T} \qquad (3)$$

where

$\overline{\theta}^*$ is the original system mean life requirement.

$\overline{\lambda}^*$ is the original system failure rate requirement.

Equation (1) is then used to determine the system reliability requirement.

▶ 2. Determine the feasibility of the reliability requirement according to the appropriate procedure of Appendix B

▶ 3. Obtain the total system failure index, K, by the formula

$$K = K_1 + K_2 + \cdots + K_j + \cdots + K_n \tag{4}$$

and form the failure index ratios

$$\begin{aligned} w_1 &= K_1/K \\ w_2 &= K_2/K \\ &\ \ \vdots \\ w_n &= K_n/K \end{aligned} \tag{5}$$

▶ 4. Compute allocated unit reliabilities from the equation

$$\hat{R}(t_j) = 1 - \frac{1-R^*(T)^{w_j}}{E_j} \tag{6}$$

where

$\hat{R}(t_j)$ is the allocated reliability of the j^{th} unit.

t_j is the average operating time of the j^{th} unit during T hours of system operation

E_j is the essentiality of the j^{th} unit. (E_j must be greater than $1 - R^*(T)^{w_j}$. Units which violate this requirement shall be excluded from the allocation and new values of the w's obtained.)

NOTE: If the system is of the unmodified serial type $\hat{R}(t_j) = R^*(T)^{w_j}$. Figures 1 to 4 give the solution to equation (6) for values of $R^*(T) = 0.95, 0.90, 0.85,$ and 0.80, for values of $E = 1.0, 0.9, 0.8, 0.7, 0.6, 0.5$, and for the complete range of w.

▶ 5. If unit requirements are desired in terms of mean life or failure rate and constant failure rates are assumed,

$$\hat{\theta}_j = - \frac{t_j}{\log \hat{R}(t_j)} \qquad \text{(natural logarithms)} \tag{7}$$

$$\hat{\lambda}_j = - \frac{\log \hat{R}(t_j)}{t_j} \tag{8}$$

where $\hat{\theta}_j$ and $\hat{\theta}^{`}_j$ are the allocated mean life and failure rate, respectively, of the jth unit. Figures 5 to 8 can be used to obtain $\hat{\theta}_j$ for values of $R^*(T)$ equal to 0.95, 0.90, 0.85, 0.80. The vertical axis represents the value of $\hat{\theta}/t$. Hence, for the jth unit, multiply the appropriate value on this axis by t_j to convert to hourly units. Since $\hat{\lambda}_j = 1/\hat{\theta}_j$, the same graphs can be used if failure rates are to be allocated.

Approximate Formulas:

Follow Steps 1 to 3;

For A. Unit mean life requirements;

$$\hat{\theta}_j = - \frac{E_j t_j}{w_j \log R^*(T)} \tag{9}$$

For B. Unit failure rate requirements:

$$\hat{\lambda}_j = - \frac{w_j \log R^*(T)}{E_j t_j} \tag{10}$$

For units known to have a failure rate which is not constant over time, an average failure rate during the unit's t hours of operation can be obtained by the equation

$$\hat{\lambda}_j = \frac{1-\hat{R}(t_j)}{t_j} \tag{11}$$

3.3 Final Steps for Redundant Systems

This section describes the remaining steps of the allocation procedure for systems which contain a single redundant configuration consisting of two units, not necessarily identical, each of which is equally effective in performing the required function -- i.e., modal design adequacies are equal. The two types of redundancy considered are:

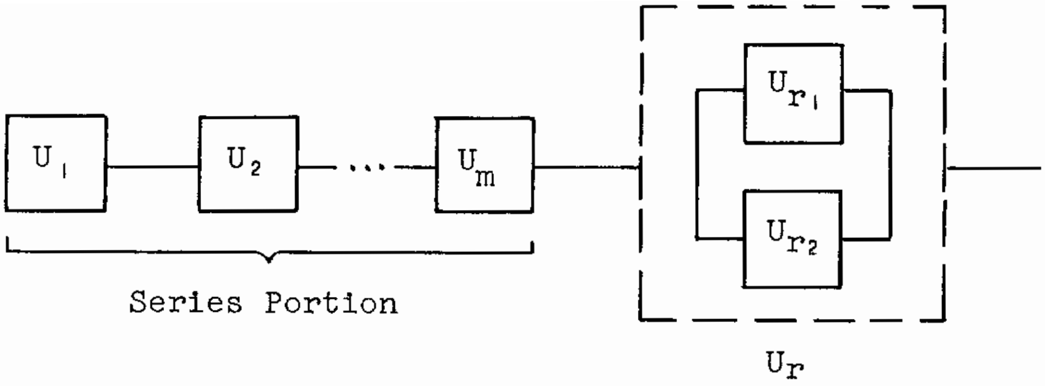
(a) Active-parallel, where both redundant units are constantly energized, and

(b) Standby, where one redundant unit is not energized until the operating redundant unit fails.

The approach used for allocating reliability in these cases is to determine an equivalent failure index for the redundant configuration from the failure indices of each redundant unit. This method will permit utilization of the basic allocation formulas. If switching is involved, it shall be assumed that the probability of premature switching (switching when not required) is relatively small as compared to the probability of failure to switch when required. The switching mechanism, therefore, can be considered as a series unit.

In the development of the formulas, it was assumed in several cases that both redundant units fail according to the exponential law. Reference 2 gives the more general formulas which should be used if this assumption is known to be in serious error.

The following unit identification notation will be used in this section.



U_1 to U_m are the series units

U_{r_1} and U_{r_2} are the two redundant units

U_r is the redundant configuration

The remaining steps of the procedure appear in sequence below:

▶ 1. Determine the system reliability requirement by first estimating the design adequacy of the system. (Follow Step 1 of Section 3.2 except that system design adequacy, D_s, is taken to be the design adequacy of each of the two modes of operation.)

▶ 2. Obtain the total failure index of all series units from the equation

$$K_a = \sum_{j=1}^{m} K_j \qquad (12)$$

where

K_a is the total failure index of the series portion

m is the number of series units.

▶ 3. If the redundant units are not identical, calculate an equivalent failure index for each redundant unit from the formula

$$K_r' = \left[K_{r_1} \cdot K_{r_2}\right]^{1/2} \tag{13}$$

where

K_r' is the equivalent failure index of redundant units having failure indices of K_{r_1} and K_{r_2}. (If U_{r_1} and U_{r_2} are identical, K_r' is the failure index for each of them.)

▶ 4. Determine the feasibility of the reliability requirement according to the appropriate procedures of Appendix B.

▶ 5. Calculate the ratio

$$\alpha = \frac{K_a}{K_r'} \tag{14}$$

▶ 6. To obtain an equivalent series failure index for the redundant configuration,

A. <u>With Active Parallel Redundancy</u>:

Obtain the value $Z(\alpha, R^*)$ from Figure 9 or 10 for the appropriate set of α and R^*,

B. <u>With Standby Redundancy</u>:

Obtain the value $Z(\alpha, R^*)$ from Figure 11 or 12 for the appropriate set of α and R^*,

and compute

$$K_r = Z(\alpha, R^*)K_a \tag{15}$$

where K_r is the equivalent series failure index of the redundant configuration.

▶ 7. Obtain the total system failure index from the equation

$$K = K_a + K_r \tag{16}$$

and calculate the series unit and redundant configuration failure index ratios:

$$\begin{aligned} w_1 &= K_1/K \\ w_2 &= K_2/K \\ &\cdot \\ &\cdot \\ &\cdot \\ w_m &= K_m/K \\ w_r &= K_r/K \end{aligned} \tag{17}$$

as well as each redundant unit failure index ratio:

$$\begin{aligned} w_{r_1} &= K_{r_1}/K \\ w_{r_2} &= K_{r_2}/K \end{aligned} \tag{18}$$

▶ 8. Allocated reliabilities for T system hours of operation can be computed from the equations given below.

A. Series Units

The allocated reliability for the jth series unit is

$$\hat{R}(t_j) = 1 - \frac{1-R^*(T)^{w_j}}{E_j} \qquad (j = 1, 2, \ldots, m) \tag{19}$$

B. Redundant Configuration

The allocated reliability for the redundant configuration is

$$\hat{R}(t_r) = 1 - \frac{1-R^*(T)^{w_r}}{E_r} \tag{20}$$

where

t_r is the average operating time of the redundant configuration during T hours of system operation.

E_r is the essentiality of the redundant configuration. (E_r must be greater than $1-R^*(T)^{w_r}$.)

C. Redundant Units

The allocated reliability for each redundant unit is

$$\hat{R}_{ri}(t_r) = R^*(T)^{w_{ri}} \qquad (i = 1,\ 2) \tag{21}$$

(Figures 1 to 4 can be used to solve for $\hat{R}$ in each of the above equations.)

▶ 9. If unit requirements are desired in terms of mean life or failure rate, Step 5 of Section 3.2 applies for the series and redundant units, provided the assumption of constant unit failure rate is good. If a constant failure rate is a poor assumption, average-unit or redundant configuration failure rate can be allocated by the equation

$$\hat{\lambda}_j = \frac{1-\hat{R}(t_j)}{t_j} \tag{22}$$

For the redundant configuration, mean life requirements are computed from the following equations:

Active Parallel

$$\hat{\theta}_r = \hat{\theta}_{r1} + \hat{\theta}_{r2} - \frac{\hat{\theta}_{r1}\,\hat{\theta}_{r2}}{\hat{\theta}_{r1} + \hat{\theta}_{r2}} \tag{23}$$

Standby

$$\hat{\theta}_r = \hat{\theta}_{r_1} + \hat{\theta}_{r_2} \qquad (24)$$

[NOTE: The time-to-failure distribution of a redundant configuration is not truly exponential. $\hat{\theta}_r$, therefore, cannot be interpreted as the mean life associated with a constant failure rate.]

The average failure rate of the redundant configuration is given by

$$\hat{\lambda}_r = \frac{1 - \hat{R}(t_r)}{t_r} \qquad (25)$$

3.4 Final Steps for Bimodal Systems

The final steps of the allocation procedures are described in this section for systems which contain a single redundant (bimodal) configuration consisting of two modal units which are not equally effective in performing the required function, i.e., the modal design adequacies differ.†

The method of allocating reliability for bimodal systems is very similar to that used for redundant systems; namely, an equivalent (series) failure index is found which allows the use of basic allocation equations. There are some important differences, however, in the number of different system designs possible, in the assumptions required, and in the allocation procedure.

Because different modal design adequacies prohibit an exact determination of the system reliability requirement, $R^*(T)$, from a knowledge of the system effectiveness requirement $S^*(T)$, an approximate method is presented for estimating $R^*(T)$ by first estimating an average design adequacy.

3.4.1 Types of Bimodal Operation

Two types of bimodal operation are considered: the uncommitted case (Type I) and the committed case (Type II). They are described below.

3.4.1.1 Type I - Uncommitted Case

In the uncommitted case, the operator or decision maker can determine if a reliable mode (all modal design functions are satisfactory) will result in mission success. If there is an indication of mission failure in a selected mode, it is possible to switch sequentially to alternate modes until the desired objective is attained. Failure occurs only when the objective is not attained after all modes are exhausted. Note

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† If the design adequacies of the two modes are approximately equal (e.g., they differ by less than 0.05), an average of the two can be taken to represent the design adequacy of each mode and the procedures described for redundant systems apply.

that this case also includes the situation in which all modes are simultaneously being used to perform the function. An example of a Type I case is a communication system in which the amount of time permitted to get a message through is long enough to allow trial of all possible transmitting modes.

(The term "continuous operation" will be used to denote the situation in which both modes are continuously energized. If the secondary mode is not activated until required, the term "sequential operation" will apply.)

3.4.1.2 Type II - Committed Case

The committed case occurs when the operator or decision maker has no way of assessing whether reliable operation in a given mode will lead to mission success, or, even if assessment is possible, it is too late to switch to an alternate mode. (This situation does not preclude modal switching if a component in the modal unit fails to perform as specified.) An example would be a reconnaissance satellite with a mission to obtain information over a particular area. If an optical mode is selected and cloud cover exists over the area, the mission might fail even though no modal failure was experienced.

NOTE: For both the uncommitted and committed cases, modal design adequacy may depend on when a mode is initially activated, e.g., a dead-reckoning mode in an aircraft navigation system. These cases are not considered.

3.4.2 Assumptions and Conditions

Below are listed the assumptions and conditions by which the effectiveness equations for bimodal systems were derived. It is believed that most systems do follow these restrictions to a great extent, and for the purpose of design stage allocation they are not unreasonable.

(a) If the system has more than one possible mission, a primary mission can be selected.

(b) A system effectiveness requirement exists for the selected primary mission.

(c) Average modal design adequacies can be estimated for the primary mission.

(d) The mode with the greater design adequacy is selected initially; the alternate mode is not activated unless

  (1) the primary mode fails, or

  (2) it can be determined that the primary mode will not yield satisfactory results (Type I case), or

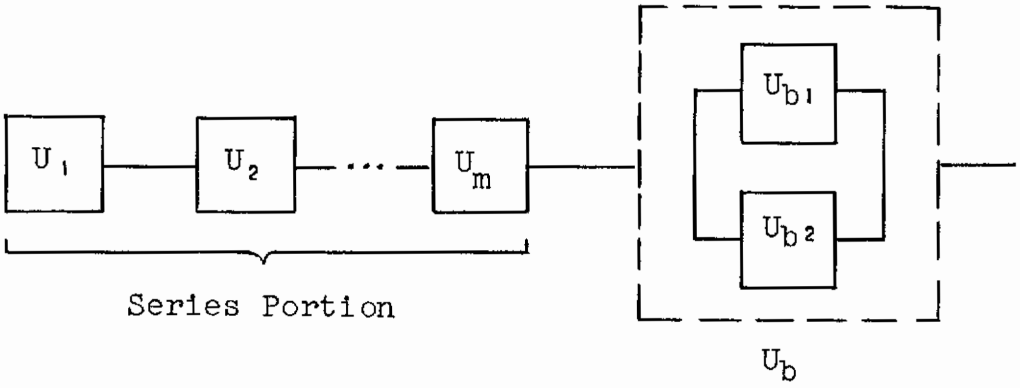
  (3) both modes operate continuously.

(e) modal switching is in one direction only -- from the primary to the alternate mode.

(f) modal switching is failure-free, or the switching mechanism is considered to be another series unit (see discussion of switching failure in Section 3.3).

3.4.3 Definition of Symbols

The unit identification notation used in this section is as shown below.



U_1 to U_m are the series units

U_{b1} and U_{b2} are the primary and alternate modal units, respectively.

U_b is the bimodal configuration.

3.4.4 Procedural Steps

The remaining procedural steps required for Type I and Type II bimodal designs and for continuous and sequential operation are shown below.

▶ 1. Estimate the design adequacy of each mode of operation over T system hours of operation. D_1 shall represent the design adequacy of the primary mode and D_2 the design adequacy of the alternate mode. Note that if $D_1 = 1.0$, a Type I design can be assumed.

▶ 2. Determine the system reliability requirement as follows:

A. Obtain the total system failure index of an equivalent series system, K_s, from the equation

$$K_s = K_1 + K_2 + \dots + K_m + K_{b_1} \tag{26}$$

where

K_1 to K_m are the failure indices of the series units

K_{b_1} is the failure index of the primary modal unit.

B. Obtain a preliminary estimate of the allocated reliability of the primary modal unit, $\hat{r}_{b_1}$, from the equation

$$\hat{r}_{b_1} = \left[\frac{S^*(T)}{D_1}\right]^{\frac{K_{b_1}}{K_s}} \tag{27}$$

C. The average design adequacy of the system is then

$$\overline{D} = \hat{r}_{b_1} D_1 + (1-\hat{r}_{b_1}) D_2 \tag{28}$$

D. The system reliability requirement is given by

$$R^*(T) = \frac{S^*(T)}{\overline{D}} \tag{29}$$

▶ 3. Estimate the average audio AEG rate required to meet the system requirement from the equation

$$\lambda = -\frac{\log R^*(T)}{K_s T} \qquad (30)$$

▶ 4. Obtain a failure index for each modal unit, K_b, that will reflect the same relative contribution to system effectiveness as the actual modal unit failure indices of K_{b_1} and K_{b_2} that were obtained in the preliminary steps. This is accomplished as follows:†

Given D_1 and D_2, the modal design adequacies of the two modes, calculate the values

$$\begin{aligned} d_1 &= -\frac{\log D_1}{\lambda T} \\ d_2 &= -\frac{\log D_2}{\lambda T} \end{aligned} \qquad (31)$$

A. Type I Case - Uncommitted

Obtain the equivalent failure index of each modal unit from the equation

$$K_b' = \frac{1}{2}\left[-(d_1+d_2) + \sqrt{(d_1+d_2)^2 + 4(K_{b_1}K_{b_2} + d_1K_{b_2} + d_2K_{b_1})}\right] \quad (32)$$

B. Type II Case, Committed

Obtain the equivalent failure index of each modal unit from the equation

$$K_b' = \frac{1}{2}\left[(d_1-d_2) + \sqrt{(d_1-d_2)^2 - 4K_{b_1}\,(d_1-d_2-K_{b_2})}\right] \qquad (33)$$

---

† If the values of K_{b_1} and K_{b_2} are nearly equal, a good approximation formula for K_b is

$$K_b' = \left[K_{b_1} \cdot K_{b_2}\right]^{1/2}$$

▶ 5. Obtain the total failure index of all series units from the equation

$$K_a = \sum_{j=1}^{m} K_j \tag{34}$$

where

K_a is the total failure index of the series portion

m is the number of series units.

▶ 6. Determine the feasibility of the reliability requirement according to the procedure in Appendix B.

▶ 7. Calculate the ratio

$$\alpha = \frac{K_a}{K_b'} \tag{35}$$

▶ 8. To obtain an equivalent series failure index for the bimodal configuration,

A. Continuous Operation:

Obtain the value of $Z(\alpha, R^*)$ from Figure 9 or 10 for the appropriate set of α and R^*,

B. Sequential Operation:

Obtain the value of $Z(\alpha, R^*)$ from Figure 11 or 12 for the appropriate set of α and R^*,

and compute

$$K_b = Z(\alpha, R^*)\, K_a \tag{36}$$

where

K_b is the equivalent series failure index of the bimodal configuration.

▶ 9. Obtain the total system failure index from the equation

$$K = K_a + K_b \tag{37}$$

and calculate the series unit failure index ratios:

$$\begin{aligned} w_1 &= K_1/K \\ w_2 &= K_2/K \\ &\ \vdots \\ w_m &= K_m/K \end{aligned} \tag{38}$$

as well as each modal unit failure index ratio

$$\begin{aligned} w_{b1} &= K_{b1}/K \\ w_{b2} &= K_{b2}/K \end{aligned} \tag{39}$$

▶ 10. Allocated reliabilities for T system hours of operation can be computed from the equations given below.

A. Series Units:

The allocated reliability for the j^{th} series unit is

$$\hat{R}(t_j) = 1 - \frac{1 - R^*(T)^{w_j}}{E_j} \quad (j = 1, 2, \ldots, m) \tag{40}$$

B. Modal Units:

The allocated reliability for each modal unit is

$$\hat{R}_{bi}(t_b) = R^*(T)^{w_{bi}} \quad (i = 1, 2) \tag{41}$$

(Figures 1 to 4 can be used to solve for $\hat{R}$ in each of the above equations.)

C. Bimodal Configuration:

An allocated reliability for the bimodal configuration can be computed from the allocated modal unit reliabilities as follows:

For continuous operation:

$$\hat{R}(t_b) = \hat{R}_{b1}(t_b) + \hat{R}_{b2}(t_b) - \hat{R}_{b1}(t_b)\,\hat{R}_{b2}(t_b) \tag{42}$$

For sequential operation:

$$\hat{R}(t_b) = \frac{L_2}{L_2 - L_1}\hat{R}_{b1}(t_b) - \frac{L_1}{L_2 - L_1}\hat{R}_{b2}(t_b) \tag{43}$$

where $L_1 = \log \hat{R}_{b1}(t_b)$ and $L_2 = \log \hat{R}_{b2}(t_b)$

▶ 11. If unit requirements are desired in terms of mean life or failure rate, Step 5 of Section 3.2 applies for the series and modal units, provided the assumption of constant unit failure rate is good. For the bimodal configuration, Equations 23 and 25 of Section 3.3 apply for continuous operation, and Equations 24 and 25 apply for sequential operation.

If a constant failure rate is a poor assumption, average unit failure rates can be allocated by the equation

$$\hat{\lambda}_j = \frac{1 - \hat{R}(t_j)}{t_j} \tag{44}$$

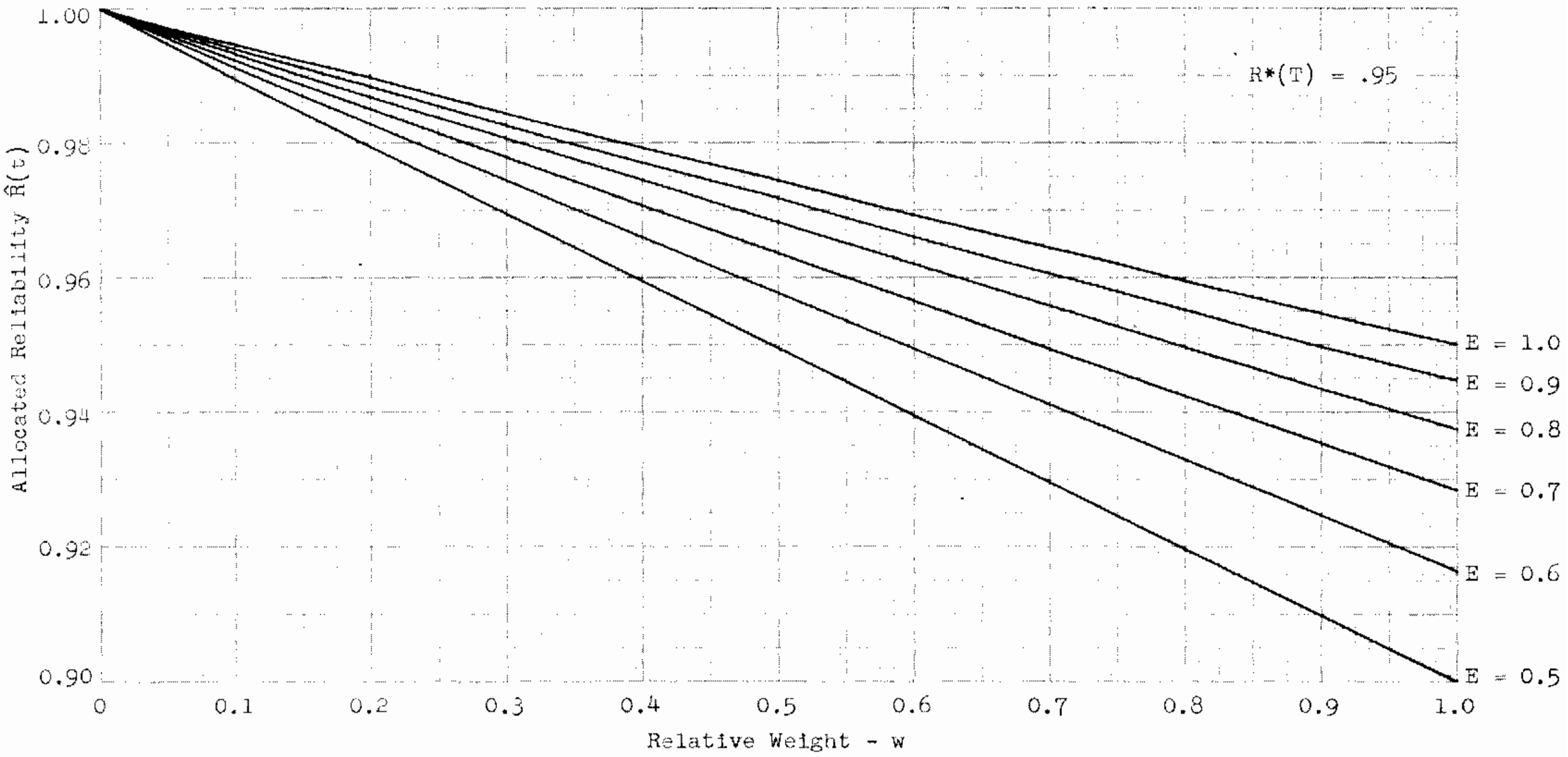


FIGURE 1

CHART FOR DETERMINING ALLOCATED COMPONENT RELIABILITY
WHEN SYSTEM RELIABILITY REQUIREMENT IS EQUAL TO .95

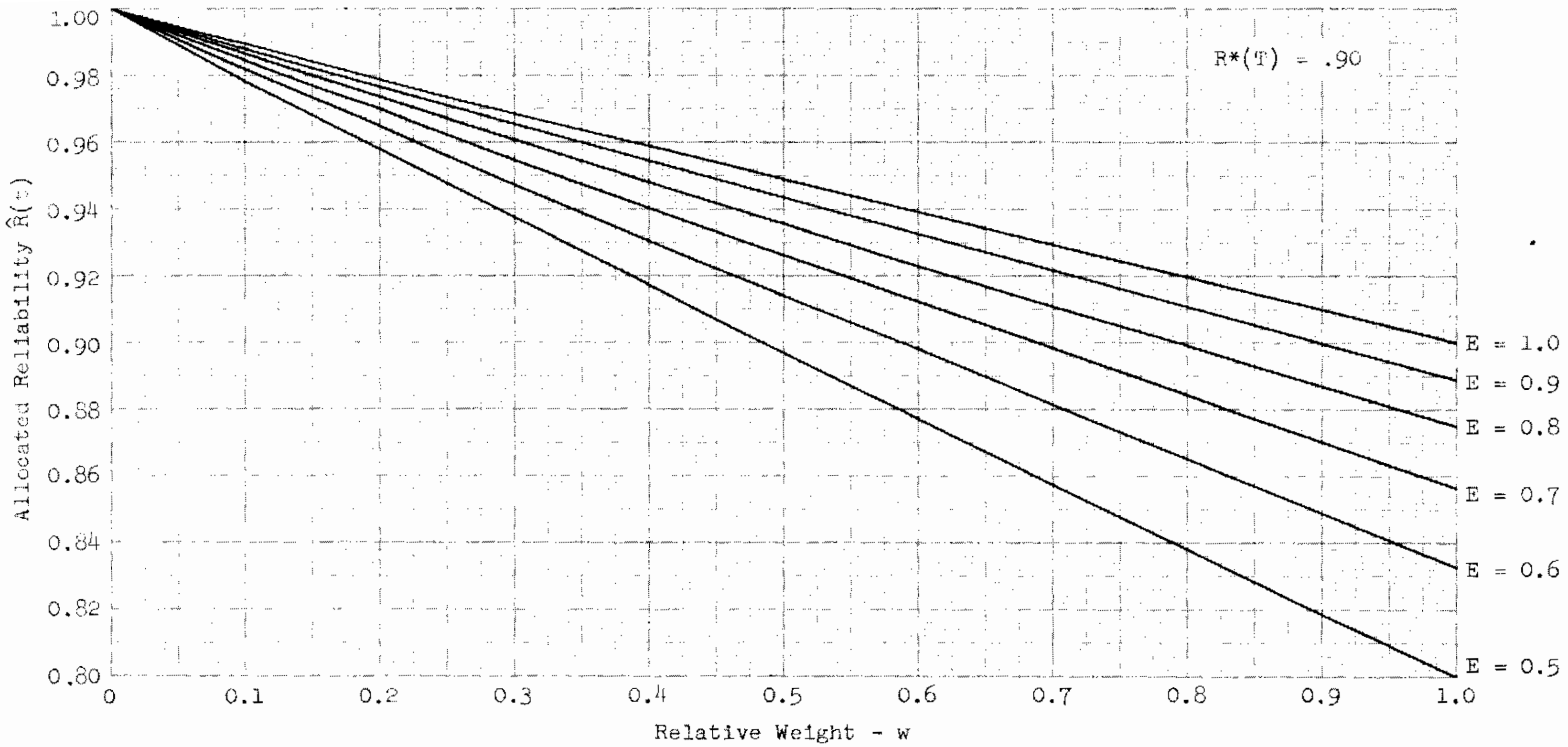


FIGURE 2

CHART FOR DETERMINING ALLOCATED COMPONENT RELIABILITY
WHEN SYSTEM RELIABILITY REQUIREMENT IS EQUAL TO .90

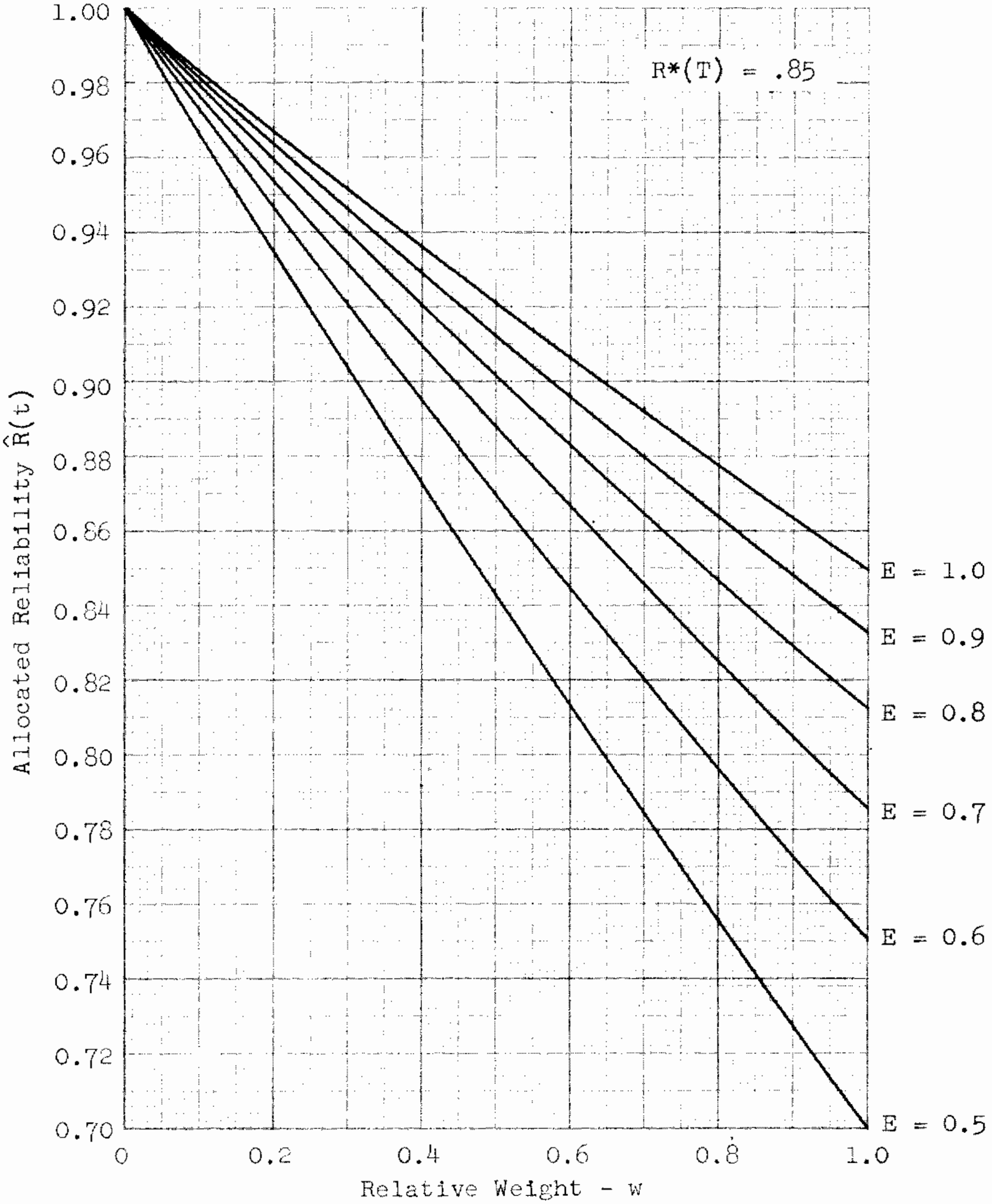


FIGURE 3

CHART FOR DETERMINING ALLOCATED COMPONENT RELIABILITY
WHEN SYSTEM RELIABILITY REQUIREMENT IS EQUAL TO .85

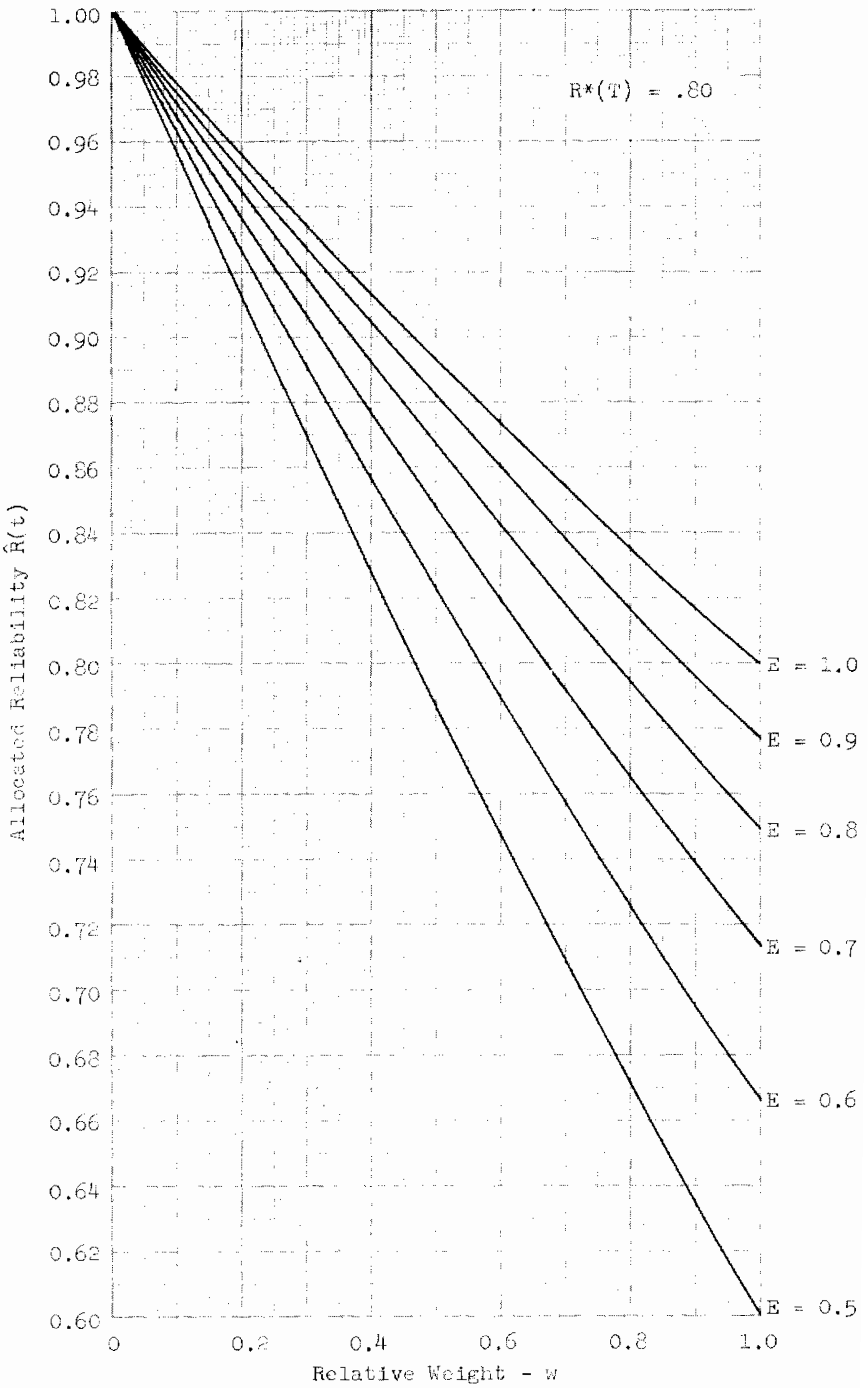


FIGURE 1

CHART FOR DETERMINING ALLOCATED COMPONENT RELIABILITY
WHEN SYSTEM RELIABILITY REQUIREMENT IS EQUAL TO .80

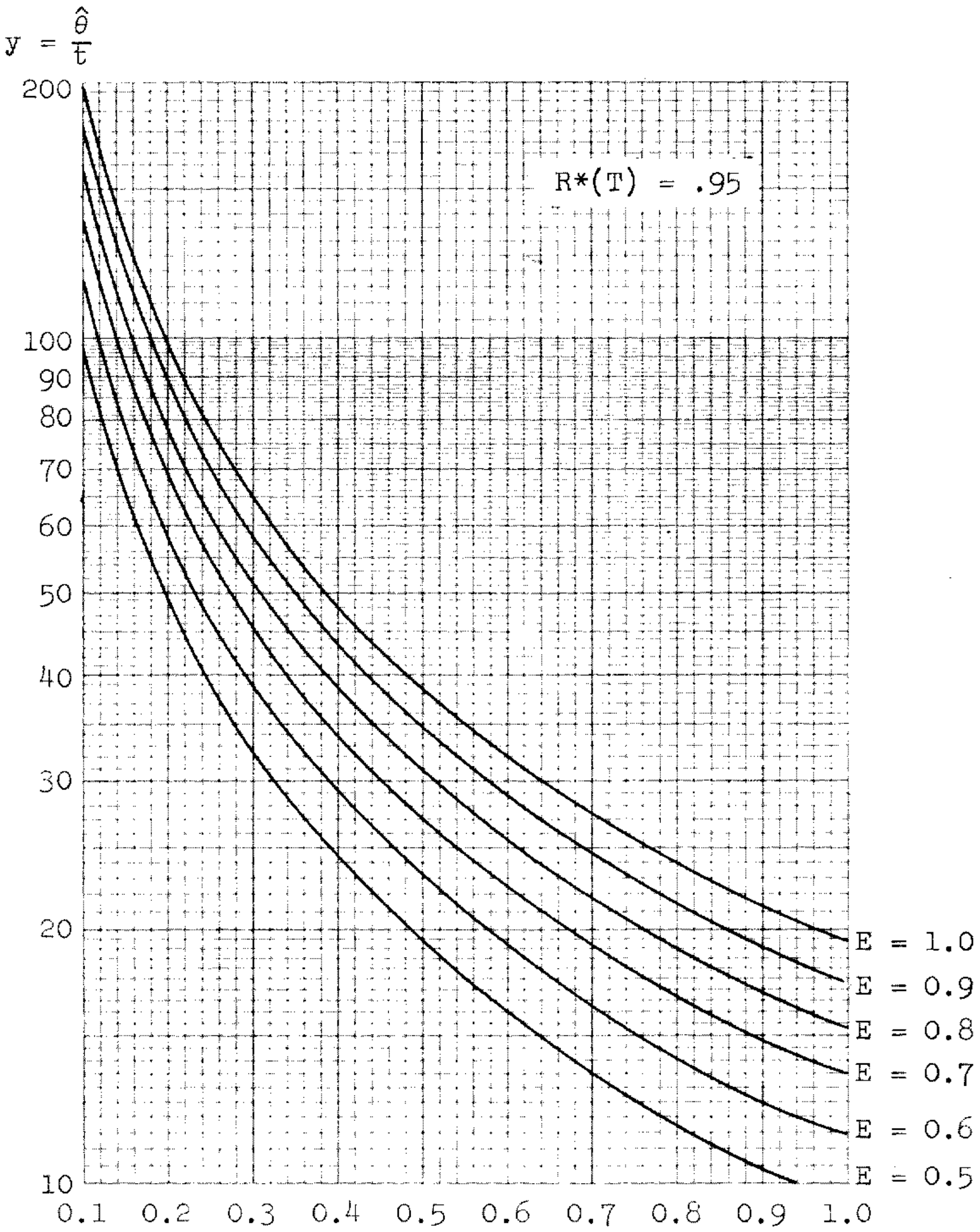


FIGURE 5

CHART FOR DETERMINING ALLOCATED COMPONENT MEAN LIFE WHEN SYSTEM RELIABILITY REQUIREMENT IS EQUAL TO .95

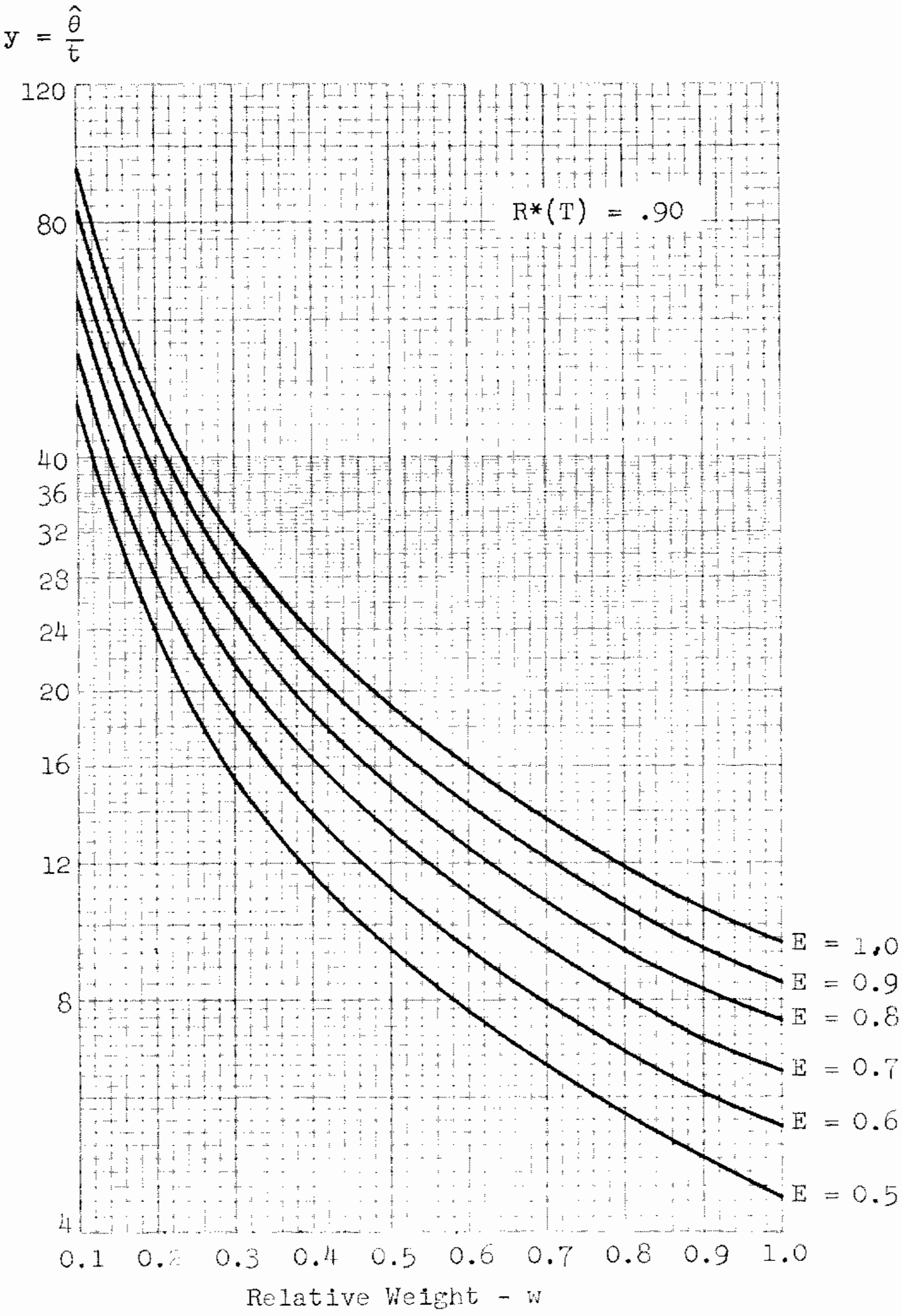


FIGURE 6

CHART FOR DETERMINING ALLOCATED COMPONENT MEAN LIFE WHEN SYSTEM RELIABILITY REQUIREMENT IS EQUAL TO .90

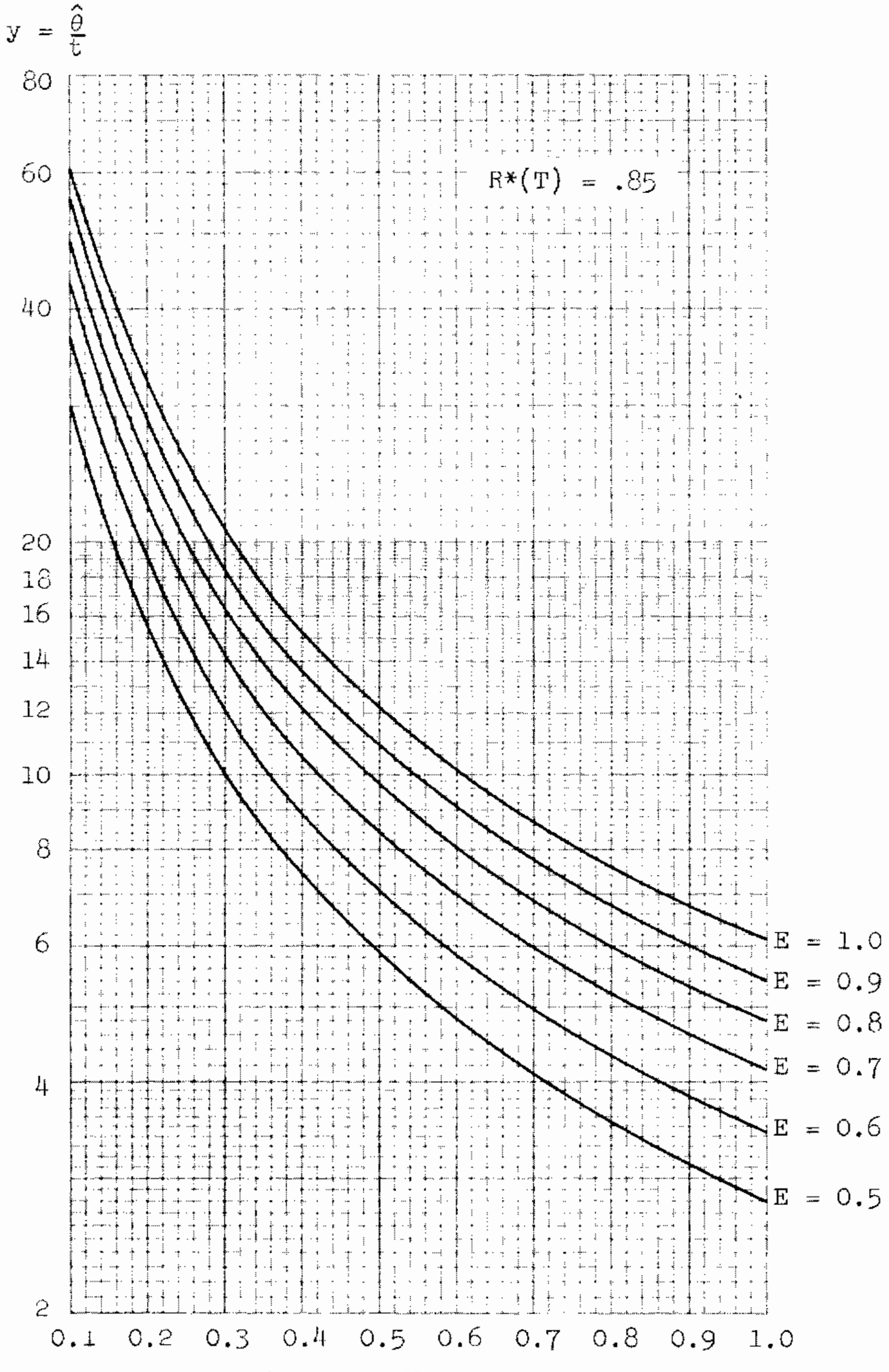


FIGURE 7

CHART FOR DETERMINING ALLOCATED COMPONENT MEAN LIFE WHEN SYSTEM RELIABILITY REQUIREMENT IS EQUAL TO .85

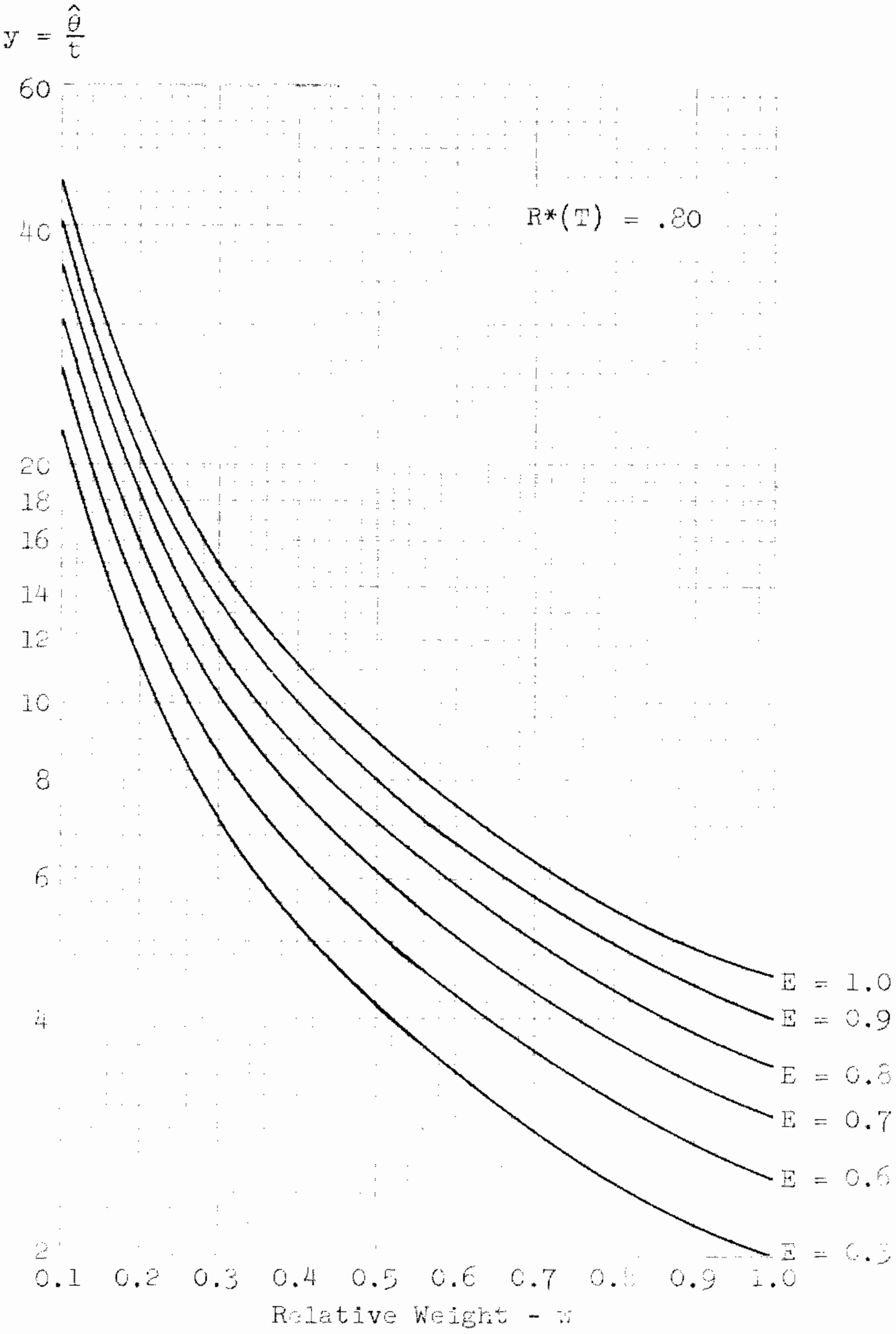


FIGURE 8

CHART FOR DETERMINING ALLOCATED COMPONENT MEAN LIFE WHEN SYSTEM RELIABILITY REQUIREMENT IS EQUAL TO .80

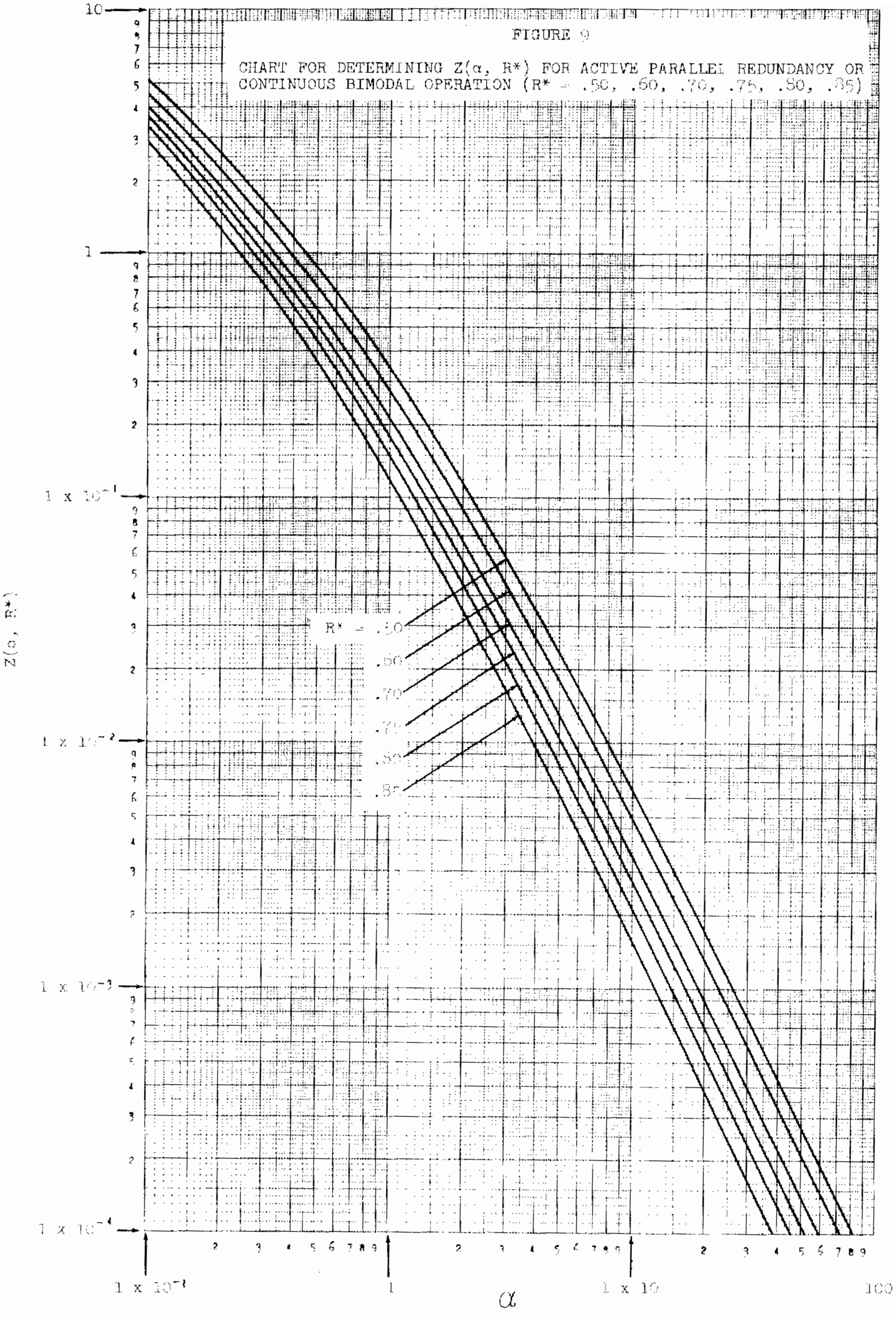


FIGURE 9

CHART FOR DETERMINING Z(α, R*) FOR ACTIVE PARALLEL REDUNDANCY OR CONTINUOUS BIMODAL OPERATION (R* = .50, .60, .70, .75, .80, .85)

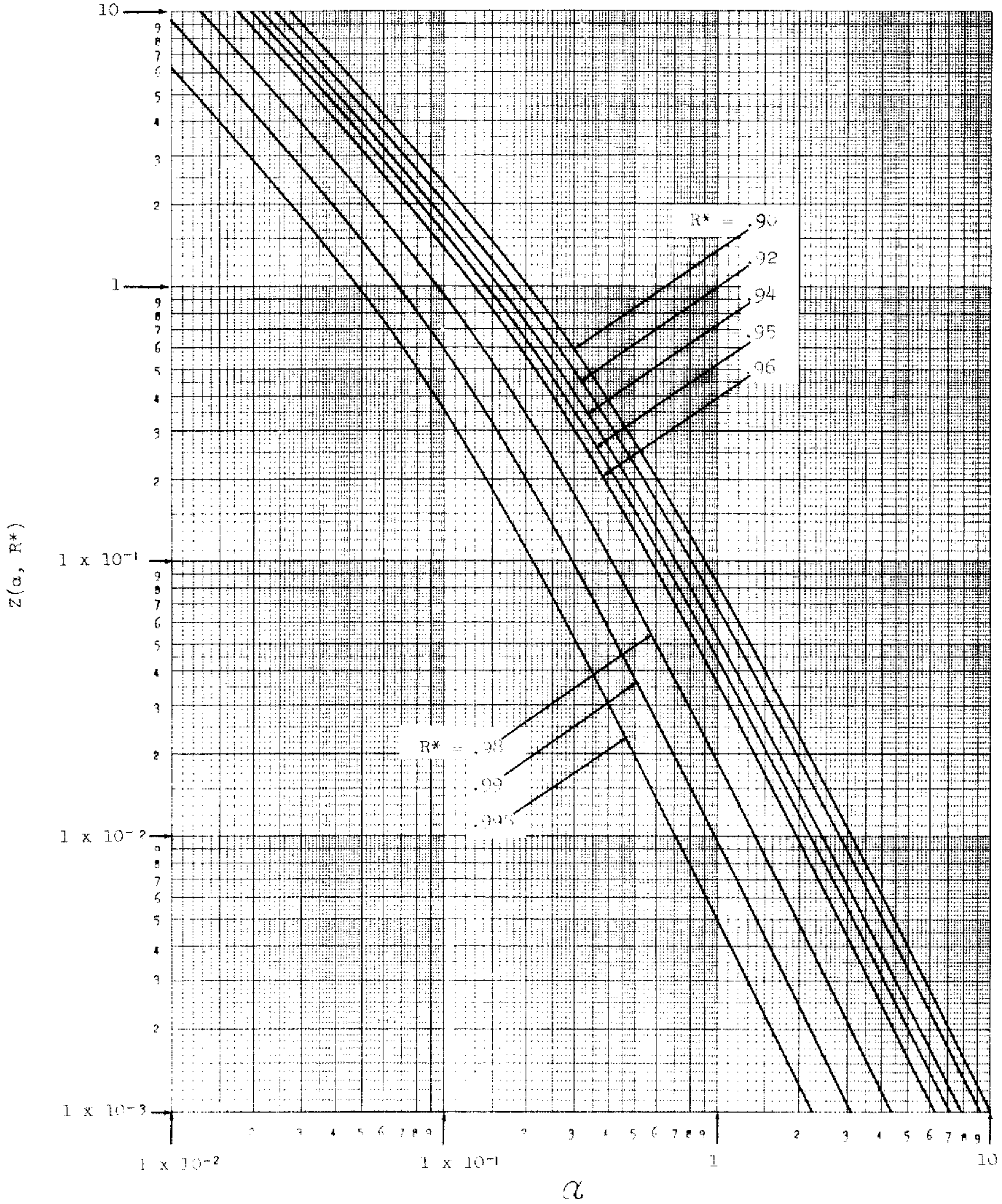


FIGURE 10

CHART FOR DETERMINING Z(α, R*) FOR ACTIVE PARALLEL REDUNDANCY OR CONTINUOUS BIMODAL OPERATION (R* = .90, .92, .94, .95, .96, .98, .99, .995)

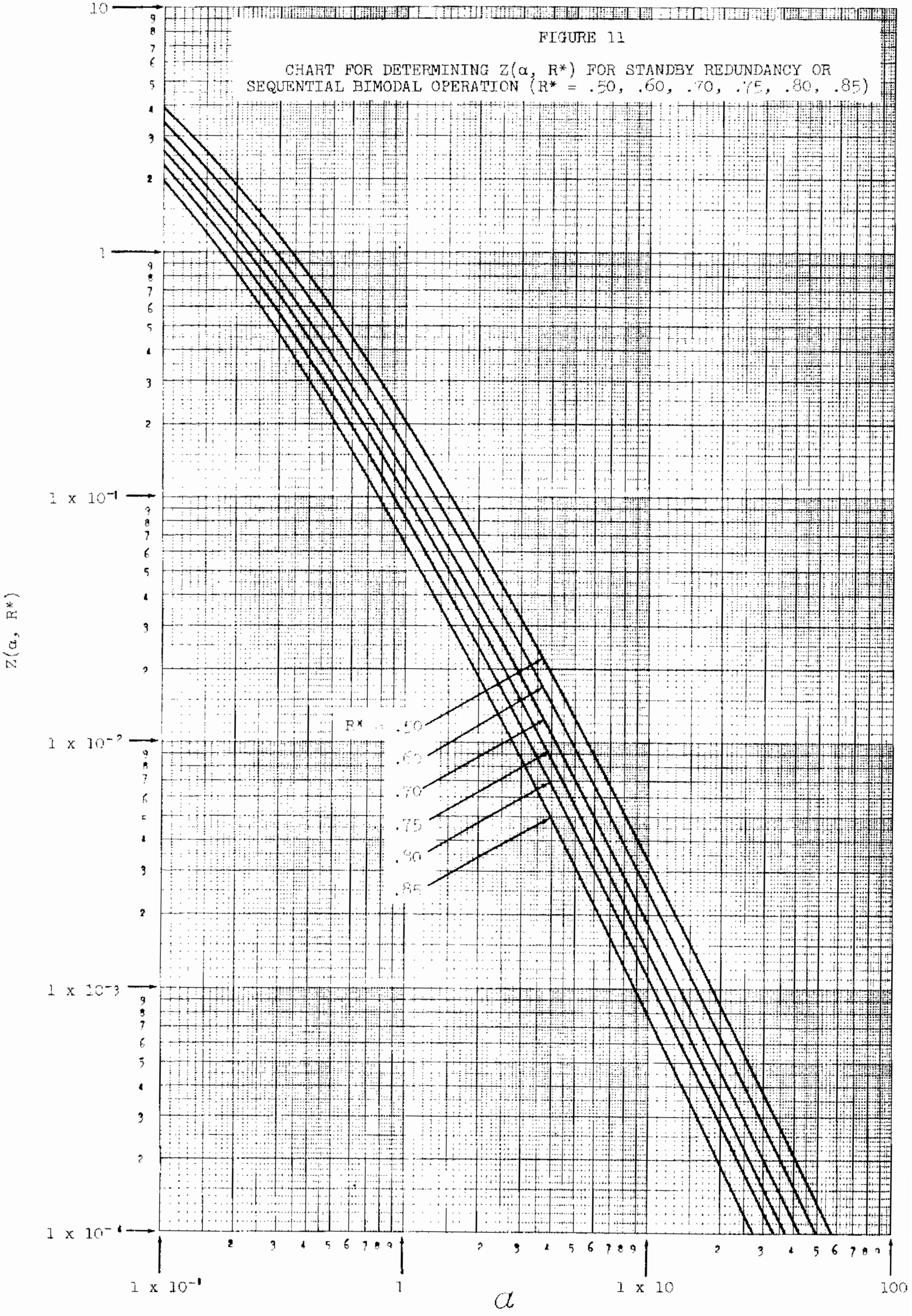


FIGURE 11

CHART FOR DETERMINING Z(α, R*) FOR STANDBY REDUNDANCY OR SEQUENTIAL BIMODAL OPERATION (R* = .50, .60, .70, .75, .80, .85)

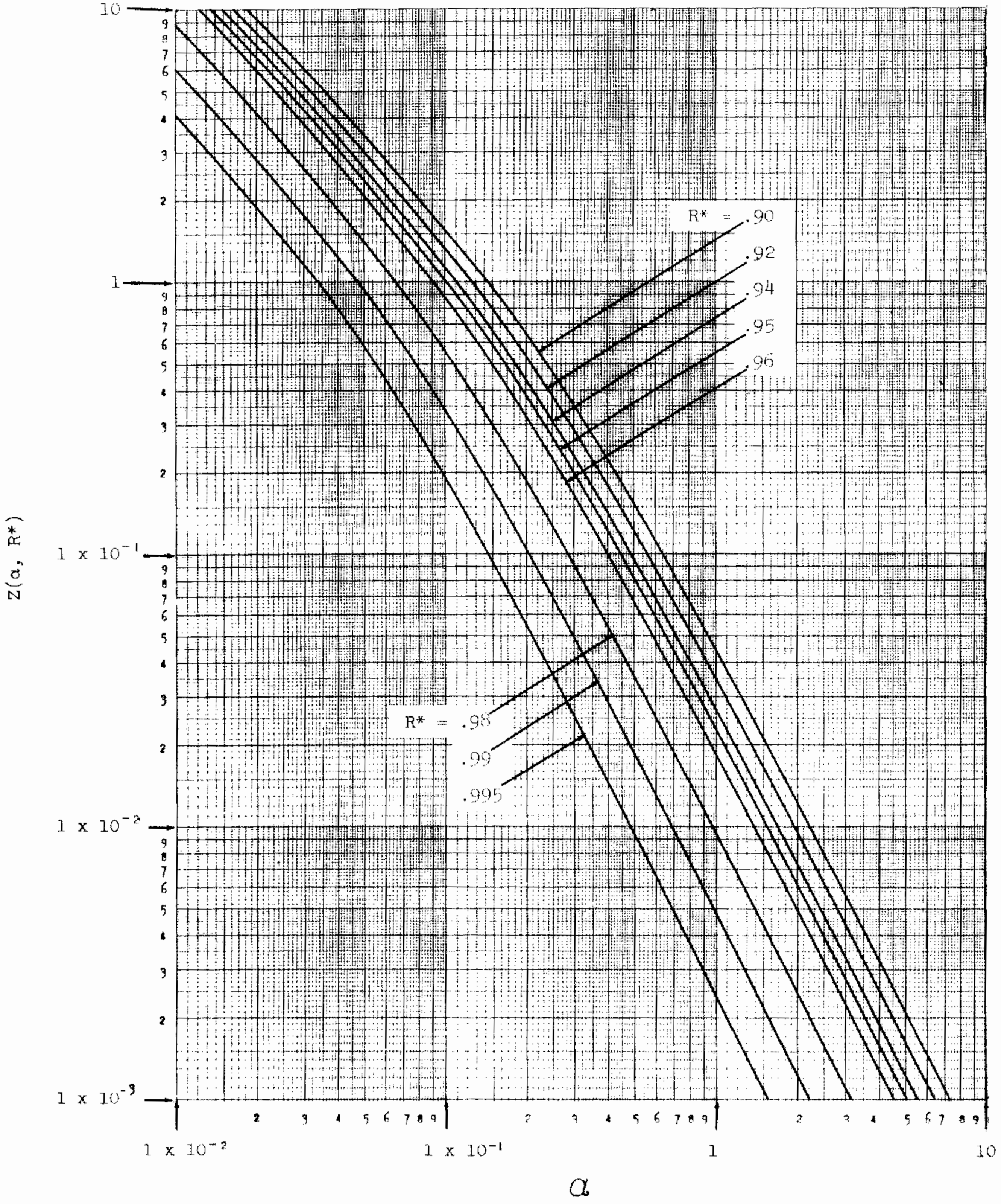


FIGURE 12

CHART FOR DETERMINING $Z(\alpha, R^*)$ FOR STANDBY REDUNDANCY OR SEQUENTIAL BIMODAL OPERATION ($R^* = .90, .92, .94, .95, .96, .98, .99, .995$)

4. ANALYSIS AND INTERPRETATION OF THE ALLOCATION

4.1 Introduction

To establish reliability as a design parameter, reliability requirements and methods of demonstrating compliance with them must be specified in each contract. The allocation of reliability during the initial stages of system evaluation permits reliability to be specified and provides a basis upon which demonstration and acceptance tests can be prepared and costed.

The methods described in Section 3 of this document permit a numerical allocation of the over-all reliability requirement. Within the restrictions of time and funding, the numbers so derived can be used in solving management problems associated with development of an effective system for the ultimate user. The extent to which they will serve this purpose depends upon the manner in which they are interpreted and applied.

An allocation obtained by the above-described procedures is based on the following fundamental assumptions:

(1) The allocation process by itself gives no assurance or guarantee that the reliability so assigned will materialize in service operation of the units or system. The allocation procedure takes an assigned over-all reliability and apportions the allowable unreliability to the various units of the system. If the system reliability requirement exceeds the state-of-the-art (see Appendix B), each unit's allocation will reflect its appropriate share of the required increase in the state-of-the-art.

(2) The allocation process assumes that the system development program is a uniform effort, i.e., that each unit will receive its equivalent share of the development funds and calendar time. The relative weighting of functional units is based on past experience in a wide variety of development and production programs; thus, the relative weighting is based on an "average" development and production background.

(3) The allocations arrived at by the above procedure can be further modified through study of trade-offs between reliability, other performance requirements, weight and space, calendar time, and monetary limitations. The initial allocation is made on the basis of factors which can be quantified at this time. Interpretation and use of the allocated numbers can be effective only if they are related to the more subtle factors in R&D programs.

4.2 Development Program Factors Affecting Allocations

The procedures described in this document have been concerned only with those allocations based upon previous experience with system components and their anticipated use in future applications (essentiality). Ideally, these should be the only factors influencing reliability allocations. Practically, however, development program managers are faced with a continual series of compromises. Brief discussions of some types of compromises or trade-offs which frequently occur are given below. Since no two situations are identical, these trade-offs cannot practicably be reduced to numerical quantities in the allocation equations. To the extent that individual unit reliabilities are compromised by such trade-offs, it is essential that, when recombined, these reliabilities equal the system requirement.

4.2.1 System Requirements Versus State-of-the-Art

Perhaps the most common problem encountered by the program manager is that in which system requirements -- and, thus, unit requirements -- are too high relative to the current state-of-the-art and the calendar time and funding permitted for the R&D program. Several possible solutions must be investigated. One solution is to lower the system requirement; another is to extend the R&D schedule; a third is to simplify the operational requirements; and a fourth is to reallocate reliability, weight, space, time, and funds. These alternatives are discussed below.

(1) Lower the System Requirement. Operational commands tend to demand much higher reliabilities than are in reality required. Thus, it is appropriate to determine whether it is more vital to provide the operational command with a system having a lower reliability by a specified

calendar time or to delay delivery until equipment with the required reliability is available. (National defense needs are often most dependent upon the time elements.)

(2) Extend The R&D Schedule. This solution is the complement of (1), above. In effect, it requires a decision that the system concept is not sufficiently advanced for fruitful developmental effort until preceded by considerable research which will increase the time and funding required. Although additional research before development may appear to cost more initially, the price of failure, rework, "late-in-the-program" research, and unusual use problems may very well overcome the initial cost differential.

(3) Simplify The Operational Requirements. It is common practice to attempt over-sophistication in present-day systems. Requirements for auxiliary functions, for accuracy, and for versatility are often more rigorous than is actually required. It thus becomes necessary to review original operational needs in relation to planned system performance. If the system can be simplified, its reliability will show a definite increase -- assuming other factors remain constant. Figure B-1, in Appendix B, clearly shows the effects of complexity.

(4) Reallocate Reliability, Weight, Space, Time, and Funds. This solution builds imbalance into the program in the interest of concentrating improvements in selected units. Specific methods employed are:

   (a) Use of redundancy, which will require reconsideration of weight allowances, volume limits, and available power sources. Redundancy entails consideration of switching devices, types of maintenance, and interaction effects that must be carefully treated if redundancy is indeed to effect an apparent increase in system reliability.

   (b) Extensive R&D efforts on selected units, with an attendant reduction in effort on other units.

The risks associated with a concentration of effort are great. It is possible to concentrate on units providing no apparent gain and possibly suffer loss through reduction of effort on others. Conversely, if the right units are selected, the gain could be great.

4.2.2 Unit-Allocated Reliability Trade-Offs Versus Time and Funds

Section 4.2.1 (d), above, established the point that it is not usually desirable to concentrate effort on only a few areas in the interest of overcoming system deficiencies. Within any program, however, development time and monetary allocations often become out of balance relative to the initial reliability allocation. Circumstances which can contribute to such imbalance include the following:

(1) Unique or radically new approaches to functional design may dictate additional funding in a particular area.

(2) If procurement of relatively standard units from other programs is intended, these programs must be considered in conjunction with the initial allocation. Increased reliability allocations could perhaps be given to units produced under the more intensified reliability efforts.

(3) Development time has a marked effect upon reliability. The more design and test time provided, the more mature -- and thus reliable -- the design is likely to become. Here, too, an increase in the reliability apportionment may be permissible. A unit produced under conditions which provide relatively little time for testing and, consequently, for correction or improvement of deficiencies, should not receive the same reliability allocation as a unit which requires little design time and thus permits more than adequate time for testing and improvement.

4.2.3 Unit-Allocated Reliability Trade-Offs Versus State of Design

The state of the design of a particular unit relative to other units affects the allocation. A unit which requires only modification and restudy from the viewpoint of reliability can generally be expected to achieve a greater reliability improvement than a unit which must be newly designed, if the same amount of effort is applied to each. When accurate reliability data is available on a standard unit, it should, of course, be utilized in lieu of the initial reliability allocation.

4.2.4 Unit-Allocated Reliability Trade-Offs Versus Type of R&D Effort

Within a system program it is not unusual to find development efforts on different units varying in scope from straight fabrication to research on individual parts. A review of the R&D efforts will result in a redistribution of the allocation based on (1) part selection criteria, (2) the type of engineering effort (tolerance studies, stability studies, derating policies, packaging, etc.), (3) the quantity and type of development and reliability testing planned, and (4) the provisions made for correction of deficiencies. Units developed under programs specifically oriented toward reliability and employing good design practice should be assigned a higher reliability than the initial allocation would normally provide. Materials handling techniques, process controls, assembly techniques, and inspection methods all influence the quality of the deliverable product. Thus, the R&D effort and the fabrication process can be considered in the reallocation of unit reliability.

4.3 Updating the Allocation

Reliability as a designed performance parameter does not remain static throughout a development program. Therefore, the allocated requirements must be periodically reviewed for their current applicability. The review should take account of circumstances such as those described below.

(1) Changes in design philosophy may affect the initial allocation determined pursuant to this document.

(2) Changes in program plans -- e.g., in funding, in scheduling, or in design and test emphasis -- will necessitate a restudy of the trade-offs made in the establishment of unit requirements.

(3) Acquisition of applicable use data on units may permit substitution of more specific, current experience for estimates incorporated in the allocation.

(4) Reliability prediction and analysis studies may indicate that individual units can achieve more or less reliability than allocated. (A note of caution: Present-day predictions often show reliability potential rather than a realistic state of the design.)

(5) Availability of test data from the program will give early indications of how well units are complying with requirements. Valid test data can provide a current base for reallocation of reliability.

LIST OF IMPORTANT SYMBOLS

Symbol	Definition
$\overline{D}$	Average design adequacy of a bimodal system
D_i	Design adequacy of the i^{th} mode
D_s	System design adequacy
E_j	Essentiality of the j^{th} unit
f_{ij}	Number of AEG's for the i^{th} function in the j^{th} unit
K	Total system failure index
K_a	Total failure index of all series units
K_b	Equivalent failure index of each modal unit
K_b	Equivalent series failure index of a bimodal configuration
K_{bi}	Failure index of the i^{th} modal unit
k_i	Relative failure rate of the i^{th} functional category
K_j	Failure index of the j^{th} unit
K_r	Equivalent failure index of each redundant unit
K_r	Equivalent series failure index of a redundant configuration
K_{ri}	Failure index of the i^{th} redundant unit
K_s	Total system failure index excluding duplicate components or units.
$R^*(T)$	System reliability requirement for T hours of system operation
$\hat{R}(t_j)$	Allocated reliability of the j^{th} unit over t_j hours
$S^*(T)$	System effectiveness requirement for T hours of system operation

LIST OF IMPORTANT SYMBOLS (continued)

Symbol	Definition
t_j	Operating time of the j^{th} unit during T hours of system operation
w_j	Failure index ratio of the j^{th} unit
$Z(\alpha, R^*)$	Factor for obtaining the failure index of a redundant or bimodal configuration
α	Ratio of K_a to K_r
$\hat{\theta}_j$	Allocated mean life of the j^{th} unit
λ	Average audio AEG failure rate
$\hat{\lambda}_j$	Allocated failure rate of the j^{th} unit

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2. "The Allocation of System Reliability, Volume I; Development of Reliability Allocation and Testing Procedures," H.S. Balaban, H.R. Jeffers, ARINC Research Corporation, November 1961, Publication No. 152-2-274.

3. WADD Technical Report 60-330, "A Compilation of Component Field Reliability Data Useful in Systems Preliminary Design," SECRET, D.E. Johnston, T.S. Durand, Wright Air Development Division, March 1961.

APPENDIX A

RELATIVE FUNCTIONAL FAILURE RATES FOR RELIABILITY ALLOCATION

APPENDIX A

RELATIVE FUNCTIONAL FAILURE RATES FOR RELIABILITY ALLOCATION

1. Scope

1.1 Content of Appendix

This appendix presents failure data for use as basic inputs to standard Air Force reliability allocation procedures. Section 5 contains step-by-step instructions for using the data to develop the relative unit failure indices as required by the procedures. Methods are also presented for adjusting the data to account for developmental and environmental conditions that preclude direct data application. In addition a sample of a typical data work sheet for implementing these data handling procedures is included.

1.2 Application

This document shall be applicable to all Air Force components, equipments, and systems having design-stage reliability requirements which are to be allocated among lower levels. For generality, the term "system" shall apply to the level at which an overall requirement exists. Any component, equipment or other subdivision of a system to which a reliability requirement is being allocated will be called a "unit". The data presented in this appendix shall be used in conjunction with the procedures in the main body of this report, or with any subsequent procedural document or Air Force specification originating from Contract AF 33(616)-7468.

2. General Description of Failure Data

2.1 Relative Failure Rates

The data presented in this appendix have been derived from analysis of field failure data on various types of ground, shipborne, and airborne equipments and systems. They are presented in terms of relative failure rates as described in subsections 2.1.1 and 2.1.2.

2.1.1 Electronic functions. Nine electronic functional categories covering all standard electronic functions have been established. Failure rates for each function relative to the audio functional failure rate are presented. They are designated by the symbol k_1. The electronic functions are described by active element groups (AEG's), an AEG being defined as a group of electronic parts consisting of an active element (one part, such as a tube, capable of performing a valving or controlling action) plus the associated passive parts within the group.

2.1.2 Non-electronic functions. The data for non-electronic functions are given in terms of functional failure rates relative to the failure rate of an average electronic AEG. The functional failure rate is designated by the symbol k_1'. Relative rates are given for those significant non-electronic functions which can be identified at the design stage and which are performed by individual parts or components. (For ease in presentation, the term AEG is also used to represent non-electronic parts or components which are to be utilized in the allocation.)

2.2 Correction Factors

Factors and methods are provided to permit adjustment of the relative rates for certain system and environmental states that preclude direct application of the rates presented in this appendix.

2.3 Precautions

2.3.1 "Average" rates. The relative failure rates presented are averages obtained through analysis of many equipment and system types over millions of hours of operation. For all the electronic categories and many of the non-electronic functions, it can be assumed that the rates apply to AEG's which are under "average" stress and local environmental conditions. There are non-electronic functions, however, for which the relative failure rates presented have been obtained from only one or two systems and therefore may be highly unrepresentative. Where possible, the raw data have been adjusted to account for situations which are unlikely to occur in the future and which have a biasing effect.

2.3.2 Recognition of uncommon factors. It is strongly recommended that, in the use of the data presented in this first issue of the appendix, care and good engineering judgment be exercised to account for any uncommon factors pertaining to the particular system and function under consideration and to make appropriate adjustments to the listed relative failure rates. Section 4 contains a discussion of several types of adjustments that may be required.

2.3.3 Use of other data. If accurate failure data are available on any particular unit to which the system requirement is to be allocated, such data should be utilized in lieu of the data presented in this appendix.

2.4 Data Insufficiencies

Relative rates are not given on functional categories for which little or no failure data are available. Section 5, step 5 presents methods for estimating relative failure rates for these categories.

3. Relative Failure Rate Data

3.1 Electronic Functions

3.1.1 Functional categories. Table I shall be used to identify the functional category to which an electronic AEG is assigned. Each AEG shall consist of a tube, transistor, or power diode with dissipation greater than one watt as the active element. (Solid state diodes of power dissipation less than 1 watt are considered as passive elements in the standard electronic functional categories.) Electronic AEG's that do not come within this definition or are of novel nature, shall be treated in accordance with the procedures described in Section 5, step 5.

3.1.2 Functional failure rates. Table II lists the basic relative functional failure rates (k_1) of each of the nine electronic functional categories. Any adjustments to the rates listed in this table that are dictated by unusual factors shall be made in accordance with the procedures described in Section 5 steps 8 and 9. Digital computers shall be treated separately in accordance with the procedures described in Section 5, step 5B.

3.2 Non-electronic Functions

3.2.1 Functional failure rates. Relative failure rates with respect to an average electronic AEG failure rate are presented for the following categories in Tables III, IV, and V:

(a) Electro-mechanical
(b) Mechanical
(c) Other non-electronic.

TABLE I

DEFINITIONS OF FUNCTIONAL CATEGORIES OF STANDARD ELECTRONIC FUNCTIONS

Functional Categories [1]	Definition	Examples
Audio	Active element groups acting on or supplying signals of an audio range used as audio output without further detection	Detectors Audio amplifiers
Primary Power	Active element groups acting to supply, modify, or control electrical power (as opposed to signals) in a form suitable to act as a power input for other active element groups	Rectifiers Rectifier bridges Voltage regulator tubes
Pulse, High Power (> 1 watt)	Active element groups acting on or developing signals of a pulse nature greater than one watt	Trigger circuits
Pulse, Low Power [2] (≤ 1 watt)	Active element groups acting on or supplying signals of a pulse nature equal to or less than one watt on the average	Trigger circuits Blocking oscillators
RF, High Power (> 1 watt)	Active element groups acting on or supplying high-frequency signals greater than one watt -- i.e., those not presentable as audio, servo, or video without detection or an equivalent operation	RF output stages
RF, Low Power (≤ 1 watt)	Active element groups acting on or supplying high-frequency signals equal to or less than one watt -- i.e., those not presentable as audio, servo, or video without detection or an equivalent operation	IF amplifiers RF amplifiers Local oscillators
Servo	Active element groups acting on or supplying signals used to perform a servo (electromechanical) function, drive a servo element, or transmit electromechanical information. (Low frequency servo carrier AEG's are classed as servo.)	Servo amplifiers
Special [3]	Non-normal AEG types containing klystrons, magnetrons, or hydrogen thyratrons, etc. Sometimes classed as exotic tube types	
Video	Active element groups acting on or supplying signals of a form to be presented as video output without further detection or equivalent operation, and active element groups which serve to present video information	Video amplifiers Cathode ray tubes

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[1] No separation according to power has been made for categories other than RF and Pulse. In any case, however, it is advisable to assign a higher relative failure rate to AEG's handling extremely high power.

[2] The 'Pulse, Low Power' category is not considered to include computer digital applications. The functional categories and relative rates of digital computers are discussed in Section 5, Step 5.

[3] The 'Special' category is quite general with respect to the nature of the AEG's involved, and greater variation in relative failure rates of AEG's can be expected in this category than in any of the other standard categories. When possible, the data for the 'Special' category should be supplemented by experimental observation of relative failure rates for the specific AEG type.

TABLE II

RELATIVE FAILURE RATES FOR ELECTRONIC FUNCTIONS[1]

Functional Category[2]	Relative Failure Rate (k_1)
Audio (12)	1.0
Primary Power (15)	4.3
Pulse, High Power (> 1 watt) (2)	27.5
Pulse, Low Power (≤ 1 watt) (5)	3.0
RF, High Power (> 1 watt) (6)	50.0
RF, Low Power (≤ 1 watt) (14)	3.4
Servo (11)	2.1
Special (4)	58.0
Video (5)	5.4

[1] The failure rates in this table are relative to the failure rate of an average tubed audio AEG.

[2] Numbers in parentheses indicate the number of equipment or system types used in deriving the corresponding relative failure rate.

TABLE III

RELATIVE FAILURE RATES FOR ELECTROMECHANICAL FUNCTIONS[1]

Function[2]	Relative Failure Rate (k_1')
Accelerometer (1)	12.40
Brush-Commutator (2)	0.25
Chopper or Vibrator (1)	3.20
Circuit Breaker (2)	0.10
Clutch, Instrument Servo (5)	0.65
Clutch, Power Transfer (1)	3.90
Counter (2)	0.01
Dynamotor (4)	0.95
Gyro (8)	12.10
Instruments - Meters (4)	0.30
Inverter (2)	3.50
Motor (16)	1.50
Relay, (Non-Stepping) (25)	0.70
Relay, Stepping (2)	8.55
Servo, Instrument (4)	2.40
Servo, Power (2)	14.85
Slip Ring (2)	0.80
Strain Gage (1)	1.95
Synchros, Resolvers (6)	0.70
Tachometer (1)	3.00
Thermostat (4)	0.20

[1] The failure rates in this table are relative to the failure rate of an average tubed electronic AEG.

[2] Numbers in parentheses indicate the number of data sources used for deriving the corresponding relative failure rate.

TABLE IV

RELATIVE FAILURE RATES FOR MECHANICAL FUNCTIONS[1]

Function[2]	Relative Failure Rate (k_i')
Antenna Assembly (1)	36.00
Brake, Axial (1)	1.90
Clutch, Slip (2)	0.35
Cooler, Electron Tube (3)	3.60
Counter (2)	0.15
Crystal Mount (1)	0.06
Dehydrator (1)	19.20
Duplexer (3)	41.20
Gear Drive Mechanism (4)	0.35
Interconnecting Cables and Wave Guides (1)	19.20
Artificial Line - Tuned Cavity (1)	0.25
Rigid Coax (1)	0.30
Waveguide (3)	10.00
Rotary joint (1)	5.60
Pendulum (1)	12.15
Prism (2)	0.55

[1] The failure rates in this table are relative to the failure rate of an average tubed electronic AEG.

[2] Numbers in parentheses indicate the number of data sources used for deriving the corresponding relative failure rate.

TABLE V

RELATIVE FAILURE RATES FOR OTHER NON-ELECTRONIC FUNCTIONS[1]

Function[2]	Relative Failure Rate (k_i')
Fuel and Power Plant	
Flow Meter (1)	2.50
Lines and Fittings (1)	0.12
Valves (1)	1.80
Vibrator, Ignition (1)	9.90
Hydraulic and Electro-Hydraulic	
Accumulator (1)	14.90
Connectors - Fittings (1)	0.05
Gage, Pressure (1)	9.90
Power Supply (1)	24.80
Pump and Motor Combination (2)	4.35
Wing Lock (1)	2.00
Pnuematic	
Bellows with Potentiometer Pick-off (1)	23.50
Compressor, Air (1)	16.50

[1] The failure rates in this table are relative to the failure rate of an average tubed electronic AEG.

[2] Numbers in parentheses indicate the number of data sources used for deriving the corresponding relative failure rate.

These basic rates shall be adjusted for non-typical factors in accordance with the procedures given in Section 5, steps 8 and 9. All non-electronic relative rates must also be adjusted to convert them to failure rates relative to an audio electronic AEG. This adjustment is described in Section 5, step 6. Many of the non-electronic functions listed exhibit a constant failure rate for only a limited period of time. Valid application of their relative failure rates requires that the operating time of each part or component in the system under consideration be no greater than the constant failure rate period of the part or component. If this requirement is not met, the footnote to Section 5, step 5A3 shall apply.

3.2.2 Source of data. The relative failure rates listed in Tables III, IV, and V have been derived primarily from the data presented in the following report:

> WADD Technical Report 60-330. "A Compilation of Component Field Reliability Data Useful in Systems Preliminary Design", SECRET, D. E. Johnston, T. S. Durand, Aeronautical Systems Division, (formerly Wright Air Development Division), W. P. A. F. B., Ohio

This report should be referred to for specific information concerning individual rates.

4. Data Adjustments

Adjustment factors that apply to the listed relative failure rates or the rates obtained by the methods described in Section 5, step 5 are set forth in this section for the following conditions:

(a) Major differences in electronic active elements.
(b) Differences in gross environment, and
(c) Unusual conditions.

Section 5, steps 8 and 9 presents procedures for incorporating these adjustment factors.

4.1 Adjustments for Differences in Electronic Active Elements

When a system is composed primarily of tube active elements, no adjustment is necessary. If a portion of a system contains classes of electronic active elements other than tubes, the relative failure rates of the appropriate electronic functional categories shall be multiplied by the correction factors given below:

TABLE VI

ADJUSTMENT FACTORS FOR DIFFERENT TYPES OF ACTIVE ELEMENTS

Active Element Type	Adjustment Factor*
Tube	1.0
Transistor	0.3
Solid state power rectifier	0.4
Tubed modular assembly	0.6

* These factors are based on limited data and should be used with caution. Factors for other active element types, such as magnetic and parametric amplifiers, masers, and molecular elements are not presently available.

4.2 Adjustments for Differences in Gross Environment

If the gross environment differs for various parts of the system (e.g., the ground-airborne telemetry combination), the relative failure rates must be adjusted by multiplying by the factors given below:

TABLE VII

ADJUSTMENT FACTORS FOR GROSS ENVIRONMENT

Gross Environment	Adjustment Factor
Ground	1.0
Airborne	8.5
Satellite	0.5*

* Based on limited data.

4.3 Other Adjustments

To permit adjustment for "uncommon factors", identification is necessary of those factors associated with units, AEG combinations, or individual AEG's, which will give rise to failure rates appreciably different from those expected in typical units of the system. Factors to be considered include: operating stress, R and D effort, design care and maturity, part procurement and testing programs, and reliability and quality assurance programs. Methods for estimating and applying these factors are given in Section 5, steps 8 and 9.

5. Procedures for Use of Data[†]

Step-by-step procedures for developing the data inputs to the allocation procedures are contained in this section. These inputs are based primarily on the relative failure rates given in Tables II through V of this appendix, with appropriate adjustments for special conditions. Procedures for obtaining relative rates for unlisted functional categories, or for obtaining appropriate modifying factors, are also given.

A work sheet similar to that illustrated on page 83 should be used to obtain the data inputs. The following step-by-step procedures describe how the work sheet should be completed.

▶ (1) The unit columns of the work sheet are headed by the units identified in step (a) of Section 3.1 of the main report text. Redundant and bimodal units should be identified accordingly. U_j will be used to identify the j^{th} unit.

▶ (2) The functional category (AEG type) column is divided into three groups: (a) the standard electronic functions which are listed in Table I; (b) the non-electronic functions for which relative failure rates are given in Tables III to V; and (c) those functions or functional categories in the system for which no relative rates are available. Within each group, the description of the AEG types contained in the system should be listed. If standard electronic AEG's contain active elements other than tubes they should be listed on a separate line, e.g., audio, tube on one line and audio, transistor on the next line (if both types exist). The number of entries in group (c) should be minimized as much as possible by classifying AEG types into one of the categories listed in Tables II to V.

▶ (3) The relative failure rates from Tables II to V should be listed in appropriate columns of the work sheet. The failure rates for the standard electronic functions are relative to an audio failure rate and are listed under the column labeled k_1. No adjustment to k_1 is made at this time for type of active element. The non-electronic failure rates are relative to an average electronic AEG and are listed under the column labeled k_1'.

[†] These procedures fulfill the requirements of Section 3.1, common step (c) of the main text.

▶ (4) The number of estimated AEG's of each category within each unit should be listed. These numbers are symbolized by f_{ij}, the number of AEG's of type i in the j^{th} unit. Fill in the f_i column by summing the f_{ij}'s for group (a) across each line, i.e., $f_i = f_{i_1} + f_{i_2} \cdots$

▶ (5) For functional categories in group (c), procedure (5)A may be used to estimate the relative failure rate k_i'. Digital computers are discussed separately in (5)B.

(5)A <u>Estimates Based on Part Failure Rates -- Non-electronic Functions, Group (c)</u>

(5)A.1 Estimate an average part-class distribution for the standard electronic AEG's in the system (see report referenced in Section 3.2.2 of this appendix for part-class distributions of several equipment or system types). Cognizance should be taken of the number of tube and transistor active elements in the system. Let n_v represent the average number of parts of v^{th} part-class type excluding active elements (e.g., resistors but not tubes) in an AEG. From the total number of tube and transistor active elements, compute w_1, the proportion of tubes, and w_2, the proportion of transistors, $(w_1 + w_2 = 1.0)$.

(5)A.2 Estimate the average part-class distribution required to perform the function (AEG type) being analyzed in the system. Let n_{ix} represent the number of parts in the x^{th} class for this i^{th} function.

(5)A.3 Using a standard set of part failure rates, compute the following:

(a) The average electronic AEG failure rate for the system corrected to a tube active element.

$$\bar{\lambda}_e = w_1\left[\lambda_{w1} + \sum_{v=1}^{V} n_v\lambda_v\right] + 3.3w_2\left[\lambda_{w2} + \sum_{v=1}^{V} n_v\lambda_v\right]$$

where V is the number of non-active element part classes

λ_v is the average part failure rate of the v^{th} part class

λ_{w1} is the average failure rate of a tube in the system

λ_{w2} is the average failure rate of a transistor in the system.

(b) The average failure rate of the function being analyzed in the system †

$$\bar{\lambda}_1 = \sum_{x=1}^{X} n_{1x}\lambda_x$$

where X is the total number of part classes in the function

λ_x is the average failure rate of the x^{th} part class.

---

† For functions which do not have approximately constant failure rates, an average failure rate over the operating time period involved can be estimated by

$$\bar{\lambda}_1(t) = \frac{1-R_1(t)}{t}$$

t is the average number of hours the function will be required to operate over T system hours

$R_1(t)$ is the reliability estimate of the function for t hours obtained by a standard method of reliability prediction.

(5)A.4 The failure rate of the i^{th} function relative to the failure rate of the average electronic AEG is then

$$k_i' = \frac{\bar{\lambda}_i}{\bar{\lambda}_e}$$

which is entered in the k_i' column.

(5)B <u>Digital Computer Functions</u>

The functional categories within a digital computer that are to be considered are:

a) Gating-logic (gate)

b) Pulse shaping, inverting, restoring (inverter)

c) Registering, counting (flip-flop)

d) Pulse storage, memory (memory unit)

e) Analog functions (e.g., basic computer clock)

No relative failure data is available for the first four categories. The analog functions in a computer are consistent with the pulse-low power electronic functional category and should be treated as such. For the digital categories, the procedures described in 5A above will apply except that no modifier should later be applied to correct for non-tubed electronic AEG's, since $\bar{\lambda}_i$ will have already been based on transistor (digital) failure rates. If the part failure set being used contains failure rates for analog transistors only, the listed rates should be multiplied by 0.06, and for diodes, the analog diode rates are to be multiplied by 0.008.

▶ (6) If there are entries in the k_i' column, compute the following

(a) Total Unadjusted Electronic Failure Index

$$K_e = \sum_{i=1}^{n(a)} f_i k_i$$

where n(a) is the number of entries in functional group (a)

(b) Number of Electronic AEG's in Group (a)

$$F_e = \sum_{i=1}^{n(a)} f_i$$

(c) Average Electronic Failure Index

$$\overline{K}_e = \frac{K_e}{F_e}$$

▶ (7) Convert each k_i' to a failure rate relative to audio by the formula

$$k_i = \overline{K}_e k_i'$$

These converted values are to be listed in the appropriate place under the k_i column.

▶ (8) Adjustment Factors

Each unit shall be assigned a modifying factor (M_j) which is a product of appropriate adjustment factors for gross unit environment and for other unique factors discussed in Section 4.3 of this appendix. Thus, if two factors such as environment and extremely low stress are appropriate for the j^{th} unit, $M_j = c_{1j}c_{2j}$ where c_{1j} is the environment modifier and c_{2j} is the stress modifier.

(a) Gross Unit Environment -- If the gross environment differs for various units within the system (e.g., the ground-airborne telemetry combination), the factors given in Table VII are to be included.

(b) Other Factors -- When appropriate data are available, adjustment factors can be obtained for the conditions enumerated in Section 4.3 of this appendix, and for any other unique condition, from the ratio

$$c_p = \frac{\text{Failure rate with unique condition p existing}}{\text{Failure rate under typical conditions}}$$

▶ (9) Adjusted Relative Failure Rates - $\bar{k}_{ij}$

(a) Functional Category Groups (b) and (c)

For each unit, each k_i in groups (b) and (c) should be multiplied by M_j and listed in the appropriate $\bar{k}_{ij}$ column.

(b) Functional Category Group (a)

For each unit, each k_i in group (a) should be multiplied by M_j and by the appropriate adjustment factor for the active element type. These latter factors are listed in Table VI. The product is to be listed in the appropriate $\bar{k}_{ij}$ column of the work sheet.

▶ (10) Unit Failure Indexes - K_j

The failure index for the j^{th} unit, as required by the allocation procedure, is obtained from the equation

$$K_j = f_{1j}\bar{k}_{1j} + f_{2j}\bar{k}_{2j} + \cdots$$

$$K_j = \sum_i f_{ij}\bar{k}_{ij} \quad .$$

DATA WORKSHEET FOR DETERMINING UNIT FAILURE INDICES, K_j

Functional Group	i	Functional Category and Type of Active Element	Relative Failure Rates		Unit 1 M_1 = ______		Unit 2 M_2 = ______		Total No. of Electronic AEG's	Electronic Failure Index
				k_i	No. of AEG's f_{i1}	Adjusted Relative Rates $\overline{k}_{i1}$	No. of AEG's f_{i2}	Adjusted Relative Rates $\overline{k}_{i2}$	$f_i = \sum_{j=1}^{2} f_{ij}$	$f_i k_i$
Group (a) - Standard Electronic Functions	1	Audio, tube		1.0	f_{11}	$\overline{k}_{11}$	f_{12}	$\overline{k}_{12}$	f_1	$f_1 k_1$
	2	Primary Power, tube		4.3	f_{21}	$\overline{k}_{21}$	f_{22}	$\overline{k}_{22}$	f_2	$f_2 k_2$
	3	Primary Power, transistor		4.3	f_{31}	$\overline{k}_{31}$	f_{32}	$\overline{k}_{32}$	f_3	$f_3 k_3$
	. . .	. . .								
									$F_e = \sum_i f_i$	$K_e = \sum_{i=1} f_i k_i$
			k'_i	$k_i = \overline{K}_e k'_i$					$\overline{K}_e = K_e/F_e$	
Group (b) - Listed Non-Electronic Functions	i_a	Motor	1.50	k_{i_a}	$f_{i_a 1}$	$\overline{k}_{i_a 1}$	$f_{i_a 2}$	$\overline{k}_{i_a 2}$		
	i_b	Relay, stepping	8.55	k_{i_b}	$f_{i_b 1}$	$\overline{k}_{i_b 1}$	$f_{i_b 2}$	$\overline{k}_{i_b 2}$		
	. . .	. . .								
Group (c) - Functions Not Listed	i_c	Computer gate, diode	k'_{i_c}	k_{i_c}	$f_{i_c 1}$	$\overline{k}_{i_c 1}$	$f_{i_c 2}$	$\overline{k}_{i_c 2}$		
	. . .	. . .								
Unit Failure Indices					$K_1 = \sum_i f_{i1} \overline{k}_{i1}$		$K_2 = \sum_i f_{i2} \overline{k}_{i2}$			

APPENDIX B

PROCEDURES FOR DETERMINING FEASIBILITY OF THE SYSTEM RELIABILITY REQUIREMENT

APPENDIX B

PROCEDURES FOR DETERMINING FEASIBILITY OF THE SYSTEM RELIABILITY REQUIREMENT

1. General

This appendix describes procedures for determining the feasibility of the system reliability requirement for the system types for which allocation procedures are presented.

It is important to note that these procedures give only a gross indication of feasibility because they are based on coarse historical data and simplifying assumptions. Improvements in reliability over that experienced in the past can and should be expected. If the system reliability requirement differs by more than one order of magnitude from the feasible reliability indicated through these procedures, a reconsideration of goals, functions, design, and program management may be necessary. This problem is discussed in Section 4 of the main body of this document.

2. Procedural Steps

The steps of the feasibility-determination procedure are set forth below.

▶ 1. Estimate the feasible system mean life of the basic series system only (i.e., those components or units that would be in the system if no redundant or alternate modes of operation existed).† This estimate is accomplished as follows:

A. Electronic Portion

(a) Count the number of series active elements (N_e) in the electronic portion of the system. Consider ten digital diodes equivalent to one electronic active element. Duplicate elements in redundant or bimodal systems are to be excluded.

(b) Determine the gross system environment in which the system will operate (ground, airborne, or satellite).

---

† In this appendix, the term "series elements" will be used to denote these units.

(c) From Figure B-1, determine the expected mean life of the electronic portion $\tilde{\theta}_{et}$ for the appropriate environment.† This represents the expected mean life of an equivalent tubed system. If other types of active elements are employed, a correction factor according to step (d) must be applied.

(d) List the electronic active elements of the system (including digital computers) by types of active element as listed below and number of each type. For each type determine the correction factor B_i from the following table.

i	Type	B_i
1	Transistor, Analog	0.3
2	Transistor, Digital	0.02
3	Diode, Digital (AEG)	0.06†
4	Tube	1.0

† Based on ten digital diodes.

The corrected mean life $\tilde{\theta}_e$ is given by:

$$\tilde{\theta}_e = \frac{\sum_{i=1}^{4} N_i}{\sum_{i=1}^{4} N_i B_i} \tilde{\theta}_{et}$$

---

† Recent failure data indicates that satellite environment is less severe than ground environment. The upper bound of the ground band can therefore be used as a tentative mid-line for a satellite band.

where N_1 is the number of series electronic active elements (N_3 = 1/10 x number of digital diodes) and $N_1 + N_2 + N_3 + N_4 = N_e$.

(This correction is not required if all active elements in the electronic portion of the system are tubes.)

B. Non-Electronic Portion†

Three alternative methods are given below. Choose the method which is most appropriate.

B.1 Use Experience With Similar Non-Electronic Elements

(a) Identify all non-electronic elements (parts or components) which are required for operation in the primary mode, i.e., only series elements.

(b) For each element, use failure data from similar element applications in past systems to obtain the observed element failure rate. The systems chosen should be of complexity similar to that of the system under consideration and should have operated in similar environments.

(c) The feasible mean life estimate of the non-electronic portion, $\tilde{\theta}_{ne}$, is then

$$\tilde{\theta}_{ne} = \frac{1}{\sum_i \lambda_i} \tag{B-1}$$

where λ_i is the observed system failure rate due to a non-electronic element of the i^{th} class.

---

† If the system is almost wholly electronic and the non-electronic portion is primarily electro-mechanical, this step can be ignored and the mean life of Step A will be used for the system as a whole.

B.2 Use Experience With Similar Systems

(a) Obtain failure data on a system that is similar to the system under consideration.

(b) Estimate the mean life of the series non-electronic portion of the system, $\tilde{\theta}'_{ne}$.

(c) If there is a large difference in total non-electronic complexity, an approximate adjustment can be obtained from the equation

$$\tilde{\theta}_{ne} = \beta\tilde{\theta}'_{ne} \qquad \text{(B-2)}$$

where

$\tilde{\theta}'_{ne}$ = observed mean life of the system being used as the basis of estimate.

β = ratio of total non-electronic complexity of the new system to that of the older system.

NOTE: Estimates made in Steps B.1 and B.2 can be improved by repeating the procedures for several older systems and averaging the results.

B.3 Use Relative Failure Data Given in Appendix A

(a) Compute the failure index of the non-electronic portion of the system (series elements only) from the formula

$$K_{ne} = \sum_{i} f'_i k'_i$$

where

k'_i is the relative unadjusted failure rate of non-electronic active element type i (excluding digital computers)

f'_i is the total number of AEG's of the i^{th} type for series elements

f'_i and k'_i are given in the worksheet developed in Appendix A for Step c of Section 3.1.

(b) The mean life of the non-electronic series portion is then

$$\tilde{\theta}_{ne} = \frac{N_e \tilde{\theta}_{et}}{K_{ne}} \qquad \text{(B-3)}$$

where

N_e = number of electronic active elements

$\tilde{\theta}_{et}$ = feasible electronic mean life obtained in Step 1 A (c).

K_{ne} = failure index of the non-electronic portion

C. Feasible mean life of an equivalent series system is then

$$\tilde{\theta}_s = \frac{\tilde{\theta}_e \, \tilde{\theta}_{ne}}{\tilde{\theta}_e + \tilde{\theta}_{ne}} \qquad \text{(B-4)}$$

If the system is serial or modified serial, to determine feasibility compare $\tilde{\theta}_s$ to θ_s^* or compare

$$\tilde{R}(T) = e^{-T/\tilde{\theta}_s} \quad \text{to } R^*(T).$$

For redundant or bimodal systems proceed as follows.

▶ 2. Compute the degree of redundancy (or component duplication), γ , as follows:†

A. Redundant System:

$$\gamma = \frac{K_r'}{K_a + K_r'} \qquad \text{(B-5)}$$

where

K_a is defined in Step 2 of Section 3.3

K_r' is defined in Step 3 of Section 3.3.

---

† Note that $\gamma = 0$ for serial or modified serial systems.

B. Bimodal System:

$$\gamma = \frac{K_b'}{K_a + K_b'} \qquad \text{(B-6)}$$

where K_a is defined in Step 5 of Section 3.4

K_b' is defined in Step 4 of Section 3.4 for the two types of bimodal configurations.

▶ 3. The feasible system reliability requirement for T system hours of operation is then given as follows:†

A. Active Parallel Redundancy or Continuous Bimodal Operation

$$\tilde{R}(T) = 2\, e^{-T/\tilde{\theta}_s} - e^{-(1+\gamma)T/\tilde{\theta}_s} \qquad \text{(B-7)}$$

(Figure B-2 gives the solution to this equation for $\gamma = 0$, 0.25, 0.50, 0.75, 1.0 for time units of $T/\tilde{\theta}_s$. Multiplying the horizontal axis by $\tilde{\theta}_s$ converts it to hourly units.)

B. Standby Redundancy or Sequential Bimodal Operation

$$\tilde{R}(T) = e^{-T/\tilde{\theta}_s}\left[1 + \gamma T/\tilde{\theta}_s\right] \qquad \text{(B-8)}$$

(The value of $e^{-T/\tilde{\theta}_s}$ can be obtained from Figure B-2 by use of the curve for $\gamma = 0$.)

NOTE: Steps 2 and 3 permit an estimate of the degree of redundancy required to meet a given reliability requirement if the reliability of a series design with feasible mean life $\tilde{\theta}_s$ is inadequate.

▶ 4. The feasibility of the system reliability requirement is determined by comparing $\tilde{R}(T)$ to $R^*(T)$.

---

† Note that $\gamma = 0$ for serial or modified serial systems.

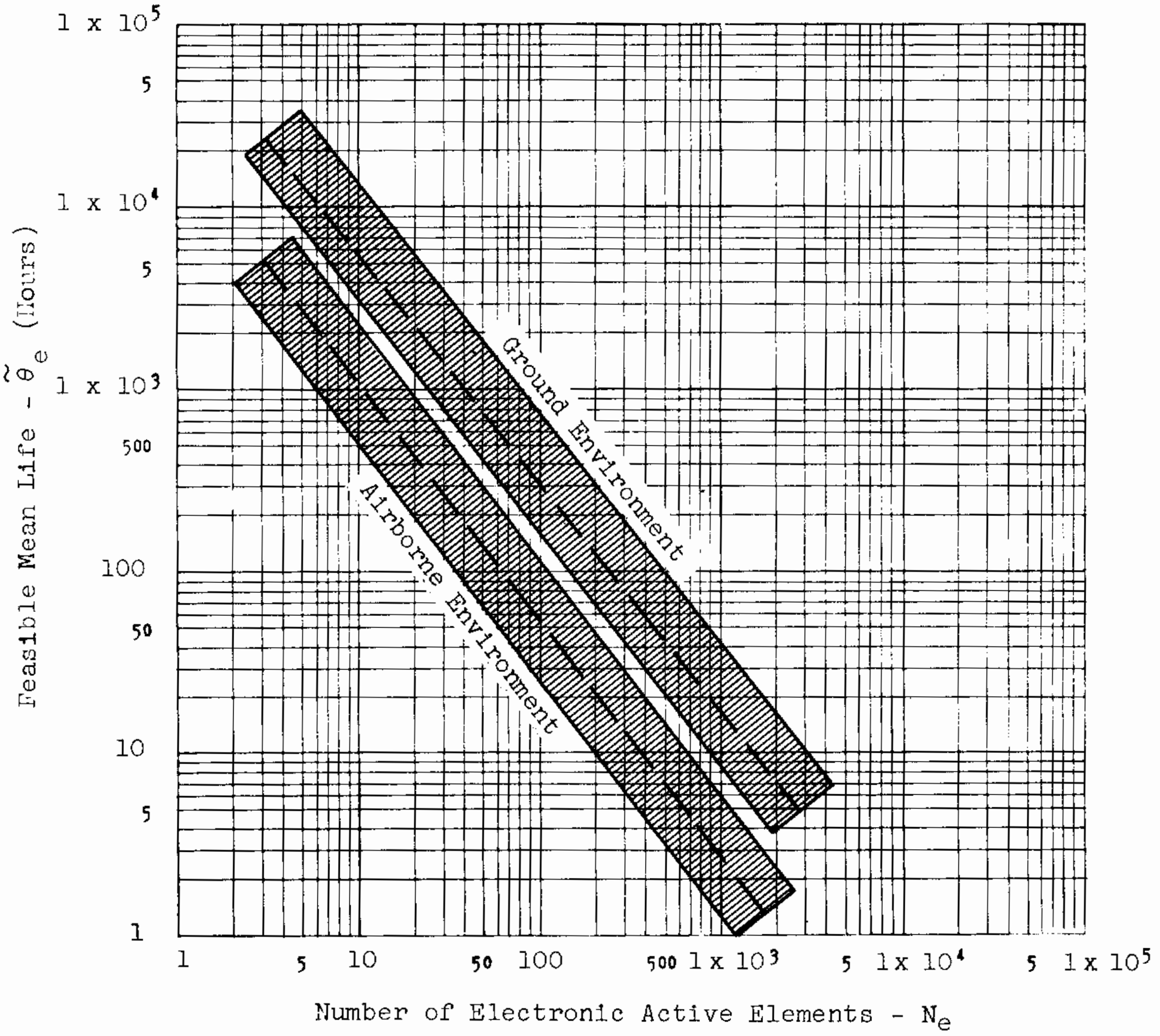


FIGURE B-1

CHART FOR DETERMINING FEASIBLE MEAN LIFE
OF NON-REDUNDANT ELECTRONIC SYSTEMS

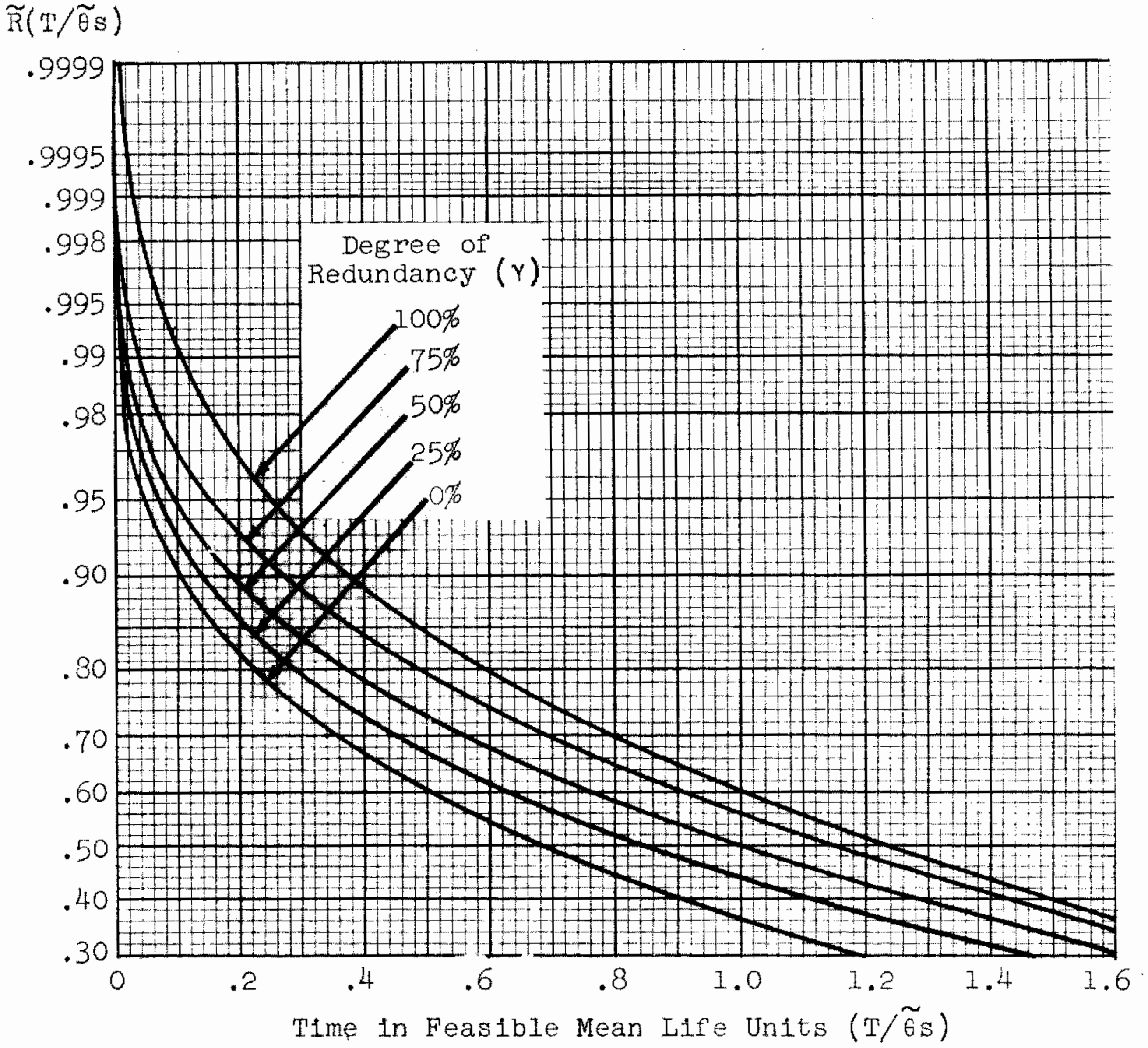


FIGURE B-2

CHART FOR DETERMINING FEASIBLE SYSTEM RELIABILITY
FOR VARYING DEGREES OF REDUNDANCY
GIVEN A FEASIBLE SERIES-SYSTEM MEAN LIFE $(\tilde{\theta}s)$

APPENDIX C

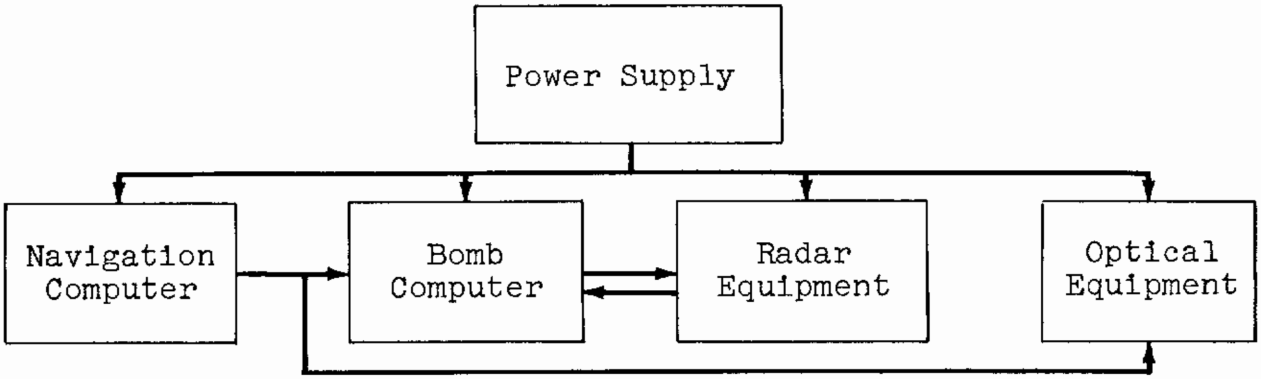
EXAMPLES OF ALLOCATION PROCEDURES APPLICATIONS

APPENDIX C

EXAMPLES OF ALLOCATION PROCEDURES APPLICATIONS

This appendix gives an analysis of a typical system during several stages of evolution. The analysis is presented in a manner which illustrates the application of allocation procedures to different system types. The method of handling design adequacy, essentiality, operating time, redundancy, and bimodal operation is shown, and an example of suballocation within a unit is given.

As the result of continued development over a number of years, a bomb navigation system of the type shown in the following drawing may have evolved.



This indicates that power is supplied to all units. The navigation computer is always required for mission success. However, the bomb computer and radar equipment can be used without the optical equipment, or the optical equipment may be used without the bomb computer and radar equipment. In either case, the system is required to have an 0.80 probability of successful operation for six hours. When the optical equipment is used, failure of the mission will occur 15 per cent of the time due to cloud cover, inaccuracies in readings, and lack of range. Navigation to the target vicinity is provided by the navigation computer; final bomb run navigation information is provided by both the navigation and the bomb computers. The bomb computer is used, on an average, for thirty minutes during the six hours of required system operation.

Assumptions are that enough design information is available so that a count of the number of AEG's in the functional categories described in Appendix A can be made. The number of AEG's per unit in each functional category is given in Table C-1, which also shows the radar equipment divided into "sub-units."

TABLE C-1

NUMBER OF AEG'S PER UNIT IN EACH FUNCTIONAL CATEGORY

Function	Power Unit	Navigation Computer	Optical Equip	Bomb Computer	Radar Equip	Radar Equipment Subdivisions			
						Power Supply	Modulator Receiver Transmitter	Radar Data Display	Antenna
Primary Power	40	10		1	30*	30*			
Pulse, Low Power		230*		20*	200*		50*	150*	
RF, High Power					2		2		
RF, Low Power					4		4		
Servo									2
Special					3		3		
Video					3 200*			3 200*	
Servo, Power									1
Synchro, Resolver		35	5	4	5			5	2
Gyro		1							
Gear Drive Mechanism									1
Prism			3						
Antenna Assembly					1				
Thermostat		25	3	3	12	3	4	5	1
Motor		40	2	3	4		1	3	2
Dehydrator		1	1						
Relay	3	70		10					
Relay, Stepping	1								
Slip Rings									1
Rotary Joint									1
Waveguide									1
Counter		12		3					

* AEG's contain active elements other than tubes.

From inspection of Tables II to IV (Appendix A), it is seen that relative failure rates are available for all functions or functional categories that exist in the system. An example of the procedures used to obtain relative failure rates for functions not listed appears in the example described in Appendix A of Volume I.

Using the system description and the foregoing list of AEG's, the following examples can be cited:

I. Serial System: Reliability allocation to a simple serial system with system design adequacy less than one.

II. Modified Serial System: Reliability allocation to a system similar to the system of Example I, but modified so that a unit has an essentiality of less than one.

III. Sub-Allocation: The system of Example I is redesigned so that the design adequacy is unity. An example of unit operating time less than system operating time is given. A unit in the system is further subdivided, and the reliability allocation of the unit is sub-allocated within the unit.

IV. Redundant System: The system of Example III is made redundant by duplicating one of its most failure-prone units.

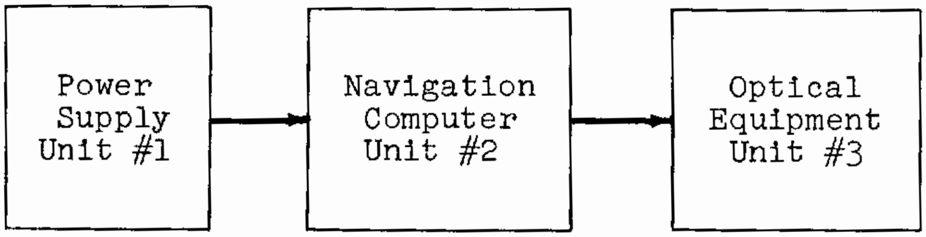
V. Bimodal System: The systems of Examples I and III are combined to provide two modes of operation.

In the discussions presented in this Appendix, each of the examples herein before mentioned will be identified by its number and title, and all subsequent format divisions will be identified by the appropriate paragraph of the main body of this document.

EXAMPLES

I. Serial System -- Early in its history, the system may have consisted of the navigation computer and the optical equipment, both of which receive power from the power supply. The requirement of an 0.80 probability of successful operation for 6 hours will be allocated to such a system.

Step 3.1-a[†] -- The system consists of three units. The reliability block diagram below defines each unit and assigns a unit number to be used for identification of the unit during this allocation.



Step 3.1-b -- The system is a serial system.

Step 3.1-c -- Referring to Appendix A for the procedures to determine the unit failure indices, K_j, the following steps are performed:

Step A-1 -- The attached worksheet (Table I) is set up for three serial units.

[†] The step identification in this Appendix refers to both the subsection (3.1) and the step number (a) in Section 3 or to the appropriate sections of Appendix A or B (e.g., A-1 means Appendix A, Step 1.

TABLE I

DATA WORKSHEET FOR DETERMINING UNIT FAILURE INDICES, K_j

Group	i	Function		k_i	Unit #1 $M_1 = 1$		Unit #2 $M_2 = 1$		Unit #3 $M_3 = 1$		Functional Totals	
					$\bar{K}_{i1}$	f_{i1}	$\bar{K}_{i2}$	f_{i2}	$\bar{K}_{i3}$	f_{i3}	f_i	$f_i k_i$
Group a	1	Primary Power		4.3	4.3	40					40	172
	2	Pulse, Low Power		3.0			3.0	10			10	30
	3	Pulse, Low Power (Trans.)		3.0			0.9	230			230	690
											$F_e = 280$	$K_e = 892$
			k'_i	$k_i = \bar{K}_e k'_i$							$\bar{K}_e = 3.186$	
Group b	4	Synchro, Resolver	0.70	2.23			2.23	35	2.23	5		
	5	Gyro	12.10	38.55			38.55	1				
	6	Prism	0.55	1.75					1.75	3		
	7	Thermostat	0.20	0.64			0.64	25	0.64	3		
	8	Motor	1.50	4.78			4.78	40	4.78	2		
	9	Dehydrator	19.20	61.17			61.17	1	61.17	1		
	10	Relay	0.70	2.23	2.23	3	2.23	70				
	11	Relay, Stepping	8.55	27.24	27.24	1						
	12	Counter	0.01	0.03			0.03	12	0.03	3		
		Unit Failure Index, $K_j = \sum_{i=1}^{12} \bar{K}_{ij} f_{ij}$			$K_1 = 205.9$		$K_2 = 778.4$		$K_3 = 89.1$			

Step A-2 -- The functional category column is divided into electronic and non-electronic groups. Pulse, Low Power (Transistor) is listed on a separate line from Pulse, Low Power. There are no AEG's for which relative failure rates are not listed.

Step A-3 -- The relative failure rates for each functional category is entered in the appropriate column.

Step A-4 -- The number of estimated AEG's of each category within each unit is entered in the column headed $f_{ij}(j=1,\ 2,\ 3)$ and the electronic category rows are summed to obtain the entries in the column headed f_i.

Step A-5 -- Not applicable, since there are no functional categories in the system for which relative rates are not given.

Step A-6 -- The average electronic failure index is computed in the following manner:

(a) Form the total unadjusted electronic failure index

$$K_e = 40(4.3) + 10(3.0) + 230(3.0) = 892.$$

(b) Determine the number of electronic AEG's in group (a).

$$F_e = 40 + 10 + 230 = 280.$$

(c) Form the average electronic failure index:

$$\overline{K}_e = 892/280 = 3.186.$$

Step A-7 -- Convert each k_i' to a failure rate relative to audio by the formula $k_i = \overline{K}_e k_i'$.

$$k_4 = (0.70)(3.186) = 2.23$$
$$k_5 = (12.10)(3.186) = 38.65$$
$$k_6 = (0.55)(3.186) = 1.75$$
$$k_7 = (0.20)(3.186) = 0.64$$
$$k_8 = (1.50)(3.186) = 4.78$$
$$k_9 = (19.20)(3.186) = 61.17$$
$$k_{10} = (0.70)(3.186) = 2.23$$
$$k_{11} = (8.55)(3.186) = 27.24$$
$$k_{12} = (0.01)(3.186) = 0.03.$$

These values are listed in the worksheet under the k_i column.

Step A-8 -- The entire system is in the same gross environment and there are no unique factors affecting any of the units, so the modifiers M_j are all equal to 1.

Step A-9 -- Adjusted relative failure rates, $\overline{k}_{ij}$

(a) Since the modifier for each unit is equal to one, the k_i's in group (b) are transfered, unaltered, to the appropriate unit column;

(b) k_1 and k_2 remain unaltered for Group (a), but since k_3 in Unit 2 has a transistor active element, the adjustment factor of 0.3 given in Table VI of Appendix A is appropriate; hence $\overline{k}_{32} = (0.3)k_3 = 0.9$.

Step A-10 -- Unit failure indices

Using the formula $K_j = \sum_{i=1}^{12} f_{ij}\overline{k}_i$ for the failure index of the j^{th} unit,

$$K_1 = 40(4.30) + 3(2.23) + 1(27.24) = 205.9$$

$$K_2 = 10(3.0) + 230(0.9) + 35(2.23) + 38.55 + 25(0.64) + 40(4.78) + 61.17 + 70(2.23) + 12(0.03) = 778.4$$

$$K_3 = 5(2.23) + 3(1.75) + 3(0.64) + 2(4.78) + 61.17 + 3(0.03) = 89.1$$

The values for the unit failure indices are entered in the allocation worksheet (see Section 3.1) which is illustrated after Step 3.2-4.

Step 3.1-d -- The system is unmodified serial. The essentiality is one for each unit.

Step 3.1-e -- The average operating time for each unit is 6 hours, the same as the average system operating time. Since the system is serial, the remainder of the procedures will be found in Section 3.2.

Step 3.2-1 -- The system description indicates that, with the system operating perfectly, mission failure will occur 15 per cent of the time due to cloud cover, lack of range, etc., so the design adequacy of the system is 0.85. The reliability requirement, then, is

$$R^*(T) = \frac{S^*(T)}{D_s} = \frac{0.80}{0.85} = 0.94.$$

Step 3.2-2 -- The feasibility of the reliability requirement is determined using the method outlined in Appendix B.

Step B-1A-- Electronic Portion

(a) The number of active elements in the electronic portion obtained from the data worksheet, is $N_e = 40 + 10 + 230 = 280$, [Note that this would include electronic **AEG's** in group c (unlisted categories) if any existed in this system.]

(b) The system is in an airborne environment.

(c) From Figure B-1, the expected mean life, $\tilde{\theta}_{et}$, of an equivalent tubed system can be found using $N_e = 280$, and reading the chart at the lower, middle, and upper lines of the airborne band. Letting subscripts 1, 2 and 3 represent the lower, middle, and upper lines of the band respectively, then

$$\tilde{\theta}_{1et} = 7.5 \text{ hours}$$
$$\tilde{\theta}_{2et} = 14 \text{ hours}$$
$$\tilde{\theta}_{3et} = 32 \text{ hours}$$

(d) There are active elements other than tubes, so the following corrections are made:

i	Type	N_i	B_i	N_iB_i
1	Tube	50	1.0	50
2	Transistor, Analog	230	0.3	69
	$N_e = \Sigma N_i = 280$		$\Sigma N_iB_i = 119$	

The actual expected mean life for this system is, in general, $\tilde{\theta}_e = \tilde{\theta}_{et} \frac{280}{119}$, and for each of the lines on the airborne band,

$$\tilde{\theta}_{1e} = (7.5)\frac{280}{119} = 17.64 \text{ hours}$$

$$\tilde{\theta}_{2e} = (14.0)\frac{280}{119} = 32.94 \text{ hours}$$

$$\tilde{\theta}_{3e} = (32.0)\frac{280}{119} = 75.30 \text{ hours}$$

Step B-1B Feasible Mean life of non-electronic portion.

Step B-1B.3 Method of using relative failure data.

(a) The failure index of the non-electronic portion is

$$K_{ne} = \sum_{4}^{12} k_i' f_i$$

$$K_{ne} = 40(0.70) + 1(12.10) + 3(0.55) + 28(0.20) + 42(1.50) + 2(19.20) + 73(0.70) + 1(8.55) + 15(0.01)$$

$$K_{ne} = 208.55$$

Notice that this would not include AEG's in unlisted categories of group c if any existed in this system.

(b) The mean life of the non-electronic portion is

$$\tilde{\theta}_{ne} = \frac{\tilde{\theta}_e \; \Sigma N_i B_i}{K_{ne}} ,$$

so that for the three lines on the airborne band,

$$\tilde{\theta}_{1ne} = (17.64) \frac{119}{208.55} = 10.07 \text{ hours}$$

$$\tilde{\theta}_{2ne} = 32.94 \; (0.571) = 18.8 \text{ hours}$$

$$\tilde{\theta}_{3ne} = 75.30 \; (0.571) = 43.0 \text{ hours}$$

(c) The feasible mean life of the system is then $\tilde{\theta}_s = \frac{\tilde{\theta}_e \tilde{\theta}_{ne}}{\tilde{\theta}_e + \tilde{\theta}_{ne}}$, and for the three lines on the airborne band,

$$\tilde{\theta}_{1s} = \frac{(17.64)(10.07)}{(17.64) + (10.07)} = 6.4 \text{ hours}$$

$$\tilde{\theta}_{2s} = \frac{(32.94)(18.8)}{32.94 + 18.8} = 11.97 \text{ hours}$$

$$\tilde{\theta}_{3s} = \frac{(75.3)\ (43.0)}{75.3 + 43.0} = 27.37 \text{ hours}$$

Since no redundancy or alternate modes of operation exist, computation of the feasible reliability requirement [which is $\tilde{R}(t_1) = 0.39$, $\tilde{R}(t_2) = 0.60$ and $\tilde{R}(t_3) = 0.80$] is unnecessary. The feasible system mean life can be studied instead. The required mean life of the system is $\theta_s^* = \frac{-T}{\ln R^*(T)} = \frac{-6}{\ln 0.94} = 97$ hours. As noted in paragraph 1 of Appendix B, if the system reliability requirement, in terms of mean life, differs by more than one order of magnitude from the feasible reliability determined through these procedures, a reconsideration of goals, functions, design, and program management may be necessary. If little attention if given to reliability during development, the mean life of 6.4 hours indicated by the lower line of the airborne band of Figure B-1 may be the best possible achievement. Conversely, if great emphasis is placed on reliability during development, the mean life of 27.37 hours, indicated by the upper line of the airborne band of Figure B-1, may be achieved or exceeded. This would cause the reliability requirement to enter the realm of feasibility.

Recognizing that steps can be taken to make the requirement feasible, the allocation will be completed using the given overall requirement.

Step 3.2-3--Total system failure index and unit failure index ratios. The total system failure index is

$$K = K_1 + K_2 + K_3 = 205.9 + 778.4 + 89.1 = 1073.4.$$

The unit failure index ratios are $w_j = K_j/K$:

$$w_1 = 205.9/1073.4 = 0.192$$

$$w_2 = 778.4/1073.4 = 0.725$$

$$w_3 = 89.1/1073.4 = 0.083$$

Step 3.2-4--The allocated unit reliability for the j^{th} unit is $\hat{R}(t_j) = 1 - \frac{1-R^*(T)^{w_j}}{E_j}$. Since the essentiality of each unit is one (see Step 3.1-d), the allocated reliability becomes $\hat{R}(t_j) = R^*(T)^{w_j}$. For the three units of the system,

$$\hat{R}(t_1) = (0.94)^{0.192} = \text{antilog } [0.192(\log .94)]$$

$$\hat{R}(t_1) = 0.988$$

$$\hat{R}(t_2) = (.94)^{0.725} = \text{antilog } [0.725(\log .94)]$$

$$\hat{R}(t_2) = 0.956$$

$$\hat{R}(t_3) = (.94)^{0.083} = \text{antilog } [0.083(\log .94)]$$

$$\hat{R}(t_3) = 0.995.$$

The allocation worksheet would have the following form:

DESIGN STAGE RELIABILITY ALLOCATION

System I
Mission: Bomb-navigation

Date: ________________
System Operating Time: 6 Hours
Reliability Requirement: 0.94

Unit U_j	U_1	U_2	U_3
Essentiality, E_j	1	1	1
Operating Time, t_j	6 Hr.	6 Hr.	6 Hr.
Failure Index, K_j	205.9	778.4	89.1
Failure Index Ratio, w_j	0.192	0.725	0.083
Allocated Reliability, $\hat{R}(t_j)$	0.988	0.956	0.995

Step 3.2-5 -- Assuming constant unit failure rates, the mean life and failure rate of the jth unit is

$$\hat{\theta}_j = -\frac{t_j}{\ln \hat{R}(t_j)} \quad \text{and} \quad \hat{\lambda}_j = -\frac{\ln \hat{R}(t_j)}{t_j} .$$

For the three units,

$$\hat{\theta}_1 = -\frac{6}{-0.01188} = 505 \text{ hours and } \hat{\lambda}_1 = 1980 \times 10^{-6}/\text{hour}$$

$$\hat{\theta}_2 = -\frac{6}{-0.04486} = 134 \text{ hours and } \hat{\lambda}_2 = 7477 \times 10^{-6}/\text{hour}$$

$$\hat{\theta}_3 = -\frac{6}{-0.00514} = 1167 \text{ hours and } \hat{\lambda}_3 = 857 \times 10^{-6}/\text{hour}$$

II. Modified Serial System -- In an effort to make the system more reliable, the capability of manually controlling the optical equipment may be made available. Power would still be necessary to operate the optical equipment, but the inputs normally supplied to the optical equipment by the navigation computer would be supplied manually.

Reliability allocation to such a system would closely parallel Example I. The procedural steps necessary for such a system are shown below, with references to Example I where applicable.

Step 3.1-a -- (Same as Example I)

Step 3.1-b -- (Same as Example I)

Step 3.1-c -- (Same as Example I)

Step 3.1-d -- The essentiality of the power supply and the optical equipment is unity. On the basis of performance of similar systems estimates are that for every 100 missions in which the navigation computer fails, 57 mission failures will result, i.e., the optical equipment is successfully controlled manually 43 times in 100 computer failures. The essentiality of the navigation computer, then, is $\frac{57}{100} = 0.57$.

Step 3.1-e -- (Same as Example I)

Step 3.2-1 -- (Same as Example I)

Step 3.2-2 -- (Same as Example I, and the same comments concerning feasibility apply to this example.)

Step 3.3-3 -- (Same as Example I)

Step 3.3-4 -- $\hat{R}(t_1) = 0.988$ and $\hat{R}(t_3) = 0.995$ as in Example I. Unit 2 has essentiality of $E_2 = 0.57$, so

$$\hat{R}(t_2) = 1 - \frac{1 - R^*(T)^{w_2}}{E_2} = 1 - \frac{1 - 0.956}{0.57} = 1 - \frac{0.044}{0.57}$$

$$\hat{R}(t_2) = 0.923.$$

Step 3.3-5 -- The unit mean life and unit failure rate are the same as Example I for units 1 and 3. For unit 2,

$$\hat{\theta}_2 = -\frac{6}{\ln 0.923} = \frac{6}{0.08013}$$

$$\hat{\theta}_2 = 74.9 \text{ hours}$$

$$\hat{\lambda}_2 = 13{,}355 \times 10^{-6}/\text{hour}$$

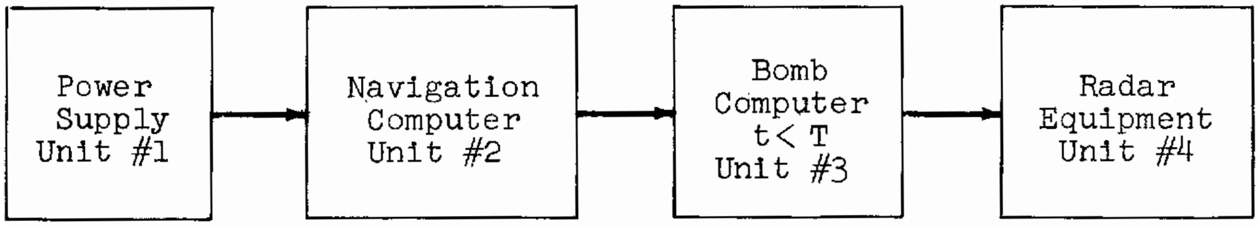
The effect of unit essentiality less than unity is to lower the allocated reliability from 0.956 to 0.923, thereby almost doubling the allocated allowable unit failure rate and halving the allocated required unit mean life.

III. Sub-Allocation -- Example I showed that system design adequacy less than unity caused the system reliability requirement to increase. For purposes of this discussion, the system has been altered by replacing the optical equipment with radar equipment in conjunction with a bomb computer.

The overall requirement for the altered system is 0.80 probability of successful operation for 6 hours, with operation of the bomb computer required for only 30 minutes. The method for handling unit operating times that are less than the system operating time is described in Subsection III(a). When the reliability allocation to the units of the system has been completed, the radar equipment will be sub-divided, and its reliability allocation will be internally allocated to illustrate sub-allocation. This procedure is described in Subsection III(b).

III(a) Basic Serial System --

Step 3.1-a -- The system consists of a power supply, navigation computer, bomb computer and radar equipment as shown in the following reliability block diagram.



Step 3.1-b -- The system is a serial system.

Step 3.1-c -- Unit failure indices are determined using the procedures outlined in Appendix A as discussed below:

Step A-1 -- The attached data worksheet (Table IIIa) is set up for four serial units.

Step A-2 -- Three categories (primary power; pulse, low power; and video) have active elements other than tubes and require separate lines on the worksheet.

Step A-3 -- Relative failure rates are entered in the worksheet from the tables .

Step A-4 -- The number of estimated AEG's of each category within each unit is entered in the worksheet and the category totals (f_i) are determined. The product $f_i k_i$ is formed and entered in the worksheet.

Step A-5 -- Since relative rates are listed for all functional categories that exist in the system, this step is not applicable.

Step A-6 -- The average electronic failure index is computed in the following manner:

TABLE III a

SUB-ALLOCATION -- BASIC SYSTEM DATA WORKSHEET FOR DETERMINING UNIT FAILURE INDICES, K_j

	i	Function			Unit #1 $M_1 = 1$		Unit #2 $M_2 = 1$		Unit #3 $M_3 = 1$		Unit #4 $M_4 = 1$		Functional Totals	
				k_i	$\bar{k}_{i1}$	f_{i1}	$\bar{k}_{i2}$	f_{i2}	$\bar{k}_{i3}$	f_{i3}	$\bar{k}_{i4}$	f_{i4}	f_i	$f_i k_i$
Group a	1	Primary Power		4.3	4.3	40							40	172
	2	Primary Power (Trans.)		4.3							1.29	30	30	129
	3	Pulse, Low Power		3.0			3.0	10	3.0	1			11	33
	4	Pulse, Low Power (Trans.)		3.0			0.9	230	0.9	20	0.9	200	450	1350
	5	RF, High Power		50.0							50.0	2	2	100
	6	RF, Low Power		3.4							3.4	4	4	13.6
	7	Special		58.0							58.0	3	3	174
	8	Video		5.4							5.4	3	3	16.2
	9	Video (Transistor)		5.4							1.62	200	200	1080
													$\Sigma f_i = 743$	$\Sigma f_i k_i = 3067.8$
			k_i'	$k_i = \bar{K}_e k_i'$									$\bar{K}_e = \frac{3067.8}{743} = 4.129$	
Group b	10	Synchro, Resolver	0.70	2.89			2.89	35	2.89	4	2.89	5		
	11	Gyro	12.10	49.96			49.96	1						
	12	Antenna, Assembly	36.00	148.64							148.64	1		
	13	Thermostat	0.20	0.83			0.83	25	0.83	3	0.83	12		
	14	Motor	1.50	6.19			6.19	40	6.19	3	6.19	4		
	15	Dehydrator	19.20	79.28			79.28	1						
	16	Relay	0.70	2.89	2.89	3	2.89	70	2.89	10				
	17	Relay, Stepping	8.55	35.30	35.30	1								
	18	Counter	0.01	0.04			0.04	12	0.04	3				
		Unit Failure Index, $K_j = \sum_{i=1}^{18} f_{ij}\bar{k}_{ij}$			$K_1 = 216.0$		$K_2 = 938.5$		$K_3 = 82.6$		$K_4 = 1044.3$			

(a) The total unadjusted electronic failure index is

$$K_e = \sum_{i=1}^{9} f_i k_i$$

$$= 40(4.3) + 30(4.3) + 11(3.0) + 450(3.0) + 2(50.0)$$

$$+ 4(3.4) + 3(58.0) + 3(5.4) + 200(5.4)$$

$$K_e = 3067.8$$

(b) The number of electronic AEG's in group (a) is as shown under column f_i of the worksheet

$$F_e = \sum_{i=1}^{9} f_i = 743$$

(c) The average electronic failure index is
$\overline{K}_e = 3067.8/743 = 4.129$

Step A-7 -- Each k_i' is converted to a failure rate relative to audio by the formula $k_i = \overline{K}_e k_i'$:

$k_{10} = (0.70)(4.13) = 2.89$
$k_{11} = (12.1)(4.13) = 49.96$
$k_{12} = (36.0)(4.13) = 148.64$
$k_{13} = (0.2)(4.13) = 0.83$
$k_{14} = (1.50)(4.13) = 6.19$
$k_{15} = (19.2)(4.13) = 79.28$
$k_{16} = (0.70)(4.13) = 2.89$
$k_{17} = (8.55)(4.13) = 35.30$
$k_{18} = (0.01)(4.13) = 0.04$

These values are entered in the k_i column of the worksheet.

Step A-8 -- The entire system is in the same gross environment, and none of the units are affected by any unique factors, therefore all modifiers (M_j) are equal to one.

Step A-9 -- Adjusted relative failure rates, $\bar{k}_i$

(a) Since the modifiers for each unit are equal to one, the k_i in group (b) are transferred unaltered to the appropriate unit column.

(b) For group (a) the adjustment factor of 0.3 for transistor active elements must be applied to the appropriate AEG types, hence $k_2 = (4.3)(0.3) = 1.29$, $k_4 = (3.0)(0.3) = 0.9$, and $k_9 = (5.4)(0.3) = 1.62$. All other k_i in this group are transferred to the appropriate unit column without modification.

Step A-10 -- Using the formula $K_j = \sum_i f_{ij}\bar{k}_i$ for the failure index of the j^{th} unit,

$$K_1 = 40(4.3) + 3(2.89) + 1(35.30) = 216.0$$

$$K_2 = 10(3.0) + 230(0.9) + 35(2.89) + 1(49.96) + 25(0.83) + 40(6.19) + 1(79.28) + 70(2.89) + 12(0.04) = 939.5$$

$$K_3 = 1(3.0) + 20(0.9) + 4(2.89) + 3(0.83) + 3(6.19) + 10(2.89) + 3(0.04) = 82.6$$

$$K_4 = 30(1.29) + 200(0.9) + 2(50.0) + 4(3.4) + 3(58.0) + 3(5.4) + 200(1.62) + 5(2.89) + 1(148.64) + 12(0.83) + 4(6.19) = 1044.3$$

These values for the unit failure indices are entered in the allocation worksheet shown after Step 3.2-5.

Step 3.1-d -- The essentiality of each unit is one.

Step 3.1-e -- Units 1, 2, and 4 have average operating time equal to the system operating time of 6 hours. Unit 3 has average operating time of 0.5 hours. (As the system is serial, the rest of the procedure is in Section 3.2.)

Step 3.2-1 -- The design adequacy of the system is one, hence the overall requirement of 0.80 is equal to the system reliability requirement.

Step 3.2-2 -- The feasibility of the reliability requirement is determined using the method outlined in Appendix B.

Step B-1A -- Electronic Portion

(a) $N_e = \sum_{i=1}^{9} f_i = 743$

(b) The system is in an airborne environment.

(c) The subscripts 1, 2 and 3 will be used to denote values depending on the lower middle and upper lines of the airborne band of Figure B-1. The expected mean lives, $\tilde{\theta}_{et}$, of an equivalent tubed system are

$$\tilde{\theta}_{1et} = 1.9 \text{ hours}$$

$$\tilde{\theta}_{2et} = 4.5 \text{ hours}$$

$$\tilde{\theta}_{3et} = 9 \text{ hours}$$

(d) Active elements other than tubes are in the system, so the above mean lives must be modified. There are 63 tubes and 680 analog transistors; thus, $\Sigma N_i = 63 + 680 = 743$, and $\Sigma N_i B_i = 63(1) + 680(0.3) = 63 + 204 = 267$. Then the actual mean lives of the electronic portion are

$$\tilde{\theta}_{1e} = \left(\frac{743}{267}\right) \tilde{\theta}_{1et} = 5.3 \text{ hours}$$

$$\tilde{\theta}_{2e} = 12.5 \text{ hours}$$

$$\tilde{\theta}_{3e} = 25.0 \text{ hours}$$

Step B-1B -- Non-electronic Portion

Step B-1B-3 -- Method of Using Relative Failure Data

(a) The failure index of the non-electronic portion of the system is

$$\begin{aligned} K_{ne} = {} & 0.07(35+4+5) + 12.10 + 36.00 \\ & + 0.20(25+3+12) + 1.50(40+3+4) \\ & + 19.20 + 0.70(3+70+10) + 8.55 \\ & + 0.01(12+3) \end{aligned}$$

$$K_{ne} = 243.4$$

(b) The mean lives of the non-electronic portion of the system are

$$\tilde{\theta}_{1ne} = \frac{N_e \tilde{\theta}_{1et}}{K_{ne}} = \frac{(743)(1.0)}{243.4} = 5.8 \text{ hours}$$

$$\tilde{\theta}_{2ne} = \frac{N_e \tilde{\theta}_{2et}}{K_{ne}} = 13.7 \text{ hours}$$

$$\tilde{\theta}_{3ne} = \frac{N_e \tilde{\theta}_{3et}}{K_{ne}} = 27.5 \text{ hours}$$

(c) The mean life of the system is given by

$$\tilde{\theta}_s = \frac{\tilde{\theta}_e \tilde{\theta}_{ne}}{\tilde{\theta}_e + \tilde{\theta}_{ne}} \text{, so that}$$

$$\tilde{\theta}_{1s} = \frac{(5.3)(5.8)}{5.3 + 5.8} = 2.8 \text{ hours}$$

$$\tilde{\theta}_{2s} = 6.6 \text{ hours}$$

$$\tilde{\theta}_{3s} = 13.1 \text{ hours}$$

The required mean life, θ_s^*, of the system is given by $R^*(T) = e^{-T/\theta_s^*}$; hence $\theta_s^* = \frac{-T}{\ln 0.80}$

$= \frac{6}{0.22314} = 26.9$ hours. Each of the above

mean lives are within an order of magnitude of the required mean life, so the reliability requirement can be considered feasible. However, since even the most optimistic estimate is less than half the required mean life, emphasis should be placed on reliability during the development stages to insure that at least the most optimistic estimate is achieved.

Step 3.2—3 -- The total system failure index is

$$K = K_1 + K_2 + K_3 + K_4$$

$$= 216.0 + 938.5 + 82.6 + 1044.3$$

$$K = 2281.4$$

The unit failure index ratios are $w_j = K_j/K$; therefore

$$w_1 = 216.0/2281.4 = 0.0946$$

$$w_2 = 938.5/2281.4 = 0.4113$$

$$w_3 = 82.6/2281.4 = 0.0362$$

$$w_4 = 1044.3/2281.4 = 0.4577$$

Step 3.2-4 -- Since the essentiality of each unit is one, the allocated reliability is simply $\hat{R}(t_j) = R^*(T)^{w_j}$. Then

$$\hat{R}(t_1) = 0.80^{0.0946} = \text{antilog } [0.0946(-0.22314)]$$

$$= \text{antilog } (0.0211) = 0.979$$

$$\hat{R}(t_2) = 0.80^{0.4113} = \text{antilog } (0.0918) = 0.912$$

$$\hat{R}(t_3) = 0.80^{0.0362} = \text{antilog } (0.0081) = 0.992$$

$$\hat{R}(t_4) = 0.80^{0.4577} = \text{antilog } (0.1021) = 0.903$$

Step 3.2-5 -- Assuming constant unit failure rates, the j^{th} unit has an allocated mean life of

$$\hat{\theta}_j = -\frac{t_j}{\ln \hat{R}(t_j)}$$ and an allocated failure

rate of $\hat{\lambda}_j = -\frac{\ln \hat{R}(t_j)}{t_j}$. For the four units,

$$\hat{\theta}_1 = -\frac{6}{-0.0211} = 284.4 \text{ hours and } \hat{\lambda}_1 = 3{,}516 \times 10^{-6}/\text{hour}$$

$$\hat{\theta}_2 = -\frac{6}{-0.0918} = 65.36 \text{ hours and } \hat{\lambda}_2 = 15{,}300 \times 10^{-6}/\text{hour}$$

$$\hat{\theta}_3 = -\frac{0.5}{-0.0081} = 61.7 \text{ hours and } \hat{\lambda}_3 = 16{,}200 \times 10^{-6}/\text{hour}$$

$$\hat{\theta}_4 = -\frac{6}{-0.1021} = 58.8 \text{ hours and } \hat{\lambda}_4 = 17{,}017 \times 10^{-6}/\text{hour}$$

The completed allocation worksheet is shown below.

DESIGN STAGE RELIABILITY ALLOCATION WORKSHEET

System: Bomb-Navigation
Primary mission: Bomb-navigation

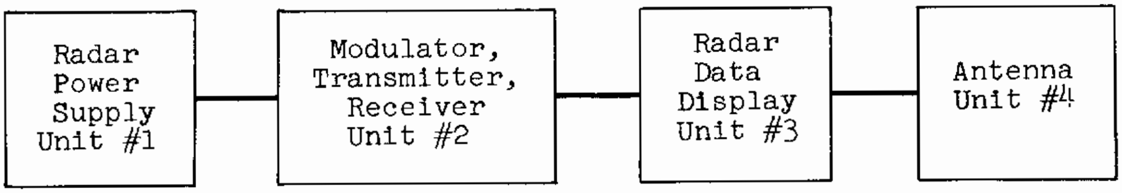
System Op. Time: 6 Hrs.
System Reliability Requirement: 0.80

Unit Identification U_j	U_1	U_2	U_3	U_4
Essentiality E_j	1	1	1	1
Operating Time t_j	6 Hrs.	6 Hrs.	0.5 Hrs.	6 Hrs.
Failure Index K_j	216.0	939.5	82.6	1044.3
Failure Index Ratio, w_j	0.0946	0.4113	0.0362	0.4577
Allocated Reliability, $\hat{R}(t_j)$	0.979	0.912	0.992	0.903

III(b) Sub-allocation, Radar Equipment

The radar equipment, as a unit in a serial system, was allocated a reliability requirement of 0.903 probability of successful operation for 6 hours. This requirement will be allocated to the units within the radar equipment.

Step 3.1-a -- The radar equipment, considered as a system, consists of a radar power supply, a modulator-receiver-transmitter unit, a radar data display unit and an antenna assembly. The reliability block diagram and identification of units by numbers is shown below:



Step 3.1-b -- The units within the radar equipment form a serial system.

Step 3.1-c -- Unit failure indices are determined using the procedures outlined in Appendix A. Since this is a simple serial system, the procedural steps followed in obtaining the unit failure indices will not be individually discussed. The attached data worksheet (Table IIIb) indicates the average electronic failure index, $\overline{K}_e$. Each non-electronic relative failure rate, k_i', is adjusted to obtain $k_i = \overline{K}_e k_i'$. The unit modifiers, M_i, are all equal to one. Three relative failure rates, k_1, k_2 and k_8, are modified for transistor active elements. The unit failure indices are formed as indicated by $K_j = \sum_i f_{ij} \overline{k}_{ij}$, and are entered in the allocation worksheet as shown following Step 3.3-4.

Step 3.1-d -- The system is unmodified serial, so the essentiality of each unit is equal to one.

TABLE IIIb

SUB-ALLOCATION: RADAR EQUIPMENT

	i	Function		k_i	UNIT 1		UNIT 2		UNIT 3		UNIT 4		Functional Totals	
					$M_1 = 1$		$M_2 = 1$		$M_3 = 1$		$M_4 = 1$			
					$\overline{K}_{11}$	f_{11}	$\overline{K}_{12}$	f_{12}	$\overline{K}_{13}$	f_{13}	$\overline{K}_{14}$	f_{14}	f_i	$f_i k_i$
Group a	1	Primary Power (Trans)		4.3	1.29	30							30	129.0
	2	Pulse, Low Power (Trans)		3.0			0.9	50	0.9	150			200	600.0
	3	RF, High Power		50.0			50.0	2					2	100.0
	4	RF, Low Power		3.4			3.4	4					4	13.6
	5	Servo		2.1							2.1	2	2	4.2
	6	Special		58.0			58.0	3					3	174.0
	7	Video		5.4					5.4	3			3	16.2
	8	Video (Transistor)		5.4					1.62	200			200	1080.0
			k_i'	$k_i = k_i' \overline{K}_e$									$\Sigma f_i = 444$	$\Sigma f_i k_i = 2117$
Group b	9	Servo, Power	14.85	70.80							70.80	1	$\overline{K}_e = \frac{2117}{444} = 4.768$	
	10	Synchro, Resolver	0.70	3.34					3.34	5	3.34	2		
	11	Gear Drive Mechanism	0.35	1.67							1.67	1		
	12	Thermostat	0.20	0.95	0.95	3	0.95	4			0.95	1		
	13	Motor	1.50	7.15			7.15	1			7.15	2		
	14	Slip Rings	0.80	3.81							3.81	1		
	15	Rotary Joints	5.60	26.70							26.70	1		
	16	Waveguides	10.00	47.68							47.68	1		
Unit Failure Index, $K_j = \sum_{i=1}^{16} f_{ij} \overline{K}_{ij}$					$K_1 = 41.6$		$K_2 = 343.6$		$K_3 = 491.9$		$K_4 = 176.8$			

Step 3.1-e -- The average operating time for each unit within the radar equipment is equal to six hours, the operating time of the radar equipment. (The system is serial, so the remainder of the procedure is in Section 3.2.)

Step 3.2-1 -- The design adequacy of the radar equipment is unity, so the reliability requirement is equal to the allocated reliability of 0.903.

Step 3.2-2 -- The feasibility of the reliability requirements on the radar equipment can be determined using the procedures of Appendix B. This requirement, however, was determined from an allocation procedure performed for the Bomb/Nav system which includes the radar equipment. If the requirement on the overall system is feasible, a feasibility determination of allocated requirements is not, in general, necessary.

Step 3.2-3 -- The total system failure index is

$$K = \sum_{j=1}^{4} K_j = 1053.9.$$ The unit failure index ratios are

$$w_1 = 41.6/1053.9 = 0.0394$$
$$w_2 = 343.6/1053.9 = 0.3260$$
$$w_3 = 491.9/1053.9 = 0.4667$$
$$w_4 = 176.8/1053.9 = 0.1677$$

Step 3.2-4 -- The allocated reliabilities are found from $\hat{R}(t_j) = R^*(T)^{w_j}$, since the essentiality of each unit is equal to one:

$$\hat{R}(t_1) = 0.996$$
$$\hat{R}(t_2) = 0.967$$
$$\hat{R}(t_3) = 0.954$$
$$\hat{R}(t_4) = 0.983$$

The completed allocation worksheet follows.

DESIGN STAGE RELIABILITY ALLOCATION WORKSHEET

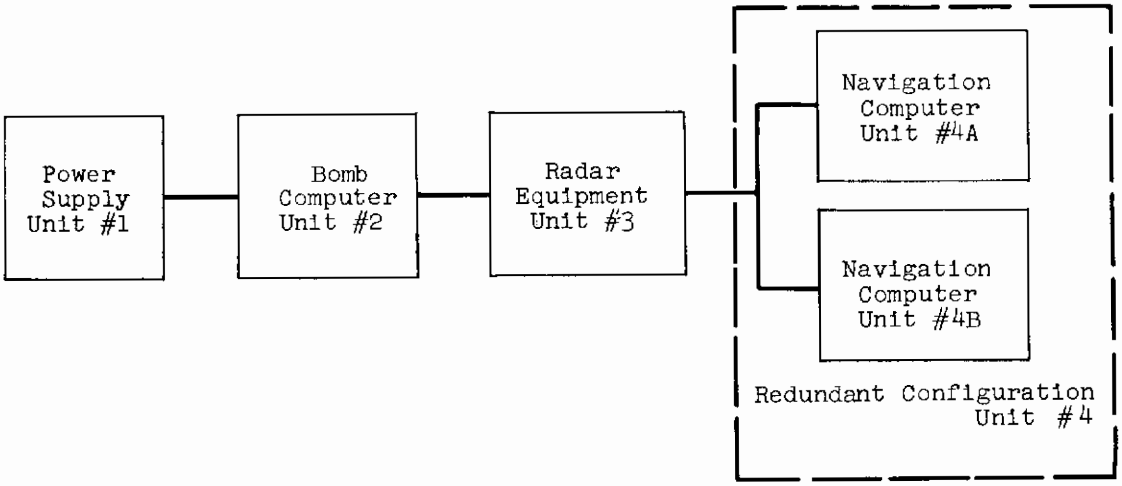
System: Radar Equipment
Primary mission: Bomb-navigation

System Op. Time: 6 Hrs.
System Reliability Requirement: 0.903

Unit Identification U_j	U_1	U_2	U_3	U_4
Essentiality E_j	1	1	1	1
Operating Time t_j	6 Hrs.	6 Hrs.	6 Hrs.	6 Hrs.
Failure Index K_j	41.6	343.6	491.9	176.8
Failure Index Ratio w_j	0.0394	0.3260	0.4667	0.1677
Allocated Reliability $\hat{R}(t_j)$	0.996	0.967	0.954	0.983

IV. Redundant Systems -- The system of Example III had a design adequacy equal to one, but it was more complex than the system of Examples I and II. The effects of this increased complexity may be partially overcome by introducing redundancy of units. For the purposes of illustrating redundancy it will be assumed that the navigation computer is duplicated in an active-parallel redundant configuration. The power supply, bomb computer, and radar equipment would remain as in Example III. The illustration of allocation to such a system follows:

Step 3.1-a -- The system, as shown in the following block diagram, consists of a power supply, bomb computer, radar equipment and a parallel redundant configuration of two navigation computers exactly like the navigation computer of Example III.



Step 3.1-b -- The system is a redundant type.

Step 3.1-c -- The unit failure indices are determined in the same manner as in Example III, and the steps of Appendix A will not be discussed in detail. The worksheet for this system is shown in Table IV. Because of the additional navigation computer, the value of $\overline{K}_e$ is changed. This in turn yields different values for each of the $k_i = \overline{K}_e k_i'$ in group (b) and the resulting values for the unit failure indices are

$$K_1 = 213.0$$

$$K_2 = 78.8$$

$$K_3 = 1031.1$$

$$K_{4A} = K_{4B} = 892.0$$

Step 3.1-d -- The essentiality of each series unit, 1, 2 and 3, and of the redundant configuration, Unit 4, is one.

Step 3.1-e -- Series units 1 and 3 and the redundant configuration, unit 4 have average operating time equal to the system operating time of 6 hours. Unit 2 has average operating time of 0.5 hours.

The system is a redundant system; therefore, the rest of the procedure is given in Section 3.3 of the main body of this document.

TABLE IV. REDUNDANT SYSTEM

DATA WORKSHEET FOR DETERMINING UNIT FAILURE INDICES, K_j

	i	Function		k_i	UNIT 1		UNIT 2		UNIT 3		UNIT 4A		UNIT 4B		Functional Totals	
					$M_1 = 1$		$M_2 = 1$		$M_3 = 1$		$M_{4A} = 1$		$M_{4B} = 1$			
					$\bar{k}_{i1}$	f_{i1}	$\bar{k}_{i2}$	f_{i2}	$\bar{k}_{i3}$	f_{i3}	$\bar{k}_{i4A}$	f_{i4A}	$\bar{k}_{i4B}$	f_{i4B}	f_i	$f_i k_i$
Group a	1	Primary Power		4.3	4.3	40									40	172.0
	2	Primary Power (Trans)		4.3					1.29	30					30	129.0
	3	Pulse, Low Power		3.0			3.0	1			3.0	10	3.0	10	21	63.0
	4	Pulse, Low Power (Trans)		3.0			0.9	20	0.9	200	0.9	230	0.9	230	680	2040.0
	5	RF, High Power		50.0					50.0	2					2	100.0
	6	RF, Low Power		3.4					3.4	4					4	13.6
	7	Special		58.0					58.0	3					3	174.0
	8	Video		5.4					5.4	3					3	16.2
	9	Video (Transistor)		5.4					1.62	200					200	1080.0
			k_i'	$k_i = \bar{K}_e k_i'$											$\Sigma f_i = 983$	$\Sigma f_i k_i = 3787.8$
															$\bar{K}_e = \frac{3787.8}{983} = 3.853$	
Group b	10	Synchro, Resolver	0.70	2.70			2.70	4	2.70	5	2.70	35	2.70	35		
	11	Gyro	12.10	46.62							46.62	1	46.62	1		
	12	Antenna Assembly	36.00	138.71					138.71	1						
	13	Thermostat	0.20	0.77			0.77	3	0.77	12	0.77	25	0.77	25		
	14	Motor	1.50	5.78			5.78	3	5.78	4	5.78	40	5.78	40		
	15	Dehydrator	19.20	73.98							73.98	1	73.98	1		
	16	Relay	0.70	2.70	2.70	3	2.70	10			2.70	70	2.70	70		
	17	Relay, Stepping	8.55	32.94	32.94	1										
	18	Counter	0.01	0.04			0.04	3			0.04	12	0.04	12		
Unit Failure Index, $K_j = \sum_{i=1}^{18} \bar{k}_{ij} f_{ij}$					$K_1 = 213.0$		$K_2 = 78.6$		$K_3 = 1031.1$		$K_{4A} = 892.0$		$K_{4B} = 892.0$			

Step 3.3-1 -- The design adequacy is one, so the reliability requirement is 0.80 probability of successful operation for 6 hours.

Step 3.3-2 -- The total failure index of the series units is

$$K_a = K_1 + K_2 + K_3$$

$$= 213.0 + 78.6 + 1031.1$$

$$K_a = 1322.7$$

Step 3.3-3 -- The redundant units are identical, thus $K_4' = 892.0$

Step 3.3-4 -- The feasibility determination exactly parallels the system of Example III, with the following additions:

Step B-1C -- The mean life of the equivalent series system is

$$\tilde{\theta}_{1s} = 2.8 \text{ hours (minimum)}$$

$$\tilde{\theta}_{2s} = 6.6 \text{ hours (average)}$$

$$\tilde{\theta}_{3s} = 13.1 \text{ hours (maximum)}$$

Step B-2A -- The degree of redundancy is $\gamma = \dfrac{K_4'}{K_a + K_4'}$

$$= \frac{892}{2214.7} = 0.4027.$$

Step B-3A -- The feasible reliability requirement is

$$\tilde{R}(6) = 2\,e^{-T/\tilde{\theta}_s} - e^{-(1+\gamma)T/\tilde{\theta}_s} .$$

$$\tilde{R}_1(6) = 0.185$$

$$\tilde{R}_2(6) = 0.526$$

$$\tilde{R}_3(6) = 0.739$$

Step B-4 -- The feasible system reliability requirement of 0.739 for 6 hours of operation is slightly lower than the required system reliability of 0.80. This could be considered to be feasible, but emphasis on reliability in the design and development stages is necessary, since the 0.739 probability came from using the upper line of the airborne band.

Step 3.3-5 -- The ratio $\alpha = K_a/K_4'$ is $\alpha = 1322.7/892.0$

$= 1.48$

Step 3.3-6 -- The relative failure rate for the redundant configuration is determined in the following manner:

A) From Figure 9, $Z(\alpha,R^*)$ for $\alpha = 1.48$ and $R^* = 0.80$ is $Z(1.48,\ 0.80) = 0.076$, and the relative failure rate for the redundant configuration (Unit 4) is $K_4 = (K_a)[Z(1.48,\ 0.80)] = 100.5$.

Step 3.3-7 -- The total system failure index is

$$K = K_a + K_r = 1322.7 + 100.5 = 1423.2$$

and the failure index ratios for the series units and the redundant configuration are

$$w_1 = 213.0/1423.2 = 0.1497$$

$$w_2 = 78.6/1423.2 = 0.0552$$

$$w_3 = 1031.1/1423.2 = 0.7245$$

$$w_4 = 100.5/1423.2 = 0.0706 \ (\text{redundant configuration})$$

$$w_{4A} = w_{4B} = 892.0/1423.2 = 0.6268$$

Step 3.3-8 -- Allocated Reliabilities

A) The allocated reliability for the series units is $\hat{R}(t_j) = R^*(T)^{w_j}$, since the essentiality of each unit is one. Therefore,

$$\hat{R}(t_1) = (0.80)^{0.1497} = \text{antilog}(-.03340) = .967$$

$$\hat{R}(t_2) = \text{antilog}\ (-.01232) = .988$$

$$\hat{R}(t_3) = \text{antilog}\ (-.16167) = .851$$

B) For the redundant configuration with essentiality of one,

$$\hat{R}(t_4) = (0.80)^{0.0706} = \text{antilog}\ (-.01576) = .984$$

C) The allocated reliability for each redundant unit is

$$\hat{R}(t_{4A}) = \hat{R}(t_{4B}) = R^*(T)^{w_{4A}} = (0.80)^{0.6268}$$

$$= \text{antilog}\ (-.13987) = .870$$

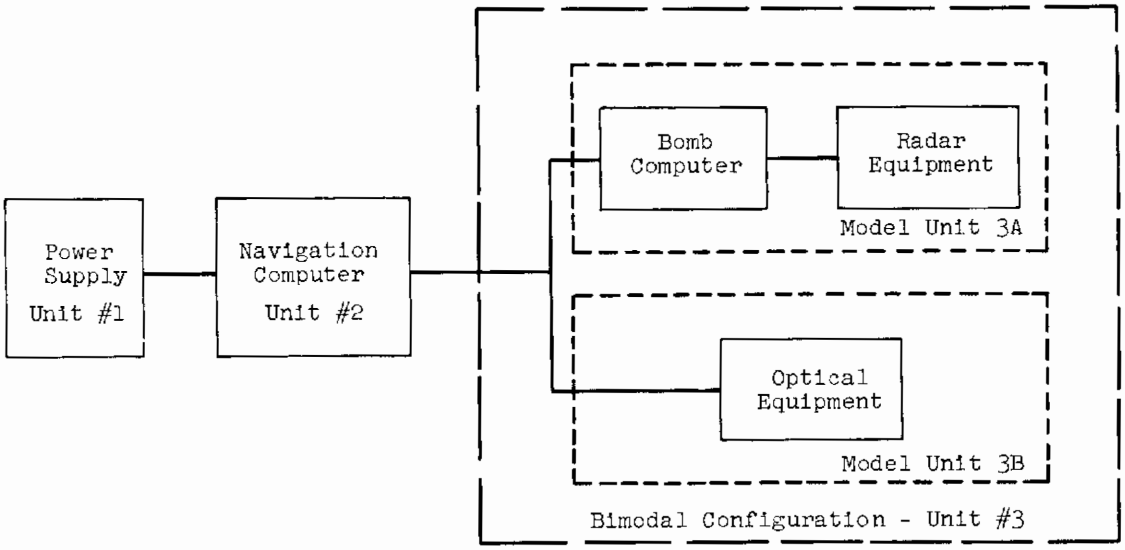
The completed allocation worksheet is shown below.

DESIGN STAGE RELIABILITY ALLOCATION WORKSHEET

System: Redundant System System Operating Time: 6 Hours
Primary Mission: Bombing System Reliability Requirement: 0.80

Unit and Configuration Identification U_j	U_1	U_2	U_3	U_4	U_{4A}	U_{4B}
Essentiality E_j	1	1	1	1	-	-
Operating Time t_j	6 Hrs.	0.5Hrs.	6 Hrs.	6 Hrs.	-	-
Failure Index K_j	213.0	78.6	1031.1	100.5	892.0	892.0
Failure Index Ratio w_j	0.1497	0.0552	0.7245	0.0706	0.6268	0.6268
Allocated Reliability $\hat{R}(t_j)$	0.967	0.988	0.851	0.984	0.870	0.870

V. Bimodal System -- The final system illustrated is a bimodal system consisting of a power supply, a navigation computer, and an optical system that is used as a secondary mode to a bomb computer and radar equipment. The reliability block diagram is shown below to identify the units and the bimodal configuration. An overall system effectiveness requirement of 0.80 for 6 hours of operation exists.



The steps common to all systems will not be discussed in detail, since they closely parallel the basic serial system of Example III. The system is bimodal and, according to the worksheet, the adjustment factor for the relative rates, ($\overline{K}_e$), is the same as that for Example III. The failure indices are the same, except that Unit 3B has been added and Unit 3A includes the bomb computer and radar equipment from Example III. The essentiality of the series units and of the bimodal configuration is one, and their operating time is six hours. Assuming that the common steps in Section 3.1 have been completed and the necessary information entered in the attached worksheet, the procedural steps for bimodal systems outlined in Section 3.4 will be discussed.

Step 3.4-1 -- $D_1 = 1.00$ and $D_2 = 0.85$, as determined in Example I. Because $D_1 = 1.0$ a Type I design can be assumed. The primary mode contains Modal Unit 3A.

TABLE V. BIMODAL SYSTEM

DATA WORKSHEET FOR DETERMINING UNIT FAILURE INDICES, K_j

	i	Function		k_i	UNIT 1		UNIT 2		UNIT 3A		UNIT 3B		Functional Totals	
					$M_1 = 1$		$M_2 = 1$		$M_{3A} = 1$		$M_{3B} = 1$			
					$\bar{k}_{11}$	f_{11}	$\bar{k}_{12}$	f_{12}	$\bar{k}_{13A}$	f_{13A}	$\bar{k}_{13B}$	f_{13B}	f_i	$f_i k_i$
Group a	1	Primary Power		4.3	4.3	40							40	172.0
	2	Primary Power (Trans)		4.3					1.29	30			30	129.0
	3	Pulse, Low Power		3.0			3.0	10	3.0	1			11	33.0
	4	Pulse, Low Power (Trans)		3.0			0.9	230	0.9	220			450	1350.0
	5	RF, High Power		50.0					50.0	2			2	100.0
	6	RF, Low Power		3.4					3.4	4			4	13.6
	7	Special		58.0					58.0	3			3	174.0
	8	Video		5.4					5.4	3			3	16.2
	9	Video (Transistor)		5.4					1.62	200			200	1080.0
			k_i'	$k_i = \bar{K}_e k_i'$									$\Sigma f_i = 743$	$\Sigma f_i k_i = 3067.8$
													$\bar{K}_e = \frac{3067.8}{743} = 4.129$	
Group b	10	Synchro, Resolver	0.70	2.89			2.89	35	2.89	9	2.89	5		
	11	Gyro	12.10	49.96			49.96	1						
	12	Antenna Assembly	36.00	148.64					148.64	1				
	13	Thermostat	0.20	0.83			0.83	25	0.83	15	0.83	3		
	14	Motor	1.50	6.19			6.19	40	6.19	7	6.19	2		
	15	Dehydrator	19.20	79.28			79.28	1			79.28	1		
	16	Relay	0.70	2.89	2.89	3	2.89	70	2.89	10				
	17	Relay, Stepping	8.55	35.30	35.30	1								
	18	Counter	0.01	0.04			0.04	12	0.04	3				
	19	Prism	0.55	2.27							2.27	3		
Unit Failure Index, $K_j = \sum_{i=1}^{19} \bar{k}_{ij} f_{ij}$					$K_1 = 216.0$		$K_2 = 938.5$		$K_{3A} = 1126.9$		$K_{3B} = 115.4$			

Step 3.4-2 -- System reliability requirement

A. The total system failure index of an equivalent series system is $K_s = K_1 + K_2 + K_{3A} = 2281.4$

B. A preliminary estimate of the allocated reliability of the primary modal unit (3A) is

$$\hat{r}_{3A} = \frac{S^*(T)}{D_1}^{K_{3A}/K_s} = (0.80)^{0.4938} = 0.896$$

C. The average design adequacy is $\overline{D} = \hat{r}_{3A} D_1 + (1-\hat{r}_{3A})D_2$,

$\overline{D} = 0.896 + 0.104(0.85) = 0.984$

D. The system reliability requirement is $R^*(T) =$

$S^*(T)/\overline{D} = 0.813$.

Step 3.4-3 -- An estimate of the required average audio AEG rate is

$$\lambda = -\frac{\ln R^*(T)}{K_s T} = \frac{0.20702}{(6)2281.4} = 15.1 \times 10^{-6}/ \text{ hours}$$

Step 3.4-4 -- Since $d_1 = -\frac{\log D_1}{\lambda T} = 0$ and $d_2 = -\frac{\log D_2}{\lambda T}$

$$= -\frac{\log 0.85}{15.1 \times 10^{-6} \times 6} = 1793.82,$$ an

equivalent failure index for the modal unit can be found for the Type I case from the equation

$$K_b' = \frac{1}{2}\left[-(d_1+d_2) + \sqrt{(d_1+d_2)^2 + 4(K_{3A}K_{3B}+d_1 K_{3B}+d_2 K_{3A})}\right].$$

Therefore,

$$K_b' = \frac{1}{2}\Bigg[(-1793.82)$$

$$+\sqrt{(1793.8)^2 + 4\left[(1126.9)(115.4) + (1793.8)(1126.9)\right]}\Bigg]$$

$$K_b' = 822.4 \ .$$

Step 3.4-5 -- The total failure index of all series units is

$$K_a = K_1 + K_2 = 1154.5$$

Step 3.4-6 -- The feasibility of the reliability requirement is determined using the procedures outlined in Appendix B. The first step is to determine the feasible mean life of the basic series system, which is the primary mode of a bimodal system. But the primary mode of our system is simply the series system of Example III for which the maximum mean life, based on the upper line of the airborne band of Figure B-1, is $\tilde{\theta}_s = 13.1$ hours. With this mean life for the basic series system, the following steps are performed:

B-2.B -- $\gamma = \dfrac{K_b'}{K_a + K_b'} = \dfrac{822.4}{1976.9} = 0.42.$

B-3.A -- Using Figure B-2, for $T/\tilde{\theta}_s = 6/13.1 = 0.46$ and estimating the $\gamma = 0.42$ curve, the feasible system reliability requirement for 6 hours of operation is 0.74. This compares favorably with the system reliability requirement of 0.813.

Step 3.4-7 -- $\alpha = K_a/K_b' = 1.40$

Step 3.4-8 -- Referring to Figure 9, for $\alpha = 1.40$ and $R^*(T) = 0.813$, the value of $Z(1.40, 0.813)$ is $Z = 0.078$. Then the equivalent series failure index for the bimodal configuration is $K_3 = (Z)(K_a) = 90.0$.

Step 3.4-9 -- The total system failure index is $K = Ka + K_3 = 1244.5$ and the unit failure indices are

$$w_1 = K_1/K = 216/1244.5 = 0.1736$$

$$w_2 = 938.5/1244.5 = 0.7541$$

for the series units, and for the modal units,

$$w_{3A} = 1126.9/1244.5 = 0.9055$$

$$w_{3B} = 115.4/1244.5 = 0.0927$$

Step 3.4-10-- Allocated Reliabilities

A. For the series units, the essentiality is unity so that the allocated reliabilities are

$$\hat{R}(t_1) = 0.813^{0.1736} = \text{antilog}(-0.0359) = 0.965$$

$$\hat{R}(t_2) = 0.813^{0.7541} = \text{antilog}(-0.1561) = 0.855$$

B. The allocated reliabilities for the modal units are

$$\hat{R}(t_{3A}) = 0.813^{0.9055} = \text{antilog}(-0.1874) = 0.829^{\dagger}$$

$$\hat{R}(t_{3B}) = 0.813^{0.0927} = \text{antilog}(-0.0192) = 0.981$$

C. If needed, the allocated reliability of the bimodal configuration can be calculated from

$$\hat{R}(t_3) = \hat{R}(t_{3A}) + \hat{R}(t_{3B}) - \hat{R}(t_{3A})\hat{R}(t_{3B})$$
$$= 0.829 + 0.981 - (0.829)(0.981) = 0.997.$$

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† Unit 3A consists of the radar equipment and the bomb computer. This reliability allocation of 0.829 probability of successful operation for 6 hours could be sub-allocated to the radar equipment and the bomb computer in the manner shown in Example III.b.

Step 3.4-11 -- For the series units and the modal units, the allocated mean life and failure rates are given by $\hat{\theta}_j = -\frac{t_j}{\ln \hat{R}(t_j)}$ and $\hat{\lambda}_j = 1/\hat{\theta}_j$. For the bimodal configuration, the allocated mean life is given by $\hat{\theta}_3 = \hat{\theta}_{3A} + \hat{\theta}_{3B} - \frac{\hat{\theta}_{3A}\hat{\theta}_{3B}}{\hat{\theta}_{3A} + \hat{\theta}_{3B}}$ and the allocated failure rate is given by $\hat{\lambda}_3 = \frac{1-\hat{R}(t_3)}{t_3}$.

The completed allocation worksheet is shown below.

DESIGN STAGE RELIABILITY ALLOCATION WORKSHEET

System: Bimodal System	System Operating Time: T = 6 Hrs.
Primary Mission: Bombing	System Reliability Requirement: $R^*(T) = 0.813$

Unit and Configuration Identification U_j	U_1	U_2	U_3	U_{3A}	U_{3B}
Essentiality E_j	1	1	1	-	-
Operating Time t_j	6 Hrs.	6 Hrs.	6 Hrs.	-	-
Failure Index K_j	216.0	938.5	90.0	1126.9	115.4
Failure Index Ratio w_j	0.1736	0.7541	-	0.9055	0.0927
Allocated Reliability $\hat{R}(t_j)$	0.965	0.855	0.997	0.829	0.981

APPENDIX D

LIST OF FIGURES AND TABLES

APPENDIX D

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