DESIGN AND ANALYSIS OF VISCOELASTIC STRUTS FOR LARGE SPACE STRUCTURES

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ABSTRACT

Use of the Modal Strain Energy Method for the optimization of viscoelastic structures requires extensive modal extraction and modal strain energy distribution analyses. For large structural systems, this iterative procedure becomes very inefficient and expensive. Based on the Rayleigh-Ritz principle and engineering assumptions, a method is developed for the optimization a class of viscoelastic struts. Dimensionless ratios of the system parameters of the modified design and the baseline design are derived, in closed form algebraic equations, to enable design optimization to be performed without iterative computer analyses. A finite element substructure analysis method is also derived for the efficient analysis of complex stiffness matrices due to the presence of viscoelastic materials in the detailed stress model. The results from simple design predictions and the dynamic substructuring finite element analysis show excellent correlation.

INTRODUCTION

Large space structures require efficient structural design with high strength and light weight materials to reduce the launch weight. Truss type structures are prevalent. High stiffness and damping are required for on-orbit performance to minimize the dynamic response due to operational disturbances. However, materials with high strength and stiffness do not normally process high damping characteristics. Therefore, a combination of different materials must be used to provide the desired system response. The design of high performance structure must balance the stiffness and damping characteristics to minimize the weight of the structure.

The Modal Strain Energy (MSE) Method[1] has been used extensively in the design of passively damped structures using viscoelastic materials (VEM). Tests on a demonstration article[2] showed excellent correlation between predicted results using MSE and measured results. Therefore, within the limits of its applicability, the MSE method can be used to achieve a design with the desired damping characteristics.

When designing a large precision structure, large finite element models with high fidelity to capture the structural behavior is often required. Furthermore, for passive damping design with VEM, the MSE Method requires a detailed structural model so that the modal strain energy distribution in the VEM can be computed accurately. The procedure for designing structures with passive damping is basically an iterative modal extraction and strain energy distribution analysis. However, this type of iterative computer analysis on a large finite element model to optimize the design parameters is prohibitively expensive and inefficient. Therefore, improved efficiency of the analysis is vital to the optimal design of large damped structures.

This paper presents a method, based on the Rayleigh-Ritz principle, such that a class of viscoelastic struts can be designed and optimized efficiently for large space trusses. This method provides a desired viscoelastic strut design using the initial baseline design and simple algebraic equations. A dynamic substructuring procedure is also presented for large finite element models with complex stiffness matrices due to the presence of viscoelastic materials. This combination of design procedure and analytical technique enables better design of large space structures with viscoelastic struts with less effort and better understanding. This method is also applicable to other large structural systems.

VIBRATION ATTENUATION

For a linear elastic single degree of freedom system, the governing differential equation for dynamic loading is :

$$m\ddot{x} + c\dot{x} + kx = p(t) \tag{1}$$

If the structure is modified to enhance its damping characteristic, the system parameters are changed to :

$$\tilde{m} = r_m m \tag{2}$$

$$\tilde{k} = r_k k \tag{3}$$

$$\tilde{f}_n = \sqrt{\frac{r_k}{r_m}}\, f_n \tag{4}$$

$$\tilde{\xi} = r_\xi \, \xi \tag{5}$$

where r_m, r_k and r_ξ represent the ratios of the modified values to the respective baseline values. The effect of such changes on dynamic response is different for different types of vibration environments. The mass, stiffness and damping of the system must be balanced for an optimum design.

Random Vibration

If p(t) is described by the power spectral density function S(f) and it is relatively constant around the natural frequency, f_n, the response for a lightly damped system ($\xi < 0.3$) can be expressed as :

$$x_{rms} \approx \frac{S(f_n)^{0.5}}{2\sqrt{2}\, m^{0.25}\, k^{0.75}\, \xi^{0.5}} \tag{6}$$

Therefore, the response of the modified structure can be expressed as :

$$\tilde{x}_{rms} \approx \frac{S(\tilde{f}_n)^{0.5}}{2\sqrt{2}\, \tilde{m}^{0.25}\, \tilde{k}^{0.75}\, \tilde{\xi}^{0.5}} \tag{7}$$

If S(f) is relatively constant over f_n and $\tilde{f}_n$, then the response attenuation factor, α, can be defined as :

$$\alpha = \frac{\tilde{x}_{rms}}{x_{rms}}$$

$$= \frac{1}{r_m^{0.25}\, r_k^{0.75}\, r_\xi^{0.5}} \tag{8}$$

Therefore the attenuation factor of the modified structure can be expressed in terms of the dimensionless ratios of mass, stiffness and damping. The attenuation factor is least sensitive to mass change and most sensitive to stiffness change. However, the change in stiffness of the modified structure is normally not very large while the change in damping ratio can be significant.

For a multiple degree of freedom system with negligible modal coupling, the modal attenuation factors can also be determined in a similar way. The modal attenuation factor can be defined as :

$$\alpha_i = \frac{1}{r_{mi}^{0.25}\, r_{ki}^{0.75}\, r_{\xi i}^{0.5}} \tag{9}$$

Very often, passive damping mechanisms increase mass and damping but reduce the stiffness. An optimum design balances the contribution from these three ratios to minimize the attenuation factor. The objective function for optimization is therefore α_i.

The key to optimization of structural response to a random vibration environment is therefore the ability to estimate the modal mass, stiffness and damping ratios accurately. The following design procedure fully exploits these modal ratios to achieve an optimized design.

Other Types of Vibration

A similar approach can be used for other type of dynamic loading. For sinusoidal input, the attenuation factor can shown to be :

$$\alpha_i = \frac{1}{r_{ki}\, r_{\xi i}} \tag{10}$$

If f(t) is transient in nature, it is best to characterize it by shock spectrum curves. The maximum modal response is related by the ratios of the shock spectrum values at the system frequency and damping values.

APPROACH

In order to successfully design passive damping into a large structure to control vibration response, the behavior of the structural system and disturbance characteristics must be thoroughly investigated. A finite element model of the baseline elastic structure is analyzed for its frequency characteristics, modal strain energy characteristics and dynamic response characteristics. The type and extent of the damping treatment must be identified using the baseline model. In order to accomplish an optimum viscoelastic strut design to control the system level response, the relationship between the viscoelastic materials, component design parameters and system level behavior must be understood. To implement any practical design concept, the number of design parameters must be reduced to a manageable size. Engineering assumptions must be made to simplify the analytical design process and develop a direct algebraic relationship between the design parameters and system level response. Optimization can then proceed expeditiously and a preliminary design can be developed.

Design Optimization Assumptions

To illustrate the design procedure described herein, the following design and analysis assumptions are made :

1. Analysis Tool - The Modal Strain Energy Method is used as an analysis tool to identify candidate locations where VEM can be most effectively placed. Uncoupled modal analysis is used to analyze the performance of the design.

2. Design Parameters - Two assumptions are made to reduce the total number of design parameters. Firstly, it is desirable to use only one VEM. For a given operating environment and dominant natural frequency, only two material parameters, G and η, are required to be optimized. Secondly, only one viscoelastic strut design is used throughout the structure. This limits the geometric parameters to only the length and thickness of the VEM.

3. Component/System Behavior Assumption - For the modes of interest, the stiffness of the system is governed by the axial deformation of truss members. This identifies the most important behavior of the strut member and allows simplified equations to be developed to predict the stiffness and strain energy distribution of the struts and the overall system.

4. System Analysis Assumption - It is assumed that the structural system has already been optimized for mass and stiffness, and the overall system performance would be acceptable if higher damping is provided. The purpose of the viscoelastic struts is only to enhance the damping characteristic of the system. It is therefore further assumed that the modes shapes of the viscoelastically damped structure are not substantially different from the baseline structure and the mode shapes of the baseline structure are a good set of generalized coordinates for the modified structure.

5. Dynamic Loading Assumptions - It is assumed that the dynamic response of the structure is governed by a random disturbance. Reduction of the modal root mean square response is used as the objective function for optimization. The same technique can be used for other types of dynamic loading conditions.

DESIGN AND ANALYSIS PROCEDURE

The procedure to design and analyze the structure with viscoelastic struts, as outlined in Table 1, is comprised of three basic parts. The first part involves the analysis of the baseline model. The steps are outlined in steps 1 through 4 of Table 1. The modal contributions to the system responses are identified. A modal strain energy distribution analysis of these modes is performed. Struts are then ranked according to modal strain energy level. The second part, outlined in steps 5 through 9 of Table 1, involves the use of simplified equations to optimize a viscoelastic strut design and predict the overall system performance. This bypasses the iterative analysis using the finite element models. The third part involves finite element verification of the analysis. A detailed finite element model of the viscoelastic strut is constructed. A reduced mass matrix and complex stiffness matrix of the strut are computed. These matrices are assembled into the global matrices in place of the basline strut members. Modal extraction is performed and equivalent viscous damping is extracted. Response analysis is performed to verify the system performance. This is outlined in steps 10 through 16 of Table 1.

ANALYSIS OF BASELINE STRUCTURE

The baseline elastic structure system satisfies the following matrix differential equation in the frequency domain :

$$[-\omega^2 \mathbf{M} + i\,\omega\,\mathbf{C} + \mathbf{K}]\,\mathbf{u}(\omega) = \mathbf{F}(\omega) \tag{11}$$

The structural system has an undamped eigensolution, Λ and Φ. The orthogonality conditions for the system are :

$$\Phi^T \mathbf{M}\,\Phi = \mathbf{I} \tag{12}$$

$$\Phi^T \mathbf{K}\,\Phi = \Lambda \tag{13}$$

The modal differential equation of motion is :

$$[-\omega^2 \mathbf{I} + i\,\omega\,\Phi^T \mathbf{C}\,\Phi + \Lambda]\,\mathbf{Q}(\omega) = \Phi^T \mathbf{F}(\omega) \tag{14}$$

The uncoupled modal damping ratio, ζ_i, is often assumed in the computation of dynamic responses. The uncoupled modal differential equation is :

Table 1. Summary of Design and Analysis Procedures

Baseline Model Analysis

1. Construct baseline structural finite element model.
   Perform a real eigenvalue analysis.
2. Perform dynamic response analysis.
   Identify modes contributing significantly to the responses.
3. For the modes of interest, perform strain energy distribution analysis of the elements in the baseline model.
4. Select a group of highly strained elements to be replaced by viscoelastic struts.
   Compute the modal strain energy ratio, ε_i, and modal mass ratio, μ_i, of the group. Eq. (17), (22)

Design Procedure

5. Analyze the viscoelastic strut using strength of materials and structural analysis methods.
   Express the strut stiffness ratio, r_a, and strain energy ratio in the viscoelastic material, r_W, in terms of design parameters, G_v, t_v and l_v. Eq. (26), (29)
6. Express the modal parameters, $\tilde{k}_i$, $\tilde{m}_i$, $\tilde{f}_i$ and $\tilde{\xi}_i$, of the modified structure in terms of the design variables. Eq. (34), (36), (37), (42)
   Express modal attenuation factor, α_i, in term of r_{mi}, r_{ki}, $r_{\xi i}$. Eq. (9)
7. Select a viscoelastic material - $\eta^v(\omega)$ and $G^v(\omega)$.
8. Compute modal attenuation factors for the range of feasible design parameters and obtain the optimum values of design parameters.
9. Update dynamic responses based on design parameters from steps 7 and 8.
   Iterate steps 4, 7 and 8, if necessary, to obtain the desired response level.

Finite Element Substructuring Analysis

10. Construct a finite element model of the viscoelastic strut.
11. Using the substructuring method, form the mass matrix and the real part and imaginary part of the condensed component stiffness matrix of the viscoelastic strut. Eq. (47), (48)
12. Assemble the complex global stiffness matrix and mass matrix.
    Perform a real eigenvalue analysis of the modified structure.
13. Repeat steps 10 to 12 for as many frequency dependent stiffness matrices as necessary.
14. Extract modal damping ratios from the imaginary part of the stiffness matrix. Eq. (53),(54)
15. Compute dynamic response based on modal parameters from finite element results.
16. If results are not satisfactory, use the model of step 12 as the baseline model and iterate.

$$[-\omega^2 + i\,2\,\zeta_i\,\omega_i\,\omega + \omega_i^2]\,q_i(\omega) = \phi_i^T\,\mathbf{F}(\omega) \tag{15}$$

A dynamic response analysis is performed to identify the modes contributing significantly to the response. A modal strain energy analysis of the baseline structural model is performed to compute the strain energy, $(w_j)_i$, of each element of the i-th mode. The locations with the highest strain energy density are identified as a group to be replaced by viscoelastic struts. The ratio of the modal strain energy of the selected group to the strain energy of the entire structure is the modal strain energy ratio, ε_i. If the strain energy ratio of the viscoelastic material to the viscoelastic strut is known, the system level modal loss factor can be estimated readily by Equations (40) and (A8). The level of damping can be increased by including more viscoelastic struts, higher strain energy ratio in the VEM and higher material loss factor. A practical level of passive damping can therefore be estimated once the modal strain energy distribution is known.

In order to analyze and fully understand the effect of viscoelastic struts on the structural stiffness, strain energy distribution and mass, it is best to separate the structure into two groups, the unmodified group and the to-be-modified group. Therefore, the global elastic stiffness matrix can be visualized as being contributed to by two matrices, $\mathbf{K_1}$ and $\mathbf{K_2}$.

$$\mathbf{K} = \mathbf{K_1} + \mathbf{K_2} \tag{16}$$

$\mathbf{K_1}$ represents the stiffness matrix of the unmodified elastic elements and $\mathbf{K_2}$ represents the stiffness matrix of the to-be-modified elastic elements. Although the modal strain energy ratio of the to-be-modified members, ε_i, is computed by the summation of individual elements, it can also be expressed as :

$$\varepsilon_i = \frac{\frac{1}{2}\phi_i^T\,\mathbf{K}_2\,\phi_i}{\frac{1}{2}\phi_i^T\,\mathbf{K}\,\phi_i}. \tag{17}$$

Using the orthogonality condition, $\phi_i^T\,\mathbf{K}\,\phi_i = \omega_i^2$, Equation (17) can be expressed as :

$$\phi_i^T\,\mathbf{K_2}\,\phi_i = \varepsilon_i\,\omega_i^2 \tag{18}$$

and therefore

$$\phi_i^T\,\mathbf{K_1}\,\phi_i = (1 - \varepsilon_i)\,\omega_i^2 \tag{19}$$

Similarly, an analysis of the effect of viscoelastic struts on the modal mass can be performed if the mass change is significant. The mass matrix is also separated into two parts.

$$\mathbf{M} = \mathbf{M_1} + \mathbf{M_2} \tag{20}$$

$\mathbf{M_1}$ is the mass matrix of the unmodified elastic elements and $\mathbf{M_2}$ is the mass matrix of the to-be-modified elastic elements. The modal mass ratio, μ_i, can also be defined as :

$$\mu_i = \frac{\phi_i^T\,\mathbf{M}_2\,\phi_i}{\phi_i^T\,\mathbf{M}\,\phi_i} \tag{21}$$

Since ϕ_i is mass normalized, then

$$\phi_i^T M_2 \phi_i = \mu_i \tag{22}$$

and

$$\phi_i^T M_1 \phi_i = (1 - \mu_i) \tag{23}$$

DERIVATION OF DESIGN EQUATIONS

Based on the stated engineering assumptions, the viscoelastic strut has only four design parameters. It is necessary to derive approximate closed form equations relating the material constants to the strut component parameters and then to the system performance parameters. Consequently, trade studies and optimization of the viscoelastic strut can be performed expeditiously by using simple tools such as a spreadsheet. This allows a comprehensive design optimization to be performed in a very short time and without substantial computer cost. It also offers a better physical insight into the effect of each parameter on the component and system behavior.

Analysis of Viscoelastic Strut

It is assumed that the stiffness of the truss, for the modes of interest, is governed by the axial deformation of the truss members. The axial stiffness and strain energy distribution of the damped strut can be derived based on a strength of materials approach. The ratio of the stiffness of the damped strut to the baseline strut, r_a, and the strain energy ratio of the VEM to the strut, r_w, are two dimensionless ratios at the strut component level which are important to the optimization process. The component stiffness ratio, r_a, affects the system stiffness and natural frequencies while the component strain energy ratio, r_w, controls the amount of damping introduced by the VEM into the system. In addition, the mass ratio, r_m, of the strut can also be computed if necessary.

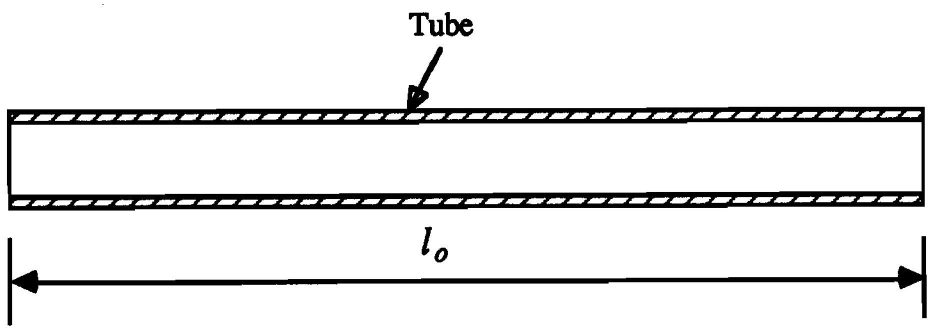
Different viscoelastic strut design concepts have been proposed[3] and some were built and tested[2]. A simple strut design concept, as show in Figure 1, is used to illustrate the design and analysis procedure. When the strut is deformed axially, the VEM carries the force in shear. Consequently, it provides the strain energy necessary for damping. It is further assumed, for the sake of simplicity, that the material constants and cross sectional areas of the inner and outer tubes are the same.

The axial stiffness of the original strut without viscoelastic material is :

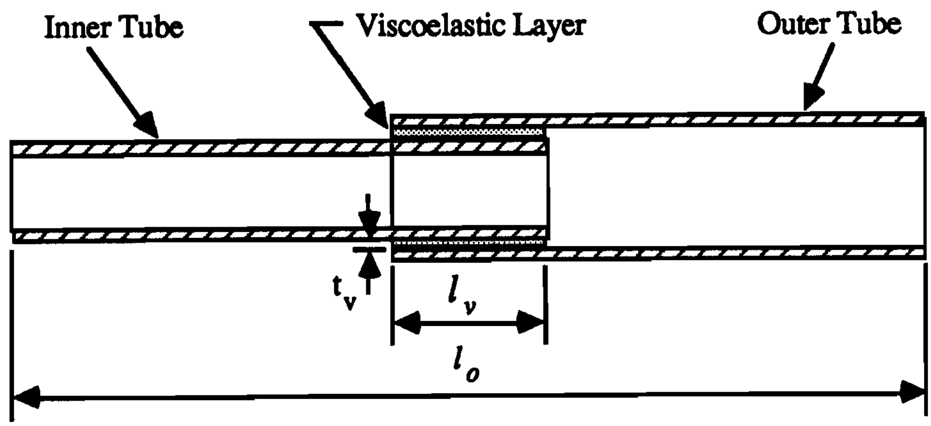
$$k_o = \frac{EA}{l_o} \tag{24}$$

The axial stiffness of the strut with viscoelastic material is :

$$k_v = \frac{1}{\frac{l_o}{EA} - \frac{l_v}{3EA} + \frac{t_v}{2\pi\bar{r} l_v G_v}} \tag{25}$$



a. Baseline Strut



b. Conceptual Viscoelastic Strut

Figure 1. Conceptual Strut Design

The ratio of the damped strut stiffness to the original strut stiffness is :

$$r_a = \frac{1}{1 - \frac{l_v}{3\, l_o} + \frac{E\,A\,t_v}{2\pi \bar{r} l_v G_v l_o}} \tag{26}$$

The strain energy in the viscoelastic material is :

$$W_v = \frac{t_v}{4\pi \bar{r} l_v G_v} \tag{27}$$

The total strain energy in the damped strut is :

$$W_t = \frac{1}{2k_v} \tag{28}$$

The ratio of the viscoelastic material strain energy to the total strain energy in the damped strut is :

$$r_W = \frac{t_v k_v}{2\bar{r} l_v G_v} \tag{29}$$

This procedure for component analysis is applicable to one class of viscoelastic strut design. When a different viscoelastic strut design is used, different expressions for k_v and W_v are obtained. However, the design and analysis procedure is the same.

Analysis of Viscoelastic Structures

In order to predict the system level response due to the introduction of viscoelastic struts, the three important modal ratios of the modified structure must be expressed in terms of the design parameters. The modified structural system satisfies the following matrix differential equation :

$$[-\omega^2 \tilde{M} + i\,\omega\,\tilde{C} + \tilde{K}]\,U(\omega) = F(\omega) \tag{30}$$

However, using this equation for design purposes in conjunction with large finite element models is impractical. Simple algebraic equations must be derived for optimization purposes.

The Modal Strain Energy Method uses mode shapes from the real eigenvalue solution and assumes that the modal equations are uncoupled. In addition, the mode shapes of the modified structure are assumed to be insignificantly affected by the modification of a small group of truss members. By the Rayleigh-Ritz principle, using the mode shapes of the baseline structure as the generalized coordinates, the approximate modal equation of the modified structure can be written as :

$$[-\omega^2\, \phi_i^T \tilde{M} \phi_i + i\,\omega\, \phi_i^T \tilde{C} \phi_i + \phi_i^T \tilde{K} \phi_i]\, Q_i(\omega) = \phi_i^T F(\omega) \tag{31}$$

Assuming $\tilde{C}$ can be diagonalized to $\tilde{c}_i$, then

$$[-\omega^2\, \tilde{m}_i + i\,\omega\,\tilde{c}_i + \phi_i^T \tilde{K}^R \phi_i + i\,\phi_i^T \tilde{K}^I \phi_i]\, Q_i(\omega) = \phi_i^T F(\omega) \tag{32}$$

If the behavior of the structural system is governed by the axial deformation of the truss members, the modified global stiffness matrix can be approximated by :

$$\tilde{K}^R \approx K_1 + r_a K_2 \tag{33}$$

and the generalized modal stiffness can be written as :

$$\tilde{k}_i = \phi_i^T K_1 \phi_i + r_a\, \phi_i^T K_2 \phi_i$$

$$\tilde{k}_i = (1 - \varepsilon_i + r_a \varepsilon_i)\, \omega_i^2 \tag{34}$$

Therefore, the ratio of the generalized stiffness of the modified structure to the baseline structure is:

$$r_{ki} = \frac{\tilde{k}_i}{\omega_i^2} = (1 - \varepsilon_i + r_a \varepsilon_i) \tag{35}$$

The corresponding generalized modal mass and the ratio of generalized mass can be written as :

$$\tilde{m}_i = r_{mi} = 1 - \mu_i + r_m \mu_i \tag{36}$$

The (Rayleigh-Ritz) frequency of the modified structure can therefore be expressed as :

$$\tilde{f}_i = \frac{1}{2\pi}\sqrt{\frac{\tilde{k}_i}{\tilde{m}_i}} \tag{37}$$

The modal strain energy ratio in the viscoelastic struts is :

$$\varepsilon_i = \frac{\frac{1}{2} r_a \, \phi_i^T \, K_2 \, \phi_i}{\frac{1}{2} r_a \, \phi_i^T \, \tilde{K}^R \, \phi_i} = \frac{r_a \varepsilon_i \, \omega_i^2}{\tilde{k}} \tag{38}$$

The modal strain energy ratio in the VEM of the viscoelastic struts is therefore,

$$\varepsilon^v{}_i = \frac{r_w \, r_a \, \varepsilon_i}{r_{ki}} \tag{39}$$

The modal loss factor can be expressed as :

$$\eta_i = \eta^v \varepsilon^v{}_i = \frac{\eta^v \, r_w \, r_a \, \varepsilon_i}{r_{ki}} \tag{40}$$

The equivalent viscous damping introduced into the i-th mode based on the modal strain energy method is :

$$\tilde{\zeta}_i = \frac{\eta^v \, r_a \, r_w \, \varepsilon_i}{2 \, r_{ki}} \tag{41}$$

If the inherit damping in the i-th mode is ζ_i, then the modal damping of the modified structure is :

$$\tilde{\xi}_i = \tilde{\zeta}_i + \zeta_i \tag{42}$$

The ratio of the modal damping of the modified structure to the baseline structure is :

$$r_{\xi i} = \frac{\tilde{\xi}_i}{\zeta_i} \tag{43}$$

Therefore, the modal ratios, r_{mi}, r_{ki} and $r_{\xi i}$, are all expressed in terms of the four design parameters, η, G, t_v and l_v, in simple algebraic form as shown in Equations (35), (36) and (43). The minimization of the modal attenuation factor, α_i, is therefore quite simple as shown in the example.

SUBSTRUCTURING ANALYSIS OF VISCOELASTIC STRUCTURES

Once a set of optimum design parameters is obtained, a finite element analysis should be performed to verify the performance of the system. The use of a substructuring method is ideal in this case since only one strut design is used. Since the model has to contain enough refinement to compute the stress distribution, substructuring allows a detailed damped strut model to be added to the large FEM without increasing the size of the model, and in this case without major modification of the existing model.

Finite Element Analysis Assumptions

1. Modal Strain Energy Method - Since the viscoelastic material is characterized by a complex modulus, the element stiffness matrix and global stiffness matrix are also complex. Real eigenvectors of the real part of the stiffness matrix are used to span the solution space. The eigenvectors are used to extract equivalent modal damping from the complex part of the global stiffness matrix. Uncoupled modal analysis is then performed.

2. Dynamic Substructuring Method - Dynamic substructuring is used to reduce the total number of degrees of freedom of the problem. Boundary node static vectors and constrained normal modes are used to condense the matrices of the substructure. Consistent with the global analysis, the set of real vectors is used to span the solution space of the complex part of the substructure stiffness matrix.

Dynamic Substructuring Method for Complex Stiffness Matrix

A detailed finite element model of the optimized viscoelastic strut is used to represent an elastic strut element. Plate elements are used to model the elastic tube. Solid elements are used to model the VEM so that the shear energy can be computed. Rigid offset transformations in the computer code, if available, should be used to model connections between plate elements and solid elements to reduce the number of degrees of freedom and improve the numerical performance. Stiff spoke systems are used at both ends of the strut model to allow for boundary connectivity to the beam elements. The material modulus is chosen at the dominant structural response frequency and operating temperature but iteration may be necessary. The strut component stiffness matrix, $\mathbf{k}$, is composed of the real part, $\mathbf{k}^R$, and the imaginary part, $\mathbf{k}^I$.

$$\mathbf{k}(\omega) = \mathbf{k}^R(\omega) + i\,\mathbf{k}^I(\omega) \tag{44}$$

The real part of the component stiffness matrix, $\mathbf{k}^R$, has stiffness contribution from two types of elements - $\mathbf{k}^R{}_e$ from the elastic elements and $\mathbf{k}^R{}_v$ from the viscoelastic elements.

$$\mathbf{k}^R = \mathbf{k}^R{}_e + \mathbf{k}^R{}_v \tag{45}$$

By the definition of the material loss factor, η^v, the imaginary part, $\mathbf{k}^I$, can be expressed as:

$$\mathbf{k}^I(\omega) = \eta^v(\omega)\,\mathbf{k}^R{}_v(\omega) \tag{46}$$

Since the strut substructure is a structure by itself, consistent with the system level assumptions, a set of real vectors is used to span the solution. This set of real vectors, $\mathbf{v}$, is comprised of the static boundary vectors and the constrained normal mode vectors of the mass and real stiffness matrix. The condensed stiffness matrix, κ, also consists of two parts, the real part,

κ^R, and the imaginary part, κ^I. The real part of the condensed element stiffness matrix, κ^R can be computed in a straightforward way :

$$\kappa^R(\omega) = v(\omega)^T k^R(\omega) v(\omega) \quad (47)$$

The imaginary part, κ^I, can be computed in a similar fashion :

$$\kappa^I(\omega) = v(\omega)^T k^I(\omega) v(\omega)$$

$$\kappa^I(\omega) = \eta^v(\omega) v(\omega)^T k^R{}_v(\omega) v(\omega) \quad (48)$$

If only boundary vectors are used, this becomes an extension of the static condensation (Guyan Reduction) procedure applied to the complex stiffness matrix problem. For the strut model, a reduced twelve by twelve matrix can be used as the element stiffness matrix in lieu of the beam stiffness matrix. The global matrix therefore has the same size and connectivities.

For practical implementation, κ^R may be computed using the static condensation method to calculate the condensed stiffness matrix of the finite element model. In order to construct $\eta^v k^R{}_v$, the same model with zero material stiffness for the elastic elements is used. However, v from the model with both elastic and viscoelastic elements must be used to extract a consistent κ^I by matrix triple product or other computational procedure. Then κ^R and κ^I are used as element stiffness matrices to assemble the global $\tilde{K}^R$ and $\tilde{K}^I$ matrices.

$$\tilde{K} = \tilde{K}^R + i\,\tilde{K}^I \quad (49)$$

$\tilde{K}^I$ is quite sparse with nonzero entries only at those degrees of freedom connected to viscoelastic struts.

The element mass matrix can be assembled using consistent formulation but normally a lumped mass procedure will suffice and hence it is not elaborated upon here.

$$[-\omega^2 \tilde{M} + i\,\omega\,\tilde{C} + \tilde{K}^R + i\,\tilde{K}^I\,]\,u(\omega) = F(\omega) \quad (50)$$

By the Modal Strain Energy Method, $\tilde{M}$ and $\tilde{K}^R$ matrices are used to extract the real eigenvectors. However, $\tilde{K}^R$ is not a constant coefficient matrix. This makes the solution of the exact eigenvalue problem extremely difficult. Different methods[4] can be used to compute an approximate solution to this frequency dependent problem. For dynamic response determined by a single mode or a limited number of modes with no modal interference, this is not a major difficulty.

If $\tilde{\phi}_i$ is the i-th mode shape vector of the viscoelastic structure, the modal equation is therefore:

$$[-\omega^2\, \tilde{\phi}_i^T \tilde{M}\, \tilde{\phi}_i + \tilde{\phi}_i^T \tilde{K}^R\, \tilde{\phi}_i + i\,\omega\,\tilde{\phi}_i^T \tilde{C}\, \tilde{\phi}_i + i\,\tilde{\phi}_i^T \tilde{K}^I \tilde{\phi}_i]\; q_i(\omega) = \tilde{\phi}_i^T F(\omega) \quad (51)$$

$$[-\omega^2 + \tilde{\omega}_i^2 + i\,\omega\,\tilde{c}_i + i\,\tilde{\phi}_i^T \tilde{K}^I \tilde{\phi}_i]\;\; q_i(\omega) = \tilde{\phi}_i^T F(\omega) \quad (52)$$

From Equation (A12), the modal loss factor contributed by the VEM can be approximated by :

$$\tilde{\eta}_i = \frac{\tilde{\phi}_i^T \tilde{K}^I \tilde{\phi}_i}{\tilde{\omega}_i^2} \tag{53}$$

So, the equivalent modal viscous damping ratio contributed by the VEM is :

$$\tilde{\zeta}_i = \frac{\tilde{\eta}_i}{2} \tag{54}$$

and the total equivalent modal viscous damping of the structural system is :

$$\tilde{\xi}_i = \tilde{\zeta}_i + \zeta_i \tag{55}$$

Modal dynamic response analysis can then be performed using $(\tilde{\omega}_i, \tilde{\xi}_i, \tilde{\phi}_i)$ as the system modal quantities.

EXAMPLE

Analysis of Baseline Structure

The structure used as an illustrative example is a large truss type structure shown in Figure 2. The model consists of 2052 nodes and the EAL[5] finite element code was used. To reduce the computation required, a selective modal extraction[6] was performed for the large finite element model.

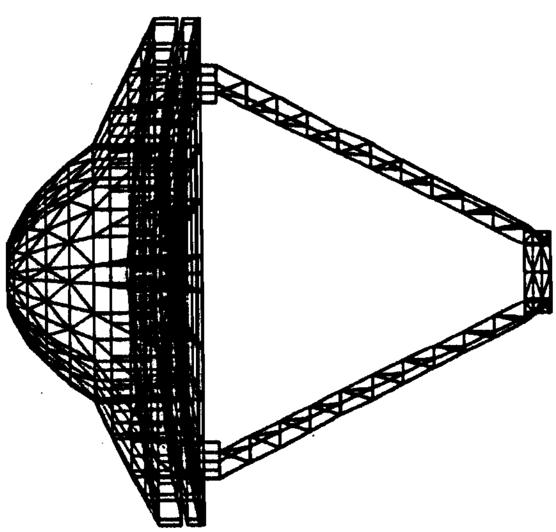


Figure 2
Finite Element Model of the Baseline Structure

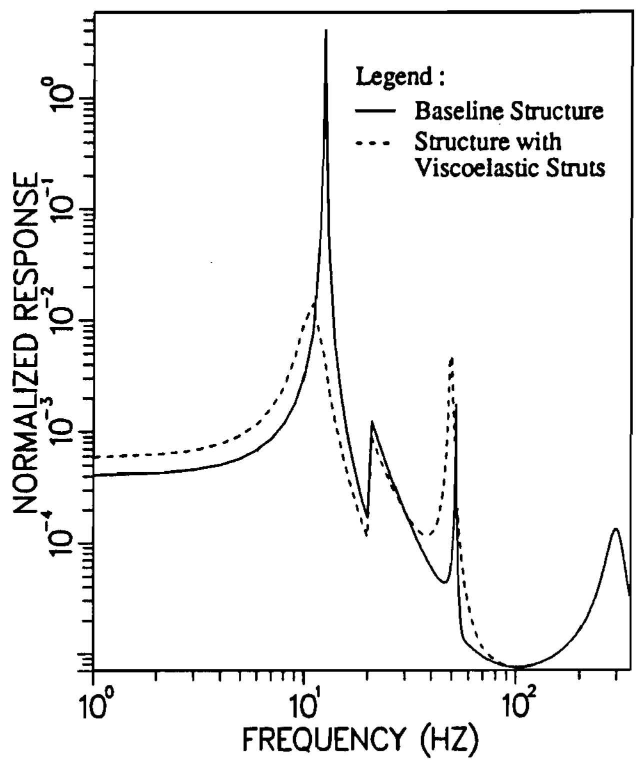


Figure 3
Response Power Spectral Density

The Line of Sight jitter response to a power spectral density input was used to evaluate the performance of the system. The response power spectral density curve in Figure 3 shows that the jitter response is dominated by the first truss bending mode of the structure.

Modal strain energy distribution analysis was performed to identify a group of members with high strain energy. These elements with high modal strain energy are the most efficient locations for application of VEM for passive damping. In this example, the longerons were found to contain the largest percentage, 26.9%, of strain energy in the dominant mode. Within this group of members, the 28 struts selected for replacement contain 54.0% of the strain energy in all of the longeron struts.

Design and Analysis of Viscoelastic Struts and Structure

The viscoelastic material reduces the stiffness of the strut while increasing the damping. It is desirable to concentrate most of the strain energy in the VEM to increase the damping which demands that the rest of the strut act as rigid links. However, this degrades the strut stiffness significantly. With these two opposing trends, optimization of the strut design can only be determined in the system response level and not at the component level.

The derived equations were coded in a spreadsheet. The VEM material with the best modulus and loss factor at the reference temperature of 25° F and frequency of 10 Hz was chosen. The effect of the VEM parameters on the component ratios is shown in Figure 4. The effect of the VEM on the system attenuation factor is shown in Figure 5. These two figures show that the use of more VEM in some design regions actual degrades the overall system performance. Therefore an optimization of the design at the system level is absolutely necessary. The predicted system parameters and attenuation factor for the optimized design are summarized in Table 2.

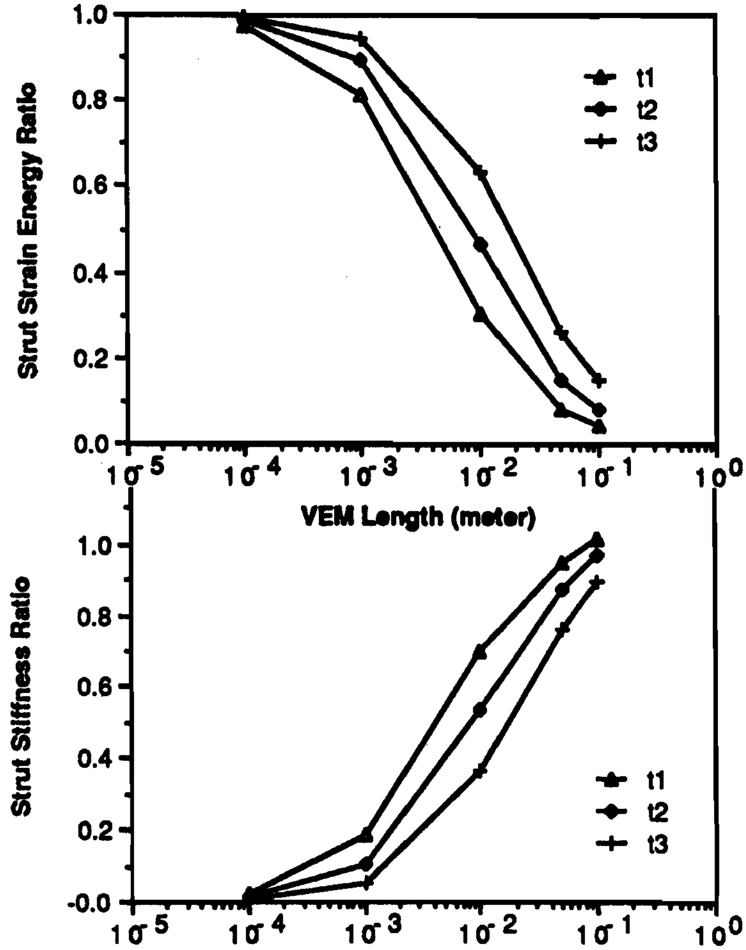


Figure 4
Strain Energy and Stiffness Ratios of Viscoelastic Strut

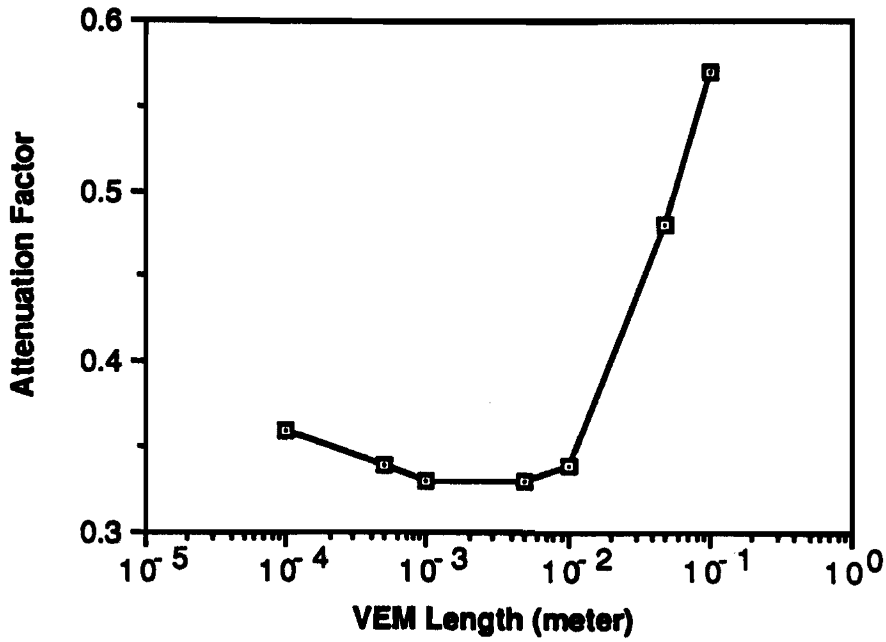


Figure 5
System Attenuation Factor due to Viscoelastic Struts

Table 2
Results of Derived Equations and Finite Element Method for Viscoelastic Structure

Structural Parameters	Derived Equations	Finite Element Model
Dominate Modes Frequency (Hz)	11.7	10.9
Modal Viscous Damping Ratio	0.100	0.097
Modal Attenuation Factor	0.24	0.27

Substructuring Analysis of Viscoelastic Structure

A detailed finite element model of the viscoelastic strut is shown in Figure 6. The condensed matrices of the optimized strut design were assembled into the baseline model as a directly specified spring matrix. This substructure method reduces the 210 node model of the strut into a two node element which can replace the existing element in the large finite element model. This includes the stiffness information in the model without requiring any modification of the geometry or connectivity of the existing baseline model. The new mode shapes were compared with the original shapes to verify that there were no significant changes in the mode shapes. The results of the finite element analysis are summarized in Table 2. The comparison with the design predictions is extremely favorable.

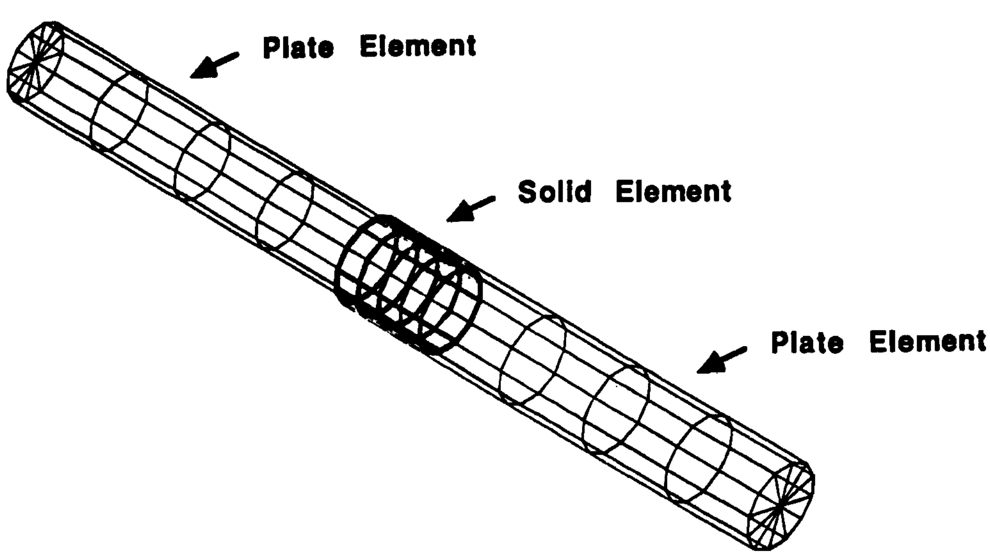


Figure 6
Finite Element Model of the Viscoelastic Strut

CONCLUSION

The design procedure presented enables the structural analysts/designers to develop viscoelastic struts which optimize the global response of very complex structures with a minimum amount of computer analysis. Information from the baseline design can be used to the maximum extent. The knowledge of the system overall behavior enables accurate assumptions to be made to optimize the design. The design optimization is based a set of closed form algebraic equations derived from Rayligh-Ritz principle. A substructuring method for the viscoelastic materials with complex stiffness matrices was derived and proven to be very efficient. The excellent comparison of the results from these two methods reinforces the soundness of the basic approach in both the design and analysis procedures. This method can easily be extended to the design and analysis of similar components and structures under different dynamic environments.

ACKNOWLEGEMENTS

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NOMENCLATURE

Symbols

c = viscous damping coefficient
f_n = natural frequency
i = imaginary unit, $\sqrt{-1}$
I = identity matrix
K, k = stiffness
M, m = mass
r = ratio
$v(\omega)$ = unit boundary displacement vectors
W, w = strain energy
α = attenuation factor
ε_i = strain energy ratio of selected group, i-th mode
Φ = matrix of eigenvectors
η = loss factor
κ = condensed stiffness
μ_i = mass ratio, i-th mode
ξ = damping ratio
Λ = matrix of eigenvalues
ζ = viscous modal damping
ω = frequency, radian/second
~ = denoting modified elements

Subscripts

a = strut axial stiffness
e = elastic
i = for the i-th mode
j = the group selected for modification
k = stiffness
m = mass
o = baseline design
v = viscoelastic
ξ = damping ratio

Superscripts

I = Imaginary
R = Real
T = matrix transpose
v = viscoelastic material constant

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APPENDIX

1. VEM Characteristics

The VEMs are strongly frequency and temperature dependent. Most often the energy dissipation is through the shear energy in the VEM. The energy dissipation property of the VEMs is conveniently modelled by the complex modulus of the material. The shear stress and shear strain constitutive relationship is often measured and expressed as :

$$\tau(\omega) = G^*(T, \omega)\, \gamma(\omega) \tag{A1}$$

$$G^*(T, \omega) = G(T, \omega)\, [\, 1 + i\, \eta(T, \omega)\,] \tag{A2}$$

This relationship is generalized to the general stress and strain constitutive relationship by :

$$\sigma_{ij} = C^*_{ijkl}\, \varepsilon_{kl} \tag{A3}$$

$$C^*_{ijkl} = C_{ijkl}\, [\, 1 + i\, \eta(T, \omega)\,] \tag{A4}$$

2. Finite Element of Viscoelastic Material

Based on the complex modulus material characterization of the viscoelastic material, the element stiffness matrix at a constant temperature is therefore also complex with both real and imaginary parts.

$$\mathbf{k}(\omega) = \mathbf{k}^R(\omega) + i\, \mathbf{k}^I(\omega) \tag{A5}$$

$$\mathbf{k}(\omega) = \mathbf{k}^R(\omega) + i\, \eta(\omega)\, \mathbf{k}^R(\omega) \tag{A6}$$

Consequently, the global structural stiffness matrix is also complex :

$$\mathbf{K}(\omega) = \mathbf{K}^R(\omega) + i\, \mathbf{K}^I(\omega) \tag{A7}$$

3. Modal Strain Energy Method

The Modal Strain Energy Method provides an efficient alternative to direct frequency response methods. The MSE method is the first tool which enables the analyst to design high modal damping into a structure by using viscoelastic materials. This allows deliberate design of passive damping into the structure. By using real normal modes of the undamped structure and assuming that modal coupling is negligible, the viscoelastic system can be characterized by modal equations of motion with modal hysteretic (structural) damping. Modal loss factors are used to approximate the damping contribution from the imaginary part of the complex stiffness. The modal loss factor for the i-th mode, η_i, can be approximated by:

$$\eta_i = \frac{\sum_j \eta^v{}_j (w_j)_i}{\sum_k (w_k)_i} \qquad \text{(A8)}$$

η_i = modal loss factor
$\eta^v{}_j$ = material loss factor of the j-th viscoelastic element
ω_j = strain energy in the j-th viscoelastic element
ω_k = strain energy in the k-th element of the structure

If $\varepsilon^v{}_{ji}$ is defined as the strain energy ratio of the j-th element of the i-th mode, then

$$\varepsilon^v{}_{ji} = \frac{(w_j)_i}{\sum_k (w_k)_i} \qquad \text{(A9)}$$

and if the sum of the the viscoelastic elements is ε_i, then

$$\varepsilon^v{}_i = \sum_j \varepsilon^v{}_{ji} \qquad \text{(A10)}$$

If only one material is used, then

$$\eta_i = \eta^v \varepsilon^v{}_i \qquad \text{(A11)}$$

In matrix form,

$$\eta_i = \frac{\frac{1}{2}\phi_i{}^T K^I \phi_i}{\frac{1}{2}\phi_i{}^T K^R \phi_i}$$

$$\eta_i = \frac{\phi_i{}^T K^I \phi_i}{\phi_i{}^T K^R \phi_i} \qquad \text{(A12)}$$