THE ROLE OF COMPUTER GRAPHICS IN THE STRUCTURAL DESIGN PROCESS

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This paper discusses Lockheed-Georgia Company research and development programs in the application of man-computer graphics to the structural design process. The results of three years effort in this specialty area are outlined. The computer graphic system currently in use by project stress personnel for analysis and design of two-dimensional structures is presented. The key feature in this system is the interactive man-computer relationship wherein the graphic console guides the user through the analysis and design process. Primary emphasis is placed on ease of input and usefulness of output. In addition, the current status of an interactive three-dimensional structural analysis program, which is specifically designed for a graphic interface, is discussed. Interactive graphic methods for setting up structural networks involving 6000-8000 node points are considered. A method for solving large banded systems of linear equations is examined with relation to computer response time.

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SECTION I

INTRODUCTION

Within the past few years there has been tremendous growth in computer technology. New and simplified computer languages have been developed, and various software systems for time-sharing a computer are still being examined. However, one of the most interesting developments has been the concept of real-time graphic input-output frequently referred to as Man-Computer Graphics. This basic concept dates back to work done by I. Sutherland on the TX-2 computer at Lincoln Laboratory (Reference 10). Sutherland showed the feasibility of supplying graphical or geometrical information to the computer through a cathode ray display. Through the use of a computer-driven display and light pen, he was able to interact graphically with the computer; that is, pictures were drawn on the display that were understood by the computer. Cathode ray displays had been used frequently, but this two-way geometric communication was a revolutionary advancement. A second important milestone was T. E. Johnson's Three-Dimensional Sketchpad (Reference 6) again developed on the Lincoln TX-2 computer at Massachusetts Institute of Technology.

Lockheed-Georgia pioneered computer graphics within the aerospace industry when it acquired a dedicated interactive graphic system (References 2, and 8). In December 1964, a Univac 418 computer, connected to a Digital Equipment Corporation (DEC) 340 Scope, became operational in the Lockheed Georgia Research Laboratory. With this system, studies were initiated to investigate software problems associated with computer graphics. At the same time, surveys were conducted to determine the application programs which should be attempted first. Since then, many prototype programs have been developed and several have been successfully converted into production programs. A two-dimensional structural analysis and design program, operational since March 1968, is discussed in detail, and near-term objectives for three-dimensional structural analysis are cited.

SECTION II

PROTOTYPE STUDIES

It was determined from various prototype programs that relatively few graphic subroutines are necessary for an application program. The three most important subroutines from a programming standpoint enable: 1. a line to be drawn from point to point, 2. a light pen control to be displayed on the scope, and 3. the program to exit from a ''wait status'' with the proper interrupt-code. Simple Fortran subroutines were developed to perform the above functions on the Univac 418.

One of the first application programs to be developed was a section properties program. With this program, it is possible to draw any singly or multiply connected region bounded by straight lines and immediately examine the plane section properties. For a multiply connected region, it is necessary to specify the path of integration with the light pen. Equation 1 is a typical line integration formula for computing the moment of inertia about the reference X-axis.

$$I_{XX} = \frac{1}{12} \sum_{i=1}^{N} (X_1 - X_2)_i (Y_2 + Y_1)_i (Y_2^2 + Y_1^2)_i \qquad (1)$$

The subscripts refer to the beginning and terminal points of straight lines bounding the cross section.

A more complex section properties program was then undertaken. This program enables a designer to recall standard structural sections from a large data file and arrange them on the scope. Structural shapes not in the file can be rapidly defined by modifying basic shapes with the light pen. Figure 1 illustrates a cross section defined in this manner. Certain constraint curves can be displayed to facilitate the arrangement of these sections. These techniques are being explored in order to develop rapidly methods for defining the section property data for structural analysis programs.

The purpose of the next research program was to demonstrate that a two-dimensional structural network could be sketched on the cathode ray display with the light pen, that the computer could interpret the topology of the network, formulate and solve the necessary equations, and display the results. In addition to the requirement of graphical input and output, the prototype program was to demonstrate that, with the ''man in-the-loop'', parametric studies could be accomplished more efficiently. These objectives were first successfully

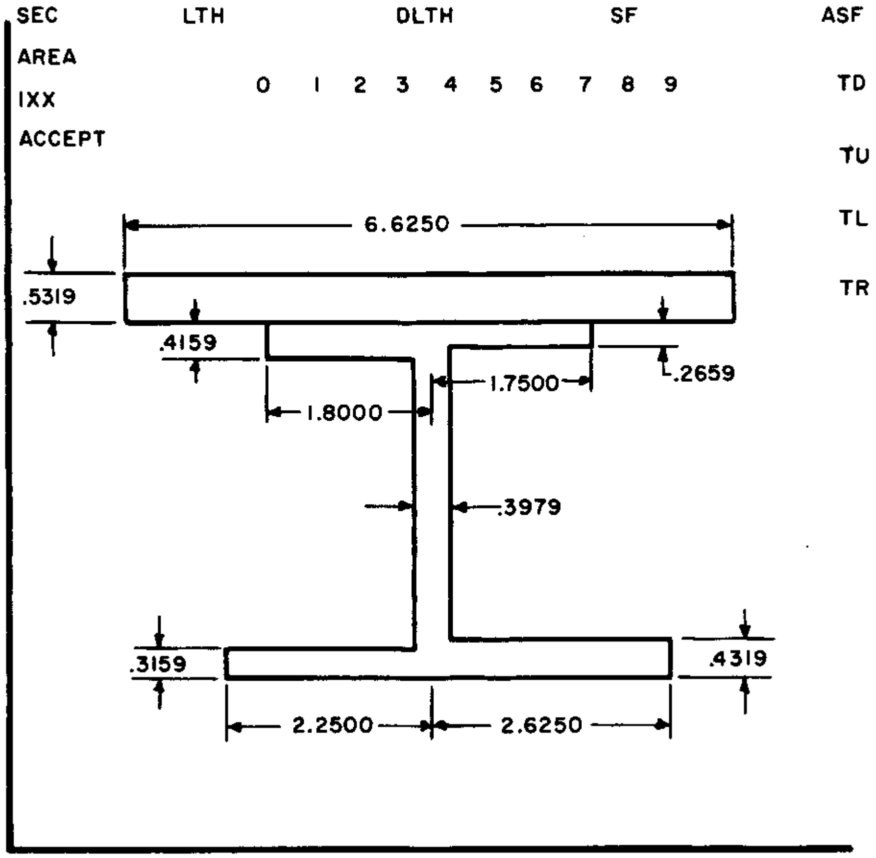


Figure 1. A complex cross section defined by modifying a basic shape with the light pen.

demonstrated with a Univac 418 Graphics Program for Structural Analysis. This program, based on the direct stiffness method, was limited to the analysis of two-dimensional structures with less than thirty node points. Two discrete elements, a uniform axial element, and a uniform beam element, were incorporated as the basic elements for idealizing a structure. The program consists of three overlays: graphic subrountines for defining and displaying the structural network, the analysis program, and subroutines for computing the appropriate output displays. The engineer at the console can define the structural network either with the light pen or enter appropriate data from cards. At any time during the construction phase, changes or corrections can be made directly on the cathode ray display.

In this program the light pen controls (light pen controls are written mneumonic symbols or words displayed on the scope), the tracking cross, and the node points are sensitive to the light pen. To define a structural network with the light pen it is first necessary to create node points. The structural elements and restraints are positioned by pointing to the appropriate nodes. Figure 2 illustrates a structural network being drawn with the light pen. The small diamonds represent node points; the double lines are the schematic representation for beam elements; and the single straight lines between node points represent axial elements. Twelve basic light pen controls (LPC) are used to create the model. For example, to create a node point with the light pen, the operator sequence is: CREATE, NODE, Position the Tracking Cross, Point to the SET (LPC). To delete a beam element, the sequence is: DELETE, BEAM, Identify the Element with the Light Pen. To create a restraint, corresponding to some specific boundary condition, the operator sequence is simply: CREATE, RESTRAINT, Select Restraint, and Identify the Node Point.

Control is transferred to the analysis program by pointing to the calculate LPC. Automatic methods for banding the stiffness matrix are incorporated into the program. At the completion of the calculation phase, the prototype program automatically initializes the output display programs. With the six LPC's in the third overlay, it is possible to display the basic model, the relative magnitudes for the structural displacements, the axial loads, and the shear and bending moment curves.

Figure 3 and 4 are output displays for the displacements and the bending moments in a complex frame. No attempt was made to display the associated alpha-numeric data in the prototype system because of computer limitations; however, the results of all calculations are listed on the printer. The operator then can transfer control to the input phase and delete or rearrange structural elements before requesting calculations the second time.

The prototype program was evaluated by various project engineers engaged in the stress analysis of the C-5A. Because of their enthusiasm, it was decided that a more comprehensive two-dimensional structural analysis program be developed. During August 1967, computer evaluations were made relative to a graphic time-sharing system for this particular application. During the computer evaluation phase, the prototype two-dimensional structures program was converted to operate on the IBM System/360. Due to the fact that only five subroutines were intimately associated with the DEC display, and because the program was written entirely in Fortran, the conversion of the program from the Univac 418 (24-bit word machine) to the IBM System/360 (time-shared graphic system) took only about three weeks. Zoom features were added to the prototype program during the evaluation phase, and it was demonstrated that background batch processing could be accomplished while the computer system was time-sharing three graphic consoles.

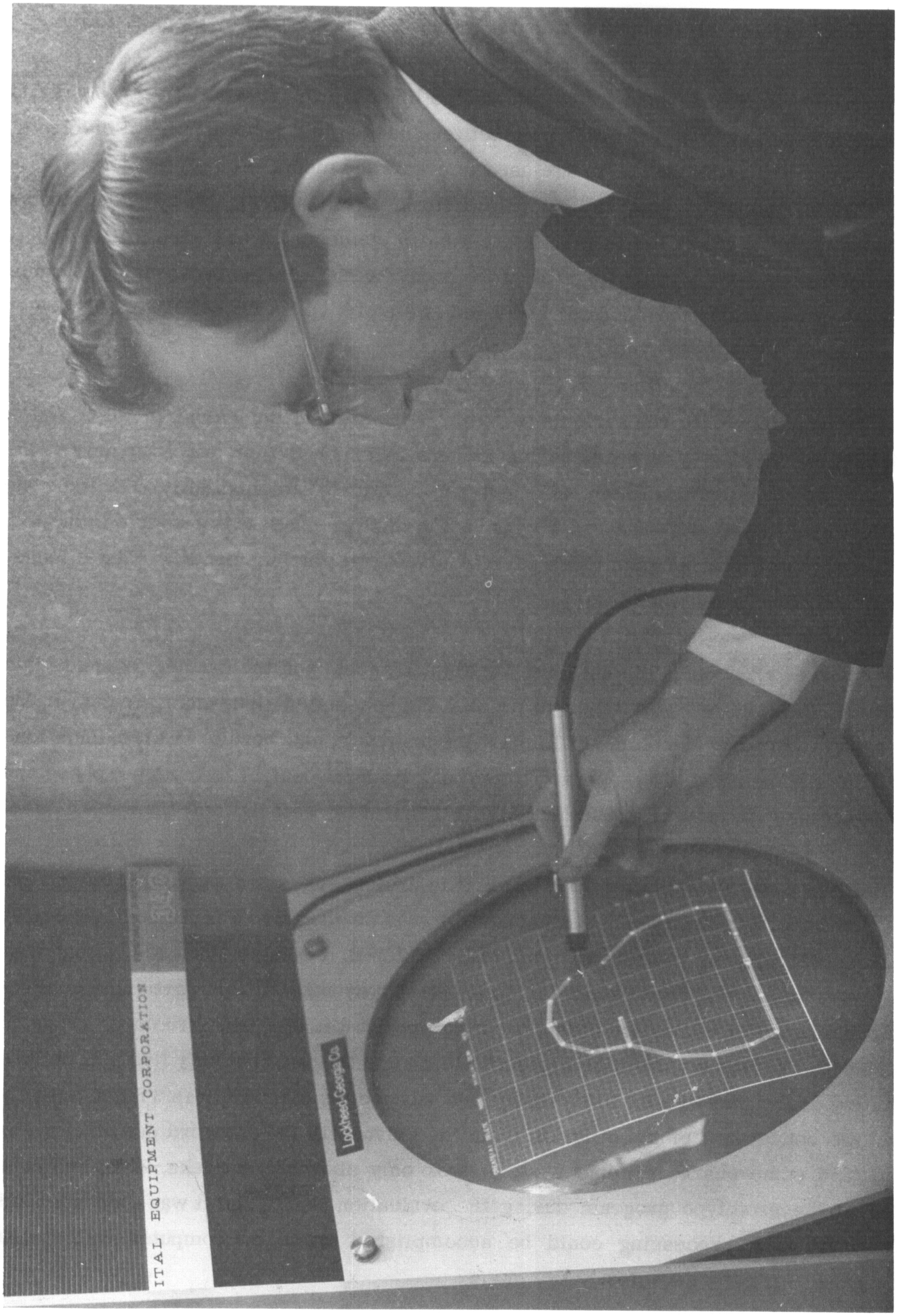


Figure 2. A complex frame being drawn with the light pen. Man and computer act as a team.

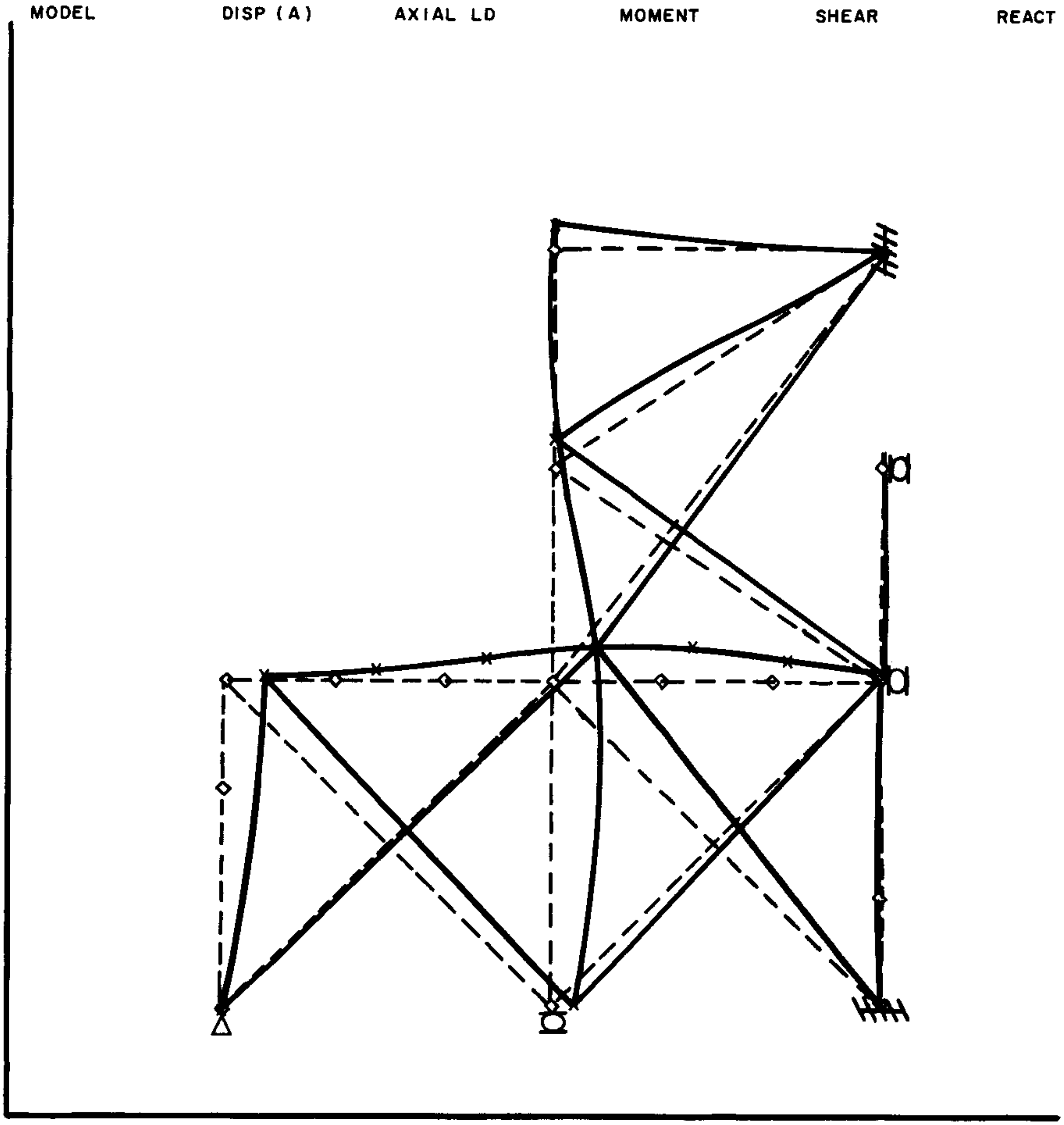


Figure 3. An output display of the relative deflections in a frame structure.

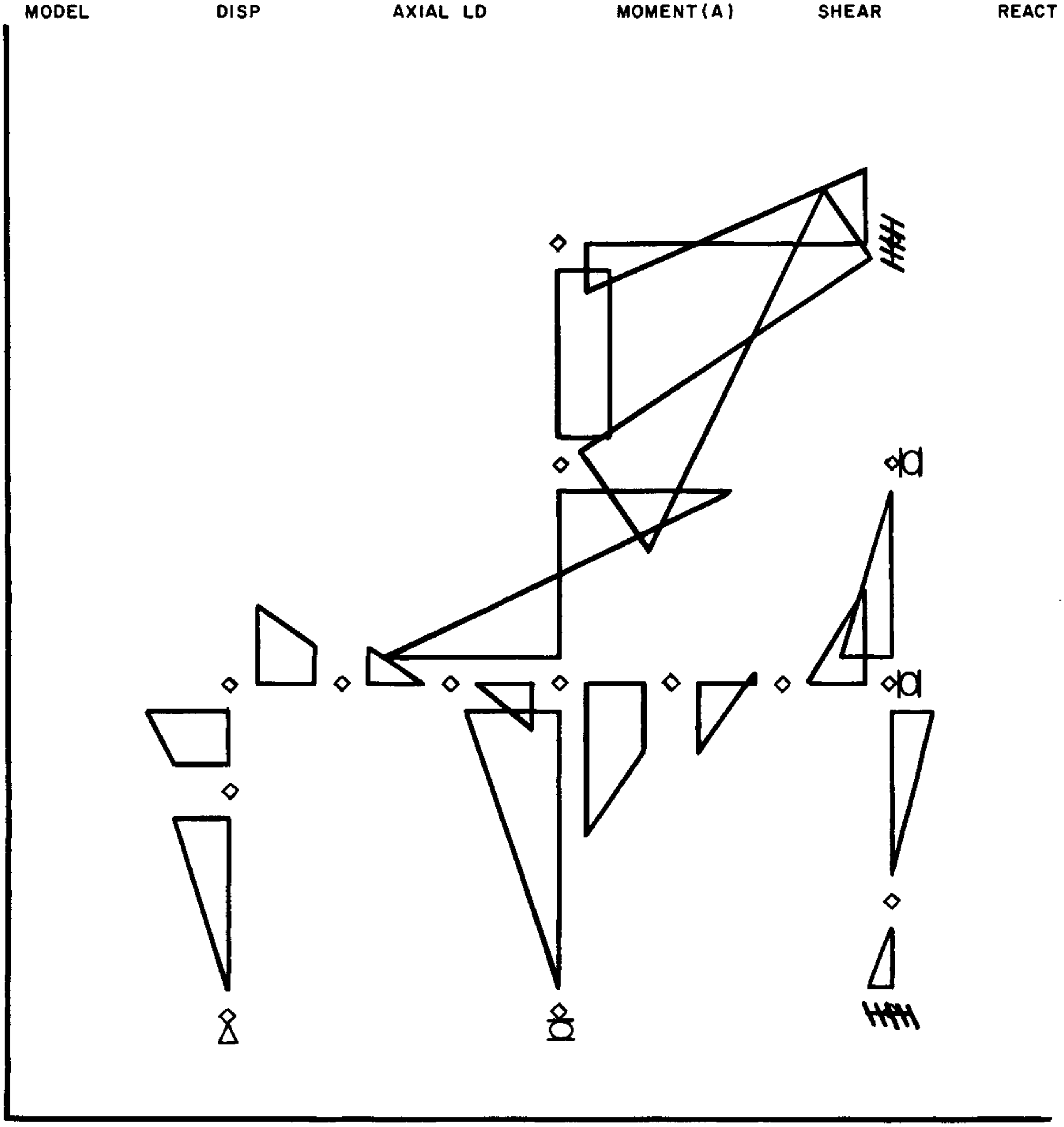


Figure 4. An exploded view of the frame structure showing a plot of the bending moment in each beam.

SECTION III

PRODUCTION PROGRAM FOR TWO-DIMENSIONAL STRUCTURAL ANALYSIS

Despite the fact that large three-dimensional structural models are employed for the analysis of all major airframe components, many two-dimensional stress problems also are solved every day. It was for this reason that a more sophisticated two-dimensional structural analysis program was undertaken. This program has been in operation at Lockheed-Georgia since March 1968. The scope of the program and an explanation of the LPC is outlined in Reference 9. Details concerning the computer system and its utilization will now be considered.

COMPUTER GRAPHIC SYSTEM

As illustrated in Figure 5, the graphic system consists of an IBM System/360 Model 50 computer with three 2250 Model 3 display units, a control console, four 2311 disc storage drives, two tape drives, and a card reader. The display units consist of a keyboard, function box, cathode ray tube, and a light pen. These display units are located in the engineering center and are connected by cable to the computer located in an adjacent building. Each disc has a storage capacity of 7.25 million bytes. The average access time to the disc is 75 milliseconds and the transfer rate is 156K bytes per second.

Programming systems provided by IBM are utilized and consist of Operating System (OS)/360, Basic Graphics Access Methods, and Graphic Subroutine Package (GSP). OS/360 basically provides the capability for operating the three display units in an essentially independent multi-programming environment. Basic Graphics Access Method is an OS/360 component which provides control of the 2250 Model 3 displays and performs those functions dependent on the characteristic of the display device. Some of these functions include transfer of display orders and data to and from the 2250 display buffers. GSP is a set of Fortran callable subroutines which perform the functions of display buffer generation, display management, and interrupt correlation with displayed elements. The GSP subroutines provide the ''bookkeeping'' functions required for graphics and allow the programmer to concentrate on the application program.

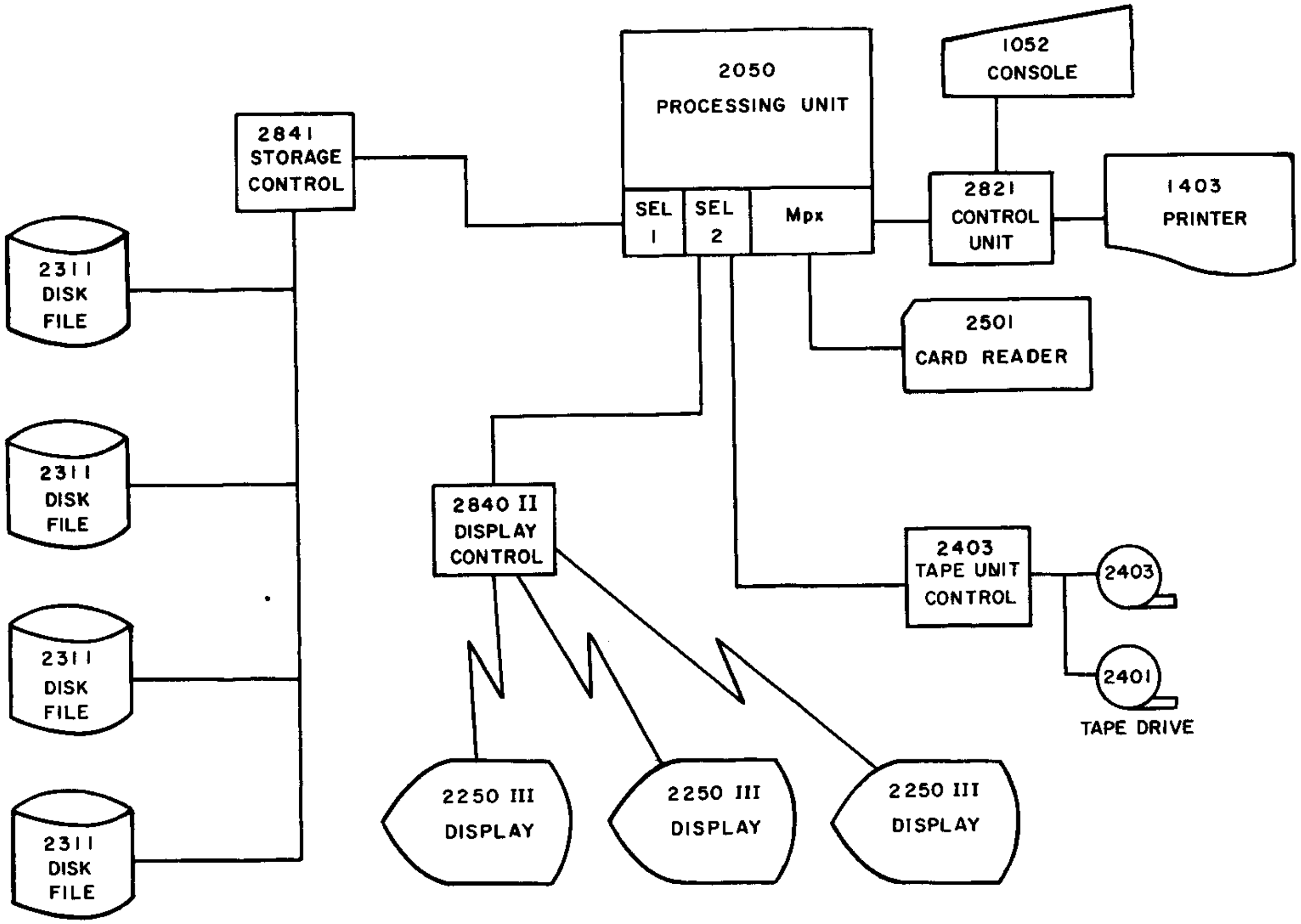


Figure 5. IBM System/360 Model 50 with three 2250 Model 3 Display Units.

PROGRAM ORGANIZATION

The entire working core of 512,000 bytes (8 bits) is divided into five areas. Each scope is assigned 80,000 bytes of working space, and, in addition, all three scopes share the same compute partition (112,000 bytes). The remaining area (160,000 bytes) is used by the library and system subroutines. Each scope partition contains the data files associated with the problem on that scope and is protected from destruction by other users. Simple calculations associated with the manipulation of the display are performed in the scope partitions. Only the matrix operations necessary for the analysis of the structure are executed in the compute partition. Each of the 2250 Model 3 scopes has its own display buffer. The scope displays are automatically sustained at about 30 to 40 cycles per second, depending upon the size of the display buffer.

Input and output data is stored in a preassigned file on the disc and cannot be destroyed unless the file's lock-code is known. Each user is assigned a lock-code or code number to identify his input file. Because of limited disc space, only forty input files and four output files are presently permitted. Each input and output file is large enough to accommodate all the data corresponding to the largest problem allowable. Each night the discs are updated and the jobs which are expected to be run the next day are made resident. If, during the day, a job must be run which is not on the disc, a utility program is used to update a file on the disc. Work in progress at the console is not interrupted, because the utility program is temporarily stored in the compute partition. An output file contains all computed results for a given problem. The operator at the console can request that this information be printed if he wishes to retain hard copy, or he can request that certain portions of the data be plotted. These plots will correspond to the displays that were on the scope and are made with a CALCOMP 835 plotter. The hard copy is available the following day. If more than four output files are needed, one of the output files must first be transferred to tape.

A priority system for interrupts gives the computer operator primary control. The scope and compute partitions receive second and third priorities. The relatively slow response of the user at the display console permits the compute partition to function efficiently despite its low priority. When the compute partition is idle, the OS/360 permits the operator to schedule background ''batch'' jobs.

At present, the MCG structures program is limited to two hundred node points. Three types of structural elements are incorporated into the program: tapered axial elements, tapered beam elements, and a uniform shear panel. The term ''tapered,'' as used here, implies a linear stiffness distribution over the element. As in the prototype program, the axial elements are schematically represented with single lines, and a beam element with a double line. The shear panels are repesented with an X, Figure 6. In addition to the restraints used in the prototype program (pins, horizontal and vertical rollers, and walls), a shell support boundary condition was incorporated into the production program. This support condition is used to approximate the shear flows in a shell structure. The symbol "S", Figure 6, indicates a shell support. When the analyst uses this type restraint, a shear panel and axial element is attached to the model but in a plane perpendicular to the network on the scope. A length, panel thickness, and effective area must be specified for the shell support.

Two types of externally applied loads are permitted. Forces and moments can be applied at the node points, and uniform transverse and shearing loads can be applied along the members. The node point forces are represented by small vectors at the node points, and the distributed loads are represented as vectors perpendicular or parallel to the element, depending on whether the applied load represents a pressure or shear distribution. Up to twenty loading conditions can be solved at one time.

Data can be input either from cards or from the scope. For large problems data is usually submitted on cards and stored on an input file on the disc. The operator at the console must first specify an identification number and lock-code for each problem. If input data is already on the disc, the problem is immediately displayed on the scope. All data entered from cards is checked for errors. An error list is displayed for each problem, and if necessary, prompting messages instruct the operator at the console to correct the data. A status panel on the scope provides the means for entering the input, calculation, or output phase of the program. During the input phase any modification to the structural model is permitted. Nodes, structural elements, restraints, and applied loads can be modified or deleted. Prompting messages displayed on the scope assist the operator in selecting the proper light pen controls. After the stress calculations are completed, the operator can display the relative magnitude plots for axial load, bending moment and shear, and also any of the alpha-numeric data associated with these plots. It is also possible to display a single structural element and examine the primary loads and stresses on that element alone.

EXAMPLE PROBLEM

Program control at the console can be further explained in conjunction with a problem recently solved with the computer graphic system. It was decided that an aircraft be fitted with a spin

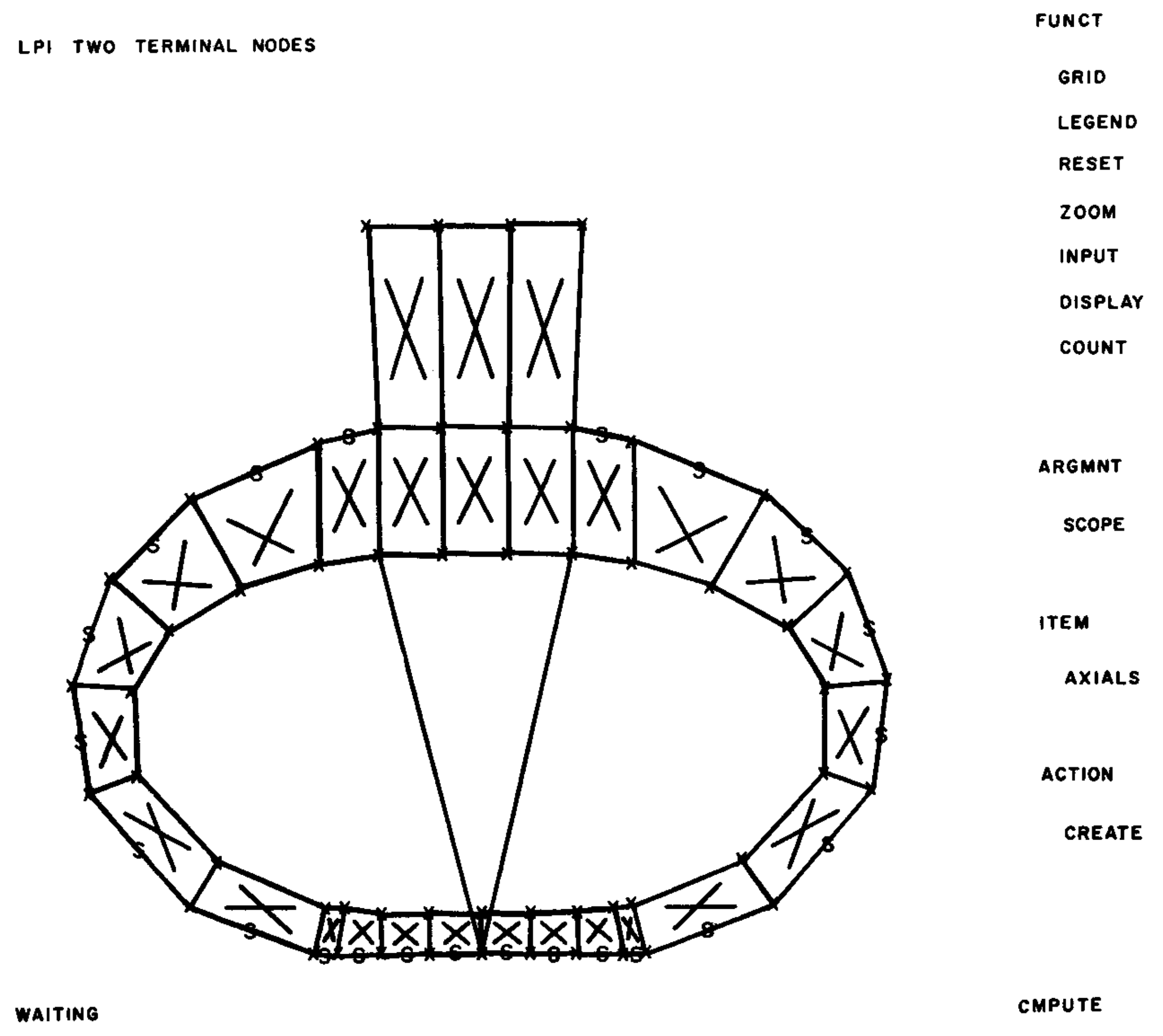


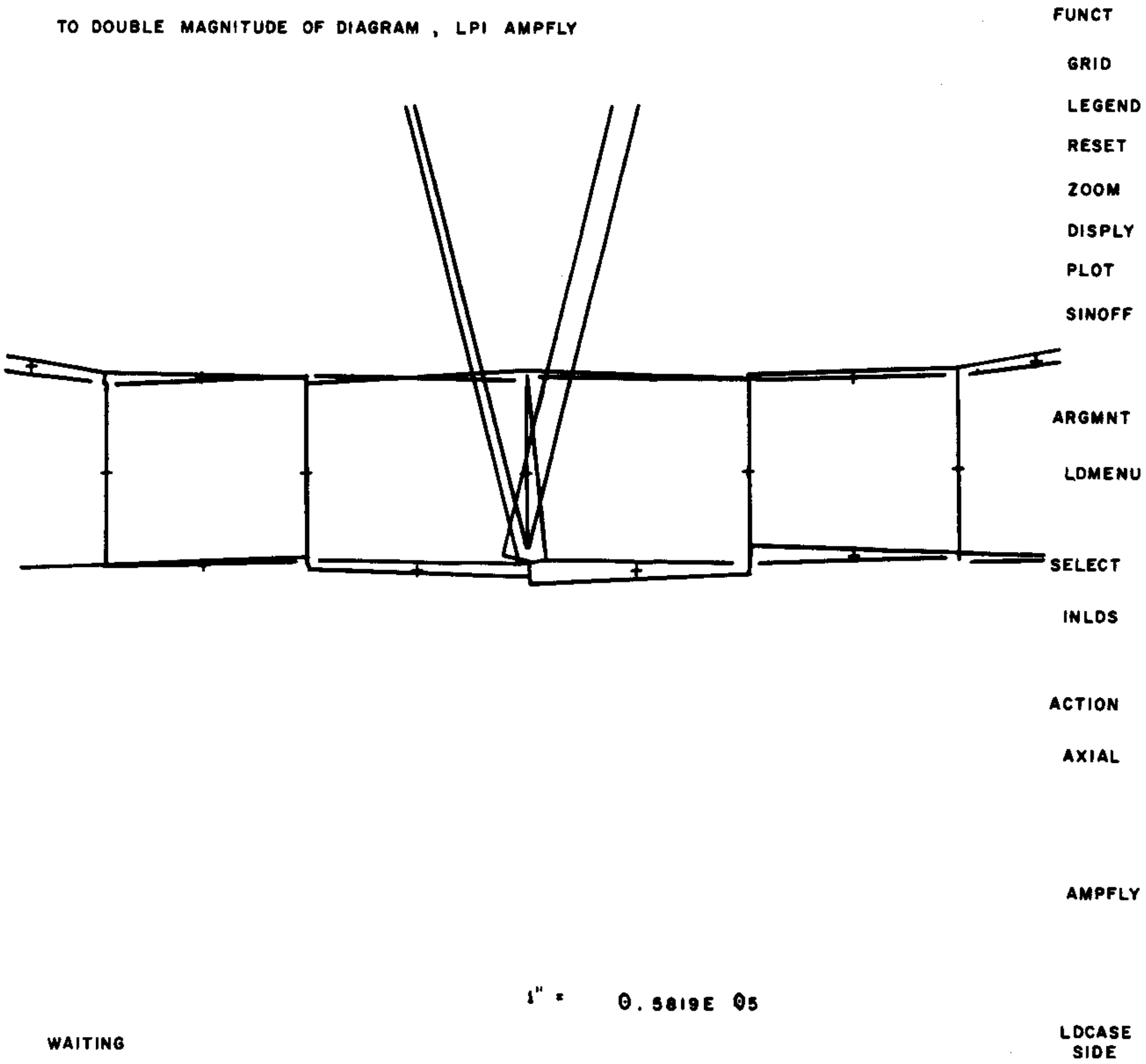
Figure 6. Structural model of typical fuselage frame displayed on IBM 2250 Model 3 display unit.

parachute. A truss structure was attached to one of the fuselage frames and a "V" type brace was installed in the plane of the ring frame to support the truss loads, Figure 6. As standard procedure, the structural arrangement was checked prior to approval of the installation. At this time the question was raised as to whether the added structure could withstand the normally induced loads from other critical flight conditions. Previously, the production frame without the V-brace had been analyzed for critical loading conditions with the computer graphic system, and it was easily redisplayed on the scope. The V-brace was then added to the structural model. As the frame was examined, it became evident that an asymmetrical loading condition on the tail could cause not only the truss structure to fail but also the frame itself, Figure 7. Within several hours, stress and design engineers examined at least six different configurations. The types of proposals investigated varied from modifications in the attachment position of the truss to changes in the depth of the floor beam. The ultimate solution was the removal of the V-brace, Figure 8, and the modification of portions of the outer cap areas of the frame along with some shear panels and vertical stiffeners, Figure 9. Even though normal analysis procedures would have pointed out the design deficiency in the spin chute frame, the value of the graphic system for error detection and rapid redesign was clearly demonstrated.

Figure 6 shows the fuselage frame with the V-brace added to the model. The model consists of axial elements and shear panels. The outer caps of the frame are shown with the shear panel supports. The V-brace was added to the frame by selecting with the light pen the words: INPUT, SCOPE, ELMNTS, AXIALS, CREATE. Prompting messages then requested the operator to select two node points to define the position of the new structural element and to specify the section properties. The members appeared on the structure automatically once this action was completed.

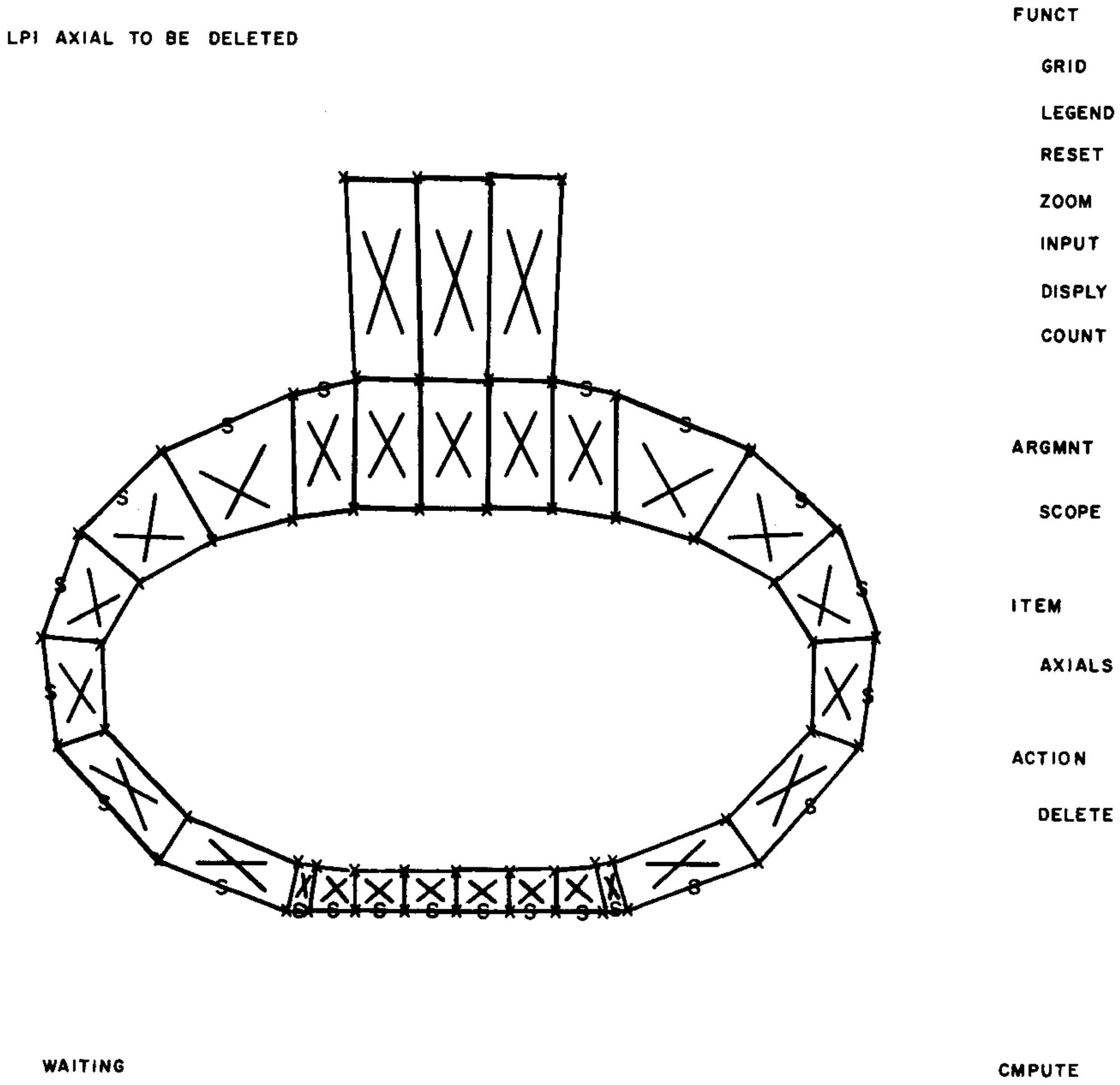
Figure 7 is a display of the relative axial loads in the frame. This display is achieved by selecting: DISPLAY, LDMENU (the names of the various loading conditions are displayed and the operator must choose a particular one, INLDS (infers the operator wishes to examine the general distribution of internal loads as opposed to detailed stresses). Any portion of the frame can be enlarged by selecting the word ZOOM and entering a viewing circle center and radius. The illustration indicates a compressive load in the V-brace which would have caused a column failure.

Figure 8 shows the frame after the removal of the V-brace. These elements were removed by selecting: INPUT, SCOPE, ELMNTS, AXIALS, DELETE, and pointing the light pen to the elements in question.



TO DOUBLE MAGNITUDE OF DIAGRAM , LPI AMPFLY

Figure 7. A display of the relative axial loads in the lower portion of the fuselage frame.



LPI AXIAL TO BE DELETED

WAITING

Figure 8. Structural model after removal of the V-brace with the light pen.

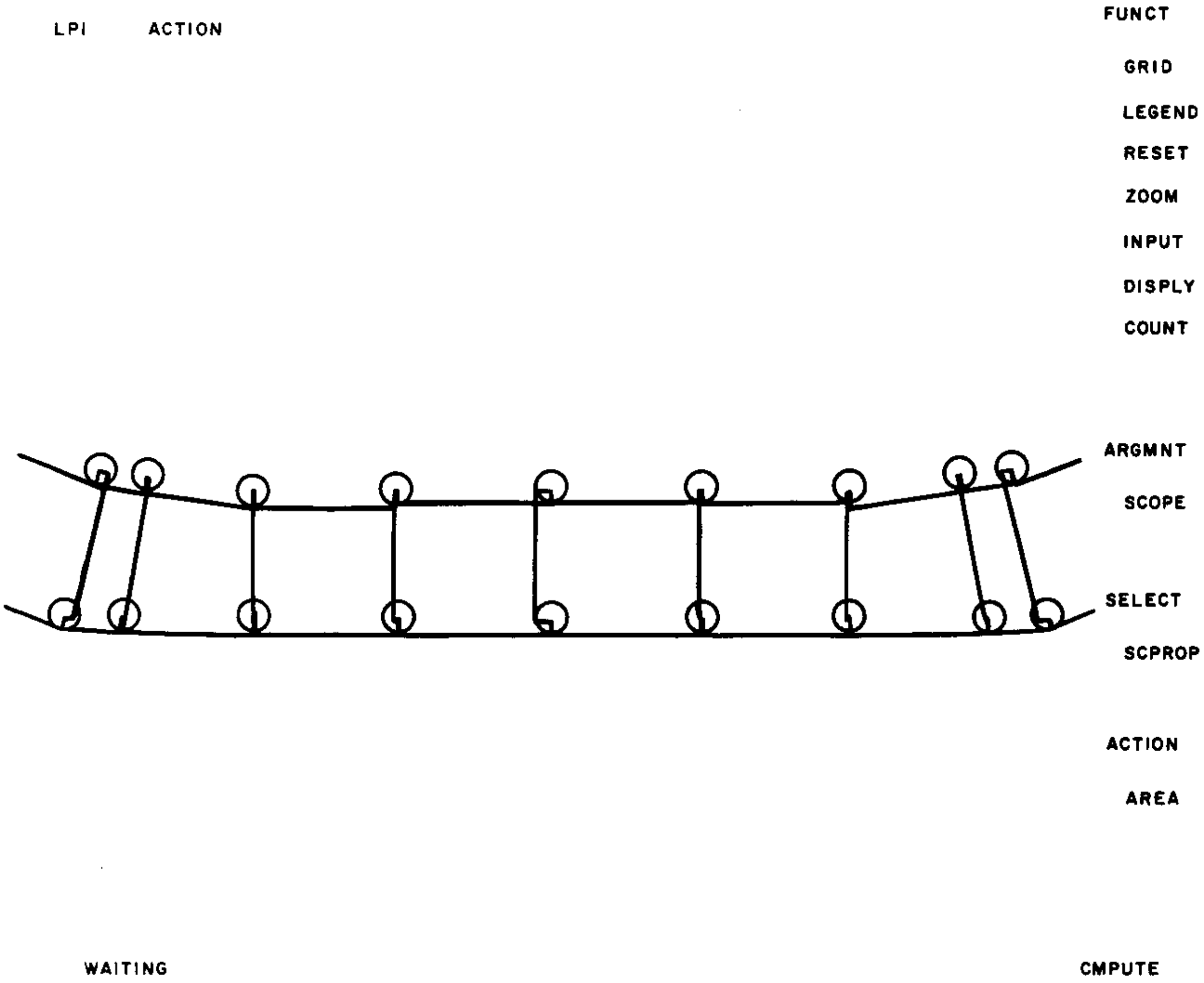
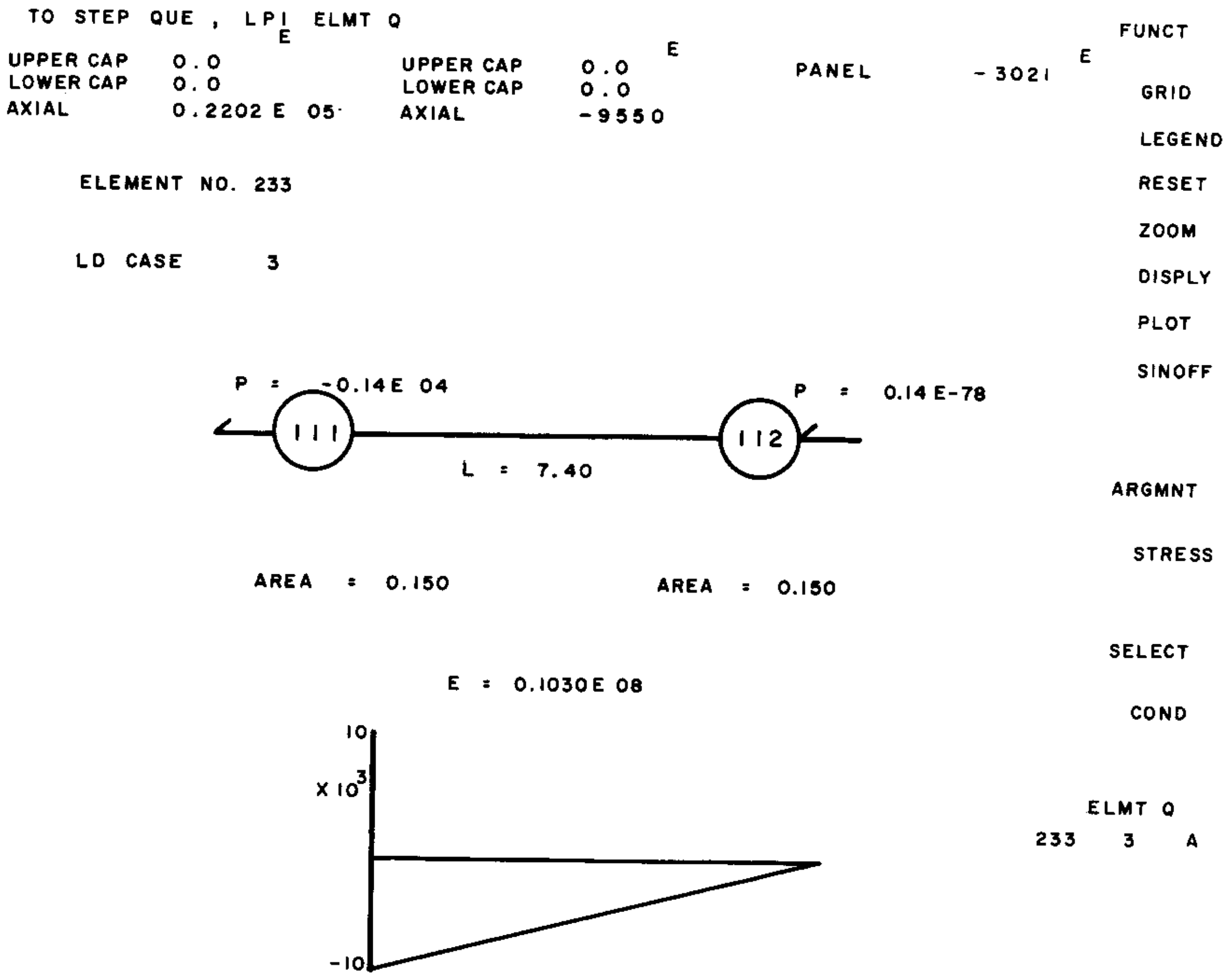


Figure 9. A plot of the cap areas in the lower portion of the fuselage frame. The circles indicate node points.

Figure 9 is a magnified view of the lower portion of the frame. Plotted on the scope are the relative magnitudes of the cap areas. The circles in this figure are node points. No structural elements are displayed with the area plots in order to clarify the picture. The operator can display, at will, any combination of quantities. For example, just node points and shear panels. The area plots were obtained by selecting: DISPLAY, SCOPE, SCPROP, AREA. These are the cap and stiffener areas as modified.

The program also permits the operator to specify an allowable stress. Structural elements exceeding the allowable stress levels are marked by arrows, and these elements can be displayed separately. Figure 10 shows a single axial element displayed on the scope with the primary loads and stresses indicated. Figure 11 shows a single beam element where the combined stresses are plotted at both ends, at the quarter points, and at the mid-point of the element. If more than one loading condition was analyzed, a stress envelop can be displayed, and the corresponding critical loading conditions are indicated.



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 TO  STEP  QUE  ,  LPI  ELMT  Q                                              FUNCT
                   E
                                                  E                        E
UPPER CAP     0.0              UPPER CAP     0.0          PANEL      -3021
LOWER CAP     0.0              LOWER CAP     0.0                                GRID
AXIAL         0.2202 E  05     AXIAL        -9550
                                                                                LEGEND
     ELEMENT  NO.  233
                                                                                RESET

                                                                                ZOOM
     LD  CASE        3
                                                                                DISPLY

                                                                                PLOT

                                                                                SINOFF

                                                                              ARGMNT

                                                                                STRESS
           AREA  =  0.150               AREA  =  0.150


                                                                              SELECT
                      E  =  0.1030E 08
                                                                               COND



                                                                            ELMT  Q
                                                                       233     3     A
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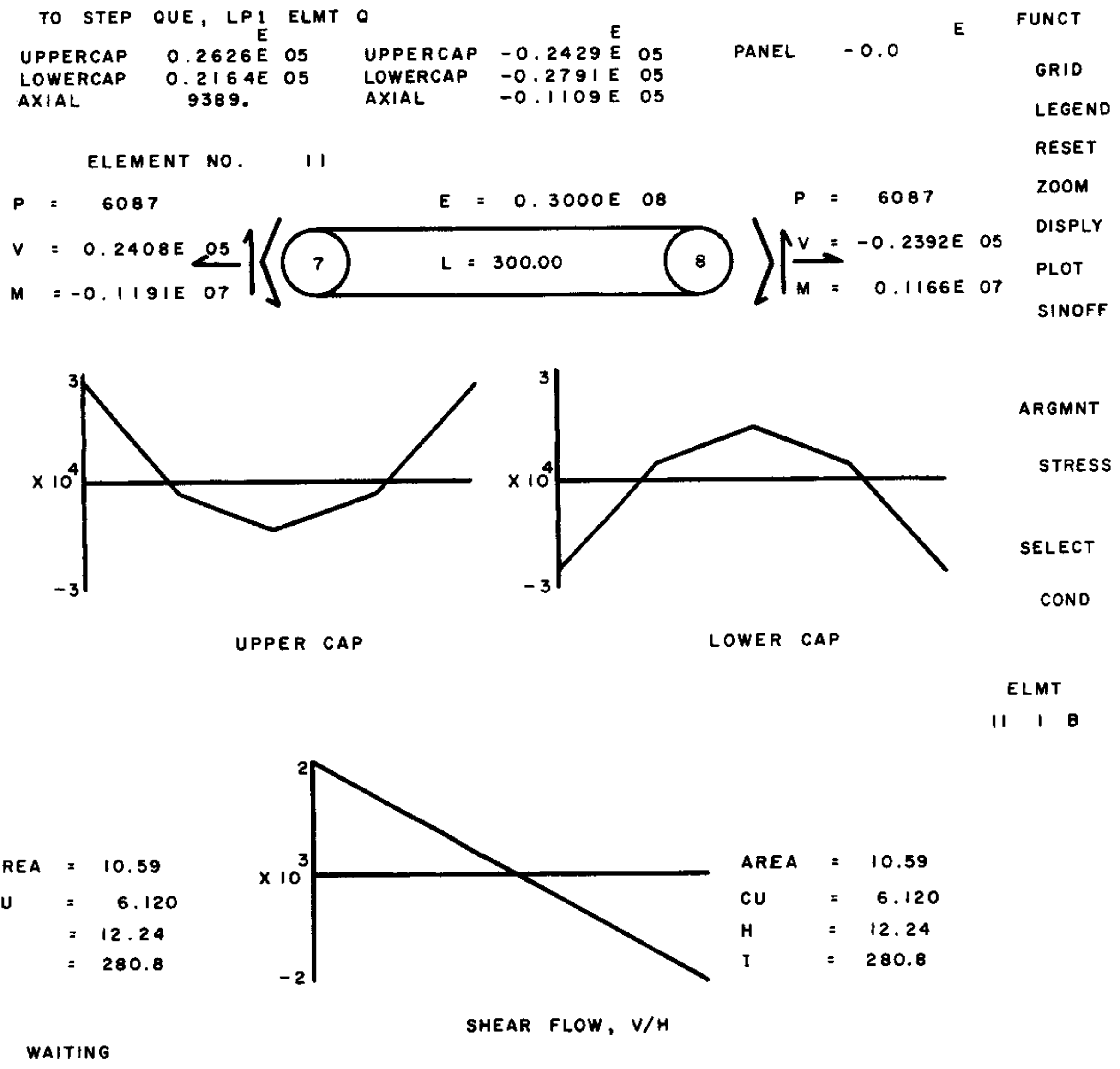
Figure 10. Output display for a single axial element. The primary load, stress, and section properties are indicated.



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TO  STEP  QUE,  LP1  ELMT  Q
                       E                            E                            E
UPPERCAP     0.2626E 05     UPPERCAP   -0.2429 E 05     PANEL      -0.0
LOWERCAP     0.2164E 05     LOWERCAP   -0.2791 E 05
AXIAL          9389.        AXIAL      -0.1109 E 05

        ELEMENT NO.     11

P  =    6087                         E  =  0.3000E 08              P  =   6087

V  =  0.2408E 05                                                   V  =  -0.2392E 05
                                     L  =  300.00
M  =-0.1191E 07                                                    M  =   0.1166E 07

AREA  =  10.59                                                     AREA  =  10.59
CU    =   6.120                                                    CU    =   6.120
H     =  12.24                                                     H     =  12.24
I     =  280.8                                                     I     =  280.8

WAITING
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Figure 11. Output display for a single beam element. Stresses in the upper and lower caps are indicated for various loading conditions.

SECTION IV

THREE-DIMENSIONAL STRUCTURAL ANALYSIS

An analysis program is currently under development in conjunction with research in the interactive analysis of large three-dimensional structures. The analysis portion of the program is operational on the Control Data Corporation (CDC) 3300 computer, and in the near future, the program will be operational on the Univac 1108. The goal of this program is to make it feasible for an engineer to analyze structures with as many as 8000 node points.

In the initial version two types of structural elements are permitted: an axial element and a shear panel. However, the program is flexible enough to allow any type of element to be added later without changing the existing program. There is practically no restriction imposed on the number of node points or the number of elements in the structural model. The only restriction is determined by the structure's topology, which governs the bandwidth of the stiffness matrix.

The numbering system for the degrees of freedom is related to the way the node points are numbered. Various features have been planned to insure a reasonable numbering system. The analyst can use an option which allows him to renumber any of the original node points. This is particularly helpful if new node points are added to the structural model. In the final version of the program, it is planned to provide an additional option whereby the analyst can input the node points in any order and direct the computer to do the numbering for him.

BASIC FORMULATION

Both the 2-D and 3-D analysis programs are based on the direct stiffness method. In matrix form, the uncoupled force-displacement equations for the composite structure can be written as

$$\left\{ P \right\} = \left[k \right] \left\{ p \right\} \tag{2}$$

From compatibility considerations

$$\left\{ p \right\} = \left[\psi \right] \left\{ q \right\} \tag{3}$$

where $\{q\}$ is a vector of node point displacements and $[\psi]$ is a matrix based on the topology of the network. The force-displacement relation for the coupled structure is then

$$\{Q\} = [\psi' k \psi] \{q\} \tag{4}$$

Equation (4) can be partitioned as follows:

$$\begin{bmatrix} Q_a \\ Q_b \end{bmatrix} = \begin{bmatrix} K_{aa} & K_{ab} \\ K_{ba} & K_{bb} \end{bmatrix} \begin{bmatrix} q_a \\ q_b \end{bmatrix} \tag{5}$$

where $\{Q_a\}$ and $\{q_a\}$ are the applied loads and the corresponding displacements. $\{Q_b\}$ and $\{q_b\}$ represent reaction forces and known displacements. The equations to be solved are then

$$\{Q_a - K_{ab} q_b\} = [K_{aa}] \{q_a\} \tag{6}$$

In terms of the node point displacements, the internal loads are

$$\{P\} = [k \ \psi] \{q\} \tag{7}$$

The reactions and equilibrium checks can be computed from Equation (8):

$$\{Q\} = [\psi'] \{P\} \tag{8}$$

The stiffness matrices used for the tapered axial and beam elements are as follows:

Tapered Axial Elements - For an axial element with a linear (EA) variation and the following sign convention

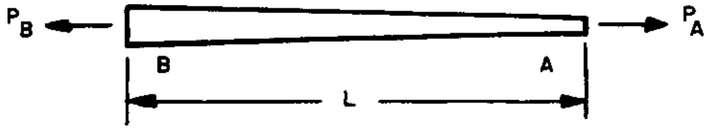

Figure 12

the stiffness matrix is

$$k = \frac{(EA)_B}{L} \left(\frac{r-1}{\ell n r} \right) \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

where

$$r = \frac{(EA)_A}{(EA)_B}$$

Tapered Beam Element - For a beam element with a linear variation in the moment of inertia and the following sign convention

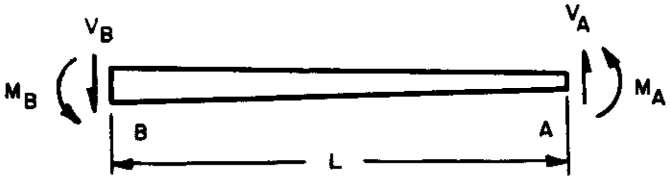

Figure 13

the stiffness matrix is

$$\mathbf{k} = \frac{EI_A}{LD}\begin{bmatrix} c_{22} & & \text{Symmetric} & \\ -c_{21} & c_{11} & & \\ -c_{22} + Lc_{21} & c_{12} - Lc_{11} & c_{22} - L(c_{12} + c_{21}) + L^2 c_{11} & \\ -c_{21} & c_{11} & c_{21} - Lc_{11} & c_{11} \end{bmatrix}$$

where

$$c_{11} = \frac{\ln r}{r-1}$$

$$c_{12} = \frac{L}{(r-1)^2}(1 - r + r\ln r) = c_{21}$$

$$c_{22} = \frac{L^2}{(r-1)^3}\left(r^2 \ln r - \frac{3}{2} r^2 + 2r - \frac{1}{2}\right)$$

$$D = c_{11}\, c_{22} - c_{12}\, c_{21}$$

PROGRAM ORGANIZATION

The problems involved in finding the displacements of a large structure are concentrated in two areas: (i) the formation of the stiffness matrix and (ii) the solution of the resulting system of linear equations. In both of these areas, the basic difficulty is the same; namely, to find an efficient way of handling large quantities of data.

The technique used in the formation of the stiffness matrix is to subdivide the given structure, if necessary, into overlapping substructures in such a way that all the data needed for a particular substructure fits inside the available core. To accomplish this, all input is brought into core, subdivided, and stored on a random access device. The renumbering of node points, if desired, is done simultaneously. Then, the data for each substructure are brought back into core. While it is being input, all the necessary connection and degree of freedom tables are set up. When this operation is complete, the node points are taken in sequence and the corresponding portion of the stiffness matrix is formed and stored on a peripheral device.

Throughout the analysis program, the basic philosophy is to overlap, as nearly as possible, computations with input and output phases. That is, calculations are sandwiched between readings of bufferfuls of data in order to utilize at least part of the time taken by the hardware to locate the next buffer. Even though it makes the organization of the program slightly more complicated, it is well worth the total time saved.

SOLUTION OF BANDED LINEAR SYSTEMS

The method presently employed for solving the linear equations is designed for a banded stiffness matrix. The stiffness matrix, $\mathbf{K}$, is symmetric and positive definite, and its decomposition can be achieved as the product of two triangular matrices. Only the lower half of the matrix need be available. The band form of $\mathbf{K}$ also enables one to restruct the decomposition process to elements within the band, since all elements external to the band remain zero throughout the reduction process. From an organizational point of view, the square root method was found to have a slight advantage over the Cholesky method. The square root method for band matrices can be stated by the following theorem: If $\mathbf{K}$ is a positive definite, symmetric matrix with a semi-band width of m, then there exists a lower triangular matrix $\mathbf{T}$, also with a semi-band width m, such that,

$$\mathbf{K} = \mathbf{T}\mathbf{T}' \tag{9}$$

The matrix $\mathbf{K}$ is said to be of band form, and m is the semi-band width, if $\mathbf{K}_{ij} = 0$ wherever $|i-j| \geq m$.

The elements of $\mathbf{T}$ can be readily computed by equating the two sides in equation (9). For $i \geq j$,

$$K_{ij} = \sum_{\ell=j_1}^{j} t_{i\ell} \cdot t_{j\ell} \tag{10}$$

where

$$j_1 = \max(j - m + 1, 1)$$

Thus,

$$t_{ii} = \left[K_{ii} - \sum_{\ell=i_1}^{i-1} t_{i\ell}^{2} \right]^{1/2} \tag{11}$$

$$t_{ij} = \left[K_{ij} - \sum_{\ell=j_1}^{j-1} t_{i\ell}\, t_{j\ell} \right] / t_{jj} \tag{12}$$

where

$$i = 1, 2, \cdots n$$
$$j = i_1, i_1 + 1 \cdots, i-1$$

and

$$i_1 = \max(i - m + 1, 1)$$

For large problems, the matrix **K** cannot be stored in core, and peripheral devices are needed to achieve its decomposition. The outstanding characteristic of band matrix decomposition is that the process may be completed in a single sweep, regardless of the order of the matrix, provided the bandwidth is sufficiently small. This results from the fact that the decomposition process for a particular row requires only the elements which lie directly above it, and, because of symmetry, below the main diagonal.

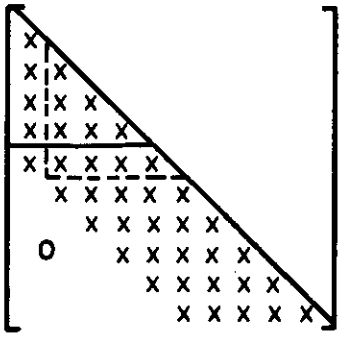

Figure 14

For a band matrix, the elements lie inside a triangle, Figure 14, which contains at most m(m-1)/2 elements, where m is the semibandwidth as defined above. Thus, if the triangle of elements (plus a row which is to be reduced using these elements) can be stored in core, decomposition can proceed as follows: (i) read a row of the matrix core, (ii) reduce this row using the elements of the triangle already stored in core, and (iii) update this triangle to correspond to the next row.

Updating of the stored triangle and subsequent index calculations may adversely affect execution time unless the elements of the triangle are carefully arranged. This organization is the principal feature of the algorithm presently being used. As each row is decomposed, updating is achieved by storing the reduced elements in predetermined locations.

As a result, the elements of the next triangle are already properly arranged, and decomposition of the next row can begin immediately. The resulting index calculation using this mode of storage is no more complicated than the one in equations 11 and 12. Because of the manner in which the algorithm makes use of each extra core location available, it is desirable to have the array containing the triangle slightly larger than the maximal size of the triangle. Even though this requirement is not essential for the success of the algorithm, it is usually met. The requirement results from the necessity of having to shift elements in core. Seldom is this necessary; for example, if 30,000 locations are assigned for the matrix and the semi-bandwidth is 200; 10,100 rows can be processed without shifting elements.

CALCULATION OF DISPLACEMENT VECTORS

To calculate the displacement vectors, the available core is frequently filled with as many load vectors as possible. Then, two sweeps through the triangular matrix $\mathbf{T}$, one forward and the other backward, are made to yield the solution. Thus, if 30 K of core is available, and $N = 5000$, six load vectors may be processed at one time. However, it can be shown that if the load matrix $\mathbf{Q}$ is stored in rows instead of columns, then the number of load vectors handled at one time can be made independent of the order of the matrix. This is because of the fact that during both forward and backward sweeps, only m rows of the load matrix need to be stored at one time. Thus, with $m = 150$, up to 200 load cases may be handled simultaneously. This process has a further advantage, in that it allows efficient use of overlap between the I/O and the computational phases of the program.

TIME HISTORY STUDIES

The present method should be used only when $m << n$, and in this case approximately $(1/2)n(m^2+7m+2)$ multiplications are necessary to obtain the solution. Thus, the computational time varies linearly with respect to order of the matrix and quadratically with respect to its bandwidth.

The following graph displays the relationship between the computational time and the bandwidth. The computational time includes the I/O and the calculation times which are, of course, overlapped. It is further based on the assumption that the coefficient matrix does not fit in core even in cases where it actually can.

The coefficient matrices were generated using test matrices from reference 7. The matrices were fully packed with nonzero elements inside the band. In practice however, the matrix is very sparse inside the band which substantially reduces the number of multiplications necessary. The test runs were made on the Univac 1108 computer with 65K of core memory. All I/O was done on the Fastrand drum with maximum transfer rate of 156K characters per second and maximum access time of 70 milliseconds.

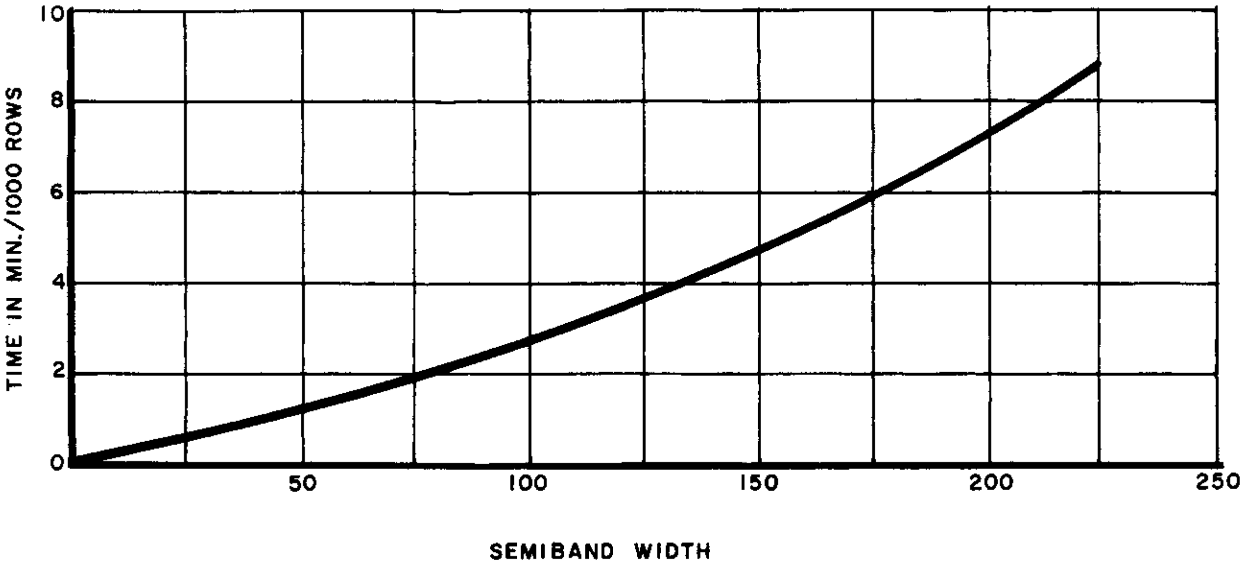

Figure 15

SECTION V

FUTURE DEVELOPMENTS

Programs for automatically determining the physical properties of large structural networks, with respect to a prescribed set of loading conditions, are at times impractical. Use of the computer graphic system for studying stiffness variations is therefore justified. However, advancements in optimization techniques may well preclude this necessity.

With respect to studying geometric arrangements the computer graphic system is outstanding, and it will probably be employed in this capacity for many years. For example, consider the following research program which is directed toward the rapid definition of three-dimensional structural networks.

A wide variety of basic substructures or entities are stored on a data file. These entities may be frames, wing ribs, or groups of shell elements. Each particular substructure is defined by a series of parameters that permit rapid modification, if necessary, of an individual substructure.

Early in the design phase, the chief structural engineer may request that several configurations be examined. An analyst, using the computer graphic system, then assembles substructures into the desired structural arrangements. In addition to rapidly defining the proper arrangements, the substructures are defined in a manner consistent with the constraints imposed by the loft definition of the aircraft.

To illustrate the concept, consider the prototype system developed on the Lockheed-Georgia Control Data Corporation (CDC) 3300 computer graphic system by T. E. Dasher. Figure 16 illustrates one of several basic substructures stored on the disc. This particular substructure, composed only of axial and shear panel elements, represents a fuselage frame with shear panels normal to the frame. In the prototype program, each substructure has its own local coordinate system defined within an overall hierarchical coordinate system definition. This allows the coordinate system for any substructure to be located with respect to any other defined coordinate system. It is, therefore, possible to recall an entity or substructure several times and attach one to another in a matter of seconds. The operator simply indicates the parent coordinate system, and the relative translation and rotation for the new coordinate system. It is also possible to view the assembly of structural elements from any previously defined coordinate system.

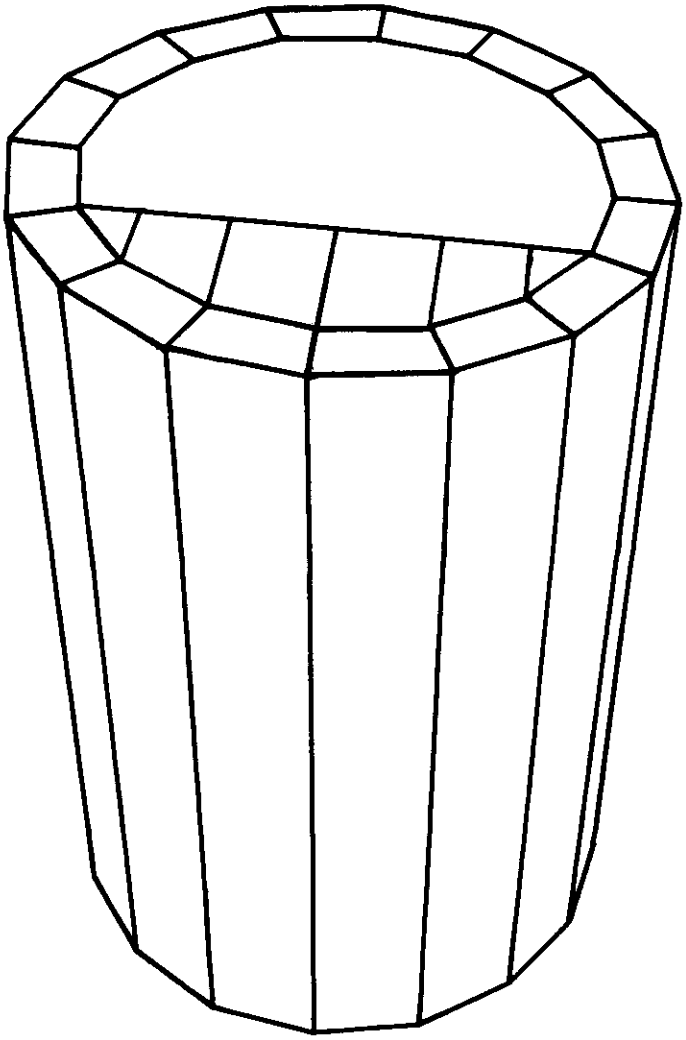

Figure 16. Typical aircraft fuselage substructure displayed on the CDC 273 Display Console.

Figure 17 illustrates two of the fuselage frame substructures of Figure 16 "locked together". A view from four separate vantage points is illustrated. Figure 18 is an anterior "close-up" of the fuselage ring frame. (The basic geometric definition of the entity is stored in the computer only once.)

Figure 19 illustrates another basic substructure, namely a wing rib with shear panel attachments. Figure 20 shows a wing box idealization rapidly defined by connecting ten wing rib substructures. Each wing rib section contains sixteen axial elements (not shown). An aft view of the wing box would show all the intermediate wing ribs.

The sophistication of the prototype program can be ascertained by noting that the hidden lines are automatically deleted from the display, and that the display is presented as a one point perspective drawing. A feature for dynamically viewing the model will be incorporated in a more complex program.

Various surface definition methods are being examined for lofting an aircraft using the computer graphic concept. Figure 21 illustrates loft cuts of a fuselage cross section generated with the aircraft preliminary design program described in reference (1). One of the goals of this program is to exploit the computer graphic system for creating, displaying, and manipulating the loft contours. Eventually, this type of geometric data will form the basis for defining the node points for each substructure.

The three-dimensional static analysis program will also be extended to include the analysis and displays for observing the dynamic response of the structure.

The two greatest research needs at the present time (based on the authors' experience and judgment) are to develop systems which enable an engineer to create his own interactive graphics programs; and to define a common data base for all specialty areas. Past objectives have been to define graphic programs that can stand alone and at the same time be economical. In the future, much emphasis will be placed on the integrated data base approach.

There is little doubt that computer graphics will become increasingly important. However, the development of acceptable interactive programs for the aerospace industry will be a slow and prodigious task.

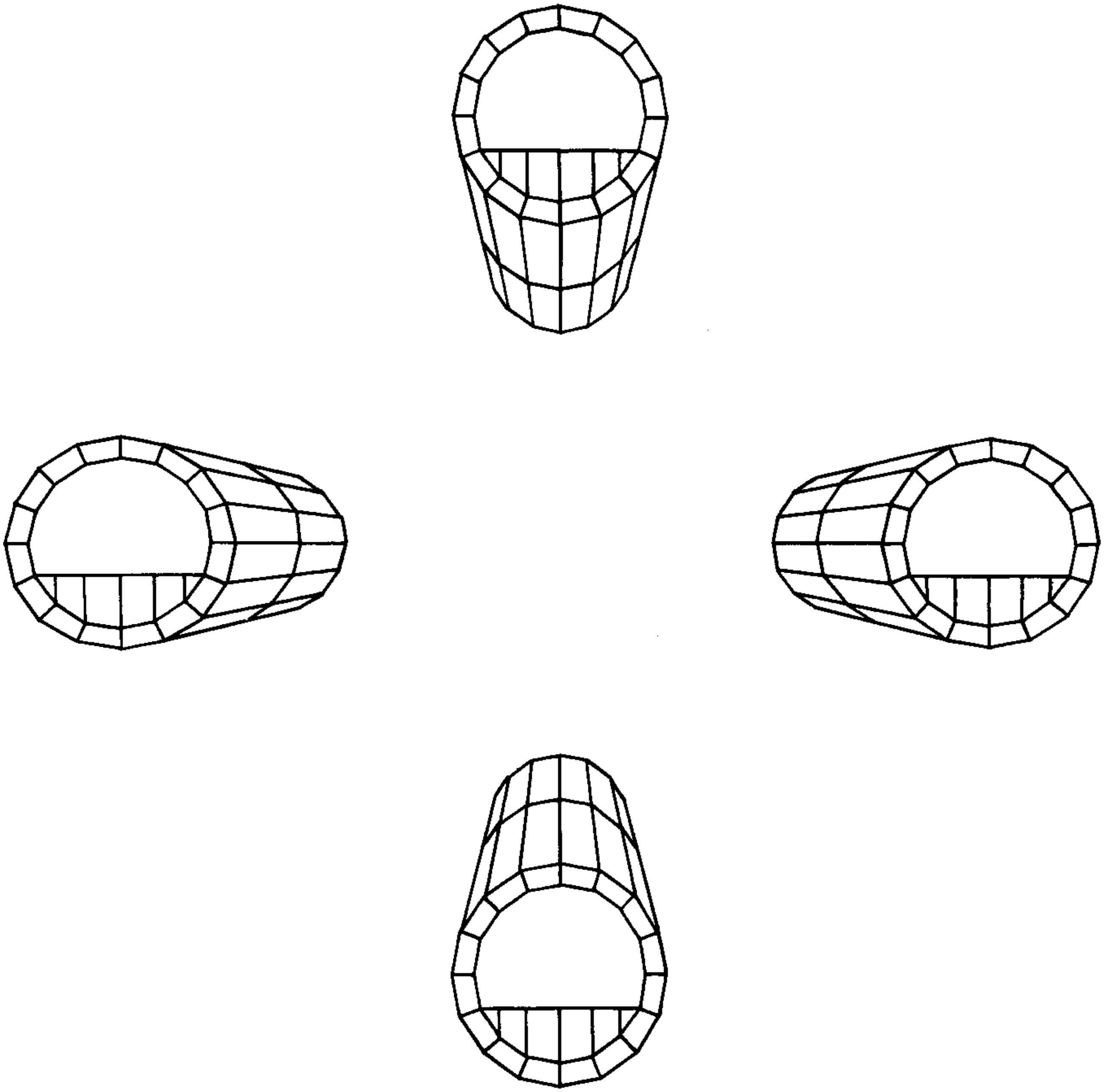

Figure 17. Two fuselage substructures "locked together". Four different views are displayed on the scope.

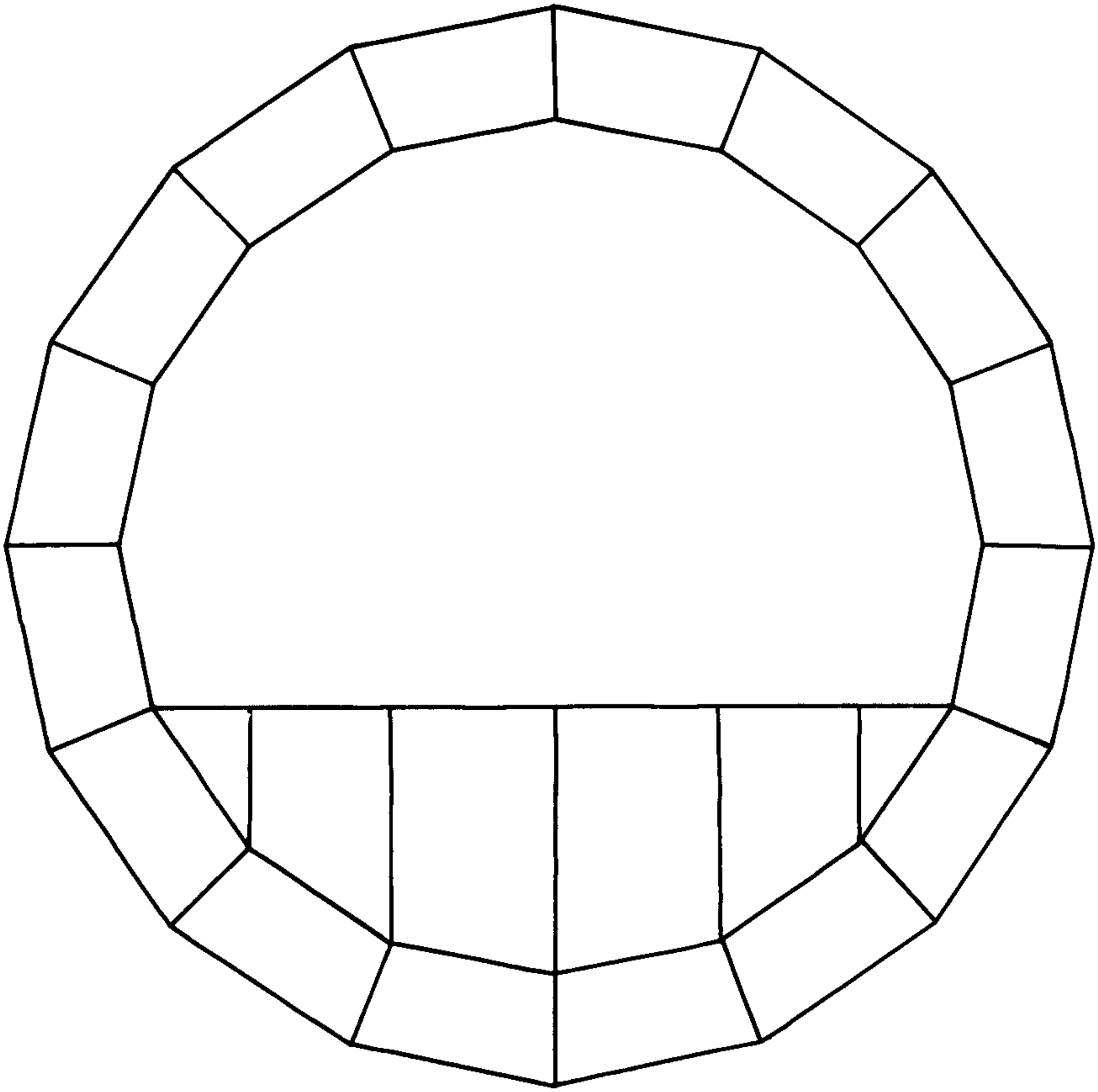

Figure 18. Front view of the fuselage substructure displayed in Figure 16.

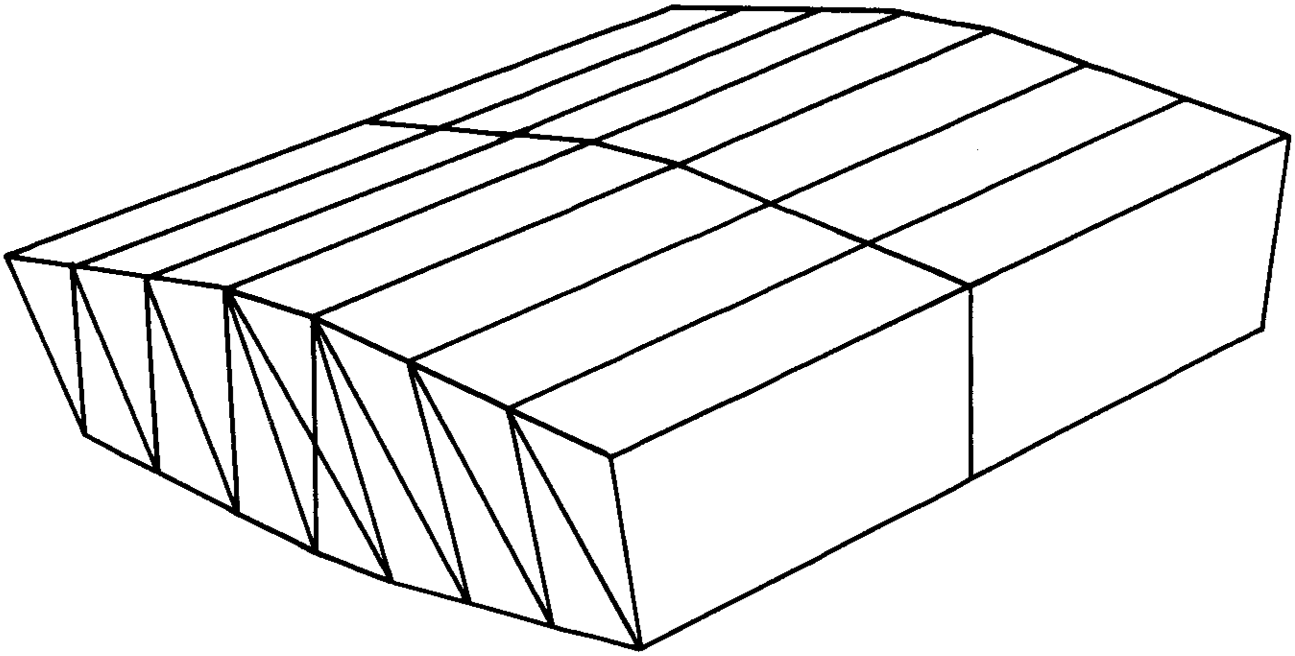

Figure 19. Two typical wing box substructures.
Each includes wing rib with shear panel attachments.

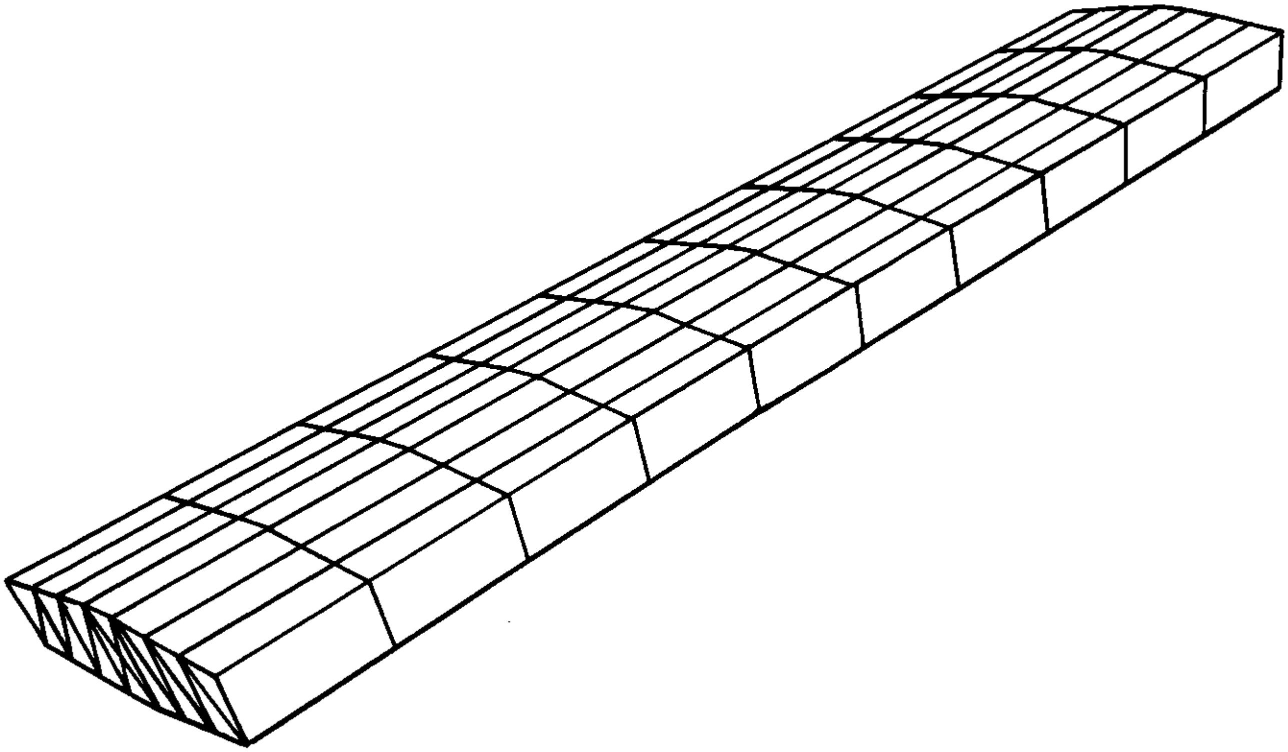

Figure 20. Ten wing box substructures "locked together."

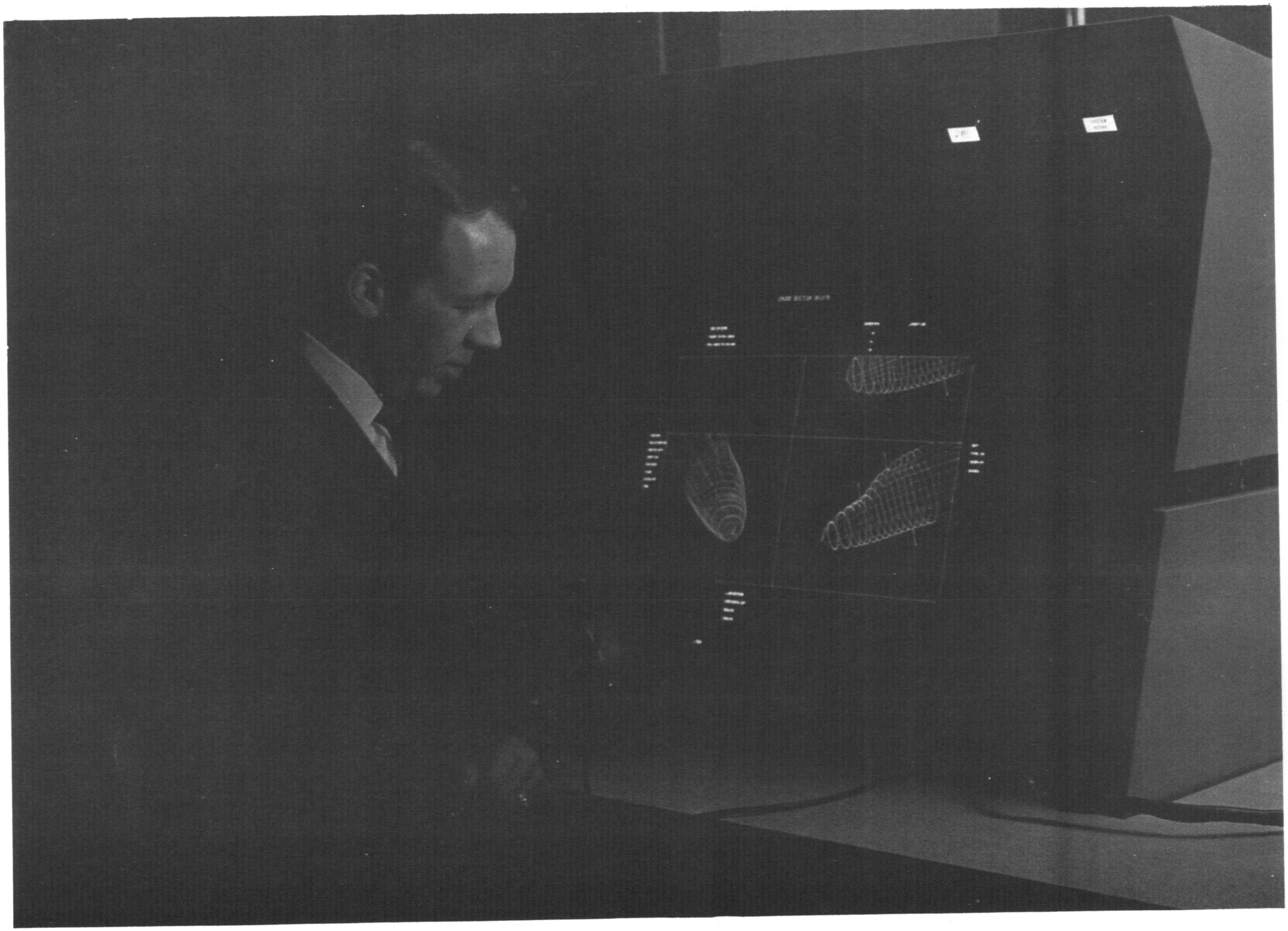

Figure 21. Display of loft cuts generated with the MCG Preliminary Design Program.

SECTION VI

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