CONSIDERATIONS OF SYNTHESIZED SYSTEM DAMPING IN DYNAMIC ANALYSIS OF SPACE STRUCTURES

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ABSTRACT

In considerations of space structures, a great number of degree-of-freedoms (DOFs) are modelled and thousands of them are retained for dynamic analysis. Natural frequencies of the structure represented by the retained DOFs may be very close to each other and the system is easily over-excited by the applied forcing function when it contains frequencies in the vicinity of the natural frequencies. In order to bring the excessive excitement down to a somewhat more realistic response level, viscous damping is usually applied. Since the damping coefficient for a flight system can not be directly obtained from ground tests, no test derived system damping is available. Damping coefficient for substructure constrained at the interface DOFs to a rigid base is obtained instead. The coefficients obtained from this test are applied to appropriate DOFs of the discrete substructure and the coefficients related to the interface DOFs are assumed to be zero. The damping matrix so constructed, upon releasing the constrained DOFs of the discrete substructure, is then transformed into an equivalent matrix for flight system analysis. Known as triple-matrix-product (TMP), this method of constructing a damping matrix by neglecting the off-diagonal elements has been widely adopted in aerospace industries. This paper is first to assess the validity of the above stated damping matrix of a discrete structure and the TMP approach, and then to propose a new method in constructing the system damping matrix by using the damping coefficient obtained from ground test. Specifically, the proposed damping matrix is synthesized by a diagonal matrix in the free-free system coordinates. Its corresponding damping elements in the substructural coordinates are best fitted to the test derived damping by using Gaussian least square technique. Applicability of the result is illustrated and assesed.

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INTRODUCTION

In dynamic analyses of a large structural system, geometric and material characteristics are represented by a system of mass and stiffness matrices. Together with a damping matrix, they constitute a complete set of governing differential equations for structural response analyses when applied forcing functions and appropriate initial conditions are given. Both mass and stiffness matrices are derived from analytical means. The damping matrix which can not be determined by analysis is usually obtained in conjunction with generalized modes of the structural system. Specifically, the analytically derived mass and stiffness matrices are used to establish a transformation matrix. Through this, the mass matrix can be transformed into an identity matrix and the stiffness matrix into a diagonal matrix in which the diagonal elements are the square of circular frequencies. Thus, a damping matrix, called system damping, is defined by multiplying a set of coefficients to a diagonal matrix consisting of circular frequencies. At this point, the set of equations in the generalized coordinate system consists of many independent differential equations. Each is a single degree-of-freedom (DOF). This set of equations can be solved by using the TRD module of the NASTRAN computer program [1]. Structural responses are then obtained by inverse transformation of the generalized DOFs.

It is known that coefficients of system damping are different for each mode. Their magnitudes can be derived from modal survey test results of the complete structural system. Since a space structure consists of a great number of DOFs and is actually operated in space of near zero gravity environment, it is very difficult, if not impossible, to establish system damping values through testing of the complete structure on the ground before a flight. In order to estimate the damping coefficients, an alternate method using modal survey test of the substructures constrained at their boundary DOFs is usually performed. The damping matrices obtained from modal survey tests of all the substructures together with assumed damping values in their boundary DOFs are then coupled into a Craig-Bampton (C-B) form [2] in the same manner as that for coupling the mass and stiffness matrices. However, a mathematical difficulty arises now. This newly coupled damping matrix, the discrete damping, can only be transformed into a fully populated damping matrix in the generalized coordinates. Thus, all generalized DOFs are still coupled to each other through the transformed damping matrix. The advantage for reducing computational time by using the TRD module is lost and the cost to solve these equations for a large structural system can not be saved. In order to take the advantage of using TRD module, an approximation by removing the off-diagonal elements from the transformed damping matrix has been commonly practiced. When this is done, the damping matrix becomes a diagonal, known as the triple matrix product (TMP) damping [3]. The set of generalized equations now become independent. The TRD module can then be readily applied to perform loads analyses with low computer cost.

The TMP damping technique has been proved to be a good approximation. Usually, results within an acceptible range of error can be obtained. Occasionally, unexplainable reponses occur. In a study of loads analysis for space transportation system payload, a larger response at a larger damping coefficient has been seen for a particular DOF when the damping value is within a particular range. The cause for this type of behavior is yet unclear. It may be partially induced by the use of TMP damping, since the practice of neglecting the off-diagonal elements is arbitrary. A new approach is proposed to refine the damping matrix used in system analyses. The proposed method applies Gaussian least square technique to synthesize the system damping. The condition is that the synthesized damping yields a smallest error between the converted and the given discrete damping values.

As an introductory development, the paper starts with a brief review of the TMP method. Derivations for the proposed approach follows. The goals of this approach are: (1) the synthesized system damping is a diagonal matrix; (2) the converted values of the synthesized damping are best fitted to the discrete damping; and (3) structural responses using the proposed method are at least as good as the results of TMP method. An example of a uniform beam is used to illustrate the characteristics of the proposed method. Results are compared to those obtained from both direct integration and TMP methods.

BRIEF REVIEW OF TRIPLE MATRIX PRODUCT DAMPING METHOD

To simplify matrix formulations for a structural system, let the mass, damping, and stiffness matrices of substructures be expressed in the C-B form. Explicitly, the boundary DOFs are kept in physical coordinates. The interior DOFs are represented by modal coordinates while the boundary DOFs are assumed to be completely constrained for each substructure. Upon coupling several substructures together to form a complete structural system, the governing differential equations for the system is given by

$$M\ddot{y} + D\dot{y} + Ky = P \qquad (1)$$

where M, D, and K are respectively the mass, damping, and stiffness matrice, y the displacement vector consisting of physical components at boundary DOFs and modal components at interior DOFs, $\dot{y}$=dy/dt, and P the forcing vector associated with the y component coordinates. Explicitly expressed into the C-B form, M, D, and K appeaar

$$M = \left[\begin{array}{c|c} M_{bb} & M_{bi} \\ \hline M_{ib} & M_{ii} \end{array}\right], \quad K = \left[\begin{array}{c|c} K_{bb} & K_{bi} \\ \hline K_{ib} & K_{ii} \end{array}\right] \tag{2a,b}$$

$$D = \left[\begin{array}{c|c} D_{bb} & D_{bi} \\ \hline D_{ib} & D_{ii} \end{array}\right] \tag{2c}$$

In these matrices, the subscripts b and i are respectively associated with boundary and interior DOFs of substructures. The submatrices with subscripts ii are diagonal. D_{ii} is the diagonal discrete damping obtained from substructural modal test. D_{bi}, D_{ib}, and D_{bb} are usually left empty due to the lack of test data. This assumption is believed to be conservative. Occasionally, D_{bi} and D_{ib} are assumed to be empty and D_{bb} is given by a set of nonzero values associated with a subsystem damping when boundary DOFs alone are treated as an independent subsystem [4].

The exact method of transforming Eq.(1) into a generalized coordinate system is through the use of complex variable modes. This method of analysis has been shown in many publications, for instance [5]. However, a real variable transformation appears to be more popularly accepted even though it is an approximate approach. The procedure of the approximation is as follows. Let η be the generalized DOFs corresponding to y by

$$y = \phi\eta \tag{3}$$

where ϕ is the transformation matrix satisfying

$$\phi' M\phi = I, \qquad \phi' K\phi = W \tag{4a,b}$$

In Eqs.(4a,b), ϕ' is the transpose of ϕ, I an identity matrix, and W a diagonal matrix. The diagonal elements of W are the square of circular frequencies. Introduction of Eqs.(3,4) into Eq.(1) gives

$$\ddot{\eta} + C\dot{\eta} + W\eta = Q \tag{5}$$

where $Q=\phi' P$ and

$$C = \phi' D\phi \tag{4c}$$

C is a fully populated matrix. Since the off-diagonal elements are generally smaller than the diagonal elements, the response using the fully populated matrix, C, makes little difference from that using the diagonalized TMP damping, C_d=diag(C). Using this matrix, the generalized DOFs are approximately computed from Eq.(5) upon replacing C by C_d. Namely,

$$\ddot{\eta} + C_d\dot{\eta} + W\eta = Q \tag{5*}$$

Effectively, the responses obtained from Eq.(5*) are actually not associated with the provided discrete damping matrix D, but with a fully populated equivalent damping matrix, D_d. Namely;

$$D_d = (\phi^{-1})' C_d (\phi^{-1}) \qquad (6)$$

Using the responses determined from Eq.(5*), the physical responses of structures are then computed from Eq.(3).

SYNTHESIS OF SYSTEM DAMPING

Let B be the diagonal matrix to be synthesized in the generalized coordinates and F the converted matrix of B in the C-B coordinates. The correlation between B and F is

$$F = (\phi^{-1})' B (\phi^{-1}) \qquad (7)$$

By letting b_k be the diagonal elements of B and f_{ij} be the elements of F, the element correlations in Eq.(7) can be written by

$$f_{ij} = \sum_{k=1}^{n} b_k \, g_{kij} \qquad (8)$$

where g_{kij} is the coefficient associated with f_{ij} when the kth element alone in matrix B is a unity, all other elements are zero.

Now, let d_{ij} be the elements of discrete damping matrix D. The sum of the square of the differences between converted damping f_{ij} and provided discrete damping d_{ij} is

$$S = \sum_{j=1}^{n_b} \sum_{i=j}^{n_b} (f_{ij} - d_{ij})^2 + \sum_{j=n_b+1}^{n} (f_{jj} - d_{jj})^2 \qquad (9)$$

where n_b is the number of boundary DOFs. Upon substituting Eq.(8) into Eq.(9), the sum becomes a function of the diagonal elements b_k. Thus, a set of Gaussian least square functions is formed when S is minimized by the condition

$$\frac{\partial S}{\partial b_m} = 0, \qquad m = 1,2,\ldots,n \qquad (10)$$

Explicitly, Eqs.(8-10) give a system of linear algebraic equations for elements b_k in the form

$$\sum_{k=1}^{n} \left(\sum_{j=1}^{n_b} \sum_{i=j}^{n_b} g_{kij} g_{mij} + \sum_{j=n_b+1}^{n} g_{kjj} g_{mjj} \right) b_k$$

$$= \sum_{j=1}^{n_b} \sum_{i=j}^{n_b} d_{ij} g_{mij} + \sum_{j=n_b+1}^{n} d_{jj} g_{mjj} \quad , \; m=1,2,\ldots,n. \qquad (11)$$

Using Eq.(11), the synthesized elements of system damping matrix B are obtained. The generalized DOFs can then be determined by using Eq.(5) upon replacing C by the synthesized damping matrix B. The structural responses can then be evaluated by using Eq.(3).

It is noted that the best fit shown in Eq.(9) includes only the specified discrete damping elements, those belonging to the boundary DOFs and the diagonal elements of internal DOFs. The rest of elements are not included in the fitting. In fact, it is uncertain if the off-diagonal discrete damping elements are really zero [6]. Therefore, it may be acceptable for practical applications to neglect fitting of elements which are not really obtained from testing but from assumptions.

ILLUSTRATION

To illustrate the performance of TMP method, let us consider a uniform beam of 25 translational DOFs. The total length is 120 inches and it is equally spaced into 24 segments. The beam properties are A=.0974 sq. in., I=.0480 in^4., $E=10^6$ psi, and ρ=.0318 lb-sec $^2/in^4$. The boundary nodes are 1, 6, 13, 18, and 25 (Fig. 1). Heavisides step forcing functions are applied to three points of the beam, 5 lbs at both ends (nodes 1 and 25) and 10 lbs at the middle point (node 13). A nominal damping coefficient of 10% for the interior modal DOFs of C-B form is assumed in order to easily illustrate the contribution of damping. Two cases of damping values at boundary DOFs are considered, $D_{bb}=0$ and $D_{bb}\neq 0$. For the case of $D_{bb}\neq 0$, a set of damping value equivalent to 10% is applied when M_{bb} and K_{bb} are assumed to be an independent subsystem. Explicitly, a transformation is first performed to generalize the boundary DOFs alone into a subsystem. A subsystem damping of 10% is then obtained. Inverse transformation of the subsystem damping, the boundary damping matrix D_{bb} is thus defined. To simplify the analysis, no other substructure is coupled to the beam. Therefore, the problem is simply the transformations between C-B form and the generalized system.

Based on the material and geometric properties, the matrices of mass, stiffness, and discrete damping are established. Through the use of Eqs.(3-5*), structural responses are obtained. To verify the damping values actually used in TMP response analysis, Eq.(6) is applied to transform the TMP damping back to C-B coordinates. The diagonal elements of the converted matrix are then compared to the corresponding elements of the discrete damping. The results shown in Table 1 indicates that the converted damping are significantly different from the provided discrete damping. The maximum error is up to 23% in the 6th mode for the case without boundary damping. When the synthesized damping is applied, the maximum error of converted diagonal damping are less than 1% in all modes for both cases of $D_{bb}=0$ and $D_{bb}\neq 0$. Therefore, the synthesized system damping are much more accurate than TMP damping when the discrete damping values are compared.

Despite the significant discrepancies between the provided discrete damping and the damping values actually used in the TMP method, the responses are in good agreement with the results of direct integrations of using the discrete damping. As shown in Table 2, the largest response error which happens to be at the negative value of node 6 is only over-estimated by 1.6% with respect to the peak acceleration 1.672 g, for the case of $D_{bb} \neq 0$. The corresponding maximum error by using the synthesized system damping method is 2.9% at the negative value of node 18, for the case $D_{bb}=0$. It must be noted that if the errors are computed by using local values, the maximum errors become very large. They are respectively 11.2% for TMP and 20.3% for synthesized system damping methods, both at the negative value of node 18 for the case of $D_{bb} \neq 0$. However, due to ignorance of accumulated numerical error acquired by using these approximate methods, this way of numerical comparison may not be fair. Generally, the results obtained from both TMP and synthesized damping methods are about in the same degree of accuracy. TMP may be slightly more accurate than the synthesized damping in this illustration.

Nevertheless, the proposed approach provides room for future improvement that TMP method does not. One of the possible improvement in syntheses of system damping is to weigh the importance of certain particular DOFs by using participation factors of the associated forcing functions. Although formulations using the factors are yet to be derived, one can capture the concept by studying the correlations between system mode shapes and the distribution of forcing functions. For this purpose, the damping values at generalized DOFs must be considered. Table 3 shows that the values for both TMP and synthesized damping are fairly close for some modes, but are significantly different for others. This indicates that the responses by using TMP damping and synthesized damping may be significantly different, depending upon the frequency of the applied forcing function. For a set of three point loads applied to the beam, the distribution of the forcing function is close to the 5th mode if it is expanded into mode shapes of the beam. Explicitly, the forcing function is more sensitive to frequency 5.2 Hz of the 5th mode. The successive important frequencies are 23.5 Hz for the 9th mode, 54.4 Hz for the 13th mode, etc., since these mode shapes, as shown in Fig. 1 for the first 9 modes, are closer to the distribution of the applied forcing function. Using these modes, it is shown by Table 3 that the coefficients of both TMP and synthesized damping are fairly close. Therefore, the responses due to both methods are little different. It may be expected that structural responses become significantly different if the distribution of forcing functions coincides with a mode for which the damping coefficients in TMP and synthesized approaches are significantly different.

The results between the cases $D_{bb}=0$ and $D_{bb} \neq 0$ must also be noted. These results are independent of the methods of analysis. For the case of $D_{bb}=0$, Fig. 2 shows that the amplitude of oscillations are fairly uniform after the early time spikes. However, Fig. 3 for the case of $D_{bb} \neq 0$ reveals that the responses decay considerably at later time. Therefore, boundary damping is important to structural responses. Due to the lack of test data, it may be difficult to establish a perfect boundary damping. Until a better approach is available, a uniform subsystem damping of 1% or 2% in the form similar to that illustrated earlier for $D_{bb} \neq 0$ may be acceptable for practical applications.

DISCUSSIONS AND CONCLUSIONS

The purpose of this study is to synthesize a system damping matrix that may best simulate the true damping behavior of a structural system. Using a given discrete damping matrix, a new approach in synthesis of system damping by using Gaussian least square is proposed. The synthesized damping matrix is always a diagonal and is readily applicable to the TRD module in NASTRAN. As illustrated by a uniform beam subjected to three point step function forces, the synthesized damping are in excellent agreement with the provided discrete damping in C-B coordinates. The structural responses using the synthesized damping also correlate very well with those using direct integration. Therefore, the new approach is worthy of further investigation for developing an improved method which can best represent the true system damping.

Many possible syntheses can be made to upgrade the proposed method, depending upon the goal of an analysis. Specifically, a set of participation factors can be assigned to weigh a class of interest. Using the factors as a weighting function, a best fit can be performed for the interest of certain structural components. For instance, the damping values can be best estimated for certain DOFs that are closest to the modes and frequencies of forcing functions as explained earlier.

In addition to the potential of the proposed method in synthesizing system damping, several properties found from the illustration may be useful for future refinement of the method. (1) Variations of boundary damping may significantly influence structural responses. The influence is more expressive for late time than for early time responses. (2) Discrepancies between the provided discrete damping and the converted values of TMP and synthesized damping appear to be not an important factor to structural responses. It is known that this may not be a correct statement. Further study is required to determine the true correlations. (3) The 10% constant discrete damping in the C-B coordinates are different from those in the generalized coordinates. It is particularly significant in the low frequency modes.

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Table 1. COMPARISONS OF DAMPING VALUES FOR 10% DISCRETE DAMPING

MODE	DISCRETE DAMPING*	$D_{bb} = 0$		$D_{bb} \neq 0$	
		TMP	SYN.	TMP	SYN.
1	(.0423)	.0023	.0027	.0405	.0426
2	(.1306)	.0104	.0127	.1247	.1316
3	(.2000)	.0127	.0146	.1929	.1999
4	(.1518)	.0079	.0082	.1486	.1567
5	(.0271)	.0006	.0006	.0260	.0272
6	7.603	5.867	7.660	7.289	7.603
7	9.195	8.282	9.223	10.38	9.195
8	15.91	15.15	15.92	15.57	15.91
9	19.48	19.64	19.51	20.38	19.48
10	29.05	30.30	29.18	32.52	29.05
11	33.21	36.55	33.25	38.14	33.21
12	52.83	56.35	52.85	57.63	52.83
13	56.96	62.53	57.08	65.06	56.96
14	65.10	72.69	65.13	74.51	65.10
15	72.76	80.26	72.77	82.46	72.76
16	102.9	108.7	102.9	109.6	102.9
17	107.6	123.1	107.7	125.9	107.6
18	120.6	131.8	120.6	133.3	120.6
19	129.6	141.2	129.6	143.0	129.6
20	156.2	173.6	156.2	174.5	156.2
21	161.7	182.5	161.7	183.1	161.7
22	185.8	200.4	185.8	201.1	185.8
23	190.1	212.1	190.1	213.2	190.1
24	200.9	230.1	200.9	231.4	200.9
25	203.5	229.7	203.5	232.6	203.5

* Values in () are for the case $D_{bb} \neq 0$.

Table 2. COMPARISONS OF ACCELERATION RESPONSES FOR 10% DAMPING

ITEMS	NODAL NUMBER							
	1 (END)		6		13 (MIDDLE)		18	
	MIN	MAX	MIN	MAX	MIN	MAX	MIN	MAX
$D_{bb}=0$								
DIR. INT.	-.389	1.672	-.374	.481	-.259	1.672	-.263	.355
TRD-TMP	-.390	1.672	-.399	.471	-.260	1.672	-.282	.354
TRD-SYN.	-.387	1.672	-.414	.478	-.265	1.672	-.303	.354
$D_{bb}\neq 0$								
DIR. INT.	-.148	1.672	-.343	.412	-.174	1.672	-.242	.290
TRD-TMP	-.147	1.672	-.370	.412	-.174	1.672	-.269	.286
TRD-SYN.	-.153	1.672	-.389	.418	-.189	1.672	-.291	.290

Table 3. COMPARISONS OF TMP AND SYNTHESIZED DAMPING COEFFICIENTS (%) IN GENERALIZED COORDINATES FOR 10% DISCRETE DAMPING

MODE	SYSTEM FREQ. (Hz)	$D_{bb} = 0$		$D_{bb} \neq 0$	
		TMP	SYN.	TMP	SYN.
1	0	-	-	-	-
2	0	-	-	-	-
3	.9675	.009	.001	9.90	10.1
4	2.657	.193	.146	9.44	10.1
5	5.191	.484	.536	9.55	9.98
6	8.550	12.3	16.7	14.0	15.0
7	12.73	16.5	17.5	18.3	13.9
8	17.71	14.1	15.1	14.2	15.3
9	23.49	14.9	13.9	15.1	13.9
10	30.06	13.5	13.2	13.9	12.2
11	37.41	14.5	12.6	14.9	12.4
12	45.51	11.2	10.5	11.3	10.5
13	54.36	12.5	11.4	12.9	11.1
14	63.91	12.9	11.3	13.2	11.3
15	74.13	13.4	11.9	13.6	12.0
16	84.94	10.6	10.1	10.7	10.1
17	96.25	12.1	10.4	12.4	10.3
18	107.9	11.7	10.7	11.8	10.8
19	119.7	12.1	11.3	12.2	11.3
20	131.3	11.1	9.99	11.2	9.99
21	142.4	12.0	10.5	12.0	10.5
22	152.4	10.5	10.1	10.5	10.0
23	161.2	11.8	10.4	11.8	10.4
24	167.9	11.6	9.99	11.7	9.99
25	172.1	11.1	10.2	11.8	10.2

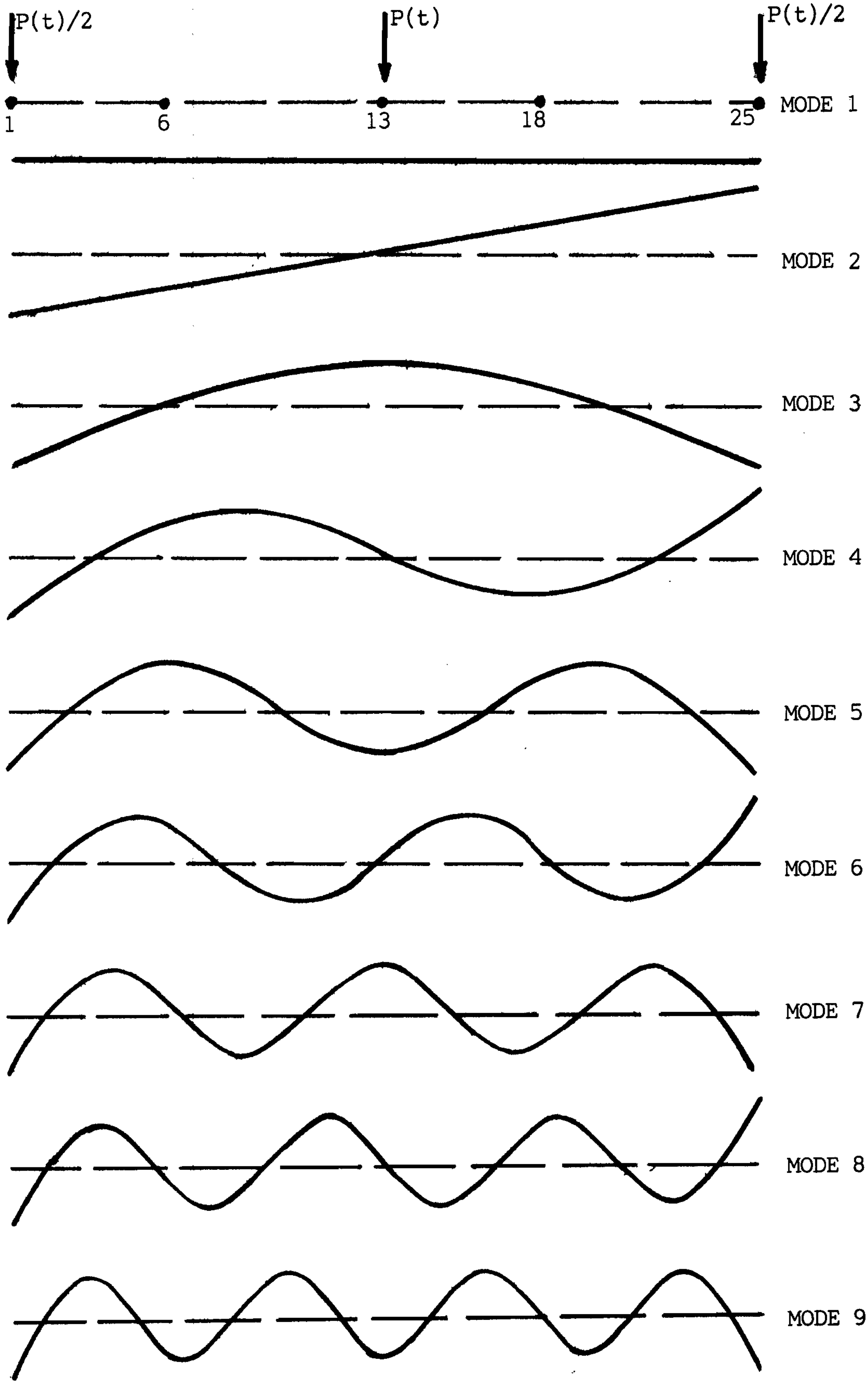

Fig. 1. MODE SHAPES OF FREE-FREE BEAM

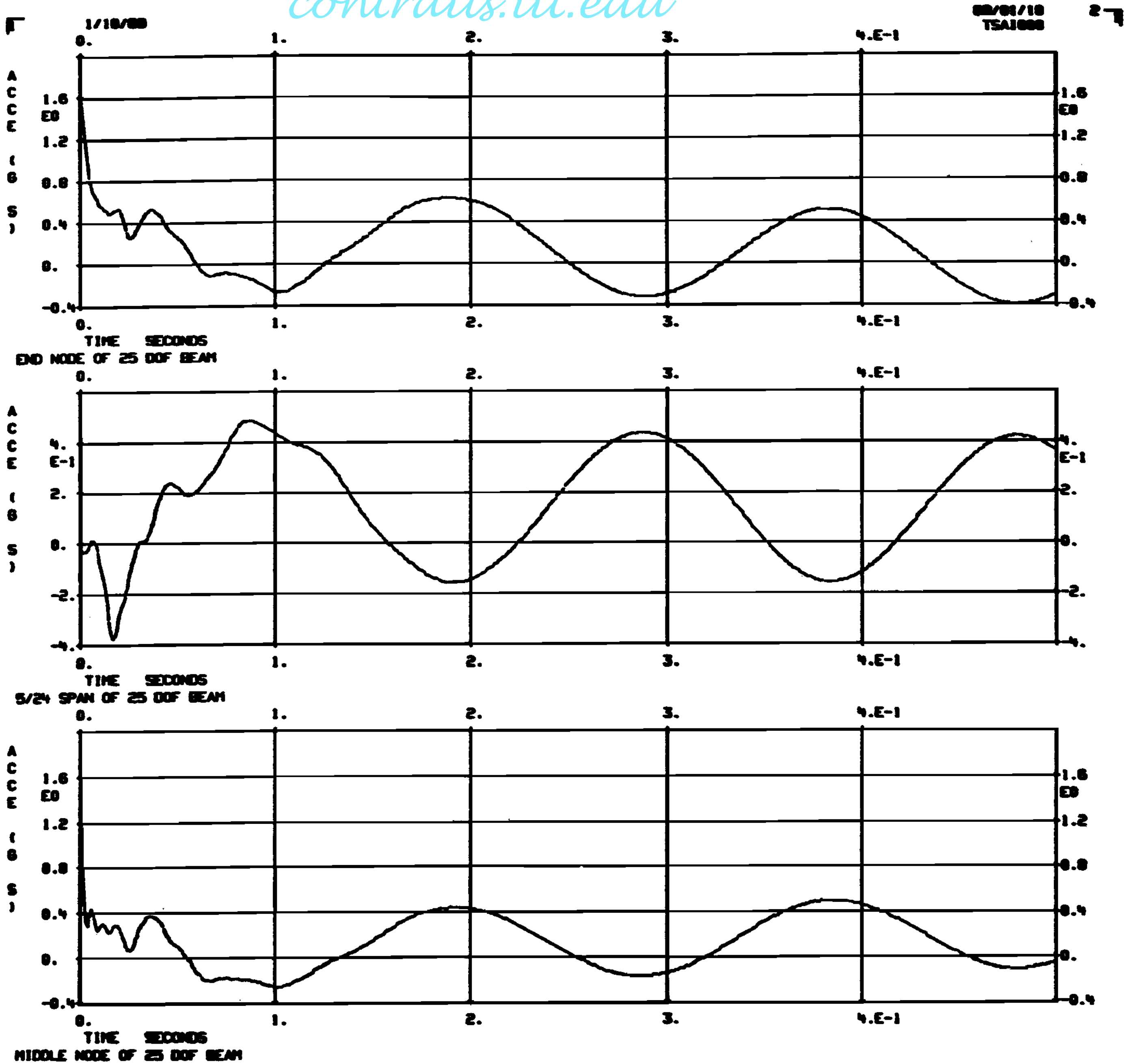

Fig. 2. RESPONSE TIME HISTORY WITHOUT BOUNDARY DAMPING

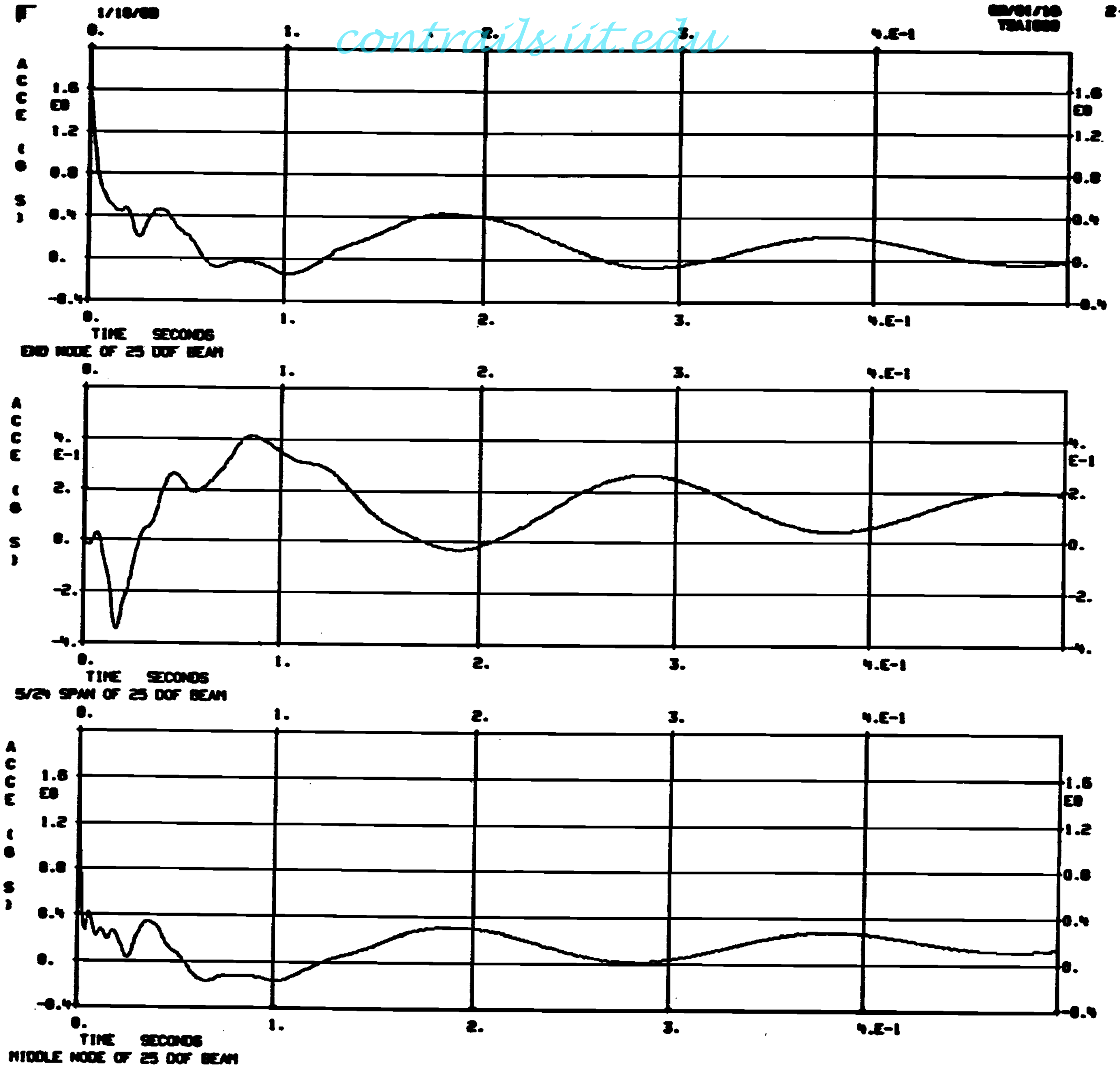

Fig. 3. RESPONSE TIME HISTORY WITH BOUNDARY DAMPING