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**A STUDY OF THE PRACTIBILITY OF ACTIVE
VIBRATION ISOLATION APPLIED TO
AIRCRAFT DURING THE TAXI CONDITION**

CHARLES D. CORSETTI, 1/LT, USAF
JAMES D. DILLOW, MAJOR, USAF

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FOREWORD

This report describes the results of a joint Air Force Flight Dynamics Laboratory-Air Force Institute of Technology investigation into the feasibility of using active landing gear control to reduce fatigue damage to an aircraft due to ground induced vibration during taxiing. The report is based on a Masters thesis project undertaken by Charles Corsetti in partial fulfillment of the requirements for a Master of Science degree at the Air Force Institute of Technology. This research effort was sponsored by James Dillow of the Air Force Flight Dynamics Laboratory. With the exception of Appendix F, this report is essentially the same as Charles Corsetti's Master thesis for the Air Force Institute of Technology, of the same title, GGC/EE/71-6, June 1971.

The work was performed under Project 8219, Task 821904, Work Unit 027. This project is a joint Air Force Flight Dynamics Laboratory and Air Force Institute of Technology investigation of problems relating to control theory and pilot-vehicle analysis. It is primarily used to identify laboratory supported research by Air Force Institute of Technology graduate students and faculty. The basic material given in this report was prepared over the period of September 1970 through June 1971.

Several Air Force Flight Dynamics Laboratory personnel contributed to this effort: R. O. Anderson, B. H. Groomes, and V. R. Schmitt provided consultation, guidance, and assistance; J. N. Marble provided invaluable support in the digital computer efforts required to perform the study and made available the required computer time on the Laboratory's CDC 1604B digital computer.

Russell Hannen, Air Force Institute of Technology, was Charles Corsetti's thesis advisor.

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This technical report has been reviewed and is approved.



C. B. WESTBROOK
Chief, Control Criteria Branch
Flight Control Division
Air Force Flight Dynamics Laboratory

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ABSTRACT

The feasibility of using an active control in the landing gear system of an aircraft to reduce wing fatigue damage resulting from ground induced vibrations during taxiing is considered. The characteristics of three vehicle models are discussed: a single landing gear system, a tricycle landing gear system, and a system of five landing gears. Mathematical expressions for the runway inputs to each vehicle model are obtained in the form of random inputs represented by Gauss-Markov processes. The model for a linear hydraulic actuator, which is used as the active control element in the landing gear system, is presented.

The approach used in the study is to determine an optimal control law which is a proportional feedback of the measurements. The measurements, in turn, are assumed to be both a linear transformation of the states and noiseless. The feedback gains in the optimal control law are obtained in such a way as to minimize a cost criterion, which is a measure of the controller's ability to reduce wing fatigue resulting from runway imposed vibrations. The methodology for obtaining the optimal solution for the given cost criterion is developed and solutions for the three different models and for various measurement schemes are obtained.

The results indicate that the combined optimal active control and landing gear system can provide a substantial improvement in reducing wing fatigue over that of the landing gear system alone. Also, the control parameters that are necessary and desirable in the optimal system, together with the physical demands placed on the actuator, are determined.

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LIST OF SYMBOLS

SYMBOL	DESCRIPTION
A	$A = F + \psi KH$ (Chapter VI).
A_o	Runway spectral density amplitude, (in. ² /rad/ft).
A_p	Area of the main piston of actuator, in. ²
C_b	$C_b = A_p$ (in this study), in. ²
C_{fg}	Aerodynamic damper on M_f , lb-sec/in.
C_p	$C_p \equiv \left(- \frac{\partial q}{\partial P_L} \right)_o$, (in. ⁵ /lb-sec).
C_{sgj}	Aerodynamic damper on M_{sj} , lb-sec/in. (Subscript j omitted in Single and Tricycle Gear Model)
C_{sj}	Damping coefficient between M_{sj} and M_f , lb-sec/in. (Subscript j omitted in Single and Tricycle Gear Model)
C_x	$C_x \equiv \left(\frac{\partial q}{\partial x_A} \right)_o$, (in. ³ /sec-ma).
D_i	Coefficient of linearized damping term, i th gear, lb-sec/in. (Subscript omitted in Single Gear Model)
$E(\cdot)$	Expected value operator.
e_j	Distance along x-axis from the center of gravity to j th elastically connected mass (+ forward), in. (Subscript omitted for Tricycle Gear Model)
F	nxn constant state matrix.
F_{Ai}	Force developed by actuator located in i th strut, in/sec. (Subscript i omitted for Single Gear Model)
G	n x p constant disturbance matrix.
H	rxn constant measurement matrix.
$h_i(t)$	Runway vertical amplitude, i th gear, in. (Subscript omitted for Single Gear Model)
I_{yy}	Total amount of inertia of airplane in pitch about the center of gravity, lb-sec ² -in.

LIST OF SYMBOLS (CONTD)

SYMBOL	DESCRIPTION
I_{xx}	Total moment of inertia of airplane in roll about the center of gravity, lb-sec ² -in.
J	Cost function.
K	m _{xr} feedback gain matrix.
K_{ai}	Linearized strut air spring constant, i th gear, lb/in. (Subscript i omitted in Single Gear Model)
K_B	Bulk modulus of the hydraulic oil in actuator, lb/in. ²
K_{sj}	Spring constant for the elastically connected mass, M_{sj} , lb/in. (Subscript j omitted in Single and Tricycle Gear Models)
K_{ti}	Linear tire spring constant, i th gear, lb/in. (Subscript i omitted in Single Gear Model)
L	Leakage coefficient of entire actuator system, in. ⁵ /lb-sec.
L_{wj}	Distance along y-axis from center of gravity to j th elastically connected mass (+ outward on right side of aircraft), in.
L_{xi}	Distance along x-axis from center of gravity to i th landing gear attachment point (+ forward), in.
L_{yi}	Distance along y-axis from center of gravity to i th landing gear attachment point (+ outward on right side of aircraft), in.
M_f	Main mass of linearized model, lb-sec ² /in.
M_{sj}	The j th elastically connected mass, lb-sec ² /in. (Subscript j omitted for Single and Tricycle Gear Models)
M_{ui}	Unsprung mass of i th landing gear, lb-sec ² /in. (Subscript i omitted for Single Gear Model)
P	n _{xn} covariance matrix (Chapter VI).
P_L	Load induced pressure developed by actuator, lb/in. ²
Q	Constant positive semidefinite p _{xp} matrix (Chapters I and VI).
q	Total flow of fluid into the cylinder of the actuator, in. ³ /sec.
R	Symmetric m _{xm} positive definite matrix (Chapter VI).

LIST OF SYMBOLS (CONTD)

SYMBOL	DESCRIPTION
R_C	Scalar constant, weighting on control.
s	Laplace operator.
t	Time
\underline{u}	m vector, the control.
u_i	Actuating signal to actuator located in i^{th} strut, ma. (Subscript omitted for Single Gear Model)
V	Effective volume of fluid under compression in actuator, in. ³
V_{Ai}	$V_{Ai} = \dot{y}_{Ai}$, velocity of actuator located in i^{th} strut, in./sec. (Subscript i omitted for Single Gear Model.)
V_H	Horizontal velocity of aircraft, fps.
\underline{x}	n vector, the state of the system models.
x_A	Displacement of the valve of the actuator, in.
$\overline{\underline{x}^T \underline{bb}^T \underline{x}}$	Variance of stress per wing, single and tricycle gear model, in. ²
$\frac{1}{2} \overline{\underline{x}^T \underline{bb}^T \underline{x}}$	Variance of stress per wing, five gear model, in. ²
\underline{y}	r vector, the measurement.
y_{Ai}	Displacement of actuator located in i^{th} strut, in. (Subscript i omitted in Single Gear Model)
\underline{z}	State vector of the vehicle model.
z_f	Vertical displacement of M_f , in.
z_{sj}	Vertical displacement of M_{sj} , in. (Subscript j omitted in Single and Tricycle Gear Models)
z_{ui}	Vertical displacement of M_{ui} , in. (Subscript i omitted in Single Gear Model)
θ	Angular displacement in pitch about the airplane center of gravity, rad.

LIST OF SYMBOLS (CONTD)

SYMBOL	DESCRIPTION
ξ	p vector, the disturbance.
ϕ	Angular displacement in roll about the airplane center of gravity, rad.
Φ	Airfield power spectral density.
ψ	$n \times m$ constant control matrix.
Ω	Spatial frequency, rad/ft
ω	Time frequency, rad/sec.
(')	Derivative with respect to time, d/dt.
(T)	Matrix transpose.
*	Denotes optimal value.

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SECTION I

INTRODUCTION

1. BACKGROUND

The combination of increased structural flexibility and increased aircraft size has produced unique problems with respect to the structural fatigue life of the large flexible aircraft. It is commonly recognized, therefore, that structural design philosophy for aerospace vehicles should include considerations of fatigue life as well as concepts of critical maximum loading. In this respect, the effects of runway unevenness on the design and operation of airplanes has been subject to scrutiny for a number of years, and considerable attention has been paid to the rational determination of ground induced loads and associated design criteria. As a result of enormous attention to the loads developed during landing, it now appears that the majority of the unsolved problems are concerned with taxiing or ground operations, since present aircraft design criteria are usually somewhat arbitrary regarding taxi loads.

2. PROBLEM

The problem considered in this study is to determine an optimal active control capable of reducing wing fatigue damage resulting from ground induced vibrations experienced by an aircraft during the taxi condition. This reduction in fatigue damage must be achieved while retaining existing in-flight handling qualities of the aircraft. Therefore attention will be focused upon improving the landing gear system of the aircraft which, although useless in flight, is designed purely to resist ground loads.

Modern aircraft landing gear systems employ oleo-pneumatic shock struts to isolate the aircraft from runway imposed shock and vibration. These struts are designed for maximum efficiency at the initial point of landing where rather large initial conditions of velocity are encountered which in turn generate rather high rates of strut closure. However, anticipated strut velocities are considerably lower for normal terrain

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operational conditions, with the result that strut orifices are usually too large for effective damping during ground operations.

The deficiency indicated above arises because the characteristics of the landing gear system are fixed or passive and cannot vary during operation. Insertion of an active control into the landing gear system appears to offer great advantages. Such an active control may consist of a hydraulic, pneumatic, electromagnetic, or other forcing transducer which would be placed between the aircraft and the unsprung mass of the landing gear system, and could be controlled so as to improve and even optimize the performance of the landing gear system in reducing wing fatigue resulting from runway shock and vibration.

3. DEFINITIONS

a. Active Control

An active control is characterized by the fact that it is subject to some kind of external control.

b. Optimal Active Control

An optimal active control is the best possible active control capable of minimizing a performance criterion.

4. OBJECTIVE

The objective of the study is to determine whether the combined optimal active control and landing gear system provide a substantial improvement in reducing wing fatigue over that of the landing gear system alone. Also, the control parameters that are necessary and desirable in the optimal system, together with the physical demands placed on the controlling element, must be determined.

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5. SUMMARY OF CURRENT KNOWLEDGE

The principal field of study in which this problem may be classified is known as active shock and vibration isolation. Despite the extensive literature in the field of vibration isolation and vehicle suspension systems in general (Reference 17), references dealing with the problem of optimizing the performance of vibration isolation systems or suspensions are limited. Of course, the response of the normal linear vibration isolator model to sinusoidal and other common types of inputs is well established (Reference 3), and the choice of parameters to minimize various constraints or performance indicators has been discussed by several authors (References 3 and 20). Various parametric studies for linear passive suspensions with statistically described guideway unevenness and simulations using nonlinear passive suspensions with measured elevations have been studied. Most of these studies assume some form of nonlinear characteristic for one of the suspension elements and then vary the nonlinear characteristics to optimize a system using this element (References 8 and 9). Various performance feedback schemes have been used to control the characteristics of adaptive or active control elements.

Although many improvements in certain aspects of suspension performance have been achieved by studies such as those referenced in the foregoing, the literature pertaining to the fundamental limitations of suspension performance and general optimization schemes is relatively sparse. This is particularly true in the design of the landing gear of an aircraft which demonstrates the advance in the technology of vibration isolation. One study in this area, of which this study is a continuation, is given in (Reference 5).

6. SUB-PROBLEMS

Three sub problems are considered in this study:

1. Establishing a vehicle model which will adequately represent the combined aircraft and landing gear system.

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2. Establishing an input model which will adequately represent the runway input to the vehicle model.

3. Establishing an actuator model which will adequately represent the dynamics of the active control element used in the landing gear system.

With respect to the first sub-problem, in order to adequately represent the action of the combined aircraft and landing gear structure the vehicle model must account for the stresses experienced by the wings and the effects of aircraft pitch and roll. Since all forces due to ground induced loads will be assumed to act only in the vertical direction, the effects of yaw are not considered. Three vehicle models were studied: a single landing gear system, a tricycle landing gear system, and a system of five landing gears.

The single landing gear system was developed by the Lockheed-Georgia Company and represents the landing gear system for the C-130 aircraft. The results of the report published under Air Force contract (Reference 19) indicate that linearization of the landing gear system is possible provided horizontal velocities of the aircraft during the taxi condition are less than 88 fps. The linearized three mass system for this model yields a three degree of freedom model. While this model accounts for the stresses experienced by an elastically connected mass, such as the mass of a wing station of the aircraft, it does not permit the study of the effects of pitch and roll.

The tricycle landing gear system represents an extension of the single landing gear system of the example airplane (C-130). The linearized five mass system for this model yields a six degree of freedom model. The model accounts for both the stresses experienced by an elastically connected mass and the effects of pitch angle. It does not permit the study of the effects of roll.

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Finally, the five landing gear model represents an extension of the tricycle landing gear model in which the effects of roll may be studied. Thus, the five landing gear model permits the study of the motion of the example aircraft in its entirety. The linearized eight mass system for this model yields a ten degree of freedom model.

The second sub-problem considered in this study involves the determination of a Gauss-Markov model which will adequately represent the runway inputs to the vehicle models mentioned above. This problem is considered in Chapter III. Here it is shown that the runway input to the single landing gear system may be modelled by a linear first order stochastic differential equation driven by Gaussian white noise. Appropriate extension of this model, employing the use of Pade approximations to time delays, results in a Gauss-Markov model which describes the runway inputs to the tricycle landing gear system. Finally, a method is proposed for correlating the runway inputs to the nose gear and the main gears of the five landing gear system, resulting in a Gauss-Markov model which describes the inputs to that system.

The final sub-problem considered in this study involves the determination of an actuator model which will adequately represent the dynamics of the active control element used in the landing gear system. The problem is considered in Chapter IV, where the model for a linear hydraulic actuator is presented. The model takes into account the effects of external load reactions on the actuator's dynamics, the pressure drop across the orifice of the actuator, the leakage of oil around the piston, and the compressibility of the oil. A linear first order differential equation results which relates the force developed by the actuator to the actuating signal.

7. APPROACH

In Chapter VI of the study the above models are appropriately combined to yield three systems models each of whose dynamical behavior may be described by a Gauss-Markov process of the form:

$$\dot{\underline{x}} = \underline{F}\underline{x} + \underline{\psi}\underline{u} + \underline{G}\underline{\xi} \quad (1)$$

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where \underline{x} - n vector, the state

F - nxn constant matrix

G - nxp constant matrix

Ψ - nxm constant matrix

\underline{u} - m vector, the control

$\underline{\xi}$ - p vector, the disturbance

and

$$E[\underline{\xi}(t)] = 0$$

$$E[\underline{\xi}(t_1) \underline{\xi}^T(t_2)] = Q\delta(t_1 - t_2)$$

where Q is a constant positive semidefinite matrix.

It is then assumed that a measurement model exists of the form

$$\underline{y} = H\underline{x} \quad (2)$$

where \underline{y} - r vector, the measurement

H - rxn constant matrix.

It is noted that Equation 2 assumes a noiseless measurement model, and therefore places a limitation on the types of measurements that may be made.

The approach to be used in this study is to use a control law which is a proportional feedback of the measurement, i.e.

$$\underline{u} = K\underline{y} \quad (3a)$$

or

$$\underline{u} = KH\underline{x} \quad (3b)$$

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where the elements of the K matrix are obtained in such a way as to minimize a cost criterion, which is a measure of the controller's ability to reduce wing fatigue resulting from runway imposed shock and vibration. Therefore, the control law given in Equation 3 is an optimal control. It is noted that the structure of the controller in Equation 2 is fixed a priori and its parameters are then adjusted to optimize a relevant set of performance characteristics. Hopefully, this will facilitate the optimization and lead to a simple and efficient control law. The mathematical development proceeds as given in Chapter VI of the study.

After the problem is presented in its mathematical formulation, a Newton-Raphson (second variation) method was implemented on the digital computer to obtain the feedback matrix K for the different system models and for various measurement schemes.

8. SCOPE

From the above study, a useful insight into the physical demands placed on the actuator for the optimal control system may be obtained. These demands would include the requirements on force, power, actuating signal, displacement, and velocity that must be supplied by the actuator. Therefore, attention will be directed at obtaining data on these requirements from the various models used in the analysis. This data will provide the designer with a tool for a rational comparison of the performance of a landing gear system with that of the best possible system. Thus, from the data, the designer can assess the practical utility of trying to improve the performance of any given concept or searching for other designs which would approach or actually duplicate the performance of the best possible system.

SECTION II

VEHICLE MODELI. SINGLE LANDING GEAR SYSTEMa. General Description

The linearized vehicle model used in the single landing gear analysis was adopted from (Reference 19). This linear model, shown in Figure 1, represents a three degree of freedom system which was developed from the nonlinear differential equations governing the dynamics of the aircraft and the landing gear system. These equations included the rigid modes of pitch and translation, coupled with four flexible wing modes, and up to five landing gears for ground loads application. Four nonlinear properties of the landing gear struts were incorporated: polytropic air compression force accounting for two nonlinear properties, strut damping force, and strut friction force. The effects of change in wing lift, non-steady aerodynamic damping, and structural damping were considered, although down-wash effects and tail aerodynamics were neglected.

b. Physical Characteristics of the System

The physical characteristics of the system are best explained in terms of the equivalent three mass system shown in Figure 2. While these characteristics are here being applied to the single landing gear system, similar characteristics may be defined for the multi-gear analyses which are to follow in subsequent sections of this study.

Each of the three masses shown in Figure 2 is assumed to have freedom only in vertical translation designated by a coordinate z_s , z_f , or z_u . The main mass of the system, M_f , corresponds to the landing gear attachment location on the airplane. The elastically connected mass is referred to as M_s , with a spring constant K_s , and a damper C_s between M_s and M_f as illustrated. The steady-state lift force, L , of the aircraft must be considered as applied to both the masses, M_f and M_s , by forces L_f and L_s . The combined structural and aerodynamic damping effects are accounted for through the parameters C_s , C_{sg} , and C_{fg} . Thus, the

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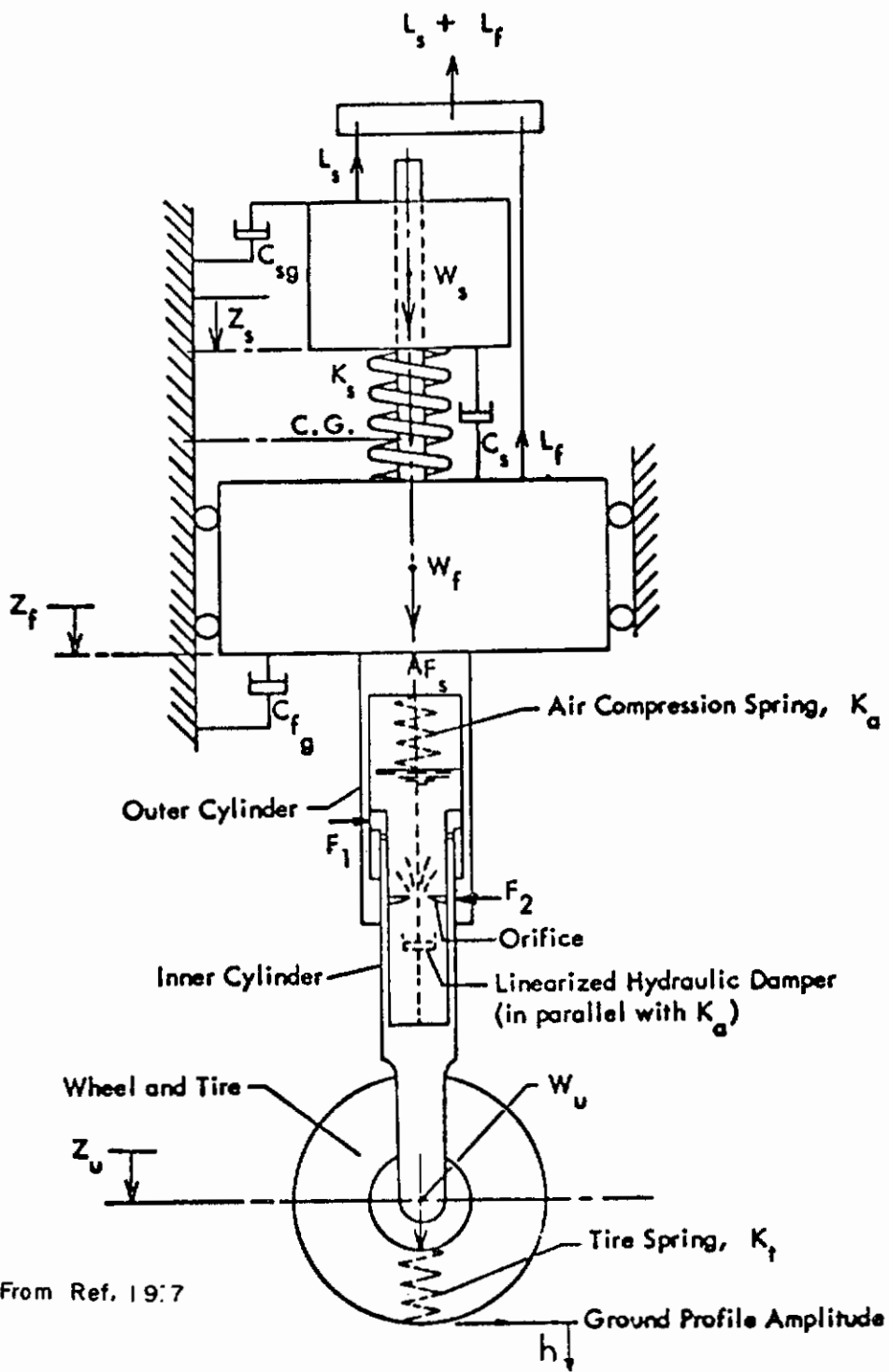


Figure 1. Three Degree of Freedom System Considered in Single Gear Analysis

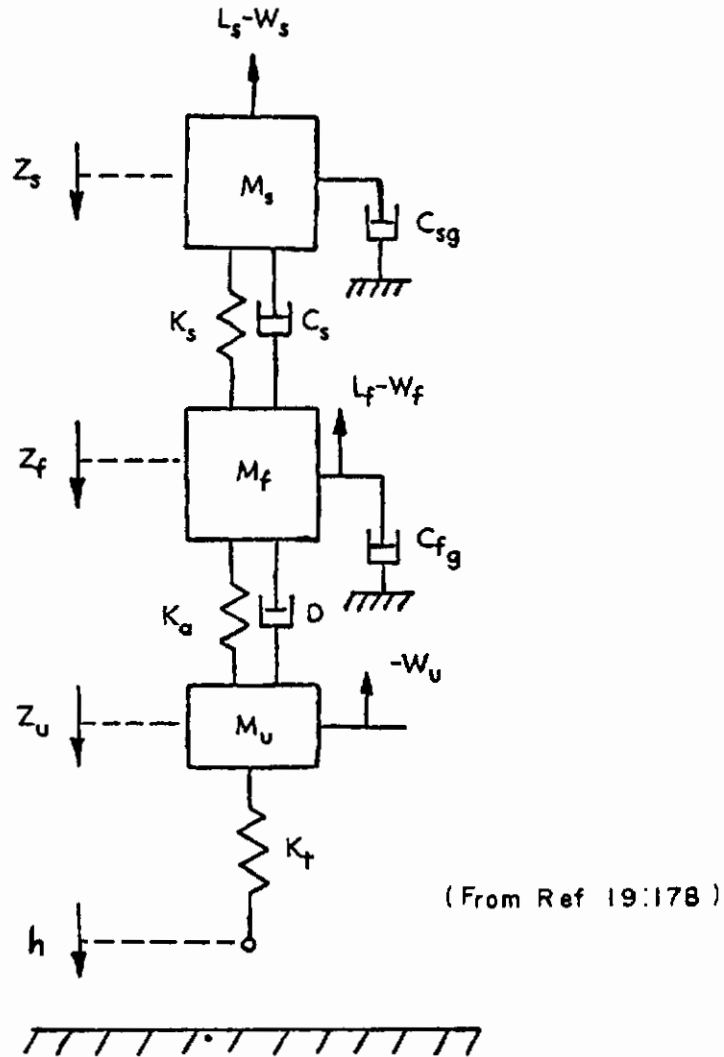


Figure 2. Equivalent Three Mass System for Single Gear Analysis

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three-mass system, with physically constrained values of M_s , M_f , C_{sg} , C_{fg} , C_s , K_s , L_s , and L_f , will represent the airplane responding in its first two free-free modes while taxiing.

The lower mass, M_u , lies below the oleo strut and consists principally of the wheel, tire, axle, and strut inner cylinder assembly (as shown in Figure 1). The strut is assumed to be infinitely rigid in bending. Horizontal forces at the axle, usually called drag or spring back loads, are not considered. Therefore, the lower mass has freedom only in vertical translation designated by the coordinate z_u , as was previously mentioned.

The parameters K_a and D are characteristics of the landing gear system. K_a is the linearized strut spring coefficient and D is the linearized damping coefficient. Finally K_t is a characteristic of the tire and is the linearized tire spring constant.

c. Equations of Motion

Summation of the forces on each of the three masses separately will result in the following equations of motion:

$$\begin{aligned}
 M_s \ddot{z}_s &= K_s z_f - K_s z_s + C_s \dot{z}_f - (C_s + C_{sg}) \dot{z}_s - (L_s - W_s) \\
 M_f \ddot{z}_s &= K_s z_s - (K_s + K_a) z_f + K_a z_u + C_s \dot{z}_s - (C_{fg} + C_s + D) \dot{z}_f \\
 &\quad + D \dot{z}_u - (L_f - W_f) \\
 M_u \ddot{z}_u &= K_a z_f - (K_a + K_t) z_u + D \dot{z}_f - D \dot{z}_u + K_t h + W_u
 \end{aligned} \tag{4}$$

In the above equations the lift forces, L_f and L_s , and the weights of the individual masses, W_s , W_f , and W_u are considered to be deterministic or known inputs to the system. Since the effects of runway unevenness, which will be modelled as a zero mean Gauss-Markov process (Chapter III), are of primary interest in this study, the forces L_f , L_s , W_f , W_s , and W_u may be omitted from the analysis with the understanding that they only affect the mean values of the states. This procedure will also be used in the multi-gear analyses.

Contrails

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Therefore Equation 4 is reformulated as follows:

$$\begin{aligned}
 M_s \ddot{z}_s &= -K_s z_s + K_s z_f - (C_s + C_{sg}) \dot{z}_s + C_s \dot{z}_f \\
 M_f \ddot{z}_f &= K_s z_s - (K_s + K_o) z_f + K_o z_u + C_s \dot{z}_s - (C_{fg} + C_s + D) \dot{z}_f + D \dot{z}_u \\
 M_u \ddot{z}_u &= K_o z_f - (K_o + K_t) z_u + D \dot{z}_f - D \dot{z}_u + K_t h
 \end{aligned}
 \tag{5}$$

To describe the behavior of the system the following state vector is chosen:

$$\underline{z} = \begin{bmatrix} z_s \\ z_f \\ z_u \\ \dot{z}_s \\ \dot{z}_f \\ \dot{z}_u \end{bmatrix} = \begin{bmatrix} z_s \\ z_f \\ z_u \\ v_s \\ v_f \\ v_u \end{bmatrix}
 \tag{6}$$

from which the following state equations are obtained:

$$\begin{aligned}
 \dot{z}_s &= v_s \\
 \dot{z}_f &= v_f \\
 \dot{z}_u &= v_u \\
 \dot{v}_s &= -\frac{K_s}{M_s} z_s + \frac{K_s}{M_s} z_f - \frac{(C_s + C_{sg})}{M_s} v_s + \frac{C_s}{M_s} v_f \\
 \dot{v}_f &= \frac{K_s}{M_f} z_s - \frac{(K_s + K_o)}{M_f} z_f + \frac{K_o}{M_f} z_u + \frac{C_s}{M_f} v_s \\
 &\quad - \frac{(C_{fg} + C_s + D)}{M_f} v_f + \frac{D}{M_f} v_u \\
 \dot{v}_u &= \frac{K_o}{M_u} z_f - \frac{(K_o + K_t)}{M_u} z_u + \frac{D}{M_u} v_f - \frac{D}{M_u} v_u + \frac{K_t}{M_u} h
 \end{aligned}
 \tag{7}$$

where h is the runway input to the vehicle.

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2. TRICYCLE LANDING GEAR SYSTEM

a. General Description

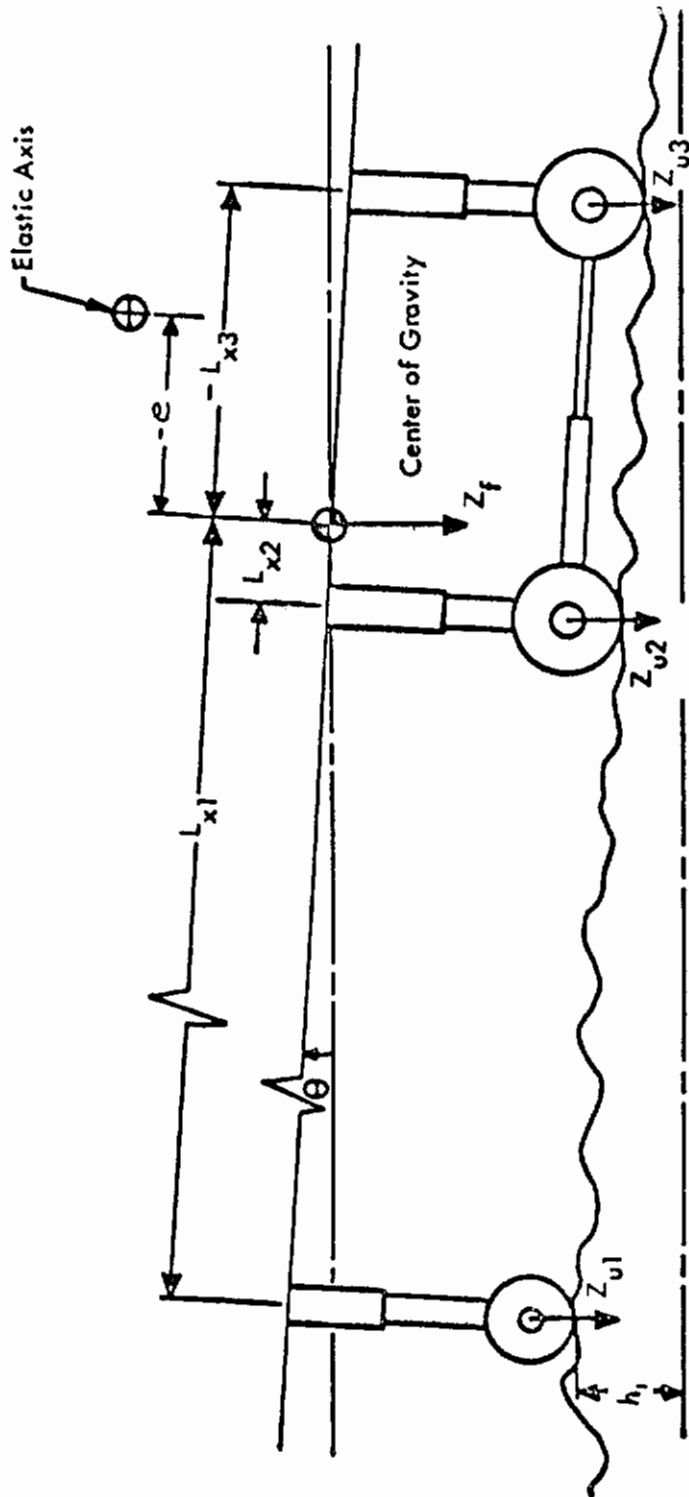
The tricycle landing gear model was obtained by extending the single landing gear model described above in a manner similar to that suggested in Reference 19. If symmetry about the airplane centerline is assumed, the tandem landing gear arrangement of the example airplane can be represented as shown in Figures 3 and 4 by three gears in sequence. The nose gear precedes the main gears by approximately 30 feet, and the main landing gears are independently suspended with a spacing of approximately 5 feet between fore-and-aft struts.

b. Physical Characteristics of the System

The tricycle landing gear arrangement yields a six degree of freedom model. The equivalent five mass system for this model is shown in Figure 4. The main mass, M_f , in the model is located at the airplane's center of gravity as in the single landing gear analysis, while the elastically connected mass is set back to its location on the elastic axis, a distance, $-e$, from the center of gravity. The landing gears are located at distances L_{x1} , L_{x2} , and $-L_{x3}$ from the center of gravity and in the x - z plane of the aircraft.

As in the single gear model, the masses M_s , M_{ui} ($i=1,2,3$) are assumed to have freedom only in vertical translation designated by a coordinate z_s , or z_{ui} ($i=1,2,3$). The main mass M_f , however, in addition to being free to translate in the vertical direction is also capable of pitch about the pitch axis of the aircraft. It is assumed that this pitch angle, θ , is small.

The parameters M_s , M_f , I_{yy} , C_{sg} , C_{fg} , C_s , and K_s are characteristics of the inertial, elastic, and aerodynamic properties of the aircraft. Descriptions of these parameters given in the single landing gear model are equally applicable to the tricycle landing gear model. The only additional definition needed is for I_{yy} , which is defined as the moment of inertia of the airplane about the pitch axis.



(From Ref 19:5)

Figure 3. Sketch of Landing Gear Arrangement Studied in Tricycle Gear Analysis

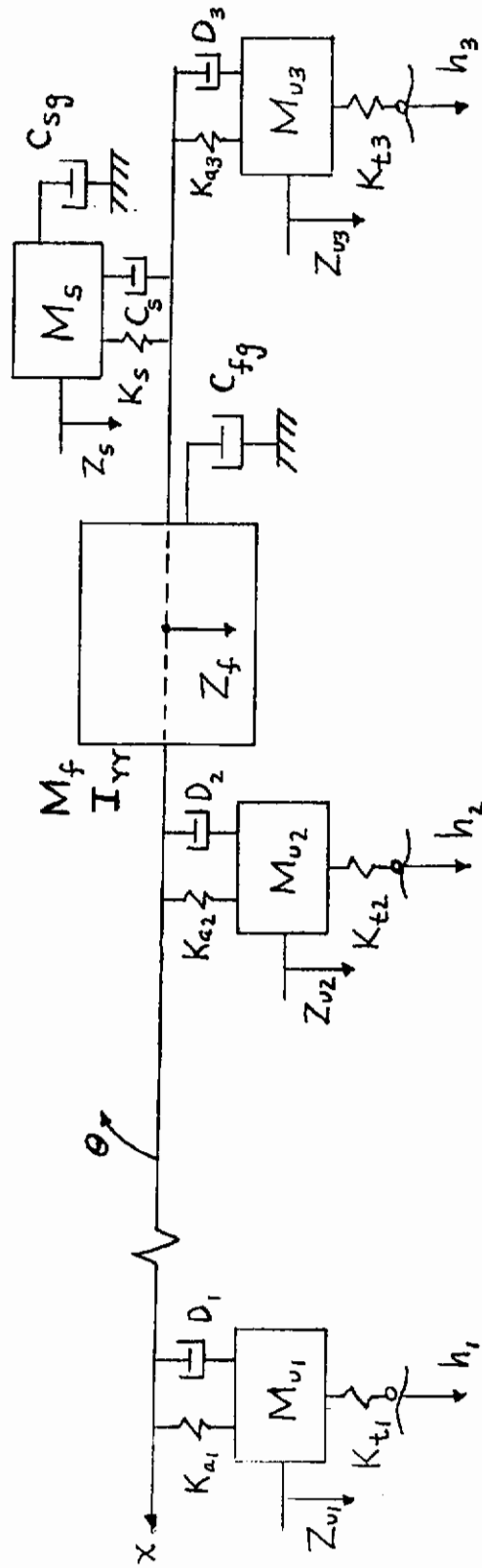


Figure 4. Equivalent Five Mass System for Tricycle Gear Analysis

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The parameters M_{ui} , K_{ai} , D_i ($i=1,2,3$) are characteristics of the landing gear system. Description of these parameters which are given in the single landing gear model are equally applicable to the tricycle landing gear model.

Finally, K_{ti} ($i=1,2,3$) is the linearized tire spring constant of the i^{th} strut.

c. Equations of Motion

Summation of forces on each of the five masses separately will result in the following equations:

$$M_s \ddot{z}_s = -K_s z_s + K_s z_f + K_s e \theta - (C_s + C_{sg}) \dot{z}_s + C_s \dot{z}_f + C_s e \dot{\theta}$$

$$\begin{aligned} M_f \ddot{z}_f = & K_s z_s - (K_{a1} + K_{a2} + K_{a3} + K_s) z_f \\ & + K_{a1} z_{u1} + K_{a2} z_{u2} + K_{a3} z_{u3} \\ & + (K_{a1} L_{x1} + K_{a2} L_{x2} - K_{a3} L_{x3} - K_s e) \theta \\ & + C_s \dot{z}_s - (D_1 + D_2 + D_3 + C_s + C_{fg}) \dot{z}_f \\ & + D_1 \dot{z}_{u1} + D_2 \dot{z}_{u2} + D_3 \dot{z}_{u3} \\ & + (D_1 L_{x1} + D_2 L_{x2} - D_3 L_{x3} - C_s e) \dot{\theta} \end{aligned} \tag{8}$$

$$\begin{aligned} M_{u1} \ddot{z}_{u1} = & K_{a1} z_f - (K_{a1} + K_{t1}) z_{u1} - K_{a1} L_{x1} \theta \\ & + D_1 \dot{z}_f - D_1 \dot{z}_{u1} - D_1 L_{x1} \dot{\theta} + K_{t1} h_1 \end{aligned}$$

$$\begin{aligned} M_{u2} \ddot{z}_{u2} = & K_{a2} z_f - (K_{a2} + K_{t2}) z_{u2} - K_{a2} L_{x2} \theta \\ & + D_2 \dot{z}_f - D_2 \dot{z}_{u2} - D_2 L_{x2} \dot{\theta} + K_{t2} h_2 \end{aligned}$$

$$\begin{aligned} M_{u3} \ddot{z}_{u3} = & K_{a3} z_f - (K_{a3} + K_{t3}) z_{u3} + K_{a3} L_{x3} \theta \\ & + D_3 \dot{z}_f - D_3 \dot{z}_{u3} + D_3 L_{x3} \dot{\theta} + K_{t3} h_3 \end{aligned}$$

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Summation of moments about the pitch axis of M_f results in the following equation

$$\begin{aligned}
 I_{yy} \ddot{\theta} = & eK_s z_s + (L_{x1} K_{a1} + L_{x2} K_{a2} - eK_s - L_{x3} K_{a3}) z_f \\
 & - L_{x1} K_{a1} z_{u1} - L_{x2} K_{a2} z_{u2} + L_{x3} K_{a3} z_{u3} \\
 & - (L_{x1}^2 K_{a1} + L_{x2}^2 K_{a2} + e^2 K_s + L_{x3}^2 K_{a3}) \theta \\
 & + eC_s \dot{z}_s + (L_{x1} D_1 + L_{x2} D_2 - eC_s - L_{x3} D_3) \dot{z}_f \\
 & - L_{x1} D_1 \dot{z}_{u1} - L_{x2} D_2 \dot{z}_{u2} + L_{x3} D_3 \dot{z}_{u3} \\
 & - (L_{x1}^2 D_1 + L_{x2}^2 D_2 + e^2 C_s + L_{x3}^2 D_3) \dot{\theta}
 \end{aligned} \tag{9}$$

The state vector is identified as

$$\begin{aligned}
 \underline{z}^T = & (z_s, z_f, z_{u1}, z_{u2}, z_{u3}, \theta, \\
 & \dot{z}_s, \dot{z}_f, \dot{z}_{u1}, \dot{z}_{u2}, \dot{z}_{u3}, \dot{\theta}) \\
 \underline{v}^T = & (z_s, z_f, z_{u1}, z_{u2}, z_{u3}, \theta, \\
 & v_s, v_f, v_{u1}, v_{u2}, v_{u3}, v_\theta)
 \end{aligned} \tag{10}$$

for which the state equations are obtained as

$$\dot{z}_s = v_s$$

$$\dot{z}_f = v_f$$

$$\dot{z}_{u1} = v_{u1}$$

$$\dot{z}_{u2} = v_{u2}$$

$$\dot{z}_{u3} = v_{u3}$$

$$\dot{\theta} = v_\theta$$

$$\dot{v}_s = -\frac{K_s}{M_s} z_s + \frac{K_s}{M_s} z_f + \frac{K_s}{M_s} e \theta - \frac{(C_s + C_{sg})}{M_s} v_s$$

$$\begin{aligned}
 & + \frac{C_s}{M_s} v_f + \frac{C_s}{M_s} e v_\theta \\
 \dot{v}_f = & \frac{K_s}{M_f} z_s - \frac{(K_{a1} + K_{a2} + K_{a3} + K_s)}{M_f} z_f \\
 & + \frac{K_{a1}}{M_f} z_{u1} + \frac{K_{a2}}{M_f} z_{u2} + \frac{K_{a3}}{M_f} z_{u3} \\
 & + \frac{(K_{a1} L_{x1} + K_{a2} L_{x2} - K_{a3} L_{x3} - K_s e)}{M_f} \theta \\
 & + \frac{C_s}{M_f} v_s - \frac{(D_1 + D_2 + D_3 + C_s + C_{fg})}{M_f} v_f \\
 & + \frac{D_1}{M_f} v_{u1} + \frac{D_2}{M_f} v_{u2} + \frac{D_3}{M_f} v_{u3} \\
 & + \frac{(D_1 L_{x1} + D_2 L_{x2} - D_3 L_{x3} - C_s e)}{M_f} v_\theta \\
 \dot{v}_{u1} = & \frac{K_{a1}}{M_{u1}} z_f - \frac{(K_{a1} + K_{t1})}{M_{u1}} z_{u1} - \frac{K_{a1}}{M_{u1}} L_{x1} \theta \\
 & + \frac{D_1}{M_{u1}} v_f - \frac{D_1}{M_{u1}} v_{u1} - \frac{D_1 L_{x1}}{M_{u1}} v_\theta + \frac{K_{t1}}{M_{u1}} h_1 \\
 \dot{v}_{u2} = & \frac{K_{a2}}{M_{u2}} z_f - \frac{(K_{a2} + K_{t2})}{M_{u2}} z_{u2} - \frac{K_{a2}}{M_{u2}} L_{x2} \theta \\
 & + \frac{D_2}{M_{u2}} v_f - \frac{D_2}{M_{u2}} v_{u2} - \frac{D_2 L_{x2}}{M_{u2}} v_\theta + \frac{K_{t2}}{M_{u2}} h_2 \\
 \dot{v}_{u3} = & \frac{K_{a3}}{M_{u3}} z_f - \frac{(K_{a3} + K_{t3})}{M_{u3}} z_{u3} + \frac{K_{a3}}{M_{u3}} L_{x3} \theta \\
 & + \frac{D_3}{M_{u3}} v_f - \frac{D_3}{M_{u3}} v_{u3} + \frac{D_3 L_{x3}}{M_{u3}} v_\theta + \frac{K_{t3}}{M_{u3}} h_3 \\
 \dot{v}_\theta = & \frac{eK_s}{I_{yy}} z_s + \frac{(L_{x1} K_{a1} + L_{x2} K_{a2} - eK_s - L_{x3} K_{a3})}{I_{yy}} z_f \\
 & - \frac{L_{x1} K_{a1}}{I_{yy}} z_{u1} - \frac{L_{x2} K_{a2}}{I_{yy}} z_{u2} + \frac{L_{x3} K_{a3}}{I_{yy}} z_{u3} \\
 & - \frac{(L_{x1}^2 K_{a1} + L_{x2}^2 K_{a2} + e^2 K_s + L_{x3}^2 K_{a3})}{I_{yy}} \theta \\
 & + \frac{eC_s}{I_{yy}} v_s + \frac{(L_{x1} D_1 + L_{x2} D_2 - eC_s - L_{x3} D_3)}{I_{yy}} v_f \\
 & - \frac{L_{x1} D_1}{I_{yy}} v_{u1} - \frac{L_{x2} D_2}{I_{yy}} v_{u2} + \frac{L_{x3} D_3}{I_{yy}} v_{u3} \\
 & - \frac{(L_{x1}^2 D_1 + L_{x2}^2 D_2 + e^2 C_s + L_{x3}^2 D_3)}{I_{yy}} v_\theta
 \end{aligned} \tag{11}$$

where h_1 , h_2 , and h_3 are the runway inputs to the vehicle.

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3. FIVE LANDING GEAR SYSTEM

a. General Description

The five landing gear model represents an extension of the three landing gear model in which the effects of roll may be studied. The idea to perform such an analysis resulted from an examination of a model given in Reference 10, Volume II, and shown in Figure 5. Thus the five landing gear model is a combination of the models presented in both Reference 19 and Reference 10, Volume II. This model permits the study of the motion of the aircraft in its entirety.

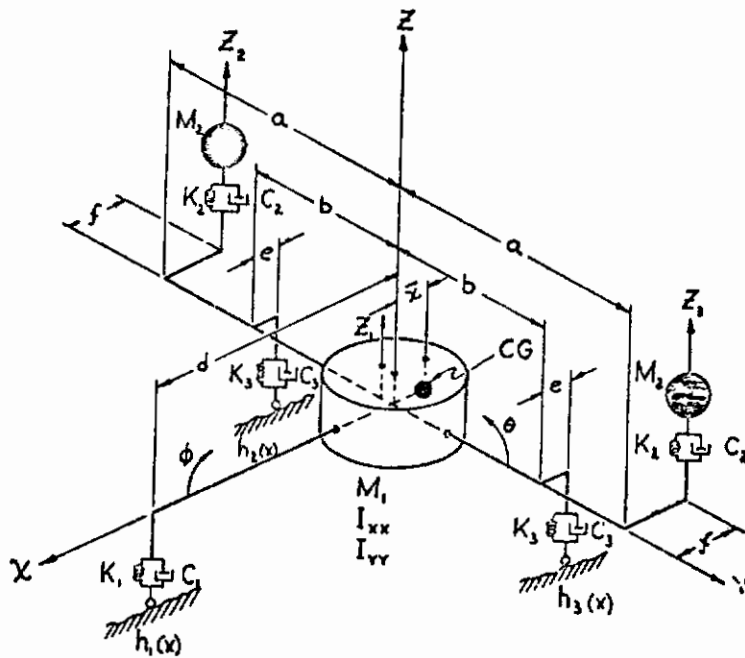


Figure 5. Five Degree of Freedom Model Studied in Ref 10: Vol II

b. Physical Characteristics of the System

The five landing gear model results in a ten degree of freedom model. The equivalent eight mass system for this model is shown in Figure 7. The main mass, M_f , in the model is again located at the

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airplane's center of gravity as in the single and tricycle landing gear arrangements. However, now there are two elastically connected masses, M_{s1} and M_{s2} , each representing the vertical motion of a wing, and each located at its proper wing station along both the x- and y-axes, as shown in Figure 7. All five landing gears are located at their true physical location along the x- and y-axes of the aircraft and with respect to the center of gravity.

As in both the single and tricycle landing gear models, the masses M_{s1} , M_{s2} , and M_{ui} ($i=1, \dots, 5$) are assumed to have freedom only in vertical translation designated by a coordinate z_{s1} , z_{s2} , or z_{ui} ($i=1, \dots, 5$). The main mass M_f is also assumed to have freedom in vertical translation designated by a coordinate z_f . However, in addition to being free to translate in the vertical direction, the mass M_f is also capable of both pitch and roll about the pitch and roll axes of the aircraft, respectively. It is assumed that the pitch angle, θ , and the roll angle, ϕ , are small.

The parameters M_{sj} , C_{sj} , K_{sj} , C_{sgj} ($j=1, 2$), M_f , C_{fg} , I_{yy} , and I_{xx} are characteristics of the inertial, elastic, and aerodynamic properties of the aircraft. Descriptions of these parameters given in the single landing gear model are equally applicable to the five landing gear model. In addition, I_{yy} is again defined as the moment of inertia of the aircraft about the pitch axis (as in the tricycle landing gear model), while I_{xx} is defined as the moment of inertia of the aircraft about the roll axis.

The parameters M_{ui} , K_{ai} , D_i ($i=1, \dots, 5$) are characteristics of the landing gear system. Descriptions of these parameters which are given in the single landing gear model are equally applicable to the five landing gear model.

Finally, K_{ti} ($i=1, \dots, 5$) is the linearized tire spring constant of the i^{th} strut.

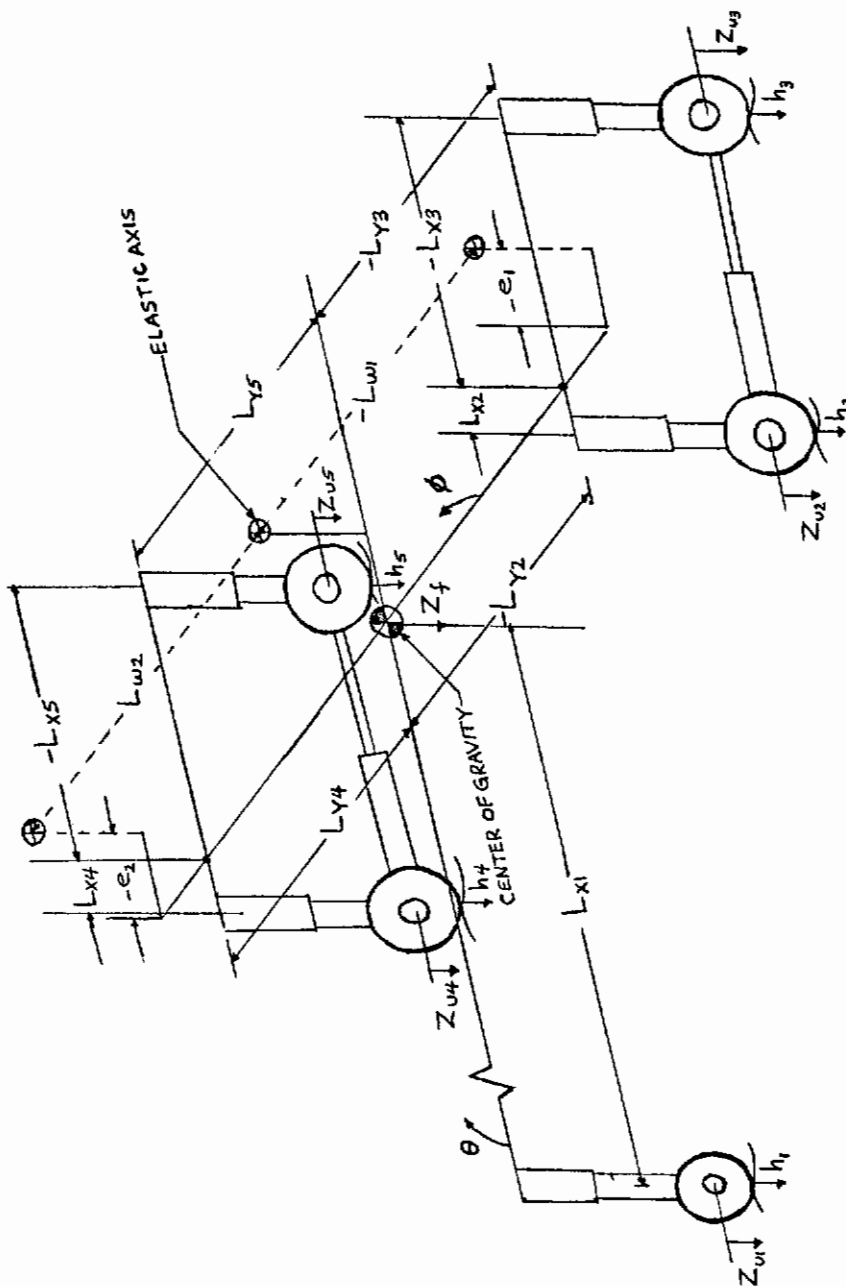


Figure 6. Sketch of Landing Gear Arrangement Studied in Five Gear Analysis

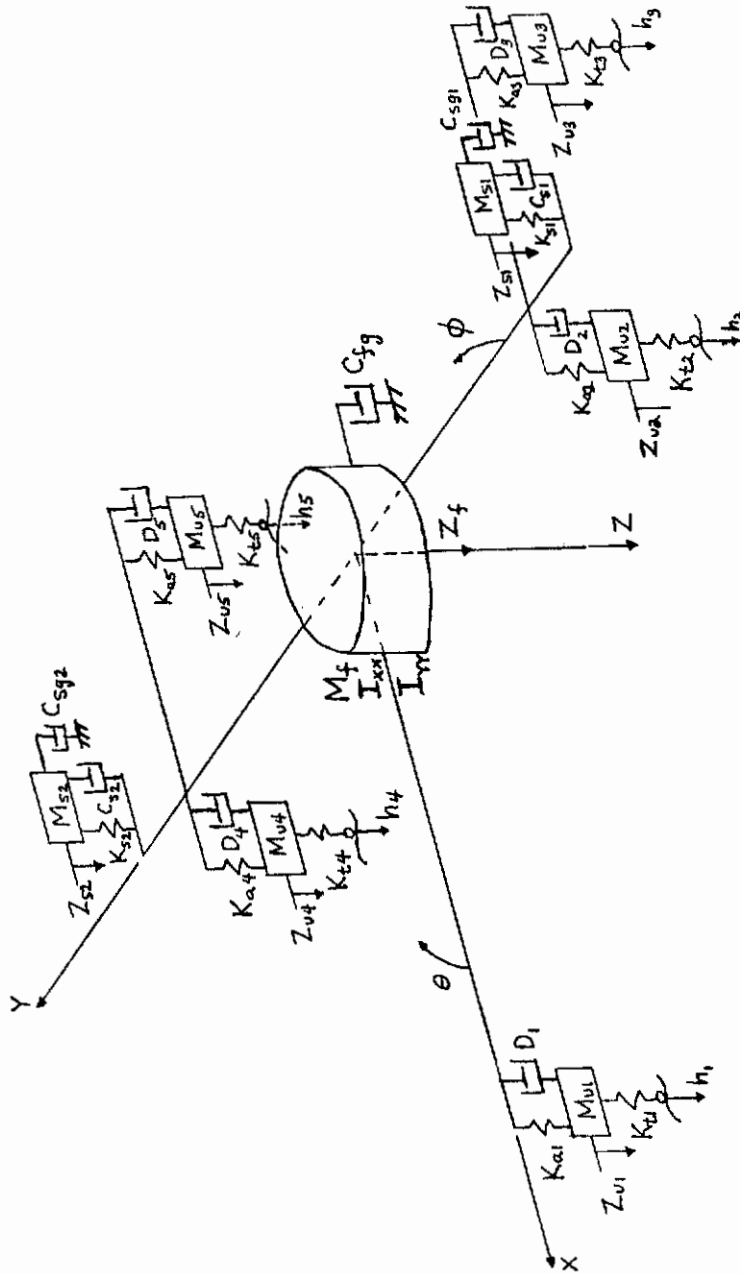


Figure 7. Equivalent Eight Mass System for Five Gear Analysis

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c. Equation of Motion

Since the equations of motion are lengthy (consisting of 20 state equations) they are given in Appendix A. The derivation of these equations is not given here; they are derived in a similar manner as the previous state equations: summation of the forces on each of the masses separately and summation of the moments about the pitch and roll axes of M_f .

SECTION III

RUNWAY MODEL1. RUNWAY SPECTRAL DENSITY

Spectral data pertaining to vehicle disturbances are generally given in terms of a spacial frequency, Ω (radians/foot) rather than a time frequency, ω (radians/second). Since it is convenient to look at vehicle dynamics as a function of time, it is appropriate at this point to discuss the transformation from a distance-based spectrum $\Phi_x(\Omega)$ to a spectrum based on time $\Phi_f(\omega)$. The following formulation is taken from Reference 15:12. First, it is noted that the spacial and time frequencies are uniquely related by the vehicle velocity, V_H :

$$\Omega = \frac{\omega}{V_H} \quad (12)$$

If a narrow strip of the spacial spectrum bounded by $\Omega, \Omega + d\Omega$, and $\Phi_x(\Omega)$ is compared with the corresponding strip in the time spectrum bounded by $\omega, \omega + d\omega$, and $\Phi_f(\omega)$, the areas of each strip must be equal since the mean squared disturbance must be invariant under the space to time transformation (Reference 15:12). Thus

$$\begin{aligned} \Phi_f(\omega) d\omega &= \Phi_x(\Omega) d\Omega \\ &= \Phi_x\left(\frac{\omega}{V_H}\right) \frac{d\omega}{V_H} \end{aligned} \quad (13)$$

from which the following result is obtained:

$$\Phi_f(\omega) = \frac{1}{V_H} \Phi_x\left(\frac{\omega}{V_H}\right) \quad (14)$$

2. RUNWAY INPUT FOR SINGLE LANDING GEAR MODEL

It has been found (Reference 15:17) that the profile spectrum of guideways (runways, highways, etc.) can be conveniently represented by an equation of the form

$$\Phi_x(\Omega) = \frac{A_0}{\Omega^2} \text{ or } \Phi_f(\omega) = \frac{A_0 V_H}{\omega^2} \quad (15)$$

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It is noted that at the low frequency end of the spectrum, Equation 15 becomes infinite, whereas elevation spectrum must level off due to the finite height of runways. The break frequency, ω_B , is apparently below that which has thus far been measured.

However, if the runway spectrum is modified as

$$\Phi_x(\Omega) = \frac{A_0}{\Omega^2 + \left(\frac{2\pi}{\lambda_0}\right)^2} \quad (16)$$

where $\lambda_0 = \frac{2\pi}{\Omega_B}$, the wavelength at the break frequency, and $\Omega_B = \frac{\omega_B}{V_H}$, then the relation between the rms elevation h_{rms} and the wavelength λ_0 is

$$h_{rms} = \sqrt{\frac{A_0 \lambda_0}{4\pi}} \quad (17)$$

The rms elevation h_{rms} must still be selected in order to obtain λ_0 . For the state-of-the-art guideways and most practical suspensions, a runway elevation of approximately 1/2 ft is selected as a representative value for the model (Reference 15:47). Figure 8 shows the spectral density for the runway used in this study. Also shown is the asymptotic curve

$$\Phi_x(\Omega) = \frac{A_0}{\Omega^2} \quad (18)$$

The parameter A_0 for the runway is about 10^{-1} . Therefore $\lambda_0 = 4.5 \times 10^3$ ft or about 0.86 mile. This corresponds to an input frequency of 0.01465 cps for a 66 fps vehicle, and would not significantly affect vehicle dynamic response (Reference 15:47).

The time-based density $\Phi_f(\omega)$ corresponding to the distance-based spectrum $\Phi_x(\Omega)$ given in Equation 16 is

$$\Phi_f(\omega) = \frac{A_0 A_H}{\omega^2 + \omega_B^2} \quad (19)$$

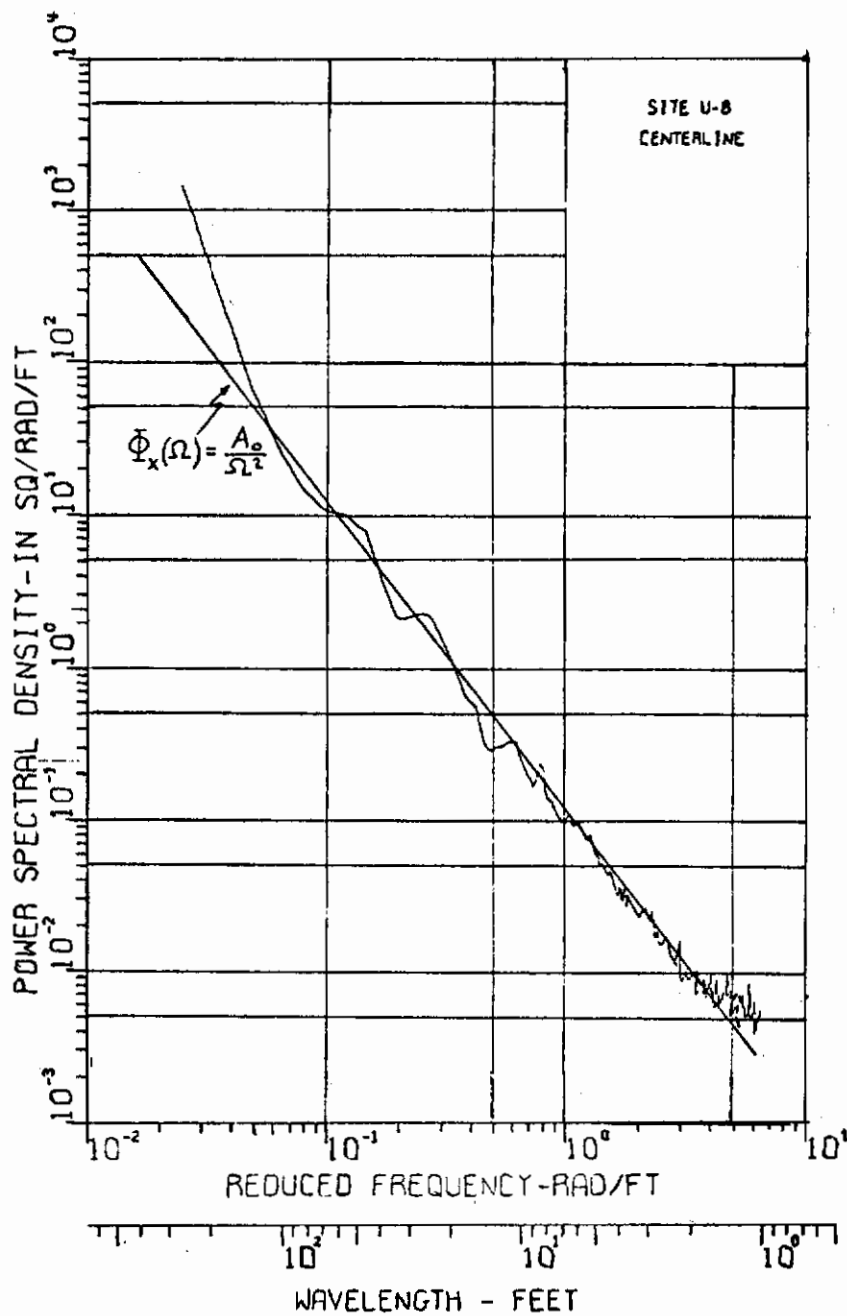


Figure 8. Airfield Power Spectral Density

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where

$$\omega_B = \frac{2\pi V_H}{\lambda_0}$$

Equation 19 is the power spectrum for the Gauss-Markov process modelled by the following differential equation

$$\dot{h} = -\omega_B h + \xi \quad (20)$$

where $\xi(t)$ is white Gaussian noise with

$$E[\xi(t)] = 0$$

and

$$\begin{aligned} E[\xi(t_1)\xi(t_2)] &= \sigma_\xi^2 \delta(t_1 - t_2) \\ &= A_0 V_H \delta(t_1 - t_2) \end{aligned}$$

Thus the runway input to the single landing gear system is given by Equation 20.

3. RUNWAY INPUT FOR TRICYCLE LANDING GEAR MODEL

While the model for the runway input given by Equation 20 conveniently represents the input to a single landing gear, this model must be extended or modified in such a way as to take care of the tricycle and ultimately the five landing gear model.

In the tricycle landing gear arrangement, all motion is considered to take place in a vertical plane, since symmetry about the airplane centerline is assumed, and the tandem landing gear arrangement of the example airplane can be represented as shown in Figures 3 and 4 by three gears in sequence. If h_1 represents the runway input to the nose gear, then the input to the front main gear, h_2 , is h_1 delayed τ_{12} seconds. This time delay may be conveniently modelled by the Padé approximation

$$\frac{h_2}{h_1} = e^{-\tau_{12}S} = -\frac{(S - 2/\tau_{12})}{(S + 2/\tau_{12})} \quad (21)$$

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Similarly, the input to the rear main gear, h_3 , is h_2 delayed τ_{23} seconds and may be modelled as

$$\frac{h_3}{h_2} = e^{-\tau_{23}S} = -\frac{(S - 2/\tau_{23})}{(S + 2/\tau_{23})} \quad (22)$$

If the input h_1 takes the form given by Equation 20

$$\dot{h}_1 = \omega_B h_1 + \xi_1 \quad (23)$$

then the stochastic differential equations for h_2 and h_3 are

$$\dot{h}_2 = (\omega_B + \frac{2}{\tau_{12}}) h_1 - \frac{2}{\tau_{12}} h_2 - \xi_1 \quad (24)$$

and

$$\dot{h}_3 = -(\omega_B + \frac{2}{\tau_{12}}) h_1 + 2(\frac{1}{\tau_{12}} + \frac{1}{\tau_{23}}) h_2 - \frac{2}{\tau_{23}} h_3 + \xi_1 \quad (25)$$

For the example airplane and for a constant taxi speed of $V_H = 66$ fps the time delays τ_{12} and τ_{23} , are obtained as

$$\begin{aligned} \tau_{12} &= 4.48 \times 10^{-1} \text{ SEC} \\ \tau_{23} &= 7.64 \times 10^{-2} \text{ SEC} \end{aligned}$$

thereby justifying the use of the Padé approximation.

4. RUNWAY INPUT FOR FIVE LANDING GEAR MODEL

As with the runway inputs to the tricycle landing gear model, the relationships among the inputs to the five landing gear model must be determined. However, in this case the input to the nose gear and to the front main gear cannot be represented by a simple time delay as was done in the tricycle landing gear analysis. This results from the fact that in the five landing gear analysis the main gears are set in their proper location, which is a given distance from the centerline of the aircraft; that is

$$L_{y2} = L_{y3} = L_{y4} = L_{y5} = l_x \quad (26)$$

One report (Reference 10: Volume 11, page 32) assumes the inputs to be completely uncorrelated. The justification of the tricycle landing gear model, however, seems to imply that the inputs are totally correlated. The following analysis presents a model in which the correlation of the inputs to the nose gear and the front main gears become a function of the distance of the main gears from the centerline of the aircraft and the horizontal velocity of the aircraft.

In Figure 9a, let l_x be the distance of the front main gear from the centerline of the aircraft, and let h_1 be the runway input to the nose gear given by the first order stochastic differential equation

$$\dot{h}_1 = -\omega_B h_1 + \xi_1 \quad (27)$$

Furthermore, let the statistics at a distance l_x to the left and right of the nose gear be described by the random processes h_2^* and h_4^* respectively (see Figure 9). If h_2^* and h_4^* have the same power spectrum as h_1 , then they may also be modelled by a first order stochastic differential equation of the form

$$\dot{h}_2^*(t) = -\omega_B h_2^*(t) + \xi_2(t) \quad (28)$$

and

$$\dot{h}_4^*(t) = -\omega_B h_4^*(t) + \xi_4(t) \quad (29)$$

where

and $E \left[\xi_2(t_1) \xi_2(t_2) \right] = A_0 V_H \delta(t_1 - t_2) = \sigma_\xi^2 \delta(t_1 - t_2)$

and $E \left[\xi_4(t_1) \xi_4(t_2) \right] = A_0 V_H \delta(t_1 - t_2) = \sigma_\xi^2 \delta(t_1 - t_2)$

and where A_0 and ω_B were previously defined. Justification of this procedure is based upon data which indicates that the power spectrum of the runway does not change significantly at a short distance to the left or right of the runway centerline (Reference 10; Volume 1). The problem now reduces to correlating the random processes h_1 , h_2^* , and h_4^* .

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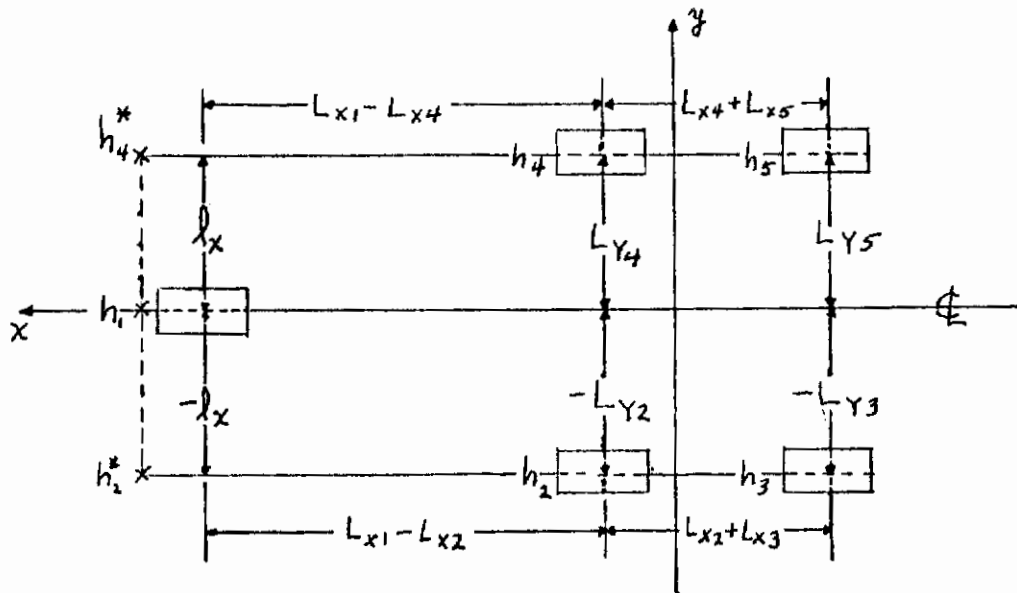


Figure 9a. Correlation of Runway Inputs for Five Gear Analysis (Distance Based)

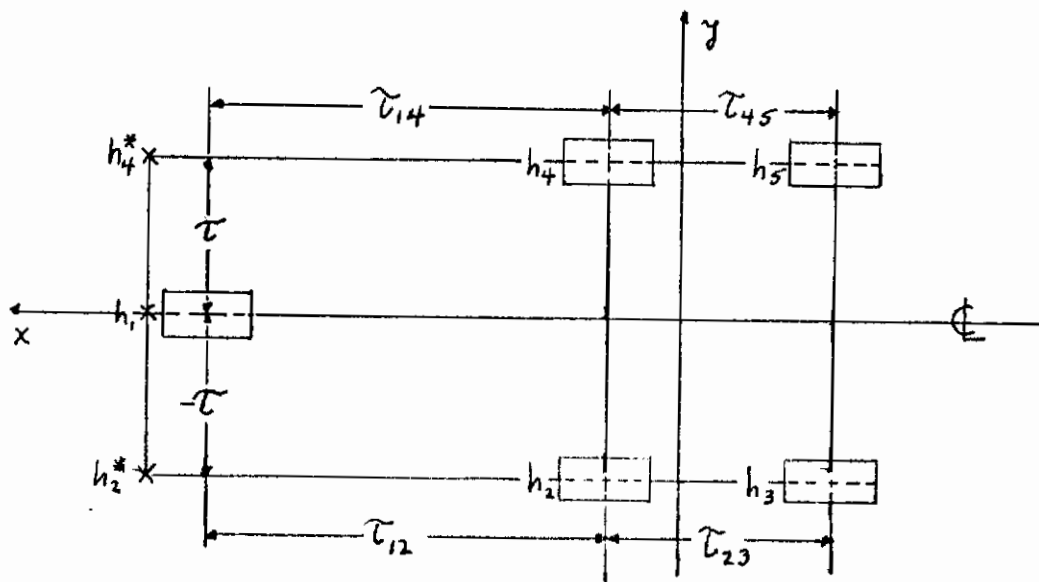


Figure 9b. Correlation of Runway Inputs for Five Gear Analysis (Time Based)

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If it is assumed that the distribution of the runway is the same in all directions to a vehicle moving at constant velocity, V_H , then at a point which is a distance l_x to the left of the nose gear the random process h_2^* is actually

$$h_2^*(x) = h_1(x - l_x) \quad (30)$$

Similarly, at a point which is a distance l_x to the right of the nose gear the random process h_4^* is actually

$$h_4^*(x) = h_1(x + l_x) \quad (31)$$

As a function of time, Equations 30 and 31 may be written as

$$h_2^*(t) = h_1(t - \tau) \quad (32)$$

$$h_4^*(t) = h_1(t + \tau)$$

where $t = \frac{x}{V_H}$ and $\tau = \frac{l_x}{V_H}$, with the result that

$$\begin{aligned} E[h_2^*(t) h_1(t)] &= E[h_1(t - \tau) h_1(t)] \\ E[h_1(t) h_4^*(t)] &= E[h_1(t) h_1(t + \tau)] \\ E[h_2^*(t) h_4^*(t)] &= E[h_1(t - \tau) h_1(t + \tau)] \end{aligned} \quad (33)$$

But since h_1 is a Gauss-Markov process given by Equation 27, the following relationship is true

$$E[h_1(t_2) h_1(t_1)] = e^{-\omega_B(t_2 - t_1)} E[h_1(t_1) h_1(t_1)] \quad (34)$$

for $t_2 \geq t_1$

Hence,

$$\begin{aligned} E[h_2^*(t) h_1(t)] &= e^{-\omega_B \tau} E[h_1(t_1) h_1(t_1)] \\ E[h_1(t) h_4^*(t)] &= e^{-\omega_B \tau} E[h_1(t_1) h_1(t_1)] \\ E[h_2^*(t) h_4^*(t)] &= e^{-2\omega_B \tau} E[h_1(t_1) h_1(t_1)] \end{aligned} \quad (35)$$

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Now, if h_1^* , h_2^* , and h_4^* are jointly Gaussian distributed with

$$\begin{aligned} E[\xi_1(t_1) \xi_2(t_2)] &= \sigma_{12}^2 \delta(t_1 - t_2) \\ E[\xi_1(t_1) \xi_4(t_2)] &= \sigma_{14}^2 \delta(t_1 - t_2) \\ E[\xi_2(t_1) \xi_4(t_2)] &= \sigma_{24}^2 \delta(t_1 - t_2) \end{aligned} \quad (36)$$

then, using Equations 27, 28, and 25, the steady-state covariance matrix for the Gauss-Markov process

$$\underline{h}^*(t) = \begin{bmatrix} h_1^*(t) \\ h_2^*(t) \\ h_4^*(t) \end{bmatrix} \quad (37)$$

is obtained as

$$AP + PA^T + Q = 0 \quad (38)$$

where

$$A = \begin{bmatrix} -\omega_B & 0 & 0 \\ 0 & -\omega_B & 0 \\ 0 & 0 & -\omega_B \end{bmatrix}$$

$$P = E[\underline{h}^*(t) \underline{h}^{*T}(t)]$$

and

$$Q = \begin{bmatrix} \sigma_{\xi}^2 & \sigma_{12}^2 & \sigma_{14}^2 \\ \sigma_{12}^2 & \sigma_{\xi}^2 & \sigma_{24}^2 \\ \sigma_{24}^2 & \sigma_{24}^2 & \sigma_{\xi}^2 \end{bmatrix}$$

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Therefore, from Equation 38 the following results are obtained

$$\begin{aligned}
 2 \omega_B E [h_1(t) h_1(t)] &= \sigma_{\xi}^2 \\
 2 \omega_B E [h_1(t) h_2^*(t)] &= \sigma_{12}^2 \\
 2 \omega_B E [h_1(t) h_4^*(t)] &= \sigma_{14}^2 \\
 2 \omega_B E [h_2^*(t) h_4^*(t)] &= \sigma_{24}^2
 \end{aligned}
 \tag{39}$$

Using Equations 35 and 39 yields

$$\begin{aligned}
 \sigma_{12}^2 &= e^{-\omega_B \tau} \sigma_{\xi}^2 \\
 \sigma_{14}^2 &= e^{-\omega_B \tau} \sigma_{\xi}^2 \\
 \sigma_{24}^2 &= e^{-2\omega_B \tau} \sigma_{\xi}^2
 \end{aligned}
 \tag{40}$$

where ω_B , τ , and σ_{ξ}^2 are already known.

To summarize, the Gauss-Markov process

$$\underline{h}^*(t) = \begin{bmatrix} h_1(t) \\ h_2^*(t) \\ h_4^*(t) \end{bmatrix}
 \tag{37}$$

is defined by the relationships

$$\begin{aligned}
 \dot{h}_1(t) &= -\omega_B h_1(t) + \xi_1(t) \\
 \dot{h}_2^*(t) &= -\omega_B h_2^*(t) + \xi_2(t) \\
 \dot{h}_4^*(t) &= -\omega_B h_4^*(t) + \xi_4(t)
 \end{aligned}
 \tag{41}$$

where

$$\begin{aligned}
 E [\xi_1(t) \xi_1(t)] &= E [\xi_2(t) \xi_2(t)] = E [\xi_4(t) \xi_4(t)] = \sigma_{\xi}^2 \\
 E [\xi_1(t) \xi_2(t)] &= E [\xi_1(t) \xi_4(t)] = e^{-\omega_B \tau} \sigma_{\xi}^2 \\
 E [\xi_2(t) \xi_4(t)] &= e^{-2\omega_B \tau} \sigma_{\xi}^2
 \end{aligned}$$

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For the example airplane and for a constant taxi speed of $V_H = 66$ fps, $\tau = 1.08 \times 10^{-1}$ sec. In this manner the correlation among $h_1(t)$, $h_2^*(t)$, and $h_4^*(t)$ has been completely defined.

Now for the main gears on the left side of the aircraft the input to the front main gear, h_2 , is h_2^* delayed τ_{12} seconds, while the input to the rear main gear, h_3 , is h_2 delayed τ_{23} seconds. These time delays may be conveniently modelled by the Padé approximation, as in the tricycle landing gear model, to obtain the stochastic differential equations for h_2 and h_3 as

$$\dot{h}_2 = (\omega_B + \frac{2}{\tau_{12}}) h_2^* - \frac{2}{\tau_{12}} h_2 - \xi_2 \quad (42)$$

$$\dot{h}_3 = -(\omega_B + \frac{2}{\tau_{12}}) h_2^* + 2(\frac{1}{\tau_{12}} + \frac{1}{\tau_{23}}) h_2 - \frac{2}{\tau_{23}} h_3 + \xi_2$$

Similarly, for the main gears on the right side of the aircraft the input to the front main gear, h_4 , is h_4^* delayed τ_{14} seconds while the input to the rear main gear, h_5 , is h_4 delayed τ_{45} seconds. The stochastic differential equations for h_4 and h_5 are then obtained as

$$\dot{h}_4 = (\omega_B + \frac{2}{\tau_{14}}) h_4^* - \frac{2}{\tau_{14}} h_4 - \xi_4 \quad (43)$$

$$\dot{h}_5 = -(\omega_B + \frac{2}{\tau_{14}}) h_4^* + 2(\frac{1}{\tau_{14}} + \frac{1}{\tau_{45}}) h_4 - \frac{2}{\tau_{45}} h_5 + \xi_4$$

Note that for the aircraft of this study $\tau_{12} = \tau_{14}$ and $\tau_{23} = \tau_{45}$.

Thus, the runway input to the five landing gear model is completely defined by the following equations

$$\dot{h}_1 = -\omega_B h_1 + \xi_1$$

$$\dot{h}_2^* = -\omega_B h_2^* + \xi_2$$

$$\dot{h}_2 = (\omega_B + \frac{2}{\tau_{12}}) h_2^* - \frac{2}{\tau_{12}} h_2 - \xi_2$$

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$$\begin{aligned}
 \dot{h}_3 &= -\left(\omega_B + \frac{2}{\tau_{12}}\right) h_3^* + 2\left(\frac{1}{\tau_{12}} + \frac{1}{\tau_{23}}\right) h_2 - \frac{2}{\tau_{23}} h_3 + \xi_2 \\
 \dot{h}_4^* &= -\omega_B h_4^* + \xi_4 \\
 \dot{h}_4 &= \left(\omega_B + \frac{2}{\tau_{14}}\right) h_4^* - \frac{2}{\tau_{14}} h_4 - \xi_4 \\
 \dot{h}_5 &= -\left(\omega_B + \frac{2}{\tau_{14}}\right) h_4^* + 2\left(\frac{1}{\tau_{14}} + \frac{1}{\tau_{45}}\right) h_4 - \frac{2}{\tau_{45}} h_5 + \xi_4
 \end{aligned}
 \tag{44a}$$

where

$$\begin{aligned}
 E\left[\underline{\xi}(t_1)\underline{\xi}^T(t_2)\right] &= A_0 V_H \begin{bmatrix} 1 & e^{-\omega_B \tau} & e^{-\omega_B \tau} \\ e^{-\omega_B \tau} & 1 & e^{-2\omega_B \tau} \\ e^{-\omega_B \tau} & e^{-2\omega_B \tau} & 1 \end{bmatrix} \delta(t_1 - t_2) \\
 &= \sigma_{\xi}^2 \begin{bmatrix} 1 & e^{-\omega_B \tau} & e^{-\omega_B \tau} \\ e^{-\omega_B \tau} & 1 & e^{-2\omega_B \tau} \\ e^{-\omega_B \tau} & e^{-2\omega_B \tau} & 1 \end{bmatrix} \delta(t_1 - t_2)
 \end{aligned}
 \tag{44b}$$

and

$$\underline{\xi}(t) = \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \\ \xi_3(t) \end{bmatrix}
 \tag{44c}$$

SECTION IV

ACTUATOR MODEL

The actuator model used in this study is that developed in References 4 and 11. This model, a diagram of which is shown in Figure 10, takes into account the effects of external load reactions on the actuator's dynamics, the pressure drop across the orifice, the leakage of oil around the piston, and the compressibility of the oil.

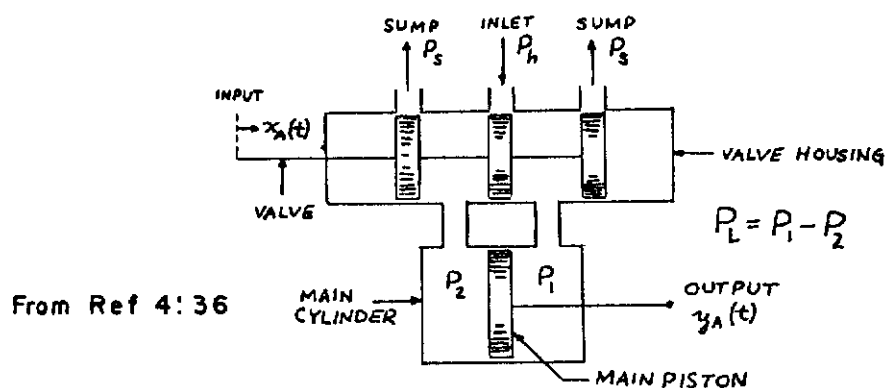


Figure 10. Hydraulic Linear Actuator

The pressure drop across the orifice, ΔP , is a function of the source pressure P_h and the load pressure P_L . If P_h is assumed constant, the flow equation is a function of valve displacement x_A and load pressure P_L :

$$q = f(x_A, P_L) \tag{45}$$

The differential of q expressed in terms of partial derivatives, is

$$dq = \frac{\partial q}{\partial x_A} dx_A + \frac{\partial q}{\partial P_L} dP_L \tag{46}$$

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If q , x_A , and P_L are measured from zero values as reference points and if the partial derivatives are constant at the values they have at zero, the integration of Equation 46 gives

$$q = \left(\frac{\partial q}{\partial x_A} \right)_0 x_A + \left(\frac{\partial q}{\partial P_L} \right)_0 P_L \quad (47)$$

By defining

$$C_x \equiv \left(\frac{\partial q}{\partial x_A} \right)_0,$$

and

$$C_p \equiv \left(- \frac{\partial q}{\partial P_L} \right)_0$$

the flow equation becomes

$$q = C_x x_A - C_p P_L \quad (48)$$

The flow of fluid into the cylinder must satisfy the conditions of equilibrium. This flow is equal to the following components:

$$q = q_o + q_l + q_c \quad (49)$$

where q_o = incompressible component (causes motion of piston)

q_l = leakage component

q_c = compressible component

The component q_o produces a motion y_A of the main piston which is given by

$$q_o = C_b \dot{y}_A \quad (50)$$

where the constant of proportionality C_b is usually taken to be equal to A_p , the area of the main actuator piston.

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The compressible component is derived in terms of the bulk modulus of elasticity, which is defined as the ratio of incremental stress to incremental strain. Thus

$$K_B = \frac{\Delta P_L}{\Delta V/V} \quad (51)$$

Solving for ΔV and dividing both sides of the equation by Δt gives

$$\frac{\Delta V}{\Delta t} = \frac{V}{K_B} \frac{\Delta P_L}{\Delta t} \quad (52)$$

Taking the limit as Δt approaches zero and letting $q_c = \frac{dV}{dt}$ results in

$$q_c = \frac{V}{K_B} \dot{P}_L \quad (53)$$

where V is the effective volume of fluid under compression and K_B is the bulk modulus of the hydraulic oil.

The leakage component is

$$q_l = L P_L \quad (54)$$

where L is the leakage coefficient of the whole system.

Combining the above equations gives

$$\begin{aligned} q &= C_x x_A - C_p P_L \\ &= C_b \dot{y}_A + \frac{V}{K_B} \dot{P}_L + L P_L \end{aligned} \quad (55)$$

Rearranging terms,

$$C_b \dot{y}_A + \frac{V}{K_B} \dot{P}_L + (L + C_p) P_L = C_x x_A \quad (56)$$

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The force developed by the main piston is

$$F_A = \eta_F A_p P_L = C_A P_L \quad (57)$$

where η_F is the force conversion efficiency of the unit (assumed to be unity in this study) and A_p is the area of the main actuator piston. From Equations 56 and 57, the relationship between actuating signal, u , and actuator force, F_A , is obtained as

$$\dot{F}_A = -\frac{C_b C_A K_B}{V} \dot{y}_A - \frac{K_B}{V} (L + C_p) F_A + \frac{C_x C_A K_B}{V} u \quad (58)$$

where $u = x_A$. To proceed any further with Equation 58, the landing gear equations must be used (see Chapter V).

Values for the parameters in the above equations were obtained from Reference 11: Part 4 and are given below

$$\begin{aligned} C_b &= A_p = 0.96 \text{ (IN.)}^2 \\ \frac{V}{K_B} &= 2.0 \times 10^{-5} \text{ (IN.}^5\text{/LB)} \\ L + C_p &= 7.0 \times 10^{-4} \text{ (IN.}^5\text{/LB - SEC)} \\ C_x &= 4.0 \text{ (IN.}^3\text{/SEC-MA)} \\ \eta_F &= 1.0 \end{aligned}$$

It is noted that the units for C_x are in.³/sec-ma. Therefore, the actuating signal, $u = x_A$, is a current input rather than a displacement input. In this connection, it was considered to model the current input, i_A , to the displacement input, x_A , by a first order lag (Reference 11: Part 7, pp. 5-6)

$$\frac{x_A}{i_A} = \frac{1}{1 + \tau_A S} \quad (59)$$

However, typical values for τ_A were 1.43×10^{-3} sec and 9.35×10^{-2} sec. Since these values are less than 0.1 sec, their effects were assumed to

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be negligible. Therefore in this study the control input u is taken to be an input current (ma), or equivalently, $u = x_A = i_A$.

It should be stressed that the values given above represent typical values, and not the values for any one particular actuator.

SECTION V
SYSTEM MODELS

1. ASSUMPTIONS

The system models for the single, tricycle, and five landing gear arrangements are obtained by appropriately combining the vehicle, runway, and actuator models described in the preceding chapters of this study. In order to accomplish this the following assumptions are made:

1. The runway is assumed to be rigid and the vehicle for each system is constrained to follow the runway profile for all time. Therefore, the deflection at the bottom of each tire is the same as the runway input to that tire, as developed in Chapter III. Hence, the effects of wheel hop cannot be studied.

2. All net forces due to ground induced loads are assumed to act only in the vertical direction. This assumption precludes any attempt to study the effects of yaw or side-slip.

3. It is assumed that the aircraft taxis at a constant horizontal speed, V_H (In this study $V_H = 66\text{fps}$). This allows the use of stationary statistics to describe the runway inputs to each vehicle model.

4. The weight of the actuator is negligible, and the actuator model developed in Chapter IV may be placed in each landing gear system without affecting the original physical characteristics of the system. Each actuator would be placed between the landing gear attachment location on the aircraft and the unsprung mass of each landing gear, and would be capable of supplying an additional force only in the vertical direction.

5. The force supplied by each actuator is constrained only by the differential equation given by Equation 58. Otherwise, no other constraints are placed on the force developed.

6. It is assumed that there exists enough clearance space or "rattle space" within the landing gear system to absorb the energy due to runway imposed vibration. This will, in general, present no problem since the landing gear system of large aircraft have as much as two to three feet allowable rattle space to absorb most runway imposed vibration energy (Reference 5:13). However, the primary concern here is that if the displacements become too large, the assumption of linearity is invalid.

7. It is assumed that there exists enough clearance space for the elastically connected mass. Again, if the displacements become too large, the assumption of linearity is invalid.

With these assumptions, the system models for each landing gear configuration are now developed.

2. SINGLE LANDING GEAR SYSTEM

In order to account for the additional force supplied by the actuator (see Figure 11), the state equations for the vehicle model given by Equation 7 are reformulated as follows:

$$\begin{aligned}
 \dot{z}_s &= v_s \\
 \dot{z}_f &= v_f \\
 \dot{z}_u &= v_u \\
 \dot{v}_s &= -\frac{K_s}{M_s} z_s + \frac{K_s}{M_s} z_f - \frac{(C_s + C_{sg})}{M_s} v_s + \frac{C_s}{M_s} v_f \\
 \dot{v}_f &= \frac{K_s}{M_f} z_s - \frac{(K_s + K_d)}{M_f} z_f + \frac{K_d}{M_f} z_u + \frac{C_s}{M_f} v_s \\
 &\quad - \frac{(C_{fg} + C_s + D)}{M_f} v_f + \frac{D}{M_f} v_u - \frac{F_A}{M_f} \\
 \dot{v}_u &= \frac{K_d}{M_u} z_f - \frac{(K_d + K_t)}{M_u} z_u + \frac{D}{M_u} v_f - \frac{D}{M_u} v_u + \frac{F_A}{M_u} + \frac{K_t}{M_u} h
 \end{aligned} \tag{60}$$

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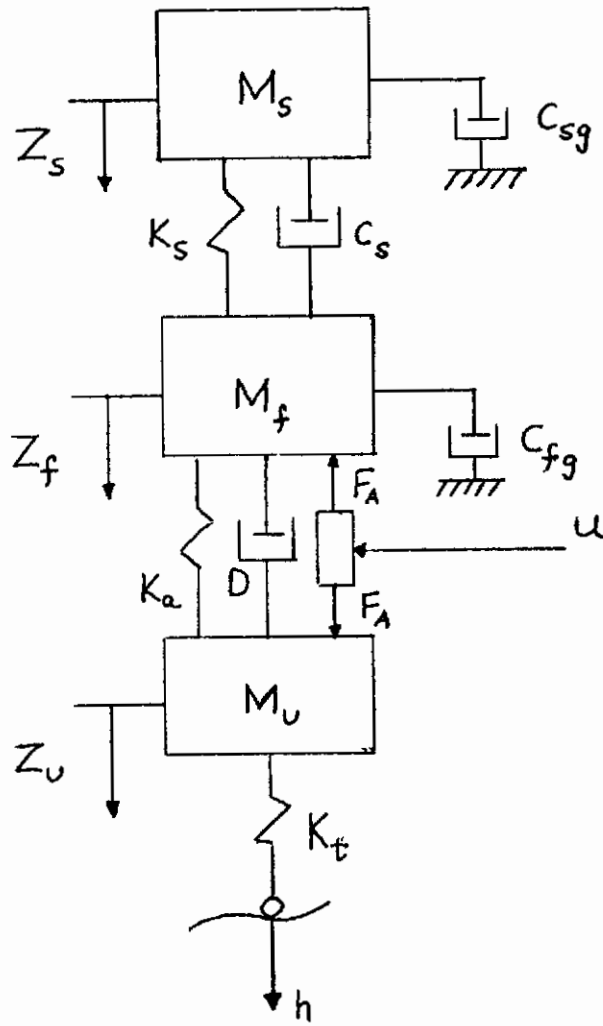


Figure 11. Single Landing Gear Model with Actuator Included

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where F_A is the force developed by the actuator and is given by Equation 58:

$$\dot{F}_A = -\frac{C_b C_A K_B}{V} \dot{y}_A - \frac{K_B}{V} (L + C_p) F_A + \frac{C_x C_A K_B}{V} u \quad (58)$$

The displacement of the actuator, y_A , which is needed to use Equation 58 is identified as

$$y_A = -Z_f + Z_u \quad (61a)$$

or

$$\dot{y}_A = \dot{v}_A = -v_f + v_u \quad (61b)$$

The displacement, h , in Equation 60 is the runway input to the single landing gear model obtained in Chapter III and is restated here as

$$\dot{h} = -\omega_B h + \xi \quad (20)$$

Equations 60, 58, and 20 are now combined under the assumptions stated at the beginning of this chapter to obtain the resulting state equations for the system model:

$$\begin{aligned} \dot{Z}_s &= v_s \\ \dot{Z}_f &= v_f \\ \dot{Z}_u &= v_u \\ \dot{v}_s &= -\frac{K_s}{M_s} Z_s + \frac{K_s}{M_s} Z_f - \frac{(C_s + C_{sg})}{M_s} v_s + \frac{C_s}{M_s} v_f \\ \dot{v}_f &= \frac{K_s}{M_f} Z_s - \frac{(K_s + K_d)}{M_f} Z_f + \frac{K_d}{M_f} Z_u + \frac{C_s}{M_f} v_s \\ &\quad - \frac{(C_{fg} + C_s + D)}{M_f} v_f + \frac{D}{M_f} v_u - \frac{F_A}{M_f} \\ \dot{v}_u &= \frac{K_d}{M_u} Z_f - \frac{(K_d + K_t)}{M_u} Z_u + \frac{D}{M_u} v_f - \frac{D}{M_u} v_u + \frac{F_A}{M_u} + \frac{K_t}{M_u} h \\ \dot{F}_A &= \frac{C_b C_A K_B}{V} v_f - \frac{C_b C_A K_B}{V} v_u - \frac{K_B}{V} (L + C_p) F_A + \frac{C_x C_A K_B}{V} u \\ \dot{h} &= -\omega_B h + \xi \end{aligned} \quad (62)$$

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where the state vector is identified as

$$\underline{x}^T = (Z_s, Z_f, Z_u, V_s, V_f, V_u, F_A, h) \quad (63)$$

3. TRICYCLE LANDING GEAR SYSTEM

The development of the system equations for the tricycle landing gear arrangement proceeds in the same manner as the single landing gear arrangement. However, since each main landing gear in this arrangement actually represents two corresponding landing gears, one on either side of the centerline of the aircraft, the parameters for the vehicle model are double those of the single gear model. Hence, the force developed by a single actuator given by Equation 58 must also be doubled. This may be conveniently handled by reformulating the state equations for the vehicle model given by Equation 11 as follows:

$$\dot{Z}_s = V_s$$

$$\dot{Z}_f = V_f$$

$$\dot{Z}_{u1} = V_{u1}$$

$$\dot{Z}_{u2} = V_{u2}$$

$$\dot{Z}_{u3} = V_{u3}$$

$$\dot{\theta} = V_\theta$$

$$\dot{V}_s = -\frac{K_s}{M_s} Z_s + \frac{K_s}{M_s} Z_f + \frac{K_s}{M_s} e\theta - \frac{(C_s + C_{sg})}{M_s} V_s + \frac{C_s}{M_s} V_f + \frac{C_s}{M_s} e V_\theta$$

$$\begin{aligned} \dot{V}_f = & \frac{K_s}{M_f} Z_s - \frac{(K_{a1} + K_{a2} + K_{a3} + K_s)}{M_f} Z_f + \frac{K_{a1}}{M_f} Z_{u1} + \frac{K_{a1}}{M_f} Z_{u2} + \frac{K_{a3}}{M_f} Z_{u3} \\ & + \frac{(K_{a1} L_{x1} + K_{a2} L_{x2} - K_{a3} L_{x3} - K_s e)}{M_f} \theta + \frac{C_s}{M_f} V_s - \frac{(D_1 + D_2 + D_3 + C_s + C_{fg})}{M_f} V_f \\ & + \frac{D_1}{M_f} V_{u1} + \frac{D_2}{M_f} V_{u2} + \frac{D_3}{M_f} V_{u3} + \frac{(D_1 L_{x1} + D_2 L_{x2} - D_3 L_{x3} - C_s e)}{M_f} V_\theta \end{aligned}$$

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$$\begin{aligned}
 \dot{v}_{u1} &= -\frac{1}{M_f} F_{A1} - \frac{2}{M_f} F_{A2} - \frac{2}{M_f} F_{A3} \\
 &+ \frac{K_{a1}}{M_{u1}} Z_f - \frac{(K_{a1} + K_{t1})}{M_{u1}} Z_{u1} - \frac{K_{a1}}{M_{u1}} L_{x1} \theta \\
 &+ \frac{D_1}{M_{u1}} v_f - \frac{D_1}{M_{u1}} v_{u1} - \frac{D_1}{M_{u1}} L_{x1} v_\theta + \frac{F_{A1}}{M_{u1}} + \frac{K_{t1}}{M_{u1}} h_1 \\
 \dot{v}_{u2} &= \frac{K_{a2}}{M_{u2}} Z_f - \frac{(K_{a2} + K_{t2})}{M_{u2}} Z_{u2} - \frac{K_{a2}}{M_{u2}} L_{x2} \theta \\
 &+ \frac{D_2}{M_{u2}} v_f - \frac{D_2}{M_{u2}} v_{u2} - \frac{D_2}{M_{u2}} L_{x2} v_\theta + \frac{2}{M_{u2}} F_{A2} + \frac{K_{t2}}{M_{u2}} h_2 \\
 \dot{v}_{u3} &= \frac{K_{a3}}{M_{u3}} Z_f - \frac{(K_{a3} + K_{t3})}{M_{u3}} Z_{u3} + \frac{K_{a3} L_{x3}}{M_{u3}} \theta \\
 &+ \frac{D_3}{M_{u3}} v_f - \frac{D_3}{M_{u3}} v_{u3} + \frac{D_3}{M_{u3}} L_{x3} v_\theta + \frac{2}{M_{u3}} F_{A3} + \frac{K_{t3}}{M_{u3}} h_3 \\
 \dot{v}_\theta &= \frac{eK_s}{I_{yy}} Z_s + \frac{(L_{x1} K_{a1} + L_{x2} K_{a2} - eK_s L_{x3} K_{a3})}{I_{yy}} Z_f \\
 &- \frac{L_{x1} K_{a1}}{I_{yy}} Z_{u1} - \frac{L_{x2} K_{a2}}{I_{yy}} Z_{u2} + \frac{L_{x3} K_{a3}}{I_{yy}} Z_{u3} \\
 &- \frac{(L_{x1}^2 K_{a1} + L_{x2}^2 K_{a2} + e^2 K_s + L_{x3}^2 K_{a3})}{I_{yy}} \theta \\
 &+ \frac{eC_s}{I_{yy}} v_s + \frac{(L_{x1} D_1 + L_{x2} D_2 - eC_s - L_{x3} D_3)}{I_{yy}} v_f \\
 &- \frac{L_{x1} D_1}{I_{yy}} v_{u1} - \frac{L_{x2} D_2}{I_{yy}} v_{u2} + \frac{L_{x3} D_3}{I_{yy}} v_{u3} \\
 &- \frac{(L_{x1}^2 D_1 + L_{x2}^2 D_2 + e^2 C_s + L_{x3}^2 D_3)}{I_{yy}} v_\theta \\
 &+ \frac{L_{x1}}{I_{yy}} F_{A1} + 2 \frac{L_{x2}}{I_{yy}} F_{A2} - 2 \frac{L_{x3}}{I_{yy}} F_{A3}
 \end{aligned} \tag{64}$$

where the values used for M_s , M_f , M_{u2} , M_{u3} , D_2 , D_3 , C_s , C_{sg} , C_{fg} , K_s , K_{a2} , K_{a3} , K_{t2} , and K_{t3} are twice the values used for the corresponding parameters for the single gear system. The values used for M_{u1} , D_1 , K_{a1} , and K_{t1} are the values for the parameters obtained for a single

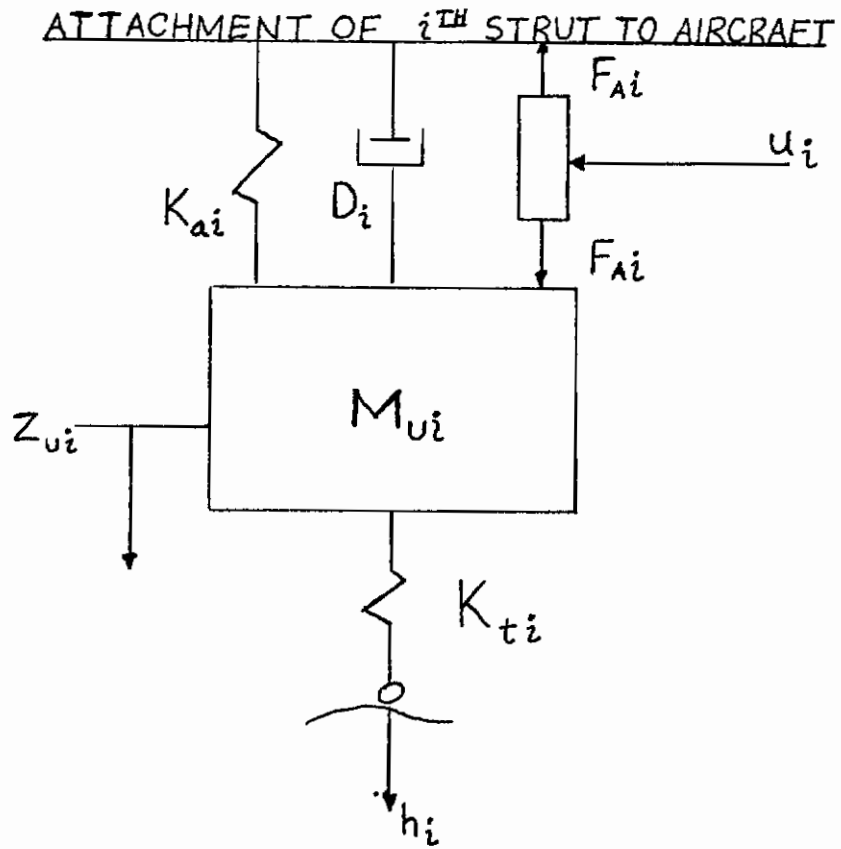


Figure 12. i^{th} Strut of Multi-Gear System with Actuator Included

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nose gear and two nose gear tires (since the example aircraft has two nose gear tires). The rationale used in selecting the values for the tricycle gear model in this manner is based, in part, on the information contained in Reference 19. In that particular report, the values for M_s , M_f , C_s , C_{sg} , C_{fg} , and K_s were given per half airplane, while the values for M_u , D , K_a , K_t were given per main gear. This also, in part, accounts for the way in which the values for the parameters for the five landing gear model were selected. For the five gear model (given in the next section) the values for M_f , C_{fg} were the only values doubled, while the values used for the other corresponding parameters are the same as those given in Reference 19:183, i.e., the same as those used for the single gear model. Final justification for using this procedure is based upon the observation that, by selecting the vehicle parameters in this manner, the results obtained for each of the three system models appear to be in the best possible agreement with each other. The actual values used for the single, tricycle, and five landing gear models are given in Appendices C, D, and E, respectively. For further information the reader is referred to Reference 19.

In Equation 64, F_{Ai} is the force developed by a single actuator located in the i^{th} strut and is given by Equation 58, that is

$$\dot{F}_{A2} = -\frac{C_D C_A K_B}{V} \dot{y}_{A2} - \frac{K_B}{V} (L + C_D) F_{A2} + \frac{C_x C_A K_B}{V} u_2 \quad (58)$$

The displacements of each actuator which are needed to use Equation 58 are identified as

$$\begin{aligned} y_{A1} &= -Z_f + Z_{u1} + L_{x1} \theta \\ y_{A2} &= -Z_f + Z_{u2} + L_{x2} \theta \\ y_{A3} &= -Z_f + Z_{u3} - L_{x3} \theta \end{aligned} \quad (65a)$$

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or

$$\begin{aligned}
 \dot{y}_{A1} &= v_{A1} = -v_f + v_{u1} + L_{x1} v_\theta \\
 \dot{y}_{A2} &= v_{A2} = -v_f + v_{u2} + L_{x2} v_\theta \\
 \dot{y}_{A3} &= v_{A3} = -v_f + v_{u3} - L_{x3} v_\theta
 \end{aligned}
 \tag{65b}$$

The displacements, h_1 , h_2 , and h_3 in Equation 64 are the runway inputs to the tricycle landing gear model obtained in Chapter III, and are restated here as

$$\begin{aligned}
 \dot{h}_1 &= -\omega_B h_1 + \xi_1 \\
 \dot{h}_2 &= (\omega_B + \frac{2}{\tau_{12}} h_1 - \frac{2}{\tau_{12}} h_2 - \xi_1 \\
 \dot{h}_3 &= -(\omega_B + \frac{2}{\tau_{12}}) h_1 + 2(\frac{1}{\tau_{12}} + \frac{1}{\tau_{23}}) h_2 - \frac{2}{\tau_{23}} h_3 + \xi_1
 \end{aligned}
 \tag{66}$$

Equations 64, 58, and 66 are now combined under the assumptions stated at the beginning of this chapter to obtain the resulting state equations for the system model:

$$\begin{aligned}
 \dot{z}_s &= v_s \\
 \dot{z}_f &= v_f \\
 \dot{z}_{u1} &= v_{u1} \\
 \dot{z}_{u2} &= v_{u2} \\
 \dot{z}_{u3} &= v_{u3} \\
 \dot{\theta} &= v_\theta \\
 \dot{v}_s &= -\frac{K_s}{M_s} z_s + \frac{K_s}{M_s} z_f + \frac{K_s}{M_s} e\theta \\
 &\quad - \frac{(C_s + C_{sg})}{M_s} v_s + \frac{C_s}{M_s} v_f + \frac{C_s}{M_s} e v_\theta \\
 \dot{v}_f &= \frac{K_s}{M_f} z_s - \frac{(K_{a1} + K_{a2} + K_{a3} + K_s)}{M_f} z_f \\
 &\quad + \frac{K_{a1}}{M_f} z_{u1} + \frac{K_{a2}}{M_f} z_{u2} + \frac{K_{a3}}{M_f} z_{u3}
 \end{aligned}$$

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$$\begin{aligned}
 & + \frac{(K_{a1} L_{x1} + K_{a2} L_{x2} - K_{a3} L_{x3} - K_s e)}{M_f} \theta \\
 & + \frac{C_s}{M_f} V_s - \frac{(D_1 + D_2 + D_3 + C_s + C_{fg})}{M_f} V_f \\
 & + \frac{D_1}{M_f} V_{u1} + \frac{D_2}{M_f} V_{u2} + \frac{D_3}{M_f} V_{u3} \\
 & + \frac{(D_1 L_{x1} + D_2 L_{x2} - D_3 L_{x3} - C_s e)}{M_f} V_\theta \\
 & - \frac{1}{M_f} F_{A1} - \frac{2}{M_f} F_{A2} - \frac{2}{M_f} F_{A3} \\
 \dot{V}_{u1} = & \frac{K_{a1}}{M_{u1}} Z_f - \frac{(K_{a1} + K_{t1})}{M_{u1}} Z_{u1} - \frac{K_{a1}}{M_{u1}} L_{x1} \theta \\
 & - \frac{D_1}{M_{u1}} V_f - \frac{D_1}{M_{u1}} V_{u1} - \frac{D_1}{M_{u1}} L_{x1} V_\theta + \frac{F_{A1}}{M_{u1}} + \frac{K_{t1}}{M_{u1}} h_1 \\
 \dot{V}_{u2} = & \frac{K_{a2}}{M_{u2}} Z_f - \frac{(K_{a2} + K_{t2})}{M_{u2}} Z_{u2} - \frac{K_{a2}}{M_{u2}} L_{x2} \theta + \frac{D_2}{M_{u2}} V_f \\
 & - \frac{D_2}{M_{u2}} V_{u2} - \frac{D_2}{M_{u2}} L_{x2} V_\theta + \frac{2}{M_{u2}} F_{A2} + \frac{K_{t2}}{M_{u2}} h_2 \\
 \dot{V}_{u3} = & \frac{K_{a3}}{M_{u3}} Z_f - \frac{(K_{a3} + K_{t3})}{M_{u3}} Z_{u3} + \frac{K_{a3}}{M_{u3}} L_{x3} \theta + \frac{D_3}{M_{u3}} V_f \\
 & - \frac{D_3}{M_{u3}} V_{u3} + \frac{D_3}{M_{u3}} L_{x3} V_\theta + \frac{2}{M_{u3}} F_{A3} + \frac{K_{t3}}{M_{u3}} h_3 \\
 \dot{V}_\theta = & \frac{eK_s}{I_{yy}} Z_s + \frac{(L_{x1} K_{a1} + L_{x2} K_{a2} - eK_s - L_{x3} K_{a3})}{I_{yy}} Z_f \\
 & - \frac{L_{x1} K_{a1}}{I_{yy}} Z_{u1} - \frac{L_{x2} K_{a2}}{I_{yy}} Z_{u2} + \frac{L_{x3} K_{a3}}{I_{yy}} Z_{u3} \\
 & - \frac{(L_{x1}^2 K_{a1} + L_{x2}^2 K_{a2} + e^2 K_s + L_{x3}^2 K_{a3})}{I_{yy}} \theta \\
 & + \frac{eC_s}{I_{yy}} V_s + \frac{(L_{x1} D_1 + L_{x2} D_2 - eC_s - L_{x3} D_3)}{I_{yy}} V_f \\
 & - \frac{L_{x1} D_1}{I_{yy}} V_{u1} - \frac{L_{x2} D_2}{I_{yy}} V_{u2} + \frac{L_{x3} D_3}{I_{yy}} V_{u3}
 \end{aligned}$$

$$\begin{aligned}
 & - \frac{(L_{x1}^2 D_1 + L_{x2}^2 D_2 + e^2 C_s + L_{x3}^2 D_3)}{I_{yy}} V_\theta \\
 \dot{F}_{A1} & = \frac{L_{x1}}{I_{yy}} F_{A1} + 2 \frac{L_{x2}}{I_{yy}} F_{A2} - 2 \frac{L_{x3}}{I_{yy}} F_{A3} \\
 & + \frac{C_b C_A K_B}{V} V_f - \frac{C_b C_A K_B}{V} V_{u1} - \frac{C_b C_A K_B}{V} L_{x1} V_\theta \\
 & - \frac{K_B}{V} (L + C_D) F_{A1} + \frac{C_x C_A K_B}{V} u_1 \\
 \dot{F}_{A2} & = \frac{C_b C_A K_B}{V} V_f - \frac{C_b C_A K_B}{V} V_{u2} - \frac{C_b C_A K_B}{V} L_{x2} V_\theta \\
 & - \frac{K_B}{V} (L + C_D) F_{A2} + \frac{C_x C_A K_B}{V} u_2 \\
 \dot{F}_{A3} & = \frac{C_b C_A K_B}{V} V_f - \frac{C_b C_A K_B}{V} V_{u3} + \frac{C_b C_A K_B}{V} L_{x3} V_\theta \\
 & - \frac{K_B}{V} (L + C_D) F_{A3} + \frac{C_x C_A K_B}{V} u_3 \\
 \dot{h}_1 & = -\omega_B h_1 + \xi_1 \\
 \dot{h}_2 & = (\omega_B + \frac{2}{\tau_{12}}) h_1 - \frac{2}{\tau_{12}} h_2 - \xi_1 \\
 \dot{h}_3 & = -(\omega_B + \frac{2}{\tau_{12}}) h_1 + 2(\frac{1}{\tau_{12}} + \frac{1}{\tau_{23}}) h_2 - \frac{2}{\tau_{23}} h_3 + \xi_1
 \end{aligned} \tag{67}$$

where the state vector is identified as

$$\begin{aligned}
 \underline{x}^T & = (Z_s, Z_f, Z_{u1}, Z_{u2}, Z_{u3}, \theta, \\
 & V_s, V_f, V_{u1}, V_{u2}, V_{u3}, V_\theta \\
 & F_{A1}, F_{A2}, F_{A3}, h_1, h_2, h_3)
 \end{aligned} \tag{68}$$

4. FIVE LANDING GEAR SYSTEM

The state equations for the system model of the five landing gear arrangement are given in Appendix B. The derivation of these equations

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is not given here; they are derived in a manner similar to the two system models previously described. The state equations for the vehicle model were first reformulated to account for the additional forces produced by an actuator in each strut:

$$\dot{F}_{A\dot{z}} = -\frac{C_b C_A K_B}{V} \dot{y}_{A\dot{z}} - \frac{K_B}{V} (L + C_D) F_{A\dot{z}} + \frac{C_x C_A K_B}{V} u_{\dot{z}} \quad (58)$$

where the displacements, y_{A1} , are identified as

$$\begin{aligned} y_{A1} &= (-Z_f + Z_{u1} + L_{x1} \theta) \\ y_{A2} &= (-Z_f + Z_{u2} + L_{x2} \theta + L_{y2} \phi) \\ y_{A3} &= (-Z_f + Z_{u3} - L_{x3} \theta + L_{y3} \phi) \\ y_{A4} &= (-Z_f + Z_{u4} + L_{x4} \theta - L_{y4} \phi) \\ y_{A5} &= (-Z_f + Z_{u5} - L_{x5} \theta - L_{y5} \phi) \end{aligned} \quad (69)$$

The runway inputs h_1, h_2, h_3, h_4 , and h_5 are those derived in Chapter III for the five landing gear model and are completely defined by the equations:

$$\begin{aligned} \dot{h}_1 &= -\omega_B h_1 + \xi_1 \\ \dot{h}_2^* &= -\omega_B h_2^* + \xi_2 \\ \dot{h}_2 &= (\omega_B + \frac{2}{\tau_{12}}) h_2^* - \frac{2}{\tau_{12}} h_2 - \xi_2 \\ \dot{h}_3 &= -(\omega_B + \frac{2}{\tau_{12}}) h_2^* + 2(\frac{1}{\tau_{12}} + \frac{1}{\tau_{23}}) h_2 - \frac{2}{\tau_{23}} h_3 + \xi_2 \\ \dot{h}_4^* &= -\omega_B h_4^* + \xi_4 \\ \dot{h}_4 &= (\omega_B + \frac{2}{\tau_{14}}) h_4^* - \frac{2}{\tau_{14}} h_4 - \xi_4 \\ \dot{h}_5 &= -(\omega_B + \frac{2}{\tau_{14}}) h_4^* + 2(\frac{1}{\tau_{14}} + \frac{1}{\tau_{45}}) h_4 - \frac{2}{\tau_{45}} h_5 + \xi_4 \end{aligned} \quad (44)$$

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SECTION VI

MATHEMATICAL FORMULATION

1. SYSTEM DYNAMICS

Each of the combined system models obtained in Chapter V of this study is described by an equation of the form

$$\dot{\underline{x}} = \underline{F}\underline{x} + \underline{\psi}\underline{u} + \underline{G}\underline{\xi} \quad (1)$$

where \underline{x} - n vector, the state
F - nxn constant matrix
G - nxp constant matrix
 $\underline{\psi}$ - nxm constant matrix
 \underline{u} - m vector, the control
 $\underline{\xi}$ - P vector, the disturbance

and

$$E[\underline{\xi}(t)] = 0$$

$$E[\underline{\xi}(t_1)\underline{\xi}^T(t_2)] = Q\delta(t_1 - t_2)$$

where Q is a constant positive semidefinite p x p matrix.

Furthermore, the measurement model is assumed to be noise free and of the form

$$\underline{y} = \underline{H}\underline{x} \quad (2)$$

where \underline{y} - r vector, the measurement
H - rxn constant matrix.

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Since the approach used in this study is to determine a control which is a proportional feedback of the measurement, that is:

$$\underline{u} = H \underline{y} \quad (3a)$$

we have by Equation 2 that

$$\underline{u} = K H \underline{x} \quad (3b)$$

where the mxr constant feedback matrix K may be written as

$$K = \begin{bmatrix} K_{11} & K_{12} & - & - & - & K_{1r} \\ K_{21} & K_{22} & - & - & - & K_{2r} \\ | & | & & & & | \\ | & | & & & & | \\ | & | & & & & | \\ K_{m1} & K_{m2} & - & - & - & K_{mr} \end{bmatrix}$$

Thus the control is constrained to be a linear transformation of the measurements.

Substitution of Equation 3b into Equation 1 yields the result:

$$\dot{\underline{x}} = A \underline{x} + G \underline{\xi} \quad (71)$$

where

$$A \equiv [F + \psi K H]$$

The process modelled by Equation 71 is Gauss-Markov and the nxn covariance matrix P(t) for the process is given by the differential equation.

$$\dot{P}(t) = A P(t) + P(t) A^T + G G^T \quad (72)$$

where

$$P(t) = E \left[\underline{x}(t) \underline{x}^T(t) \right]$$

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Since A, G, and Q are constant matrices it is possible, under the condition that the system

$$\dot{\underline{x}} = \underline{A}\underline{x} \quad (73)$$

is asymptotically stable, that $P(t) \rightarrow P$, a constant matrix, as $t \rightarrow \infty$. If so, $\dot{P} \rightarrow 0$, so that P may be determined from the linear relation

$$\underline{A}P + PA^T + GQG^T = 0 \quad (74)$$

Such a process is statistically stationary; essentially, the random forcing function $\underline{\xi}(t)$ is balanced by the damping of the system as expressed in A of Reference 2:334. Stationary statistics will be assumed throughout the following analysis.

Therefore, in order to work with Equation 74, stability of the A matrix for each of the system models must be determined. The test used is based upon the following necessary and sufficient condition for stability.

a. Liapunov's Theorem

The system $\dot{\underline{x}} = \underline{A}\underline{x}$ is asymptotically stable if and only if the Liapunov equation

$$\underline{A}^T \underline{X} + \underline{X} \underline{A} = - \underline{I} \quad (75)$$

has a solution in X which is positive definite, where I is the nxn identity matrix.

Proof of the theorem is given in Reference 13:81-84. If X exists, it is symmetric and the eigenvalues, which are real, are easily computed using the Jacobi method described in Reference 18:Chap. 7. X is positive definite if all the eigenvalues are greater than zero.

2. THE COST CRITERION

Since the problem is to be formulated in terms of an optimal control problem, a cost or performance criterion must be established. Wing fatigue

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results from the stresses experienced by the wing. These stresses are proportional to the relative displacement between the wing and the fuselage of the aircraft. Therefore, a reasonable performance criterion to be followed here is to minimize the square of the relative displacement between the main mass in the system models and the elastically connected mass. However, the control effort must also be considered in the performance criterion. This term should result in a penalty due to any excessive effort on the control. Therefore, the following performance criterion is adopted:

$$J = \lim_{t \rightarrow \infty} E \left\{ \underline{x}^T \underline{b} \underline{b}^T \underline{x} + \underline{u}^T R \underline{u} \right\} \quad (76)$$

where \underline{b} is an n vector chosen such that $\underline{b}^T \underline{x}$ is the relative displacement between the main mass and the elastically connected mass; R is a symmetric $m \times m$ positive definite matrix; and E denotes the expected value. The limit as t approaches infinity is taken in Equation 76 to permit the use of stationary statistics. Furthermore, if the $n \times n$ symmetric matrix B is defined as

$$B \equiv \underline{b} \underline{b}^T \quad (77)$$

Equation 76 becomes

$$J = \lim_{t \rightarrow \infty} E \left\{ \underline{x}^T B \underline{x} + \underline{u}^T R \underline{u} \right\} \quad (78)$$

From Equation 3b, the form of the control is given as

$$\underline{u} = K H \underline{x} \quad (3b)$$

Therefore Equation 78 may be rewritten as

$$J = \lim_{t \rightarrow \infty} E \left\{ \underline{x}^T B \underline{x} + \underline{x}^T H^T K^T R K H \underline{x} \right\} \quad (79)$$

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To simplify the analysis, it is assumed that the R matrix is of the form

$$R_{ij} = R_c \delta_{ij} \quad i, j = 1, \dots, m$$

or

(80)

$$R = R_c I_{m \times m}$$

where R_c is a scalar constant and $I_{m \times m}$ is the $m \times m$ identity matrix. This results in a diagonal R matrix with equal weightings on each control and no cross-coupling among the controls. In an actual problem where a particular actuator is being used, the R_c is chosen so that the 3σ capability of the actuator is not exceeded; i.e., the requirements placed on the actuator are within the maximum capability of the actuator 99.7% of the time. In the problem of this study, since no specific actuator is being used, the range of values of R_c is selected such that no significant reduction in wing fatigue results by further decreasing R_c .

Equation 79 may now be rewritten as

$$J = \lim_{t \rightarrow \infty} E \left\{ \underline{x}^T B \underline{x} + R_c \underline{x}^T H^T K^T K H \underline{x} \right\} \quad (81)$$

Since the expression within the expectation operator is a quadratic form in \underline{x} , the expression for J may be written as

(82a)

$$J = \lim_{t \rightarrow \infty} \left\{ \text{trace} [C P(t)] \right\}$$

or

$$J = \lim_{t \rightarrow \infty} \left\{ C \left[\lim_{t \rightarrow \infty} P(t) \right] \right\}$$

$$= \text{trace} [C P] \quad (82b)$$

where the $n \times n$ matrix C is defined as

$$C \equiv B + R_c H^T K^T K H \quad (83)$$

and P is the covariance matrix obtained from Equation 74.

Equations 82 and 83 express the cost as a real valued function of the feedback matrix K . It is also noted that since this matrix enters the state equations through Equation 71 the statistics on the states become functions of the matrix K . In particular, the covariance matrix P given in Equation 74 is a function of K ; that is

$$P = P(K) \tag{84}$$

The problem now reduces to one of finding a feedback matrix K^* such that the cost given by Equation 82 is minimum:

$$J(K^*) < J(K) \quad \text{for all } K \tag{85}$$

A necessary condition for the minimum in Equation 85 is obtained by setting the gradients of J with respect to each of the elements of the K matrix equal to zero:

$$\frac{\partial J}{\partial K_{ij}} = 0 \quad \begin{matrix} i = 1, \dots, m \\ j = 1, \dots, r \end{matrix} \tag{86}$$

Using Equation 82 the gradients of J with respect to each element of the matrix K given by

$$\frac{\partial J}{\partial K_{ij}} = \text{trace} \left[\frac{\partial C}{\partial K_{ij}} P + C \frac{\partial P}{\partial K_{ij}} \right] \tag{87}$$

The $\frac{\partial C}{\partial K_{ij}}$ may be obtained by taking the partial derivative of both sides of Equation 83 with respect to K_{ij} :

$$\frac{\partial C}{\partial K_{ij}} = R_c H^T \left[\frac{\partial K^T}{\partial K_{ij}} K + K^T \frac{\partial K}{\partial K_{ij}} \right] H \tag{88}$$

Likewise, the $\frac{\partial P}{\partial K_{ij}}$ may be obtained by taking the partial derivative of both sides of Equation 74 with respect to K_{ij} and solving for $\frac{\partial P}{\partial K_{ij}}$, knowing A , P , and $\frac{\partial A}{\partial K_{ij}}$:

$$A \frac{\partial P}{\partial K_{ij}} + \frac{\partial P}{\partial K_{ij}} A^T + \frac{\partial A}{\partial K_{ij}} P + P \frac{\partial A^T}{\partial K_{ij}} = 0 \tag{89a}$$

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or

$$A \frac{\partial P}{\partial K_{ij}} + \frac{\partial P}{\partial K_{ij}} A^T = - \frac{\partial A}{\partial K_{ij}} P - P \frac{\partial A^T}{\partial K_{ij}} \quad (89b)$$

where the $\frac{\partial A}{\partial K_{ij}}$ may be obtained from Equation 71 as

$$\frac{\partial A}{\partial K_{ij}} = \psi \frac{\partial K}{\partial K_{ij}} H \quad (90)$$

While the vanishing of the gradients given by Equation 87 is only a necessary condition for $J(K^*)$ to be a minimum, it is intuitively felt that the choice for the control \underline{u} given by

$$\underline{u} = K^* H \underline{x} \quad (91)$$

does minimize the cost criterion since J can be made arbitrarily large by choosing \underline{u} arbitrarily large. However, a sufficient condition that a minimum J has been obtained results if the Hessian matrix of the second partial derivatives of J with respect to the K_{ij} 's is positive semi-definite for all possible K . No attempt will be used in this study to take this approach to validate the fact that the extremal is indeed a local minimum. Rather, intuition and the results which are obtained will serve to indicate whether or not a minimum J has been obtained.

Since a Newton-Raphson or second variation method is used to carry out the minimization, the second partial derivatives of J , $\frac{\partial^2 J}{\partial K_{kl} \partial K_{ij}}$ must also be determined. Taking the partial derivative of Equation 87 with respect to K_{kl} , results in

$$\begin{aligned} \frac{\partial^2 J}{\partial K_{kl} \partial K_{ij}} = \text{trace} \left[\frac{\partial^2 C}{\partial K_{kl} \partial K_{ij}} P + \frac{\partial C}{\partial K_{ij}} \frac{\partial P}{\partial K_{kl}} + \frac{\partial C}{\partial K_{kl}} \frac{\partial P}{\partial K_{ij}} \right. \\ \left. + C \frac{\partial^2 P}{\partial K_{kl} \partial K_{ij}} \right] \quad (92) \end{aligned}$$

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The $\frac{\partial^2 C}{\partial K_{kl} \partial K_{ij}}$ may be obtained by taking the partial derivative of both sides of Equation 88 with respect to K_{kl} :

$$\frac{\partial^2 C}{\partial K_{kl} \partial K_{ij}} = R_c H T \left[\frac{\partial K^T}{\partial K_{ij}} \frac{\partial K}{\partial K_{kl}} + \frac{\partial K^T}{\partial K_{kl}} \frac{\partial K}{\partial K_{ij}} \right] H \quad (93)$$

where the fact that

$$\frac{\partial^2 K}{\partial K_{kl} \partial K_{ij}} = 0 \quad (94a)$$

and

$$\frac{\partial^2 K^T}{\partial K_{kl} \partial K_{ij}} = 0 \quad (94b)$$

were used in obtaining Equation 93.

The $\frac{\partial^2 P}{\partial K_{kl} \partial K_{ij}}$ may be obtained by taking the partial derivative of both sides of Equation 89 with respect to K_{kl} :

$$\begin{aligned} A \frac{\partial^2 P}{\partial K_{kl} \partial K_{ij}} + \frac{\partial^2 P}{\partial K_{kl} \partial K_{ij}} A^T + \frac{\partial A}{\partial K_{ij}} \frac{\partial P}{\partial K_{kl}} + \frac{\partial P}{\partial K_{kl}} \frac{\partial A^T}{\partial K_{ij}} \\ + \frac{\partial A}{\partial K_{kl}} \frac{\partial P}{\partial K_{ij}} + \frac{\partial P}{\partial K_{ij}} \frac{\partial A^T}{\partial K_{kl}} = 0 \end{aligned} \quad (95a)$$

or

$$\begin{aligned} A \frac{\partial^2 P}{\partial K_{kl} \partial K_{ij}} + \frac{\partial^2 P}{\partial K_{kl} \partial K_{ij}} A^T = - \left[\frac{\partial A}{\partial K_{ij}} \frac{\partial P}{\partial K_{kl}} + \frac{\partial P}{\partial K_{kl}} \frac{\partial A^T}{\partial K_{ij}} \right. \\ \left. + \frac{\partial A}{\partial K_{kl}} \frac{\partial P}{\partial K_{ij}} + \frac{\partial P}{\partial K_{ij}} \frac{\partial A^T}{\partial K_{kl}} \right] \end{aligned} \quad (95b)$$

where the fact that

$$\frac{\partial^2 A}{\partial K_{kl} \partial K_{ij}} = 0 \quad (96)$$

was used, which is obvious from Equations 90 and 94.

SECTION VII

NEWTON RAPHSON ALGORITHM

1. PROBLEM

Consider n functions $f_i(k_1, \dots, k_n)$ ($i=1, \dots, n$) of n variables, k_1, \dots, k_n . For notational convenience, the vector \underline{k} is defined as

$$\underline{k}^T = (k_1, \dots, k_n) \quad (97a)$$

and

$$f_i(k_1, \dots, k_n) = f_i(\underline{k}) \quad (97b)$$

The problem is to find the vector \underline{k}^* such that

$$f_i(\underline{k}^*) = 0 \quad i = 1, \dots, n. \quad (98)$$

Suppose that the functions $f_i(\underline{k})$ are expanded about a given vector \underline{k}^P in a Taylor series expansion, assuming that the functions are analytic. Then retaining only the first order terms of the series results in

$$f_i(\underline{k}^P + \Delta \underline{k}^P) = f_i(\underline{k}^P) + \sum_{j=1}^n \frac{\partial f_i}{\partial k_j}(\underline{k}^P) \Delta k_j^P \quad (99)$$

where

$$\Delta \underline{k}^P = \begin{bmatrix} \Delta k_1^P \\ \vdots \\ \Delta k_n^P \end{bmatrix}$$

Since it is desired that $f_i(\underline{k}^P + \Delta \underline{k}^P) = 0$, the right hand side of Equation 99 is set equal to zero:

$$f_i(\underline{k}^P) + \sum_{j=1}^n \frac{\partial f_i}{\partial k_j}(\underline{k}^P) \Delta k_j^P = 0 \quad (100a)$$

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or

$$\sum_{j=1}^n \frac{\partial f_i}{\partial k_j} (\underline{k}^P) \Delta k_j^P = - f_i (\underline{k}^P) \quad (100b)$$

Equation 100 represents simultaneous linear equations in n unknowns,

Δk_j^P ($j=1, \dots, n$). These equations can be solved by the Gaussian

Elimination Method to obtain the values of the Δk_j^P . Then

$$\underline{k}^{P+1} = \underline{k}^P + \Delta \underline{k}^P \quad (101)$$

and the process can be repeated. Assuming that the process remains stable, i.e., converges, the following results are obtained

$$\lim_{P \rightarrow \infty} \underline{k}_j^P = \underline{k}^* \quad (102a)$$

and

$$\lim_{P \rightarrow \infty} f_i (\underline{k}^P) = f_i (\underline{k}^*) = 0 \quad i = 1, \dots, n \quad (102b)$$

For the cost criterion developed in the preceding chapter, the following identifications are made:

$$\begin{aligned} (m \times r) &\longleftrightarrow n \\ \frac{\partial J}{\partial K_{ij}} (K) &\longleftrightarrow f_i (\underline{k}) \\ \frac{\partial^2 J}{\partial K_{k\ell} \partial K_{ij}} (K) &\longleftrightarrow \frac{\partial f_i}{\partial k_j} (\underline{k}) \end{aligned} \quad (103)$$

The resulting equations corresponding to Equation 100 are

$$\sum_{k=1}^m \sum_{\ell=1}^r \frac{\partial^2 J}{\partial K_{k\ell} \partial K_{ij}} (K^P) \Delta K_{k\ell}^P = - \frac{\partial J}{\partial K_{ij}} (K^P) \quad \begin{matrix} i=1, \dots, m \\ j=1, \dots, r \end{matrix} \quad (104)$$

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Equation 104 represents $m \times r$ simultaneous linear equations in $m \times r$ unknowns, ΔK_{kl}^P , which may be solved to obtain

$$K^{P+1} = K^P + \Delta K^P \quad (105)$$

where

$$\Delta K^P = \begin{bmatrix} \Delta K_{11}^P & \dots & \Delta K_{1r}^P \\ \vdots & & \vdots \\ \Delta K_{m1}^P & \dots & \Delta K_{mr}^P \end{bmatrix}$$

and the process can be repeated. Assuming that the process converges, the following results are obtained

$$\lim_{P \rightarrow \infty} K^P = K^* \quad (106a)$$

and

$$\lim_{P \rightarrow \infty} \frac{\partial J}{\partial K_{ij}} (K^P) = 0 \quad \begin{array}{l} i = 1, \dots, m \\ j = 1, \dots, r \end{array} \quad (106b)$$

But from Chapter VI the above K^* minimizes the cost criterion, J . Hence,

$$\lim_{P \rightarrow \infty} J(K^P) = J(K^*) \quad (107a)$$

and

$$J(K^*) < J(K) \quad \text{for all } K \quad (107b)$$

Since the second partial derivatives of J , $\frac{\partial^2 J}{\partial K_{kl} \partial K_{ij}}$, are used in the minimization, the method is often referred to as a second variation method. The algorithm for the entire minimization process proceeds as follows.

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2. ALGORITHM

1. Calculate the matrices F, ψ, G and H in Equations 1 and 2.
2. Using an initial or previously calculated K^P , calculate A .
3. Test for the stability of A using Liapunov's Theorem.
4. If A is asymptotically stable calculate and store $P(K^P)$, the value of P obtained for K^P , from Equation 74. If A is not asymptotically stable, the process must be either terminated or initialized again to obtain a stable A matrix.
5. Using Equation 82, calculate $J(K^P)$.
6. Using Equations 88, 89, and 87, calculate and store $\frac{\partial C}{\partial K_{ij}}(K^P)$, $\frac{\partial P}{\partial K_{ij}}(K^P)$, and $\frac{\partial J}{\partial K_{ij}}(K^P)$.
7. For a given $\frac{\partial J}{\partial K_{ij}}(K^P)$ obtained in Step 6, calculate and store $\frac{\partial^2 J}{\partial K_{kl} \partial K_{ij}}$ using Equation 92. Note that the $\frac{\partial C}{\partial K_{kl}}(K^P)$, $\frac{\partial P}{\partial K_{kl}}(K^P)$, $\frac{\partial^2 C}{\partial K_{kl} \partial K_{ij}}(K^P)$, and $\frac{\partial^2 P}{\partial K_{kl} \partial K_{ij}}(K^P)$ which are needed for this calculation may be obtained from Equations 88, 89, 93, and 95, respectively.
8. Repeat Step 7 for $k=1, \dots, m$ and $l=1, \dots, r$.
9. Repeat Steps 6, 7, and 8 for $i=1, \dots, m$ and $j=1, \dots, r$.
10. Set up and solve the $m \times r$ linear equations given by Equation 104 to obtain ΔK^P , and $K^{P+1} = K^P + \Delta K^P$.
11. Continue the process until $\left| \frac{\partial J}{\partial K_{ij}}(K^P) \right| < \epsilon$, where $\frac{\partial J}{\partial K_{ij}}$ is obtained in Step 6 and ϵ is a tolerance used in the test for convergence. The cost obtained in Step 5 is then taken to be the minimum J , $J(K^*)$.

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A computer flow chart of the algorithm is given in Figure 13.

Computational time for each of the models was as follows:

1. Single Landing Gear System: 5 minutes on the IBM 7094 Direct Coupled System.
2. Tricycle Landing Gear System: 20 minutes on the IBM 7094 Direct Coupled System or 5 minutes on CDC 6600 System.
3. Five Landing Gear System: 16 hours on the CDC 1604B System.

The value of ϵ currently being used for each of the models is 10^{-6} . However, in many cases this convergence criterion was not met in the computer times stated above. In these cases, the convergence criteria came out on the range between 10^{-3} and 10^{-5} .

For some of the results obtained in this study, the Conjugate Gradient Method of Fletcher and Reeves (Reference 7:149-154) was used to initialize the Newton Raphson Method. In other cases, the results of a previous minimization was used to initialize the Newton Raphson Method for the next minimization.

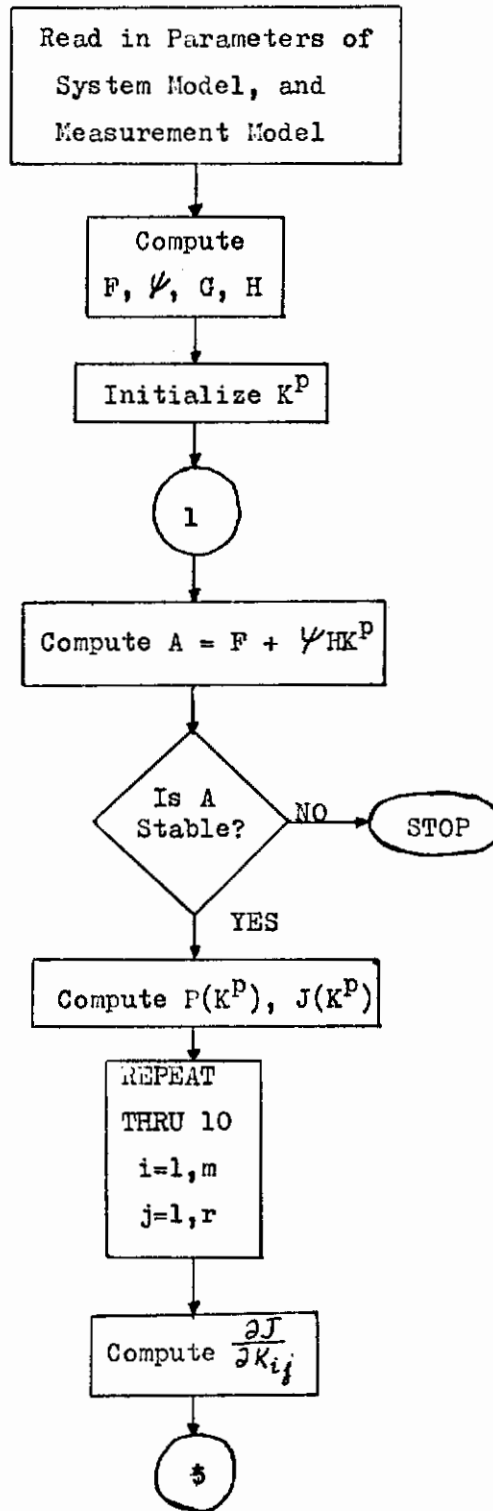


Figure 13. Computer Flow Diagram

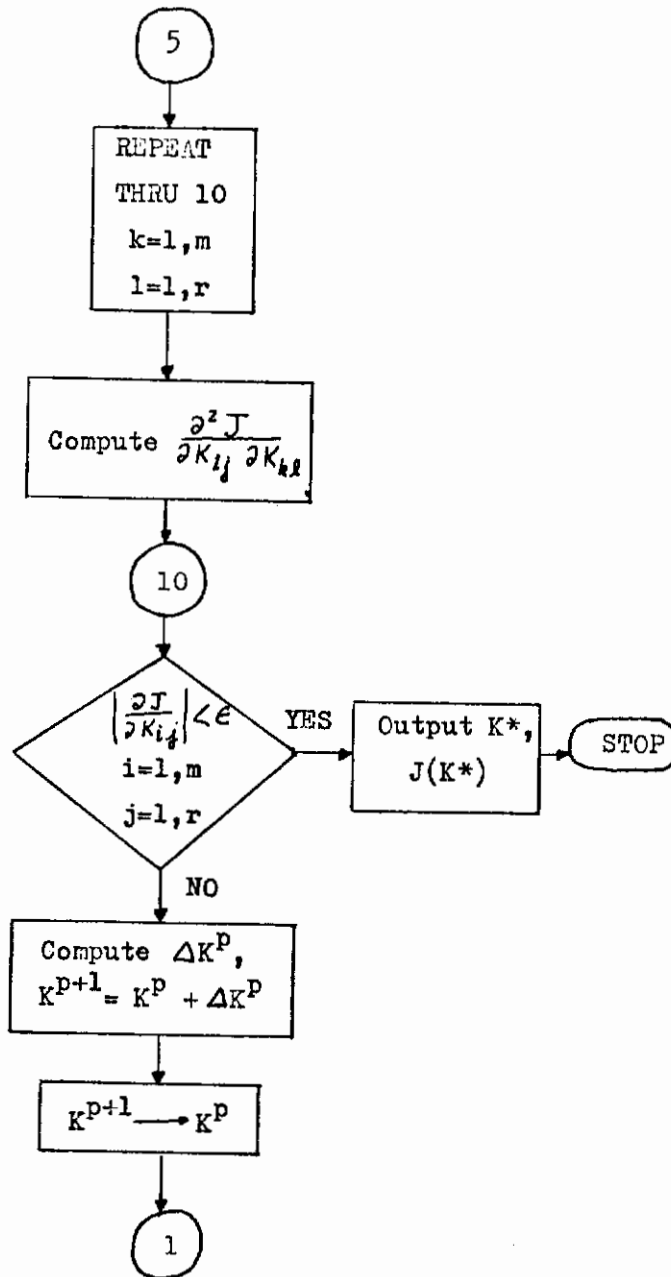


Figure 13. (Contd) Computer Flow Diagram

SECTION VIII

RESULTS1. GENERAL REMARKS

Appendices C, D, and E present the results obtained for the single, tricycle, and five landing gear systems, respectively. The data includes the requirements on force, power, control signal, displacement and velocity that must be supplied by the actuators in each configuration studied and for various values of control weighting R_c . It is noted that all of the above results are given in terms of the variances of the various physical requirements with the exception of power, for which the average value, or covariance of actuator force and velocity, is given. Also given in the above appendices are the results of the minimization procedure described in Chapter VII. These later results include the variance of the stress per wing and the corresponding $J(K^*)$ as a function of the control weighting R_c for each configuration studied. It is noted here (and also shown in the appropriate tables of Appendices C, D, and E) that the stress per wing for each system without the actuators included in the landing gears was obtained as:

Single Landing Gear System: 9.768 (in.)^2

Tricycle Landing Gear System: 8.838 (in.)^2

Five Landing Gear System: 8.770 (in.)^2

With the actuators included in each system, but without any control signal applied (i.e. $\underline{u}=0$), this value of stress per wing was reduced to the following values:

Single Landing Gear System (for all feedback laws studied): 5.508 (in.)^2

Tricycle Landing Gear System:

Case I (Actuators were placed in all three Landing gears): 7.304 (in.)^2

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Case II (Actuators were placed only in the two main landing gears): 6.124 (in.)^2

Five Landing Gear System: 7.240 (in.)^2

Thus for each configuration studied a reduction in stress per wing occurs when the actuators are placed in the landing gear system.

A brief description now follows on the highlights of the results obtained for each system.

2. SINGLE LANDING GEAR SYSTEM

The primary purpose of the single landing gear systems was to test the effectiveness of various feedback laws. A total of four feedback laws were studied for this purpose (see Appendix C):

Feedback Law I: Consisted of measuring the relative displacement between M_s and M_f , and measuring the acceleration of M_f .

Feedback Law II: Consisted of measuring the acceleration of both M_f and M_u .

Feedback Law III: Consisted of measuring the relative displacement between M_s and M_f , and the relative displacement between M_f and M_u .

Feedback Law IV: Consisted of measuring the acceleration of both M_s and M_f .

From the results presented in Appendix C, it was found that Feedback Laws I and IV proved to be very effective in reducing the stress on the wing and yielded almost identical results. This is not unusual when it is considered that the model used is linear thus providing a direct relationship between the acceleration of M_s and the relative displacement between M_s and M_f . The results indicate that by using either Feedback Law I or IV the stress on the wing may be made arbitrarily small (asymptotically approaching zero).

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Results for Feedback Law II indicate that this control law is incapable of reducing the stress on the wing below a value of 5.2 in.^2 , and thus is an ineffective control law.

Finally, results for Feedback Law III indicate that this control law was incapable of reducing the stress on the wing below a value of 2.6 in.^2 .

3. TRICYCLE LANDING GEAR SYSTEM

Because of the effectiveness of Feedback Law I for the single landing gear system, this control law was also used for the tricycle landing gear system. It is noted that the selection of Feedback Law I over Feedback Law IV was arbitrary as they both had the same performance. Results for two cases were obtained for the tricycle landing gear system (see Appendix D):

Case I: Actuators were placed in all three landing gears.

Case II: Actuators were placed only in the two main landing gears; i.e., the actuator was removed from the nose gear of Case I.

From the results given in Appendix D, it can be seen that the feedback law used proved to be very effective for each of the above cases. These results indicate that the stress on the wing may be made arbitrarily small for each case. However, the arrangement used in Case II has the obvious advantage that it uses one less actuator than Case I, thus reducing the cost of implementing the arrangement. This advantage is further brought out when it is observed that the characteristics of the actuator in the nose gear vary greatly from those of the two main gears. In particular, it is noted that the velocity of the actuator in the nose gear varies from 142.2 to $236.5 \text{ (in./sec)}^2$ while the maximum velocity of the actuators in either main gear is $64.13 \text{ (in./sec)}^2$. Also it is noted that there is a change in the sign of the feedback constants K_{11} and K_{12} (see Table XIV) from minus to plus with decreasing weighting on the

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control vector \underline{u} . On the other hand, the characteristics of the actuators in either main gear are essentially the same in either Case I or Case II.

As far as any agreement between the results obtained for the single landing gear system and the results obtained for the tricycle landing gear system is concerned, it can be said that the results for the single gear system present the trend in the characteristics of the actuator which may also be observed in the characteristics of the actuators in the main gears in the tricycle gear system. That the discrepancies do exist should come as no surprise, since the single gear arrangement is a much more simplified model of the actual landing gear configuration of the aircraft than the tricycle gear configuration. In this respect it was noted during the course of the study that the pitch angles were small (with variances of 10^{-5} (rad)²). Therefore, the discrepancies are not likely to result from the effect of aircraft pitch but rather from the actual location of the landing gears themselves. Hence, it should be expected that there will also be discrepancies between the tricycle gear arrangement and the five landing gear arrangement. This is indeed the case.

4. FIVE LANDING GEAR SYSTEM

Because of the effectiveness of Feedback Law I for both the single and tricycle landing gear systems, this control law was also used for the five landing gear system.

From the results given in Appendix E, it can be seen that the data obtained for the tricycle landing gear system is very much the same as the data obtained for the five landing gear system. However, there are discrepancies. The most noteworthy of these discrepancies occurs in the feedback constants (see Tables XXIII to XXVII). Unlike the case of the tricycle system where only the relative displacement between one elastically connected mass and the main mass is measured and fed back to the controllers, in the five landing gear system, the relative displacements between two elastically connected masses and the main mass are fed back to the controllers, each independent of the other, and each being

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multiplied by a different feedback constant. As it turns out, for each of the four main gears, the feedback constant multiplying the measurement from one wing is not numerically the same as the feedback constant multiplying the measurement from the other wing, thus indicating that each of the main gears sees two different wings. However, the feedback constants multiplying the measurements from each wing for the nose gear are the same, thus indicating that the nose gear sees two identical wings. From this result it can be said that since each main gear is displaced a given distance from the centerline of the aircraft, it no longer sees the aircraft as being symmetrical about the roll axis. But the overall effect on the entire system model is symmetrical about the roll axis; that is to say that the actuators in the main gears on one side of the aircraft will weight the signal from each wing in exactly the reverse order as the corresponding main gears on the opposite side of the aircraft. This results in the net effect of having the same physical demands placed on the corresponding actuators on either side of the aircraft: the actuators in the front main gears have identical physical demands placed on them, while those in the rear main gears have identical physical demands placed on them (see Tables XXIX and XXX).

Finally, it is noted that the pitch and roll angles that were observed during the course of the study were small: variance of pitch angle 10^{-5} (rad)²; variance of roll angle 10^{-4} (rad)². Therefore, it is concluded that the effects of aircraft pitch and roll have little to do with the discrepancies between the results obtained for the tricycle gear arrangement and the results of the five gear arrangement. Rather, the discrepancies result from the actual physical locations of the landing gears themselves.

5. TRADEOFF DIAGRAMS

Since the actuators used in this study are limited by the output velocities they are capable of attaining (Reference 12:3), the results in Appendices C, D, and E include tradeoff diagrams which relate the maximum wing stress with the output velocities required for each actuator for the optimal system. For the single landing gear system, the

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output velocity and maximum wing stress is plotted against R_c on the same diagram. For the multiple gear analyses, since more than one actuator is used, the output velocities and maximum wing stress are plotted against R_c on separate diagrams. The use of the diagrams is simple. For a desired value of wing stress, a designer would obtain the corresponding value of R_c . For this value of R_c , the designer would then refer to the other diagrams (for the single landing gear model, the same diagram) to find the required output velocities of each actuator. Therefore, these diagrams, together with the other information contained in the appendices, can provide the designer with a tool for a rational comparison of the performance of a landing gear system with that of the best possible system. From the results given, the designer can assess the practical utility of trying to improve the performance of any given concept or of searching for other designs which would approach or actually duplicate the performance of the best possible system. Appendix F contains an example application of this procedure for the three system models. In Appendix F, actuator requirements are determined for a 30%, 50%, and 70% reduction in fatigue damage for each system model. Also, the results of the one, three, and five landing gear systems are compared.

Where possible, the designer should use the results available for the highest order system (the five gear system), since it is expected that these results give the best indication of the performance of the optimal system. Where these results are not available, the data for the tricycle landing gear system can be used. Where data does not exist for either the five or tricycle gear system, the data for the single landing gear system may be used with the understanding that these results only give a fair approximation to the performance of the optimal system and the control parameters that are necessary for the optimal system.

Finally, the technique used in this study was applied to only one aircraft to demonstrate its applicability to the design of the optimal

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system to reduce wing fatigue. Therefore, results of this effort serve primarily as a guide for the design engineer. In using the results, the designer must evaluate and classify the data with respect to the aircraft being considered.

SECTION IX

CONCLUSIONS AND RECOMMENDATIONS

1. CONCLUSIONS

From the results obtained in the preceding chapter the following conclusions can be made:

1. The discrepancies that exist among the three system models are due primarily to the actual landing gear arrangements used in each model.

2. The effects of aircraft pitch and roll are negligible.

3. The tricycle landing gear arrangement does provide results which agree well with the five landing gear arrangement. While the results for the single landing gear arrangement are not in total agreement with the results of the multiple gear arrangements, the single gear analysis does provide the trend in the characteristics of the actuator which may also be observed in the characteristics of the actuator in the main gears of the multiple gear analyses.

4. The technique used in this study is applicable to the design of the optimal system for reducing wing fatigue due to ground induced vibrations experienced by the aircraft during the taxi condition. In particular, by employing Feedback Law I and IV for the single landing gear system and Feedback Law I for both the tricycle and five landing gear systems, the stress on the wing may be made arbitrarily small.

5. Results given in Appendices C, D, and E indicate that the requirements demanded of the actuators used in the landing gear system appear to be within the capabilities of current actuator design and technology.

6. The results of this study can provide the designer with a tool for a rational comparison of the performance of a landing gear system with that of the best possible system.

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2. RECOMMENDATIONS

The following recommendations for further investigation are made:

1. Additional data should be obtained for the five landing gear model. This data should include the case where the actuator is removed from the nose gear. In addition, for both the tricycle and five gear model, the actuators may be removed from the nose gear and either the front or rear main gears in order to investigate the possibility of implementing such an arrangement.

2. The case where there is no dynamic equation relating the control signal to the actuator force should be investigated. Here the force developed by the actuator and the actuator signal are taken to be the same, thus representing a very simplified model for the actuator. The validity of this simplified model could be determined.

3. The application of the technique used in this study to other aircraft, runway, and actuator models. One such aircraft which should be investigated is the C-5A. Other runway models and statistics are available (Reference 10: Vol 1). An actuator model which should be investigated is that of an electro-hydraulic system in which a hydraulic fluid whose viscosity is sensitive to electric fields is used (Reference 6).

4. The optimal stochastic linear regulator solution for the models used in this study should be obtained (Reference 14: Chap. 10). The comparison of the results of this solution with the results presented in this study should be made. Preliminary investigation in this direction performed by the author indicates that the physical requirements placed on the actuators may be reduced by employing the optimal stochastic linear regulator solution. However, the feasibility of implementing such a solution is questionable.

5. The use of preview control may be investigated, where a sensor is used to obtain data of the runway profile ahead of the vehicle. Further discussion of this technique is given in Reference 16.

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6. In the course of this study, it was found that information on the physical characteristics, such as relative displacements, velocities, accelerations and jerk, of the masses used in each model may be readily obtained. In future investigations, these characteristics should be given along with the data on actuator requirements. This additional information can further serve as a guide to the designer in evaluating the performance of the optimal system.

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Contrails

APPENDIX A

STATE EQUATIONS FOR THE VEHICLE MODEL OF THE FIVE LANDING GEAR SYSTEM

This appendix contains the state equations for the vehicle model of the five landing gear system. They were obtained in the manner described in Chapter II of the report, and are given as

$$\dot{Z}_{s1} = V_{s1}$$

$$\dot{Z}_{s2} = V_{s2}$$

$$\dot{Z}_f = V_f$$

$$\dot{Z}_{u1} = V_{u1}$$

$$\dot{Z}_{u2} = V_{u2}$$

$$\dot{Z}_{u3} = V_{u3}$$

$$\dot{Z}_{u4} = V_{u4}$$

$$\dot{Z}_{u5} = V_{u5}$$

$$\dot{\theta} = V_\theta$$

$$\dot{\phi} = V_\phi$$

$$\begin{aligned} \dot{V}_{s1} = & -\frac{K_{s1}}{M_{s1}} Z_{s1} + \frac{K_{s1}}{M_{s1}} Z_f + \frac{K_{s1} e_1}{M_{s1}} \theta - \frac{K_{s1} L_{w1}}{M_{s1}} \phi \\ & - \frac{(C_{s1} + C_{sg1})}{M_{s1}} V_{s1} + \frac{C_{s1}}{M_{s1}} V_f + \frac{C_{s1} e_1}{M_{s1}} V_\theta - \frac{C_{s1} L_{w1}}{M_{s1}} V_\phi \end{aligned}$$

$$\begin{aligned} \dot{V}_{s2} = & -\frac{K_{s2}}{M_{s2}} Z_{s2} + \frac{K_{s2}}{M_{s2}} Z_f + \frac{K_{s2} e_2}{M_{s2}} \theta + \frac{K_{s2} L_{w2}}{M_{s2}} \phi \\ & - \frac{(C_{s2} + C_{sg2})}{M_{s2}} V_{s2} + \frac{C_{s2}}{M_{s2}} V_f + \frac{C_{s2} e_2}{M_{s2}} V_\theta + \frac{C_{s2} L_{w2}}{M_{s2}} V_\phi \end{aligned}$$

$$\begin{aligned} \dot{V}_f = & \frac{K_{s1}}{M_f} Z_{s1} + \frac{K_{s2}}{M_f} Z_{s2} - \frac{(K_{a1} + K_{a2} + K_{a3} + K_{a4} + K_{a5} + K_{s1} + K_{s2})}{M_f} Z_f \\ & + \frac{K_{a1}}{M_f} Z_{u1} + \frac{K_{a2}}{M_f} Z_{u2} + \frac{K_{a3}}{M_f} Z_{u3} + \frac{K_{a4}}{M_f} Z_{u4} + \frac{K_{a5}}{M_f} Z_{u5} \end{aligned}$$

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$$\begin{aligned}
 & + \frac{(K_{a1} L_{x1} + K_{a2} L_{x2} - K_{a3} L_{x3} + K_{a4} L_{x4} - K_{a5} L_{x5} - K_{s1} e_1 - K_{s2} e_2)}{M_f} \theta \\
 & + \frac{(K_{a2} L_{y2} + K_{a3} L_{y3} - K_{a4} L_{y4} - K_{a5} L_{y5} + K_{s1} L_{w1} - K_{s2} L_{w2})}{M_f} \phi \\
 & + \frac{C_{s1}}{M_f} V_{s1} + \frac{C_{s2}}{M_f} V_{s2} - \frac{(D_1 + D_2 + D_3 + D_4 + D_5 + C_{s1} + C_{s2} + C_{fg})}{M_f} V_f \\
 & + \frac{D_1}{M_f} V_{u1} + \frac{D_2}{M_f} V_{u2} + \frac{D_3}{M_f} V_{u3} + \frac{D_4}{M_f} V_{u4} + \frac{D_5}{M_f} V_{u5} \\
 & + \frac{(D_1 L_{x1} + D_2 L_{x2} - D_3 L_{x3} + D_4 L_{x4} - D_5 L_{x5} - C_{s1} e_1 - C_{s2} e_2)}{M_f} V_\theta \\
 & + \frac{(D_2 L_{y2} + D_3 L_{y3} - D_4 L_{y4} - D_5 L_{y5} + C_{s1} L_{w1} - C_{s2} L_{w2})}{M_f} V_\phi
 \end{aligned}$$

$$\begin{aligned}
 \dot{V}_{u1} &= \frac{K_{a1}}{M_{u1}} Z_f - \frac{(K_{a1} + K_{t1})}{M_{u1}} Z_{u1} - \frac{K_{a1} L_{x1}}{M_{u1}} \theta + \frac{D_1}{M_{u1}} V_f \\
 & - \frac{D_1}{M_{u1}} V_{u1} - \frac{D_1 L_{x1}}{M_{u1}} V_\theta + \frac{K_{t1}}{M_{u1}} h_1
 \end{aligned}$$

$$\begin{aligned}
 \dot{V}_{u2} &= \frac{K_{a2}}{M_{u2}} Z_f - \frac{(K_{a2} + K_{t2})}{M_{u2}} Z_{u2} - \frac{K_{a2} L_{x2}}{M_{u2}} \theta - \frac{K_{a2} L_{y2}}{M_{u2}} \phi \\
 & + \frac{D_2}{M_{u2}} V_f - \frac{D_2}{M_{u2}} V_{u2} - \frac{D_2 L_{x2}}{M_{u2}} V_\theta - \frac{D_2 L_{y2}}{M_{u2}} V_\phi + \frac{K_{t2}}{M_{u2}} h_2
 \end{aligned}$$

$$\begin{aligned}
 \dot{V}_{u3} &= \frac{K_{a3}}{M_{u3}} Z_f - \frac{(K_{a3} + K_{t3})}{M_{u3}} Z_{u3} + \frac{K_{a3} L_{x3}}{M_{u3}} \theta - \frac{K_{a3} L_{y3}}{M_{u3}} \phi \\
 & + \frac{D_3}{M_{u3}} V_f - \frac{D_3}{M_{u3}} V_{u3} + \frac{D_3 L_{x3}}{M_{u3}} V_\theta - \frac{D_3 L_{y3}}{M_{u3}} V_\phi + \frac{K_{t3}}{M_{u3}} h_3
 \end{aligned}$$

$$\begin{aligned}
 \dot{V}_{u4} &= \frac{K_{a4}}{M_{u4}} Z_f - \frac{(K_{a4} + K_{t4})}{M_{u4}} Z_{u4} - \frac{K_{a4} L_{x4}}{M_{u4}} \theta + \frac{K_{a4} L_{y4}}{M_{u4}} \phi \\
 & + \frac{D_4}{M_{u4}} V_f - \frac{D_4}{M_{u4}} V_{u4} - \frac{D_4 L_{x4}}{M_{u4}} V_\theta + \frac{D_4 L_{y4}}{M_{u4}} V_\phi + \frac{K_{t4}}{M_{u4}} h_4
 \end{aligned}$$

$$\begin{aligned}
 \dot{V}_{u5} &= \frac{K_{a5}}{M_{u5}} Z_f - \frac{(K_{a5} + K_{t5})}{M_{u5}} Z_{u5} + \frac{K_{a5} L_{x5}}{M_{u5}} \theta + \frac{K_{a5} L_{y5}}{M_{u5}} \phi \\
 & + \frac{D_5}{M_{u5}} V_f - \frac{D_5}{M_{u5}} V_{u5} + \frac{D_5 L_{x5}}{M_{u5}} V_\theta + \frac{D_5 L_{y5}}{M_{u5}} V_\phi + \frac{K_{t5}}{M_{u5}} h_5
 \end{aligned}$$

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$$\begin{aligned}
 \dot{V}_\theta = & \frac{e_1 K_{s1}}{I_{yy}} Z_{s1} + \frac{e_2 K_{s2}}{I_{yy}} Z_{s2} \\
 & + \frac{(L_{x1} K_{a1} + L_{x2} K_{a2} - L_{x3} K_{a3} + L_{x4} K_{a4} - L_{x5} K_{a5} - e_1 K_s - e_2 K_{s2})}{I_{yy}} Z_f \\
 & - \frac{L_{x1} K_{a1}}{I_{yy}} Z_{u1} - \frac{L_{x2} K_{a2}}{I_{yy}} Z_{u2} + \frac{L_{x3} K_{a3}}{I_{yy}} Z_{u3} - \frac{L_{x4} K_{a4}}{I_{yy}} Z_{u4} + \frac{L_{x5} K_{a5}}{I_{yy}} Z_{u5} \\
 & - \frac{(L_{x1}^2 K_{a1} + L_{x2}^2 K_{a2} + L_{x3}^2 K_{a3} + L_{x4}^2 K_{a4} + L_{x5}^2 K_{a5} + e_1^2 K_s + e_2^2 K_{s2})}{I_{yy}} \theta \\
 & + \frac{(L_{x2} L_{y2} K_{a2} + L_{x3} L_{y3} K_{a3} + L_{x4} L_{y4} K_{a4} - L_{x5} L_{y5} K_{a5} + e_1 L_{w1} K_{s1} - e_2 L_{w2} K_{s2})}{I_{yy}} \phi \\
 & + \frac{e_1 C_{s1}}{I_{yy}} V_{s1} + \frac{e_2 C_{s2}}{I_{yy}} V_{s2} \\
 & + \frac{(L_{x1} D_1 + L_{x2} D_2 - L_{x3} D_3 + L_{x4} D_4 - L_{x5} D_5 - e_1 C_{s1} - e_2 C_{s2})}{I_{yy}} V_f \\
 & - \frac{L_{x1} D_1}{I_{yy}} V_{u1} - \frac{L_{x2} D_2}{I_{yy}} V_{u2} + \frac{L_{x3} D_3}{I_{yy}} V_{u3} - \frac{L_{x4} D_4}{I_{yy}} V_{u4} + \frac{L_{x5} D_5}{I_{yy}} V_{u5} \\
 & - \frac{(L_{x1}^2 D_1 + L_{x2}^2 D_2 + L_{x3}^2 D_3 + L_{x4}^2 D_4 + L_{x5}^2 D_5 + e_1^2 C_{s1} + e_2^2 C_{s2})}{I_{yy}} V_\theta \\
 & + \frac{(-L_{x2} L_{y2} D_2 + L_{x3} L_{y3} D_3 + L_{x4} L_{y4} D_4 - L_{x5} L_{y5} D_5 + e_1 L_{w1} C_{s1} - e_2 L_{w2} C_{s2})}{I_{yy}} V_\phi \\
 \\
 \dot{V}_\phi = & - \frac{L_{w1} K_{s1}}{I_{xx}} Z_{s1} + \frac{L_{w2} K_{s2}}{I_{xx}} Z_{s2} \\
 & + \frac{(L_{y2} K_{a2} + L_{y3} K_{a3} - L_{y4} K_{a4} - L_{y5} K_{a5} + L_{w1} K_{s1} - L_{w2} K_{s2})}{I_{xx}} Z_f \\
 & - \frac{L_{y2} K_{a2}}{I_{xx}} Z_{u2} - \frac{L_{y3} K_{a3}}{I_{xx}} Z_{u3} + \frac{L_{y4} K_{a4}}{I_{xx}} Z_{u4} + \frac{L_{y5} K_{a5}}{I_{xx}} Z_{u5} \\
 & + \frac{(-L_{y2} L_{x2} K_{a2} + L_{y3} L_{x3} K_{a3} + L_{y4} L_{x4} K_{a4} - L_{y5} L_{x5} K_{a5} + L_{w1} e_1 K_{s1} - L_{w2} e_2 K_{s2})}{I_{xx}} \theta \\
 & - \frac{(L_{y2}^2 K_{a2} + L_{y3}^2 K_{a3} + L_{y4}^2 K_{a4} + L_{y5}^2 K_{a5} + L_{w1}^2 K_{s1} + L_{w2}^2 K_{s2})}{I_{xx}} \phi \\
 & - \frac{L_{w1} C_{s1}}{I_{xx}} V_{s1} + \frac{L_{w2} C_{s2}}{I_{xx}} V_{s2} \\
 & + \frac{(L_{y2} D_2 + L_{y3} D_3 - L_{y4} D_4 - L_{y5} D_5 + L_{w1} C_{s1} - L_{w2} C_{s2})}{I_{xx}} V_f \\
 & - \frac{L_{y2} D_2}{I_{xx}} V_{u2} - \frac{L_{y3} D_3}{I_{xx}} V_{u3} + \frac{L_{y4} D_4}{I_{xx}} V_{u4} + \frac{L_{y5} D_5}{I_{xx}} V_{u5} \\
 & + \frac{(-L_{y2} L_{x2} D_2 + L_{y3} L_{x3} D_3 + L_{y4} L_{x4} D_4 - L_{y5} L_{x5} D_5 + L_{w1} e_1 C_{s1} - L_{w2} e_2 C_{s2})}{I_{xx}} V_\theta \\
 & - \frac{(L_{y2}^2 D_2 + L_{y3}^2 D_3 + L_{y4}^2 D_4 + L_{y5}^2 D_5 + L_{w1}^2 C_{s1} + L_{w2}^2 C_{s2})}{I_{xx}} V_\phi
 \end{aligned}$$

(108)

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where $h_1, h_2, h_3, h_4,$ and h_5 are the runway inputs to the five landing gear system and where the state vector is identified as:

(109)

$$\underline{z}^T = (z_{s1}, z_{s2}, z_f, z_{u1}, z_{u2}, z_{u3}, z_{u4}, z_{u5}, \theta, \phi, \\ v_{s1}, v_{s2}, v_f, v_{u1}, v_{u2}, v_{u3}, v_{u4}, v_{u5}, v_\theta, v_\phi)$$

APPENDIX B

STATE EQUATIONS FOR THE SYSTEM MODEL OF THE FIVE LANDING GEAR SYSTEM

This appendix contains the state equations for the system model of the five landing gear system. They were obtained in the manner described in Chapter V of the report, and are given as

$$\dot{Z}_{s1} = V_{s1}$$

$$\dot{Z}_{s2} = V_{s2}$$

$$\dot{Z}_f = V_f$$

$$\dot{Z}_{u1} = V_{u1}$$

$$\dot{Z}_{u2} = V_{u2}$$

$$\dot{Z}_{u3} = V_{u3}$$

$$\dot{Z}_{u4} = V_{u4}$$

$$\dot{Z}_{u5} = V_{u5}$$

$$\dot{\theta} = V_{\theta}$$

$$\dot{\phi} = V_{\phi}$$

$$\begin{aligned} \dot{V}_{s1} = & - \frac{K_{s1}}{M_{s1}} Z_{s1} + \frac{K_{s1}}{M_{s1}} Z_f + \frac{K_{s1}e_1}{M_{s1}} \theta - \frac{K_{s1}L_{w1}}{M_{s1}} \phi \\ & - \frac{(C_{s1} + C_{sg1})}{M_{s1}} V_{s1} + \frac{C_{s1}}{M_{s1}} V_f + \frac{C_{s1}e_1}{M_{s1}} V_{\theta} - \frac{C_{s1}L_{w1}}{M_{s1}} V_{\phi} \end{aligned}$$

$$\begin{aligned} \dot{V}_{s2} = & - \frac{K_{s2}}{M_{s2}} Z_{s2} + \frac{K_{s2}}{M_{s2}} Z_f + \frac{K_{s2}e_2}{M_{s2}} \theta + \frac{K_{s2}L_{w2}}{M_{s2}} \phi \\ & - \frac{C_{s2} + C_{sg2}}{M_{s2}} V_{s2} + \frac{C_{s2}}{M_{s2}} V_f + \frac{C_{s2}e_2}{M_{s2}} V_{\theta} + \frac{C_{s2}L_{w2}}{M_{s2}} V_{\phi} \end{aligned}$$

$$\begin{aligned} \dot{V}_f = & \frac{K_{s1}}{M_f} Z_{s1} + \frac{K_{s2}}{M_f} Z_{s2} - \frac{K_{a1} + K_{a2} + K_{a3} + K_{a4} + K_{a5} + K_{s1} + K_{s2}}{M_f} Z_f \\ & + \frac{K_{a1}}{M_f} Z_{u1} + \frac{K_{a2}}{M_f} Z_{u2} + \frac{K_{a3}}{M_f} Z_{u3} + \frac{K_{a4}}{M_f} Z_{u4} + \frac{K_{a5}}{M_f} Z_{u5} \end{aligned}$$

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$$\begin{aligned}
 & + \frac{(K_{a1}L_{x1} + K_{a2}L_{x2} - K_{a3}L_{x3} + K_{a4}L_{x4} - K_{a5}L_{x5} - K_{s1}e_1 - K_{s2}e_2)}{M_f} \theta \\
 & + \frac{(K_{a2}L_{y2} + K_{a3}L_{y3} - K_{a4}L_{y4} - K_{a5}L_{y5} + K_{s1}L_{w1} - K_{s2}L_{w2})}{M_f} \phi \\
 & + \frac{C_{s1}}{M_f} V_{s1} + \frac{C_{s2}}{M_f} V_{s2} - \frac{(D_1 + D_2 + D_3 + D_4 + D_5 + C_{s1} + C_{s2} + C_{fg})}{M_f} V_f \\
 & + \frac{D_1}{M_f} V_{u1} + \frac{D_2}{M_f} V_{u2} + \frac{D_3}{M_f} V_{u3} + \frac{D_4}{M_f} V_{u4} + \frac{D_5}{M_f} V_{u5} \\
 & + \frac{(D_1 L_{x1} + D_2 L_{x2} - D_3 L_{x3} + D_4 L_{x4} - D_5 L_{x5} - C_{s1} e_1 - C_{s2} e_2)}{M_f} V_\theta \\
 & + \frac{(D_2 L_{y2} + D_3 L_{y3} - D_4 L_{y4} - D_5 L_{y5} + C_{s1} L_{w1} - C_{s1} L_{w2})}{M_f} V_\phi \\
 & - \frac{F_{A1}}{M_f} - \frac{F_{A2}}{M_f} - \frac{F_{A3}}{M_f} - \frac{F_{A4}}{M_f} - \frac{F_{A5}}{M_f} \\
 \dot{V}_{u1} = & \frac{K_{a1}}{M_{u1}} Z_f - \frac{(K_{a1} + K_{t1})}{M_{u1}} Z_{u1} - \frac{K_{a1} L_{x1}}{M_{u1}} \theta + \frac{D_1}{M_{u1}} V_f - \frac{D_1}{M_{u1}} V_{u1} \\
 & - \frac{D_1 L_{x1}}{M_{u1}} V_\theta + \frac{F_{A1}}{M_{u1}} + \frac{K_{t1}}{M_{u1}} h_1 \\
 \dot{V}_{u2} = & \frac{K_{a2}}{M_{u2}} Z_f - \frac{(K_{a2} + K_{t2})}{M_{u2}} Z_{u2} - \frac{K_{a2} L_{x2}}{M_{u2}} \theta - \frac{K_{a2} L_{y2}}{M_{u2}} \phi \\
 & + \frac{D_2}{M_{u2}} V_f - \frac{D_2}{M_{u2}} V_{u2} - \frac{D_2 L_{x2}}{M_{u2}} V_\theta - \frac{D_2 L_{y2}}{M_{u2}} V_\phi \\
 & + \frac{F_{A2}}{M_{u2}} + \frac{K_{t2}}{M_{u2}} h_2 \\
 \dot{V}_{u3} = & \frac{K_{a3}}{M_{u3}} Z_f - \frac{(K_{a3} + K_{t3})}{M_{u3}} Z_{u3} + \frac{K_{a3} L_{x3}}{M_{u3}} \theta - \frac{K_{a3} L_{y3}}{M_{u3}} \phi \\
 & + \frac{D_3}{M_{u3}} V_f - \frac{D_3}{M_{u3}} V_{u3} + \frac{D_3 L_{x3}}{M_{u3}} V_\theta - \frac{D_3 L_{y3}}{M_{u3}} V_\phi \\
 & + \frac{F_{A3}}{M_{u3}} + \frac{K_{t3}}{M_{u3}} h_3
 \end{aligned}$$

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$$\begin{aligned} \dot{V}_{u4} = & \frac{K_{a4}}{M_{u4}} Z_f - \frac{(K_{a4} + K_{t4})}{M_{u4}} Z_{u4} - \frac{K_{a4}L_{x4}}{M_{u4}} \theta + \frac{K_{a4}L_{y4}}{M_{u4}} \phi \\ & + \frac{D_4}{M_{u4}} V_f - \frac{D_4}{M_{u4}} V_{u4} - \frac{D_4L_{x4}}{M_{u4}} V_\theta + \frac{D_4L_{y4}}{M_{u4}} V_\phi \\ & + \frac{FA_4}{M_{u4}} + \frac{K_{t4}}{M_{u4}} h_4 \end{aligned}$$

$$\begin{aligned} \dot{V}_{u5} = & \frac{K_{a5}}{M_{u5}} Z_f - \frac{(K_{a5} + K_{t5})}{M_{u5}} Z_{u5} + \frac{K_{a5}L_{x5}}{M_{u5}} \theta + \frac{K_{a5}L_{y5}}{M_{u5}} \phi \\ & + \frac{D_5}{M_{u5}} V_f - \frac{D_5}{M_{u5}} V_{u5} + \frac{D_5L_{x5}}{M_{u5}} V_\theta + \frac{D_5L_{y5}}{M_{u5}} V_\phi \\ & + \frac{FA_5}{M_{u5}} + \frac{K_{t5}}{M_{u5}} h_5 \end{aligned}$$

$$\begin{aligned} \dot{V}_\theta = & \frac{e_1K_{s1}}{I_{yy}} Z_{s1} + \frac{e_2K_{s2}}{I_{yy}} Z_{s2} \\ & + \frac{(L_{x1}K_{a1} + L_{x2}K_{a2} - L_{x3}K_{a3} + L_{x4}K_{a4} - L_{x5}K_{a5} - e_1K_{s1} - e_2K_{s2})}{I_{yy}} Z_f \\ & - \frac{L_{x1}K_{a1}}{I_{yy}} Z_{u1} - \frac{L_{x2}K_{a2}}{I_{yy}} Z_{u2} + \frac{L_{x3}K_{a3}}{I_{yy}} Z_{u3} - \frac{L_{x4}K_{a4}}{I_{yy}} Z_{u4} \\ & + \frac{L_{x5}K_{a5}}{I_{yy}} Z_{u5} \\ & - \frac{(L_{x1}^2K_{a1} + L_{x2}^2K_{a2} + L_{x3}^2K_{a3} + L_{x4}^2K_{a4} + L_{x5}^2K_{a5} + e_1^2K_{s1} + e_2^2K_{s2})}{I_{yy}} \theta \\ & - \frac{(-L_{x2}L_{y2}K_{a2} + L_{x3}L_{y3}K_{a3} + L_{x4}L_{y4}K_{a4} - L_{x5}L_{y5}K_{a5} + e_1L_{w1}K_{s1} - e_2L_{w2}K_{s2})}{I_{yy}} \phi \\ & + \frac{e_1C_{s1}}{I_{yy}} V_{s1} + \frac{e_2C_{s2}}{I_{yy}} V_{s2} \\ & + \frac{(L_{x1}D_1 + L_{x2}D_2 - L_{x3}D_3 + L_{x4}D_4 - L_{x5}D_5 - e_1C_{s1} - e_2C_{s2})}{I_{yy}} V_f \end{aligned}$$

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$$\begin{aligned}
 & - \frac{L_{x1}D_1}{I_{yy}} V_{u1} - \frac{L_{x2}D_2}{I_{yy}} V_{u2} + \frac{L_{x3}D_3}{I_{yy}} V_{u3} - \frac{L_{x4}D_4}{I_{yy}} V_{u4} + \frac{L_{x5}D_5}{I_{yy}} V_{u5} \\
 & - \frac{(L_{x1}^2D_1 + L_{x2}^2D_2 + L_{x3}^2D_3 + L_{x4}^2D_4 + L_{x5}^2D_5 + e_1^2C_{s1} + e_2^2C_{s2})}{I_{yy}} V_{\theta} \\
 & + \frac{(-L_{x2}L_{y2}D_2 + L_{x3}L_{y3}D_3 + L_{x4}L_{y4}D_4 - L_{x5}L_{y5}D_5 + e_1L_{w1}C_{s1} - e_2L_{w2}C_{s2})}{I_{yy}} V_{\phi} \\
 & + \frac{L_{x1}}{I_{yy}} F_{A1} + \frac{L_{x2}}{I_{yy}} F_{A2} - \frac{L_{x3}}{I_{yy}} F_{A3} + \frac{L_{x4}}{I_{yy}} F_{A4} - \frac{L_{x5}}{I_{yy}} F_{A5} \\
 \dot{V}_{\phi} = & - \frac{L_{w1}K_{s1}}{I_{xx}} Z_{s1} + \frac{L_{w2}K_{s2}}{I_{xx}} Z_{s2} \\
 & + \frac{L_{y2}K_{a2} + L_{y3}K_{a3} - L_{y4}K_{a4} - L_{y5}K_{a5} + L_{w1}K_{s1} - L_{w2}K_{s2}}{I_{xx}} Z_f \\
 & - \frac{L_{y2}K_{a2}}{I_{xx}} Z_{u2} - \frac{L_{y3}K_{a3}}{I_{xx}} Z_{u3} + \frac{L_{y4}K_{a4}}{I_{xx}} Z_{u4} + \frac{L_{y5}K_{a5}}{I_{xx}} Z_{u5} \\
 & + \frac{(-L_{y2}L_{x2}K_{a2} + L_{y3}L_{x3}K_{a3} + L_{y4}L_{x4}K_{a4} - L_{y5}L_{x5}K_{a5} + L_{w1}e_1K_{s1} - L_{w2}e_2K_{s2})}{I_{xx}} \theta \\
 & - \frac{(L_{y2}^2K_{a2} + L_{y3}^2K_{a3} + L_{y4}^2K_{a4} + L_{y5}^2K_{a5} + L_{w1}^2K_{s1} + L_{w2}^2K_{s2})}{I_{xx}} \phi \\
 & - \frac{L_{w1}C_{s1}}{I_{xx}} V_{s1} + \frac{L_{w2}C_{s2}}{I_{xx}} V_{s2} \\
 & + \frac{(L_{y2}D_2 + L_{y3}D_3 - L_{y4}D_4 - L_{y5}D_5 + L_{w1}C_{s1} - L_{w2}C_{s2})}{I_{xx}} V_f \\
 & - \frac{L_{y2}D_2}{I_{xx}} V_{u2} - \frac{L_{y3}D_3}{I_{xx}} V_{u3} + \frac{L_{y4}D_4}{I_{xx}} V_{u4} + \frac{L_{y5}D_5}{I_{xx}} V_{u5} \\
 & + \frac{(-L_{y2}L_{x2}D_2 + L_{y3}L_{x3}D_3 + L_{y4}L_{x4}D_4 - L_{y5}L_{x5}D_5 + L_{w1}e_1C_{s1} - L_{w2}e_2C_{s2})}{I_{xx}} V_{\theta} \\
 & - \frac{(L_{y2}^2D_2 + L_{y3}^2D_3 + L_{y4}^2D_4 + L_{y5}^2D_5 + L_{w1}^2C_{s1} + L_{w2}^2C_{s2})}{I_{xx}} V_{\phi} \\
 & + \frac{L_{y2}}{I_{xx}} F_{A2} + \frac{L_{y3}}{I_{xx}} F_{A3} - \frac{L_{y4}}{I_{xx}} F_{A4} - \frac{L_{y5}}{I_{xx}} F_{A5}
 \end{aligned}$$

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$$\begin{aligned} \dot{F}_{A1} &= \frac{C_b C_A K_B}{V} V_f - \frac{C_b C_A K_B}{V} V_{u1} - \frac{C_b C_A K_B}{V} L_{x1} V_\theta \\ &\quad - \frac{K_B (L + C_p)}{V} F_{A1} + \frac{K_B C_A C_x}{V} u_1 \\ \dot{F}_{A2} &= \frac{C_b C_A K_B}{V} V_f - \frac{C_b C_A C_B}{V} V_{u2} - \frac{C_b C_A K_B}{V} L_{x2} V_\theta \\ &\quad - \frac{C_b C_A K_B}{V} L_{y2} V_\phi - \frac{K_B (L + C_p)}{V} F_{A2} + \frac{K_B C_A C_x}{V} u_2 \\ \dot{F}_{A3} &= \frac{C_b C_A K_B}{V} V_f - \frac{C_b C_A K_B}{V} V_{u3} + \frac{C_b C_A K_B}{V} L_{x3} V_\theta \\ &\quad - \frac{C_b C_A K_B}{V} L_{y3} V_\phi - \frac{K_B (L + C_p)}{V} F_{A3} + \frac{K_B C_A C_x}{V} u_3 \\ \dot{F}_{A4} &= \frac{C_b C_A K_B}{V} V_f - \frac{C_b C_A K_B}{V} V_{u4} - \frac{C_b C_A K_B}{V} L_{x4} V_\theta \\ &\quad + \frac{C_b C_A K_B}{V} L_{y4} V_\phi - \frac{K_B (L + C_p)}{V} F_{A4} + \frac{K_B C_A C_x}{V} u_4 \\ \dot{F}_{A5} &= \frac{C_b C_A K_B}{V} V_f - \frac{C_b C_A K_B}{V} V_{u5} + \frac{C_b C_A K_B}{V} L_{x5} V_\theta \\ &\quad + \frac{C_b C_A K_B}{V} L_{y5} V_\phi - \frac{K_B (L + C_p)}{V} F_{A5} + \frac{K_B C_A C_x}{V} u_5 \\ \dot{h}_1 &= -w_B h_1 + \xi_1 \\ \dot{h}_2^* &= -w_B h_2^* + \xi_2 \\ \dot{h}_2 &= \left(w_B + \frac{2}{\tau_{12}}\right) h_2^* - \frac{2}{\tau_{12}} h_2 - \xi_2 \\ \dot{h}_3 &= -\left(w_B + \frac{2}{\tau_{12}}\right) h_2^* + 2\left(\frac{1}{\tau_{12}} + \frac{1}{\tau_{23}}\right) h_2 - \frac{2}{\tau_{23}} h_3 + \xi_2 \\ \dot{h}_4^* &= -w_B h_4^* + \xi_4 \\ \dot{h}_4 &= \left(w_B + \frac{2}{\tau_{14}}\right) h_4^* - \frac{2}{\tau_{14}} h_4 - \xi_4 \\ \dot{h}_5 &= -\left(w_B + \frac{2}{\tau_{14}}\right) h_4^* + 2\left(\frac{1}{\tau_{14}} + \frac{1}{\tau_{45}}\right) h_4 - \frac{2}{\tau_{45}} h_5 + \xi_4 \end{aligned}$$

(110)

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where the state vector is identified as

$$\begin{aligned} \underline{x}^T = & (z_{s1}, z_{s2}, z_f, z_{u1}, z_{u2}, z_{u3}, z_{u4}, z_{u5}, \theta, \phi, \\ & v_{s1}, v_{s2}, v_f, v_{u1}, v_{u2}, v_{u3}, v_{u4}, v_{u5}, v_\theta, v_\phi, \\ & F_{A1}, F_{A2}, F_{A3}, F_{A4}, F_{A5}, h_1, h_2, h_2^*, h_3, \\ & h_4^*, h_4, h_5) \end{aligned} \quad (111)$$

APPENDIX C

RESULTS FOR SINGLE LANDING GEAR MODEL

This appendix gives the results obtained for the single landing gear system model described in Chapter V. The state equations for this system are given by Equation 62. The state vector is identified as

$$\underline{x}^T = (Z_s, Z_f, Z_u, V_s, V_f, V_u, F_A, h) \quad (63)$$

and the system matrix F is readily identified as an 8x8 matrix. The control is the scalar u. Results for four feedback laws were obtained for this model:

Feedback Law 1: Consisted of measuring the relative displacement between M_s and M_f , and measuring the acceleration of M_f (see Figure 2). Thus, the measurement vector \underline{y} is identified as

$$\underline{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (112)$$

and

$$\begin{aligned} y_1 &= -Z_s + Z_f \\ y_2 &= \ddot{Z}_f \\ &= \frac{K_s}{M_f} Z_s - \frac{(K_s + K_0)}{M_f} Z_f + \frac{K_0}{M_f} Z_u + \frac{C_s}{M_f} V_s \\ &\quad - \frac{(C_{fg} + C_s + D)}{M_f} V_f + \frac{D}{M_f} V_u - \frac{F_A}{M_f} \end{aligned} \quad (113)$$

The control u is then given as:

$$u = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (114)$$

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Feedback Law II: Consisted of measuring the acceleration of both M_f and M_u . Thus the measurement vector \underline{y} is identified as

$$\underline{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (115)$$

and

$$\begin{aligned} y_1 &= \dot{V}_f \\ &= \frac{K_s}{M_f} Z_s - \frac{(K_s + K_a)}{M_f} Z_f + \frac{K_a}{M_f} Z_u + \frac{C_s}{M_f} V_s \\ &\quad - \frac{(C_{fg} + C_s + D)}{M_f} V_f + \frac{D}{M_f} V_u - \frac{F_A}{M_f} \end{aligned} \quad (116)$$

$$\begin{aligned} y_2 &= \dot{V}_u \\ &= \frac{K_a}{M_u} Z_f - \frac{(K_a + K_t)}{M_u} Z_u + \frac{D}{M_u} V_f - \frac{D}{M_u} V_u \\ &\quad + \frac{F_A}{M_u} + \frac{K_t}{M_u} h \end{aligned}$$

and the control u is given as

$$\underline{u} = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (117)$$

Feedback Law III: Consisted of measuring the relative displacement between M_s and M_f , and the relative displacement between M_f and M_u . Thus, the measurement vector \underline{y} is identified as:

$$\underline{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (118)$$

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and

$$y_1 = -Z_s + Z_f \quad (119)$$

$$y_2 = -Z_f + Z_u$$

The control u is then given as

$$u = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (120)$$

Feedback Law IV: Consisted of measuring the acceleration of both M_s and M_f . Thus, the measurement vector \underline{y} is identified as

$$\underline{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (121)$$

and

$$\begin{aligned} y_1 &= \dot{V}_s \\ &= -\frac{K_s}{M_s} Z_s + \frac{K_s}{M_s} Z_f - \frac{(C_s + C_{sg})}{M_s} V_s + \frac{C_s}{M_s} V_f \\ y_2 &= \dot{V}_f \\ &= \frac{K_s}{M_f} Z_s - \frac{(K_s + K_a)}{M_f} Z_f + \frac{K_a}{M_f} Z_u + \frac{C_s}{M_f} V_s \\ &\quad - \frac{(C_{fg} + C_s + D)}{M_f} V_f + \frac{D}{M_f} V_u - \frac{F_A}{M_f} \end{aligned} \quad (122)$$

The control u is then given as

$$u = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (123)$$

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The cost criterion used for each of the above feedback laws is given by Equation 76 and is restated as

$$J = \lim_{t \rightarrow \infty} E \left[\underline{x}^T \underline{b} \underline{b}^T \underline{x} + R_c u^2 \right] \quad (76)$$

where the relative displacement between the main mass and the elastically connected mass is identified as

$$\underline{b}^T \underline{x} = -Z_s + Z_f \quad (124)$$

The parameters for the vehicle model that were used for each of the feedback laws were obtained from data in Reference 19:183 and are given here (for $V_H=66\text{fps}$) as:

$$M_s = 43.97 \text{ LB} - \text{SEC}^2 / \text{IN.}$$

$$M_f = 122.02 \text{ LB} - \text{SEC}^2 / \text{IN.}$$

$$M_u = 1.68 \text{ LB} - \text{SEC}^2 / \text{IN.}$$

$$D = 185.0 \text{ LB} - \text{SEC} / \text{IN.}$$

$$C_s = 2.27 \text{ LB} - \text{SEC} / \text{IN.}$$

$$C_{sg} = 10.87 \text{ LB} - \text{SEC} / \text{IN.}$$

$$C_{fg} = 30.2 \text{ LB} - \text{SEC} / \text{IN.}$$

$$K_s = 2055.0 \text{ LB} / \text{IN.}$$

$$K_g = 14,170 \text{ LB} / \text{IN.}$$

$$K_f = 8330.0 \text{ LB} / \text{IN.}$$

The parameters for the runway model that were used for each of the feedback laws were obtained in Chapter III and are restated here (for $V_H=66\text{fps}$) as:

$$A_0 = 10^{-1} (\text{IN.}^2 / \text{RAD} / \text{FT})$$

$$\lambda_0 = 4.5 \times 10^5 \text{ FT}$$

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The parameters for the actuator model that were used for each of the feedback laws are given in Chapter IV and are restated here as

$$C_b = A_p = 0.96 \text{ (IN.)}^2$$

$$\frac{V}{K_B} = 2.0 \times 10^{-5} \text{ (IN.}^5\text{/LB)}$$

$$L + C_p = 7.0 \times 10^{-4} \text{ (IN.}^5\text{/LB-SEC)}$$

$$C_x = 4.0 \text{ (IN.}^3\text{/SEC-MA)}$$

$$n_F = 1$$

The optimization procedure described in Chapter VII was carried out on each of the four feedback laws for various values of R_c . The results for the single landing gear system now follow.

TABLE I
RESULTS OF MINIMIZATION FOR FEEDBACK LAW I
(SINGLE GEAR MODEL)

Weighting R_c	Stress per Wing $\frac{T \cdot T}{x \cdot b \cdot x}$ (in.) ²	Cost J(K*)
**	9.768	9.768
INFINITE	5.508	5.508
1.0×10^2	5.464	5.486
4.0×10^1	5.399	5.453
1.0×10^1	5.107	5.300
6.0	4.883	5.177
4.0	4.639	5.037
2.0	4.075	4.688
1.0	3.364	4.188
4.0×10^{-1}	2.377	3.350
1.0×10^{-1}	1.209	2.052
4.0×10^{-2}	7.256×10^{-1}	1.369
1.0×10^{-2}	3.132×10^{-1}	6.789×10^{-1}
1.0×10^{-3}	6.525×10^{-2}	1.766×10^{-1}

** ACTUATOR REMOVED FROM LANDING GEAR.

TABLE II
FEEDBACK CONSTANTS FOR FEEDBACK LAW I

Weighting R_c	K_1^*	u	K_2^*
INFINITE	0.0		0.0
1.0×10^2	1.898×10^{-4}		3.549×10^{-3}
4.0×10^1	4.716×10^{-4}		8.827×10^{-3}
1.0×10^1	1.833×10^{-3}		3.442×10^{-2}
6.0	2.983×10^{-3}		5.618×10^{-2}
4.0	4.354×10^{-3}		8.222×10^{-2}
2.0	8.104×10^{-3}		1.541×10^{-1}
1.0	1.449×10^{-2}		2.780×10^{-1}
4.0×10^{-1}	2.910×10^{-2}		5.665×10^{-1}
1.0×10^{-1}	7.259×10^{-2}		1.454
4.0×10^{-2}	1.238×10^{-1}		2.552
1.0×10^{-2}	2.575×10^{-1}		5.774
1.0×10^{-3}	7.682×10^{-1}		2.489×10^1

TABLE III
DEMANDS ON ACTUATOR FOR FEEDBACK LAW I

Weighting R_c	Signal $E(u^2)$ (ma) ²	Displacement $E(y_A^2)$ (in.) ²	Velocity $E(v_A^2)$ (in./sec) ²	Force $E(F_A^2)$ (lb) ²	Power $E(F_A v_A)$ (lb-in./sec)
INFINITE	0.0	2.451×10^{-1}	25.73	2.063×10^7	-1.567×10^4
1.0×10^2	2.202×10^{-4}	2.473×10^{-1}	25.81	2.042×10^7	-1.570×10^4
4.0×10^1	1.345×10^{-3}	2.506×10^{-1}	25.94	2.013×10^7	-1.575×10^4
1.0×10^1	1.930×10^{-2}	2.667×10^{-1}	26.55	1.901×10^7	-1.596×10^4
6.0	4.907×10^{-2}	2.804×10^{-1}	27.07	1.841×10^7	-1.611×10^4
4.0	9.970×10^{-2}	2.967×10^{-1}	27.70	1.806×10^7	-1.626×10^4
2.0	3.068×10^{-1}	3.417×10^{-1}	29.47	1.873×10^7	-1.655×10^4
1.0	8.237×10^{-1}	4.188×10^{-1}	32.60	2.374×10^7	-1.675×10^4
4.0×10^{-1}	2.431	5.962×10^{-1}	40.16	4.508×10^7	-1.644×10^4
1.0×10^{-1}	8.424	1.124	64.63	1.353×10^8	-1.321×10^4
4.0×10^{-2}	1.609×10^1	1.739	95.20	2.537×10^8	-8.110×10^3
1.0×10^{-2}	3.656×10^1	3.305	178.08	5.686×10^8	6.713×10^3
1.0×10^{-3}	1.114×10^2	8.590	501.01	1.656×10^9	6.598×10^4

TABLE IV
RESULTS OF MINIMIZATION FOR FEEDBACK LAW II

Weighting R_c	Stress per Wing $\frac{T}{x} \frac{T}{bb} \frac{T}{x}$ (in.) ²	Cost J(K*)
**	9.768	9.768
INFINITE	5.508	5.508
1.0×10^1	5.471	5.488
4.0	5.434	5.466
1.0	5.368	5.411
4.0×10^{-1}	5.336	5.374
1.0×10^{-1}	5.294	5.326
4.0×10^{-2}	5.269	5.298
1.0×10^{-2}	5.240	5.262
4.0×10^{-3}	5.228	5.244
1.0×10^{-3}	5.217	5.227
1.0×10^{-4}	5.210	5.213
1.0×10^{-5}	5.207	5.208
1.0×10^{-6}	5.207	5.207

**ACTUATOR REMOVED FROM LANDING GEAR

TABLE V
FEEDBACK CONSTANTS FOR FEEDBACK LAW II

Weighting R_c	K_1^*	K_2^*
INFINITE	0.0	0.0
1.0×10^1	3.794×10^{-4}	9.300×10^{-6}
4.0	8.032×10^{-4}	2.331×10^{-5}
1.0	1.818×10^{-3}	8.944×10^{-5}
4.0×10^{-1}	2.450×10^{-3}	1.957×10^{-4}
1.0×10^{-1}	3.049×10^{-3}	4.700×10^{-4}
4.0×10^{-2}	3.270×10^{-3}	6.722×10^{-4}
1.0×10^{-2}	3.458×10^{-3}	9.083×10^{-4}
4.0×10^{-3}	3.523×10^{-3}	1.008×10^{-3}
1.0×10^{-3}	3.577×10^{-3}	1.098×10^{-3}
1.0×10^{-4}	3.613×10^{-3}	1.161×10^{-3}
1.0×10^{-5}	3.623×10^{-3}	1.181×10^{-3}
1.0×10^{-6}	3.627×10^{-3}	1.187×10^{-3}

TABLE VI
DEMANDS ON ACTUATOR FOR FEEDBACK LAW II

Weighting R_c	Signal $E(u^2)$ (ma) ²	Displacement $E(y_A^2)$ (in.) ²	Velocity $E(\dot{v}_A^2)$ (in./sec) ²	Force $E(F_A^2)$ (lb) ²	Power $E(F_A \dot{v}_A)$ (lb-in./sec)
INFINITE	0.0	0.2451	2.573×10^1	2.063×10^7	-1.567×10^4
1.0×10^1	1.755×10^{-3}	0.2483	2.584×10^1	2.075×10^7	-1.568×10^4
4.0	7.895×10^{-3}	0.2520	2.601×10^1	2.098×10^7	-1.567×10^4
1.0	4.305×10^{-2}	0.2610	2.687×10^1	2.190×10^7	-1.555×10^4
4.0×10^{-1}	9.468×10^{-2}	0.2673	2.850×10^1	2.271×10^7	-1.527×10^4
1.0×10^{-1}	3.185×10^{-1}	0.2748	3.487×10^1	2.373×10^7	-1.412×10^4
4.0×10^{-2}	7.344×10^{-1}	0.2785	4.377×10^1	2.453×10^7	-1.249×10^4
1.0×10^{-2}	2.212	0.2827	7.013×10^1	2.688×10^7	-7.645×10^3
4.0×10^{-3}	4.113	0.2847	1.017×10^2	2.980×10^7	-1.811×10^3
1.0×10^{-3}	9.473	0.2878	1.886×10^2	3.799×10^7	1.425×10^4
1.0×10^{-4}	3.298×10^1	0.2970	5.659×10^2	7.391×10^7	8.404×10^4
1.0×10^{-5}	1.072×10^2	0.3237	1.756×10^3	1.874×10^8	3.042×10^5
1.0×10^{-6}	3.346×10^2	0.4044	5.398×10^3	5.348×10^8	9.781×10^5

TABLE VII
RESULTS OF MINIMIZATION FOR FEEDBACK LAW III

Weighting R_c	<u>Stress per Wing</u> $\frac{\pi}{x} \frac{\pi}{bb} \frac{\pi}{x}$ (in.) ²	Cost J(K*)
**	9.768	9.768
INFINITE	5.508	5.508
100.00	5.424	5.466
40.00	5.303	5.404
10.00	4.785	5.125
4.00	4.078	4.691
2.00	3.449	4.208
1.00	2.955	3.682
0.40	2.635	3.119
0.10	2.531	2.695
0.04	2.524	2.594
0.01	2.523	2.541

** ACTUATOR REMOVED FROM LANDING GEAR

TABLE VIII
 FEEDBACK CONSTANTS FOR FEEDBACK LAW III

Weighting R_c	K_1^*	K_2^*
INFINITE	0.0	0.0
100.00	-1.721×10^{-3}	4.384×10^{-2}
40.00	-4.296×10^{-3}	1.075×10^{-1}
10.00	-1.679×10^{-2}	3.914×10^{-1}
4.00	-3.790×10^{-2}	8.219×10^{-1}
2.00	-5.962×10^{-2}	1.279
1.00	-7.312×10^{-2}	1.747
0.40	-6.628×10^{-2}	2.218
0.10	-4.196×10^{-2}	2.532
0.04	-3.277×10^{-2}	2.576
0.01	-2.687×10^{-2}	2.583

TABLE IX
DEMANDS ON ACTUATOR FOR FEEDBACK LAW III

Weighting R_c	Signal $E(u^2)$ (ma) ²	Displacement $E(y_A^2)$ (in.) ²	Velocity $E(v_A^2)$ (in./sec) ²	Force $E(F_A^2)$ (lb) ²	Power $E(F_A v_A)$ (lb-in./sec)
INFINITE	0.0	0.2451	25.73	2.063×10^7	-1.567×10^4
100.00	4.194×10^{-4}	0.2456	25.74	2.066×10^7	-1.577×10^4
40.00	2.530×10^{-3}	0.2464	25.75	2.075×10^7	-1.592×10^4
10.00	3.404×10^{-2}	0.2493	25.77	2.175×10^7	-1.655×10^4
4.00	1.532×10^{-1}	0.2518	25.74	2.525×10^7	-1.745×10^4
2.00	3.794×10^{-1}	0.2521	25.62	3.174×10^7	-1.830×10^4
1.00	7.265×10^{-1}	0.2513	25.40	4.164×10^7	-1.903×10^4
0.40	1.210	0.2515	25.06	5.539×10^7	-1.961×10^4
0.10	1.639	0.2574	24.78	6.762×10^7	-1.988×10^4
0.04	1.747	0.2644	24.74	7.076×10^7	-1.991×10^4
0.01	1.779	0.2676	24.73	7.169×10^7	-1.992×10^4

TABLE X
RESULTS OF MINIMIZATION FOR FEEDBACK LAW IV

Weighting R_c	Stress per Wing $\frac{x^T b b^T x}{(in.)^2}$	Cost J(K*)
**	9.768	9.768
INFINITE	5.508	5.508
100.00	5.464	5.486
40.00	5.399	5.453
10.00	5.107	5.300
4.00	4.637	5.037
2.00	4.070	4.686
1.00	3.354	4.183
0.40	2.359	3.339
0.10	1.188	2.033
0.04	7.099×10^{-1}	1.351
0.01	3.102×10^{-1}	6.683×10^{-1}

** ACTUATOR REMOVED FROM LANDING GEAR.

TABLE XI
FEEDBACK CONSTANTS FOR FEEDBACK LAW IV

Weighting R_c	u K_1^*	K_2^*
INFINITE	0.0	0.0
100.00	7.598×10^{-5}	1.894×10^{-4}
40.00	1.891×10^{-4}	4.707×10^{-4}
10.00	7.391×10^{-4}	1.828×10^{-3}
4.00	1.773×10^{-3}	4.337×10^{-3}
2.00	3.340×10^{-3}	8.060×10^{-3}
1.00	6.069×10^{-3}	1.437×10^{-2}
0.40	1.251×10^{-2}	2.868×10^{-2}
0.10	3.266×10^{-2}	7.029×10^{-2}
0.04	5.772×10^{-2}	1.175×10^{-1}
0.01	1.289×10^{-1}	2.336×10^{-1}

TABLE XII
DEMANDS ON ACTUATOR FOR FEEDBACK LAW IV

Weighting R_c	Signal $E(u^2)$ (ma) ²	Displacement $E(y_A^2)$ (in.) ²	Velocity $E(v_A^2)$ (in./sec) ²	Force $E(F_A^2)$ (lb) ²	Power $E(F_A v_A)$ (lb-in./sec)
INFINITE	0.0	2.451×10^{-1}	25.73	2.063×10^7	-1.567×10^4
100.00	2.202×10^{-4}	2.474×10^{-1}	25.81	2.042×10^7	-1.570×10^4
40.00	1.345×10^{-3}	2.508×10^{-1}	25.94	2.012×10^7	-1.575×10^4
10.00	1.933×10^{-2}	2.673×10^{-1}	26.56	1.898×10^7	-1.595×10^4
4.00	9.997×10^{-2}	2.982×10^{-1}	27.72	1.800×10^7	-1.624×10^4
2.00	3.081×10^{-1}	3.446×10^{-1}	29.50	1.867×10^7	-1.653×10^4
1.00	8.289×10^{-1}	4.245×10^{-1}	32.63	2.380×10^7	-1.672×10^4
0.40	2.450	6.091×10^{-1}	40.14	4.578×10^7	-1.641×10^4
0.10	8.451	1.162	63.80	1.393×10^8	-1.331×10^4
0.04	1.602×10^1	1.809	92.24	2.625×10^8	-8.585×10^3
0.01	3.581×10^1	3.481	164.79	5.941×10^8	4.374×10^3

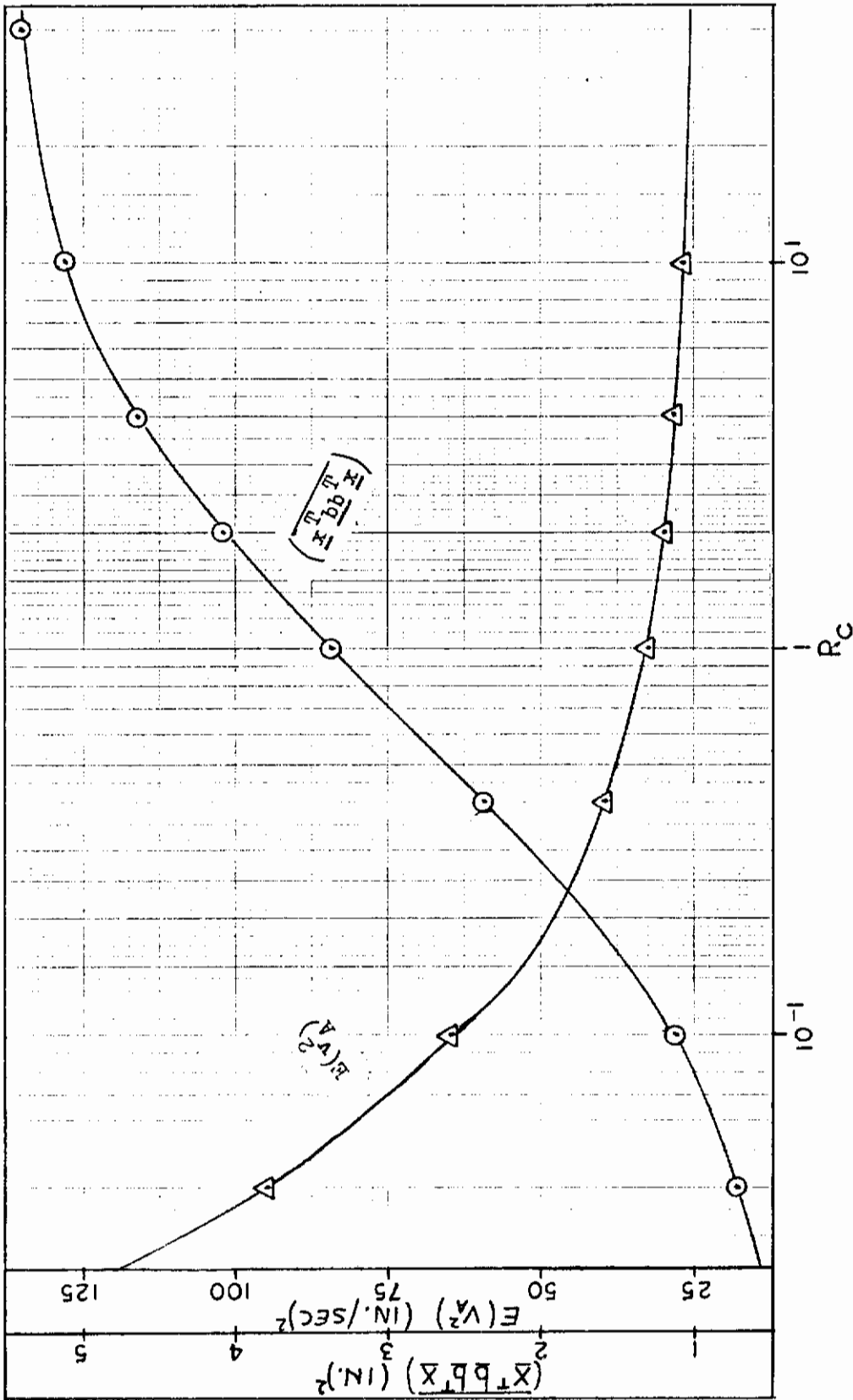


Figure 14. Plot of $\underline{x}^T \underline{b} \underline{b}^T \underline{x}$ and $E(v_A^2)$ vs R_C , Feedback Law 1 (Single Gear Model)

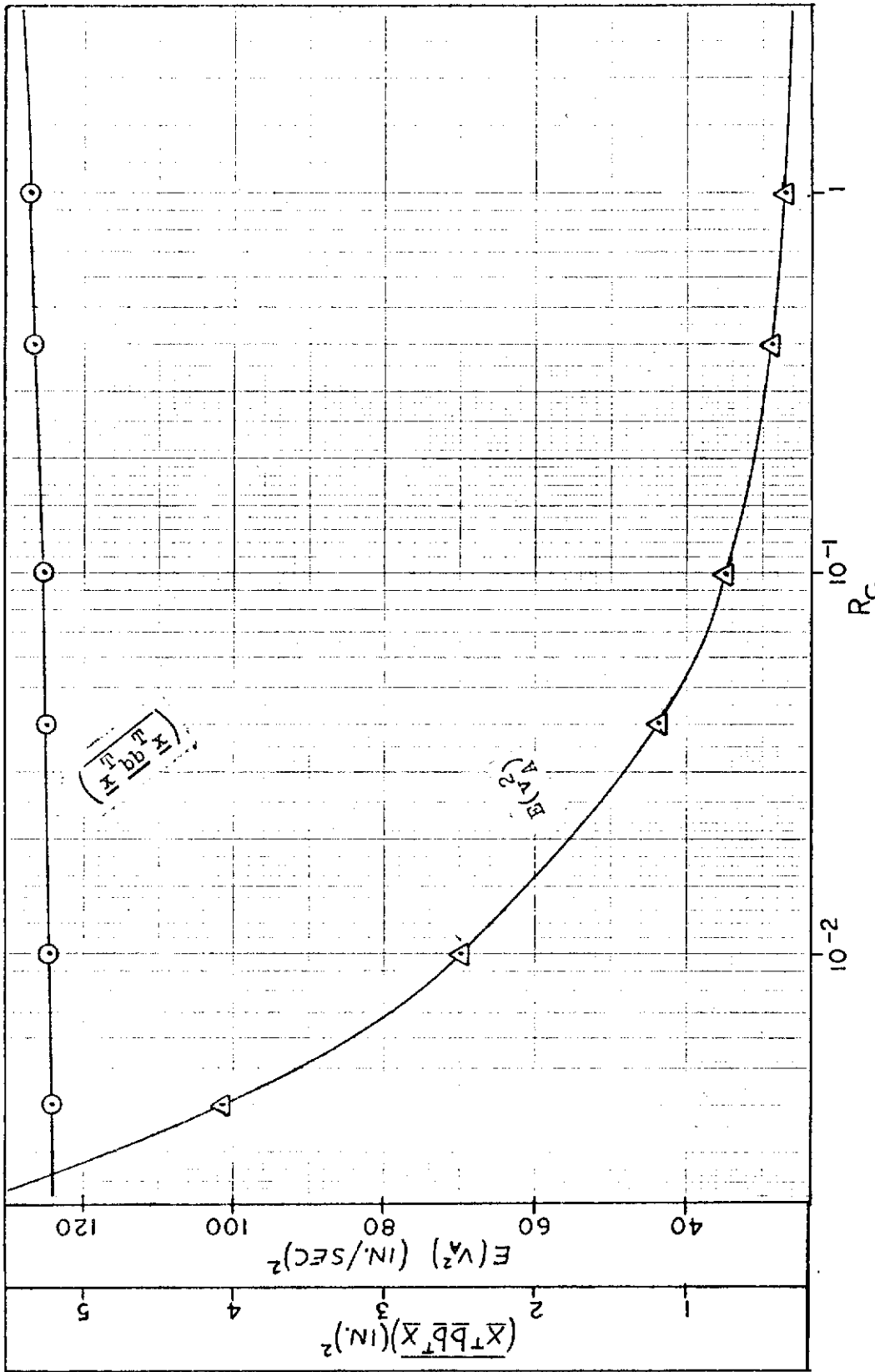


Figure 15. Plot of $\frac{(x^T b b^T x)}{(x^T x)}$ and $E(v_A^2)$ vs R_C , Feedback Law II (Single Gear Model)

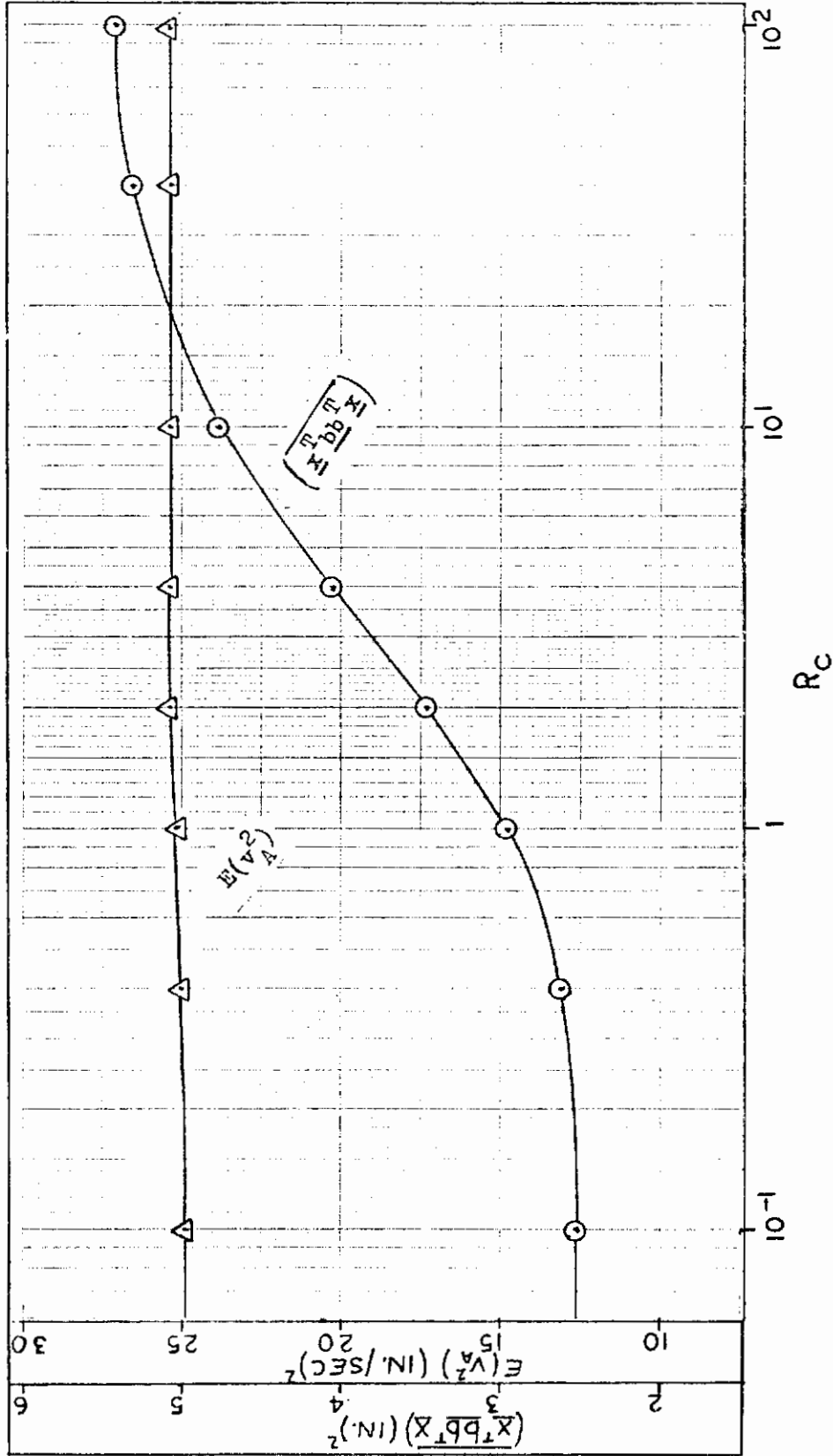


Figure 16. Plot of $\underline{x}^T \underline{b} \underline{b}^T \underline{x}$ and $E(v_A^2)$ vs R_C , Feedback Law III (Single Gear Model)

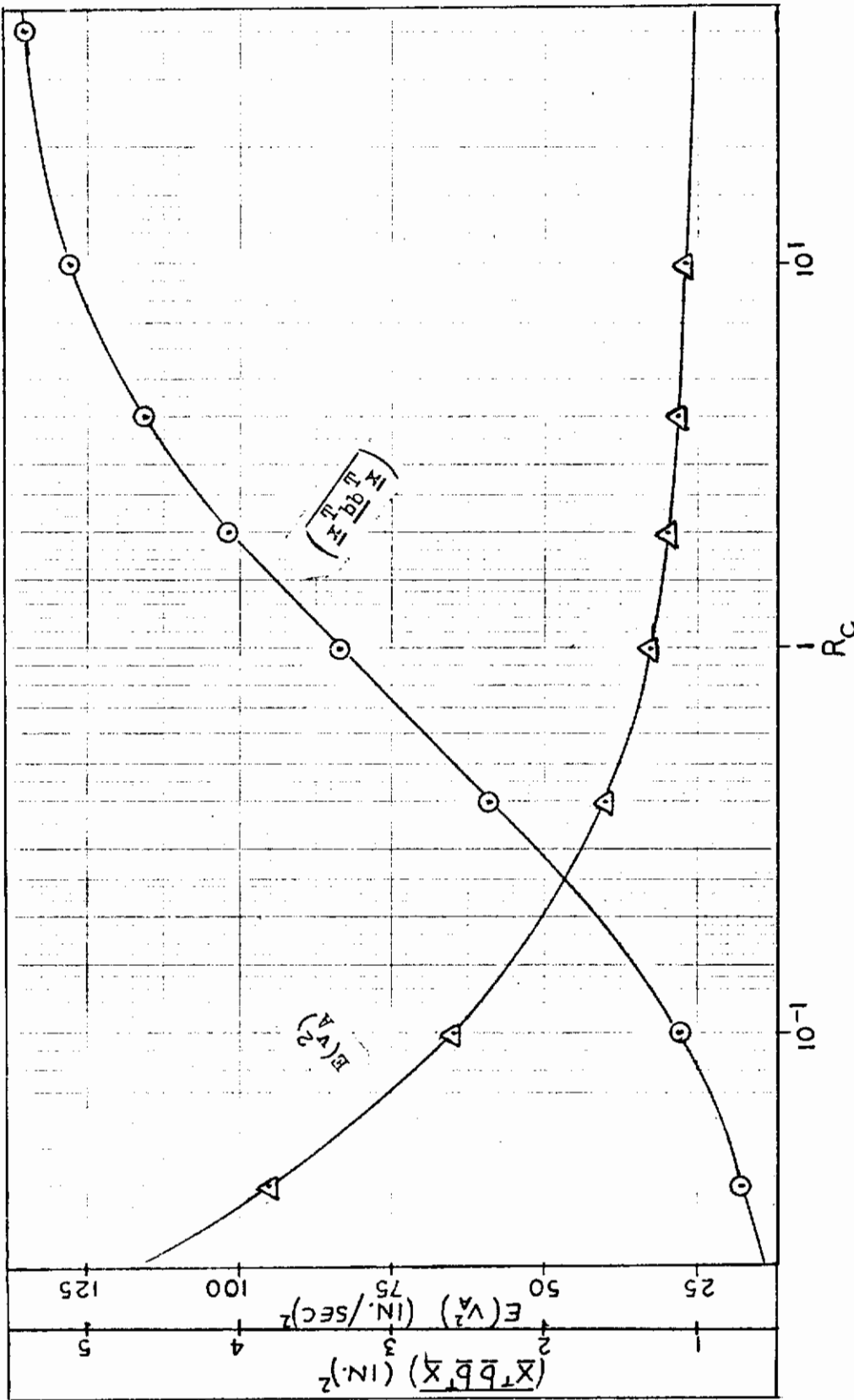


Figure 17. Plot of $x^T b b^T x$ and $E(v_A^2)$ vs R_C , Feedback Law IV (Single Gear Model)

APPENDIX D

RESULTS FOR TRICYCLE LANDING GEAR MODEL

This appendix gives the results obtained for the tricycle landing gear system model described in Chapter V. Results for two cases were obtained:

Case I: Actuators were placed in all three landing gears. The state equations for this system are given by Equation 67. The state vector is identified as

$$\underline{x}^T = (z_s, z_f, z_{u1}, z_{u2}, z_{u3}, \theta, v_s, v_f, v_{u1}, v_{u2}, v_{u3}, v_\theta, F_{A1}, F_{A2}, F_{A3}, h_1, h_2, h_3) \quad (68)$$

and the system matrix F is readily identified as an 18x18 matrix. The control vector \underline{u} is identified as

$$\underline{u}^T = [u_1 \quad u_2 \quad u_3] \quad (125)$$

Case II: Actuators were placed only in the two main landing gears, i.e., the actuator was removed from the nose gear of Case I. While the state equations for this model are not exactly those given by Equation 67, the necessary modifications are obvious. There is no longer any additional force F_{A1} applied by the actuator in the nose gear. The state vector is then identified as

$$\underline{x}^T = (z_s, z_f, z_{u1}, z_{u2}, z_{u3}, \theta, v_s, v_f, v_{u1}, v_{u2}, v_{u3}, v_\theta, F_{A2}, F_{A3}, h_1, h_2, h_3) \quad (126)$$

and the state equations are given as

$$\begin{aligned} \dot{z}_s &= v_s \\ \dot{z}_f &= v_f \\ \dot{z}_{u1} &= v_{u1} \end{aligned}$$

$$\begin{aligned} \dot{z}_{u2} &= v_{u2} \\ \dot{z}_{u3} &= v_{u3} \\ \dot{\theta} &= v_{\theta} \\ \dot{v}_s &= -\frac{K_s}{M_s} z_s + \frac{K_s}{M_s} z_f + \frac{K_s}{M_s} e \cdot \theta \\ &\quad - \frac{(C_s + C_{sg})}{M_s} v_s + \frac{C_s}{M_s} v_f + \frac{C_s}{M_s} e \cdot v_{\theta} \\ \dot{v}_f &= \frac{K_s}{M_f} z_s - \frac{(K_{a1} + K_{a2} + K_{a3} + K_s)}{M_f} z_f \\ &\quad + \frac{K_{a1}}{M_f} z_{u1} + \frac{K_{a2}}{M_f} z_{u2} + \frac{K_{a3}}{M_f} z_{u3} \\ &\quad + \frac{(K_{a1} L_{x1} + K_{a2} L_{x2} - K_{a3} L_{x3} - K_s e)}{M_f} \theta \\ &\quad + \frac{C_s}{M_f} v_s - \frac{(D_1 + D_2 + D_3 + C_s + C_{fg})}{M_f} v_f \\ &\quad + \frac{D_1}{M_f} v_{u1} + \frac{D_2}{M_f} v_{u2} + \frac{D_3}{M_f} v_{u3} \\ &\quad + \frac{(D_1 L_{x1} + D_2 L_{x2} - D_3 L_{x3} - C_s e)}{M_f} v_{\theta} - \frac{2}{M_f} F_{A2} - \frac{2}{M_f} F_{A3} \\ \dot{v}_{u1} &= \frac{K_{a1}}{M_{u1}} z_f - \frac{(K_{a1} + K_{t1})}{M_{u1}} z_{u1} - \frac{K_{a1}}{M_{u1}} L_{x1} \theta \\ &\quad + \frac{D_1}{M_{u1}} v_f - \frac{D_1}{M_{u1}} v_{u1} - \frac{D_1}{M_{u1}} L_{x1} v_{\theta} + \frac{K_{t1}}{M_{u1}} h_1 \\ \dot{v}_{u2} &= \frac{K_{a2}}{M_{u2}} z_f - \frac{(K_{a2} + K_{t2})}{M_{u2}} z_{u2} - \frac{K_{a2}}{M_{u2}} L_{x2} \theta \\ &\quad + \frac{D_2}{M_{u2}} v_f - \frac{D_2}{M_{u2}} v_{u2} - \frac{D_2}{M_{u2}} L_{x2} v_{\theta} + \frac{2}{M_{u2}} F_{A2} + \frac{K_{t2}}{M_{u2}} h_2 \\ \dot{v}_{u3} &= \frac{K_{a3}}{M_{u3}} z_f - \frac{(K_{a3} + K_{t3})}{M_{u3}} z_{u3} + \frac{K_{a3}}{M_{u3}} L_{x3} \theta \\ &\quad + \frac{D_3}{M_{u3}} v_f - \frac{D_3}{M_{u3}} v_{u3} + \frac{D_3}{M_{u3}} L_{x3} v_{\theta} + \frac{2}{M_{u3}} F_{A3} + \frac{K_{t3}}{M_{u3}} h_3 \end{aligned}$$

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$$\begin{aligned}
 \dot{v}_\theta &= \frac{eK_s}{I_{yy}} Z_s + \frac{(L_{x1}K_{a1} + L_{x2}K_{a2} - eK_s - L_{x3}K_{a3})}{I_{yy}} Z_f \\
 &\quad - \frac{L_{x1}K_{a1}}{I_{yy}} Z_{u1} - \frac{L_{x2}K_{a2}}{I_{yy}} Z_{u2} + \frac{L_{x3}K_{a3}}{I_{yy}} Z_{u3} \\
 &\quad - \frac{(L_{x1}^2K_{a1} + L_{x2}^2K_{a2} + e^2K_s + L_{x3}^2K_{a3})}{I_{yy}} \theta \\
 &\quad + \frac{eC_s}{I_{yy}} V_s + \frac{(L_{x1}D_1 + L_{x2}D_2 - eC_s - L_{x3}D_3)}{I_{yy}} V_f \\
 &\quad - \frac{L_{x1}D_1}{I_{yy}} V_{u1} - \frac{L_{x2}D_2}{I_{yy}} V_{u2} + \frac{L_{x3}D_3}{I_{yy}} V_{u3} \\
 &\quad - \frac{(L_{x1}^2D_1 + L_{x2}^2D_2 + e^2C_s + L_{x3}^2D_3)}{I_{yy}} v_\theta \\
 &\quad + 2 \frac{L_{x2}}{I_{yy}} F_{A2} - 2 \frac{L_{x3}}{I_{yy}} F_{A3} \\
 \dot{F}_{A2} &= \frac{C_b C_A K_B}{V} V_f - \frac{C_b C_A K_B}{V} V_{u2} - \frac{C_b C_A K_B}{V} L_{x2} v_\theta \\
 &\quad - \frac{K_B}{V} (L + C_p) F_{A2} + \frac{C_x C_A K_B}{V} u_2 \\
 \dot{F}_{A3} &= \frac{C_b C_A K_B}{V} V_f - \frac{C_b C_A K_B}{V} V_{u3} + \frac{C_b C_A K_B}{V} L_{x3} v_\theta \\
 &\quad - \frac{K_B}{V} (L + C_p) F_{A3} + \frac{C_x C_A K_B}{V} u_3 \\
 \dot{h}_1 &= -\omega_B h_1 + \xi_1 \\
 \dot{h}_2 &= (\omega_B + \frac{2}{\tau_{12}}) h_1 - \frac{2}{\tau_{12}} h_2 - \xi_1 \\
 \dot{h}_3 &= -(\omega_B + \frac{2}{\tau_{12}}) h_1 + 2(\frac{1}{\tau_{12}} + \frac{1}{\tau_{23}}) h_2 - \frac{2}{\tau_{23}} h_3 + \xi_1
 \end{aligned}$$

(127)

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The system matrix F is readily identified as a 17×17 matrix. The control vector \underline{u} is identified as

$$\underline{u} = \begin{bmatrix} u_2 & u_3 \end{bmatrix} \quad (128)$$

The feed back law used for each of the above cases is equivalent to Feedback Law I for the single landing gear system (see Appendix C). It consisted of measuring the relative displacement between M_s and M_f , and the acceleration of M_f . Thus, the measurement vector \underline{y} is identified as

$$\underline{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (129)$$

and

$$\begin{aligned} y_1 &= -Z_s + Z_f + e\theta \\ y_2 &= \dot{V}_f \\ &= \frac{K_s}{M_f} Z_s - \frac{K_{a1} + K_{a2} + K_{a3} + K_s}{M_f} Z_f \\ &\quad + \frac{K_{a1}}{M_f} Z_{u1} + \frac{K_{a2}}{M_f} Z_{u2} + \frac{K_{a3}}{M_f} Z_{u3} \\ &\quad + \frac{(K_{a1} L_{x1} + K_{a2} L_{x2} - K_{a3} L_{x3} - K_s e)}{M_f} \theta \\ &\quad + \frac{C_s}{M_f} V_s - \frac{(D_1 + D_2 + D_3 + C_s + C_{fg})}{M_f} V_f \\ &\quad + \frac{D_1}{M_f} V_{u1} + \frac{D_2}{M_f} V_{u2} + \frac{D_3}{M_f} V_{u3} \\ &\quad + \frac{(D_1 L_{x1} + D_2 L_{x2} - D_3 L_{x3} - C_s e)}{M_f} V_\theta \\ &\quad - \frac{1}{M_f} F_{A1} - \frac{2}{M_f} F_{A2} - \frac{2}{M_f} F_{A3} \end{aligned} \quad (130)$$

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for Case I, while

$$\begin{aligned}
 y_1 &= -Z_s + Z_f + e\theta \\
 y_2 &= \dot{V}_f \\
 &= \frac{K_s}{M_f} Z_s - \frac{(K_{a1} + K_{a2} + K_{a3} + K_s)}{M_f} Z_f \\
 &\quad + \frac{K_{a1}}{M_f} Z_{u1} + \frac{K_{a2}}{M_f} Z_{u2} + \frac{K_{a3}}{M_f} Z_{u3} \\
 &\quad + \frac{(K_{a1} L_{x1} + K_{a2} L_{x2} - K_{a3} L_{x3} - K_s e)}{M_f} \theta \\
 &\quad + \frac{C_s}{M_f} V_s - \frac{(D_1 + D_2 + D_3 + C_s + C_{fg})}{M_f} V_f \\
 &\quad + \frac{D_1}{M_f} V_{u1} + \frac{D_2}{M_f} V_{u2} + \frac{D_3}{M_f} V_{u3} \\
 &\quad + \frac{(D_1 L_{x1} + D_2 L_{x2} - D_3 L_{x3} - C_s e)}{M_f} V_\theta \\
 &\quad - \frac{2}{M_f} F_{A2} - \frac{2}{M_f} F_{A3}
 \end{aligned} \tag{131}$$

for Case II.

The control vector for each of the two cases is then given by

$$\underline{u} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \\ K_{31} & K_{32} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{132}$$

for Case I, while

$$\underline{u} = \begin{bmatrix} K_{21} & K_{22} \\ K_{31} & K_{32} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{133}$$

for Case II.

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The cost criterion used for both cases is given by Equation 76 and is restated as

$$J = \lim_{t \rightarrow \infty} E \left[\underline{x}^T \underline{b} \underline{b}^T \underline{x} + R_c \underline{u}^T \underline{u} \right] \quad (76)$$

where the relative displacement between the main mass and the elastically connected mass is identified as:

$$\underline{b}^T \underline{x} = -z_s + z_f + e\theta \quad (134)$$

The parameters for the vehicle model that were used for both cases were obtained from Reference 19:183 and Reference 1:13, and are given here (for $V_H=66$ fps) as:

$$M_s = 87.94 \text{ LB} - \text{SEC}^2/\text{IN.}$$

$$M_f = 244.04 \text{ LB} - \text{SEC}^2/\text{IN.}$$

$$M_{u1} = 1.68 \text{ LB} - \text{SEC}^2/\text{IN.}$$

$$M_{u2} = M_{u3} = 3.36 \text{ LB} - \text{SEC}^2/\text{IN.}$$

$$D_1 = 185.0 \text{ LB} - \text{SEC}/\text{IN.}$$

$$D_2 = D_3 = 370.0 \text{ LB} - \text{SEC}/\text{IN.}$$

$$C_s = 4.54 \text{ LB} - \text{SEC}/\text{IN.}$$

$$C_{sg} = 21.74 \text{ LB} - \text{SEC}/\text{IN.}$$

$$C_{fg} = 60.4 \text{ LB} - \text{SEC}/\text{IN.}$$

$$K_s = 4,110.0 \text{ LB}/\text{IN.}$$

$$K_{a1} = 1,941.0 \text{ LB}/\text{IN.}$$

$$K_{a2} = K_{a3} = 28,340 \text{ LB}/\text{IN.}$$

$$K_{11} = 12,632 \text{ LB}/\text{IN.}$$

$$K_{12} = K_{13} = 16,660.0 \text{ LB}/\text{IN.}$$

$$I_{yy} = 1.0195 \times 10^7 \text{ LB} - \text{SEC}^2 - \text{IN.}$$

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$$L_{x1} = 360.515 \text{ IN.}$$

$$L_{x2} = 6.015 \text{ IN.}$$

$$L_{x3} = 54.485 \text{ IN.}$$

$$e = 2.13220 \text{ IN.}$$

The parameters for the runway model that were used for both cases were obtained in Chapter III and are restated here (for $V_H=66\text{fps}$) as:

$$A_0 = 10^{-1} (\text{IN.}^2/\text{RAD}/\text{FT})$$

$$\lambda_0 = 4.5 \times 10^3 \text{ FT}$$

$$\tau_{12} = 4.48 \times 10^{-1} \text{ SEC}$$

$$\tau_{23} = 7.64 \times 10^{-2} \text{ SEC}$$

The parameters for the actuator model that were used for both cases are given in Chapter IV and are restated here as

$$C_b = A_p = 0.96 (\text{IN.})^2$$

$$\frac{V}{K_B} = 2.0 \times 10^{-5} (\text{IN.}^3/\text{LB})$$

$$L + C_p = 7.0 \times 10^{-4} (\text{IN.}^3/\text{LB} - \text{SEC})$$

$$C_x = 4.0 (\text{IN.}^3/\text{SEC} - \text{mA})$$

$$\eta_F = 1.0$$

Note that the same parameters were used for each actuator.

The optimization procedure described in Chapter VII was carried out for both cases and for various values of R_C . The results for the tricycle landing gear system now follow.

TABLE XIII
RESULTS OF MINIMIZATION FOR CASE I

Weighting R_c **	Stress per Wing $\frac{T}{x} \frac{T}{bb} x$ (in.) ²	Cost (J(K*))
**	8.838	8.838
INFINITE	7.304	7.304
100.00	7.175	7.239
50.00	7.053	7.177
10.00	6.257	6.740
5.00	5.556	6.313
2.00	4.328	5.447
1.00	3.325	4.607
0.50	2.401	3.703
0.20	1.431	2.571
0.10	9.218×10^{-1}	1.851
0.05	5.818×10^{-1}	1.286
0.01	1.875×10^{-1}	4.994×10^{-1}

** ACTUATORS REMOVED FROM ALL LANDING GEARS.

TABLE XIV
FEEDBACK CONSTANTS FOR CASE I

Weighting R_c	u_1		u_2		u_3	
	K_{11}^*	K_{12}^*	K_{21}^*	K_{22}^*	K_{31}^*	K_{32}^*
INFINITE	0.0	0.0	0.0	0.0	0.0	0.0
100.00	-4.541×10^{-4}	-5.999×10^{-5}	5.863×10^{-3}	4.621×10^{-5}	6.974×10^{-3}	6.461×10^{-5}
50.00	-8.715×10^{-4}	-1.188×10^{-4}	1.163×10^{-2}	9.196×10^{-5}	1.383×10^{-2}	1.285×10^{-4}
10.00	-3.088×10^{-3}	-5.535×10^{-4}	5.486×10^{-2}	4.422×10^{-4}	6.506×10^{-2}	6.153×10^{-4}
5.00	-3.704×10^{-3}	-1.028×10^{-3}	1.033×10^{-1}	8.436×10^{-4}	1.222×10^{-1}	1.170×10^{-3}
2.00	3.072×10^{-3}	-2.165×10^{-3}	2.270×10^{-1}	1.867×10^{-3}	2.666×10^{-1}	2.574×10^{-3}
1.00	2.895×10^{-2}	-3.512×10^{-3}	3.954×10^{-1}	3.205×10^{-3}	4.604×10^{-1}	4.392×10^{-3}
0.50	1.018×10^{-1}	-5.158×10^{-3}	6.691×10^{-1}	5.228×10^{-3}	7.703×10^{-1}	7.083×10^{-3}
0.20	3.524×10^{-1}	-6.812×10^{-3}	1.297	9.574×10^{-3}	1.467	1.258×10^{-2}
0.10	7.436×10^{-1}	-6.613×10^{-3}	2.078	1.509×10^{-2}	2.320	1.920×10^{-2}
0.05	1.404	-4.677×10^{-3}	3.214	2.365×10^{-2}	3.542	2.924×10^{-2}
0.01	5.028	8.931×10^{-3}	7.911	6.184×10^{-2}	8.431	7.367×10^{-2}

TABLE XV
DEMANDS ON ACTUATOR IN NOSE GEAR FOR CASE I

Weighting R_c	Signal $E(u_1)$ (ma) ²	Displacement $E(v_{Al}^2)$ (in.) ²	Velocity $E(\dot{v}_{Al}^2)$ (in./sec) ²	Force $E(F_{Al}^2)$ (lb) ²	Power $E(F_{Al} v_{Al})$ (lb-in./sec)
INFINITE	0.0	4.354×10^{-1}	236.5	6.806×10^7	-4.120×10^4
100.00	1.652×10^{-5}	4.366×10^{-1}	236.1	6.798×10^7	-4.124×10^4
50.00	6.414×10^{-5}	4.377×10^{-1}	235.7	6.790×10^7	-4.129×10^4
10.00	1.310×10^{-3}	4.459×10^{-1}	233.1	6.727×10^7	-4.159×10^4
5.00	4.340×10^{-3}	4.546×10^{-1}	230.4	6.655×10^7	-4.190×10^4
2.00	1.901×10^{-2}	4.741×10^{-1}	223.7	6.477×10^7	-4.257×10^4
1.00	5.458×10^{-2}	4.942×10^{-1}	215.5	6.266×10^7	-4.334×10^4
0.50	1.455×10^{-1}	5.127×10^{-1}	204.1	6.041×10^7	-4.438×10^4
0.20	4.282×10^{-1}	5.223×10^{-1}	185.8	5.927×10^7	-4.590×10^4
0.10	8.032×10^{-1}	5.497×10^{-1}	171.8	6.060×10^7	-4.666×10^4
0.05	1.367	7.164×10^{-1}	159.4	6.307×10^7	-4.697×10^4
0.01	4.424	3.567	142.2	7.824×10^7	-4.998×10^4

TABLE XVI
DEMANDS ON ACTUATOR IN FRONT MAIN GEAR FOR CASE I

Weighting R_c	Signal $E(u_2^2)$ (ma) ²	Displacement $E(y_{A2}^2)$ (in.) ²	Velocity $E(v_{A2}^2)$ (in./sec) ²	Force $E(F_{A2}^2)$ (lb) ²	Power $E(F_{A2}^2 v_{A2}^2)$ (lb-in./sec) ⁴
INFINITE	0.0	9.115×10^{-2}	22.70	1.315×10^7	-1.051×10^4
100.00	2.557×10^{-4}	9.222×10^{-2}	22.71	1.313×10^7	-1.054×10^4
50.00	9.884×10^{-4}	9.327×10^{-2}	22.73	1.313×10^7	-1.056×10^4
10.00	1.940×10^{-2}	1.014×10^{-1}	22.86	1.340×10^7	-1.072×10^4
5.00	6.082×10^{-2}	1.109×10^{-1}	23.02	1.412×10^7	-1.083×10^4
2.00	2.253×10^{-1}	1.361×10^{-1}	23.48	1.721×10^7	-1.098×10^4
1.00	5.165×10^{-1}	1.714×10^{-1}	24.15	2.288×10^7	-1.102×10^4
0.50	1.048	2.299×10^{-1}	25.28	3.347×10^7	-1.096×10^4
0.20	2.289	3.638×10^{-1}	27.86	5.907×10^7	-1.081×10^4
0.10	3.736	5.265×10^{-1}	30.78	9.014×10^7	-1.077×10^4
0.05	5.652	7.537×10^{-1}	34.39	1.331×10^8	-1.081×10^4
0.01	1.216×10^1	1.573	44.43	2.870×10^8	-1.046×10^4

TABLE XVII
DEMANDS ON ACTUATOR IN REAR MAIN GEAR FOR CASE I

Weighting R_c	Signal $E(u_3^2)$ (ma) ²	Displacement $E(y_{A3}^2)$ (in.) ²	Velocity $E(\dot{y}_{A3}^2)$ (in./sec) ²	Force $E(F_{A3}^2)$ (lb) ²	Power $E(F_{A3} v_{A3})$ (lb-in./sec)
INFINITE	0.0	1.102×10^{-1}	19.35	1.440×10^7	-6.930×10^3
100.00	3.666×10^{-4}	1.117×10^{-1}	19.40	1.436×10^7	-6.928×10^3
50.00	1.416×10^{-3}	1.132×10^{-1}	19.45	1.434×10^7	-6.926×10^3
10.00	2.766×10^{-2}	1.243×10^{-1}	19.86	1.455×10^7	-6.885×10^3
5.00	8.623×10^{-2}	1.372×10^{-1}	20.34	1.538×10^7	-6.801×10^3
2.00	3.152×10^{-1}	1.713×10^{-1}	21.61	1.921×10^7	-6.475×10^3
1.00	7.109×10^{-1}	2.185×10^{-1}	23.37	2.635×10^7	-5.891×10^3
0.50	1.411	2.945×10^{-1}	26.17	3.967×10^7	-4.814×10^3
0.20	2.981	4.624×10^{-1}	31.99	7.140×10^7	-2.358×10^3
0.10	4.756	6.588×10^{-1}	37.99	1.093×10^8	3.253×10^2
0.05	7.061	9.235×10^{-1}	44.94	1.609×10^8	3.564×10^3
0.01	1.461 $\times 10^1$	1.790	64.13	3.413×10^8	1.453×10^4

TABLE XVIII
RESULTS OF MINIMIZATION FOR CASE II

Weighting R_c	Stress per Wing $\frac{T}{x} \frac{T}{bb^2 x}$ (in.) ²	Cost J(K*)
**	8.838	8.838
INFINITE	6.124	6.124
100.00	6.041	6.082
50.00	5.961	6.042
10.00	5.432	5.755
5.00	4.952	5.467
2.00	4.079	4.867
1.00	3.339	4.267
0.50	2.629	3.603
0.20	1.816	2.730
0.10	1.312	2.128
0.05	9.116×10^{-1}	1.604
0.01	3.557×10^{-1}	7.419×10^{-1}

** ACTUATORS REMOVED FROM ALL LANDING GEARS.

TABLE XIX
FEEDBACK CONSTANTS FOR CASE II

Weighting R_c	u_2			u_3	
	K_{21}^*	K_{22}^*	K_{31}^*	K_{32}^*	K_{32}^*
INFINITE	0.0	0.0	0.0	0.0	0.0
100.00	5.630×10^{-3}	6.595×10^{-5}	6.129×10^{-3}	7.313×10^{-5}	
50.00	1.117×10^{-2}	1.312×10^{-4}	1.216×10^{-2}	1.455×10^{-4}	
10.00	5.262×10^{-2}	6.316×10^{-4}	5.735×10^{-2}	6.994×10^{-4}	
5.00	9.884×10^{-2}	1.212×10^{-3}	1.078×10^{-1}	1.341×10^{-3}	
2.00	2.142×10^{-1}	2.750×10^{-3}	2.340×10^{-1}	3.036×10^{-3}	
1.00	3.643×10^{-1}	4.910×10^{-3}	3.984×10^{-1}	5.411×10^{-3}	
0.50	5.932×10^{-1}	8.465×10^{-3}	6.482×10^{-1}	9.305×10^{-3}	
0.20	1.081	1.665×10^{-2}	1.170	1.819×10^{-2}	
0.10	1.682	2.718×10^{-2}	1.791	2.943×10^{-2}	
0.05	2.614	4.363×10^{-2}	2.713	4.678×10^{-2}	
0.01	6.967	1.192×10^{-1}	6.801	1.272×10^{-1}	

TABLE XX
DEMANDS ON ACTUATOR IN FRONT MAIN GEAR FOR CASE II

Weighting R_c	Signal $E(u_2^2)$ (ma) ²	Displacement $E(v_{A2}^2)$ (in.) ²	Velocity $E(\dot{v}_{A2}^2)$ (in./sec) ²	Force $E(F_{A2}^2)$ (lb) ²	Power $E(\dot{K}_{A2} v_{A2})$ (lb-in./sec)
INFINITE	0.0	8.521×10^{-2}	23.77	1.558×10^7	-1.214×10^4
100.00	1.887×10^{-4}	8.628×10^{-2}	23.80	1.555×10^7	-1.216×10^4
50.00	7.322×10^{-4}	8.733×10^{-2}	23.83	1.552×10^7	-1.219×10^4
10.00	1.470×10^{-2}	9.547×10^{-2}	24.09	1.553×10^7	-1.235×10^4
5.00	4.693×10^{-2}	1.049×10^{-1}	24.40	1.587×10^7	-1.250×10^4
2.00	1.792×10^{-1}	1.300×10^{-1}	25.25	1.769×10^7	-1.276×10^4
1.00	4.218×10^{-1}	1.650×10^{-1}	26.45	2.141×10^7	-1.298×10^4
0.50	8.854×10^{-1}	2.219×10^{-1}	28.42	2.889×10^7	-1.317×10^4
0.20	2.096	3.523×10^{-1}	32.81	4.958×10^7	-1.326×10^4
0.10	3.796	5.218×10^{-1}	38.08	8.035×10^7	-1.305×10^4
0.05	6.555	7.897×10^{-1}	45.57	1.330×10^8	-1.240×10^4
0.01	1.880×10^1	2.010	73.03	3.885×10^8	-8.334×10^3

TABLE XXI
DEMANDS ON ACTUATOR IN REAR MAIN GEAR FOR CASE II

Weighting R_c	Signal $E(u_3^2)$ $ma)^2$	Displacement $E(y_{A3}^2)$ $(in.)^2$	Velocity $E(v_{A3}^2)$ $(in./sec)^2$	Force $E(F_{A3}^2)$ $(lb)^2$	Power $E(F_{A3}^v A_3)$ $(lb-in./sec)$
INFINITE	0.0	8.015×10^{-2}	20.25	1.292×10^7	-7.192×10^3
100.00	2.247×10^{-4}	8.162×10^{-2}	20.28	1.279×10^7	-7.235×10^3
50.00	8.718×10^{-4}	8.308×10^{-2}	20.31	1.268×10^7	-7.277×10^3
10.00	1.753×10^{-2}	9.433×10^{-2}	20.52	1.203×10^7	-7.559×10^3
5.00	5.606×10^{-2}	1.075×10^{-1}	20.79	1.172×10^7	-7.828×10^3
2.00	2.147×10^{-1}	1.427×10^{-1}	21.54	1.221×10^7	-8.344×10^3
1.00	5.063×10^{-1}	1.926×10^{-1}	22.66	1.462×10^7	-8.781×10^3
0.50	1.061	2.752×10^{-1}	24.57	2.080×10^7	-9.060×10^3
0.20	2.474	4.628×10^{-1}	29.08	4.039×10^7	-8.459×10^3
0.10	4.368	6.917×10^{-1}	35.00	7.113×10^7	-6.404×10^3
0.05	7.297	1.020	44.09	1.238×10^8	-2.472×10^3
0.01	1.982×10^1	2.358	79.71	3.726×10^8	1.144×10^4

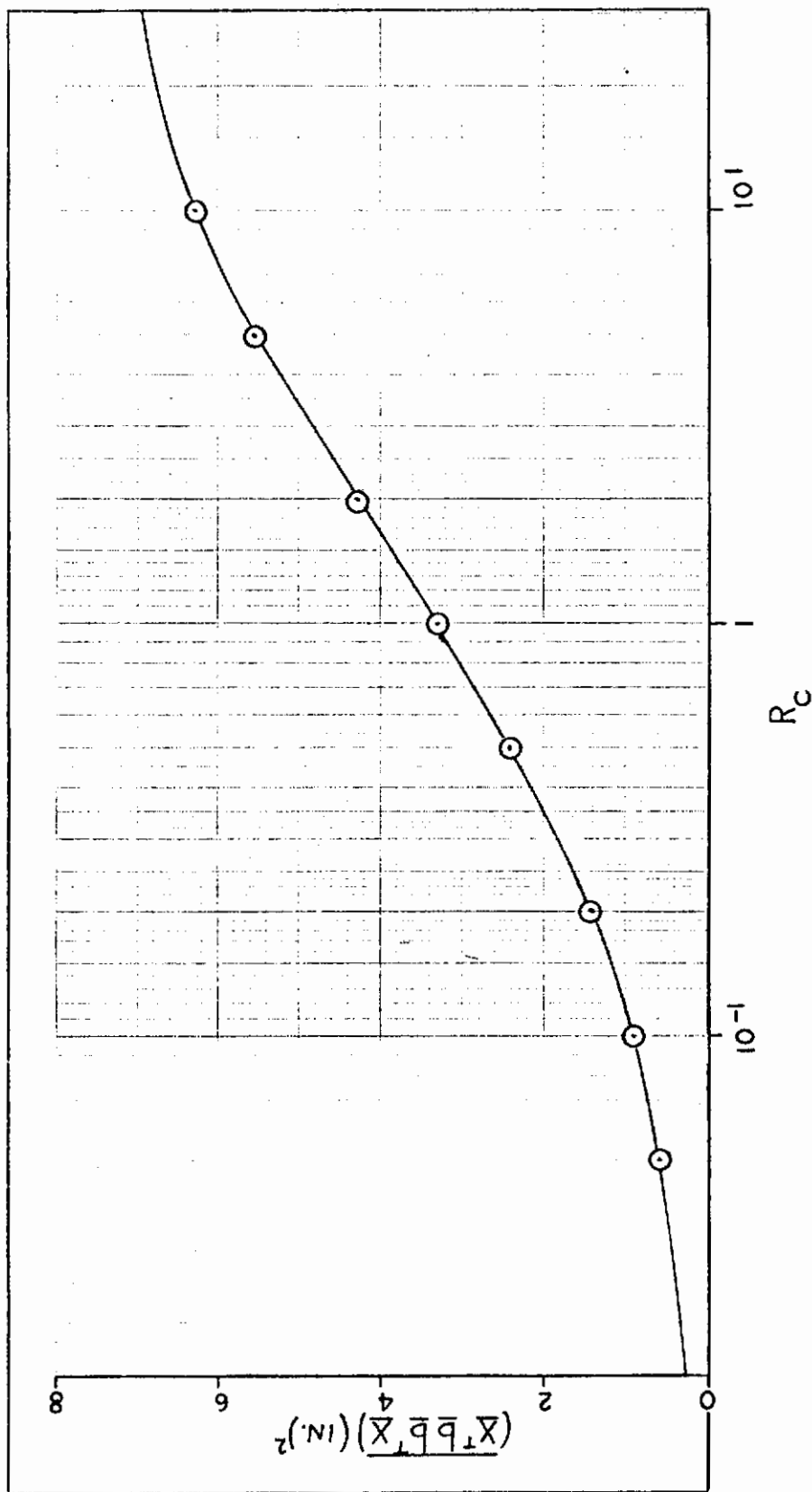


Figure 18. Plot of $\overline{x^T b b^T x}$, Case 1 (Tricycle Gear Model)

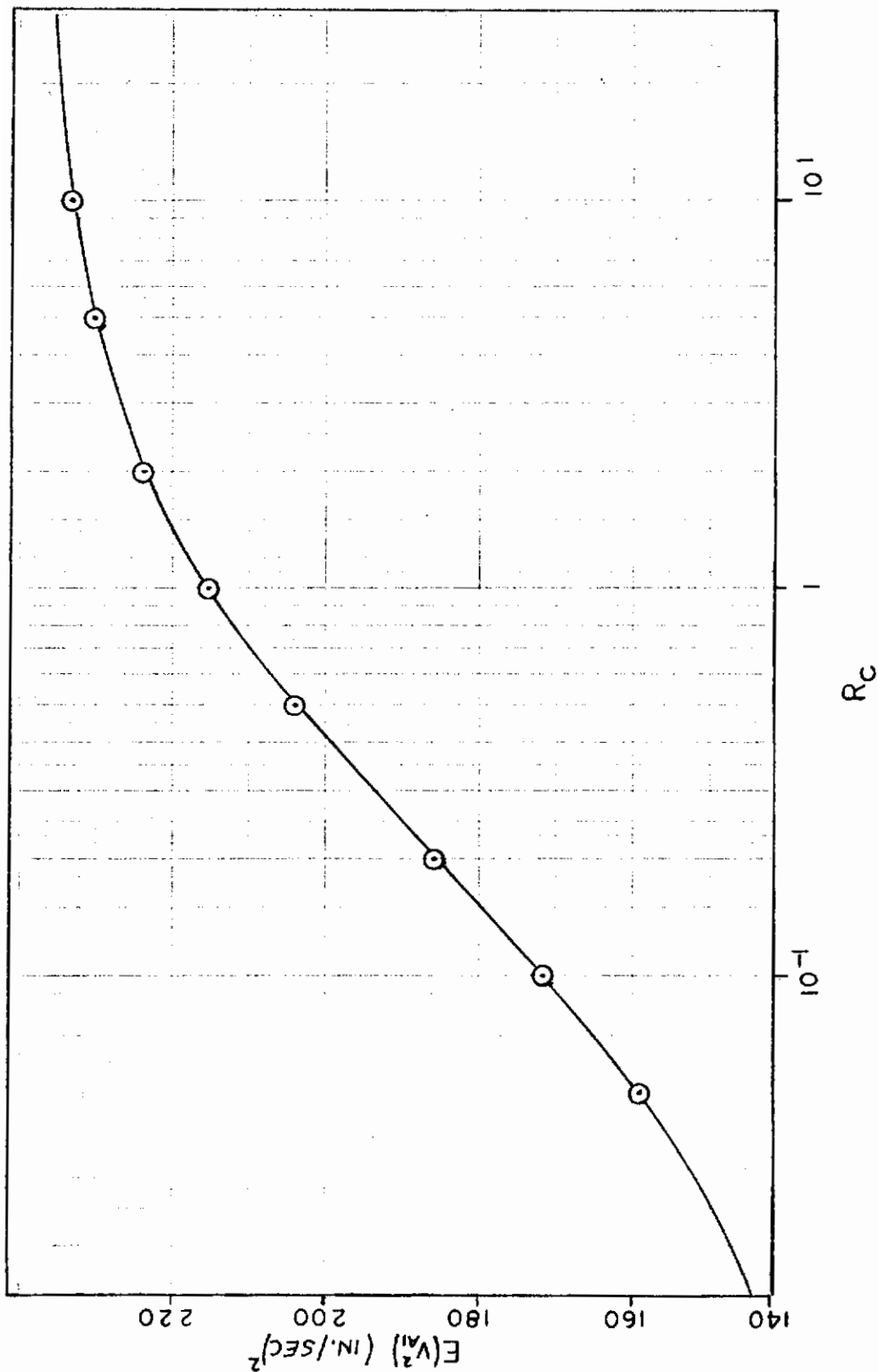


Figure 19. Plot of $E(v_{A1}^2)$ vs R_C , Case I (Tricycle Gear Model)

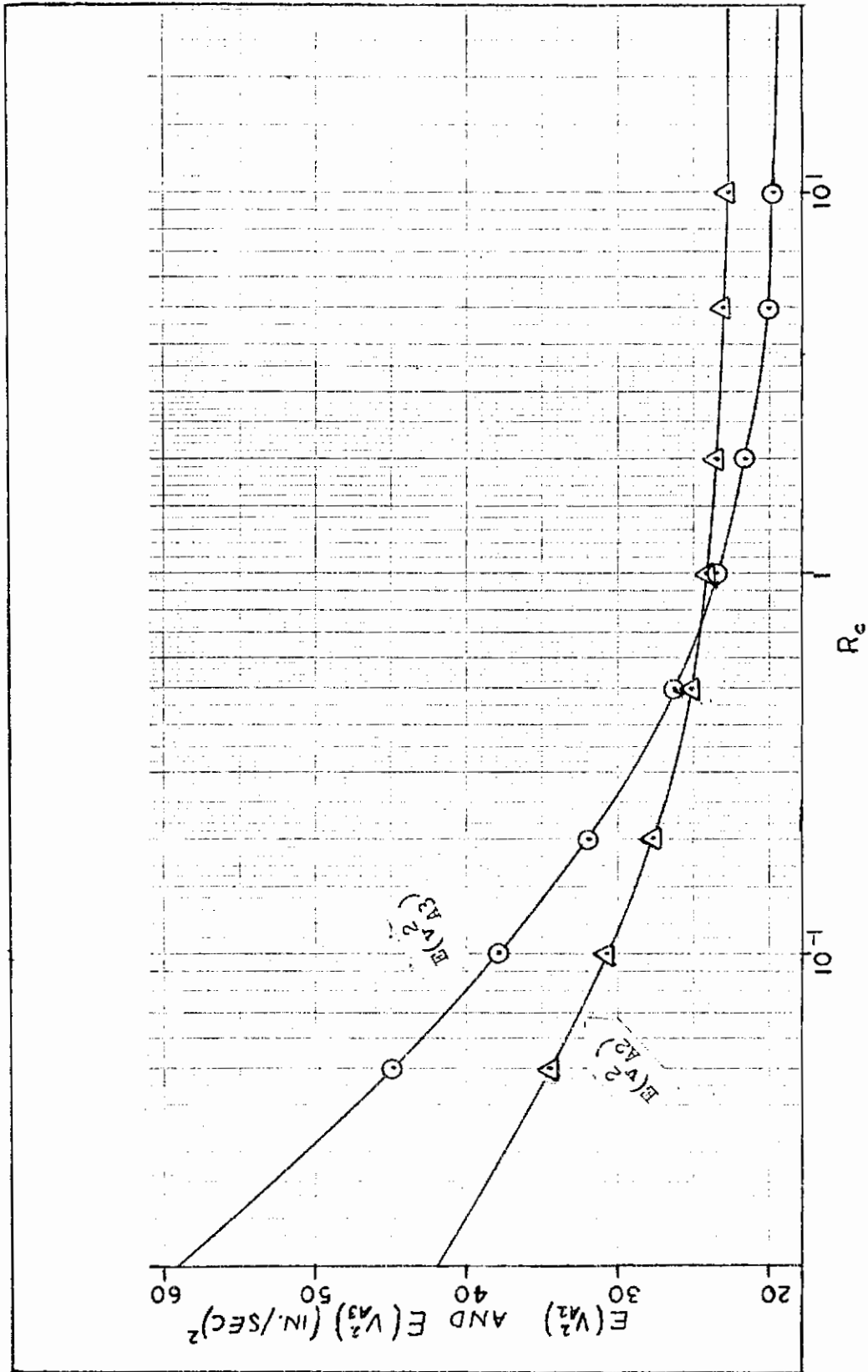


Figure 20. Plot of $E(v_{A2}^2)$ and $E(v_{A3}^2)$ vs R_e , Case I (Tricycle Gear Model)

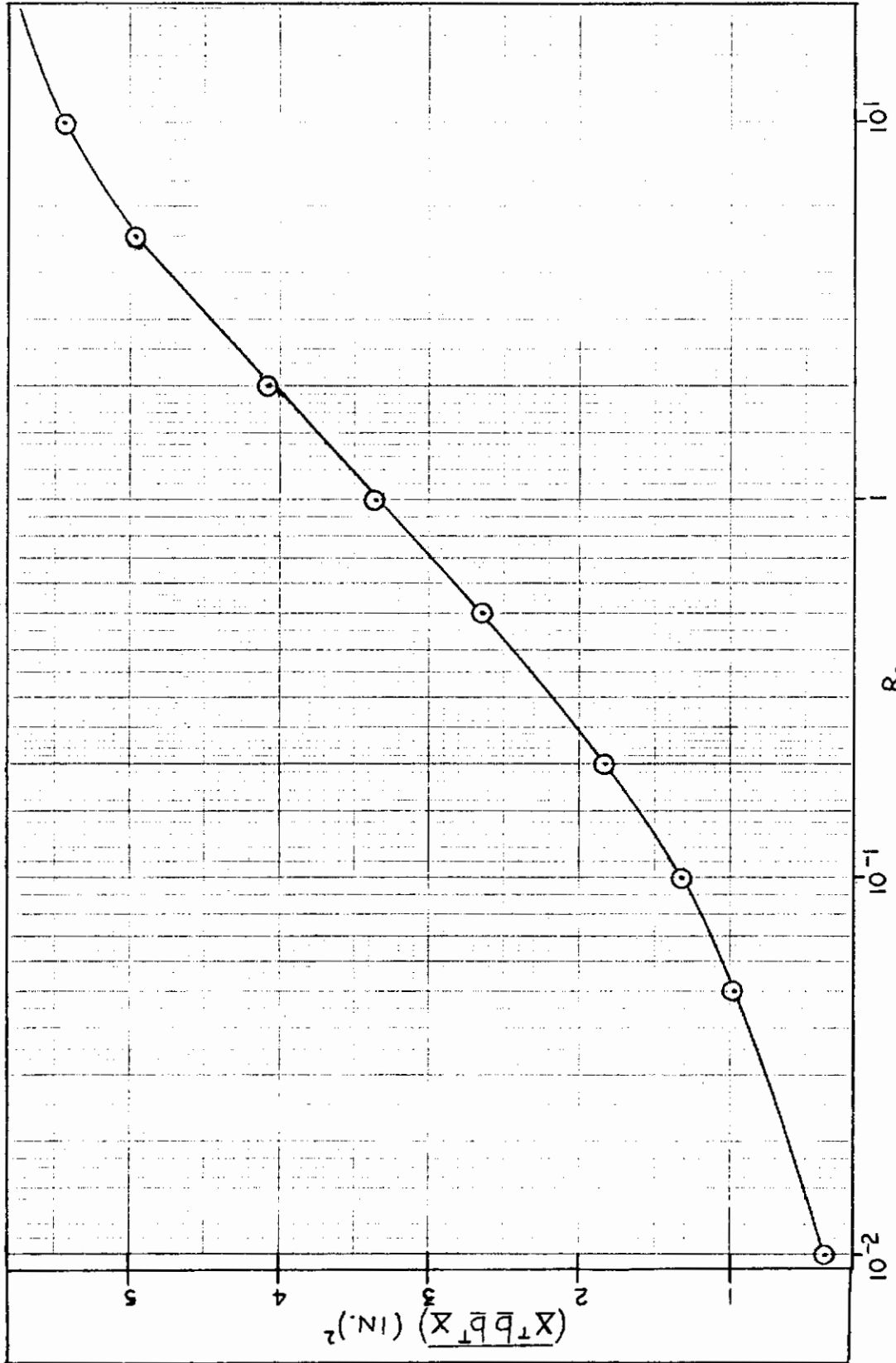


Figure 21. Plot of $\frac{x^2 + b^2}{1}$, Case II (Tricycle Gear Model)

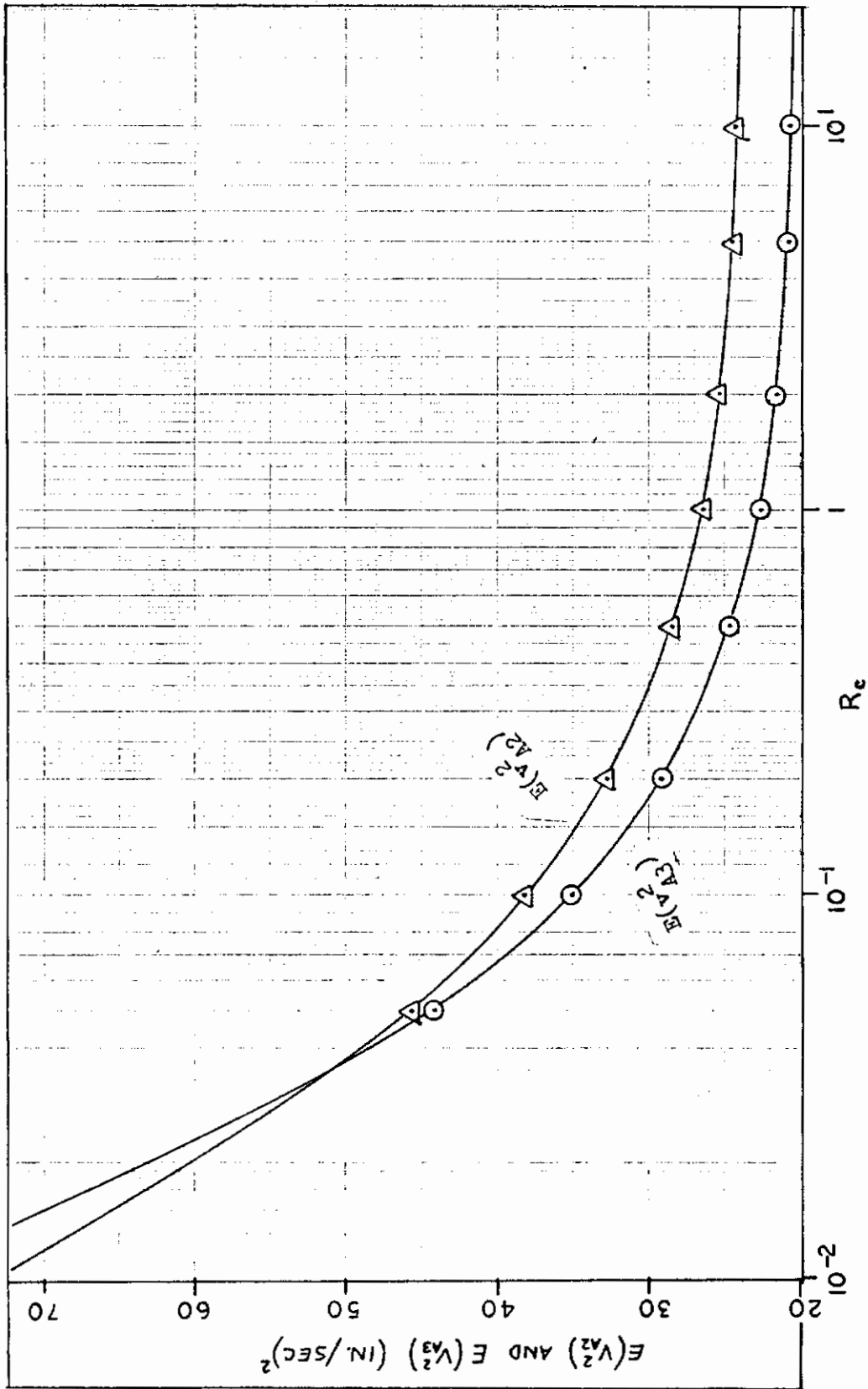


Figure 22. Plot of $E(v_{A2}^2)$ and $E(v_{A3}^2)$ vs R_c , Case II (Tricycle Gear Model)

APPENDIX E

RESULTS FOR THE FIVE LANDING GEAR MODEL

This appendix gives the results obtained for the five landing gear system model described in Chapter V. The state equations for the system are given in Appendix B. The state vector is identified as

$$\begin{aligned} \underline{x}^T = & (Z_{s1}, Z_{s2}, Z_f, Z_{u1}, Z_{u2}, Z_{u3}, Z_{u4}, Z_{u5}, \\ & \theta, \phi, V_{s1}, V_{s2}, V_f, V_{u1}, V_{u2}, V_{u3}, V_{u4}, V_{u5}, \\ & V_\theta, V_\phi, F_{A1}, F_{A2}, F_{A3}, F_{A4}, F_{A5}, h_1, \\ & h_2^*, h_2, h_3, h_4^*, h_4, h_5) \end{aligned} \quad (111)$$

and the system matrix F is readily identified as a 32x32 matrix. The control vector \underline{u} is identified as

$$\underline{u}^T = [u_1 \ u_2 \ u_3 \ u_4 \ u_5] \quad (135)$$

Only one feedback law was used for this system. It is equivalent to Feedback Law I for the single landing gear system (see Appendix C) and consisted of measuring the relative displacement between M_{s1} and M_f , the relative displacement between M_{s2} and M_f , and the acceleration of M_f . Thus, the measurement vector is identified as

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad (136)$$

and

$$y_1 = -Z_{s1} + Z_f + e_1 - L_{w1} \phi$$

$$y_2 = -Z_{s2} + Z_f + e_2 \theta + L_{w2} \phi$$

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$$\begin{aligned}
 y_3 &= \dot{V}_f \\
 &= \frac{K_{s1}}{M_f} Z_{s1} + \frac{K_{s2}}{M_f} Z_{s2} - \frac{(K_{a1} + K_{a2} + K_{a3} + K_{a4} + K_{a5} + K_{s1} + K_{s2})}{M_f} Z_f \\
 &+ \frac{K_{a1}}{M_f} Z_{u1} + \frac{K_{a2}}{M_f} Z_{u2} + \frac{K_{a3}}{M_f} Z_{u3} + \frac{K_{a4}}{M_f} Z_{u4} + \frac{K_{a5}}{M_f} Z_{u5} \\
 &+ \frac{(K_{a1}L_{x1} + K_{a2}L_{x2} - K_{a3}L_{x3} + K_{a4}L_{x4} - K_{a5}L_{x5} - K_{s1}e_1 - K_{s2}e_2)}{M_f} \theta \\
 &+ \frac{(K_{a2}L_{y2} + K_{a3}L_{y3} - K_{a4}L_{y4} - K_{a5}L_{y5} + K_{s1}L_{w1} - K_{s2}L_{w2})}{M_f} \phi \\
 &+ \frac{C_{s1}}{M_f} V_{s1} + \frac{C_{s2}}{M_f} V_{s2} - \frac{(D_1 + D_2 + D_3 + D_4 + D_5 + C_{s1} + C_{s2} + C_{fg})}{M_f} V_f \\
 &+ \frac{D_1}{M_f} V_{u1} + \frac{D_2}{M_f} V_{u2} + \frac{D_3}{M_f} V_{u3} + \frac{D_4}{M_f} V_{u4} + \frac{D_5}{M_f} V_{u5} \\
 &+ \frac{(D_1L_{x1} + D_2L_{x2} - D_3L_{x3} + D_4L_{x4} - D_5L_{x5} - C_{s1}e_1 - C_{s2}e_2)}{M_f} V_\theta \\
 &+ \frac{(D_2L_{y2} + D_3L_{y3} - D_4L_{y4} - D_5L_{y5} + C_{s1}L_{w1} - C_{s2}L_{w2})}{M_f} V_\phi \\
 &- \frac{F_{A1}}{M_f} - \frac{F_{A2}}{M_f} - \frac{F_{A3}}{M_f} - \frac{F_{A4}}{M_f} - \frac{F_{A5}}{M_f}
 \end{aligned}
 \tag{137}$$

The control vector is then given by:

$$\underline{u} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \\ K_{41} & K_{42} & K_{43} \\ K_{51} & K_{52} & K_{53} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}
 \tag{138}$$

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The cost criterion used is given by Equation 76 and is restated as

$$J = \lim_{t \rightarrow \infty} E \left[\underline{x}^T \underline{b} \underline{b}^T \underline{x} + R_c \underline{u}^T \underline{u} \right] \quad (76)$$

where the sum of the relative displacements between the main mass and the two elastically connected masses (M_{s1} and M_{s2}) is identified as

$$\begin{aligned} \underline{b}^T \underline{x} = & (-Z_{s1} + Z_f + e_1 \theta - L_{w1} \phi) \\ & + (-Z_{s2} + Z_f + e_2 \theta + L_{w2} \phi) \end{aligned} \quad (139)$$

The parameters for the vehicle model were obtained from Reference 19: 183 and Reference 1:13, and are given here (for $V_H = 66\text{fps}$) as:

$$\begin{aligned} M_{s1} &= M_{s2} = 43.97 \text{ LB} - \text{SEC}^2 / \text{IN.} \\ M_f &= 244.04 \text{ LB} - \text{SEC}^2 / \text{IN.} \\ C_{s1} &= C_{s2} = 2.27 \text{ LB} - \text{SEC} / \text{IN.} \\ C_{sg1} &= C_{sg2} = 10.87 \text{ LB} - \text{SEC} / \text{IN.} \\ C_{fg} &= 60.4 \text{ LB} - \text{SEC} / \text{IN.} \\ K_{s1} &= K_{s2} = 2055.0 \text{ LB} / \text{IN.} \\ M_{u1} &= M_{u2} = M_{u3} = M_{u4} = M_{u5} = 1.68 \text{ LB} - \text{SEC}^2 / \text{IN.} \\ K_{a1} &= 1941.0 \text{ LB} / \text{IN.} \\ K_{t1} &= 12,632 \text{ LB} / \text{IN.} \\ K_{a2} &= K_{a3} = K_{a4} = K_{a5} = 14,170 \text{ LB} / \text{IN.} \\ K_{t2} &= K_{t3} = K_{t4} = K_{t5} = 8330.0 \text{ LB} / \text{IN.} \\ D_1 &= D_2 = D_3 = D_4 = D_5 = 185.0 \text{ LB} - \text{SEC} / \text{IN.} \\ I_{xx} &= 3.0527 \times 10^7 \text{ LB} - \text{SEC}^2 - \text{IN.} \\ I_{yy} &= 1.0195 \times 10^7 \text{ LB} - \text{SEC}^2 - \text{IN.} \\ L_{x1} &= 360.515 \text{ IN.} \\ L_{x2} &= L_{x4} = 6.015 \text{ IN.} \\ L_{x3} &= L_{x5} = 54.485 \text{ IN.} \end{aligned}$$

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$$L_{y2} = L_{y3} = L_{y4} = L_{y5} = 85.625 \text{ IN.}$$

$$e_1 = e_2 = 2.13220 \text{ IN.}$$

$$L_{w1} = L_{w2} = 87.5 \text{ IN.}$$

The parameter for the runway model were obtained in Chapter III, and are restated here (for $V_H=66\text{fps}$) as:

$$A_0 = 10^{-1} (\text{IN.}^2 / \text{RAD} / \text{FT})$$

$$\lambda_0 = 4.5 \times 10^3 \text{ FT}$$

$$\tau_{12} = \tau_{14} = 4.48 \times 10^{-1} \text{ SEC}$$

$$\tau_{23} = \tau_{45} = 7.64 \times 10^{-2} \text{ SEC}$$

$$\tau = 1.08 \times 10^{-1} \text{ SEC}$$

The parameters for the actuator model are given in Chapter IV, and are restated here as:

$$C_b = A_p = 0.96 (\text{IN.})^2$$

$$\frac{V}{K_B} = 2.0 \times 10^{-5} (\text{IN.}^5 / \text{LB})$$

$$L + C_p = 7.0 \times 10^{-4} (\text{IN.}^5 / \text{LB} - \text{SEC})$$

$$C_x = 4.0 (\text{IN.}^3 / \text{SEC} - \text{MA})$$

$$n_F = 1.0$$

Note that the same parameters were used for each actuator.

The optimization procedure described in Chapter VII was carried out for various values of R_c , and the results for the five landing gear system now follow.

TABLE XXII
RESULTS OF MINIMIZATION

Weighting R_c	<u>Stress per Wing</u> $\frac{1}{2}x^{\text{II}}bb^{\text{II}}x$ (in.) ²	Cost J(K*)
**	8.770	1.754×10^1
INFINITE	7.240	1.448×10^1
10.00	6.180	1.334×10^1
5.00	5.475	1.248×10^1
2.00	4.242	1.073×10^1
1.00	3.237	9.046
0.50	2.318	7.236
0.10	8.854×10^{-1}	3.569

** ACTUATORS REMOVED FROM ALL LANDING GEARS.

TABLE XXIII

FEEDBACK CONSTANTS FOR ACTUATOR IN NOSE GEAR

Weighting		u_1		
R_c	K_{11}^*	K_{12}^*	K_{13}^*	
INFINITE	0.0	0.0	0.0	0.0
10.00	-3.149×10^{-3}	-3.149×10^{-3}	-1.082×10^{-3}	
5.00	-3.936×10^{-3}	-3.936×10^{-3}	-1.975×10^{-3}	
2.00	1.678×10^{-3}	1.678×10^{-3}	-3.962×10^{-3}	
1.00	2.389×10^{-2}	2.389×10^{-2}	-5.982×10^{-3}	
0.50	8.540×10^{-2}	8.540×10^{-2}	-7.842×10^{-3}	
0.10	5.816×10^{-1}	5.816×10^{-1}	-6.178×10^{-3}	

TABLE XXIV

FEEDBACK CONSTANTS FOR ACTUATOR IN LEFT FRONT MAIN GEAR

Weighting		u_2		
R_c	K_{21}^*	K_{22}^*	K_{23}^*	
INFINITE	0.0	0.0	0.0	0.0
10.00	2.675×10^{-2}	2.806×10^{-2}	4.385×10^{-4}	
5.00	5.033×10^{-2}	5.293×10^{-2}	8.313×10^{-4}	
2.00	1.103×10^{-1}	1.167×10^{-1}	1.821×10^{-3}	
1.00	1.919×10^{-1}	2.043×10^{-1}	3.123×10^{-3}	
0.50	3.243×10^{-1}	3.478×10^{-1}	5.168×10^{-3}	
0.10	9.876×10^{-1}	1.068	1.526×10^{-2}	

TABLE XXV

FEEDBACK CONSTANTS FOR ACTUATOR IN LEFT REAR MAIN GEAR

Weighting		u_3		
R_c	K_{31}^*	K_{32}^*	K_{33}^*	
INFINITE	0.0	0.0	0.0	0.0
10.00	3.185×10^{-2}	3.316×10^{-2}	6.088×10^{-4}	
5.00	5.977×10^{-2}	6.238×10^{-2}	1.148×10^{-3}	
2.00	1.302×10^{-1}	1.366×10^{-1}	2.489×10^{-3}	
1.00	2.249×10^{-1}	2.373×10^{-1}	4.205×10^{-3}	
0.50	3.766×10^{-1}	4.001×10^{-1}	6.795×10^{-3}	
0.10	1.118	1.199	1.877×10^{-2}	

TABLE XXVI

FEEDBACK CONSTANTS FOR ACTUATOR IN RIGHT FRONT MAIN GEAR

Weighting		u_4		
R_c	K_{41}^*	K_{42}^*	K_{43}^*	
INFINITE	0.0	0.0	0.0	0.0
10.00	2.806×10^{-2}	2.675×10^{-2}	4.385×10^{-4}	
5.00	5.293×10^{-2}	5.033×10^{-2}	8.313×10^{-4}	
2.00	1.167×10^{-1}	1.103×10^{-1}	1.821×10^{-3}	
1.00	2.043×10^{-1}	1.919×10^{-1}	3.123×10^{-3}	
0.50	3.478×10^{-1}	3.243×10^{-1}	5.168×10^{-3}	
0.10	1.068	9.876×10^{-1}	1.526×10^{-2}	

TABLE XXVII

FEEDBACK CONSTANTS FOR ACTUATOR IN RIGHT REAR MAIN GEAR

Weighting		u_5		
R_c	K_{51}^*	K_{52}^*	K_{53}^*	
INFINITE	0.0	0.0	0.0	0.0
10.00	3.316×10^{-2}	3.185×10^{-2}	6.088×10^{-4}	
5.00	6.238×10^{-2}	5.977×10^{-2}	1.148×10^{-3}	
2.00	1.366×10^{-1}	1.302×10^{-1}	2.489×10^{-3}	
1.00	2.373×10^{-1}	2.249×10^{-1}	4.205×10^{-3}	
0.50	4.001×10^{-1}	3.766×10^{-1}	6.795×10^{-3}	
0.10	1.199	1.118	1.877×10^{-2}	

TABLE XXVIII
DEMANDS ON ACTUATOR IN NOSE GEAR

Weighting R_c	Signal $E(u_1^2)$ (ma) ²	Displacement $E(v_{A1}^2)$ (in.) ²	Velocity $E(\dot{v}_{A1}^2)$ (in./sec) ²	Force $E(F_{A1}^2)$ (lb) ²	Power $E(F_{A1} v_{A1})$ (lb-in./sec)
INFINITE	0.0	4.336×10^{-1}	235.0	6.773×10^7	-4.101×10^4
10.00	5.013×10^{-3}	4.548×10^{-1}	230.0	6.611×10^7	-4.151×10^4
5.00	1.610×10^{-2}	4.723×10^{-1}	225.9	6.479×10^7	-4.188×10^4
2.00	6.410×10^{-2}	5.085×10^{-1}	216.7	6.201×10^7	-4.269×10^4
1.00	1.601×10^{-1}	5.384×10^{-1}	206.3	5.964×10^7	-4.357×10^4
0.50	3.558×10^{-1}	5.547×10^{-1}	193.9	5.835×10^7	-4.461×10^4
0.10	1.518	7.340×10^{-1}	165.6	6.318×10^7	-4.743×10^4

TABLE XXIX
DEMANDS ON ACTUATOR IN EITHER FRONT MAIN GEAR

Weighting R_c	Signal	Displacement	Velocity	Force	Power
	$E(u_2^2)$ or $E(u_4^2)$ (ma)	$E(y_{A2}^2)$ or $E(y_{A4}^2)$ (in.) ²	$E(v_{A2}^2)$ or $E(v_{A4}^2)$ (in./sec)	$E(F_{A2}^2)$ or $E(F_{A4}^2)$ (lb) ²	$E(F_{A2}^v v_{A2}^4)$ or $E(F_{A4}^v v_{A4}^4)$ (lb-in./sec) ⁴
INFINITE	0.0	1.130×10	23.02	1.330×10^7	-1.024×10^4
10.00	1.911×10^{-2}	1.228×10^{-1}	23.16	1.352×10^7	-1.042×10^4
5.00	5.976×10^{-2}	1.318×10^{-1}	23.31	1.422×10^7	-1.052×10^4
2.00	2.209×10^{-1}	1.563×10^{-1}	23.74	1.722×10^7	-1.065×10^4
1.00	5.068×10^{-1}	1.913×10^{-1}	24.42	2.75×10^7	-1.070×10^4
0.50	1.029	2.505×10^{-1}	25.61	3.315×10^7	-1.073×10^4
0.10	3.591	5.421×10^{-1}	31.02	8.708×10^7	-1.085×10^4

TABLE XXX
DEMANDS ON ACTUATOR IN EITHER REAR MAIN GEAR

Weighting R_c	Signal $E(u_2)$ or $E(u_4)$ (ma)	Displacement $E(v_{A2})$ or $E(v_{A4})$ (in.) ²	Velocity $E(v_{A2})$ or $E(v_{A4})$ (in./sec)	Force $E(F_{A2})$ or $E(F_{A4})$ (lb) ²	Power $E(F_{A2} v_{A2})$ or $E(F_{A4} v_{A4})$ (lb-in./sec)
INFINITE	0.0	1.312×10^{-1}	19.73	1.454×10^7	-6.699×10^3
10.00	2.725×10^{-2}	1.445×10^{-1}	20.22	1.461×10^7	-6.622×10^3
5.00	8.474×10^{-2}	1.569×10^{-1}	20.67	1.537×10^7	-6.522×10^3
2.00	3.093×10^{-1}	1.899×10^{-1}	21.93	1.903×10^7	-6.172×10^3
1.00	6.991×10^{-1}	2.366×10^{-1}	23.73	2.605×10^7	-5.570×10^3
0.50	1.394	3.134×10^{-1}	26.70	3.944×10^7	-4.447×10^3
0.10	4.641	6.691×10^{-1}	38.95	1.090×10^8	1.330×10^3

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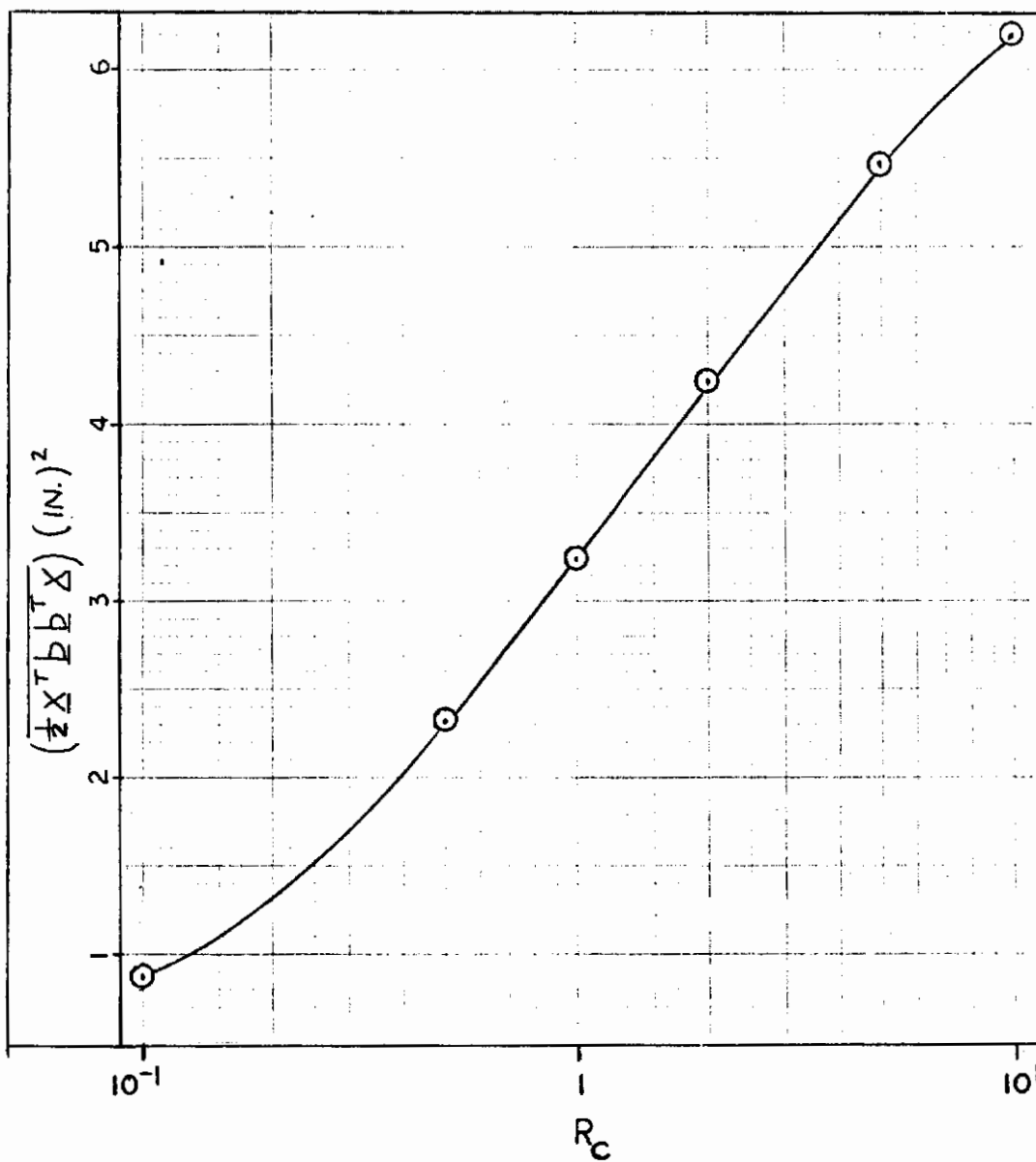


Figure 23. Plot of $\frac{1}{2} \underline{x}^T \underline{b} \underline{b}^T \underline{x}$ (Five Gear Model)

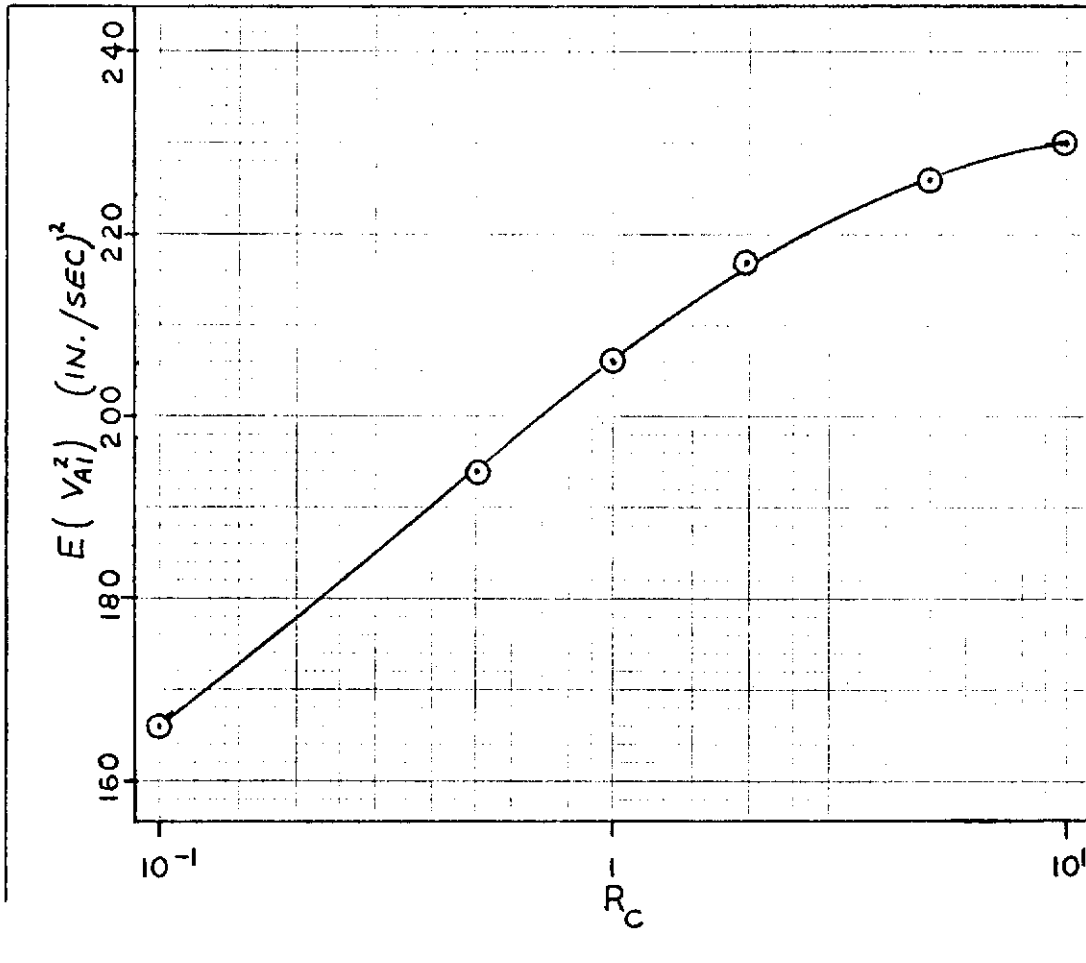


Figure 24. Plot of $E(v_{AI}^2)$ vs R_c (Five Gear Model)

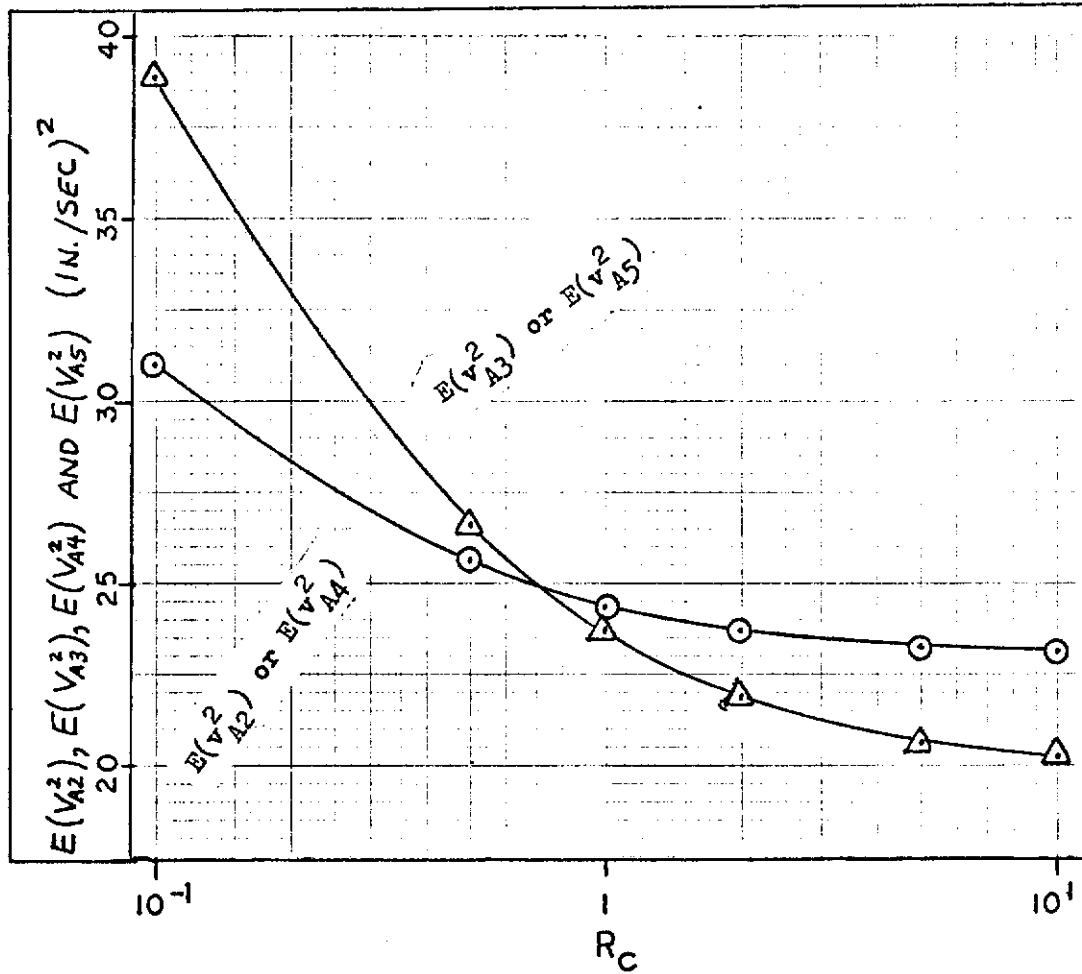


Figure 25. Plot of $E(v_{A2}^2)$, $E(v_{A3}^2)$, $E(v_{A4}^2)$, and $E(v_{A5}^2)$ vs R_c (Five Gear Model)

APPENDIX F

EXAMPLE APPLICATION OF TRADEOFF DIAGRAMS

In this appendix an example application of the tradeoff diagrams from Appendices C, D, and E is given. The actuator requirements are determined for a 30%, 50%, and 70% reduction in fatigue damage for each system model. The results of the one, three, and five landing gear systems are compared.

It is assumed that fatigue damage is directly proportional to root mean square stresses experienced by the wing which are again proportional to the root mean square relative displacement between the wing and the fuselage of the aircraft. With this assumption, the fatigue damage is reduced 30% by reducing the variance of the stress in each wing by 51%, the variance of the stress being $\underline{x}^T \underline{bb}^T \underline{x}$ in the one and three gear model and $1/2 \underline{x}^T \underline{bb}^T \underline{x}$ in the five gear model. Similarly the fatigue damage is reduced by 50% by reducing the variance of the stress in each wing by 75%, and the fatigue damage is reduced 70% by reducing the variance of the stress in each wing by 91%.

For the one landing gear model, Feedback Law I was considered. This case used a feedback law which was based upon the measured value of the relative displacement between M_s and M_f and the measured acceleration of M_f . These are also the sensed variables used to define the feedback law for the three and five gear models.

For the three landing gear model, Case I and Case II were considered. In Case I all three landing gears are actively controlled, including the nose gear. This corresponds with the five landing gear model where active control of the nose gear is also considered. In Case II, it was assumed that the nose gear is not actively controlled.

The results are tabulated in Tables XXXI, XXXII, and XXXIII. It is clear from the results that the one landing gear model is a poor approximation to an actual five gear system, particularly in those cases where large control authorities are considered feasible (i.e., large stress

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reductions are realized). The one gear example does provide trend data; and of course, as pointed out in the conclusions section, it does provide a simple case for evaluating different control schemes or feedback laws. The three landing gear model, Case I, yields almost identical results as the more complicated five landing gear model, indicating that it is a quite adequate approximation for the five gear model in an analysis of this type. The three landing gear model, Case II, shows the increased actuator requirements to achieve a comparable reduction in stress without active control of the nose gear.

TABLE XXXI
COMPARISON OF ACTUATOR REQUIREMENTS FOR 30% REDUCTION IN FATIGUE DAMAGE

Model	"Stress" per Wing Without Actuator (in) ²	"Stress" per Wing With Active Control (in) ²	R _c	Nose Gear E(vA1) ² (in/sec) ²	Front Main Gear E(vA2) ² (in/sec) ²	Rear Main Gear E(vA3) ² (in/sec) ²
One Landing Gear Feedback Law I	9.768	4.78	4.7	-	27.5	-
Three Landing Gear, Case I	8.838	4.32	2.0	224	23.5	21.6
Three Landing Gear, Case II	8.838	4.32	2.7	-	25.0	21.5
Five Landing Gear	8.77	4.3	2.2	218	23.6	21.7

TABLE XXXII
COMPARISON OF ACTUATOR REQUIREMENTS FOR 50% REDUCTION IN FATIGUE DAMAGE

Model	"Stress" per Wing Without Actuator (in) ²	"Stress" per Wing With Active Control (in) ²	R _c	Nose Gear E(vA1) ² (in/sec) ²	Front Main Gear E(vA2) ² (in/sec) ²	Rear Main Gear E(vA3) ² (in/sec) ²
One Landing Gear Feedback Law I	9.768	2.44	.44	-	39.	-
Three Landing Gear, Case I	8.838	2.21	4.1	199	25.6	27.5
Three Landing Gear, Case II	8.838	2.21	3.1	-	30.5	27.0
Five Landing Gear	8.77	2.14	.44	192	26.0	27.5

TABLE XXXIII
COMPARISON OF ACTUATOR REQUIREMENTS OF 70% REDUCTION IN FATIGUE DAMAGE

Model	"Stress" per Wing Without Actuator (in) ²	"Stress" per Wing With Active Control (in) ²	R _c	Nose Gear E(v _{A1}) ² (in/sec) ²	Front Main Gear E(v _{A2}) ² (in/sec) ²	Rear Main Gear E(v _{A3}) ² (in/sec) ²
One Landing Gear Feedback Law I	9.768	.88	.058	-	82.	-
Three Landing Gear, Case I	8.838	.795	.08	168	32.	40.
Three Landing Gear, Case II	8.838	.795	.033	-	51.5	51.5
Five Landing Gear	8.77	.79	.09	164	31.5	40.

Contrails

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13. ABSTRACT <p>The feasibility of using an active control in the landing gear system of an aircraft to reduce wing fatigue damage resulting from ground induced vibrations during taxiing is considered. The characteristics of three vehicle models are discussed: a single landing gear system, a tricycle landing gear system and a system of five landing gears. Mathematical expressions for the runway inputs to each vehicle model are obtained in the form of random inputs represented by Gauss-Markov processes. The model for a linear hydraulic actuator which is used as the active control element in the landing gear system is presented.</p> <p>The approach used in the study is to determine an optimal control law which is a proportional feedback of the measurements. The measurements, in turn, are assumed to be both a linear transformation of the states and noiseless. The feedback gains in the optimal control law are obtained in such a way as to minimize a cost criterion which is a measure of the controller's ability to reduce wing fatigue resulting from runway imposed vibrations. The methodology for obtaining the optimal solution for the given cost criterion is developed and solutions for the three different models and for various measurement schemes are obtained.</p> <p>The results indicate that the combined optimal active control and landing gear system can provide a substantial improvement in reducing wing fatigue over that of the landing gear alone. Also, the control parameters that are necessary and desirable in the optimal system, together with the physical demands placed on the actuator, are determined.</p>		

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