EFFECTS OF RISE TIME AND DAMPING ON FINITE ELEMENT ANALYSIS OF RESPONSE OF STRUCTURES

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This paper discusses the results of some sensitivity studies on the effects of rise time and viscous damping on the output of a linear finite element code. The results are interpreted by means of modal analysis of the finite element model. Approximate methods are presented for the determination of the highest modal frequency of a uniform grid and spurious oscillations at the highest modal frequency are correlated to the shock spectrum of the forcing function. The effect of damping on shock propagation is described in modal terms. Finally, the effects of modal participation of the forcing function upon finite element models of continua are mentioned.

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INTRODUCTION

There has been considerable interest in recent years, in the determination of dynamic response of half buried and/or buried structures to air blast and ground shock loading induced by the detonation of nuclear weapons or high explosives. These structures are generally of the thick wall construction as they are "hardened" against the blast and shock environments. The important task of determining the response of such structures is not only in the evaluation of structural integrity under the severe blast and shock environments but also in the understanding of transfer of shock environment from the surrounding medium into the structures as various mechanical, electrical and electronic components are mounted (either hard mounted or shock isolated) to the interior of the structures. Consequently, the response analysis must include both the gross response of the structure and the stress wave propagation in the material. In other words, stress in the thickness direction cannot be ignored.

Various numerical methods (e.g., spring-mass model and finite difference methods) have been used in the past in the study of this problem. The finite element approach has been used, in recent years, in the response analysis of this structure-medium interaction problem. In using the finite element method for analyzing such a problem, two questions were frequently raised:

- What are the criteria for selecting the grid size?
- How should damping be considered?

This paper will attempt to address itself to both of these questions.

References 1) and 2) which refer to spring-mass models, provided the perspective from which the finite element discretization effects were assessed. Reference 3) describes the effects of proportional damping without particular reference to shock propagation.

CONSIDERATION OF RISE TIME IN THE SELECTION OF GRID SIZE

Typically, the blast and shock loading considered in the response analysis, as shown in Figure 1, has a short rise time, usually on the order of a fraction of a millisecond to a few milliseconds. When a shock load of this type is applied to a finite element model of a soil-structure complex as shown in Figure 2, spurious oscillations always occur. Such oscillations are spurious because they do not represent the physical motion of the continuum. Generally, they are caused by the dispersion and the filtering effects of the discrete elements. Such oscillations may be minized selecting the mesh size intelligently. Oscillations in one part of a mesh propagate into other regions as time passes. Consequently, the usual static procedure of placing more elements in regions of stress concentrations is not always the best method of selecting a mesh. This means that the grid of finite elements must be nearly uniform. Also, shock fronts in uniform grids tend to be "smeared out" across several elements. Such a filtering effect of the force pulse seems to indicate that if the rise time is short compared with the transit time across an element, similar response will be obtained when impulses of these pulses are nearly equal.

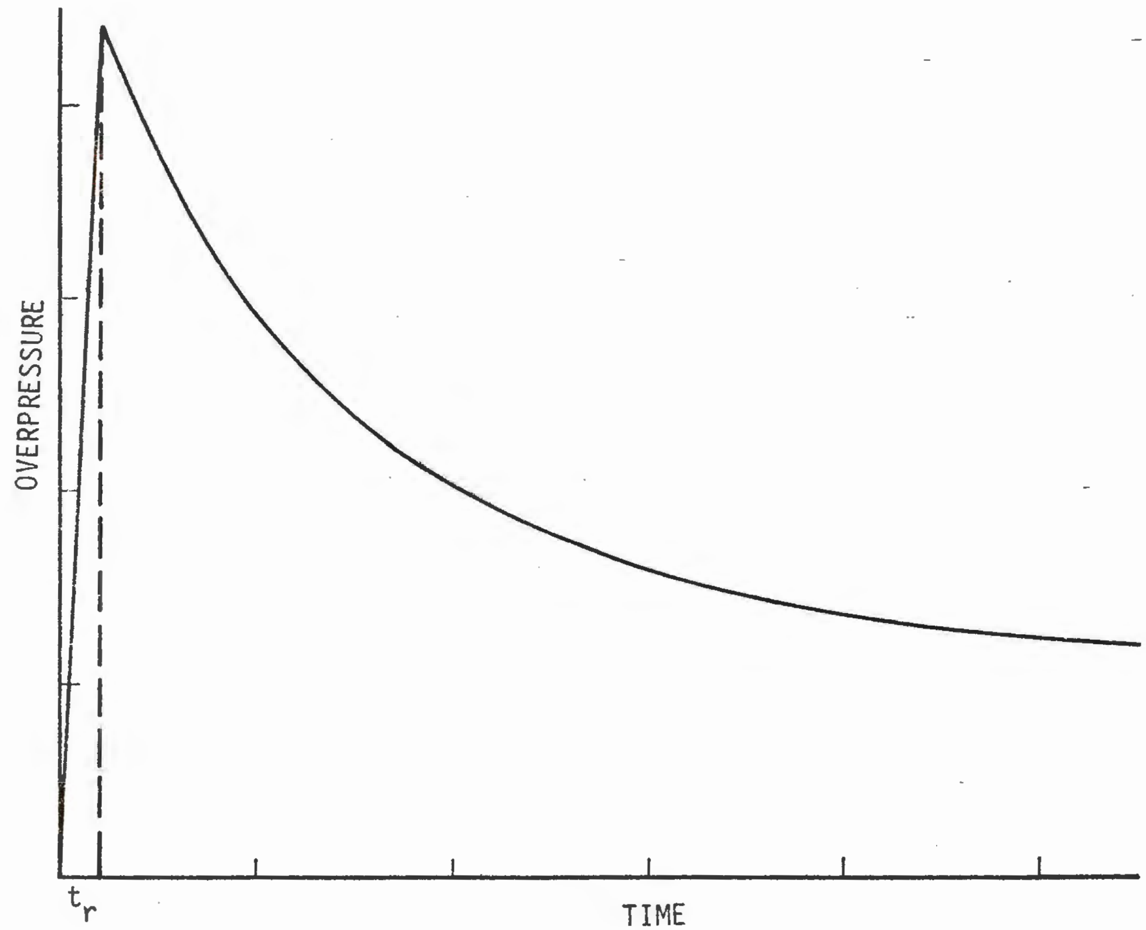

FIGURE 1. TYPICAL TIME HISTORY OF BLAST WAVE

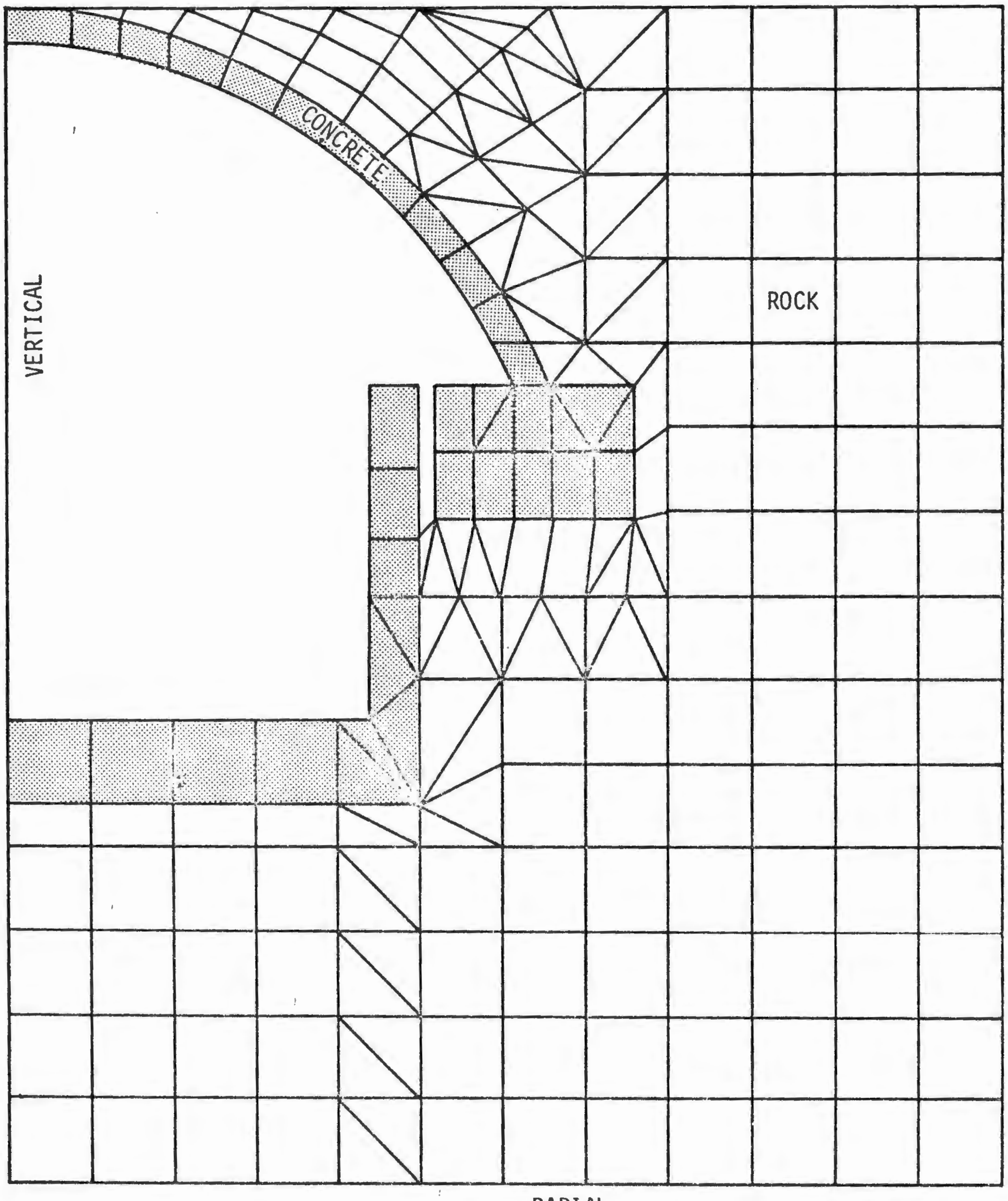

FIGURE 2. TYPICAL FINITE MODEL OF UNDERGROUND STRUCTURE

The frequencies associated with spurious oscillations are the highest modal frequencies of a grid of finite elements. For convenience of discussion, one-dimensional problems are used as examples. For a uniform one-dimensional grid, the mode shape corresponding to the highest frequency is approximately:

$$u = \{ 1 \ -1 \ \ 1 \ -1 \ \ldots\ldots \ 1 \ \ -1 \}$$

The Rayleigh quotient for the one-dimensional system can be computed to be:

$$Q(u) = \frac{\tilde{u}Ku}{\tilde{u}Mu} = \frac{4k}{m} \left(1 - \frac{1}{4n-2}\right)$$

where

k = stiffness of one element, $\frac{1-\nu}{(1+\nu)(1-2\nu)} \frac{EA}{L}$

m = mass at each node, ρAL

n = number of masses in the chains

ρ = density

ν = Poisson's ratio

E = Young's modulus

L = length of an element

A = cross-sectional area

As n becomes large, u becomes a good approximation to the highest mode shape and Q(u) approaches the value ω^2 of the highest mode. Consequently,

$$\omega^2 \simeq 4 \ k/m$$

or

$$\omega = 2 \ c_d/L$$

where c_d is the velocity of the dilatational wave (p-wave)

$$c_d = \sqrt{\frac{E}{\rho} \frac{(1-\nu)}{(1+\nu)(1-2\nu)}}$$

The transit time of the P-wave across an element is given by

$$t_e = L/c_d$$

and the period for the highest mode becomes:

$$\tau = \pi t_e$$

A series of calculations were made to consider a step pressure pulse with a rise time (t_r) varying from 0 to 2π. It was observed that relative minima for spurious oscillations occur approximately at the minima for the shock spectra of the highest frequency mode of a grid of finite elements. A strong correlation between the shock spectrum of a pressure pulse and the extent of the spurious oscillation can then be established. This enables one to "tune" the grid size to the relative minima of the shock spectra of the applied forces. For example, Figure 4 shows shock spectra for the calculation on a one-dimensional finite element grid as shown in Figure 3. It can be seen that if element sizes (t_r/τ) are selected in such a way that relative minima of shock spectra can be obtained the spurious oscillations can be minimized.

For good numerical accuracy, the time step for numerical integration can be taken to be less than or equal to 1/20 of the shortest element transit time. Figures 5, 6, and 7 show the time histories of stress which were computed. These figures illustrate two trends: First overshoot due to discretization is a function of depth; second, depending upon rise time overshoot may be greater at depth than at the surface. One explanation of these effects is that deeper elements participate more strongly in slightly lower frequency modes and hence, may have either higher or lower overshoot depending upon the ratio $\frac{t_r}{\tau}$ where τ is the most strongly represented mode. Another possibility which these figures suggest is that other rise conditions, e.g., versed sine or cycloid could be considered when waveforms other than step functions are to be modelled.

Figure 8 shows velocity waveforms. In general, velocity overshoots are less than stress overshoot.

DAMPING CONSIDERATION

Many of the today's finite element structural and continuum dynamics analyses assume the proportional damping, that is

$$C = \alpha M + \beta K$$

in

$$M\ddot{x} + Cx + Kx = F$$

Using the modal approach, one reduces the equation to

$$\ddot{q}_n + 2\gamma_n q_n + \omega_n^2 q_n = \frac{Q_n}{m_n}$$

where

$$\gamma_n = 0.5 \; (\; \alpha + \beta \; \omega_n^2 \;)$$

This indicates that if $\beta \neq 0$, there is a possibility that low frequency modes might be underdamped while high frequency modes were being overdamped.

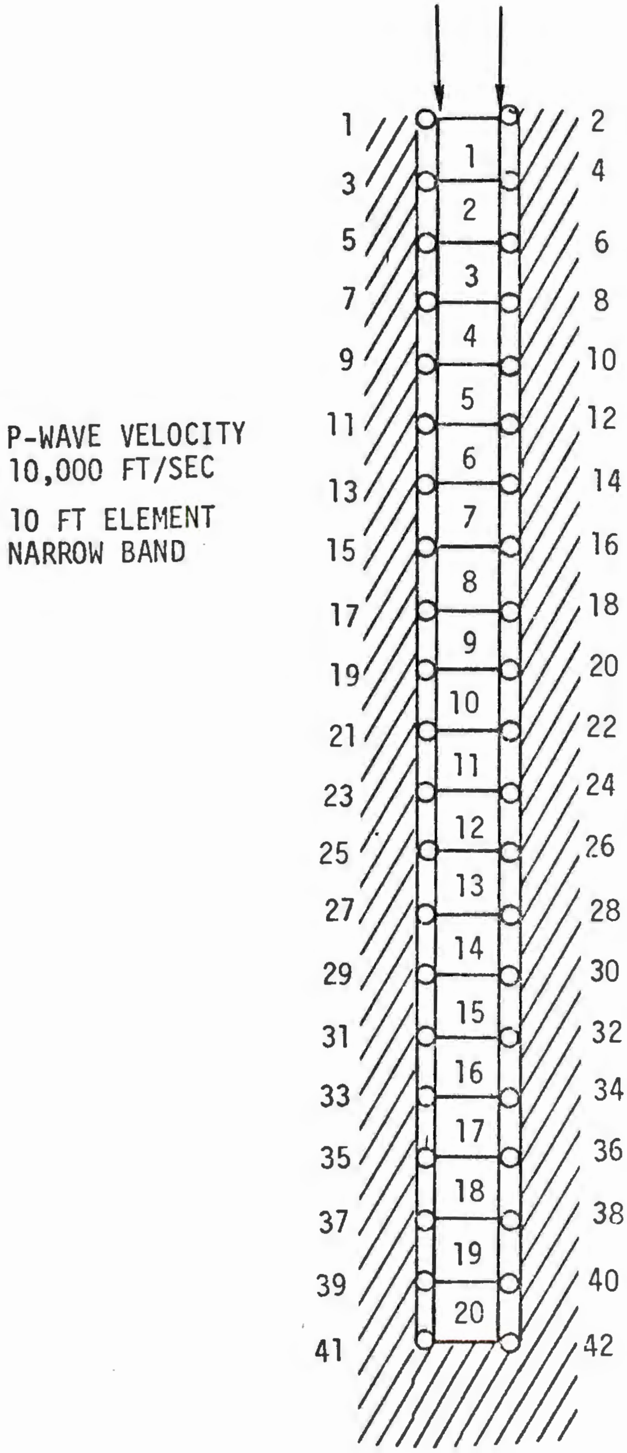

FIGURE 3. GEOMETRY OF GRID FOR RISE-TIME SENSITIVITY STUDY

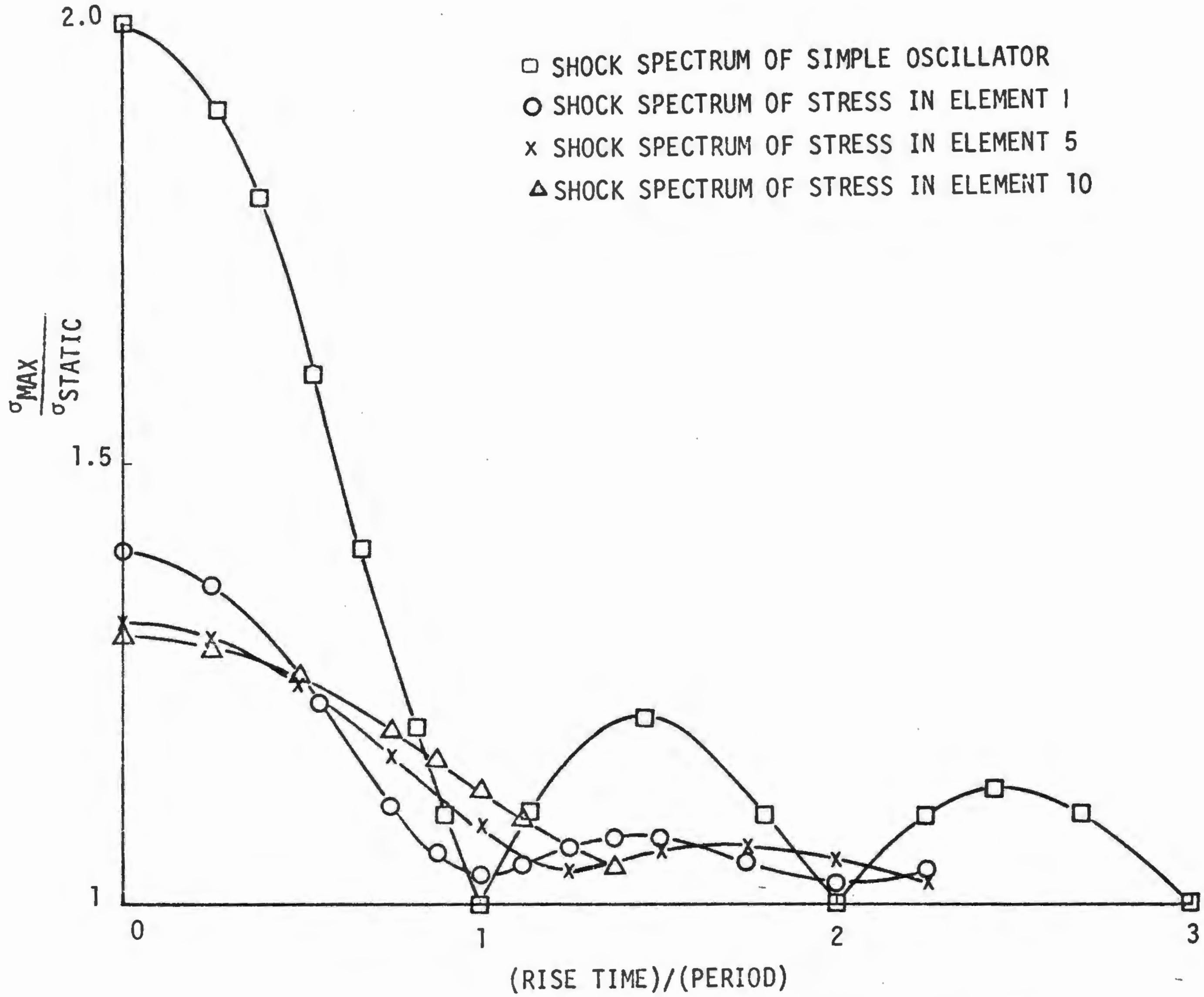

FIGURE 4. SHOCK SPECTRA FOR RISE-TIME SENSITIVITY STUDY

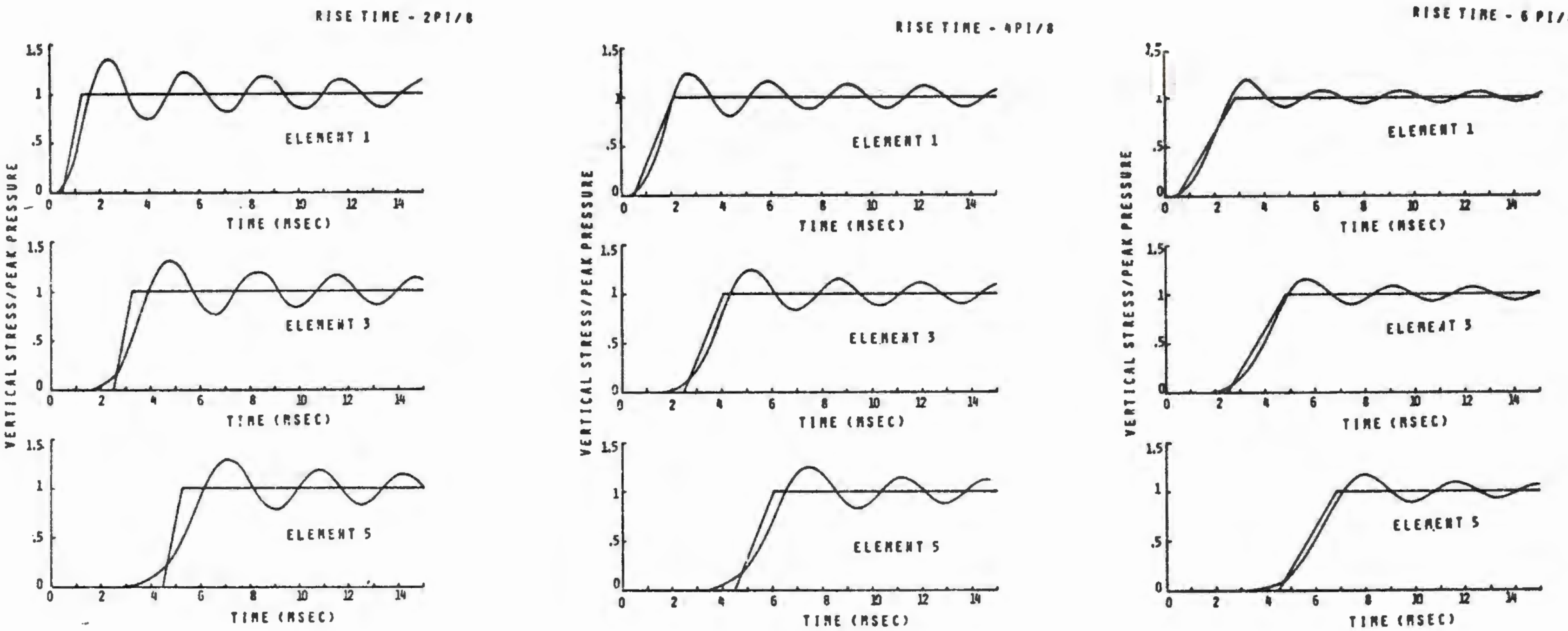

FIGURE 5. STRESS TIME HISTORIES RISE TIMES: $2\pi/8$, $4\pi/8$, $6\pi/8$

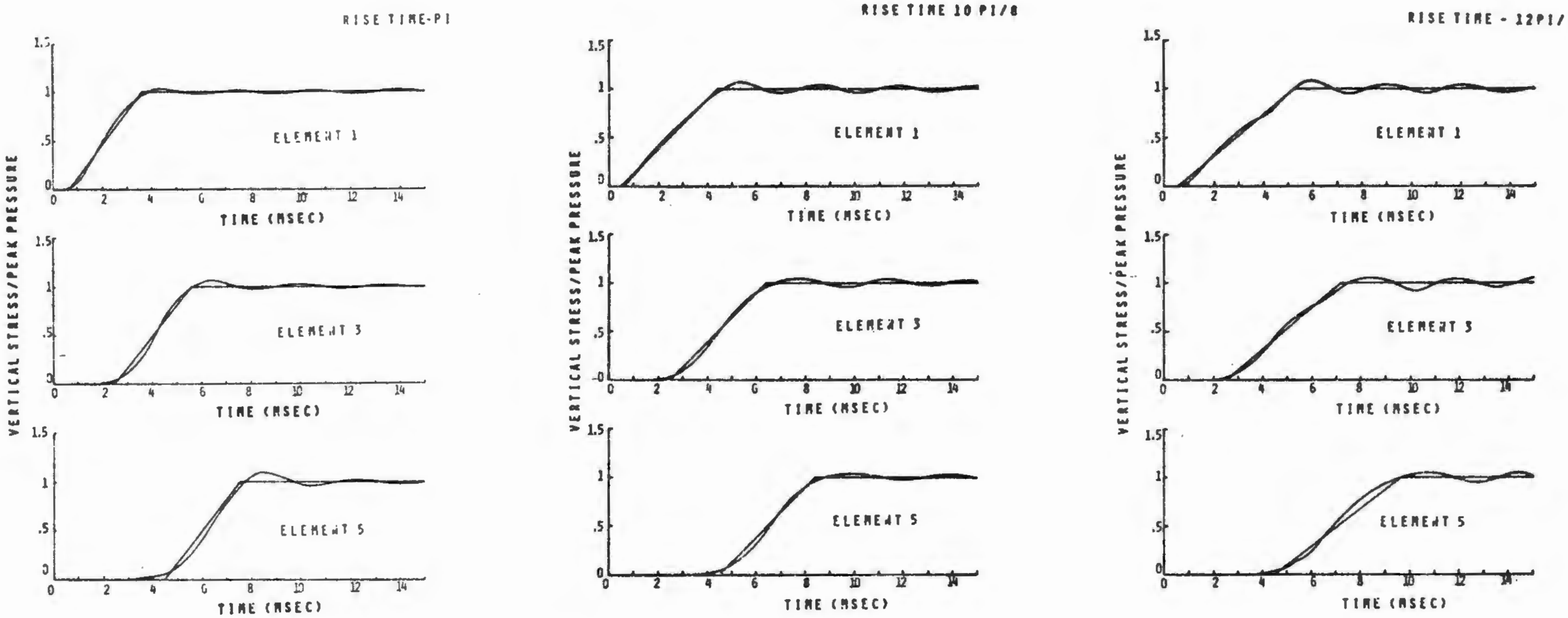

FIGURE 6. STRESS TIME HISTORIES RISE TIMES: π, $10\pi/8$, $12\pi/8$

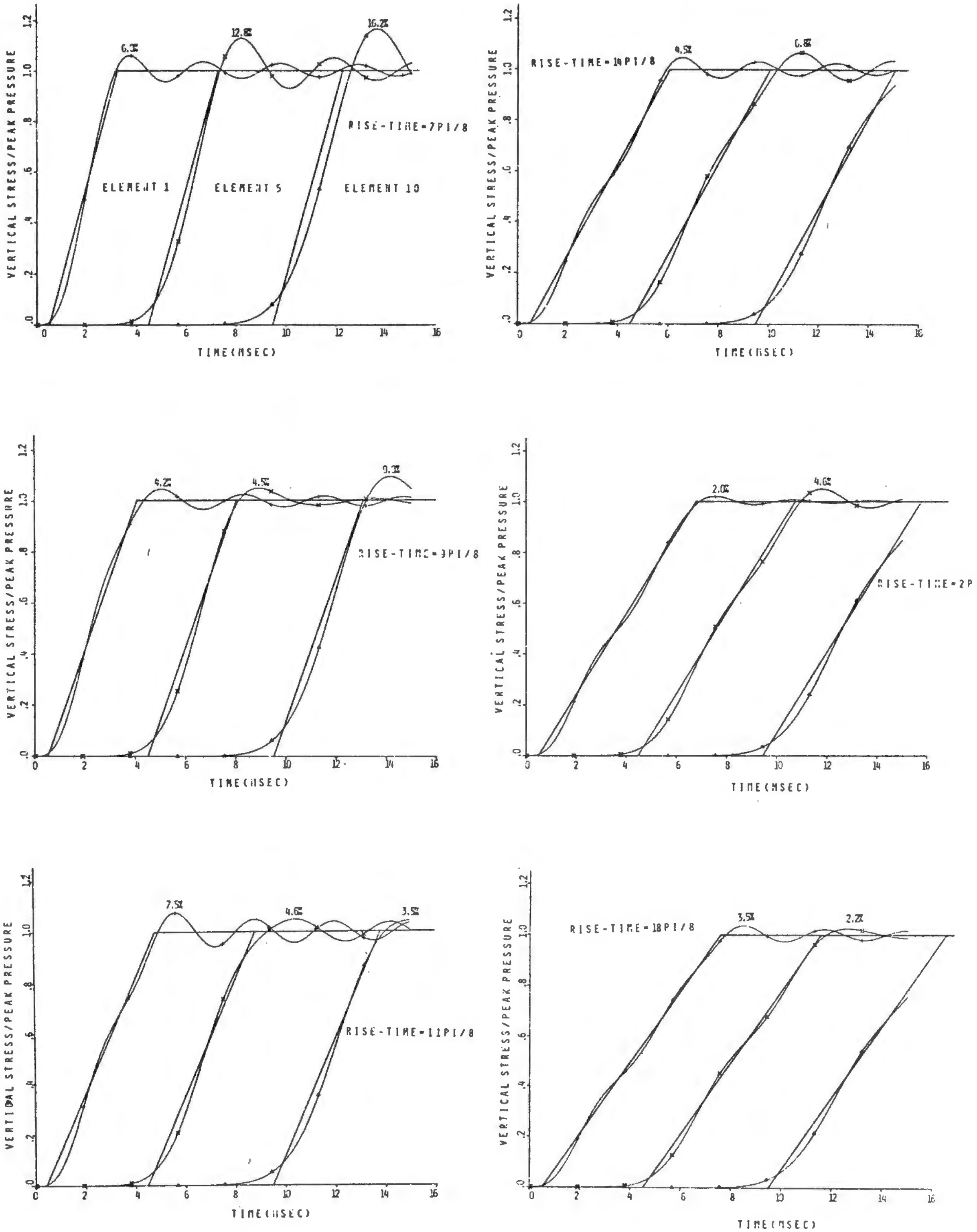

FIGURE 7. STRESS TIME HISTORIES
RISE TIMES: $7\pi/8$, $9\pi/8$, $11\pi/8$, $14\pi/8$, 2π, $18\pi/8$

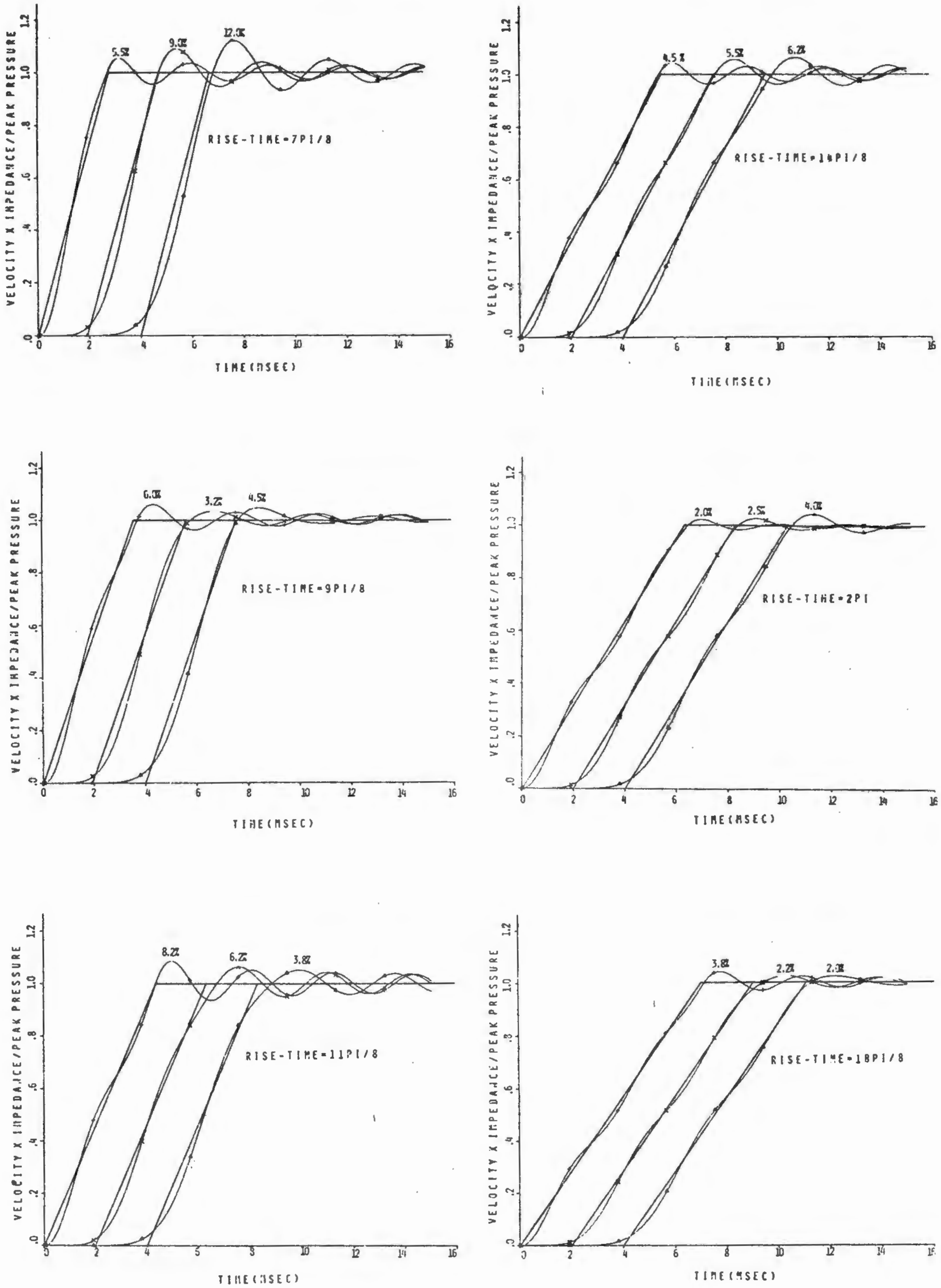

FIGURE 8. VELOCITY TIME HISTORIES
RISE TIMES: $7\pi/8$, $9\pi/8$, $11\pi/8$, $14\pi/8$, 2π, $18\pi/8$

Critical damping occurs for the n-th mode when

$$\omega_n = \gamma_n$$

three cases were examined. If $\beta = 0$, $\alpha = 2\omega_n$ yields the critical damping. This indicates that for $\beta = 0$, modes with frequencies less than $\alpha/2$ will be overdamped and those with frequencies greater than $\alpha/2$ will be underdamped. If $\alpha = 0$, $\beta = 2/\omega_n$ yields the critical damping, and modes with $\omega_n < (2/\beta)$ are underdamped while higher modes, $\omega_n > (2/\beta)$ are overdamped. If α and β are both nonzero, regions are defined for α and β where the system is either overdamped or underdamped. It is obvious that of the damping coefficients α and β, only β has the effect of damping high frequency components. This means that if physical effects actually cause a high frequency cut off then β can be used to simulate such effects. For most structural analyses, one finds it appropriate to consider $C = \beta K$ if β can be chosen properly. A series of finite element calculations were made to study the sensitivity of finite element solution to the selection of coefficient β. Figure 9 presents some of the results. Three observations were made:

- Wave propagation phenomena can not be properly simulated by a grid of finite elements whose highest mode is critically damped.

- Structural damping is not effective in reducing peak spurious oscillations due to finite element discretization of a continuum.

- Calculation of high frequency structural response using the finite element method requires an accurate choice of the damping coefficient, β.

Although above discussions apply primarily to linear elastic one-dimensional P-wave propagation, similar observations have been made on the spurious shear oscillations, that is, such oscillations can also be minimized by using rise times proportional to the transit time of a shear wave across an element. Moreover, the elastic P-wave rise-time criteria have been successfully applied to axisymmetric and plane strain shock propagation in nonlinear media.

Ideally, one would prefer to choose a very fine mesh of elements so that all pertinent wave phenomena (dilitation, shear, Rayleigh, etc.) could be modelled with little spurious oscillation. Unfortunately, many blast and shock problems do not lend themselves to very fine grids. Sometimes one may use a priori knowledge of physics or test results to reduce the number of finite elements. For example, explosions in underground cavities are known to cause shock which has longer rise times at greater ranges. Hence, a grid having larger elements at greater range is appropriate. On the other hand, the modelling of the high frequency response

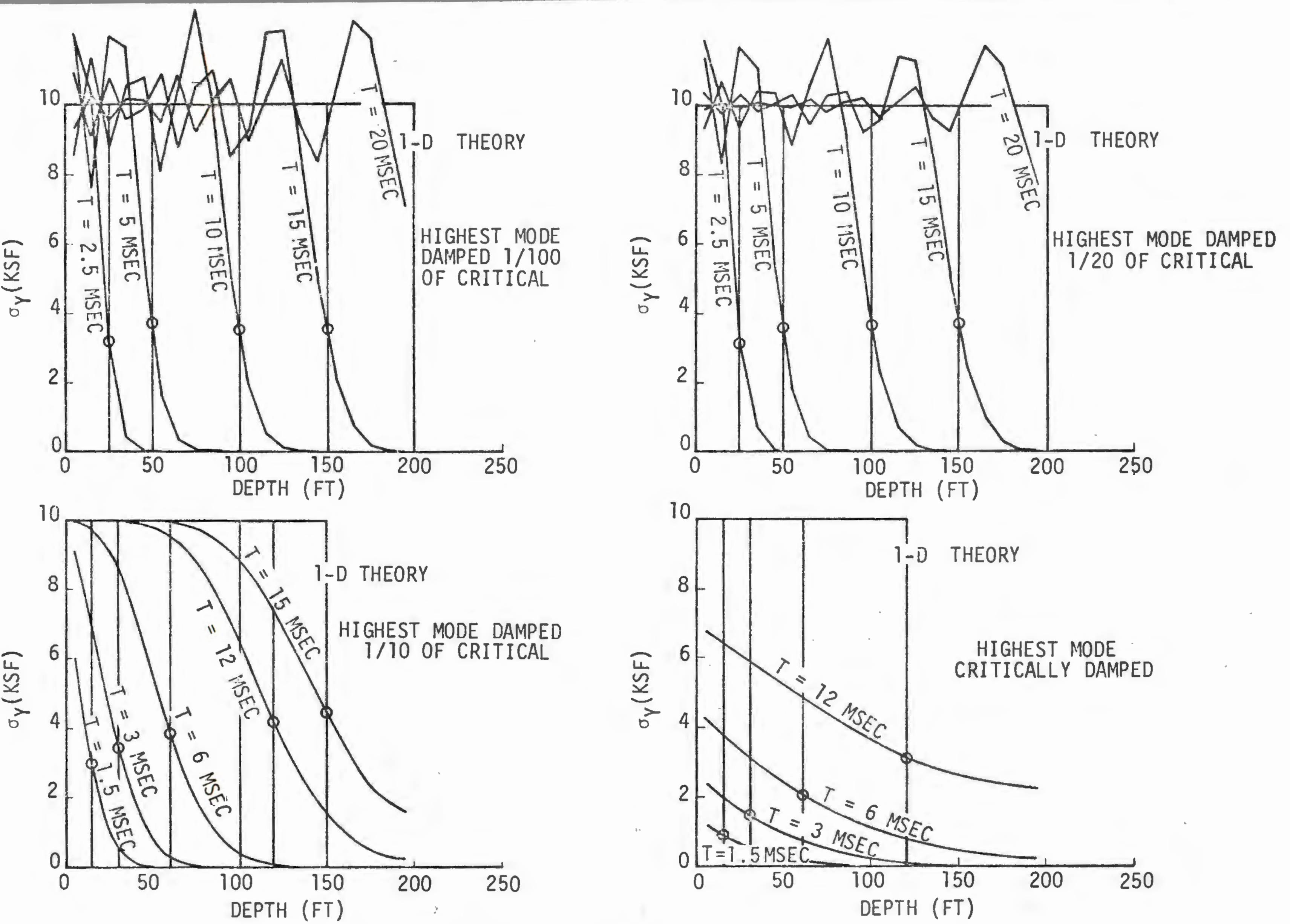

FIGURE 9. STRESS SPATIAL DISTRIBUTIONS FOR VISCOUS EFFECTS

of a stress gage in a cavity requires a different approach. Figure 10 shows these two problems which are modelled by the same grid under different loading conditions. In one case (the contained explosion) the grid is appropriate. In the other case (the high frequency stress gage) the grid is inappropriate. The reasons for the differences are threefold.

- The physics of underground explosions, as verified by test, is such that the high frequency content of the groundshock is attenuated with range (by both linear and nonlinear effects).
- The goals of the two analyses could be different i.e., low frequency displacements for the underground explosion vs high frequency changes in stress.
- The loading conditions are such that the grid adequately models the participation of the near cavity elements for the underground explosion while the coarse exterior grid limits the high frequency participation of the cavity under an external load.

CONCLUSION

If one considers the following effects, one has a better chance of achieving ones computational goals economically by finite element analyses:

- Finite element grids cannot propagate shocks whose frequency content is higher than the highest modal frequency of the grid.
- Spurious oscillation associated with the highest modal frequencies of a finite element mesh may be minimized by tuning the grid to the forcing function by shock spectral considerations.*
- The modal participations of the forcing functions and critical elements should be weighed carefully against computational goals when grids are made nonuniform for economic reasons.
- Structural damping becomes highly important for high frequency shocks; sharp shocks cannot propagate in and overdamped mode.

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* Vector plots showing the spatial velocity field or stress fields are most easily interpreted if spurious oscillations are suppressed. Moreover spurious oscillations on codes with nonlinear material finite elements can cause irreversible nonlinear effects.

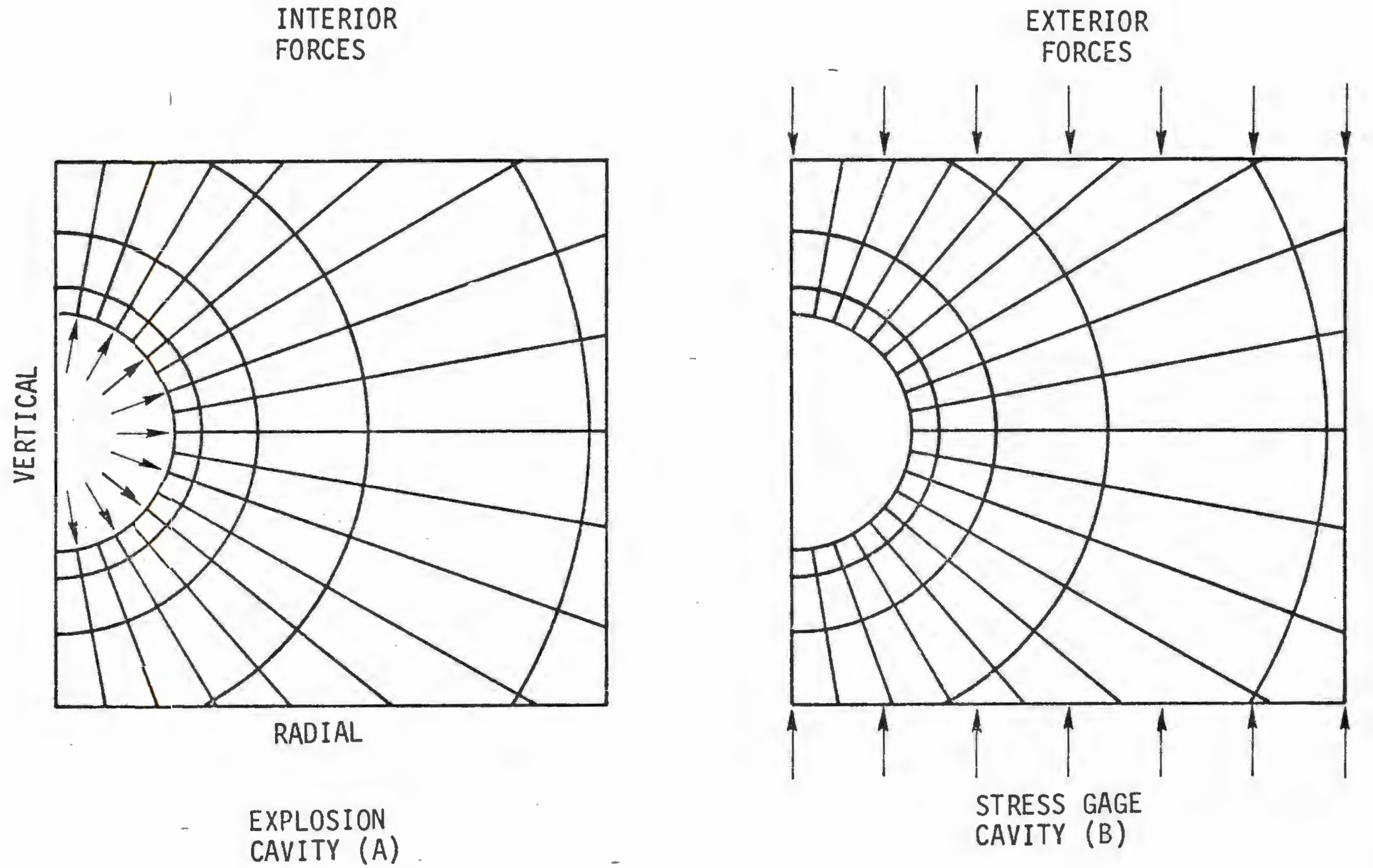

FIGURE 10. EXAMPLES OF MODAL PARTICIPATIONS OF LOAD

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