SPECIAL SEMINAR PAPERS

A NEW LOOK AT STRUCTURAL PEAK DISTRIBUTIONS UNDER RANDOM VIBRATION

by

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ABSTRACT

The response of typical aircraft structures to sonic loading is investigated from a fatigue point of view. A deficiency in the Rayleigh peak distribution is illustrated by statistical analysis of experimental data. The variability of mean stress is considered in this analysis. Results indicate that the response of a typical structure can be completely characterized for fatigue analysis by its power spectrum and root mean square stress.

I. INTRODUCTION

Many papers (1-8) have been written concerning the fatigue of structure caused by the noise of jet engines. All have used the Rayleigh distribution of peak amplitudes in their consideration of fatigue damage. In some cases (8), a reasonable correlation between test results and the Rayleigh distribution is illustrated.

At the 56th Conference of the Acoustical Society of America, November 1958, much controversy on the subject of peak distribution was instigated by the author's paper (7) on Structural Acoustic Proof Testing. Although part of the discussion was concerning the noise itself, the questions and interest in the subject of stress peak distributions caused the author to investigate this subject further.

II. ANALYSIS OF PEAK DISTRIBUTIONS

Following a series of sonic fatigue tests conducted at the Long Beach Division of Douglas Aircraft, the strain gage response of various structural parts was statistically analyzed (9). All structures studied were of the sheet rib type of construction and stress data were analyzed for the structure exposed to both a jet engine on an aircraft and within a test cell. Contrary to expectation, absolutely no statistical difference in peak response distributions could be differentiated between a structure on the aircraft or in the test cell, even though the noise spectra between the two environments were considerably different (7). For all stress response data analyzed, a comparison between the observed values and the Rayleigh distribution by a Chi-squared goodness of fit test (10) failed to reject at the 95 percent confidence level.

At this point, the subject of peak distribution may be closed with the conclusion that the Rayleigh distribution is sufficient. However, since the need for a knowledge of peak distribution is to apply to fatigue analysis, a further look from a fatigue point of view must be made.

Under the ideal conditions that a structure is a lightly damped single-degree-of-freedom system, theory and experiment show that the response to random input could well be described as sinusoidal with varying amplitude, the amplitude excursions being described by the Rayleigh distribution. However, under real conditions the structure contains many modes of response and can only be described as a complex wave form. The fact that theory still predicts a Rayleigh distribution of peaks is not sufficient to compensate for the fatigue damage associated to the higher frequency components.

Powell (5) shows an expression for an equivalent frequency, Equation (1), in terms of the power spectrum S(f) and the root mean square $\bar{s}$ of the response.

$$f_e = \left| \frac{\int_0^{f_{max}} f^2 \, S(f) \, df}{\int_0^{f_{max}} S(f) \, df} \right|^{1/2} \qquad \text{Equation (1)}$$

This equation will compensate timewise for the higher frequency components if we assume that fluctuations that do not cause a zero crossing are negligible. This is probably a reasonable assumption since any significant fluctuation of stress will cause a zero unless it is superimposed on another frequency component of much greater magnitude. The usefulness of Equation (1) is limited because it depends upon a knowledge of the stress power spectrum, but it is still an important tool for estimating the effects of high frequency components in preliminary fatigue studies.

Knowing the frequency of occurrence is but the first step in fatigue analysis; of much more importance is the distribution of stress fluctuations. A glance at typical stress response records from a structure driven by jet noise, illustrates a significant point. That is, the negative stress peak immediately following a positive peak very seldom is equal in magnitude. If one chooses to use the Rayleigh distribution for fatigue analysis, it is assumed implicitly that the negative peak is of equal magnitude. Since this is not true, an inherent degree of conservation is immediately proven. Even though this conservation may be a good virtue in design practice, it does not add to an understanding of the problem.

In order to take a more realistic view of peak distributions and their influence on fatigue, consider two parameters in describing the peaks: the range of stress and the mean stress. Data are then reduced by not counting positive and/or negative peaks but by pairing positive and negative peaks as illustrated in Figure 1.

Tables I and II show the results of peak analysis of the response of various aircraft structural members. Approximately 1000 cycles were reduced for each case and data were grouped into ten or eleven class intervals. Distribution tails were grouped to contain at least 5 occurrences. Typical histograms are shown in Figures 2a and 2b.

Table I shows the results of Chi-squared goodness of fit tests compared to a normal distribution of mean and alternating stress. As would be expected, as a result of the Central Limit Theorem of Statistics, the distribution of mean stress failed to reject at the 95 percent confidence level when compared to a normal distribution.

Although the Chi-squared goodness of fit check in most cases rejects the distribution of the alternating stress at the 95 percent confidence level, when compared to a normal distribution, a similar check of the alternating stress with the Rayleigh distribution produces even greater errors.

Table II is a summary of the statistics calculated from the experimental data. After normalization by the measured root mean square stress, an uncanny consistency between these statistics is noted.

In fact, a comparison of the various normalized histograms that were plotted from the data showed much similarity.

To derive an empirical distribution at this point would not be wise until more data have been considered. However, in order to continue this investigation and show the effects of these revised distributions on fatigue, assume both the distribution of mean stress and alternating stress is normal. These distributions may then be expressed as Equations (2) and (3)

$$f_1(\sigma_m) = \frac{1}{\sigma_a\sqrt{2\pi}} \; e^{-1/2 \left(\frac{\sigma_m}{\sigma_a}\right)^2} \qquad \text{Equation (2)}$$

$$f_2(\sigma_A) = \frac{1}{\sigma_b\sqrt{2\pi}} \; e^{-1/2 \left(\frac{\sigma_A-\sigma_c}{\sigma_b}\right)^2} \qquad \text{Equation (3)}$$

where σ is the normalized stress $\frac{s}{\bar{s}}$, σ_a is the normalized standard deviation of the mean stress, σ_b is the normalized standard deviation of the alternating stress, and σ_c is the normalized mean alternating stress.

III. FATIGUE DAMAGE

In order to estimate the effect on fatigue damage caused by using a normal rather than Rayleigh distribution, and to estimate the effect of the fluctuation of mean stress, cumulative damage calculations can be made in a manner similar to previous derivations.

Make the following assumptions:

1. For comparative purposes, Minor's Cumulative Damage is valid.

2. There is no correlation between the distribution of mean and alternating stress.

3. Equations (2) and (3) describe the distribution of mean and alternating stress.

4. The standard SN diagram may be extrapolated by the Modified Goodman Diagram.

Then the probability of occurrence of mean stress σ_m and alternating stress σ_A is given by Equation (4),

$$P(\sigma_m,\sigma_A) = f_1(\sigma_m)\, f_2(\sigma_A)\, d\sigma_m\, d\sigma_A \qquad \text{Equation (4)}$$

The number of occurrences in N cycles of mean stress σ_m and alternating stress σ_A is given by Equation (5),

$$dn = NP\ (\sigma_m,\sigma_A) \qquad \text{Equation (5)}$$

and the incremental damage is given by Equation (6),

$$dD = \frac{NP(\sigma_m,\sigma_A)}{N(\sigma_m,\sigma_A)} \qquad \text{Equation (6)}$$

where $N(\sigma_m,\sigma_A)$ is the number of cycles to fatigue failure at alternating stress σ_A and mean stress σ_m.

The total number of cycles to failure N is that which produces a damage of unity and is given by Equation (7),

$$\frac{1}{N} = \int_{\sigma_A}\int_{\sigma_m} \frac{f_1(\sigma_m)f_2(\sigma_A)d\sigma_m\, d\sigma_A}{N(\sigma_m,\sigma_A)} \qquad \text{Equation (7)}$$

In order to complete the integration of Equation (7), the function $N(\sigma_m,\sigma_A)$ must be defined. A method which may be used to define this function is outlined in a paper by the author (11) in which the concepts of the Modified Goodman Diagram are applied. The function $N(\sigma_m,\sigma_A)$ is then expressed,

$$N(\sigma_m, \sigma_A) = N_o \left(\frac{\sigma A}{1 - \frac{\sigma_m}{\sigma_\nu}} \right) \qquad \text{Equation (8)}$$

where the function N_o is the SN curve for completely reversed stress cycles and σ_ν is the normalized ultimate stress.

Equation (7) may now be solved by numerical integration and a "random" fatigue curve constructed. Inspection of the tabulations involved in this integration shows that peak damage is localized around an alternating stress radia of 1.75 and a mean of zero. This region of peak damage occurs at a much lower stress ratio than that shown by the Rayleigh distribution (8).

Since the numerical integration of Equation (7) is time consuming, a number of calculations were made using Equation (9) where the fluctuation of mean stress is disregarded.

$$\frac{1}{N} = \int_{\sigma_A} \frac{f_2(\sigma_A)dA}{N_o(\sigma_A)} \qquad \text{Equation (9)}$$

The results of the above calculations as well as results of similar calculations using a Rayleigh distribution of peaks are shown in Figure 3.

As would be expected, the Rayleigh distribution is the most severe and shows the least life for a given root mean square stress.

The results of calculations using Equation (9) are shown by squares and are compared to the circles. No significant difference between the results of Equations (8) and (9) is evident which shows that the additional computations caused by considering the mean stress is unwarranted. However, the difference between results of the normal and Rayleigh distribution is considerable.

IV. CONCLUSIONS

The preceding has shown that the Rayleigh distribution of peaks does not provide an adequate description of the stress fluctuations caused by jet noise if fatigue analysis is of the prime concern. Although insufficient data have been presented to prove that a normal distribution of stress fluctuations is correct, the consistency in the statistics which describe both the variation of mean stress and alternating stress indicates that a single empirical distribution could be derived.

Once this distribution has been proven, then simple mechanically programmed fatigue tests could be substituted for costly sonic tests and correlated with the sonic response through the root mean square stress and power spectrum.

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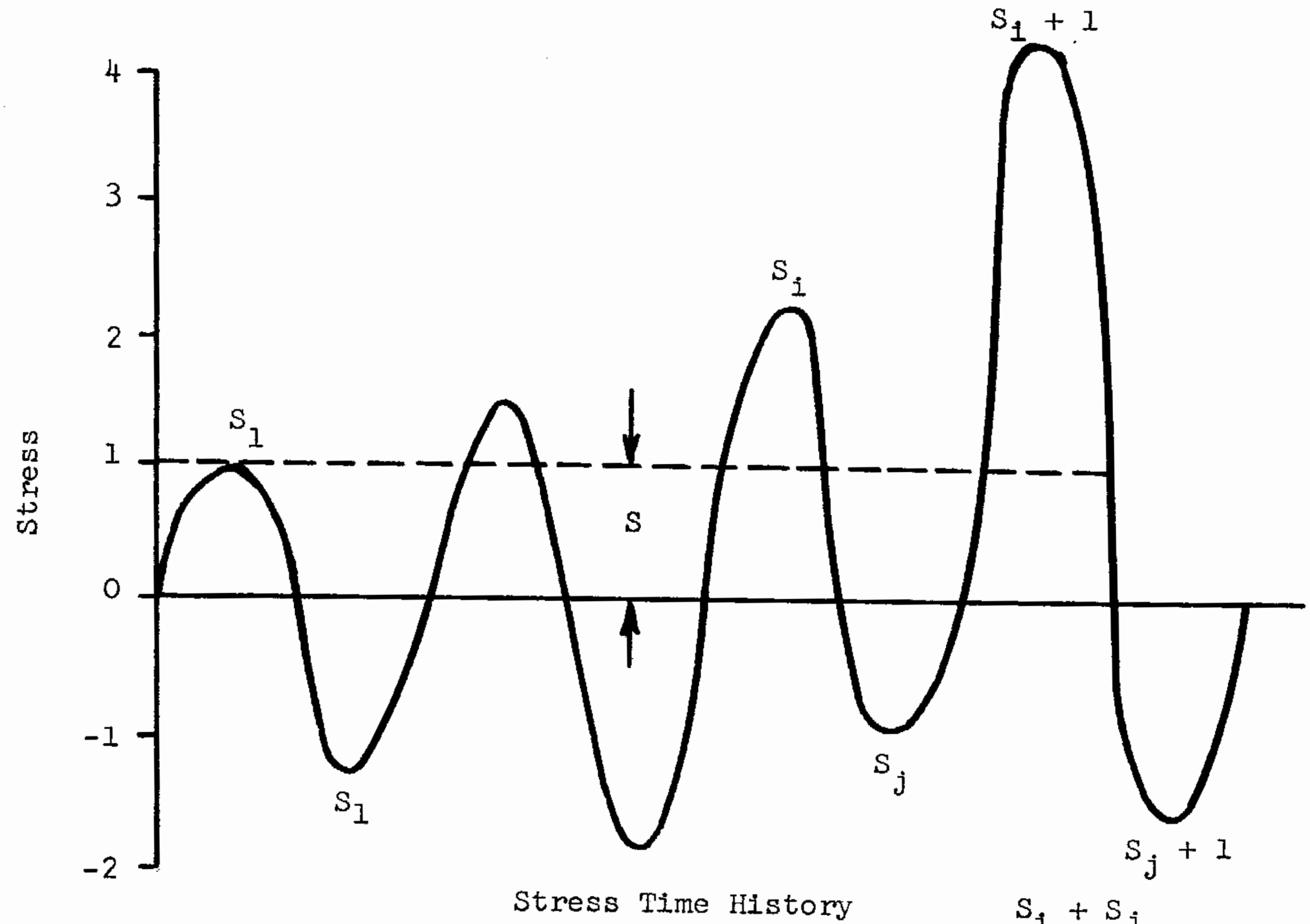
TABLE I - CHI-SQUARED GOODNESS OF FIT TEXT

Strain Gage No.	Type of Structure	Gage Position	X^2 Mean Stress S_M	X^2 Alternating Stress S_A	$X^2_{.95}$
1	SKIN	Skin	3.23	13.8	12.6
2	SUPPORTED	Skin	6.22	16.6	
3	BY	Longeron	5.99	16.4	
4	LONGERONS	Longeron	8.3	37.8	
5_a		Longeron	8.75	39.4	
5_b			4.42	41.6	
5_c			14.15	38.6	
5_d			13.84	116.6	12.6
6	Skin Supported by Ribs	Skin	——	——	——

TABLE II - A SUMMARY OF STATISTICS CALCULATED FROM EXPERIMENTAL DATA.

Strain Gage No.	% Engine RPM	RMS Measured Stress psi $\bar{x}$	MEAN STRESS S_M PSI				ALTERNATING STRESS S_A PSI				
			Mean $\bar{x}$	$\frac{\bar{x}}{\bar{s}}$	Std Dev s	$\frac{s}{\bar{s}}$	Mean $\bar{x}$	$\frac{\bar{x}}{\bar{s}}$	Std Dev s	$\frac{s}{\bar{s}}$	$\frac{s}{\bar{x}}$
1	100.7	481	1.4	.0029	140	0.208	638	1.325	221	0.460	0.347
2	100.7	749	44.9	.06	239	0.320	1205	1.61	367	0.491	0.304
3	100.7	1951	-22.7	-0.0116	392	0.201	2000	1.025	768	0.394	0.384
4	100.7	1584	13.7	0.0087	312	0.192	1660	1.049	692	0.436	0.416
5a	100.7	1620	-7.9	-0.0048	225	0.139	1362	0.844	571	0.353	0.481
5b	99.5 (-2db)	1250	4.0	0.0032	220	0.176	1120	0.90	477	0.381	0.426
5c	98 (-3.5db)	1000	6.2	0.006	194	0.194	903	0.903	373	0.373	0.413
5d	80 (-4.5db)	770	16	0.022	151	0.196	625	0.814	284	0.370	0.455
6	100.7	890	25.6	0.029	189	0.212	684	0.769	295	0.331	0.432
					Mean	0.204		1.027		0.399	
					Standard Dev.	0.0485		0.225		0.0525	
					95% Conf.	±0.031		±0.14		±0.032	



$$S_m = \frac{S_i + S_j}{2} \quad i = j$$

$$S_A = \frac{S_i + S_j}{2} \quad i = j$$

Figure 1 - The Method of Peak Reduction

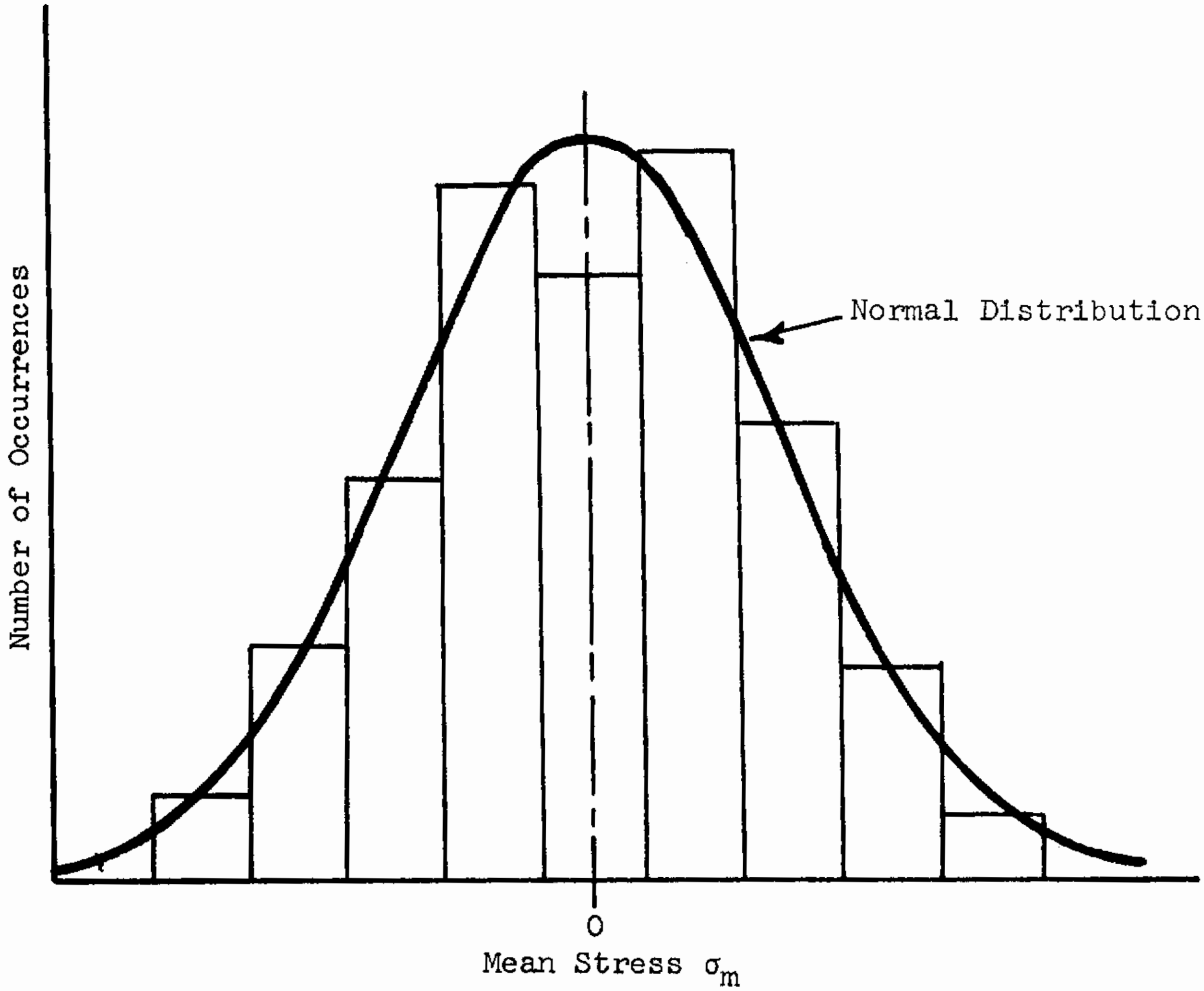


Figure 2a - Typical Histogram of Peak Distributions

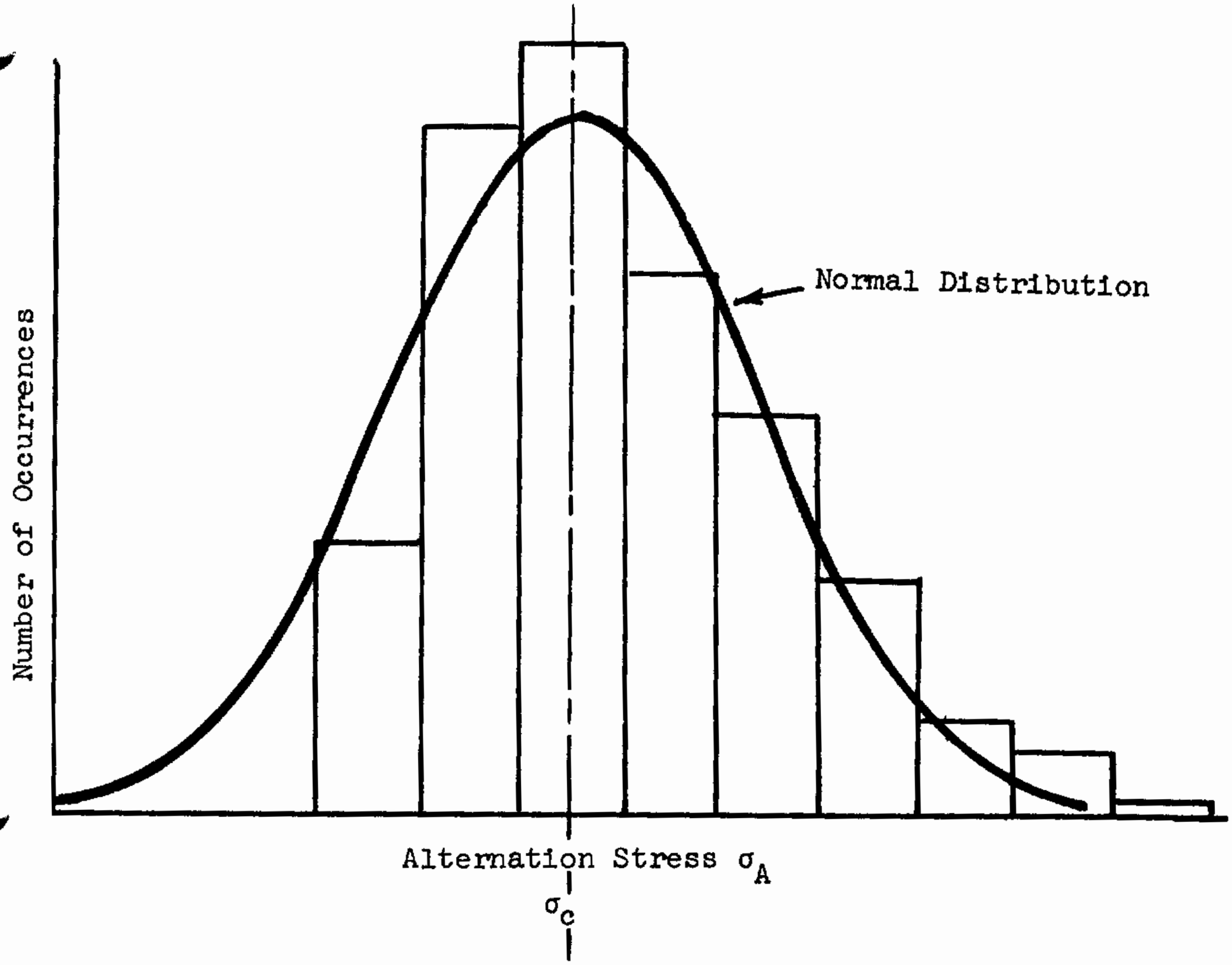


Figure 2b - Typical Histogram of Peak Distributions

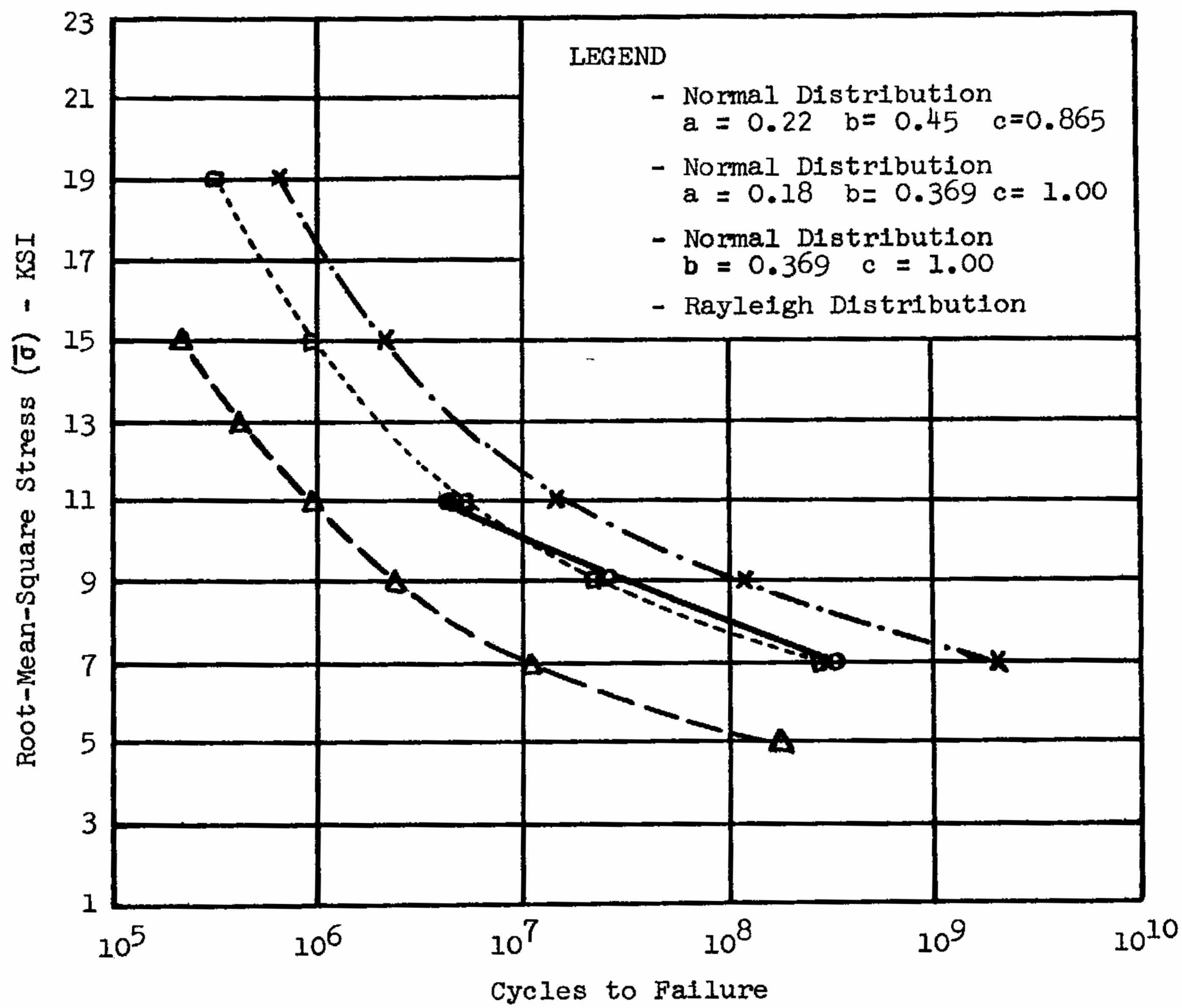


Figure 3 - Random Fatigue Curves for Alclad 7075-T6 Aluminum