WADC TECHNICAL REPORT 59-4
ASTIA DOCUMENT NO. AD 209388

REAL GAS FLOW TABLES FOR NONDISSOCIATED AIR

LOUIS G. KAUFMAN II
AERONAUTICAL RESEARCH LABORATORY

JANUARY 1959

WRIGHT AIR DEVELOPMENT CENTER

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TASK 70135, PROJECT 7065

WRIGHT AIR DEVELOPMENT CENTER
AIR RESEARCH AND DEVELOPMENT COMMAND
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

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FOREWORD

This report was initiated by the Fluid Dynamics Research Branch of the Aeronautical Research Laboratory, Wright Air Development Center, Wright-Patterson Air Force Base, Ohio. The report was written by 1/Lt. Louis G. Kaufman II, Ph.D. under Task 70135, entitled "Hypersonic Flow Techniques", of Project 7065, entitled "Experimental Techniques in Aero-Mechanics". Lieutenant Kaufman was both Task Scientist and Project Scientist during the writing of this report.

The author is indebted to Maj. Charles A. Scolatti for initiating interest in compiling the tables given herein. The sincere gratitude of the author is also extended to Dr. Miles S. Edwards and Mr. Harry E. Petersen of the Digital Computation Branch of the Aeronautical Research Laboratory for programming the equations for calculation by a large digital computer (UNIVAC, Remington Rand "Scientific" 1103-A).

Louis G. Kaufman II

ABSTRACT

One-dimensional, high-speed air flow characteristics are tabulated herein. Stagnation pressure and temperature; density ratio across a normal shock wave; and Mach number, static pressure and temperature, and stagnation pressure and temperature behind a normal shock wave are tabulated for several sets of given conditions which are free-stream Mach number and static pressure and temperature. The Beattie-Bridgeman equation of state, widely accepted as one of the best, empirical, pressure-temperature-density relationships for air, is used in the calculations. The equation includes the effects of molecular vibrational energy which become important for high speed flows; the neglect of these effects in perfect gas law tables leads to considerable errors, even when so-called correction factors are included. The tables herein cover the range of flight and wind tunnel conditions wherein the vibrational effects are important but before dissociation effects become important.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER:

Roscoe H Mills
ROSCOE H. MILLS
Chief, Fluid Dynamics Research Branch
Aeronautical Research Laboratory

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LIST OF SYMBOLS

The principal symbols used herein are defined below. All other symbols are fully defined when used in the text. Digital computation machine notation for decimal point location is used in the tables; e.g.: $1.234\text{-}07 = 1.234 \cdot 10^{-7}$, $3.541\ 02 = 3.541 \cdot 10^{2}$, $9.876 = 9.876 \cdot 10^{0} = 9.876$.

Symbol	Definition	Page first used
a, b, c, d, f, g and h	Functions of only t defined in Appendix I	9
D	Density ratio across normal shock wave, $D = \rho_1/\rho_2$	3
I, J, K and L	Functions of P and T defined in Appendix I	3
M	Mach number	3
P	Pressure (in atmospheres)	2
R	Gas constant, R = 0.73024(atm.ft.^{3}/lb.mol.°R.)	2
t	The inverse of the temperature, t = 1/T	9
T	Temperature (in degrees Rankine)	2
X	$X = IM^2/(5J + IM^2)$	3
Y and Z	Functions defined in Appendix I	9
γ	Ratio of specific heats	9
ρ	Density (in pound moles per cubic foot)	2

Subscripts*

01	Indicates free-stream stagnation conditions	3
1	Indicates free-stream static conditions	3
02	Indicates stagnation conditions behind normal shock	3
2	Indicates static conditions behind a normal shock wave	3

* Digital computation machine notation for subscripts is used in the column headings of Tables I and II; e.g.: P_{01} = P01, T_2 = T2.

INTRODUCTION

Air no longer acts as a perfect gas under the conditions of high pressures and high temperatures encountered in high-speed flows; the use of the perfect gas law in calculating high-speed flow characteristics leads to significant errors. Correction factors have been applied to perfect-gas-law tables but these leave much to be desired in both accuracy and ease of use (Ref. 1). One of the main failings of the perfect gas law is that it excludes the effects of molecular vibrational energy which become important for high-speed air flows. The Beattie-Bridgeman thermal equation of state includes the vibrational energy effects and is widely accepted as one of the best empirical gas laws for air as long as no appreciable dissociation occurs (Refs. 2 - 5). The value of using the Beattie-Bridgeman equation for high-speed air flow calculations has long been acknowledged (Refs. 3 - 5) but, perhaps because of its complexity, it has not previously been utilized in tabulating high-speed air flow characteristics. The following tables are compiled for the flight and wind-tunnel conditions for which vibrational energy effects are important but before dissociation effects, which appear at higher speeds and altitudes, become dominant.

DEVELOPMENT AND USE OF TABLES

The equations and calculation procedures used in obtaining the following tables and examples of the use of the tables are shown below. The Beattie-Bridgeman equation of state and its range of applicability are given. The gas law and one-dimensional isentropic expansion and normal shock wave flow equations are embodied in an iterative numerical procedure used in obtaining the tables. Examples are provided of the engineering use of the tables and the results of using the Beattie-Bridgeman gas equation are compared with those of the perfect gas equation with and without correction factors.

BEATTIE-BRIDGEMAN EQUATION FOR AIR

The Beattie-Bridgeman equation of state relates pressure, density

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Manuscript released by the author in January 1959 for publication as a WADC Technical Report.

and temperature and may be written for air as (Refs. 2 - 5):

$$P = \rho RT + \left(0.73859 - \frac{333.85}{RT} - \frac{4,054,300}{T^3}\right)\rho^2 RT + \left(0.13026 + \frac{103.26}{RT} - \frac{2,994,500}{T^3}\right)\rho^3 RT - \frac{528,110}{T^3}\rho^4 RT \qquad (1)$$

where:
P is the pressure in atmospheres;
ρ is the density in pound moles per cubic foot;
R is the gas constant, $R = 0.73024$(atm.cubic ft./lb.mol.°R.); and
T is the temperature in degrees Rankine.

The Beattie-Bridgeman equation has been well verified experimentally within the range of applicability discussed below. The pressure and temperature must be such that neither condensation nor dissociation occurs and the pressure should never exceed about 250 atmospheres (Ref. 2, p. 253). Hence, throughout the flow, the pressure and temperature values must remain within the area shaded in Fig. 1. Condensation would occur to the left of the region, dissociation would occur to the right of the region and the pressure would exceed 250 atmospheres above the region. The following tables are restricted to cases for which all values of the pressure and temperature throughout the entire flow process remain within the region of validity shown in Fig. 1.

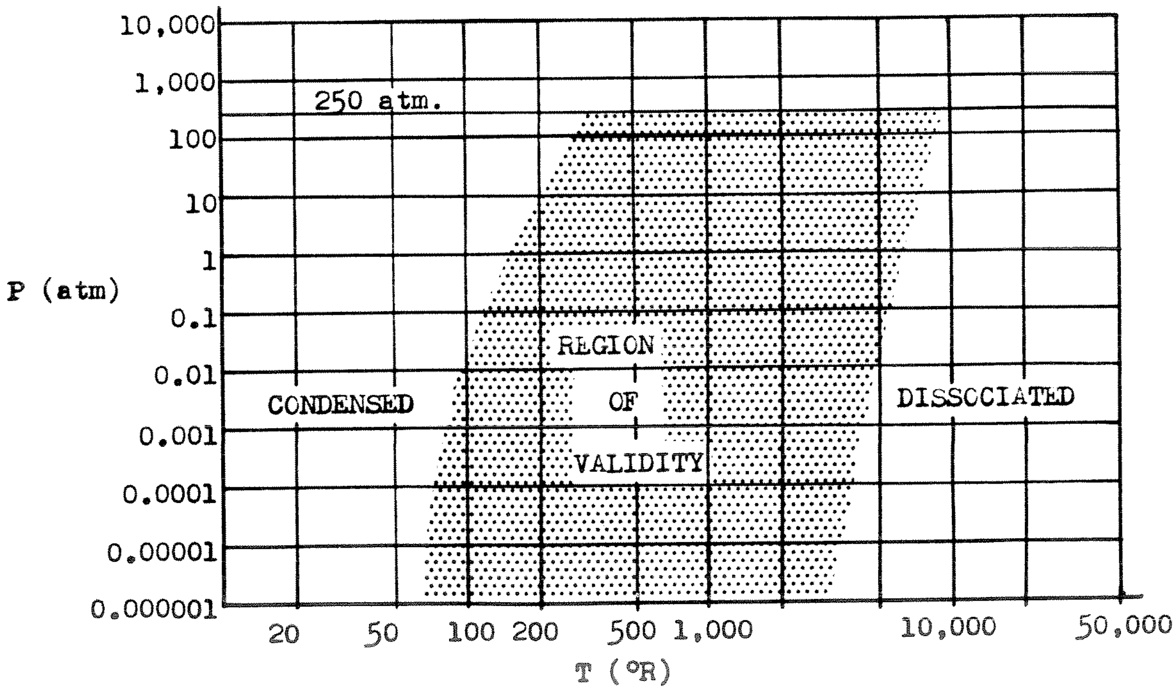

Figure 1. Range of Validity of Beattie-Bridgeman Gas Equation

CALCULATION PROCEDURE

The Beattie-Bridgeman equation and the compressible one-dimensional flow equations for isentropic expansion and normal shock waves are used in obtaining the following, numerical, iterative procedure used in calculating the tables herein. For given free-stream conditions of Mach number and static pressure and temperature the procedure yields: isentropic stagnation pressure and temperature; density ratio across a normal shock wave; and the Mach number, static pressure and temperature, and stagnation pressure and temperature downstream of the normal shock wave.

The following symbols and subscripts are used in the calculation procedure.
D is the density ratio across a normal shock wave, $D = \rho_1/\rho_2$
I, J, K and L, functions of P and T, are shown in Appendix I
M is the Mach number
P is the pressure in atmospheres
T is the temperature in degrees Rankine
$X = IM^2/(5J + IM^2)$

Subscripts
01 indicates free-stream stagnation conditions
1 indicates free-stream static conditions
02 indicates stagnation conditions behind a normal shock wave
2 indicates static conditions behind a normal shock wave

The subscripts on I, J, K and L indicate the P and T values to be used in calculating I, J, K and L. For example, I_2 is a function of only P_2 and T_2 and L_{01} is a function of only P_{01} and T_{01}.

1. P_1, T_1 and M_1 are given
2. I_1, J_1, K_1 and L_1 are calculated using the equations shown in Appendix I
3. X_1 is calculated using $X_1 = I_1M_1^2/(5J_1 + I_1M_1^2)$
4. T_{01} is calculated using $T_{01} = T_1(J_1 + 0.2I_1M_1^2)$
5. P_{01} is calculated using $P_{01} = \frac{P_1}{K_1}\left(\frac{T_{01}}{T_1}\right)^{3.5}$
6. J_{01} and K_{01} are calculated using the equations shown in Appendix I
7. A new T_{01} value is calculated using $\frac{T_{01}}{T_1} = \frac{J_1 + 0.2I_1M_1^2}{J_{01}}$

8. A new P_{01} value is then calculated using $\frac{P_{01}}{P_1} = \frac{K_{01}}{K_1}\left(\frac{T_{01}}{T_1}\right)^{3.5}$

9. New J_{01} and K_{01} values are calculated using the T_{01} and P_{01} values of steps 7 and 8 in the equations shown in Appendix I

10. Steps 7, 8 and 9 are iterated until succeeding pairs of values of J_{01}, K_{01}, P_{01} and T_{01} agree to four or more significant figures.

11. First approximations to the static pressure and temperature downstream of a normal shock wave are calculated using the perfect-gas-law flow equations

$$P_2 = (7M_1^2 - 1)\frac{P_1}{6} \qquad \text{and} \qquad T_2 = (7M_1^2 - 1)(M_1^2 + 5)\frac{T_1}{36M_1^2}$$

12. J_2 and L_2 are calculated using the equations shown in Appendix I

13. D is calculated using the following quadratic, the smaller of the two roots is used as the value of D

$$X_1(L_2 - 7)D^2 + X_1(7 - L_1)D + L_1D - L_2 = 0$$

14. X_2 is calculated using $X_2 = X_1D^2$

15. P_2 is calculated using $P_2 = \frac{P_1L_2(1 - X_2)}{DL_1(1 - X_1)}$

16. T_2 is calculated using $T_2 = \frac{T_{01}J_{01}(1 - X_2)}{J_2}$

17. Steps 12 through 16 are iterated until succeeding pairs of values of J_2, L_2, D, X_2, P_2 and T_2 agree to four or more significant figures

18. I_2 and K_2 are calculated using the equations shown in Appendix I

19. M_2 is calculated using $M_2^2 = \frac{5J_2X_2}{I_2(1 - X_2)}$

20. As a first approximation, T_{02} is calculated using $T_{02} = J_{01}T_{01}$

21. A first P_{02} value is calculated using $P_{02} = \frac{P_2}{K_2}\left(\frac{T_{02}}{T_2}\right)^{3.5}$

22. J_{02} and K_{02} are calculated using the equations shown in Appendix I

23. T_{02} is calculated using $T_{02} = J_{01}T_{01}/J_{02}$

24. P_{02} is calculated using $\frac{P_{02}}{P_2} = \frac{K_{02}}{K_2}\left(\frac{T_{02}}{T_2}\right)^{3.5}$

25. Steps 22, 23 and 24 are iterated until succeeding pairs of values of P_{02} and T_{02} agree to four or more significant figures

The results of the above calculation procedure: P_{01} and T_{01} from step 10; D, P_2 and T_2 from step 17; M_2 from step 19; and P_{02} and T_{02} from step 25; are tabulated for several sets of the given conditions P_1, T_1 and M_1.

USE OF TABLES

The one-dimensional flow characteristics calculated in the manner described above are tabulated in Tables I and II for several sets of given free-stream conditions corresponding to those encountered in free flight and those encountered in wind tunnels. Standard digital-computation machine notation for decimal point location is used in the tables; the notation is best explained by considering a few examples: $1.234\text{-}07 = 1.234 \cdot 10^{-7}$, $-5.432\ 02 = -5.432 \cdot 10^{2}$, $9.876\text{-}01 = 0.9876$. The tables cover the pressure-temperature region of validity for the Beattie-Bridgeman equation shown in Fig. 1. The first table lists free flight conditions for altitudes from sea level up to 310,000 feet. The ARDC Model Atmosphere (Ref. 6) is used to relate the altitude with the static pressure and temperature. At greater altitudes the air in the atmosphere is dissociated sufficiently to invalidate the use of the Beattie-Bridgeman equation (Ref. 7). The second table lists several sets of free-stream conditions covering those that would occur in supersonic or hypersonic wind tunnels. Some of the relatively small Mach number values are excluded from the second table because perfect-gas-law flow tables (Ref. 1) are valid for these cases. Free-stream Mach number increments of one tenth are used throughout both tables. The range of free-stream Mach-number values and the free-stream static pressure and temperature values used in the two tables are shown in Appendix II. It is contemplated that the usual use of the tables for the free-stream stagnation conditions will be in determining the stagnation conditions required for isentropic expansion to given static conditions and Mach number in the test section of a wind tunnel or else what static conditions may be obtained in a wind tunnel with given stagnation conditions.

Since the tables herein involve three independent variables rather than the single independent variable M_1 of perfect-gas-law flow tables (Ref. 1), it is necessary to use a more complex interpolation scheme if the values of flow characteristics are desired for intermediary values of those listed for the given conditions. Triple interpolation is generally necessary for three independent variables.

The first table relates the static pressure and temperature to altitude. Thus there are actually just two independent variables, altitude and free-stream Mach number, and so only double interpolation is necessary. An example is considered to demonstrate the procedure to be followed. Say it is desired to find the value of M_2 at an altitude of 86,000 ft. for $M_1 = 4.73$. The table shown below in Fig. 2(a) is completed by reading the indicated M_2 values directly from Table I. The first interpolation is then made with respect to M_1. At an altitude of 80,000 ft., ordinary interpolation indicates that, for $M_1 = 4.73$, the value of M_2 is: $M_2 = 0.4151 + 0.3(0.4129 - 0.4151) = 0.4144$.

Values of M_2

Alt. \ M_1	4.7	4.8
80,000	0.4151	0.4129
90,000	0.4146	0.4124

(a)

$M_1 = 4.73$

Altitude	M_2
80,000	0.4144
90,000	0.4139

(b)

Figure 2. Example of First Step of Double Interpolation

Similarly for 90,000 ft. the value of M_2 for $M_1 = 4.73$ is found to be 0.4139. These two calculated values of M_2 for $M_1 = 4.73$ are tabulated in a second table as shown in Fig. 2(b). Ordinary interpolation is then used to find the desired value of M_2 at an altitude of 86,000 ft. for $M_1 = 4.73$: $M_2 = 0.4144 + 0.6(0.4139 - 0.4144) = 0.4141$.

The second table, for wind tunnel conditions, has three independent variables and so triple interpolation is necessary. Say it is desired to find T_{01} for $P_1 = 0.005$, $T_1 = 160$ and $M_1 = 8.89$. The two tables shown below in Fig. 3(a) are completed by reading the indicated T_{01} values directly from Table II. Ordinary interpolation for the values

Values of T_{01} for $M_1 = 8.8$

P_1 \ T_1	150	175
0.003	2,308	2,654
0.010	2,293	2,636

Values of T_{01} for $M_1 = 8.9$

P_1 \ T_1	150	175
0.003	2,353	2,705
0.010	2,336	2,685

(a)

Values of T_{01} for $M_1 = 8.89$

P_1 \ T_1	150	175
0.003	2,349	2,700
0.010	2,332	2,680

(b)

$T_1 = 160$ and $M_1 = 8.89$

P_1	T_{01}
0.003	2,489
0.010	2,471

(c)

Figure 3. Example of First Two Steps of Triple Interpolation

of T_{01} for M_1 = 8.89 then yields the table shown in Fig. 3(b). The second interpolation, similar to the one discussed in the preceding paragraph, yields the table shown in Fig. 3(c). For example, the value of T_{01} for P_1 = 0.003, T_1 = 160 and M_1 = 8.89 shown in the table in Fig. 3(c) is calculated using: $T_{01} = 2{,}349 + \frac{10}{25}(2{,}700 - 2{,}349) = 2{,}489$. A third interpolation yields the desired result for the value of T_{01} for P_1 = 0.005, T_1 = 160 and M_1 = 8.89: $T_{01} = 2{,}489 + \frac{0.005 - 0.003}{0.010 - 0.003}(2{,}471 - 2{,}489) = 2{,}484$.

COMPARISONS WITH OTHER GAS LAW FLOW TABLES

The increased accuracy of the tables presented herein as compared to tables using the perfect gas law, both with and without correction factors, is illustrated below in Fig. 4. Two example sets of conditions, chosen at random, are considered; the first set, from Table I, is for sea level conditions and M_1 = 4.0; the second set, from Table II, is for P_1 = 0.001, T_1 = 500 and M_1 = 8.0. The perfect gas law results differ most from those obtained using the Beattie-Bridgeman gas law. Correction factors have been applied to the perfect gas law results (Ref. 1) but even these differ by as much as five percent from those obtained using the Beattie-Bridgeman gas law for the above examples.

	Altitude = 0 M_1 = 4.0			P_1 = 0.001 T_1 = 500 M_1 = 8.0		
GAS LAW	PERFECT	CORRECTED PERFECT	BEATTIE - BRIDGEMAN	PERFECT	CORRECTED PERFECT	BEATTIE - BRIDGEMAN
P_{01}	151.8	162.9	156.0	9.766	16.90	16.68
T_{01}	2,178	2,068	2,047	6,900	5,918	5,888
D	0.2188	0.2058	0.2067	0.1797	0.1493	0.1498
M_2	0.4350	0.4298	0.4291	0.3929	0.3678	0.3669
P_2	18.50	18.80	18.77	0.07450	0.07726	0.07713
T_2	2,099	2,009	1,999	6,695	5,764	5,778
P_{02}	21.07	21.21	21.20	0.08289	0.08415	0.08407
T_{02}	2,178	2,068	2,060	6,900	5,918	5,891

Figure 4. Comparison of Three Different-Gas-Law Flow Tables

CONCLUSIONS

One-dimensional flow parameters are tabulated herein using the Beattie-Bridgeman equation of state which includes the effects of molecular vibrational energy. These tables extend over the entire pressure-temperature region wherein the Beattie-Bridgeman equation is valid. The flow parameters presented are those most frequently required for analyses of supersonic and hypersonic air flows.

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APPENDIX I

I, J, K AND L CALCULATION PROCEDURE

The Beattie-Bridgeman equation in the form shown on page 2 yields the pressure in terms of the density and temperature. In aerodynamic studies the pressure and temperature are usually given rather than the density and temperature. Therefore the equation is rewritten to give the density in terms of the pressure and temperature (Refs. 3 - 5); this conversion leads to the functions I, J, K and L which are evaluated using the calculation procedure shown in this appendix. I, J, K and L are functions of only P and T that result from rewriting the gas law Eq. (1); thus the same subscript must apply to I, J, K and L as applies to P and T.

1. P and T are given
2. The inverse of the temperature, t, is calculated using $t = 1/T$
3. The seven functions a, b, c, d, f, g and h, functions of only t, are calculated using the following equations

$$a = 0.73859 + t(-457.18 - 4{,}054{,}300t^2)$$

$$b = 0.13026 + t(141.41 - 2{,}994{,}500t^2)$$

$$c = -528{,}100t^3$$

$$d = -1.3694(at)$$

$$f = 1.8753t^2(2a^2 - b)$$

$$g = 2.5680t^3\left[5a(b - a^2) - c\right]$$

$$h = 3.5167t^4\left\{3b^2 + a\left[6c + 7a(2a^2 - 3b)\right]\right\}$$

4. The density ρ, a function of P and T, is then calculated using

$$\rho = 1.3694Pt\left\{1 + P\left\{d + P\left[f + P(g + hP)\right]\right\}\right\}$$

5. The ratio of specific heats, γ, is calculated using

$$\gamma = 1 + \frac{0.068589\left\{(1 + 8{,}108{,}600\rho t^3)\left[1 + \rho(0.73859 + 0.13026\rho)\right]\right\}^2}{Y\left\{1 + \rho\left[2a + \rho(3b + 4c\rho)\right]\right\}}$$

where

$$Y = 0.17147 + 504{,}760\left(\frac{t}{\sinh 3033t}\right)^2 + 55{,}256\left(\frac{t}{\sinh 2007t}\right)^2$$

$$+ (\rho t^3)\left[1{,}668{,}500 + \rho(616{,}160 + 72{,}444\rho)\right]$$

6. The functions I, J, K and L are then calculated using

$$I = \frac{\gamma}{1.4}\left\{1 + \rho\left[2a + \rho(3b + 4c\rho)\right]\right\}$$

$$J = 1 + t\left[\frac{1,386.5}{e^{6066t} - 1} + \frac{229.37}{e^{4014t} - 1}\right] + \rho\left[0.21103 - 261.24t - 4,633,500t^3\right.$$
$$\left. + \rho(0.037216 + 60.604t - 4,277,800t^3)\right] - 301,770(\rho t)^3$$

$$K = \frac{1.3694Pt}{\rho e^Z}\left[\frac{e^{\frac{6066t}{e^{6066t} - 1}}}{1 - e^{-6066t}}\right]^{0.8}\left[\frac{e^{\frac{4014t}{e^{4014t} - 1}}}{1 - e^{-4014t}}\right]^{0.2}$$

where

$$Z = \rho(0.73859 + 8,108,600t^3) + \rho^2\left[0.065129 + t^3(2,994,500 + 352,070\rho)\right]$$

$$L = \frac{1}{J\left\{1 + P\left\{d + P\left[f + P(g + hP)\right]\right\}\right\}}$$

APPENDIX II

SETS OF INDEPENDENT VARIABLES FOR TABLES

The sets of the independent variables, P_1, T_1 and M_1, for which the flow characteristics are tabulated in Tables I and II are listed in this Appendix. The P_1 and T_1 values and the range of values of M_1 are shown; M_1 takes on all values increasing by increments of 0.1 in the range shown for each pair of P_1 and T_1 values.

SETS OF INDEPENDENT VARIABLES FOR TABLE I

ALTITUDE (ft.)	P_1 (atm.)	T_1 (°R.)	RANGE OF M_1
0	1.0000	518.69	1.1 thru 4.3
10,000	0.68783	483.04	1.1 " 4.6
20,000	0.45991	447.43	1.1 " 4.9
30,000	0.29754	411.86	1.1 " 5.3
40,000	0.18577	389.99	1.1 " 5.7
50,000	0.11512	389.99	1.1 " 6.1
60,000	0.071366	389.99	1.1 " 6.6
70,000	0.044264	389.99	1.1 " 6.9
80,000	0.027467	389.99	1.1 " 7.5
90,000	0.017149	402.48	1.1 " 8.0
100,000	0.010909	418.79	1.1 " 8.4
110,000	0.0070629	435.09	1.1 " 8.9
120,000	0.0046485	451.37	1.1 " 9.4
130,000	0.0031063	467.63	1.1 " 9.8
140,000	0.0021053	483.88	1.1 " 9.9
150,000	0.0014458	500.11	1.1 " 10.7
160,000	0.0010040	508.79	1.1 " 9.0
170,000	0.00069863	508.79	1.1 " 9.0
180,000	0.00048550	499.00	1.1 " 9.0
190,000	0.00033299	477.98	1.1 " 9.0
200,000	0.00022454	456.95	1.1 " 9.0
210,000	0.00014882	435.99	1.1 " 9.0
220,000	0.000096646	415.03	1.1 " 8.0
230,000	0.000061403	394.09	1.1 " 8.0
240,000	0.000038075	373.16	1.1 " 8.0
250,000	0.000022983	354.35	1.1 " 8.0
260,000	0.000013715	354.35	1.1 " 8.0
270,000	0.0000081880	354.35	1.1 " 6.9
280,000	0.0000048909	354.35	1.1 " 6.9
290,000	0.0000029229	354.35	1.1 " 6.9
300,000	0.0000017477	354.40	1.1 " 6.9
310,000	0.0000012585	360.48	1.1 " 6.9

SETS OF INDEPENDENT VARIABLES FOR TABLE II

RANGE OF M_1

T_1 \ P_1	0.10	0.03	0.010	0.003	0.0010	0.0003
100			4.0-9.0	5.0-10.9	6.0-11.9	7.0-15.0
125	4.0-6.0	4.0-6.9	"	"	"	7.0-14.7
150	"	"	"	"	"	7.0-14.7
175	"	"	"	"	"	7.0-14.0
200	"	"	"	5.0-10.5	"	7.0-14.0
250	"	"	"	5.0-10.5	"	7.0-12.0
300	"	"	4.0-8.9	5.0-10.0	"	7.0-11.0
350	"	"	4.0-8.9	5.0-10.0	6.0-11.0	7.0- 9.9
400	"	"	4.0-8.5	5.0- 9.0	6.0-10.0	7.0- 9.0
450	"	"	"	5.0- 9.0	6.0- 8.9	7.0- 8.0
500	"	"	"	5.0- 7.9	6.0- 8.0	

RANGE OF M_1

T_1 \ P_1	0.00010	0.00003	0.000010	0.000003	0.0000010
75	8.0-16.9	10.0-20.0	12.0-22.0	14.0-22.0	16.0-21.9
100	8.0-16.9	10.0-18.0	12.0-17.9	14.0-18.0	16.0-18.0
125	8.0-16.0	10.0-15.9	12.0-16.0	14.0-16.0	
150	8.0-16.0	10.0-15.9	12.0-14.9		
175	8.0-15.0	10.0-14.0	12.0-14.0		
200	8.0-13.9	10.0-12.9	12.0-13.0		
250	8.0-12.0	10.0-11.0			
300	8.0-10.9				
350	8.0- 9.0				
400	8.0- 9.0				