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ANALYTICAL STUDIES
OF
AIRCRAFT STRUCTURES EXPOSED TO TRANSIENT EXTERNAL HEATING

Volume I

Thermal Response of a "Thin" Plate Under the Influence of a Constant Temperature Edge

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FOREWORD

This report was prepared by A. Ambrosio and T. Ishimoto of the Department of Engineering, University of California, Los Angeles, under Contract No. AF 33(616)-293. The contract was initiated under Project No. 1350, "Effects of Atomic Weapons on Aircraft Systems," and was administered by the Aircraft Laboratory, Directorate of Laboratories, Wright Air Development Center, with Lt. Leonard C. Pincus acting as Project Engineer.

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ABSTRACT

This report is one of a series on analytical studies of aircraft structures exposed to transient external heating. The present study has been limited to skin regions between the underlying structures such as spars and stringers. By reducing the physical system to a simplified mathematical model, the conditions under which the temperature of the skin area is beyond the thermal influence of the spars and stringers were determined.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER:

Carl E Reichert

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CONTENTS

		Page
SECTION I.	INTRODUCTION	1
SECTION II.	MATHEMATICAL MODEL OF PHYSICAL SYSTEM	3
SECTION III.	ANALYSIS	9
SECTION IV.	RESULTS	15
SECTION V.	DISCUSSION OF RESULTS	23
SECTION VI.	CONCLUSIONS	25
BIBLIOGRAPHY		27
APPENDIX A.	SAMPLE USAGE OF RESULTS	29
APPENDIX B.	ISOLATED "THIN" PLATE	31

LIST OF FIGURES

Figure	Title	Page
1	A Physical System and Its Reduction to a Mathematical Model	4
2	The Transient Heating Function	5
3	Graphical Formulation of the Thermal Isolation Criterion	14
4	Thermal Isolation Criterion with a 5% Temperature Ratio	16
5	Thermal Isolation Criterion with a 10% Temperature Ratio	17
6	Thermal Isolation Criterion with a 15% Temperature Ratio	18
7	Thermal Isolation Criterion with a 20% Temperature Ratio	19
8	Thermal Isolation Criterion, Independent of Time	20
B-1	Maximum "Thin" Skin Temperature Rise	30
B-2	Time to Maximum "Thin" Skin Temperature Rise . .	31

NOMENCLATURE

a	Thermal diffusivity	ft^2/hr
A	Area	ft^2
b	Thickness	ft
C_p	Heat capacity	Btu/lb °F
h	Unit thermal conductance	Btu/lb ft^2 °F
k	Thermal conductivity	Btu/hr ft^2(°F/ft)
q	Heat rate per unit area	Btu/hr ft^2
Q	Total heat $\left(\int_0^\infty q(t)\,dt\right)$	Btu/ft^2
t	Time	sec or hr
T	Temperature	°F or °R
x	Distance	ft
I	Integral	dimensionless
γ	Density	lb/ft^3
Δ	Difference	dimensionless
η	Reference time	sec or hr
τ	Time ratio	dimensionless

Subscripts

c	Convective	m	Maximum
ref	Reference	∞	Ambient

Superscript

$+$ Dimensionless quantity

SECTION I

INTRODUCTION

The outer skin of an aircraft exposed to an external heat source undergoes a temperature rise that is dependent upon both the surface boundary values and the thermal influence of underlying structures such as spars and stringers. A previous temperature study[1*] has shown that certain areas of the outer skin may be considered to be thermally isolated from the attachments. The distance from these supporting structures at which the influence of the temperature of attachments is negligible in its effect on the skin temperature was partially found in an analysis of such a system. This system was analyzed for specified analytical heat source functions but did not include the effect of convective heat losses. The present study includes the effect of convective cooling.

From the present analysis of this system a criterion is established from which the skin area may be separated into two regions: (1) areas whose temperatures are within the influence of the temperatures of the underlying structures and, (2) areas whose temperatures are beyond the influence of the temperatures of the underlying structures. This division of skin areas simplifies considerably the thermal analysis of the physical system.

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[*] Numbers in superscripts indicate references in the bibliography at the end of the report.

SECTION II

MATHEMATICAL MODEL OF PHYSICAL SYSTEM

Physical System

A section of the type of aircraft structures which are under consideration in this report is illustrated in Figure 1. The upper surface of the skin is exposed to the interior conditions of the structure whereas the lower skin surface is exposed to both an external heat source and convective cooling. A complete analysis of this section was not attempted but was limited to areas of the skin between the underlying structures.

Heating Source. The heating function is represented in Figure 2 as a curve that increases very rapidly at the outset, reaches a peak value in the elapsed time, η, and then decreases more slowly. The time, η, is used as a reference time and is incorporated in the dimensionless parameter, τ, as well as in the dimensionless time variable, t^+.

Convective Cooling. Forced convective cooling resulting from the flow of fluid over a heated surface is denoted by the unit thermal conductance, h_c. Equations predicting these conductances over isothermal and non-isothermal surfaces are available.[2,3,4,5]

Mathematical Model. The mathematical model of the physical system illustrated in Figure 1 represents the system that is used to predict the behavior of the real system.

The three dimensional structure was reduced to a unidimensional one by postulating that heat flow in the spanwise direction is negligible and that the skin is "thin." The latter states that the temperature differences across the skin are negligible compared to the rise in skin temperature. The validity of this condition for a specific system is established by using the "thin" plate criterion.[1][*]

Other simplifications included the postulation of insulated interior surfaces (although free convection heat losses are present) and the absence of

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[*] In addition to the referenced report, a report containing a "thin-plate" criterion that includes convective cooling is in preparation.

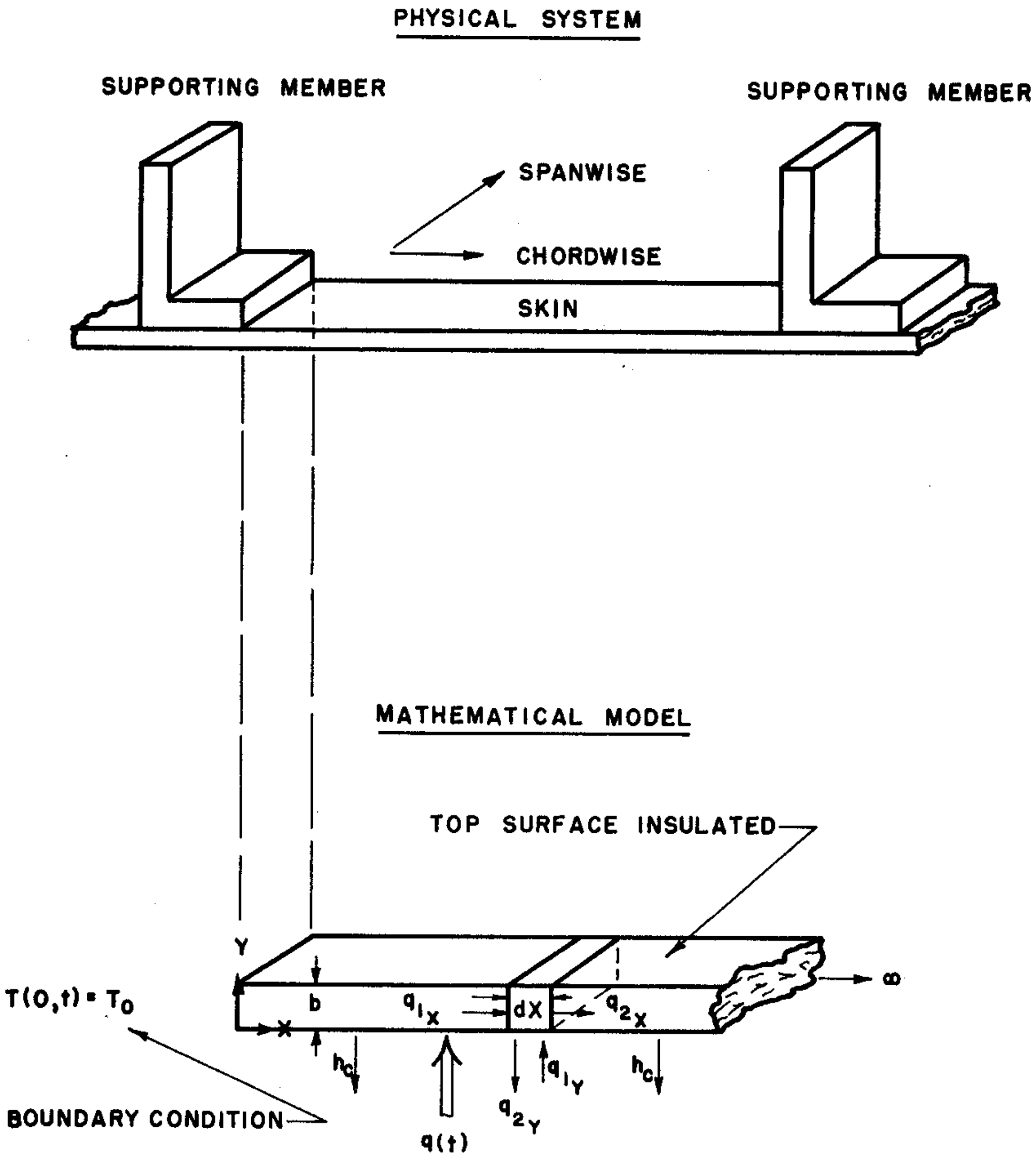

FIGURE 1 THE PHYSICAL SYSTEM AND ITS REDUCTION TO A MATHEMATICAL MODEL

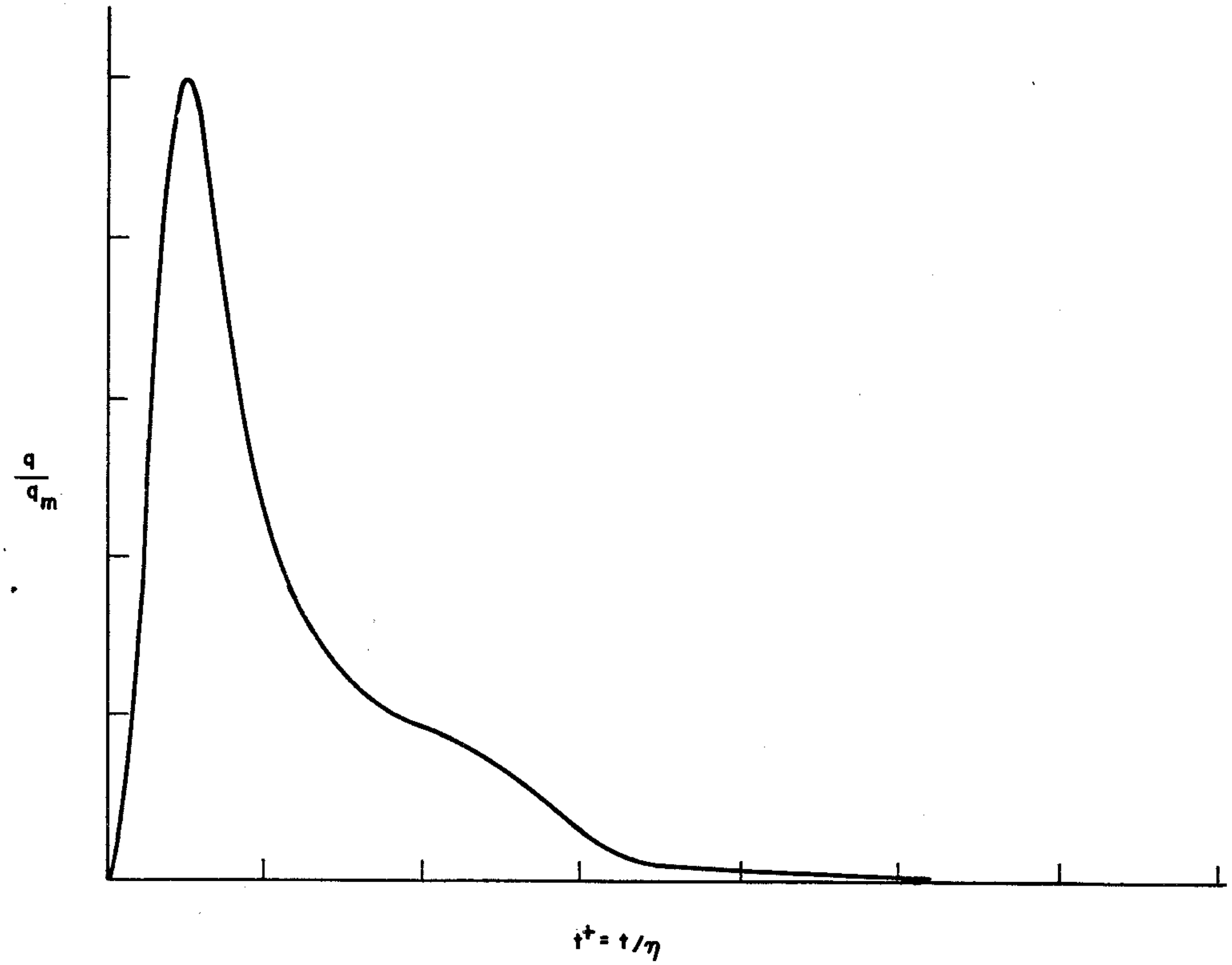

FIGURE 2 THE TRANSIENT HEATING FUNCTION

radiation from the surfaces to their surroundings. The study of the T-33[2] had shown that these free convection and radiation losses are small compared to forced convection losses.

In the physical system, the temperature of a stringer/or spar is transient, i.e., the temperature varies with time, however, this temperature is postulated to be constant and equal to the initial temperature of the structure in the mathematical model. The results from the analysis of the model with this postulation are conservative in that the largest temperature differences between the isolated and marginal regions are predicted.

In addition to the above simplifications, the following conditions are postulated:

1. The unit thermal conductance is constant and uniform along the chordwise direction. In reality the skin undergoes a non-uniform temperature rise resulting in non-uniform unit thermal conductances.
2. The material properties are independent of temperature.
3. The initial temperature is the ambient air temperature.
4. The skin is semi-infinite in extent.

A mathematical expression describing the system is determined by considering a heat balance on an element of length, *x*, in the skin (Figure 1).

In the x-direction

The heat flow **in** is:

$$q_{1_x} = -kA_x \frac{\partial}{\partial x}(T - T_\infty) \tag{1a}$$

The heat flow **out** is:

$$q_{2_x} = -kA_x \frac{\partial}{\partial x}\left[(T - T_\infty) + \frac{\partial}{\partial x}(T - T_\infty)\,\Delta x\right] \tag{1b}$$

In the y-direction

The heat flow **in** is (1c)

$$q_{1_y} = q(t)A_y$$

The heat flow out is:

$$q_{2y} = h_c (T - T_\infty) A_y \tag{1d}$$

The heat stored

$$= \gamma C_p \; A_x \frac{\partial}{\partial \cancel{x} t} (T - T_\infty) \tag{1e}$$

Equating the heat flow in to the sum of the heat flow out and heat stored,

$$A_y \, q(t) + k A_x \frac{\partial^2}{\partial x^2} (T - T_\infty) \Delta x = \gamma \, C_p A_x \Delta x \frac{\partial}{\partial t} (T - T_\infty) + h_c A_y (T - T_\infty) \tag{2}$$

But if a unit distance in the z-direction is considered,

$$A_x = b \qquad A_y = \Delta x$$

so that equation (2) becomes

$$q(t) + bk \frac{\partial^2 (T - T_\infty)}{\partial x^2} = \gamma \, C_p \, b \frac{\partial}{\partial t} (T - T_\infty) + h_c \, (T - T_\infty) \tag{3}$$

where:

$q(t)$ = heat flow (Btu/hr ft²)

t = time (hrs)

γ = density of slab (lb/ft³)

k = thermal conductivity [Btu/hr ft²(°F/ft)]

C_p = specific heat of slab (Btu/lb °F)

b = thickness of slab (ft)

T = temperature (°F)

T_∞ = ambient temperature (°F)

h_c = unit thermal conductance (Btu/hr ft² °F)

The boundary and initial conditions are:

$$\text{at} \quad x = 0, \quad T(0,t) = T_\infty \tag{4a}$$

$$\text{as} \quad x \to \infty, \quad T(\infty,t) \text{ remains finite} \tag{4b}$$

and when

$$t = 0, \quad T(x,0) = T_\infty \tag{4c}$$

Equations (3) and (4) describe the behavior of the simplified system consisting of the skin area between stringers.

SECTION III

ANALYSIS

The influence of the boundary condition (ambient temperature of the skin attached to stringer) on skin temperature can be determined by comparing the thermal response as predicted by equations (3) and (4) with the response of an isolated skin area (area beyond the thermal influence of the boundary condition). The region of skin whose thermal response is influenced by the boundary condition will be called the marginal region, and that beyond the influence of the boundary condition will be called the isolated region.

Analytical Method

General Expression (marginal region). By rearranging and manipulating the quantities in equation (3), the general temperature expression is expressible in a dimensionless form containing dimensionless parameters.

$$\frac{\partial^2 T^+}{\partial x^{+2}} = \frac{\partial T^+}{\partial t^+} + \tau T^+ - \frac{q^+(t^+)}{I} \tag{5}$$

where:

$T^+ = \Delta T/\Delta T_{ref} \quad (\Delta T = T - T_\infty)$

$x^+ = x/\sqrt{a\eta} \quad (a = k/\gamma C_p)$

$q^+ = q/q_m$

$t^+ = t/\eta$

$\tau = h_c \eta/\gamma C_p b$

$I = \int_0^\infty q^+(t^+)dt^+$

The maximum value of $q(t)$ is q_m, the elapsed time to the peak input value is η, and the reference temperature, ΔT_{ref}, is defined by the relationship,

$$\Delta T_{ref} = \frac{q_m \eta I}{\gamma C_p b} = \frac{Q}{\gamma C_p b} \quad \left(Q = \int_0^\infty q(t)dt \right)$$

and represents the maximum temperature rise in the slab if there were no heat loss. The integral, I, is a dimensionless quantity whose value is 2.14 for the heating function shown in Figure 2.

In dimensionless form, the boundary and initial conditions become:

$$\text{at} \quad x^+ = 0, \; T^+(0,t^+) = 0 \tag{6a}$$

$$\text{as} \quad x^+ \to \infty, \; T^+(\infty,t^+) \text{ remains finite} \tag{6b}$$

$$\text{and when} \quad t^+ = 0, \; T^+(x^+,0) = 0 \tag{6c}$$

Laplace transformation of the partial differential equation with respect to the dimensionless time variable, t^+, reduces the expression to an ordinary differential equation.

$$\frac{d^2T^+(x^+,s)}{dx^2} = sT^+(x^+,s) - T^+(x^+,0) + \tau T^+(x^+,s) - \frac{q^+(s)}{I} \tag{7}$$

Solving equation (7) for $T^+(x^+,s)$ results in

$$T^+(x^+,s) = C_1 e^{\sqrt{s+\tau}\, x^+} + C_2 e^{-\sqrt{s+\tau}\, x^+} + \frac{q^+(s)}{I(s+\tau)} \tag{8}$$

The constant C_1 and C_2 are evaluated by applying boundary conditions (6a) and (6b).

$C_1 = 0$, because $T^+(x^+,s)$ must remain finite as $x^+ \to \infty$

Therefore,

$$T^+(x^+,s) = C_2 e^{-\sqrt{s+\tau}\;x^+} + \frac{q^+(s)}{I(s+\tau)} \tag{9}$$

When $x^+ = 0$, $T^+(0,s) = 0$, so that

$$C_2 = -\frac{q^+(s)}{I(s+\tau)} \tag{10}$$

By substituting equation (10) into equation (9), the transformed expression for the temperature response becomes:

$$T^+(x^+,s) = \frac{q^+(s)}{I(s+\tau)} - \frac{q^+(s)e^{-\sqrt{s+\tau}\;x^+}}{I(s+\tau)} \tag{11}$$

The general temperature expression is found by determining the inverse transformation of equation (11). Because the transformed equation is a product of transforms, the inverse transformation can be represented as a convolution of functions. The inversion is denoted symbolically by the asterisk, * .

$$\mathcal{L}^{-1}[T^+(x^+,s)] = T^+(x^+,t^+) = F_1(t^+)*F_2(x^+,t^+) - F_1(t^+)*F_3\;(x^+,t^+) \tag{12}$$

where:

$$F_1(t^+) = \mathcal{L}^{-1}\left\{\frac{q^+(s)}{I}\right\} = \frac{q^+(t^+)}{I}$$

$$F_2(x^+,t^+) = \mathcal{L}^{-1}\left\{\frac{1}{s+\tau}\right\} = e^{-\tau t^+}$$

$$F_3(x^+,t^+) = e^{-\tau t^+}\mathcal{L}^{-1}\left\{\frac{e^{-\sqrt{s}\,x^+}}{s}\right\} = e^{-\tau t^+} erfc(x^+/\sqrt{4t^+}) \;.$$

Because the heating function is not readily represented by a mathematical expression, the general temperature expression is presented in the form of a Faltung (convolution) integral.

$$T^{+}(x^{+},t^{+}) = \int_{0}^{t^{+}} \frac{q^{+}(\beta)}{I} e^{-\tau(t^{+}-\beta)} d\beta - \int_{0}^{t^{+}} \frac{q^{+}(t^{+}-\beta)}{I} e^{-\tau\beta} \, erfc \, (x^{+}/\sqrt{4\beta}) \, d\beta \quad (13)$$

A more suitable expression is one that shows the temperature difference between the isolated region and the marginal region. The temperature response of the isolated region can be found by solving the ordinary differential equation describing the system or it can be reduced from the general temperature expression represented by equation (13). In the latter method, by letting the dimensionless variable, x^{+}, approach infinity, the complementary error function approaches zero, and in the limit the second integral of equation (13) becomes zero. The expression for the isolated skin area then becomes:

$$T^{+}(\infty,t^{+}) = \int_{0}^{t^{+}} \frac{q^{+}(\beta)}{I} e^{-\tau(t^{+}-\beta)} d\beta \quad (14)$$

The temperature difference is found by subtracting equation (13) from equation (14).

$$T^{+}(\infty,t^{+}) - T^{+}(x^{+},t^{+}) = \int_{0}^{t^{+}} \frac{q^{+}(t^{+}-\beta)}{I} e^{-\tau\beta} \, erfc \, (x^{+}/\sqrt{4\beta}) \, d\beta \quad (15)$$

Description of Solution and Method of Evaluation. The expression as represented by equation (15) indicates the temperature difference in terms of the reference temperature that denotes the temperature rise of the isolated skin with no heat loss.

$$T^{+}(\infty,t^{+}) - T^{+}(x^{+},t^{+}) \equiv \frac{\Delta T(\infty,t^{+}) - \Delta T(x^{+},t^{+})}{\Delta T_{ref}} \quad (16)$$

A more suitable equation is one that compares the temperature difference with the temperature rise of the isolated area with heat loss.

$$\frac{T^{+}(\infty,t^{\tau}) - T^{+}(x^{+},t^{+})}{T^{+}(\infty,t^{+})} = \frac{\int_{0}^{t^{+}} \frac{q^{+}(t^{+}-\beta)}{I} e^{-\tau\beta} \, erfc \, (x^{+}/\sqrt{4\beta}) d\beta}{\int_{0}^{t^{+}} \frac{q^{+}(\beta)}{I} e^{-\tau(t^{+}-\beta)} d\beta} \quad (17)$$

However, this expression has its limitations because a small temperature difference that may be relatively unimportant appears significant if it is compared with the corresponding temperature rise of the isolated area. To obviate this limitation, the temperature differences are compared with the maximum temperature rise of the isolated area with heat loss.

$$\frac{\Delta T^+}{T_m^+} \equiv \frac{T^+(\infty, t^+) - T^+(x^+, t^+\}}{T_m^+(\infty, t^+)} = \frac{\int_0^{t^+} \frac{q^+(t^+-\beta)}{I} e^{-\tau\beta} \, erfc\,(x^+/\sqrt{4\beta}\,)\, d\beta}{T_m^+(\infty, t^+)} \qquad (18)$$

The integral is evaluated numerically because the heating function is not readily expressible as a mathematical function. For a given $x^+$and τ, the area under the curve representing the integrand (equation (18)) is approximated by a summation of rectangles. The accuracy of the approximation increases with a decrease in interval and an interval of $t^+ = 0.10$ was found to give sufficiently accurate results for the range of τ considered in this study.

Thermal Isolation Criterion of a "Thin" Skin. The results obtained numerically are presented in a graphical form in terms of τ, x^+, and t^+ with $\Delta T^+/T_m^+$ as a constant. These graphs were obtained by several cross-plots two of which are illustrated in Figures (3a) and (3b).

For a specified temperature difference, a family of curves or functions defined by the equation of the form $F(x^+, t^+, \tau) = 0$ is obtained. This family of functions possesses an envelope that represents the limiting conditions related to the fixed temperature difference. That is, for the range of the time variable considered, conditions above the envelope represent lower temperature differences than that specified for this family of functions.

Because the results as presented in a form illustrated in (3b) is used to predict the distance, x, at which a skin region is thermally isolated from the underlying structures, the results are termed a **Thermal Isolation Criterion.**

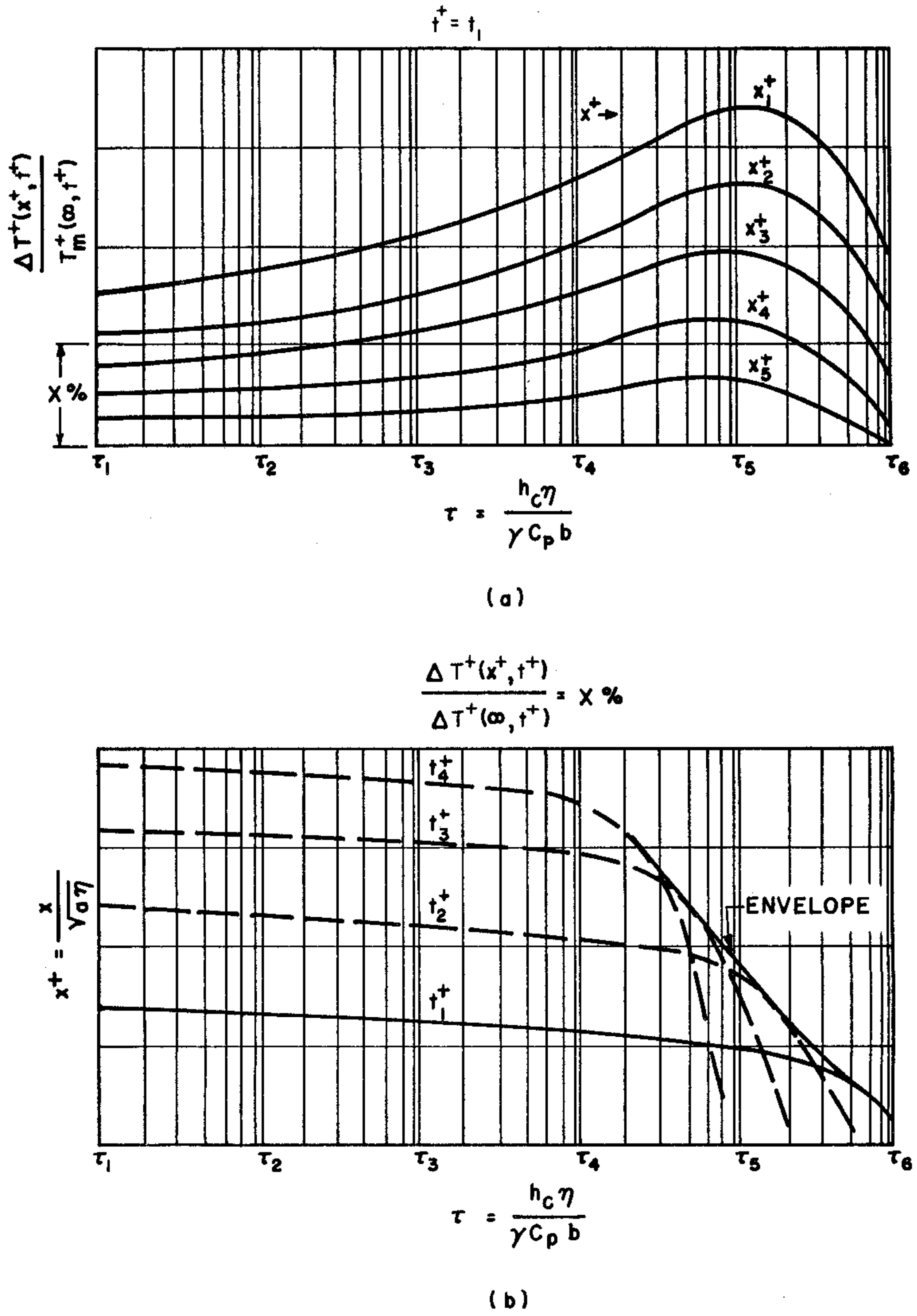

FIGURE 3 GRAPHICAL FORMULATION OF THE THERMAL ISOLATION CRITERION

SECTION IV

RESULTS

The mathematical analysis presented in Section III resulted in a family of numerically and graphically evaluated functions for each specified temperature difference. These functions of the form $F(x^+, t^+, \tau) = 0$ are graphically presented in Figures 4 through 7 by expressing x^+as a function of τ with t^+ as a parameter. Each figure contains one family of curves defined by a specified temperature difference, $\Delta T^+/T_m^+$. The specified temperature differences and corresponding figure numbers are listed below.

$\Delta T^+/T_m^+$	Figure Number
5%	4
10%	5
15%	6
20%	7

Each of the family of curves possesses an envelope that represents the limiting condition with respect to range of times, t^+, used for that family of curves. That is for values of x^+ and τ which lie above the curve of the envelope, the temperature difference is always less than the temperature difference specified. Envelopes for each family of curves shown in Figures 4 through 7 are presented in Figure 8.

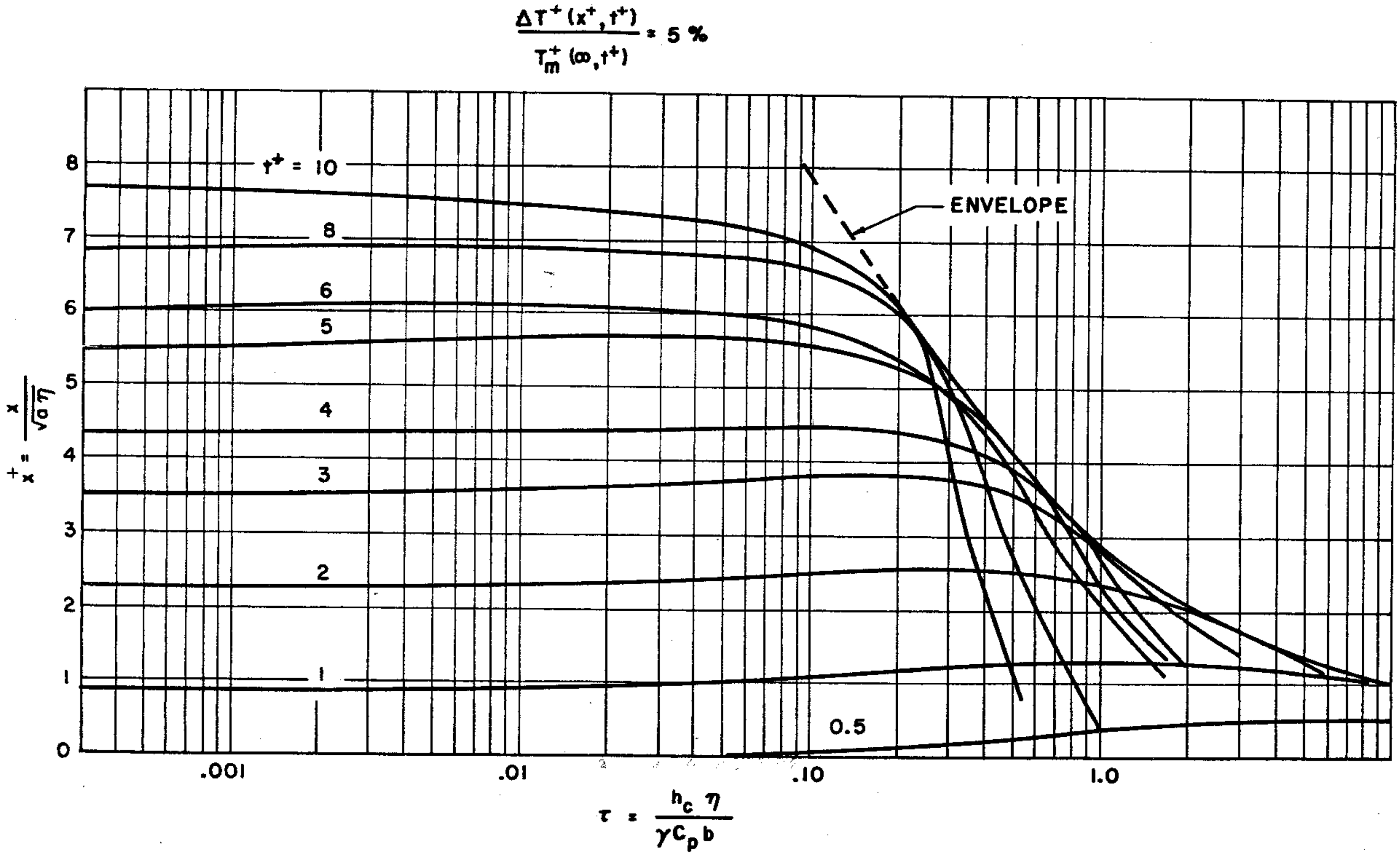

FIGURE 4 THERMAL ISOLATION CRITERION WITH A TEMPERATURE RATIO OF 5%

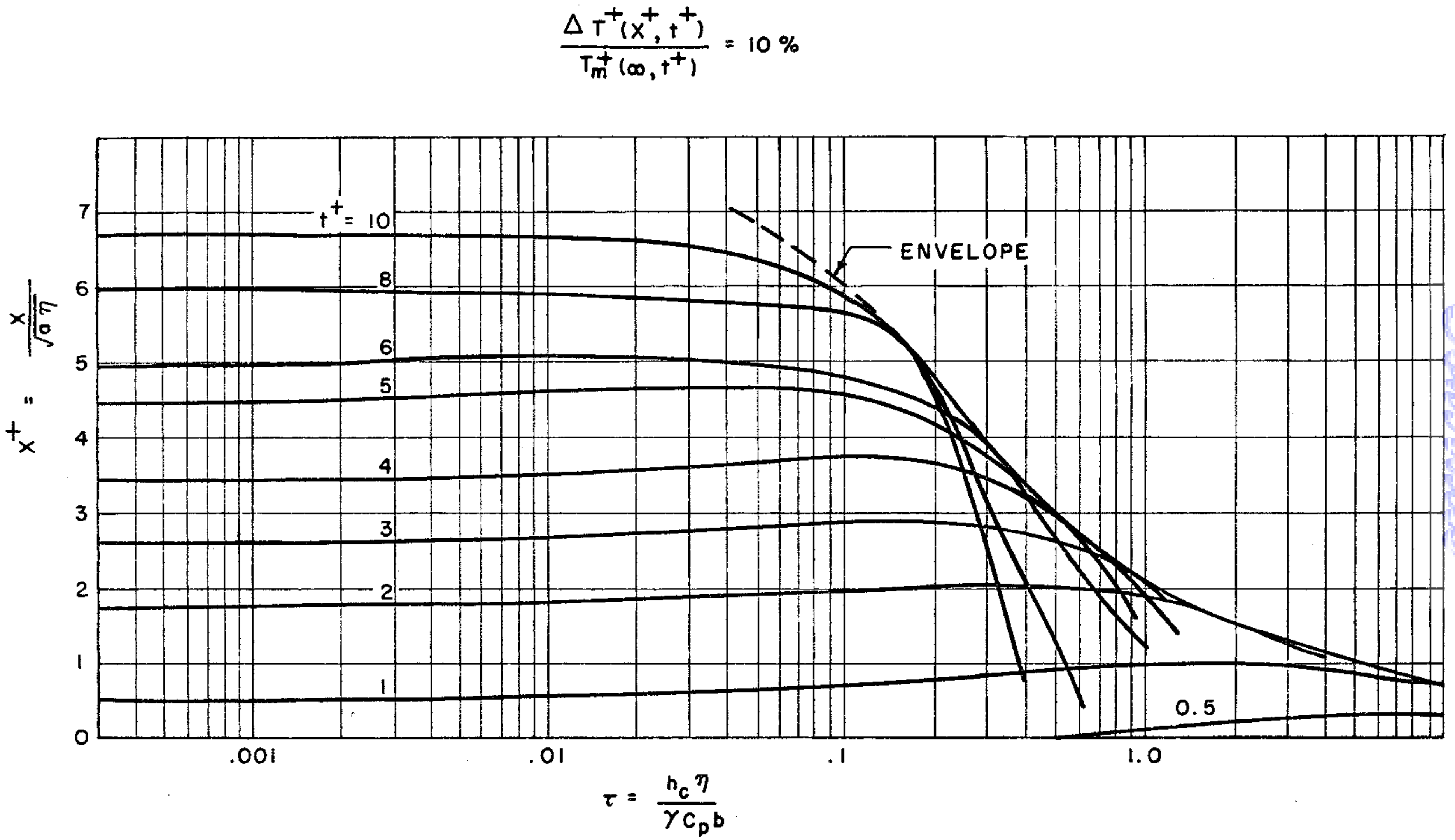

FIGURE 5 THERMAL ISOLATION CRITERION WITH A TEMPERATURE RATIO OF 10%

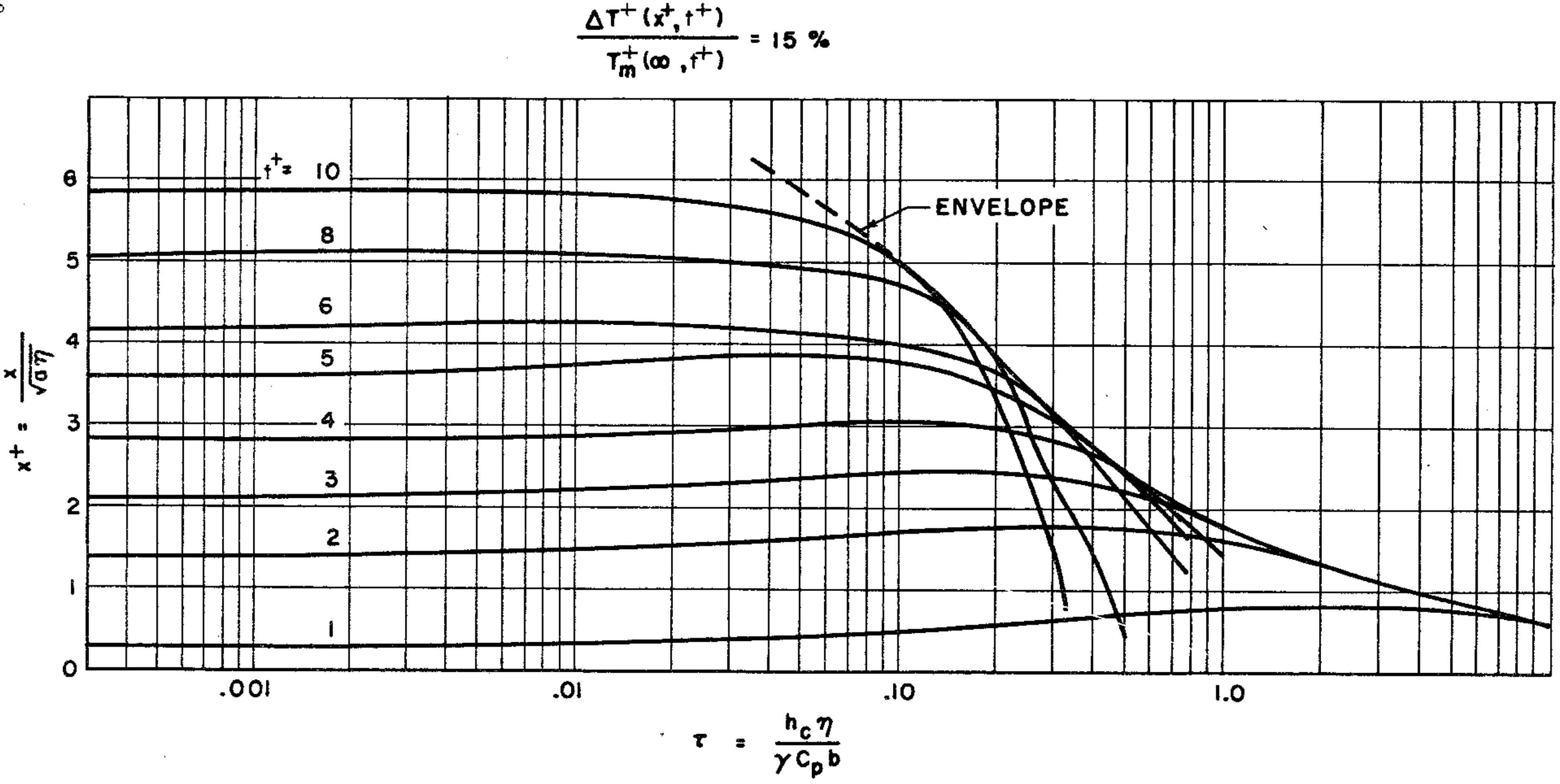

FIGURE 6 THERMAL ISOLATION CRITERION WITH A TEMPERATURE RATIO OF 15 %

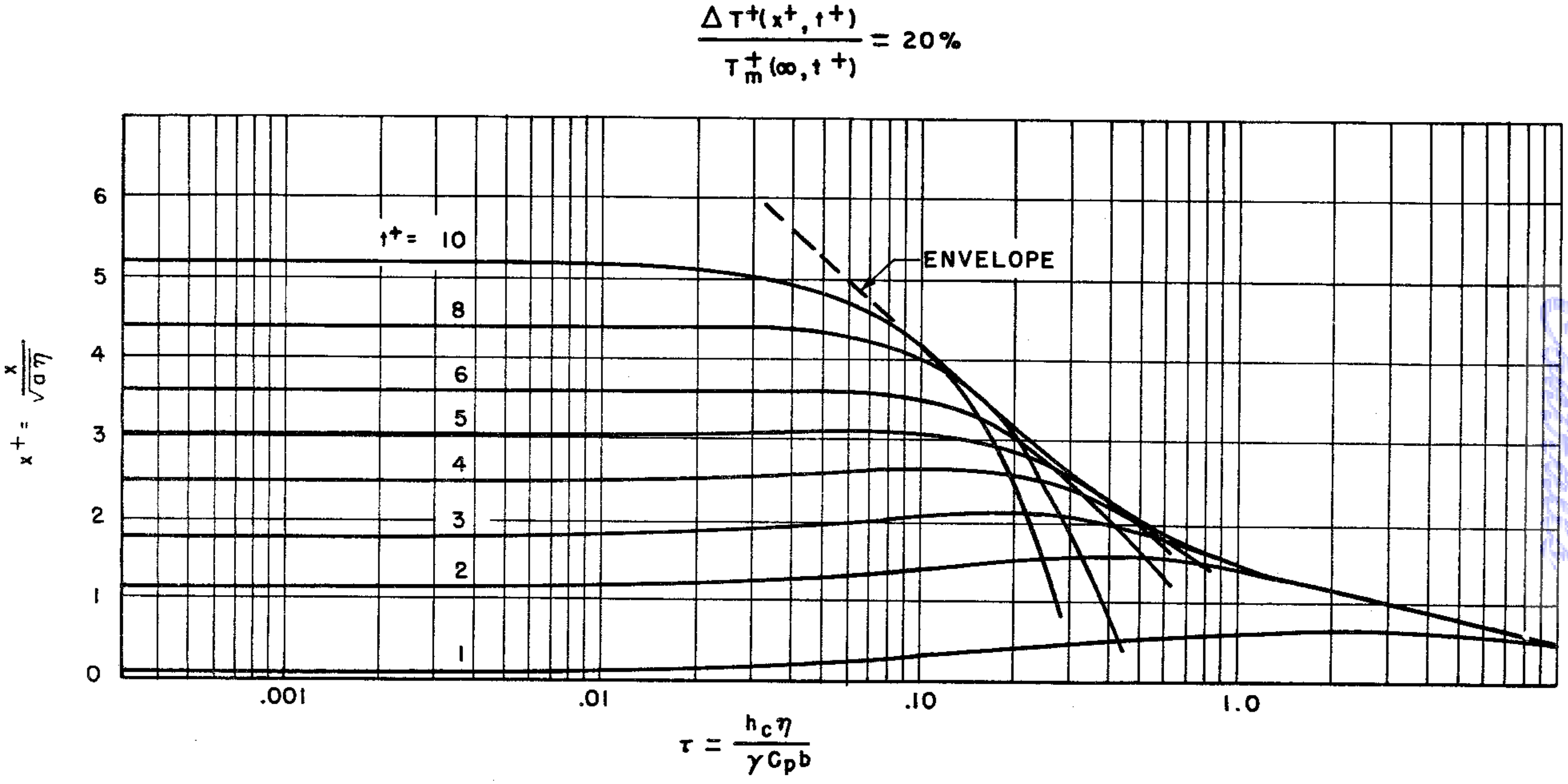

FIGURE 7 THERMAL ISOLATION CRITERION WITH A TEMPERATURE RATIO OF 20%

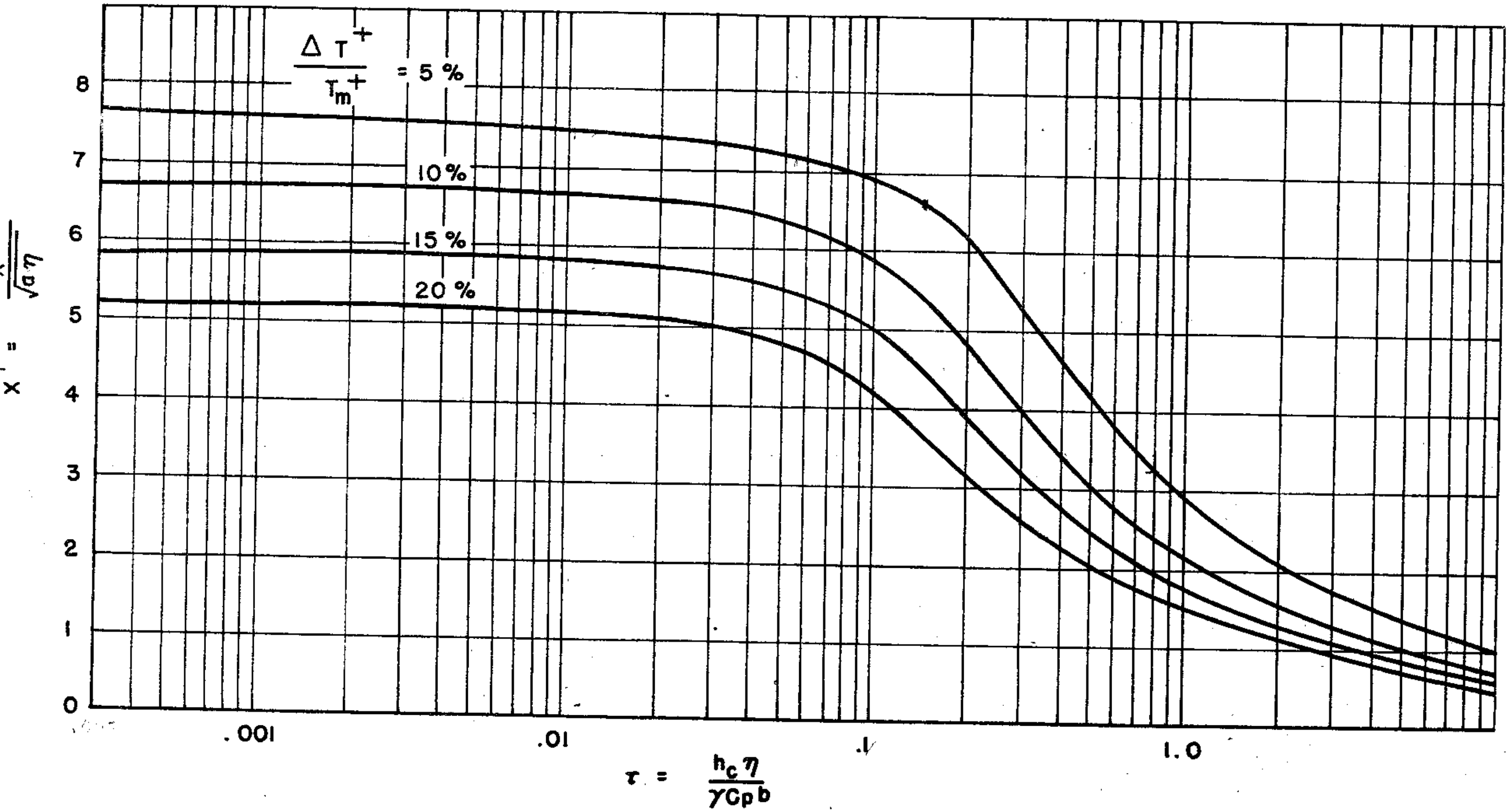

FIGURE 8 THERMAL ISOLATION CRITERION, INDEPENDENT OF TIME

SECTION V

DISCUSSION OF RESULTS

The curves show the conditions under which a "thin" skin can be considered to be beyond the thermal influence of underlying structures. By specifying a temperature difference, an elapsed time, and the dimensionless quantity, τ, the conditions for an isolated skin area are established.

From the graphs of the present investigation a criterion for predicting the isolation of a skin area not subject to convective heat losses can be compared to a criterion subject to convective cooling. For τ less than 0.10, curves of Figures 4-7 are essentially independent of τ. This implies that convective heat losses have little or no effect on the "edge" criterion within this interval because the conditions of no heat loss are approximated by the smallest τ.

It is apparent from the curves that for $\tau \leq 0.10$, a larger t^+ is associated with a larger x^+, which correspondingly means a smaller isolated area. At larger τ's the curves decrease more rapidly with a longer elapsed time, such that a longer t^+ is associated with a smaller x^+ (a larger isolated area).

SECTION VI

CONCLUSIONS

1. The Thermal Isolation Criterion is established by specifying the elapsed time, the temperature difference, and the boundary conditions contained in the dimensionless parameter, τ.
2. For τ less than 0.10, the difference in the Thermal Isolation Criterion with and without convective cooling is small.
3. For the prediction of a minimum isolated area, a larger τ results in a larger isolated area.

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APPENDIX A

SAMPLE USAGE OF RESULTS

The curves in Figures 4-8 may be used to determine aircraft skin areas which are thermally isolated from the remainder of the structure and those areas which are within the thermal influence of the underlying structure.

The procedure for determining the separation point of these regions is outlined as follows:

1. Specify a temperature ratio $(\Delta T^+/T_m^+)$.
2. Evaluate the dimensionless quantity, $\tau = h_c \eta/\gamma C_p b$.
3. Determine from the envelopes (Figure 8) the dimensionless parameter, x^+, corresponding to the calculated, τ, and the specified temperature ratio.
4. Calculate x from the relation,

$$x^+ = x/(a\eta)^{\frac{1}{2}} \quad , \text{ where } a = k/\gamma C_p$$

Because this may result in a very conservative prediction of the extent of the isolated area, a more realistic x can be found by specifying the elapsed time. These elapsed times are usually specified under conditions of maximum temperature differences between the isolated area and the boundary condition that is postulated to be the ambient temperature. The elapsed times at maximum temperature differences are approximated with elapsed times to the peak temperature rises of the isolated skin. A plot of peak time as a function of the dimensionless parameter, τ, is presented in Figure B-2.

Example

1. Physical System
   a. Material (24S-T)
      Density (γ) = 173 lb/ft^3
      Heat Capacity (C_p) = 0.23 Btu/lb °F
      Thermal Conductivity (k) = 66.7 Btu/hr ft^2 °F

b. Boundary Conditions

Heat Loss (h_c) = 25 Btu/hr ft^2 °F (specified)

(true air speed - 600 ft/sec

altitude - 40,000 ft)

Reference Time (η) = 3 sec

c. Skin Thickness (b) = 1/16 in.

2. Evaluation of x (Elapsed time not specified)

a. Temperature Ratio ($\Delta T^+/T_m^+$) = 5% (specified)

b. Dimensionless Parameter (τ) = $h_c\,\eta/\gamma\, C_p\, b$

$= 0.10$

c. The value of x^+ corresponding to $\Delta T^+/T_m^+ =$ 5% and $\tau = 0.10$ is 6.9 (Figure 8).

d. The distance, x, is then:

$$x = x^+(a\eta)^{\frac{1}{2}} = 6.9(.0374) = 0.258 \text{ ft.}$$
$$= 3.1 \text{ inches}$$

3. Evaluation of x (elapsed time specified)

a. Temperature ratio ($\Delta T^+/T_m^+$) = 5% (specified)

b. Dimensionless parameter (τ) = 0.10

c. The elapsed time corresponding to the maximum temperature rise of the isolated area for $\tau = 0.10$ is:

$$t^+_{T_m^+} = 4.9 \text{ (Figure B-2)}$$

d. The value of x^+ corresponding to $\Delta T^+/T_m^+ =$ 5% and $t^+ = 4.9$ is 5.6 (Figure 4)

e. The distance, x, then becomes:

$$x = 5.6\ (0.0374) = 0.209 \text{ ft}$$
$$= 2.51 \text{ inches}$$

Thus, for the specified physical system and the imposed boundary conditions, skin areas 3.1 inches or farther from underlying structures may be considered as thermally isolated areas at all times. That is, these areas will always have temperatures within 5% of the maximum temperature attained by an area physically isolated from any other structure. Areas 2.51 inches or farther away from underlying structures may be considered as thermally isolated areas during the time interval from the start of the thermal input until maximum skin temperature obtains.

APPENDIX B

ISOLATED "THIN" PLATE

An expression predicting the temperature response of isolated skin area was found in Section I. This expression,

$$T^{+}(\infty, t^{+}) = \int_{0}^{t^{+}} \frac{q^{+}(\beta)}{I} e^{-\tau(t^{+}-\beta)} d\beta \qquad \textbf{(B-1)}$$

shows that the temperature is a function of the dimensionless time, t^{+}, and the dimensionless time ratio, τ. A series of curves, $T^{+}(\infty, t^{+})$ vs t^{+} with τ as a parameter result from integration of equation (B-1).

Because only the conditions at maximum temperature rise are pertinent to this investigation, plots of $T_{m}^{+}(\infty, t^{+})$ vs τ and $t_{T_{m}^{+}}^{+}$ vs τ are presented.

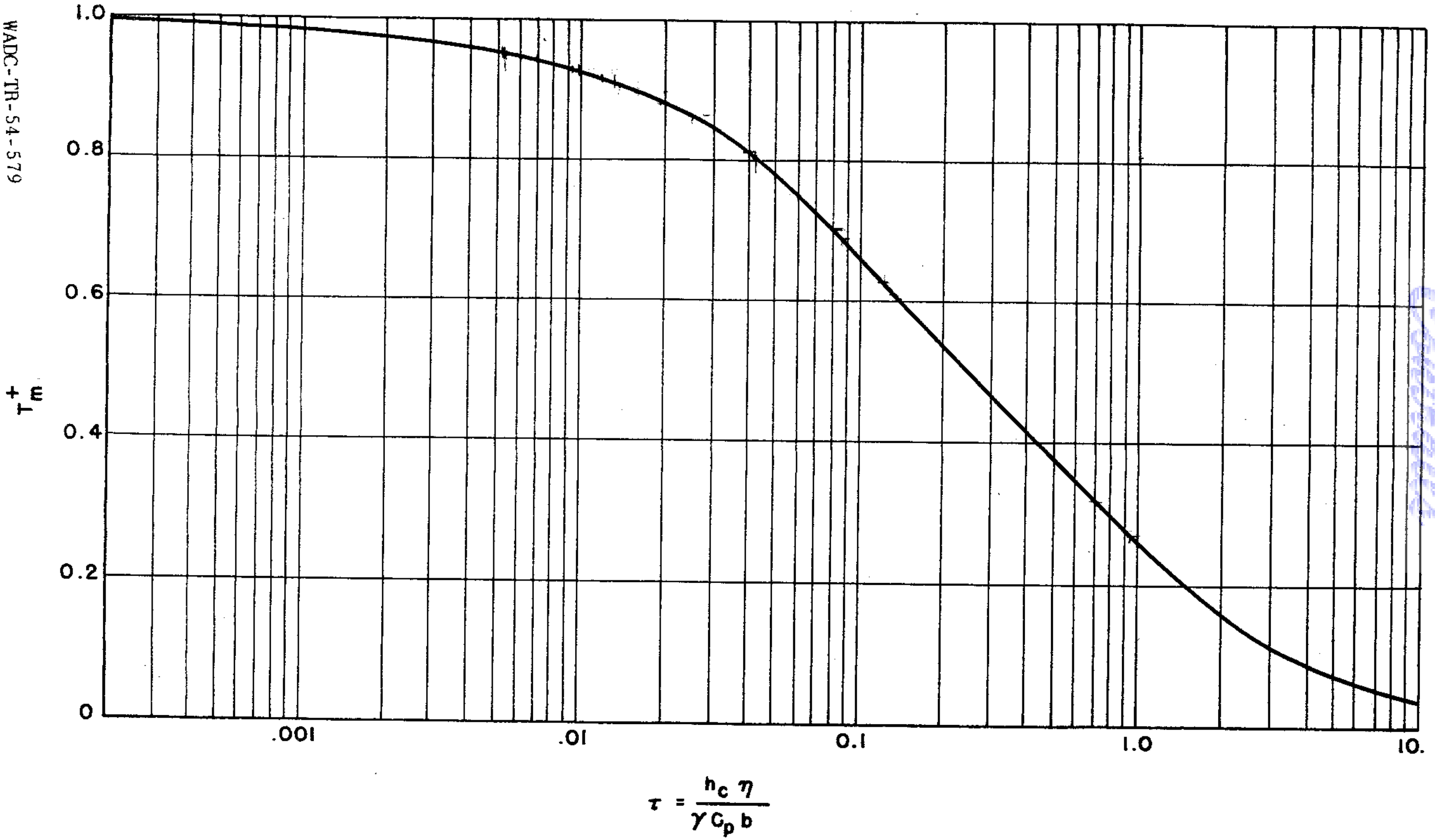

FIGURE B-1 MAXIMUM "THIN" SKIN TEMPERATURE RISE

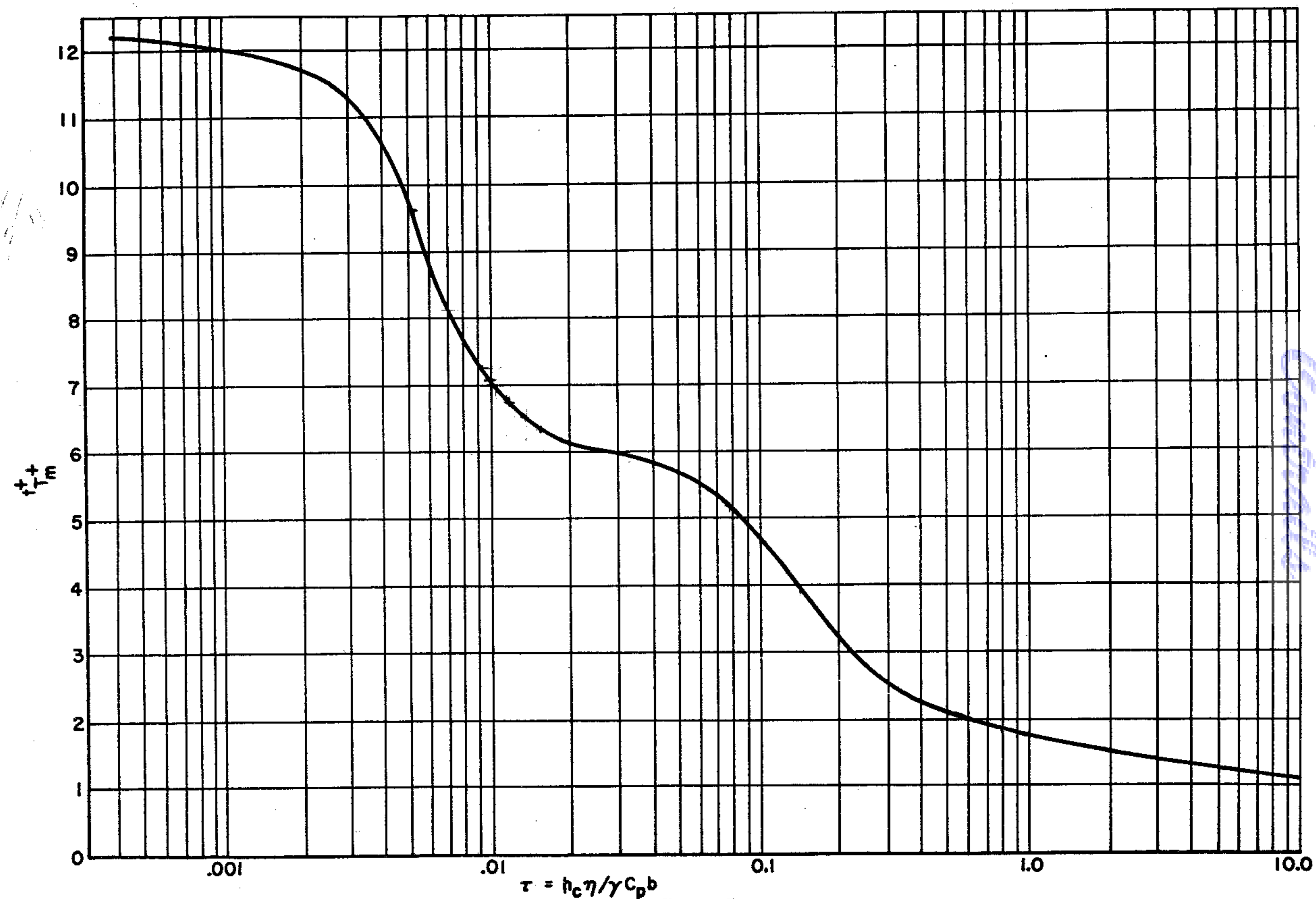

FIGURE B-2 TIME TO PEAK "THIN" SKIN TEMPERATURE RISE

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