WHAT GOOD IS OPERATIONS RESEARCH AFTER AN EMERGENCY?

Laurence L. George
Lawrence Livermore National Laboratory
P. O. Box 808, L-140
Livermore, CA 94550 U.S.A.

What is Operations Research?

Operations Research is the application of mathematics to solve problems. The problems are "word problems", real problems converted into mathematical form and solved. The mathematics are "applications" according to real mathematicians.

Operations Research is needed after emergencies because there will be plenty of problems. There may not be much time to solve them, but Operations Research already has solutions to fit the problems.

Operations Researchers want to solve problems. They will be attracted to and challenged by problems that arise after emergencies because of their motivation to restore order, because the problems are different from everyday problems, and because of the challenge to get quick solutions.

Operations Research requires computers because the days of simple solutions to simple problems have gone. There are still simple solutions, but they are applied to so much data that computers must store inputs and execute the simple solutions. The availability of personal computers makes Operations Research convenient after emergencies.

This is how an Operations Researcher solves a problem. The Manager and Operations Researcher formulate the problem and describe alternatives. Then the Operations Researcher abstracts the problem into a mathematical or computer model, solves the model and suggests the solution to the Manager. That solution may not solve the manager's problem because it is a solution to an abstract model. With some iteration, the Manager and the Operations Researcher can usually improve the solution. They might even find alternatives that had been overlooked. This problem solving process can be employed during and after emergencies

The Operations Researcher usually forces the problem into a standard form and uses standard solutions. Some solutions are preprogrammed so that problem solving amounts to gathering the data, applying the standard solution and reporting the results. The difficulties lie in problem formulation and data collection.

Operations Research, Management Science and Systems Science are similar. The publications are similar. Table 1 lists the main topics of the core journals in each field. Table 2 describes some typical Operations Research problems.

How does Operations Research help after emergencies? The obvious answer is it solves problems. Some solutions are already available. They were obtained in the course of designing systems for emergencies (Section 2) or allocating resources after emergencies (Section 3). But applications are scarce judging from the small number of references. This is due to the infrequency of emergencies relative to the frequency of everyday problems.

For example, a characteristic of mathematical programming makes it an attractive problem solving method after emergencies. The first step in mathematical programming is to find feasible solutions. After an emergency, feasible solutions may be scarce and all that is needed.

Table 1: Divisions of Some Leading Journals

Operations Research (Area Editors)	Management Science (Departments)	International Journal of Systems Science
Mathematical Programming & Optimization	Mathematical Programming and Networks	Math. Modeling
Distribution Networks & Facilities	Logistics, Distribution & Inventory	Optimization
Production	Production and Operations Management	Industry
Simulation	Simulation	Simulation
Decision Analysis	Decision Analysis	Control
Stochastic Processes & Queuing	Applied Stochastic Processes	
Social Systems, Health & Service	Public Sector Applications	Bio.Systems
Natural Resources & Energy	Finance; Information Systems and Accounting	Environment
Reliability	Marketing; Organization Analysis	
Military Oper. Research	Planning and Design; R & D and Innovation; Planning, Forecasting and Applied Game Theory	

Table 2. Typical Problems in Some Divisions

1. Mathematical Programming
   Find the best alternative to maximize one function subject to constraints on other functions of the alternatives.
2. Probability and Stochastic Processes
   Find the probabilities of some events or find the values of some functions of these probabilities.
3. Simulation
   Build a computer model of the problem and see what happens to the model when we change it.
4. Game Theory
   Find strategies to satisfy some objective in competition with adversaries.
5. Multi-objective Decision Analysis
   Satisfy several objectives simultaneously as well as possible.

What Has Operations Research Been Used for Prior to Emergencies? Operations Research has been applied extensively in planning, design and operation of production, distribution and service industries. Some of these applications are planning for emergencies:

1. design power generation and transmission systems for reliability in case of earthquake or fire (54, 55, 62),
2. allocate resources, plan inventories, and distribute supplies (blood, food, etc.) (24, 44, 45, 47, 48, 65, 66, 67),
3. design queuing, service and communication systems to handle overload or withstand electromagnetic pulse (63, 67, 63),
4. estimate reliability of lifeline networks (16, 17, 18, 19, 20, 21, 22, 48),
5. construct models of contagion, epidemics and disasters for assessment of their effects (25, 26, 37, 38, 39, 43, 61, 68, 69),
6. locate and dispatch emergency services such as health, police, fire and shelter (1, 23, 28, 29, 30, 31, 33, 41, 42),
7. estimate insurance premiums and the value of life (53, 56, 57, 58, 59, 60),
8. plan fire control strategies (2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 26, 35, 36, 46),

The design for reliability of power, water and communication systems recently shifted emphasis. Systems have always been designed to avoid breakdown due to wearout or internal malfunction. But the need to withstand external shocks has been recently emphasized. Shocks could be earthquakes which shutdown power plants, knock over transmission towers, and break water mains. They could be electromagnetic pulses which scramble circuits. They could be storms which interrupt communications and cause damage. They could be fires inside nuclear power plants which can't be fought aggressively because of inaccessibility. Operations Research helps design for reliability in case of shocks and emergencies by estimating reliability of components and systems.

The typical model of component failure due to shock is that some load, mechanical, electromagnetic, or thermal, exceeds the component's capacity to withstand the load. The load and the component's capacity to withstand the load are modeled as random variables to represent inherent randomness. Component failure probability is

$$P[\text{Load} > \text{Strength}] = P[X > Y] = \int_0^\infty (1-F_Y(x))f_X(x)dx \qquad (1)$$

where $F_Y(x) = P[\text{Strength} < x]$ and $f_X(x)dx \cong P[x < \text{Load} < x+dx]$.

The probability a system survives (reliability) is the probability that some combinations of components survive. System reliability is not the product of component reliabilities because components are dependent. They are dependent because the component loads are all caused by the same external load.

For example, suppose two identical relays are used for redundancy in a circuit. Circuit failure probability is the probability both relays fail. Suppose the relays are in two different cabinets to reduce the probability that both are damaged by a fire. Suppose a fire occurs in the room containing the cabinets. Assume the relays fail if the peak temperatures in their cabinets exceed the capacities of their wiring insulations to withstand heat. Denote X_1 and X_2 the peak temperatures and Y_1 and Y_2 the capacities. Assume they are independent pairs of correlated normal random variables. The peak temperatures are correlated because they are due to the same fire. The probability of circuit failure increases with the correlation between X_1 and X_2. Incorrectly assuming X_1 and X_2 are uncorrelated underestimates circuit failure probability.

The recent shift in emphasis of reliability analysis has required that dependence be accounted for. This is done (64). In addition to handling dependence, the shift in emphasis requires that the effect of secondary threat following a shock, such as fire following earthquake, be accounted for.

For example, suppose earthquake load and subsequent fire can cause relay failures. Assume a relay fails if either the earthquake load exceeds the mechanical stsrength or if the fire temperature exceeds the insulation capacity. Let E and F denote earthquake and fire random variables. Then component failure probability is

$$P[X_1(E) > Y_1(E) \text{ or } X_1(F) > Y_1(F)]. \qquad (2)$$

If there is deterioriation due to earthquake, component failure probability is

$$P[X_1(E)>Y_1(E)] + P[X_1(F)>Y_1(F) \mid X_1(E)<Y_1(E)]\ P[X_1(E) < Y_1(E)], \qquad (3)$$

earthquake failure probability plus earthquake survival and fire failure probability. The probability $P[X_1(F) > Y_1(F) \mid X_1(E) < Y_1(E)]$ conditions fire failure on earthquake survival allowing earthquake deterioriation to affect fire failure probability.

What Good is Operations Research After an Emergency?

There are many Operations Research methods useful after an emergency:

1. Management of resources (50,65,52),
   a. perishable inventories (24),
   b. transportation and distribution (27, 29, 34, 40, 41, 65, 67),
   c. rationing (47),
   d. cannibalization of spare parts (44, 45),
   e. feasible resource allocation (66).
2. Models of contagion, epidemic, and fire,
   a. stochastic process models of epidemics (68),
   b. Markov process models of fire spread in 2D and 3D (26, 68),
3. Models of control of emergencies
   a. Markov decision processes for control of fires (4, 5, 6, 7, 8, 9, 11),
   b. dynamic lifeline analysis to construct the optimal network after damage (16, 17, 18, 19, 20, 21, 22),
   c. bottleneck transportation problems (27),
4. Search (70).

Because there are so many methods, I'll describe only feasible resource allocation.

The typical resource allocation problem is to find the levels of activities x_j, $j=1,2,\ldots,n$, to maximize some profit function where p_j is the profit per unit of activity j

$$z = \underline{p}^T x = \sum_{j=1}^{n} p_j x_j \tag{4}$$

subject to constraints

$$\sum_{j=1}^{n} a_{ij}\, x_j \leq b_i \quad i=1,2,\ldots m \tag{5}$$

$$x_j \geq 0. \tag{6}$$

The b_i are available resource quantities, the a_{ij} are the consumption rates of resource i per unit of activity j, and the product $a_{ij}\, x_j$ is the amount of resource i used by activity x_j. This problem is called a linear program. The constraints define a convex set in the vector space spanned by x. The set may be empty if resources are insufficient to satisfy intermediate production constraints.

For example, there may be a limited amount of unpolluted water which can be used in three activities, drinking by humans, watering agriculture, and drinking by livestock. The variable x_1 represents the amount of water for drinking by humans so its input-output coefficient $a_{11}=1$ and its profit may be assumed to be $p_1=1$. The variable x_2 represents the level of agricultural activity. Its input output coefficient a_{12} is the water consumption per unit of agricultural activity. The profit p_2 per unit of agriculture is greater than from livestock, p_3.

There may be other constraints which depend on population size that define the minimum levels of combinations of activities required for survival; e.g.

$x_1 \geq b_1$ (water required by population)

$a_{22}x_2 + a_{23}x_3 \geq b_2$ (calories required by population)

After an emergency, the problem may be to find a feasible set of activity levels to support the population.

The data for feasible resource allocation exists except for the amounts of resources and the population. As soon after an emergency as resource and population estimates become available, the feasibility problem can be solved and solutions recommended to resource managers. If no feasible solutions exist, sensitivity analyses can show which resources are most needed and how much.

Conclusions

Operations Research is ready and waiting to help after emergencies. One reason it has been used so little is no longer applicable, the lack of computer support. I recommend that emergency managers prepare the data, the programs, the operations researchers and themselves to use the support Operations Research can provide after emergencies.

References

1. D. L. Mitchell, An Optimization Study of Blast Shelter Deployment, Vol. I-III, Lambda, OCD-PS-66-113, 6609 AD--659377-9
2. W. S. Jewell and R. W. Shephard, Operations Research in Forest Fire Problems, National Academy of Sciences - National Research Council Pub. 949, O.R. Center #172-19, U.C. Berkeley (1961).
3. P. Casamajor, Wildland Fire Problems, O.R. Center #172-26, U. C. Berkeley, (1962).
4. G. Parks, Analytical Models for Attack and Control of Wildland Fires, O.R. Center #63-6, U.C. Berkeley, (1963).
5. R. J. Swersey, "Parametric and Dynamic Programming in Forest Fire Control Models," Ops. Res. 12(5), 781-783, (1964).
6. G. Parks, Computer Simulation of the Attack and Control of Wildland Fires, O.R. Center #63-16*, U. C. Berkeley, (1963).
7. D. P. Heyman, Least Cost Allocation of Fire Suppression Equipment, O.R. Center #64-4, U.C Berkeley, (1964).
8. G. Parks, "Mathematical Investigation of The Attack and Control of Wildland Fires," Man. Sci., 10(4), (1964).
9. G. Parks, "The Development and Application of an Analytical Model for Initial Attack of Wildland Fires," Man. Sci., 10(4), 760-766, (1964).
10. W. S. Jewell, Forest Fire Fighting Models, O. R. Center #65-29*, U. C. Berkeley, (1965).
11. A. McMasters, Wildland Fire Control with Limited Suppression Forces, O. R. Center #66-5, (1966).
12. R. Chandrasekaran and R. W. Shephard, "Optimal Strategies of Fire Fighting by Fire Breaks", Proc. 4th International Conf. on Ops. Res., Boston, (1966).

13. R. Chandrasekaran, Optimal Control Policies for Fire Fighting in Fire Breaks, O. R. Center #67-35, U. C. Berkeley, (1967).
14. C. R. Glassey and R. M. Karp, Optimal Search for Pollution Sources, O. R. Center #72-1, U. C. Berkeley, (1972).
15. G. Aumann, Catastrophes on Lattices, O. R. Center #77-4, U. C. Berkeley, (1977).
16. R. E. Barlow, A. Der Kiureghian and V. Satyanarayana, "New Methodologies for Analyzing Pipeline and Other Lifeline Networks Relative to Seismic Risk", Proc. ASME Century II Pressure Vessels and Piping Conf., San Francisco, O. R. Center #80-5*, U. C. Berkeley, (1980).
17. V. Satyanarayna, Multi Terminal Network Reliability, O. R. Center #80-6*, U. C. Berkeley, (1980).
18. V Satyanarayana and J. N. Hagstrom "A New Formula and an Algorithm for the Reliability Analysis of Multiterminal Networks", IEEE Trans. on Rel, R-30(4), 325-334, (1981).
19. R. K. Wood, Efficient Calculation of the Reliability of Lifeline Networks Subject to Seismic Risk, O. R. Center #80-13, U. C. Berkeley, (1980).
20. V Satyanarayana "A Unified Formula for the Analysis of Some Network Reliability", IEEE Trans. on Rel., (R-31), O. R. Center #81-8*, U. C. Berkeley, (1982).
21. Panoussis, G., Seismic Reliability of Lifeline Networks, No. R74-57, Department of Civil Engineering Research, Massachusetts Institute of Technology, Cambridge, Massachusetts, (1974).
22. Taleb-Agha, G., Seismic Risk Analysis of Lifeline Networks, No. R75-49, Department of Civil Engineering Research, Massachusetts Institute of Technology, Cambridge, Massachusetts, (1975).
23. G. O. Weslovs and R. F. Love, "The Optimal Location of New Facilities Using Rectangular and Distances", Opns. Res. (19)1, 124-130, (1971).
24. S. Nahmias, "Perishable Inventory Theory: A Review", Opns. Res. (30)4, 680-708, (1982).
25. W. S. Jewell and A. B. Willoughby, A Study to Analyze and Improve Procedures for Fire Damage Assessment Following Nuclear Attack, Parts I & II, AD 286943, (1960).
26. P. V. Phung and A. B. Willoughby, Prediction Models for Fire Spread Following Nuclear Attacks, AD 613359, (1965).
27. U. Derigs "On Three Basic Methods of Solving Bottleneck Transportation Problems", Nav. Res. Log. Quart, (29)3, 505-515, (1982).
28. Alexander, Michael N., J. Brooks Ferebe, Paul J. Grim, Leonard S. Lebow, Stephen D. Senturia, and Ann M. Singleterry. "Effect of Population Mobility on the Location of Communal Shelters," Opns. Res. (6), 207-231.
29. Kolesar, Peter, and Edward H. Blum. "Square Root Laws for Fire Engine Response Distrances, Management Sci., (19), 207-231.
30. Toregas, Constantine, Charles ReVelle, and Ralph Swain, "Reply to Rao's 'Note on the Location of Emergency Service Facilities'", Opns. Res. (22), 1262-1267.
31. Toregas, Constantine, Ralph Swain, Charles ReVelle, , and Lawrence Bergman, "The Location of Emergency Service Failities", Opns. Res. (19), 1363-1373.

32. Bentz, Richard, et al. "Some Civil Defense Problems in the Nation's Capital Following Widespread Thermonuclear Attack", Opns. Res. (5), 319-350.
33. Kelleher, Grace J. "A Damage-Limiting Shelter-Allocation Strategy", Opns. Res. (15), 211-220.
34. Faulkner, "Determining Ship Routes to Minimize Fallout Dosage", Opns. Res. (10), 799-807.
35. Chaiken, Jan M., and Richard C. Larson," Methods for Allocating Urban Emergency Units: A Survey", Management Sci. (19), 110-130.
36. Valinsky, David, "A Determination of the Optimum Location of Fire-Fighting Units in New York City", Opns. Res. (3), 494-512, (1955).
37. Wilkins, C. A., J. A. Shaw. and A. Keane., "A Comparison of Estimates of Nuclear Bomb Casualities From Two Different Urban Models", Opns. Res. (17), 923-925.
38. Rivas, J. Roberto, and Dale F. Rudd., "Man-Machine Synthesis of Disaster-Resistant Operations", Opns. Res. (23), 2-21, (1974).
39. Everett, Hugh, III, and George E. Pugh. "The Distribution and Effects of Fallout in Large-Weapon Campaigns", Opns. Res. (7), 226-248, (1959).
40. Stearns, S. D., "A Mathematical Model for Strategic Movement", Opns. Res. (12), 239-254, (1964).
41. Kolesar, Peter, Warren Walker, and Jack Hausner, "Determining the Relation Between Fire-Engine Travel Times and Travel Distances in New York City", Opns. Res. (23), 614-627, (1975).
42. Kolesar, Peter, and Warren E. Walker., "An Algorithm for the Dynamic Relocation of Fire Companies", Opns. Res. (22), 249-274, (1974).
43. Hunter, Jeffrey J., An Analytical Technique for Urban Casualty Estimation from Multiple Nuclear Weapons", Opns. Res. (15), 1096-1108, (1967).
44. R. M. Simon "The Reliability of Multi-component Systems Subject to Cannibalization " Nav. Res. Log. Quart. (19)1, 1-14, (1972).
45. D. Khalifa, V. M. Hottenstein and S. Aggrawal, "Cannibalization Policies for Multistate Systems", Ops. Res. (25)6, 1032-1039, Nov. (1977).
46. M. Stade, "Cost Effectiveness of Water Bombers in Forest Fire Control", Can. Ops. Res. J. (5)1, 1-18, March (1967).
47. A. Kaplan, "Stock Rationing". Man. Sci. Theory, (15)5, 260-267, (1969).
48. Aneja, Y. P. and K. P. K. Nair, "Multicommodity Network Flows with Probabilistic Losses", Man. Sci., (28)9, 1080-1086, (1983).
49. Peterson, D. W. et. al, Development of a Dynamic Model to Evaluate Economic Recovery Following a Nuclear Attack, Vol. I & II, Pugh-Roberts Assoc., Inc., ADA 098 587 and ADA 098 588, (1980).
50. Hill, G. A. et al, Managing the U. S. Economy in a Post Attack Environment: A System Dynamics Model of Viability, Vol. II, Analytical Assessments Corp., ADA 087708, Oct. (1979).
51. J. B. Hallan, "A Simulation Model of an Emergency Medical System", Proc. Symp. on Post Attack Recovery from Nuclear War, 67-91, AD672 790, (1967).
52. Bickley, L. J. and E. S. Pearsall, "Estimates of the Potential of the United States Economy Following a Nuclear Attack", 245-273, Proc. Symp. on Post Attack Recovery from Nuclear War, 67-91, AD672 790, (1967).

53. Seal, H. L., Stochastic Theory of a Risk Business, Wiley, New York, (1969).

54. George, L. L. and J. E. Wells, "The Reliability of Systems of Dependent Components", Proc. ASQC Quality Congress, San Francisco, (1981).

55. George, L. L. and J. E. Wells, "Loss of Load Probability for Systems of Dependent Transmission and Generation Components", Proceedings, Reliability Conference for the Electric Power Industry, IEEE, Portland, April (1981).

56. Ramachandran, G. "Extreme Value Theory and Earthquake Insurance", Trans. 21st Int'l Congress of Actuaries, (I), 337-355, Zurich and Lausanne (1980).

57. Ramachandran, G., "Extreme Value Theory and Fire Insurance", Trans. 21st Int'l Congress of Actuaries, (III), 695-707, Zurich and Lausanne, (1980).

58. Johansen, P. "Mathematical Models Regarding Fire and Consequential Loss", Skandinavian Actuarial Journal, 229-234, (1978).

59. Lemaire, J. "Exchange de Risques Entre Assureurs et Theoree des Jeux", ASTIN Bulletin, (9), 181-190, (1977).

60. Ramachandran, G. "Extreme Value Theory and Large Fire Losses", ASTIN Bulletin, (7) 293-310, (1975).

61. Shpilberg, D. C. "The Probability Distribution of Fire Loss Amount", J. of Risk and Insurance, (64)1, 103-115, (1977).

62. Morrison, G. N., J. M. Kallis, L. A. Stratton, I. R. Jones, and A. L. Lena, RADC Thermal Guide for Reliability Engineers, RADC-TR-82-172, Rome Air Development Center, AFSC, Griffiss AFB, NY 13441, June (1982).

63. Bevensee, R. M., H. S. Cabayan, F. J. Deadrick, L. C. Martin and R. W. Mensing, Characterization of Errors Inherent in System EMP Vulnerability Assessment Programs, UCRL-52954, Lawrence Livermore National Laboratory, Livermore, CA 94550, (1980).

64. Wells, J. E., L. L. George, and G. E. Cummings, Seismic Safety Margins Research Program, Phase 1 Final Report, Systems Analysis (Project VII), NUREG/CR2015, Vol. 8, UCRL-53021 Vol. 8, Lawrence Livermore National Laboratory, Livermore, CA, (1981).

65. Hillier, F. S. and G. J. Lieberman, Operations Research, Holden-Day, San Francisco, (1967).

66. Dantzig, G. B., Linear Programming and Extensions, Princeton University Press, (1963).

67. Gaver, D. P., J. P. Lehoczky and M. P. Perlas, "Service Systems with Transitory Demand", in Logistics, Vol. 1, M. A. Geisler, Editor, North Holland/American Elsevier, New York, (1973)

68. Cox, D. R. and H. D. Miller, The Theory of Stochastic Processes, Wiley, New York, (1965).

69. Schmidt, Jr, L. A., A Parametric Study of Probabilistic Fire Spread Effects, Inst. for Defense Analyses Paper P-1372, AD-e500113, (1979).

70. Stone, L. D. The Process of Search Planning: Current Approaches and Continuing Problems", Opns. Res. 31(2), pp207-233, (1983).

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